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RADIOACTIVE COLLOID ADVECTION IN A SORBING POROUS MEDIUM: ANALYTICAL SOLUTION

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Colloids are capable of increasing rates of hydrogeologic transport because (1) they can move faster on average than solute due to hydrodynamic chromotography, (2) they can be retarded less than solute by sorption onto rock, and (3) they can increase the apparent liquid-phase radionuclide concentration.

Colloid formation might occur as water carrying radionuclide solute leaves the immediate vicinity of the waste package and encounters changing geochemical conditions. Also, colloids of less soluble waste species might form as the waste matrix around them dissolves and migrates away.

We restrict our analysis to "true colloids" that are a solid-phase of the radionuclide itself. We neglect "pseudocolloids", colloids of a separate material onto which radiounuclides are sorbed.

To predict one-dimensional transport of colloids through a sorbing porous medium with coupling to the solute phase, we assume a concentration step function in time at the inlet boundary. We neglect dispersion and assume local sorption equilibrium with distinct retardation coefficients for the colloid and solute phases. We also assume local equilibrium between colloid and solute phase concentrations, resulting in solute concentration of the colloid solubility limit whenever the colloid phase is present. Colloid filtration is neglected.

We expect the colloid front to travel faster than that of the solute as a result of disparities in retardation. The solution involves mathematical difficulties. A moving boundary forms with discontinuous colloid and solute concentrations. At this front a localized (delta function) reaction occurs between colloid and solute with colloid instantaneously dissolving to maintain solute saturation. Furthermore, the speed at which the front moves is dependent on both colloid and solute concentrations at the front. Such an idealized problem is ill-suited to numerical techniques. The analytical solution described in this paper provides unique insight to the fundamental behavior of a colloid system.

A solution is obtained with two distinct time periods. Early in time we see a sharp front moving away from the source with discontinuities in both colloid and solute concentrations. The front velocity depends on the relative colloid and solute concentrations at the front and on the individual transport velocities and retardation coefficients. As the front progresses, decay of solute and colloids causes the colloid concentration to decrease with distance, while the solute concentration is maintained at its solubility limit. Therefore, the front velocity steadily decreases with time and distance. At the critical time t^*, the colloid concentration at the front vanishes. At t^*, the front location x^* defines the distance beyond which no colloid penetrates.

Beyond x^*, the solute front continues at a steady speed with the concentration decaying exponentially with distance.

The conservation form (mass balance) of the governing equations is given by

$$R_1 \left[\frac{\partial}{\partial t} + \lambda + \frac{v_1}{R_1} \frac{\partial}{\partial x} \right] C_1(x,t) + R_2 \left[\frac{\partial}{\partial t} + \lambda + \frac{v_2}{R_2} \frac{\partial}{\partial x} \right] C_2(x,t) = 0, \quad x, t > 0 \tag{1}$$

where R_1 is the colloid retardation coefficient, λ is the decay constant, v_1 is the linear colloid velocity, $C_1(x,t)$ is the colloid concentration in the liquid phase, R_2 is the solute retardation coefficient, v_2 is the linear solute velocity, and $C_2(x,t)$ is the solute concentration. The boundary conditions at $x = 0$ are given by $C_1(0,t) = C_0$ and $C_2(0,t) = C_S$ the solubility for $t > 0$. At $t = 0$, $C_1(x,0) = C_2(x,0) = 0$ for $x > 0$. The early time solution is

$$C_1(x,t) = \left[\left(C_0 + \frac{R_2}{R_1} C_S \right) e^{-\lambda x R_1 / v_1} - \frac{R_2}{R_1} C_S \right] h(x_f(t) - x), \quad x \geq 0, \ 0 \leq t \leq t^* \tag{2}$$

$$C_2(x,t) = C_S h(x_f(t) - x), \quad x \geq 0, \ 0 \leq t \leq t^* \tag{3}$$

where $h(x)$ is the unit step function, $x_f(t)$ is the front location given by

$$x_f(t) = \frac{-v_1}{\lambda R_1} \log \left[1 - \frac{1 + \frac{v_2 C_S}{v_1 C_0}}{1 + \frac{R_2 C_S}{R_1 C_0}} \left(1 - e^{-\lambda t} \right) \right], \quad 0 \leq t \leq t^* \tag{4}$$

and t^* is given by

$$t^* = \frac{1}{\lambda} \log \left(1 + \frac{v_1 C_0}{v_2 C_S} \right) \tag{5}$$

The uniqueness of this solution is under study.

To illustrate the effect of dissolution interaction on transport, Figure 1 shows concentration profiles at 1000 years with and without colloid/solute coupling. Without coupling, the colloid front is at 1000 meters (off the scale) when the solute front is at 10 meters. With coupling the colloid/solute front is at 26 meters. The maximum colloid penetration distance x^* is 28 meters and t^* is 1160 years.

Colloids can significantly increase the transport rates of contaminants under certain conditions, thereby reducing the containment ability of geologic disposal sites. Neglecting colloid/solute interaction is conservative but unrealistic. This study gives simple formulas for estimating the velocity of a colloid front and the maximum penetration distance of a colloids that are in local dissolution equilibrium with solute.
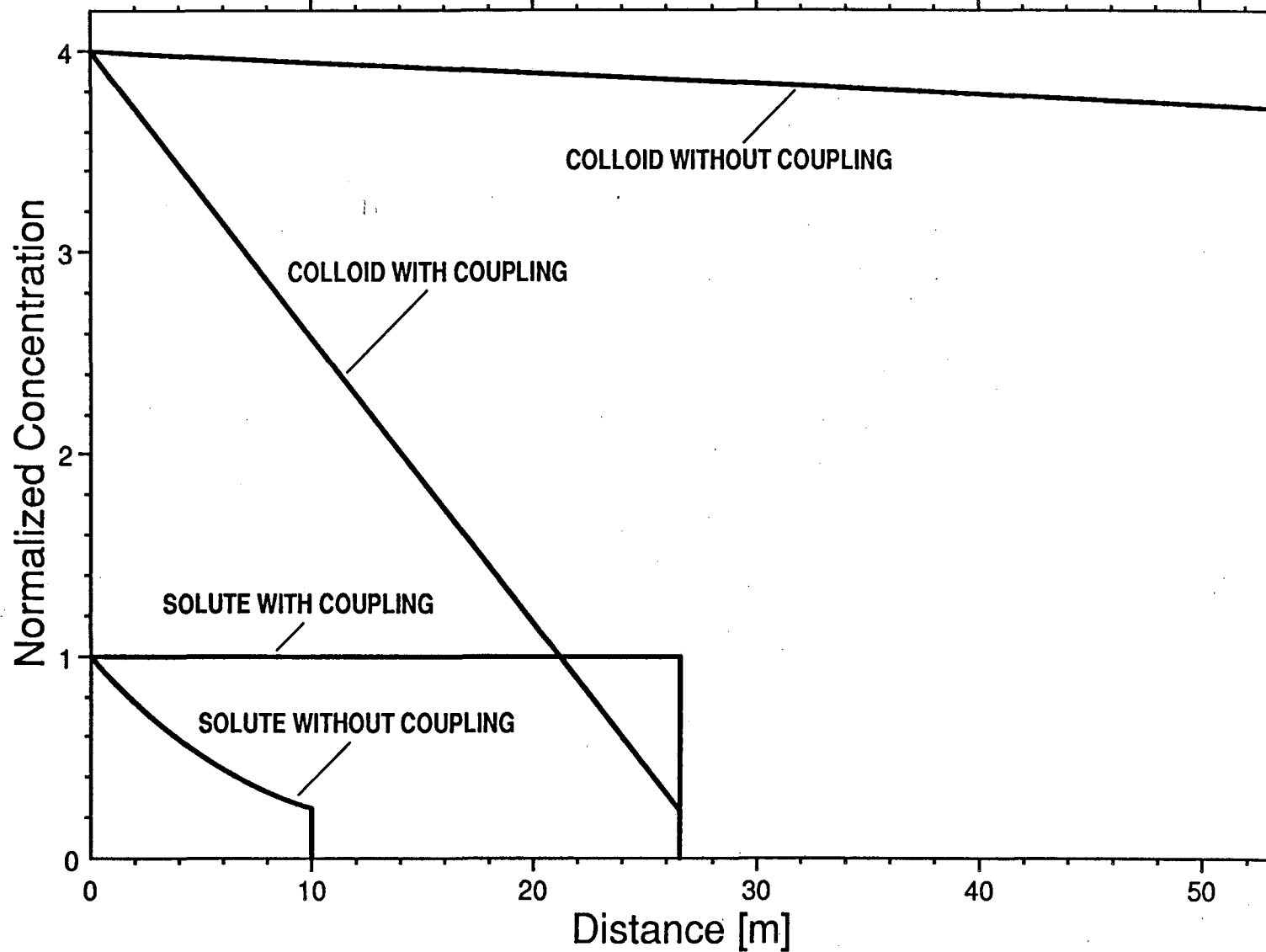
FIGURE 1. Concentration profiles for colloids and solute at 1000 years, with and without dissolution coupling. ($v_1 = v_2 = 1$ m/y, $R_1 = 1$, $R_2 = 100$, $C_O/C_S = 4$, $t_{\frac{1}{2}} = 500$ y)

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