UC Irvine

UC Irvine Previously Published Works

Title

Bd⁰—Bd⁰ Mixing, Flavor-Changing Rare Processes, and the Electric Dipole Moment of the Neutron

Permalink

<https://escholarship.org/uc/item/6qz2j0cs>

Journal

Annals of the New York Academy of Sciences, 578(1)

ISSN

0077-8923

Authors

SHIN, M
BANDER, M
SILVERMAN, D

Publication Date

1989

DOI

10.1111/j.1749-6632.1989.tb50626.x

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at

<https://creativecommons.org/licenses/by/4.0/>

Peer reviewed

$B_d^0 - \bar{B}_d^0$ Mixing, Flavor-Changing Rare Processes, and the Electric Dipole Moment of the Neutron^a

MICHAEL SHIN, MYRON BANDER,
AND DENNIS SILVERMAN

*Department of Physics
University of California
Irvine, California 92717*

INTRODUCTION

In our present understanding of the weak interaction sector, there is one potentially important piece of experimental data whose explanation may require new physics beyond the standard model (SM). It is the surprisingly large $B_d^0 - \bar{B}_d^0$ mixing, first reported here at this conference two years ago by the ARGUS¹ collaboration, which has been further confirmed by the CLEO² collaboration.

Although such a large mixing ($r_d = 0.18 \pm 0.05 \pm 0.05$) can be understood (in principle) within the (3-family) standard KM model (see FIG. 1) by assuming³ a large t -quark mass ($m_t > 100$ GeV) or certain KM matrix elements in the neighborhood of their present upper limits, this will fail to provide the solution if the t -quark is discovered below 100 GeV or so in the near future and the $|V_{ub}|$ element is measured to be well below⁴ (e.g. $|V_{ub}|/|V_{cb}| \approx 0.06 - 0.08$) the present upper limit,⁵ $|V_{ub}|/|V_{cb}| \leq 0.21$. In such a case, one may try to rescue the SM by introducing the fourth family t' -quark; the t' -quark contribution to the usual box-diagram could account for the magnitude of $B_d^0 - \bar{B}_d^0$ mixing. This would require special values for $m_{t'}$ and for the four family KM element $V_{t'd}V_{t'b}^*$. While there exist practically no experimental constraints on these parameters, recent investigations⁶ on this matter, using "realistic" estimates on the four family KM angles, suggest that this may not be so. This is because the t' -quark exchange diagrams, which are required to bring the $B_d^0 - \bar{B}_d^0$ mixing to the observed level, inevitably enhance the CP -violating amplitude in $K^0 - \bar{K}^0$ mixing, giving too large a contribution⁶ to $\text{Re}\epsilon_K (= 1.62 \times 10^{-3})$.

In this paper, we propose that the observed large $B_d^0 - \bar{B}_d^0$ mixing is the first signal for the new physics beyond the SM. The general physical picture that we have is as follows: Suppose some new physics (e.g. heavy exotic fermions, technicolor, compositeness) is present at the mass scale M , which we presume to be just above m_t or the electroweak scale $v' = 250$ GeV. The presence of such new physics will in general affect the low-energy world of the standard model quarks and leptons. The most affected ones would be the members of the heaviest family (say the third family), since their mass gap with the new physics at M is the smallest. This effect of new physics is expected to

^aThis research is supported by the NSF under Grant NSF-PHY-8605552.

FIGURE 1. Box-diagram contribution to $B_d^0 - \bar{B}_d^0$ mixing in KM model.

become smaller as we move on to lighter families, with the first family being the least affected. This is schematically shown in FIGURE 2.

Among such effects of the new physics on the standard model quarks and leptons, one interesting possibility is the tree-level neutral flavor changing couplings (NFCC) of the SM Higgs scalar, H^0 , or the neutral gauge boson Z^0 , induced by this new physics. Here, we propose that the observed large $B_d^0 - \bar{B}_d^0$ mixing is indeed mediated by this tree-level NFCC of H^0 or Z^0 (see FIG. 3), and $B_d^0 - \bar{B}_d^0$ mixing is the first signal of this NFCC seen in the laboratory.

AN ILLUSTRATIVE EXAMPLE: VECTOR SINGLET MODEL

As a simple, illustrative example of the general class of models, in which tree-level NFCC of H^0 and Z^0 between ordinary quarks and leptons are generated through the effect of mixings with heavy exotic fermions, we consider a model^b with an $SU(2)_L$ vector singlet of charge $-1/3$ quarks, D_L and D_R , plus the three standard families of quarks and leptons.

In the basis of weak-eigenstates d_{iL}^0 and d_{jR}^0 , the mass and the Yukawa couplings of the charge $-1/3$ quarks are given by

$$-L_Y = \{\bar{d}_{iL}^0 (M^d)_{ij} d_{jR}^0 + \bar{d}_{iL}^0 (y^d)_{ij} d_{jR}^0 H^0 / \sqrt{2}\} + \text{h.c.} \quad (1)$$

FIGURE 2. Effect of New Physics on the low-energy world of ordinary quarks and leptons.

^bGeneralizations to the case with a leptonic vector singlet with charge -1 (E_L^- and R_R^-) and/or charge $2/3$ quarks (U_L and U_R) is straightforward. For earlier works on models with exotic heavy fermions, see ref. 7 and references therein.

where

$$(y^d) = \begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (2)$$

$$(M^d) = (y^d)v/\sqrt{2} + (M'), \quad v \equiv (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV} \quad (3)$$

with

$$(M') = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M \end{bmatrix}. \quad (4)$$

In equation 1, $i, j = 1, 2, 3$ correspond to the three families of ordinary d -type quarks (d, s, b), while $i, j = 4$ corresponds to the heavy exotic ones, D_L and D_R . (M') of

FIGURE 3. $B_d^0 - \bar{B}_d^0$ mixing mediated by the tree-level NFCC of H^0 and/or Z^0 .

equation 4 is due to the bare mass term M and takes the given form without any loss of generality. The mass matrix (M^d) can be diagonalized by the unitary matrices V_L and V_R ,

$$(V_L)^\dagger (M^d) (V_R) \equiv M_{\text{diag}}^d. \quad (5)$$

Defining the mass-eigenstates d_{iL} and d_{iR} , for $i = 1, 2, 3, 4$, by

$$\begin{aligned} d_{iL}^0 &\equiv (V_L)_{ij} d_{jL}, & d_{iL} &\equiv (V_L^\dagger)_{ij} d_{jL}^0 \\ d_{iR}^0 &\equiv (V_R)_{ij} d_{jR}, & d_{iR} &\equiv (V_R^\dagger)_{ij} d_{jR}^0, \end{aligned} \quad (6)$$

it is easy to show that the flavor-changing couplings of H^0 and Z^0 are

$$L_{\text{F.C.}}^{H^0} = y'_{ij} \bar{d}_{iL}^* d_{jR} H^0 + \text{h.c.} \quad \text{for} \quad i \neq j \quad (7)$$

$$y'_{ij} \equiv (M/v) (V_L)_{4i}^* (V_R)_{4j} \quad \text{for} \quad i \neq j \quad (8)$$

$$L_{\text{F.C.}}^{Z^0} = (e/2 \sin \theta_w \cos \theta_w) \hat{g}_{ij}^L \bar{d}_{iL} \gamma^\mu d_{jL} Z_\mu^0 \quad \text{for} \quad i \neq j \quad (9)$$

$$\hat{g}_{ij}^L \equiv (V_L)_{4i}^* (V_L)_{4j} \quad \text{for} \quad i \neq j, \quad (10)$$

from the mismatch between the Yukawa coupling matrix (y^d) and the mass matrix (M^d), and the isospin structure of the electroweak gauge coupling. The main results⁶ of this discussion show that the flavor-changing couplings of H^0 and Z^0 are given by equations 7, 8, 9, and 10, with strengths proportional to the product of the mixing angles $(V_L)_{4i}^*$ and $(V_{L,R})_{4j}$ of the unitary matrices V_L and V_R .

PRESENT EXPERIMENTAL CONSTRAINTS ON THE FLAVOR-CHANGING COUPLINGS OF H^0 AND Z^0

In order to discuss the physics of flavor-changing couplings of H^0 and Z^0 we need to know their present experimental constraints. We shall use the notation and conventions described below. The most general form of couplings of the Z^0 to ordinary quarks and leptons is

$$\begin{aligned} L_{Z^0} &= \sum_{i,j} (g_{ij}^L \bar{f}_{iL} \gamma^\mu f_{jL} + g_{ij}^R \bar{f}_{iR} \gamma^\mu f_{jR}) Z_\mu^0 + \text{h.c.} \\ &= \frac{e}{2 \sin \theta_w \cos \theta_w} \left\{ \sum_{i,j} (\hat{g}_{ij}^L \bar{f}_{iL} \gamma^\mu f_{jL} + \hat{g}_{ij}^R \bar{f}_{iR} \gamma^\mu f_{jR}) Z_\mu^0 + \text{h.c.} \right\}. \quad (11) \end{aligned}$$

From hermiticity

$$\hat{g}_{ji}^L = \hat{g}_{ij}^{L*} \quad \text{and} \quad \hat{g}_{ji}^R = \hat{g}_{ij}^{R*}. \quad (12)$$

The indices i and j stand for flavors of quarks and leptons. Similarly, the most general form of couplings of the Higgs scalar, H^0 , to the ordinary quarks and leptons is given by

$$\begin{aligned} L_{H^0} &= \sum_{i,j} (\bar{f}_{iL} y_{ij}^R f_{jR} + \bar{f}_{iR} y_{ij}^L f_{jL}) H^0 + \text{h.c.} \\ &= \frac{e}{2 \sin \theta_w \cos \theta_w} \left\{ \sum_{i,j} (\bar{f}_{iL} \hat{y}_{ij}^R f_{jR} + \bar{f}_{iR} \hat{y}_{ij}^L f_{jL}) H^0 + \text{h.c.} \right\} \quad (13) \end{aligned}$$

Similar tree-level flavor changing couplings of H^0 and Z^0 exist for vector doublet models ($\hat{g}_{ij}^R, y_{ij}$) and mirror fermion models ($\hat{g}_{ij}^L, \hat{g}_{ij}^R, y_{ij}$); generalizations of our discussions to these models are straightforward. Two prototype theories involving these heavy exotic fermions are models based upon gauge groups E_6 and $SO(18)$. For E_6 , the fermion representation 27 contains $16 + 10 + 1$ (in $SO(10)$ notation), where the rep. 10 contains a quark vector singlet (D_L and $\bar{D}_R$) of charge $-1/3$, and a lepton vector doublet $(E^-, N)_L$ and $(E^-, N)_R$. For $SO(18)$,⁹ the entire fermion family unification is achieved by a single spinor rep. 256, which predicts 4 families of ordinary quarks and leptons and 4 additional families of mirror fermions in the low-energy sector.

with

$$\hat{y}_{ji}^R = \hat{y}_{ij}^{L*} \quad \text{and} \quad \hat{y}_{ji}^L = \hat{y}_{ij}^{R*}. \quad (14)$$

The unknown mass of the Higgs scalar, M_H , will be expressed in terms of $\hat{M}_H$, where

$$\hat{M}_H \equiv M_H/M_Z = M_H/(92 \text{ GeV}) \quad (15)$$

We have investigated a variety of flavor-changing neutral processes which are likely to provide the most stringent constraints on the flavor-changing couplings of H^0 and Z^0 . Our results are summarized in the first three columns of TABLE 1 and TABLE 2^d; details will be published elsewhere.¹⁴ Note that the results for $B_d^0 - \bar{B}_d^0$ mixing are not to be taken as a bound, but, in the spirit of this talk, as a positive result fixing the parameters coupling the b -quark to the d -quark.

THEORETICAL EXPECTATIONS ON THE FLAVOR DEPENDENCE OF THE FLAVOR-CHANGING COUPLINGS

In equations 8 and 10, we have seen that, in the context of a simple vector singlet model, the flavor-changing couplings are proportional to the product of mixing angles $(V_{L,R})_{4i}^*(V_{L,R})_{4j}$. We expect similar results to hold in other models involving heavy exotic fermions. Let us now consider how such mixing angles $((V_{L,R})_{4j}$'s) should depend on the generation (family) index j . From experience with KM angles, we expect

$$|(V_{L,R})_{41}| \ll |(V_{L,R})_{42}| \ll |(V_{L,R})_{43}| \ll 1. \quad (16)$$

Lighter fermions are expected to have smaller mixing with the heavy exotic ones in order to keep their masses small; too much mixing would spoil this smallness. This may be the reason why the flavor-changing neutral processes between the first two lightest families (i.e., $d \leftrightarrow s$, $e \leftrightarrow \mu$) have not been observed thus far, and the GIM¹⁵ mechanism has been so successful, since these are the ones that are likely to be the most suppressed in terms of the mixing angles. However, as one moves on to heavier families, one expects larger flavor violations in $\mu^- \leftrightarrow \tau^-$, $s \leftrightarrow b$, $c \leftrightarrow t$, $e^- \leftrightarrow \tau^-$, $d \leftrightarrow b$, and $u \leftrightarrow t$.

To make a reasonable estimate on these mixing angles, we shall be guided by the mixing angle vs. the mass-eigenvalue ratio in simple case of 2×2 mixing, which should shed some light on the general relation between the mixing angles and the mass-eigenvalues. Consider a 2×2 real symmetric matrix,

$$(m) = \begin{bmatrix} \alpha & \beta \\ \beta & \gamma \end{bmatrix} \quad (17)$$

^dThe full expressions, of course, involve the coupling constants in a complicated manner. For example, in the case of H^0 exchange, the constraint from ΔM_K is $(1/\hat{M}_H)|(\hat{y}_{sd}^L)^2 + (\hat{y}_{sd}^R)^2 - (2/5)(6 + \beta_K^{-1})(\hat{y}_{sd}^L)(\hat{y}_{sd}^R)|^{1/2} \leq 9.5 \times 10^{-5} (10/\beta_K)^{1/2}$, with $\beta_K \equiv (m_K/(m_s + m_d))^2 \simeq (10 \pm 5)$. Since we cannot disentangle with various parts of this expression, we assume that there are no fortuitous cancellations and present results for a generic coupling $|\hat{y}_{sd}^{L,R}|$.

TABLE 1. Present Experimental Constraints on the Flavor-Changing Couplings (Times Higgs Mass Factor) of the Higgs Scalar, H^0 , and Their Predicted Values for $p = \frac{1}{2}$ and $p = 1$ by Equation 21^a

Source	Coupling	Experimental Upper Bound	Predictions: $p = \frac{1}{2}$	Predictions: $p = 1$
$\Delta M_K = 3.52 \times 10^{-15}$ GeV [5] ^b	$\left(\frac{1}{M_H}\right) \hat{y}_{td}^{L,R} $	$9.0 \times 10^{-5} \sqrt{\frac{10}{\beta_K}}$	$(1.6 \pm 0.4) \times 10^{-4}$	$(2.6 \pm 0.7) \times 10^{-5}$
$D^0 - \bar{D}^0$ Mixing [10]	$\left(\frac{1}{M_H}\right) \hat{y}_{cd}^{L,R} $	$7.2 \times 10^{-4} \left(\frac{0.16 \text{ GeV}}{f_D}\right)$	$(3.9 \pm 0.9) \times 10^{-4}$	$(1.6 \pm 0.4) \times 10^{-4}$
$B_S^0 - \bar{B}_S^0$ Mixing: ARGUS, CLEO [1, 2]	$\left(\frac{1}{M_H}\right) \hat{y}_{cd}^{L,R} $	$(9.5 \pm 2.3) \times 10^{-4} \left(\frac{0.15 \text{ GeV}}{f_B}\right)$	input	input
$B R(B^+ \rightarrow \mu^+ \mu^- X) \leq 1.2 \times 10^{-3}$ [11]	$\left(\frac{1}{M_H}\right)^2 \hat{y}_{bc}^{L,R} $	$10.3 \left(\frac{1}{0.043}\right) \left(\frac{V_{cb}}{f_B}\right)$	$\left(\frac{1}{M_H}\right) (4.1 \pm 1.0) \times 10^{-3}$	$\left(\frac{1}{M_H}\right) (1.8 \pm 0.4) \times 10^{-2}$
—	$\left(\frac{1}{M_H}\right) \hat{y}_{tc}^{L,R} $	—	$(4.2 \pm 1.0) \times 10^{-2} \sqrt{\frac{m_t}{60 \text{ GeV}}}$	$(1.9 \pm 0.5) \left(\frac{m_t}{60 \text{ GeV}}\right)$
—	$\left(\frac{1}{M_H}\right) \hat{y}_{bc}^{L,R} $	—	$(7.5 \pm 1.8) \times 10^{-2} \sqrt{\frac{m_b}{50 \text{ GeV}}}$	$(5.9 \pm 1.4) \left(\frac{m_b}{50 \text{ GeV}}\right)$
$B R(\mu^+ \rightarrow e^+ \gamma) \leq 4.9 \times 10^{-11}$ [12]	$\left(\frac{1}{M_H}\right) \hat{y}_{\mu e}^{L,R} $	1.6×10^{-2}	$(1.1 \pm 0.3) \times 10^{-5}$	$(1.2 \pm 0.3) \times 10^{-7}$
$\delta a_\mu < 2 \times 10^{-10}$ [13]	$\left(\frac{1}{M_H}\right) \hat{y}_{\mu e}^{L,R} \hat{y}_{\mu e}^{L,R} ^{1/2}$	0.38	$(1.4 \pm 0.3) \times 10^{-4}$	$(2.0 \pm 0.5) \times 10^{-5}$
$B R(\mu \rightarrow e^+ \gamma) \leq 4.9 \times 10^{-11}$ [12]	$\left(\frac{1}{M_H}\right)^2 \hat{y}_{\mu e}^{L,R} \hat{y}_{\mu e}^{L,R} $	1.9×10^{-6}	$(2.8 \pm 0.7) \times 10^{-7}$	$(8.4 \pm 4.0) \times 10^{-8}$
$\delta a_\mu < 3 \times 10^{-8}$ [13]	$\left(\frac{1}{M_H}\right) \hat{y}_{\mu e}^{L,R} \hat{y}_{\mu e}^{L,R} ^{1/2}$	23.8	$(2.0 \pm 0.5) \times 10^{-3}$	$(4.2 \pm 1.0) \times 10^{-3}$
$B R(\tau \rightarrow \mu^+ \mu^- \gamma) \leq 2.9 \times 10^{-5}$ [12]	$\left(\frac{1}{M_H}\right)^2 \hat{y}_{\tau \mu}^{L,R} $	35.3	$\left(\frac{1}{M_H}\right) (2.0 \pm 0.5) \times 10^{-3}$	$\left(\frac{1}{M_H}\right) (4.2 \pm 1.0) \times 10^{-3}$
—	$\left(\frac{1}{M_H}\right) \hat{y}_{\tau \tau}^{L,R} $	—	$(3.9 \pm 0.9) \times 10^{-2} \sqrt{\frac{m_\tau}{40 \text{ GeV}}}$	$(1.6 \pm 0.4) \left(\frac{m_\tau}{40 \text{ GeV}}\right)$

^aIn the first row, $\beta_K = (m_K/(m_s + m_d))^2 \simeq (10 \pm 5)$. For p between $\frac{1}{2}$ and 1, the predicted values are between the values for $p = \frac{1}{2}$ and $p = 1$.

^bReference numbers appear within square brackets.

TABLE 2. Present Experimental Constraints on the Flavor-Changing Couplings of the Z^0 , and Their Predicted Values for $p = 1/2$ and $p = 1$ by Equation 21^a

Source	Coupling	Experimental Upper Bound	Predictions: $p = 1/2$	Predictions: $p = 1$
$\Delta M_K - 3.52 \times 10^{-15}$ GeV [5] ^b	$ \hat{g}_{sd}^{L,R} $	2.2×10^{-4}	$(1.3 \pm 0.3) \times 10^{-4}$	$(2.1 \pm 0.5) \times 10^{-5}$
$BR(K_L \rightarrow \mu^+ \mu^-) = 9.1 \times 10^{-9}$ [5]	$ \text{Re}(\hat{g}_{sd}^R - \hat{g}_{sd}^L) $	1.34×10^{-5}	$(1.3 \pm 0.3) \times 10^{-4}$	$(2.1 \pm 0.5) \times 10^{-5}$
$D^0 - \bar{D}^0$ Mixing [10]	$ \hat{g}_{ss}^{L,R} $	$7.9 \times 10^{-4} \left(\frac{0.16 \text{ GeV}}{f_D} \right)$	$(3.0 \pm 0.7) \times 10^{-4}$	$(1.2 \pm 0.3) \times 10^{-4}$
$B_d^0 - \bar{B}_d^0$ Mixing, ARGUS, CLEO [1, 2]	$ \hat{g}_{bd}^{L,R} $	$(7.5 \pm 1.8) \times 10^{-4} \left(\frac{0.15 \text{ GeV}}{f_B} \right)$	input	input
$BR[B_d^0 \rightarrow \mu^+ \mu^- X] \leq 1.2 \times 10^{-3}$ [11]	$ \hat{g}_{bv}^{L,R} $	$1.2 \times 10^{-2} \left(\frac{ V_{cs} }{0.043} \right)$	$(3.2 \pm 0.8) \times 10^{-3}$	$(1.4 \pm 0.3) \times 10^{-2}$
	$ \hat{g}_{tc}^{L,R} $	—	$(3.3 \pm 0.8) \times 10^{-2} \sqrt{\frac{m_t}{60 \text{ GeV}}}$	$(1.5 \pm 0.4) \times 10 \left(\frac{m_t}{60 \text{ GeV}} \right)$
	$ \hat{g}_{bb}^{L,R} $	—	$(5.9 \pm 1.4) \times 10^{-2} \sqrt{\frac{m_b}{50 \text{ GeV}}}$	$(4.7 \pm 1.1) \left(\frac{m_b}{50 \text{ GeV}} \right)$
$BR(\mu^+ \rightarrow e^+ e^- e^-) \leq 1.0 \times 10^{-12}$ [12]	$ \hat{g}_{\mu\mu}^{L,R} $	2.3×10^{-6}	$(8.4 \pm 2.0) \times 10^{-6}$	$(9.5 \pm 2.3) \times 10^{-8}$
$BR(\tau^- \rightarrow e^- \mu^+ \mu^-) \leq 3.3 \times 10^{-5}$ [12]	$ \hat{g}_{\tau\tau}^{L,R} $	3.9×10^{-2}	$(1.1 \pm 0.3) \times 10^{-4}$	$(1.6 \pm 0.4) \times 10^{-5}$
$BR(\mu^+ \rightarrow e^+ \gamma) \leq 4.9 \times 10^{-11}$ [12]	$ \hat{g}_{\tau\tau}^{L,R}, \hat{g}_{\mu\mu}^{L,R} $	7.5×10^{-6}	$(1.7 \pm 0.7) \times 10^{-7}$	$(5.3 \pm 2.4) \times 10^{-8}$
$BR(\tau^- \rightarrow \mu^- \mu^+ \mu^-) \leq 2.9 \times 10^{-5}$ [12]	$ \hat{g}_{\tau\mu}^{L,R} $	2.9×10^{-2}	$(1.6 \pm 0.4) \times 10^{-3}$	$(3.3 \pm 0.8) \times 10^{-3}$
	$ \hat{g}_{\tau\tau}^{L,R} $	—	$(3.1 \pm 0.7) \times 10^{-2} \sqrt{\frac{m_\tau}{40 \text{ GeV}}}$	$(1.3 \pm 0.3) \left(\frac{m_\tau}{40 \text{ GeV}} \right)$

^aFor p between $1/2$ and 1, the predicted values are between the values for $p = 1/2$ and $p = 1$.

^bReference numbers appear in square brackets.

which is diagonalized by an orthogonal matrix $R(\theta)$,

$$R(\theta)^T(m)R(\theta) \equiv m_{\text{diag.}} \equiv \begin{bmatrix} -m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$R(\theta) \equiv \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (18)$$

In such a case, one can easily show¹⁶ that the most natural relation between the mixing angle and the mass ratio is

$$\sin \theta \simeq (m_1/m_2)^p, \quad 1/2 \lesssim p \lesssim 1 \quad (19)$$

from the argument of naturality (e.g. no fine-tuned cancellations between parameters). Moreover, $p \simeq 1/2$ is expected to be more realistic than $p \simeq 1$, and a particularly interesting one is the one, in which the mixing angle is exactly the square root of the mass ratio. Several examples of this relation of mixing angles as square roots of mass ratios already exist in the literature.⁶

From this discussion of the 2×2 case, we see that the most reasonable estimate on $(V_{L,R})_{4j}$ is

$$(V_{L,R})_{4j} \simeq \left(\frac{m_j}{M} \right)^p \text{ with } \frac{1}{2} \lesssim p \lesssim 1 \quad (20)$$

and the generation dependence of the flavor-changing couplings is expected to be

$$\frac{\hat{y}_{ij}}{\hat{y}_{kl}} \simeq \left(\frac{m_i m_j}{m_k m_l} \right)^p \simeq \frac{\hat{g}_{ij}}{\hat{g}_{kl}} \text{ with } \frac{1}{2} \lesssim p \lesssim 1 \quad (21)$$

Moreover, $p \simeq 1/2$ is expected to be more realistic than $p \simeq 1$.

COMPARISON WITH EXISTING DATA AND PREDICTIONS FOR FUTURE EXPERIMENTS

Considering $B_d^0 - \bar{B}_d^0$ mixing as an anchor for the NFCC, we use the results of the previous discussion as summarized in equation 21 to predict the expected values for other coupling constants and compare these with the present experimental data on positive results or on bounds. The results for $p = 1/2$ and for $p = 1$ are shown in the last

⁶The charge $-1/3$ quark mass matrix of the form of equation 17 with $\alpha = 0$ for the two family case is well known to give the phenomenologically successful relation $\sin \theta_C \simeq \sqrt{m_d/m_s} = \sqrt{1/20} = 0.22$, where θ_C is the Cabibbo angle. A similar mass matrix for the neutrinos gives rise to the well known seesaw mechanism. For three families of quarks, a generalization of this matrix gives phenomenologically successful relations^{4,17} between their mixing angles and masses, where the entire set of KM angles are expressible in terms of the square roots of the quark mass ratios. For the four family case, see ref. 6.

two columns of TABLE 1 and TABLE 2. Before looking at the details, we should discuss two caveats. First, we use equation 21 to extend the coupling constants, not only to systems made out of charge $-1/3$ quarks, but also to those of charge $2/3$ quarks, and to leptons. This would be valid if the mass scale responsible for the breaking of the GIM mechanism would be the same for all three of the above systems. Even though this may be unlikely, we do not expect these masses to be orders apart; thus there may be a rescaling by a small factor as we go from group to group. Second, as discussed in footnote *d*, we are presenting results for a common coupling constant for each process, while the detailed expressions may involve complicated sums of products of left- and right-handed couplings. Thus we are ignoring possible detailed cancellations or enhancements. Owing to these two caveats, all of our results should be viewed as valid only up to a factor not too different from one. With these remarks in mind we see that we have no gross violations of any present experimental bounds. We also note that predictions for several, as yet unobserved processes are close to their present bounds. We shall discuss these in some detail.

In case the observed $B_d^0 - \bar{B}_d^0$ mixing is due to flavor-changing couplings of the Higgs scalar H^0 , TABLE 1 predicts two flavor-changing couplings which are slightly below the present experimental upper limit. These are the ones for $D^0 - \bar{D}^0$ mixing and $BR(\mu^- \rightarrow e^- \gamma)$. Predictions on these quantities are given in TABLE 3. If we also take equation 20 literally, the required strength of $|\hat{y}_{bd}^{L,R}|$ (of TABLE 1) for $B_d^0 - \bar{B}_d^0$ mixing predicts

$$M_H \simeq (2.4 \pm 0.6)(f_B/(0.15 \text{ GeV})) M_Z \simeq (200 - 300) \text{ GeV} \quad (22)$$

for $p = 1/2$, and $\hat{M}_H M \simeq (0.5 \pm 0.1)(f_B/0.15 \text{ GeV}) \text{ GeV}$ for $p = 1$. Thus the latter case predicts a rather unrealistic, low value of $M \lesssim 5 \text{ GeV}$ (since we know $\hat{M}_H \gtrsim 0.1$) and thus we conclude that $p = 1/2$ would be much closer to reality than $p = 1$.

In case the observed $B_d^0 - \bar{B}_d^0$ mixing is due to the flavor-changing coupling of Z^0 , TABLE 2 predicts several flavor-changing couplings of Z^0 which may, in the near future, have observable consequences: namely, $BR(\mu^- \rightarrow e^- e^+ e^-)$, $D^0 - \bar{D}^0$ mixing, $BR(B_s^0 \rightarrow \mu^+ \mu^- X)$, $BR(\tau^- \rightarrow \mu^- \mu^+ \mu^-)$, $BR(\mu^- \rightarrow e^- \gamma)$ and the flavor-changing decay modes^f of Z^0 which can be tested with the $10^7 Z^0$'s expected at LEP. The predictions on these quantities are likewise given in TABLE 3. Again, if one takes equation 20 literally, one finds from the required strength of $|\hat{g}_{sd}^{L,R}|$ (of TABLE 2) for $B_d^0 - \bar{B}_d^0$ mixing

$$M \simeq (275 \pm 66) \text{ GeV} \times (f_B/0.15 \text{ GeV}) \quad (23)$$

for $p = 1/2$, while $p = 1$ gives $M \simeq (7.5 \pm 0.9) \text{ GeV} \times \sqrt{f_B/0.15 \text{ GeV}}$. The latter value of M (= the scale of the heavy exotic fermion masses) is, again, rather too low to be realistic. It is interesting to note that the observed strength of $B_d^0 - \bar{B}_d^0$ mixing implies that the values of M_H in equation 22 and of M in equation 23 to be $O(v = 250 \text{ GeV})$, the scale of the electroweak symmetry breaking; this may not be a numerical coincidence.

^fThese flavor-changing Z^0 decay mode branching ratios are much larger than the ones expected¹⁸ in the SM.

TABLE 3. Summary of the Predictions on the Neutral Flavor-Changing Rare Processes

Mediating Boson	Present Experimental Upper Limits	Predictions for Future Experiments	
		$p = 1/2$	$p = 1$
Higgs Scalar	$D^0 - \bar{D}^0$ Mixing: $r_D \leq 5.6 \times 10^{-3}$	$D^0 - \bar{D}^0$ Mixing of $O(10\%)$ of Present Upper Limit $(1.1 \pm 0.6) \times 10^{-12}$	$D^0 - \bar{D}^0$ Mixing of $O(0.1\%)$ of Present Upper Limit $(1.1 \pm 0.8) \times 10^{-13}$
H^0	$BR(\mu^- \rightarrow e^- \gamma) \leq 4.9 \times 10^{-11}$		
	$BR(\mu^- \rightarrow e^- e^+ e^-) \leq 1.0 \times 10^{-12}$	Any value in the neighborhood of Present Upper Limit	$(1.7 \pm 0.8) \times 10^{-15}$
	$D^0 - \bar{D}^0$ Mixing: $r_D \leq 5.6 \times 10^{-3}$	$D^0 - \bar{D}^0$ Mixing of $O(1 - 10)\%$ of Present Upper Limit $(8.5 \pm 4.2) \times 10^{-5}$ $(8.8 \pm 4.8) \times 10^{-8}$ $(2.5 \pm 2.0) \times 10^{-14}$	$O(0.01 - 0.1)\%$ of Present Upper Limit $(1.6 \pm 0.7) \times 10^{-2}$ $(3.8 \pm 1.8) \times 10^{-7}$ $(2.4 \pm 2.0) \times 10^{-15}$
	$BR(B_s^0 \rightarrow \mu^+ \mu^- X) \leq 1.2 \times 10^{-3}$ $BR(\tau^- \rightarrow \mu^- \mu^+ \mu^-) \leq 2.9 \times 10^{-5}$ $BR(\mu^- \rightarrow e^- \gamma) \leq 4.9 \times 10^{-11}$		
Z^0			
Flavor-Changing Z^0 Decay Mode Branching Ratios $\times 10^7$			
	$BR(Z^0 \rightarrow b\bar{s} + s\bar{b}) \times 10^7$	14 ± 7	270 ± 120
	$BR(Z^0 \rightarrow t\bar{c} + c\bar{t}) \times 10^7$	$(1500 \pm 700) (m_t/60 \text{ GeV})$	$O(10^4)$
	$BR(Z^0 \rightarrow b'\bar{b} + b\bar{b}') \times 10^7$	$(4800 \pm 2300) (m_b/50 \text{ GeV})$	$O(10^5)$
	$BR(Z^0 \rightarrow \mu^+ \tau^- + \mu^- \tau^+) \times 10^7$	3.6 ± 1.8	15 ± 7
	$BR(Z^0 \rightarrow \tau^+ \tau^- + \bar{\tau}^+ \tau^-) \times 10^7$	$(1300 \pm 600) (m_\tau/40 \text{ GeV})$	$O(10^4)$

THE ELECTRIC DIPOLE MOMENT OF THE NEUTRON

The flavor-changing couplings of H^0 and Z^0 give rise to the electric dipole moment (EDM) of the d -quark (and similarly for other quarks and leptons) at the one-loop level¹⁹ with the internal fermions f ,

$$D_d^H = \left(-\frac{1}{3} \right) [5.3 \times 10^{-24} \text{ ecm}] \sum_f \left(\frac{m_f}{250 \text{ GeV}} \right) \cdot \left[\frac{|y_{df}^L y_{df}^{R*}|}{(8 \text{ MeV}/250 \text{ GeV})} \right] \sin \delta_f^y \left[\frac{F(M_H, m_f)}{F(250 \text{ GeV}, 250 \text{ GeV})} \right] \quad (24)$$

where $\delta_f^y \equiv$ phase of $(y_{df}^L y_{df}^{R*})$, and

$$F(M_H, m_f) \equiv [(M_H^2)^2 \ln(M_H^2/m_f^2) - (1/2)(3M_H^2 - m_f^2)(M_H^2 - m_f^2)]/(M_H^2 - m_f^2)^3 \quad (25)$$

for the Higgs exchange case, where $F(M_H, M_H) = 1/(3M_H^2)$, and

$$D_d^Z = \left(-\frac{1}{3} \right) [2.8 \times 10^{-24} \text{ ecm}] \sum_f \left(\frac{m_f}{250 \text{ GeV}} \right) \cdot \left[\frac{|\hat{g}_{df}^L \hat{g}_{df}^{R*}|}{(8 \text{ MeV}/250 \text{ GeV})} \right] \sin \delta_f^g \left[\frac{G(M_Z, m_f)}{G(M_Z, 250 \text{ GeV})} \right] \quad (26)$$

where $\delta_f^g \equiv$ phase of $(\hat{g}_{df}^L \hat{g}_{df}^{R*})$, and

$$G(M_Z, m_f) \equiv [(M_Z^2 m_f^2) \ln(m_f^2/M_Z^2) + (1/2)(M_Z^2 + m_f^2)(M_Z^2 - m_f^2)]/(M_Z^2 - m_f^2)^3 \quad (27)$$

for the Z^0 exchange case.

The largest contribution in each case comes from the heaviest internal fermion F with mass M_F (e.g. the vector singlets D_L and D_R in the vector singlet model), unless the CP-violating phase factor $\sin \delta_F^{y,g}$ is much smaller than those for the lighter internal ordinary quarks. Using $|y_{dF}^{LR}| \simeq \sqrt{m_d M_F}/v$, $m_d = 8 \text{ MeV}$, and $m_H \simeq 250 \text{ GeV}$ from the observed strength of the $B_d^0 - \bar{B}_d^0$ mixing, the EDM²⁰ of the neutron, $D_N (= (4D_d - D_u)/3)$ from the quark model) is estimated to be

$$|D_N^H| \simeq 3 \times 10^{-24} \text{ ecm} \sin \delta_F^y \cdot (M_F/250 \text{ GeV})^2 [F(250 \text{ GeV}, M_F)/F(250 \text{ GeV}, 250 \text{ GeV})] \quad (28)$$

for the Higgs exchange case. For the Z^0 exchange case, one finds

$$|D_N^Z| \simeq 1.6 \times 10^{-24} \text{ ecm} \sin \delta_F^g, \quad (29)$$

using $|\hat{g}_{dF}^{LR}| \simeq \sqrt{m_d/M_F}$, and $M_F \simeq 250 \text{ GeV}$ from the observed strength of $B_d^0 - \bar{B}_d^0$ mixing. Thus, if the recently reported²¹ two preliminary values, $D_N = -(1.4 \pm 0.6) \times 10^{-25} \text{ e-cm}$ and $D_N = -(1.1 \pm 0.7) \times 10^{-25} \text{ e-cm}$ turn out to be true, it can be easily

understood if $\sin \delta_F^{sg} \simeq O(0.1)$. Moreover, as we do not expect¹⁹ $\sin \delta_F^{sg}$ to be unnaturally small, positive results on the measurement of D_N may appear in the near future, even as large as 10^{-25} e-cm.

CONCLUSIONS

(i) The observed large $B_d^0 - \bar{B}_d^0$ mixing of ARGUS and CLEO may be an indication of the small tree-level flavor-changing couplings of the SM Higgs scalar, H^0 , or the Z^0 , induced by new physics at an energy scale M beyond the standard model.

(ii) The observed strength of $B_d^0 - \bar{B}_d^0$ mixing indicates that this scale M and/or the mass of the Higgs scalar M_H seem to coincide with the Higgs VEV, $v = 250$ GeV. The latter is interesting, because in our present understanding of gauge field theories, there is *a priori* no reason why the mass of the Higgs scalar should coincide with the VEV ($v = 250$ GeV) of itself; this corresponds to $M_H^2 = \lambda v^2$, with the scalar quartic coupling constant $\lambda = 1$. If it is indeed so, there must be a more fundamental theoretical reason and this may provide us with some important piece of information on the structure of new physics beyond the SM (Perhaps technicolor or compositeness may be relevant). This may indicate that new physics will indeed show up at a mass scale of 250 GeV.

(iii) The tree-level flavor-changing couplings of H^0 and Z^0 postulated in this talk,

$$|y_{ij}^{LR}| \simeq \sqrt{m_i m_j}/v, \quad |\hat{g}_{ij}^{LR}| \simeq \sqrt{m_i m_j}/M \quad (30)$$

are shown to be consistent with all the known aspects of weak interaction phenomenology. Flavor violating couplings may indeed increase with the masses of the fermions involved!

(iv) The phenomenological implications of such NFCC of H^0 and Z^0 on the flavor changing neutral rare processes are very rich and are well summarized in TABLE 3.

(v) The EDM of the neutron is seen to arise at the one-loop level, with the magnitude compatible with the present upper limit of 10^{-25} e-cm, and may take any value around or below this value.

SUMMARY

The observed large $B_d^0 - \bar{B}_d^0$ mixing is proposed to be due to the tree-level flavor-changing neutral coupling of the standard model Higgs scalar, H^0 , or the Z^0 , induced by new physics with a mass scale beyond the standard model. The strengths of the flavor-changing couplings of H^0 and Z^0 are shown to be increasing with the masses of the fermion flavors involved. If the observed $B_d^0 - \bar{B}_d^0$ mixing is due to the flavor-changing coupling of H^0 , the key predictions are $D^0 - \bar{D}^0$ mixing of $O(10\%)$ of the present experimental upper limit and $BR(\mu^- \rightarrow e^- \gamma) \simeq (1.1 \pm 0.6) \times 10^{-12}$, and the mass of the Higgs scalar $M_H \simeq (200 - 300)$ GeV. In case the observed $B_d^0 - \bar{B}_d^0$ mixing is due to the flavor-changing coupling of the Z^0 , the rare decay mode $\mu^- \rightarrow e^- e^+ e^-$ is predicted to be observable at any time in the near future with the branching ratio in the neighborhood of the present experimental upper limit, while other predictions include: $D^0 - \bar{D}^0$ mixing of $O(1 - 10)\%$ of the present upper limit, $BR(B_s^0 \rightarrow \mu^+ \mu^- X) \simeq (8.5 \pm 4.2) \times 10^{-5}$, $BR(\tau^- \rightarrow \mu^- \mu^+ \mu^-) \simeq (8.8 \pm 4.8) \times 10^{-8}$,

and the branching ratios for the flavor-changing decay modes of Z^0 , $BR(Z^0 \rightarrow b\bar{s} + s\bar{b}) \times 10^7 \simeq (14 \pm 7)$, $BR(Z^0 \rightarrow t\bar{c} + c\bar{t}) \times 10^7 \simeq (1500 \pm 700)$ ($m_t/60$ GeV), $BR(Z^0 \rightarrow b'\bar{b} + b\bar{b}') \times 10^7 \simeq (4800 \pm 2300)$ ($m_{b'}/50$ GeV), $BR(Z^0 \rightarrow \mu^-\tau^+ + \mu^+\tau^-) \times 10^7 \simeq 3.6 \pm 1.8$, and $BR(Z^0 \rightarrow \tau^+\tau^- + \bar{\tau}^+\tau^-) \times 10^7 \simeq (1300 \pm 600)$ ($m_{\tau'}/40$ GeV). These flavor-changing branching ratios of Z^0 can be tested at LEP with $10^7 Z^0$'s. From the observed strength of $B_d^0 - \bar{B}_d^0$ mixing the scale of new physics can be inferred to be $M \simeq 250$ GeV. The electric dipole moment of the neutron is seen to arise at the one-loop level, with the magnitude compatible with the present upper limit of 10^{-25} e-cm.

ACKNOWLEDGMENTS

We thank Gordon Shaw and Bill Molzon for the valuable discussions when the original work¹⁶ on this subject was done. We also thank the organizers of this conference.

REFERENCES

1. ARGUS Collab., H. ALBRECHT *et al.* 1987. Phys. Lett. **192B**: 245.
2. CLEO Collab. 1988. XXIV Int. Conf. on High Energy Physics, Munich.
3. For a recent review, see FRANZINI, P. J. 1988. CERN-TH-5083, and references therein.
4. SHIN, M. 1984. Phys. Lett. **145B**: 285; *ibid.* **191B**: 464.
5. Particle Data Group. 1988. Phys. Lett. **204B**: 1.
6. SHIN, M. 1988. Phys. Lett. **201B**: 559; SHIN, M., R. S. CHIVUKULA & J. FLYNN. 1986. Nucl. Phys. **271B**: 509; See also SANDA, A. in this volume.
7. FISHBANE, P. M., K. GAEMERS, S. MESHKOV & R. E. NORTON. 1985. Phys. Rev. **32D**: 1186, and references therein.
8. For a recent review, see HEWETT, J. M. & T. G. RIZZO. 1989. Phys. Rep. In press, and references therein.
9. GELL-MANN, M., P. RAMOND & R. SLANSKY. 1979. In Supergravity P. VAN NIEUWENHUIZEN & D. Z. FREEDMAN, Eds. Amsterdam: North-Holland Pub. Co.; WILCEK, F. & A. ZEE. 1982. Phys. Rev. **25D**: 553; BAGGER, J. & S. DIMOPOULOS. 1984. Nucl. Phys. **244B**: 247.
10. LOUIS, W. C. *et al.* 1986. Phys. Rev. Lett. **56**: 1027.
11. BEAN, A. *et al.* 1987. Phys. Rev. **35D**: 3533.
12. EICHLER, R. 1987. Nucl. Phys. Proc. Suppl. **3B**: 39.
13. KINOSHITA, T., B. NIZIC & Y. OKAMOTO. 1984. Phys. Rev. Lett. **52**: 717.
14. SHIN, M., M. BANDER & D. SILVERMAN. 1989. In preparation.
15. GLASHOW, S. L., J. ILIOPOULOS & L. MAIANI. 1970. Phys. Rev. **2D**: 1285.
16. SHIN, M., M. BANDER & D. SILVERMAN. 1989. Phys. Lett. **219B**: 381.
17. FRITZSCH, H. 1978. Phys. Lett. **73B** 317; 1979. Nucl. Phys. **155B**: 189.
18. GANAPATHI, V., T. WEILER, E. LAERMANN, I. SCHMITT & P. M. ZERWAS. 1983. Phys. Rev. **27D**: 579; CLEMENTS, M., C. FOOTMAN, A. KRONFELD, S. NARASIMHAN & D. PHOTIADIS. 1983. *ibid.* **27D**: 570; AXELROD, A. 1982. Nucl. Phys. **209B**: 349.
19. SHIN, M., M. BANDER, G. L. SHAW & D. SILVERMAN. 1989. In preparation. Constraints on the CP-violating phases will be considered from ϵ_K , ϵ'/ϵ , etc.
20. For a recent review, see HE, X-G., B. H. J. MCKELLAR & S. PAKVASA. 1989. Preprint UH-511-666-89.
21. ALTAREV, I. S., *et al.* 1986. JEPT Lett. **44**: 460; HECKEL, B. R. 1988. Bull. Am. Phys. Soc. **33**: 1549.