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What Makes Children Change Their Minds?
Changes in Problem Encoding Lead to Changes in Strategy Selection

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Abstract

This study examined how changes in children's problem encoding influenced their strategy selection. Fourth-grade students (N=51) solved six mathematical equivalence problems (e.g., 3+4+5=___+5) in a pretest. Children's problem encoding was then manipulated in one of two ways, or was not manipulated in a Control group. In the Subtle group, children solved four additional problems with the equal sign highlighted in red. In the Direct group, children solved the same four problems, and were directed to notice the equal sign in each problem. Children then solved six problems in a posttest, and did so again four weeks later in a follow-up test. The strategies children conveyed in their spoken and gestured explanations were assessed. Children in the Direct group considered multiple strategies for the posttest problems more often than children in the other groups, as reflected in their spoken and gestured explanations. Children in the Direct group were also most likely to generate gestured strategies and to abandon verbal strategies over the course of the study. These findings suggest that changes in problem encoding lead to changes in strategy selection.

People often use incorrect or inefficient strategies to solve problems. Over time, people sometimes abandon these inadequate strategies and begin to use better ones. Where do these new strategies come from? Understanding the origin of new strategies is key to understanding both learning and developmental change. The present study examines the origin of new strategies in children learning to solve mathematical equations.

Previous studies have shown that the path of strategy change in children is typically gradual rather than abrupt (Alibali, in press; Siegler, 1995). That is, in most cases, children gradually expand and then contract their strategy repertoires, rather than switch suddenly from one strategy to another. New strategies are used infrequently at first, and come to be used more consistently over time (Kuhn & Phelps, 1982; Miller & Seier, 1994).

Thus, there is growing consensus in the literature that new strategies enter the repertoire gradually. However, the genesis of these gradually-emerging strategies remains a puzzle. In this paper, we suggest that, in order to understand where new strategies come from, it is essential to consider not only the strategies that children use, but also the features of the problems that they encode, or mentally represent.

We propose that, when solving a problem, children encode selected features of the problem, and then apply a set of operations to those encoded features. Under this view, a *strategy* is a set of operations applied to a particular set of problem features. This view implies that, when children encode new features of problems, they can then use those features in new problem solving strategies. Thus, one potential source of new problem solving strategies is changes in the problem features that children encode. Changes in children's problem encoding may provide them with "building blocks" for use in constructing new problem solving strategies.

We propose that changes in children's problem encoding may be one source of variability in their strategy use. Previous studies have shown that children often exhibit variable strategy use, sometimes considering multiple strategies for individual problems (e.g., Alibali, in press; Alibali & Goldin-Meadow, 1993; Siegler, 1995). This within-problem variability is evident in gesture-speech *mismatches*, in which children express one strategy in speech and another in gesture (Church & Goldin-Meadow, 1986; Perry, Church, & Goldin-Meadow, 1988). We suggest that children's mismatching gestures reflect information that they have encoded about the problems, but have not used in solving the problems (see Siegler, 1984). Thus, strategies expressed uniquely in gesture are strategies that children consider but do not use to derive problem solutions (see also Garber, Alibali, & Goldin-Meadow, 1998).

The goal of the present study is to investigate the role of changes in problem encoding in the process of strategy change. To this end, we experimentally manipulated children's problem encoding, and examined the resulting changes in their strategy use.

As our experimental task, we selected mathematical equivalence problems (e.g., 3+4+5=___+5). We chose these problems for several reasons. First, previous work has shown that children often use incorrect strategies for solving such problems, and that children express their problem-solving strategies in both gesture and speech (Alibali & Goldin-Meadow, 1993; Perry et al., 1988). Second, previous work using a reconstruction task (similar to those used by Chase & Simon, 1973, and Siegler, 1976) has shown that children who solve equivalence problems incorrectly often encode the problems incorrectly (Alibali, 1998). Specifically, children who solve equivalence problems incorrectly often fail to properly encode the position of the equal sign. In the present study, we manipulated children's encoding of equivalence problems by drawing their attention to the position of the equal sign.

We hypothesized that changes in children's problem encoding would lead to increased variability in their strategy use, which would be manifested in increased production of gesture-speech mismatches. Further, we hypothesized that changes in children's problem encoding would lead to changes in their strategy use. Specifically, we predicted that changes in encoding would cause children to generate new strategies, and that these new strategies would often be expressed uniquely in gesture.

Method

Participants

Sixty-four fourth-grade students (ages 9 and 10) participated in the study. All attended parochial schools in the greater Pittsburgh area. Thirteen of the students solved one or more problems correctly on the experimental pretest. These students were eliminated from the analyses presented in this paper. Thus, the final sample consisted of 51 students (34 girls and 17 boys), all of whom solved six mathematical equivalence problems incorrectly on the pretest.[1]

Procedure

Students were asked to solve and explain six mathematical equivalence problems (e.g., 3+4+5=__+5) as part of a pretest. Students were then randomly assigned to one of three interventions, two of which were designed to modify their encoding of the equivalence problems. All three interventions used a set of four mathematical equivalence problems, comparable to those presented in the pretest. In the *Subtle* condition, participants solved and explained four problems in which the equal sign was printed in red. In the *Direct* condition, participants solved and explained the same four problems with the red equal signs, and in addition, the experimenter directed children's attention to the equal sign, saying "Make sure you notice where the equal sign is in the problem." In the *Control* condition, participants solved and explained the same four problems, with ordinary black equal signs. Note that children did not receive any feedback about whether their solutions were correct in any of the groups. Following the intervention, each child was asked to solve and explain another set of six equivalence problems as part of a posttest. Approximately four weeks later, each child was again asked to solve and explain a set of six problems as part of a follow-up test.

Coding

For each of the problems, we coded the strategy that the child expressed in speech and the strategy that the child expressed in gesture. Examples of problem-solving strategies expressed in speech and gesture are presented in Table 1.

Table 1

Examples of Strategies Expressed in Speech and in Gesture for the Problem 3+4+5=___+5

Strategy	*Speech*	*Gesture*
Add All	"I added 3 plus 4 plus 5 plus 5, and it made 17."	Right-hand point to 3, 4, left 5, right 5, solution.
Add to Equal Sign	"3 plus 4 is 7, plus 5 is 12."	Right-hand point to 3, 4, left 5, solution.
Grouping	"I just added 3 and 4 to make 7."	Left-hand point to 3, 4, solution.
Equalize	"3 and 4 and 5 is 12, and 7 plus 5 is also 12."	Left-hand point to 3, 4, left 5, hand down. Right-hand point to right 5, solution.

We then identified all of the strategies that each child expressed in speech and/or in gesture during the problem explanations on the pretest, posttest and follow-up test. Note that this method for assessing a child's strategy repertoire includes any strategy that the child ever mentioned during the problem explanations (in either modality). We then compared the sets of strategies that children expressed on each test. Any strategy expressed on the pretest but not on a later test (posttest or follow-up) was considered to be

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[1] The unequal distribution of boys and girls was unexpected, and was due to the fact that more girls than boys returned permission slips with parental consent to participate in the study.

Abandoned. Similarly, any strategy that was not expressed on the pretest but was expressed on a later test was considered to be *Generated.*

Finally, for each problem explanation, we coded the relationship between gesture and speech. If the child expressed the same strategy in both modalities, the explanation was coded as a gesture-speech *Match.* If the child expressed different strategies in speech and gesture, the explanation was coded as a *Mismatch.* For example, consider a child who, for the problem 4+5+7=___+7, said, "I added the 4 and the 5 and the 7", and pointed to the 4, the 5, and the left 7. In this example, the child expressed the Add to Equal Sign strategy in both speech and gesture, so the response would be coded as a Match. Next, consider a child who said, "I added the 4 and the 5 and the 7", and pointed to the 4, the 5, the left 7, and then the right 7. In this example, the child expressed the Add to Equal Sign strategy in speech, and the Add All strategy in gesture, so the response would be coded as a Mismatch (see Perry et al., 1988).

Results

We assessed the effects of changes in children's encoding on two types of dependent measures: (1) changes in the number of strategies that children considered for individual problems, and (2) changes in the repertoires of strategies that children expressed in speech and in gesture. We hypothesized that, even in the absence of feedback about correctness, changes in children's problem encoding would lead them to more frequently consider multiple strategies for individual problems. We further hypothesized that changes in encoding would lead children to generate new strategies. Based on previous work (Alibali & Goldin-Meadow, 1993), we expected that such new strategies would often be expressed uniquely in gesture.

Did changes in encoding lead to changes in the variability of strategy choices?

We first examined whether the encoding manipulation influenced the number of strategies that children considered in solving individual problems. We hypothesized that changes in encoding would lead children to more frequently consider alternative strategies. We have argued in previous work that when children consider multiple strategies in solving a problem, they produce gesture-speech mismatches (Alibali & Goldin-Meadow, 1993; Goldin-Meadow, Alibali, & Church, 1993). Thus, we predicted that the manipulation would lead children to produce more mismatches on the posttest and follow-up test problems.

We first considered changes from the pretest to the posttest. As seen in Figure 1 (white and dotted bars), children in the Control and Subtle groups produced *fewer* mismatches on the posttest than on the pretest, with the greatest decrease in the Control group. In contrast, children in the Direct group produced slightly *more* Mismatches on the posttest than they did on the pretest. Repeated measures ANOVA revealed a significant interaction between condition (Control, Subtle, Direct) and test (Pre, Post), $F(2,48)=4.49$, $p<.02$. Planned contrasts comparing pretest-to-posttest change scores across groups revealed that the Direct group differed significantly from the Control group ($F(1,48)=9.36$, $p<.005$), and the Subtle group differed marginally from the Control group ($F(1,48)=2.96$, $p<.10$). Thus, as predicted, the manipulation led children to consider multiple strategies for individual problems more frequently.

We next considered the effects of the encoding manipulation on changes from the pretest to the follow-up test. Recall that the follow-up session took place approximately four weeks after the initial session. Six of the children were not available to be tested at the follow-up session, so the overall N for all follow-up analyses is reduced to 45. As seen in Figure 1, in all three groups, children produced fewer mismatches at the follow-up than in the pretest. The drop-off from pretest to follow-up was greatest in the Control group (–1.33), smaller in the Subtle group (–0.80), and smallest in the Direct group (–0.67). However, the interaction between condition (Control, Subtle, Direct) and test (Pre, Follow-up) did not reach significance ($F(2,42)<1$).

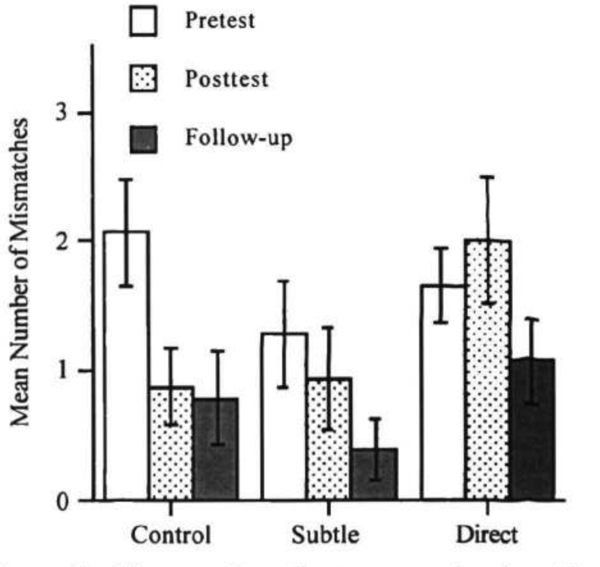

Figure 1. Mean number of gesture-speech mismatches produced by children in each group on the pretest, posttest, and follow-up test. Pretest and posttest data are based on the full sample ($N=51$); follow-up data is based on the reduced sample ($N=45$). The errors bars depict standard errors.

In sum, children the Direct group more frequently considered multiple strategies for individual problems immediately following the encoding manipulation. However, at the follow-up, four weeks later, this effect had dissipated.

Did changes in encoding lead to changes in strategy use?

We next examined whether the encoding manipulation led children to solve the posttest and follow-up problems differently from the pretest problems. To address this question, we compared the repertoire of strategies that each child expressed in speech and gesture on each test. We then assessed whether children generated new strategies in speech or in gesture, and whether children abandoned old strategies from speech or from gesture.

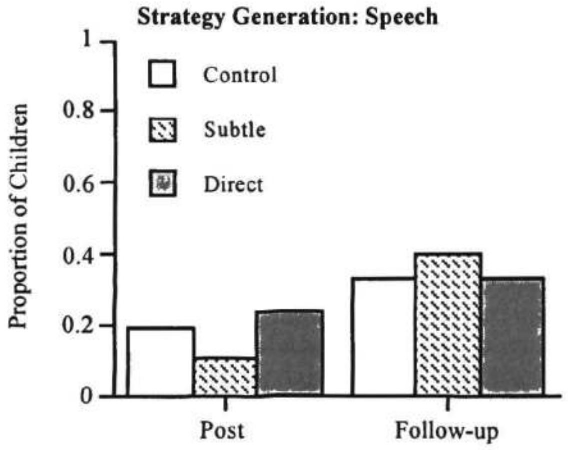

Figure 2. Proportion of children in each condition who generated new strategies in *speech* (with or without matching gesture) on the posttest and follow-up test.

We first consider strategy generation. We assessed the proportion of children who expressed any new strategies either in speech or uniquely in gesture on the posttest and the follow-up test. As shown in Figure 2, the proportion of children who expressed new strategies in *speech* did not differ systematically across conditions, for either the posttest or the follow-up test. However, as seen in Figure 3, the proportion of children who expressed new strategies *uniquely in gesture* was greatest in the Direct group, both at the posttest and at the follow-up test. More children in the Direct group than in the Control and Subtle groups generated gestured strategies from pretest to posttest ($X^2(1)=5.80$, $p<.02$), and from pretest to follow-up ($X^2(1)=2.95$, $p<.10$). Thus, the Direct encoding manipulation led children to generate new strategies that they expressed uniquely in gesture.

Although many children generated new problem-solving strategies over the course of the study, it should be noted that almost all of the strategies children generated were incorrect. In order for children to generate correct strategies for solving equivalence problems, instructional input or feedback about correctness may be required.

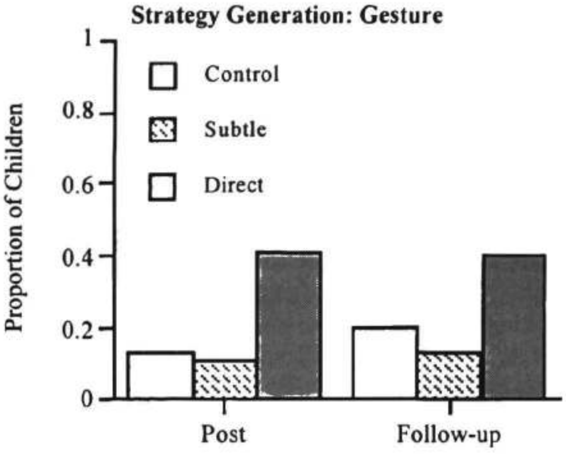

Figure 3. Proportion of children in each condition who generated new strategies *uniquely in gesture* on the posttest and follow-up test.

We next considered strategy abandoning. We assessed the proportion of children in each condition who abandoned one or more of the strategies expressed on the pretest, either in speech or uniquely in gesture. As shown in Figure 4, the proportion of children who abandoned strategies from *speech* was greatest in the Direct group. More children in the Direct group than in the Control and Subtle groups abandoned spoken strategies both from pretest to posttest ($X^2(1)=3.98$, $p<.05$), and from pretest to follow-up ($X^2(1)=4.60$, $p<.05$). Thus, the Direct encoding manipulation led children to abandon spoken strategies.

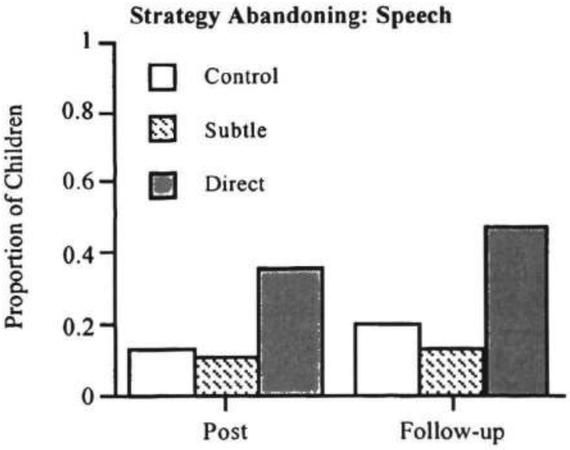

Figure 4. Proportion of children in each condition who abandoned strategies from *speech* from pretest to posttest, and from pretest to follow-up.

The manipulation may have helped children to zero in on pretest strategies were "worth keeping." Presumably, children maintained strategies that were consistent with their new encodings, and abandoned strategies that were inconsistent with their new encodings. However, it should be noted that all of the strategies that children maintained were incorrect. This is because our sample included only children who solved the pretest problems incorrectly.

Finally, we assessed the proportion of children in each group who abandoned strategies that they expressed *uniquely in gesture* on the pretest. Children who expressed no strategies uniquely in gesture on the pretest were excluded from this analysis. As seen in Figure 3, the proportion of children who abandoned such strategies was high overall. Furthermore, more children in the Control group than in the Subtle and Direct groups abandoned strategies from gesture. This pattern was non-significant at the posttest, but significant at the follow-up test ($X^2(1)=7.37$, $p<.01$). Thus, children whose encoding was manipulated were *less* likely to abandon gestured strategies.

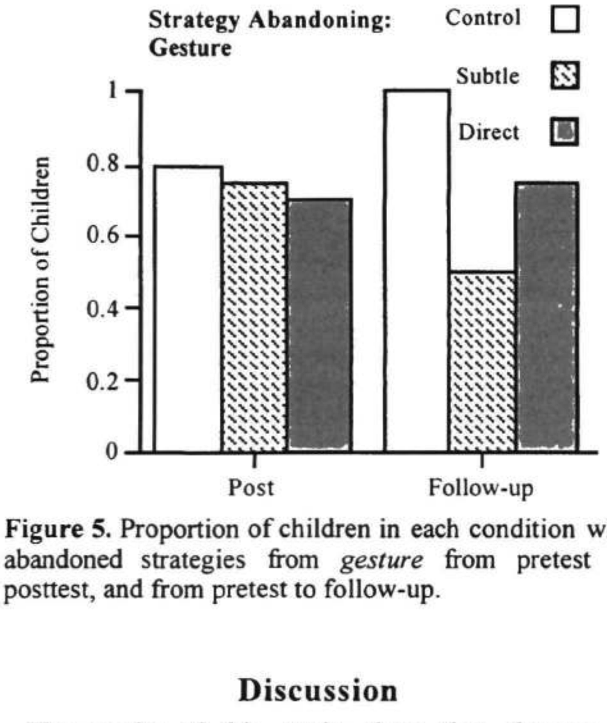

Figure 5. Proportion of children in each condition who abandoned strategies from *gesture* from pretest to posttest, and from pretest to follow-up.

Discussion

The results of this study show that changes in children's problem encoding influenced their strategy choices. Specifically, guiding children's attention to the equal sign led them (1) to more frequently consider multiple strategies, as reflected in their continued production of gesture-speech mismatches, (2) to generate new strategies that they expressed uniquely in gesture, (3) to abandon strategies that they initially expressed in speech, and (4) to maintain strategies that they initially expressed uniquely in gesture. These changes occurred even though children received no instruction and no feedback that their problem-solving strategies were incorrect. Overall, the data suggest that changes in children's problem encoding influence their strategy choices.

Because we did not directly measure children's problem encoding, we cannot be certain that the manipulation actually caused children to improve their encoding. We also cannot evaluate *how* children's encoding changed as a result of the manipulation. However, the results strongly suggest that children in the Direct group did indeed change their encoding of the problems. Further, anecdotal evidence suggests that children in the Direct group specifically changed their encoding of the equal sign, which was the focus of the manipulation. For example, on the posttest, one child described a new strategy in which he "switched" the positions of the equal sign and the plus from the right-hand side of the equation (i.e., changing the problem from 3+4+5=__+5 to 3+4+5+5=__), and then added all four addends. This strategy makes explicit mention of the position of the equal sign. In a follow-up study that we are currently conducting, we are directly measuring children's encoding both before and after an encoding manipulation, in order to establish whether the manipulation actually causes changes in encoding.

Why might changes in encoding lead to increased variability, and to new strategies that were expressed in gesture and not in speech? We suggest that, when gesture conveys information that is not expressed in speech, it reveals information that children have encoded about the problems, but have not used in their problem solutions (see Siegler, 1984). This encoded-but-not-used information is active in the children's reasoning about the problems, but does not play a role in the final solution procedures that they select. In contrast, information that is both encoded *and* used to solve a problem is expressed in speech (with or without matching gesture) rather than uniquely in gesture. The present results suggest that the Direct manipulation led to changes in children's encoding, but that children did not use their new encodings in the procedures that they ultimately selected to solve the problems.

The present results underscore that, to detect the first signs of new strategies, it is essential to use measures that are sensitive to subtle changes in children's reasoning about problems. In this study, a relatively coarse-grained measure (new strategies expressed in speech) did not reveal changes due to the encoding manipulation, while more subtle measures (e.g., new strategies expressed in gesture, the frequency of gesture-speech mismatches) did. By using both gesture and speech to assess children's knowledge, we were able to identify problems for which children considered multiple problem-solving strategies, and we were able to identify strategies that were not expressed in speech.

Our results demonstrate that assessing performance with such fine-grained measures can help us to understand the process of change in children's thinking.

We began this paper by raising the question of where new strategies come from. Although there are probably many correct answers to this question, in the present study we have focused on one particular source of new strategies: changes in children's problem encoding. Our data suggest that changes in encoding can indeed be a source of new problem-solving strategies—specifically, new strategies that are expressed in gesture. Our data further suggest that changes in encoding also contribute to the process of strategy abandoning. In future work, we will examine whether feedback about correctness will encourage children to construct and use new strategies based on their new encodings.

Of course, knowing that changes in encoding can be a source of new strategies does not solve the problem of the genesis of new forms—one must then ask where new encodings come from. Nevertheless, we submit that the present findings are important because they can guide and constrain theorizing about the mechanisms that underlie strategy generation. Indeed, the mechanisms by which new encodings are generated are likely to be quite different from the mechanisms by which new strategies are generated. We suggest that new encodings may arise as a result of implicit learning processes that apprehend structure in a stimulus environment (Reber, 1993). These new encodings can then be used as raw material for strategy construction. Thus, we suggest that, in order to fully understand the process of strategy change, it is essential to understand when and why problem solvers change their problem encoding.

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