Title

Multiresolution spatiotemporal mechanical model of the heart as a prior to constrain the solution for 4D models of the heart

Authors

Gullberg, Grant T
Veress, Alexander I
Shrestha, Uttam M
et al.

Multiresolution spatiotemporal mechanical model of the heart as a prior to constrain the solution for 4D models of the heart

Grant T. Gullberg[a,*], **Alexander I. Veress**[b,c], **Uttam M. Shrestha**[a], **Jing Liu**[a], **Karen Ordovas**[a], **W. Paul Segars**[d], **Youngho Seo**[a]

[a]Department of Radiology and Biomedical Imaging, University of California, San Francisco, California, USA

[b]Department of Mechanical Engineering, University of Washington, Seattle WA, USA

[c]Numeric Design Engineering, LLC. Seattle, WA, USA

[d]Department of Radiology, Duke University, Raleigh-Durham, North Carolina, USA

Abstract

In several nuclear cardiac imaging applications (SPECT and PET), images are formed by reconstructing tomographic data using an iterative reconstruction algorithm with corrections for physical factors involved in the imaging detection process and with corrections for cardiac and respiratory motion. The physical factors are modeled as coefficients in the matrix of a system of linear equations and include attenuation, scatter, and spatially varying geometric response. The solution to the tomographic problem involves solving the inverse of this system matrix. This requires the design of an iterative reconstruction algorithm with a statistical model that best fits the data acquisition. The most appropriate model is based on a Poisson distribution. Using Bayes Theorem, an iterative reconstruction algorithm is designed to determine the maximum a posteriori estimate of the reconstructed image with constraints that maximizes the Bayesian likelihood function for the Poisson statistical model. The a priori distribution is formulated as the joint entropy (JE) to measure the similarity between the gated cardiac PET image and the cardiac MRI cine image modeled as a FE mechanical model. The developed algorithm shows the potential of using a FE mechanical model of the heart derived from a cardiac MRI cine scan to constrain solutions of gated cardiac PET images.

Keywords

Bayesian reconstruction; joint entropy; finite element cardiac mechanical model; positron emission tomography; cardiac; hybrid PET/MRI

1   Introduction

PET/MRI technology holds the potential to provide a wealth of diagnostic information from functional analysis of the myocardium as well as biochemical and physiological

---

[*]Grant T Gullberg, gtgullberg@lbl.gov.

characterization of disease processes. One example (see Figure 1), involves using PET and MRI together to develop technologies for calculating, evaluating, and studying *myocardial efficiency,* particularly in patients with heart failure. Cardiac work of specific tissue regions can be calculated by measuring cardiac strain (distance) from MRI tag and cine pulse sequences and stress (force) from an image derived finite element (FE) mechanical model of the patient heart. Combine this with measures of oxygen utilization, measures of tissue efficiency and its inhomogeneity can be ascertained. Imaging and analytical methods to measure myocardial work and oxygen consumption will provide methods to quantify cardiac efficiency and relationships between myocardial perfusion, structure, and wall dynamics. A key aspect of this effort is the use of a mechanical model of the imaged heart not only to measure cardiac efficiency but also to provide improved PET images of the cardiac motion that can be used as priors in a maximum a posteriori reconstruction algorithm.

The Dassault Systèmes FE Living Heart model[1] provides technology for developing patient specific electromechanical cardiac models. The model consists of four chambers: the left and right atria and the left and right ventricles, connected by four valves. A FE model of the whole heart is created from assumed circulatory models and from anatomic information that might be obtained from PET and MRI imaging data. All four deformable chambers are electrically excitable with hyperelastic material properties connected by in- and out-flow conditions. Kinematic equations, boundary conditions, and excitation contraction coupling provide a realistic electromechanical model of the heart. The living heart model was designed to aid in device design and treatment planning in cardiac diseases. Using the Dassault Systèmes FE cardiac modeling technology, we obtain patient specific mechanical models of the heart by fitting it to cardiac MRI cine data acquired from a PET/MRI scanner at the University of California San Francisco (UCSF).

We hypothesize that the FE mechanical model of the patient myocardium will better constrain the reconstructed solution of the patient's cardiac deformation from data acquired during the acquisition of gated cardiac PET data. In our application the patient specific FE mechanical model is obtained from cardiac MRI cine data. Tang and Rahmim[2] provide an excellent review of work using CT and MRI images to design priors to constrain solutions in the application of maximum *a posteriori* (MAP) reconstruction algorithms. Some work has attempted to provide constraints by specifying deformation based upon optical flow; however, no work has used a FE mechanical model to more accurately model the deformation of the heart over the cardiac cycle. In our work here we consider the 4D problem of space and cardiac deformation motion using Gaussian basis functions to model the continuous deformation of the heart.[3] We use the joint entropy (JE)[4] to measure the similarity between the gated cardiac PET image and the mechanical model specified from the cardiac MRI cine images. The JE measure provides the prior for our Bayesian PET image reconstruction.

2 Algorithm

The 4D model of the activity distribution of a radionuclide in the image space for a deforming myocardium is represented by a function $A(x, \tau)$, where x is the spatial

coordinate and τ is the cardiac phase coordinate due to beating of the heart. The activity distribution can be written as a tensor product of the spatiotemporal basis functions:[3]

$$A(\boldsymbol{x}, \tau) = \sum_{m, q} a_{mq} S^m(\boldsymbol{x}) W^q(\tau),$$

where $S^m(\boldsymbol{x})$, $m = 1, 2, \ldots M$, are the spatial basis functions and $W^q(\tau)$, $q = 1, 2, \ldots Q$, are basis functions corresponding to the cardiac phases. The reconstruction problem thus involves the estimation of the expansion coefficients a_{mq}.

For the projection p with elements p_i, the activity at phase coordinate τ for detector element d_i is given by

$$p_i(\tau) = \int_{\chi} F[\boldsymbol{x}, d_i] A(\boldsymbol{x}, \tau) d\boldsymbol{x},$$

where the spatiotemporal distribution of the activity is integrated along the line of projection i in the image space χ. The weighting function $F[\boldsymbol{x}, d_i]$ incorporates the physical factors of the image detection process and provides a mapping of the activity from a position $\boldsymbol{x}$ and cardiac phase coordinate τ in the image space into the projection at the detector position d_i. It is assumed that the detection of photons at each detector element d_i is governed by a Poisson distribution.

The random number of photons recorded in projection p_{ik} during the gate or phase interval $[\tau_k, \tau_k + \Delta\tau_k]$ is $p_{ik} = \int_{\tau_k}^{\tau_k + \Delta\tau_k} p_i(\tau) d\tau$ where p_{ik} is a Poisson distributed random variable. The mean of P_{ik} is

$$\bar{p}_{ik} = \sum_{m, q} a_{mq} \Lambda_{ikmq}. \quad (1)$$

where $\Lambda_{ikmq} = \int_{\tau_k}^{\tau_k + \Delta\tau} U^m(d_i) W^q(\tau) d\tau$ and $U^m(d_i) = \int_{\chi} F[\boldsymbol{x}, d_i] S^m(\boldsymbol{x}) d\boldsymbol{x}$.

Hence, assuming the detection of photons p_{ik} in each projection bin i during the phase interval k is an independent process, the likelihood function is given by

$$\mathcal{L}(p|A) = \prod_{i, k} \frac{e^{-\bar{p}_{ik}} (\bar{p}_{ik})^{p_{ik}}}{p_{ik}!},$$

where A is the vector of basis function coefficients $A = \{a_{mq}\}$. Equivalently, one can write the log of the likelihood function as

$$\ln[\mathcal{L}(p|A)] = \sum_{i, k} \left\{ -\bar{p}_{ik} + p_{ik} \ln \bar{p}_{ik} - \ln(p_{ik}!) \right\}.$$

From Equation (1), the log-likelihood function is

$$ln[\mathscr{L}(p\,|\,A)] = -\sum_{i,k}\left\{\sum_{m,q}a_{mq}\Lambda_{ikmq}\right\} + \sum_{i,k}p_{ik}ln\left\{\sum_{m,q}a_{mq}\Lambda_{ikmq}\right\} - \sum_{i,k}ln[p_{ik}!]\,.$$

(2)

Now using Bayes theorem, the Bayesian log-likelihood is

$$ln\!\left(\mathscr{L}_B(A\,|\,p)\right) = ln(\mathscr{L}(p\,|\,A)) + ln(P(A)) - ln(P(p))\,.$$

Substituting the expression for the likelihood function in Equation (2), we have

$$ln\!\left(\mathscr{L}_B(A\,|\,p)\right) = -\sum_{i,k}\left[\sum_{mq}a_{mq}\Lambda_{ikmq}\right] + \sum_{i,k}p_{ik}ln\left[\sum_{m,q}a_{mq}\Lambda_{ikmq}\right] - \sum_{i,k}ln[p_{ik}!] + ln(P(A)) - ln(P(p))\,.$$

Constraining the solution by a prior $ln(P(A))$ equal to the JE between the gated cardiac PET images A and a mechanical cardiac model M determined from the cardiac MRI cine data (dropping terms independent of a_{mq}), the objective function is given by:[4]

$$ln\!\left(\mathscr{L}_B(A\,|\,p)\right) = -\sum_{i,k}\left[\sum_{mq}a_{mq}\Lambda_{ikmq}\right] + \sum_{i,k}p_{ik}ln\left[\sum_{m,q}a_{mq}\Lambda_{ikmq}\right] - \beta\Phi_{JE}(A,M) - ln(P(p))$$

Where $ln(P(A)) = \Phi_{JE}(A,M) = -\int p(A(x,\tau),M(x',\tau'))log[p(A(x,\tau),M(x',\tau'))]dxd\tau dx'd\tau'$

$$p(A(x,\tau)) = \frac{1}{N_xN_\tau}\sum_{i=1}^{N_x}\sum_{j=1}^{N_\tau}\phi\!\left(\frac{A(x,\tau) - \sum_{m,q}a_{mq}S^m(x_i)W^q(\tau_j)}{\sigma_{x\tau}}\right)$$

$$p(A(x,\tau),M(x',\tau')) = \frac{1}{N_xN_\tau}\sum_{i=1}^{N_x}\sum_{j=1}^{N_\tau}\phi\!\left(\frac{A(x,\tau) - \sum_{m,q}a_{mq}S^m(x_i)W^q(\tau_j)}{\sigma_{x\tau}}\right)\phi\!\left(\frac{M(x',\tau') - M(x'_i,\tau'_j)}{\sigma_{x'\tau'}}\right)$$

and ϕ are Gaussian functions with means $\sum_{m,q}a_{mq}S^m(x_i)W^q(\tau_j)$ and x'_i, τ'_j, and variances $\sigma_{x\tau}$ and $\sigma_{x'\tau'}$, respectively.

The gradient of the joint entropy is

$$\frac{\partial \Phi_{JE}(A, M)}{\partial a_{mq}} = -\frac{1}{N_x N_\tau} \sum_{k, l, k', l'}^{N_p} \left(1 + log\left(p\left(A\left(x_k, \tau_l\right), M\left(x'_{k'}, \tau'_{l'}\right)\right)\right)\right)\phi\left(\frac{M(x', \tau') - M\left(x'_{k'}, \tau'\right)}{\sigma_{x'\tau'}}\right)\frac{\partial \phi}{\partial a_{mq}}$$
$$\left(\frac{A(x, \tau) - \sum_{m, q} a_{mq} S^m\left(x_k\right)W^q\left(\tau_l\right)}{\sigma_{x\tau}}\right),$$

where N_p is the number of points for which ϕ is computed and

$$\frac{\partial \phi}{\partial a_{mq}}\left(\frac{A(x, \tau) - \sum_{m, q} a_{mq} S^m\left(x_k\right)W^q\left(\tau_l\right)}{\sigma_{x\tau}}\right) = \phi\left(\frac{A(x, \tau) - \sum_{m, q} a_{mq} S^m\left(x_k\right)W^q\left(\tau_l\right)}{\sigma_{x\tau}}\right)\left(\frac{\left(A(x, \tau) - S^m\left(x_k\right)W^q\left(\tau_l\right)\right)}{\sigma_{x\tau}^2}\right).$$

The 4D maximum a posteriori iterative expectation maximization reconstruction algorithm is[4]

$$a_{mq}^{new} = \left[\frac{a_{mq}^{old}}{S_{\mathbf{mq}} + \beta \frac{\partial \Phi_{JE}(A, M)}{\partial a_{mq}}\bigg|_{a_{mq} = a_{mq}^{old}}}\right]\left[\sum_{j, k} \Lambda_{jkmq}\left(\frac{p_{jk}}{p_{jk}^{old}}\right)\right],$$

where the sensitivity term is $S_{\mathbf{mq}} = \Sigma_{j,k}\Lambda_{jkmq}$.

3   Preliminary Results

Here we present some current results from two patient PET/MRI studies. In both studies a simultaneous PET/MRI cardiac patient study was performed using the GE PET/MRI 3.0T scanner (GE Healthcare, Milwaukee, WI) at UCSF. **Patient Study 1.** The patient was first injected with 9.9 mCi of [18]FDG 20 mins before a PET/CT scan. The patient was then brought to the PET/MRI suite with ½ the original activity. During a 10 min PET acquisition, a 3D whole heart cine with compressed sensing was simultaneously performed.[5] The time-of-flight PET images in Figs. 2A & 2C give better resolution compared with the non time-of-flight reconstructions in Figs. 2B & 2D. The gated reconstructions (Figs. 2C & 2D) show more noise than the ungated reconstructions as one would expect. **Patient Study 2.** A dynamic cardiac PET study began immediately prior to the injection of 15.1 mCi of ^{11}C-acetate and subsequently acquires PET data for 30 minutes while simultaneously acquiring MRI data from cine pulse sequences. One of the time-of-flight reconstructed transaxial images of ^{11}C-acetate uptake is shown in Fig. 2G. The Dassault FE mechanical model (upper frame, Fig. 2H) was warped to fit the 3D cine results to provide a patient specific FE mechanical model of the heart (lower frame, Fig. 2H) that models the cardiac deformation for the entire cardiac cycle. We expect that using a patient specific FE cardiac mechanical model as prior in a maximum a posteriori reconstruction algorithm will improve the noise in the images in Figs. 2C & 2D.

4   Summary

The developed algorithm has the potential of using a FE mechanical model of the heart derived from a cardiac MRI cine scan to constrain the solution using a joint entropy prior. This work implements new mathematical models from simultaneously acquired PET and MRI data with several firsts: **1)** First to use 4D modeling of cardiac deformation using a FE mechanical model; **2)** First to use human specific FE mechanical model of the heart as a prior in a MAP reconstruction algorithm; **3)** First to use basis functions to represent 4D modeling in a joint entropy MAP reconstruction algorithm; **4)** First to use joint entropy mutual prior to reconstruct PET data constrained by a joint entropy prior between PET and MRI data.

References

[1]. Baillargeon B, Rebelo N, Fox DD, Taylor RL, and Kuhl E, "The Living Heart Project: A robust and integrative simulator for human heart function," Eur. J. Mech. A Solids 48, 38–47 (2014). [PubMed: 25267880]

[2]. Tang J, and Rahmim A, "Anatomy assisted PET image reconstruction incorporating multiresolution joint entropy," Phys. Med. Biol 60(1), 31–48 (2015). [PubMed: 25479422]

[3]. Shrestha UM, Seo Y, Botvinick EH, and Gullberg GT, "Image reconstruction in higher dimension: Myocardial perfusion imaging of tracer dynamics with cardiac motion due to deformation and respiration," Phys. Med. Biol 60, 8275–301 (2015). [PubMed: 26450115]

[4]. Somayajula S, Panagiotou C, Rangarajan A, Li Q, Arridge SR, and Leahy RM, "PET image reconstruction using information theoretic anatomical priors," IEEE Trans. Med. Imag 30(3), 537–49 (2011).

[5]. Liu J, and Saloner D, "Accelerated MRI with CIRcular Cartesian UnderSampling (CIRCUS): a variable density Cartesian sampling strategy for compressed sensing and parallel imaging," Quantitative Imaging in Medicine and Surgery 4, 57–67 (2014). [PubMed: 24649436]

[6]. Segars W, Sturgeon G, Mendonca S, Grimes J, and Tsui B, "4D XCAT phantom for multimodality imaging research," Med. Phys 37, 4902 (2010). [PubMed: 20964209]
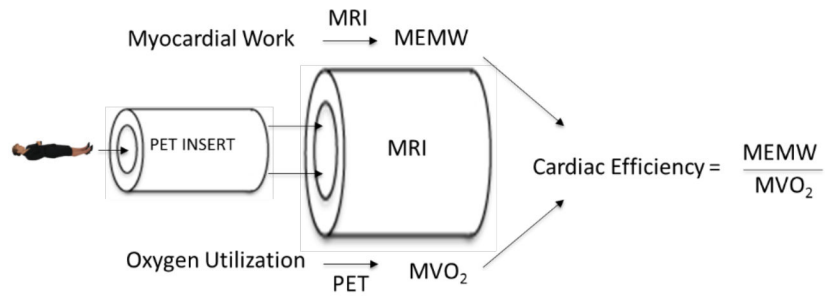
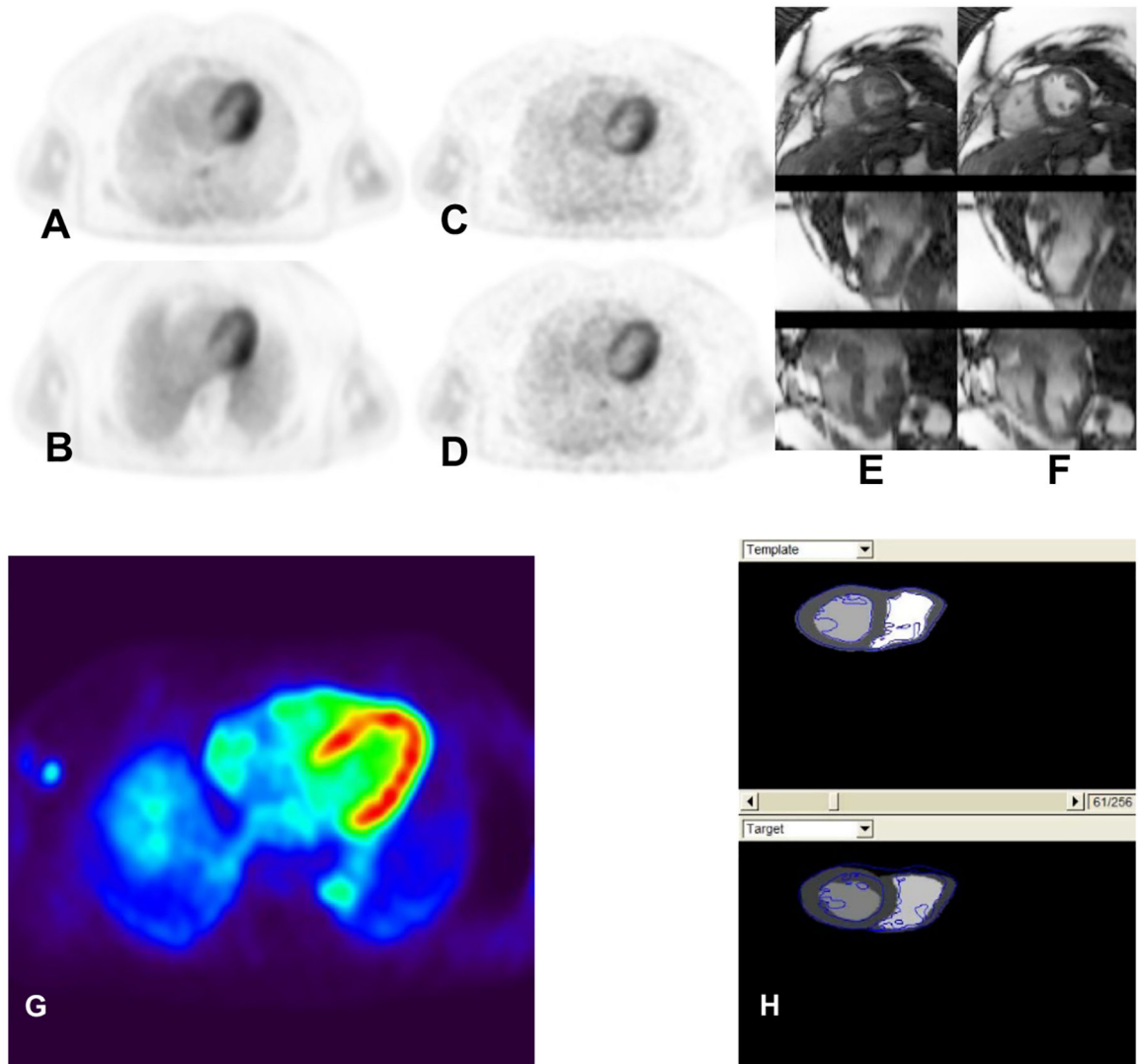**Figure 1. Concept for using PET/MRI for measuring cardiac efficiency.**

Figure 2. PET/MRI studies of the human heart.
Patient 1: A) Non ECG gated, time-of-flight reconstruction. **B)** Non ECG gated, non time-of-flight reconstruction. **C)** End diastole, time-of-flight reconstruction. **D)** End diastole, non time-of-flight reconstruction. **E)** 3D Cine MRI at systole. **F)** 3D Cine MRI at diastole.
Patient 2: G) Time-of-flight reconstruction of a transaxial image of ^{11}C-acetate uptake in the heart. **H)** A patient specific FE mechanical model of the heart is formed in the lower frame by warping the Dassault FE model in the upper frame to fit the cine MRI data.