Title

BELL'S THEOREM WITHOUT HIDDEN VARIABLES

Permalink

https://escholarship.org/uc/item/6qp4f35x

Author

Eberhard, P.H.

Publication Date

1976-05-01

BELL'S THEOREM WITHOUT HIDDEN VARIABLES

P. H. Eberhard

May 1976

DISCLAIMER

BELL'S THEOREM WITHOUT HIDDEN VARIABLES[*]

P. H. Eberhard

Physics Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720
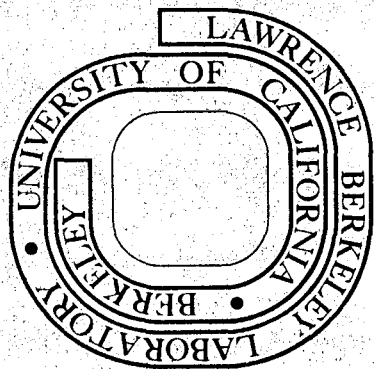
MAY 1976

BELL'S THEOREM WITHOUT HIDDEN VARIABLES

P. H. Eberhard

Introduction As stated by Prof. d'Espagnat at this Conference, hidden variables

theories are not the only theories in trouble with locality. Following ideas

expressed in Refs. 1 and 2,   I will demonstrate the CHSH inequality[3] from

locality alone without using either determinism or the concept of hidden

variables. Then, I will comment about the violation of this inequality by

quantum theory.

Let us consider two measuring apparatus located in two different places

A and B. There is a knob a on apparatus A and a knob b on apparatus B.

Since A and B are separated in space, it is natural to think that what will

happen in A is independent of the setting of the knob b and vice versa.

The principles of relativity seem to impose this point of view if the time at

which the knobs are set and the time of the measurements are so close that,

in the time laps, no light signal can travel from A to B or vice versa. Then,

no signal can inform a measurement apparatus of what the knob setting of the

other is. However, there are cases where the predictions of quantum theory

make that independence assumption impossible. If quantum theory is true, there

are cases where the results of the measurements in A will depend on the setting of

knob b and/or the results of the measurements in B will depend on the setting of

knob a.

Definitions   A simple experiment consists of the recording of N events and each

one of these events involves a measurement in both apparatus A and B. There

are only two possibilities for the outcome of the measurement in each apparatus

and we label these measurement results as +1 and -1. The j'th event corre-

sponds to a response α_j in A and β_j in B. Each term α_j and β_j is either +1

or -1. We define the correlation by a statistical mean.

$$C = \frac{1}{N} \sum_j \alpha_j \beta_j = <\alpha_j \beta_j> \qquad (1)$$

where the symbol $<\ >$ around a quantity designates the statistical mean of

that quantity over j. C is equal to the fraction of events where α_j and β_j

have the same signs minus the fraction where they have opposite signs.

There are two positions $a^{(1)}$ and $a^{(2)}$ of the knob a and two possible

positions $b^{(1)}$ and $b^{(2)}$ of the knob b. C will depend on a and b. We define

$$
\left.
\begin{aligned}
C^{(1,1)} &= <\alpha_j \beta_j> \text{ if } a = a^{(1)}, \ b = b^{(1)} \\
C^{(2,1)} &= <\alpha_j \beta_j> \text{ if } a = a^{(2)}, \ b = b^{(1)} \\
C^{(1,2)} &= <\alpha_j \beta_j> \text{ if } a = a^{(1)}, \ b = b^{(2)} \\
C^{(2,2)} &= <\alpha_j \beta_j> \text{ if } a = a^{(2)}, \ b = b^{(2)}
\end{aligned}
\right\} \qquad (2)
$$

<u>The Locality Assumption</u>   We first have to suppose that it makes sense to consider

the different results of a future experiment for different settings of the knobs

a and b, although only one setting, at most, will actually be chosen for the

actual experiment.[4]   Then, for a given position of b, we can write that α_j will

be equal to $\alpha_j^{(1)}$ if $a = a^{(1)}$ and α_j will be equal to $\alpha_j^{(2)}$ if $a = a^{(2)}$, for the

same event j but different settings of knob a. Similarly, for a given value of

a, we define the values $\beta_j^{(1)}$ and $\beta_j^{(2)}$ of β_j if $b = b^{(1)}$ or $b^{(2)}$. Let's suppose,

in addition, that the measurement α_j in A will not depend on b, and the measure-

ment β_j in B will not depend on a. Then, for each event j, we can write

$$
\left.
\begin{aligned}
\left.
\begin{aligned}
\alpha_j &= \alpha_j^{(1)} \quad \text{if } a = a^{(1)} \\
\alpha_j &= \alpha_j^{(2)} \quad \text{if } a = a^{(2)}
\end{aligned}
\right\} \text{ whatever b is} \\
\left.
\begin{aligned}
\beta_j &= \beta_j^{(1)} \quad \text{if } b = b^{(1)} \\
\beta_j &= \beta_j^{(2)} \quad \text{if } b = b^{(2)}
\end{aligned}
\right\} \text{ whatever a is}
\end{aligned}
\right\} \qquad (3)
$$

We refer to this assumption as the locality assumption.

THE CHSH Inequality   Now we can define the following mathematical expression

for event #j

$$\gamma_j = \alpha_j^{(1)} \beta_j^{(1)} + \alpha_j^{(2)} \beta_j^{(1)} + \alpha_j^{(1)} \beta_j^{(2)} - \alpha_j^{(2)} \beta_j^{(2)} \quad (4)$$

Each product $\alpha_j \beta_j$ is equal to ± 1, therefore γ_j is an even integer. Furthermore, γ_j cannot be equal to 4 because $\alpha_j^{(2)} \beta_j^{(2)}$ is equal to the product of the first three terms of the right hand side of equation (4), and it is equal to +1 if the first three terms are positive. Therefore,

$$\gamma_j \leq 2 \quad (5)$$

It follows that

$$\frac{1}{N} \sum_j \gamma_j = <\gamma_j> \; = \; <\alpha_j^{(1)} \beta_j^{(1)}> + <\alpha_j^{(2)} \beta_j^{(1)}> + <\alpha_j^{(1)}\beta_j^{(2)}>$$
$$- <\alpha_j^{(2)} \beta_j^{(2)}> \leq 2 \quad (6)$$

From equations (2), (3) and (6), we derive

$$c^{(1,1)} + c^{(2,1)} + c^{(1,2)} - c^{(2,2)} \leq 2 \quad (7)$$

Equation (7) is the CHSH inequality[3] that is a generalization of Bell's inequality.[5]

Quantum Theoretical Predictions   Let's consider an idealized experiment

where each event consists of the detection in A and B of two photons emitted

in the direction of A and B by an atom located between A and B. The detection

is made with the help of a polarization analyzer based on an idealized birefringent

prism so that the polarization of the photon is measured with respect to the

axis of the analyzer. We count $\alpha_j = +1$ when the photon is measured with one

polarization in A and $\alpha_j = -1$ with the other. We count $\beta_j = \pm 1$ similarly,

depending on the polarization of the photon measured in B.

Let a and b be the angles of the analyzers in A and B. The ef-
ficiencies are all 100%. There is a 100% correlation between the planes of
polarization of the two photons at emission.

Quantum theory makes a prediction for the quantity C of Eq. (1):
if the number N of events is large enough

$$C \cong \cos(2a - 2b) \tag{8}$$

in almost all cases. (The sign $\cong$ means approximately equal.)
Then, for

$$\left.\begin{array}{l} a^{(1)} = 45° \\ a^{(2)} = 0° \\ b^{(1)} = 22.5° \\ b^{(2)} = 67.5° \end{array}\right\} \tag{9}$$

quantum theory predicts

$$C^{(1,1)} + C^{(2,1)} + C^{(1,2)} - C^{(2,2)} \cong 2\sqrt{2} > 2 \tag{10}$$

in almost all cases. This prediction is in contradiction with (7). Therefore,
the predictions of quantum theory are incompatible with our locality assumptions.

Conclusions   If quantum theory is correct, it is not allowed to think about
the options of the future as resulting in a sequence α_j depending on the choice
made for knob a only and a sequence β_j depending on the choice of knob b alone.
The α_j's will be different for different settings of b and/or the β_j's will be
different for different settings of a.

This demonstration has been performed without invoking either the
concept of hidden variables, or the concept of determinism. We used only the
results α_j and β_j's of the measurements, the hypothetical values $\alpha_j^{(1)}$, $\alpha_j^{(2)}$,
$\beta_j^{(1)}$ and $\beta_j^{(2)}$ that α_j and β_j would take for different knob settings[4] and the

locality concepts as defined by Eqs. (3). The theorem affects more theories

then just the hidden variable theories.[6]

Alternate Formulation   There is an alternate way to formulate this theorem with-

out using the concept of hypothetical results of experiments that will not be

performed, but involving only the results of actual experiments. Let's consider

the following thought experiment. The two photon experiment with N events is

repeated a lot of times in all the four combinations of a and b defined by Eq. (9).

What is the probability that one can find a set of four experiment, i.e., one for

each of the four knob combinations, such that Eqs. (3) are valid?

We have to select experiment for different settings of knob b such that

the lists of α_j's are identical from j = 1 to j = N. Then we have to make a

similar selection for β_j and knob a. N is finite. The probability to find a

set of experiments associated to such sequences $\alpha_j^{(1)}$, $\alpha_j^{(2)}$, $\beta_j^{(1)}$ and $\beta_j^{(2)}$

is very small but, because of all the possible statistical fluctuations, it is

not zero. However, if in addition we require that each correlation defined by

Eq. (2) differs from the expectation value predicted by quantum theory by less

than the quantity $\frac{\sqrt{2}-1}{2}$, then the probability to find such a set of four experiments

is an absolute zero. Indeed, any set of sequences $\alpha_j^{(1)}$, $\alpha_j^{(2)}$, $\beta_j^{(1)}$ and $\beta_j^{(2)}$

that one introduces in Eqs. (3) will generate correlations C that satisfy in-

equality (7), and inequality (7) cannot be satisfied if all the correlations are

different from cos(2a - 2b) by less than $\frac{\sqrt{2}-1}{2}$.

If we perform a lot of experiments with the different knob settings of

Eqs. (9) and look for experiments that have identical α_j's for different settings

of knob b, and the same β_j's for different settings of knob a, the only set of

experiments one may find will contain experiments whose results are very different

from the prediction of quantum theory. Actually, the differences between the

statistical mean C of Eq. (1) and the quantum theoretical predictions (Eq. (8)),

in absolute value, will add up to at least $2(\sqrt{2}-1)$.

LBL-4888

REFERENCES

1. See for instance H. P. Stapp, Phys. Rev. D $\underline{3}$, 1303 (1971) and H. P. Stapp,
    Lectures on Bell's Theorem, Trieste (1975).

2. J.S. Bell, Res. Th. 2053 - CERN The Theory of Local Beables, communication
    at the 6th Gift conference, Jaca, 2-7 June 1975; J. F. Clauser and M. A.
    Horne, Phys. Rev. D $\underline{10}$, 526 (1974); N. Herbert Am. Jour. Phys. $\underline{43}$, 315 (1975).

3. CHSH stands for J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt,
    Phys. Rev. Lett $\underline{23}$, 880 (1969).

4. That consideration is related to Stapp's concept of "contrafactual defin-
    iteness" (See Ref. 1). It would not make sense to talk about non-actual
    experiments if, for instance, we believed that there is no free will, and
    the choice by an experimenter is determined anyway. Otherwise, I believe
    that considering different possible results of a future experiemnt, depending
    on one's decision, is a rather natural way of thinking.

5. J. S. Bell, Physics (N.Y.), $\underline{1}$, 195 (1964).

6. Note that the conventional formalism of quantum theory also involves non-
    local evolutions of the wave function when the wave function collapses.

0 0 0 0 4 5 0 2 8 2 5