Chapter 9
Time Allocation, Induction, and Matching Theory

William M. Baum

For a species to evolve and not go extinct, its members must reproduce. Not all its members need to reproduce. The rate of reproducing just needs to equal or exceed the death rate. Some may produce many offspring, some few, and others none. That is the basis of natural selection. If the members that produce more offspring carry a different genetic variant, the variation covaries with fitness (i.e., reproductive success), and the ones that produce more offspring will increase in the population at the expense of the others. Thus, any population that has evolved for a long enough time will include many organisms that tend to reproduce at the highest rate possible.

Natural selection explains not only why organisms tend to reproduce but also why organisms exist in the first place. Cells or groups of cells (organisms) that reproduce more effectively than genetically divergent rivals are selected through time. These individuals eventually predominate in any species. Among the traits that natural selection selects are those that enable more effective interaction with the world around, the environment. Even in a high-reproducing population, success varies: some members may not reproduce, but all members will have been selected to behave in ways that potentially facilitate reproducing.

Reproducing usually requires surviving, although exceptions exist, as when a male spider or praying mantis allows the female to eat him after copulation, or when a parent risks or sacrifices their life for the sake of offspring, or when warriors in cooperative-breeding species risk or sacrifice their lives for the sake of the social group. Apart from such exceptions, surviving is usually a major occupation in individual organisms (Baum, 2024).

Surviving (i.e., staying alive) is a process that includes multiple component parts, some of which are physiological and some behavioral. The behavioral parts all interact with the environment around. Surviving includes gaining resources—food, water, and shelter—avoiding predators and other dangerous situations, maintaining

W. M. Baum (✉)
University of California, Davis, CA, USA

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health, sleeping, resting, and, in social species like humans, maintaining relationships. These all function together to ensure surviving, and if any one should fail, the result is death.

Each process part of staying alive—gaining resources, maintaining health, and maintaining social bonds—is itself composed of component processes. For example, maintaining health includes grooming, avoiding injury, resting, and sleeping (Baum, 2018, 2024).

Induction

The present concept of induction builds on the concept of induction proposed by Segal (1972). Instead of elicitation, with its connotations of momentary one-to-one correspondence between events, Segal proposed that induced behavior might be temporally extended and consist of whole patterns of activities. The present concept adds that unconditional inducing events are events that affect evolutionary fitness (reproductive success)—*phylogenetically important events* (PIE; Baum, 2012). PIEs include events like encountering food, water, a potential mate, predators, injury, cold, and illness. All of these must be dealt with adequately. Selection favors individuals that respond effectively to PIEs, to enhance or produce the supportive ones, and to avoid or mitigate the threatening ones, because these individuals tend to leave more offspring. Thus, populations today consist of individuals who respond vigorously to PIEs. They build nests, protect offspring, court potential mates, capture prey, avoid predators, and so on. In other words, PIEs reliably induce specific activities in these individuals.

An extension to Segal's (1972) conception is that PIEs induce not only activities selected in phylogeny but also the activities selected in ontogeny—so-called *operant* activities (Baum, 2012). If a rat is trained to press a lever to produce food, the food induces the lever pressing. If a rat is trained to press a lever to avoid electric shock, electric shocks induce lever pressing. The general principle of operant behavior is that activity that produces a fitness-enhancing PIE is induced by that PIE, and activity that avoids a fitness-threatening PIE is induced by that PIE.

Time Allocation

The suite of activities and allocation of time among them varies, depending on many factors: timeframe (e.g., milliseconds, minutes, years, or lifetimes; Baum, 1997, 2016), the physiological state of the organism (e.g., whether it is well-fed or starving), and features of the environment (e.g., whether hot or cold, dry or wet, abundant or sparse). Which of these to consider will depend on whether one is concerned with the components of a courtship display, the movements in a dance step, foraging for scarce food, holding a job, or work–life balance.

Despite all the variation, certain points will always be true. First, behavior, like any process, takes time. To measure the prevalence or rate of any activity, one must measure the time it takes up. Second, staying alive requires the organism to be continuously active. To be alive is to behave (Baum, 2013, 2024). Thus, an organism's activities in any time period take up all the time available. Therefore, third, an organism's activities must compete with one another for time.

The following equation, sometimes called the generalized matching law (GML; Baum, 1974) or the Law of Allocation (Baum, 2018), summarizes these necessarily true points:

$$\frac{T_j}{\sum_{i=1}^{N} T_i} = \frac{V_j}{\sum_{i=1}^{N} V_i}, \tag{9.1}$$

where T_i is the time taken by Activity i, and V_i is the competitive weight of Activity i out of the N activities that occur. The equation states that the proportion of time taken by any one activity, out of the total time available ($\sum T$), equals (i.e., matches) the competitive weight of the activity relative to the total competitive weight of all the activities ($\sum V$). If the competitive weight of any activity increases or decreases, other activities' times also must decrease or increase. An increase in one means a decrease in others, and a decrease in one means an increase in others, because the activities together take up all the time. If I spend more time working, I may have to spend less time with family or sleeping, because a day contains only 24 hours.

Two points about Eq. 9.1 are worth noting. First, researchers commonly measure the time spent in an activity in the laboratory by attaching a switch to a lever, key, or button, and counting the number of switch operations. If each switch operation indicates a certain amount of time in the activity of interacting with the lever, key, or button, then the number of switch operations is a good estimate of the activity time (Baum, 1976). When time is measured in switch operations, we make this clear by substituting B for T:

$$\frac{B_j}{\sum_{i=1}^{N} B_i} = \frac{V_j}{\sum_{i=1}^{N} V_i}. \tag{9.2}$$

One pitfall of measuring time taken by an activity as switch operations is that the matching relation becomes constrained at low activity levels. In the extreme, if activity levels were so low that each switch operation produced a PIE delivery, matching would be forced, because the PIE rate would equal the switch-operation rate. When activity levels are high, this problem may be ignored, but as switch--operation rates fall, this constraint may force matching as an artifact. The solution is simple: one need only subtract the number of PIE deliveries from the total of switch operations at each alternative, thus leaving only unconstrained activity and eliminating the forced relation.

Second, as noted above, we expect that the competitive weight of an activity will depend on a variety of factors. We require an expression that will allow all possible

variables to affect competitive weight. Considerable evidence supports induction according to a power function (e.g., Baum, 2020, 2021; Baum & Aparicio, 2020):

$$V = Cx^S, \tag{9.3}$$

where x is an environmental variable that is a measure of a PIE, such as its rate, amount, or immediacy, s is characteristic of x but subject to variation according to timeframe and physiological factors, and c is a coefficient that amplifies or diminishes the activity's competitive weight.

More than one of the dimensions of a PIE might vary in the same timeframe. Then each dimension conforms to Eq. 9.3, and competitive weight depends on the product of the power functions:

$$V_i = \prod_{j=1}^{m} c_j x_{ij}^{s_j}. \tag{9.4}$$

In Eq. 9.4, x_{ij} is the level of x_j for Activity i, c_j and s_j are the parameters for x_j, and m is the number of dimensions. The equation has been tested only in a limited way, for rate and amount (e.g., Baum, 2023). Further research would help clarify whether it is generally correct.

In an experiment that focuses on one activity, Eqs. 9.1 and 9.2 provide a specific equation:

$$\frac{T_1}{T_1 + T_0 + T_N} = \frac{B_1}{B_1 + B_0 + B_N} = \frac{V_1}{V_1 + V_0 + V_N}, \tag{9.5}$$

where the leftmost and middle terms equal the proportion of time taken by the activity of focus (Activity 1), T_0 and B_0 equal time taken by other PIE-induced activities treated as a single activity (Activity 0), and T_N and B_N equal time in activities not induced by the PIE under study. V_1, V_0, and V_N represent the corresponding competitive weights.

Since most research has focused on rates of various activities and, with a few exceptions (e.g., Baum, 1975), measured time as number of switch operations, rewriting Eq. 9.5 leads to the following:

$$\frac{B_1}{B_1 + B_0 + B_N} = \frac{c_1 r^{S_1}}{c_1 r^{S_1} + c_0 r^{S_0} + V_N}. \tag{9.6}$$

We simplify Eq. 9.6 into a useful equation for fitting data by assuming that $B_1 + B_0 + B_N$ take up all the time available and setting the sum equal to K, the total time in the units of B_1. In addition, we may assume that V_N is generally negligible, except if we are dealing with extremely low PIE rate or extinction, when B_N would be extreme (Podlesnik & Baum, 2024). Finally, we divide top and bottom on the right by $c_1 r^{S_1}$. The result is:

$$B_1 = \frac{K}{1 + cr^{-S}}, \tag{9.7}$$

where $c = \frac{c_0}{c_1}$ and $S = S_1 - S_0$. This equation allows fitting to time measured as switch operation rate as it varies with PIE (e.g., food) rate. One feature to note is that whether B_I increases or decreases as r increases depends on whether S_I is greater or less than S_0. If S in Eq. 9.7 is positive, B_I increases with increasing r, but if S is negative, B_I decreases with increasing r. The activity with the larger exponent may be said to dominate over the other activity. When B_N is large because PIE rate is too low, the PIE induces B_I and B_0 only weakly, and the two activities do not compete. Each increases according to its competitive weight, because the competitive weight V_N is negligible. In this unusual situation, Eq. 9.6 reduces to B equals V.

Induced Non-Operant Activities

As noted above, PIEs induce activities that are connected to them by phylogeny. Although a PIE induces multiple activities, research typically focuses on just one. To apply the model, let B_I be the focal activity and B_0 other activities the food induces. For example, if a rat receives a small food pellet every 30 s, it will drink water early in the interval, run in a wheel midway through the interval, and nose around in the food hopper late in the interval (Staddon, 1977). A pigeon so treated will attack another pigeon nearby early in the interval and peck at things and approach the food hopper later in the interval. Two experiments by Flory (1969, 1971) illustrate these phenomena. His results appear in the top two panels of Fig. 9.1.

In the results depicted on the top left in Fig. 9.1, a pigeon received bits of food at intervals with a stuffed bystander pigeon present. The bystander was attached to a switch that operated whenever the experimental pigeon moved it. The bottom left panel shows how the model fitted attack rate as a function of food rate. In the increasing leg of the graph (squares), food rate was too low to induce B_I and B_0 sufficiently for them to compete, and these attack (B_I) rates conformed to a power function (squares). When food rate was 0.5 per minute or higher, B_I (attacking) and B_0 competed, with B_0 dominating by having the larger exponent (2.46 versus 1.340) and driving attack rate down (Eq. 9.7). The results depicted on the top right in Fig. 9.1 show parallel results for a rat drinking water when periodically fed on a fixed-interval schedule. In the bottom right panel, the increasing leg at low food rates fits a power function because B_I (drinking) and B_0 did not compete (squares), and the decreasing leg at food rates 0.25 per minute or greater results from competition with B_0, which dominated with a higher exponent (2.10 versus 1.364).

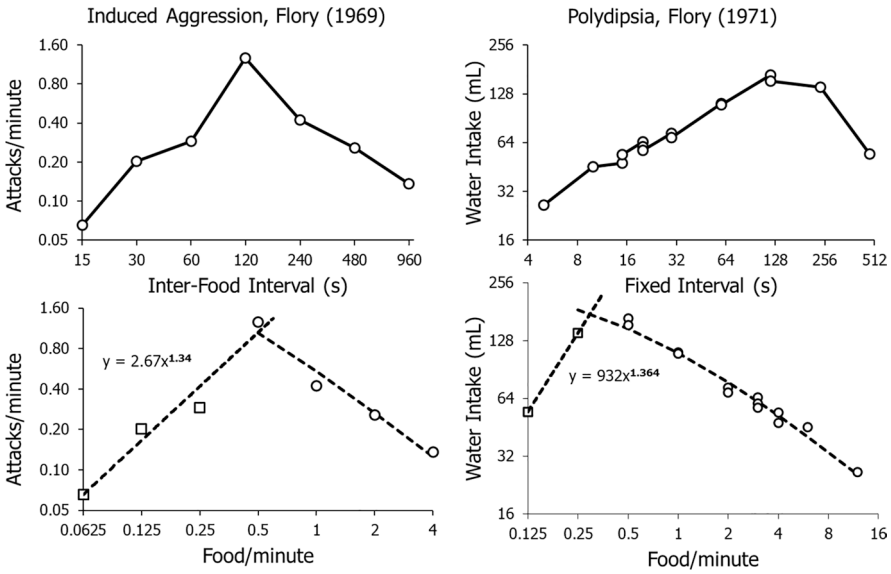

Fig. 9.1 Food-induced non-operant activities. Top left: attacks by a pigeon on a bystander pigeon at various inter-food intervals. Top right: drinking by a rat under various fixed-interval schedules of food. Bottom left: rate of attack as a function of food rate; broken lines fitted by the model. Bottom right: drinking as a function of food rate; broken lines fitted by the model

Induced Operant Activity

The same considerations apply to operant activity, because the PIE induces the operant activity that produces it in the same way as the PIE induces non-operant activity. In Eq. 9.7, the operant activity is represented by B, and the accompanying non-operant activity as B_0. Figure 9.2 shows some examples. In the left panel, operant nose-poking produces food, and the food induces nose-poking at different rates as food rate varies. In the fit of Eq. 9.7, the asymptote K equaled 59.2. In the right panel, a pigeon produced food by operating a key in one situation (circles) and a treadle in another situation (squares). In the fits of Eq. 9.7, the asymptote K equaled 59.1 for key pecking and 23.9 for treadle pressing. Each treadle operation represented more than twice as much time as each key operation. Operant activity always dominated over B_0, with S equal to 0.525 for key operation, and 2.38 for treadle operation.

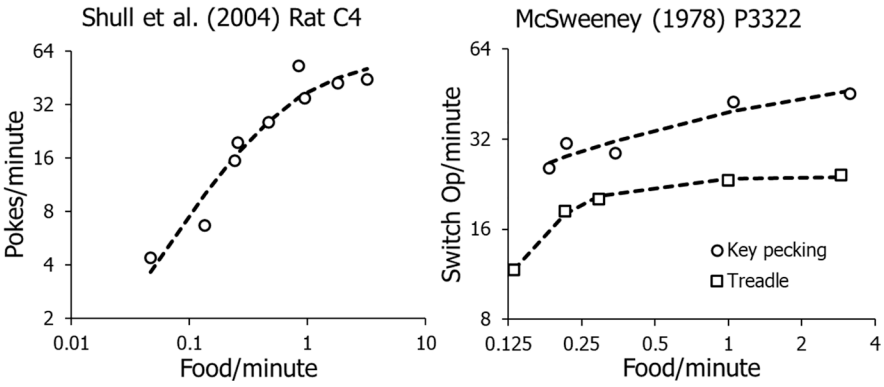

Fig. 9.2 Induction of operant activity. Left: rate at which a rat operating a key by poking it with its nose varied with obtained food rate. Right: rate at which a pigeon operating a switch by pecking (circles) or by pressing a treadle (squares) varied with obtained food rate. Broken lines show fits of Eq. 9.7

Pair Comparisons

Most research on operant choice has studied two alternative activities (Baum, 1979; Wearden & Burgess, 1982). If we write two equations for the two alternatives, B_1 and B_2, using Eq. 9.6, and take the ratio of the two equations, we get:

$$\frac{B_1}{B_2} = \frac{c_1 x_1^{S_1}}{c_2 x_2^{S_2}}. \tag{9.8}$$

For analyzing data, Eq. 9.8 is written in logarithmic form:

$$\log\left(\frac{B_1}{B_2}\right) = S \cdot \log\left(\frac{x_1}{x_2}\right) + \log(b), \tag{9.9}$$

where b represents $\frac{c_1}{c_2}$ and any extrinsic bias factor, and S_1 is assumed to equal S_2. To the best of my knowledge, no data set has been produced that requires S_1 and S_2 to be unequal. What circumstances might cause them to be unequal remain unknown. Figure 9.3 shows some examples with S_1 equal to S_2.

More than Two Alternatives

Although the form of Eqs. 9.2 and 9.4 is fine for theoretical exposition and deriving Eq. 9.9 for the special case of two-alternative choice or pair comparison, a general form for data analysis should allow rates of any number of alternatives to be

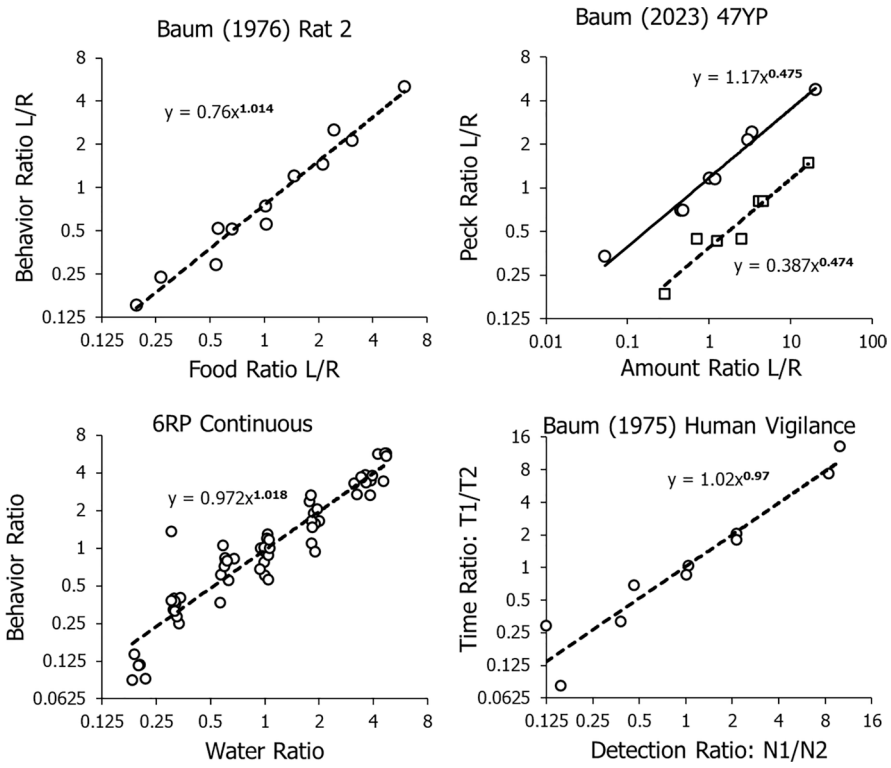

Fig. 9.3 Two-choice experiments. Upper left: a rat pressing two levers producing food at different rates. Upper right: a pigeon pecking two keys producing different amounts of food at two different levels of relative rate. Lower left: a pigeon living in the experiment operating two keys producing water at different rates. Lower right: a person holding two keys in a vigilance game producing targets at different rates

displayed in logarithmic coordinates. If we consider one alternative out of N alternatives and multiply all its N ratios together, we obtain:

$$\frac{B_j^N}{\prod_{i=1}^N B_i} = \frac{V_j^N}{\prod_{i=1}^N V_i}. \tag{9.10}$$

If we apply Eq. 9.4 for one variable (e.g., rate) to Eq. 9.10, we obtain:

$$\frac{B_j}{GM(B_i)} = \frac{c_j}{GM(c_i)} \bullet \frac{x_j^S}{GM(x_i)^S}, \tag{9.11}$$

where *GM* stands for geometric mean. An example of applying Eq. 9.11 for 4 alternatives may be found in Schneider and Davison (2005).

Power-Function Induction

Can one validate the power-function induction of Eq. 9.3 that underlies all the discussion so far? The equation for rate of either of two concurrent alternatives is:

$$\frac{B_1}{B_1 + B_2 + B_0 + B_N} = \frac{c_1 r_1^{s_1}}{c_1 r_1^{s_1} + c_2 r_2^{s_2} + c_0 r^{s_0} + V_N}, \tag{9.12}$$

where r equals overall PIE (food) rate, $r_1 + r_2$, assuming we need not partition B_0 between Alternatives 1 and 2. A parallel equation applies to Alternative 2.

Examining Eq. 9.12, one sees that if the sum of B on the left equals K and the denominator on the right were to remain constant as r_1 and r_2 varied, we would arrive at a simple power function:

$$B_1 = Q r_1^{s_1}, \tag{9.13}$$

where Q equals $\frac{Kc_1}{D}$, a constant if the denominator on the right of Eq. 9.12, D, remains constant. If one could hold overall food rate fixed while varying relative rates, one might verify Eq. 9.13. Figure 9.4 shows some results consistent with this.

One way to hold the denominator D constant is to fix the overall food rate while varying the proportion of food deliveries that are contingent on operant activity relative to those not contingent on operant activity. In that situation, B_1 is operant activity, r_1 is rate of contingent food, r_2 is rate of noncontingent food, B_0 is all non-operant activity, and other operant activity is absent. The top two panels in Fig. 9.4 show results from such experiments. On the left, Pigeon R30 was exposed to a series of different VI schedules (squares) and a series of conditions in which a single VI 90 s programmed food deliveries and the proportion of deliveries that were noncontingent varied across conditions (circles). The solid curve fitted to the activity rates maintained by the various VI schedules represents Eq. 9.7. The dashed line, a power function in these logarithmic coordinates, was fitted by least-squares regression, and its equation appears next to it. This dashed line validates power-function induction.

The top right panel in Fig. 9.4 shows results from a similar experiment with rats (Baum & Aparicio, 2020). Rat LE2 was exposed to a series of conditions in which a VI 63 s programmed 64 food deliveries in a session, and the proportion of noncontingent deliveries varied across conditions. The dashed line, a power function, shows that operant activity rate (lever pressing) conformed to Eq. 9.13, again confirming power-function induction. The solid curve is fitted to rates of food-hopper entry, which may be thought of as non-operant activity, part of B_0. That it decreases as lever pressing increases confirms the expectation based on Eq. 9.6, that B_0 competes with operant activity B_1.

The reasoning leading to Eq. 9.13, that if overall food rate and the denominator on the right of Eq. 9.12 (D) are constant, power-function induction should be

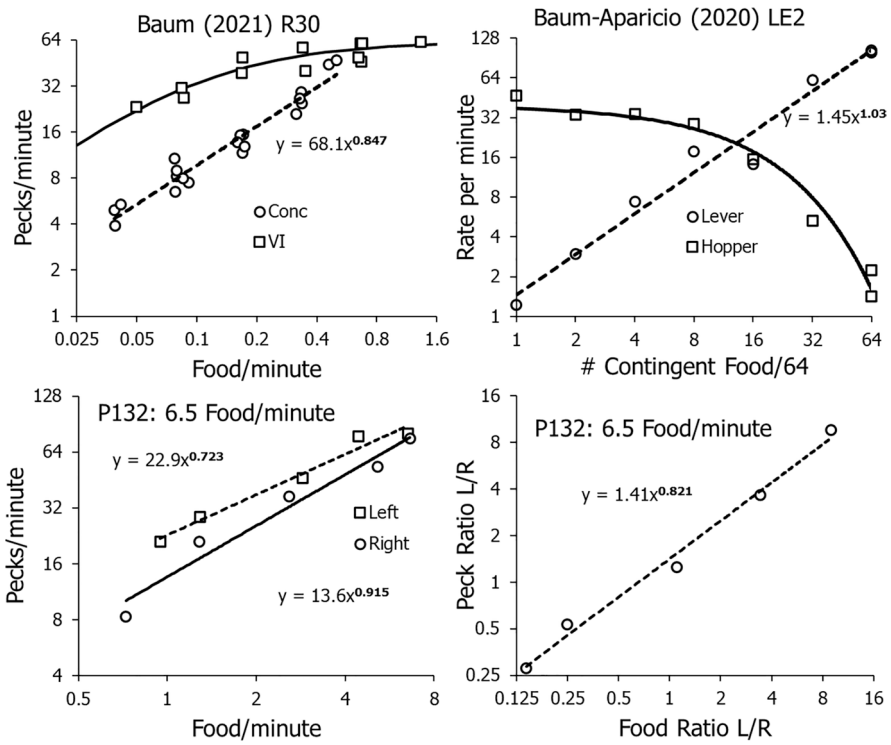

Fig. 9.4 Empirical verification of power-function induction. Top left: Pigeon R30 pecking at a key producing food according to VI schedules (squares) and situations in which overall food rate remained constant while proportion of noncontingent food deliveries varied. Data from Baum (2021). Top right: Rat LE2 pressing a lever in situations with constant overall food rate and varying proportion of noncontingent food deliveries (circles) while food-hopper entries were recorded (squares). Data from Baum and Aparicio (2020). Lower left: Pigeon P132 pecking at two keys producing food at different rates. Lower right: pair comparison of the two alternatives at the lower left. Data from Alsop and Elliffe (1988)

revealed in two-alternative choice, is supported by the two lower panels in Fig. 9.4. The lower left panel shows results from an experiment in which both relative and overall food rates were varied across conditions (Alsop & Elliffe, 1988). The graph shows peck rates at each key as a function of food rate at the key. The two lines (power functions) show power-function induction in this experiment at the highest overall food rate (6.5 food/minute). At lower overall food rates, power functions were less visible, presumably because D was not constant enough. The high overall rate may have rendered variation in B_0 and V_0 negligible.

The lower right panel in Fig. 9.4 shows the pair-comparison plot of the rates in the lower left panel. The dashed line shows the power-function fit to the points (Eq. 9.8), a line in these logarithmic coordinates (Eq. 9.9). The equation of the power function appears next to the line. The exponents of the power functions in the lower left panel are unequal (0.72 versus 0.91). The exponent of the pair-comparison

power function (0.82) falls between these two, close to their mean (0.81). The coefficient (1.41) falls a bit short of the ratio of the two coefficients in the lower left panel (1.68), possibly indicating a small position bias in favor of the right key.

Single Schedules: Variable Interval and Variable Ratio

Up to this point, all empirical findings were based on time-based scheduling, particularly variable interval (VI) schedules, and behavior-based schedules, such as variable ratio (VR) schedules, have received no attention. Prior research has identified two canonical differences between performance on VI schedules and VR schedules: (1) VR schedules maintain higher activity rates than VI schedules at low or moderate food rates; and (2) whereas VI schedules maintain activity at low food rates, extremely lean VR schedules do not maintain any operant activity—a cut-off exists for VR schedules (e.g., Ferster & Skinner, 1957). Figure 9.5 shows some illustrative feedback functions.

If we model operant activity as a feedback system in which the activity interacts with the environment to produce PIEs at some rate, and that PIE rate feeds back to induce the activity, one sees that the feedback either may push the activity further or may not suffice to induce it at its present level. The curvature of the VI feedback function indicates that induction is high at low activity rates but diminishes at high activity rates. This shift allows VI performance to stabilize at a moderate activity rate. VR schedules, being linear, will tend to drive activity higher and higher or lower and lower.

We can model both types of schedule using Eq. 9.6. B_N and V_N are negligible except when food rate r is low. To be complete and incorporate performance at low

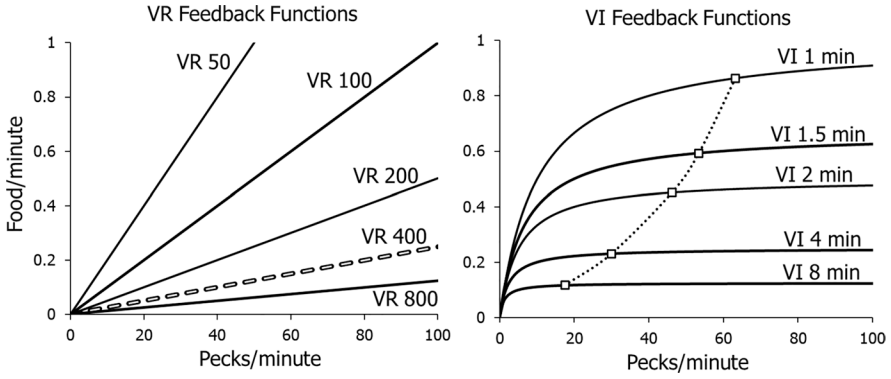

Fig. 9.5 Feedback functions for VI and VR schedules. Left: VR feedback functions are straight lines. The dashed line represents the threshold (VR 400) below which operant activity is not maintained. In the graph, VR 800 would not maintain any operant activity. Right: VI feedback functions are curves that approach an asymptote given by the programmed food (PIE) rate. The dotted lines connect squares showing typical performances across VI schedules

food rate, we model the relation between B_N and V_N as interval-like, considering that V_N likely would approach an upper limit when food was absent and B_N would take up all the time:

$$V_N = \frac{1}{t + \frac{1}{B_N}}. \tag{9.14}$$

The asymptotic level of V_N equals $\frac{1}{t}$. Past experience indicates that t equal to about 3.0 works (Baum & Davison, 2014), but the estimate of t will generally depend on the scaling of all variables. We can estimate B_N with the following equation, derived from Eqs. 9.6 and 9.14:

$$B_N = \frac{K - \left(V + V_0\right)}{t\left(V + V_0\right) + 1}. \tag{9.15}$$

Using Eqs. 9.14 and 9.15 to calculate V_N, we arrive at an equation for a single schedule:

$$B = \frac{Kcr^S}{cr^S + c_0 r^{S_0} + V_N}, \tag{9.16}$$

where K is the sum of $B + B_0 + B_N$, as before. With parameters K, c, c_0, S, S_0, and t, we can calculate predictions of activity rate B.

If an equilibrium point exists for a schedule, it will be where the feedback function intersects Eq. 9.16. Figure 9.6 shows such equilibrium points, the circles. They are equilibrium points because induction acts to correct deviations of activity rate away from them in either direction. If activity rate falls short, the disparity between the dashed curve and line means an excessive food rate for the activity rate, and activity rate increases. If the activity rate exceeds the equilibrium rate, the disparity between the curve and the feedback function means that the food rate is too low to induce that activity rate, and the activity rate falls back toward equilibrium.

Figure 9.6 shows the contrast between VR and VI schedules. Equilibrium points exist for all VI schedules, no matter how lean, because the feedback functions flatten, never falling beneath the dashed curve. In contrast, equilibrium points exist only for VR feedback lines above a certain level. The line for VR 320 predicts low activity rate, but the line for VR 640 falls below the dashed curve. No equilibrium point exists for VR 640, because its feedback line falls below threshold, as shown in Fig. 9.5. In Fig. 9.6, the threshold falls between VR 320 and VR 640.

The panel on the left of Fig. 9.7 shows the equilibrium points from Fig. 9.6, illustrating the predictions made by the model. The panel on the right shows fits to actual data. In Baum's (1993) experiment, VR and VI components alternated in a multiple schedule. The components were widely separated in time to minimize interaction. The VR peck rates were corrected by subtracting 0.17 s from the time

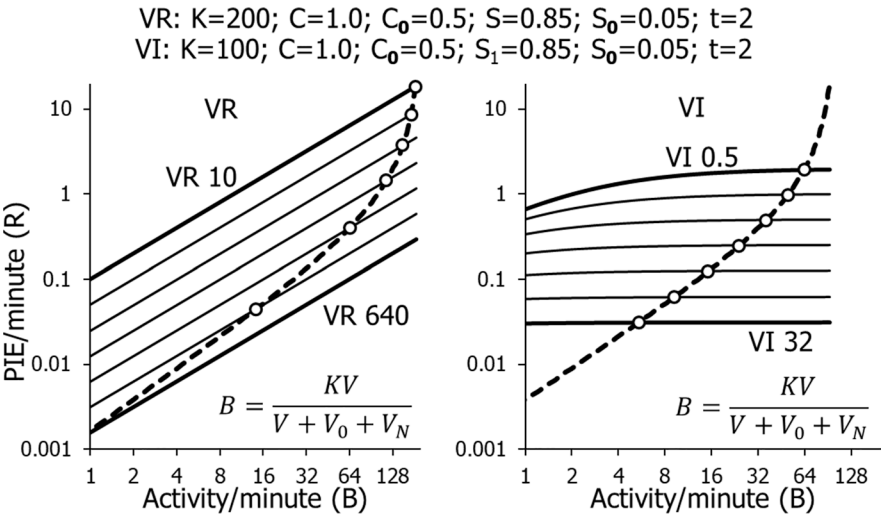

Fig. 9.6 Predicted equilibrium points for VR and VI schedules. The dashed curve represents Eq. 9.16 and the equation shown. Left: various VR feedback lines, ranging from VR 10 to VR 640, intersecting the curve at different equilibrium points (circles). Right: various VI feedback functions, ranging from VI 0.5 minute to VI 32 minutes, intersecting the curve at different equilibrium points. Note logarithmic axes

base for each food delivery to remove an artifact due to the physical separation between the food hopper and the pecking key (see Baum & Grace, 2020, for more explanation). The VI data are the same as in Fig. 9.4, from Baum (2021). As reflected in the estimates of K in Fig. 9.7, the VR schedules maintained higher activity rates—higher switch-operation rates—for the same food rates. As VI schedules become richer and food rate increases beyond the lower range shown, pigeons' interaction with the key shifts toward patterns that operate the key at higher rates, approaching the switch-operation rates for VR schedules (Baum, 1993; Baum & Grace, 2020).

Concurrent Variable Ratio and Variable Interval Schedules

Compared with concurrent VI schedules, relatively few experiments have included choice between a VI schedule and a VR schedule. Different results might occur for at least two reasons: (a) VR schedules constrain the possible relation between PIE rate and activity, because PIE rate must always be proportional to activity rate; and (b) VR schedules induce activity that operates the recording switch at extremely high rates, as evidenced by the high level of K in Fig. 9.7. Although results from experiments with concurrent VR VI appear to be comparable to those from concurrent VI VI, these two characteristics of VR activity appear in the results (Baum, 1981).

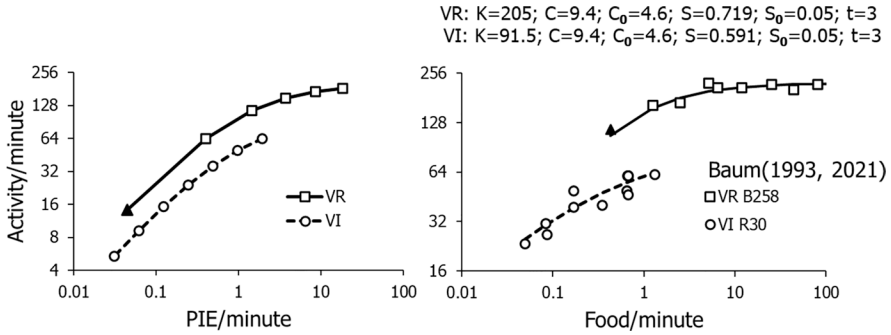

Fig. 9.7 VR and VI Performance Predictions and Data. Left: Equilibrium points from Fig. 9.6. The triangle represents predicted performance for VR 320; VR 640 is predicted to fail to maintain operant activity. Right: Squares show one pigeon's VR data from an experiment in which VR and VI food rates were equated in a multiple schedule (Baum, 1993). Circles show one pigeon's data from a series of VI schedules (Baum, 2021; Fig. 9.4). Parameters for VR and VI were all the same except K and S. The triangle indicates the leanest VR that maintained the operant activity

The constraint that VR schedules impose becomes clear if we rewrite Eq. 9.8 with S_1 and S_2 equal to 1.0, and c_1/c_2 equal to b:

$$\frac{B_{VI}}{r_{VI}} = \frac{B_{VR}}{br_{VR}} = \frac{VR}{b}. \tag{9.17}$$

This equation says that matching is entirely determined by the VR schedule, because it fixes the time per PIE delivery for that alternative. For matching to occur, time on the VI alternative per PIE delivery must equal the VR divided by bias b. If allocation is unbiased, b equals 1.0, and time per PIE delivery must equal the VR for matching to occur. If S_1 and S_2 are not equal to 1.0, the constraint is less obvious, but the closer the exponents are to 1.0, the more obvious is the constraint.

The higher rate of switch operation results in a bias in favor of the VR alternative when activity rate is measured as switch operations, but this bias vanishes when activity time is measured (Herrnstein & Heyman, 1979). In a study on concurrent VR VI with rats, Baum and Aparicio (1999) found that the VR constraint resulted in bias in lever presses toward the VR, regardless of the position of the VR and regardless of the degree of preference.

Apart from the matching theory developed here, other attempts at explaining concurrent performance have often appealed to optimality theory. Matching on concurrent VI VI is readily shown to be optimal (Baum, 1981), but matching on concurrent VR VI conflicts with optimality theory. For example, experiments with pigeons that show matching on concurrent VR VI tell against optimality. Bell and Baum (2017) found that even when the VI alternative was rich and the VR lean, so that any activity at the VR alternative was non-optimal, because it could only reduce overall food rate, pigeons still visited the VR alternative anyway. Baum and Aparicio (1999) found a tendency toward a pattern in rats that might be optimal. Performance in their

experiment moved toward what has been labeled "fix and sample": activity predominantly at the VR alternative with occasional brief visits (samples) at the VI alternative (Aparicio & Baum, 2006; Baum et al., 1999).

Variation in methods may produce variation in results of experiments with both concurrent VI VI and concurrent VR VI. Apart from species, always important, how long conditions last may be one important factor. Performance in rapidly changing conditions, when several conditions occur within each session, stabilizes quickly, but may stabilize at a performance different from an experiment in which each condition lasts for many sessions (Aparicio et al., 2016; Baum et al., 1999; Davison & Baum, 2000).

Concurrent Variable Ratio Variable Ratio Schedules

Only a few experiments have examined performance with choice between two VR schedules. Herrnstein and Loveland (1975) exposed pigeons to a variety of VR-VR pairs. Although the pigeons' activity generally focused on the richer (smaller) VR, it did not usually move all the way to exclusive preference.

One would expect that choice between VRs would be highly constrained, because rate of PIE delivery r is proportional to activity rate B at both alternatives:

$$\frac{B_1}{B_2} = \frac{VR_1}{VR_2} \bullet \frac{r_1}{r_2}. \tag{9.18}$$

This equation shows that simple matching between two VR schedules can only occur when the two VRs are equal. When the VRs are unequal, the behavior ratio cannot equal the PIE ratio, because the ratio of the VRs differs from 1.0 and is different for each VR-VR pair. For some kind of matching to occur, the behavior ratio must be sensitive to the ratio of the VR schedules. One way to express this necessity is to recast a VR as a probability—P equals $\frac{1}{VR}$, which equals $\frac{r}{B}$. Suppose the behavior ratio matches the ratio of probabilities:

$$\frac{B_1}{B_2} = \frac{P_1}{P_2} = \frac{r_1}{B_1}\frac{B_2}{r_2}. \tag{9.19}$$

Multiplying both sides by $\frac{B_1}{B_2}$ and taking the square root, we arrive at:

$$\frac{B_1}{B_2} = \frac{P_1}{P_2} = \left(\frac{r_1}{r_2}\right)^{0.5}. \tag{9.20}$$

This equation tells us that probability matching is identical with rate matching, only with an exponent of 0.5—a square root law.

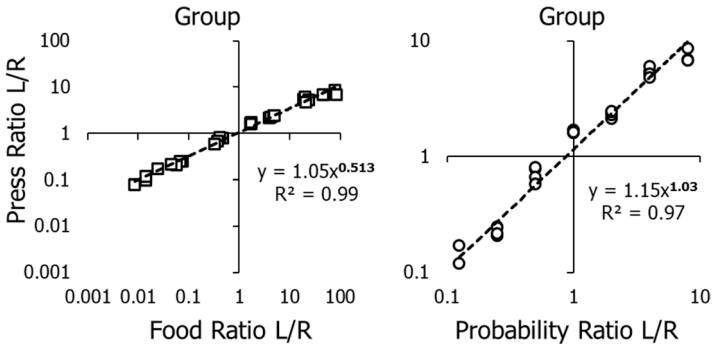

Fig. 9.8 Probability matching in concurrent VR VR schedules in rats. Left: Behavior ratio as a function of food ratio. Right: Behavior ratio as a function of probability ratio. Dashed lines were fitted to the logarithms by least-squares. Data are from Baum et al. (2022). Note logarithmic axes

Figure 9.8 shows an example of probability matching in rats (Baum et al., 2022). The procedure presented 7 different concurrent VR VR schedules within each session—a dynamic environment. Each component lasted until ten food deliveries had occurred, and the data from the last 4 inter-food intervals were taken as stable. Three conditions were run, in which 2, 4, or 8 lever presses were treated as a unit of activity. Figure 9.8 shows results from the 2-press-unit condition. These group results were representative of the individual rats' results. The individual rats' food-rate exponents varied from 0.51 to 0.58, with a mean of 0.54, and their food-probability exponents were all close to 1.0. In the 4-press-unit condition, individual food-rate exponents ranged from 0.45 to 0.55, with a mean of 0.5. When the unit was 8 presses, individual exponents ranged from 0.42 to 0.56, with a mean of 0.48. Although some variation occurred, the food-rate exponents all tended to be close to 0.5.

The signature of probability matching is a PIE-rate exponent equal to 0.5, because that matching relation is algebraically identical with probability matching with an exponent of 1.0. The question arises, however, of distinguishing whether an exponent of 0.5 represents a process of probability matching or just strong undermatching to rate ratio. The exponent alone cannot answer. Other features of performance, such as visit durations at the two alternatives, may decide. See Baum et al. (2022) for further discussion.

Avoidance and Punishment

Up to now, we have been dealing only with PIEs, such as food, water, mates, nest, and offspring, that are fitness-promoting and induce activities that approach and enhance them. Other PIEs tend to threaten fitness, such as predators, cold, and

injury. These fitness-threatening PIEs induce activities that mitigate or avoid them. A predator induces flight, hiding, or defense. An injury induces avoidance.

The first study of free-operant avoidance employed mild electric shock as a proxy for injury. Sidman (1962) invented a procedure that maintained avoidance in the form of lever pressing in rats. His procedure depends on two intervals: the shock-shock interval (S-S) and the response-shock interval (R-S). In the absence of lever pressing, shocks occur regularly at a rate determined by the S-S. Every time the lever is pressed, shock is postponed by the R-S. For example, with S-S equal to 10 s and R-S equal to 20 s, in the absence of presses, shock after shock occurs every 10 s, but every press is followed by shock only after 20 s without a press. In principle, lever pressing could occur at a high enough rate to reduce the shock rate to zero. This rarely happens, because when the shock rate is zero, pauses to avoidance occur that produce a shock after the R-S. If the pause continues, shocks resume at the S-S. Sidman studied a wide variety of S-S and R-S combinations and observed a range of avoidance (press) rates.

Sidman (1962, 1966) proposed to explain the avoidance activity he observed as due to the reduction in shock rate produced by the lever pressing. Analyzing Sidman's data, Baum (2020) found that shock-rate reduction failed to account for the pressing, but that all the various conditions could be explained by the shock rate received, that avoidance was induced by the shocks received.

Figure 9.9 shows two examples of shock rate inducing avoidance. On the left, press rates in one of Sidman's (1962) rats are plotted as a function of rate of shocks received. The line fitted to the points represents the power function shown. On the right, a smaller data set (Clark & Hull, 1966) consisting of procedures with equal S-S and R-S also shows power-function induction by received shocks. For more examples and more discussion, see Baum (2020).

To explain avoidance, we might apply a version of Eq. 9.16:

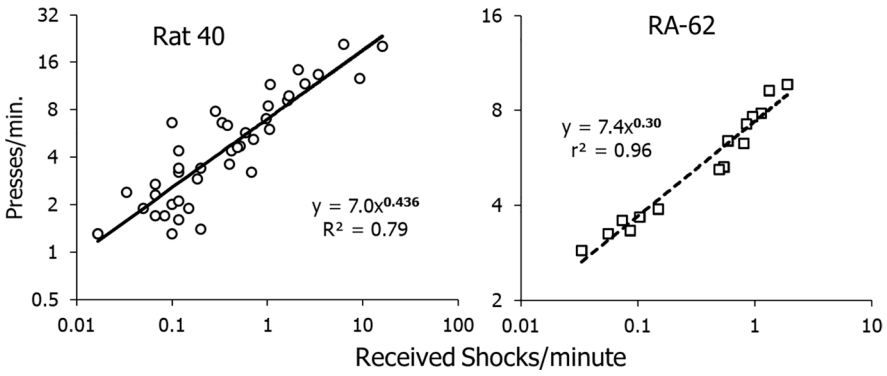

Fig. 9.9 Avoidance induced according to power functions. Left: avoidance (press) rates in one of Sidman's (1962) rats across a wide variety of conditions, showing power-function induction of avoidance by received shocks. Right: avoidance induced by received shocks in a smaller data set (Clark & Hull, 1966). Lines represent the power functions shown. Note logarithmic coordinates

$$B = \frac{KV}{V + V_0 + V_N} = \frac{Kcz^S}{cz^S + c_0 z^{S_0} + V_N}. \tag{9.21}$$

In Eq. 9.21, B is avoidance activity such as lever pressing, z is shock rate, and K equals the sum of all activities: $B + B_0 + B_N$. Since Fig. 9.9 shows that B is directly related to z by a power function, the denominator on the right must remain constant as z varies, just as was true in Eq. 9.13. We may speculate that as z increases and decreases, V increases and decreases, but V_0 and V_N also may decrease and increase, offsetting the variation in V.

We are now in a position to address punishment, which is the relation in which a single activity is both induced by a fitness-enhancing PIE and also produces a fitness-threatening PIE. Criminal activities are induced by their benefits but may produce incarceration. Boasting about one's achievements lets people know about them, but produces disapproval as well. In the laboratory, electric shocks may be superimposed on a schedule that produces food. For example, a rat may press a lever that produces food on a VI schedule and also produces shock according to a VR schedule. When the punishment is effective, the shocks induce activities that compete with lever pressing, lowering the press rate.

Punishment may be seen as the complement of avoidance. In Sidman's procedure, for example, all activities other than lever pressing are punished, because they result in shock. The only difference between avoidance and punishment is that in avoidance a specific activity avoids shock and all other activities are punished, whereas in punishment a specific activity produces shock and all other activities avoid shock.

How to model punishment is not obvious. One approach is to think of the activities (B_z) induced by the threats ("punishers"; shock) as cutting into the activity (B) maintained by the enhancers ("rewards"; food). If the activity that would be maintained if the punishment were ineffective is B', then B equals B'-B_z. We write equations for B' and B_z:

$$B' = \frac{KV}{V + V_z + V_0 + V_N}, \tag{9.22}$$

$$B_z = \frac{KV_z}{V + V_z + V_0 + V_N}, \tag{9.23}$$

$$B = B' - B_z = \frac{K\left(V - V_z\right)}{V + V_z + V_0 + V_N}. \tag{9.24}$$

I was unable to find a data set to test Eq. 9.24, but some experiments with concurrent schedules have programmed both food and electric shock on two choice alternatives. For each choice alternative, Eq. 9.24 becomes:

$$B_1 = \frac{K\left(V_1 - V_{z1}\right)}{V_1 + V_{z1} + V_2 + V_{z2} + V_0 + V_N}. \tag{9.25}$$

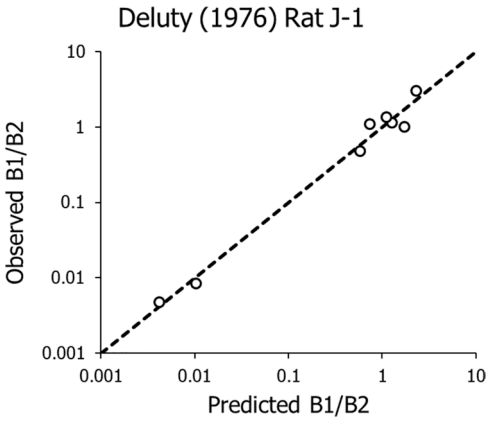

Fig. 9.10 Equation 9.26 applied to the results from one of Deluty's (1976) rats studying choice between alternatives producing both food and electric shock. Note logarithmic axes

When we take the ratio of the two equations for the two alternatives, we obtain:

$$\frac{B_1}{B_2} = b\frac{V_1 - V_{z1}}{V_2 - V_{z2}}, \tag{9.26}$$

and this equation is testable with at least one data set from Deluty (1976). To reduce the number of free parameters in Eq. 9.26, I set all the c coefficients to 1.0, and the exponent for food (S_r) also to 1.0, leaving only the exponent for shock (S_z) and bias (b) as free parameters. The bias b should reflect any position bias between the two levers. The result for one of Deluty's rats appears in Fig. 9.10. With S_z equal to 0.905 and b equal to 0.865, the predictions of Eq. 9.26 are near perfect. The bias indicates a position preference for the right lever.

We can try predicting the activity rates, using Eq. 9.25, if we simplify by assuming the last two items in the denominator are negligible in comparison with V_1, V_2, V_{z1}, and V_{z2}. Then we may use the same parameters as for Fig. 9.10, but adding separate estimates of K for Alternatives 1 and 2: K_1 and K_2. Thus, Eq. 9.25 becomes:

$$B_1 = \frac{K_1\left(V_1 - V_{z1}\right)}{V_1 + V_{z1} + V_2 + V_{z2} + V_0 + V_N}, \tag{9.27}$$

and a parallel equation applies to B_2. Figure 9.11 shows the results of applying those two equations.

Equation 9.27 describes the activity rates moderately well. The estimates of K are radically unequal: K_1 equaled 114, and K_2 equaled 74. The ratio of K_2 to K_1 equaled 0.647, which is considerably smaller than b in the analysis in Fig. 9.10. The difference might reflect a position bias in addition to the unequal estimates of K. Some residual variance remains unaccounted for, more for B_2 (0.413) than for B_1 (0.115).

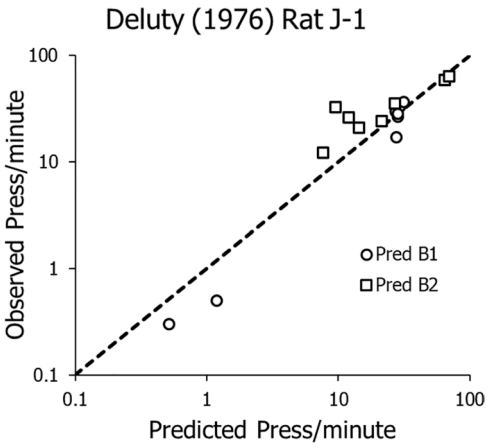

Fig. 9.11 Predicting activity rates in choice between alternatives producing both good and bad PIEs. Equation 9.27 predicts the rates. The broken line is the locus of equality between predicted and observed rates

Conclusion

Equations 9.1, 9.2, and 9.4 provide the basis for a theoretical framework that may be used to explain a broad range of behavioral phenomena. Here, we have only touched on ones of these that have been explored to a significant extent. These are questions of rates of various activities in adaptation to environmental relations. We have not addressed conflicts of time scale, when short-term consequences conflict with long-term consequences. Some discussion of these conflicts has appeared in two earlier papers (see Baum, 2016, 2022). No discussion of dynamics beyond explaining activity rates on VI and VR schedules was possible, and dynamics in this framework remain to be explored. An initial attempt appears in a paper by Podlesnik and Baum (2024). Effects of varying covariance between an activity and a PIE have been largely unexplored, although one study studied varying covariance (Baum, 2022). Another topic barely explored and omitted here is what has been traditionally called "stimulus control." In this framework, covariance between a stimulus (signal) and a PIE causes the stimulus to induce activities that the PIE induces, particularly operant activity. This approach replaces associative models like the Rescorla-Wagner model that rely on hidden variables like response "strength" (de Haan & Simon, 2024). This matching and evolutionary framework, called the "multiscale molar view," remains to be applied to many other behavioral phenomena (Baum, 2018, 2024).

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