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Publication Date

1952-07-21

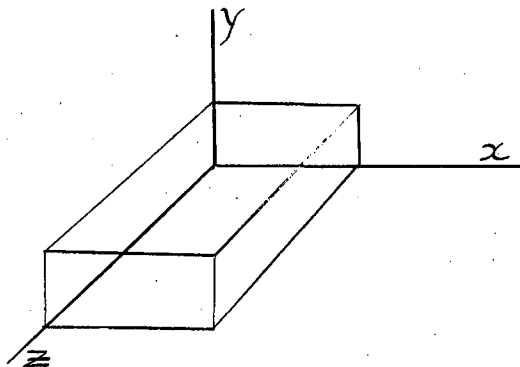
ELECTRICAL ENGINEERING REVIEW COURSE

Lecture XVI
 July 21, 1952

E. Martinelli
 (Notes by: R. Byrns, R. Burleigh)

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Maxwell's Equations Applied to Rectangular Tube



Consider a sinusoidal wave transmitted in +Z direction and having only a y component $E = jE_y$. This is known as the transverse electric mode. We look for a mode in which:

$$E = E_y(x,y)e^{j(\omega t - kz)}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

The symbol $\vec{}$ may be removed as this is essentially a scalar equation, then:

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$

substituting:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$

or:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - K^2 E_y + \frac{W^2}{c^2} E_y = 0$$

Then:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} - (K^2 - \frac{W^2}{c^2}) E_y = 0$$

(This is the differential equation of a wave in a rectangular tube.)

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Solving the above equation:

$$\text{let } E_y = X(x) Y(y)$$

$$X''Y + Y''X - (K^2 - \frac{w^2}{c^2})XY = 0$$

$$\frac{X''}{X} = -\frac{Y''}{Y} + K^2 - \frac{w^2}{c^2} = -\lambda^2$$

$$X = A \sin \lambda x + B \cos \lambda x$$

$$Y = C \sin By + D \cos By$$

where

$$B = \sqrt{-\lambda^2 - K^2 + \frac{w^2}{c^2}}$$

(K is an arbitrary propagation constant)

Putting in boundary condition:

tangential field at edges = 0

$$\lambda a = n\pi \text{ or } \lambda = \frac{n\pi}{a}$$

$$E_y = 0 \text{ at } X = a$$

$$\text{div } E = 0$$

Then, as we have only a y component:

$$\frac{\partial E_y}{\partial y} = 0, \quad E_y = \boxed{f(x)}$$

since there is only E_y , $B = 0$

$$B = \sqrt{-\lambda^2 - K^2 + \frac{w^2}{c^2}} = 0$$

then:

$$-\left(\frac{n\pi}{a}\right)^2 - K^2 + \left(\frac{2\pi}{\lambda}\right)^2 = 0$$

$$K = \frac{2\pi}{\lambda y}$$

$$\lambda y = \frac{\lambda}{\sqrt{1 - \left(\frac{n\lambda}{2a}\right)^2}}$$

Therefore, it may be seen that a number of wavelengths maybe propagated down the guide, i.e., for $n = 1, 2$, etc. (n represents type of mode.)

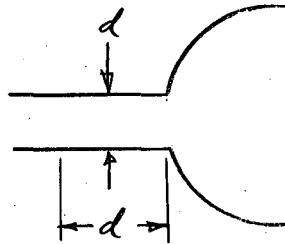
For cut-off:

$$\frac{n}{2a} = 1$$

$$K^2 = \left(\frac{2\pi}{\lambda}\right)^2 - \left(\frac{n\pi}{a}\right)^2$$

$$K^2 = \left(\frac{\lambda}{2\pi}\right)^2 = 1 - \left(\frac{n\lambda}{2a}\right)^2$$

Wave is attenuated if wave length is greater than cut-off (if λ is too large). This is utilized in attenuators:



In one diameter the field will be attenuated by $e^{-\pi}$ (where length of pipe = diameter.)

For cut-off:

$$\lambda = \frac{2a}{n}$$

Therefore, lowest frequency that can be propagated:

$$\lambda = 2a$$

Magnetic Field

Knowing the electric field we can apply Maxwell's Equations and investigate the magnetic field.

$$\text{curl } \vec{E} = -\frac{1}{c} \frac{d\vec{B}}{dt}$$

$$\vec{E} = \hat{j} E_y e^{j(\omega t - kz)}$$

$$\vec{B} = \vec{B}_{x,y} e^{j(\omega t - kz)}$$

Take the curl of the vector with only one component and get two terms.

$$\text{curl } \vec{E} = \hat{i} \frac{dE_y}{dz} + \hat{k} \frac{dE_y}{dx} \quad (E_y = E_{\max} \sin \frac{\pi x}{a})$$

$$= \hat{i} (jk E_{\max} \sin \frac{\pi x}{a}) + \hat{k} \frac{\pi}{a} E_{\max} \cos \frac{\pi x}{a}$$

$$= -j \frac{\omega}{c} \vec{B}_x - j \frac{\omega}{c} \vec{B}_z \quad (\text{Only two components, } z \text{ and } x, \text{ of magnetic field}).$$

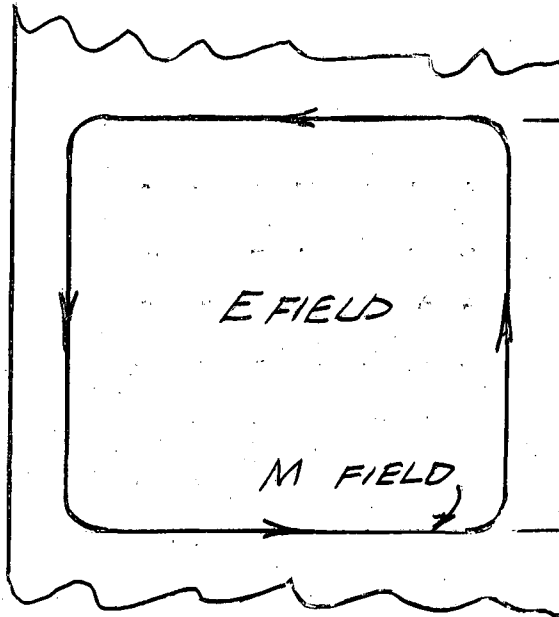
Solving:

$$B_x = \frac{kc}{w} E_{\max} \sin \frac{\pi x}{a}$$

$$B_z = -j \frac{\pi}{a} \frac{c}{w} \cos \frac{\pi x}{a}$$

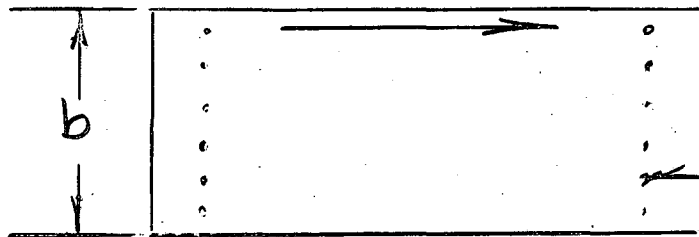
At $X = 0$, only z component is present, at middle only X component as \sin approaches zero.

PATTERN MOVES DOWN



λ_g WAVE LENGTH INSIDE GUIDE

PLAN VIEW

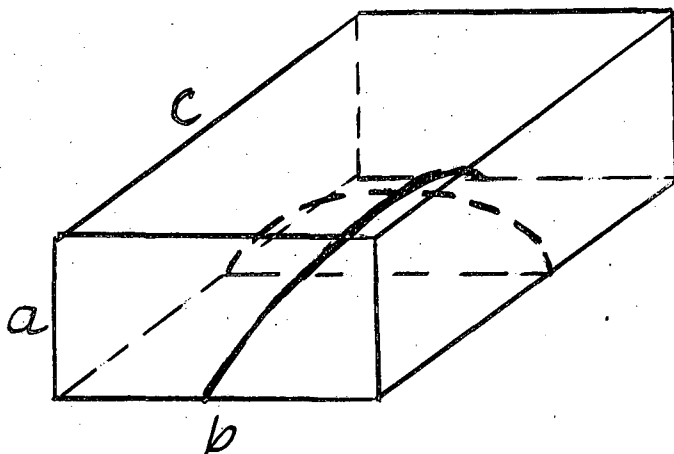


END VIEW

$$\oint H \cdot dl = 4\pi I$$

CURRENT IN LOOP

$$I = \frac{10bc}{4aw} E_{\max} \text{ gives current flowing in sides of guide.}$$



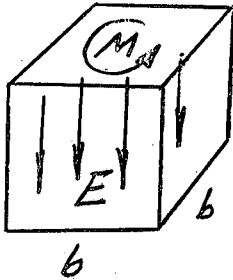
Make $C = \frac{\lambda_g}{2}$ to get 0 at both ends

Solve for C :

$$2C = \frac{\lambda}{1 - \left(\frac{\lambda}{2b}\right)^2}$$

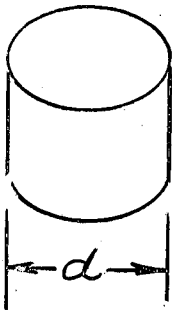
$$\lambda = C \frac{1}{\sqrt{1 + \frac{c^2}{b^2}}}$$

We thus can get an infinite number of cavities of many shapes all resonant at the same frequency.



$$\lambda = \frac{b}{2} \text{ (similar to Linac, Bldg. 10)}$$

In the square resonator, the resonant frequency is lowered slightly by the corner effect.



The round cavity, similar to square, frequency is given by:

$$\lambda = \frac{d}{1.31}$$