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THE EFFECTIVE W APPROXIMATION

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## THE EFFECTIVE W APPROXIMATION

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### Abstract

We generalize the effective photon approximation to include the massive  $W^\pm$  and  $Z$  gauge bosons of the Weinberg-Salam model. The  $W^\pm$  and  $Z$  bosons are treated as partons in the proton and we present predictions for the structure functions of both transversely and longitudinally polarized  $W$ 's and  $Z$ 's. Our results are valid only at high energies, ( $\sqrt{s} \gtrsim 20$  TeV), and greatly simplify calculations involving vector bosons in the intermediate state of a scattering process. As examples, we use our treatment of the  $W$  and  $Z$  as partons to calculate hadronic production of heavy Higgs bosons from  $WW$  scattering and also to compute the associated production of a heavy charge  $2/3$  and charge  $-1/3$  quark from  $W$ -gluon scattering.

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## I. Introduction

The concept of the proton as containing partons which interact with each other independently has allowed the direct computation of hadronic cross sections with a reasonable degree of accuracy.<sup>1</sup> As higher and higher energies are reached experimentally, it becomes important to include more and more constituents in the parton picture of the proton. As the charm, bottom, and top quark thresholds are reached, the heavy quarks contribute to the parton sea. In interactions with energies in the TeV range, the parton sea also contains the  $W$  and  $Z$  gauge bosons of the Weinberg-Salam  $SU(2) \times U(1)$  model. By considering these bosons as partons, calculations involving gauge bosons in the intermediate states can be considerably simplified.

The treatment of the  $W$  and  $Z$  bosons as constituents of the proton is analogous to the effective photon method in which the "effective number of photons" in an electron is calculated.<sup>2,3</sup> We consider processes of the type,

$$\begin{aligned} p p &\rightarrow W^+ W^- X \rightarrow X' \\ p p &\rightarrow Z Z X \rightarrow X' \end{aligned} \quad (1.1)$$

in which the incident protons radiate  $W$  or  $Z$  bosons which then annihilate to form the state  $X'$ . The cross sections for such processes are suppressed by at least  $(\alpha/\sin^2\theta_W)$  relative to processes involving a single gauge boson such as,

$$\begin{aligned} p p &\rightarrow W X \rightarrow X' \\ p p &\rightarrow Z X \rightarrow X' \end{aligned} \quad (1.2)$$

The two-W (or two-Z) process will obviously be completely negligible at low energies. At high energies,  $\sqrt{s} \gg M_W, M_Z$ , the case is radically different. The protons can radiate transverse W's and Z's so easily that cross sections involving transverse W's are enhanced by factors of  $[\ln s/M_W^2]^2$  or  $[\ln s/M_Z^2]^2$ . Processes involving longitudinal W's (or Z's) which are scattered into small forward angles are also enhanced. This enhancement is clearly seen in the calculation of Ref. 4 in which the cross section for  $pp \rightarrow WWX \rightarrow HX'$  is compared with that for  $pp \rightarrow WX \rightarrow HX'$ . (H is the Higgs boson of the Weinberg-Salam model). At high energies,  $\sqrt{s} \gtrsim 20$  TeV, and for Higgs masses much greater than the W boson mass, the cross section involving two W's in the intermediate state is several orders of magnitude larger than that with a single W.

In Section II, we present our results for the probability distributions of transverse and longitudinally polarized  $W^\pm$  and Z gauge bosons in the proton at energies much greater than the gauge boson masses. Our derivation of the distribution functions follows closely the calculation of Brodsky, Kinoshita, and Terazawa<sup>3</sup> for the distribution of photons in an electron. Some care is needed, however, because the intermediate vector bosons, unlike the photon, have both transverse and longitudinal degrees of freedom. Our approach neglects the effects of the interference between transverse and longitudinal vector bosons in a process such as that of Eq. (1.1) and we discuss the limitations of this approximation in Section II.

In Section III, we present two applications of our effective W approximation -- heavy quark production from Wg scattering and Higgs boson production from  $W^+W^-$  scattering. Using the effective W

approximation, the calculation of these cross sections is greatly simplified. Higgs boson production from  $W^+W^-$  scattering has been previously calculated in Ref. 4 in a small angle approximation and the result is in good agreement with that obtained here from the effective W approximation.<sup>4,5</sup>

## II. The Effective W Approximation

### (a) Polarization Tensors

In this section, we generalize the effective photon method to the case of massive gauge bosons. We consider separately the contribution of transverse and longitudinal gauge bosons to a given cross section, (such as that of Eq. (1.2)). From this, we then extract the gauge boson distribution functions. In part (c) of this section, we discuss the effects of the interference between the longitudinally and transversely polarized gauge bosons in a particular scattering process – this interference cannot be included in the model of the  $W^\pm$  and  $Z$  bosons as partons.

We define four orthogonal polarization tensors for a gauge boson  $V$  with four-momentum  $k = (k_0, 0, 0, |\vec{k}|)$  and mass  $M_V$ :

$$\begin{aligned}\epsilon_0 &= \frac{1}{M_V} (k_0, 0, 0, |\vec{k}|) \\ \epsilon_1 &= (0, 1, 0, 0) \\ \epsilon_2 &= (0, 0, 1, 0) \\ \epsilon_3 &= \frac{1}{M_V} (|\vec{k}|, 0, 0, k_0)\end{aligned}\quad (2.1)$$

These polarization tensors satisfy the relations,

$$\begin{aligned}\epsilon_i \cdot \epsilon_j &= -\delta_{ij} \\ \sum_i \epsilon_{i\mu} \epsilon_{i\nu} &= -g_{\mu\nu}\end{aligned}\quad (2.2)$$

$\epsilon_1$  and  $\epsilon_2$  correspond to transversely polarized gauge bosons and  $\epsilon_3$  is the longitudinal polarization vector. For on-shell gauge bosons,  $k^2 = M_V^2$ , and  $\epsilon_3$  can be written,

$$\epsilon_3 = \frac{k}{M_V} + \frac{|\vec{k}| - k_0}{M_V} (1, 0, 0, -1) \quad (2.3)$$

For  $W^\pm$  and  $Z$  bosons which are coupled to massless fermions, the contribution to  $\epsilon_3$  proportional to  $k$  gives zero when sandwiched between fermion spinors and so,

$$\epsilon_3 = \frac{M_V}{k_0 + \sqrt{k_0^2 - M_V^2}} (-1, 0, 0, 1) \quad (2.4)$$

In order to treat the  $W^\pm$  and  $Z$  bosons as partons, we must consider them as on-shell, physical bosons. We work in unitarity gauge and everywhere neglect light quark masses. This means that we will always use Eq. (2.4) as the longitudinal polarization tensor in Section II. We make the approximation that the partons have zero transverse momentum which ensures that the longitudinal and transverse projections of the  $W$  and  $Z$  partons are uniquely specified.

### (b) Transverse Gauge Bosons

The derivation of the distribution function for massive transversely polarized vector bosons is almost identical to the effective photon calculation of the distribution of transverse photons in an electron.<sup>6</sup> We begin by calculating the number of transverse  $W$ 's (or  $Z$ 's)

in a quark,  $f_{q/v}(x)$ , where  $x$  is the momentum fraction of the quark carried by the gauge boson. This distribution function must then be folded in with the quark distribution functions in order to find the number of vector bosons in a proton. All of our distribution functions are normalized to satisfy the momentum sum rule,

$$\sum_{\substack{i=\text{parton} \\ \text{types}}} \int dx x f_i(x) = 1 . \quad (2.5)$$

The amplitude for a transversely polarized vector boson  $V_t$  to scatter from a quark  $q_2$  into the final state  $X$ , (see Figure 1a), is

$$i A_T(V_t + q_2 \rightarrow X) = \epsilon_i \cdot M \sqrt{E_q} \quad i=1,2 , \quad (2.6)$$

where  $E_q$  is the quark energy and we have replaced the  $V_\mu - q_2 - X$  vertex by an effective coupling  $M_\mu \sqrt{E_q}$ . Averaging over the quark spin and the two transverse polarizations of the vector boson, the cross section for the process  $V_t + q_2 \rightarrow X$  is,

$$\begin{aligned} d\sigma_T(V_t + q_2 \rightarrow X) &= 1/(16 k_0 E_q \eta) |A_T|^2 \frac{\pi}{x} 2\pi/(2E_x) \delta^4(k+P_2-P_x) d^3P_x \\ &= 1/(16 k_0 E_q \eta) |A_T|^2 d\Gamma , \end{aligned} \quad (2.7)$$

where we have defined the Lorentz invariant phase space of the state  $X$  to be  $d\Gamma$ . We work in the lab frame of the quark and  $k_0$  is the vector boson energy. Using Eq. (2.6), the cross section is,

$$d\sigma_T(V_t + q_2 \rightarrow X) = 1/(16 k_0 \eta) \sum_{i=1}^2 |\epsilon_i \cdot M|^2 d\Gamma . \quad (2.8)$$

$$\text{and } \eta = \sqrt{1 - M_V^2 / k_0^2} .$$

We now consider the two-body scattering process shown in Figure (1b),

$$q_1 + q_2 \rightarrow V_t + q_1' + X . \quad (2.9)$$

The approximation we will make is that only the transversely polarized vector bosons contribute to the amplitude of Eq. (2.9). The amplitude for this process is,

$$\begin{aligned} i A_T(q_1 + q_2 \rightarrow V_t + q_1' + X) \\ = \sum_{i=1}^2 \overline{u(p')} (C_V + C_A \gamma_5) \not{\epsilon}_i u(p) \epsilon_i \cdot M \sqrt{E_q} \left( \frac{1}{(p-p')^2 - M_V^2} \right) , \end{aligned} \quad (2.10)$$

where the momenta are labeled in Figure (1b). For  $V = W$  we have,

$$C_V = -C_A = g/(2\sqrt{2}) , \quad (2.11a)$$

and for  $V = Z$ ,

$$\begin{aligned} C_V &= g/\cos \theta_w (1/2 T_{3L} - Q \sin^2 \theta_w) \\ C_A &= -g/\cos \theta_w (1/2 T_{3L}) , \end{aligned} \quad (2.11b)$$

where  $g = e/\sin \theta_w$ ,  $T_{3L}$  is the third component of weak isospin of  $q_1$ , and  $Q$  is its electric charge.

The spin averaged total cross section, (keeping only the contributions from the transverse polarization vectors), is,

$$\sigma_T(q_1 + q_2 \rightarrow v_t \rightarrow q_1' + X) = \left( \frac{1}{32 E E_q} \right) \int \frac{d^3 p'}{(2\pi)^3} \frac{|A_T|^2}{E'} d\Gamma \quad (2.12)$$

$k = p - p'$  is the vector boson four momentum and  $E(E')$  is the energy of  $q_1(q_1')$ . Substituting Eq. (2.10) into Eq. (2.12), we find,

$$\sigma_T(q_1 + q_2 \rightarrow v_t \rightarrow q_1' + X) = \int \frac{d^3 p'}{(2\pi)^3} \left( \frac{C_V^2 + C_A^2}{8 E E'} \right) \left( \frac{d\Gamma}{(k^2 - M_V^2)^2} \right) \cdot \sum_{i,j=1}^2 (2p_i^k \epsilon_i \cdot p \cdot \epsilon_j - \frac{1}{2} k^2 \delta_{ij}) (\epsilon_i \cdot M) (\epsilon_j \cdot M)^+ \quad (2.13)$$

We average over the azimuthal angle,  $\phi'$ , in Eq. (2.13), ( $\hat{p} \cdot \hat{p}' = \cos\theta'$ ), which is equivalent to making the replacement,

$$2p_i^k \epsilon_i \cdot p \cdot \epsilon_j \rightarrow \delta_{ij} \frac{E^2 E'^2}{|\vec{k}|^2} \sin^2 \theta' \quad (2.14)$$

This has the effect of eliminating the off-diagonal terms in Eq. (2.12),

$$\sigma_T(q_1 + q_2 \rightarrow v_t \rightarrow q_1' + X) = \int \frac{d^3 p'}{(2\pi)^3} \left( \frac{C_V^2 + C_A^2}{8 E E'} \right) \left( \frac{d\Gamma}{(k^2 - M_V^2)^2} \right) \cdot \left[ \sin^2 \theta' \frac{E^2 E'^2}{|\vec{k}|^2} - \frac{k^2}{2} \right] \sum_{i=1}^2 |\epsilon_i \cdot M|^2 \quad (2.15)$$

The effective W approximation consists of replacing the transverse current  $|\epsilon_i \cdot M|$  and the phase space  $d\Gamma$  by their values at  $k^2 \rightarrow M_V^2$  and  $\theta' = 0$ . (The effective W approximation is thus equivalent to a small angle approximation. We will return to this point later.) With these approximations,

$$\sigma_T(q_1 + q_2 \rightarrow v_t \rightarrow q_1' + X) = \left( \frac{C_V^2 + C_A^2}{2\pi^2} \right) \int dk_0 \sin \theta' d\theta' \sigma_T(v_t + q_2 \rightarrow X) \eta \left( \frac{k_0 E'}{E} \right) \left( \frac{1}{(k^2 - M_V^2)^2} \right) \left\{ \sin^2 \theta' \frac{E^2 E'^2}{|\vec{k}|^2} - \frac{k^2}{2} \right\} \quad (2.16)$$

where  $k^2 = 2EE'(\cos\theta' - 1)$ .

In order to interpret V as a parton within a quark, we define the distribution function  $f_{q/v^t}(x)$  such that,

$$\sigma_T(q_1 + q_2 \rightarrow v_t \rightarrow q_1' + X) = \int_{M_V/E}^1 dx f_{q/v^t}(x) \sigma_T(v_t + q_2 \rightarrow X) \quad (2.17)$$

The probability distribution of transverse W's or Z's in a quark can now be found from Eq's. (2.16) and (2.17),

$$f_{q/v^t}(x) = \left( \frac{C_V^2 + C_A^2}{8\pi^2 x} \right) \left[ \frac{-x^2}{1 + M_V^2/(4E^2(1-x))} + \frac{2x^2(1-x)}{M_V^2/E^2 - x^2} + \left\{ x^2 + \frac{x^4(1-x)}{(M_V^2/E^2 - x^2)^2} \left( 2 + \frac{M_V^2}{E^2(1-x)} \right) - \frac{x^2}{(M_V^2/E^2 - x^2)^2} \frac{M_V^4}{2E^4} \right\} \cdot \log \left( 1 + \frac{4E^2(1-x)}{M_V^2} \right) + x^4 \left( \frac{2-x}{M_V^2/E^2 - x^2} \right)^2 \log \left( \frac{x}{2-x} \right) \right] \eta \quad (2.18)$$

$$\eta = \left[ 1 - \frac{M_V^2}{x^2 E^2} \right]^{1/2}$$

In the very high energy limit in which the quark sub-energy  $E$  is much greater than the vector boson mass  $M_V$ , Eq. (2.18) reduces to the simpler form,

$$f_{q/v^t}(x) \xrightarrow{E \gg M_V} \left( \frac{C_V^2 + C_A^2}{8\pi^2 x} \right) (x^2 + 2(1-x)) \log(4E^2/M_V^2) \quad (2.19)$$



Since it is the quark sub-energy  $E$  and not the total hadronic energy  $s$  which enters Eqs. (2.18) and (2.19), it is not a priori clear if  $E \gg M_v$  is a good approximation. This question must be addressed numerically.

To obtain the distribution of vector bosons in a proton,  $f_{p/v}^t(x)$ , it is necessary to integrate Eq. (2.17) or (2.18) with the quark distribution functions,  $f_i(x)$

$$f_{p/v}^t(x) = \sum_i \int_{x_{\min}}^1 \frac{dx'}{x'} f_i(x') f_{q_i/v}^t\left(\frac{x}{x'}\right), \quad (2.20)$$

where the sum on  $i$  runs over all relevant quarks and anti-quarks. Since the parton model treats the  $W$  and  $Z$  bosons as physical, on-shell bosons, we have the restriction  $x > M_v/E$  in Eq. (2.20). Figure 2 shows the probability distribution  $f_{p/v}^t$  for finding a transversely polarized  $W_t$  or  $Z_t$  with momentum fraction  $x$  in a proton at hadronic energies of 20 and 40 TeV.

The  $W_t^+W_t^-$  (or  $Z_tZ_t$ ) luminosity in a two quark system can be found from Eq. (2.18) or (2.19),

$$\begin{aligned} \frac{dL}{d\tau} \Big|_{qq/v}^t = & \int_{\tau}^1 f_{q/v}^t(x) f_{q/v}^t(\tau/x) \frac{dx}{x} \\ & \left[ \frac{C_V^2 + C_A^2}{8\pi^2} \frac{1}{\tau} \log\left(\frac{\hat{s}}{M_v^2}\right)^2 \left[ (2+\tau) 2\ln(1/\tau) \right. \right. \\ & \left. \left. - 2(1-\tau)(3+\tau) \right] \right], \quad (2.21) \end{aligned}$$

where  $\sqrt{\hat{s}}$  is the total energy in the  $q_1 - q_2$  system.

The luminosity of transversely polarized vector bosons in a proton proton system is then,

$$\frac{dL}{d\tau} \Big|_{pp/v}^t = \sum_{i,j} \int_{\tau}^1 \frac{d\tau'}{\tau'} \int_{\tau'}^1 \frac{dx}{x} f_i(x) f_j\left(\frac{\tau'}{x}\right) \frac{dL}{d\tau} \Big|_{qq/v}^t, \quad (2.22)$$

where  $\zeta = v/\tau'$ . Our luminosities are defined such that a hadronic cross section is given by,

$$\sigma_{pp \rightarrow VV \rightarrow X}(s) = \int_{\tau_{\min}}^1 d\tau \frac{dL}{d\tau} \Big|_{pp/vv} \sigma_{VV \rightarrow X}(\tau s). \quad (2.23)$$

Our results for the luminosities of transversely polarized  $W$ 's and  $Z$ 's are shown in Fig. (3) for hadronic energies of  $\sqrt{s} = 20$  and 40 TeV. The quark distribution functions we use are those of Eichten et al.,<sup>7</sup> which are guaranteed to be sensible at TeV energies. The running  $Q^2$  dependence of the quark structure functions can safely be neglected, since the treatment of the vector bosons as partons fixes the relevant  $Q^2$  to be  $Q^2 \approx M_v^2$ .

### (b) Longitudinal Gauge Bosons

The computation here follows the same method as for the transverse case and so we will only sketch our derivation. The scattering of a quark from a longitudinally polarized vector boson (Figure (1a)), has the spin averaged cross section,

$$\sigma_L(q_2 + V_\ell \rightarrow X) = 1/(8k_0 \eta) |\epsilon_3 \cdot M|^2 d\Gamma, \quad \eta = \sqrt{1 - M_V^2/k_0^2}, \quad (2.24)$$

where  $k_0$  is the boson energy,  $\epsilon_3$  is the longitudinal polarization vector, and  $d\Gamma$  is the Lorentz invariant phase space of the final state X. The spin averaged contribution of longitudinally polarized  $W_\ell^\pm$  or  $Z_\ell$  bosons to the process  $q_1 + q_2 \rightarrow V_\ell \rightarrow q_1' + X$  (Figure 1b) is,

$$\sigma_L(q_1 + q_2 \rightarrow V_\ell \rightarrow q_1' + X) = \left( \frac{C_V^2 + C_A^2}{8} \right) \int \frac{d^3 p'}{(2\pi)^3} \frac{d\Gamma}{EE'} |\epsilon_3 \cdot M|^2 \cdot \frac{(2p' \cdot \epsilon_3 p \cdot \epsilon_3 - p \cdot p' \epsilon_3 \cdot \epsilon_3)}{(k^2 - M_V^2)^2} \quad (2.25)$$

where the energy and momenta are as labeled in Figure (1b). We again make a small angle approximation and evaluate  $|\epsilon_3 \cdot M|^2 d\Gamma$  by its value on mass shell,  $k^2 \rightarrow M_V^2$  and at a small scattering angle,  $\theta' \rightarrow 0$ . With these assumptions equation (2.25) becomes,

$$\sigma_L(q_1 + q_2 \rightarrow V_\ell \rightarrow q_1' + X) = \left( \frac{C_V^2 + C_A^2}{4 \pi^2} \right) \int dk_0 \sin \theta' d\theta' \left( \frac{E' k_0}{E} \right) \eta \cdot \sigma_L(q_2 + V_\ell \rightarrow X) \frac{(2p' \cdot \epsilon_3 p \cdot \epsilon_3 - p \cdot p' \epsilon_3 \cdot \epsilon_3)}{(k^2 - M_V^2)^2} \quad (2.26)$$

$V_\ell$  is interpreted as a parton contained in the quark  $q_1$  and the probability distribution function  $f_{q/v,1}(x)$  is defined such that,

$$\sigma_L(q_1 + q_2 \rightarrow V_\ell \rightarrow q_1' + X) = \int \frac{dx}{M_V/E} f_{q/v,1}(x) \sigma_L(q_2 + V_\ell \rightarrow X) \quad (2.27)$$

Since we neglect all quark masses, the longitudinal polarization tensor is given by Eq. (2.4). (We can take the W and Z on-shell since they are being treated as partons.) After some tedious algebra, the distribution of longitudinally polarized gauge bosons in a quark is found from Eqs. (2.26) and (2.27) to order  $M_V^2/E^2$ ,

$$f_{q/v,1}(x) = \left( \frac{C_V^2 + C_A^2}{\pi^2} \right) \left( \frac{1-x}{x} \right) \left( \frac{\eta}{(1+\eta)^2} \right) \left\{ \frac{1-x-M_V^2/8/E^2}{1-x+M_V^2/4/E^2} - \frac{M_V^2}{4E^2} \frac{1+2(1-x)^2}{(1-x)+M_V^2/4/E^2} \frac{1}{M_V^2/E^2-x^2} - \frac{M_V^2}{4E^2} \left( \frac{x^2}{2(1-x)(x^2-M_V^2/E^2)^2} \right) \right. \\ \cdot \left[ (2-x)^2 \log \left( \frac{x}{2-x} \right) - \left( x - \frac{M_V^2}{E^2 x} \right)^2 - (2(1-x)+x^2) \log \left( 1 + \frac{4E^2(1-x)}{M_V^2} \right) \right] \\ - \frac{M_V^2}{8E^2} \frac{x}{\sqrt{x^2-M_V^2/E^2}} \left[ \frac{2}{x^2-M_V^2/E^2} + \frac{1}{1-x} \right] \left[ \log \frac{2-x-\sqrt{x^2-M_V^2/E^2}}{2-x+\sqrt{x^2-M_V^2/E^2}} \right. \\ \left. \left. - \log \frac{x-\sqrt{x^2-M_V^2/E^2}}{x+\sqrt{x^2-M_V^2/E^2}} \right] \right\} \quad (2.28)$$

where  $E$  is the sub-energy of the quark  $q_1$  and  $x$  is the fraction of the quark momentum carried by the  $W^\pm$  or  $Z$  boson. For very high energies, Eq. (2.28) is considerably simplified,

$$f_{q/v,1}(x) \xrightarrow{E \gg M_V} \left( \frac{C_V^2 + C_A^2}{4 \pi^2} \right) \left( \frac{1-x}{x} \right) \quad (2.29)$$

The probability distributions for longitudinally polarized gauge bosons,  $f_{p/v,i}(x)$ , in a proton are shown in Fig. 4 for  $\sqrt{s} = 20$  and 40 TeV. Note the extremely weak dependence of these distribution functions on the hadronic energy scale. Integrating  $f_{q/v,i}(x)$ , the  $V_\ell V_\ell$  luminosity

can be found. In the case  $M_V \gg E$ , the luminosity can be found analytically,

$$\left. \frac{dL}{d\tau} \right|_{qq/v^1 v^1} = \left( \frac{C_V^2 + C_A^2}{4\pi^2} \right)^2 \frac{1}{\tau} \left[ (1+\tau) \ln(1/\tau) + 2(\tau-1) \right] . \quad (2.30)$$

This result has also been found by Chanowitz and Gaillard.<sup>5</sup>

In Figure 5, we present the luminosities for longitudinally polarized  $W^+W^-$  and  $ZZ$  pairs in proton-proton collisions at 40 TeV. We use Eqs. (2.30) and (2.21) to obtain these results. From Eqs. (2.28) and (2.30), it is clear that to leading order in  $M_W^2/\hat{s}$ , the luminosities of longitudinal  $W_\rho$ 's and  $Z_\rho$ 's are independent of  $\hat{s}$ .

### (c) Uncertainties of the Calculation

The effective W approximation requires taking the vector bosons in the intermediate state of a scattering process on mass shell, assuming that the vector bosons are produced at small angles to the incoming quarks, and ignoring the interference between the transverse and longitudinal degrees of freedom. We will deal with each of these approximations in turn.

The approximations of the effective W method require,

$$x \gg 2M_V / \sqrt{\hat{s}}, \quad (2.31)$$

where  $x$  is the momentum fraction carried by either quark in a pp collision. In our numerical calculations, we have abruptly cut off all

integrals at this minimum value of  $x$ . For resonant production of a particle with mass  $M$  from  $VV$  scattering, our approximations are only valid for  $M \gg 2M_W$ .

The effective W method is intrinsically a small angle approximation. From Eqs. (2.16) and (2.26), it is clear that for gauge bosons produced in a small angular region, the transverse cross section,  $\sigma_T$ , depends on the angle  $\theta'$  of the scattered quark  $q_1'$  as,

$$\sigma_T \sim \int \frac{\theta'^2 d\theta'^2}{(\theta'^2 + M_V^2/E E')^2} \left( \frac{1}{2} + \frac{1}{\theta'^2 + k_o'^2/E E'} \right), \quad (2.32)$$

while the longitudinal cross section,  $\sigma_L$ , has the angular dependence

$$\sigma_L \sim \int \frac{d\theta'^2}{(\theta'^2 + M_V^2/E E')^2}. \quad (2.33)$$

Approximately half of the transverse cross section comes from angles  $\theta' < [M_V/\sqrt{\hat{s}}]^{1/2}$  where  $\hat{s}$  is the sub-energy in the quark sector.<sup>3</sup> On the other hand, because of the presence of the  $\theta'^2$  in the numerator of Eq. (2.32), the longitudinal cross section is collimated into even smaller forward angles than  $\sigma_T$ .

The equivalent photon method is known to be quite accurate in  $e^+e^-$  scattering at energies greater than a few GeV.<sup>8</sup> In this case, the two photon scattering cross sections, (which are purely transverse), are enhanced by factors of  $[\ln(E/m_e)]^2 \sim 60$  for  $E = 1$  GeV. For  $W^+W^-$  scattering, however, the transverse cross section is proportional to  $[\ln \sqrt{\hat{s}}/M_W]^2$ . Even for a very high energy quark,  $\sqrt{\hat{s}} = 1$  TeV, this enhancement factor is only six. The longitudinal luminosities are typically a factor of one hundred smaller than the transverse

luminosities, so unless there is a large enhancement at the V-V-final state vertex, these cross sections will be small.

Finally, we consider the effects of ignoring the interference between the longitudinal and transverse currents. The cross section for the scattering process of Figure (1b),  $q_1 + q_2 \rightarrow V \rightarrow q_1' + X$ , can be written,

$$\sigma(q_1 + q_2 \rightarrow q_1' + X) = \sigma_T + \sigma_L + \sigma_{LT},$$

where  $\sigma_L$  and  $\sigma_T$  can be calculated in the equivalent W approximation and  $\sigma_{LT}$  represents the interference between the longitudinal and transverse gauge bosons. Direct computation shows that  $\sigma_{LT}$  is not suppressed relative to  $\sigma_T$  and  $\sigma_L$ . This means that the effective W approximation is only useful for calculating processes where the scattering amplitude is clearly dominated by the contribution of either the longitudinal or the transverse gauge bosons.

We begin by considering a scattering process of the type described by Eq. (1.1). We show the  $W^\mu W^\nu X$  vertex schematically in Fig. 6 and represent the amplitude by the tensor  $T_{\mu\nu}(q_1, q_2)$ , where  $q_1$  and  $q_2$  are the momenta of the W's. If both of the gauge bosons are longitudinally polarized, the scattering amplitude for  $pp \rightarrow W_L W_L X \rightarrow X'$  will be proportional to,

$$A_{LL} = \epsilon_{3\mu} T^{\mu\nu} \epsilon_{3\nu} \\ \sim \frac{\hat{s}^2}{M_W^2} \quad (2.34)$$

where  $\hat{s}$  is the center-of-mass energy of the WW system and we have taken the W bosons to be on-shell since we are treating them as partons. On the other hand, the scattering amplitude for  $pp \rightarrow W_L W_L X \rightarrow X'$  is proportional to,

$$A_{TT} = \epsilon_{i\mu} T^{\mu\nu} \epsilon_{i\nu} \quad i=1,2 \\ \sim \frac{\hat{s}}{M_W^2} \quad (2.35)$$

At very high energies  $\hat{s} \gg M_W^2$ , the longitudinal contribution clearly dominates,

$$\frac{A_{LL}}{A_{TT}} \sim \frac{\hat{s}}{M_W^2} \quad (2.36)$$

For  $\sqrt{\hat{s}} \gg M_W$ , the contributions to a scattering cross section from the transverse gauge bosons and from the interference between the transverse and longitudinal bosons are negligible. At intermediate energy scales, however, the relative importance of  $\sigma_L$  and  $\sigma_T$  is not clear.

In Section III we examine the interplay between transversely polarized and longitudinally polarized gauge bosons for the associated production of a heavy charge 2/3 and -1/3 quark.

### III. Applications of the Effective W Approximation

We consider here two applications of the effective W approximation -- the associated production of a heavy flavor quark pair and Higgs boson production from  $W^+W^-$  (or ZZ) scattering.

#### (a) Heavy Quark Production

The calculation of production rates for heavy quarks of different charges is greatly simplified by using the effective W approximation. As an example, we calculate the rate for  $W^+ g \rightarrow U\bar{D}$ , where U and  $\bar{D}$  are heavy quarks with charges 2/3 and  $-1/3$  respectively. (See Fig. 7.) The scattering amplitude for a W boson with polarization tensor  $\epsilon_i$  is,

$$A(W_i^+ g \rightarrow U\bar{D}) = \frac{g g_s}{2\sqrt{2}} \overline{u(p')} (1-\gamma_5) \not{\epsilon}_i(p) \left( \frac{\not{p}' - \not{p} + M}{(p'-p)^2 - M^2} \right) \gamma^\nu v(q') T^a \epsilon_\nu(q) , \quad (3.1)$$

where both of the heavy quarks have mass M and  $T^a = \lambda^a/2$ , where  $\lambda^a$  is a Gell-Mann matrix with index  $a = 1, \dots, 8$ . The amplitude squared is then,

$$|A(W_i^+ g \rightarrow U\bar{D})|^2 = g_s^2 g^2 \left( \frac{1}{\hat{t} - M^2} \right)^2 \left\{ \epsilon_i \cdot \epsilon_i \left( \hat{t} \left( \frac{\hat{s}}{2} + \frac{\hat{t}}{2} - 2M^2 \right) - M^2 \frac{\hat{s}}{2} - \frac{M^4}{2} \right) + 2\epsilon_i \cdot p' \left( -2M^2 \epsilon_i \cdot q' + \epsilon_i \cdot q (3M^2 - \hat{t}) \right) \right\} , \quad (3.2)$$

where  $\hat{t}$ ,  $\hat{u}$ , and  $\hat{s}$  are the kinematic variables of the  $W^+g$  system and  $\epsilon_i$  is defined in Eq. (2.1)

Treating the W boson as a parton the effective W approximation gives the hadronic cross section,

$$\sigma(s)_{pp \rightarrow U\bar{D}} = \int dx_1 dx_2 \left\{ g(x_1) f_{p/w}^t(x_2) \sigma_T(\tau s)_{Wg \rightarrow U\bar{D}} + g(x_1) f_{p/w}^1(x_2) \sigma_L(\tau s)_{Wg \rightarrow U\bar{D}} + \dots \right\} , \quad (3.3)$$

where  $\tau = x_1 x_2$ ,  $f_{p/w}^t$  and  $f_{p/w}^1$  are given by Eqs. (2.20) and (2.28), and  $g(x)$  is the gluon distribution function.

The spin averaged longitudinal cross section in the parton system can be found using Eq. (2.1),

$$\frac{d\sigma_L}{d\hat{t}} = \left( \frac{\alpha_s \alpha}{8 \sin^2 \theta_W \hat{s}^2} \right) \left( \frac{1}{\hat{t} - M^2} \right)^2 \frac{\hat{s}}{M_V^2} \left\{ -4M^2(M^2 + \hat{s}\beta^2) + (3M^2 - \hat{t})\hat{s} \right. \\ \left. \cdot (1 - (1 - \frac{4M^2}{\hat{s}} - 4\beta^2)^{1/2}) \right\} , \quad (3.4a)$$

where we have calculated to leading order in  $M_W/\sqrt{\hat{s}}$ . From Eq. (3.2) the transverse cross section to  $O(M_W/\sqrt{\hat{s}})$  is

$$\frac{d\sigma_T}{d\hat{t}} = \left( \frac{\alpha_s \alpha}{4 \sin^2 \theta_W \hat{s}^2} \right) \left( \frac{1}{\hat{t} - M^2} \right)^2 \left\{ -\hat{t}(\hat{s} + \hat{t} - 4M^2) + \hat{s}M^2 + M^4 + 4M^2\beta^2\hat{s} \right\} , \quad (3.4b)$$

where  $\beta = 1/\hat{s} (\hat{u}\hat{t} - M^4)^{1/2} = (1/2\sqrt{1 - 4M^2/\hat{s}}) \sin\theta'$ . Performing the angular integrations in a small angle approximation,

$$\sigma_L = \frac{\alpha_s \alpha}{8 \sin^2 \theta_W M_V^2} \left\{ (2 - 4M^2/\hat{s}) \sqrt{1 - 4M^2/\hat{s}} - 2(1 - \frac{4M^2}{\hat{s}}) \right. \\ \left. + (1 - \sqrt{1 - \frac{4M^2}{\hat{s}}}) \ln \left[ \frac{1 + \sqrt{1 - 4M^2/\hat{s}}}{1 - \sqrt{1 - 4M^2/\hat{s}}} \right] \right\} \quad (3.5a)$$

$$\sigma_T = \frac{\alpha_s \alpha}{4 \sin^2 \theta_W \hat{s}} \left\{ - (1-4M^2/\hat{s})^{3/2} + (1-2M^2/\hat{s}) \ln \left[ \frac{1+\sqrt{1-4M^2/\hat{s}}}{1-\sqrt{1-4M^2/\hat{s}}} \right] \right\} \quad (3.5b)$$

Using Eqs. (3.5) and (3.3) and the W structure functions derived in Section II, the hadronic cross section for  $pp \rightarrow U\bar{D} + \text{anything}$  can be found. Our results are shown in Fig. 8a for  $\sqrt{s} = 20$  TeV and in Fig. 8b for  $\sqrt{s} = 40$  TeV. The contributions of the longitudinally and transversely polarized W's are shown separately.

### (b) Heavy Higgs Production

The Higgs boson of the Weinberg-Salam model can be produced from resonant  $W^+W^-$  (or ZZ) scattering. The cross section for this process, (Fig.9), has been calculated in Ref. 4 in a small angle approximation,

$$\sigma(q_1+q_2 \rightarrow VVX+HX) = \frac{g_{VVH}^2}{64\pi^4 \hat{s}^2} (C_V^2 + C_A^2)^2 \int d\theta^2 J(\theta^2, \frac{4M_V^2}{s}) \left[ \log \frac{2}{\sqrt{\theta^2 + 4M_H^2/\hat{s}}} \right] \quad (3.6)$$

where  $M_H$  is the Higgs mass,  $\hat{s}$  is the center of mass energy of the  $q_1-q_2$  system, and

$$J(\theta^2, A_V) = \frac{128\pi}{\theta^3} \left\{ \frac{\theta^2 + A_V}{(\theta^2 + 4A_V)^{5/2}} \log \left( \frac{\sqrt{\theta^2 + 4A_V} + \theta}{\sqrt{\theta^2 + 4A_V} - \theta} \right) + \frac{\theta}{4A_V} \frac{\theta^2 - 2A_V}{(\theta^2 + 4A_V)^2} \right\} \quad (3.7)$$

For  $V = W$  we have,

$$g_{WWH} = g M_W \quad (3.8)$$

and for  $V = Z$  we have,

$$g_{ZZH} = \frac{g M_W}{\cos^2 \theta_W} \quad (3.9)$$

The Higgs production cross section of Eq. (3.6) can be obtained easily in the effective W approximation. The resonance cross section for  $WW \rightarrow H$  is,<sup>5</sup>

$$\begin{aligned} \sigma_{\text{eff}} &= \frac{16\pi^2}{M_H^3} \Gamma(H \rightarrow W^+W^-) \tau \left. \frac{dL}{d\tau} \right|_{pp/WW} \\ &= \frac{\alpha \pi^2}{\sin^2 \theta_W} \frac{\tau}{M_W^2} \left. \frac{dL}{d\tau} \right|_{pp/WW} \quad (3.10) \end{aligned}$$

where  $\tau = M_H^2/\hat{s}$  and

$$\Gamma(H \rightarrow W^+W^-) = \frac{\alpha}{16\sin^2 \theta_W} \frac{M_H^3}{M_W^2} \quad (3.11)$$

At high energies,  $M_W^2 \ll \hat{s}$ , the Higgs production cross section is dominated by the contribution from longitudinal W's and so we will use only the longitudinal contribution to  $dL/d\tau$  in evaluating Eq. (3.10).

Figure 10 shows the cross section of Ref. 4 and that of the effective W approximation as a function of Higgs mass for various center of mass energies. The two calculations agree to within 30% for Higgs masses less than 500 GeV. For larger Higgs masses, the discrepancy is larger.

rising to about a 40% difference for  $M_H = 1 \text{ TeV}$  and  $\sqrt{s} = 40 \text{ TeV}$ .

#### IV. Conclusion

At high energies,  $\sqrt{s} \gtrsim 20 \text{ TeV}$ , the W and Z bosons may be treated as partons in the proton and we have presented distribution functions and luminosities for these bosons at various energies. It is clear, however, that our results may only be used to calculate amplitudes for processes in which the major contribution comes from quark sub-energies much greater than the vector boson mass. A detailed numerical study is needed to test the accuracy of the equivalent W approximation at intermediate energy scales, ( $\sqrt{s} \sim 1 - 20 \text{ TeV}$ ).

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## Figure Captions

Fig. 1. (a) Feynman diagram for a massive gauge boson  $V_i^\mu$  with polarization  $\varepsilon_i$  to scatter from an incident quark  $q_2$  to the final state X.

(b) Feynman diagram for the process  $q_1 + q_2 \rightarrow V_i \rightarrow q_1' + X$ .

Fig. 2. Probability distributions,  $f_{p/v}(x)$ , for transversely polarized  $W^+$  and  $Z$  bosons in a proton. The solid line is the  $W_t^+$  distribution from Eq. (2.18) and the dot-dashed line the  $W_t^+$  distribution using the approximation of Eq. (2.19). The dashed (dotted) line is  $f_{p/z}(x)$  from Eq. 2.18 (Eq. 2.19). Figure 2a has a hadronic center of mass energy  $\sqrt{s}$  of 20 TeV. Fig. 2b has  $\sqrt{s} = 40$  TeV.

Fig. 3. Luminosity of transversely polarized gauge bosons in a proton proton system using the approximation of Eq. (2.21). The solid and the dot-dashed lines are the  $W_t^+ W_t^-$  luminosities at 40 TeV and 20 TeV, respectively. The dashed and dotted lines are the  $Z_t Z_t$  luminosities at 40 TeV and 20 TeV.

Fig. 4. Probability distributions,  $f_{p/v}(x)$ , for longitudinally polarized  $W^+$  and  $Z$  bosons in a proton. The solid line is the  $W_\ell^+$  distribution from Eq. (2.28) and the dot-dashed line the  $W_\ell^+$  distribution using the approximation of Eq. (2.29). The dashed (dotted) line is  $f_{p/z}$  for Eq (2.28) (Eq. 2.29). Figure 4a has  $\sqrt{s} = 20$  TeV. Figure 4b has  $\sqrt{s} = 40$  TeV.

Fig. 5. Luminosity of longitudinally polarized gauge bosons in a proton proton system using the approximation of Eq. (2.30). The solid (dashed) line is the  $W_\ell^+ W_\ell^-$  ( $Z_t Z_t$ ) luminosity.

(Note that Eq. 2.30 is independent of the center of mass energy).

Fig. 6. Vector boson scattering for the process  $V^\mu + V^\nu \rightarrow X$ .

Fig. 7. Feynman diagram for the process  $W^+ + g \rightarrow U + \bar{D}$ .  $U$  and  $\bar{D}$  are new heavy quarks with charges  $2/3$  and  $-1/3$  respectively.

Fig. 8. Total cross section for  $W^+ + g \rightarrow U + \bar{D}$  at  $\sqrt{s} = 20$  TeV (Fig. 8a) and  $\sqrt{s} = 40$  TeV (Fig. 8b) as a function of the heavy quark mass  $M$ . The solid line is the total cross section and the dashed (dot-dashed) line the contributions from  $W_\ell^+$  ( $W_t^+$ ).

Fig. 9. Feynman diagram for the resonant process  $W^+ W^- \rightarrow H$ .

Fig. 10. Total cross section for the process  $W^+ W^- \rightarrow H$  as a function of the Higgs boson mass  $M_H$ . The solid and the dashed lines are the results using the effective  $W$  approximation of Eq. (3.10) at  $\sqrt{s} = 20$  TeV and 40 TeV respectively. The dotted and dot-dashed lines are the results of Ref. 4 at  $\sqrt{s} = 20$  TeV and 40 TeV.

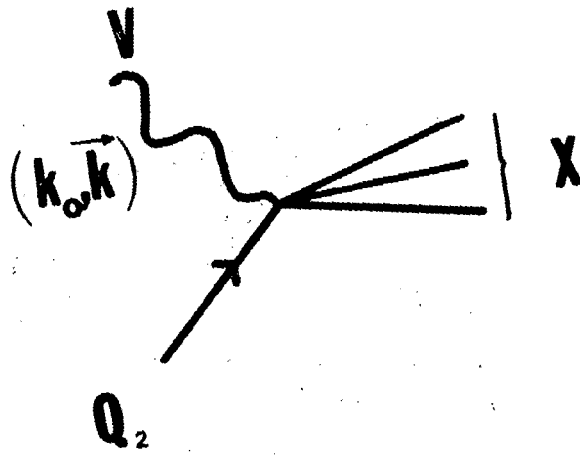


Fig. 1a

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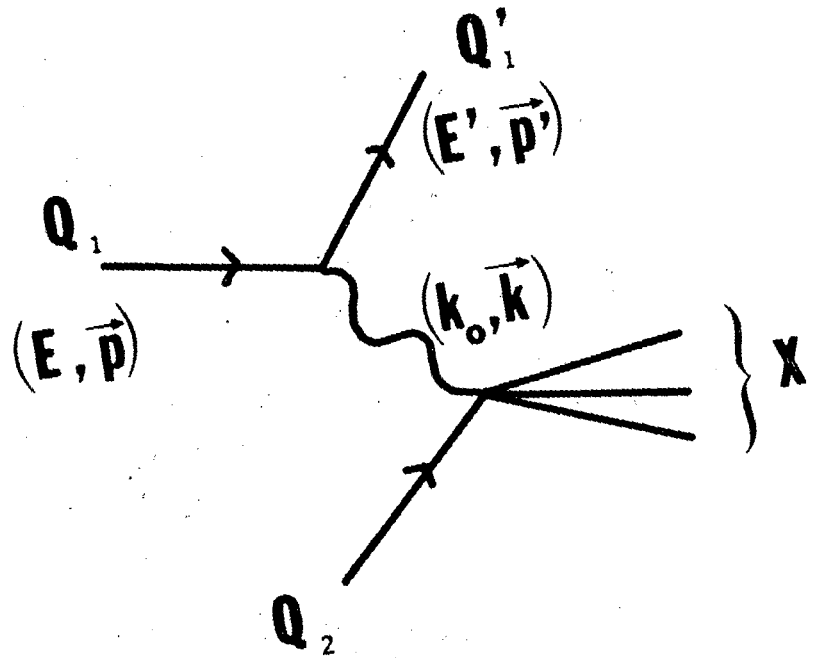


Fig. 1b

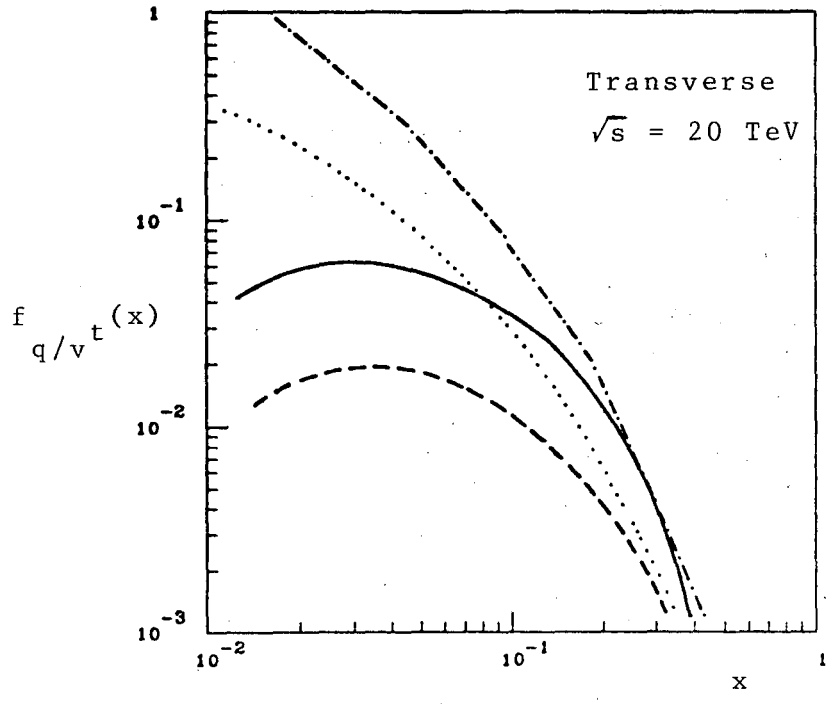


Fig. 2a

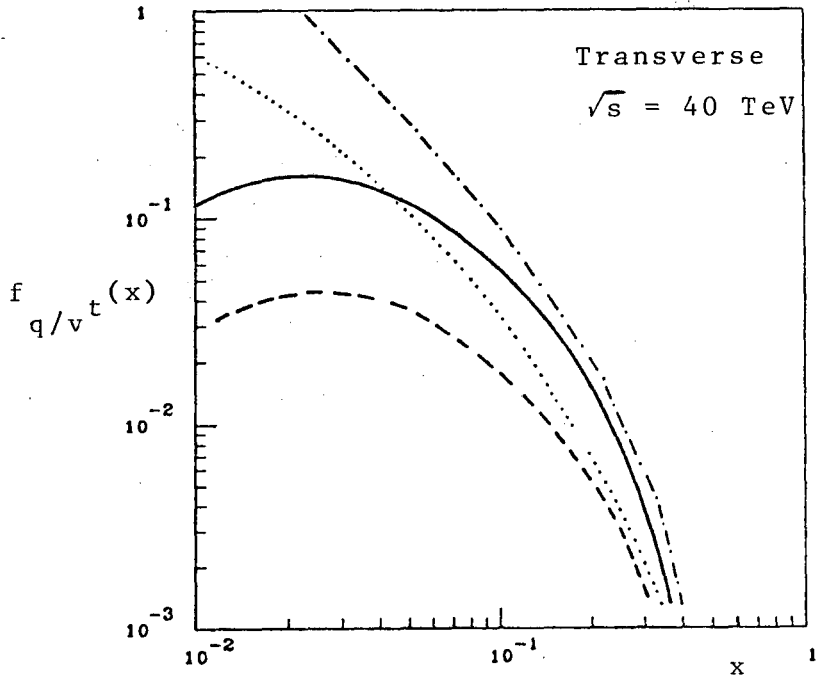


Fig. 2b

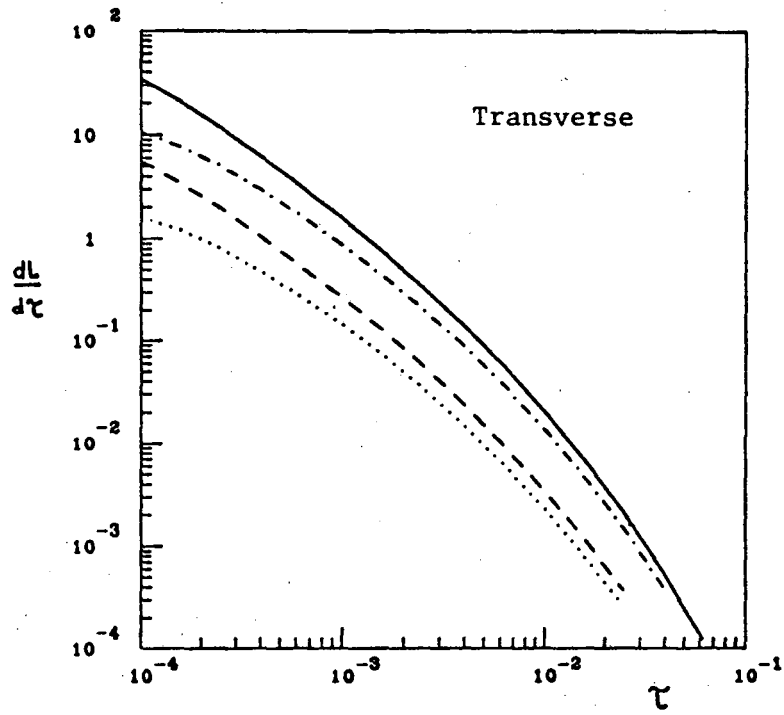


Fig. 3

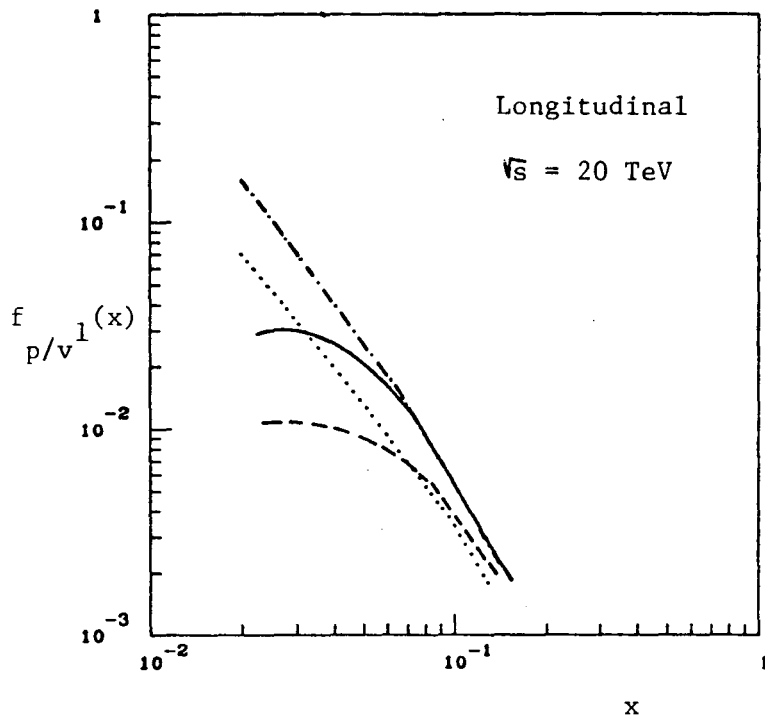


Fig. 4a

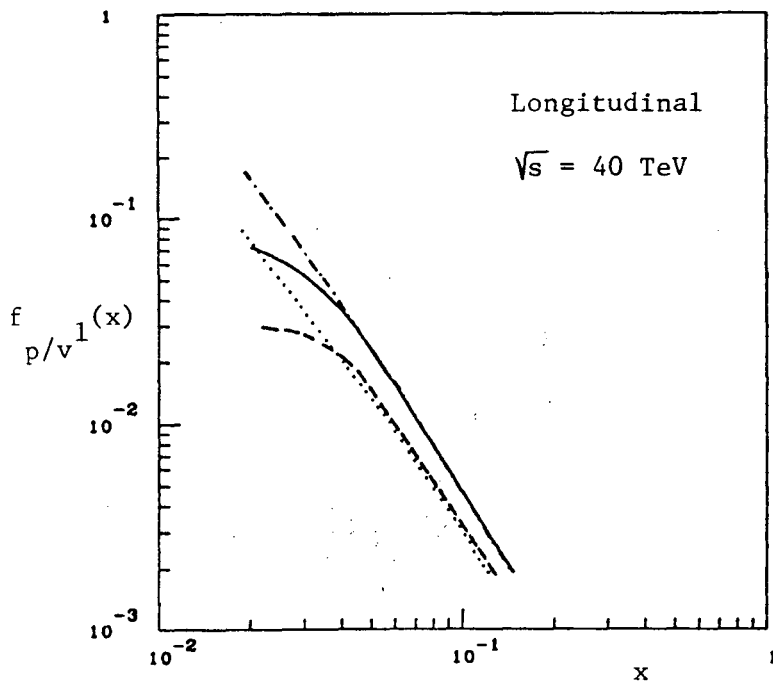


Fig. 4b

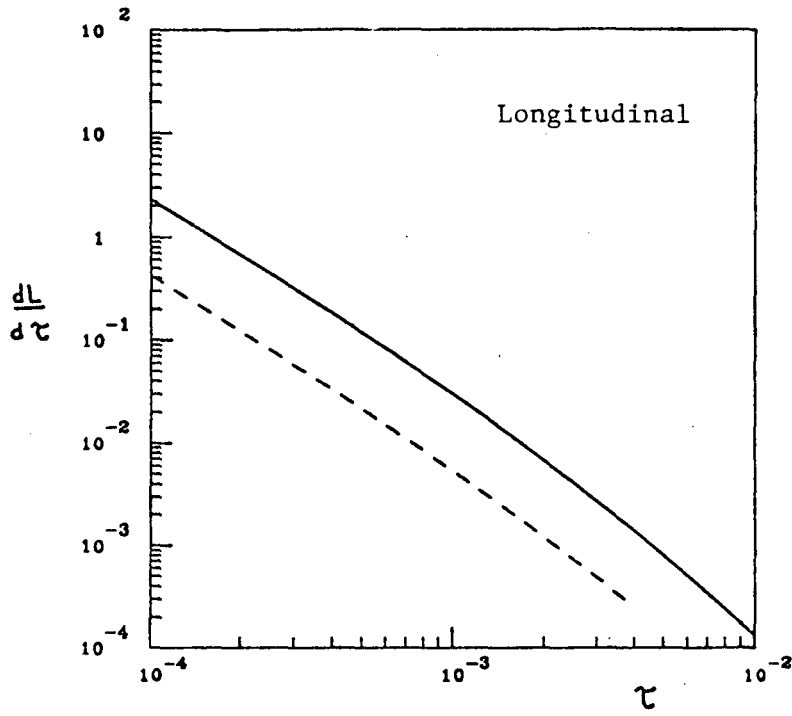


Fig. 5

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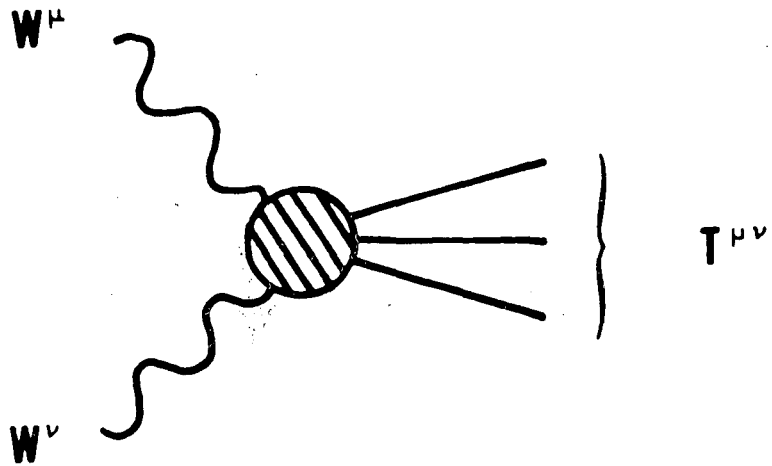


Fig. 6

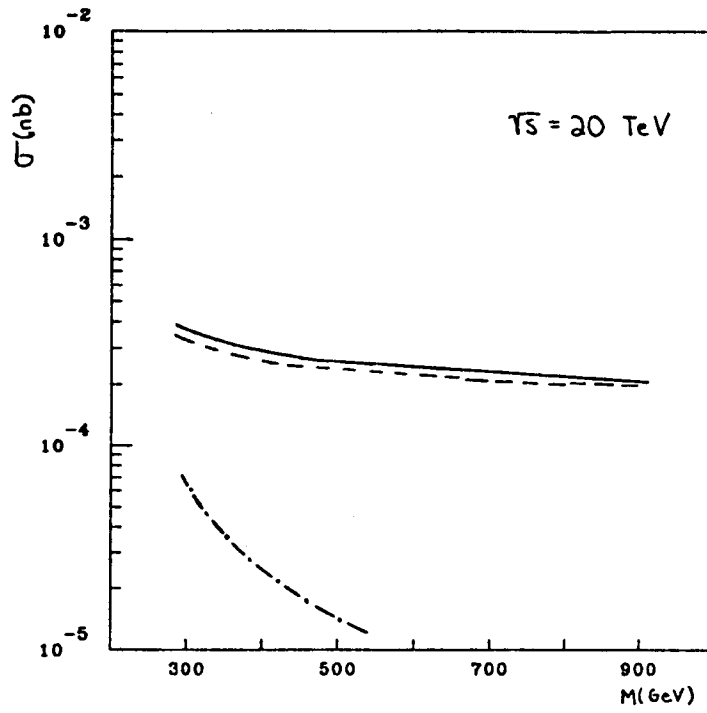


Fig. 8a

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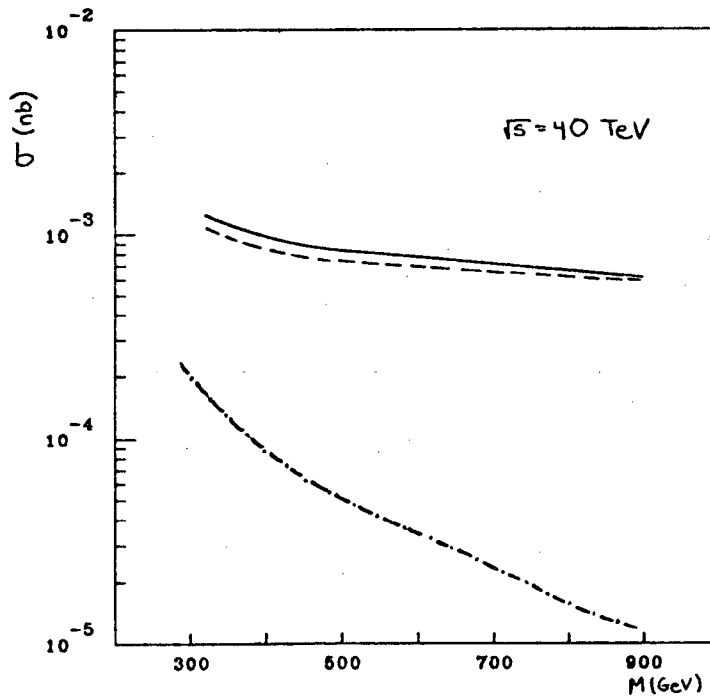


Fig. 8b

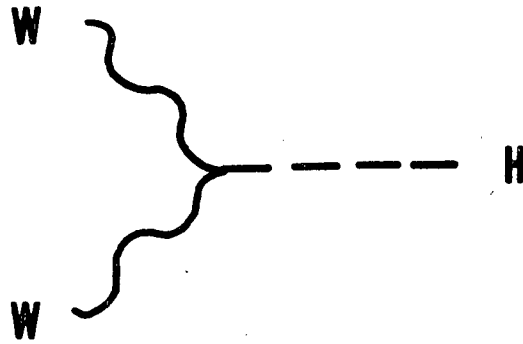


Fig. 9

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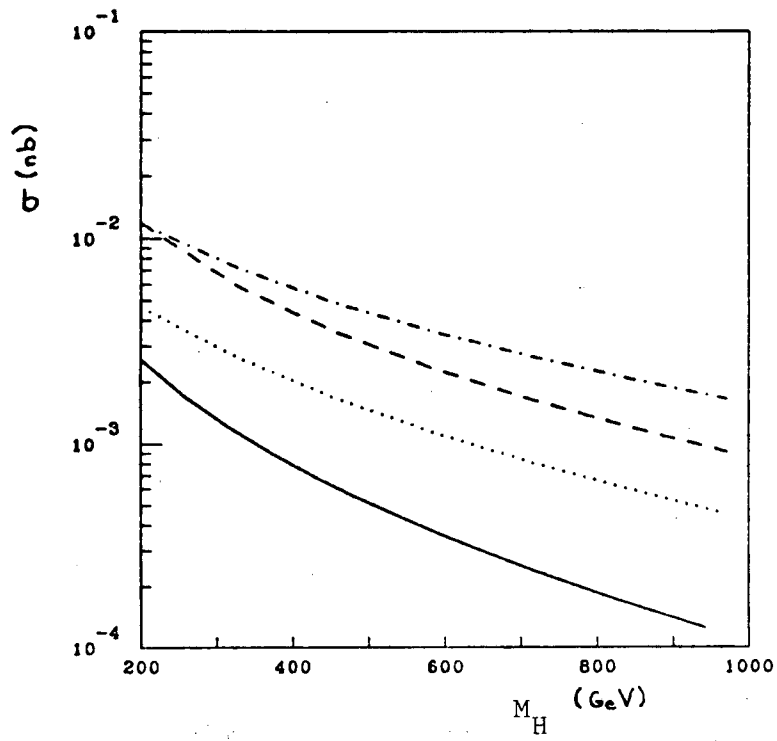


Fig. 10



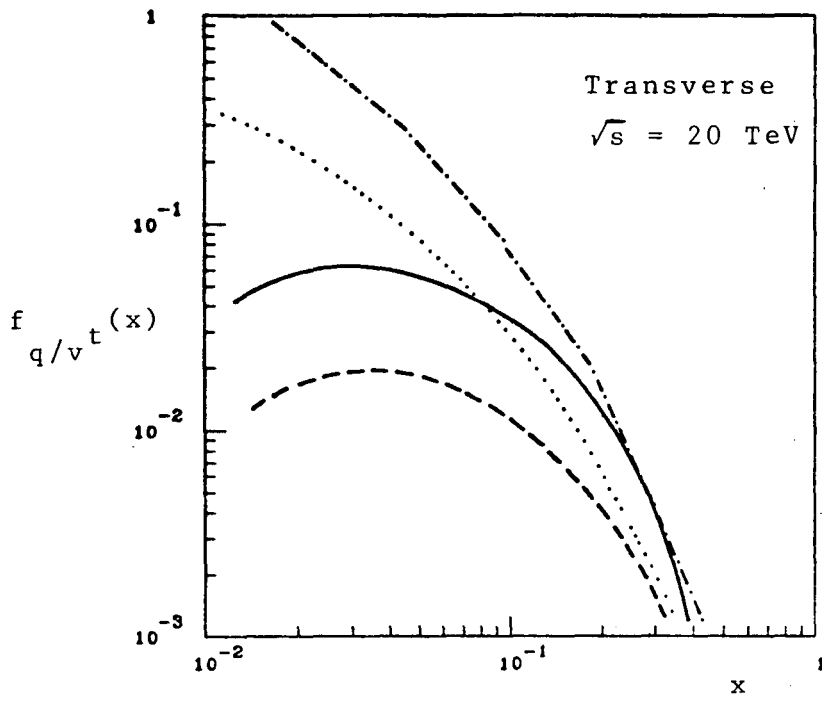


Fig. 2a

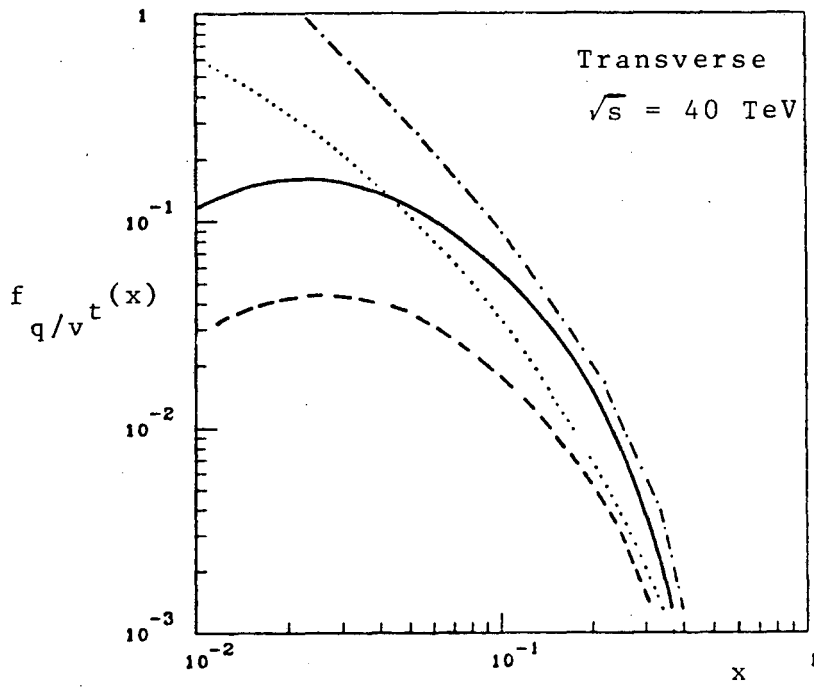


Fig. 2b

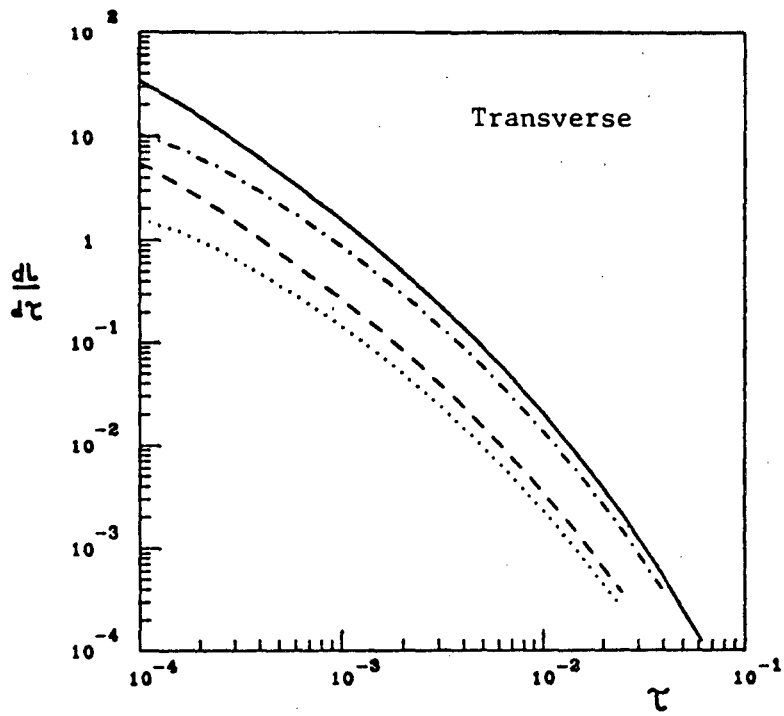


Fig. 3

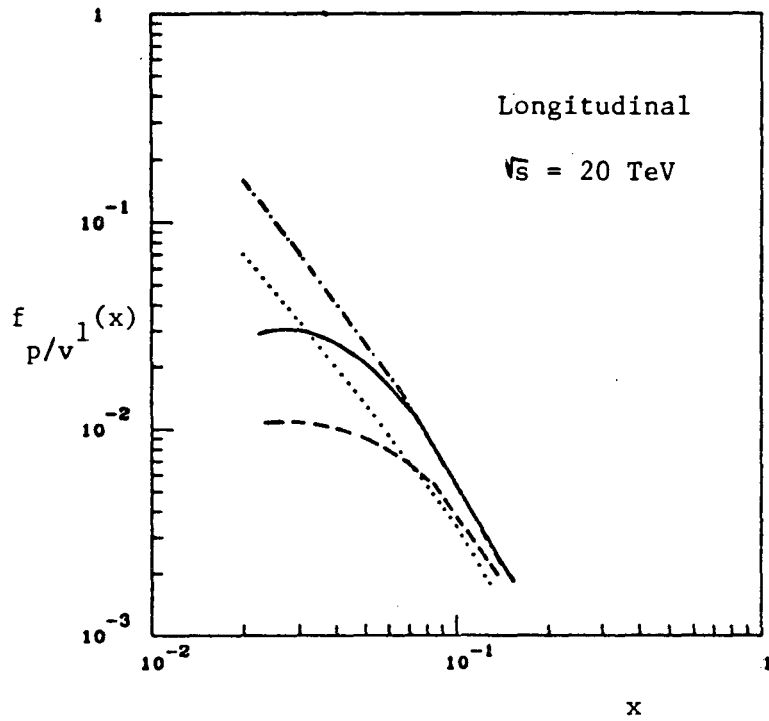


Fig. 4a

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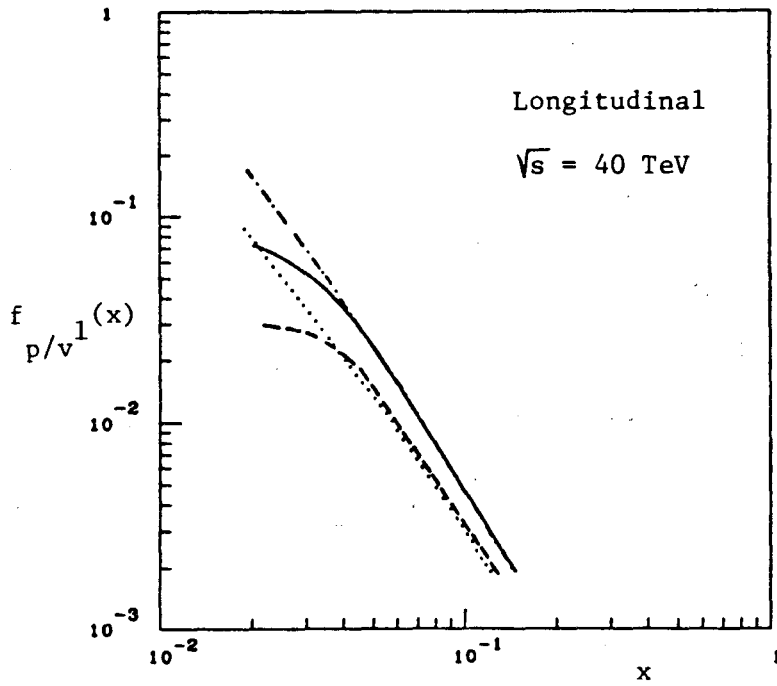


Fig. 4b

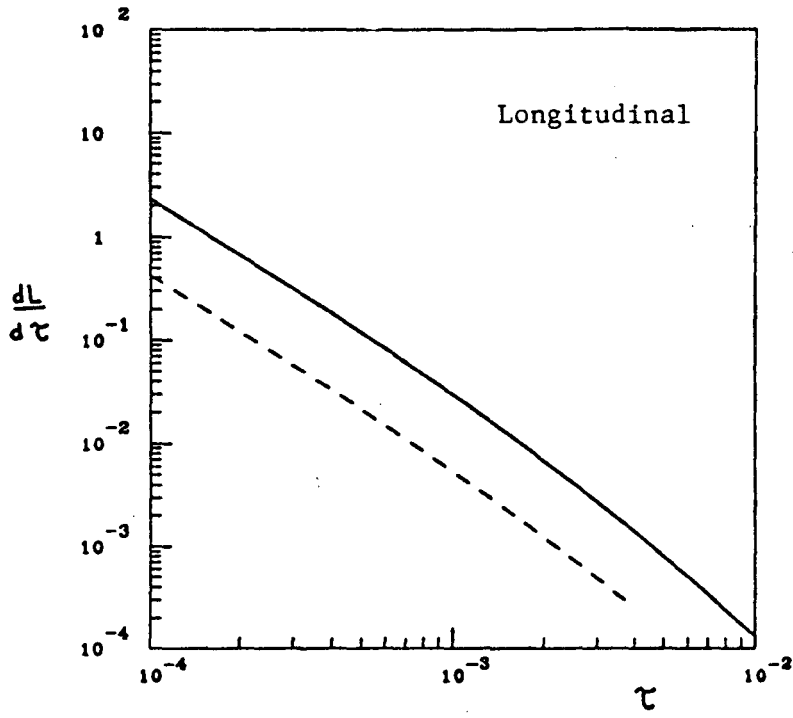


Fig. 5

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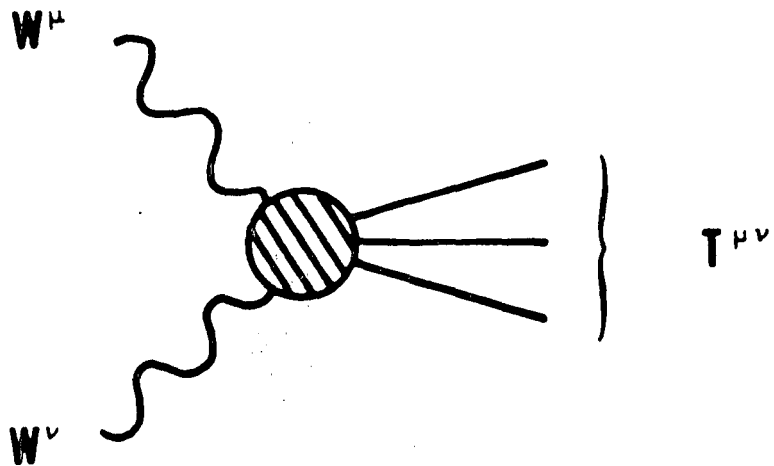


Fig. 6

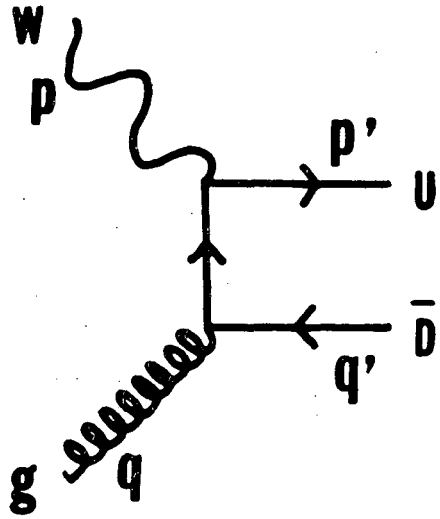


Fig. 7

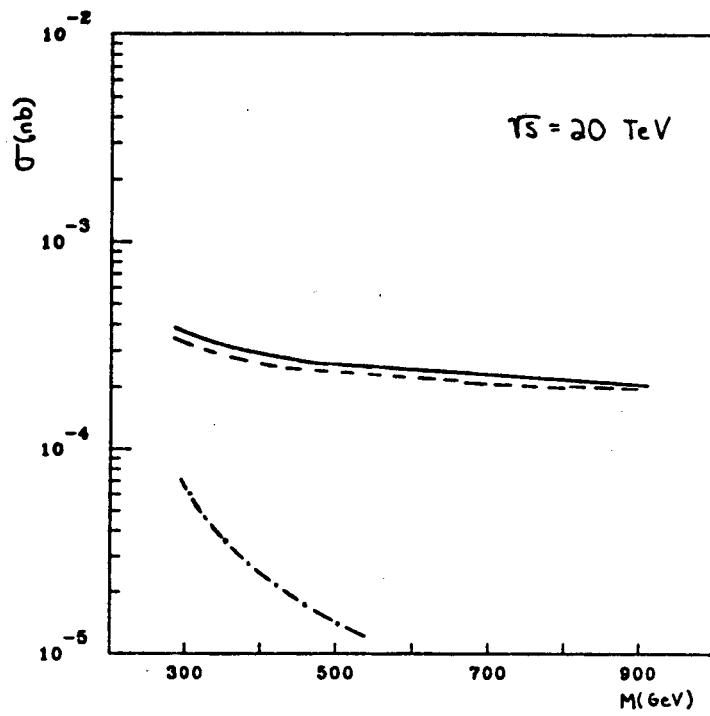


Fig. 8a

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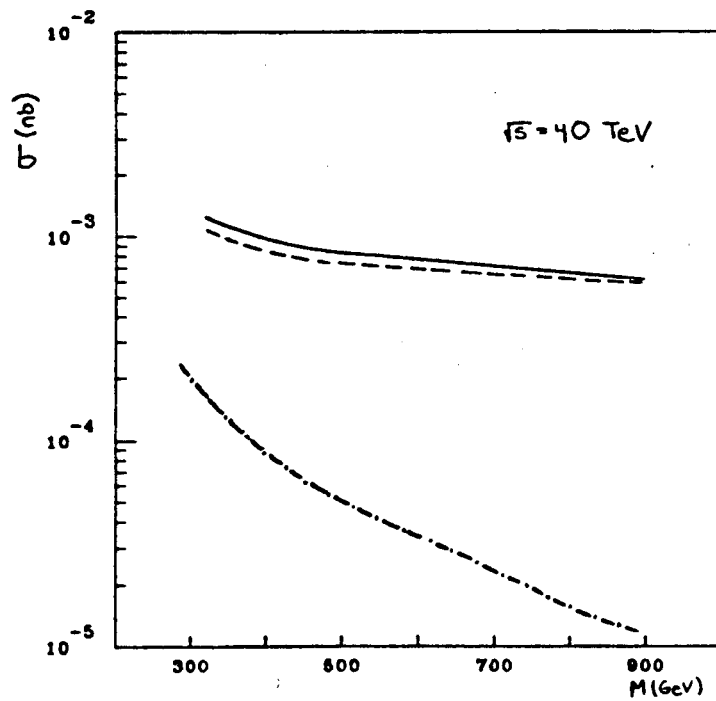


Fig. 8b

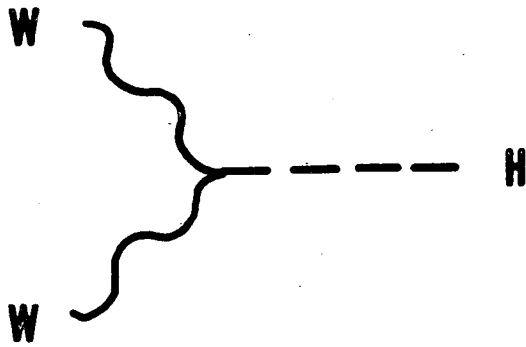


Fig. 9

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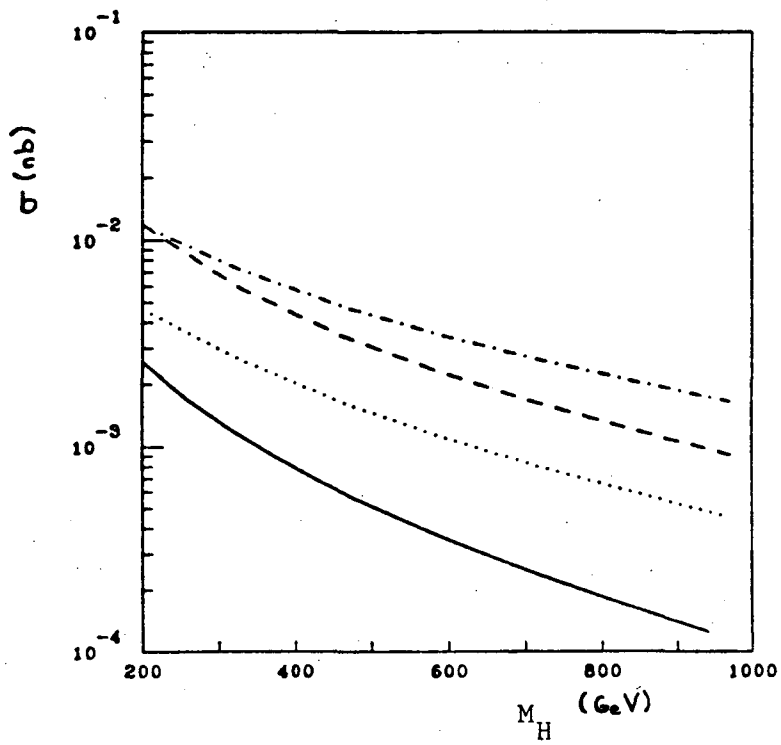


Fig. 10

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