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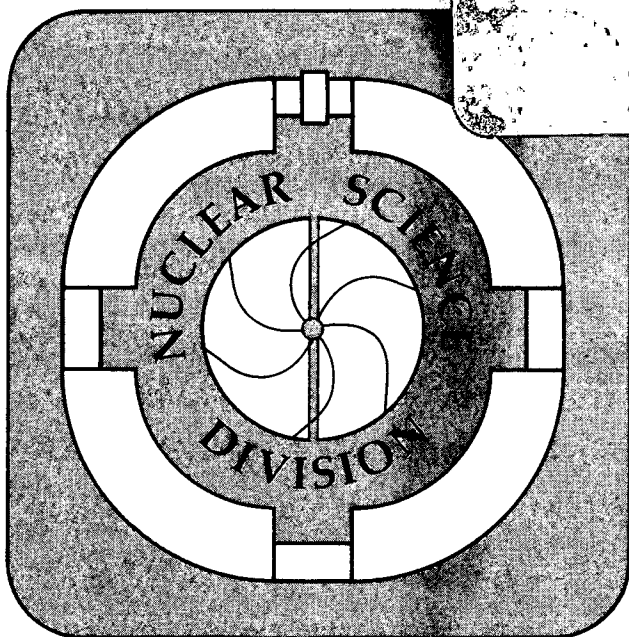
A MICROSCOPIC DESCRIPTION OF BOSON- AND FERMION-  
ALIGNMENT IN OCTUPOLE BANDS OF ACTINIDE NUCLEI

L.M. Robledo, J.L. Egido and P. Ring

June 1985

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A Microscopic Description of Boson- and Fermion-Alignment  
in Octupole Bands of Actinide Nuclei\*

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Abstract:

The rotational alignment of the octupole Boson in negative parity bands of Actinide nuclei is investigated in the framework of Cranked Random Phase Approximation (CRPA).

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## 1. Introduction.

Modern heavy-ion accelerators allow us to study the behavior of rapidly rotating nuclei. Much effort has been devoted in the last fifteen years both on the experimental as on the theoretical side to understand the rotational spectra of bands in the vicinity of the yrast line. A number of anomalous phenomena have been observed in this area and it turned out that they can be explained as an interplay of collective and single particle degrees of freedom: Alignment phenomena of one or several particles caused by the strong Coriolis field of the rotating core give us the essential key for a understanding of these phenomena. The most famous example is backbending in the discovered in the early seventies in the Rare earth region<sup>1)</sup>, which can be understood by the Stephens-Simon effect<sup>2)</sup> as a sudden alignment of two neutrons in the  $i_{13/2}$  orbit. At the same time one has seen strongly disturbed bands<sup>3)</sup> and decoupled bands<sup>4)</sup> in odd mass nuclei, which are characterized by the alignment of a single particle in the intruder orbit. Later on one has observed additional alignment phenomena, such as the alignment of  $h_{11/2}$  protons<sup>5)</sup>, the alignment pattern in triaxial shapes<sup>6,7)</sup> or the alignment of one of the particles in an excited two-quasiparticle band (hybrid bands<sup>8,9)</sup>).

In all these cases the alignment is caused by Fermions, namely neutrons or protons in intruder orbits with a large single particle angular momentum. On the other side collective nuclear excitations have also been described in terms of Bosons. In negative parity excitations, which we concentrate on in this paper, octupole Bosons play a crucial role. Most of the low lying low spin negative parity states in heavy deformed nuclei have been described as collective octupole vibrations, i.e. one-Boson excitations on the groundstate. Octupole Bosons carry 3 units of angular momentum. In the

Coriolis field of the rotating core they therefore feel a tendency to align, just as the single Fermions in the high spin intruder orbits<sup>10</sup>). Because of the smaller amount of angular momentum the corresponding Coriolis matrix elements are nearly a factor 2 smaller as in the case of  $i_{13/2}$  particles, but they are even at small angular velocities on the order of the splitting of the band heads with the quantum numbers  $K = 0, 1, 2, 3$ , i.e. the Coriolis force, which drives alignment overcomes already in the region of not too high spins the residual interaction of the deformed field of the core, which tends to couple the octupole vibration along the symmetry axis. In a pure Boson picture one therefore finds pretty soon a value of  $3\hbar$  for the aligned angular momentum<sup>11,12</sup>).

In real nuclei, however, collective excitations consist of linear combinations of two-quasiparticle states. In a negative parity state, one of these Fermions sits in the high spin intruder orbit, i.e. in the Actinides in the  $i_{13/2}$  orbit for protons or in the  $j_{15/5}$  orbit for neutrons. It therefore feels itself a strong Coriolis force, which tries to align it parallel to the rotational axis. At low spins this individual particle alignment is prevented by the relatively strong octupole correlations in the pair. It couples the two particles to spin  $3\hbar$ . Obviously the spins of the two partners are then antiparallel, similar to the particles in a Cooper-pair, where the partners are coupled to spin zero. With increasing spin, however, the Coriolis force will become strong enough, to break the  $3^-$  pair, just as one Cooper-pair is broken in the backbending region. The question arises therefore, at which spin values this individual pair alignment will occur. Are the octupole correlations strong enough, to keep the octupole Boson together up to spin values large enough that we can observe the rotational alignment of the Boson,

or does the octupole pair break already before this alignment is achieved? Recent experiments in the Actinides<sup>13,14</sup>), where it is well known, that octupole correlations are strong, show indeed in some cases first alignment of the Boson and only later on individual pair alignment.

The purpose of this paper is to present a unified microscopic theory for these complex alignment phenomena. For a microscopic theory of octupole excitations based on the ground state the random phase approximation (RPA) has been used by a number of authors<sup>15-20</sup>). In this theory the octupole Bosons are treated on a microscopic basis as superposition of a large number of two-quasiparticle configurations. They are kept together by the correlations induced through a residual octupole-octupole force. In a deformed field one finds a splitting between the different K-quantum numbers. For the description of the rotational bands on top of these band heads several methods have been used: Neergard and Vogel<sup>21,22</sup>) couple the RPA-Boson determined microscopically at spin zero to a rotor just in the sense of the particle-plus-rotor model. They diagonalize the corresponding Coriolis interaction and describe in this way the alignment of the Boson. This method is mixed microscopic-macroscopic: vibrational degrees of freedom are treated microscopically and rotational degrees of freedom are treated macroscopically. Apart from the fact, that it is not clear, if such a model can describe the alignment process properly, since it is well known that the particle-plus-rotor model is not able to describe the particle alignment quantitatively because of the missing attenuation of the Coriolis force<sup>3,23</sup>), this method does not include the degree of freedom of individual particle alignment, i.e. the collective pair is never broken. P.Vogel therefore introduced a different method<sup>10</sup>): he coupled two particles to the rotor



in the sense of the Stephens-Simon model<sup>2</sup>). Octupole correlations are introduced by a residual octupole-octupole force, which is diagonalized in the two-particle space. This method includes in principle all the important degrees of freedom, the attenuation problem is however not solved and it requires a considerable numerical effort, since the number of two-particle configurations, which is needed for a microscopic description of the aligning Boson grows rapidly. Severe restrictions have to be made and no quantitative comparison with experiment is possible.

In the meantime Cranked RPA theory has been developed<sup>24-26</sup>) and has been applied with great success for the description of rotational bands with positive parity<sup>27,28</sup>). In this paper we apply this method to negative parity bands in the Actinide region. At spin zero it corresponds precisely to the old RPA theory, which is very successful in the description of the band heads. For finite spins it treats the alignment mechanisms for Bosons and Fermions on the same microscopic level and it is well known, that the Cranking model has the proper amount of Coriolis attenuation<sup>23,29</sup>). We use the parameter space of ref. 30, which provides an excellent description of the yrast levels in the entire deformed Actinide region. The only additional parameters are the force parameters of the octupole-interaction, which are adjusted at spin zero and one additional parameter, which determines the alignment at high spin. We then are able to describe the high spin region for a series of Actinide nuclei, which show a rather different alignment pattern, practically with only one parameter. We find rather good, sometimes even quantitative agreement for the spectra and the alignment plots.

The RPA formalism in the context of negative parity is briefly outlined in section 2 and the details of our model are discussed. In section 3 we present numerical results for the nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ . A conclusion is given in section 4.

## 2. Microscopic Theory of Excited Rotational Bands of Negative Parity

We start with the microscopic Hamiltonian

$$H = \epsilon - \frac{1}{2} \sum_{\rho} \chi_{\rho} D_{\rho}^{\dagger} D_{\rho} \quad (1)$$

It describes the motion of the valence particles in the spherical oscillator shells  $N = 5, 6$  for protons and  $N = 6, 7$  for neutrons.  $\epsilon_i$  are spherical single particle energies. They are calculated in the Nilsson model with the usual parameters  $\kappa_{\pi} = 0.0577$ ,  $\kappa_{\nu} = 0.0635$  and  $\mu_{\pi} = 0.65$ ,  $\mu_{\nu} = 0.325$ . The effective residual interaction is assumed to be of separable form. It contains the following essential degrees of freedom: quadrupole, octupole and hexadecupole in the ph-channel and monopole pairing in the pp-channel. We use operators symmetrized with respect to signature, namely the multipole operators:

$$Q_{\lambda\mu} = \sum_{mm'} \frac{\alpha_{\tau}}{\langle r^2 \rangle_N} \left\langle m \left| \frac{r^2 (-)^{\mu}}{\sqrt{2(1 + \delta_{\mu 0})}} (Y_{\lambda|\mu|} + r_{\mu} (-)^{\lambda} Y_{\lambda-|\mu|}) \right| m' \right\rangle a_m^{\dagger} a_{m'} \quad (2)$$

where  $r_{\mu} = \pm 1$  for  $\mu \geq 0$  and  $r_0 = (-)^{\lambda}$

with  $\alpha_{\pi} = (5 + 3/2)(2Z/A)^{1/3}$  ;  $\alpha_{\nu} = (6 + 3/2)(2N/A)^{1/3}$

for  $\lambda = 2, 4$ , a similar operator for  $\lambda = 3$  (see eq.(21)) and the pairing operators

$$P_{\tau}^{\dagger} = \sum_{m>0} (a_m^{\dagger} a_{-m}^{\dagger})_{\tau} \quad (3)$$

for protons and neutrons ( $\tau = \pi, \nu$ )

The first step is the calculation of the mean field. We assume that it preserves parity, i.e. we do not consider stable octupole deformations, which is a reasonable assumption in the region of nuclei under consideration. We thus find for the groundstate a deformed and superfluid mean field of the form:

$$H_0 = \epsilon - \beta_2 Q_{20} - \beta_4 Q_{40} + \sum_{\tau} \Delta_{\tau} (P_{\tau}^{\dagger} + P_{\tau}) \quad (4)$$

The parameters  $\beta_2$  and  $\beta_4$  for quadrupole and hexadecupole deformation and the gap parameters  $\Delta_p$  and  $\Delta_n$  depend in principle on the strength parameters of the residual interaction. In order to avoid additional parameters in our model, we use, however, the experimental values given in Table 1. We also neglect stretching and Coriolis-anti-pairing effect, i.e. we use the same deformation- and gap-parameters for all spins. As has been shown in ref. 30 this is a very reasonable approximation for the Actinides in the mass region  $230 < A < 250$  and for spin values  $I < 30\%$ . For finite angular momenta we then only have to add the Coriolis term

$$H_{\omega} = H_0 - \omega J_x \quad (5)$$

We thus end up with a rotating basis. The wave functions of the levels at the yrast line are obtained by constructing a generalized Slater determinant in filling up this rotating deformed mean field from the bottom. Excellent agreement with experimental data on the yrast line is obtained in this way in ref. 27. In the present paper we use the same basis and calculate vibrations of small amplitude around this quasistatic rotating solution. We

concentrate on states with negative parity. Therefore only the octupole-octupole part of the residual interaction in eq.(1) plays a role in the following considerations.

The Cranked RPA approximation is obtained most easily by representing the Hamiltonian in the rotating frame by Bosons<sup>24</sup>). For an arbitrary single particle operator D we have the quasiparticle representation in the basis which diagonalizes  $H_\omega$ .

$$D = \langle D \rangle + \sum_{k < k'} (D_{kk', \alpha_k^\dagger \alpha_{k'}}^{20} + D_{kk', \alpha_{k'} \alpha_k}^{02}) + \sum_{kk'} D_{kk', \alpha_k^\dagger \alpha_{k'}}^{11} \quad (6)$$

$\langle D \rangle$  is the expectation value of D in the rotating shell model. In terms of Bosons operators  $A_{kk'}^\dagger$ , it has the form

$$D = \langle D \rangle + \sum_{k < k'} D_{kk', A_{kk'}^\dagger}^{20} + D_{kk', A_{kk'}}^{02} + \sum_{kk'l} D_{kk', A_{k'l}^\dagger A_{k'l}}^{11} + \dots \quad (7)$$

Inserting these representation for the operators  $\epsilon$ ,  $J_x$  and  $D_\rho$  in the Hamiltonian (1) and taking into account only up to quadratic terms in the Boson operators  $A_{kk'}^\dagger$ , we find in the rotating frame:

$$H' = H - \omega J_x = \langle H' \rangle + \sum_{kk'l} (E_k + E_{k'}) A_{k'l}^\dagger A_{k'l} \quad (8)$$

$$+ \frac{1}{2} \sum_{\rho} \sum_{kk'l'l'} (D_{\rho kk', A_{kk'}^\dagger}^{20*} + D_{\rho kk', A_{kk'}}^{02}) \chi_{\rho} (D_{\rho' l'l', A_{l'l'}^\dagger}^{20} + D_{\rho' l'l', A_{l'l'}}^{02})$$

where  $E_k$  are the quasiparticle energies, the eigenvalues of the operator  $H_\omega$  (eq.(5)). In this approximation the Hamiltonian is a quadratic expression in the operators  $A_{kk'}$ ,  $A_{kk'}^\dagger$ . It can be diagonalized

$$H' = E_{RPA} + \sum_{\mu} \Omega_{\mu} B_{\mu}^{\dagger} B_{\mu} \quad (9)$$

in terms of collective Bosons by the ansatz

$$B_{\mu}^{\dagger} = \sum_{k < k'} X_{kk'}^{\mu} A_{kk'} - Y_{kk'}^{\mu} A_{kk'} \quad (10)$$

$E_{RPA}$  is the RPA-energy of the corresponding Boson-vacuum, which represents a correlated wave function for the levels at the yrast line<sup>26)</sup>.  $\Omega_{\mu}$  are the excitation energies of the one-Boson excitations. They describe collective bands above the yrast line. The amplitudes  $X_{kk'}^{\mu}$  and  $Y_{kk'}^{\mu}$  are obtained by the solution of the RPA-equation. Since we violate nearly all symmetries the number of two-quasi-particle configurations is extremely large. However, the diagonalization of the RPA-equation is extremely simplified by the fact, that we use a separable interaction. Using the Brown-Bolsterli trick<sup>31)</sup> we end up with a nonlinear eigenvalue problem, whose dimension is just the number of separable terms in the Hamiltonian:

$$d_{\rho}^{\mu} = \sum_{\rho'} S_{\rho\rho'}(\Omega_{\mu}) X_{\rho\rho'}^{\mu} d_{\rho'}^{\mu} \quad (11)$$

where the energy dependent matrix  $S$  is given by

$$S_{\rho\rho'}(\Omega) = \sum_{k < k'} \frac{D_{\rho k k'}^{20*} D_{\rho k k'}^{02}}{\Omega - E_k - E_{k'}} - \frac{D_{\rho k k'}^{02*} D_{\rho k k'}^{20}}{\Omega + E_k + E_{k'}} \quad (12)$$

and the quantities  $d_{\rho}^{\mu}$  are defined as the  $D_{\rho}$ -strength of the corresponding eigenstate:

$$d_{\rho}^{\mu} = \langle 0 | D_{\rho} | \mu \rangle = \sum_{k < k'} D_{kk'}^{02} X_{kk'}^{\mu} - D_{kk'}^{20} Y_{kk'}^{\mu} \quad (13)$$

In order to find the proper normalization we have to calculate the RPA-amplitudes:

$$\chi_{kk'}^\mu = \frac{\sum_p D_{\rho kk'}^{20} d_p^\mu}{\Omega_\mu - E_k - E_{k'}} \quad \text{and} \quad \gamma_{kk'}^\mu = \frac{\sum_p D_{\rho kk'}^{02} d_p^\mu}{\Omega_\mu + E_k + E_{k'}} \quad (14)$$

which are normalized to one

$$\sum_{k < k'} |X_{kk'}^\mu|^2 - |Y_{kk'}^\mu|^2 = 1 \quad (15)$$

In the present case we are only interested in excitations with negative parity. Therefore we have only the octupole operators  $Q_{3\mu}$  in eq.(12). Signature symmetry decomposes the matrix  $S$ , whose dimension is  $7 \times 7$  into a  $3 \times 3$  matrix with positive signature ( $K = +1, +2, +3$ ) and a  $4 \times 4$  matrix with negative signature ( $K = 0, -1, -2, -3$ ). We calculate in each case the few lowest eigenstates.

For each value of the cranking frequency  $\omega$  we obtain in this way a number of excitations, which form as a function of  $\omega$  rotational bands "parallel" to the yrast line. In order to obtain discrete energies for fixed values of the angular momentum we have to use the cranking condition:

$$\langle \mu | J_x | \mu \rangle + \omega \mathcal{I}_c = \sqrt{I(I+1) - \langle \mu | J_z^2 | \mu \rangle} \quad (16)$$

for each one-Boson state  $|\mu\rangle$ .  $\mathcal{I}_c$  is a core moment of inertia, which takes care of the fact, that our configuration space is too small as to reproduce the full experimental moment of inertia. It has been adjusted to the

excitation energy of the 2-level on the yrast line in ref. 30. We have  $\int_c = 17.2 \text{ (MeV)}^{-1}$  for  $^{232}\text{Th}$ ,  $\int_c = 14.0 \text{ (MeV)}^{-1}$  for  $^{236}\text{U}$  and  $\int_c = 13.6 \text{ (MeV)}^{-1}$  for  $^{238}\text{U}$ . In all these cases the quantity  $\langle \mu | J_z^2 | \mu \rangle \approx K^2$  for small I-values. For large angular momenta, where K-mixing occurs it is small against  $I(I+1)$  and can be neglected. We are therefore never faced with the problem to calculate the expectation value of this two-body-operators.

The Cranking condition (16) has been derived from an approximate angular momentum projection.<sup>32,33</sup>). In the case of even parity bands this derivation also shows that even values of the angular momentum correspond to positive signature and odd values correspond to negative signature<sup>34</sup>). An extension of these considerations to negative parity bands is straightforward since we consider the parity as a good quantum number.

Great care has to be taken in the calculation of the expectation value  $\langle J_x \rangle$  of the RPA-states in eq.(16). As has been shown in ref. 26 we do not have to worry about this condition in linear order: Starting at a point at the yrast line with a fixed I-value, determined from the usual cranking condition

$$\langle J_x \rangle_{\text{yrast}} + \omega \int_c = \sqrt{I(I+1)} \quad (17)$$

we obtain the excitation energy of any one-Boson state with the same I-value as the eigenvalue  $\Omega_\mu$  of the CRPA-equation at the same cranking frequency, because in linear order there is a cancellation between two effects: i) the one-Boson state has in general a average angular momentum somewhat different from that of the yrast state (the Boson vacuum) and we therefore should in principle choose a slightly different  $\omega$ -value for the solution of the



CRPA-equation. ii) The excitation energy  $\Omega_\mu$  at the new  $\omega$ -value is different from the excitation energy of the old  $\omega$ -value. This cancellation, however, is true only in linear order. If the Boson carries a large amount of angular momentum, as we find it in aligning configurations, the linear approximation is not good enough. We have to calculate in each case the full expectation value of the one-Boson configurations:

$$\langle \mu | J_x | \mu \rangle = \langle J_x \rangle + J_{\mu\mu}^{11} \quad (18)$$

Using this expression in eq.(16) we find to each angular momentum I for the one-Boson state a cranking frequency different from the value at the yrast line. The laboratory energy of the one-Boson configuration is given by the expression

$$\begin{aligned} E(I) &= \langle \text{RPA} | B_\mu^\dagger H B_\mu | \text{RPA} \rangle \quad (19) \\ &= \langle \text{RPA} | B_\mu^\dagger (H - \omega J_x) B_\mu | \text{RPA} \rangle + \omega \langle \text{RPA} | B_\mu^\dagger J_x B_\mu | \text{RPA} \rangle \end{aligned}$$

We neglect ground state correlations and replace the expectation values  $\langle \text{RPA} | \dots | \text{RPA} \rangle$  by the corresponding RSM-results at the corrected angular velocity. We thus find

$$E(I) = E_{\text{yrast}} + \Omega_\mu + \omega J_{\mu\mu}^{11} \quad (20)$$

In fact it turned out, that this correction is crucial for a understanding of the experimental alignment patterns. Without it we would

obtain in all cases a very early individual particle alignment, we would not find in our theory the alignment of the unbroken octupole Boson.

A final remark has to be made about the violation of translational symmetry. It is well known that, in cases where the static mean-field approximation violates symmetries, one finds spurious excitations in the theoretical spectra. In our case the mean field violates translational invariance as well as rotational symmetry and number symmetry for protons and neutrons. However only translational symmetry are essential, because the spurious excitations connected with breaking of rotational and number symmetry have positive parity. The spurious excitation connected with translational symmetry is the center of mass motion. Its operator is the linear momentum  $\vec{P}$ , which has negative parity. It should show up in the negative parity channel. In fact one finds in text books<sup>34)</sup>, that the RPA has the advantage of separating the corresponding spurious excitations exactly. They should show up with zero excitation energy and should be easily removable. However, this is only true, if the mean field is calculated selfconsistently with a translational invariant force. In our case we do not use translational invariant forces. Even our entire model is not translational invariant, because we restrict our calculation to a limited configuration space of a few shells in a translational symmetry violating oscillator potential. The center of mass motion therefore does not separate exactly, we can have spurious admixtures in all low lying one-Boson excitations. A number of recipes have been invented to deal with such a situation<sup>35-38)</sup>. We used the method of refs. 36 and 37 and solved the RPA-equations for an effective Hamiltonian, which restores translational symmetry. This procedure, however, produced only very small corrections, which means that in the cases under consideration the

spurious center of mass motion is relatively well separated from the octupole excitation. In Fig.1 we show the overlap of the octupole state with the spurious states characterized by  $P_y$  and  $P_z$  for the  $K = 0^-$  bands (the corresponding quantity for  $P_x$  vanishes identically in this case). It stays in all cases below 5%. At zero angular momentum only  $P_z$  contributes. With increasing angular momentum we find because of the K-mixing also contributions of  $P_y$ . They stay however for all angular momenta smaller than 1%. We therefore understand, why the correction for spurious admixtures has so little influence on our results. In general such a correction turned out to be unnecessary.

### 3. Numerical Results

As we discussed in section 2, the calculation has been carried out in two steps:

In the first step the basis of the rotating shell model (RSM) is determined. We used for this purpose the HFB-wave functions and the quasiparticle energies determined in ref. 30. There the RSM-equations are solved by the gradient method. We therefore have for each angular momentum at the yrast line  $I_{\text{yrast}}$ , the corresponding angular velocity  $\omega$  (Table 2). In the following we use  $I_{\text{yrast}}$  as reference. We have to keep in mind, however, that the angular momentum of the corresponding one-Boson states can differ from this value.

In the second step we transform the octupole operators to the rotating quasiparticle bases. In particular we use the following form of the residual interaction in the negative parity channel

$$V = -\frac{1}{2} \sum_{\mu} \kappa_{3\mu} \tilde{Q}_{3\mu}^{\dagger} \tilde{Q}_{3\mu} \quad (21)$$

with the operators

$$\tilde{Q}_{3\mu} = \sigma_{\pi} (Q_{3\mu})_{\pi} + \sigma_{\nu} (Q_{3\mu})_{\nu} \quad (22)$$

where  $(Q_{3\mu})_{\tau}$  is defined by:

$$(Q_{3\mu})_{\tau} = \sum_{mm'} \left\langle m \left| \frac{r^3 (-)^{\mu}}{\sqrt{2(1 + \delta_{\mu 0})}} (Y_{3|\mu|} - r_{\mu} Y_{3-|\mu|}) \right| m' \right\rangle a_m^{\dagger} a_{m'} \quad (23)$$

The interaction (21) depends on two parameters: The factors  $\sigma_{\tau}$  determine the

relative strength of the octupole correlations for protons and neutrons. They are defined in the spirit of Baranger and Kumar<sup>39</sup>):

$$\sigma_{\pi} = \left(\frac{2Z}{A}\right)^n, \quad \sigma_{\nu} = \left(\frac{2N}{A}\right)^n \quad (24)$$

Different values for  $n$  were used:  $n = 0, 1, 2$ . In Table 3 we compare the results obtained in this ways. We find in agreement with earlier investigations of Neergard and Vogel<sup>21</sup>) that the parameter  $n$  has little influence on the results for small spins. For high spins this is no longer true. In the region, where the octupole pairs break and a pair of individual particles aligns, this parameter determines, which type of particles align first. This parameter determines in some extension the relative amount of protons and neutrons in the pair. A large value of  $n$  ( $\sigma_{\nu} > 1$ ) favours the presence of neutrons and disfavours the protons. We therefore use  $n$  as a parameter to reproduce the experimental alignment patterns as good as possible. Best agreement is found for  $n = 2$  and we use in the following only this value for all three nuclei under consideration.

In order to reproduce the band heads properly  $K$ -dependent coupling constants have to be used<sup>20-22</sup>). The bandheads are determined for angular velocity  $\omega = 0$ . We therefore have no  $K$ -mixing and can determine the coupling constants  $x_{3K}$  for each  $K$ -value separately in such a way that the experimental bandhead energy  $\Omega_K$  is obtained as solution of the RPA-equation (11).

$$2x_{3K} \sum_{k < k'} \frac{(E_k + E_{k'}) |\tilde{Q}_{3K}^{20}|^2}{(E_k + E_{k'})^2 - \Omega_K^2} = 1 \quad (25)$$

The values for  $\bar{x}_{3K} = x_{3K} A^3$  are given in Table 4. For all three nuclei we

find roughly  $\bar{\chi}_{31} \approx \bar{\chi}_{30}$ ,  $\bar{\chi}_{32} \approx 1.6 \bar{\chi}_{30}$  and  $\bar{\chi}_{33} \approx 2.4 \bar{\chi}_{30}$ . Qualitatively we can understand this K-dependence of the force parameters by the arguments of selfconsistency applied by Neergard<sup>21</sup>). Using his method we would obtain for a deformation  $\beta = 0.25$  the relations  $\bar{\chi}_{31} = 0.68 \bar{\chi}_{30}$ ,  $\bar{\chi}_{32} = 1.37 \bar{\chi}_{30}$ ,  $\bar{\chi}_{33} = 2.17 \bar{\chi}_{30}$  and for  $\beta = 0.3$   $\bar{\chi}_{31} = 0.61 \bar{\chi}_{30}$ ,  $\bar{\chi}_{32} = 1.45 \bar{\chi}_{30}$ ,  $\bar{\chi}_{33} = 2.50 \bar{\chi}_{30}$ . These values are in qualitative agreement with the parameters found by our fit to the experimental band head energies.

In Fig.2 we show theoretical and experimental spectra for the low lying bands with negative parity in the nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ . The theoretical energies are obtained according to the prescription given in eq.(20). In view of the fact, that we adjusted only the band heads and use only one parameter  $n = 2$  for all three nuclei the overall agreement between theory and experiment is rather satisfactory.

In order to see the alignment pattern more clearly we have to go more into the details: In Fig.3 we show as an example the low lying uncorrelated two-quasiparticle energies in the RSM for negative signature as a function of the Yrast angular momentum (the corresponding angular velocities are given in Table 2). For the characterization of the quasiparticle components the following notation is used: p,n means proton or neutron with signature +i, p(dash), n(dash) means proton or neutron with signature -i. The first of the two indices gives the number of corresponding quasiparticle energy (1 is the lowest, 2 the next, and so on). The second index (+ or -) indicates the parity. For increasing angular momentum most of these two-quasiparticle energies decrease with rather different slopes, indicating a rather different amount of alignment.

In Fig.4 we show the low lying eigenvalues of the RPA-equation (eq.(11)) for the three nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ . We observe a considerable lowering of the collective states characterized by the quantum numbers  $K = 0, 1, 2$  and  $3$ . For spin zero these are exact quantum numbers. For higher spin values we have some  $K$ -mixing by the Coriolis force.

The alignment pattern in the lowest band with negative parity ( $K = 0^-$ ) for the different nuclei is given in Fig.5. The theoretical aligned angular momentum  $i$  is the angular momentum component parallel to the rotational axis carried by the Boson. It is given by

$$i_{\text{th}} = \langle \text{RPA} | B_{\mu} J_x B_{\mu}^{\dagger} | \text{RPA} \rangle - \langle \text{RPA} | J_x | \text{RPA} \rangle = J_{\mu\mu}^{11} \quad (25)$$

In Fig.5 we show these values as a function of the cranking frequency. We also indicate the contributions coming from neutrons and protons. The three nuclei show a rather different behavior: in the two U-nuclei we first observe Boson-alignment: a value close to  $3\hbar$  is reached rather soon. The neutron contribution is definitely larger as the proton contribution, but for all angular velocities smaller than 200 keV no particle alignment is observed. In  $^{232}\text{Th}$  the situation is different. We see in the theory only a rather weak indication of Boson alignment. At an angular velocity of  $\omega \approx 140$  keV the alignment increases strongly. The octupole-pair is broken and the proton in the  $j = 13/2$  shell decouples.

We also compare in Fig.5 our theoretical results with the experiment. For the definition of the experimental alignment the yrast line was used as reference. A VMI-model was fitted to the low lying members of the yrast line and for constant angular velocity the experimental alignment is defined as the

difference between the full angular momentum and the VMI-value for the angular momentum of the yrast line. Although this definition of alignment is somewhat different from the theoretical alignment in Fig.5, we still observe a surprisingly good qualitative agreement in the alignment pattern for the three nuclei: In  $^{232}\text{Th}$  very soon particle alignment sets in and Boson alignment cannot develop. In  $^{236}\text{U}$  and  $^{238}\text{U}$  we see only Boson alignment. Although we can reproduce these pattern rather nicely in our theory, we do not obtain quantitative agreement. As often observed in calculations<sup>30)</sup> Cranking theory produces somewhat too much of alignment.

In order to demonstrate even more clearly the different behavior of the  $K = 0^-$  bands in these three nuclei we show in Fig.6 to what extend the maximally aligned configuration ( $i_{13/2}$  with signature  $+i$  for the protons and  $j_{15/2}$  with signature  $-i$  for the neutrons) are occupied in the "collective pair" obtained as the lowest solution of the RPA-equation. We find a steep proton alignment in the nucleus  $^{232}\text{Th}$ , but nearly no individual particle alignment in the two other cases.



#### 4. Conclusions

Based on calculations within the Rotating Shell Model in ref. 30, which give a very satisfying description of the yrast configurations in Actinide nuclei, the Cranked RPA equations are solved for low lying negative parity bands in the three nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ . A separable octupole-octupole interaction is used, whose strength parameters are adjusted to the experimental band head energies.

We are thus able to describe in a unique microscopic framework the complicated interplay between collective degrees of freedom, as static multipole deformations and pairing, rotation and dynamic octupole vibrations on one side, and the single particle degrees of freedom dominated by the large angular momentum of the intruder orbits on the other side.

In detail we observe in some cases rather strongly bound collective pairs, which align together as a octupole Boson in the Coriolis field of the rotating core, in other cases the collective nature disappears rather soon, the octupole Bosons breaks up and we find the alignment of single particles in the intruder orbits. Cranked RPA theory includes all these degrees of freedom and we find nice agreement with the experimental results. Depending on the nature of the quasiparticles in the vicinity of the Fermi surface and the strength of the residual forces it can describe as well pure two-quasiparticle configurations as collective Bosons, whose internal correlation energy is large enough to avoid a too early break up in individual particles.

In detail we find, that we have to take the Cranking condition fully. Because of the large single particle angular momenta involved in aligning configurations the linear approximation commonly used in CRPA theory<sup>26)</sup> breaks down here.

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## Tables

Table 1 Deformation and gap parameters of the RSM-potential in eq.(4)

The experimental values are taken from ref. 40

	$^{232}\text{Th}$	$^{236}\text{U}$	$^{238}\text{U}$
$\beta_2$	0.238	0.259	0.283
$\beta_4$	0.130	0.124	0.059
$\Delta_p$	1.075	0.952	0.864
$\Delta_n$	0.750	0.665	0.644

Table 2 Cranking frequencies corresponding to fixed angular momentum values at the yrast line  $I_{\text{yrast}}$

$I_{\text{yrast}}$	$^{232}\text{Th}$	$^{236}\text{U}$	$^{238}\text{U}$
0	0.0	0.0	0.0
2	0.040	0.037	0.037
4	0.071	0.067	0.067
6	0.102	0.095	0.096
8	0.130	0.122	0.124
10	0.156	0.147	0.149
12	0.180	0.169	0.171
14	0.200	0.188	0.186
16	0.218	0.204	0.195
18	0.234	0.216	0.202
20	0.247	0.225	0.207
22	0.246	0.233	0.213
24	0.246	0.240	0.221
26	0.245	0.247	0.231
28	0.246	0.254	0.244
30	0.254	0.263	0.253

Table 3 Influence of the parameter  $n$  of eq.(24) on excitation energies  $\Omega$  and aligned angular momenta  $J_x^{11}$  for different spin values  $I = 4$  and  $I = 18$  in the low lying  $K = 0^-$  band of the nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ . Only relative quantities are given.

n	$^{232}\text{Th}$				$^{236}\text{U}$				$^{238}\text{U}$			
	I = 4		I = 18		I = 4		I = 18		I = 4		I = 18	
	$\Omega$	$J_x^{11}$	$\Omega$	$J_x^{11}$	$\Omega$	$J_x^{11}$	$\Omega$	$J_x^{11}$	$\Omega$	$J_x^{11}$	$\Omega$	$J_x^{11}$
0	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
1	1.005	0.973	1.26	0.949	0.994	0.969	9.974	1.011	0.992	0.965	0.894	1.1
2	1.009	0.951	1.492	0.906	0.997	0.952	0.912	1.049	0.983	0.913	0.813	1.20

Table 4 The coupling constants  $\bar{\chi}_{3K} = \chi_{3K} \cdot A^3$  determined by eq.(25) in units of  $\text{MeV fm}^{-6}$

	$^{232}\text{Th}$	$^{236}\text{U}$	$^{238}\text{U}$
$\bar{\chi}_{30}$	158.1	166.5	164.7
$\bar{\chi}_{31}$	161.6	153.3	149.2
$\bar{\chi}_{32}$	245.6	231.4	221.0
$\bar{\chi}_{33}$	393.2	413.7	391.5

## Figure Captions

- Fig.1 Overlaps between the  $3^-$  octupole excitation and the spurious center of mass motions characterized by  $P_y$  and  $P_z$  for the three nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ . The overlaps are normalized in the following way:  $S_{p_{y,z}} = \langle \text{RPA} | P_{y,z} B^\dagger | \text{RPA} \rangle / \langle \text{HFB} | P_{y,z} P_{y,z} | \text{HFB} \rangle$
- Fig.2 Theoretical and experimental spectra for low lying bands with negative parity in the nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ . The experimental data are from ref. 40 ( $^{232}\text{Th}$ ) and ref. 41 in  $^{238}\text{U}$ .
- Fig.3 Two-quasiparticle energies of the rotating shell model (RSM) for the negative signature in  $^{232}\text{Th}$ . Details are explained in the text
- Fig.4 Low lying eigenfrequencies of the RPA-equation (eq.(11)) with negative parity as a function of the angular momentum on the yrast line. The collective octupole bands are characterized by the quantum numbers  $K = 0, 1, 2$  and  $3$ , which are exact quantum numbers for spin zero. 2qp indicates a rather pure two-quasiparticle configuration.
- Fig.5 Aligned angular momentum  $i$  for the low lying negative parity band ( $K = 0^-$ ) in the nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$  as a function of the cranking frequency. The full lines are theoretical values obtained from the RPA-calculation according to eq.(5). Proton- and neutron contributions to these theoretical results are given by dashed lines. The theoretical values are compared with experimental aligned angular momenta as defined in ref. 11 (dashed-dotted lines).

Fig.6 Structure of the lowest lying RPA solution ( $K = 0^-$  band) as a function of the angular momentum for the three nuclei  $^{232}\text{Th}$ ,  $^{236}\text{U}$  and  $^{238}\text{U}$ .  $\sum = \sum_{kk'} |x_{kk'}^\mu|^2$  are the RPA-amplitudes of eq.(10) and the index  $k$  runs over all levels in the  $i_{13/2}$  orbit with signature  $\pm i$  for the protons and over all levels in the  $j_{15/2}$  orbit with signature  $\pm i$  for the neutrons.



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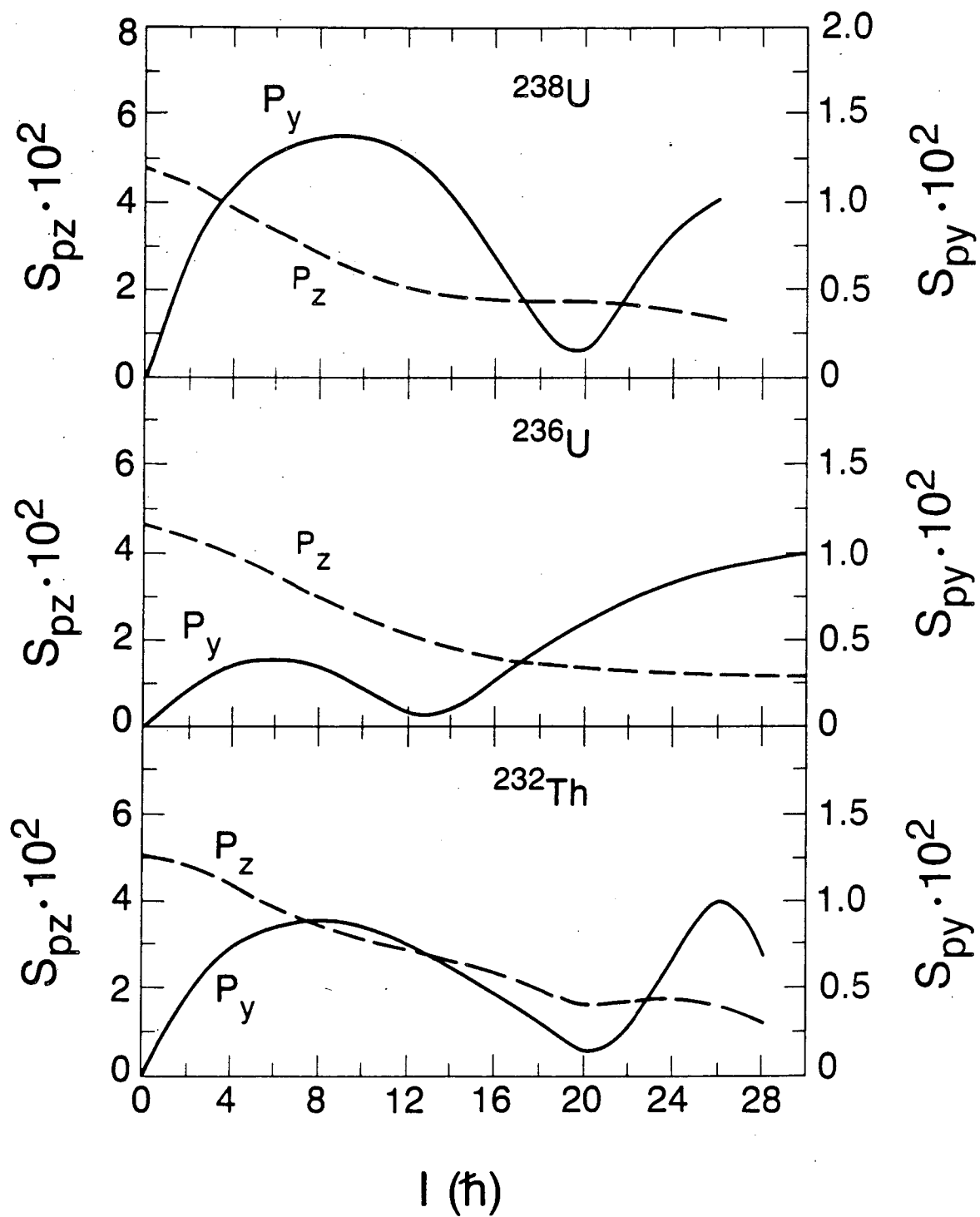


Fig. 1

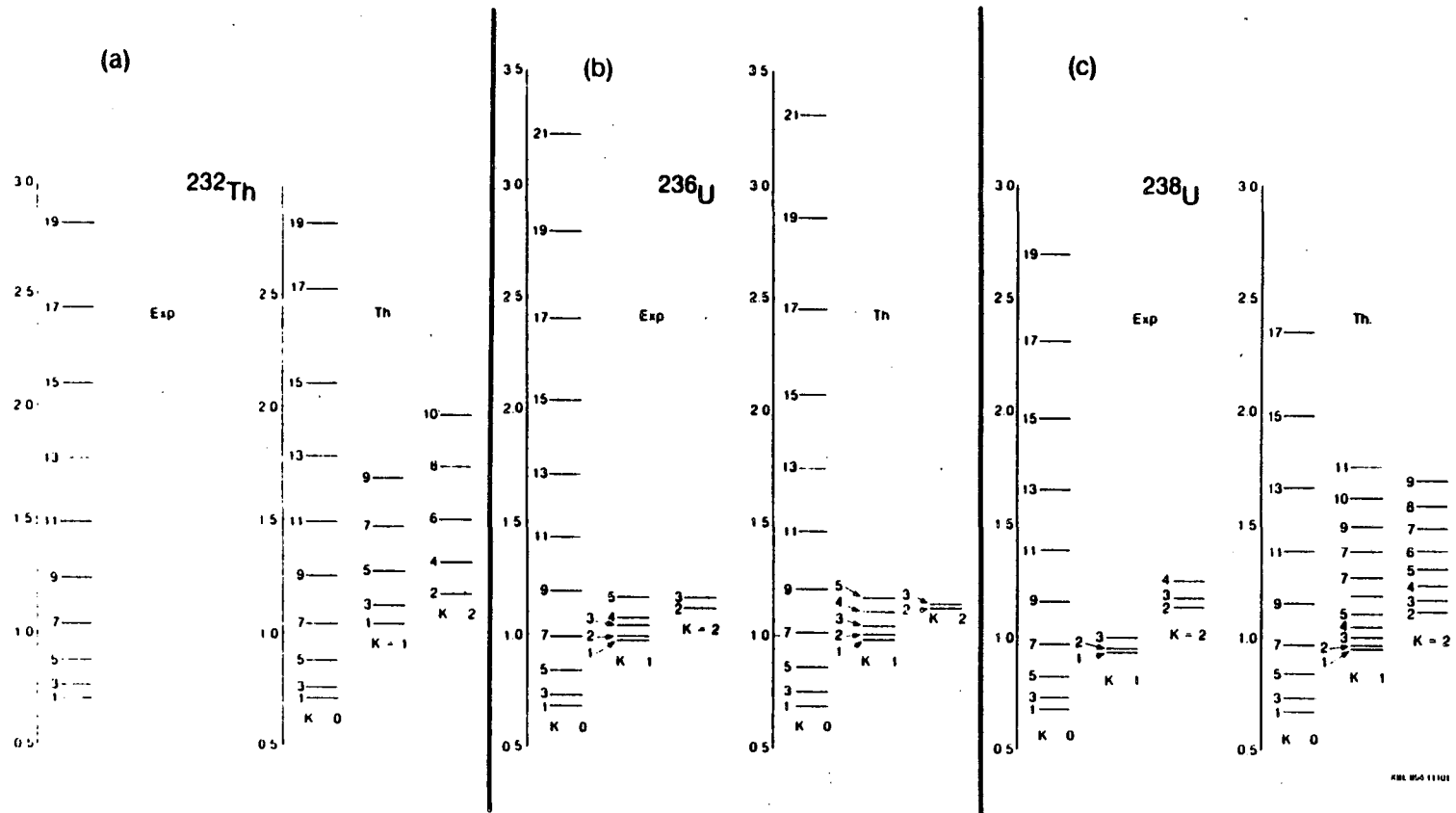


Fig. 2

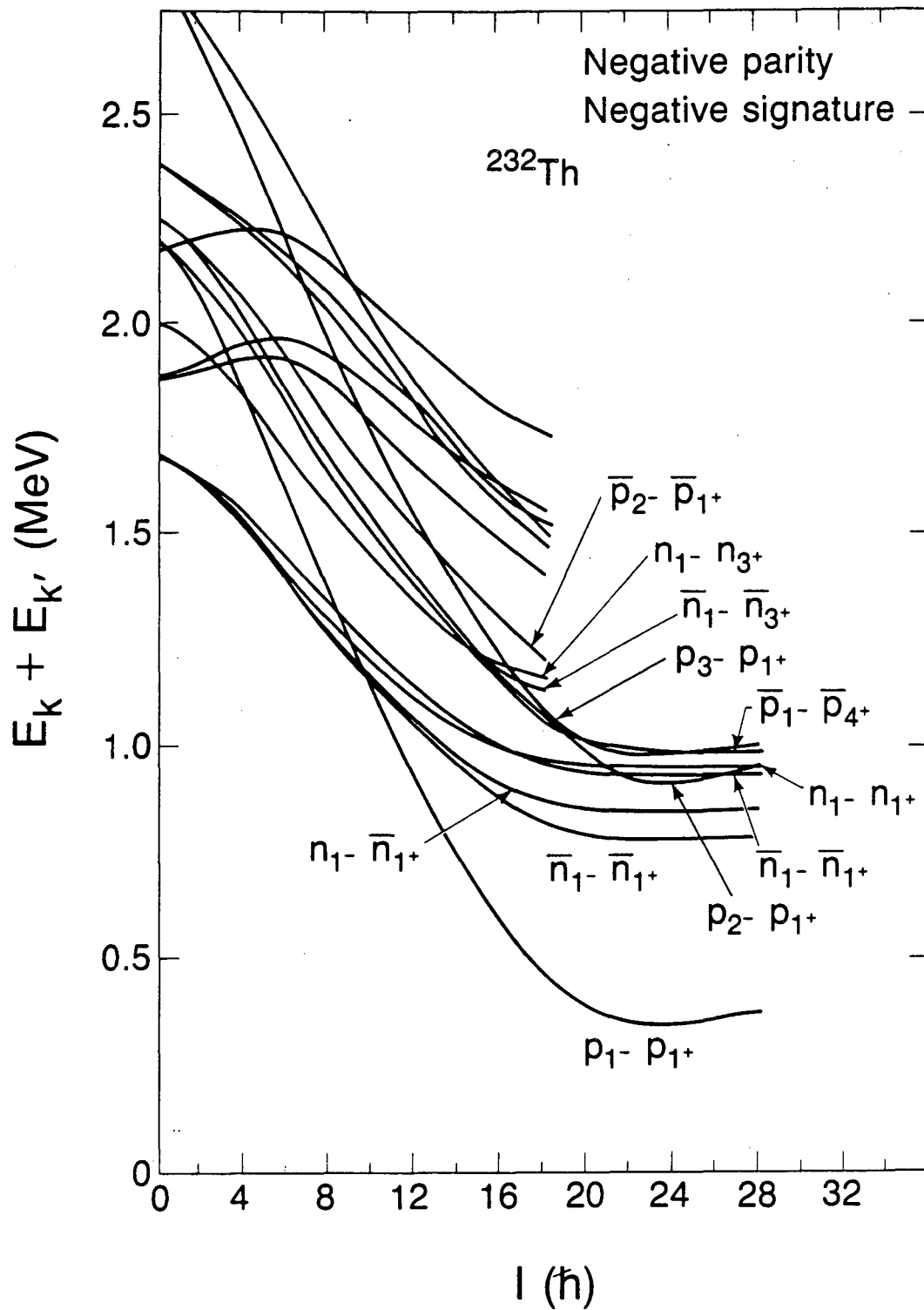


Fig. 3

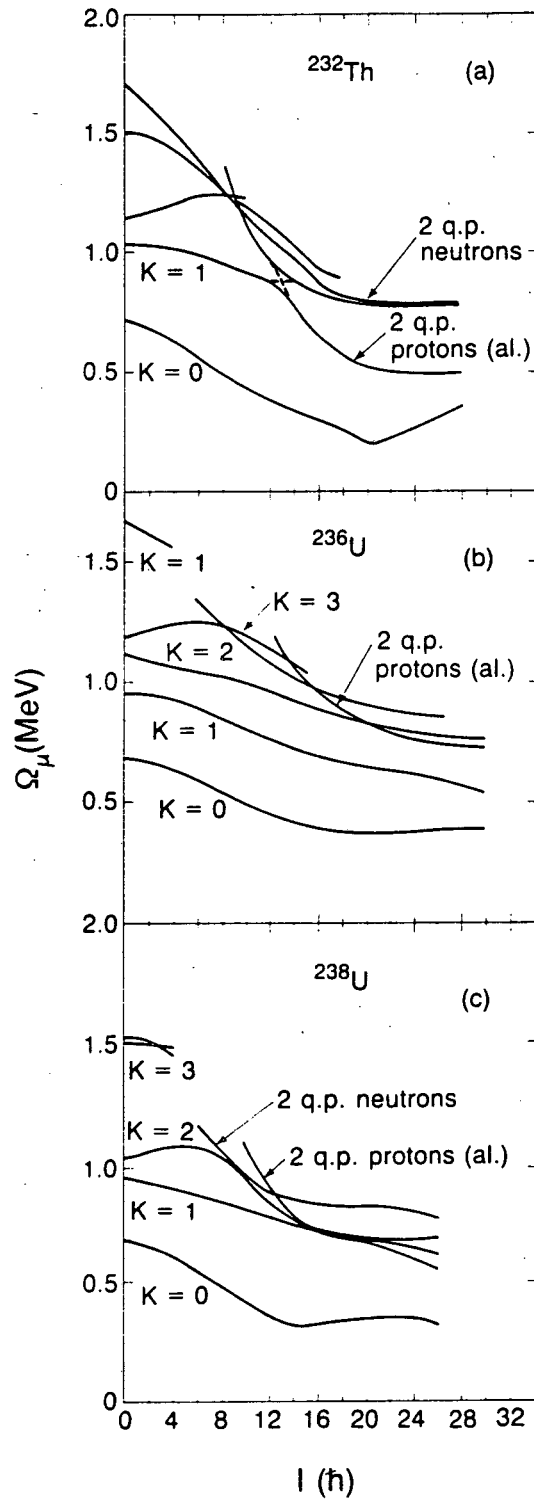


Fig. 4

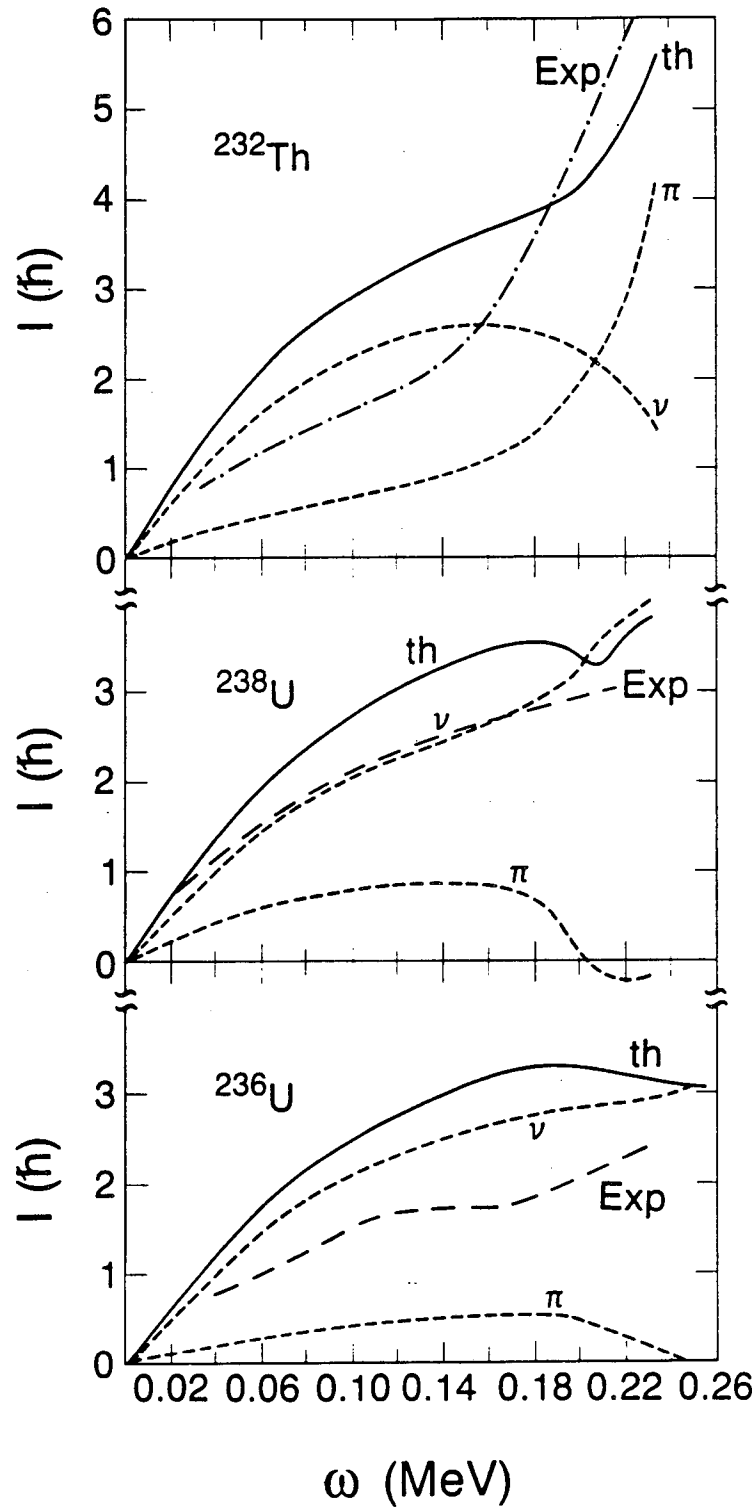


Fig. 5

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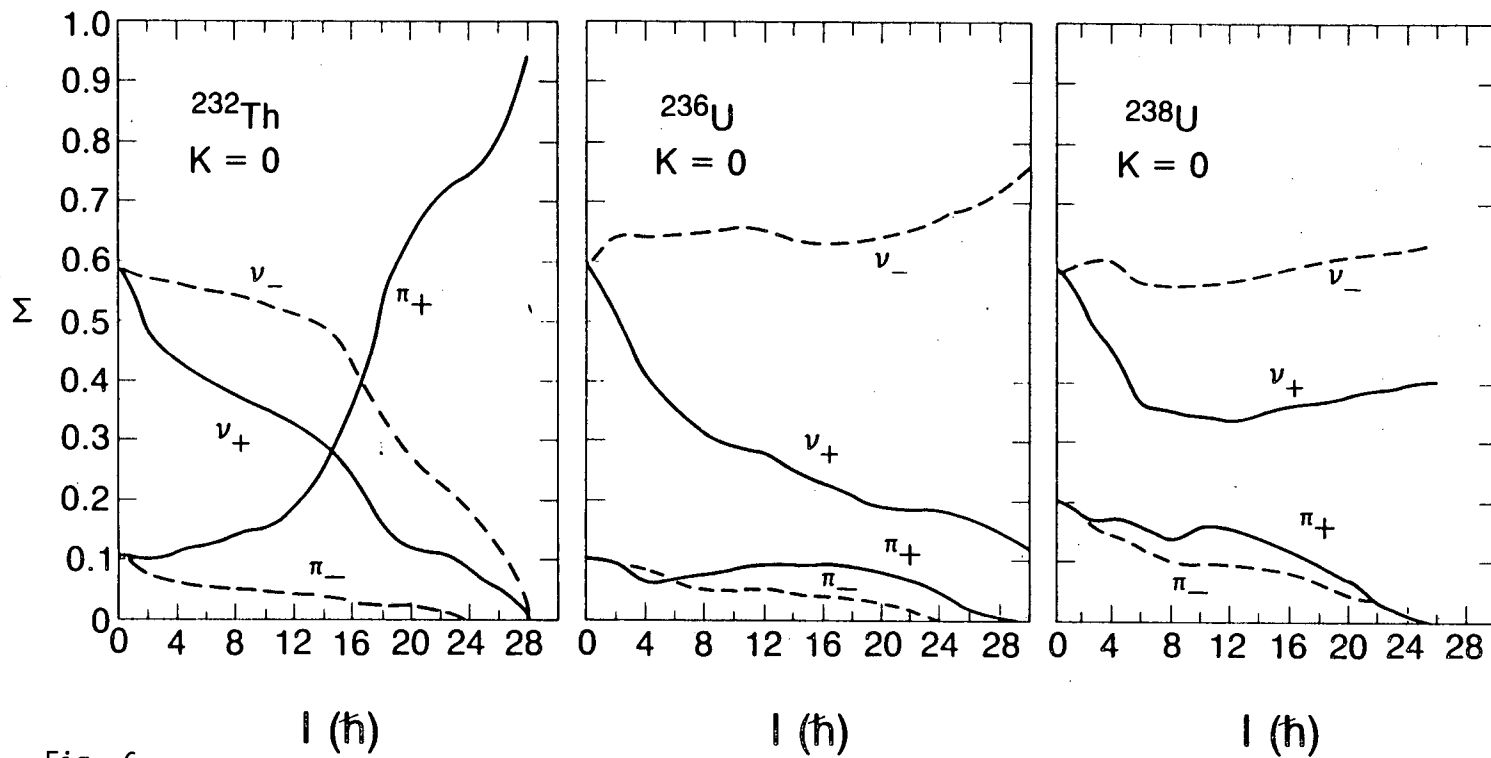


Fig. 6

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