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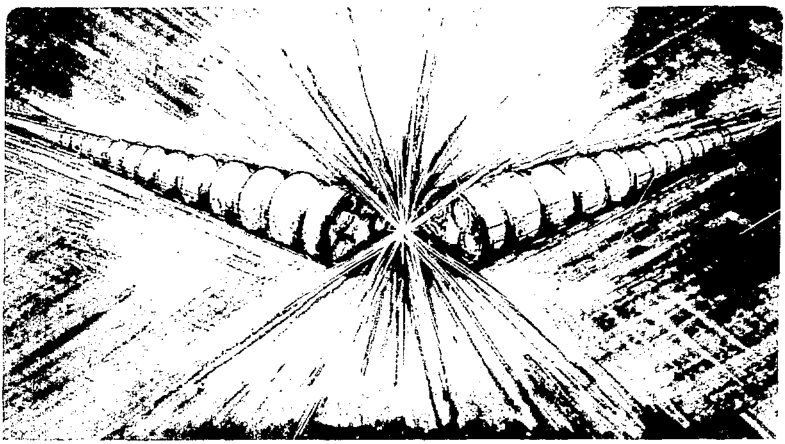

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HAMILTONIAN KINETIC THEORY OF PLASMA PONDEROMOTIVE PROCESSES

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ABSTRACT

The nonlinear nonresonant interaction of plasma waves and particles is formulated in a Hamiltonian kinetic theory which treats the wave-action and particle distributions on an equal footing, thereby displaying reciprocity relations. In the quasistatic limit, a nonlinear wave-kinetic equation is obtained. The generality of the formalism allows for applications to arbitrary geometry, with the nonlinear effects expressed in terms of the linear susceptibility.

Plasma dynamics utilizes a number of models which are self-consistent and obey conservation laws. Heretofore these properties have been deduced from the evolution equations of each model, rather than from the underlying structure of the model. The recent realization that a Hamiltonian field theory could be based on a Poisson structure, without the need for conjugate pairs of fields, has led to the Hamiltonian formulation of several standard models, namely MHD[1], Vlasov-Coulomb[2], Vlasov-Maxwell[3], and two-fluid[4].

One great advantage of a Hamiltonian formulation is that it points the way to new self-consistent models. In the present work, we extend the Poisson structure of the Vlasov-Coulomb model to the kinetic ponderomotive model[5], i.e., the nonresonant nonlinear interaction of high-frequency plasma waves with the low-frequency dynamics of a Vlasov plasma. The resulting evolution equations extend previous studies to arbitrary geometry and waves, and are inherently self-consistent.

We recall[2] that a Vlasov distribution f(z), i.e., a six-dimensional phase-space density of particles, allows a Poisson structure [,] for functionals A(f):

$$[A_1(f), A_2(f)] = \int d^6z\, f(z)\, \{\delta A_1/\delta f(z), \delta A_2/\delta f(z)\}, \qquad (1)$$

where { , } is the Poisson bracket for functions on z-space. With the evolution equation for any functional:

$$\dot{A}(f) = [A(f), H(f)], \qquad (2)$$

for a given Hamiltonian functional H(f), we find that f itself evolves as

$$\dot{f}(z) = -\{f(z), h(z;f)\}, \qquad (3)$$

where

$$h(z;f) = \delta H(f)/\delta f(z) \qquad (4)$$

is the self-consistent particle Hamiltonian. We recognize (3) as the standard Vlasov equation; note that the over-dot means evolution at fixed z. (We suppress species labels for simplicity.)

It is well-known that waves evolve as a Hamiltonian system, in the eikonal (or ray) approximation. Here the six-dimensional phase-space is $y = (\underline{x},\underline{k})$, and the Hamiltonian is the dispersion relation for frequency $\bar{\omega}(\underline{x},\underline{k})$. By analogy to the formulas above, the standard wave-kinetic equation

$$\dot{J}(y) = - \{J(y),\omega(y)\}, \tag{5}$$

for the action distribution $J(\underline{x},\underline{k})$, can be obtained from a Hamiltonian functional $H(J)$, with $\omega(y) = \delta H(J)/\delta J(y)$. In (5) $\{\ ,\ \}$ is the standard Poisson bracket for functions on y-space; for simplicity, wave-branch labels are suppressed.

To couple particles and waves, we consider functionals $A(f,J)$ of distributions $f(z)$ and $J(y)$. Their Poisson bracket is taken as

$$[A_1(f,J),\ A_2(f,J)] = \int d^6z\ f(z)\ \{\delta A_1/\delta f(z),\ \delta A_2/\delta f(z)\}_z + \int d^6y\ J(y)\ \{\delta A_1/\delta J(y),\ \delta A_2/\delta J(y)\}_y\ ; \tag{6}$$

note that there are now three kinds of Poisson bracket: $[\ ,\ ]$ $\{\ ,\ \}_z$ $\{\ ,\ \}_y$. With the evolution equation

$$\dot{A}(f,J) = [A(f,J),\ H(f,J)], \tag{7}$$

we obtain the coupled equations:

$$\dot{f}(z) = - \{f(z),\ h(z;f,J)\}_z, \tag{8a}$$

$$\dot{J}(y) = - \{J(y),\ \omega(y;f,J)\}_y, \tag{8b}$$

with

$$h(z;f,J) = \delta H(f,J)/\delta f(z), \tag{9a}$$

$$\omega(y;f,J) = \delta H(f,J)/\delta J(y). \tag{9b}$$

The choice of Hamiltonian $H(f,J)$ (and of Poisson structure) determines the physical processes included in the model. We wish to study the dynamics of particles acted on by the ponderomotive forces of the waves, and the dynamics of the waves resulting from changes in the particle distribution. From previous work in this area[6], we identify z as oscillation-center variables, and f as the distribution of oscillation centers.

Thus, in (9a), h represents the oscillation-center Hamiltonian in the presence of the wave field J, while, in (9b), ω is the local wave dispersion relation for oscillation-center distribution f. A remarkable reciprocity relation follows on cross-differentiation of (9):

$$\delta h(z)/\delta J(y) = \delta\omega(y)/\delta f(z). \tag{10}$$

The left side is the ponderomotive contribution to the Hamiltonian, while the right side is essentially the linear susceptibility, as we show below. This relation was discovered several years ago by one of us and Cary[7], by comparing explicit expressions for the ponderomotive Hamiltonian and the susceptibility. The present derivation uncovers its significance: each expression is the same functional second derivative of $H(f,J)$.

To be more explicit, we choose

$$H(f,J) = \int d^6z\, h_0(z) f(z) + \int d^6y\, \omega(y;f) J(y), \tag{11}$$

where $h_0(z)$ is the unperturbed particle Hamiltonian, and $\omega(y;f)$ is the dispersion relation, determined as a root of the dispersion function:

$$D(y,\omega;f) = 0. \tag{12}$$

From (12), we have

$$\delta\omega(y)/\delta f(z) = - [\delta D/\delta f(z)]\, [\partial D/\partial\omega]^{-1}, \tag{13}$$

with ω replaced by $\omega(y)$ on the right after differentiation. The dispersion function D of (12) is obtained from the Hermitian part of the dispersion tensor D and the local polarization $\hat{e}$:

$$D = \hat{e}^{*}\cdot \underset{\sim}{D} \cdot \hat{e}, \tag{14}$$

while

$$\underset{\sim}{D} = \underset{\sim}{I} - c^2(k^2\underset{\sim}{I} - \underline{k}\underline{k})/\omega^{-2} + \underset{\sim}{\chi}(y;f), \tag{15}$$

where $\underset{\sim}{\chi}$ is the linear susceptibility. Thus (13) becomes

$$\delta\omega(y)/\delta f(z) = - [\partial D/\partial\omega]^{-1}\, \hat{e}^{*}\cdot \delta\underset{\sim}{\chi}(y)/\delta f(z)\cdot\hat{e}. \tag{16}$$

Returning to (9a) and (10), and using (16), we have

$$h = h_0 - \int d^6y\, J(y)\, [\partial D/\partial\omega]^{-1}\, \hat{e}^{*}\cdot\delta\underset{\sim}{\chi}/\delta f(z)\cdot\hat{e}. \tag{17}$$

Finally we express J in terms of the electric field[8]:

$$J(y) = (\partial D/\partial\omega)\, |\hat{e}^{*}\cdot\underline{\tilde{E}}(y)|^2/4\pi, \tag{18}$$

to obtain the previously known relation:

$$h(z) = h_0(z) - \int d^6y\, \underline{\tilde{E}}^{*}(y)\cdot\delta\underset{\sim}{\chi}(y)/\delta f(z)\cdot\underline{\tilde{E}}(y)/4\pi, \tag{19}$$

somewhat generalized.

Before proceeding, we should stress that the evolution equations obtained here are not yet complete, as they ignore the effects of the low-frequency fields. A fuller treatment, incorporating them, is in preparation, and will be published elsewhere.

Except for this omission, the present equations may be considered as the generalization of ponderomotive kinetic theory[5] to arbitrary geometry, e.g., to nonuniform plasma and magnetic field. As our next step, we consider the quasistatic limit for Eq. (8a), in order to eliminate f and to obtain a nonlinear equation for J, thereby generalizing the "nonlinear Schrödinger equation."

In (8a), we set $\dot{f} = 0$, and satisfy $\{f,h\} = 0$ by choosing

$$f(z) = a \exp(-\beta) h(z;f,J). \tag{20}$$

Thus f is a functional of J, and we have (ignoring the weak dependence of h on f):

$$\delta f(z)/\delta J(y) = -\beta f(z)\ \delta h(z)/\delta J(y) = -\beta f(z)\ \delta\omega(y)/\delta f(z), \tag{21}$$

by Eq. (10). Thus $\omega(y;f)$ has an implicit dependence on J:

$$\delta\omega(y)/\delta J(y') = \int d^6 z\,[\delta\omega(y)/\delta f(z)]\ [\delta f(z)/\delta J(y')] = -\beta\int d^6 z\ f(z)\,[\delta\omega(y)/\delta f(z)]\ [\delta\omega(y')/\delta f(z)]. \tag{22}$$

Noting the symmetry in (22), we obtain another reciprocity relation:

$$\delta\omega(y)/\delta J(y') = \delta\omega(y')/\delta J(y), \tag{23}$$

governing nonlinear frequency shifts. This relation enables us to formulate the Hamiltonian functional for wave-action:

$$H(J) = \int d^6 y\ \omega_0(y)J(y) + \frac{1}{2}\iint d^6 y\ d^6 y'\ J(y)J(y')\omega_2(y,y'), \tag{24}$$

with the coupling coefficient $\omega_2(y,y')$ given by the right side of (22). This Hamiltonian yields the desired nonlinear frequency, by (9b):

$$\omega(y;J) = \omega_0(y) + \int d^6 y'\ J(y')\omega_2(y,y'). \tag{25}$$

To illustrate these formulas, we may consider the simplest case, Langmuir waves in an unmagnetized plasma. Here (16) reads

$$\delta\omega(\underline{k},\underline{x})/\delta f(\underline{r},\underline{p}) = -(\omega/2)(4\pi e^2/m)\delta(\underline{x}-\underline{r})(\omega - \underline{k}\cdot\underline{v})^{-2}, \tag{26}$$

and the wave-wave coupling is

$$\omega_2(\underline{k},\underline{x};\underline{k},\underline{x}') = -\delta(\underline{x}-\underline{x}')\beta(\omega\omega'/4)(4\pi e^2/m)^2\ \cdot \int d^3 p\ f(\underline{x},\underline{p})[(\omega - \underline{k}\cdot\underline{v})(\omega' - \underline{k}'\cdot\underline{v})]^{-2}, \tag{27}$$

with $\omega = \omega_0(\underline{k},\underline{x})$, $\omega' = \omega_0(\underline{k}',\underline{x}')$.

Applications of this formulation to more interesting situations will be the subject of future publications.

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