Title
ELECTRICAL ENGINEERING REVIEW COURSE - LECTURE XVI

Authors
Martinelli, E.
Byrns, R.
Burleigh, R.

Publication Date
1952-07-21

UNIVERSITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

UCRL 2030

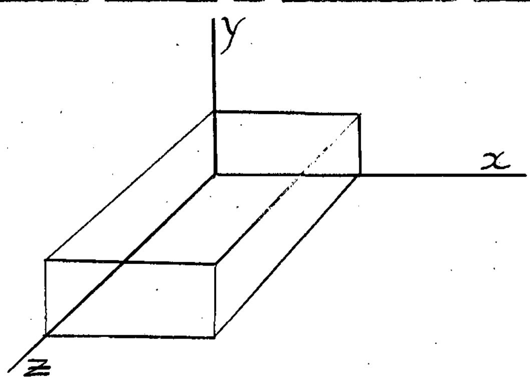
ELECTRICAL ENGINEERING REVIEW COURSE

Lecture XVI
July 21, 1952

E. Martinelli
(Notes by: R. Byrns, R. Burleigh)

- - - - - - -

Maxwell's Equations Applied to Rectangular Tube



Consider a sinusoidal wave transmitted in +Z direction and having only a y component $E = jEy$. This is known as the transverse electric mode. We look for a mode in which:

$$E = Ey(x_1 y)e^{j(wt - kz)}$$

$$\nabla^2 \vec{E} - \frac{1}{C^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

The symbol $\longrightarrow$ may be removed as this is essentially a scaler equation, then:

$$\nabla^2 E - \frac{1}{C^2} \frac{\partial^2 E}{\partial t^2} = 0$$

substituting:

$$\frac{\partial^2 E}{\partial X^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{C^2} \frac{\partial^2 E}{\partial t^2} = 0$$

or:

$$\frac{\partial^2 Ey}{\partial X^2} + \frac{\partial^2 Ey}{\partial y^2} - K^2 Ey + \frac{W^2}{C^2} Ey = 0$$

Then:

$$\frac{\partial^2 Ey}{\partial X^2} + \frac{\partial^2 Ey}{\partial y^2} - (K^2 - \frac{W^2}{C^2})Ey = 0$$

(This is the differential equation of a wave in a rectangular tube.)

Solving the above equation:

$$\text{let } Ey = X(x)\ Y(y)$$

$$X''Y + Y''X - (K^2 - \frac{w^2}{c^2})XY = 0$$

$$\frac{X''}{X} = - \frac{Y''}{Y} + K^2 - \frac{w^2}{c^2} = - \lambda^2$$

$$X = A \sin \lambda x + B \cos \lambda x$$

$$Y = C \sin By + D \cos By$$

where

$$B = \sqrt{- \lambda^2 - K^2 + \frac{w^2}{c^2}}$$

(K is an arbitrary propagation constant)

Putting in boundary condition:

tangential field at edges = 0

$$\lambda a = n\pi \text{ or } \lambda = \frac{n\pi}{a}$$

$$Ey = 0 \text{ at } X = a$$

$$\text{div } E = 0$$

Then, as we have only a y component:

$$\frac{\partial Ey}{\partial y} = 0, \quad Ey = \left[f(x) \right]$$

since there is only E_y, $\beta = 0$

$$\beta = \sqrt{-\lambda^2 - K^2 + \frac{w^2}{c^2}} = 0$$

then:

$$- (\frac{n}{a}\pi)^2 - K^2 + (\frac{2\pi}{\lambda})^2 = 0$$

$$K = \frac{2\pi}{\lambda y}$$

$$\lambda y = \frac{\lambda}{\sqrt{1 - (\frac{n\lambda}{2a})^2}}$$

Therefore, it may be seen that a number of wavelengths maybe propagated down the guide, i.e., for n = 1, 2, etc. (n represents type of mode.)

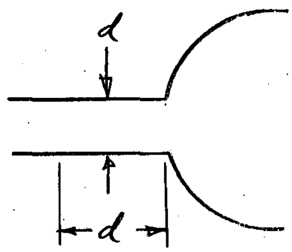
For cut-off:

$$\frac{n}{2a} = 1$$

$$K^2 = (\frac{2\pi}{\lambda})^2 - (\frac{n\pi}{a})^2$$

$$K^2 = (\frac{\lambda}{2\pi})^2 = 1 - (\frac{n\lambda}{2a})^2$$

Wave is attenuated if wave length is greater than cut-off (if λ is too large). This is utilized in attenuators:



In one diameter the field will be attenuated by $e^{-\pi}$ (where length of pipe = diameter.)

For cut-off: $\lambda = \frac{2a}{n}$

Therefore, lowest frequency that can be propagated:

$$\lambda = 2a$$

<u>Magnetic Field</u>

Knowing the electric field we can apply Maxwell's Equations and investigate the magnetic field.

$$\text{curl } \vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt}$$

$$\vec{E} = \hat{j}E_y e^{j(wt - kz)}$$

$$\vec{B} = \vec{B}_{x,y} e^{j(wt - kz)}$$

Take the curl of the vector with only one component and get two terms.

$$\text{curl } \vec{E} = \hat{i} - \frac{dE_y}{dz} + \hat{k}\frac{dE_y}{dx} \quad (E_y = E_{max} \text{ Sin } \frac{\pi x}{a})$$

$$= \hat{i}(jkE_{max} \sin\frac{\pi x}{a}) + \hat{k}\frac{\pi}{a} E_{max} \cos\frac{\pi x}{a}$$

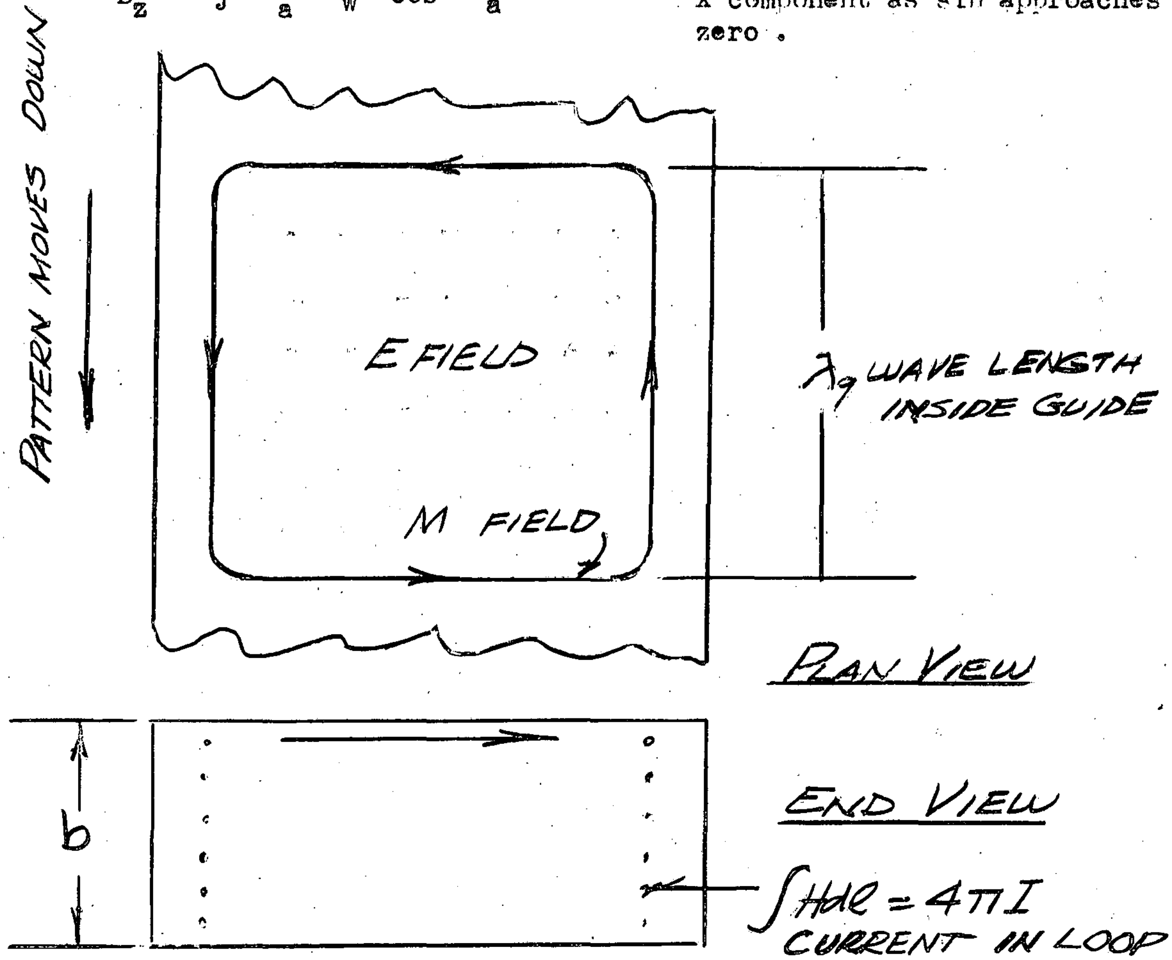
$$= -j\frac{w}{c}\vec{B}_x - j\frac{w}{c}\vec{B}_z$$ (Only two components, z and x, of magnetic field).

Solving:

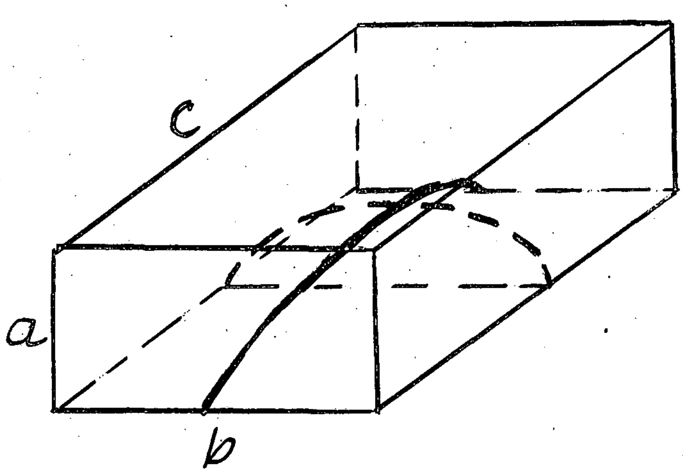
$$B_x = \frac{kc}{w} E_{max} \sin \frac{\pi x}{a}$$

$$B_z = -j \frac{\pi}{a} \frac{c}{w} \cos \frac{\pi x}{a}$$

At X = 0, only z component is present, at middle only X component as sin approaches zero .



$I = \frac{10bc}{4aw} E_{max}$ gives current flowing in sides of guide.



Make $C = \frac{\lambda g}{2}$ to get 0 at both ends

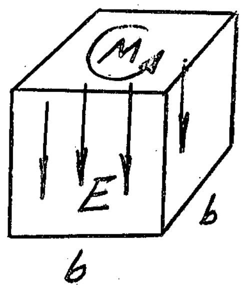
Solve for C:

$$2C = \frac{\lambda}{1 - (\frac{\lambda}{2b})^2}$$

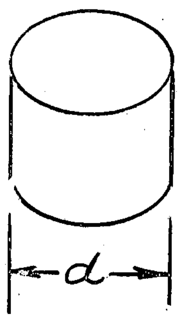
$$\lambda = C \frac{1}{\sqrt{1 + \frac{c^2}{b^2}}}$$

We thus can get an infinite number of cavities of many shapes all resonant at the same frequency.

In thesquare resonator, the reson-ant frequency is lowered slightly by the corner effect.



$\lambda = \frac{b}{2}$ (similar to Linac, Bldg. 10)



The round cavity, similar to square, frequency is given by:

$$\lambda = \frac{d}{1.31}$$