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Author
Chew, G.F.

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G.F. Chew
Physics Division

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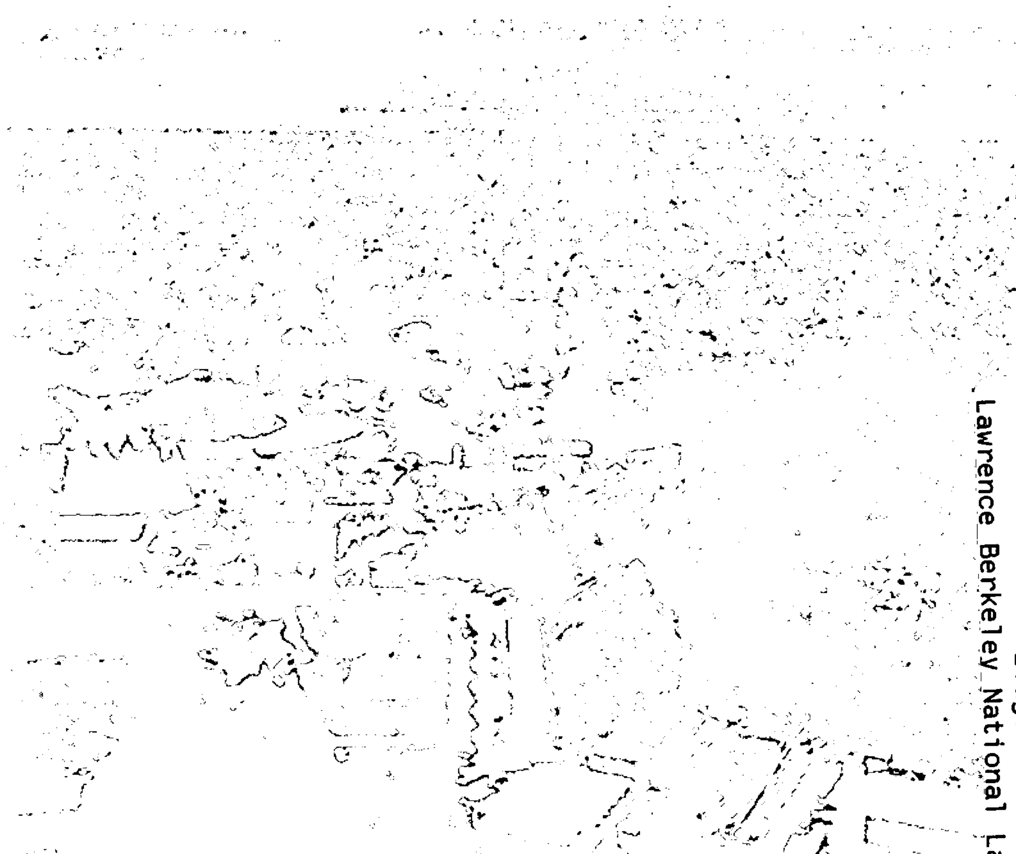

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G.F. Chew

Theoretical Physics Group
Physics Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

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OBJECTIVITY IN MULTI-TIME COSMOLOGY

Geoffrey Chew

Theoretical Physics Group, Lawrence Berkeley Laboratory
One Cyclotron Road
Berkeley, California 94720, U.S.A.

ABSTRACT

Geodesics with particle-trajectory interpretation, that on Hubble scale closely approach standpoint spatial location, are studied within the homogeneous-universe approximation to multi-time cosmology and shown to be pseudo-Riemannian up to an error related to angular momentum. Matter flowing along such geodesics is correspondingly objectified. Energy density, including an important vacuum contribution, is deduced. Also inferred, as functions of redshift, are luminosity distance and angular-diameter distance. These increase less rapidly than in Milne cosmology but more rapidly than for Einstein-de Sitter.

I. Introduction

Reference (1) describes a single-parameter multi-time standpoint-based cosmological model that is demonstrating phenomenological promise. The present paper studies those homogeneous-universe geodesics that on Hubble scale come "close" to standpoint. It will be found that such "almost-radial"–or "low angular-momentum"–geodesics are describable within a pseudo-Riemannian approximation to the model's Finsler metric. The model correspondingly displays "approximate objectivity." Spacetime curvature in standpoint neighborhood will translate into an energy density determined by the model's sole parameter–standpoint age, with an important "vacuum" contribution to energy density emerging.

A cosmological constant of motion related to angular momentum will lead not only to an ordinary second-order differential equation satisfied by geodesics but to a Hamiltonian that is constant for almost-radial light propagation. This Hamiltonian will allow deduction of luminosity distance as function of redshift, with the "standard" connection to angular-diameter distance. These "optical distances" increase with redshift more slowly than in the Milne model but more rapidly than for Einstein-de Sitter. Our concluding section discusses the model's correlation of "low angular momentum" with objective reality–an association that complements the celebrated quantum-mechanical correlation between objectivity and "large action."

II. Gravitational Action Expressed Through Polar Coordinates

The reader is referred to Reference (1) for definition of "spacetime belonging to standpoint i." Rotational invariance confines any homogeneous-universe geodesic within i spacetime to a plane containing standpoint i. To the extent we are concerned with a single standpoint (and not the relation between different standpoints), we shall in this paper often omit the index i. Let us then here introduce polar coordinates r, ψ for the geodesic–containing plane, the distance r from standpoint having the significance accorded the symbol r_i in Reference (1). The angle symbol ψ appears here for the first time; in Reference (1) the 2-dimensional direction symbol $\vec{n}_i$ was employed. Relating the two notations, confinement of motion to a plane means that

$$d\vec{n}_i^{\,2} = d\psi^2. \tag{1}$$

According to the Lagrangian-defining relation,

$$ds_i = -L_i \, dt_i, \tag{2}$$

the homogeneous-universe Finsler metric given by Formula (24) of Reference (1) is equivalent to the gravitational Lagrangian

$$L(t, r, \dot{r}, \dot{\psi}) \; = -(1 - \dot{r}^2)^{1/2} \mathcal{L}(t, r, u), \tag{3}$$

with

$$\dot{r} \equiv \frac{dr}{dt} \quad , \quad \dot{\psi} \equiv \frac{d\psi}{dt} \quad , \quad u \equiv \frac{r^2 \dot{\psi}^2}{1 - \dot{r}^2}, \tag{4}$$

and

$$\mathcal{L}(t, r, u) \; \equiv \frac{1}{\sqrt{2}} \{ (\mathcal{G} - gu)^2 + (\mathcal{G}' - g'u)^2 \}^{1/4}. \tag{5}$$

The coefficients $\mathcal{G}, g, \mathcal{G}', g'$ depend on t and r but not on ψ, the explicit dependence being given in Formulas (25), (26), (27) of Reference (1).

The 7-dimensional Finsler phase space (4 spacetime coordinates plus 3 velocities) has by symmetry been reduced to 5 dimensions with one ignorable coordinate ψ (on which L does not depend). The physical phase space is limited by

$$\left.\begin{array}{l} 0 \le t \pm r \le 2R \\ \\ 0 \le \psi < 2\pi \end{array}\right\} \text{“double cone interior”} \\ 0 \le \dot{r}^2 \le 1 \\ 0 \le u \le u_{\max}(r, t), \tag{6}$$

where the upper limit on u, as determined below, will be found to approach 1 as r^2/R^2 approaches zero. The positive parameter R has in Formula (37) of Reference (1) been related to “age” of standpoint and thereby to Hubble time.

Two special regions of phase space, where the Finsler metric becomes approximately pseudo-Riemannian, will receive emphasis in this paper:

1. The region spatially close to standpoint, where $r^2/R^2 << 1$.

2. The velocity-space region where $u << 1$.

A "nearly-radial"–or "small impact-parameter"–geodesic along its entire extent occupies at least one of the foregoing regions. Characterization of such a geodesic will be achieved through low value of a constant of motion stemming from rotational symmetry and related to angular momentum. The following two sections expose general dynamical considerations related to this constant. A radial Hamiltonian will be defined. For nearly-radial lightlike propagation, approximate constancy of this Hamiltonian will in Section VIII facilitate calculation of luminosity distance.

III. Geodesic Differential Equation

Rotational invariance allows any geodesic interpretable as "particle trajectory" (see Reference (1) for the meaning of "particle") to be described as a solution of a radial second-order ordinary differential equation whose coefficients depend on t, r and a conserved parameter J of length dimension, defined by

$$J \equiv \frac{\partial L(t, r, \dot{r}, \dot{\psi})}{\partial \dot{\psi}}. \tag{7}$$

For particles of nonzero rest mass, J is loosely describable as "angular momentum per unit rest mass." With respect to light propagation, such language is inappropriate.

In view of (3) and (4) one may rewrite (7) as

$$\frac{J}{r} = -\frac{\partial \mathcal{L}(t, r, (u^{1/2})^2)}{\partial u^{1/2}}. \tag{8}$$

Note absence of $\dot{r}$ from (8); definition of u was designed to achieve this absence. Assuming it possible to invert (8) so as to define a function $u_J(t, r)$ that allows $\dot{\psi}$ to be expressed in terms of $t, r, \dot{r}, J$ through

$$\dot{\psi} = (1 - \dot{r}^2)^{1/2} \, \frac{u_J^{1/2}(t, r)}{r}, \tag{9}$$

we define a "radial Hamiltonian,"

$$H_J(t, r, \dot{r}) \equiv \dot{r}\frac{\partial L}{\partial \dot{r}} + \dot{\psi}\frac{\partial L}{\partial \dot{\psi}} - L, \tag{10}$$

in terms of which the geodesic equation may be written

$$\frac{d}{dt}H_J(t, r, \dot{r}) = -\frac{\partial}{\partial t}L(t, r, \dot{r}, \dot{\psi}), \tag{11}$$

exploiting standard techniques of classical dynamics. From (3), (4) and (10) one finds

$$H_J(t,r,\dot{r}) = \frac{\mathcal{H}_J(t,r)}{(1-\dot{r}^2)^{1/2}}, \tag{12}$$

where

$$\mathcal{H}_J = \mathcal{L}(t,r,u_J(t,r)) + \frac{J u_J^{1/2}(t,r)}{r}. \tag{13}$$

In view of the relation

$$\frac{\partial}{\partial t}\mathcal{H}_J(t,r) = \frac{\partial \mathcal{L}(t,r,u_J)}{\partial t}, \tag{14}$$

which follows from (8) and (13), Equation (11) may be written

$$\frac{d}{dt}[\frac{\mathcal{H}_J}{(1-\dot{r}^2)^{1/2}}] = (1-\dot{r}^2)^{1/2}\frac{\partial \mathcal{H}_J}{\partial t}, \tag{15}$$

or, evaluating $\frac{d}{dt}$ in (15) as $\ddot{r}\frac{\partial}{\partial \dot{r}} + \dot{r}\frac{\partial}{\partial r} + \frac{\partial}{\partial t}$,

$$(\frac{\ddot{r}}{1-\dot{r}^2} + \dot{r}\frac{\partial}{\partial t} + \frac{\partial}{\partial r})\mathcal{H}_J(t,r) = 0. \tag{15'}$$

Yet another way of writing the geodesic differential equation is

$$\ddot{r} = -(1-\dot{r}^2)\frac{(\dot{r}\frac{\partial}{\partial t} + \frac{\partial}{\partial r})\mathcal{H}_J(t,r)}{\mathcal{H}_J(t,r)}. \tag{15''}$$

This latter form exhibits the stability of radial "lightlike" geodesics– characterized by $\dot{r}^2 = 1$.

More generally, the radial geodesics discussed in Reference (1) correspond to the special case of $J = 0$ with $u_J = 0$, where according to (13), $\mathcal{H}_{J=0} = \mathcal{L}(t,r,0) = \frac{1}{\sqrt{2}}(\mathcal{G}^2 + \mathcal{G}'^2)^{1/4}$. Let us now examine in detail the relation between J and u_J.

IV. Physical Interval of u

Evaluating (8) in view of (5), one finds

$$\frac{J}{r} = \frac{(g^2+g'^2)^{1/4}}{\sqrt{2}} \frac{u_J^{1/2}(\omega - u_J)}{[(\omega-u_J)^2+\epsilon^2]^{3/4}}, \tag{16}$$

where ω and ϵ are functions of t and r given by

$$\omega \equiv \frac{g\mathcal{G} + g'\mathcal{G}'}{g^2+g'^2}, \tag{17}$$

$$\epsilon \equiv \frac{g'\mathcal{G} - g\mathcal{G}'}{g^2+g'^2}. \tag{18}$$

Using the forms for $g, \mathcal{G}, g', \mathcal{G}'$ given in Reference (1), one finds $\omega \geq 1$ and $\epsilon \geq 0$ everywhere within the double light-cone interior that constitutes standpoint spacetime. The lower limits on ϵ and ω are seen to be reached at $r = 0$.

In terms of ω and ϵ, the function $\mathcal{L}(t, r, u)$ becomes

$$\mathcal{L} = \frac{1}{\sqrt{2}}(g^2 + g'^2)^{1/4}\{(\omega - u)^2 + \epsilon^2\}^{1/4} \tag{19}$$

so, from (13) after a short calculation,

$$\mathcal{H}'_J \equiv \frac{\mathcal{H}_J}{J} = \frac{1}{ru_J}(\omega + \frac{\epsilon^2}{\omega - u_J}). \tag{20}$$

Because of J constancy along a geodesic, this latter function may, for $J \neq 0$, replace $\mathcal{H}_J$ in the differential equation (15''). The simple form of (20) is somewhat illusory because we still require the function $u_J(t, r)$–defined by inversion of (16). Is this inversion unambiguous?

Consider J as a function of u_J at any fixed spacetime point for which $r > 0$. According to (16), J vanishes at $u_J = 0$ and initially grows with increasing u_J until a maximum is reached at $u_J = u_{\max}(t, r)$, where

$$\omega - u_{\max} = \epsilon(2 + \frac{9}{4}\frac{\epsilon^2}{\omega^2})^{1/2} - \frac{3}{2}\frac{\epsilon^2}{\omega}. \tag{21}$$

Note that $u_{\max}$ is always smaller than ω. The order of magnitude of the maximum J is R throughout most of standpoint spacetime. Because of the need to invert (16), the physical interval of u is bounded from above by $u_{\max}$. Within the interval $0 \leq u_J \leq u_{\max}$, the relation between u_J and J is monotonic and invertible.

It is interesting to evaluate $u_{\max}(t, r)$ in the neighborhood of standpoint–where $r/R << 1$ and $| t - R | << R$. One finds, from (21), (17), (18) and the formulas for $g, \mathcal{G}, g', \mathcal{G}'$ in Reference (1),

$$\omega(t, r) = 1 + (\frac{r}{R})^2 + \ldots \tag{22}$$

$$\epsilon(t, r) = \frac{1}{2}(\frac{r}{R})^2 + \ldots \tag{23}$$

$$\omega - u_{\max}(t, r) = \frac{1}{\sqrt{2}}(\frac{r}{R})^2 + \ldots \tag{24}$$

We see that $u_{\max}$ approaches 1 as $r/R \to 0$, differing from 1 by order $(r/R)^2$. Even when $| t - R |$ is not much smaller than R, the foregoing order of magnitude for $1 - u_{\max}$ prevails in standpoint spatial neighborhood.

V. Pseudo-Riemannian Approximation

We now show how a pseudo-Riemannian approximation to the Finsler metric of standpoint spacetime results *either* from expanding $\mathcal{L}^2(t,r,u)$ in powers of u *or* from expanding $\mathcal{L}^2/1-u$ in powers of $\frac{u}{1-u}(\frac{r}{R})^2$ and keeping terms of zeroth and first order. These expansions both relate to an expansion in powers of $(J/R)^2$. Referring to Formula (16), one sees that for $r << R$, $J \approx r(\frac{u}{1-u})^{1/2}$, whereas for $r \sim R$, J is of order $u^{1/2}R$ when u is small.

Consider first the straightforward small $-u$ expansion:

$$\begin{aligned}\mathcal{L}^2(t,r,u) &= \frac{1}{2}\{\mathcal{G}^2+\mathcal{G}'^2-2u(g\mathcal{G}+g'\mathcal{G}')+u^2(g^2+g'^2)\}^{1/2}\\ &\approx \frac{1}{2}\{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}-\frac{g\mathcal{G}+g'\mathcal{G}'}{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}}u \quad + \quad \ldots\}. \end{aligned} \tag{25}$$

Multiplication of (25) by $(1-\dot{r}^2)dt^2$ leads to

$$ds^2 \approx \frac{1}{2}\{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}(dt^2-dr^2)-\frac{g\mathcal{G}+g'\mathcal{G}'}{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}}r^2d\psi^2\}, \tag{26}$$

once one remembers that $(1-\dot{r}^2)u = r^2\dot{\psi}^2$. Because $g\mathcal{G}+g'\mathcal{G}'$ is positive throughout standpoint spacetime, (26) is pseudo-Riemannian. Error in (26) is of order u^2.

A more powerful expansion is achievable by recognizing that the t,r functions

$$\begin{aligned}\epsilon &\equiv \mathcal{G}-g\\ \epsilon' &\equiv \mathcal{G}'-g'\end{aligned} \tag{27}$$

both admit power series expansion in $(r/R)^2$ and $\frac{t-R}{R}$, with the leading terms being of *first* order in $(r/R)^2$. That is, ϵ and ϵ' vanish linearly as $(r/R)^2 \to 0$. It follows that zeroth and first-order terms of an expansion in $(r/R)^2$ coincide with zeroth and first-order terms of an expansion in ϵ and ϵ'. Rewriting $\mathcal{L}^2$ as

$$\mathcal{L}^2(t,r,u) = \frac{1}{2}\{[\mathcal{G}(1-u)+\epsilon u]^2+[\mathcal{G}'(1-u)+\epsilon' u]^2\}^{1/2}, \tag{28}$$

we expand $\mathcal{L}^2/1-u$ in powers of $\frac{\epsilon u}{1-u}$ and $\frac{\epsilon' u}{1-u}$ to find

$$\begin{aligned}\frac{\mathcal{L}^2(t,r,u)}{1-u} &= \frac{1}{2}\{\mathcal{G}^2+\mathcal{G}'^2+2\frac{u}{1-u}(\epsilon\mathcal{G}+\epsilon'\mathcal{G}')+(\frac{u}{1-u})^2(\epsilon+\epsilon'^2)\}^{1/2}\\ &\approx \frac{1}{2}\{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}+\frac{u}{1-u}\frac{\epsilon\mathcal{G}+\epsilon'\mathcal{G}'}{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}}+\ldots\}.\end{aligned} \tag{29}$$

Multiplication by $(1-u)(1-\dot{r}^2)dt^2$ then gives

$$ds^2 \approx \frac{dt^2}{2}\{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}(1-\dot{r}^2)+r^2\dot{\psi}^2[-(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}+\frac{\epsilon\mathcal{G}+\epsilon'\mathcal{G}'}{(\mathcal{G}^2+\mathcal{G}'^2)^{1/2}}]\} \quad , \tag{30}$$

which becomes identical with (26) after rearrangement of terms in the square bracket. The error in (26) is now seen to be of order $(\frac{u}{1-u})^2(\frac{r}{R})^4$, so this pseudo-Riemannian metric is accurate over the entire span of geodesics that have $J/R << 1$. The error is uniformly of order $(J/R)^4$.

So long as u is not close to 1, $(J/R)^2$ is small when $(r/R)^2$ is small, but according to (22) and (24), $1 - u_{\mathbf{max}}$ is of order $(r/R)^2$. In standpoint spatial neighborhood we must correspondingly make a distinction between "ordinary" and "fast" geodesics.

At a geodesic's point of closest spatial approach to standpoint, u equals square of velocity because here $\dot{r} = 0$. For ordinary geodesics at "impact", u is (by definition) not close to 1. But for a fast geodesic at impact, $1 - u$ is close to zero. Were $1 - u$ as small in order of magnitude as $(r/R)^2$, J/R would not be small and non-Riemannian higher-order terms in (29) would be non-negligible. We shall see in Section VII that, for light at impact, $1 - u$ is not as small as $(r/R)^2$. It is nevertheless incorrect to characterize error in *fast-geodesic* standpoint-neighborhood pseudo-Riemannian metric as of order $(r/R)^4$.

For our deduction of energy density in Section VI it suffices that ordinary-geodesic acceleration in standpoint neighborhood is described by general relativity up to an error of order $(r/R)^4$; Einstein-metric curvature of order R^{-2} is then meaningful. Our treatment of light propagation in Section VIII requires only that J/R be small.

VI. Energy Density Near Standpoint

The pseudo-Riemannian approximation (26), when applied to "ordinary" geodesics in standpoint neighborhood, allows calculation of the Ricci curvature tensor at standpoint and, correspondingly, the energy density implied by Einstein theory for homogeneous universe. One finds

$$\begin{aligned} \mathcal{R}_{0\mu} &= \mathcal{R}_{\mu 0} = 0, \qquad && \mu = 0,1,2,3 \\ \mathcal{R}_{ij} &= \delta_{ij}\frac{9}{2R^2}, && i,j = 1,2,3 \end{aligned} \tag{31}$$

(Note that i, j here are *not* standpoint indices.) The Einstein equation for energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{8\pi G}(\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu}), \tag{32}$$

with

$$\mathcal{R} \equiv g^{\alpha\beta}\mathcal{R}_{\alpha\beta}, \tag{33}$$

then yields at standpoint

$$T_{00} = \frac{1}{8\pi G}\frac{27}{4}\frac{1}{R^2}, \tag{34}$$

$$T_{ij} = -\frac{1}{3}T_{00}\delta_{ij}. \tag{35}$$

The negative sign in (35) implies an important contribution to T_{00} from "vacuum energy density." Taking the relation

$$T_{ij}^{\mathrm{vac}} = -T_{00}^{\mathrm{vac}}\delta_{ij}, \tag{36}$$

to be a defining characteristic of "vacuum," [(2)] the decomposition

$$T_{\mu\nu} = T_{\mu\nu}^{\mathrm{matter}} + T_{\mu\nu}^{\mathrm{vac}}, \tag{37}$$

leads from (35) to

$$\begin{aligned} T_{00}^{\mathrm{matter}} &= (\frac{2}{3} - \frac{\sigma}{6})\rho, \quad & T_{ij}^{\mathrm{matter}} &= \frac{\sigma}{6}\delta_{ij}\rho, \\ T_{00}^{\mathrm{vac}} &= (\frac{1}{3} + \frac{\sigma}{6})\rho, \quad & T_{ij}^{\mathrm{vac}} &= -(\frac{1}{3} + \frac{\sigma}{6})\delta_{ij}\rho, \end{aligned} \tag{38}$$

where $\rho = T_{00}$ is total energy density at standpoint and $0 \leq \sigma \leq 1$ is a parameter left undetermined by the gravitational action. If all matter is "hot," $\sigma = 1$ whereas if all matter is "cold," $\sigma = 0$.[(2)]

Employing as a convenient unit the standard-model critical density, $\frac{3}{8\pi}\frac{H^2}{G}$, the Reference (1) relation between R and Hubble time leads to

$$\begin{aligned} \Omega &= (\frac{3}{1+\sqrt{2}})^2 \approx 1.54 \\ \Omega^{\mathrm{matter}} &= (\frac{2}{3} - \frac{\sigma}{6})\Omega \\ \Omega^{\mathrm{vac}} &= (\frac{1}{3} + \frac{\sigma}{6})\Omega. \end{aligned} \tag{39}$$

Note that for a very young standpoint where, as discussed in Reference (1), matter is presumably hot so $\sigma \approx 1$, $\Omega^{\mathrm{matter}} \approx \Omega^{\mathrm{vac}} \approx .77$.

VII. Nearly-Radial Light Propagation

Apart from exactly-radial propagation with $\dot{r}^2 = 1$, "distance" or "proper time" along a geodesic according to Equation (24) of Reference (1) cannot vanish. How then is "nearly radial"—small impact-parameter — light propagation characterized? At distance from standpoint much larger than impact parameter it is convenient to consider the (small) angle between propagation direction and radial direction

$$\chi \equiv \tan^{-1} \left| \frac{r\dot{\psi}}{\dot{r}} \right|. \tag{40}$$

Radial motion corresponds to $\chi = 0$, almost-radial to $\chi << 1$. Although for χ small and positive, $u^{1/2}$ also is small and positive, for lightlike motion the "degree of smallness" for $u^{1/2}$ is less that for χ because $1-\dot{r}^2$ also is small and positive. From (4) it follows generally that

$$1 - \dot{r}^2 = \frac{1}{1 + u/\tan^2\chi}. \tag{41}$$

In order that $\dot{r}^2 \to 1$ as $\chi \to 0$, (41) implies for light propagation that

$$\lim_{\chi \to 0} \frac{\chi}{u^{1/2}} = 0. \tag{42}$$

Note, by contrast, that for "ordinary"–nonzero rest-mass—matter the limit (42) is *nonvanishing.* For light, even though u vanishes in the radial-propagation limit, $u^{1/2}$ decreases *less* rapidly than linearly in χ as this limit is approached. Linear behavior characterizes ordinary matter. For *infinitesimally-small* χ, light propagation exhibits the relation

$$\chi = (1 - \dot{r}^2)^{1/2} u^{1/2}, \tag{43}$$

with *each* of the two positive factors on the right-hand side of (43) *separately* approaching zero as $\chi \to 0$.

The relation (43) between 3 infinitesimally-small quantities, without specifying the limiting ratio of $1 - \dot{r}^2$ to u, will allow us to calculate luminosity distance as a function of redshift. What seems missing information plausibly relates to light *wavelength.* The approach here corresponds to geometrical optics, with only wavelength *ratios* being considered. Photons, like "ordinary" particles, have been presumed in the foregoing to move along geodesics and, for wavelengths that are not small compared to impact parameter, our model requires extension. Ambiguity surrounding zero-impact-parameter limit in the case of light may be unavoidable.

Considerations paralleling those above apply at spatial locations close to standpoint. Here, although the angle χ becomes large, we may consider instead the dimensionless ratio b/R, where b is impact parameter. Smallness of b/R evidently correlates with smallness of χ when r is of order R. Later, in fact, we shall define luminosity distance through the limiting ratio b/χ.

Using the subscript I to designate "impact," $\dot{r}_I = 0$ and $r_I = b$. As discussed in Section V, $1 - u_I$ for light differs from zero by an amount that vanishes as $b/R \to 0$. Also vanishing in this limit is J. Formula (16) reveals an impact relation similar to (43) between the three infinitesimally-small quantities $b/R, (1-u_I)^{1/2}$ and J/R:

$$\frac{b}{R} = (1 - u_I)^{1/2} \frac{J}{R}. \tag{44}$$

It is important that b/R is smaller than either $(1-u_I)^{1/2}$ or J/R (ambiguity in the ratio of the latter two quantities parallels that discussed above). Because $1 - u_I$ approaches zero *less* rapidly than $(b/R)^2$, the pseudo-Riemannian approximation of

Section V is relevant to light. For light, as for "ordinary matter," the standpoint-neighborhood metric expansion is in powers of $(J/R)^2$. Nevertheless, because the physical meaning of J for light is obscure, we shall proceed cautiously.

For almost-radial light propagation, one may infer from (15) that both H_J and

$$H'_J \equiv \frac{H_J}{J} = \frac{1}{(1-\dot{r}^2)^{1/2} r u}(\omega + \frac{\epsilon^2}{\omega - u}) \tag{45}$$

are constants of motion. The inference rests on two complementary considerations: (1) When $r >> b$, the angle χ is small so, according to (43), $(1-\dot{r}^2)^{1/2}$ is small and the right hand side of (15) is negligible. (2) When r is of order b, so $r/R << 1$, computation of $\mathcal{H}_J$ from (13) and (16) reveals a negligibly-weak dependence on t (exemplified by (22) and (23)).

Constancy of H_J and H'_J (implying constancy of J) allows interesting calculations without explicit solution of the nonlinear equation (15″). What do we learn from equating the value of H'_J at light source to value at standpoint impact? Using the subscript s to designate "source" and the subscript I to designate "impact," and invoking (43) together with the limits $\omega_I \to 1$, $\epsilon_I^2(\omega_I - u_I)^{-1} \to 0, u_I \to 1$ as $b/R \to 0$, we find the limiting relation

$$\frac{1}{r_s \chi_s} \frac{\mathcal{G}_s^2 + \mathcal{G}_s'^2}{g_s \mathcal{G}_s + g_s' \mathcal{G}_s'} = \frac{1}{b} \tag{46}$$

in "observer" coordinates (observer standpoint is near impact). For evaluation of coefficients at source, the value of t_s (time of source as measured by observer), determined by the limiting condition $\dot{r} = -1$ for $r >> b$, is $t_s = R - r_s$.

Conservation of H_J, on the other hand, yields the limiting relation

$$\frac{1}{\sqrt{2}}(\mathcal{G}_s^2 + \mathcal{G}_s'^2)^{1/4} \frac{1}{(1-\dot{r}_s^2)^{1/2}} = \frac{1}{(1-u_I)^{1/2}}, \tag{47}$$

once (44) is recognized. Formula (47) depicts a "propagational redshift" in the observer's coordinate system because the factor

$$\frac{1}{\sqrt{2}}(\mathcal{G}_s^2 + \mathcal{G}_s'^2)^{1/4} = \frac{1}{(1+\frac{2r_s}{R})^{1/4}} \tag{48}$$

is smaller than 1. Even though both $(1-\dot{r}_s^2)^{1/2}$ and $(1-u_I)^{1/2}$ approach zero in the limit as χ and b/R approach zero, the limiting *ratio* of the former quantities is inversely proportional to the ratio of photon energies at source and impact (in "observer" coordinates; still to be considered is "Doppler shift" due to motion of source).

VIII. Luminosity Distance as Function of Redshift

The definition of luminosity distance d_L is[3]

$$d_L \equiv (1+z)\frac{b}{\chi_0}, \tag{49}$$

where z is the usual redshift parameter and χ_0 is angle of emission in the coordinate system belonging to a standpoint labeled "0" that is located *close to source* along the straight line connecting source to observer standpoint. We assume that in this "0" system the source is at rest and emits light isotropically. Although $\chi_0 > \chi_s$, we suppose $\chi_0 << 1$.

Reference (1) relates z to a Lorentz boost that in turn relates to r_s/R. The Reference (1) relation $HR = \frac{1}{\sqrt{2}} + \frac{1}{2}$ leads to the formula

$$Hr_s = \frac{1+\sqrt{2}}{4} - \frac{1}{\sqrt{2}}\frac{1}{(1+z)^2} - \frac{1-\sqrt{2}}{4}\frac{1}{(1+z)^4}. \tag{50}$$

For $z << 1$, (50) implies $Hr_s \approx z$ while for $z >> 1$, $r_s \approx R/2$. Provided we can calculate the angle ratio χ_s/χ_0, Formula (46) together with (50) will allow computation from (49) of luminosity distance as function of redshift.

Substituting b from (46) into (49) yields

$$d_L = (1+z)\frac{\chi_s}{\chi_0} r_s \frac{g_s\mathcal{G}_s + g'_s\mathcal{G}'_s}{\mathcal{G}_s^2 + \mathcal{G}_s'^2}. \tag{51}$$

The angle ratio χ_s/χ_0 is deducible from the relation between infinitesimals,

$$\chi_0 = (1-\dot{r}_0^2)^{1/2} u_0^{1/2}, \tag{52}$$

that parallels (43). The factors $(1-\dot{r}_0^2)^{1/2}$ and $u_0^{1/2}$ are separately small (compared to 1) although not as small as χ_0. It follows that

$$\frac{\chi_s}{\chi_0} = [\frac{(1-\dot{r}_s^2)u_s}{(1-\dot{r}_0^2)u_0}]^{1/2}. \tag{53}$$

The ratio $(\frac{1-\dot{r}_s^2}{1-\dot{r}_0^2})^{1/2}$ may be computed from the ratio $1+z$ between $(1-u_I)^{1/2}$ and $(1-\dot{r}_0^2)^{1/2}$ together with the propagational redshift (48). One finds

$$(\frac{1-\dot{r}_s^2}{1-\dot{r}_0^2})^{1/2} = \frac{1+z}{(1+2r_s/R)^{1/4}}. \tag{54}$$

The ratio $(u_s/u_0)^{1/2}$ may be deduced from the Riemannian approximation (26) to the metric of standpoint spacetime. Metric invariance implies (see Section X)

$$\frac{u_s}{u_0} = \frac{\mathcal{G}_s^2 + \mathcal{G}_s'^2}{g_s\mathcal{G}_s + g'_s\mathcal{G}'_s}. \tag{55}$$

Combining the foregoing generates the final result,

$$Hd_L = \frac{(1+z)^2}{(1+2r_s/R)^{1/4}}\left[\frac{g_s\mathcal{G}_s + g'_s\mathcal{G}'_s}{\mathcal{G}_s^2+\mathcal{G}_s'^2}\right]^{1/2} Hr_s, \tag{56}$$

plotted in Figure 1 for $0 < z < 1$. A small-z approximation to (56) is

$$Hd_L = z + \frac{z^2}{2} - 3\left(\frac{3}{2} - \sqrt{2}\right)z^3 + \ldots \tag{57}$$

while the asymptotic behavior is

$$Hd_L \underset{z\to\infty}{\sim} .359 z^2. \tag{58}$$

Figure 1 shows for comparison the Milne and Einstein-de Sitter predictions; the multi-time model prediction is seen to fall between.

IX. Angular-Diameter Distance

Angular-diameter distance follows directly from the pseudo-Riemannian approximation (26) and metric invariance. To the extent that "radius of source" corresponds to ordinary-matter displacement from "center of source"' we can ignore fast-matter subtleties. Angular-diameter distance is defined as radius of source in source ("0") coordinates divided by angle subtended in observer coordinates. From (26) using observer coordinates, a transverse displacement at distance $r_s(dt_s = dr_s = 0)$ by an angle $d\psi_s$ corresponds to a distance in *either* coordinate system equal to

$$\frac{1}{\sqrt{2}}\frac{(g_s\mathcal{G}_s + g'_s\mathcal{G}'_s)^{1/2}}{(\mathcal{G}_s^2+\mathcal{G}_s'^2)^{1/4}} r_s d\psi_s \quad . \tag{59}$$

Thus angular-diameter distance is

$$d_A = \frac{1}{\sqrt{2}}\frac{(g_s\mathcal{G}_s + g'_s\mathcal{G}'_s)^{1/2}}{(\mathcal{G}_s^2+\mathcal{G}_s'^2)^{1/4}} r_s. \tag{60}$$

The formula,

$$\mathcal{G}_s^2 + \mathcal{G}_s'^2 = \frac{4}{1+2r_s/R}, \tag{61}$$

valid on the observer's backward light cone, shows from comparison of (60) with (56) that

$$\frac{d_L}{d_A} = (1+z)^2, \tag{62}$$

a well-known relation deducible in the most-general pseudo-Riemannian cosmology.[3] We have in (62) some verification that, in multi-time cosmology, luminosity distance as well as angular-diameter distance is calculable within pseudo-Riemannian approximation to small-J/R propagation. Plausibly, *all* physical observables are correspondingly calculable, although we do not claim to have so demonstrated.

X. Low J/R and Objective Reality

General relativity ascribes no importance to choice of coordinate system. "Objective reality" resides in locally-Minkowskian representation of matter moving along geodesics that, expressed in one coordinate system may unambiguously be represented in any other through coordinate transformation that preserves pseudo-Riemannian metric. Any such metric may locally be transformed to Minkowski form. "Observer" lacks status; matter is "really there," even though connection is acknowledged between "measurement" and locally-Minkowskian coordinates. Such objective reality prevails up to the largest scales in standard cosmology (despite comoving coordinates being recognized as convenient).

Quantum mechanics on the other hand, by paying attention to measurement, relegates the objective reality of matter trajectories at small scales to the status of large-action *approximation*, where the unit of action is $\hbar$. In this concluding section we emphasize that multi-time cosmology allows (approximate) pseudo-Riemannian geodesic mapping between different standpoint spacetimes *only* for spacetimes where the geodesic has a *low* value of J/R (Pseudo-Riemannian metric is necessary and sufficient for local energy-momentum conservation). There is then a large-scale correlation between objective reality and *low* J/R that complements the small-scale local-physics correlation of objectivity with *large* action. Objectivity is a feature of an *intermediate* level of reality that regards $\hbar$ as negligibly small and R as infinite.

The appendix displays, on the basis of Reference (1), exact mappings of $J = 0$ geodesics between those standpoint spacetimes whose spatial origins are intersected by the geodesics. The body of the present paper provides basis for *approximate* mapping of geodesics that carry $J/R << 1$ for the standpoint spacetimes being compared, error in pseudo-Riemannian approximation to the Finsler metric having been shown to be of order $(J/R)^4$ both in standpoint neighborhood and at Hubble-scale distance. In the neighborhood of the straight line connecting spatial origins of a pair of standpoint spacetimes, a geodesic with small J/R in both spacetimes, even if spatial distance between standpoints is large, may be mapped from one spacetime onto the other in a locally-Minkowskian approximation. Loosely speaking, any standpoint spacetime whose spatial origin locates "close" to a portion of the geodesic (i.e., impact parameter small on Hubble scale) provides a locally-Minkowskian representation of geodesic along geodesic entirety. Associated unambiguous geodesic mapping between different spacetimes means that matter flowing along the geodesic is "objectively described." The matter may be characterized as "really there" up to an error of order $(J/R)^4$.

Explicitly, when standpoint spatial separation is large compared to the geodesic's impact parameter with respect to either standpoint, it is convenient to choose one spatial coordinate axis in each spacetime parallel to the direction connecting the spatial locations of the two standpoints. Designating the corresponding "longitudinal" spatial coordinates by x_i, x_j, while "transverse" coordinates are designated $\vec{x}_{i,tr}$, $\vec{x}_{j,tr}$, the "cylindrical" mapping is given to first order in transverse coordinates by the

$(t_i, x_i) \leftrightarrow (t_j, x_j)$ longitudinal mapping of the appendix, together with the following transverse mapping dictated by (26)*:

$$[\frac{g_i \mathcal{G}_i + g_i' \mathcal{G}_i'}{(\mathcal{G}_i^2 + \mathcal{G}_i'^2)^{1/2}}]^{1/2} \vec{x}_{i,tr} = [\frac{g_j \mathcal{G}_j + g_j' \mathcal{G}_j'}{(\mathcal{G}_j^2 + \mathcal{G}_j'^2)^{1/2}}]^{1/2} \vec{x}_{j,tr}. \tag{63}$$

Here the coefficients are earlier-introduced functions of $(t_i, | x_i |)$ and $(t_j, | x_j |)$. The result (60) for angular-diameter distance may be regarded as an application of (63).

The foregoing (cylindrical) Minkowskian mapping between spacetimes of widely-spaced standpoints is confined to a range of transverse coordinates that is small compared to distance between standpoints–being restricted to a "spatially-tubular" portion of each spacetime. Curvature of either spacetime within its tube is neglected by the mapping.

On the other hand when standpoint spacing both spatially and in time is small on Hubble scale, there can be mapping of ordinary geodesics in the general relativistic sense that includes curvature. Such mapping extends throughout the (common) neighborhood of the two standpoints as defined in Section VI of Reference (1), not being restricted to thin tubes. Although *all* geodesics passing through such a neighborhood have small J/R for both spacetimes, for *fast* geodesics we have failed to show that homogeneous-universe metric curvature (of order R^{-2}) is meaningful in Einstein's sense.

What about fast geodesics with J/R of order 1? If such geodesics cannot be "objectified," physical interpretation will be difficult. Nevertheless, in quantum mechanics small action has proved to be interesting even while defying objective interpretation, so it would be premature to declare geodesics with $J/R \sim 1$ as devoid of interest. This paper has presented the ordinary differential equation governing *all* homogeneous-universe geodesics in multi-time cosmology.

Appendix: Mapping of $J = 0$ Geodesics

Exploiting results from Reference (1), we here make explicit a $1+1$ dimensional Minkowski mapping between spacetimes belonging to standpoints i and j. The single spatial coordinate corresponds to displacement along any $J = 0$ geodesic connecting the spatial locations of i and j. Mapping between (t_i, x_i) and (t_j, x_j) is expedited by auxiliary coordinates (τ_i, ζ_i) and (τ_j, ζ_j). Corresponding to Formula (29) of Reference(11), the auxiliary coordinates satisfy

$$\begin{aligned} 1 - \frac{\tau_i \pm \zeta_i}{4R_i} &= (1 - \frac{t_i \pm x_i}{2R_i})^{1/2}, \\ 1 - \frac{\tau_j \pm \zeta_i}{4R_j} &= (1 - \frac{t_j \pm x_j}{2R_j})^{1/2}. \end{aligned} \tag{A.1}$$

*Implicit in (63) is that angles *within* the transverse plane are preserved.

The auxiliary coordinates relate *to each other* by the boost Δ_{ij} defined in Reference (1):

$$\tau_i \pm \zeta_i = e^{\pm\Delta_{ij}}(\tau_j \pm \zeta_j). \tag{A.2}$$

Note that the mapping applies only to $J = 0$ geodesics. Notice also that the double-cone restriction $t \pm x \leq 2R$ means that, if $R_i > R_j$, only a portion of the geodesic in i spacetime can be mapped onto j spacetime.

According to (A.1) and (A.2), infinitesimal displacements along a $J = 0$ geodesic satisfy

$$\frac{dt_i \pm dx_i}{(1 - \frac{t_i \pm x_i}{2R_i})^{1/2}} = \frac{dt_j \pm dx_j}{(1 - \frac{t_j \pm x_j}{2R_j})^{1/2}} e^{\pm\Delta_{ij}} \quad , \tag{A.3}$$

implying

$$(\mathcal{G}_i^2 + \mathcal{G}_i'^2)^{1/2}(dt_i^2 - dx_i^2) = (\mathcal{G}_j^2 + \mathcal{G}_j'^2)^{1/2}(dt_j^2 - dx_j^2), \tag{A.4}$$

once it is remembered that

$$\mathcal{G}^2 + \mathcal{G}'^2 = \frac{1}{(1 - \frac{t+x}{2R})(1 - \frac{t-x}{2R})}. \tag{A.5}$$

Thus $ds_i = ds_j$.

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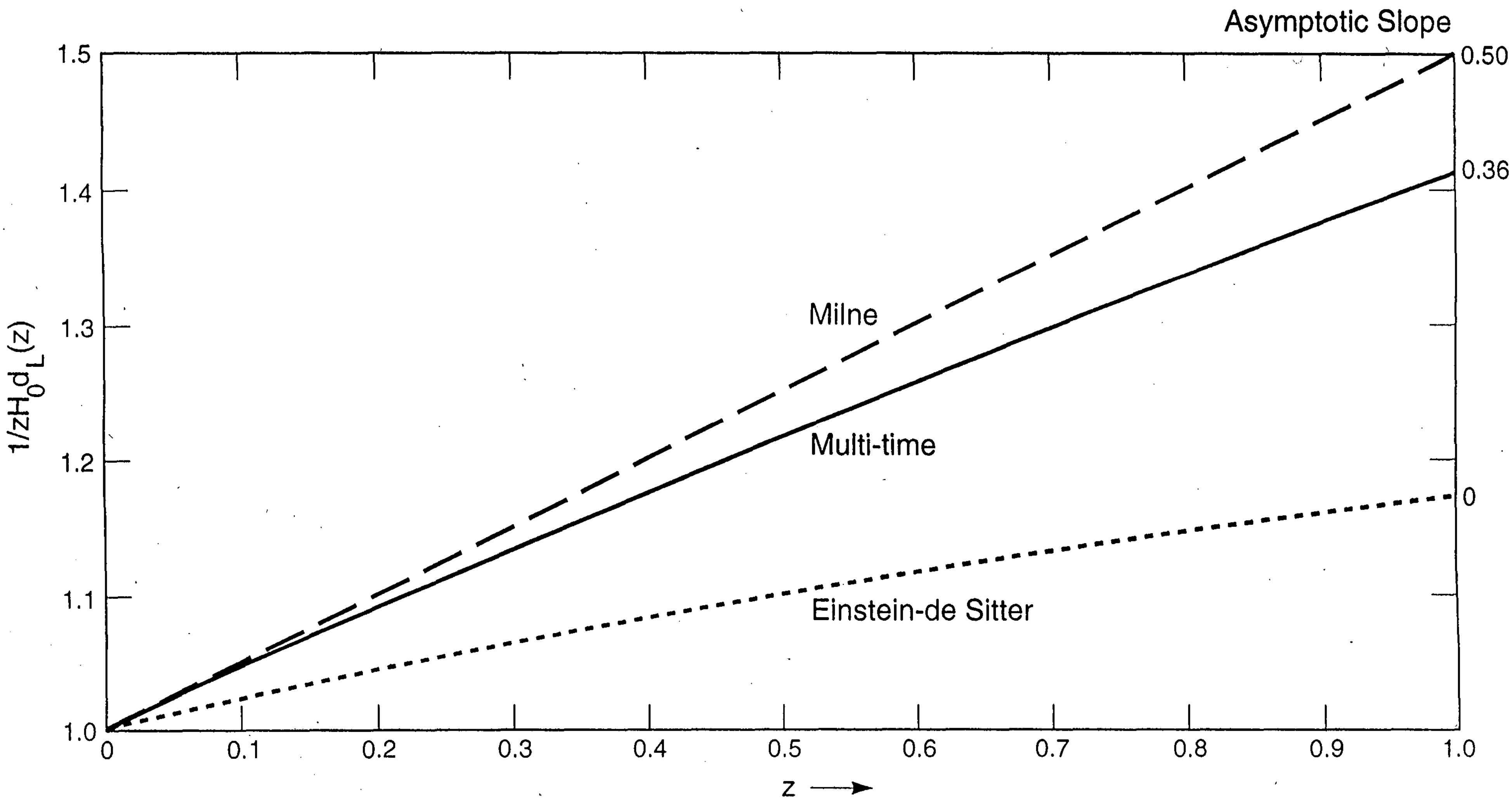

Figure 1. Luminosity distance in Hubble units, divided by the redshift parameter z, as a function of z.

ERNEST ORLANDO LAWRENCE BERKELEY NATIONAL LABORATORY
ONE CYCLOTRON ROAD | BERKELEY, CALIFORNIA 94720

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