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Extreme Astrophysical Systems as Windows on High-Energy and Gravitational Phenomena

A dissertation submitted in partial satisfaction
of the requirements for the degree

Doctor of Philosophy
in
Physics

by

Joseph R. Farah

Committee in charge:

Dr. D. Andrew Howell, Adjunct Professor, Co-Chair
Professor Lars Bildsten, Co-Chair
Professor Maxwell Millar-Blanchaer

June 2026

The Dissertation of Joseph R. Farah is approved.

Professor Maxwell Millar-Blanchaer

Professor Lars Bildsten, Co-Chair

Dr. D. Andrew Howell, Adjunct Professor, Co-Chair

May 2026

Extreme Astrophysical Systems as Windows on
High-Energy and Gravitational Phenomena

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by

Joseph R. Farah

For my father, and my family

“

FOR KNOWLEDGE IS YOUR CROWN, AND UNDERSTANDING YOUR STAFF; AND WHEN THEY
ARE WITH YOU, YOU CAN POSSESS NO GREATER TREASURES.

—Gibran Khalil Gibran, *The Prophet*, 1923



“

THOUGH MY SOUL MAY SET IN DARKNESS, IT WILL RISE IN PERFECT LIGHT;
I HAVE LOVED THE STARS TOO TRULY TO BE FEARFUL OF THE NIGHT.

— Sarah Williams, “The Old Astronomer To His Pupil”, 1868



Acknowledgements

“

DON'T WORRY, JUST DO THE SAME FOR YOUR STUDENTS.

—*Prof. Melissa Eva Bronwen Franklin*

„

In 2005, my mother and father sat me in front of our little 19" CRT television and turned on an episode of the PBS programme *NOVA* called “Welcome to Mars”. For reasons I don’t think I will ever fully come to understand, watching the women and men of the Mars missions pilot a little robot one hundred and forty-two million miles away over to some water-tossed blueberries in a crater wall inspired a deep love of science and the unknown in me. That day was the start of a lifelong dream to become a scientist, a dream that that has been held aloft by a thousand hands. I spend most of my time these days overwhelmed with gratitude for the friends, family, and mentors who have selflessly helped me pursue this wild ambition. And there is no part of this dissertation that I am more excited or eager to write than this letter of thanks to the people in my life who have helped me come to this moment.

I begin with my family. I thank my wife Sissi; my mother and father, Mai and Rachid; my brother Albert and my sister Yuna; and my brother-in-arms, Jonathan. Mom, I will never forget our days spent at the Museum of Science, as you fanned the flames of my burgeoning love of all things space with our trips to the Mugar Omni Theater and the Charles Hayden Planetarium. Dad, thank you for the undying support at every step, including in my toughest moments when I felt I had no one else. You are my rock, in all things, for all time. Sissi, thank you for teaching me about love, life, and

living, and for your selfless support so I could finish my Ph.D. All we have to decide is what to do with the time that is given us, and I will never stop being grateful that you decided to spend that time with me.

Next, I want to thank the advisors that have guided and encouraged me through my studies. First and foremost, I owe a debt of unbelievable magnitude to my doctoral advisor, Andy Howell. They say the advisor makes or breaks the Ph.D., and I know for an absolute certainty that my Ph.D. was as fun and amazing as it was because of you. You fought to give me the best chance of success every step of the way, and you were exactly the advisor I needed to grow into the best scientist I could be. There are too many instances to count, but, in particular, the extraordinary lengths to which you went to ensure my Miller Fellowship nomination was submitted on time reflect a dedication to your students I will never forget, and after everything is said and done, I would run through a brick wall for you.

I also want to thank my long-time EHT advisor, Michael Johnson. At this point we've been collaborating for almost a decade, and you remain one of the most brilliant scientists I have ever met. Thank you for sending me to possibly the most beautiful place in the world. More than once since we met, when my path forward was unclear, the hope that I could find a way to work with you again was my anchor.

Finally, I want to thank my first real research advisor, Melissa Franklin. Thank you for taking a chance on me when no one else would; for being an endless repository of wisdom and advice over the years; and for being a great friend!

“

DO THINGS WITH A SHELF LIFE OF 500 YEARS.

—*Prof. Michael Douglass Johnson*

The people and culture of Las Cumbres Observatory are incredible and are the best part of working there. In particular I'd like to thank Curtis McCully for so much advice over the years and continuously helping me stay on the right track as I navigated many challenges of the Ph.D. I'd also like to thank, in no particular order, the many wonderful people of LCO and KITP, including but not limited to: Logan, my labmates and postdocs/scientists current and former, Moira, Kathryn, Jake, Giacomo, Chris, Megan, Craig, Este, Jamie, Daichi, Griffin, Rachel, Tim, Katarzyna, and Carrie; the amazing leadership of LCO, particularly Lisa and Wayne; and the talented engineers and scientists, especially Lindy, Emily (thank you for laughing at every one of my terrible joke pitches during AoT rehearsals!), Joey, and Carolina. And of course, thank you to Sandy Seale, one of the most remarkable people I have ever met; I count myself incredibly lucky to call you a close friend and I am so grateful for your support over the years.

Throughout the last eight years, I have been very fortunate to have close collaborations, mentorships, and friendships with members of the black hole physics and VLBI community, which has allowed me to continue being involved in this exciting sphere regardless of institution. I especially want to thank Alex Lupsasca for his generous mentorship of my work on the BHEX project, despite being on the opposite side of the country. You're going to be a wonderful dad. I also want to thank, in no particular order: David James, Andrew Chael, Daniel Palumbo, Maciek Wielgus, Norbert Bartel, Katie Bouman, Dom Pesce, Michael Bietenholz, Zack Gelles, Lindy Blackburn, Sara Issaoun, Peter Galison, Janice Houston, Dan Marrone, Jim Moran, Shep Doleman, Ramesh Narayan, Kazu Akiyama, and Jordy Davelaar (especially, thank you Jordy for the many memes that kept me going through grad school!) I have also been so grateful to benefit from friends and mentors across science and university, and I want to say thank you to Ann Wang, Alex Tuna, Tony Tong, David Stuart, Gary Horowitz, as well

as Anna Theogarajan and Grace Kimball of the UCSB Grad Slam team.

//

DON'T TRY TO GAME IT. JUST DO THE RIGHT THING.

—*Prof. Dale Andrew Howell*



My personal journey to this moment has been supported by so many teachers, professors and advisors, the following non-exhaustive list of whom I want to highlight: Meghan Coyne, Rocco Cieri, Michael Wadness, Catherine Newman, Patricia Brandl, Douglass Bowen-Flynn, Barbara Chyen, Stephen Arnason, Rahul Kulkarni, Chris Fuchs, Bala Sundaram, Alfred Noel (thank you for teaching me how to “close the game”!), Katherine Newman, Alioscia Hamma, and Adolfo del Campo. I have also had the pleasure and privilege of supporting and mentoring several amazing students as well, whom I would like to thank for their hard work and for inspiring and teaching me with every interaction, especially Courtney, Frank, Songgun, Tazzy, Sanjit, and Matthew.

I am extremely lucky to have a wonderful support system of friends who have helped me keep my chin up when things get tough, as they are wont to do. Thank you to my grad school friends, Liam, Joaquin, Majid (Majid, you continue to inspire me every single day), Diego, James, Jake (thank you for being there when I needed it the most), Mary, and Ryan; my undergrad friends Will, Meg L. (thank you for introducing me to *The Good Place*, which is now my comfort show and practically got me through grad school), Steven, Sarah, Meghan C., Erjon, and especially Matt, Scott, Andrew, and Nat, whom I have known since kindergarten and will cherish forever. I also want

to thank my therapist Shuriee G.-W.; Zack K., my coach; and my barber, Michelle C., who saved me with last-minute haircuts before unexpected academic events too many times to count.

I have also been honored by the support and help of too many administration professionals to count, and I want to especially thank Muriel Hodges, Barbara Elfman, Jennifer Farrar, Sarah Rettinger, Melissa Renda, Nan Brooks, and Nicole Grenier.

Finally, I would like to thank the additional members of my committee: Lars Bildsten, Maxwell-Millar Blanchaer, and Tim Brandt.

Curriculum Vitæ
Joseph R. Farah

Full CV begins on next page.

Joseph R. Farah

CONTACT INFORMATION	Las Cumbres Observatory and Global Telescope 6740 Cortona Dr Goleta, CA 93117 USA	<i>ORCID:</i> 0000-0003-4914-5625 <i>E-mail:</i> jfarah@lco.global <i>Website:</i> http://jrfarah.com <i>Citations:</i> Google Scholar profile
RESEARCH INTERESTS	Astrophysics, supernovae, photon ring, radio astronomy, black holes, magnetars, very long baseline interferometry, space VLBI, strong gravity, Bayesian modeling, magnetohydrodynamics, frameworks, machine learning, deep learning	
EDUCATION	University of California Santa Barbara , Santa Barbara, CA, USA	2021–2026
	<i>Ph.D. in Physics (in progress)</i>	
	<i>M.A. in Physics (completed)</i>	2023
	University of Massachusetts Boston , Boston, MA, USA	2017–2021
SELECTED HONORS AND AWARDS	Miller Fellowship , Miller Institute for Basic Science, Berkeley	2026–2029
	Bhaumik Prize Postdoctoral Fellowship, UCLA (declined)	2026
	UC Santa Barbara Mananya Tantiwivat Award	2025
	Broida-Hirschfelder Fellowship	2025
	<i>CITATION: In recognition of his project uncovering the secrets of the strangest object in the universe, the black hole photon ring.</i>	
	1 st Place Award, UC Santa Barbara Grad Slam	2025
	<i>Round 1 (Physics and Chemistry)</i>	
	UC Santa Barbara GSA Excellence in Teaching Award (nominated)	2025
	UC Santa Barbara Academic Senate Outstanding Teaching Assistant Award (nominated)	2024
	Event Horizon Telescope Early Career Award	2022
	<i>CITATION: For his leadership and contribution to the dynamic imaging of the interferometric data used to image Sgr A*.</i>	
	LeRoy Apker Award	2021
	<i>CITATION: For the invention of the selective dynamical imaging method, with applications for studying rapidly-varying black holes.</i>	
	UC Santa Barbara James and Mary Jo Hartle Graduate Fellowship	2021
	National Science Foundation Graduate Research Fellowship	2021–2026
	Arthur W. Martin III Scholarship	2021
	Breakthrough Prize in Fundamental Physics (co-recipient)	2020
	Northrop Grumman Scholarship	2020
	Barry M. Goldwater Scholarship	2019
	National Science Foundation Diamond Achievement Award [‡]	2019
	Alton J. Brann Endowed Scholarship	2019

	Smithsonian Fellowship	2018, 2019
	Oracle Fellowship	2018, 2019
	AJAS/AAAS Lifetime Fellowship	2018
	Chancellor's Scholarship, University of Massachusetts Boston	2017–2021
	2 nd Place Award, Massachusetts State Science and Engineering Fair	2017
	1 st Place Award, Massachusetts State Science and Engineering Fair	2016
SELECTED WORK AND RESEARCH EXPERIENCE	Las Cumbres Observatory	2021 - Present
	<i>NSF Graduate Research Fellowship — Goleta, CA</i>	
	I use the LCO global network of telescopes to perform optical and spectroscopic follow-up of a wide variety of supernovae and transients. Using rapid follow-up observations combined with new shock cooling physics, I seek to better characterize and understand the mass stripping mechanism of Type IIb supernovae, and the underlying mechanisms powering superluminous supernovae. Additionally, I help with infrastructure maintenance and development for the robotic LCO platform.	
	<i>Supervisor: Andy Howell</i>	
	Harvard-Smithsonian Center for Astrophysics	
	<i>Member of the Black Hole Explorer Collaboration — Cambridge, MA</i>	2020 - Present
	I develop analytic and deep learning techniques for measuring physical properties from the black hole's photon ring. Along with the BHEX collaboration, I assist in the preparation and development of the BHEX spacecraft, an orbiting radio telescope which will interface with the EHT and use space VLBI to make high resolution images and movies of black holes.	
	<i>Supervisor: Alex Lupsasca</i>	
	<i>Smithsonian Fellowship with the Event Horizon Telescope — Cambridge, MA</i>	2018 - 2021
	I image supermassive black holes and assist in observations using the Event Horizon Telescope in order to help take the first ever pictures of black holes. I explored alternative parametric representations of shadows in the Kerr metric and non-standard GR theories, and methods constraining the angular momentum of rotating black holes in EHT data using image- and Fourier-domain feature extraction methods. As an undergraduate, I was the first and only junior collaborator of the EHT Consortium.	
	<i>Supervisor: Michael Johnson</i>	
	Quantum Computing Research Group	2019 - 2021
	<i>Research Intern with Dr. Alioscia Hama — Boston, MA</i>	
	Harvard Laboratory for Particle Physics and Cosmology	2017 - 2018
	<i>Research Intern with Dr. Melissa Franklin — Cambridge, MA</i>	
	QBism Research Group	2017 - 2019
	<i>Oracle Fellow with Dr. Christopher Fuchs — Boston, MA</i>	
	Harvard School for Engineering and Applied Sciences	Summer 2016

Summer Research Intern mentored by Dr. Chinwendu Enyioha — Cambridge, MA

Tufts Department of Astrophysics Summer 2015
Summer Research Intern mentored by Dr. Anna Sajina — Medford, MA

STUDENTS
MENTORED

Undergraduate

Courtney Duong (UC Santa Barbara) 2024 - Present
 Project: Convolutional neural network for extraction of the $n = 1$ black hole photon ring
 Award: *Gene and Susan Lucas Undergraduate Research Scholarship*
 Award: *Edison STEM Research Scholarship*

Franklin Myhre (UC Santa Barbara) 2024 - Present
 Project: Development of a `python` library for automated image-domain subring extraction

Sanjit Masanam (UC Santa Barbara) 2024 - Present
 Award: *2025 Goldwater Scholarship (nominated)*
 Award: *2025 Ernest F. Hollings Scholarship*

Songgun Lee (UC Santa Barbara) 2024 - Present
Co-founder at Masterminding

Tazzy Imbabi (UC Santa Barbara) 2024
 Project: Exploration of selective dynamical imaging applications to BHEX

INVITED TALKS,
PANELS, AND
FILMOGRAPHY

(last updated: Sep 2025)

NB: for a list of interviews, please see [my online CV](#).

General relativity beats the heart of a dying star. Santa Barbara Museum of Natural History. Invited speaker for public lecture. March 2026. [Event URL](#).

Presentation presentation: a seminar on seminars. Las Cumbres Observatory. Invited to lead workshop. Jan 2026.

General relativity meets a supernova for the first time. Las Cumbres Observatory. Invited speaker for colloquium. Jan 2026. [Event URL](#).

The first 4 years of SN 1993J revisited. UC Berkeley, Berkeley, CA. Invited seminar speaker. September 2025.

Lense-Thirring precessing magnetar engine drives a superluminous supernova. UC Berkeley, Berkeley, CA. Invited speaker for Multi-RAPTOR colloquium. September 2025.

Extragalactic science with the Vera C. Rubin Observatory. Santa Barbara Museum of Natural History. Invited speaker for public lecture. August 2025. [Event URL](#).

Probing fundamental physics using high-energy astrophysical phenomena. Goleta, CA. Invited speaker for the Las Cumbres Observatory Science Advisory Committee. August 2025.

Machine- and deep-learning-driven angular momentum inference from Black Hole Explorer observations of the $n=1$ photon subring. *Topics in Gravitational Physics*. Presentation at the APS Global Physics Summit. March 2025. [Event URL](#).

Current and Future Impacts of Survey Capabilities on the Stripped Envelope Problem. Kavli Institute for Theoretical Physics, Santa Barbara, CA. February 2025.

Shock-cooling Constraints via Early-time Observations of the Type IIb SN 2022hnt. Kavli Institute for Theoretical Physics, Santa Barbara, CA. February 2025.

The Black Hole Explorer and the Edge of the Universe. Santa Barbara Museum of Natural History. April 2025. Invited speaker for public lecture. [Event URL](#).

The Black Hole Explorer. Santa Barbara, CA. Invited speaker for *Astronomy on Tap*. December 2024. [Event URL](#).

What does an astronomer do? Canalino Elementary School. October 2024. Invited educational outreach speaker.

Inferring black hole spin from images of the photon ring. Vanderbilt University VandyGRAF initiative. September 2024. Invited speaker.

The Biggest Telescopes! Aliso Elementary School. October 2023. Invited educational outreach speaker.

Imaging the Black Hole at Our Galaxy's Center. Santa Barbara Museum of Natural History. May 2023. Invited speaker for public lecture. [Event URL](#).

LCO's Top 9 Tips For Taking Your Best Black Hole Photo. Medford High School, Medford, Massachusetts. Invited speaker for honors and AP physics classes. 2022.

LCO's Top 9 Tips For Taking Your Best Black Hole Photo. Santa Barbara, CA. Invited speaker for *Astronomy on Tap*. November 2022. [Event URL](#).

Selective Dynamical Imaging of Interferometric Data and the Second Image of a Black Hole. University of California, Santa Barbara, California. Invited speaker for *Astro Tea*. June 2022.

AskScience AMA Series: We're Event Horizon Telescope scientists with groundbreaking results on our own galaxy; Ask Us Anything. *Reddit*. May 2022. [Article URL](#).

What Lies Within: Imaging the Galactic Center. April 2022. April APS Meeting, New York City, New York. Invited speaker for American Physical Society.

The First Image of a Black Hole. Seagrave Memorial Observatory, North Scituate, Rhode Island. Invited speaker for Skyscrapers, Inc. 2021.

Galison, P. L. (Director). 2021. *Black Holes: The Edge of All We Know*. Film. Sandbox Films. [View on Netflix](#).

Very Long Baseline Interferometry and the Event Horizon Telescope. University of Massachusetts, Boston, Massachusetts. Invited speaker for Astronomy 121. 2019 (2 times), 2020 (2 times).

Galison, P. L. (Director). 2019. *Portrait of a Shadow*. Film. Sandbox Films.

Seeing the Unseeable: The First Image of a Black Hole. University of Massachusetts Boston, Boston, Massachusetts. May 2019.

Imaging a Black Hole with the Event Horizon Telescope. Harvard University, Cambridge, MA. April

2019.

PUBLICATIONS
(FIRST-AUTHOR)

Stats (all papers):
CITATIONS: 19,100+
REF'D PAPERS: 118+
h-INDEX: 37
i10-INDEX: 73

8. **J. R. Farah**, Lupsasca, A., Quataert, E., Johnson, M. D. 2025. *Interferometric inference of black hole spin from photon ring size and brightness*. Submitted to ApJ, positive referee report received. [arXiv:2509.23628](#).
7. **J. R. Farah**, Prust, L. J., Howell, D. A., *et al.*. 2025. *Lense-Thirring precessing magnetar engine drives a superluminous supernova*. *Nature*, accepted. [arXiv:2509.08051](#).
6. **J. R. Farah**, Howell, D. A., Hiramatsu, D., *et al.*. 2025. *When IIb Ceases To Be: Bridging the Gap Between IIb and Short-plateau Supernovae*. Submitted to ApJ, positive referee report received. [arXiv:2509.12470](#).
5. **J. R. Farah**, Prust, L. J., Terreran, G., *et al.*. 2025. *The First 4 Years of SN 1993J Revisited: Geometric m-ring Modeling of the Radio Shell with Closure Quantities Only*. arXiv e-prints, [arXiv:2509.20601](#).
4. **J. R. Farah**, Davelaar, J., Palumbo, D., Johnson, M., Delgado, J. 2025. *Machine and Deep Learning-driven Angular Momentum Inference from BHEX Observations of the $n = 1$ Photon Ring*. ApJ, 983, [185](#). [arXiv:2411.01060](#).
3. **J. R. Farah**, Howell, D. A., Terreran, G., *et al.*. 2025. *Shock-cooling Constraints via Early-time Observations of the Type IIb SN 2022hnt*. ApJ, 984, [60](#). [arXiv:2501.17221](#).
2. ^S**J. R. Farah**, Galison, P., Akiyama, K., *et al.*. 2022. *Selective Dynamical Imaging of Interferometric Data*. ApJL, 930, [L18](#). [arXiv:2409.08321](#).
1. **J. R. Farah**, Pesce, D. W., Johnson, M. D., Blackburn, L. 2020. *On the Approximation of the Black Hole Shadow with a Simple Polar Curve*. ApJ, 900, [77](#). [arXiv:2007.06732](#).

PUBLICATIONS
(MAJOR OR
NORMAL
CONTRIBUTION)

(last updated: Sep 2025)

110. Wang, Z. Y., Pastorello, A., Cai, Y. Z., ..., **J. R. Farah**, *et al.*. 2025. *Massive stars exploding in a He-rich circumstellar medium: XI. Diverse evolution of five Ibn SNe 2020nxt, 2020taz, 2021bbv, 2023utc, and 2024aej*. A&A, 700, [A156](#). [arXiv:2506.15139](#).
109. Bostroem, K. A., Valenti, S., Sand, D. J., ..., **J. R. Farah**, *et al.*. 2025. *Late-time Hubble Space Telescope Ultraviolet Spectra of SN 2023ixf and SN 2024ggi Show Ongoing Interaction with Circumstellar Material*. arXiv e-prints, [arXiv:2508.11756](#).
108. Kwok, L. A., Singh, M., Jha, S. W., ..., **J. R. Farah**, *et al.*. 2025. *JWST and Ground-based Observations of the Type Iax Supernovae SN 2024pxl and SN 2024vjm: Evidence for Weak Deflagration Explosions*. ApJL, 989, [L33](#). [arXiv:2505.02944](#).
107. Gagliano, A., Villar, V., Matsumoto, T., ..., **J. R. Farah**, *et al.*. 2025. *Evidence for an Instability-induced Binary Merger in the Double-peaked, Helium-rich Type II_n Supernova 2023zkd*. ApJ, 989, [182](#). [arXiv:2502.19469](#).
106. Śniegowska, M., Trakhtenbrot, B., Makrygianni, L., ..., **J. R. Farah**, *et al.*. 2025. *AT 2019aalc*:

^SThis was the seventh paper of the 10-paper first Galactic Center results series from the Event Horizon Telescope, presenting the second image of a black hole and the first horizon-scale images of Sgr A*.

- A Bowen Fluorescence Flare with a Precursor Flare in an Active Galactic Nucleus.* ApJ, 989, 173. [arXiv:2505.00083](#).
105. Makrygianni, L., Arcavi, I., Newsome, M., ..., **J. R. Farah**, *et al.*. 2025. *The Double Tidal Disruption Event AT 2022dbl Implies that at Least Some “Standard” Optical Tidal Disruption Events Are Partial Disruptions.* ApJL, 987, L20. [arXiv:2505.16867](#).
104. Dahale, R., Cho, I., Moriyama, K., ..., **J. R. Farah**, *et al.*. 2025. *Origin of the ring ellipticity in the black hole images of M87*.* A&A, 699, A279. [arXiv:2505.10333](#).
103. Park, S. H., Rho, J., Yoon, S. C., ..., **J. R. Farah**, *et al.*. 2025. *Near-Infrared Spectroscopy and Detection of Carbon Monoxide in the Type II Supernova SN 2023ixf.* arXiv e-prints, [arXiv:2507.11877](#).
102. Goddi, C., Carlos, D. F., Crew, G. B., ..., **J. R. Farah**, *et al.*. 2025. *First polarization study of the M87 jet and active galactic nuclei at submillimeter wavelengths with ALMA.* A&A, 699, A265. [arXiv:2505.10181](#).
101. Jiang, S. Q., Xu, D., van Hoof, A. P., ..., **J. R. Farah**, *et al.*. 2025. *EP240801a/XRF 240801B: An X-Ray Flash Detected by the Einstein Probe and the Implications of Its Multiband Afterglow.* ApJL, 988, L34. [arXiv:2503.04306](#).
100. Sun, H., Li, W. X., Liu, L. D., ..., **J. R. Farah**, *et al.*. 2025. *A fast X-ray transient from a weak relativistic jet associated with a type Ic-BL supernova.* Nature Astronomy, 9, 1073-1085. [arXiv:2410.02315](#).
99. Kumar, H., Berger, E., Blanchard, P. K., ..., **J. R. Farah**, *et al.*. 2025. *A Near-infrared Search for Helium in the Superluminous Supernova SN 2024ahr.* ApJ, 987, 127. [arXiv:2501.01485](#).
98. Singh, M., Kwok, L. A., Jha, S. W., ..., **J. R. Farah**, *et al.*. 2025. *Photometry and Spectroscopy of SN 2024pxl: A Luminosity Link Among Type Ia_x Supernovae.* arXiv e-prints, [arXiv:2505.02943](#).
97. Jacobson-Galán, W., Dessart, L., Davis, K., *et al.*. 2025. *Final Moments III: Explosion Properties and Progenitor Constraints of CSM-Interacting Type II Supernovae.* arXiv e-prints, [arXiv:2505.04698](#).
96. Liu, M., Zhang, Y., Wang, Y., ..., **J. R. Farah**, *et al.*. 2025. *Detection of an Orphan X-Ray Flare from a Blazar Candidate EP240709a with the Einstein Probe.* ApJ, 984, 5. [arXiv:2412.18463](#).
95. Shrestha, M., DeSoto, S., Sand, D. J., ..., **J. R. Farah**, *et al.*. 2025. *Spectropolarimetry of SN 2023ixf Reveals Both Circumstellar Material and an Aspherical Helium Core.* ApJL, 982, L32. [arXiv:2410.08199](#).
94. Ravi, A. P., Valenti, S., Dong, Y., ..., **J. R. Farah**, *et al.*. 2025. *Luminous Type II Short-plateau SN 2023ufx: Asymmetric Explosion of a Partially Stripped Massive Progenitor.* ApJ, 982, 12. [arXiv:2411.02493](#).
93. Röder, J., Wielgus, M., Lobanov, A. P., ..., **J. R. Farah**, *et al.*. 2025. *A multifrequency study of sub-parsec jets with the Event Horizon Telescope.* A&A, 695, A233. [arXiv:2501.05518](#).
92. Event Horizon Telescope Collaboration, ..., **J. R. Farah**, *et al.*. 2025. *The persistent shadow of the supermassive black hole of M87: II. Model comparisons and theoretical interpretations.* A&A, 693, A265.

91. Yesmin, N., Pellegrino, C., Modjaz, M., ..., **J. R. Farah**, *et al.*. 2025. *Spectral dataset of young type Ib supernovae and their time evolution*. A&A, 693, A307. [arXiv:2409.04522](#).
90. Onori, F., Nicholl, M., Ramsden, P., ..., **J. R. Farah**, *et al.*. 2025. *The case of AT2022wtn: a tidal disruption event in an interacting galaxy*. MNRAS, 540, 498-520. [arXiv:2504.21686](#).
89. Ma, X., Wang, X., Mo, J., ..., **J. R. Farah**, *et al.*. 2025. *Supernovae at distances ≤ 40 Mpc: II. Supernova rate in the local Universe*. A&A, 698, A306. [arXiv:2504.04507](#).
88. Ma, X., Wang, X., Mo, J., ..., **J. R. Farah**, *et al.*. 2025. *Supernovae at distances ≤ 40 Mpc: I. Catalogues and fractions of supernovae in a complete sample*. A&A, 698, A305. [arXiv:2504.04393](#).
87. Zeng, X., Luo, Y., Wang, X., ..., **J. R. Farah**, *et al.*. 2025. *SN 2023xqm: A gradually fading Ia supernova exhibiting isolated high-velocity signatures*. A&A, 698, A70.
86. Zeng, X., Hu, L., Zheng, S., ..., **J. R. Farah**, *et al.*. 2025. *SN 2023ehl: A Normal Type Ia Supernova with High-velocity Features*. ApJ, 986, 68.
85. Hsu, B., Smith, N., Goldberg, J. A., ..., **J. R. Farah**, *et al.*. 2025. *One Year of SN 2023ixf: Breaking through the Degenerate Parameter Space in Light-curve Models with Pulsating Progenitors*. ApJ, 990, 148. [arXiv:2408.07874](#).
84. Palit, B., Śniegowska, M., Markowitz, A., ..., **J. R. Farah**, *et al.*. 2025. *Markarian 590: the AGN awakens*. MNRAS, 540, L14-L20. [arXiv:2501.07225](#).
83. Subrayan, B. M., Sand, D. J., Bostroem, K. A., ..., **J. R. Farah**, *et al.*. 2025. *Early Shock Cooling Observations and Progenitor Constraints of Type IIb Supernova SN 2024uwq*. ApJL, 990, L68. [arXiv:2505.02908](#).
82. Shu, X., Yang, L., Yang, H., ..., **J. R. Farah**, *et al.*. 2025. *EP241021a: A Months-duration X-Ray Transient with Luminous Optical and Radio Emission*. ApJL, 990, L29. [arXiv:2505.07665](#).
81. Andrews, J. E., Shrestha, M., Bostroem, K. A., ..., **J. R. Farah**, *et al.*. 2025. *Asymmetries and Circumstellar Interaction in the Type II SN 2024bch*. ApJ, 980, 37. [arXiv:2411.02497](#).
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Abstract

Extreme Astrophysical Systems as Windows on High-Energy and Gravitational Phenomena

by

Joseph R. Farah

The most extreme phenomena in the universe present the most intriguing opportunities to test our understanding of physics in environments no laboratory on Earth can simulate. In this dissertation, I present a series of observational and theoretical studies of extreme astrophysical systems, spanning infant supernovae and horizon-scale black hole physics. First, I investigate the earliest phases of core-collapse supernovae in both optical and radio wavelengths, describing and applying a framework for the robust application of shock-cooling models across the hydrogen-rich envelope mass spectrum. Next, I consider theoretical properties of the black hole shadow in general relativity, and extend these to perform angular momentum inference using modern very-long-baseline interferometers in dynamically evolving horizon-scale systems. These results directly contribute to the second Event Horizon Telescope image of a black hole (the first image of Sagittarius A*) and help explore the science case for the future Black Hole Explorer mission. Finally, this dissertation bridges the domains of supernovae astrophysics and general relativity by identifying and characterizing the first supernova to show evidence of a relativistic frame-dragging central engine in its light curve, providing observational confirmation of the magnetar model for superluminous supernovae.

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Part I

Infant supernova physics and progenitor inference

Chapter 1

Introduction to infant core-collapse supernovae

“

I WAS CONTEMPLATING THE STARS IN A CLEAR SKY...I NOTICED THAT A NEW AND UNUSUAL STAR, SURPASSING THE OTHERS IN BRILLIANCY, WAS SHINING ALMOST DIRECTLY ABOVE MY HEAD; AND...IT WAS QUITE EVIDENT TO ME THAT THERE HAD NEVER BEEN ANY STAR IN THAT PLACE IN THE SKY. I WAS SO ASTONISHED AT THIS SIGHT...A MIRACLE INDEED, ONE THAT HAS NEVER BEEN PREVIOUSLY SEEN BEFORE OUR TIME, IN ANY AGE SINCE THE BEGINNING OF THE WORLD.

— Tycho Brahe, De Nova Stella, 1573 (observing the supernova of 1572)

The vicinity of an infant supernova is one of the most extreme environments in the Universe, with temperatures regularly exceeding $\sim 10^{10}$ K in the first few milliseconds

[6], expanding shock velocities approaching weakly relativistic speeds [$v_s \sim 0.05c$; 7], central density exceeding nuclear saturation [$\rho_c \gtrsim 10^{14} \text{ g cm}^{-3}$ 6; 8], and $\sim 10^{53}$ ergs of energy released almost exclusively via neutrinos [9]. Their interiors may be illuminated by neutron stars [10; 11] with magnetic fields approaching or exceeding the Schwinger limit [12; 13], or rapidly accreting black holes [14; 15]. Due to this, supernovae provide environments that no laboratory on Earth could possibly reproduce, and thus represent unique opportunities to probe fundamental physics. In particular, the neutrino burst accompanying core collapse encodes the equation-of-state of dense nuclear matter at beyond-nuclear densities, while the resulting explosion sets the chemical yields that drive the enrichment of subsequent stellar generations. Indeed, significant theoretical and observational work has been organized towards achieving a complete understanding of the behavior and evolution of supernovae at their earliest phases. In this chapter, we review the post-explosion mechanisms and corresponding literature for the flavors of core-collapse supernovae investigated in this dissertation.

1.1 Overview of supernovae channels

Broadly, supernovae can be organized into two pathways to explosion, or “channels”: (i) the thermonuclear explosion of a white dwarf star in a binary system, and (ii) the core-collapse of an end-of-life massive star. This work is primarily concerned with the second category.

A massive late-stage star achieves hydrostatic equilibrium through the pressure of its hot gas and radiation, with nuclear fusion in the core providing the power that matches that lost from the surface due to radiation. Once the core reaches the iron binding energy peak, further nuclear burning becomes endothermic. At the temperatures required for silicon burning ($T \gtrsim 3 \times 10^9 \text{ K}$), photodisintegration dissociates iron

nuclei back into free nucleons, absorbing thermal energy and reducing the pressure support of the core. Simultaneously, electron capture onto disassociated protons and iron nuclei further depletes the electron degeneracy pressure. Together, these processes trigger a rapid and catastrophic collapse of the core. The stellar material falls inwards on the dynamical timescale ($\sim 1/\sqrt{G\rho} \sim 30\text{--}120$ ms for the iron core with density $\rho \sim 10^{9-10}$ g cm $^{-3}$) and “bounces” off the core. The immense pressure from the “bounce” compacts the core further and can create a hyperdense stellar remnant known as a proto-neutron star; depending on the mass of the progenitor, this may survive as a neutron star or collapse further into a black hole [6]. The shockwave reflects off the core and traverses back through the star on a timescale of roughly R_0/v (where R_0 is the radius of the progenitor and v is the velocity of the shock) superheating the envelope of the star until it reaches the “edge” of the star, where the optical depth τ drops significantly and the shock is no longer supported [7]. This picture is somewhat simplified; in special cases, surrounding material (the circumstellar medium, or CSM) may prolong the shock breakout by raising the optical depth enough for the shock to be communicated beyond the edge of the star [see e.g., 16, for a review]. Additionally, emission from decay of radioactive elements (e.g., the unstable Nickel-56 (^{56}Ni) isotope) synthesized during the supernova contributes luminosity to the light curve, but this emission is usually dominated at early times ($\lesssim$ a few weeks) by emission from the shock and interaction.

The envelope of the star, now shocked and unbound, begins to expand (it is now *ejecta*) and cool. Radiation deposited by the passing shock is trapped by the high density of the ejecta immediately after shock breakout; only photons at the very edge of the star ($\tau \sim 1$, the photosphere) can escape. As the ejecta expands, the transition between the adiabatic and non-adiabatic region of the star ($\tau \sim c/v \sim 30$) moves inwards (in comoving coordinates, e.g., mass coordinates) through the ejecta along

with the photosphere, exposing more radiation that was previously trapped [17, and citations within]. Depending on the amount of hydrogen in the envelope at the time of explosion, the amount of radiation released can vary quite significantly, partially contributing to the wide diversity of core-collapse supernovae. We expand on some of these variations below in [section 1.2](#).

Throughout the supernova, ^{56}Ni synthesized in the initial explosion decays into ^{56}Co (on a half-life timescale of ≈ 6.1 days) and then finally ^{56}Fe (on a half-life timescale of ≈ 77.7 days). The emission from this decay contributes to the overall luminosity, but is significantly dwarfed in core-collapse supernovae by the emission produced from energy deposited by the shock. At later times, as the ejecta becomes sufficiently optically thin for radiation from ^{56}Ni decay to escape and deposit energy in the observable layers, radioactive decay becomes the dominant driver of the luminosity. However, during the plateau phase of hydrogen-rich supernovae, this contribution is subdominant to the release of shock-deposited radiation from the extended hydrogen envelope [18; 19].

1.2 Type II supernovae

Supernovae are classified as Type II based on the presence of hydrogen in their spectra [20]. Here, we briefly review the diversity of Type II supernovae, which is largely driven by the amount of hydrogen remaining in the stellar envelope ($M_{\text{H}_{\text{env}}}$) prior to explosion ([Figure 1.1](#)), and consider open questions driven by this diversity. The boundaries of the different classifications are not universally agreed upon; however, we refer to [3] for the mass ranges used below.

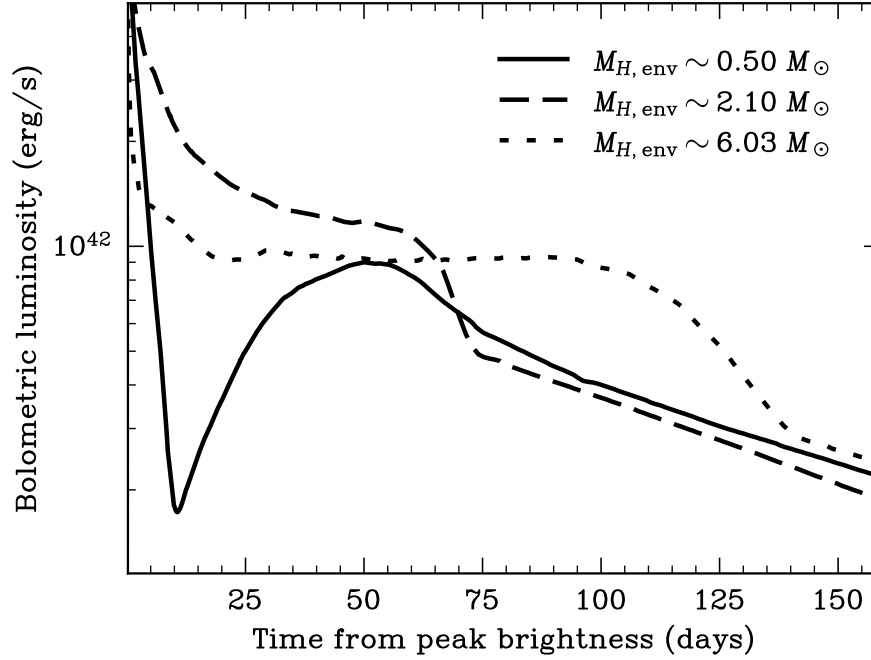


Figure 1.1: Comparison of bolometric luminosity light curves for core-collapse supernovae. Keeping all other parameters the same and only varying the hydrogen-rich envelope mass, we can reproduce the typical morphologies of the Type II-plateau, short-plateau and IIb subclasses. A large amount of hydrogen ($M_{H,env} \gg 2 M_{\odot}$) creates an extended plateau of ~ 100 days; as the hydrogen is reduced ($M_{H,env} \lesssim 2 M_{\odot}$) the plateau gets shorter until it disappears altogether, leaving the primarily ^{56}Ni -driven peak and early excess emission. These simulations are reproduced using the library generated in [3]. All light curves are aligned so $t = 0$ corresponds to the peak luminosity.

1.2.1 Type II-Plateau (IIP) and short-plateau

When the progenitor star has $M_{H,env} \gtrsim 4.5 M_{\odot}$, there is sufficient hydrogen to sustain a ~ 100 day plateau, during which the photosphere recedes through the envelope at the recombination front, releasing the radiation deposited by the shock [21]. The photosphere recedes inwards in comoving coordinates at approximately the same rate as the supernova is expanding, leading to a roughly flat plateau in the light curve. The plateau luminosity is set by the energy of the explosion, the radius of the progenitor

star, and the mass of the envelope; the canonical relationship is [22; 23],

$$L \propto \frac{E^{5/6} R_0^{2/3}}{\kappa^{1/3} M^{1/2}}. \quad (1.1)$$

Intuitively, adding energy to the system increases the energy available to radiate and therefore the luminosity. Similarly, increasing the radius both increases the area over which to radiate while also reducing adiabatic losses during expansion, both of which increase the luminosity. By contrast, increasing the hydrogen-rich envelope mass increases the timescale over which the energy is distributed (as the photosphere must work through a more extended envelope), lowering the luminosity. The duration of the plateau can be understood via the Popov relation [23],

$$t_p \propto \kappa^{1/6} M_{\text{Henv}}^{1/2} R_0^{1/6} E^{-1/6}, \quad (1.2)$$

where we can see that a greater presence of hydrogen leads to a longer plateau duration, with weaker dependencies on radius and explosion energy. A wide range of plateau durations ($\sim 80 - 120$ days) have been observed in nature, well-reproduced by $5 M_\odot \lesssim M_{\text{Henv}} \lesssim 16 M_\odot$ [3]. Type IIP supernovae are readily classified by the presence of a significant and persistent $\text{H}\alpha$ P Cygni profile in their spectra [20].

If the progenitor star of a core-collapse supernova experiences mass loss prior to explosion, it may have a less-extended envelope post-explosion, leading to a short plateau as the photosphere takes a shorter amount of time to completely recede. In recent years, short-plateau supernovae have been identified as a distinct category, with some characteristics typical to the Type II class but with plateau durations of $\sim$ tens of days instead of the usual ~ 100 -day plateaus characteristic of Type IIP supernovae. Numerical modeling of candidate short-plateau events such as SN 2006Y and SN

2016egz suggest that they produce large amounts of ^{56}Ni ($\gtrsim 0.05 M_{\odot}$) and originate from high-mass ($M \gtrsim 18 M_{\odot}$) red supergiant progenitors that have undergone unusually high mass-loss prior to stellar death. The shorter plateau duration is driven by significantly lower M_{Henv} than typical Type IIP supernovae. This behavior is used to define the boundaries of the short-plateau class as $0.9 M_{\odot} \lesssim M_{\text{Henv}} \lesssim 2.1 M_{\odot}$ [3]. The inferred lower M_{Henv} for candidate events are supported by spectra which feature P Cygni $\text{H}\alpha$ features with smaller absorption-to-emission component ratios, indicative of unusually low hydrogen content in the outer envelope of the progenitor [24].

1.2.2 Type IIb

If the mass loss experienced by the progenitor prior to stellar death is sufficiently large, it may result in the stripping of all of the hydrogen and even the helium from the outer envelope, leading to a Type Ib and Type Ic supernova, respectively [25]. These supernovae are identified by their spectra, which lack hydrogen (Ib) or lack both hydrogen and helium (Ic) [20]. However, in the last several decades, some supernovae have been identified that have spectral characteristics of both the Type II and Type Ib classes [26; 27]. These supernovae show a clear $\text{H}\alpha$ for the first few weeks of their evolution, often leading to their classification as Type II; however, the $\text{H}\alpha$ line is weak and is eventually overwhelmed by a coincident He absorption feature, before eventually disappearing altogether $\sim$ weeks or $\sim$ months into the lifetime of the supernova (see Figure 1.2). At this point, due to the presence of helium and no $\text{H}\alpha$ line, these spectra resemble that of a Type Ib supernova; hence, supernovae with these characteristics are classified as Type IIb ('II' for the initial hydrogen line, 'b' for the disappearance later).

Type IIb supernovae have their own rich diversity, with approximately half ($\sim 40\%$)

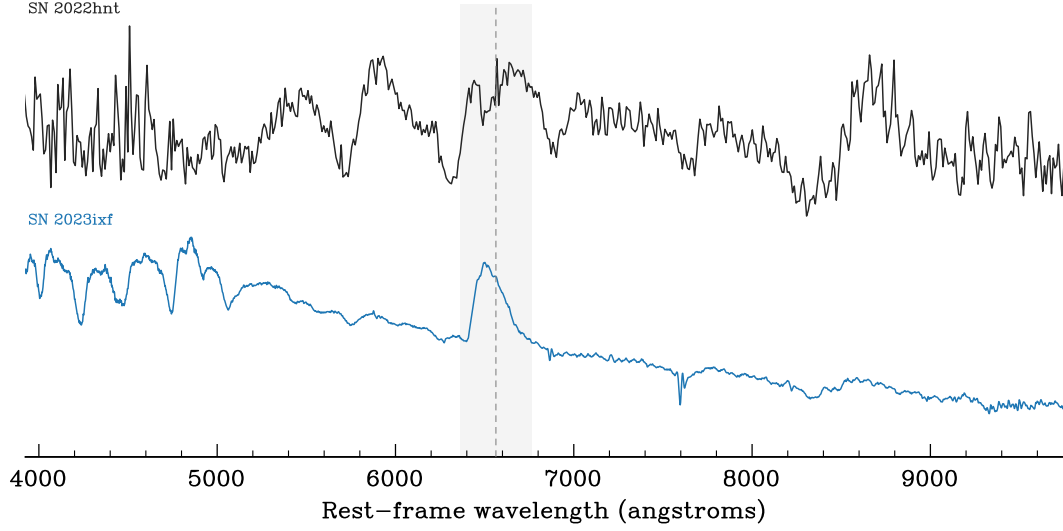


Figure 1.2: Spectra of a Type IIb (SN 2022hnt, black) and Type IIP (SN 2023ixf, blue) supernova approximately one month into evolution. At earlier stages, both objects displayed a clear $H\alpha$ feature at $6563 \text{ \AA}$, marked by the dashed line and shaded region. However, after a few weeks, the Type IIP supernova (which has not depleted its hydrogen) retains its clear $H\alpha$ feature. By contrast, the Type IIb (which has depleted most of its hydrogen) sees its $H\alpha$ fading, rapidly becoming dominated by a coincident helium absorption feature, leading to a characteristic “flat-top”. Data for SN 2022hnt and SN 2023ixf are reproduced from Las Cumbres Observatory observations, courtesy of the Global Supernova Project.

of Type IIb supernovae displaying an unusual, sharp early peak in their light curve, prior to the primary peak understood to be driven by ^{56}Ni decay [28]. The height and duration of this excess emission varies significantly, but the duration ($\sim$ days) is always significantly shorter than the duration of the primary ^{56}Ni peak ($\sim$ weeks). The origin of this excess emission is attributed to a small amount of hydrogen ($\ll 1 M_{\odot}$) remaining in the envelope prior to explosion, which shock cools and produces luminosity but is depleted quickly [29].

1.2.3 Open questions

Above, we have described the morphology of hydrogen-rich supernovae. We now consider open questions raised by this diversity, questions toward which progress has been made within this dissertation.

Continuum or category? The seemingly distinct behaviors and characteristics of core-collapse supernovae have left an open question about the origin of this diversity [20; 25; 30]. Are different supernovae produced by distinctly different mechanisms, or are they part of some broad, smoothly varying continuum of properties, of which categorization is merely an observational illusion? There are arguments to be made for both cases. On the one hand, there clearly exist in nature completely distinct explosion mechanisms that result in different observational categories (e.g., Type II vs. Type Ia supernovae). Even with the subset of core-collapse supernovae, some categories seem to be over-represented. There are significantly more stripped-envelope core-collapse supernovae (Type Ib/c, $\sim 20\%$ of core-collapse supernovae) and very hydrogen-rich events (Type IIP/L, $\sim 65\%$) versus transitional objects such as Type IIb SNe and short-plateau supernovae ($\lesssim 15\%$ combined) [31]. These over-representations may be connected to the “islands of explodability” [32], indicating that very separate progenitors and explosion channels are required to produce different event classes.

On the other hand, new observations repeatedly seem to defy the strict categorizations we might otherwise impose [26; 3]. Transitional objects—which display attributes of more than one categorization—are increasingly commonly identified, aligning with predictions from numerical simulations that vary progenitor properties smoothly along a continuum to predict new sub-types. In [chapter 2](#), we demonstrate that the most heavily-stripped Type II supernovae (Type IIb) can be self-consistently described with the same models of shock cooling calibrated for Type IIP/L super-

novae, a unification that favors the continuum hypothesis. We also present evidence of several new transitional objects that display both characteristics of Type IIb and short-plateau supernovae, blurring the categorization between the classes and favoring a continuum explanation (see [chapter 3](#)). Beyond taxonomy, resolving this question constrains the dominant mass-loss mechanisms operating in massive stars in their final evolutionary stages. A smooth continuum would favor a gradual process such as binary mass transfer or steady stellar winds, while distinct over-represented categories may instead reflect discrete progenitor structures tied to the islands of explodability [32], pointing to fundamental sensitivity of the explosion mechanism to the detailed core architecture of the progenitor.

Modeling the early excess emission? A significant open question regards the origin of the early excess emission seen in some Type IIb supernovae [see, e.g., 28]. Namely, we wish to understand (i) what causes the early excess emission? (ii) what controls the size and duration? and (iii) why does it sometimes not appear at all? The emission is currently believed to be the result of a small amount of hydrogen left over in the envelope releasing energy deposited by the shock, briefly producing luminosity above and beyond the decay of ^{56}Ni synthesized during the explosion, before being rapidly depleted [29]. Efforts have been made to model this behavior from first principles, with some success. There currently exist two primary classes of models which have been used to model this early-time emission:

- *Piro+ model class*: [33] investigated the occurrence of a double-peak in supernovae light curves, first finding that standard progenitors cannot reproduce the observed behavior, then using a preliminary one-zone (i.e., compact core surrounded by a large low-mass envelope) model to successfully create a secondary early peak in the light curve. [34] condensed and formalized the model. [35]

extended the model to two zones (a secondary envelope with a different density profile is included outside of the first envelope) and showed it could approximately reproduce a broader range of light curves than the initial model.

- *SKW+ model class*: [36] calibrated the canonical Type IIP plateau luminosity to numerical simulations of the explosions of red supergiants, producing a light curve evolution model sensitive to progenitor properties that was then extended to ~few days post-explosion by [37] and [1]. Recently, [2] interpolated the spherical phase solution of [38] and [1] with approximate solutions for the very early planar phase presented in [36], providing a single cohesive model of the light curve from explosion to post-peak.

These models have been used to infer progenitor properties from a large sample of core-collapse supernovae. The different construction methods of the models (Piro+, totally analytic versus SKW+, calibrated against numerical simulations) provides a useful self-consistency check of our analytic and numerical methods. Unfortunately, this is a test that is frequently failed, with different models (even iterations of models of the same class) providing highly variable estimates of parameters such as radius, $M_{\text{H}_{\text{env}}}$, and shock velocity for the same objects [e.g., 39–41]. It remains an open question whether this is due to improper model application or a fundamental conflict between the physics assumed from the analytic approach and numerical simulations. In [chapter 2](#), we present a framework for the application of shock cooling models that accounts for often-neglected regimes of model validity to help reduce intra-class inconsistencies. In [chapter 3](#), we find that even with this more careful treatment, analytic-based models will still disagree with inferences from numerical simulations, particularly in radius, and characterize this effect.

Driver of diversity? Core-collapse supernovae are undeniably diverse in both pho-

tometric and spectroscopic evolution. While the diversity of Type IIP/L supernovae is relatively well-understood, the diversity of subclasses such as Type IIb is significantly more poorly explored, in part due to the vastly lower number of discovered objects. Despite originating from arguably a simple requirement—some mass loss process leaves a very small ($M_{\text{Henv}} \ll M_{\odot}$) amount of hydrogen in the progenitor envelope—Type IIb show a striking amount of diversity, including:

- *The H α P Cygni*: despite all Type IIb supernovae nominally having very similar amounts of hydrogen, we observe significant diversity in the appearance of the H α P Cygni at early times [i.e., first few weeks [42](#)], as well as the flat-top that appears as the supernova evolves [[20](#)]. What controls this diversity?
- *Inferred progenitor properties*: do Type IIb supernovae have preferred progenitors or systems in which they are more likely to form? Sample studies of Type IIb supernovae report a wide array of progenitor characteristics, especially radius; however, it is well-documented that different models applied to the same object can also give a wide range of plausible properties. Is this diversity a result of insufficient modeling, or is it intrinsic to the population?
- *Early excess emission*: approximately half ($\sim 40\%$) of Type IIb supernovae show early peaks attributed to shock cooling, but some do not show any early emission at all. There must be some hydrogen present, evidenced by the spectra; why do some objects show evidence of extended hydrogen envelopes in the light curve and others don't?

Addressing these open questions and more is the subject of active and ongoing work, including the work presented in this dissertation. Answering these questions is ultimately a question about how massive stars end their lives: the inferred progenitor-to-remnant mapping across the full diversity of core-collapse supernovae constrains

the neutron star birth property distribution, the fraction of stellar deaths that produce black holes rather than neutron stars, and the mass and kinetic energy returned to the interstellar medium per stellar generation. We primarily attempt to make progress on these questions by investigating the application of shock cooling models to hydrogen-rich supernovae as well as cataloging new and unusual events which contribute to our overall picture of the diversity of these objects.

1.3 Type I supernovae

Supernovae are classified as Type I based on the lack of hydrogen in their spectra, in direct contrast with Type II. Here, we briefly review the literature on Type I superluminous supernovae in particular, and consider open questions regarding their unusual overbrightness and behavior.

1.3.1 Superluminous supernovae

Superluminous supernovae (SLSNe) are a recently discovered subclass of supernovae which have peak luminosities orders of magnitude brighter than ordinary supernovae (Figure 1.3) [43–46]. There is no clearly defined cutoff for what constitutes a supernova being “superluminous”; initial work suggested an absolute magnitude cutoff of ~ -21 , but since then objects have been found with similar spectroscopic and photometric qualities fainter than ~ -21 absolute magnitude but brighter than ~ -20 .

Spectroscopically, young Type I superluminous supernovae (SLSNe-I) consistently show strong O II absorption features between 3000 Å and 5000 Å. By definition, they do not have any visible broad hydrogen lines in their spectra; however, interacting Type I SLSNe may show narrow hydrogen lines as the shock runs into circumstellar material distributed around the progenitor [47]. Most SLSNe-I also do not show helium

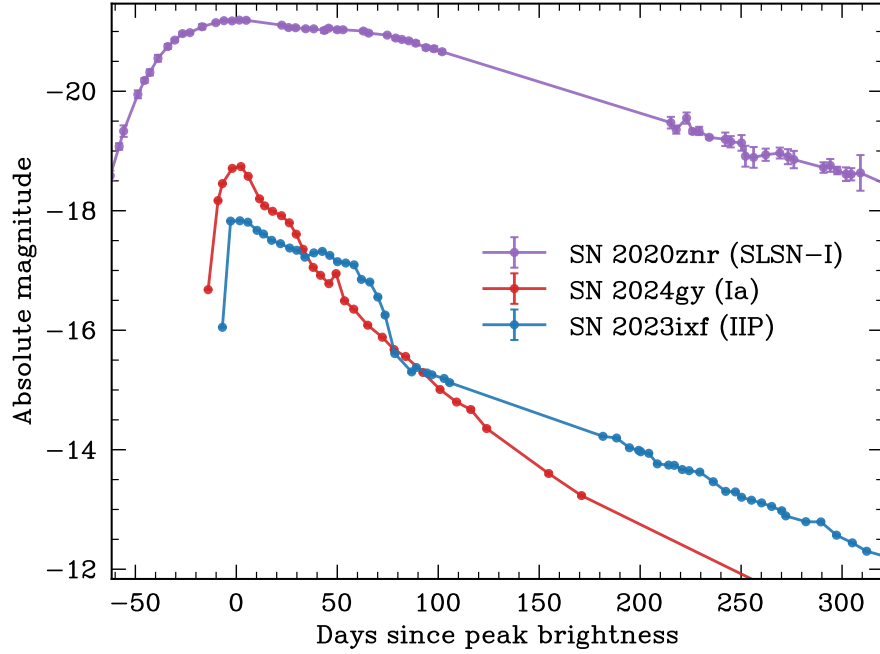


Figure 1.3: Comparison of r -band light curves for an SLSN-I, IIP, and Ia. While ordinary supernovae have peak absolute magnitudes approximately between -17 and -19, superluminous supernovae can be several orders of magnitude brighter, and they maintain that luminosity for significantly longer. Data for SN 2024gy and SN 2023ixf are reproduced from Las Cumbres Observatory observations, courtesy of the Global Supernova Project. Data for SN 2020znr are reproduced from publicly-available datasets provided courtesy of the Zwicky Transient Facility. All light curves are aligned so $t = 0$ corresponds to the peak brightness in r -band.

features [48; 49]. Spectral absorption lines most interior to the ejecta (e.g., iron) reveal extremely fast-moving ($v \gtrsim 10^4$ km/s) layers. Some superluminous supernovae are observed to have line velocities that stay relatively constant with time (especially pre-peak) [50; 51], in contrast with almost all other supernovae which show velocities that decrease steadily with time.

Photometrically, SLSNe have several identifiable characteristics. The key property of SLSNe are their extreme peak luminosities, typically $\gtrsim 10^{43}$ erg/s up to 10^{45} erg/s, several orders of magnitude beyond ordinary supernovae. Rise times are typically longer than ordinary supernovae as well; recent studies estimate an average rise time

for SLSNe-I to be ~ 40 days [50; 52]. Unlike, e.g., Type Ia supernovae, SLSNe-I also frequently feature chaotic, bumpy light curves at all stages of their evolution. Modulations and excess emission have been observed in SLSNe-I prior to explosion (as precursors) [53–55], immediately following explosion, during the entire duration of the main event, and even on the late-phase ($\gtrsim 150$ days) declines in the few objects where we have data at sufficiently late times [56; 57].

In part due to the recency of their discovery (last ~ 20 years), the origin of both the overbrightness and modulations in SLSNe-I is a major open question. Below, we explore proposed scenarios for both.

1.3.2 Power source of SLSNe-I

The source of the extreme peak luminosities of SLSNe-I has been hotly debated. There are currently four primary hypotheses proposed, of which two are most favored, and none of which can simultaneously explain the diverse behavior of the SLSNe-I class.

The first and least plausible mechanism is that the overbrightness is driven by excess ^{56}Ni production and decay. Most core-collapse supernovae generate $\lesssim 0.1 M_{\odot}$ of ^{56}Ni , which is insufficient to explain the peak luminosities of superluminous supernovae [58; 59]. The minimum amount of ^{56}Ni required to explain most of the observed luminosity distribution of SLSNe-I is $3 M_{\odot} \lesssim M_{^{56}\text{Ni}} \lesssim 10 M_{\odot}$. This range already presents a significant problem as synthesizing more than $\sim$ few solar masses of ^{56}Ni is an open problem not yet addressed in core-collapse simulations. However, a non-trivial percentage of SLSNe-I are sufficiently bright to require $M_{^{56}\text{Ni}} \gtrsim 10 M_{\odot}$ [60; 45]. Studies of ^{56}Ni synthesis in core-collapse simulations found that, while the ^{56}Ni mass produced scales with explosion energy (up to $\sim 5 M_{\odot}$ for $\sim 10^{52}$ erg), producing more than $\sim 10 M_{\odot}$ required unphysical explosion energies [e.g., 61]. Such a large

amount of ^{56}Ni practically requires exceptionally rare explosion channels such as pair-instability supernovae, which itself poses a problem as the expected light curves of pair-instability supernovae do not match observations of the majority of SLSNe-I nor their spectra [62; 60].

The second (and second least-plausible) hypothesized mechanism is fallback accretion onto a remnant black hole formed during the core-collapse [14; 63]. If the core of the progenitor star is sufficiently massive, the collapse may produce a stellar-mass black hole instead of a neutron star. This black hole could be fed by material that failed to be ejected during the supernova, generating an astrophysical jet or collimated disk wind. This energy outflow illuminates the interior wall of the ejecta, contributing luminosity to the expanding supernova. The parameterized light curves of this scenario are fairly flexible and can indeed accommodate many of the SLSNe-I observed, which is why we consider it more plausible than the ^{56}Ni power scenario. However, recent studies have calculated that the accreted material required to sustain SLSNe-I luminosities for hundreds of days often exceeds the expected mass of the star [15], making this channel not viable for any but the faintest SLSNe.

The final two scenarios we discuss are equally viable. The third proposed mechanism is interaction with circumstellar material (CSM) around the progenitor star. In this mechanism, the progenitor underwent mass loss prior to exploding [30], depositing shells of matter that the supernova shock later runs into [16]. The shock-matter interaction produces significant extra luminosity in the form of high-energy photons (e.g., X-ray) which are then reprocessed by the ejecta and released into the optical regime on the diffusion timescale of the ejecta. CSM interaction is a highly flexible model and the freedom to distribute shells of matter at varying radii and with varying densities allows this model to reproduce a significant fraction of SLSNe-I light curves [64; 65]. Many SLSNe-I are also found to have signatures of CSM interaction in their

spectra [e.g., multiple post-peak modulations, distinct helium signatures, bluer spectra 66; 67], which this model can explain. Furthermore, only reasonable mass-loss rates [e.g., $\sim 10^{-2} M_{\odot} \text{ yr}^{-1}$; 16] and velocities [e.g., $\lesssim 1000 \text{ km/s}$; 57] are required to explain the energy budget required for the majority of SLSNe-I, making this scenario highly plausible [47; 68]. The primary concerns with the CSM hypothesis are related to applicability; a significant fraction of SLSNe-I appear to not be strongly compatible with the predictions made by the model [51; 69]. For example, CSM interaction with multiple shells creates light curves that are irregular and bumpy; yet we often observe SLSNe-I that have no bumps at all, or in rare cases quite periodic or quasiperiodic modulations, which seems to belie a stochastic explanation [70; 71]. Further, not all SLSNe-I show signs of interaction, and indeed interaction with large amounts of (primarily hydrogen) CSM raises questions about SLSNe-I that appear to have no signs of hydrogen–narrow or broad–in their spectra.

The final scenario we describe is the magnetar model, proposed independently by [10] and [11]. In this scenario, the collapse of the star produces a neutron star remnant, which inherits and amplifies the magnetic field and angular momentum of the progenitor. The magnetar radiates away its angular momentum via magnetic dipole radiation, which illuminates the inner wall of the ejecta as in the fallback accretion scenario. However, unique to this mechanism, the wind from the magnetar is also expected to produce a hole in the cavity, compacting the ejecta into a dense shell. This particular feature of the magnetar model may be able to explain the unusual constant-velocity spectral line behavior observed in many SLSNe-I [50; 51]. Further, the energy liberated from the spindown of the magnetar is sufficient to explain the entire space of SLSNe-I luminosities. Like the CSM interaction model, the magnetar model has great flexibility in that varying the magnetar properties (especially, spin period $1 \text{ ms} \lesssim P \lesssim 10 \text{ ms}$ and magnetic field strength $10^{12} \text{ G} \lesssim B \lesssim 10^{15} \text{ G}$) can explain

almost all the rise times, peak luminosities, and late-time decays observed in the current SLSNe-I sample [52]. The primary weakness of the magnetar model is that there is no native way to explain the presence of modulations as observed in a majority of SLSNe-I events, as the magnetar energy injection is unmodulated. However, observationally confirming the magnetar engine would provide direct constraints on the distribution of spin periods and magnetic field strengths at neutron star birth, quantities otherwise only accessible through the present-day pulsar population after significant spin-down evolution.

Both the CSM interaction and magnetar models have been used for parameter inference, but even systematic attempts to apply them to all SLSNe-I find no compelling evidence favoring either scenario [57; 56].

1.3.3 Origin of the modulations

The two most successful theories for SLSNe-I (CSM interaction and the magnetar model, described above), while presenting explanations for much of the SLSNe-I morphology, also raise several important questions as to the origin of the modulations observed in the majority of SLSNe-I light curves. Examples of these modulations are provided in [chapter 4](#); see especially [Figure 4.4](#).

CSM interaction provides the most natural explanation for the modulations. Interaction with randomly distributed shells of matter can produce the singular, irregular bumps seen in many SLSNe-I, and even the behavior of some SLSNe-I that have multiple irregular modulations, such as SN 2017egm, which in particular seems to be only describable using CSM interaction [72; 73]. As mentioned above, most SLSNe-I show signatures of interaction in the spectra and light curve at some level. However, a small sample of SLSNe-I show variability in the light curves that seems more structured

than would be expected from CSM interaction [70; 71]. These include SLSNe-I with multiple, possibly periodic modulations, patterns which are not easy to explain via random mass-loss episodes. Alternative explanations to describe these edge cases have been invoked, such as pulsational pair-instability supernovae (PPISNe), which occur when a massive star ($\sim 70\text{--}140 M_{\odot}$) undergoes repeated pair-instability-driven (i.e., rendered unstable by electron-positron production) explosive burning episodes that eject mass in discrete pulses [74–76]. PPISNe offer a compelling patch to the CSM interaction model, as they provide a way for shells of matter to be distributed around the progenitor in a more structured, sequential fashion, leading to the appearance of “periodic” light curves. Indeed, until SN 2024afav [77], no SLSN had been observed to have more than 2 periodic modulations, making claims of periodicity challenging to verify.

The magnetar model struggles significantly more with the presence of modulations, as there is no immediate way to explain them using the energy budget justification alone [10]. Significant work has been conducted in recent years to modify the magnetar to support singular or multiple modulations in an SLSN light curve. Singular modulations may be explained via magnetar flare events, generated by the wind-up of the magnetic field [e.g., 78]. These flaring events match SLSN light curves well, and have been directly observed in other astrophysical events, such as gamma ray bursts. Multiple, possibly periodic modulations have been described using modifications to the magnetar model that use misalignments of the magnetic inclination angle to produce oscillations in the overall energy output [71]. This modified magnetar model has been applied to several SLSNe-I with moderate success. However, no existing version of the magnetar model can explain SLSNe-I light curves with multiple, irregular bumps (e.g., SN 2017egm).

Both the magnetar model and the CSM interaction model have an additional con-

cern, which is that neither has been observationally confirmed beyond adequate fit performance. Both models are sufficiently flexible to explain most morphologies without making additional, independent predictions that can be verified [57]. The work in this dissertation (particularly, [chapter 4](#)) enters this ongoing conversation in an attempt to make progress on some of these unresolved questions. In [chapter 4](#), we present observations of a new supernova (SN 2024afav) with a unique light curve feature that presents an unprecedented opportunity to verify the magnetar model as the engine behind at least some superluminous supernovae.

1.4 Observations of supernovae

To address many of the theoretical questions above, we will confront various models with detailed data on infant supernovae. We provide a brief review of these observation modes and associated challenges below, and note that specific observation parameters are provided in greater detail in each chapter.

1.4.1 Discovery

The discovery of infant supernovae is primarily driven by wide-field all-sky surveys designed to monitor large fractions of the sky on nightly or sub-nightly cadences. Prominent (but not exhaustive) examples include the Asteroid Terrestrial-impact Last Alert System [ATLAS; 79], the Zwicky Transient Facility [ZTF; 80], and the anticipated Legacy Survey of Space and Time [LSST; 81]. These surveys collectively provide near-complete coverage of the extragalactic sky, enabling the rapid identification of new transients and their prompt submission to the Transient Name Server for community follow-up. In some cases, nearby supernovae are discovered independently by amateur astronomers prior to or concurrent with professional surveys.

A key advantage of modern all-sky surveys is the availability of pre-discovery constraints through forced photometry. Even when a transient has not yet been flagged by automated detection pipelines, surveys such as ATLAS and ZTF maintain photometric measurements at fixed sky coordinates across archival images [e.g., 82; 83]. These forced-photometry stacks can recover flux below nominal detection thresholds, providing constraints on the explosion epoch and early light curve behavior that would otherwise be inaccessible.

A central challenge for infant supernovae is that the most physically informative early phases are often the most observationally elusive. For objects such as Type IIb supernovae, the early excess emission attributed to shock cooling of the residual hydrogen envelope rises and fades on timescales of only $\sim$ days [e.g., 35; 28], meaning that a delay of even 24–48 hours between discovery and the initiation of follow-up can result in the loss of the most constraining portion of the light curve. Well-sampled observations during this first rise place the strongest constraints on the envelope mass and progenitor radius, yet are frequently absent from the literature precisely because this window is so narrow. This challenge motivates the rapid-response follow-up strategy described below.

1.4.2 Photometric follow-up

Photometric follow-up involves obtaining time-series brightness measurements of a supernova across multiple broadband filters, producing a multi-color light curve that traces the evolving spectral energy distribution of the source. The primary scientific goal of photometric follow-up is to probe the powering mechanisms driving the supernova by comparing the resulting light curve morphology against model predictions; for the objects studied in this dissertation, this encompasses shock cooling models

[e.g., 1; 35], radioactive decay [e.g., 18], and magnetar spin-down [10; 11], each of which imprints a characteristic shape and timescale on the light curve.

Photometric follow-up in this dissertation is primarily conducted through the Las Cumbres Observatory [LCO; 84], a global robotically-enabled network of telescopes ranging from 0.4m to 2m designed for rapid follow-up of active transients, as part of the Global Supernova Project [GSP; 85]. Photometric observations were conducted in a wide array of bands, including U , B , V , g , r , and i , using a combination of the 1m and 2m telescopes in the network. All LCO data featured in this dissertation is reduced using the `lcogtsnpipe` photometric reduction pipeline [86]. We typically obtained observations at daily cadences, initiated as close to discovery as possible and continued through the radioactive tail to ensure coverage of all major light curve features. In some cases, additional photometry was obtained via forced photometry from ATLAS or ZTF archival imaging [82; 83], or from supplementary sources such as KeplerCAM.

A particular observational challenge for infant supernovae is the reliable measurement of the source temperature at early epochs. Broadband optical photometry can be used to fit a blackbody spectral energy distribution to constrain temperature and photospheric radius; however, this approach becomes unreliable when the available filters probe only the Rayleigh-Jeans tail of the blackbody, as is the case for hot, young supernovae observed only in optical bands. Without ultraviolet (UV) coverage, blackbody temperature fits at early epochs can be biased by as much as $\sim 20,000$ K [87], severely compromising inferences about the progenitor radius and explosion energy. UV photometry from facilities such as the *Neil Gehrels Swift* Observatory Ultraviolet/Optical Telescope [Swift-UVOT; 88] directly samples the peak of the spectral energy distribution at early times and substantially reduces this bias. Where available, we incorporate Swift-UVOT UV photometry into our analysis; however, UV coverage

is not uniformly available for all objects in this dissertation, and we account for the resulting temperature uncertainty explicitly in those cases.

A further complication is that the blackbody approximation itself becomes increasingly unreliable as the supernova evolves and strong line features develop in the spectrum. At early times, when the photosphere is hot and the ejecta relatively featureless, the blackbody approximation is reasonable. At later epochs, line blanketing and complex opacity structure can cause the broadband photometry to deviate significantly from a smooth blackbody, and temperature and radius estimates derived from photometry alone should be interpreted with caution without spectroscopic support.

1.4.3 Spectroscopic follow-up

While photometric follow-up tracks the integrated brightness of a supernova across broad wavelength bins, spectroscopic follow-up resolves the brightness per unit wavelength, providing a far richer view of the evolving source. Time-series spectra serve several complementary scientific purposes. The shape of the spectral continuum can be fit with a blackbody to constrain the photospheric temperature and emitting radius as a function of time, providing an independent check on photometrically-derived quantities. Superimposed on this continuum, spectral line features encode the chemical composition, velocity structure, and spatial distribution of the ejecta. Spectroscopic observations are also highly useful for constraining the distance to the host galaxy via recession velocity, and for estimating line-of-sight extinction using features such as the Na I D doublet [89].

Spectroscopic follow-up in this dissertation is primarily conducted using the FLOYDS spectrograph [90], a low-resolution cross-dispersed instrument mounted on the 2m telescopes at Haleakala Observatory and Siding Spring Observatory within the

LCO network. FLOYDS provides continuous wavelength coverage from approximately 350 nm to 1 μm in a single exposure, spanning the full optical window and enabling simultaneous observation of features from the near-ultraviolet through the near-infrared. Spectra were reduced using the `floydsspec` pipeline, which handles cosmic ray rejection, spectrum extraction, and flux and wavelength calibration. We typically obtained spectra on weekly to biweekly cadences, with higher cadence early in the evolution of each object where spectral features change most rapidly.

1.4.4 Radio interferometric follow-up

For sufficiently nearby and radio-bright supernovae, very-long-baseline interferometry (VLBI) can spatially resolve the expanding ejecta, directly imaging the shock structure at milliarcsecond scales. VLBI achieves this resolution by combining signals from geographically distributed arrays of radio telescopes to synthesize an effective aperture spanning intercontinental baselines [91] (for a more detailed overview of VLBI, please see [chapter 6](#)). At the distance of nearby supernovae, this resolution enables direct measurement of the ejecta diameter, width, and azimuthal brightness distribution as a function of time, providing geometric constraints that are entirely inaccessible at optical wavelengths.

A central challenge in VLBI analysis is the sensitivity of standard imaging and model-fitting approaches to calibration systematics: phase errors, complex gain variations, and evolving (u, v) -coverage can all introduce artifacts into reconstructed images or bias model parameter estimates. In [chapter 5](#), we present a re-analysis of the first ~ 2500 days of SN 1993J—the prototypical Type IIb supernova and best-studied radio supernova—using an analysis framework based entirely on closure quantities, which are derived interferometric observables that are by construction insensitive to these

dominant sources of systematic error [92].

Chapter 2

A framework of considerations for modeling of shock cooling in Type IIb supernovae



EVERY ACT OF CREATION IS FIRST AN ACT OF DESTRUCTION.

—Pablo Picasso, c. 1960



We report the results of a rapid follow-up campaign on the Type IIb Supernova (SN) 2022hnt. We present a daily, multi-band, photometric follow-up using the Las Cumbres Observatory, the Zwicky Transient Facility, the orbiting *Swift* observatory, and the Asteroid Terrestrial-impact Last Alert System (ATLAS). A distinctive feature in the light curve of SN 2022hnt and other IIb SNe is an early narrow peak prior to

the ^{56}Ni peak caused by rapid shock cooling of the hydrogen envelope, which can serve as an important probe of the properties of the massive progenitor star in the moments before explosion. Using SN 2022hnt as a case study, we demonstrate a framework of considerations for the application of shock cooling models to type IIb SNe, outlining a consistent procedure for future surveys of Type IIb SNe progenitor and explosion properties. We fit several recent models of shock-cooling emission and obtain progenitor radii between ~ 50 and $\sim 100 R_{\odot}$, as well as hydrogen-enriched envelope masses between ~ 0.01 and $\sim 0.1 M_{\odot}$, both consistent with values for other IIb SNe. One of these models is the model of [2], marking the first time this model has been applied to a Type IIb SN. We evaluate contrasting predictions between shock-cooling models to construct a fiducial parameter set which can be used for comparison to other SNe.

2.1 Permissions and Attributions

1. The content of [chapter 2](#) is the result of a collaboration with the Global Supernova Project as well as Dr. Jonathan Morag, Dr. Ido Irani, and Dr. Logan Prust, and has previously appeared in the *Astrophysical Journal* as Joseph R. Farah *et al.* (2025). It is reproduced here with the permission of the *Astrophysical Journal*: <https://iopscience.iop.org/article/10.3847/1538-4357/adad5a>. Full acknowledgements are available on the journal page.

2.2 Introduction

Type IIb supernovae (SNe) represent a subclass of core-collapse supernovae characterized by the presence of hydrogen lines in their early spectra, which subsequently

disappear [27; 20]. They are believed to originate from massive stars that have shed a substantial portion of their outer envelopes [e.g., 93, and references therein]. This extensive mass loss prior to explosion results in the removal of most, but not all, of the hydrogen-rich material in the progenitor’s outer shell. The mass loss mechanism in Type IIb SNe progenitors is not well-understood, but is thought to occur via multiple channels as a result of stellar wind stripping, accretion onto a binary companion, or rotation effects [30; 94]. Observational and theoretical studies suggest that SNe IIb may arise from a variety of progenitor scenarios, including single massive stars and binary systems [e.g., 95–100]. The diversity of progenitor and mass-loss channels for Type IIb SNe makes them a useful probe of the characteristics of near-death massive stars. This population of stars plays an important role in the evolution of galaxies and the metal enrichment of the interstellar medium [101]. Ongoing studies continue to refine our understanding of the progenitor systems and explosion mechanisms of SNe IIb, contributing to our broader comprehension of stellar evolution and supernova diversity.

Many Type IIb supernovae have a characteristic “double-peak” in their early light curves, presenting as a sharp rise within a few days of explosion followed by a rapid ($\sim$ hours-days) dimming, and then a gradual ($\sim$ weeks) re-brightening to a second peak before experiencing a final, long-lasting ($\sim$ months) decay [e.g., 102–108; 40]. The secondary peak, similar in appearance and duration to those found in Type Ib/c SNe, is attributed to luminosity generated from the decay of nickel synthesized in the supernova explosion [22]. By contrast, the initial sharp rise to peak and subsequent rapid decay is attributed to the emission generated by the supernova shock heating and then cooling (by expansion) the extended envelope of the progenitor [29]. Due to the short timescale and very early occurrence of the shock-cooling peak, it has only been observed in detail in a small number of previous objects [e.g., 39; 108; 40; 109].

Analytical calculations of the behavior of the extended progenitor envelope following deposition of the supernova energy predict a dependence on both the properties of the progenitor and explosion [36; 33]. This dependence makes analytical modeling of the shock-cooling light curve a powerful and effective probe for the stellar properties of the progenitor prior to explosion, even providing clues about the mass-loss history of the progenitor [see e.g., 110]. The shock-cooling modeling approach has been used to create a small sample of progenitor estimates for supernovae on a case-by-case basis [see e.g., 39; 108; 40], but larger surveys with a consistent model application are needed to properly probe the space of progenitor channels and mass-loss modes.

In the last decade, multiple models for shock-cooling emission have been presented and tested on applicable supernova events. [34] built on the earlier work of [33] to model the shock-cooling behavior pre- and post-peak. [35] then extended [34] to include a two-zone model of variable power-law density in order to better model early shock-cooling emission behavior observed in several supernovae. Unlike [34], which did not assume a specific polytropic structure for the stellar model, other approaches (e.g., [1]) assume a polytropic density profile and calibrate analytically derived expressions using numerical models. A more recent model [2] interpolated between the solutions of [1] and the previously developed [37] model, resulting in an earlier validity than existing models. As the model of [2] is not calibrated specifically for Type IIb SNe and has never been applied to a Type IIb SN, the question of whether such a model can be applied to Type IIb SNe to take advantage of early-time data is of great interest.

Here, we present early-time observations of the Type IIb SN 2022hnt coupled with a dense, multi-band, months-long follow-up campaign. We take advantage of the exceptionally early constraint on the shock-cooling rise of the light curve and infer the progenitor properties of the system using a variety of analytical shock-cooling mod-

els. Using SN 2022hnt as an example, we seek to present a framework for consistent shock-cooling model application to Type IIb SNe, taking into account: (i) regimes of validity, (ii) robustness of highly-constraining data points, (iii) temperature evolution uncertainties, (iv) intra- and inter-model variation, and (v) fiducial parameter construction. The chapter is organized as follows. We provide a broad overview of our proposed modeling framework in [section 2.3](#). Next, we apply the framework to the new object, SN 2022hnt. We summarize the follow-up campaign and resulting data and reduction steps in [section 2.4](#). In [section 2.5](#) we characterize the spectral evolution of the transient over the first five weeks. In [section 2.6](#), we characterize the light curve evolution and choose shock-cooling models from the literature to fit to the SN 2022hnt data. In [section 2.7](#), we report the results of fits to the described models and compare and contrast the inferred progenitor properties both in the context of different model choices and different supernovae. Finally, in [section 2.8](#), we summarize our results.

2.3 Overview of the framework

Here, we propose a framework of considerations for the application of shock cooling models to Type IIb SNe. The goal of this framework is to provide a consistent procedure which maximizes the correctness and reliability of the resulting parameter estimates. The overall steps of this framework are:

- †1. Assess photometric and spectroscopic evolution of the transient,
- †2. Validate inference of shock cooling,
- †3. Select models for parameter estimation,
- †4. Compute and restrict data to regimes of model validity,

- †5. Perform parameter estimates, varying model settings,
- †6. Reject estimates based on *a priori* information, and
- †7. Construct fiducial estimates from non-rejected models.

We expand on these items below.

Assess photometric and spectroscopic evolution of the transient, †1: In general, the parameters we seek to estimate from models of shock cooling are: (i) the mass of the hydrogen-rich envelope, (ii) the initial velocity of the radiation-mediated shock, (iii) the radius of the progenitor star prior to explosion, and (iv) the explosion epoch. Properties of the transient evolution can be used to develop expectations for these parameters, which can in turn be used to reject models that are clearly incorrect based on these expectations. For example, [3] showed that the hydrogen-rich envelope mass had an effect on the early light curve, with more hydrogen resulting in a flatter light curve (the “short-plateau”). Additionally, the presence of non-detections and other tight *a priori* constraints on the explosion epoch can provide a straightforward validation of fits.

Validate inference of shock cooling, †2: Models of shock cooling are only applicable when the assumption of shock cooling is itself valid. Alternative light curve drivers (e.g., wind breakout or other heating) can produce similar photometric evolution while resulting in incorrect and misleading fits as a result of the invalid cooling assumption. During this step, photometry and spectra should be used to gauge the temperature evolution of the transient. Data for shock cooling modeling should be restricted exclusively to times when the envelope is cooling, and in particular where the temperature evolution falls off as $T \propto t^{-0.45}$, the nominal prediction of shock cooling [36]. If the envelope is being heated e.g., due to a wind breakout, estimates and/or upper limits can be placed on the extent of the wind and the mass enclosed (see [110] for an overview,

or [subsection 2.6.1](#) for a demonstration on SN 2022hnt).

Select models for parameter estimation, †3: There are multiple models of shock cooling available, divided into two classes: the Piro+ models [33–35] and the SW models [36; 37; 1; 2]. The two model classes differ substantially in assumptions, implementation, and calibration; as a result, we recommend fitting at least one model from each class when possible for cross-model validation of parameter estimates. Generally, each model iteration improved upon the last, making it only necessary to fit the newest iteration. For example, [35] extended [34] from a one-zone to a two-zone model, while [2] extended [1] to describe the spherical phase of the ejecta expansion. However, we recommend fitting previous model iterations if: (i) a model is so new that it has not been yet well established, making a comparison to previous iterations a useful sanity check, or (ii) when comparing parameter estimates to other objects in the literature which have used the previous model for analysis.

Compute and restrict data to regimes of model validity, †4: Performing fits using a model that is not valid for particular data points will lead to erroneous and possibly misleading parameter estimates. Once a suite of models have been selected, the data selection must be tailored to the regimes of validity for each model. This consideration is particularly important when fitting a model which has been calibrated for one object class (e.g., Type II SNe) to another object class (e.g., Type IIb SNe). Some models provide analytic validity timescales which vary with the model parameters. These validity timescales should be implemented in the fitting routine, so that no iteration of the fit extends the model beyond its regimes of validity.

Perform parameter estimates, varying model settings, †5: Many models have *a priori* assumptions (e.g., polytropic index, emission suppression, etc.) which can be varied, resulting in different parameter estimates. The spread of values resulting from varying

model assumptions will help characterize the true robustness of the final parameter estimates in a more comprehensive way than an uncertainty from a single fit would.

Reject estimates based on a priori information, †6: Using the expectations set in †1, incorrect or clearly erroneous model fits can be rejected to improve the accuracy and robustness of the fiducial parameter set. For example, a model which predicts an envelope mass $\gg 5M_{\odot}$ can be discounted if the light curve shows a prominent double peak, based on the results of [3, Fig. 5]. Similarly, fits which suggest a very early explosion epoch can be rejected if the estimated explosion epoch is incompatible with the light curve (e.g., there are multiple robust non-detections after the modeled explosion epoch).

Construct fiducial estimates from non-rejected models, †7: The result of the previous steps is a curated set of parameter estimates from a variety of model classes and *a priori* assumptions, probing both the uncertainty in the data and the uncertainties of the model assumptions. Construct a fiducial estimate using a method which incorporates both the variation of the parameter estimates and the cross-model parameter spread into a single estimate and uncertainty (e.g., a weighted mean and uncertainty).

2.4 Observations and data

SN 2022hnt was discovered at *g*-band magnitude 18.1 by the Zwicky Transient Facility (ZTF) on April 14, 2022 [111]. Here, we present a summary of the follow-up campaign organized by ZTF, our own follow-up campaign using Las Cumbres Observatory (LCO) via the Global Supernova Project (GSP), early detections by ATLAS [ATLAS; 79], and publicly-available ultraviolet (UVOT) observations from *Swift* [88].

SN 2022hnt was detected at right ascension (R.A.) 11:36:59.751 and declination (decl) +55:09:50.26 in the young spiral galaxy NGC 3759A. An image of the object com-

posed of stacked LCO exposures from ~ 2 days post-explosion is shown in Figure 2.1. The object was classified as a Type IIb SNe at redshift $z \sim 0.0192$ in the Transient Name Server (TNS) using a spectrum obtained by the Spectral Energy Distribution Machine (SEDm) instrument mounted on the 1.5m Palomar telescope operated by ZTF [111]. Based on this redshift, and the distance to the host reported in the NASA Extragalactic Database (NED), we adopt a distance of 84 Mpc to the supernova. From the [112] dust map, we adopt an extinction along the line-of-sight to SN 2022hnt of $E(B - V)_{\text{MW}} \sim 0.0130$ mag. All light curve and photometric data presented in this chapter are corrected using this extinction assumption.

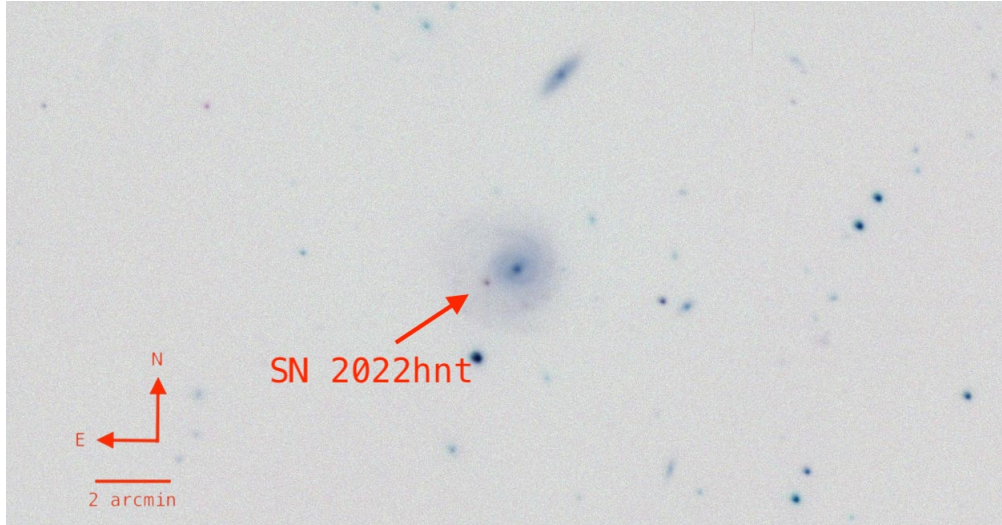


Figure 2.1: An image of SN 2022hnt ~ 2 days after explosion. This image was made by stacking 80 minutes of LCO exposures in U , V , B , g , r and i bands. Colors are inverted to more clearly show the host and supernova.

LCO is a global robotically-enabled network of telescopes ranging from 0.4m to 2m designed for rapid follow-up of active transients, as part of the GSP. We organized an intense follow-up campaign within 24 hours of discovery. Photometric observations in the U , B , V , g , r , and i bands were made using the 1m and 2m telescopes in the network. We reduced all LCO data using the `lcogtsnpipe` photometric reduction

pipeline, which incorporates point-spread-function fitting and color term calculation prior to extraction and magnitude measurement [86]. We calibrated photometric observations in the U , B , and V bands to the Landolt standard fields, which uses Vega magnitudes [113]. Observations in all other bands were calibrated to the Sloan Digital Sky Survey (SDSS) catalog, which uses AB magnitudes [114]. Though the supernova was detected in close proximity to its host, the apparent magnitude of the host spiral arms is relatively low, and as a result, host subtraction was not required for the first 150 days of the observation.

In addition to photometry, we also used LCO to monitor the spectroscopic evolution of SN 2022hnt. We obtained five spectra between day 5 and day 35 using the FLOYDS spectrograph mounted on the 2m telescope at Haleakala Observatory. The FLOYDS spectrograph covers wavelengths from 350 nm to 1 micron. To process the data, we employed the specialized LCO floydsspec pipeline, which handles cosmic ray flagging, spectrum extraction, and calibration [90]. The spectroscopic observations were used to estimate extinction due to the host, following the Na I D equivalent width method described in [89]. We estimate host extinction of $E(B - V)_{\text{host}} \sim 0.0109$ mag. We apply this extinction correction globally to all photometry collected on SN 2022hnt.

As part of the LCO follow-up campaign, we triggered the Ultraviolet and Optical Telescope instrument on the *Swift* observatory [88]. We obtained several epochs of UV photometry spanning the first few weeks of the supernova evolution. The reduction of the ultraviolet photometry was performed using the *Swift* Optical and Ultraviolet Supernova Archive (SOUSA) pipeline, with the most up-to-date zero points and calibrated sensitivity [115].

ATLAS is a robotically-controlled set of two 0.5m survey telescopes located in Hawaii, on Mauna Loa and Haleakala [79]. ATLAS performed multiple observations of SN 2022hnt over a period of 90 days during the explosion, including a $\sim 3\sigma$ detection

prior to the first peak, which tightly constrains the rise time of the supernova. We discuss vetting of this detection in [subsection 2.4.1](#). The ATLAS photometry was obtained in the custom ATLAS “orange” (*o*-band) filter. Photometry was reduced using the proprietary ATLAS pipeline with calibration to the Pan-STARRS catalog.

ZTF is a wide-field, nightly astronomical transient survey camera mounted on the 48-inch Palomar Observatory robotic telescope [80]. ZTF initially discovered the object and reported non-detections down to *g*-band 20th magnitude within hours of the first detection [111]. We incorporate the publicly available ZTF detections between April 2, 2022 and June 19, 2022 into our dataset. Subtractions are automatically performed using previous template images collected by the ZTF survey. Spectra for SN 2022hnt reported by ZTF were obtained via the SEDm on the 60-inch telescope at Palomar Observatory and downloaded from TNS. The SEDm is a low-resolution ($R \sim 100$) spectrograph and reports flux on a wavelength range from ~ 350 nm to ~ 900 nm [83].

A compilation of all the photometry data described above is presented in [Figure 2.2](#). A compilation of all spectra described above is presented in [Figure 2.4](#).

2.4.1 Examination of the earliest ATLAS detection

The earliest ATLAS detection at $t_{\text{AT}} = 59681.4$ (“AT” = “ATLAS”) requires more scrutiny than the remainder of the dataset for two reasons: (i) the relatively low confidence of the detection and (ii) the impact of very early emission being probed with a robust detection. Unlike the remainder of the ATLAS detections, the highest-SNR raw detection reported by ATLAS at t_{AT} is a 2σ detection (measurement of $66 \pm 33 \mu\text{Jy}$). Given the proximity to the first unambiguous detections of the transient, the t_{AT} detection, if non-spurious, would play a significant role in constraining models of the light curve.

The observations reported by ATLAS at t_{AT} consist of three exposures, acquired

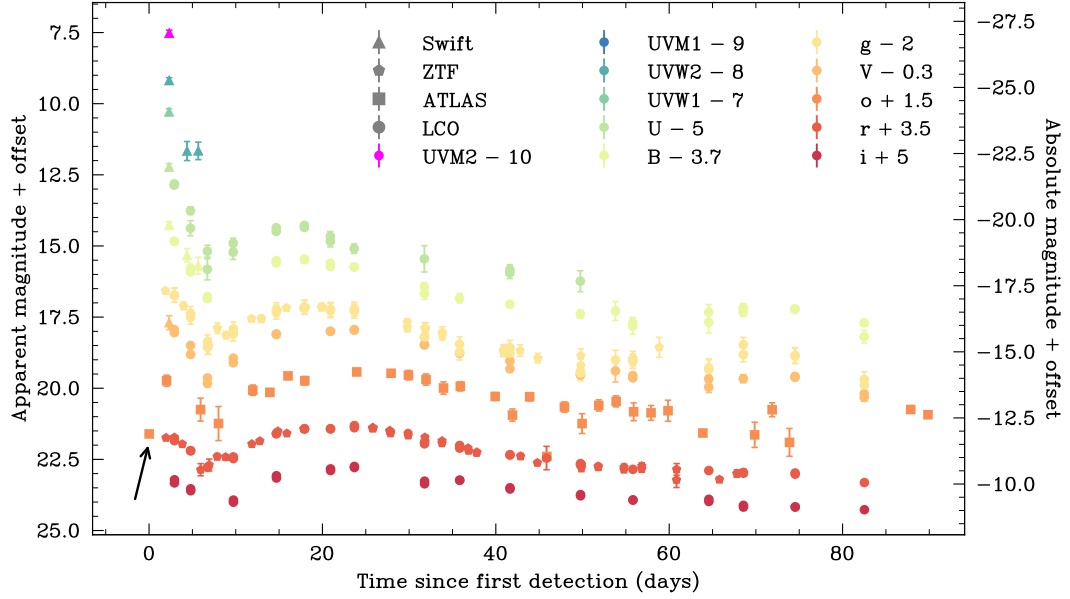


Figure 2.2: A compilation of the photometric data collected on SN 2022hnt by LCO, ZTF, *Swift*, and ATLAS follow-up programs. Apparent and absolute magnitudes are shown offset arbitrarily to improve visibility. The ATLAS detection constraining the rise is indicated with an arrow. Error bars represent uncertainties of one standard deviation.

within 15 minutes of each other. Two exposures are $\sim 1\sigma$ and the third is an exactly 2σ detection. To evaluate, we use the tool *ATclean* [116] to perform cuts based on eight control epochs, drawn from a circular region around the transient location [see 116, for further details on the procedure]. An example subset of the control epochs is shown in Figure 2.3. All three data points exceed the minimum constraints required to be considered real; namely, all three points have uncertainty $< 160 \mu\text{Jy}$ and a PSF fit $\chi^2 < 20$.

Next, we seek to improve our understanding of the emission captured at t_{AT} . We stack all three intranight exposures at this epoch using the given uncertainties and produce a co-added measure of the photometry, which results in a slight increase of the confidence from 2σ to 2.1σ . However, we suggest that the ATLAS error bar may be a mild overestimate of the uncertainty present in the measurement. Examining

the no-flux control curves after applying the cuts described above, we note that 2σ measurements appear $\ll 5\%$ of the time, less than would be expected for a true 2σ measurement. Additionally, the spread associated with no flux in the control curves is $\sim \pm 26 \mu\text{Jy}$, smaller than the errors reported by all three observations acquired at t_{AT} . Both these effects are visualized using a subset of the control curves in Figure 2.3. If we apply this estimate of the uncertainty to the observations at t_{AT} , it expands the co-added significance to $\sim 2.7\sigma$. While this confidence does not make the measurement highly robust, we choose to include it with caution in the analysis based on the above discussion and the proximity of the detection to the transient.

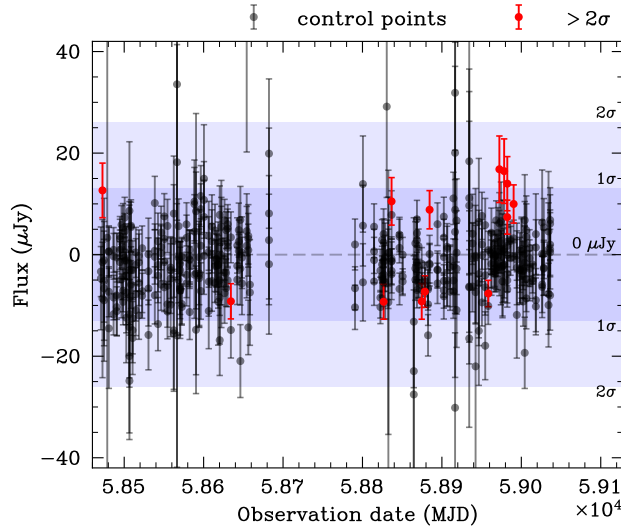


Figure 2.3: Example subset of the ATLAS control curves used to assess the validity of earliest ATLAS detection. The earliest ATLAS detection at $t_{\text{AT}} = 59861.4$ has a 2σ error. Examining the control curves, we find that 2σ detections (red) occur $< 5\%$ of the time, an indication that the errorbar may be moderately conservative. The intrinsic scatter of the measurements ($\approx 26 \mu\text{Jy}$, darker blue shaded region) is also lower than the t_{AT} measurement errorbar ($33 \mu\text{Jy}$) would suggest. By co-adding all exposures at t_{AT} and adopting the $26 \mu\text{Jy}$ errorbar, we can increase the significance of the t_{AT} detection to 2.7σ .

2.5 Evolution of the transient

2.5.1 Spectral evolution

The earliest LCO spectrum of SN 2022hnt was taken within one week of the estimated explosion date. Over the next ~ 5 weeks, we recorded a spectrum of the transient once every ~ 1 week, for a total of 5 spectra covering the first month of the evolution of the supernova. An additional ZTF spectrum was taken during the first 24 hours post-discovery [111]. The complete spectral evolution of SN 2022hnt (LCO+ZTF) is shown in Figure 2.4. The spectral evolution is broadly consistent at representative phases with other Type IIb SNe such as SN 2016gkg, also shown in Figure 2.4.

The earliest spectrum (taken by ZTF, obtained from TNS) show a largely featureless blackbody, typical for a IIb supernova in the earliest explosion stage. The high temperature blue spectrum is consistent with a very hot fireball; spectral lines will not begin to show until the ejecta have cooled down considerably [20]. The spectrum is too low-resolution to conclusively rule out flash features indicating interaction with a circumstellar medium (CSM) as are sometimes found in Type II SNe in the few hours post-explosion [e.g., 117; 118]. We do not note any such features in the SN 2022hnt spectra.

The next two spectra correspond to ~ 1 week post-explosion. At this stage, the ejecta have shock cooled and several lines are beginning to develop. We identify the presence of a broad $H\alpha$ bump in the day 6 spectrum which develops into a clear P Cygni line profile as the line-of-sight absorption becomes more prominent. Additionally, we note the presence of Ca II, definitively by day 8 at 4000 and 9000 Å, and possibly but not definitively by day 6.

By day 13, the $H\alpha$ has strengthened and the absorption feature is more clear, almost

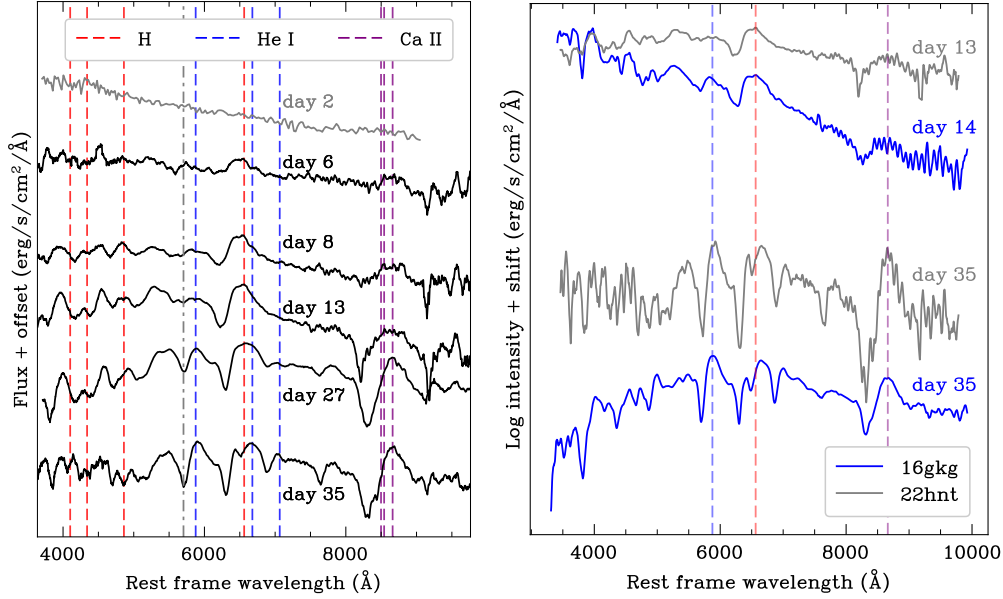


Figure 2.4: (*left*) The complete spectral evolution of SN 2022hnt (LCO, black; ZTF, gray) in rest wavelength. We highlight the following spectral lines as dashed vertical lines: hydrogen (red), helium (blue), and Calcium II (purple). The gray dot-dashed line at ~ 5700 Å shows He I blueshifted by ~ 8790 km/s. (*right*) A comparison of the spectra of 22hnt to another well-known IIb, SN 2016gkg, at two representative phases. Dashed lines show the location of rest-frame hydrogen (red), helium (blue), and calcium (purple). Both spectra are shown in their rest frame wavelength. The spectrum of SN 2022hnt is quite consistent with other objects in the Type IIb subclass.

equal in magnitude to the emission feature. A Mg I feature is clearly visible at ~ 5500 Å. This feature may have been present at day 8 as a weak bump, but is a prominent emission line by day 13. Both the blue and red Ca II lines have strengthened as well, particularly the narrower Ca II doublet at ~ 4000 Å. The $H\alpha$ emission is continuing to increase in intensity but is flattened by the presence of an increasingly blueshifted He I P Cygni profile, as is common for Type IIb SNe [20; 119; 40].

By day 27, the $H\alpha$ P Cygni, Ca II lines, and Mg I are at their maximum intensities achieved during our monitoring. We note blueshifted He I absorption features coincident with the $H\alpha$ emission as well at ~ 5800 Å. The blueshifted helium emission originates from a more interior layer of the ejecta and is an effective approximation

of initial shock velocity in the absence of emission profiles for more interior elements (e.g., Fe III). To estimate the velocity of the He I absorption lines, we fit a Gaussian profile to each feature and interpreted the best-fit mean as the location of the feature. Though the absorption line at $\sim 5800 \text{ \AA}$ is potentially coincident with a nearby Na line, we noted that both the absorption feature at $\sim 5800 \text{ \AA}$ and the He I absorption line coincident with $H\alpha$ evolved similarly, indicating either would be a suitable choice to measure the He I velocity. We find a corresponding line blueshift of $\sim 8790 \text{ km/s}$ on both the stronger absorption feature at $\sim 5800 \text{ \AA}$ and the absorption feature coincident with the $H\alpha$ P Cygni profile. We perform this fit for all days on the stronger feature when visible and find a slight decrease in velocity from $\sim 9000 \text{ km/s}$ to $\sim 7000 \text{ km/s}$ over the first 35 days of the explosion.

Following 27 days post-explosion, most emission and absorption features begin to decrease in intensity, particularly the $H\alpha$ feature, as is common for a hydrogen-stripped IIb [20].

2.5.2 Photometric evolution

The full photometric evolution of SN 2022hnt is shown in [Figure 2.2](#). This visualization includes all LCO photometry as well as photometry obtained via ATLAS, ZTF, and Swift. The early ($t < 1 \text{ week}$) luminosity is almost entirely dominated by the emission from shock cooling as the shocked envelope expands [29]. During these epochs, the emission is blue and rapidly diminishing. The rise to the first peak is constrained by an early sequence of serendipitous ATLAS detections immediately prior to the peak of the shock cooling emission. Due to the rise to peak being only $\sim$ hours in duration, it is extraordinarily difficult to organize and execute follow-up campaigns which probe the behavior during this phase. Thus, the presence of rigorous detections on the rise

represents an opportunity to test the effectiveness of existing shock cooling models at describing behavior during this phase.

After the shock cooling phase, the emission in the explosion is largely dominated by the decay of ^{56}Ni synthesized during the explosion into ^{56}Co . The onset of this emission causes a slower re-brightening of the object to a second peak ~ 1 month into the evolution of the transient [22]. The remainder of the light curve is dominated by this ^{56}Ni decay and further decays from $^{56}\text{Co} \rightarrow ^{56}\text{Fe}$, which forms an elongated “tail”.

In the following sections, we fit a series of physically-motivated models to the shock-cooling phase of SN 2022hnt. These models use the light curve behavior to directly measure physical properties of the explosion, progenitor, and ejecta.

2.6 Photometric modeling

In order to extract physical measurements from the data, we fit several physically motivated, analytic models of the shock-cooling light curve behavior. Of interest is assessing how the various models agree and conflict, which can identify limitations and inform which models may be most consistent for making measurements on a sample of Type IIb supernovae. Previous analyses reported significant disagreements in shock cooling parameter estimates between different models [40], as well as applications of the same model with different initial assumptions [e.g., polytropic index; see e.g., 39]. Additionally, shock cooling models must be applied with careful attention to regimes of validity in order to report conservative and accurate results [see e.g., 110, for a discussion].

To address these problems, we propose a framework for incorporating the most impactful considerations of shock cooling into light curve modeling attempts, with the goal of producing the most robust parameter estimates possible (see section 2.3 for a full

description). In [subsection 2.6.1](#), we discuss validation of the shock cooling assumption and argue for the placement of upper limits on the configuration of a heating scenario if cooling cannot be confirmed back to the explosion epoch. In [subsection 2.6.2](#), we present the necessary information to fit several models of shock cooling, with the intention of using consistency between the models to validate parameter inference. In [subsection 2.6.3](#), we consider the mass, temperature, and transparency regimes of validity for each model, which will be used to caveat the reliability of fiducial parameter estimates. Finally, in [subsection 2.6.4](#), we describe our fitting procedure and motivate variation of pre-fitting considerations (e.g., model assumptions, selective data exclusion, etc.), to better understand the robustness of the parameter inference within the context of the individual models.

2.6.1 Validation of shock cooling inference

The choice to fit models of shock cooling to the early time emission must first be validated by ensuring that cooling is in fact taking place. Heating mechanisms (e.g., wind breakout, where the shock breaks out of an extended CSM beyond the stellar surface) can produce similar light curves to shock cooling but will produce erroneous fits due to the underlying incorrect cooling assumption. We investigate the cooling assumption by performing blackbody luminosity, temperature, and radius fits for each epoch with multicolor photometry. The results of the blackbody fits are shown in [Figure 2.5](#). The bolometric luminosity shows a clear double peak, characteristic of the Type IIb subclass. The blackbody temperature shows monotonic decay well-approximated by a $t^{-0.45}$ power-law, the nominal theoretical prediction of shock cooling [\[36\]](#). Finally, the radius grows linearly with time, consistent with expected behavior as the supernova is in the coasting phase of its expansion. The result of these fits are consistent with a

shock cooling phenomenon occurring for least $t \gtrsim 2$ days, corresponding to the second epoch of data and beyond.

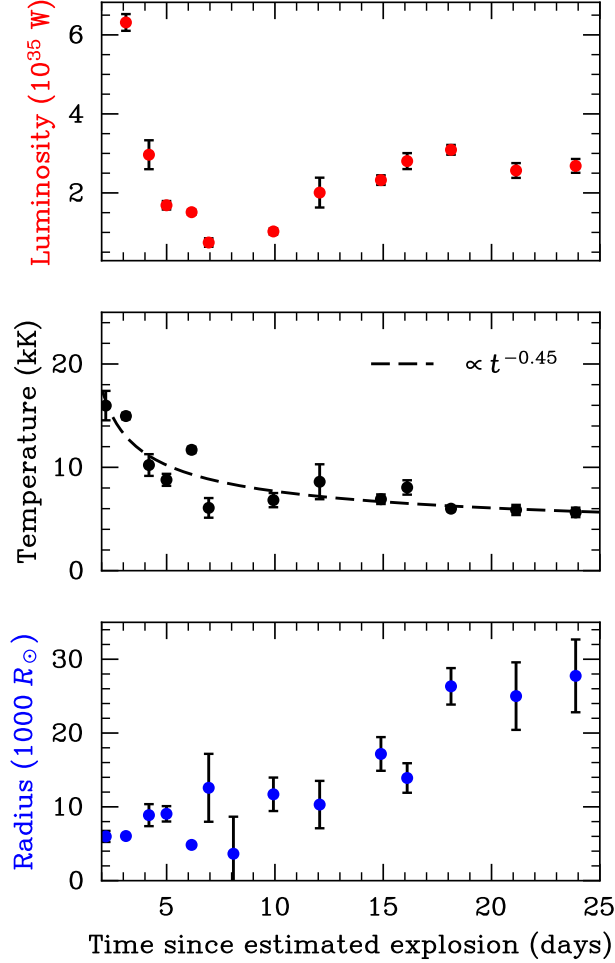


Figure 2.5: Blackbody fits of temperature, radius, and luminosity to the multicolor data of SN 2022hnt. Radial expansion is consistent with a linear coasting phase; temperature shows a decline consistent with the nominal prediction of shock cooling, a $T \propto t^{-0.45}$ power law. The luminosity fits clearly reproduce the double peak characteristic of Type IIb SNe.

The assessment of shock cooling, while valid for all data points $t \gtrsim 2$ days, can not be as easily made for the earliest data epoch (~ 59681.3 MJD). Due to the spectrum of the supernova likely peaking at UV wavelengths, a single o -band measurement is insufficient to constrain the temperature at this epoch. As a result, we cannot defini-

tively determine the temperature gradient at $t < 2$ days, which would conclusively confirm or rule out shock cooling. The worst-case scenario, where the object temperature evolution is consistent with heating due to wind breakout until $t \sim 2$ days, places an upper limit on the extent of the CSM R_{bo} being heated by the shock. Following [110], we use the relation $R_{\text{bo}} \sim t_{\text{rise}} v_s$, where $t_{\text{rise}} \lesssim 2$ days is the maximum possible rise time, and v_s is the estimated shock velocity. We estimate an upper limit on the extent of the CSM to be

$$R_{\text{bo}} \lesssim 2500 \left(\frac{t_{\text{rise}}}{2 \text{ days}} \right) \left(\frac{v_s}{10^4 \text{ km/s}} \right) R_{\odot}. \quad (2.1)$$

This estimate for R_{bo} is consistent with (i.e., less than) the earliest blackbody radius fits, $R_{\text{bo}} \lesssim 7000 R_{\odot}$. We can similarly upper bound the mass of the shocked CSM emitting prior to breakout from the extended wind. Again following [110], we approximate the characteristic bolometric luminosity at breakout L_{bo} as the total amount of kinetic energy deposited into the CSM, $L_{\text{bo}} \sim M v_s^2 / t_{\text{rise}}$, where M is the mass of the CSM interior to R_{bo} . We estimate an upper limit on the mass contained as,

$$M \lesssim 8.7 \left(\frac{L_{\text{bo}}}{10^{36} \text{ watts}} \right) \left(\frac{t_{\text{rise}}}{2 \text{ days}} \right) \left(\frac{v_s}{10^4 \text{ km/s}} \right)^{-2} 10^{-4} M_{\odot}. \quad (2.2)$$

By simply adopting R_{bo} into a volume, we can estimate a characteristic CSM density of $M/R_{\text{bo}}^3 \sim 3.3 \times 10^{-13} \text{ g cm}^{-3}$.

2.6.2 Choice of shock cooling model

[1] Model

[1] addressed the emission generated during $t \lesssim$ a few days post-explosion (when the

luminosity is primarily generated in the layers of the ejecta proximate to the stellar surface), but also extended the model of [36] to $t \gtrsim$ a few days post-explosion, when the luminosity is generated in a ρ -profile dependent manner from the innermost layers of the material.

The luminosity and resulting color temperature is dependent on the assumed polytropic index n , which itself varies depending on the assumed physical situation. For typical red supergiant stars with convective envelopes, we assume a polytropic index of $n = 3/2$. However, in stellar environments dominated by a radiative envelope (e.g., blue supergiants), a polytropic index of $n = 3$ is more appropriate. We modify the final expression for the luminosity from [1] to account for choice of polytropic index n and obtain

$$L(t) \sim (1.88 - 0.147[n - 3/2]) \times 10^{42} \times \left(\frac{v_{s,8.5}^2 t^2}{f_\rho M \kappa_{0.34}} \right)^{3 \times 10^{-3} - 5.9 \times 10^{-2} n} \times \frac{v_{s,8.5}^2 R_{13}}{\kappa_{0.34}} \times \exp \left(- \frac{[1.93nt - 1.23t]}{[19.5 \kappa_{0.34} M_e v_{s,8.5}^{-1}]^{0.5}} \right)^{0.87 - 4.7 \times 10^{-2} n} \text{ erg s}^{-1}, \quad (2.3)$$

where $v_{s,8.5}$ is the velocity of the shock in units of $10^{8.5} \text{ cm s}^{-1}$, t is the phase of the supernova in days, M is the total mass of the star (core M_c plus envelope M_e) in solar masses, $\kappa_{0.34}$ is the opacity in units of $0.34 \text{ cm}^2 \text{ g}^{-1}$, R_{13} is the radius of the extended envelope of the progenitor star R_e in units of 10^{13} cm , and

$$f_\rho \sim \begin{cases} (M_e/M_c)^{0.5} & n = 3/2 \\ 0.08 (M_e/M_c) & n = 3 \end{cases}. \quad (2.4)$$

In order to fit multiband data such as that collected for SN 2022hnt, [1] also derived a color temperature for the expanding photosphere as a function of time. We transform the temperature expression similarly to Equation 3.1 and obtain

$$T(t) \sim (2.14 - 0.06n) \times 10^4 \times \left(\frac{v_{s,8.5}^2 t^2}{f_\rho M \kappa_{0.34}} \right)^{3.8 \times 10^{-2} - 7.3 \times 10^{-3} n} \times \frac{R_{13}^{0.25}}{\kappa_{0.34}^{0.25}} t^{-0.5} \text{ K.} \quad (2.5)$$

[35] Model

The basic model of [34] was revisited and extended in [35]. The extended model incorporated low-mass extended material into a two-zone model with a broken power law dependence. The two-zone model consists of an outer region with a steep radial dependence for density and an inner region with a shallow radial dependence. The behavior was characterized before and after a time $t \approx t_d$, corresponding to the approximate time the diffusion reaches the boundary between the inner and outer zones. We reproduce the analytic expression for the luminosity $t \lesssim t_d$ as calculated in [35] here:

$$L(t) \approx \frac{\pi(n-1)}{3(n-5)} \frac{c R_e v_t^2}{\kappa} \left(\frac{t_d}{t} \right)^{4/(n-2)}. \quad (2.6)$$

After the diffusion reaches the zone boundary (i.e., $t \gtrsim t_d$) the luminosity was derived by [35] to be

$$L(t) = \Upsilon \exp \left[-\frac{1}{2} \left(\frac{t^2}{t_d^2} - 1 \right) \right]. \quad (2.7)$$

In both of these expressions, $n \approx 10$ is the steep power-law dependence of the outer zone, R_e is the radius of the extended envelope, κ is the electron scattering opacity, t

is the phase of the supernova in days, Υ is a prefactor defined as

$$\Upsilon = \frac{\pi(n-1)}{3(n-5)} \frac{cR_e v_t^2}{\kappa}, \quad (2.8)$$

and v_t is the velocity at the zone boundary, defined as

$$v_t = \left[\frac{(n-5)(5-\delta)}{(n-3)(3-\delta)} \right]^{1/2} \left(\frac{2E_e}{M_e} \right)^{1/2}, \quad (2.9)$$

where M_e is the mass of the extended envelope, $\delta \approx 0.1$ is the shallow radial dependence of the inner zone, and E_e is the fraction of the total energy of the supernova deposited in the extended material. We follow [35] and choose

$$E_e \approx 2 \times 10^{49} E_{51} \left(\frac{M_c}{3M_\odot} \right)^{-0.7} \left(\frac{M_e}{0.01M_\odot} \right)^{0.7} \text{ erg}, \quad (2.10)$$

Given the luminosity derived above the assumed radius, we use the Stefan-Boltzmann law and derive the following expression for the temperature as a function of supernova phase

$$T(t) = \left[\frac{L(t)}{4\pi R^2(t) \sigma_{\text{SB}}} \right]^{1/4}, \quad (2.11)$$

where σ_{SB} is the Stefan-Boltzmann constant.

[2] Model

[2] treats the expansion of the supernova ejecta in two phases: (i) a planar phase where the width of the emitting region is $\delta r \ll R$ and is located at $r \sim R$, and (ii) a later, spherical phase where the emitting region is located at $r \gg R$. Previous works have presented exact and approximate solutions for the luminosity and color temperature in the planar and spherical phase, respectively [36; 38]. [2] presents an

analytically-derived, numerically-calibrated interpolation between the exact solution previously derived for the planar phase and the approximate solution derived in [1] for the spherical phase of the expansion.

We reproduce the interpolation here. For simplicity, the quantities in the interpolation are normalized to the time t_{br} corresponding to the transition point between the planar and the spherical phase. [2] reported the luminosity for the planar phase as

$$L/L_{\text{br}} = \tilde{t}^{-4/3} + A \exp \left[- (at/t_{\text{tr}})^\alpha \right] \tilde{t}^{-0.17}, \quad (2.12)$$

where $\tilde{t} \equiv t/t_{\text{br}}$, $A \sim 0.9$, $a \sim 2$, $\alpha \sim 0.5$. Additionally,

$$t_{\text{tr}} = \sqrt{\frac{\kappa M_{\text{env}}}{8\pi c v_{s*}}} \quad (2.13)$$

$$= 19.5 \sqrt{M_{\text{env}} \kappa_{0.34} v_{s*,8.5}^{-1}} \text{ d}, \quad (2.14)$$

where $v_{s*} \sim 1.05 f_\rho^{-0.19} \sqrt{E/M}$ is a derivative quantity of the shock velocity, $M_{\text{env}} = M_0 - M_c$ is the envelope mass in solar masses, and

$$L_{\text{br}} = 3.69 \times 10^{42} R_{13}^{0.78} v_{s*,8.5}^{2.11} (f_\rho M_0)^{0.11} \kappa_{0.34}^{-0.89} \text{ ergs}^{-1}. \quad (2.15)$$

In order to compare this to the multiband data collected on SN 2022hnt, we cite the corresponding normalized color temperature relation from [2]:

$$T_{\text{col}}/T_{\text{col,br}} = \min \left[0.98 v_{\text{bo},9}^{-0.05} \tilde{t}^{-1/3}, \tilde{t}^{-0.45} \right], \quad (2.16)$$

where

$$v_{\text{bo}} = 3.31 M_0^{0.13} v_{*,8.5}^{0.13} R_{13}^{-0.26} \kappa_{0.34}^{0.13} f_\rho^{-0.09} \sqrt{E/M}. \quad (2.17)$$

With the modification of the t_{tr} quantity, the validity of this model was extended to $t \lesssim t_{\text{tr}}/a$. The model is more broadly valid in the shock-cooling phase up to hydrogen recombination, i.e., until the temperature in the ejecta drops to ~ 0.7 eV. However, this model is not numerically calibrated specifically for Type IIb SNe and has never been applied to Type IIb SNe. As a result, while the model may reproduce the light curve behavior to a reasonable degree, the anticipated performance of the model is uncertain and it is expected to be far less discriminatory than if it were applied to ordinary Type II SNe.

2.6.3 Regimes of validity

We consider regimes of validity for shock cooling models as applied to SNe Type IIb, which have lower envelope masses than the SNe Type II such models are normally calibrated against. We first consider the temperature regime of validity. Both [1] and [2] are only valid for $T \gtrsim 0.7$ eV ~ 8120 K. To ensure all fits are performed at valid temperatures, we restrict the data to epochs where the blackbody fits performed in [subsection 2.6.1](#) reported a bolometric temperature $T > 8120$ K. A rigid application of this rule (using a strict cutoff based on the blackbody fit values) restricts the data to $t \lesssim 7$ days. A less rigid application (using a looser cutoff based on the $t^{-0.45}$ power law fit in [Figure 2.5](#)) restricts the data $t \lesssim 10$ days.

Next, we consider the transparency regime of validity. Shock cooling models incorporate an approximate late-time bolometric luminosity suppression factor, which is valid up to the time t_{tr}/a , when the envelope becomes transparent (i.e., optical depth $\tau \sim c/v$). The approximation for [1] [[2]] is valid for mass ranges $0.1M_{\odot} \lesssim M_{\text{env}} \lesssim 10M_{\odot}$ [$1M_{\odot} \lesssim M_{\text{env}} \lesssim 10M_{\odot}$]. For the [1] model, this validity range will make the approximation applicable for a substantial range of IIb envelope masses, which

typically live between $0.01 M_{\odot} \lesssim M_{\text{env}} \lesssim 0.5 M_{\odot}$. However, we are beyond the calibration regime for the model of [2], which creates an opportunity to assess the effectiveness of the calibration for Type IIb SNe. Assuming a valid or near-valid mass estimate, the transparency timescale t_{tr} can be computed using equation (20) from [1] (or [2] equation A3) as

$$t_{\text{tr}} \sim 2 \left(\frac{M_{\text{env}}}{0.1 M_{\odot}} \right)^{1/2} \left(\frac{v_s}{3 \times 10^{8.5} \text{ cm/s}} \right)^{-1/2} \text{ days.} \quad (2.18)$$

As this timescale is sensitive to the envelope mass and shock velocity (which will be varied throughout fitting), we dynamically implement the data cutoff by restricting the likelihood calculation for a given model to data with $t \lesssim t_{\text{tr}}/a + 0.5 \text{ day}$. Photometric data were sampled on $\sim 1 \text{ day}$ cadence; thus, the additional 0.5 day overhead allows photometry just outside the validity timescale to be included with minimal complications for the fit.

Finally, we consider the homologous expansion break time. [2] interpolates between solutions for the planar and spherical phases (switching at time $t = t_{\text{br}}$, while [1] considers only the spherical phase. An expression for the break time is given by equation (40) of [2],

$$t_{\text{br}} = 0.86 \left(\frac{R}{10^{13} \text{ cm}} \right)^{1.26} \left(\frac{v_{s*}}{10^{8.5} \text{ cm}} \right)^{-1.13} (f_{\rho} M_0 \kappa_{0.34}) \text{ hrs.} \quad (2.19)$$

The break time is not a validity timescale, but it is a useful quantity for comparing the models of [1] and [2]. Provided t_{br} is substantially earlier than the first epoch of data used in the fits, [1] and [2] should report similar results. Thus, the homologous expansion break timescale forms the basis of a useful sanity check on the fits. A visualization of each of these validity ranges is shown in [Figure 2.6](#).

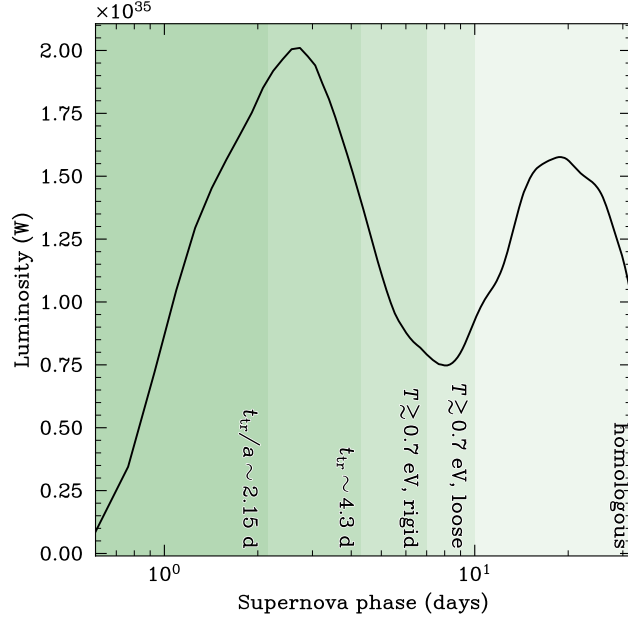


Figure 2.6: A visualization of how the regimes of model validity restrict fitting to subsets of the data for a model with $M = 0.2 M_{\odot}$, $R = 100 R_{\odot}$, and $v_s = 5000$ km/s. The light curve is a smoothed fit to the luminosities of SN 2022hnt. From right to left, the restrictions are: (i) “homologous”, a restriction to the part of the supernova evolution where the expansion can be considered homologous (effectively $t \gtrsim 1$ hr), (ii) “ $T \gtrsim 0.7$ eV, loose”, a loose restriction (see [subsection 2.6.3](#)) to the part of the supernova evolution where the temperature is above the hydrogen recombination temperature, (iii) “ $T \gtrsim 0.7$ eV, rigid”, the same as (ii) but a rigid restriction (see [subsection 2.6.3](#)), (iv) “ t_{tr} ”, a restriction to times less than the transparency timescale defined in [Equation 2.18](#), and (v) “ t_{tr}/a ”, the most conservative restriction to the subset of the evolution with phase less than the scaled transparency timescale as described in [subsection 2.6.3](#).

2.6.4 Fitting procedure and considerations

We fit the models above via the open-source package `lightcurve-fitting` [[120](#)]. This package implements a Markov-Chain Monte Carlo (MCMC) fitting routine, which we initialized with 15 walkers with $\gtrsim 10^5$ steps (10^3 -step burn-in) for posterior exploration. Fit settings (walkers, steps, etc.) were chosen based on two metrics of convergence: (i) the improved Gelman-Rubin statistics (specifically, $\hat{R} < 1.01$) [[121](#)], and (ii) chain autocorrelation (specifically, checking that the chain length is a factor of $\gtrsim 50$ longer than

the autocorrelation timescale) [122]. `lightcurve-fitting` performs the maximization and exploration of the resulting posterior using a wrapper of the MCMC algorithm implemented in the python package `emcee` [122]. All data are weighted uniformly, with no de-emphasis based on distance from explosion date as conducted by, e.g., [39]. We condition the model on the dataset using a simple Gaussian likelihood function. We investigated potential likelihood function biases (e.g., intrinsic scatter terms [123], quantifying deviations from a blackbody [124], etc.) and did not find these altered the results. Confidence intervals on the maximum a posteriori value were determined from the posteriors by computing the highest posterior density interval [as computed in, e.g., 125], as all posterior distributions (excluding the nuisance parameter $f_{\rho}M$) were unimodal and skewed-Gaussian-like. To explore the effect of modeling choices on the outcome of the fits, we vary the following:

Polytropic index n for the model of [1]: We generally do not expect the choice of polytropic index to substantially modify the results. Equation 2.5 and Equation 3.1 indicate that the temperature and luminosity (respectively) are only weakly dependent on n . However, at times close to the transparency timescale t_{tr} , emission interior to the edge of the envelope becomes visible and the emission becomes more sensitive to the density structure. Since IIb SNe are expected to have low envelope masses, t_{tr} is significantly reduced, increasing the expectation that the emission will be affected by choice of n .

UV line suppression factor A_{sup} for the model of [2]: Equation A4 of [2] presents a modification of the specific luminosity, introducing a factor $A_{\text{sup}} = 0.74$ to suppress UV line emission. This factor accounts for suppression caused by dominance of atomic transitions at shorter wavelengths. Given our dataset has multiple *Swift* bands probing the UV emission of the transient, we may be sensitive to the suppression factor. We vary A_{sup} to be either 0.74 or 1 (no suppression) for the [2] fits to test our sensitivity to

the suppression factor. We additionally explore the effect of the UV data on the fits by performing a fit with $A_{\text{sup}} = 0.74$ but with the *Swift* UVM1 and UVM2 filters removed from the dataset.

Presence of the earliest ATLAS detection: The inclusion or exclusion of the earliest ATLAS detection at $t_{\text{AT}} \sim 59681.40$ has direct consequences for the inferred explosion epoch of the transient, which in turn affects what data can be included in the likelihood calculation according to the transparency timescale. For the purposes of investigating the impact of the rise-constraining ATLAS detection on the analysis, we have performed fits where the detection was omitted (“-ATLAS”), included (“+ATLAS”) with t_{exp} restricted from varying past t_{AT} , and included with t_{exp} allowed to vary past t_{AT} (+ATLAS^a).

We vary all the above considerations simultaneously, but only report a subset of combinations which is sufficient to describe the range of observed model behaviors. In particular, we focus on the model of [2], in order to assess the performance of this new model on Type IIb SNe. By contrast, the model of [1] serves as a reference point, and a way to compare to past IIb SNe measurements made using this model [see, e.g., 39; 40]. Finally, in addition to the cross-model comparison, we compare the most plausible model fit to best-fit parameters for other objects in the literature, with the intent of contextualizing SN 2022hnt to other Type II and IIb SNe.

2.7 Analysis and physical implications

Having applied the framework of considerations discussed in [section 2.6](#), we now perform all proposed fits and assemble a landscape of parameter estimates.

Model	n	A_{sup}	ATLAS?	$v_{s,*}$ ($10^{8.5}$ cm/s)	M_{env} ($M_{\odot}$)	f_{ρ} M ($M_{\odot}$)	R_0 (10^{13} cm)	t_{exp}	$t_{\text{exp}} - t_{\text{AT}}$ (days)
SW17	3/2	N/A	Yes	$1.7^{+0.05}_{-0.04}$	$0.10^{+0.04}_{-0.02}$	110 ± 70	0.64 ± 0.09	59681.1 ± 0.05	-0.3 ± 0.06
SW17	3/2	N/A	No	$2.8^{+0.7}_{-0.6}$	$0.11^{+0.04}_{-0.02}$	110 ± 70	$0.20^{+0.12}_{-0.07}$	$59682.4^{+0.3}_{-0.8}$	$1^{+0.3}_{-0.8}$
SW17	3	N/A	Yes	$1.7^{+0.13}_{-0.1}$	$0.9^{+0.3}_{-0.2}$	90 ± 80	0.64 ± 0.06	59681.31 ± 0.02	-0.09 ± 0.02
SW17	3	N/A	No	$2.5^{+0.5}_{-0.7}$	1.1 ± 0.4	110 ± 60	$0.19^{+0.34}_{-0.06}$	$59682.5^{+0.2}_{-0.9}$	$1.1^{+0.2}_{-0.9}$
SW17	3/2	N/A	Yes ^a	$2.0^{+0.4}_{-0.2}$	$0.12^{+0.06}_{-0.03}$	80 ± 80	$0.55^{+0.09}_{-0.17}$	59681.27 ± 0.05	0.87 ± 0.05
M24	3/2	0.74	Yes	$3.6^{+0.2}_{-0.8}$	$0.03^{+0.012}_{-0.008}$	50 ± 40	0.57 ± 0.04	59681.32 ± 0.03	-0.08 ± 0.03
M24	3/2	1	Yes	$2.0^{+0.4}_{-0.3}$	$0.022^{+0.013}_{-0.007}$	110 ± 60	0.6 ± 0.1	59681.32 ± 0.03	-0.08 ± 0.03
M24	3/2	1	No	2.5 ± 0.4	$0.05^{+0.02}_{-0.01}$	30 ± 10	0.5 ± 0.2	59681.7 ± 0.05	0.3 ± 0.05
M24	3/2	1	Yes ^a	$2.3^{+0.4}_{-0.3}$	0.07 ± 0.02	24 ± 14	$0.64^{+0.07}_{-0.09}$	59681.31 ± 0.05	-0.09 ± 0.05
M24	3/2	0.74	No	$3.8^{+0.5}_{-0.5}$	$0.018^{+0.006}_{-0.003}$	40 ± 40	$0.22^{+0.07}_{-0.04}$	$59682.8^{+0.1}_{-0.3}$	$1.4^{+0.1}_{-0.3}$
M24	3/2	0.74	Yes ^a	$3.1^{+0.5}_{-0.4}$	$0.022^{+0.008}_{-0.005}$	110 ± 6	$0.5^{+0.2}_{-0.1}$	59682.6 ± 0.2	1.2 ± 0.2
M24	3/2	0.74 ^b	Yes	$2.7^{+0.4}_{-0.3}$	$0.17^{+0.09}_{-0.06}$	3 ± 2	0.4 ± 0.07	$59681.25^{+0.1}_{-0.2}$	$-0.15^{+0.1}_{-0.2}$

^a = the earliest ATLAS detection was included, but the explosion epoch was allowed to vary beyond it.

^b = fit performed with *UVM1* and *UVM2* filters removed.

Table 2.1: Numerical results from fitting the models described in [section 2.6](#).

2.7.1 Cross-model analysis

The results of each fit are shown in [Table 2.1](#). Representative fits for both the model of [\[1\]](#) and [\[2\]](#) are shown in [Figure 2.7](#). Broadly, we find a wide range of envelope masses and explosion epochs, with a narrower range of shock velocities and progenitor radii. Envelope masses range from plausible IIb masses ($0.01M_{\odot} \lesssim M_{\text{env}} \lesssim 0.1M_{\odot}$) to less plausible masses ($M \sim M_{\odot}$). These higher masses are more consistent with a Type II or short-plateau Type II explosion [e.g., [126](#); [3](#)], scenarios which are both inconsistent with the photometric and spectral evolution characteristics. Additionally, such a high mass is only estimated in models with polytropic index $n = 3$, which has two ramifications. First, the high sensitivity of the mass estimate to n indicates that we are in the low-mass ($M \lesssim 0.1M_{\odot}$) regime, as discussed above. Second, given the other evidence that the transient is a IIb with a lower envelope mass, the higher mass estimate reported by the $n = 3$ models indicates that the density profile is more consistent with a fully convective envelope ($n = 3/2$). Indeed, the $n = 3/2$ models report envelope mass estimates more plausible for a IIb SN. We discuss only the $n = 3/2$ models below, unless otherwise noted.

The model of [\[35\]](#) extends the earlier model of [\[34\]](#), separating the envelope into two zones of variable power-law density to more precisely model the resulting shock cooling emission as the photosphere recedes (in mass coordinates) into the envelope. This model has been applied to several Type IIb SNe in the literature [e.g., [39](#); [127](#); [40](#)]. However, this model typically requires granular data within $\sim$ several days of the shock cooling peak in order to produce meaningful fits. Our dataset has a significant gap of ~ 2.5 days around the presumed shock cooling peak where no observations were made. As a result, there is likely at most one epoch within the regime of validity of the [\[35\]](#) model. We attempted fits using this singular epoch (following the approach

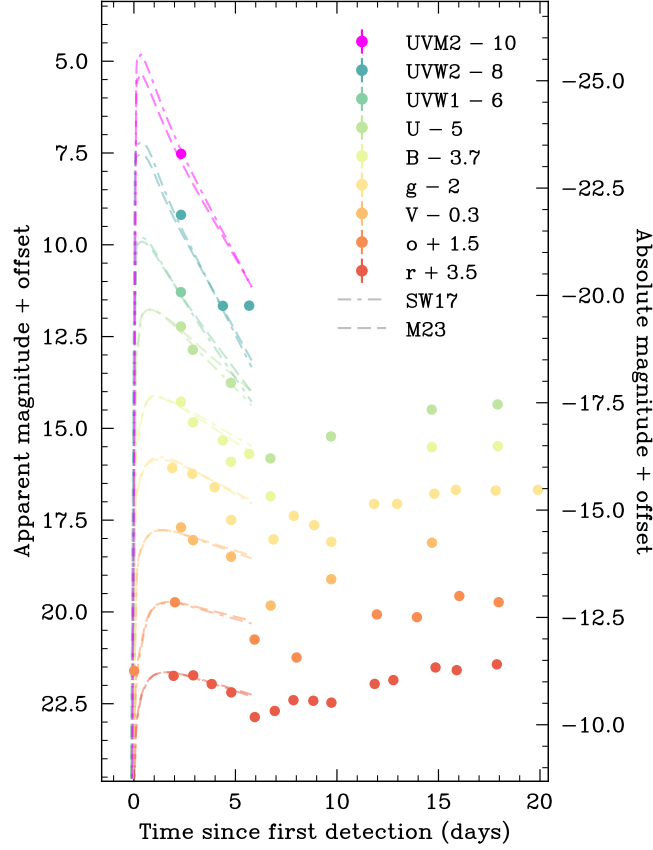


Figure 2.7: Comparison of representative maximum likelihood fits to the models of [1] (dot-dashed) and [2] (dashed). The shock cooling models are extrapolated linearly beyond their regime of validity (up to day 6) to assess the performance beyond the transparency timescale. The shock cooling behavior is well-captured in all bands.

of [40]) and found the limited data to be insufficient to constrain any of the model features. We also unsuccessfully attempted fits of the [35] model using data beyond the presumed regime of validity. As a result, we are choosing to omit the [35] model from application to this particular object.

Next, we examine shock velocity. The best-fit values from all models report a relatively narrow range of shock velocities, between $2.0 \times 10^{8.5} \text{ cm s}^{-1} \lesssim v_{s,*} \lesssim 4.0 \times 10^{8.5} \text{ cm s}^{-1}$ ($6300 \text{ km s}^{-1} \lesssim v_{s,*} \lesssim 12700 \text{ km s}^{-1}$). The velocities estimated from the model fits are consistent to 1σ (model uncertainty) with the range identified from

the He I spectra lines, which are not predictive of the initial shock velocity but are comparable. Higher shock velocities ($v_{s,*} \gtrsim 3 \times 10^{8.5} \text{ cm s}^{-1}$) are exclusively generated in the [2] model fits, which implement the UV line suppression with $A_{\text{sup}} = 0.74$. When the UV line suppression is not included (or included, but the shorter wavelength data is excluded), the shock velocity estimates drop to be more consistent with the model of [1]. Similarly, the inclusion of the UV line suppression factor in the [2] model results in mass estimates $\sim 10\times$ smaller than the [1] model or the [2] model with no suppression. Visual examination of the [2] fits both with (Figure 2.8) and without (Figure 2.9) UV line suppression do not show clear differences in the ability to reproduce the UV photometry. Both fits are able to reproduce the *UVW1* and *UVM2* photometry, while simultaneously substantially underestimating the *UVW2* photometry. However, the model including UV line suppression appears to slightly underestimate the *U*-band photometry compared to the model with no line suppression.

In addition to shock velocity, the effect of UV line suppression can be seen in the reported progenitor radius R_0 . All fits report values for the progenitor radius in the range $30R_{\odot} \lesssim R_0 \lesssim 100R_{\odot}$, a relatively narrow range consistent with a yellow supergiant, which have been identified as the progenitors for some past IIb SNe [128; 129; 100]. Models with UV line suppression mostly favor a lower ($R \lesssim 60R_{\odot}$) progenitor radius, with the sole exception being the model of [2] with the early ATLAS detection included, which favors a progenitor radius of $\sim 80R_{\odot}$. By contrast, all fits with the model of [1] favor a progenitor radius of $R_0 \sim 90R_{\odot}$. In general, we found that mild posterior degeneracies existed between the progenitor radius and the other parameters. These did not improve with increased MCMC exploration time, and likely cannot be avoided. Indeed, we compared our posteriors to those reported in the literature [e.g., 40; 41] and identified similar degeneracies between the model parameters.

Finally, we consider the best-fit explosion epoch. Most models clearly favor one

of two explosion epochs: (i) ~ 1 day after the ATLAS detection at t_{AT} , or (ii) ~ 3 hours before the ATLAS detection at t_{AT} . There is little variation in these individual subgroups. The presence of the ATLAS detection at t_{AT} is an obvious but imperfect predictor of the estimated explosion epoch. When the ATLAS detection is present and the fit is forced to find an explosion epoch prior to it (+ATLAS), the fits prefer an explosion epoch ~ 3 hours before t_{AT} . There is more variability for the other model configurations. If the ATLAS detection is excluded (-ATLAS), the explosion epoch is always preferred to be $\gtrsim 0.25$ days after t_{AT} . Results vary if the point is included and the fit is allowed to find an explosion epoch $> t_{\text{AT}}$. We discuss the ATLAS detection at t_{AT} and its impact on the inferred explosion epoch in greater detail in [subsection 2.7.3](#).

We now consider a fiducial set of best-fit parameters to use for comparison to other supernovae. Based on the above analysis involving the polytropic index, we immediately exclude all fits where $n = 3$ from consideration. Given that the early ATLAS detection is only $\sim 3\sigma$, we choose to exclude fits where the explosion epoch was forced to be $t_{\text{exp}} < t_{\text{AT}}$; i.e., we are restricting to the -ATLAS and +ATLAS^a model configurations. Our next consideration is the UV suppression factor. Model configurations where $A_{\text{sup}} = 0.74$ universally favored mass estimates $M_{\text{env}} \ll 0.1M_{\odot}$, where the transparency timescale is $\lesssim 1$ day. Such a short transparency timescale would exclude all but three data points, making a meaningful fit unlikely. By contrast, model configurations $A_{\text{sup}} = 1.0$ (i.e., no suppression) found masses consistent with a transparency timescale sufficient to achieve a meaningful fit; we therefore further restrict to these models. Of the remaining models (both [2] and [1] where $A_{\text{sup}} = 1$ and ATLAS? is No or Yes^a), the parameters of interest ($v_{s,*}$, M_{env} , R_0) are largely consistent. As described in [subsection 2.6.4](#), we take the maximum a posteriori value of each posterior as our parameter estimate and calculate an uncertainty using the highest posterior density interval. From this set of estimates μ_i and uncertainties σ_i for each

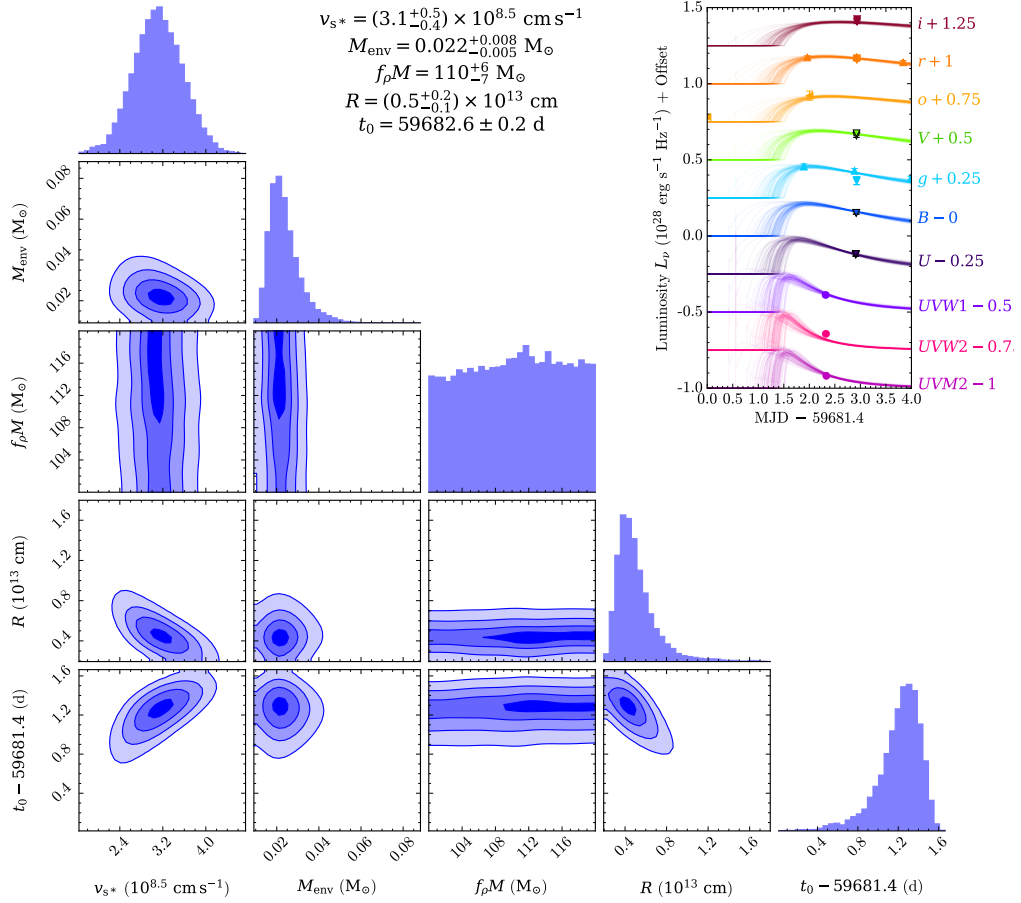


Figure 2.8: Corner plot and posterior samples for the [2] model fit with $A_{\text{sup}} = 0.74$ and inclusion of the early ATLAS detection with allowed variation past t_{AT} . The low envelope mass (and thus, short transparency timescale) results in the model clearly favoring an explosion epoch of ~ 1.25 days after the ATLAS detection. This placement of the explosion epoch is representative of the [2] model behavior when UV line suppression is included. All models seem to underestimate the UVW2 flux to the degree shown here. Note that f_{ρ} is a nuisance parameter and is not expected to be well-constrained.

parameter, we calculate a fiducial weighted mean p_{fid} and uncertainty σ_{fid} using

$$p_{\text{fid}} = \left[\sum_i \frac{1}{\sigma_i^2} \right]^{-1} \sum_i \frac{\mu_i}{\sigma_i^2}, \quad (2.20)$$

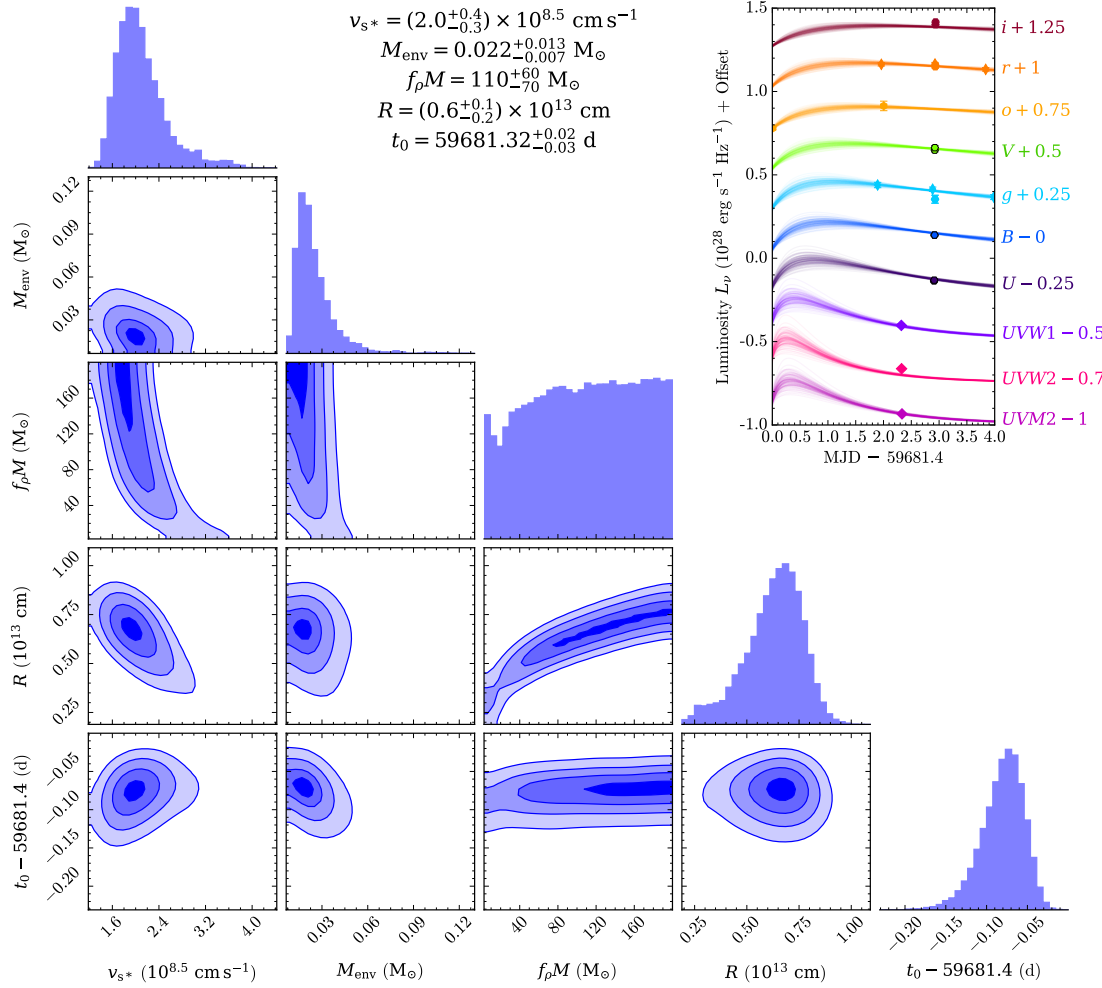


Figure 2.9: Corner plot and posterior samples for the [2] model fit with $A_{\text{sup}} = 1.0$ and inclusion of the early ATLAS detection with variation past t_{AT} not allowed. The inclusion of the ATLAS detection clearly results in a strong constraint on the rise when the explosion epoch is not allowed to vary past t_{AT} . All models seem to underestimate the UVW2 flux to the degree shown here. Note that f_{ρ} is a nuisance parameter and is not expected to be well-constrained.

and

$$\sigma_{\text{fid}} = \left[\sum_i \frac{1}{\sigma_i^2} \right]^{-1/2}. \quad (2.21)$$

Using this method, we find fiducial estimates for the parameters to be: $v_{s,*}, f = 2.32 \pm$

$0.22 \times 10^{8.5}$ cm/s ($v_{s,*,f} = 7.34 \pm 0.69 \times 10^3$ km/s), $M_{\text{env},f} = 0.068 \pm 0.013 M_{\odot}$, and $R_{0,f} = 0.493 \pm 0.063 \times 10^{13}$ cm.

2.7.2 Comparison with other supernovae

We additionally compare SN 2022hnt to samples from the spectrum of envelope-stripped SNe: specifically, SN 2016gkg (a typical stripped-envelope IIb), SN 2023ixf (a typical non-stripped-envelope Type II), and SN 2020bio (a peculiar IIb). These SNe were chosen in part due to their exceptionally early phase coverage. Both SN 2016gkg and SN 2020bio have data probing the shock cooling emission peak, and the dense SN 2023ixf follow-up campaign probed almost the entire shock cooling rise. Additionally, the SNe we have selected span from heavily-stripped-envelope objects (SN 2016gkg) to non-stripped-envelope objects (SN 2023ixf), in order to examine how the interpretation of the behavior of SN 2022hnt stands relative to similar objects.

We begin by comparing the light curve of SN 2022hnt to SN 2016gkg, a similar Type IIb with a low envelope mass. Data were obtained from LCO archives and were published in [39]. This visualization is made in [Figure 2.10](#). To perform the comparison, we calculate absolute magnitudes for both objects and shift the light curves in time so that their Ni^{56} peak in r -band is at $t = 0$. The light curve peak is calculated via the maximum of a third-order polynomial fit to the nickel peak. We compare the photometric evolution in r -band as well as providing the SN 2022hnt o -band data to understand how the early ATLAS detection impacts the estimated explosion epoch. The light curves show remarkably similar evolution, particularly post-peak. Both light curves also show a prominent dip at ~ 14 days pre-peak. However, SN 2016gkg drops significantly faster to this minimum from the shock cooling peak than SN 2022hnt does in either r - or o -band. This discrepancy could be attributed to the difference in

the envelope masses for the two objects; SN 2016gkg was found to have an envelope mass of $\sim 0.01 M_{\odot}$, while we find an envelope mass estimate for SN 2022hnt ~ 1 order-of-magnitude larger. The larger envelope mass could plausibly result in a slower luminosity decay as more hydrogen is available for recombination. The early ATLAS detection is $\lesssim 0.9$ hours away from the shifted best-fit explosion epoch for SN 2016gkg, providing a potential validation that the detection is related to the explosion and not an artifact due to random noise. Additionally, this comparison to another well-sampled IIb SN emphasizes the uniquely early constraining power provided by such an early detection.

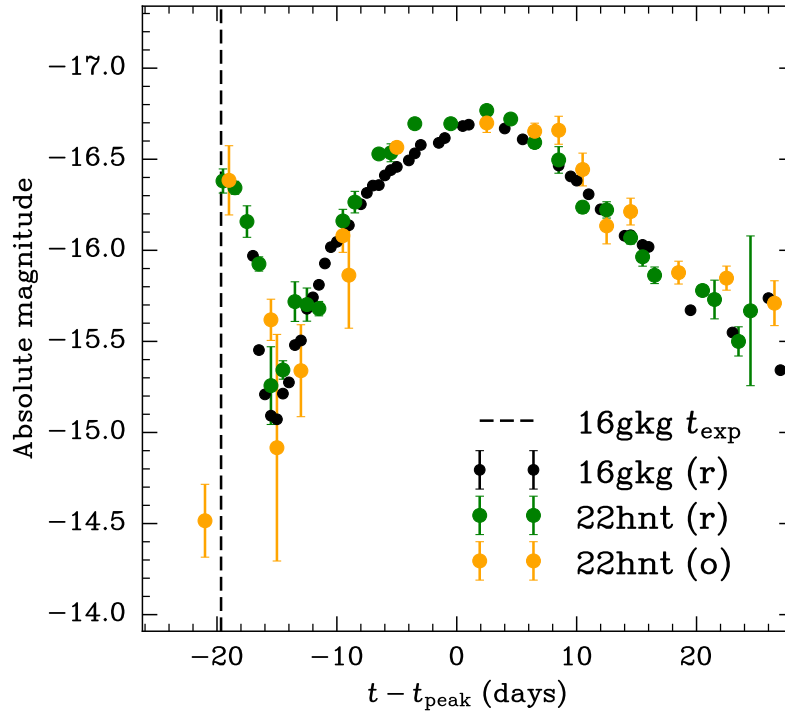


Figure 2.10: Comparison of the light curves of SN 2022hnt and SN 2016gkg. We compare the LCO data for 16gkg in r -band as well as the ATLAS data in o -band. The explosion epoch estimate (dashed line) for SN 2016gkg is based on [130]. Both light curves are shifted so the clearly-probed ^{56}Ni peaks are aligned. SN 2022hnt appears to have a slightly slower rise to and fall from the shock cooling peak, indicating a higher envelope mass, consistent with the results from the shock cooling fits.

Next, we compare the SN 2022hnt fiducial fit parameters to other objects in the literature. We visualize this comparison in [Figure 2.11](#). SN 2020bio and SN 2016gkg only have fits to the model of [\[1\]](#) available; however, we provide fits to both the model of [\[1\]](#) and [\[2\]](#) for SN 2023ixf, as presented in [\[41\]](#).

We first examine envelope mass, which is a direct probe of the pre-explosion stripping that differentiates Type IIb SNe from regular Type II SNe. The envelope of SN 2023ixf is considerably (i.e., $\gtrsim 2$ orders of magnitude) more massive than either SN 2020bio, SN 2022hnt, or SN 2016gkg, appropriate for a Type II, which typically do not experience significant envelope stripping prior to explosion. SN 2022hnt and SN 2016gkg are more typical mostly-stripped-envelope Type IIb SNe, and as a result report envelope masses $\lesssim 0.1 M_{\odot}$. SN 2020bio represents a more peculiar IIb; despite having extraordinarily weak $H\alpha$ and $H\beta$ emission features (indicating enhanced mass loss), shock cooling fits based on the model of [\[1\]](#) indicated a potentially less stripped envelope than more typical IIb SNe. However, early spectra indicated interactions with a CSM doped via significant mass loss, and models based on [\[34\]](#) and [\[35\]](#) reported envelope masses more consistent with a typical IIb.

We next examine progenitor radius. We generally find that the stripped-envelope objects in our comparison are consistent with our reported radius for SN 2022hnt, finding a progenitor radius of $\sim 100 R_{\odot}$. This is substantially (~ 1 order of magnitude) less than radii reported from fits for SN 2023ixf, which is typical for Type II SNe. We identify no clear relationship between progenitor radius and shock velocity. Aside from the model of [\[1\]](#) applied to SN 2023ixf—which reported a large but also quite uncertain progenitor radius—all progenitor radii in our comparison are relatively uniformly distributed below $\sim 500 R_{\odot}$. Small ($R \lesssim 200 R_{\odot}$) progenitor radii are identified in systems with fast (SN 2016gkg, SN 2020bio) and slow (SN 2022hnt, SN 2023ixf) estimated shock velocities.

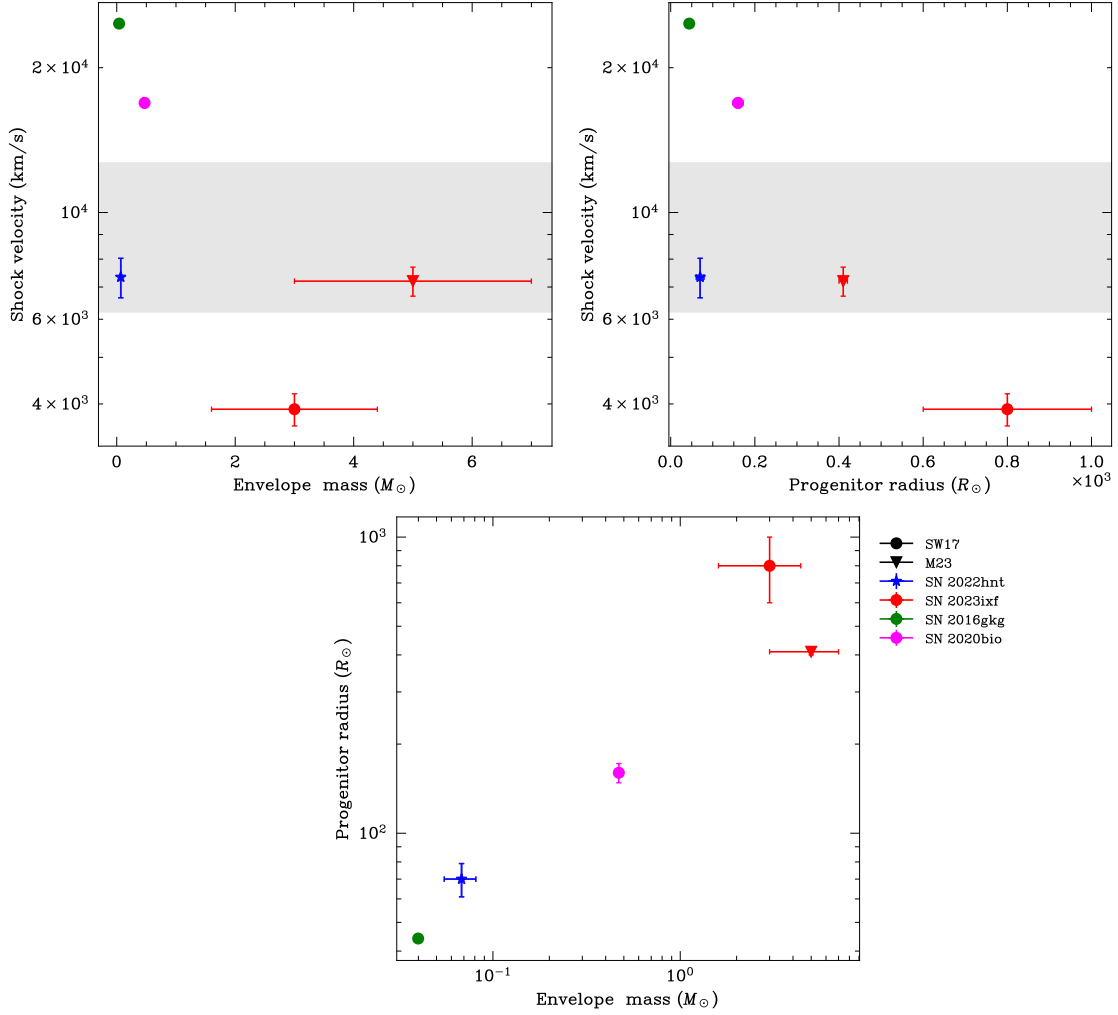


Figure 2.11: Cross-comparisons of shock velocity, envelope mass, progenitor radius, and rise time for SN 2022hnt, SN 2016gkg, SN 2023ixf, and SN 2020bio. Circles denote fits performed using the model of [1], while triangles denote fits performed using the model of [2]. The gray shaded region corresponds to the range of values recovered from the He I line velocity. Overall, we find shock velocities comparable with the He I line blueshift. The envelope mass reported by the shock cooling models is consistent with expectations from comparisons to other stripped and non-stripped envelope SNe (i.e., being less than Type II envelope masses but greater than very highly stripped envelope SNe such as 16gkg).

We similarly do not identify a clear relationship between shock velocity and envelope mass. The shock velocity we obtain for SN 2022hnt is consistent with the He I line blueshift, and similar to the SN 2023ixf measured shock velocity as well as the He I line

blueshift measured for SN 2020bio, which had a comparable progenitor radius and envelope mass. However, the shock cooling fits for both SN 2020bio and SN 2016gkg reported shock velocities a factor of ~ 2 -3 greater than SN 2022hnt. Unique among our fit parameters, the estimated shock velocity was largely statistically consistent regardless of model type or the inclusion of the earliest ATLAS detections. This consistency is in contrast to the model fits for SN 2023ixf and SN 2016gkg, which reported significant disagreement across the different classes of models used to measure shock velocity. However, despite reporting a shock velocity inconsistent with the He I line velocity, the modeled velocity measurements of SN 2020bio are self-consistent across model types.

Finally, we consider explosion epoch. Assuming the detection at t_{AT} is non-spurious, we consider where such an early detection lies in the context of the literature. We consider three IIb SNe with early detections for comparison: (i) SN 2016gkg, (ii) SN 2017jgh, and (iii) SN 2020bio. For each of these three objects, we consider: (a) time of earliest detection relative to peak of the transient in V -band (denoted δt_V , inferred from shock cooling fits if needed), (b) time of the earliest detection relative to the estimated explosion epoch (denoted δt_e , inferred from shock cooling fits if needed), and (c) magnitude change between the earliest detection and the peak in the corresponding band (denoted δm). These values are summarized in [Table 2.2](#). Immediately, we note a significant variation in the δt_V estimate. Since all considered objects were discovered relatively shortly after explosion (i.e., $\delta t_e \ll \delta t_V$), the δt_V quantity is mostly dominated by the rise time to the shock cooling peak, which is controlled by the envelope mass and shock velocity of the explosion. By contrast, the δt_e quantity is more directly informative of how early the supernova was captured. The earliest ATLAS detection probes a regime similar to that probed by the earliest amateur detections of SN 2016gkg, with a comparable change in magnitude to peak (δm) as well. We demonstrate this with a visual comparison in [Figure 2.10](#).

Event	δt_V (hours)	δt_e (hours)	δm
SN 2016gkg ^a	-10.8	2.16	2.3
SN 2017jgh	-62	$< 1^b$	4
SN 2020bio ^c	-19.2	5.76	0.9
SN 2022hnt	-28.8	2.88	2.1

^aWe refer to [130] for these numbers.

^bSN 2017jgh, remarkably, had Kepler data probing the entire rise from explosion as well as an extended sequence of non-detections immediately prior to the explosion epoch. We cite [108] for these numbers.

^cWe refer to [40] for these numbers.

Table 2.2: Comparison of the earliest observations made in SN 2016gkg, SN 2017jgh, SN 2020bio, and SN 2022hnt. δt_V considers the time between the earliest observation and the shock cooling peak in V band. δt_e considers the elapsed time between the explosion epoch and the earliest observation. δm considers the magnitude change between the earliest detection and the peak in the same band.

2.7.3 Consequences of ATLAS constraint

The earliest ATLAS detection at $t = t_{\text{AT}}$ is a $\lesssim 3$ sigma detection, which would conventionally be treated as ambiguous or even a non-detection. However, its proximity to the transient motivates an investigation into whether such emission would be plausibly predicted for a shock cooling event. The presence of the ATLAS detection has little impact on the best-fit envelope mass, progenitor radius, or shock velocity, an indicator that the emission at t_{AT} is consistent with the explosion parameters of SN 2022hnt. When the detection is removed, approximately 66% of our model fits favor an explosion epoch $\gtrsim t_{\text{AT}}$. However, the majority of the fits which favor $t_{\text{exp}} \gtrsim t_{\text{AT}}$ have $A_{\text{sup}} = 0.74$, which also produces a low envelope mass ($M_{\text{env}} \sim 0.01 M_{\odot}$) and therefore a very short transparency timescale ($t_{\text{tr}} \sim 0.5$ days). This unusually low mass estimate motivated the exclusion of the $A_{\text{sup}} = 0.74$ fits for consideration in the fiducial parameter set. Following that restriction, a significant majority of the remaining models prefer an explosion $\lesssim 3$ hours prior to t_{AT} . We therefore conclude that the emission at

this epoch is consistent with shock cooling.

2.8 Conclusions

We report the follow-up campaign of SN 2022hnt, a Type IIb SN at 84 Mpc. We organized an intense photometric and spectroscopic follow-up campaign shortly after discovery using the LCO telescope network. Additional data on SN 2022hnt were obtained using ZTF and ATLAS, including an exceptionally early detection in *o*-band by ATLAS. The presence of this early point constrains the shock cooling rise of the supernova and provides an opportunity to characterize the behavior of shock cooling models in a difficult-to-probe regime.

Recently developed models of shock cooling can probe earlier in the supernova evolution than previous models, but require careful attention to a series of model considerations which affect performance and accuracy. We describe a framework for addressing the most significant considerations of existing shock cooling models into attempts to model the light curve, with the aim of producing conservative parameter estimates. This framework considers alternatives to shock cooling (e.g., heating due to wind breakout), intra- and inter-model consistency, and regimes of validity for individual models in order to construct a robust fiducial parameter estimate.

We used the models of [1] and [2], which characterize the emission generated by the rapid shock heating and cooling of the hydrogen envelope due to the supernova in the days following explosion. Both models have been used to provide measurements of envelope mass, shock velocity, progenitor radius, and explosion epoch for other objects; however, this marks the first application of the model of [2] to a Type IIb SN. We performed fits to the photometric evolution of SN 2022hnt using the `lightcurve-fitting` package, which employs an MCMC routine to produce maximum likelihood param-

ter estimates. To explore the impact of model choices on the parameter estimation, we varied polytropic index, degree of UV line suppression, and the presence of the earliest ATLAS detections. Despite the model of [2] not being calibrated specifically for Type IIb SNe, we found that, regardless of model choice, it reports similar results to the more relevantly calibrated [1] model. We report fiducial estimates of the explosion properties of SN 2022hnt and find $v_{s,*,f} = 7.08 \pm 0.44 \times 10^3$ km/s, $M_{\text{env},f} = 0.07 \pm 0.02 M_{\odot}$, and $R_{0,f} = 0.64 \pm 0.08 \times 10^{13}$ cm. Additionally, we characterize where these parameter estimates live on the spectrum of Type II supernovae. We find the shock velocity to be unusually low, but find a progenitor radius and envelope mass consistent with other Type IIb SNe and predictably lower than non-stripped-envelope Type II SNe.

The work presented here represents a framework that can be duplicated and expanded upon with more in-depth surveys of IIb SNe using the advanced shock cooling models that have been developed in recent years. Our treatment demonstrates that shock cooling models not specifically calibrated to IIb SNe [such as 2] nevertheless can be cautiously extended to heavily stripped-envelope objects. Future population studies of IIb SNe will more deeply explore the relationships between progenitor and explosion properties that have been presented here, and will better characterize the spectrum of IIb explosions. Our investigation of the impact of the early ATLAS detection demonstrates the importance of very early ($t \lesssim 2$ days) data on Type IIb SNe, particularly in order to better constrain the effectiveness of the shock cooling assumption. The collection of early-time data as exploited here—and the rapid detection and follow-up that enables it—is crucial to test theoretical predictions and systematically study the behavior of IIb SNe.

Chapter 3

On the transition between IIb and short-plateau supernovae

“

FINALLY, THEN, VARIETIES HAVE THE SAME GENERAL CHARACTERS AS SPECIES, FOR THEY CANNOT BE DISTINGUISHED FROM SPECIES...NOTWITHSTANDING THAT INTERMEDIATE LINKING FORMS HAVE NOT BEEN DISCOVERED; BUT THE AMOUNT OF DIFFERENCE CONSIDERED NECESSARY TO GIVE TO TWO FORMS THE RANK OF SPECIES IS QUITE INDEFINITE.

— Charles Darwin, *On the Origin of Species*, 1859

Hydrogen-rich supernovae (SNe) span a range of hydrogen envelope masses at core collapse, producing diverse light curves from extended plateaus in Type IIP SNe to double-peaked Type IIb SNe. Recent simulations predict a continuous sequence of light-curve morphologies as hydrogen is removed, with short plateau SNe (plateau

durations $\approx 50\text{--}70$ days) emerging as a transitional class. However, the observational boundary between IIb and short-plateau remains poorly defined, and thus far unobserved. We report on extensive photometric and spectroscopic follow-up of SN 2023wdd and SN 2022acr, candidate transitional events on the low-mass end of the short-plateau class. Both exhibit weak, double-peaked light curves which we interpret as exceptionally short plateaus (10–20 days), and hybrid spectral features: persistent $H\alpha$ absorption with He I contamination, but without the helium dominance characteristic of IIb SNe. Using analytic shock-cooling models and numerical light curve fitting, we estimate hydrogen-rich envelope masses of $\sim 0.6\text{--}0.8 M_{\odot}$ —significantly larger than canonical IIb values ($\lesssim 0.1 M_{\odot}$) but consistent with the $\sim 0.9 M_{\odot}$ threshold predicted for short-plateau behavior. Although the progenitor radii inferred from analytic and numerical methods differ by factors of 2–5, envelope mass estimates are consistent across approaches. Comparisons to well-studied IIb (SN 2016gkg, SN 2022hnt), short-plateau (SN 2023ufx, SN 2006ai, SN 2016egz, SN 2006Y), and II SNe (SN 2023ixf, SN 2013ej) suggest a monotonic relationship between hydrogen envelope mass and plateau length consistent with analytic and numerical expectations. These findings provide additional evidence for a continuous distribution of envelope stripping in hydrogen-rich core-collapse progenitors and place SN 2023wdd and SN 2022acr along the IIb/short-plateau boundary.

3.1 Permissions and Attributions

1. The content of [chapter 3](#) is the result of a collaboration with the Global Supernova Project, in particular Prof. D. Andrew Howell and Prof. Daichi Hiramatsu, and has previously appeared in the *Astrophysical Journal* as Joseph R. Farah *et al.* (2026). It is reproduced here with the permission of the *Astrophysical Journal*:

<https://iopscience.iop.org/article/10.3847/1538-4357/ae3a71>.

2. Full acknowledgements are available on the journal page.

3.2 Introduction

Hydrogen-rich supernovae (SNe) represent some of the most diverse outcomes of massive ($> 8 M_{\odot}$) star evolution, characterized by the removal—none to partial to near-complete—of the hydrogen and/or helium shells prior to core collapse. Within this broad umbrella, Type IIb SNe retain a thin layer of hydrogen, manifesting as early-time hydrogen lines in their spectra that later give way to helium-dominated features [e.g., 27; 20; 99]. At the opposite extreme lie the Type II SNe, which hold onto a substantial hydrogen envelope and exhibit long (~ 100 days) photometric plateaus or linear declines powered by hydrogen recombination [see 131, for a review]. Historically, this distinction between unstripped Type II and mostly-stripped Type IIb events suggested that these were separate pathways, identified by spectral features [20; 44] and reflecting either minimal or extensive mass loss. Yet, an emerging body of theoretical and observational evidence indicates the possibility of a continuum in the amount of hydrogen envelope stripping, raising the question of how many physically intermediate (or “transitional”) events remain unclassified under the standard taxonomy [e.g., 132; 84; 44; 79; 80; 3].

The conventional picture of Type IIb progenitors invokes binary interaction or strong stellar winds to peel away nearly all the hydrogen above the helium core, leaving behind an envelope of only $\sim 0.01 - 0.1 M_{\odot}$ of H-rich material [30; 93; 130]. Double-peaked IIb supernovae ($M_{\text{Henv}} \lesssim 0.9 M_{\odot}$; range from [3]) often display prominent early-time excess emission in their light curves, where an initial shock-cooling peak gives way

to a re-brightening powered by radioactive nickel-56 decay [e.g., 102–108; 40; 133]. By contrast, Type II ($M_{\text{H}_{\text{env}}} \gtrsim 2.1 M_{\odot}$; range from [3]) supernovae (commonly associated with red supergiant progenitors) experience only modest envelope stripping and thus show extended plateaus (IIP) or a linear (IIL) decline for 80–120 days, followed by a transition to the radioactive tail [e.g., 134; 86; 41]. Between these two archetypes lie the so-called short-plateau ($0.9 M_{\odot} \lesssim M_{\text{H}_{\text{env}}} \lesssim 2.1 M_{\odot}$) SNe, showing plateaus of only ~ 50 –70 days, or sometimes even shorter [3]. These objects appear to occupy an intermediate regime in terms of hydrogen envelope mass, suggesting the boundaries between “IIb” and “II” do not form sharp categories, but rather reflect a distribution of envelope retention.

A central open question, therefore, is whether the observed range of hydrogen envelope masses—spanning canonical Type II ($M_{\text{H}_{\text{env}}} \gtrsim 2.1 M_{\odot}$) down to the thin envelopes of IIb supernovae—arises along a smooth continuum of stripping. Such a continuum view would help clarify the role of binary interactions, mass-transfer episodes, stellar winds, or eruptive mass loss in shaping the final envelope states of massive stars [95; 30; 94; 135; 136]. Systematic surveys targeting the earliest phases of core-collapse events are increasingly revealing transients with partial hydrogen depletion [e.g., 137; 138; 28, etc.]. Yet the supporting data are still sparse, especially around the boundary between Type IIb and the short-plateau regime.

In this work, we seek to demonstrate that the lower boundary of the short-plateau SNe subclass (i.e., between IIb and short-plateau SNe) may be part of a continuum of hydrogen stripping, and not a rigidly defined boundary. Observations of legacy supernovae have suggested that the hydrogen stripping present in Type IIb SNe may be part of a spectrum connecting to Type Ib SNe [138; 139]. Numerical models have extended this spectrum from Type IIb SNe to Type II SNe, suggesting that Type II SNe have experienced hydrogen loss to varying degrees, providing further evidence of a

continuum [3; 140].

Motivated by these considerations, we present detailed observations of two supernovae (SN 2023wdd and SN 2022acrv) that exhibit photometric and spectroscopic signatures intermediate to double-peaked IIb and short-plateau events. Their double-peaked light curves bear a qualitative resemblance to IIb SNe, but the initial drop is far more subtle, and a broad $H\alpha$ persists longer than archival IIb SNe. In [section 3.3](#), we report on the follow-up campaign and spectrophotometric reductions for both SN 2023wdd and SN 2022acrv. In [section 3.4](#) we review the spectroscopic evolution of both objects. In [section 3.5](#), we present the photometric evolution and model the light curve to measure the hydrogen-rich envelope masses of both transients, contextualizing them along the continuum of hydrogen-rich objects. In [section 3.6](#), we draw comparisons in evolution and inferred parameters to other supernovae in the continuum. Finally, in [section 3.7](#), we summarize our results.

3.3 Observations and data

3.3.1 SN 2023wdd

SN 2023wdd was discovered at *o*-band magnitude 17.9 by the Asteroid Terrestrial Last-impact Alert System [141, ATLAS] on Nov 1, 2023 (MJD 60249). This discovery was automatically reported to the Transient Name Server (TNS) on the same day as discovery. SN 2023wdd was located at right ascension (R. A.) 01:18:11.045 and declination (decl) +38:26:32.75. The SN is associated with host spiral galaxy UGC 00831. SN 2023wdd was initially classified as a young Type II SN by Li-Jiang One hour per Night observation of SNe (LiONS) [142], before being reclassified by the extended Public ESO Spectroscopic Survey of Transient Objects (ePESSTO+) on Nov 6

as a Type IIb [143]. A broad H α emission feature constrains the redshift to $z \sim 0.024$, consistent with the host galaxy redshift of $z \sim 0.024303$ (value from NED¹). Based on a redshift of $z \sim 0.024$ and assuming a flat universe ($\Omega_M = 0.286, \Omega_\Lambda = 0.714$) with $H_0 \approx 69.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$, we adopt a distance of 105.3 Mpc to the SN. Based on the [112] dust map, we adopt a line-of-sight extinction value of $E(B - V)_{\text{MW}} = 0.0373 \pm 6 \times 10^{-4}$ mag. We correct all light curve and photometric data presented in this paper with this extinction value. Next, we describe our multiband follow-up observations of SN 2023wdd.

Shortly ($\lesssim 2$ days) after discovery, we triggered a high-cadence, multiband follow-up campaign with the Las Cumbres Observatory [LCO, 84]. As part of the Global SN Project (GSP), LCO provides a global, networked array of robotic telescopes designed specifically for rapid follow-up of active transients. We obtained photometric observations in U, B, V, g, r and i bands using the LCO 1m and 2m telescopes. All photometric LCO data were reduced using the `lco_gtsnpipe` photometric reduction pipeline, which uses point-spread-function (PSF) and color term computation before magnitude extraction and measurement [86]. Extracted magnitudes were then calibrated to Landolt standards (using Vega magnitudes) for UBV filters [113] and the Sloan Digital Sky Survey (SDSS) catalog (using AB magnitudes) for the remaining bands [114].

The LCO observatory network was also used to obtain high-cadence spectra for the transient. We obtained seventeen spectra between day 2 and day 88 using the FLOYDS spectrograph, which is mounted on the northern 2m LCO site (Haleakala Observatory). This spectrograph has a wide coverage (350 nm to 1 micron), and data is processed via the custom LCO `floydsspec` pipeline [86]. `floydsspec` incorporates cosmic ray flagging, spectrum extraction, and calibration. LCO spectra were used

¹The NASA/IPAC Extragalactic Database (NED) is funded by the National Aeronautics and Space Administration and operated by the California Institute of Technology.

to identify host galaxy extinction via the method of [89], which fits the equivalent width of the Na I ($\lambda 5893$) doublet to estimate extinction. We find a best-fit extinction coefficient of $E(B - V)_{\text{host}} = 0.105 \pm 0.022$ mag. We additionally correct all light curve and photometric data presented in this paper with this host extinction value.

In addition to the LCO follow-up campaign, we also included forced photometry from the ATLAS survey [79] using the web portal [82]. ATLAS is a pair of two 0.5m all-sky survey telescopes which observe in at least one band at least once every other day. The ATLAS instruments are located in Hawaii, on Mauna Loa and Haleakala. ATLAS observed SN 2023wdd on ~dozen epochs in multiple bands simultaneously during the LCO observation campaign. ATLAS photometry was obtained in both the custom ATLAS *o*- (orange) and *c*- (cyan) band filters. We reduced all ATLAS photometry using the ATLAS forced photometry pipeline², with calibration to the Pan-STARRS catalog.

Besides LCO and ATLAS, we also incorporated publicly-available observations made by the Zwicky Transient Facility [80, ZTF]. ZTF is a nightly transient survey camera incorporated into the 48-inch Palomar Observatory telescope. We incorporate observations of SN 2023wdd (ZTF23abobwsd) made between Oct 29, 2023 and Jan 06, 2024. Subtractions and magnitude extractions are automatically performed using templates previously collected by ZTF and processed by the ZTF photometry reduction pipeline [83].

A compilation of all photometry data collected for SN 2023wdd is shown in [Figure 3.1](#). A compilation of all spectra for SN 2023wdd is shown in [Figure 3.2](#).

²<https://fallingstar-data.com/forcedphot/>

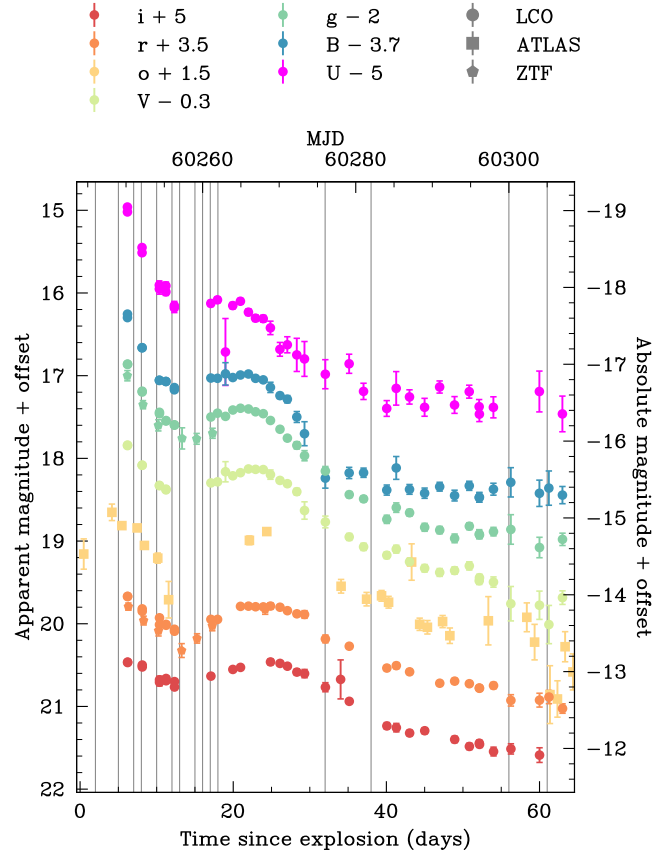


Figure 3.1: Full, multiband photometric data collected on SN 2023wdd by LCO, ATLAS, and ZTF. Follow-up programs for all three facilities clearly captured the initial decline and rise to second peak. We offset both absolute and apparent magnitudes by arbitrary amounts (shown in legend) to improve visibility. Error bars indicate 1σ uncertainty. Vertical gray lines indicate when spectra were taken. The estimated explosion epoch is based on the shock cooling fits discussed in [section 3.5](#).

3.3.2 SN 2022acr_v

SN 2022acr_v was discovered at *o*-band magnitude 18.3 by ATLAS, and reported to the Transient Name Server (TNS) on Dec 12, 2022. SN 2022acr_v was located at R.A. 05:46:44.992 and decl -20:08:58.66. The SN was not initially associated with any visible host, but may be associated with GALEXASC J054644.80-200858.1. SN 2022acr_v was classified as a young Type II SN by ePESSTO+ within two weeks of discovery [144]. A

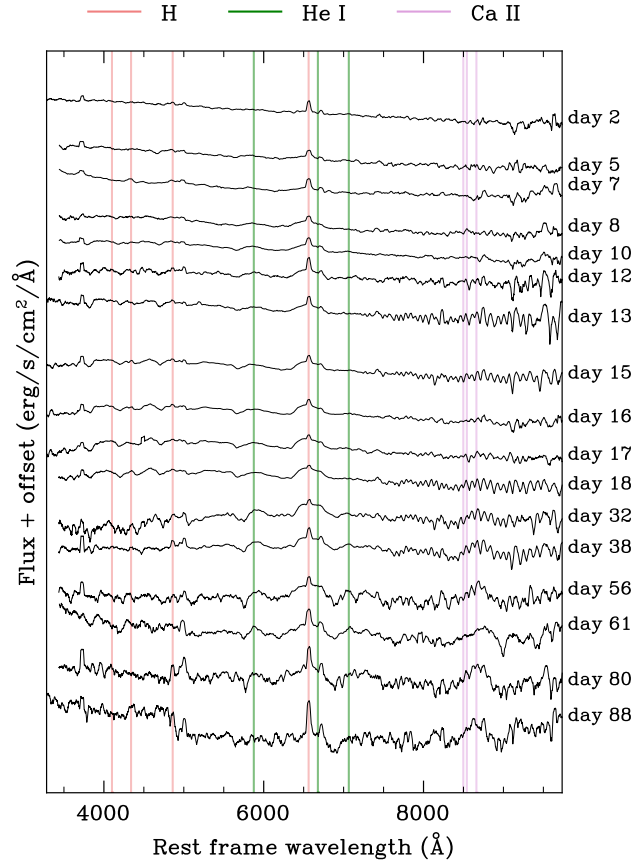


Figure 3.2: The spectral evolution of SN 2023wdd from explosion through the first 88 days. All spectra shown are from the FLOYDS instrument at LCO. We highlight the rest frame wavelengths of hydrogen (red line), helium (blue line) and calcium II (purple line). The persistent narrow feature at the hydrogen rest wavelength is contamination from the host galaxy (as evidenced by its unchanging equivalent width, see [subsection 3.5.2](#) for a discussion), superimposed over the broad SN hydrogen emission line. At later times, there is weak contamination of the broad hydrogen from a coincident helium absorption, but the hydrogen feature is still significant 88 days post-explosion.

H α emission feature constrains the redshift to $z \sim 0.028$, consistent with the candidate host galaxy redshift of $z \sim 0.029067$ (value from NED). Based on a redshift of $z \sim 0.0291$ and the cosmological parameters used above, we adopt a distance of 127.7 Mpc to the SN. Based on the [112] dust map, we adopt a line-of-sight extinction value of $E(B - V)_{\text{MW}} = 0.0441 \pm 3 \times 10^{-4}$ mag. We correct all light curve and photometric data

presented in this paper with this extinction value. Next, we describe our multiband follow-up observations of SN 2022acrv.

LCO was not triggered on this object until well into its evolution (≈ 1 month from discovery). By this point, the transient had executed a initial rise, partial decline, and secondary rise to peak, as visualized in [Figure 3.3](#). LCO obtained weekly photometry and spectra on SN 2022acrv from Jan 21, 2023 until the object faded ≈ 100 days later, providing a detailed multi-band light curve of the SN tail. LCO photometric and spectroscopic reduction details are identical to [subsection 3.3.1](#), with one exception: as the object was located in the southern sky, we employed the FLOYDS spectrograph mounted on the 2m telescope at Siding Spring Observatory in Australia. In addition to FLOYDS spectra, we also obtained spectra using the Low Dispersion Survey Spectrograph 3 [LDSS-3 [145](#)] on the Magellan Clay Telescope at the Las Campanas Observatory in Chile, which covers primarily optical wavelength range (3700 – 10060 angstroms). Spectra were processed following the procedure described in [[146](#)]: extraction and reduction were performed via PyRAF, with calibration to a standard star spectrum taken within one week of each target spectrum. The Magellan and FLOYDS spectra of SN 2022acrv are shown in [Figure 3.4](#).

As with SN 2023wdd, we used the available Magellan spectra to identify host galaxy extinction via the method of [[89](#)]. We find a best-fit extinction coefficient of $E(B-V)_{\text{host}} = 0.134 \pm 0.037$ mag. We additionally correct all light curve and photometric data presented in this paper with this host extinction value.

Finally, similarly to SN 2023wdd, we incorporate every-other-day ATLAS observations in o - and c - band, available via the forced photometry server. See [subsection 3.3.1](#) for reduction details.

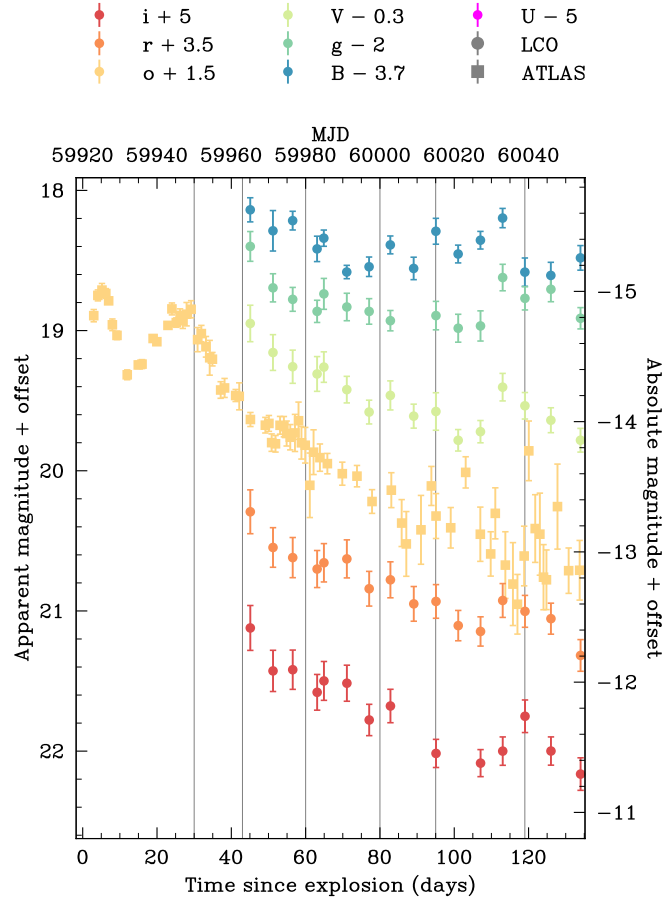


Figure 3.3: Full, multiband photometric data collected on SN 2022acrv by LCO and ATLAS. We offset both absolute and apparent magnitudes by arbitrary amounts (shown in legend) to improve visibility. Error bars indicate 1σ uncertainty. Vertical gray lines indicate when spectra were taken. The estimated explosion epoch is based on the shock cooling fits discussed in [section 3.5](#).

3.4 Spectroscopic evolution

Both SN 2023wdd and SN 2022acrv show similar spectral evolution, with particular features supporting their characterization as candidates for boundary objects between Type IIb and short-plateau SNe.

The full spectral evolution of SN 2023wdd is shown in [Figure 3.2](#). LCO spectra are available between day 2 and day 88 (days from explosion epoch measured using

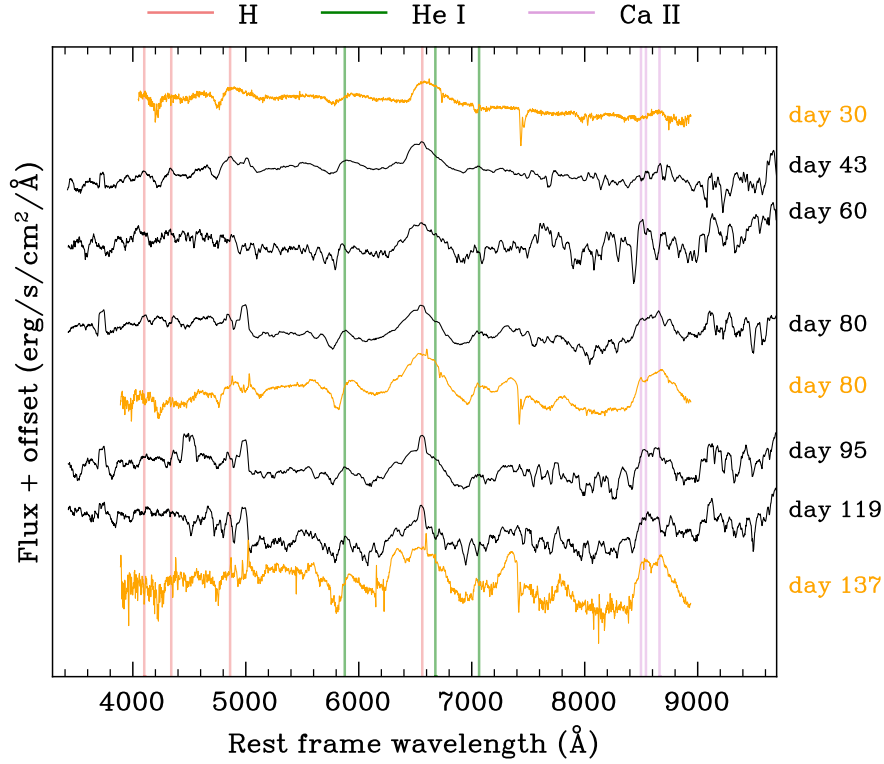


Figure 3.4: The spectral evolution during the first 137 days of the evolution of SN 2022acrv. Spectra from LCO are shown in black, while spectra from the Magellan instrument are shown in orange. We highlight the rest frame wavelengths of hydrogen (red line), helium (blue line) and calcium II (purple line). The large flat-top feature at ≈ 4500 angstroms on day 95 is due to averaging of a large, narrow cosmic-ray artifact. The persistent narrow feature at the hydrogen rest wavelength is contamination from the host galaxy, superimposed over the broad SN hydrogen emission line.

the shock cooling fits in [section 3.5](#)). The earliest spectra (within 1 week of explosion) show an almost featureless blue continuum, with the exception of a persistent narrow hydrogen feature produced by the host galaxy (see [subsection 3.5.2](#) for further discussion of the origin of this line). The lack of features is due to the extreme temperature of the ejecta [20], which we estimate from blackbody fits to photometry to be $\gtrsim 10^4$ K in the first few days of the observations. As the ejecta cool, lines begin to show, especially a broad $H\alpha$ feature at ≈ 6500 angstroms, facilitating classification of this

object as a Type II SN. The broad $H\alpha$ line does not appear to have an associated P Cygni absorption feature, which has been discussed in previous literature as evidence of interaction with circumstellar material (CSM) [20; 147; 148]—we consider this possibility in [subsection 3.5.2](#). The hydrogen line then develops a flat-top due to contamination from a coincident helium line, which, in conjunction with the double-peaked light curve, suggests a IIb SN classification [20; 44].

However, in the light curve of a typical double-peaked IIb SN, the $H\alpha$ line is usually weak, and quickly gets contaminated by the growing helium absorption feature, made visible by stripping of the hydrogen-rich outer envelope of the progenitor star [20; 119; 40; 133]. In contrast to this expectation, the broad $H\alpha$ remains fairly strong even almost 100 days post-explosion; and even though the presence of a He I line coincident with $H\alpha$ contaminates the $H\alpha$ emission somewhat, it does not dominate it as it might in a normal IIb SN. The He I feature—visible as a P Cygni at ≈ 5800 angstroms—is also weaker than expected for a IIb. To emphasize the distinction with an ordinary IIb SNe, we compare SN 2023wdd at early and late times to a well-known Type IIb SN, SN 2016gkg, in [Figure 3.5](#). At early times, both spectra show a prominent $H\alpha$ feature with moderate helium contamination, but by ~ 2 months in, SN 2023wdd has retained its $H\alpha$ feature, while the $H\alpha$ feature in SN 2016gkg almost entirely disappears, making the spectrum look more similar to a Type Ib SN.

The evolution of SN 2022acrv has some overlap in these peculiarities, and is shown in full in [Figure 3.4](#). Our hybrid (LCO + Magellan) spectroscopic coverage is available from day 30 to day 137. Despite the clear double-peak in the light curve of SN 2022acrv (suggesting a IIb SN), the spectra is dominated for at least the first 137 days by a broad $H\alpha$. Indeed, despite the presence of helium in the ejecta (visible at all epochs at ≈ 5800 angstroms), the central $H\alpha$ is not significantly contaminated, conflicting with the classification as a IIb SN. Like SN 2023wdd, this peculiar mismatch between the

light curve and spectra motivates investigation of SN 2022acrv as a boundary object between Type IIb and short-plateau SNe.

To quantitatively assess the spectral similarity of SN 2023wdd and SN 2022acrv to Type IIb versus Type II SNe, we investigate the $H\alpha$ line using the methods of [149] and [24]. [24] investigated the $H\alpha$ morphology of Type II SNe and identified general trends between the ratio of the equivalent widths (EW) of the absorption and emission components (denoted a/e) and other properties of the $H\alpha$ line and light curve. In particular, [24] identified negative correlations between a/e and both the slope of the fall from light curve peak (s_1) and the $H\alpha$ velocity in their dataset. We measure a/e , s_1 , and $v_{H\alpha}$ for both our objects following the method of [24] (see their work for procedural details) and find a/e is ≈ 0.02 and ≈ 0.10 for both SN 2023wdd and SN 2022acrv, respectively. Such a small a/e (i.e., $a/e \lesssim 0.1$) is consistent with $\approx 8\%$ of the Type II SNe in [24]. However, [24] found that smaller a/e correlated with faster inferred $H\alpha$ velocities and a steeper s_1 ; indeed we find that both objects have inferred $v_{H\alpha} \approx 1.1 \times 10^4$ km/s and $s_{1,23wdd} = 11.6 \pm 1.1$, $s_{1,22acrv} = 10.4 \pm 0.8$ mag/(100 d), which are consistent with the [24] trend. As [24] also found that a/e is positively correlated with the optically thin transition time (a proxy for hydrogen-rich envelope mass; [3]), we may also speculate that these high-velocity, low a/e , shorter-plateau objects are hydrogen-deficient.

We also compare the pseudo-equivalent width (pEW) of the hydrogen feature to the sample of Type IIb SNe investigated in [149]. [149] found that at ≈ 15 days post- B -band maximum, their sample had a pEW distribution for $H\alpha$ of 175 ± 14 Å. We find $\text{pEW}_{23wdd} = 180.7$ Å and $\text{pEW}_{22acrv} = 192.5$ Å in the nearest spectra (chronologically), which is statistically consistent with the Type IIb value. Combined with the [24] analysis, we interpret that SN 2023wdd and SN 2022acrv are spectroscopically similar to both Type IIb SNe and the lower-mass (inferred via optically thin transition timescale

proxy) Type II SNe, providing supporting evidence that these are indeed boundary objects.

We also obtained a late (364 days post-explosion) spectrum of the SN 2022acr^v host galaxy using Magellan, in order to assess metallicity of host galaxy and therefore the immediate environment of the SN. Previous works have identified that higher metallicity environments facilitate progenitor stripping, possibly suggesting that Type II SNe may occur in lower metallicity conditions [150; 151]. Following [3], we recover line fluxes by fitting a Gaussian with a vertical offset to the following lines: H α (λ 6563), H β (4861), [O II] (λ 3727), [O III] (λ 4959), [N II] (λ 6583), [S II] (λ 6717), S II (λ 6731). Using the derived expressions of [152] (specifically equations 2 and 3), we estimate an O3N2 value of ≈ 1.938 and an N2 value of ≈ -1.457 , which corresponds to a host galaxy metallicity of $Z \approx 0.255 Z_{\odot} \pm 0.01 Z_{\odot}$. We obtain a consistent host galaxy metallicity using the open-source tool PyMCZ [153]. This metallicity is consistent with the small sample of other short-plateau SNe, and is most similar to SN 2016egz [$Z \sim 0.27 Z_{\odot}$ 3]. Along with the host galaxy of SN 2023ufx, which had an extremely low metallicity ($Z \lesssim 0.05 Z_{\odot}$), short-plateau SNe seem to moderately favor lower-metallicity galaxies, though the sample of short-plateau SNe is too small to make this assertion with any confidence. The host metallicities of these boundary objects ($12 + \log(O/H) \approx 8.2$) are also consistent with previous studies of IIb SNe host galaxy metallicities [154], which found that IIb SNe are slightly preferentially found in local environments with lower metallicities ($12 + \log(O/H) \sim 8.367 \pm 0.149$), though there was not a large distinction between Type II SNe host galaxy metallicities ($12 + \log(O/H) \sim 8.447 \pm 0.126$).

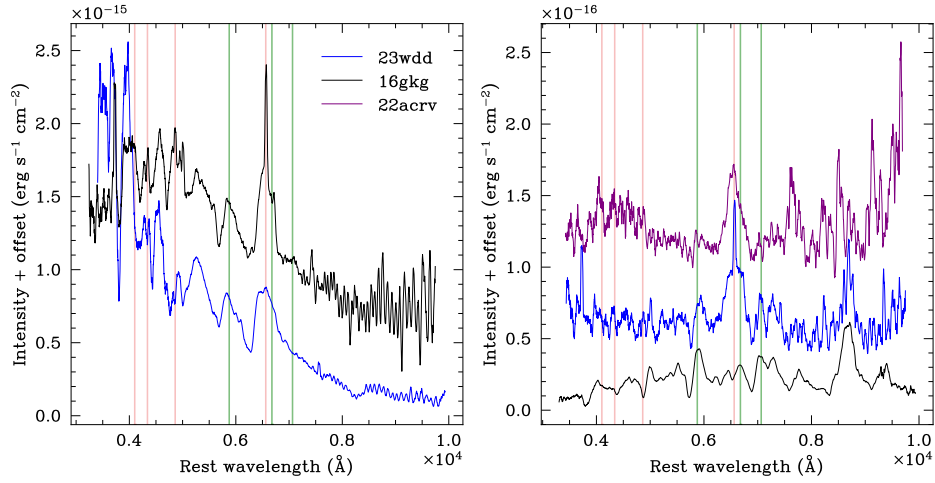


Figure 3.5: Comparison of the spectra of our boundary objects versus a well-studied Type IIb SNe, SN 2016gkg (black). Specific elements are shown using the same legend as Figure 3.2 and Figure 3.4. (left) Comparison of the spectra of SN 2023wdd and SN 2016gkg ~ 2 weeks into their evolution (SN 2022acr does not have spectra available this early). At this early time, the hydrogen line is still present, and developing the flat-top associated with helium contamination [20]. Even at this early stage, the helium contamination in SN 2016gkg is more prominent than in SN 2023wdd. (right) Comparison of the spectra of SN 2016gkg, SN 2023wdd and SN 2022acr ≈ 50 days into their respective evolutions. At this stage, the heavily-stripped envelope of SN 2016gkg has recombined its limited hydrogen, causing the previously prominent $H\alpha$ feature to be overwhelmed by the strong coincident helium line that was causing the flat-top in earlier spectra. By contrast, SN 2023wdd displays some helium contamination in the form of a flat-top, but both SN 2023wdd and SN 2022acr still have significant $H\alpha$ features, suggesting that their envelopes contain substantially more hydrogen than SN 2016gkg [$M_{\text{H}_{\text{env}}} \sim 0.01 M_{\odot}$, 129].

3.5 Light curve modeling

3.5.1 Photometric evolution

The full photometric evolution of SN 2023wdd is available in Figure 3.1. The combined LCO, ATLAS, and ZTF light curves provide a densely sampled, multiband reconstruction of the transient evolution, from a single-color constraint on the rise to the radioactive tail. However, the lack of UV data (e.g., from *Swift*) at early high-temperature

epochs may introduce a systematic bias when attempting to recover blackbody evolution (for a full discussion, see [87]). In particular, [87] identified that systematic errors of $\sim 10 \times 10^3$ K are needed when the effective temperature inferred from optical photometry is $\gtrsim 25 \times 10^3$ K. We investigate and find that both datasets for SN 2023wdd and SN 2022acrv begin at ~ 5 days post-explosion; the standard blackbody relation $L \sim 4\pi R^2 \sigma T^4$ applied to an expanding ejecta suggests that for $R \sim vt$ with $v \sim 10^4$ km/s and $L \sim 10^{42}$ erg/s (standard values for Type IIb SNe; see e.g., [3]), at $t \sim 5$ days the temperature should evolve to $T(5 \text{ days}) \sim 10^4$ K, well below the 25×10^3 K regime where [87] identified the need for additional systematic errors. We find that our blackbody fitting (described below) recovers values of around 9×10^3 K in the first epochs of data for both objects, consistent with this expectation and not requiring the systematic correction or UV data to mitigate the blackbody bias.

We fit all available epochs with multicolor data using a blackbody model to recover luminosity, temperature and radius, employing the blackbody fitter in the `lightcurve-fitting` package [120]. This fitter estimates blackbody properties from filter data by fitting a blackbody spectral energy distribution integrated over each filter’s transmission function. The luminosity and temperature evolution along the first ≈ 10 days is consistent with the nominal predictions of a shocked envelope expanding and cooling [$T \propto t^{-0.45}$, 36]. Around ≈ 10 –20 days, the light curve executes a weak double-peak, with $\lesssim 0.5$ mag difference in brightness from dip to second peak in all bands. After rising to a second peak at ≈ 22 days, the light curve relaxes onto an apparent radioactive tail. Modeling the luminosity along this tail by the radioactive decay of nickel-56 [22; 126] yields an estimated nickel mass synthesized during explosion of $\approx 0.039 M_\odot$.

The full photometric evolution of SN 2022acrv is similar to SN 2023wdd, and is shown in Figure 3.3. A $\approx$ daily reconstruction of the transient behavior is only available in ATLAS *o*- and *c*-band, retrieved via forced photometry. In the ATLAS data, two

peaks are clearly observed, with multiple data points constraining the initial rise. Due to the far more limited multicolor data at early times, only a few epochs of bolometric information are available. Nonetheless, the first peak of SN 2022acrv is consistent with the predictions of shock cooling up to ≈ 9 days [36]. After executing the shock cooling peak and decline, the transient rises to a secondary nickel peak before relaxing onto a radioactive tail, which is well-probed in multiple filters via a combination of LCO and ATLAS photometry. Modeling the luminosity along this tail by the radioactive decay of nickel-56 using the same approach as above yields an estimated nickel mass synthesized during explosion of $\approx 0.064 M_{\odot}$.

The weak double-peak light curve feature indicates both objects have experienced some stripping of their outer hydrogen-rich envelopes, supported by the helium contamination of the $H\alpha$ line discussed in [section 3.4](#). In the sections below, we attempt to use the light curve evolution described above to constrain properties of the progenitor system and explosion for each object. In particular, we seek a constraint of the hydrogen envelope mass to further solidify the placement of each supernova along the continuum of Type II/stripped-envelope objects.

3.5.2 CSM investigation

Interaction between expanding ejecta and a high-density CSM can power light curve luminosities comparable to shock cooling and radioactive nickel decay, creating degeneracies in the light curve that can complicate parameter inference. We examine possible indications of CSM in SN 2023wdd and SN 2022acrv observations to assess whether CSM interaction is significant enough to warrant dedicated modeling in our parameter inference scheme. CSM interaction can produce a variety of effects on light curve and spectra, including but not limited to delayed shock breakout [see e.g.,

155; 156], lack of a large drop onto a radioactive tail [see e.g., 157], $H\alpha$ P-cygni absorption suppression [see e.g., 148], or narrow ionization lines or early flash features [see e.g., 158–160]. Both SN 2023wdd and SN 2022acrv have rapid rises to peak, suggesting short-timescale shock breakout and therefore ruling out the presence of an extended envelope or dense CSM. However, a less dense CSM at very early times is known to produce fast rise times [161; 155; 156], and thus we cannot rule out the presence of a very early interaction with a less dense CSM. We further examine the light curves and spectra for any other indications of CSM interaction at any phase.

SN 2022acrv does not display signs of significant CSM interaction in its light curve or spectra; by contrast, SN 2023wdd has two potential indicators. These are: (i) an apparent lack of a P Cygni profile in the $H\alpha$ line, which has been linked to interaction in previous studies [20; 147; 148], and (ii) an extremely narrow and strong $H\alpha$ component, which may be an indication of CSM interaction or, alternatively, may be host-galaxy contamination. We consider both of these indicators below.

First, we consider the apparent lack of a P Cygni profile in the $H\alpha$ line. To investigate, we model the $H\alpha$ as the combination of a linear underlying continuum, a P Cygni profile and a narrow Gaussian emission line profile. We construct the P Cygni profile as separate absorption and emission components, following e.g., [162] and [163]. We find that once the ejecta have cooled sufficiently (1-2 weeks), the $H\alpha$ line indeed develops a clear P Cygni component, with an absorption feature $\sim 1/2$ the strength of the coupled emission feature. Using the method of [164] and [165], we set an upper limit of $\rho_{\text{CSM}} \sim 8 \times 10^{-16} \text{ g cm}^{-3}$ ($\dot{M} \sim 1.5 \times 10^{-4} M_{\odot} \text{ yr}^{-1}$). This density is not sufficient to warrant additional modeling, as, based on the results of [3], such a density is not large enough to impact the light curve evolution significantly.

Next, we examine the strong, narrow feature coincident with the $H\alpha$ line. This line is likely due to host contamination, given the presence of other prominent host galaxy

lines (e.g., $H\beta$, [O III], [S II]). If this narrow feature is produced via interaction with a CSM, it may vary as the ejecta expands, like in a Type IIn SN; by contrast, if the feature is produced due to host contamination, it should be unchanged as the supernova evolves. We measure the equivalent width of the narrow $H\alpha$ feature over time and do not observe statistically significant variation over the course of the observations. Fitting a line to the variation yields a slope statistically consistent with zero. Therefore, we attribute the narrow $H\alpha$ emission line to the host galaxy and not to interaction with a dense CSM. For completeness' sake, we can further consider—if the narrow $H\alpha$ feature is indeed produced by interaction, what CSM density would it imply? Following the method of [118] (which is based on the method of [166]), we use the peak flux of the narrow $H\alpha$ emission feature in the $t = 2$ day spectrum and constrain the density to be $\rho \lesssim 5 \times 10^{-15} \text{ g cm}^{-3}$, which implies a mass loss rate upper limit of $\dot{M} \lesssim 3 \times 10^{-4} M_{\odot} \text{ yr}^{-1}$. This upper limit is consistent with the mass loss rate derived from the P Cygni feature, and consistent with the ability of no-CSM shock cooling models to describe the early- and late-time light curve evolution (see subsection 3.5.4). In conclusion, we assess that the CSM interaction present in SN 2023wdd, if any at later times, is not sufficient to warrant the addition of a separate interaction model component to the analytical shock cooling models discussed below.

3.5.3 Light curve morphology and parameterization

Several previous works have analyzed the behavior of hydrogen-rich and hydrogen-poor Type II SN light curves by parameterizing the overall shape [e.g., 24; 28; 167; 168]. These analyses identified possible relationships between the photometric properties of Type II SN light curves, which may be used to infer hydrogen content in the progenitor outer envelope. Here, we quantify SN 2023wdd and SN 2022acrv within these param-

eterizations and assess implications for their location along the suggested continuum of hydrogen envelope-stripping.

We begin with the parameterization of [24], which broke the light curve into segments and characterized the duration and slope of each segment (see their Fig. 2 for a complete description of the parameterization). We fit the light curves of our boundary objects using the phenomenological model of [24], and compare the resulting parameters to the distributions identified for hydrogen-rich Type II SNe. As expected based on a visual inspection of the light curves, while the maximum magnitude at V -band peak (M_{max}) is consistent, the slopes of the s_1 and s_2 segments for SN 2023wdd ($s_1 = 11.629 \pm 1.128 \text{ mag (100 d)}^{-1}$, $s_2 = -2.046 \pm 0.302 \text{ mag (100 d)}^{-1}$) and SN 2022acr ($s_1 = 10.370 \pm 0.775 \text{ mag (100 d)}^{-1}$, $s_2 = -3.249 \pm 0.334 \text{ mag (100 d)}^{-1}$) differ quite dramatically from Type II SNe ($s_1 = 12.72 \pm 1.94 \text{ mag (100 d)}^{-1}$, $s_2 = 0.97 \pm 0.61 \text{ mag (100 d)}^{-1}$). The natural physical explanation for this deviation is the lower hydrogen mass available to power a smooth, slowly changing plateau. Instead, SN 2023wdd and SN 2022acr experience steep falls from the shock-cooling peak, and then a steep rise more consistent with a hybrid $^{56}\text{Ni} + \text{hydrogen}$ power source. We use this segment-based parameterization to characterize our wider sample in [section 3.6](#).

The fall from peak is an important light curve metric that has been suggested to disambiguate between Type IIb and Type II SNe [e.g., 167]. We adopt the dm_1 parameter from [167], which measures the earliest maximum of the first derivative of the light curve after maximum light. In the [167] sample, $dm_1 \approx 0.06$ was identified as a possible boundary between hydrogen-rich ($0.0 \lesssim dm_1 \lesssim 0.06$) and hydrogen-poor ($0.06 \lesssim dm_1 \lesssim 0.14$) Type II SNe. Measuring the dm_1 quantity for our proposed boundary objects, we find SN 2023wdd has $dm_1 = 0.08 \pm 0.02$ and SN 2022acr has $dm_1 = 0.08 \pm 0.03$. Taken at face value, SN 2023wdd and SN 2022acr are indeed consistent with the boundary between Type II and Type IIb identified by [167], favoring the

hydrogen-rich classification. A classification of hydrogen-rich based on this quantity alone would conflict with the fairly concrete classification as hydrogen-poor based on the s_1 and s_2 segment slopes (see above). We argue the classification ambiguity in objects such as SN 2023wdd and SN 2022acr_v is evidence of their transitional nature.

A recent analysis has used similar photometry parameterizations to characterize the presence of extra emission (namely, the presumed shock-cooling peak; [28]). Visually, both SN 2023wdd and SN 2022acr_v have clear early excess emission which is well-modeled by a hydrodynamic shock cooling model. To quantify this behavior, we adopt the ΔM_{15} (magnitude difference from peak to peak + 15 days) and t_{rise} (time to peak) parameters from [28]. We find SN 2022acr_v has $\Delta M_{15} = 0.59 \pm 0.09$ and $t_{\text{rise}} = 23.3$ d, while SN 2023wdd has $\Delta M_{15} = 0.92 \pm 0.05$ and $t_{\text{rise}} = 16.75$ d. The reported ΔM_{15} and t_{rise} are both consistent with the excess emission objects in the [28] sample.

3.5.4 Analytic shock-cooling modeling

Recently, analytic models of shock cooling have been presented which enable progenitor and explosion parameter estimation from early supernova light curves. As part of our investigation, we estimate progenitor and explosion parameters—particularly the hydrogen envelope mass—using the model of [1]. [1] extends the model of [36], which accurately addressed emission generated at $t \lesssim$ a few days post-explosion, when the visible emission is primarily generated close to the surface of the star. [1] extends this model to $t \gtrsim$ a few days post-explosion, when the luminosity becomes dependent on the density profile of the material interior to the surface of the star. We follow [133],

and model the luminosity using the following expression (transformed from [1]):

$$L(t) \sim (1.88 - 0.147[n - 3/2]) \times 10^{42} \times \left(\frac{v_{s,8.5}^2 t^2}{f_\rho M \kappa_{0.34}} \right)^{3 \times 10^{-3} - 5.9 \times 10^{-2} n} \times \frac{v_{s,8.5}^2 R_{13}}{\kappa_{0.34}} \times \exp \left(- \frac{[1.93nt - 1.23t]}{[19.5 \kappa_{0.34} M_e v_{s,8.5}^{-1}]^{0.5}} \right)^{0.87 - 4.7 \times 10^{-2} n} \text{ erg s}^{-1}, \quad (3.1)$$

where n is the polytropic index ($n = 3$ for a radiative envelope, $n = 3/2$ for a fully convective envelope) $v_{s,8.5}$ is the velocity of the shock in units of $10^{8.5} \text{ cm s}^{-1}$, t is the phase of the supernova in days, M is the total mass of the star (core M_c plus envelope M_e) in solar masses, $\kappa_{0.34}$ is the opacity in units of $0.34 \text{ cm}^2 \text{ g}^{-1}$, R_{13} is the radius of the extended envelope of the progenitor star R_e in units of 10^{13} cm , and

$$f_\rho \sim \begin{cases} (M_e/M_c)^{0.5} & n = 3/2 \\ 0.08 (M_e/M_c) & n = 3 \end{cases}. \quad (3.2)$$

Note that $f_\rho M$ is largely a nuisance parameter and is not expected to be well-constrained. [1] also derived a color temperature evolution for the photosphere as a function of time, which we use to fit our multiband datasets. We again refer to the transformed expression used in [133] and use,

$$T(t) \sim (2.14 - 0.06n) \times 10^4 \times \left(\frac{v_{s,8.5}^2 t^2}{f_\rho M \kappa_{0.34}} \right)^{3.8 \times 10^{-2} - 7.3 \times 10^{-3} n} \times \frac{R_{13}^{0.25}}{\kappa_{0.34}^{0.25}} t^{-0.5} \text{ K}. \quad (3.3)$$

This model has analytic regimes of validity based on the temperature and transparency timescale of the system. To be consistent with these validity criteria, we only fit data with temperature $\gtrsim 8000$ K and less than the transparency timescale [see 1, for definition] days from explosion, dynamically calculated in the fit depending on the model parameters. See [133] for further details on our shock cooling emission modeling approach.

We fit this model using the open-source package `lightcurve-fitting` [120], which conditions the [1] model on given data using a Markov-Chain Monte Carlo (MCMC) posterior exploration routine. We initialize the fit with 15 walkers and use sufficient steps (10^3 steps for burnin in and $\gtrsim 10^4$ steps for posterior exploration) to achieve convergence via the improved Gelman-Rubin statistic and chain autocorrelation length; see [133] section 5.4 for more details. As is standard, we choose a simple Gaussian likelihood function to compute the likelihood of a particular parameter configuration producing the data. All data are weighted only by their error bars. Parameter estimates and uncertainties are determined by computing the highest posterior density interval around the maximum a posteriori value. This approach is simple but motivated by the marginal parameter posteriors, which are exceedingly well-behaved (i.e., unimodal with minimal skew), with the exception of the nuisance parameter $f_{\rho}M$.

For both SN 2023wdd and SN 2022acrv, we varied the polytropic index n between $n = 3$ and $n = 3/2$ and found that $n = 3$ did not provide a satisfactory fit. The fits were unsatisfactory for one or more of the following reasons: (i) the $n = 3$ maximum likelihood models described the data significantly more poorly than the $n = 3/2$ fits; (ii) the resulting marginal posterior distributions were highly non-Gaussian, with several parameters favoring the edges of the prior, or (iii) the fit favored highly unphysical parameter values, such as envelope masses $\gg 20 M_{\odot}$. The inferior performance of the $n = 3$ models suggest that the SN 2023wdd envelope is convective. We also attempted

fits both including and excluding non-detections, which did not significantly alter the fit result for either object.

The fit results for SN 2023wdd are shown in [Figure 3.6](#). Overall, the model fits well in all bands, with a moderate deviation in ATLAS *o*-band. Best-fit values of the shock velocity ($v_{s*} \approx 5 \times 10^3$ km/s) and progenitor radius ($R \approx 200 R_\odot$) are consistent with the broad range of values for other double-peaked objects and indicate a relatively compact progenitor. The best-fit hydrogen-rich envelope mass ($M_{H_{\text{env}}} \approx 0.8 \pm 0.04 M_\odot$) is significantly larger than typical double-peaked IIb SNe ($M_{H_{\text{env}}} \lesssim 0.1 M_\odot$), but is consistent with the proposed boundary between IIb and short-plateau SNe ($M_{H_{\text{env}}} \sim 0.91 M_\odot$) [3].

The analytic fits for SN 2022acr (Figure 3.7) shows significantly more uncertainty, due to the lack of multicolor data for much of the regime of validity. Nevertheless, the results for SN 2022acr are similar to SN 2023wdd. The shock velocity ($v_{s*} \approx 8 \times 10^3$ km/s) and progenitor radius ($R \approx 100 R_\odot$) are again consistent with previous compact double-peak transient progenitors. The constraint on the hydrogen-rich envelope mass is weaker due to the limited data, but we are still able to constrain $M_{H_{\text{env}}} \approx 0.8 \pm 0.2 M_\odot$, consistent with the proposed IIb-SP SNe boundary. Thus, the analytic fits for both objects provide evidence that these objects indeed are examples of a transition between the established IIb subclass and the short-plateaus.

3.5.5 Numerical modeling via simulation grid

In addition to analytic models of shock cooling, we model the entire light curve using numerical simulations to identify a progenitor system configuration that best reproduces the observed light curve. [3] generated a dense ($N \sim 4 \times 10^4$), discrete grid of numerical simulations varying progenitor zero-age main-sequence masses, mass

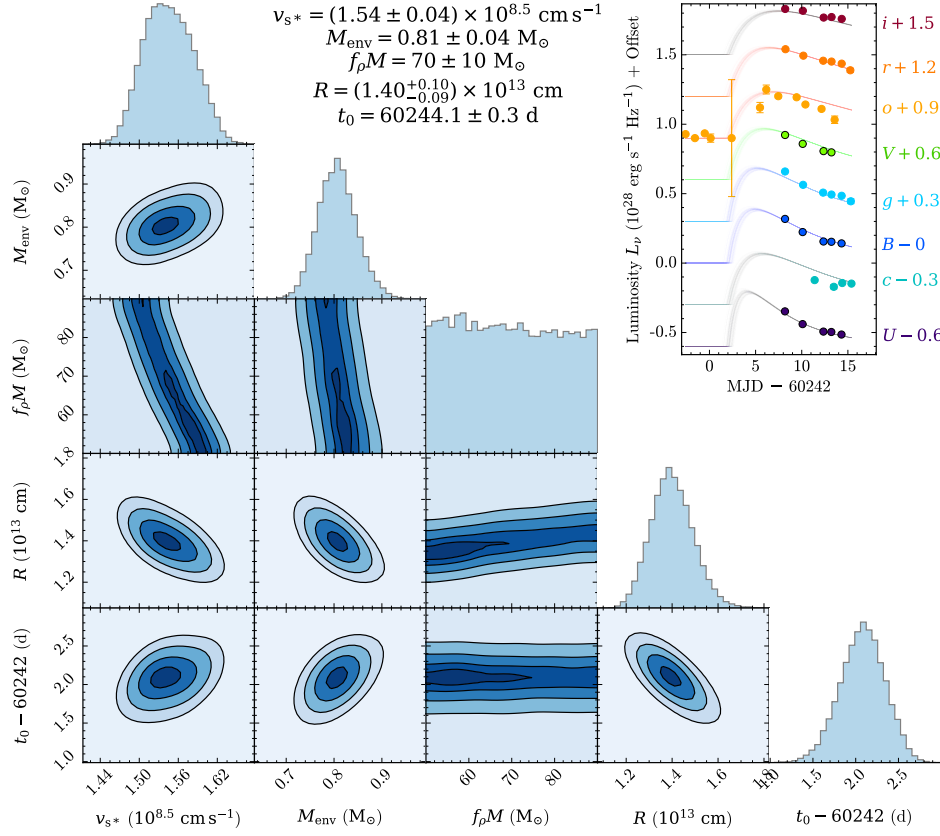


Figure 3.6: Corner plot and posterior samples for the model of [1] with $n = 3/2$ applied to the early photometry of SN 2023wdd. We include ATLAS non-detections, which narrowly constrain the explosion epoch. The data shown in the sub-panel are the full dataset included in the fit; however, for any individual likelihood calculation, a limited subset of the available data are used, based on the regime of validity for the specific realization of the model being evaluated; see [133] section 5.3 for a full discussion. Analytic modeling reports a progenitor radius of $\approx 200 R_{\odot}$ and a hydrogen-rich envelope mass of $\approx 0.8 \pm 0.04 M_{\odot}$, placing SN 2023wdd along the boundary between IIb and short-plateau SNe ($M_{H_{\text{env}}} \sim 0.91 M_{\odot}$) [3].

loss rates, synthesized nickel-56 masses, CSM densities, and explosion energies. We explore this parameter space to identify a maximum likelihood model with corresponding uncertainties.

We begin by computing the bolometric luminosity of each observed light curve, which is then compared directly to simulated light curves. To account for uncertainty

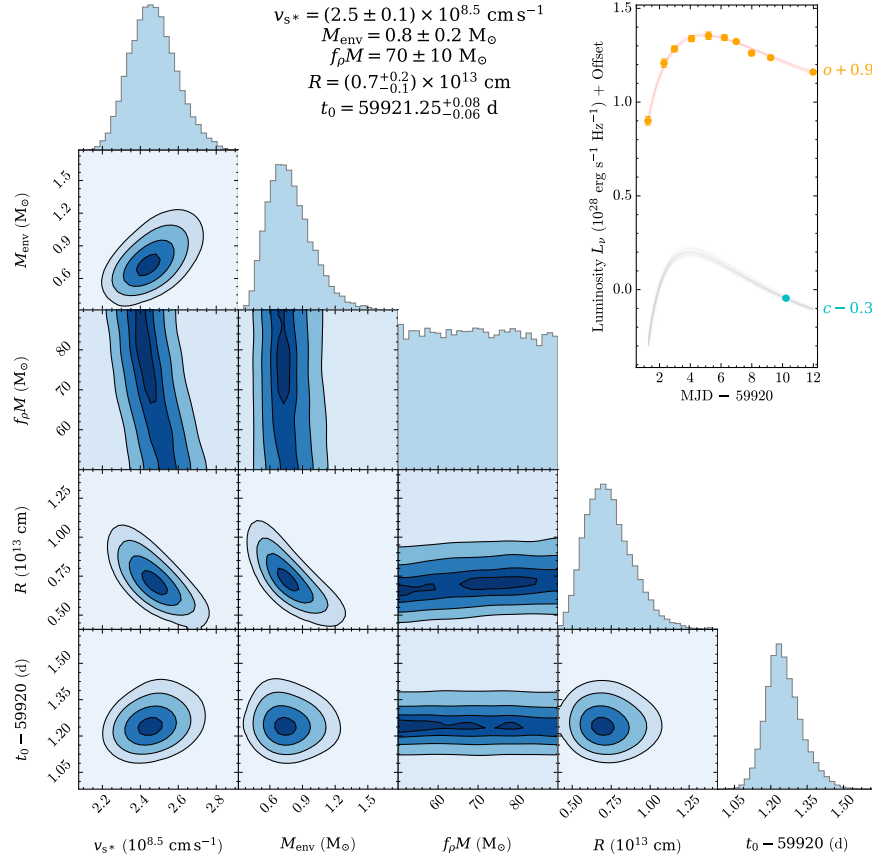


Figure 3.7: Corner plot and posterior samples for the model of [1] with $n = 3/2$ applied to the early photometry of SN 2022acrv. Only ATLAS data is available during the shock cooling phase of the light curve. As in Figure 3.6 data shown in the sub-panel are the full dataset included in the fit; however, for any individual likelihood calculation, a limited subset of the available data are used, based on the regime of validity for the specific realization of the model being evaluated; see [133] section 5.3 for a full discussion. Analytic modeling reports a progenitor radius of $\approx 100 R_{\odot}$ and (similar to SN 2023wdd) a hydrogen-rich envelope mass of $\approx 0.8 \pm 0.2 M_{\odot}$, placing SN 2022acrv along the boundary between IIb and short-plateau SNe ($M_{H_{\text{env}}} \sim 0.91 M_{\odot}$) [3]. The increased uncertainty (an order of magnitude larger than SN 2023wdd) is due to the limited multicolor coverage in this region.

in the explosion epoch, each model is allowed an arbitrary shift in time, which is absorbed into the model parameterization. For each model in our discrete parameter grid, we compute a likelihood using a simple Gaussian likelihood function based on the agreement between the model and observed data. To estimate the marginal distributions for each model parameter, we proceed as follows: for each model in the grid, we assign its computed likelihood as a contribution to the corresponding bin of each parameter’s value. Since the parameter grid is not evenly sampled—some parameter values appear more frequently than others—we normalize each bin by the number of models it contains. This yields an average likelihood for each value of each parameter. Finally, we use kernel density estimation [169; 170] to convert the discretely sampled bins for each parameter into a continuous marginal probability distribution. These can then be combined to form joint distributions for parameter pairs. Like the analytic modeling, parameter estimates and uncertainties are then determined by computing the highest posterior density interval around the maximum a posteriori value.

Results of this fitting approach for SN 2023wdd are shown in Figure 3.8. The top 100 models by likelihood have $M_{\text{Ni}} = 0.04 M_{\odot}$, consistent with our calculations from the light curve in subsection 3.5.1. Our maximum likelihood model is consistent with the light curve across all epochs, including the very early shock cooling decline. The best-fit hydrogen envelope mass ($M_{H_{\text{env}}} \approx 0.62 \pm 0.14 M_{\odot}$) is consistent up to 1σ with the results from the analytic models, and provides further evidence that SN 2023wdd is a boundary object. However, there is tension (factor of ≈ 2.5) between the progenitor photospheric radius at shock breakout estimated by the model ($R_{\text{ph}} \approx 500R_{\odot}$) and by the analytic modeling ($R \approx 200R_{\odot}$). This discrepancy may be due to (i) simplified assumptions of the stellar density structure present in the analytic models which are more realistically modeled in the numerical grid, or (ii) deviations from reality in the

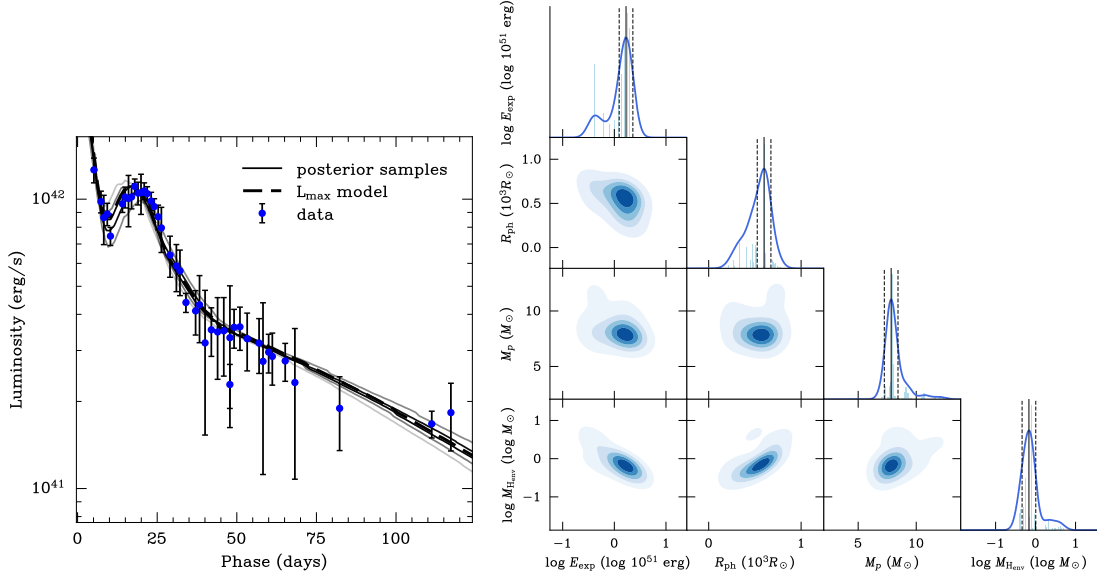


Figure 3.8: Samples and pseudo-posterior distributions for numerical modeling of SN 2023wdd using the simulation grid of [3]. (*left*) Samples from the posterior, ranked by likelihood. The maximum likelihood model is shown as a black dashed line. Sub-maximum likelihood samples are weighted in transparency by likelihood (i.e., darker indicates greater likelihood). (*right*) Psuedo-posterior parameter distributions estimated from the simulation grid. Maximum likelihood parameter estimates are shown in the 1D histograms as solid lines; $\pm 1\sigma$ uncertainties are shown as dashed lines. The maximum likelihood simulations have $M_{\text{Henv}} \approx 0.62 \pm 0.14 M_{\odot}$, consistent up to 1σ with the analytic modeling, and supporting the characterization of SN 2023wdd as a IIb/short-plateau boundary object.

underlying envelope temperature, composition, or density stratification assumed by the analytic model, which can cause order unity shifts in the estimated progenitor radius and shock cooling timescale.

Results of the fitting approach for SN 2022acrv are shown in Figure 3.9. The top 100 models by likelihood have $M_{\text{Ni}} = 0.07 M_{\odot}$, consistent with our calculations from the light curve in subsection 3.5.1. The maximum likelihood model predicts a progenitor radius ($R_{\text{ph}} \approx 500 R_{\odot}$) of similar size to SN 2023wdd and a hydrogen-rich envelope mass ($M_{\text{Henv}} \approx 0.59 \pm 0.08 M_{\odot}$) supportive of the IIb-SP SNe boundary characterization of SN 2022acrv. Consistency of the maximum likelihood model with the data is significantly

worse for SN 2022acrv as compared to SN 2023wdd. The model overestimates the object luminosity by $\approx 25\%$ on the transition from nickel peak to the radioactive tail, and predicts a dip (between shock cooling decline and nickel peak) $\approx 30\text{--}40\%$ deeper than suggested by the bolometric luminosity. The significant overestimate is limited to a $\sim$ few day period immediately after the modeled (but unobserved) peak. Discrepancies between the modeled and observed light curve in this region may be related to parameters of the explosion not explored in our numerical grid, e.g., nickel mixing [see e.g., 171; 172]. Additionally, we note that comparisons in the earlier region of the data may not be reliable, due to the low number of filters available to properly constrain the bolometric properties of the supernova. However, at later times, when significantly more multicolor data is available (LCO+ATLAS), there is more agreement of the maximum likelihood model and the object luminosity.

Despite a discrepancy between the progenitor radius estimated by the analytic versus numerical modeling channels, all modeling approaches consistently estimate the hydrogen-rich envelope mass of SN 2023wdd and SN 2022acrv to be on the boundary of the IIb SNe and short plateau SNe classifications, supporting their characterization as transition objects between these two subclasses. Having established the object characterizations via direct modeling of their progenitor envelopes, we next compare them to other occupants of the Type II/stripped-envelope continuum.

3.6 Comparison to other supernovae

We have established that SN 2023wdd and SN 2022acrv lie upon the boundary between Type IIb and short-plateau SNe. To strengthen this conclusion, we compare these boundary objects to other supernovae with both larger and smaller hydrogen-rich envelope masses. For our sample, we select SN 2016gkg and SN 2022hnt (Type IIb), SN

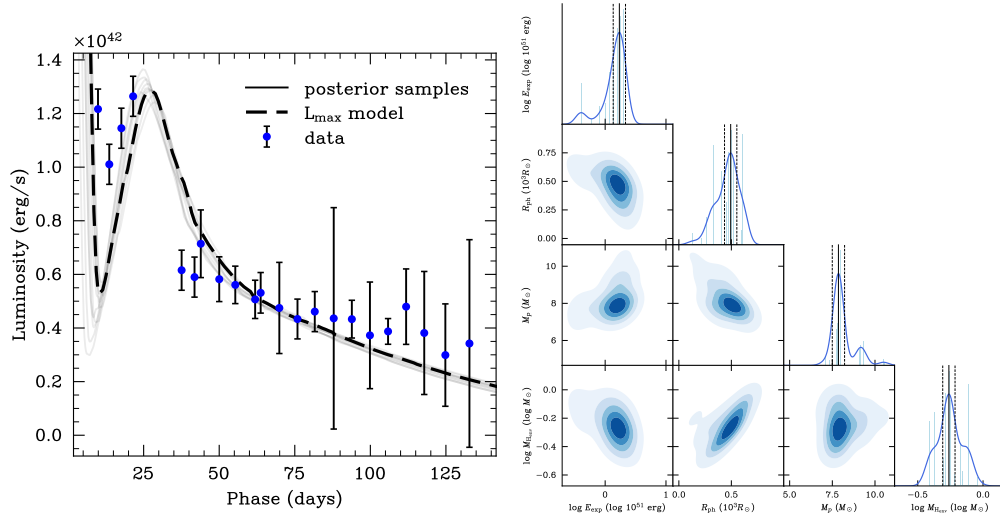


Figure 3.9: Samples and pseudo-posterior distributions for numerical modeling of SN 2022acrv using the simulation grid of [3]. (*left*) Samples from the posterior, ranked by likelihood. The maximum likelihood model is shown as a black dashed line. Sub-maximum likelihood samples are weighted in transparency by likelihood (i.e., darker indicates greater likelihood). (*right*) Pseudo-posterior parameter distributions estimated from the simulation grid. Maximum likelihood parameter estimates are shown in the 1D histograms as solid lines; $\pm 1\sigma$ uncertainties are shown as dashed lines. The maximum likelihood simulations have $M_{\text{Henv}} \approx 0.59 \pm 0.08 M_{\odot}$, consistent up to 1σ with the analytic modeling, and supporting the characterization of SN 2022acrv as a IIb-SP boundary object.

2023ufx, SN 2006ai, SN 2016egz, and SN 2006Y (short-plateau SNe), and SN 2023ixf, SN 2013ej (Type II SNe). These supernova are well-studied, with dense spectrophotometric observations and published constraints on explosion and progenitor parameters. The objects are characterized hydrodynamically using the shock-cooling model of [1], either from our fits or referencing existing fits in the literature. The photometric properties of the objects are characterized using the procedure described in subsection 3.5.3. We summarize the objects' properties in Table 3.1.

Object	Type	R_p ($R_\odot$)	$M_{\text{H}_{\text{env}}} (M_\odot)$	$s_1 + s_2^a$	s_2^b	Reference
SN 2016gkg	I Ib	310 ± 17.9	0.024 ± 0.003	N/A	N/A	[129; 39]
SN 2022hnt	I Ib	70.9 ± 9	0.068 ± 0.013	N/A	N/A	[133]
SN 2023wdd	boundary	560 ± 80	0.80 ± 0.04	19.2 ± 0.8	-2.89 ± 1.03	This work
SN 2022acr	boundary	489 ± 78	0.568 ± 0.053	26.6 ± 0.7	-3.25 ± 0.36	This work
SN 2023ufx	short-plateau	354 ± 88	1.2 ± 0.2	32.0 ± 0.4	1.13 ± 0.20	[173; 174]
SN 2006Y	short-plateau	480 ± 100	2.5 ± 1.2	40.7 ± 0.4	0.58 ± 0.13	[3]
SN 2006ai	short-plateau	600 ± 110	1.7 ± 1.1	28.6 ± 1.9	4.55 ± 0.35	[3]
SN 2016egz	short-plateau	620 ± 100	2.1 ± 1.15	59.1 ± 0.9	0.55 ± 0.17	[3]
SN 2023ixf	II	873 ± 443	4.5 ± 0.5	77.2 ± 1.1	2.22 ± 0.07	[110]
SN 2013ej	II	500 ± 100	8.55 ± 2	79.6 ± 2.0	1.70 ± 0.05	[134]

^aHydrogen-powered plateau duration, length of segment 1 and 2 in the parameterization of [24]; units of days.

^bSlope of the plateau (s_2 component) in the parameterization of [24]; units of mag (100 day)⁻¹.

Table 3.1: Hydrogen envelope and plateau properties for various SNe

We begin by comparing to the Type IIb SNe in [Figure 3.10](#), as both SN 2023wdd and SN 2022acr_v were initially classified as Type IIb SNe. Additionally, we compare to the short-plateau SN 2023ufx, described in [\[173\]](#) and [\[174\]](#). As we have previously noted, the helium-contaminated H α line which provided the basis for the IIb classification appears to be stronger than expected for a IIb SNe, with weaker contamination than expected. We find a similar behavior in the light curve comparison. Though by eye there appears to be a double peak in both objects, comparison to “true” IIb SNe reveals that the peak to be strikingly weak in some bands and arguably nonexistent in others. Particularly in the bluer bands, the extremely shallow dip of the boundary object light curve is more similar to the short plateau light curve behavior (SN 2023ufx) than the IIb light curves. SN 2023ufx is especially interesting for comparison, as with a modeled hydrogen-rich envelope mass of $M_{\text{Henv}} \approx 1.2 \pm 0.2 M_{\odot}$, it lies along (more specifically, just above) the IIb/short-plateau boundary ($M_{\text{Henv}} \approx 0.91 M_{\odot}$) as well.

The *o*-, *r*-, and *B*- band light curves of SN 2022acr_v and SN 2023wdd are compared to SN 2016gkg and SN 2022hnt in [Figure 3.10](#). We characterize all light curves based on a “dip-to-peak” distance in magnitudes, following the “lightning bolt” model of [\[175\]](#). That work characterized this phase of the evolution of Type IIb SNe using the *m3* parameter, which describes the slope of the second rise to nickel peak. Our “dip-to-peak” distance is the product of the *m3* parameter and the rise time to the second peak. We directly the “dip-to-peak” distance by taking the nickel peak absolute magnitude and subtracting it from the minimum absolute magnitude achieved during the dip demarcating the double peak. Beginning with *o*-band, we observe that SN 2022hnt experiences an absolute “dip-to-peak” distance of ≈ 1.8 mag. By contrast, SN 2023wdd and SN 2022acr_v execute almost identical “dip-to-peak” distances of ≈ 0.6 and ≈ 0.5 mag, respectively. This is a factor of ~ 3 in magnitude space less prominent of a dip in *o*-band for the boundary objects as compared to the Type IIb SNe.

Next, we compare the behavior in r -band in a similar way to o -band. While SN 2023wdd executes a “dip-to-peak” distance of ≈ 0.7 mag (comparable to o -band), the IIb SNe SN 2016gkg and SN 2022hnt display “dip-to-peak” distances of ≈ 1.5 mag. This is a factor of ~ 2 in magnitude space more prominent of a dip for the Type IIb SNe as compared to SN 2023wdd.

Finally, we compare the evolution in B -band, where the difference in behavior is stark. As in all other bands, the Type IIb SNe execute prominent dips, with “dip-to-peak” distances of ≈ 1.5 mag, similar to r - and o -bands. However, in B -band, SN 2023wdd hardly executes a dip at all, with all brightnesses during the dip consistent with a plateau at absolute mag ≈ 16.2 . In the most charitable interpretation of the data, the maximum possible “dip-to-peak” distance in this band is $\lesssim 0.1$ mag.

The weak double-peak behavior—in contrast to the more heavily hydrogen-rich Type IIb SNe—can be explained by the boundary objects possessing more hydrogen in their outer envelopes than the Type IIb SNe prior to explosion. This explanation is consistent with theoretical predictions from [3], as well as the modeled hydrogen-rich envelope masses, which found $M_{\text{H}_{\text{env}}} \lesssim 0.1 M_{\odot}$ for the Type IIb SNe and $M_{\text{H}_{\text{env}}} \approx 0.8 M_{\odot}$ for the boundary objects.

In addition to light curve behavior, we compare inferred progenitor properties to other objects along the Type II/stripped-envelope continuum. Across the supernovae in our sample, we evaluate hydrogen-rich envelope mass, progenitor radius, approximate plateau length, and plateau slope, and compare to theoretical predictions. A visualization of this comparison is shown in Figure 3.11. Proposed classifications (IIb, short plateau, II SNe) based on the hydrogen-rich envelope mass ranges presented in [3] are shown as shaded regions in the figure. We include a sample of Type II SNe from [176] and [168], which calculated progenitor properties and bolometric light curve characteristics for 74 SNe in a manner similar to this work. We find that the

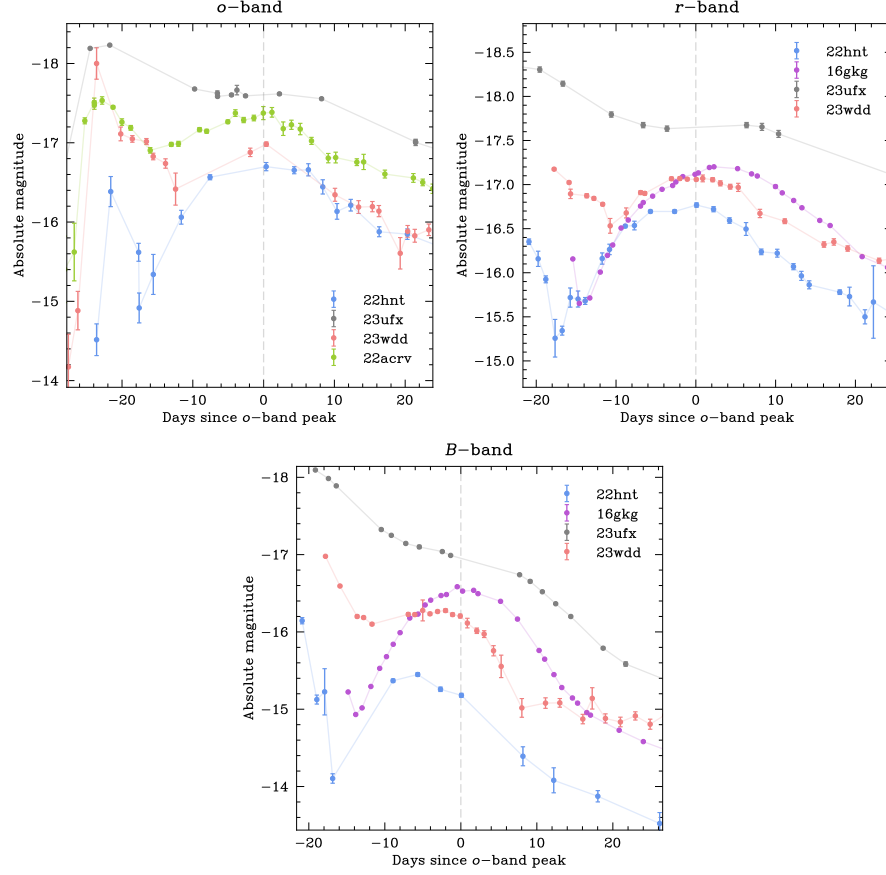


Figure 3.10: Comparison of filter photometry for the early evolution of SN 2023wdd, SN 2022acr, and two established IIb SNe (SN 2022hnt and SN 2016gkg). Despite appearing to be double-peaked by eye, SN 2023wdd and SN 2022acr have a significantly lower dip-to-peak distance than conventional IIb SNe, owing to the more hydrogen-rich envelope. This pattern becomes more apparent in r -band and B -band. Particularly in B -band, the light curve of SN 2023wdd looks almost plateau-like by comparison to SN 2016gkg and SN 2022hnt. Such behavior is reasonable for an object on the boundary between double-peaked IIb SNe and objects with short plateaus.

placement of SN 2023wdd and SN 2022acr along the Type II/stripped-envelope continuum based on their inferred hydrogen-rich envelope mass is consistent with our assessment from spectrophotometric data that they lie upon the boundary between IIb and short-plateau SNe. Further, if the weak dip feature in SN 2022acr and SN 2023wdd is instead treated as a “plateau”, we find it is in strong agreement with the lower- $M_{\text{H}_{\text{env}}}$ -shorter-plateau trend observed among other core-collapse supernovae,

as well as with predictions from analytic [23] and numerical theory [3]. For comparison to [23] across both our boundary objects and the Type II sample, we define the plateau length as the total length of segments s_1 and s_2 (in the [24] parameterization), which measures from maximum light to the “drop” onto the radioactive tail. This definition is chosen to more completely measure the duration of the hydrogen-powered portion of the light curve, especially in cases where the behavior is less conventionally “plateau”-like. Under this parameterization, we find that plateau length scales roughly with the hydrogen-rich envelope mass of the progenitor, consistent with the scaling predicted by [23] (Figure 3.11). In fact, given the unusual part-dip, part-plateau behavior of the light curve, SN 2023wdd in particular may represent a sort of “missing link”, potentially the highest $M_{\text{H}_{\text{env}}}$ Type IIb SN (or the shortest short-plateau SN) possible. This distinction can potentially be better disambiguated using the s_2 segment slope in addition to the hydrogen-rich envelope mass. We find that objects with more conventional plateaus (i.e., the short-plateau and Type II SNe) tend to have positive s_2 slopes, while our boundary objects, like Type IIb SNe, have unambiguously negative s_2 slopes, due to the presence of the dip. However, as demonstrated in Figure 3.10, the s_2 slope approaches zero for SN 2023wdd in B -band, another possible transition indicator.

There are several factors contributing to why such boundary objects have not been better characterized in past studies of hydrogen-rich supernovae. First, Type IIb supernovae are a relatively rare outcome of the core collapse of massive stars. [31] found that Type IIb SNe make up $\approx 11\%$ of their volume-limited sample, which more recent studies have corroborated [177]. Even among the already rare IIb subclass, double-peaked IIb SNe (which provide in their double peak a direct probe of $M_{\text{H}_{\text{env}}}$) occur $\lesssim 48\%$ of the time [28]. The low volume of IIb SNe generally available for study makes refining the boundaries of this subclass challenging; however, future surveys

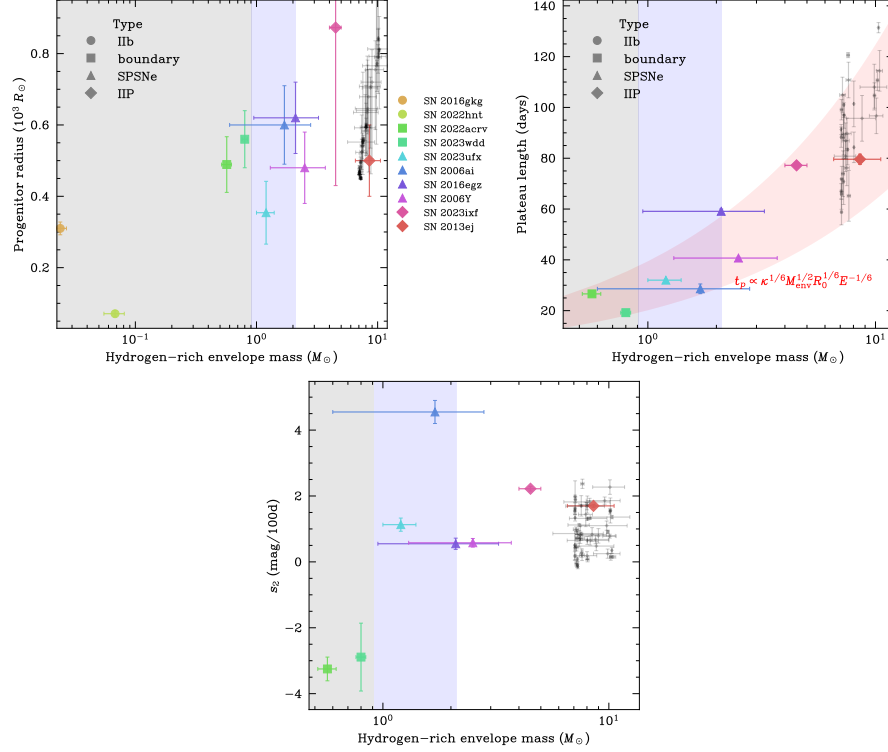


Figure 3.11: Comparison of progenitor properties across the spectrum of stripped envelope objects. In addition to our boundary objects, we examine established IIb SNe (SN 2022hnt and SN 2016gkg), candidate SPSNe (SN 2006ai, SN 2016egz, SN 2006Y), and established Type II SNe (SN 2023ixf and SN 2013ej). (See Table 3.1 for object property references.) Regions of the plot are shaded based on the predicted hydrogen-rich envelope mass ranges for IIb ($M_{\text{Henv}} \lesssim 0.91 M_\odot$), short-plateau ($0.91 M_\odot \lesssim M_{\text{Henv}} \lesssim 2.1 M_\odot$), and II ($M_{\text{Henv}} \gtrsim 2.12 M_\odot$) SNe proposed in [3]. The marker for each supernova identifies its established or proposed classification, based on spectroscopic or photometric evolution. In all panels, the faint gray points correspond to the sample of Type IIP supernovae presented in [176] and [168], reproduced from the data in those papers. (left) Comparison of hydrogen-rich envelope mass and progenitor radius for all objects. There is remarkable consistency between object classification, estimated hydrogen envelope mass, and the subtype separation proposed by [3] (colored regions). (right) Comparison of hydrogen-rich envelope mass and plateau length for all objects. The IIb SNe are not shown, as they do not have a plateau. For the boundary objects, we interpret the weak dip as a plateau for comparison purposes. There is a clear trend between plateau length and hydrogen-rich envelope mass, consistent with expectations from analytic [23, shown as red spread] and numerical theory [3]. The theoretical prediction of [23] is plotted here based on the spread of inferred explosion energies, hydrogen-rich envelope masses, and progenitor radii for the objects in the sample. (bottom) Slope of the plateau from the s_2 parameter in the [137] parameterization compared to the hydrogen-rich envelope mass.

[e.g., Legacy Survey of Space and Time; 81] are expected to greatly increase the rate of Type IIb supernovae discovery by orders of magnitude. Additionally, these boundary objects may be challenging to identify as they are often classified as Type IIb SNe (due to the presence of helium in the spectra), which typically have a larger dip-to-peak distance (or steeper m_3) than objects on the IIb/short-plateau boundary. Indeed, multiple studies have found significant diversity in the early excess emission behavior of objects classified as Type IIb SNe [175; 28], but in this view, boundary objects such as SN 2023wdd and SN 2022acr_v are seen as outliers in the Type IIb subclass, and not as probing a smooth continuum of hydrogen envelope stripping. In our analysis, we leverage SN 2023wdd and SN 2022acr_v to argue instead that much of the bulk light curve behavior of hydrogen-rich core-collapse supernovae can be explained by the outer envelope hydrogen content ($M_{\text{H}_{\text{env}}}$). In this view, increasing hydrogen stripping leads sequentially from Type IIP/L $\rightarrow$ short-plateau; further stripping leads from short-plateau $\rightarrow$ double-peaked Type IIb, where the remaining hydrogen promotes the early excess emission; and the stripping of the final remaining hydrogen leads to an insignificant first peak (IIb without early excess emission) and then eventually a completely stripped envelope (Type Ib).

If Type IIb supernovae reflect a continuum in hydrogen-envelope stripping rather than discrete subclasses, the resulting spectral diversity can be explained directly by the residual hydrogen-rich envelope mass. Population-synthesis models demonstrate that by varying only the extent of hydrogen-envelope removal—primarily through binary interactions such as Roche-lobe overflow or common-envelope evolution—a continuous spectral sequence emerges naturally, spanning Type IIP to IIL, then to IIb, and finally to Ib as residual hydrogen decreases from several solar masses down to below $10^{-3}M_{\odot}$ [178]. The identification of such a continuum observationally may imply that binary stripping, rather than single-star winds, dominates mass loss in

typical hydrogen-rich/stripped-envelope progenitors.

3.7 Conclusions

Our observations and analysis of SN 2023wdd and SN 2022acr_v reveal two objects on the boundary of the distinct domains of Type IIb and short-plateau supernovae (short-plateau). Despite sharing similarities with double-peaked IIb SNe, both show comparatively weaker early declines (or very short plateaus), and spectroscopic features in which H α remains significant even as modest He I contamination appears. By fitting both analytic shock-cooling models and the numerical simulation grid of [3], we estimate hydrogen envelope masses of $\sim 0.6\text{--}0.8 M_{\odot}$, placing these events above the typical hydrogen mass range for double-peaked IIb SNe ($\lesssim 0.1 M_{\odot}$) yet just below that of the proposed short-plateau subclass ($\gtrsim 0.91 M_{\odot}$). This quantitative constraint, along with their spectral and photometric evolution, provides supporting evidence for a continuum of hydrogen envelope stripping in massive stars before core collapse.

Notably, both supernovae show “weaker” second peaks than canonical double-peaked IIb SNe. In SN 2023wdd, the transition from the shock-cooling decline to the radioactive peak is especially subtle in bluer bands, approaching a short-plateau appearance more similar to the behavior of SNe such as SN 2023ufx rather than the pronounced dip observed in well-studied IIb SNe like SN 2016gkg or SN 2022hnt. Such behavior theoretically follows from a more hydrogen-rich outer envelope in the progenitor prior to explosion. Although the light-curves of SN 2023wdd and SN 2022acr_v could be characterized as borderline short-plateau SNe, their helium contamination and partial envelope stripping align them more closely with IIb-like progenitors, and thus place them along the boundary between Type IIb SNe and short-plateau SNe predicted by recent theoretical work.

The findings here underscore the importance of obtaining well-sampled, multi-band photometry and high-cadence spectroscopy in the first few weeks following explosion. Such observations are key to distinguishing subtle transitions between “hydrogen-poor” and “hydrogen-rich” envelope configurations, especially in this intermediate regime. As new large-scale transient surveys deliver hundreds of core-collapse supernovae each year, the detection of additional boundary objects will illuminate the physical processes enabling fine-grained variations in progenitor hydrogen-rich envelope mass. Ultimately, building a robust sample of these transitional events will help unify our understanding of how mass loss shapes the diversity of hydrogen-rich explosions.

Chapter 4

The first chirping supernova and its implications for the magnetar model of SLSNe-I

//

O CUCKOO! SHALL I CALL THEE BIRD, OR BUT A WANDERING VOICE?

NO BIRD, BUT AN INVISIBLE THING, A VOICE, A MYSTERY...

WHICH MADE ME LOOK A THOUSAND WAYS, IN BUSH, AND TREE, AND SKY.

— William Wordsworth, “To the Cuckoo”, 1807

∩

Type I superluminous supernovae (SLSNe-I) are at least an order of magnitude brighter than standard supernovae, with the power source for their luminosity still unknown [44; 179; 46]. The central engines of SLSNe-I are hypothesized to be mag-

netars [10; 11], but the majority of SLSNe-I light curves have multiple bumps that are unexplained by the standard magnetar model [180; 57; 56]. Existing explanations for the bumps either modulate the engine luminosity or invoke interactions with circumstellar material. Surveys of the limited sample of SLSN-I light curves find no compelling evidence favoring either scenario [65; 57], leaving both the nature of the light-curve fluctuations and the applicability of the magnetar model unresolved. Here, we report high-cadence multiband observations of an SLSN-I with clear “chirped” (i.e., decreasing period) light-curve bumps that can be directly linked to the properties of the magnetar central engine. Our observations are consistent with a magnetar centrally located within the expanding supernova ejecta, surrounded by an infalling accretion disk undergoing Lense-Thirring precession. Our analysis demonstrates that the light curve and bump frequency independently and self-consistently constrain the magnetar spin period to $P = 4.2 \pm 0.2$ ms and the magnetic-field strength to $B = (1.6 \pm 0.1) \times 10^{14}$ G. These results provide the first observational evidence of the Lense-Thirring effect in the environment of a magnetar, and confirm the magnetar spin-down model as an explanation for the extreme luminosity observed in SLSNe-I. We anticipate this discovery will create avenues for testing general relativity in a new regime—the violent centers of young supernovae.

4.1 Permissions and Attributions

1. The content of [chapter 4](#) is the result of a collaboration with the Global Supernova Project, in particular Dr. Logan J. Prust as well as Prof. D. Andrew Howell and Dr. Yuan Qi Ni, and has previously appeared as the cover of the March 12, 2026 issue of the flagship *Nature* journal as Joseph R. Farah *et al.* (2026). It is reproduced here with the permission of *Nature*: www.nature.com/articles/

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2. Full acknowledgements and author contributions are available on the journal page.



4.2 Overview of results

Supernova (SN) 2024afav is a nearby (~ 327 Mpc) SLSN-I which displayed the first unambiguously chirped light-curve modulations ever observed in an SN. It was discovered by the Liverpool–Gravitational-wave Optical Transient Observer (L-GOTO) Collaboration Dec 12.7, 2024 [all times in UTC, 181] and classified by the extended Public European Southern Observatory Spectroscopic Survey of Transient Objects (ePESSTO+) collaboration as an SLSN-I on Jan 24, 2025 [77] based on spectral features (see also Figure 4.5). Early observations indicated an exponential ~ 40 -day rise followed by a bumpy, ~ 50 -day quasisplateau phase peaking at absolute magnitude ~ -20.5 (Figure 4.1). Partway through the “plateau,” we triggered observations using the Las Cumbres Observatory (LCO) via the Global Supernova Project (GSP) and the Fred Lawrence Whipple Observatory (FLWO) KeplerCam to obtain photometry and spectra (see subsection 4.3.1 for full discussion). Upon the appearance of a second (Modified Julian Date; MJD 60736) and third (MJD 60779) sinusoidal modulation in the light curve, each with a period reduced by $\sim 35\%$, we phenomenologically predicted both the epochs and luminosities of subsequent bumps and dynamically adjusted our observation campaign to capture them (see subsection 4.3.1). In this way, we successfully observed a fourth (MJD 60802) and a possible fifth (MJD 60820) modulation, refining the initial period reduction fraction estimate to $\sim 29 \pm 10\%$ (see subsection 4.3.3).

In the prevailing theory to explain SLSNe, a strongly magnetized millisecond pulsar (magnetar) is created in the SN explosion. As the magnetar emits energy via magnetic dipole radiation, the spin rate decreases, and it transfers some of this energy to the expanding ejecta [10; 11]. However, while initial analytic models determined the energy budgets were sufficient to power the SLSN, they did not investigate a mechanism

of energy transfer.

The magnetar model predicts a rapid rise followed by a smooth, monotonic decline. Despite the magnetar model predicting smooth monotonic behavior in the light curve post-peak, a majority of SLSNe-I are observed to have bumps or modulations of some kind [180; 57; 56]. Within the context of a magnetar-powered model, previous studies have attempted to explain light-curve fluctuations in SLSNe-I by invoking either aperiodic [e.g., magnetar flares; 78] or completely periodic [e.g., misaligned magnetic inclination angle precession; 71] modulations of the central engine, which have been successfully modeled against light curves with ≤ 2 bumps. The standard magnetar model fails to explain the unusual chirped signal in SN 2024afav, as shown in [Figure 4.1](#) (see also, [Figure 4.7](#)). While some number of bumps are common in SLSN-I light curves [57], existing models are fundamentally incompatible with a light curve with ≥ 4 unambiguous modulations displaying a decaying period.

The display of a clear, damped, quasi-periodic component of the light curve motivates exploration of a physical mechanism that may be able to explain the timing of the modulations. A young magnetar in certain circumstances may form a small, tilted (relative to the magnetar spin axis) accretion disk from matter originating in the progenitor star that failed to eject during the SN ([Figure 4.2](#), [subsection 4.3.4](#)) [182; 63; 183]. The radius of the misaligned disk is set by the equilibrium between the radiation pressure from the magnetar-launched wind and the ram pressure of the disk [184; 185], which shifts inward as the accretion rate decreases, leading to the disk “infall” ([subsection 4.4.2](#)). Any misalignment between the accretion disk and the spin axis of the neutron star would induce a Lense-Thirring torque on the accretion disk [see, e.g., 186–190, for a review], causing it to precess (with increasing frequency due to the infall) around the magnetar spin axis. As the accretion disk precesses, it may periodically modulate the emission from the magnetar by, for example, alternately

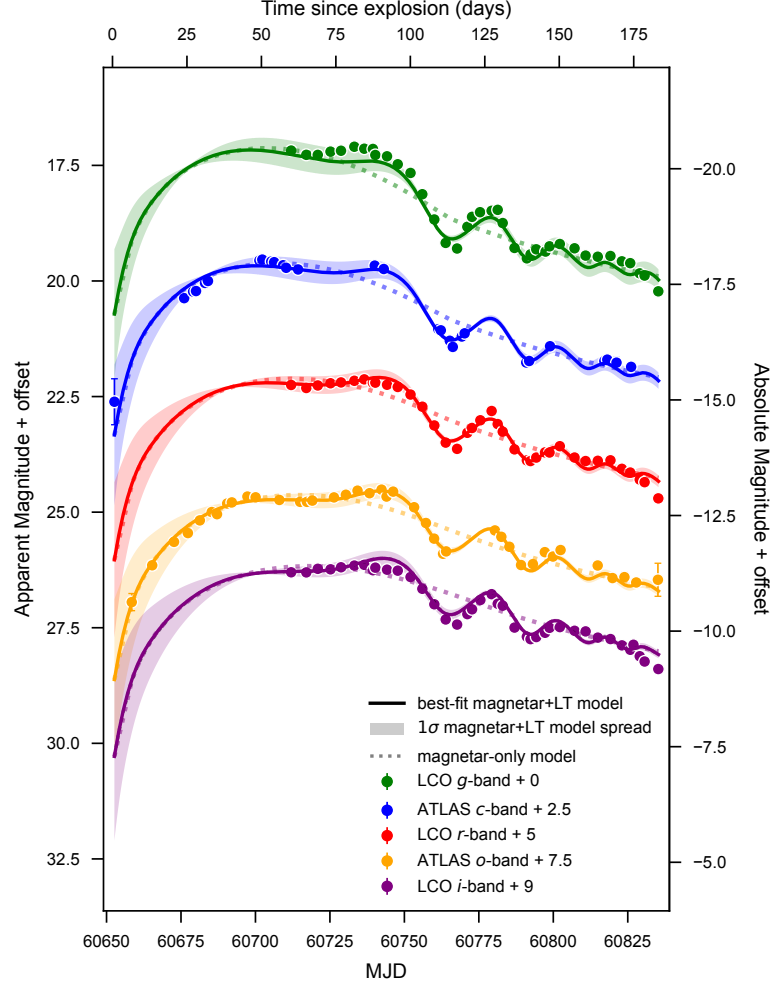


Figure 4.1: | **Multiband light curves of SN 2024afav.** The combined LCO+ATLAS+KeplerCam (KC) light curves of SN 2024afav, from shortly after explosion to ~ 125 days post-peak. Error bars represent 1σ uncertainties computed from the photometry and may be smaller than the data points. The magnetar model is shown as the dotted line (“magnetar-only”; see [subsection 4.3.2](#)). While the data on average are consistent with the magnetar model, there is a chirped signal post-peak that the model fails to explain. We model the light curve as generated by an infalling accretion disk undergoing Lense-Thirring precession (“magnetar+LT”; solid line; [subsection 4.3.4](#), $k = 15$ model degrees of freedom) and find that the model reproduces the overall behavior of the data, with reduced $\chi^2 \approx 1.5$ for the whole fit ($N = 706$ data points) and reduced $\chi^2 \approx 1.6$ for the modulations (40–180 days, $N = 618$ data points).

obscuring or reflecting the magnetar luminosity [as in, e.g., 191; 192], redirecting a jet [193; 194] along the normal of the accretion disk [as in, e.g., 195], redirecting wind from the disk itself [as in, e.g., 14], or changing the accretion rate onto the magnetar [196]. The modulating emission from the magnetar could lead to a non-constant deposition of energy in the SN ejecta [e.g., 71] that precesses with the accretion disk and appears to an observer to oscillate on the Lense-Thirring timescale of $P_{\text{LT}} \approx 25\text{--}45$ days (subsection 4.3.4). High-energy photons from the magnetar interact with the ejecta and are reprocessed into optical photons, delaying the emission and diffusing out on a timescale of $t_d \approx 15$ days during the bumps [197; 198, see also subsection 4.3.4]. The intensity of the bumps is determined not only by the energy injection [199; 14] but also the reprocessing and diffusion characteristics of the expanding ejecta [19]. However, the evolving periodicity of the modulations is solely determined by the properties of the precession (subsection 4.3.4), enabling a direct probe of strong-gravity effects in the vicinity of the magnetar. For our analysis, we propose a phenomenological model composed of a sinusoid acting on a Lense-Thirring precession phase function, modulated by a Gaussian envelope to match the observations. These modulations are added directly to the magnetar model.

We fit our combined magnetar and Lense-Thirring precession model to the SN 2024afav light curve (labeled “magnetar+LT” in Figure 4.1). Crucially, the phase component of the modulation has only two free parameters—the initial phase of the accretion disk and the accretion rate—and is otherwise entirely determined by the parameters of the magnetar (see subsection 4.3.4). We find our model reproduces the overall behavior of the transient during both the rise and the modulations (reduced $\chi^2 \approx 1.5$ for the whole fit, reduced $\chi^2 \approx 1.6$ for the modulations; subsection 4.3.4). Assuming standard neutron star mass ($M_{\text{NS}} \approx 1.4 M_{\odot}$) and size ($R_{\text{NS}} \approx 10^6$ cm), we can separately constrain the magnetar spin period P using the observed rise to peak

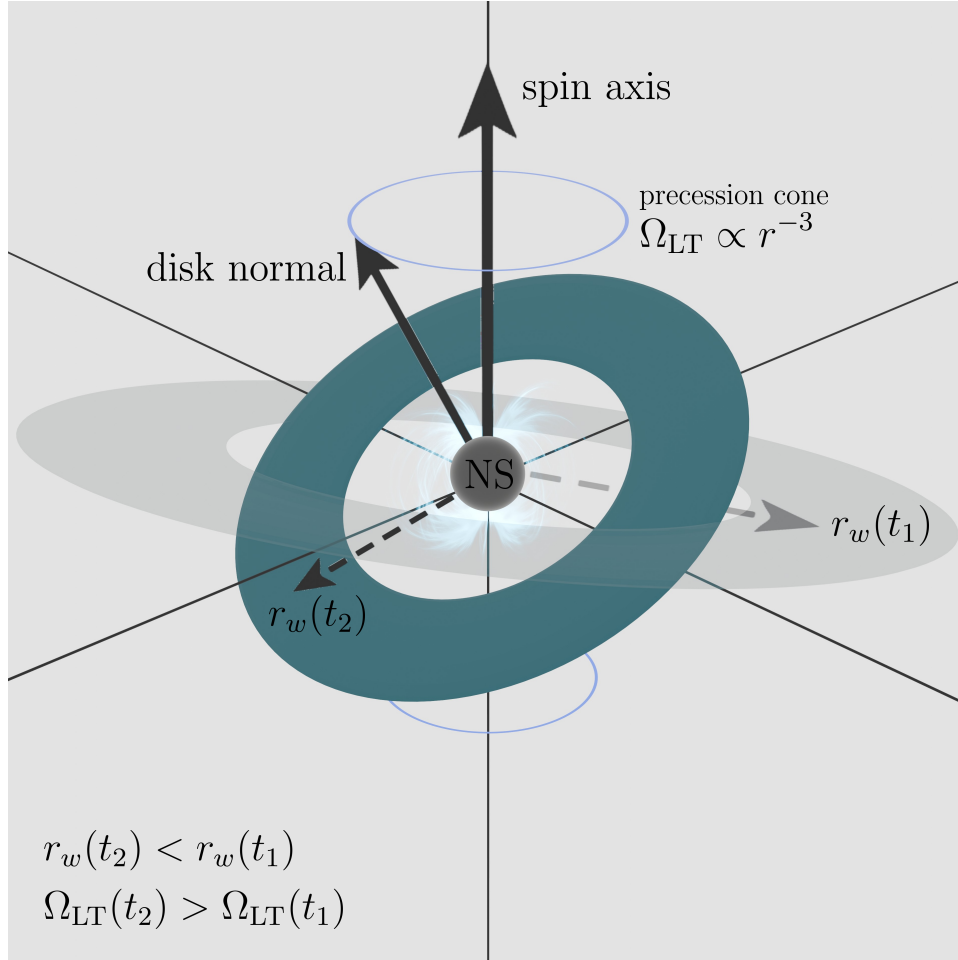


Figure 4.2: | **Diagram of disk infall and precession.** Schematic diagram showing the physical picture motivating the magnetar+LT model. The magnetar is shown in the center of the grid (gray sphere, NS), producing emission which is modulated along the observer's line of sight by an accretion disk (blue annulus). The misalignment between the accretion disk and the spin axis of the neutron star induces a Lense-Thirring torque that causes the disk to precess (with frequency $\Omega_{LT} \propto r^{-3}$) around the spin axis, producing modulations in the light curve from the perspective of the observer as the disk periodically blocks or reflects the spin-down energy from the magnetar. As the accretion disk infalls from a larger radius $r_w(t_1)$ (gray annulus) to a smaller one $r_w(t_2)$ (blue annulus), the induced Lense-Thirring torque increases and the precession (and therefore modulation frequency) increase as well.

(spin-down energy injection constraint) and the residual behavior of the modulations (Lense-Thirring precession constraint). We find the period determined from the rise in the standard magnetar model ($P = 5.5 \pm 1.25$ ms, [subsection 4.3.2](#)) and the modulations ($P = 5.4 \pm 1.8$ ms, [subsection 4.4.2](#)) are statistically self-consistent and, combined with the estimate from the full light curve (forward-modeling both the magnetar model and Lense-Thirring precession-driven modulation model simultaneously), constrain the spin of the magnetar to $P = 4.2 \pm 0.2$ ms ([subsection 4.3.4](#)). Such self-consistency is remarkable as it suggests the modulations and overall light-curve behavior can be directly derived from the same properties of the magnetar. Similarly, combining the measurements from the early light curve and the modulations, the magnetic-field strength is constrained to be $B \approx (1.6 \pm 0.1) \times 10^{14}$ G ([subsection 4.3.4](#)). Assuming the accretion disk is wind-supported [[184](#); [185](#)], we constrain the accretion rate to $\dot{M}(t = 30 \text{ days}) = \dot{M}_0 = (3.71 \pm 0.36) \times 10^{-5} M_{\odot} \text{ yr}^{-1}$ ([subsection 4.4.2](#)).

We consider alternative physical scenarios that may produce the same oscillating light-curve behavior, including various kinds of precession and input energy modulation (see [subsection 4.4.3](#)). Existing explanations that invoke precession-driven magnetic inclination angle variations of a deformed magnetar predict approximately the correct timescale for the modulations [[71](#)] but a positive precession period derivative as the magnetar spins down, contrasting with the observations which show a decreasing period. With the assumption that a precessing accretion disk is instead modulating the line-of-sight energy injection from the magnetar into the ejecta, we consider alternative origins of the precession. Apical precession is unlikely as the disk is highly circularized within minutes to hours of formation [[200](#)]. Magnetic warp precession [[201](#)] and quadrupole-driven nodal precession [[202–204](#)] can give comparable periodicities. However, neither can produce the required period derivative, and in the case of magnetic warp precession, the period is expected to increase owing to

an induced disk-threading torque [201]. By contrast, the tilted, infalling accretion disk undergoing Lense-Thirring precession gives the correct timescales and precession period derivative without deviating from the properties of the magnetar inferred from the overall behavior of the SN (see Figure 4.3).

Some explanations of modulations in SLSN-I light curves have proposed interaction with circumstellar material (CSM) as the origin [e.g., 205; 206; 57]. In particular, the modulations in SN 2017egm were found to favor a CSM interaction origin over alternative scenarios [206]. The scenario of [206] could be used to model arbitrary modulations in an SLSN-I light curve, including SN 2024afav. However, we consider the CSM modulation mechanism less likely for several reasons. First, to reproduce ≥ 4 modulations with decaying period and amplitude requires dozens of highly fine-tuned CSM parameters. Second, the shape of the modulation is firmly sinusoidal, as is visible in the residuals (see Figure 4.12), contrasting with the more irregular shape produced by the CSM model of [205]. Third, the light curve has no modulations until well beyond the diffusion timescale ($t_d \gtrsim 37$ days), consistent with a central-engine-based mechanism. Finally, our proposed description of the oscillating behavior in SN 2024afav sees its goodness-of-fit improve when a diffusion correction with no additional free parameters is incorporated (subsection 4.3.4), behavior that has been used to favor a central-engine origin over CSM interaction [57]. Thus, interaction with CSM is unlikely to be the origin of the observed light-curve modulations in SN 2024afav. However, this does not preclude CSM interactions from being present at a lower level.

The uniquely high-cadence (~ 0.5 day) and extended (~ 200 day) quality of our dataset unambiguously reveal chirped modulations in an SLSN-I light curve, behavior which is evidence of a central-engine origin. Moreover, both the overall light-curve evolution and the modulations themselves are quantitatively consistent with a magnetar-

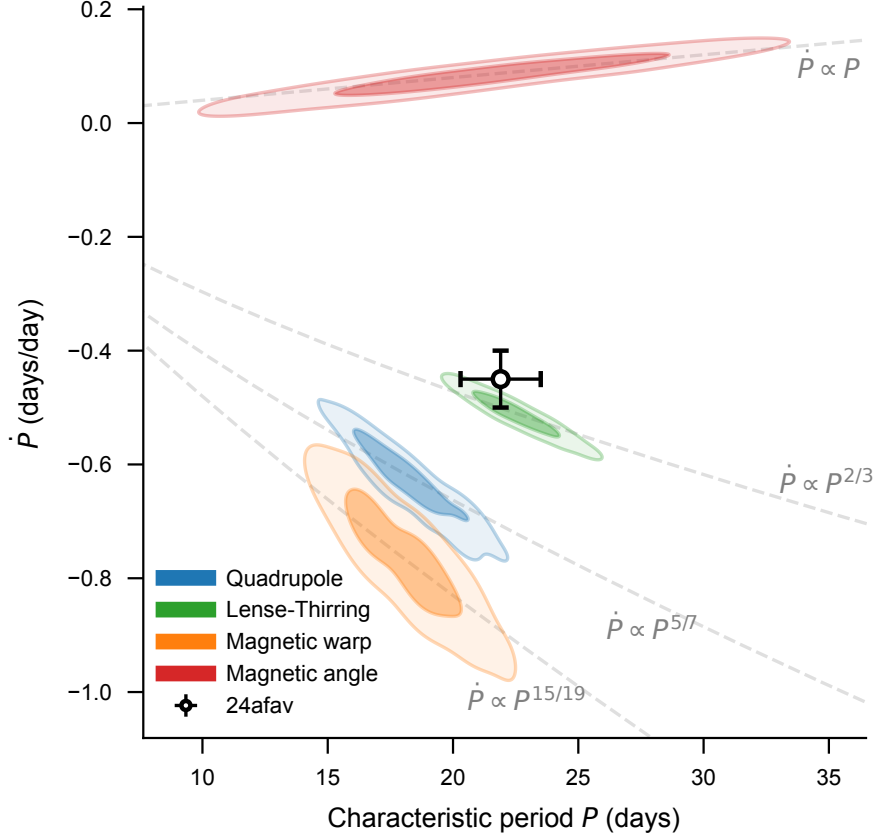


Figure 4.3: | **Alternative explanations of modulations.** Ability of alternative scenarios to explain the modulation of the SN 2024afav light curve. We analyze other possible origins of the accretion-disk precession to assess the uniqueness of the LT model’s ability to explain the observed modulations in SN 2024afav. We characterize the candidate explanations based on their characteristic period (average period over the 80 days when the bumps are visible) and precession-period derivative (see [subsection 4.4.3](#)). The corresponding location of SN 2024afav is shown as the point with error bars. 1σ and 2σ contours are shown for the candidate explanations based on the MOSFiT uncertainties (see [subsection 4.3.2](#)). For models not equipped with an $\dot{\Omega} > 0$ behavior, we invoked radial infall to explain the changing period. We consider magnetic-angle precession, quadrupole-driven nodal precession, and magnetic-warp precession. Only the Lense-Thirring effect around a magnetar produces a characteristic period and period derivative consistent with the observations. Gray dashed lines indicate the predicted $\dot{P}(P)$ behavior based on the linear radial infall assumption described in [subsection 4.3.4](#).

powered model, confirming its viability as an explanation for the extreme luminosity of SLSNe-I and providing evidence of a direct physical mechanism to communicate the magnetar spin-down power into the ejecta. Our analysis further demonstrates that Lense-Thirring precession provides the most plausible explanation for the chirped signal, constituting observational evidence of the Lense-Thirring effect in a magnetar and establishing the influence of general relativistic frame-dragging in a new astrophysical regime: young supernovae.

We also find that our framework reproduces the behavior of previously observed SLSNe-I with periodic light-curve features ([subsection 4.4.4](#)). Previous extensions of the magnetar model [[207](#); [78](#); [206](#); [71](#)] to explain periodic behavior fail to reproduce the rapidly decaying period we observe in SN 2024afav; by contrast, our model can accommodate both chirped behavior as well as the completely periodic behavior observed in some SLSNe-I. We consider a sample of SLSNe-I which have been modeled previously using CSM interaction [SN 2019unb; [57](#); [56](#)], central-engine flares [SN 2021mkr; [78](#)], and magnetar precession [SN 2018kyt; [71](#)]. We find that these objects can be satisfactorily modeled using our magnetar+LT model ([Figure 4.4](#)). For each source, the observed period follows directly from the best-fit magnetar spin and magnetic-field strength, which fix the Lense-Thirring torque and hence the precession frequency (see [subsection 4.3.4](#), [subsection 4.4.4](#)). Thus, SLSNe described by three different mutually inconsistent physical mechanisms can now be described by one.

Forthcoming wide-field surveys, particularly the Legacy Survey of Space and Time (LSST) [[81](#); [208](#)], are expected to discover thousands to tens of thousands of SLSNe-I [[209](#)], which may be followed with facilities like LCO to acquire the extended observations needed to detect the fainter modulations predicted by the magnetar+LT model. This sample would provide a powerful testbed for the magnetar model and enable population-level inference of magnetar properties. Detailed follow-up campaigns will

lead to robust constraints on magnetar properties, and—when combined with spectral diagnostics of accretion and ejecta—possibly even tests of general relativity using infant magnetars.

4.3 Methods

4.3.1 Observations and data reduction

SN 2024afav is located at right ascension (R.A.) $12^{\text{hr}}49^{\text{m}}12.050^{\text{s}}$ and declination (Dec.) $-18^{\circ}06'12.61''$ (J2000 epoch), in the vicinity of the faint galaxy GALEXASC J124911.85-180609.6. The unusually bright absolute magnitude (~ -20.5 in the g band at peak, K -correction of $K = -0.06$; [210]) and a hydrogen-deficient spectrum led to a classification by the extended Public ESO Spectroscopic Survey of Transient Objects (ePESSTO+) in January 2025 as an SLSN-I at redshift $z \approx 0.072$ [77]. Based on the redshift and assuming flat cosmological parameters ($\Omega_M = 0.286$, $\Omega_\Lambda = 0.714$, $H_0 = 69.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$), we adopt a distance of 327 Mpc to the SN. The host galaxy was exceptionally faint in our images, and SN 2024afav is significantly offset from the center ($\sim 5''$). Using the Na I D doublet method of [89], we constrain the extinction due to the host to be $E(B-V)_H \lesssim 0.04 \text{ mag}$. However, based on the [112] line-of-sight extinction map, we correct photometry for a Milky Way extinction coefficient of $E(B-V)_{\text{MW}} = 0.0527 \pm 0.0029 \text{ mag}$. We review the properties of instruments used in our campaign in [subsection 4.4.1](#).

As observations were ongoing, we predicted that our model would have a regime of validity bounded on the left by the diffusion timescale t_d and bounded on the right by the transition of the ejecta to the nebular phase. The magnetar+LT model assumes a luminosity driven by the magnetar wind interacting with the ejecta (see [subsection 4.3.4](#)). Once the ejecta expand to the point where they are no longer

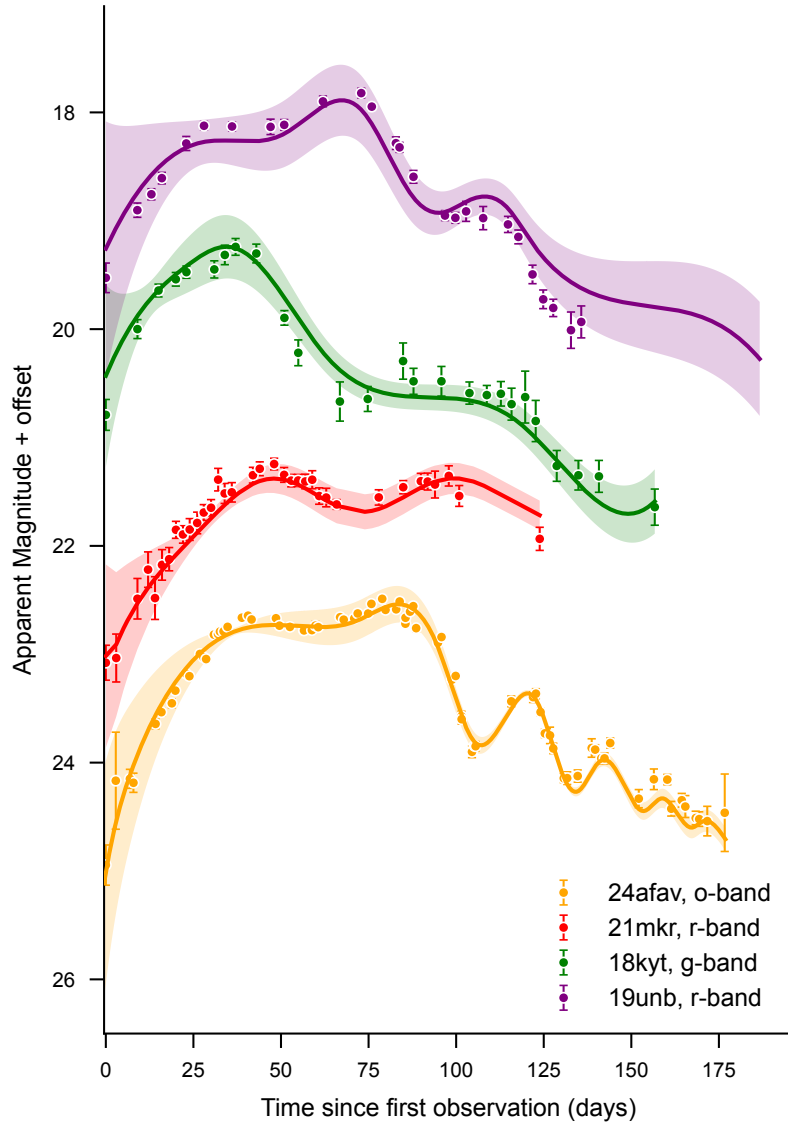


Figure 4.4: Fits to other SLSNe-I using the magnetar+LT model. Periodic fluctuations (with ≤ 2 bumps observed) in SN 2018kyt (green), SN 2019unb (purple), and SN 2021mkr (red) were explained using CSM interaction, magnetic-inclination precession, or central-engine flares. These solutions are either periodic on the timescale of the SN or aperiodic, and cannot describe the chirped behavior of SN 2024afav (orange). By contrast, we find that the magnetar+LT model can explain the behavior in these other objects as well as SN 2024afav. (Note: bands for each object were selected based on which band most clearly displayed the modulations. Error bars represent 1σ uncertainties from the photometry and may be smaller than the data points.)

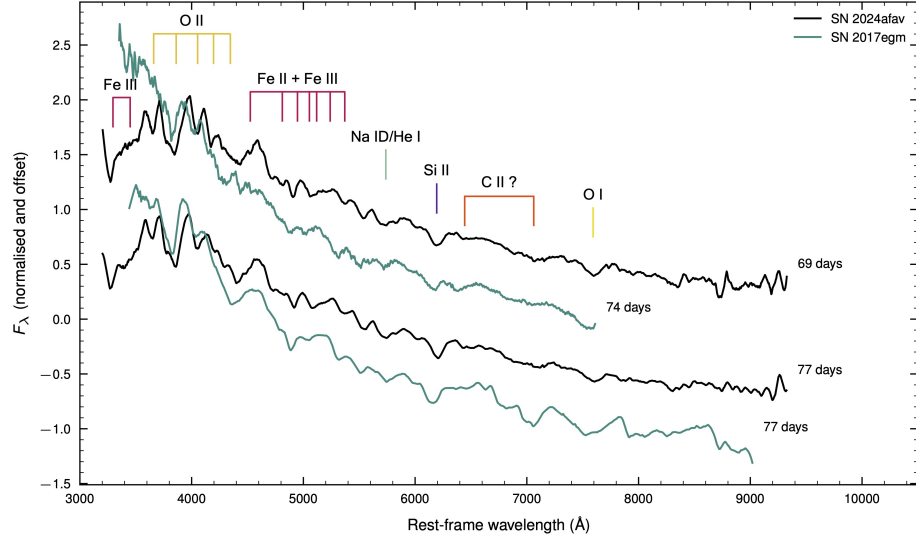


Figure 4.5: | **Comparison of SLSN spectra.** Comparison of the spectra of SN 2024afav (black) with the spectra of SN 2017egm (blue, [206]), another well-known SLSN. Lines are shown at a velocity of $\sim 8000 \text{ km s}^{-1}$. The spectra of the two objects are similar, supporting the classification as an SLSN, to which the magnetar model of [10] may be applicable. We note minor deviations from the spectra of SN 2017egm, such as variations in the strength of iron lines in the Fe II + Fe III complex around 5000 Å .

optically thick (indicated by a transition to the nebular phase, visible in the spectra, for example), this assumption (and therefore the model) is no longer applicable. Around the time the spectra transitioned to the nebular phase, we also observed a significant deviation from a luminosity power law $L(t) \propto (1 + t/t_p)^{-2}$ predicted by magnetar models (see Figure 4.6), adding supporting evidence that the ejecta are no longer sufficiently optically thick to be illuminated by the magnetar wind. For these reasons, we constrain the regime of validity of our analysis to $37 \text{ days} \lesssim t \lesssim 181 \text{ days}$.

4.3.2 Magnetar-only model fits

The unusually bright light curve (~ -20.5 peak absolute magnitude) with a long rise (> 40 days, see subsection 4.3.1) are consistent with predictions for an SN explosion

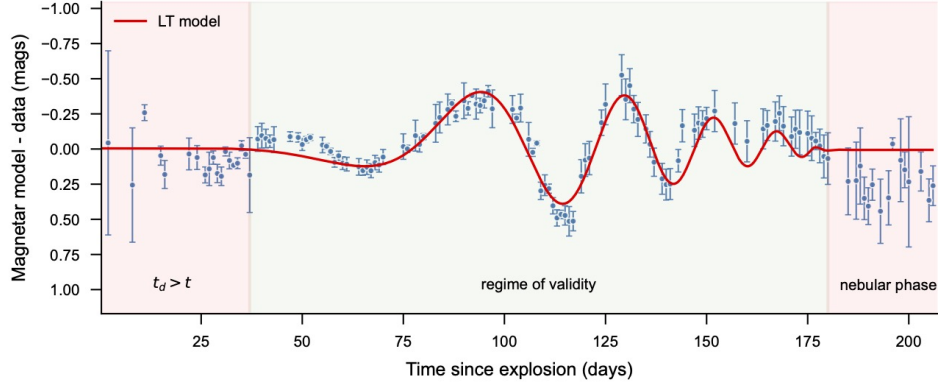


Figure 4.6: | **Regime of validity of the LT model.** Visualization of the regime of validity of the magnetar+LT model. Error bars represent the standard deviation of the photometry binned daily in all filters. The magnetar+LT model fundamentally assumes that the luminosity of the SN is driven by the emission interacting with the ejecta. At early times, when the diffusion timescale $t_d > t$, modulations are not visible and thus our model is not valid. At late times, when the ejecta are optically thin (nebular phase, indicated by the spectral evolution) and no longer illuminated by the magnetar wind, our model is similarly invalid. Between these two regimes, our model may be applied, as the ejecta are sufficiently optically thick to allow high-energy photons from the magnetar to be reprocessed but sufficiently optically thin to allow the reprocessed photons to escape.

powered by a magnetar central engine, as described by the model of [10]. The magnetar model monotonically declines post-peak and is not expected to explain the chirped modulations. Despite this, the overall behavior of the light curve may still be used to infer parameters of the magnetar system. We fit the magnetar model of [10] to the entire dataset using the Modular Open Source Fitter for Transients [MOSFiT; 211], which has been used to fit a wide variety of SLSN-I light curves [e.g., 212; 52]. We initialize the fit with 1600 walkers and use sufficient steps (~ 8000 after a ~ 2000 -step burn-in) to achieve convergence, which we measure via chain autocorrelation length and the improved Gelman-Rubin statistic, following Section 5.4 of [133]. The model includes a parameter (σ , units of mag) which scales the uncertainties so that the reduced χ^2 of the fit is approximately unity. Once convergence was achieved, parameter estimates

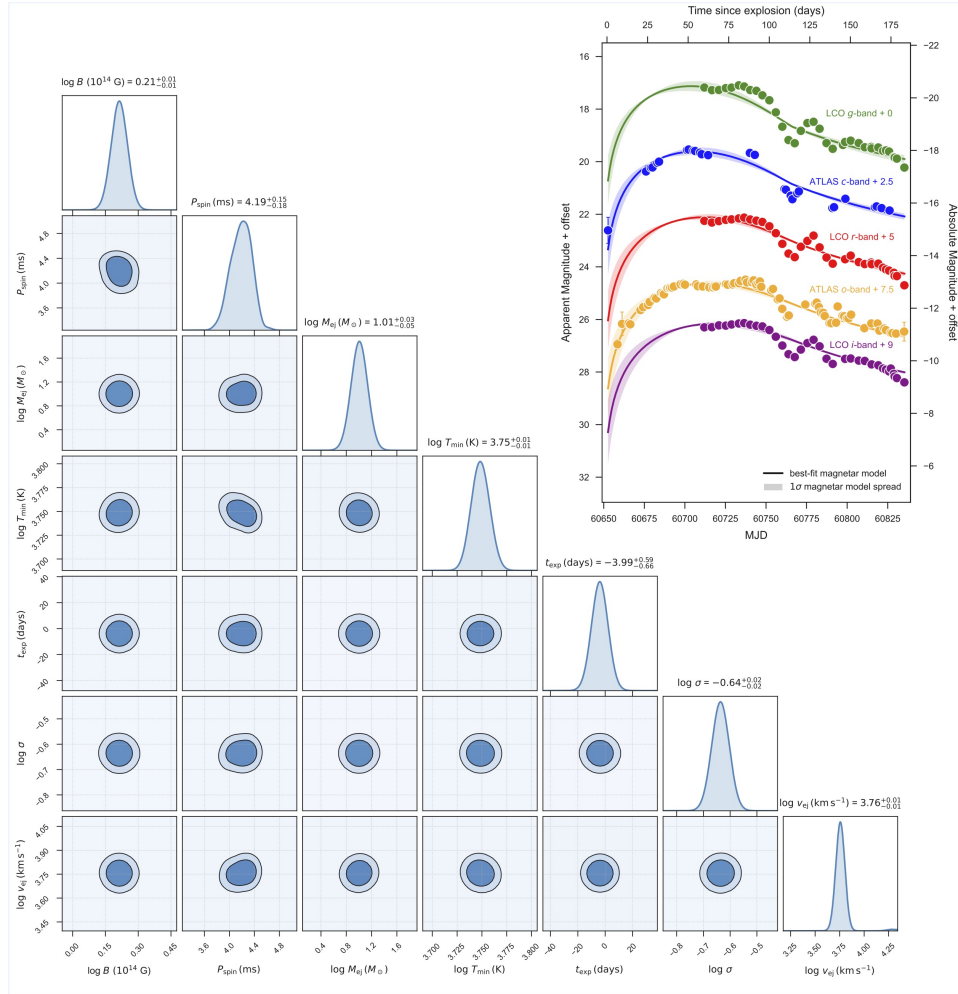


Figure 4.7: | Fit to the magnetar-only model. Corner plot and posterior samples (inset) for the magnetar-only model fit to the light curve of SN 2024afav. The σ parameter (magnitude units) is an additional systematic uncertainty the model is allowed to vary to bring the fit reduced χ^2 to 1. The posterior exploration achieved satisfactory convergence with minimal correlations between parameters and symmetric, unimodal uncertainties. The model explains the rise and overall trend of the light curve well; however, the model fails to explain the post-peak chirped modulations which appear in all bands. 1σ and 2σ regions of the joint posterior distributions are shown as contours.

and uncertainties were estimated by computing the highest posterior density interval around the maximum a posteriori value.

The results of the fit are shown in [Figure 4.7](#). All fit-parameter posteriors were unimodal with approximately symmetric uncertainties. The fit explains the rise, peak,

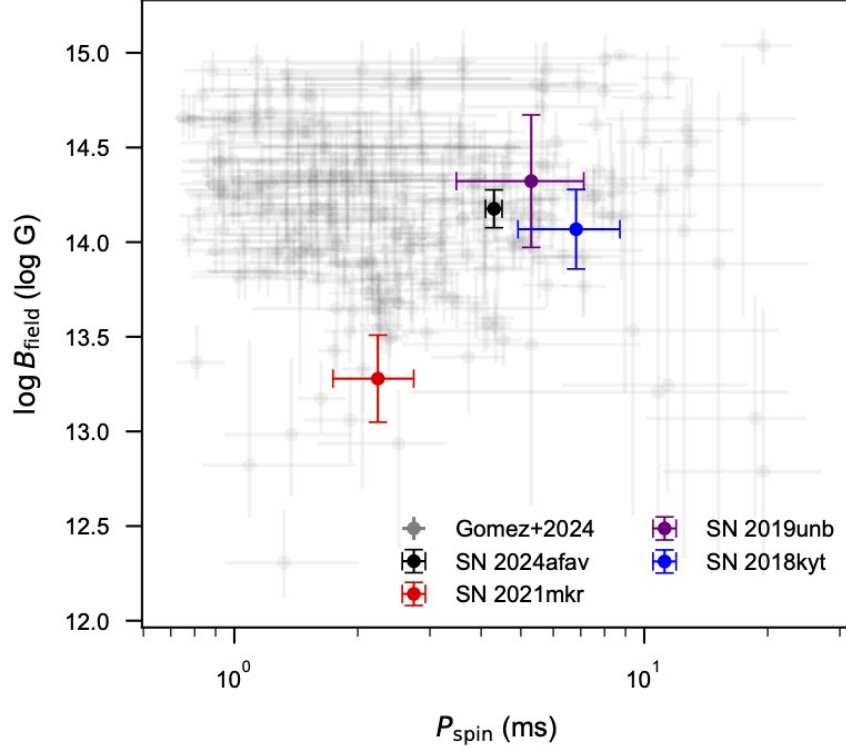


Figure 4.8: | **Landscape of inferred magnetar properties in SLSNe-I.** Comparison of P_{spin} and B_{field} between objects modeled with the magnetar+LT mechanism (SN 2024afav, SN 2021mkr, SN 2018kyt and SN 2019unb) and the sample of SLSNe-I provided by [52]. Errorbars represent 1σ uncertainties on parameter estimates from the fits. The parameters inferred by our model are consistent with the overall distribution of SLSNe-I.

and average decline well, but as expected, does not explain the modulations. The MOSFiT fit infers the presence of a magnetar with magnetic-field strength $B = (1.62 \pm 0.1) \times 10^{14}$ G and spin period $P_{\text{spin}} = 4.19 \pm 0.18$ ms, illuminating ejecta of mass $M_{\text{ej}} = 10 \pm 1.1 M_{\odot}$ expanding with velocity $(5.754 \pm 0.12) \times 10^3 \text{ km s}^{-1}$. The magnetar properties are consistent with expectations from theory [10] and surveys of other SLSNe-I [212; 52], as shown in Figure 4.8.

We consider the possibility that the bumps may be significantly altering the outcome of the fit, as the largest modulations represent ± 1 mag deviations from the mag-

netar model. To address this, we reperform the magnetar fit, using only data from $\lesssim 50$ days (approximately the diffusion timescale). We find that fitting using this reduced sample of the data moderately affects the best-fit parameters. First, the uncertainties dramatically increase across all parameters owing to the lack of data, as only ATLAS *c*- and *o*-band data were consistently available prior to peak brightness (subsection 4.3.1). The period is statistically consistent with the previous fit ($P_{\text{spin}} = 5.49^{+0.11}_{-1.25}$ ms). The magnetic-field strength and ejecta mass decrease substantially ($B = (1.0 \pm 0.7) \times 10^{14}$ G and $M_{\text{ej}} = 5.0^{+1.0}_{-4.7} M_{\odot}$). The ejecta velocity is again consistent with the previous fit and predictions from [10] at $v_{\text{ej}} = 6.03^{+1.51}_{-2.75} \times 10^3$ km s $^{-1}$. In both fits, the behavior of the light curve through the modulations is very similar, indicating the bumps are not significantly affecting the post-peak behavior of the model. However, the start of the modulations coincides approximately with the peak brightness of the SN, which is highly constraining for certain parameters (particularly the spin period, magnetic-field strength, and diffusion timescale). By trimming our dataset to before the modulations began, we also limited the ability of the fit to properly constrain the peak of the light curve, possibly exacerbating discrepancies in the parameters between the two fits.

4.3.3 Residual analysis

We characterize the modulations by computing the residual between the data and the model fit in subsection 4.3.2. We work in magnitude space and weight all filters equally as (i) the behavior is consistent enough in all bands to make this effective, and (ii) the spectral energy distribution (SED) of the object is quite complicated based on the spectra, which introduces uncertainty into (for example) a blackbody fit to convert to luminosity space. The effect of this assumption is to adopt a flat SED, which is sufficient for our analysis but introduces minor deviations in some bands as seen in

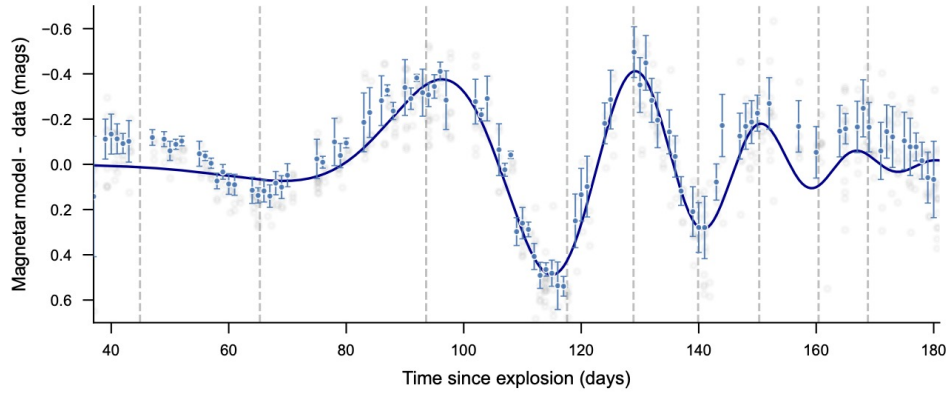


Figure 4.9: | **Magnetar-model residuals.** Residuals between the magnetar and model data across all bands phenomenologically investigated. The aggregated residuals from each band (gray points) are binned in time (blue points). Error bars represent the standard deviation of the photometry binned daily in all filters. We identify at least 5 clear bumps in the residual (gray dashed lines). To characterize the behavior, we fit a combined quadratic polynomial envelope and a cubic polynomial phase function, for a total of seven free parameters (dark-blue line). The result of this fit indicates a hybrid growing/decaying envelope peaking ~ 120 days from the explosion epoch estimated by MOSFiT and a chirped signal with periodicity declining from $P \approx 50$ days to $P \approx 20$ days over the course of ~ 80 days. The fit underestimates the first peak but produces peaks consistent with the remaining modulations.

Figure 4.1. The result of this residual and subsequent binning is shown in Figure 4.9.

We use a standard peak-extraction algorithm [213] to identify candidate extrema in each band, which we then aggregate by computing a mean location for each cluster of extrema weighted by the number of bands in which the extremum appears. Using this approach, we identify troughs at $t = 65.3, 117.6, 139.9$, and 160.4 days, and peaks at $t = 44.9, 93.6, 128.9, 153.3$ days as well as a low-confidence peak detection at $t = 168.8$ days. Note that “peak” and “trough” refer to the conventional “brighter” and “dimmer” points on the light curve residual, which are smaller and larger magnitude, respectively, due to the construction of magnitudes. To quantify our confidence in the presence of the bumps, we fit a simple Gaussian model to each modulation and use the goodness-of-fit compared to a no-signal noise model to compute the probability of false

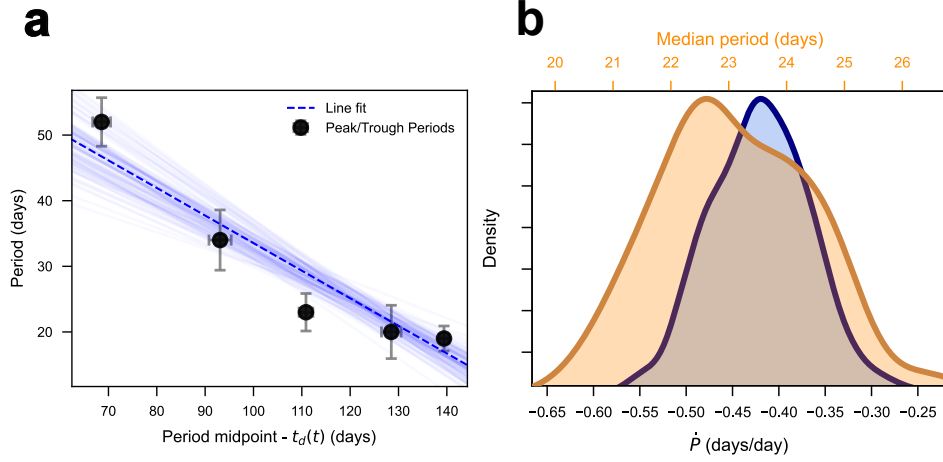


Figure 4.10: | **Characteristic parameter estimation.** We pedagogically demonstrate the procedure for estimating characteristic parameters $\dot{P}_c$ and P_c , as described in [subsection 4.3.3](#). The left panel (a) shows the periods estimated from SN 2024afav, corrected for the diffusion timescale, with 1σ uncertainties propagated from quadratic fits to the modulations used to determine extrema positions. We fit a line to the periods and use a posterior sampling of the parameters to estimate the slope $\dot{P}_c = 0.44 \pm 0.05$ (right panel, b, blue distribution). Next, we compute the median of the period measurements to get a characteristic period $P_c = 23 \pm 1.6$ days (uncertainty is estimated from the spread in the medians of the linear fits, right panel, orange distribution). For our models, which generate effectively continuous time series periods, we sample them on the same days as the period midpoints shown above, and compute their characteristic quantities using that sampling. This serves as a pedagogical tool to help understand the ability of the models to describe the data.

alarm (P_{FA}). We find that the false-alarm probability is $\ll 0.1\%$ for all except the last two modulations. The penultimate dip (modulation #7) has a false-alarm probability of $\sim 6\%$, which is acceptable. By contrast, the final modulations (#8 and #9) have a false-alarm probability of $\sim 40\%$. Such a high false-alarm probability suggests that the final peak and dip are consistent with noise and not a robust detection. However, this quantitative metric significantly strengthens the evidence for the presence and location of the remaining bumps.

Based on the high-confidence peaks and troughs present, we compute a characteristic period (median of the set of computed periods) and period derivative for the

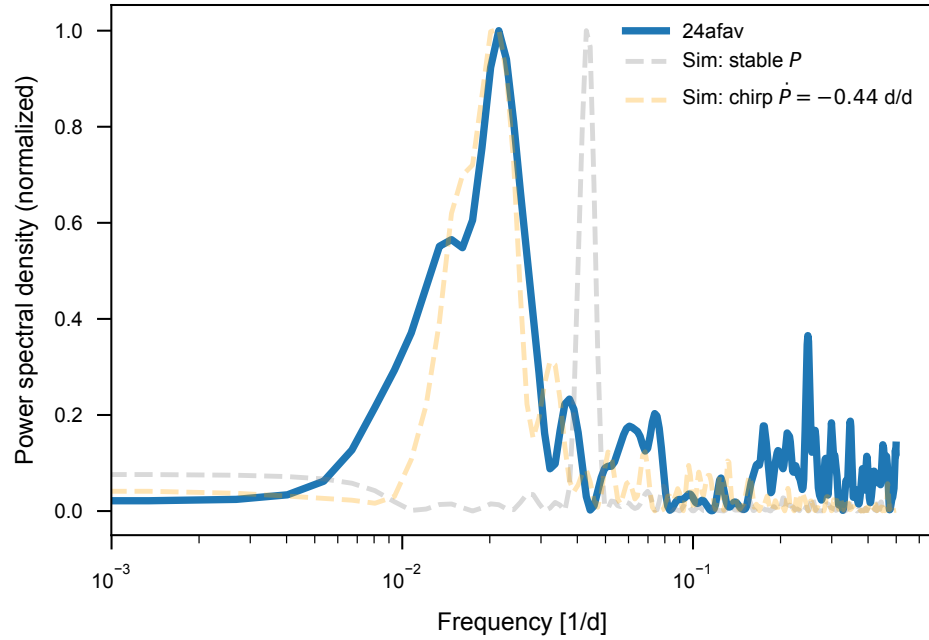


Figure 4.11: | **Power spectral density of chirp.** Power density spectrum of the SN 2024afav residual from $t = 30$ to $t = 180$ days in comparison with a simulated static period and a simulated chirp. The presence of the chirp “smears” the peak frequency in a characteristic way, visible in both the simulated chirp and the SN 2024afav data. By contrast, a stable periodicity produces a narrow peak, which is clearly inconsistent with the SN 2024afav data.

residual modulations. To be conservative, we additionally exclude the first modulation at $t \approx 40$ days, as this may be degenerate with the magnetar-model peak. Using the peak-to-peak and dip-to-dip locations to estimate periods as a function of time, we fit a linear period variation (demonstrated in Figure 4.10) and find $P_c = 23 \pm 1.6$ days and a period derivative of $\dot{P}_c = -0.44 \pm 0.05$. Uncertainties are based on errors in the peak locations, estimated by fitting a quadratic function to the local region around each extrema. We note that the choice of characteristic period and derivative estimation method does not significantly impact the results, so long as the corresponding period and period derivative are computed identically for the theoretical models.

Next, we investigate the sinusoidal behavior of the modulations and attempt to describe them phenomenologically. We construct a generic model to describe a sinusoid

Table 4.1: We compute the confidence of each modulation versus a null hypothesis of noise via a false-alarm probability P_{FA} [4] for bumps and dips. Start and end times of the modulations are approximate.

No.	Peak/dip?	t_{start}	t_{end}	Amplitude	P_{FA}
1	peak	35	60	0.43 ± 0.08	1×10^{-10}
2	trough	60	80	0.26 ± 0.04	1×10^{-7}
3	peak	80	110	0.47 ± 0.11	1×10^{-5}
4	trough	105	122	0.76 ± 0.04	6×10^{-4}
5	peak	122	136	0.49 ± 0.07	8×10^{-5}
6	trough	136	146	0.47 ± 0.03	2×10^{-7}
7	peak	146	154	0.34 ± 0.08	0.06
8	trough	154	166	0.14 ± 0.10	0.42
9	peak	166	175	0.18 ± 0.21	0.51

with varying periodicity and amplitude using

$$\Delta L(t) = \left[\sum_{a=0}^{n_a} c_a t^a \right] \cos \left[\sum_{p=0}^{n_p} c_p t^p \right], \quad (4.1)$$

which varies the periodicity and envelope using polynomials of arbitrary degree. We find that a quadratic ($n_a = 2$) polynomial envelope and a cubic ($n_p = 3$) phase function are required to describe the data to reduced $\chi^2 \approx 1$. The resulting fit (visualized in Figure 4.9) does not reproduce the very first modulation but produces peaks and troughs consistent with the remaining bumps. We use this phenomenological model to quantify our confidence in the presence of a chirp in the light curve. We compute the statistical significance of the model parameters from both $\Delta L = 0$ (no modulations) and $\phi(t) \propto t$ (linear phase indicating no chirp). Using likelihood-ratio tests ($\Delta\chi^2$) against nested null models, we find that both the pure-noise and constant-periodicity hypotheses are rejected at $\gg 5\sigma$. We repeat this test with the fifth bump excluded and find the same results.

To further investigate the presence of a chirp in the light curve, we compute a

power-density spectrum from the residuals from $t = 30$ to $t = 180$ days following [214], and compare this spectrum with a simulated static-period signal as well as a simulated chirp (contaminated with noise comparable to the SN 2024afav light curve). The results of this comparison are shown in Figure 4.11. A static period will produce a narrow peak at a single frequency, but the presence of a chirp characteristically “smears” the highest-amplitude frequencies. We confirm that the SN 2024afav light curve indeed produces a power-density spectrum profile characteristic of a chirp and visibly inconsistent with a static period. Together, the above evidence indicates the presence of four robust modulations with a clearly decreasing period. Further, the success of the phenomenological description of an envelope multiplied by a chirped sinusoid motivates our use of such a model to explore physical mechanisms, replacing the arbitrary phase function with a model-dependent one.

4.3.4 Construction and fit of the Lense-Thirring modulation model

To describe the fluctuations, we modify our phenomenological model (subsection 4.3.3) which can embed a chirped signal in a declining magnetar light curve. By replacing the arbitrary phase function in Equation 4.1 with a physically-motivated, model-dependent one, we may explore how the timings associated with different physical mechanisms compare to the observations.

We review the model presented in section 4.2, emphasizing that we cannot present a definitive model without theoretical investigation that is out of the scope of this paper; nor can we conclusively constrain all aspects of the energy injection scenario without additional data. However, we may consider plausible physical mechanisms that can lead to a chirped light curve.

In the minutes to hours following core collapse and the magnetar birth, an accretion

disk composed of fallback material from the progenitor envelope forms and circularizes [182; 215; 216; 183]. The inner radius of the disk is suspended by a wind launched from the magnetar [184; 185] or, in more extreme cases, the magnetosphere itself. Through some process during formation, the accretion disk becomes tilted or warped, perhaps due to the presence of a binary companion [217], a natal kick [218], or magnetic torques [201]. The tilted disk will experience a Lense-Thirring torque, inducing a precession around the spin axis of the magnetar [186; 217]. The modulation of the magnetar signal by the accretion disk could occur in several ways. These include (but are not limited to) reflecting and obscuring the luminosity from the magnetar [as in, e.g., 191; 192], jet redirection [193; 194] along the normal of the plane of the accretion disk [as in, e.g., 195], wind redirection originating in the disk [as in, e.g., 14], or modulating the accretion rate onto the magnetar itself [196]. The proposed methods produce an anisotropic energy injection into the ejecta [191; 192] which evolves due to the precession of the disk and results in a modulated luminosity as seen by a distant observer [219; 220]. Significant theoretical investigation into each of these scenarios will be conducted in future work but is outside the scope of this analysis. We offer an illustrative investigation into the reflection/occlusion mechanism in [section A.1](#) and demonstrate it can produce the signal observed in SN 2024afav.

The torques induced in the disk by Lense-Thirring precession are communicated through the disk by viscous forces. In the inner disk, where Lense-Thirring torques are the strongest, viscous torques may be insufficient to transport angular momentum quickly enough to prevent the shearing of the disk into distinct rings. Indeed, various groups have found that Lense-Thirring torques can lead to disk breaking within several tens of gravitational radii [e.g., 196; 221–223]. Ref. [196] equated the viscous and Lense-Thirring torques to show that breaking occurs within a radius R_{break} . Our best-fit parameters give an R_{break} value comparable to the radius of the magnetar—two orders

of magnitude below that of the light cylinder—indicating that the disk is unaffected by warping or breaking due to Lense-Thirring torques. A break radius comparable to the light cylinder would require a disk aspect ratio $H/R \lesssim 10^{-4}$, all else being equal, which is inconsistent with our fits of the disk properties (see [subsection 4.4.2](#)). Thus, the Lense-Thirring torque is likely to dominate and drive the precession, and therefore the modulations in the light curve. While the magnetar exerts a torque on the accretion disk, we do not expect the accretion disk to impact the evolution of the magnetar. First, the inner radius of the accretion disk (set by $r_w(t)$, see [subsection 4.4.2](#)) is always outside of the light cylinder of the magnetar ($R_{\text{LC}} = cP/(2\pi) \approx 10^7 \text{ cm} \ll r_w$). More rigorously, [\[224\]](#) found that magnetars with surface-dipole strength $B \approx 10^{14} \text{ G}$ or less do not have any significant impact on the evolution of their host magnetar, regardless of disk mass. As we are in this regime (see [subsection 4.3.2](#)), we do not expect the accretion disk to have any impact on the magnetar, and thus neglect this in our model.

Additionally, in order to assume that the disk precesses as a solid body, we implicitly assume that $H/R \gtrsim \alpha$ [\[225\]](#). This assumption is consistent with the model we are invoking, as a relatively thick disk would be required to meaningfully occlude line-of-sight emission from the magnetar. Furthermore, we validate this assumption by checking that best-fit solutions for α satisfy this assumption, based on the computed radial evolution (see [subsection 4.4.2](#)) and the standard α -disk prescription, following [\[226\]](#) and using relationships summarized in [\[215\]](#).

Finally, since the energy originates within an expanding SN ejecta, the observed light curve is significantly affected by the diffusion timescale t_d of the ejecta. Refs. [\[56\]](#) and [\[57\]](#) found that, even after the diffusion timescale, modulations with timescales $\lesssim t_d$ are smoothed out, but modulations that occur with characteristic timescale $> t_d$ will be phase shifted. For an expanding ejecta with $M \approx 7.5 M_\odot$ and velocity $v \approx 6 \times 10^3 \text{ km s}^{-1}$, the timescale at $t \approx 80 \text{ days}$ is $t_d \approx 30 \text{ days}$ (vs. the observed period of ~ 50

days, ratio $\approx 1.6 > 1$). Toward the end of the modulations, the diffusion timescale at $t \approx 150$ d is $t_d \approx 15$ d (vs. the observed period of ~ 20 d, ratio ≈ 1.3). Physical mechanisms producing periodic modulations on a shorter timescale than t_d may result in approximately sinusoidal oscillations as the finer structure is smoothed out [19]. The primary effect of the diffusion timescale, then, is to delay the emission by $t_d(t)$. We account for this by correcting the peak locations by $\sim t_d$ before attempting to reproduce the periodicity in the extrema-only calculations (see [subsection 4.4.2](#)). We find the results are not highly sensitive to the magnitude of this correction; we could obtain acceptable results even just assuming a constant diffusion offset equal to the average $t_d(t)$ between 80 and 160 days. In the full light-curve fit, this average offset is absorbed into the ϕ_0 parameter. The light curve modulations also appear to vary in amplitude; the physical origin of the amplitude modulation in the residual is uncertain. Plausible interpretations will vary depending on the specific assumption of anisotropic emission generator; for example, a jet-based model may see amplitude fluctuation due to the variation of the jet power, while a disk wind-based model may see variation due to changes in disk size with time. Future investigation into energy-injection scenarios will require significant theoretical work outside the scope of this analysis but yield quantitative predictions for the residual amplitude behavior, helping constrain the underlying mechanism.

Modulation reproduction with magnetar parameters

The additional signal in the light curve is well approximated as the cosine of a phase function, as motivated by the analysis of the residuals ([subsection 4.3.3](#)). For our model, we assume the frequency is driven by Lense-Thirring precession and derive an approximate phase function for the disk in [subsection 4.4.2](#). The phase function is determined by the magnetar properties, an accretion rate, and an overall offset.

By considering only the locations of the peaks of the modulations corrected for the instantaneous diffusion timescale, we can fit the phase function directly without the rest of the light curve or describing the envelope. In this way, we can directly probe the parameters of the magnetar, independent from the rest of the light curve. We perform this fit with disk outer-inner radius ratio $r_o/r_i = 25$, allowing the magnetic-field strength, α -disk parameter, accretion rate, and period to vary. We find $\alpha = 0.07 \pm 0.03$ and constrain the accretion rate through the disk at $t = 30$ days to be $\dot{M}_0 = (3.71 \pm 0.36) \times 10^{-5} M_\odot \text{ yr}^{-1}$. This value is somewhat correlated with the choice of r_o/r_i , which introduces an additional source of uncertainty. We find that varying r_o/r_i between 1 and 30 varies the accretion rate by $\sim (1.6 \pm 10^{-5}) M_\odot \text{ yr}^{-1}$, but without a meaningful constraint as r_o/r_i is almost completely degenerate with $\dot{M}_0$. When the accretion rate is allowed to vary to balance the change in r_o/r_i , we find that the resulting best-fit P_c and $\dot{P}_c$ change by $\lesssim 4\%$ and $\lesssim 6\%$, respectively. This consistency validates our simplifying assumption of constant r_o/r_i ; even if this ratio varies due to spreading or infall, our testing indicates that the resulting inferred P_c and $\dot{P}_c$ will not be significantly altered. Notably, we also find that the magnetic-field strength is well constrained to $B = (1.9 \pm 0.6) \times 10^{14} \text{ G}$, consistent with the values from the bumpless fits in [subsection 4.3.2](#); similarly, we find $P_{\text{spin}} = 5.42_{-1.81}^{+4.53} \text{ ms}$, also consistent with the bumpless fits in [subsection 4.3.2](#). The resulting agreement across disjoint observables eliminates degeneracy with external power sources (CSM interaction, flares, etc.) and rules out coincidental tuning of free parameters: single B and P_{spin} values reproduce both the modulation evolution and the total luminosity evolution. Such a result unifies timing and energetics under one physical engine.

We combine a Gaussian envelope (motivated by the shape of the modulation amplitude in the residuals; [subsection 4.3.3](#)) and the phase function in [subsection 4.4.2](#) to

form an overall fit function,

$$\Delta L(t) = \left(A_0 \exp \left[- \left(\frac{t - t_{\text{peak}}}{A_1} \right)^2 \right] \right) \times \cos \left[- \frac{7}{5} (r_o/r_i)^{-7/4} \frac{1 - (r_o/r_i)^{7/4}}{1 - (r_o/r_i)^{5/4}} \frac{GJ}{\dot{r}_i^3 c^2} (t_{\text{infall}} - t)^{-2} - \phi_0 \right]. \quad (4.2)$$

This function has the following free parameters: A_0 , setting the overall scale of the modulations; t_{peak} , setting the location of the envelope peak; A_1 , setting the rise and decay of the envelope; $\dot{M}_0$ (the accretion rate through the disk at $t = 30$ days), which sets $\dot{r}$ and t_{infall} ; and ϕ_0 , which is an initial condition on the angle of the disk. In contrast to the self-consistency check performed in [subsection 4.4.2](#), we fix all parameters that overlap with the MOSFiT magnetar model to those values identified in [subsection 4.3.2](#). Finally, we incorporate a diffusion-timescale correction based on the optical depth of the expanding ejecta following [19], motivated by previous studies which found that anisotropies in input energy could propagate through the ejecta, resulting in viewing-angle-dependent light curves [219; 220]. This correction washes out variability at early times when the diffusion timescale $t_d \gg t$, and results in a delay at later times when $t_d \lesssim 2\pi/\Omega_{\text{LT}}$. The delay is absorbed into the ϕ_0 parameter.

The results of the fit are shown in [Figure 4.12](#). The physics-incorporated model achieves a similar goodness-of-fit (reduced $\chi^2 \approx 1.6$, number of data points $N = 618$, model degrees of freedom $k = 3$) to the purely phenomenological model in [subsection 4.3.3](#). Reincorporating the model into the overall light curve as done in [Figure 4.1](#) emphasizes the ability of the model to explain the overall light curve and modulation behavior in conjunction with the magnetar model of [10]. We investigate the impact of the diffusion correction by attempting the fit with and without it; we find that without the diffusion correction, reduced $\chi^2 \approx 1.9$; by contrast, the diffusion

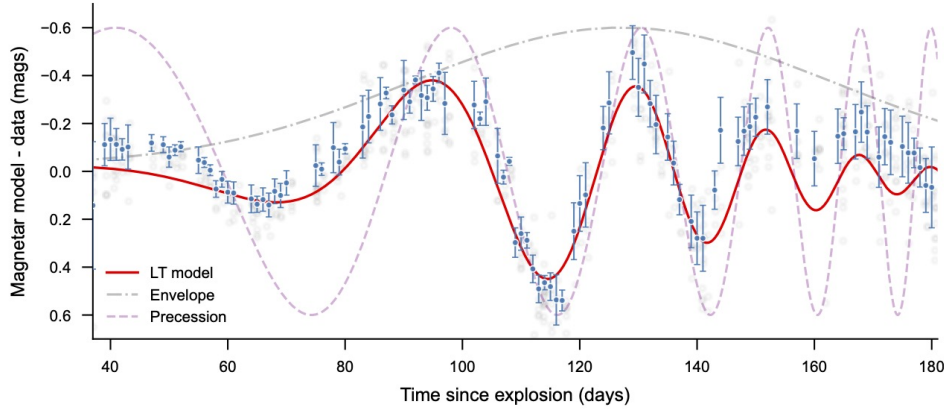


Figure 4.12: | **LT model fit to residuals.** Same as Figure 4.9, but instead investigated using the LT model. We characterize the residual behavior using an amplitude envelope modulated by spin-down power and photon diffusion through the ejecta, combined with a precession governed by the Lense-Thirring effect. Unlike the seven-parameter phenomenological fit shown in Figure 4.9, this model only requires three parameters (overall scale, accretion rate, and initial precession angle of the disk) yet explains the data to a similar degree of success. We show the decomposition of the model into the envelope and precession, which makes the shrinking period easily visible and demonstrates the consistency between the location of the four unambiguous bumps and the prediction from the Lense-Thirring effect.

correction improves the goodness-of-fit by almost 20% without adding any additional parameters. This behavior has been presented as evidence supporting a central-engine origin in previous works [e.g., 57], as processes external to the ejecta would not require a diffusion correction.

Similarly to the polynomial evolution model of subsection 4.3.3, the magnetar+LT model has difficulty reproducing the peak at $t \approx 45$ days, but more successfully reproduces the trough at $t \approx 70$ days. Decomposing the model into the precession and envelope as shown in Figure 4.12, we can immediately see that the precession model accurately reproduces the position of each peak and trough observed in the data. The inconsistency in the full model possibly originates from the amplitude component, which is likely too simple to explain the evolution of the modulations. This peak is also consistent with the expected peak of the light curve based on the magnetar

model with no modulations, resulting in a minor degeneracy regarding whether the very first peak is a result of the precession, the spin-down of the magnetar, or both. However, this does not affect interpretation; as demonstrated in [subsection 4.4.2](#), all physical quantities of interest can be estimated by fitting the extrema of our model to the dates of extrema in the light curve, without requiring a description of the amplitudes at all. There is also an inconsistency of $\sim 15\text{--}20$ days between the location of the trough at $t \approx 65$ days; however, all the other extrema are well modeled by the precession component. Incorporating estimates of the period and magnetic field from both the precession ([subsection 4.4.2](#)), early light curve ([subsection 4.3.2](#)), and full light-curve evolution (forward-modeling the light curve with both the magnetar model ([Figure 4.7](#)) and precession component ([Figure 4.12](#)) simultaneously included, we estimate $P = 4.2 \pm 0.2$ ms and $B = (1.6 \pm 0.1) \times 10^{14}$ G.

4.4 Supplementary Information

4.4.1 Observing instruments

SN 2024afav was almost continuously observed since explosion by the Asteroid Terrestrial Last-Alert System (ATLAS) [79]. ATLAS is comprised of dual 0.5 m telescopes located on Mauna Loa and Haleakala in Hawaii which survey the entire night sky in at least one band daily. We used the ATLAS online photometry pipeline¹ to obtain forced photometry in the *o* (orange) and *c* (cyan) bands for SN 2024afav over the first ~ 180 days of its evolution. Calibration of the ATLAS forced photometry was done to the Pan-STARRS catalog.

Halfway through the “plateau,” we began observing SN 2024afav approximately

¹<https://fallingstar-data.com/forcedphot/>

daily with the Las Cumbres Observatory [84] 1 m and 2 m telescopes via the Global Supernova Project, as well as obtaining roughly weekly spectra with the wide-coverage (350 nm to 1 micron) FLOYDS spectrograph located on the northern 2 m LCO instrument at Haleakala Observatory. We obtained photometry in the g , r , and i bands, reduced the data using the `lcogtsnpipe` pipeline [86], and calibrated the extracted magnitudes to AB magnitudes [227] using the Sloan Digital Sky Survey catalog [114]. Spectroscopic data were extracted and reduced using the `floydsspec` pipeline [86].

We also obtained optical images of SN 2024afav in the gri filters with the KeplerCam imager on the 1.2 m telescope at FLWO. The data were reduced using standard IRAF routines, and we performed photometry with the `daophot` package. Instrumental magnitudes were measured by modeling the point-spread function (PSF) of each image using field stars and subtracting the model PSF from the target. To separate the flux of SN 2024afav from that of its host galaxy we perform image subtraction on each image with `HOTPANTS` [228], using archival PS1/ 3π images as reference templates. We estimate individual zero-points of each image by measuring the magnitudes of field stars and comparing to photometric AB magnitudes from the PS1/ 3π catalog [229]. The uncertainties reported for KeplerCam photometry are the combination of the photometric uncertainty and the uncertainty in the zero-point determination.

In addition to the LCO FLOYDS spectra, we obtained spectra using the Magellan telescope in Chile and the MMT instrument at the Fred Lawrence Whipple Observatory in southern Arizona. The data were reduced using the `PypeIt` package [230] in a standard manner. The one-dimensional spectrum was extracted and flux-calibrated using a standard-star observation obtained with the same configuration.

4.4.2 Derivation of the precession

Following [subsection 4.3.3](#), we fit these oscillations with a two-component model, multiplying a time-evolving envelope $A(t)$ by a sinusoid,

$$\Delta L_{\text{LT}}(t) = A(t) \cos[\phi(t)]. \quad (4.3)$$

The choice of a sinusoid with a phase offset to represent the observed signal is well-motivated by the clearly sinusoidal modulations in the data. The phase function $\phi(t)$ is derived directly from the precession-frequency magnitude (Ω_p) in the weak-field approximation, which has both a Lense-Thirring component and a quadrupole-driven component. Given that the magnetar-disk system has an unknown orientation relative to the observer, we adopt a fully general prescription, following [\[188\]](#) and [\[231\]](#). We choose $\hat{z}$ as the line of sight of the observer (with sky coordinates $\hat{x}, \hat{y}$). In the context of the orbit, we construct our unit triad $(\hat{i}, \hat{m}, \hat{h})$, following Equations 2–4 of [\[231\]](#), and magnetar spin vector $\hat{k}$. The spin axis $\hat{k}$ is defined by two spherical angles: the polar angle α' (primed to differentiate from α -disk characteristic parameter) between $\hat{k}$ and the observer's $\hat{z}$ axis, and the azimuthal angle δ between the projection of $\hat{k}$ onto the $(\hat{x}, \hat{y})$ plane and the $\hat{x}$ axis. Finally, the tilt between the spin axis and the disk is defined by $\cos \beta = \hat{k} \cdot \hat{h}$, since $\hat{h}$ is the unit orbital angular momentum vector (that is, the normal of the disk). In this coordinate system, we find that the magnitude of the combined Lense-Thirring and quadrupole precession is

$$\Omega_p(r, \beta) = \Omega_{\text{LT}} + \Omega_{J_2} = \frac{2GJ}{c^2 r^3} + \frac{3}{2} J_2 \sqrt{\frac{GM}{r^3}} \left(\frac{R}{r} \right)^2 \cos \beta, \quad (4.4)$$

where J is the angular momentum of the magnetar, r is the characteristic radius of the accretion disk, R is the radius of the magnetar, and J_2 is the magnitude of the

geopotential's quadrupole deviation from spherical symmetry. In theory, J_2 , J , and β are all free parameters. However, in practice, even at the theoretical upper limit of $J_2 \approx 10^{-3}$ [203] and assuming $\cos \beta \approx 1$ (unlikely as the tilt of the disk relative to the spin axis is not expected to be large), the ratio $\Omega_{\text{LT}}/\Omega_{J_2} \approx 10^2\text{--}10^3$ for all system parameters. Additionally, $J = I\Omega = (2\pi/P_{\text{spin}})kMR^2$ with $k \approx 0.35$ [232] is completely determined by the remaining free parameters (namely, M , R , P_{spin}). Thus, for simplicity, we restrict our analytic treatment to

$$\Omega_{\text{LT}} \approx \frac{2GJ}{c^2 r^3(t)}. \quad (4.5)$$

We tested this simplification by performing all fits with the full expression, and found that we indeed cannot constrain β or J_2 , as the quadrupole term is not sufficiently dominant to affect parameter constraints. This expression is consistent with similar derivations in the literature [e.g., 186], and is valid for a test-particle orbit or for a very thin annulus; however, the accretion disk will be composed of a range of orbits, resulting in a different precession frequency for the disk as a whole. Following [233], we model the disk with surface density $\Sigma = \Sigma_i(r/r_i)^{-\zeta}$, and find that for $\zeta = 3/4$ [215], the precession frequency derived above becomes

$$\Omega_{\text{LT}} = \frac{7}{5} \frac{1 - (r_o/r_i)^{-5/4}}{(r_o/r_i)^{7/4} - 1} \frac{2GJ}{c^2 r_i^3(t)}. \quad (4.6)$$

As the outer and inner radii (r_o and r_i) of the annulus often appear as a ratio, we treat r_o/r_i as a single parameter. For the purposes of our calculation, we assume r_o/r_i is constant; however, we test the validity of this assumption below.

The accumulated phase $\phi(t)$ is the integral of the angular frequency with time. The exact form of $r_i(t)$ depends on the mechanism driving the infall; however, most infall

mechanisms can be approximated to first order as linear over the timescale of tens of days (see Figure 4.3). We assume $r_i(t) \propto t$ to first order; i.e., $r_i(t) = r_0 - \dot{r}_i(t - t_0) = \dot{r}_i(t_{\text{infall}} - t)$ where $t_{\text{infall}} = t_0 + r_0/\dot{r}_i$. Under this assumption, the phase function becomes

$$\phi(t) = \int \Omega_{\text{LT}} dt \approx -\frac{7}{5} \frac{1 - (r_0/r_i)^{-5/4}}{(r_0/r_i)^{7/4} - 1} \frac{GJ}{c^2 \dot{r}_i^3} (t_{\text{infall}} - t)^{-2} - \phi_0, \quad (4.7)$$

where the initial phase of the disk ϕ_0 is a free parameter, and t_{infall} and $\dot{r}$ are determined based on the assumed infall model. Next, we assume the accretion disk is supported by the magnetar-launched wind, which causes the disk to infall as the accretion rate decreases [184; 185]. To approximate the equilibrium radius $r_w(t)$ (which we will identify with the inner radius r_i used above to obtain $r_i(t)$) where the disk pressure balances against the magnetar wind pressure, we consider the disk gas pressure [215]

$$P_g \propto \dot{M}^A r^{-b}, \quad (4.8)$$

where $\dot{M} \propto t^{-C}$ is the accretion rate through the disk, an approximation well-motivated for $C \approx 5/3$ at $\sim 10^6$ s by [14]. We also consider the radiation pressure generated by the disk [215],

$$P_r \propto \alpha^{-0.8} \dot{M}_0^{A+0.7} t^{-C(A+0.7)} r^{-b-3/8}, \quad (4.9)$$

where $\dot{M}_0$ is a free parameter, corresponding to the accretion rate through the disk at a reference time t_0 (we take $t_0 = 30$ days). This combined disk gas and radiation pressure is balanced against the radiation pressure from the magnetar wind,

$$P_w \propto L(t)/r^2, \quad (4.10)$$

where $L(t) \propto (1 + t/t_p)^{-2}$ as given by [10], which defines t_p as the spin-down timescale of the magnetar. We numerically solve for the disk radius $r_w(t)$ which satisfies $P_g + P_r \propto P_w$, where we adopt standard values of $A = 17/20$, $b = 21/8$, and $C = 5/3$ from [215], [14], and [199]. We use this radial infall function $r(t)$ to determine the phase function in Equation 4.7.

4.4.3 Investigation of alternative scenarios

We evaluate other mechanisms that could replicate the oscillating light curve, such as different precession modes and time-variable energy injection. For each mechanism, we compute a characteristic period and average period derivative over the timescale of the modulations observed in the SN 2024afav light curve. For each mechanism, we vary model parameters to fit the characteristic period P_c and calculate from those parameters a period derivative for comparison. For accretion disk precession-based mechanisms, we invoke wind-based radial infall to produce $\dot{P}_c < 0$ if the mechanism is not equipped with an evolving periodicity. Finally, we vary the MOSFiT magnetar model parameters and compute the corresponding P_c and $\dot{P}_c$ values to generate a Monte Carlo sampling of their possible range. We compare these to the SN 2024afav values of $P_c = 23 \pm 1.6$ days and $\dot{P}_c = -0.44 \pm 0.05$. A visualization of the period/period-derivative parameter space of the below scenarios is shown in Figure 4.3. For each scenario, we compute the precession period and period derivative exactly, and compare to an analytic prediction based on assuming linear radial infall, to validate the assumption of subsection 4.4.2. The analytic prediction is visualized for each mechanism as the gray dashed lines in Figure 4.3.

Magnetic inclination angle precession: Ellipticity of the magnetar can lead to precession, causing modulations in the light curve [71]. We assume the model of [71] and

compute the period of the magnetar versus time using their Equation 6. We adopt the MOSFiT magnetar parameters and find that a high deformation parameter of $\epsilon \approx 10^{-8}$ is required to obtain $P_c \approx 23$ days. However, the combination of parameters required to reproduce the periodicity observed in the data results in $\dot{P}_c \approx 0.1$, which implies an increasing periodicity. There is no mechanism in this model to create a decreasing period, and the magnitude of the predicted $|\dot{P}_c|$ is a factor of several too small. This model is disfavored.

Quadrupole-driven nodal precession: An oblate neutron star will cause orbits in the vicinity to precess in a manner which scales with the quadrupole coefficient of the neutron star J_2 and decreases with the square of the orbital radius [202]. We obtain the following expression for the quadrupole-driven nodal precession frequency of an annulus with inner and outer radii r_i and r_o ,

$$T_{\text{QDNP,annulus}} = \frac{4\pi}{3\sqrt{GM}J_2R_{\text{mag}}^2 \cos \beta} \frac{1 + \zeta}{5/2 - \zeta} r_i^{7/2} \frac{(r_o/r_i)^{5/2-\zeta} - 1}{1 - (r_o/r_i)^{-1-\zeta}}, \quad (4.11)$$

where R_{mag} is the magnetar radius, ζ parameterizes the surface density (i.e., surface density $\Sigma(r) \propto r^{-\zeta}$), and J_2 is a dimensionless coefficient which parametrizes the oblateness of the magnetar. There is no inbuilt mechanism for quadrupole-driven nodal precession to produce a decreasing period, so we invoke the radial infall of [subsection 4.3.4](#). We find that an accretion rate of $\dot{M} = 3.1 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ coupled with a high $J_2 \approx 10^{-3}$ [203] produces $P_c = 18 \pm 2$ days. However, the accretion rate required to reproduce the period enforces $\dot{P}_c = -0.62 \pm 0.04$; thus, this model is disfavored.

Apsidal precession: If the disk has intrinsic ellipticity, it may experience apsidal precession due to relativistic effects [e.g., 234]. However, this effect is unlikely to be relevant on the timescale of the modulations observed in SN 2024afav. The disk

circularizes on a timescale of roughly seconds to hours [200; 235; 216], whereas the bumps first appear ~ 2 months into the evolution of the SN. By this point, the accretion disk has circularized and does not experience apsidal precession.

Magnetic-warp precession: A diamagnetic accretion disk experiences a precessional torque with characteristic angular frequency given by [201]. We derive the following expression for the period of the magnetic-warp precession in the case of an annulus with thickness r_o/r_i :

$$T_{\text{MW,annulus}} = \frac{2\pi^3 \Sigma(r_i) \sqrt{GM}}{\mu^2 \cos \beta (3 \cos^2 \theta - 1)} \sqrt{\frac{2H}{r_m}} \frac{9/4 + \zeta}{5/2 - \zeta} r_i^{11/2} \frac{(r_o/r_i)^{5/2-\zeta} - 1}{1 - (r_o/r_i)^{-9/4-\zeta}}, \quad (4.12)$$

where θ is the obliquity angle (between the spin axis and magnetic moment of the magnetar), r_m is the magnetosphere radius, $\Sigma(r) \propto r^{-\zeta}$ is the surface density of the disk, H is the height of the disk (we assume $H/r > \alpha$), and μ is the magnetic moment of the magnetar. This magnetic-warp precession can give comparable periodicities if the disk surface density is unusually high ($\Sigma \gtrsim 10^7 \text{ g cm}^{-2}$), but the period is again likely to increase due to an induced disk-threading torque [201], conflicting with observations. If we instead modify the period by invoking radial infall, we can obtain $P_c = 13.4 \pm 6$ days using an accretion rate of $\dot{M} = 1.3 \times 10^{-3} \text{ M}_\odot \text{ yr}^{-1}$, which is somewhat high. However, this results in a best-fit $\dot{P}_c = -0.72 \pm 0.05$, which is once again inconsistent with the observations of SN 2024afav; thus, this model is disfavored.

4.4.4 Fits to other SLSNe-I

We consider previously studied SLSNe-I to which our model may be applicable. A majority of SLSNe-I have bumps or modulations of some kind [e.g., 57], and some appear to be periodic, motivating oscillating physical processes to explain them [71].

A variety of physical origins are plausible for SLSN-I modulations; we do not expect our model will be sufficient to describe all of them, especially those with nonsinusoidal or nonperiodic bumps. The magnetar+LT model can be considered when the early light curve is well described by the magnetar model and the modulations are clearly periodic or chirped in nature. We consider the following candidates which match these criteria to attempt to describe with our model.

1. SN 2018kyt: discovered [236] 2018 Dec. 19 by the Zwicky Transient Facility [ZTF; 80] at R.A. = $12^{\text{hr}}27^{\text{m}}56.240^{\text{s}}$, Dec. = $+56^{\circ}23'35.59''$. It was classified as an SLSN-I by ZTF based on a spectrum obtained on 2019 Jan. 29 [237]. SN 2018kyt had two clear peaks and possibly two dips at the same cadence as the peaks (~ 70 days). SN 2018kyt was previously considered as a possible candidate for modulation due to magnetic-inclination precession [71].
2. SN 2019unb: discovered [238] 2019 Oct. 20 at R.A. = $09^{\text{hr}}47^{\text{m}}57.010^{\text{s}}$, Dec. = $+00^{\circ}49'35.94''$ by ZTF. It was classified as an SLSN-I by ZTF based on a spectrum obtained on 2020 Feb. 22 [239]. SN 2019unb had two clear peaks and two clear dips with similar cadences (~ 45 days). SN 2019unb was previously considered as a possible candidate for modulation due to CSM interaction [57; 56].
3. SN 2021mkr: discovered [240] 2021 May 15 at R.A. = $1456^{\text{m}}52.570^{\text{s}}$, Dec. = $+60^{\circ}50'05.10''$ by ZTF. It was classified as an SLSN-I by ZTF based on a spectrum obtained on 2021 Aug. 09 [241]. SN 2021mkr had two clear peaks and one dip with a possible cadence of ~ 60 days. SN 2021mkr was previously considered as a possible candidate for modulation due to central-engine flares [78] or alternatively magnetic-inclination precession [71].

For each object, we downloaded publicly available ZTF data. The ZTF survey observes on a nightly basis using a dedicated camera mounted on the 48-inch Oschin Schmidt

telescope at Palomar Observatory. We obtained subtracted and extracted magnitudes in the r and g bands preprocessed by the ZTF photometry reduction pipeline [83].

To model to these data, we incorporate the phenomenological magnetar+LT model constructed in [subsection 4.3.4](#). The procedure for fitting the candidate objects is identical to our method for fitting SN 2024afav.

The results of the fits are shown in [Figure 4.4](#). We find that our model is able to satisfactorily explain the modulations in the data with goodness-of-fit in the range of reduced $\chi^2 \approx 1.3\text{--}1.9$ ($N \approx 75\text{--}105$ data points, $k = 15$), and, as shown by [Figure 4.8](#), estimates values for P_{spin} and B_{field} within normal ranges for SLSNe-I [52]. The key result is that the periodicity observed in the light curve for each object is well approximated by assuming Lense-Thirring precession of an accretion disk at the equilibrium radius, given the best-fit magnetar model parameters from the bulk behavior of the light curve. Successfully fitting the magnetar+LT model to legacy objects suggests SN 2024afav may not be a fluke or unique object, and this mechanism may be relatively common in young magnetars. However, multiple SLSNe-I have been followed to hundreds of days with moderate-cadence data [e.g., [242](#)] without evidence of periodic modulations, which may have several explanations. First, the mechanism proposed here is not required to occur in every young magnetar system. Lack of fallback material or an insufficient accretion rate may prevent the formation of an accretion disk. Alternatively, an insufficient natal kick (or similar) may result in the disk forming with little to no tilt, resulting in no Lense-Thirring torque. Second, the intensity of the modulations is impacted by the diffusion timescale and inclination of the system relative to the observer line of sight, so for particular system configurations (e.g., spin axis parallel to the line of sight) no modulations will be visible. Additionally, our dataset is relatively unique in that, anticipating subsequent bumps, we maintained the intense cadence across all LCO bands to capture them with a high signal-to-noise ratio and

on a subdaily cadence. Had we only observed the first 2–3 bumps clearly—as is the case for most SLSNe-I with modulations (see, e.g., [\[57\]](#) or [\[71\]](#))—we would have been unable to disfavor completely periodic or aperiodic processes.

Chapter 5

Geometric m -ring Modeling of the Infant SN 1993J Radio Shell with Closure Quantities Only

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WHAT WE CALL THE BEGINNING IS OFTEN THE END

AND TO MAKE AN END IS TO MAKE A BEGINNING.

THE END IS WHERE WE START FROM.

— T. S. Eliot, “Little Gidding”, 1942

Ω

SN 1993J is the best-studied radio supernova, with VLBI observations spanning from weeks post-explosion through nearly three decades of ejecta evolution. Imaging and modeling have revealed an expanding shell-like structure, with diameter well-

constrained between ~ 50 and ~ 2500 days. We present a re-analysis of this period using a new VLBI technique: the m -ring model, a phenomenological alternative to the conventional spherical shell model. The m -ring simultaneously fits emission diameter, width, central opacity, and azimuthal brightness modulation. Critically, we use closure quantities only, providing calibration-insensitive constraints on ejecta geometry and largely independent comparisons with previous results from the same datasets. We find power-law expansion with exponent $\omega = 0.89 \pm 0.01$ before a break at ~ 600 days and $\omega = 0.81 \pm 0.01$ after; a single-exponent fit yields $\omega = 0.81 \pm 0.01$, both consistent with prior shell-fitting analyses. For the first time, we measure ejecta width at $t \lesssim 670$ days, as early as 175 days post-explosion, finding a mean fractional width of 0.23 ± 0.03 (in units of the radius) with no significant temporal evolution. We also quantify the azimuthal brightness modulation over time: an apparent horseshoe pattern rotates from east to south-southwest between $t = 175$ and at least 2500 days, subsequently evolving into more complex morphology, extending previous image-only results by ~ 500 days. These constraints provide a new avenue for testing advanced simulations of explosion and circumstellar interaction mechanisms.

5.1 Permissions and Attributions

1. The content of [chapter 5](#) is the result of a collaboration with Dr. Andy Howell, Dr. Giacomo Terreran, Dr. Logan J. Prust, Dr. Michael Bietenholz, Dr. Norbert Bartel, Dr. Curtis McCully, and Dr. Michael Johnson, and will be submitted to the *Astrophysical Journal* as Joseph R. Farah *et al.* (2026).

5.2 Introduction

Supernova (SN) 1993J was discovered in March 1993 in the spiral galaxy M81 [243]. As the prototypical Type IIb supernova [244], it was intensely studied across a variety of wavelengths, substantially facilitated by its proximity [3.6 Mpc; 245] and peak brightness [246]. In particular, the radio-bright nature of the explosion led to a decade-long follow-up radio campaign using VLBI [see e.g., 247, for a review], which resolved an expanding asymmetric quasi-spherical shell [248–250]. The direct observation of the radio shell expansion provides a unique opportunity to probe high-energy supernova physics at all initial phases of the explosion.

Previous works have characterized the behavior of the expanding shell using a variety of approaches. Both modeling and imaging approaches have constrained the apparent size and width of the shell-like structure, and observed a decelerating power-law expansion [248; 251; 252] with a fractional shell width of $\approx 30\%$ [253; 251; 254; 247]. In particular, the shell width at early epochs ($\lesssim 700$ days) has not been well-characterized. Images at these early epochs are beam-convolved and cannot be used to measure a shell width, while width-enabled model-fits either fixed the width prior to day 900 [252] or did not fit width and size independently [247]. More recent analyses have attempted to address these issues via careful treatment of calibration-related uncertainties as well as fitting all parameters of a more advanced spherical shell model (namely, including a radial gradient) simultaneously [255]. The reported shell width is consistent with previous simulations and theoretical predictions [see e.g., 256; 257, etc.].

Multiple imaging investigations have attempted to characterize the degree and evolution of asymmetry present in the azimuthal brightness along the ridge of the shell, which report a spherically symmetric object with an azimuthal brightness modulation

[248–250; 258]. In particular, [258] found a horseshoe-like brightness pattern in the images with the maximum rotating from east to south south-west over the first 1000 d before dissolving into a more complex structure. Concerns have been raised that the reported asymmetry and subsequent evolution may be artifacts caused by uneven and evolving coverage [259]. Existing models [e.g., a spherical shell of uniform emissivity; 251] have been symmetric; a major limitation of such models are that they cannot characterize any asymmetry in intensity or shape present in the data. Recently, the m -ring model of [260] has been developed which varies the size, width, and brightness asymmetry of a ring in the context of black hole VLBI observations. The m -ring model (an infinitesimally thin ring blurred with a Gaussian kernel) looks phenomenologically similar to the SN 1993J images, but is not as physically motivated as the spherical shell model of supernovae ejecta. However, it has the benefit of incorporating azimuthal brightness variations of arbitrary complexity while retaining an analytic Fourier transform. Such functionality allows for a more extensive investigation of the impact of arbitrary azimuthal brightness modulations on estimates of size and width, as well as novel estimates (with uncertainties) of the ejecta brightness asymmetry independent of imaging.

A major challenge in the study of SN 1993J has been assessing the impact of calibration choices and other systematics on imaging and model inference. Previous works have investigated the impacts of model assumptions [252] and data processing [247]. The effect on inferred parameters and reconstructed images is significant, resulting in conflicting measurements from different analyses. Previous analyses have attempted to address this issue, reanalyzing the dataset from day 687 onwards and attempting more careful self-calibration [255]. Recently, [92] and others explored imaging with so-called “closure” quantities, derived quantities which by construction are invariant to phase errors, complex gains, and several other systematic artifacts. Directly modeling

the SN 1993J evolution using these quantities may sidestep particular calibration and systematic error concerns entirely, although the amount of independent information would decrease and the total flux density information would be lost.

In this paper, we re-analyze selected data sets of the first four years of the evolution of the SN 1993J radio shell, using a new geometric model capable of measuring asymmetry and basing our inference solely on calibration-insensitive closure quantities. In [section 5.3](#), we summarize the observation and subsequent reduction of the data taken on the SN 1993J explosion. In [section 5.4](#), we present our modeling scheme and fitting approach, and detail how we assess fit performance and quality. In [section 5.5](#), we apply our modeling framework to the radial evolution of SN 1993J, and present the first-ever quantitative measurements of asymmetry in the azimuthal brightness variation along the ridge of the ring, as well as the first width measurements $\lesssim 900$ days post-explosion. Finally in [section 5.6](#) and [section 5.7](#), we discuss our results and methods, and summarize our conclusions.

5.3 Overview of SN 1993J data products

5.3.1 Observations and data reduction

SN 1993J was observed 22 times at 8.4 GHz over the range of interest for our analysis, between explosion in 1993 and 1997. The observations were conducted at varying times of year, multiple times each year. The earliest observation is estimated to correspond to 50 days post-explosion. The latest observation is estimated to correspond to ~ 1700 days post-explosion. For a full breakdown of the array and contributing interferometers and stations, see [\[250; 252\]](#). We list stations we reference in this paper and their associated acronyms in [Table 5.1](#).

Each observation was conducted at ≈ 8.4 GHz, corresponding to a wavelength of ≈ 3.6 cm. Between 100 and 500 scans were performed over each observation. The number of stations available to observe ranged from 12 to 18 stations, which produced $n(n-1)/2$ baselines ranging from 66 to 153 baselines, $\approx \binom{n}{3}$ closure triangles ranging from 220 to 816 triangles, and $\approx \binom{n}{4}$ closure quadrangles ranging from 495 to 3060 quadrangles.

The observations were phase-referenced using the galactic core of M81 as a calibrator, alternating between SN 1993J and M81 every few minutes. The proximity of SN 1993J to the M81 core makes M81 an excellent calibrator, enabling the production of high-fidelity images at both early epochs (when the small size of the source presents a challenge) and late epochs (when the low flux density of the source presents a challenge).

Table 5.1: A list of VLBI network sites referenced in this paper and their corresponding two-letter acronyms.

Acronym	Station Name
OV	Owens Valley Radio Observatory, California
NT	Noto Radio Observatory, Italy
RB	Robledo de Chavela, Spain
SC	Saint Croix, U.S. Virgin Islands
HN	Hancock, New Hampshire
Y	Very Large Array (VLA), New Mexico
LA	Los Alamos, New Mexico
NL	North Liberty, Iowa
PT	Pie Town, New Mexico
BR	Brewster, Washington
FD	Fort Davis, Texas
MK	Mauna Kea, Hawaii
KP	Kitt Peak, Arizona

5.3.2 Construction of closure quantities

Here, we review the construction of various VLBI data products. We primarily cite [261] for general conventions and descriptions, and [92] and [262] for expressions and descriptions related to the construction of closure quantities.

Interferometers measure a quantity known as a visibility V_{ij} , which is probed between two stations i and j , forming the baseline b_{ij} . This visibility measures the complex Fourier component of the source image by the van Cittert-Zernike theorem

$$V_{ij} = \mathcal{F}_{uv}I(x, y) = \int dx dy I(x, y) e^{2\pi i(ux+vy)}. \quad (5.1)$$

Here $I(x, y)$ is the intensity distribution of the source image and $\mathcal{F}_{uv}$ is the Fourier transform which takes spatial components (x, y) to frequency space (u, v) . The coordinates (u, v) represent the components of b_{ij} projected along the line-of-sight from the array to the source. By transforming models from image space into Fourier space, we can compare predicted (model) visibilities to observed (data) visibilities and identify models which best reproduce the observations.

The observed visibility is corrupted from Equation 5.1 by several sources of error:

- *Thermal noise*: thermal noise ϵ_{ij} is added to the true visibility from a combination of Earth's atmosphere, non-zero telescope temperature, and an astrophysical background. The thermal noise is Gaussian, and varies both with time and the specific baseline being considered.
- *Phase error*: the argument of the complex visibility is modified due to corrupting effects from the atmosphere and instrumentation/timing errors. Site-specific phase error ϕ_i is incorporated as a phase shift in the true visibility.

- *Complex gains*: the visibility is further modified by a complex gain G_i which is incorporated multiplicatively. The complex gain is time-dependent and varies on a per-site basis.

Following [92], we ignore more complex corrupting effects, such as per-station polarization mixing. Incorporating these effects into the visibility gives,

$$V'_{ij} = G_i G_j e^{i(\phi_i - \phi_j)} (V_{ij} + \epsilon_{ij}). \quad (5.2)$$

These errors on the visibility complicate model inference, as they corrupt the true visibility. Previous analyses of SN 1993J and other VLBI targets have primarily focused on self-calibration, which uses redundancy in the data to iteratively correct phase and amplitude errors. Recently, [92] demonstrated that a set of derived quantities called “closure quantities” are not affected by many of the most significant sources of error and can be employed to create robust and gain-insensitive images. In this work, we seek to exploit two closure quantities (the closure phase and the log closure amplitude) to model the SN 1993J ejecta, independent of complex gain magnitude or calibration choices.

Bispectrum and closure phases

The closure phase quantity is constructed by adding the phases of 3 baselines (forming a triangle) in closure. The closure phase is the argument of the complex bispectrum $V_B = V_{12}V_{23}V_{31}$,

$$\phi_{123} = \arg V_B = \arg V_{12} + \arg V_{23} - \arg V_{13}, \quad (5.3)$$

which contains more information than the closure phase alone but it is also more sensitive to complex gains and calibration issues. This construction has the benefit of only being sensitive to thermal noise and is robust to complex gains and phase errors. As a result, the closure phases computed for an observation will be identical to the closure phase computed for the true source image (modulo contributions from thermal noise). Since the closure phases are sensitive only to thermal noise, we use the method of [263] to estimate the present thermal noise from closure phases and appropriately rescale the measured uncertainties. We note that this procedure will underestimate the noise present in the data for several reasons: (i) the expressions in [263] are valid at high S/N ; however, much of the SN 1993J data (particularly at later days) is at low S/N ; (ii) there may be additional noise sources besides thermal noise present; and (iii) we assume Gaussian uncertainties when at least the low S/N data likely follow a Ricean distribution. Indeed, in practice we find that the uncertainties estimated from the closure phase method do not produce model fits with $\chi^2 \sim 1$ for many epochs, so we additionally rescale the uncertainties so that the model $\chi^2 \sim 1$ for all epochs. All fits are performed using these rescaled uncertainties.

Log closure amplitudes

The closure amplitude is formed by taking the ratio of the product of visibilities on one pair of baselines to the product on a second pair, and then taking the absolute value to remove phase dependence:

$$|V_{\text{LCA}}| = \left| \frac{V_{12}V_{34}}{V_{13}V_{24}} \right|. \quad (5.4)$$

All possible cyclic permutations then form the set of closure amplitudes for a single quadrangle of four antennas. The effect of this construction is to cancel out the complex

time-dependent gain terms G_i , resulting in a robust data product. At low signal-to-noise (S/N), the closure amplitude distribution becomes substantially non-Gaussian due to the $1/|V_{13}V_{24}|$ term, complicating estimates of the likelihood. The log closure amplitude (constructed by taking the natural log of the closure amplitude) addresses the non-Gaussian tail of the reciprocal visibility amplitude and has the added benefit of avoiding the preferential weighting of the denominator baselines. [92] and [262] found that the log closure amplitude was a more robust observable for imaging and compatible with a Gaussian likelihood distribution, even at lower S/N.

Closure quantity special considerations

Use of closure quantities in imaging and modeling has several nuances, to which we describe our approach here:

1. *Closure quantities formed using zero baselines:* in some interferometers, stations may be so close together geographically that the baseline between them does not meaningfully probe source image structure on scales of interest (a “zero baseline”). Triangles formed with a zero baseline have zero closure phase, which do not add new information but can be used to improve measurements via averaging [264]. Geographically co-located sites also result in trivial closure amplitudes, but baselines to co-located sites can add new information [265]. To maximize available information and robustness, we follow [92] and include closure quantities constructed using zero baselines.
2. *Diagonalized closure quantities:* recent work [262] has presented diagonalized closure phases and log closure amplitudes, which transform closure quantities into an orthogonal basis. This transformation has the effect of negating possible correlations between measurements. All fits performed in this study use diagonalized

closure quantities.

3. *Debiasing of log closure amplitudes:* in the low S/N regime, the presence of noise on positive definite amplitude measurements (from which the closure amplitude is constructed) results in a Ricean bias which biases the amplitudes to larger values. Amplitudes can be debiased using the first order correction presented in [261]:

$$|V_{ij}|_{\text{debiased}} = \left[|V_{ij}|_{\text{measured}}^2 - \sigma_{ij}^2 \right]^{1/2}, \quad (5.5)$$

where σ_{ij} is the standard deviation of the thermal noise on b_{ij} . Following [92], we debias all closure amplitudes prior to fitting. However, even with debiasing, if the data have a high fraction of data points with $S/N < 1$, model fits will still be Ricean biased. A side effect of debiasing is that points with $S/N < 1$ will be flagged. The uncertainty rescaling described above worsens this effect; we report the percentage of points with $S/N < 1$ per epoch in Table 5.3. We note, however, that a large fraction of these flagged points are points probing the suppressed flux on long baselines; these are largely non-constraining anyway. Far more impactful are the points probing the first null and the width-constraining peak as described in section 5.4; in general we find that these regions report acceptable flagged percentages on most days (also shown in Table 5.3, in parentheses).

4. *Total intensity constraint:* it is understood that the closure phase and closure amplitude cannot recover information about the total intensity of a source [92]. Therefore, we perform our model fits in this paper with a fixed total intensity. The total intensity is chosen to be the average of the intensity on the shortest baselines ($\rho \equiv \sqrt{u^2 + v^2} < 0.02 \text{ G}\lambda$).

5.4 m -ring modeling and fitting framework

5.4.1 Construction of the m -ring model

Here, we motivate and construct a model which we will use to fit the SN 1993J data quantities. Previous imaging-based investigations [see e.g., 247; 254] of the SN 1993J ejecta have identified the presence of a spherical shell expanding with time. The spherical shell morphology, when projected into the two-dimensional observer plane, can be well-approximated by a thick ring model with uniform brightness in the center. We propose a generic, single-component model based on the m -ring model presented in [260]—specifically equations (18) and (19), which we reproduce here. The m -ring considers a ring of angular diameter d , azimuthally-varying brightness, and infinitesimal width. The brightness distribution around the ring is formulated as a sum of polar Fourier modes, giving the following polar intensity function,

$$I(\rho, \varphi_\rho) = \frac{1}{\pi d} \delta\left(\rho - \frac{d}{2}\right) \sum_{m=-\infty}^{\infty} \beta_m e^{im\varphi_\rho}. \quad (5.6)$$

Here, β_m is the coefficient corresponding to the m -th Fourier mode, with $\beta_0 > 0$ representing the total flux density of the ring. The resulting visibility function is

$$V(u, \varphi_u) = \sum_{m=-\infty}^{\infty} \beta_m J_m(\pi u d) e^{im(\varphi_u - \pi/2)}. \quad (5.7)$$

Here, the function J_m refers to the m -th Bessel function of the first kind, which can be approximated by

$$J_m(\pi u d) \approx \frac{1}{\pi} \sqrt{\frac{2}{u d}} \cos\left[\pi\left(u d - \frac{2m+1}{4}\right)\right]. \quad (5.8)$$

Opacity effects (causing the brightness in the center of the shell projection to be non-zero) are reproduced in the m -ring model by the addition of a “filling floor”, a disk of constant brightness and total flux density F_d with the same radius as the m -ring. The brightness of the disk is controlled in the model input by the “fractional filling floor” parameter f , which represents the fraction of the total flux density $F_d/(F_d + \beta_0)$ the filling floor contributes. In general, the spherical shell model is well-approximated by $f \in [0.3, 0.45]$. We note that the log closure amplitude and closure phase do not constrain the total intensity of the source, and so we do not fit this parameter and instead (for visualization or calculation purposes) fix it to the average amplitude of the shortest baselines ($\rho < 0.02 \text{ G}\lambda$). The model is then convolved with a Gaussian kernel of full-width half max (FWHM) α to produce a thickness. Finally, since closure quantities are insensitive to offsets of the source image from the origin, we do not incorporate any translation parameters into our model. Examples of the model parameters are shown in [Table 5.2](#).

Param.	Explanation	Example values
I_0	Total flux of the model, split between the ring and the central floor (not constrained by closure quantities)	1 Jy, 0.1 Jy
d	Diameter of both the m -ring and the disk	1 mas, 500 μas
α	Blurring kernel that produces a ring width of α	500 μas , 0.1 mas
f	Fraction of the total flux from the central disk	0.1, 0
β_m	Complex value describing azimuthal flux distribution: $ \beta_1 $ sets asymmetry strength; $\arg \beta_1$ sets direction	0, $0.2e^{i\pi/3}$, $0.2 + 0.17i$

Table 5.2: A list of m -ring model parameters, their definitions, and sample values.

This model is convenient for several reasons. First, the visibility function is analytic, which is important for improving performance in rigorous but slow fitting algorithms such as Markov-Chain Monte Carlo. Additionally, it is a single-component model

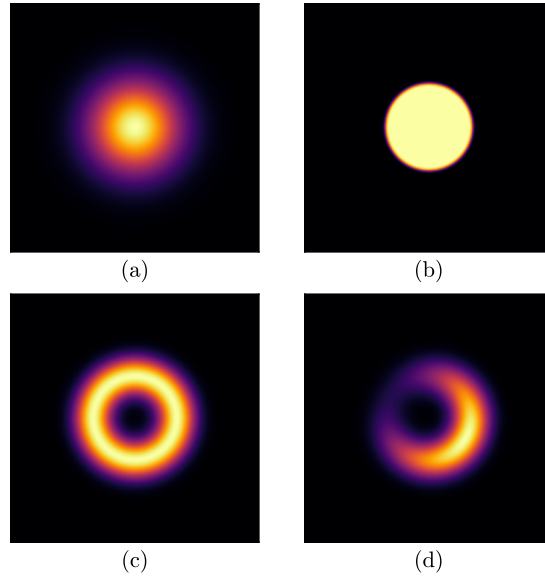


Figure 5.1: A demonstration of the different morphologies within the m -ring model. (a) A Gaussian morphology, produced by an m -ring model with a small diameter parameter d and large blurring kernel parameter $\alpha \gg d$. (b) A disc morphology, produced by an m -ring model with a small blurring kernel parameter $\alpha \ll d$ and a 100% fractional floor. (c) A ring morphology, produced by a 0% fractional floor and a blurring kernel parameter $\alpha \sim d$ comparable to the diameter. (d) A crescent morphology, produced by giving a non-zero significance to the cosine-expansion azimuthal ring distribution. Note that all these models are circularly symmetric, in that they are constructed by varying a brightness distribution on a circle; thus, even in the case of the crescent (d), though the brightness varies around the ring, the width of the Gaussian radial profile does not.

which can transition seamlessly from Gaussians to disks to rings to crescents, which covers a large range of circularly-symmetric morphologies we expect to be represented in the early SN 1993J data. Examples of the different morphologies reproducible in this model are shown in [Figure 5.1](#).

5.4.2 Variation in fit quality per epoch

There are several nuances in both the data and the model construction that result in anticipated epoch-dependent difficulties during the fitting process. First, while the

$m = 1$ m -ring model can reproduce a variety of model morphologies (namely, discs, rings, Gaussians, and crescents), it may perform poorly if the ejecta asymmetry is more complex than a simple one-mode angular brightness profile. Imaging [254; 247] indicates that for $t \lesssim 1000$ d, the ejecta are well-reproduced by the one-mode m -ring model, as demonstrated in Figure 5.1. However, for $t \gtrsim 1000$ d, the angular brightness profile becomes substantially more complex as the ejecta breaks up under the ram pressure of the surrounding gas. Thus it is important to emphasize that particularly at the later days, the m -ring β parameter should be interpreted as reporting an average azimuthal brightness asymmetry angle versus information about the location of any particular hotspot or structure. Moreover, the $m = 1$ mode provides a practical advantage beyond its simplicity: by consistently probing the same Fourier component of the azimuthal brightness at all epochs, it preserves a uniform effective angular resolution along the ring over time, allowing the azimuthal brightness pattern to be monitored consistently even as the ejecta becomes more complex. Transitioning to $m = 2$ or $m = 3$ models at later epochs may recover additional structural detail but would sacrifice this inter-epoch continuity, complicating comparisons across the dataset. Indeed, we note that the linear drift in the $\arg \beta_1$ parameter discussed in section 5.5 persists into the latest epochs examined ($t \sim 2500$ d), suggesting the $m = 1$ mode continues to track a meaningful global asymmetry even when the local ejecta structure is no longer well-described by a single cosine mode. In contrast, when the ejecta are too small to be unresolved, the β parameters are clearly unconstrained, and so we perform the fits without those parameters on those days (namely, day 50, 91, and 128).

Second, our model incorporates a width parameter α , and fits both α and d simultaneously. Previous analyses indicated difficulty fitting width and diameter simultaneously [247]; we do not find such difficulty in our fitting approach for the majority of epochs. In particular, no previous analysis has independently constrained the width

at $t \lesssim 700$ d, as the width is below the beam size of the interferometer at these times. However, it has been demonstrated in other analyses that modeling can constrain ring width well below the resolution of the interferometer, provided the diameter of the compact structure is sufficiently large [see e.g., 266]. Indeed, we find that we can constrain the width of the SN 1993J ejecta so long as the ejecta are large enough that both the first null and second peak (first peak at $\rho = 0$) are present in the visibility amplitudes, as shown in Figure 5.2. This first occurs around $t \sim 300$ d. Though we do not fit the visibility amplitudes (instead favoring the log closure amplitudes for the same information), the visibility amplitudes help visualize and confirm that at earlier times ($t \lesssim 264$ d), the ejecta structure is effectively degenerate with a Gaussian or filled disk and the width may be difficult to constrain without further assumptions. Such a degeneracy may also be present on days where the noise is sufficiently large. For days where this may be the case (indicated by the fit, particularly the width parameter, failing to converge or attempting to violate the prior boundaries) we drop the α parameter and fit the remaining parameters assuming $\alpha = 0.2d$.

5.4.3 Fitting procedure and parameter estimation

For a given dataset and model prescription, we identify a “best-fit” model by maximizing a standard Gaussian likelihood function, which [92] established as sufficient for closure phases and log closure amplitudes. The maximization and subsequent posterior exploration is performed via a Markov-Chain Monte Carlo (MCMC) procedure. We utilize the python package `emcee` in order to perform the MCMC fitting procedure [122]. `emcee` is a Bayesian parameter estimator that implements the affine invariant MCMC ensemble sampler of [267]. We implemented `emcee` using uniform priors for all parameters. For parameters which are naturally bounded (e.g., $\arg \beta \in [0, 2\pi)$,

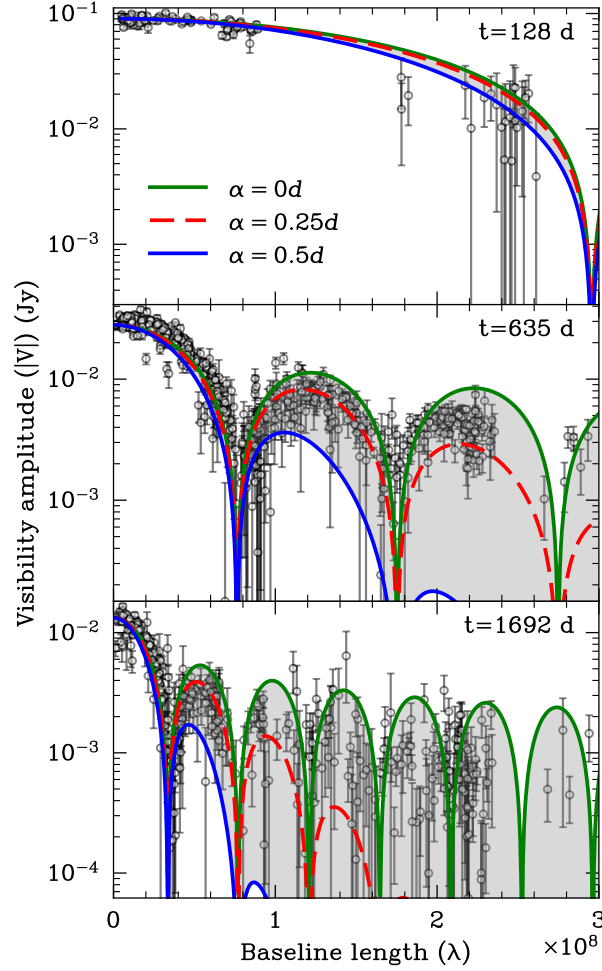


Figure 5.2: Visibility amplitudes for several days in the SN 1993J dataset, with the corresponding m -ring Fourier transform. The m -ring width is varied between infinitesimally thin (green solid line) to unphysically thick (blue solid line), with an intermediate value of $\alpha = 0.25d$ shown for demonstration purposes (red dashed line). We can only meaningfully constrain the width of the ejecta structure as long as the second peak (first peak is at $\rho = 0$) is clearly visible; earlier than this, the ejecta are effectively degenerate with a bivariate Gaussian intensity distribution. As a result, we are able to constrain the width $\gtrsim 500$ days earlier than previous works.

$f \in [0, 1]$), the prior was bounded by the full range of the parameter. However, for parameters which are not naturally bounded, conservative (i.e., range-maximizing) parameter ranges were chosen. The diameter of the model d and the width α are allowed a motivated lower bound of zero and an upper bound several times larger than the largest ejecta size reported in the literature. The fractional filling floor parameter f was difficult to constrain due to degeneracies between f , d , and α ; however, we found $f \sim 0.3$ was preferred on average throughout the observation. To simplify the posterior exploration, we fixed $f = 0.3$ for all fits.

Beyond debiasing (see [subsection 5.3.2](#)), no processing was done to the data prior to fitting. Additionally, a “bootstrapping” procedure was implemented prior to each science fit, wherein rapid fits are performed on simpler models to provide a meaningful starting point for the full complex model fit. For the m -ring models, a fit to a simple symmetric ring model was performed first, and the best-fit parameters of this fit were used to initialize the full m -ring fit. In testing, we found that the final fit solution was not sensitive to the starting point, but a meaningful initialization improved convergence times and provided more symmetric uncertainties. The bootstrapping fits were performed via a gradient-descent optimization routine. The final fits were performed via an MCMC procedure as described above, which used 10^3 walkers and was allowed to run as long as required to achieve convergence. Convergence was assessed based on the improved Gelman-Rubin statistic [121] and a check that the chain length for each parameter was at least 50x longer than the corresponding autocorrelation timescale [122]. This strategy resulted in unimodal Gaussian posterior distributions for all parameters. Due to the simplicity of the posterior distributions, we computed confidence intervals around the maximum a posteriori parameter values using the highest posterior density interval [125].

An example fit (for day 520) is shown in [Figure 5.3](#), alongside model-data compar-

isons on sample closure triangles and quadrangles. The fit selected here is performed varying f , to demonstrate the preference for $f \sim 0.3$. The fit demonstrates good convergence, with all parameters exhibiting symmetric Gaussian marginal posterior distributions. There are mild correlations between several parameters and the fractional floor. The modeled closure quantities show significant agreement with the observed closure quantities, regardless of choice of quadrangle or triangle. The model is also able to clearly reproduce trivial triangles and quadrangles, an important sanity check [92]. We emphasize that this (the fit represented in Figure 5.3) represents a meaningful width constraint ≈ 250 days earlier than any previous work; the earliest width constraints in our sample are $\gtrsim 500$ days earlier than any previous analysis. Additionally, the angle of brightness asymmetry in the shell is constrained to better than $\lesssim 40^\circ$, representing a novel statistically quantified constraint on the angular brightness distribution of the ejecta. We emphasize that interpretation of the recovered brightness asymmetry angle is model-dependent as the β_1 parameter only provides information on the location of the weighted mean of the azimuthal modulations, and not on the location of any particular hotspot or structure.

5.5 Application to SN 1993J

5.5.1 Radial evolution of the ejecta

Transformation between spherical shell and m -ring model

Due to differences in model construction, it is generally not possible to compare model parameter outputs to other models or images directly. For example, a spherical shell model was previously used in the literature to estimate the width of the SN 1993J radio emission [252; 247]. The width in this model corresponds to the true width of the

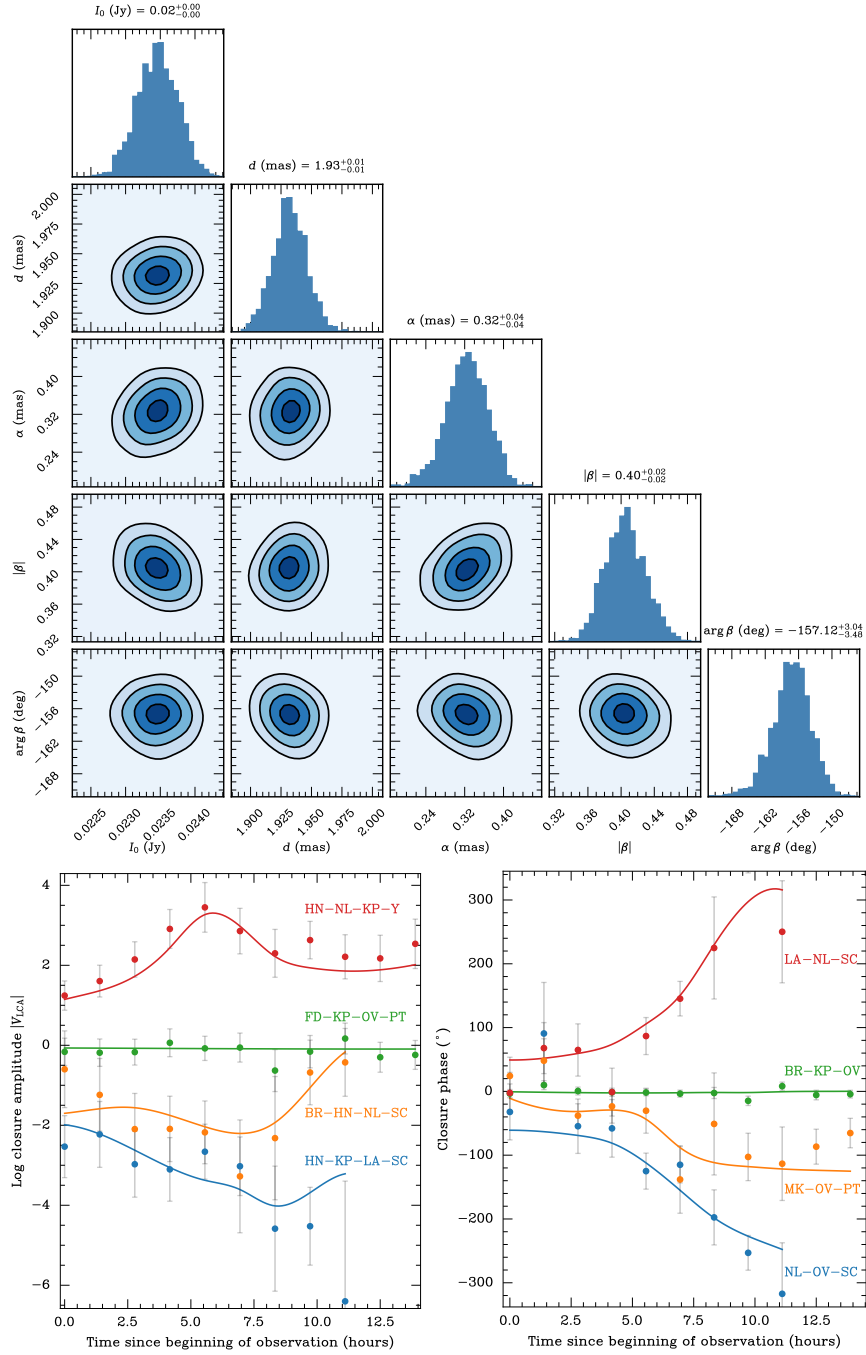


Figure 5.3: Here, we present a representative fit of an $m = 1$ ring to the data from day 520. (*top*) The corner plot resulting from the fit. All parameters have symmetric Gaussian marginal distributions, with mild correlations between the width, diameter, and fractional floor. (*bottom left*) The maximum likelihood solution identified by the fit (solid lines) compared to the log closure amplitudes of the data (points) on four example quadrangles (HN-NL-KP-Y, FD-KP-OV-PT, BR-HN-NL-SC, HN-KP-LA-SC). The fit is a convincing representation of the data. The data have been averaged in time for clarity, but fits were performed on non-averaged data. (*bottom right*) The maximum likelihood solution identified by the fit (solid lines) compared to the closure phases of the data (points) on four example triangles (LA-NL-SC, BR-KP-OV, MK-OV-PT, NL-OV-SC). The fit is a convincing representation of the data. The data have been averaged in time for clarity, but fits were performed on non-averaged data.

emission region, assuming the ejecta is indeed a spherical shell of uniform emissivity. However, this width cannot be naively compared to our model, which measures the projected width of the spherical shell in the two-dimensional observer plane.

In order to more meaningfully compare our width results with those in the literature, we transform our m -ring width α in terms of the spherical shell width θ_w by finding a best-fitting spherical shell radial profile for any given m -ring. We demonstrate this procedure in [Figure 5.4](#). The m -ring and spherical shell models have similar radial profiles, despite the m -ring not being physically motivated. Under convolution with the nominal beam (particularly at early epochs when the ejecta are small), this similarity makes them almost indistinguishable. We determine R_{out} and $R_{\text{out}}/R_{\text{in}}$ corresponding to the m -ring values of d and α as follows. For each epoch, the MCMC fit generates posterior samples, each including values of d and α . The values and uncertainties for d , α , $\arg \beta_1$ and $|\beta_1|$ given in [Table 5.3](#) are the means and standard deviations over the posterior samples. From these posterior samples for each epoch, we select 20 random instances. For each of these instances, we calculate the brightness vs. radius profile implied by the instance values of d and α . We then fit a projected shell brightness profile to this m -ring profile by least squares, thus yielding the values

of R_{out} and $R_{\text{out}}/R_{\text{in}}$ for that instance. We give the mean and standard deviation over the 20 instances as the values and uncertainties of R_{out} and $R_{\text{out}}/R_{\text{in}}$ in [Table 5.3](#).

This transformation yields values for R_{out} that are similar to best-fit values from the literature (see [Figure 5.4](#)), validating the transformation procedure. While this procedure serves as an effective method of transformation, uncertainties on R_{out} estimated using the transformation are underestimated in comparison to direct fitting of the spherical shell model. The radius and width of the best-fitting spherical shell are used to construct a transformed fractional width, which we compare to literature values in the following sections.

Evolution of the ejecta size

We report monotonic radial expansion in the ejecta. The modeled ejecta diameter for each epoch, along with the associated uncertainties, are shown in [Figure 5.5](#). The values in this table represent the maximum likelihood solution obtained during the MCMC fitting performed as described in [section 5.4](#). The supernova age reported in the table is computed relative to the explosion date of March 28, 1993 (MJD 49074).

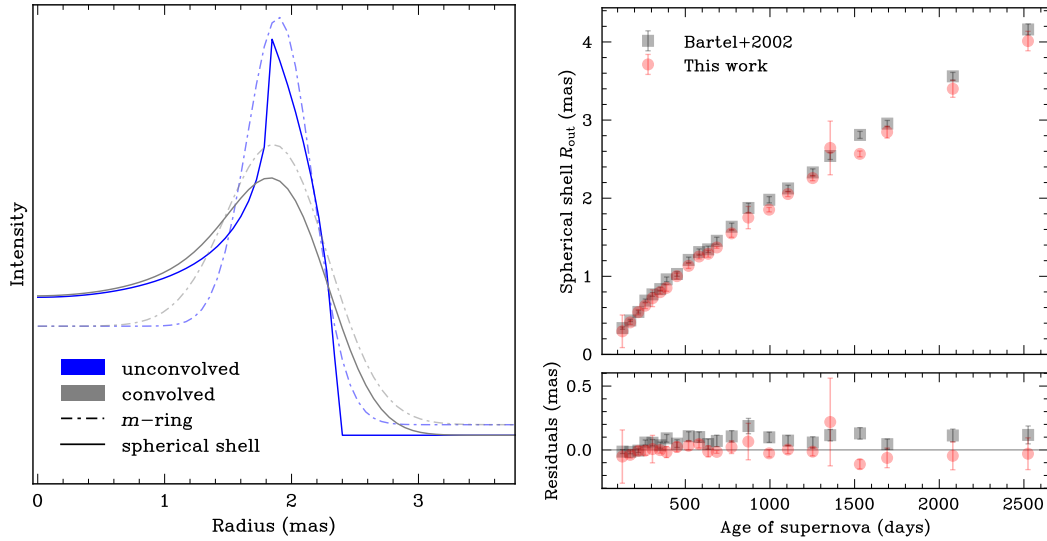


Figure 5.4: (left) Comparison of the radial profile of the m -ring model (dot-dashed line, $f = 0.3$, $d = 3.9$ mas, $\alpha = 0.6$ mas) to the best-fitting spherical shell model (solid line, $R_{\text{out}} = 2.3$ mas, $R_{\text{out}}/R_{\text{in}} = 1.25$) used in e.g., [252; 247]. The models feature different parameterizations but largely describe similar profiles. Even without convolution (blue), the m -ring model is an acceptable approximation to the spherical shell. Under convolution with an ≈ 1 mas beam, the models become nearly indistinguishable. (right) Comparison of the [252] best-fit spherical shell outer radius to the m -ring size parameter transformed according to the procedure in Figure 5.5.1, with uncertainties re-scaled to produce a reduced χ^2 of unity for a best-fit power-law. The consistency with the results from [252] (which fit a spherical shell model directly) is an important sanity check for our procedure. For most epochs, we report consistency up to 1σ with the results from [252], which validates the fitting and transformation procedure. We note a minor systematic underestimation of our diameters in comparison with the [252] results at all epochs.

Age (days)	d (mas)	α (mas)	$\arg \beta_1$ (°)	$ \beta_1 $	R_{out} (mas)	$R_{\text{out}}/R_{\text{in}}$	% SNR < 1
50	0.251 ± 0.014	—	—	—	0.149 ± 0.008	1.3041 ± 0.0005	0.0%
91	0.39 ± 0.05	—	—	—	0.23 ± 0.03	1.3039 ± 0.0014	57.8%
128	0.48 ± 0.13	—	—	—	0.3 ± 0.1	1.3038 ± 0.0015	45.0%
175	0.700 ± 0.015	0.12 ± 0.04	112 ± 27	0.30 ± 0.06	0.410 ± 0.012	1.26 ± 0.05	4.5% (4.5%)
223	0.917 ± 0.012	0.17 ± 0.04	115 ± 6	0.33 ± 0.04	0.540 ± 0.016	1.29 ± 0.05	23.0% (22.7%)
264	1.054 ± 0.028	0.18 ± 0.03	124 ± 12	0.35 ± 0.04	0.624 ± 0.017	1.277 ± 0.028	10.4% (9.9%)
306	1.22 ± 0.10	—	130 ± 23	0.35 ± 0.09	0.72 ± 0.05	1.304 ± 0.001	88.9% (88.2%)
352	1.334 ± 0.021	—	134 ± 6	0.466 ± 0.024	0.796 ± 0.011	1.3040 ± 0.0007	61.7% (54.7%)
390	1.433 ± 0.029	—	148 ± 8	0.39 ± 0.04	0.852 ± 0.020	1.3035 ± 0.0020	54.3% (42.8%)
451	1.713 ± 0.009	0.26 ± 0.06	193 ± 30	0.25 ± 0.05	1.002 ± 0.018	1.27 ± 0.04	60.1% (46.0%)
520	1.932 ± 0.014	0.32 ± 0.04	157 ± 4	0.405 ± 0.024	1.134 ± 0.015	1.274 ± 0.027	41.2% (32.9%)
582	2.121 ± 0.028	—	173 ± 6	0.22 ± 0.04	1.253 ± 0.014	1.300 ± 0.004	18.7% (13.6%)
635	2.233 ± 0.027	0.307 ± 0.030	173 ± 16	0.276 ± 0.029	1.288 ± 0.020	1.231 ± 0.011	90.6% (86.1%)
686	2.310 ± 0.008	0.429 ± 0.013	202 ± 9	0.23 ± 0.01	1.369 ± 0.005	1.296 ± 0.006	39.8% (21.4%)
774	2.595 ± 0.029	0.54 ± 0.06	167 ± 19	0.380 ± 0.029	1.549 ± 0.024	1.317 ± 0.022	16.9% (12.2%)
873	3.00 ± 0.14	—	293 ± 100	0.12 ± 0.11	1.75 ± 0.07	1.298 ± 0.003	61.7% (51.0%)
997	3.134 ± 0.021	0.57 ± 0.03	194 ± 11	0.178 ± 0.019	1.853 ± 0.013	1.292 ± 0.008	30.9% (18.0%)
1107	3.432 ± 0.023	0.691 ± 0.022	310 ± 20	0.139 ± 0.027	2.053 ± 0.015	1.311 ± 0.008	25.8% (11.9%)
1253	3.878 ± 0.028	0.59 ± 0.05	256 ± 6	0.196 ± 0.023	2.257 ± 0.015	1.251 ± 0.016	42.7% (19.3%)
1356	4.48 ± 0.27	—	300 ± 16	0.15 ± 0.06	2.64 ± 0.16	1.2981 ± 0.0020	89.2% (79.4%)
1532	4.370 ± 0.017	0.75 ± 0.06	547 ± 13	0.217 ± 0.028	2.567 ± 0.017	1.276 ± 0.014	35.2% (22.0%)
1693	4.78 ± 0.05	0.91 ± 0.12	457 ± 50	0.022 ± 0.017	2.85 ± 0.04	1.305 ± 0.023	40.2% (25.8%)
2080	5.63 ± 0.06	1.29 ± 0.15	472 ± 25	0.072 ± 0.027	3.40 ± 0.05	1.34 ± 0.03	47.2% (27.3%)
2524	6.69 ± 0.11	1.39 ± 0.22	564 ± 70	0.05 ± 0.04	4.01 ± 0.06	1.32 ± 0.04	52.3% (30.2%)

Table 5.3: Best-fit numerical values as a function of age in days for radial expansion and asymmetry. The values represent the maximum likelihood solution obtained during the MCMC fitting performed as described in [section 5.4](#). The age shown is relative to the explosion date of MJD 49074. The R_{out} and R_{in} correspond to the best-fit spherical shell model computed using the method described in [Figure 5.5.1](#). “% SNR < 1” is the percentage of data points with signal-to-noise ratio below unity after rescaling uncertainties so that the reduced χ^2 of the amplitudes relative to the best-fit model is 1; the value in parentheses is the same percentage restricted to points at distance smaller than the first peak (first maximum after the first null) in the model amplitudes. Parameters marked with — were not fit for that epoch.

The size measurement was largely constrained by data on baseline lengths comparable to the location of the first null in the amplitudes, which is the strongest proxy available for the overall size of the compact emitting region. The location of this null ρ_{null} increasingly tends towards shorter baselines as the angular size of the ejecta d increases, following $\rho_{\text{null}} \propto 1/d$. The magnitude of the uncertainty in the measurement in a noisy dataset is therefore affected by the presence of numerous and high- S/N baselines with lengths at $\approx \rho_{\text{null}}$. Since the S/N ratio does not vary monotonically with baseline length, the uncertainties on the diameter were similarly inconsistent.

The expansion as a function of supernova age is shown in [Figure 5.5](#). The expansion is consistent with a power-law of the form

$$d(t) = R_0 \left(\frac{t}{\tau_0} \right)^\omega. \quad (5.9)$$

We evaluate the deceleration parameter ω of this power-law as well as a broken power-law fit to compare to the literature. The data and power-laws are compared in [Figure 5.5](#). We find fitting [Equation 5.9](#) to the full dataset a best-fit $\omega = 0.81 \pm 0.01$, consistent with the results of [\[252\]](#), from which we find $\omega = 0.82 \pm 0.01$ after re-fitting the same days of their reported diameter measurements. We also attempt a broken power-law fit, motivated by the clear break at $t \approx 600$ d. This break time is clearly

visible in our dataset and that of [252] but is not immediately visible in the values reported by [247]. The presence and location of this break has consequences for physical models of the object, which may use the presence of a break to infer changes in the circumstellar material profile and therefore the mass loss history of the progenitor [e.g., 268; 269]. Prior to the break, our results ($\omega_1 = 0.89 \pm 0.01$) are consistent with [252] ($\omega_1 = 0.91 \pm 0.02$) and substantially higher than [247] ($\omega_1 = 0.83 \pm 0.03$). Post break, our results ($\omega_2 = 0.81 \pm 0.01$) are again consistent with [252] ($\omega_2 = 0.79 \pm 0.01$) and significantly more consistent with [247] ($\omega_2 = 0.83 \pm 0.01$). We also find a larger deceleration parameter at late times than [255] ($\omega_2 = 0.78 \pm 0.04$, though statistically consistent with our result). In general, [255] also reported larger R_o values than our analysis or [252], which may be due to the inclusion of a gradient in the radial profile of their model. We also note that while our reduced χ^2 for the pre-break evolution are less than unity, our goodness-of-fit post-break are reduced $\chi^2 \approx 2.8$, indicating significantly underestimated uncertainties. One possibility for the cause of this underestimation is that our Gaussian assumption for the uncertainties becomes less valid at the later days, when the source intensity is lower. As a result, we may be underestimating the true uncertainty present in the data, causing the diameter estimates to be overconfident. Additionally, at later times, the ejecta appear to be distinctly more complex than the *m*-ring model, introducing model uncertainty not adequately captured by our approach.

Evolution of the ejecta width

Our modeling approach recovers a largely monotonically increasing width, ranging between an angular size of ≈ 0.2 mas at the earliest epochs to ≈ 1.5 mas at the latest, corresponding to a $\approx 6\times$ increase over the first $\approx 2.5 \times 10^3$ days of the supernova radio emission expansion. We transform to a spherical shell width following the procedure

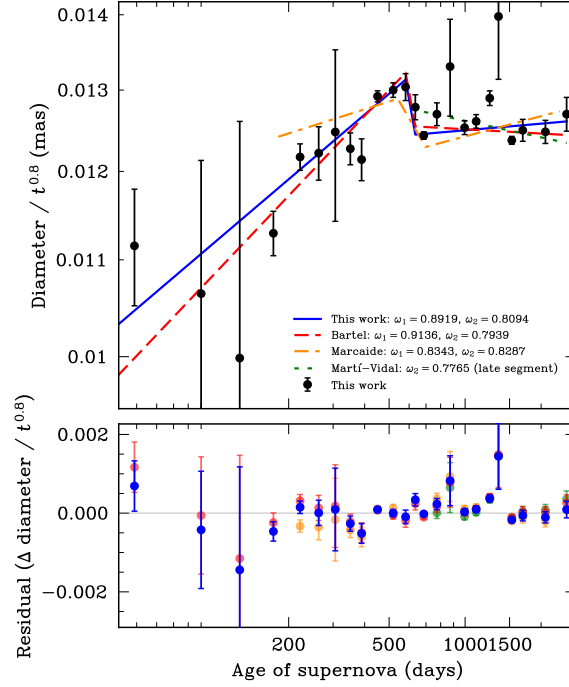


Figure 5.5: (*top*) Radial expansion recovered via our modeling approach, alongside the best-fitting broken-power law (blue line). We include a comparison with [255] (green line), [252] (red line) as well as [247] (orange line) to demonstrate that our radial expansion is in agreement with the recovered value in the literature. We report comparable agreement with [252] at all epochs. Radial expansion is shown divided by $t^{0.8}$ to more clearly highlight the changing power law behavior. (*bottom*) Residual comparison of deceleration reported by this work, [252], [247], and [255]. The magnitude of the deviation from both the power-law fit and the literature values is $\lesssim 0.2$ mas at all epochs.

laid out in subsection 5.5.1. The evolution of the width is shown in Figure 5.6. The width evolves in a manner that can be approximated by $\approx 20\%$ of the power-law radial expansion solution reported in subsection 5.5.1. Unlike the diameter, the parameter uncertainties appear to accurately represent the variability in the model fit estimates, as the reduced χ^2 of the single power-law fit is approximately unity ($\chi^2 \approx 0.99$).

For the comparison with [252], [255], and [247], we determined the mean and standard deviation of the widths given in those studies for epochs overlapping with ours (specifically, $t \gtrsim 900$ d for [252], $t \gtrsim 1500$ d for [247] and $t \gtrsim 670$ d for [255]).

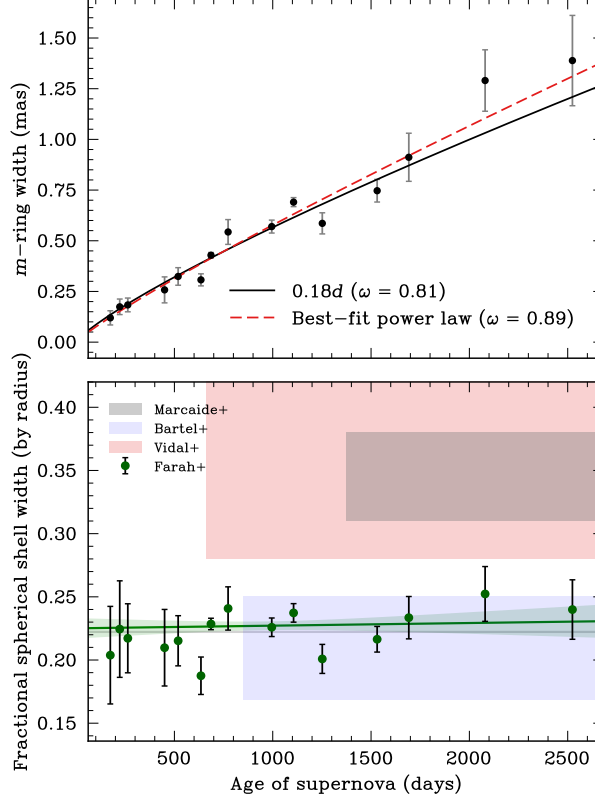


Figure 5.6: (*top*) Evolution of the ejecta width (α parameter in the m -ring) as recovered by our model. Displayed are the raw width values recovered by the model. The width evolution is well-approximated by 18% of the radial expansion ($\omega = 0.81$, shown by the solid black line) but is most consistent with a slower deceleration ($\omega = 0.89$, dashed red line). The modeled width evolution tracks the consensus diameter evolution closely. (*bottom*) Comparison of transformed (see subsection 5.5.1 and sFigure 5.5.1) widths between literature model fits (gray and blue shaded regions) and our analysis (green points). For the comparison to [252], we computed a mean and standard deviation of the widths reported in those analyses on epochs overlapping with our analysis (namely, $t \gtrsim 900$ d), visualized as the blue band. For the comparison to [247], we quote the spread of values they report, beginning around $t \sim 1500$ d (gray band). We globally recover widths consistent $\lesssim 1\sigma$ with the analysis of [252]. By contrast, our widths are universally inconsistent with the results of [247], being $\approx 30\%$ smaller on average for all epochs. More recently, [255] found a fractional width of $\approx 0.31 \pm 0.08$, which is slightly more consistent with our analysis. There does not appear to be statistically significant evidence of a drift in the fractional shell width; a linear fit finds a drift of $\sim 2 \pm 7 \times 10^{-6}$ per day, consistent with no drift in the dimensionless fraction width.

The results of our comparison are visualized in [Figure 5.6](#). For the later epochs, [\[247\]](#) reported a spherical shell model width ranging from $31 \pm 2\%$ to $37.8 \pm 1.3\%$. This range is consistent with the wider range of values reported in [\[255\]](#) but it is in substantial disagreement with both the early-time results reported by our analysis and [\[252\]](#). By contrast, we find agreement up to 1σ with the results of [\[252\]](#) for most days. The fractional shell width eventually increased after ~ 6 years [\[270\]](#), of which we potentially see an indication towards the end of the dataset examined by our analysis.

5.5.2 Evolution of the brightness asymmetry

We present a unique measurement of the asymmetry of the SN 1993J ejecta—to wit, the first measurements with statistically meaningful uncertainties. To assess the asymmetry of the ejecta, we fit the m -ring model with $m = 1$ modes to all epochs in our sample. We report an evolving asymmetry in the azimuthal structure of the ejecta with a southern bias.

The asymmetry in the m -ring model is generated by the β parameters. For each β contribution, the modulus $|\beta|$ sets the magnitude of the deviation from uniform (i.e., higher $|\beta|$ corresponds to a stronger cosine mode), while the argument $\arg \beta$ sets the direction. Under the construction of our model, $\arg \beta = 0^\circ$ corresponds to the north (up) of the map, and $\arg \beta = 90^\circ = \pi/2$ corresponds to the east (left). For simple ejecta structure with one minimum and one maximum and a largely symmetric azimuthal brightness profile, the $\arg \beta_1$ parameter approximates the location of the maximum. However, for more complex azimuthal brightness profiles with multiple maxima (e.g., for $t > 996$ d), the $m = 1$ proxies the location of the circular mean asymmetry and not necessarily the location of any particular maximum.

To assess the asymmetry, we draw samples of the complex β parameters from

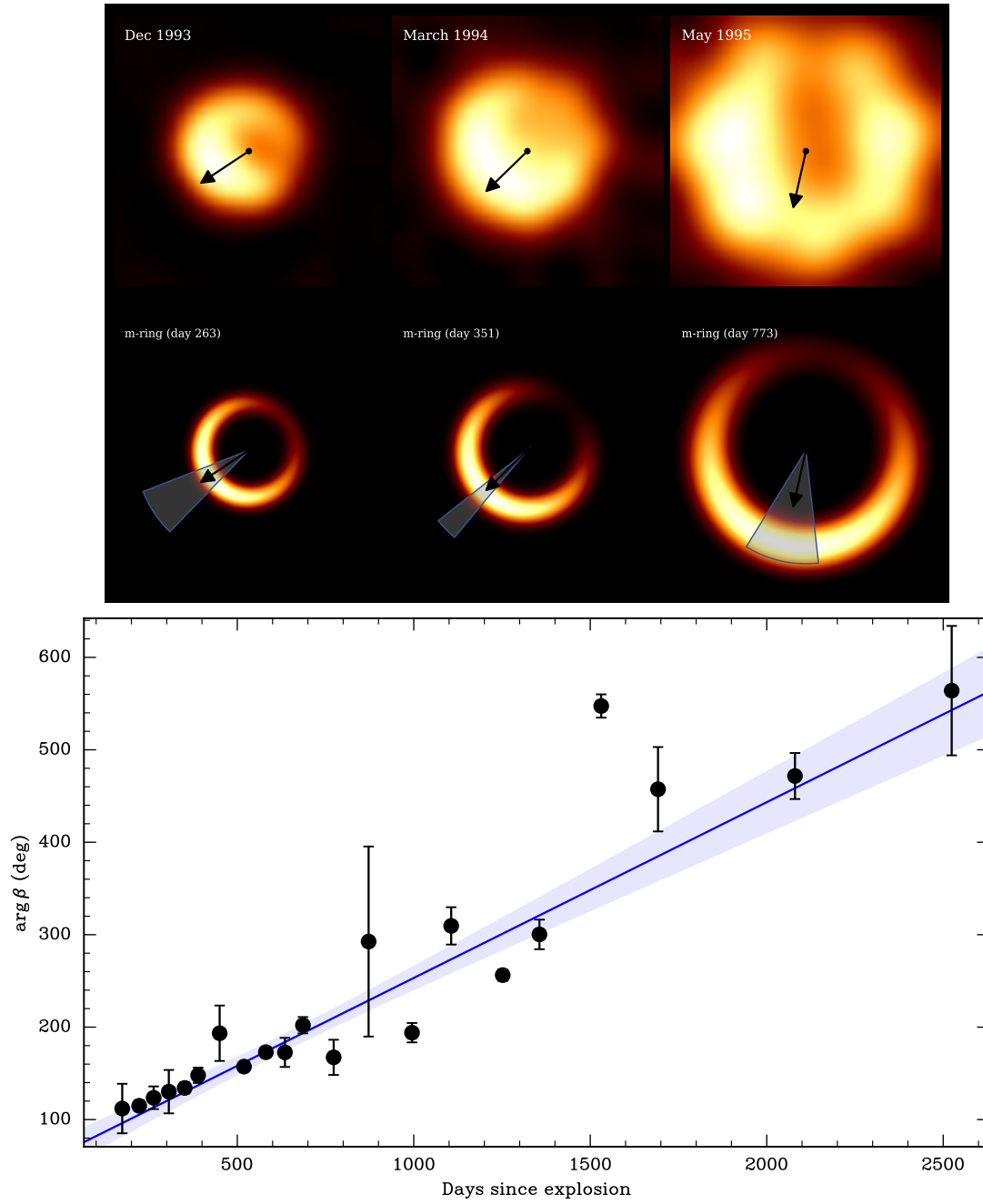


Figure 5.6: (*top*) Visual comparison of the best-fit m -ring models to images from [258]. Black arrows indicate the approximate mean flux asymmetry angle ($\arg \beta_1$ in the m -ring) recovered by our fitting approach for each epoch (measured values and uncertainties are reported in Table 5.3). The partially transparent fan shows the uncertainty in the angle recovered by the m -ring. For simpler azimuthal brightness profiles, this angle corresponds to the peak intensity angle, which is well-modeled by the m -ring. However, as the ejecta become more complex, the m -ring $\arg \beta_1$ parameter can no longer be interpreted as the location of a particular hotspot. (*bottom*) Evolution of the circular mean asymmetry as a function of time (black points). We find that on average the azimuthal asymmetry drifts approximately $0.19^\circ \pm 0.02^\circ$ per day counterclockwise (i.e., from East to West, blue dashed line), providing evidence of a drift in the ejecta brightness distribution. This linear drift persists even into the later epochs.

the posterior explored during the fit and measure an average asymmetry position from each using the method of circular mean [271]. The circular mean provides an estimate of asymmetry that is agnostic to choice of zero angle direction in the polar coordinate system. This distribution of asymmetry positions contributes to the mean (and standard deviation) which becomes our measurement (and model uncertainty) for each epoch. The evolution of the $\arg \beta_1$ parameter, processed using the above procedure, is shown in Figure 5.6. We find a bulk asymmetry toward the south centered around $\approx 240^\circ$, measured counterclockwise from north, averaged across all epochs. The data are relatively well-described by a linear drift model with $d \arg \beta_1 / dt = 0.19^\circ \pm 0.02^\circ$ per day counterclockwise (i.e., from East to West). The estimated linear drift is inconsistent with no drift to $\gg 5\sigma$, providing evidence of a drift in the ejecta brightness distribution.

We compare and contrast the imaging and modeling approaches to recovery of the azimuthal brightness modulations in the top panel of Figure 5.6. The images from [258] show a clearly evolving shell structure, resembling a horseshoe pattern rotating counterclockwise during the first several hundred days. At early times, the ejecta structure is fairly simple, with one peak brightness location approximately 180° opposite from the

least bright point along the ring. This simple modulation profile is well-approximated by the $m = 1$ m -ring, which produces a similar structure via the cosine modulation. Under these conditions, the $\arg \beta_1$ parameter of the m -ring model aligns with the peak brightness location in the images, which for simple structure is almost identical to the circular mean brightness asymmetry angle (the quantity that $\arg \beta_1$ actually probes). However, at later times, the azimuthal behavior of the ejecta becomes more complex, and as a result the m -ring is a poorer approximation. In the May 1995 example shown in [Figure 5.6](#), an additional hotspot has formed opposite the peak brightness location, moving the mean asymmetry angle partway between the two hotspots. The m -ring captures this movement of the mean asymmetry angle accurately but does not provide full information about the ejecta structure. The primary benefit of the m -ring approach, then, comes from the information afforded about the uncertainty in the best-fit mean asymmetry angle; i.e., the range of mean asymmetry angles plausibly supported by the data. While the imaging approaches can reproduce arbitrarily complex brightness profiles, there is no information available about the space of image structures that fit the data with similar success.

5.5.3 Uncertainties and systematic error

The reported uncertainties associated with quantities measured via our m -ring model fitting approach are typically inconsistent with those reported for similar quantities in the literature, likely due at least in part to procedural factors discussed in [section 5.3](#). By contrast, the uncertainties reported by our analysis on the azimuthal brightness variation are novel and have no established result available for comparison. Here, we examine possible sources of systematic error and evaluate the impact on our results.

Absorption in the center of the radio shell

[252] investigated the impact of the opacity of the radio shell (particularly at early times) on estimates of the outer radius. The spherical shell model used to perform measurements did not have an absorption component, leading to a bias of up to $\approx 10\%$ on the size at the earliest days. However, this bias was not considered an issue for several reasons. First, the calculation was performed assuming the worst-case scenario of complete absorption, which [258] found was not the case. Second, the bias decreased with resolution, so the impact was primarily on the earliest days. Third and finally, the bias was in the opposite direction of the bias created by the lack of angular variability in the chosen model, so the two systematic biases were presumed to cancel. We cannot rely on these motivations—our analysis focuses on early data (when the opacity effects may be significant) and as discussed in [subsection 5.5.3](#), the m -ring model is significantly less affected by systematic bias to azimuthal complexity. However, our analysis is indeed largely unaffected by this source of systematic error, as the fractional floor parameter f of the m -ring acts identically to an opacity parameter, and therefore we can account for varying absorption at both early and late epochs.

Size-width degeneracy

There exists a minor degeneracy between angular size and width, discussed by both [252] and [247]. [252] noted that at an assumed unresolved shell width of $\approx 25\%$ (corresponding to an m -ring $\alpha \approx 34\%$), the resulting bias in shell size at early epochs was $\approx 2\%$, and fixed the shell width at the earliest epochs ($t < \text{day } 900$). [247] reported that, due to this degeneracy, simultaneous fits of angular size and width were difficult to obtain without strong correlations and incorrect parameter recovery. To counter this, [247] fixed the model width, fit the angular size, then fixed the angular size and

re-fit the width. This approach does not constitute an independent measure of size or width, especially given the bias in width of up to 25% reported by [247] when the diameter is incorrectly estimated by just 5%.

Our analysis is largely unaffected by this source of systematic error, as we fit the size and width simultaneously for all epochs when the width is fit at all. As a result, we do not need to fix the width for these epochs as in [252] (which fixed the width between $t = 389$ and $t = 996$ days, but fit them simultaneously after) or fit width and size separately as in [247] (which fixed the width and fit the diameter, then fixed the diameter to the best-fit diameter and re-fit the width). Further, we do not reproduce the difficulties reported by [247], and recover clean, uncorrelated posteriors for simultaneous fits of width and size, as shown in Figure 5.3. Since we are fitting size and width simultaneously, any bias resulting from the size-width degeneracy will be incorporated into the uncertainty reported by the MCMC fitting algorithm. Our fits to days where the width is fixed may be affected by this 2% bias.

Complexity of the m -ring

[252] investigated the impact of azimuthal modulation along the model ridge as a potential source of systematic error in their size and width measurements. Though their images reported in [250; 258; 254] showed angular variation in the brightness profile of the compact object, the spherical shell model used to conduct measurements was circularly symmetric in projection and was unable to reproduce these features. They calculated a systematic error corresponding to a $\approx 5\%$ reduction in the size of the outer radius, assuming a fixed width, at the earliest epochs, where the effect was most pronounced. The effect diminished as the modulation increased in complexity with time.

A novel aspect of our analysis is the incorporation of azimuthal complexity via the introduction of m -ring modes with $m > 0$. As reported in [252], the properties of an asymmetric source can be misrepresented if fit to a model without asymmetry. We investigated the impact of our m -ring implementation on our size estimates by re-fitting the observations using a symmetric m -ring model ($m_{1,2,\dots} = 0$) as well as fitting a synthetic observation of an m -ring with $m = 3$ complexity. Across all epochs, we recover angular sizes for the $m = 0$ -only fits consistent with the $m = 1$ fits to $\lesssim 1\sigma$, with no evidence for a systematic offset or bias. However, we found that reducing the number of parameters improved the uncertainties. For example, on day 520, our full m -ring analysis reports a size uncertainty of $\approx \pm 1\%$. However, fitting with only I_0 , d , α , and f results in an uncertainty $\sim 2\times$ smaller, of about 0.5%. Increasing the number of m -ring modes from $m = 0$ to $m = 1$ results in an increased size uncertainty of $\approx 1\%$. Finally, we found that fitting a model with $m = 3$ complexity with the $m = 1$ model results in an approximately 2% reduction in the measured diameter (compared to the true diameter), which is consistent within the associated uncertainties. By contrast, there was $\lesssim 1\%$ bias in the width measurement, which in the m -ring model is not affected by the presence of asymmetry. We mitigate the azimuthal complexity bias by only fitting to days which have a significant $m = 1$ mode contribution (even if those with significant $m = 1$ mode contribution have more complex substructure) based on imaging [258; 247], as discussed in subsection 5.4.2.

5.6 Discussion

The results obtained using our modeling analysis yield a radial expansion that is largely consistent with the results produced by previous imaging analyses. As shown in Figure 5.5, the modeled radial expansion is consistent with results in the literature at

all epochs. We find statistically greater consistency with the width evolution modeled in [252] and [255] versus [247].

Our analysis fits size and width simultaneously [in contrast to some previous analyses, e.g. 247] and fits width at all epochs [in contrast to some previous analyses, e.g. 252], which provides greater confidence in the size and width measurements. Given that the size of the ejecta is the easiest quantity to recover in this type of analysis, our determination of a size and evolution consistent with previous analyses and simulation is a strong verification that our results are a proper description of the dataset.

As reported in subsection 5.5.2, the asymmetric maps obtained by our modeling framework are somewhat consistent with the image reconstructions attempted on this dataset previously. We find a bulk asymmetry to the south of the image across all epochs, which decreases in magnitude in the later epochs as the ejecta expands. Our approach, however, offers a unique estimate of the uncertainties on the bulk asymmetry. The sparsity problem of image reconstruction makes it challenging to assess which reconstructed features are believable, and to what degree. Forward-modeling-based imaging approaches (e.g., regularized maximum likelihood) can approximate statistical robustness by performing feature-extraction on a “top-set” of best-fitting images, somewhat akin to a posterior exploration. The uncertainties associated with our best-fit angular profiles properly characterize the space of possible azimuthal characteristics the data represent, to the extent our model is able to describe the data. As a result, while reconstructions performed in the literature seem to indicate an asymmetry in the ejecta towards the south, we can uniquely characterize the amount, and assess its statistical significance.

The ability to probe such azimuthal brightness variations directly from the data, and with well-characterized uncertainties, is critical for distinguishing real physical

evolution from artifacts. [259] argued that the apparent rotation in imaging studies [e.g., 258; 272] could arise from changing (u, v) -coverage; in contrast, our modeling incorporates coverage incompleteness into the posterior uncertainties, rendering the measurement insensitive to such effects. The persistence of the rotation signal in our fits argues against a coverage artifact and in favor of intrinsic evolution in the ejecta.

Our analysis confirms that the radio emission are persistently asymmetric, with a circular mean asymmetry angle of $\sim 240^\circ$ averaged over the full set of observations (measured counterclockwise from north). A global linear fit yields a counterclockwise shift of $0.19^\circ \pm 0.02^\circ$ per day, inconsistent with zero at $\gg 5\sigma$. From a theoretical perspective, these measurements open a new window into the physics of young supernova remnants. Detecting and quantifying azimuthal brightness variations this early after explosion provides a direct test of models in which asymmetric density or wind profiles [e.g., 273], interaction with anisotropic circumstellar material [e.g., 274], or clumpy magnetic field amplification govern the morphology of the expanding shell [e.g., 275]. The evidence for a rapid early drift that slows at later times suggests a dynamical transition in the ejecta–environment coupling, for which models of shell interaction and brightening must account.

As discussed in section 5.5, we report an expanding projected ejecta width equal to approximately 20% of the projected shell diameter. In contrast to previous analyses, we can robustly fit the width of the radio feature to as early as $t \approx 175$ days, ~ 500 days earlier than estimates from the literature (e.g., [258], which fit the width down to 996 days, or [255], which fit down to 672 days). Our fractional width is statistically significantly lower than later results reported in [247] (by a factor of ≈ 1 –2). Further, our widths are statistically consistent to $\lesssim 1\sigma$ with the results of [252] and [255]. There are several nuances about this comparison that should be emphasized.

First, the definition of “width” across model classes is not consistent, and, as a re-

sult, one model’s “width” cannot be blindly compared to another’s. The construction of width in the m -ring model (convolution of an infinitesimally thin ring with a Gaussian kernel) is similar to how width is measured in image-domain feature extraction methods, but distinct to how the width is constructed in the spherical shell model of e.g., [252] and [247]. We address this by identifying our m -ring width with the spherical shell width which best reproduces the best-fit m -ring radial profile.

Second, the m -ring model is phenomenological and not as physically-motivated as models used in other analyses (e.g., the spherical shell of uniform emissivity). As a result, there is a valid concern that, if the true underlying shape of the SN 1993J ejecta is a non- m -ring morphology, our result could be biased. This concern particularly applies to the width, which is sensitive to both data noise and model construction. We examined this concern in the case of the spherical shell of uniform emissivity, and found that: (a) for any spherical shell of uniform emissivity, there is at least one m -ring configuration that can reproduce the visibility function up to the resolution of the interferometer, and (b) the ability of the m -ring to produce Gaussians, disks, rings, and crescents (see [section 5.4](#) for a demonstration) with a single component provides it more flexibility than almost any other model type. Finally, comparing the ability of the m -ring model and a spherical shell of uniform emissivity to describe the data using a statistical f -test indicates that a spherical shell of uniform emissivity does not describe the data better on any epoch than the best-fitting m -ring. Therefore, while caution needs to be taken in direct model-to-model comparisons, we are confident that the m -ring model is sufficient to describe the data.

5.7 Conclusions

The m -ring model provides a streamlined yet flexible framework for describing the morphology of SN 1993J. Its analytic visibility function makes it computationally efficient for rigorous inference methods, while its parameterization can reproduce Gaussians, disks, rings, and crescents, spanning the circularly-symmetric morphologies expected in the data. This generality allows us to capture the projected spherical-shell geometry used in earlier analyses, while also enabling explicit characterization of azimuthal brightness variations. The construction is, however, phenomenological rather than physically motivated, and at later epochs—when the ejecta develop multiple hotspots and more complex substructure—the m -ring provides only an approximation to the mean asymmetry rather than a complete description.

We mitigate these limitations by directly comparing the radial profile of the m -ring to spherical-shell models and demonstrating that the m -ring can reproduce the most important features of the spherical shell. We exploit this similarity to map m -ring quantities to the equivalent (i.e., best-fitting) spherical shell, mitigating the unphysical construction of the m -ring by translating to physically motivated quantities. Additionally, by fitting closure quantities rather than visibilities, our analysis is rather independent of previous approaches, despite relying on the same dataset. Closure phases and log closure amplitudes make our analysis insensitive to calibration systematics that have complicated prior analyses of SN 1993J. However, as a drawback, it is not possible with this method to calibrate the data in Jansky and obtain the total flux density. A key remaining limitation of our procedure is the apparent underestimation of parameter uncertainties. We investigate possible sources for the underestimation but rely on adjustment via the reduced χ^2 metric to correct the underestimation.

We summarize our main results:

1. **The m -ring model:** this analysis represents the first use of the m -ring model for supernova VLBI investigations. This model was developed for black hole imaging as a flexible method of simultaneously constraining the structure type of a compact source (i.e., gaussian, disk, ring, m -ring) and inferring parameters. The utility of a flexible parameterization such as this is high when reconstructing an object whose source structure is unknown, as it allows the modeling to be partially structure agnostic. This flexibility is particularly useful for supernovae, whose projected radio emission evolve from Gaussian-like point sources to optically thick disks to m -ring-adjacent structures, enabling us to fit one model to all epochs.
2. **Use of closure quantities:** by relying on closure quantities for this analysis, we significantly reduce exposure to calibration concerns, which are still an open question in VLBI. The use of closure quantities in our approach improves robustness and reliability of the inferred parameters.
3. **Size characterization of the expanding ejecta.** We measure the diameter from 50 d to 2524 d and find an average power-law index of $\omega = 0.81 \pm 0.01$, consistent with previous analyses. We also find a clear break in the deceleration after ~ 600 d, with $\omega = 0.89 \pm 0.01$ at $t \lesssim 600$ d and $\omega = 0.81 \pm 0.01$ and $t \gtrsim 600$ d.
4. **Early-time measurement of the width.** For the first time, we have presented geometric measurements of the ejecta width earlier than ~ 700 d. We independently fit the ejecta diameter and width down to 175 d, ~ 500 days earlier than previous analyses. We find an average ejecta width of 0.23 ± 0.03 between $t = 264$ d and $t = 1693$ d.
5. **Azimuthal brightness modulation fit.** We identify a brightness pattern which

rotates from east to south with a rate of $0.19^\circ \pm 0.02^\circ$ per day during first few hundred days, after which the azimuthal structure becomes more complex with increasing angular resolution.

This analysis provides a proof-of-concept for interferometric closure quantity modeling of young radio supernovae and will be useful for future VLBI observations of explosive transients.

Part II

Inference and imaging in strong-field black hole spacetimes

Chapter 6

An overview of physics at the edge: the black hole shadow

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AS ONE GREAT FURNACE FLAM'D, YET FROM THOSE FLAMES

NO LIGHT, BUT RATHER DARKNESS VISIBLE

—John Milton, *Paradise Lost*, 1667



Black holes are the most extreme manifestations of general relativity (GR), making them ideal laboratories for testing both GR predictions and potential departures from the theory [276–278; 234; 279–281]. In particular, supermassive black holes are responsible for a wide range of high-energy astrophysical phenomena across the universe, yet the physics governing them is confined to the remarkably compact region at and just beyond the event horizon [282; 283]. In another extraordinary conver-

gence, the extreme gravity of any black hole, regardless of its mass, spin, or origin, produces a universal signature visible in any image—a sharp, bright ring called the photon ring [284; 285; 283; 286–288; 260]. This photon ring bounds the black hole “shadow”, a depression in the image of the accretion flow cast by the black hole itself. The first black hole images, enabled by very-long-baseline interferometry (VLBI), offered a glimpse into these horizon-scale environments, but fell short of the resolution needed to resolve the fine features imprinted by the black hole’s spacetime geometry [289; 266; 290; 291; 5; 292–298]. An upcoming mission, the Black Hole Explorer (BHEX), will extend ground VLBI arrays to space and probe the photon ring for the first time, enabling unprecedented tests of fundamental physics [299–305]. In this chapter, we review the physics of the photon ring and the shadow of the black hole, as well as the broader literature surrounding their utility in testing general relativity in the strong-field regime.

6.1 Motivation for horizon scale science

The extraordinary technical requirements of horizon-scale imaging (e.g., Earth-diameter VLBI baselines, atomic clock synchronization, supercomputing-scale correlation) are justified by the corresponding scientific returns: robust characterization of black hole properties and demographics, and direct null-hypothesis tests of general relativity. We discuss these motivations below.

6.1.1 Measurement of black hole properties

By directly resolving the apparent shadows of the largest black holes by angular diameter on the sky, the Event Horizon Telescope (EHT) and its associated extensions furnish a unique observational probe of black hole properties and the spacetime geom-

etry in their immediate vicinity [266; 295; 280; 281]. Under the no-hair theorem, which states that stationary black holes are fully described by their mass, charge, and angular momentum, an uncharged black hole is characterized by only its mass M and angular momentum J [306]. Importantly, the angular diameter of the black hole shadow is determined predominantly by the mass of the underlying black hole (degenerate with the distance to the black hole D), and is largely insensitive to the detailed structure of the surrounding accretion flows [283; 287; 279]. This feature of the shadow can be exploited to make precise, largely model-independent mass measurements, obtained by characterizing the diameter of the lensed emission at scales of just a few gravitational radii from the event horizon.

A similar inference pathway exists for angular momentum measurements. However, unlike the shadow size, which provides a clean mass observable largely decoupled from the accretion environment, spin manifests through more subtle morphological asymmetries in the shadow and photon ring that are partially degenerate with other astrophysical parameters. The most significant of these degeneracies is with the observer inclination (θ_o). At equatorial viewing angles ($\theta_o \approx 90^\circ$), spin introduces asymmetries in the shadow morphology at the level of a $\sim 5 - 10\%$ effect by diameter relative to the Schwarzschild case, with the effect scaling with the dimensionless spin parameter $a = Jc/(GM)^2$; by contrast, a face-on (pole-aligned, $\theta_o \approx 0^\circ$) observer is almost entirely insensitive to spin. Additionally, the spin-induced geometric asymmetry in the shadow is comparable in magnitude to distortions caused by the surrounding dynamical accretion flow, introducing a further source of degeneracy. These degeneracies can in principle be broken through sufficiently high-resolution imaging of the photon ring, whose sub-structure carries a cleaner gravitational imprint that is increasingly decoupled from accretion-flow contamination at finer angular scales (see [subsection 6.2.2](#)).

In general, measurements of mass can be made with angular resolution $\lesssim 3r_g$

(with the gravitational radius $r_g \equiv GM/c^2$), comparable to the size of the shadow. Indeed, the EHT provided a $\approx 10\%$ mass measurement of M87* with a resolution of $\approx 18 \mu\text{as}$, a factor of $\approx 5r_g$ for M87* and a factor of ≈ 2.2 smaller than the shadow diameter [266; 293]. This resolution proved insufficient to make any constraint on spin. The upcoming BHEX spacecraft will improve this resolution to $\approx 6 \mu\text{as}$, sufficient to constrain spin to within $\sim 10\%$ for M87* and Sgr A* [299; 300]. The robust mass measurements across a demographically representative sample of supermassive black holes (SMBHs) provided by this additional resolution can be used to directly test the empirical correlations between black hole mass and galaxy kinematics that encode the evolutionary history of galaxies and remain poorly understood from first principles.

6.1.2 Black hole demographics

Even at the impressive angular resolution achieved by the EHT, direct shadow imaging was feasible only for the two black holes with the largest apparent angular diameters on the sky (M87* and Sgr A*), with all other known systems remaining spatially unresolved at horizon scales [266; 295]. However, the proposed BHEX spacecraft will improve upon the resolution of the EHT (from $\approx 18 \mu\text{as}$ to $\approx 6 \mu\text{as}$), resolving the shadows of dozens of supermassive black holes across the local universe. This increased resolution enables a horizon-scale survey of the properties of supermassive black holes and their immediate environments [299; 300]. Beyond increased spatial resolution, the BHEX observing campaign format will provide opportunities for high-cadence, weeks-long “movies” of the horizon-scale structure, enabling estimates of accretion flow properties in the vicinity of the black hole.

The ability to measure black hole properties across a large sample (versus the two accessible with current technology) advances several lines of inquiry. The most acces-

sible and immediately robust will be mass measurements that probe the black hole initial mass function. In addition to increasing the sampling of the mass function from $\approx 10^8\text{--}10^{10} M_\odot$, the black hole masses BHEX will probe are the primary contributors to the nanohertz gravitational wave background. The observed amplitude of this background exceeds theoretical predictions from supermassive black hole binary mergers [307], and improved constraints on the high-mass end of the black hole mass function provide a direct path to resolving this tension [299].

While direct spin measurements (i.e., via photon ring) will be feasible for only a subset of BHEX targets, indirect spin inference (via, e.g., horizon-scale time-domain observations of accretion flows, polarization, etc.) remains possible across the full sample, enabling characterization of the supermassive black hole spin distribution. This spin function encodes key information about the growth histories of supermassive black holes [308; 309]: systems that grew predominantly via prolonged gas accretion are expected to be driven toward maximal spin by the sustained angular momentum torque of the accreting material, whereas those assembled primarily through mergers are expected to exhibit low, isotropically distributed spins as a result of stochastic angular momentum cancellation. Measuring the shape of the spin distribution across a demographically representative sample therefore offers a direct observational window into the dominant assembly channel of supermassive black holes across cosmic time, a question that remains unresolved by electromagnetic and gravitational-wave observations alone. Finally, probing the spin distribution also has direct implications for our understanding of the energetics of active galactic nuclei (AGN) feedback. In particular, the efficiency with which a black hole extracts energy from its accretion flow, and as a consequence its capacity to suppress star formation in its host galaxy [e.g., 310], is sensitive to spin.

6.1.3 Tests of general relativity

Among the most compelling aims of BHEX is the ability to perform strong-field tests of general relativity through direct resolution of the photon ring [260; 299]. The lensing substructure of the photon ring is a precise and universal prediction of Kerr spacetime that is largely insensitive to the astrophysical properties of the accretion environment and varies quite significantly with the metric.

The classical Kerr metric predicts a specific shape and size for the black hole shadow and its component subrings, driven by the lensing bands that are a unique feature of this spacetime [284; 287; 286]. In particular, the radius of the shadow is expected to be within 4% of $5r_g$, enabling a null hypothesis test of the Kerr metric [279; 280]. The EHT measurement of the shadow sizes of M87* and Sgr A* was consistent with these expectations [293; 298] (see Figure 6.1). However, the resolution was insufficient to rule out exotic alternative theories of gravity that can produce similar shadow sizes, such as boson stars and gravastars [266].

The angular resolution of BHEX will be sufficient to place substantially tighter constraints on deviations from general relativity, and to conclusively rule out a broad class of alternative theories. At one extreme, some exotic compact objects may not support a photon ring, producing no photon ring signature; detection of a photon ring around M87* would therefore immediately and decisively exclude this class of objects [e.g., 312; 313]. At the other extreme, metric-based modifications to Kerr spacetime (e.g., such as those arising in effective field theory extensions of GR) can produce shadow deformations that fall below the EHT sensitivity threshold but remain well within the detection capability of BHEX [314]. However, certain theories predict deviations sufficiently subtle that even BHEX resolution is inadequate to detect them; dynamical Chern-Simons modified gravity is one such example, requiring angular

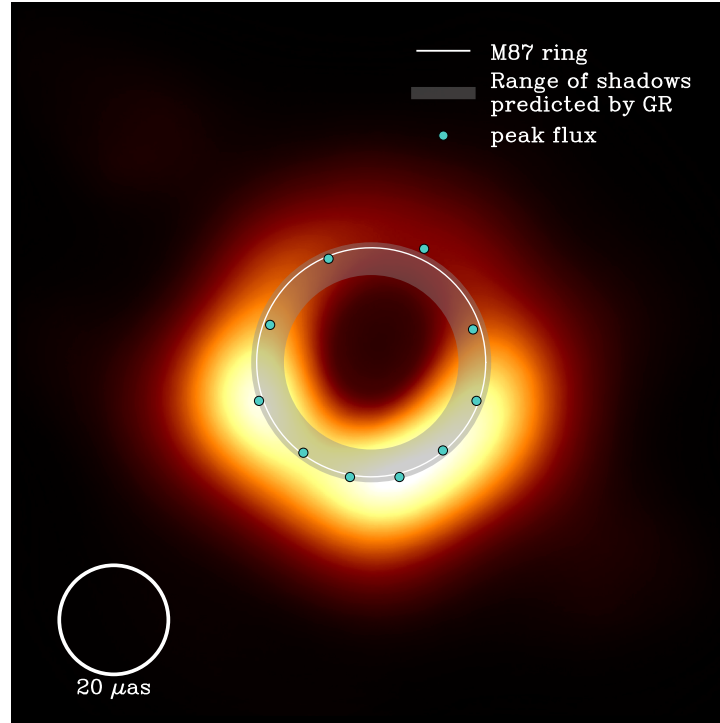


Figure 6.1: Comparison of the M87* shadow to predictions from general relativity. By extracting locations of peak flux (radial) from the EHT image of M87* (blue points), we can construct an estimated shadow profile of $42 \pm 3 \mu\text{as}$ (white line and band). This is consistent with predictions from general relativity based on expectations for the mass of M87* ($\approx 6.2 \pm 1.1 \times 10^9 M_\odot$ from gas dynamics, [311]), which results in a predicted Schwarzschild shadow diameter of $37.6 \pm 6.2 \mu\text{as}$. Combined with the $\sim 4\%$ uncertainty from the space of allowable Kerr diameters, this produces a range of shadow sizes (gray band) that are consistent with the EHT measurement, constituting a successful null hypothesis test of general relativity.

resolution beyond that of any currently proposed instrument to yield meaningful constraints [315]. More broadly, any confirmed deviation from Kerr at horizon scales would constitute evidence for new physics at the interface of general relativity and quantum mechanics, the regime where semiclassical predictions become relevant. Thus, strong-field imaging tests with BHEX directly connect to some of the deepest unresolved questions in theoretical physics.

6.2 Construction of the black hole shadow

6.2.1 The critical curve

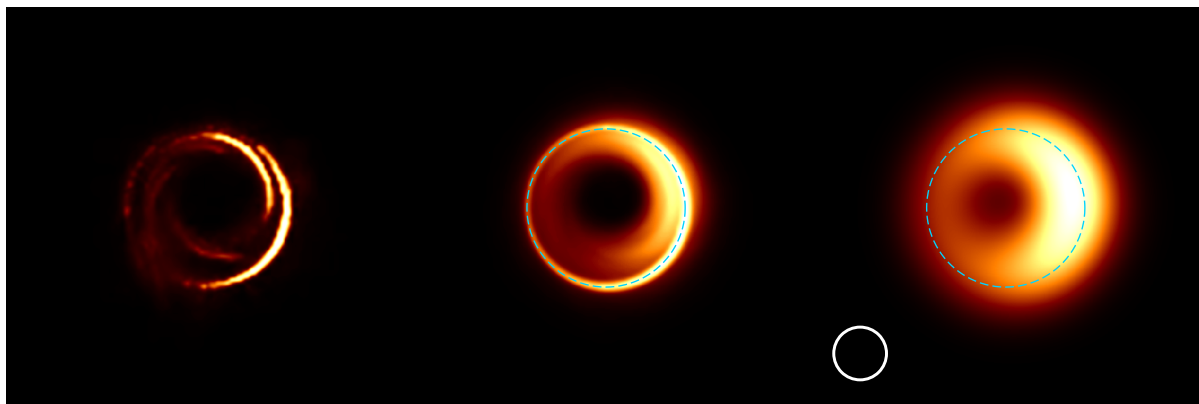


Figure 6.2: The black hole shadow is clearly visible in simulations of a black hole embedded in an optically thin emitting medium. *(left)* A snapshot of a simulation of a black hole with $a = 0.9375$ and $\theta_0 = 10^\circ$. The photon ring is present but not clearly visible among the transient flux. *(center)* After averaging over many ($\sim 10^4 M$), the transient flux is washed out and only persistent features—namely, the black hole shadow—remain. The white dashed line indicates the critical curve, which bounds the shadow. *(right)* Even under convolution with the EHT beam ($\sim 21 \mu\text{as}$), the central depression and approximate size of the black hole shadow are still clearly visible. Simulations graciously provided by Charles Gammie and students for use in [316].

When a black hole is embedded in an optically thin emitting medium, or is projected against a distant background source, the set of photon trajectories that terminate on the event horizon defines a dark central depression in the observer’s image: the black hole shadow [285; 283; 317]. Examples of this shadow for two black hole simulations are shown in Figure 6.2. The boundary of the shadow is defined by the critical curve, the locus of unstable spherical photon orbits in the Kerr spacetime, which partitions incoming photon trajectories into those captured by the event horizon and those that escape to a distant observer. These orbits lie on spheres of constant radius r in the Boyer-Lindquist coordinate system. For a spinning (Kerr) black hole, these orbits lie

on a range from fully prograde (+) to fully retrograde (-),

$$r_{\pm} = 2\frac{GM}{c^2} \left(1 + \cos \left[\frac{2}{3} \arccos(\mp|a|) \right] \right), \quad (6.1)$$

where $a \equiv Jc/(GM^2) \in [0, 1]$ is the dimensionless spin defined by the angular momentum J of the black hole. These orbits trace out a characteristic shadow shape in observer screen coordinates (α, β) given by,

$$\begin{aligned} \alpha(r) &= -\ell / \sin \theta_o \\ &= \frac{r^3 + a^2 r + a^2 - 3r^2}{a(r-1) \sin \theta_o}, \\ \beta(r) &= \pm \sqrt{\chi + a^2 \cos^2 \theta_o - \ell^2 / \tan^2 \theta_o} \\ &= \pm \frac{1}{a(r-1)} \left\{ a^4 (r-1)^2 \cos^2 \theta_o - [a^2 (r+1) \right. \\ &\quad \left. + (r-3)r^2]^2 \cot^2 \theta_o - r^3 [r(r-3)^2 - 4a^2] \right\}^{1/2}, \end{aligned} \quad (6.2)$$

where ℓ is the specific (i.e., per unit energy) angular momentum about the spin axis, and χ is the specific (i.e., per unit squared energy) Carter constant.

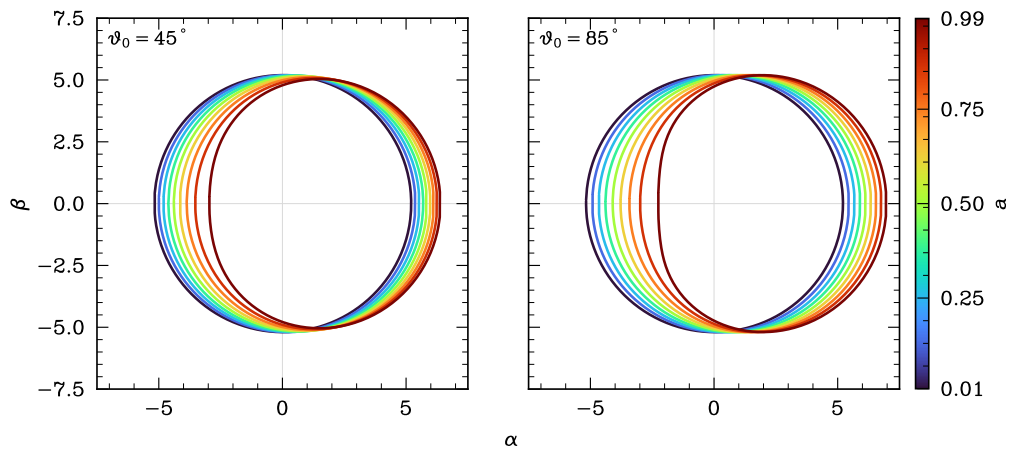


Figure 6.3: Samples of the black hole shadow's critical curve at several spins and two observer inclinations.

Samples of equatorial critical curves are shown in [Figure 6.3](#). At edge-on inclinations, the asymmetry between the impact parameters of prograde and retrograde photon orbits (the former being dragged closer to the black hole by frame-dragging, the latter pushed outward) produces a characteristic D-shaped distortion, with the shadow boundary appearing flattened on the side corresponding to the approaching limb.

While the critical curve provides a complete theoretical characterization of the black hole shadow boundary, its full two-dimensional representation in the image plane is unwieldy for practical parameter estimation. For instance, while the critical curve is formally injective over the Kerr parameter space (i.e., no two combinations of mass, spin, and inclination produce identical shadow boundaries), any realistic instrument operates at finite resolution and sensitivity, rendering shadows that are theoretically distinct effectively indistinguishable in practice. This practical degeneracy is difficult to diagnose from the full curve representation alone. In [chapter 7](#), we introduce and investigate compact analytic approximations to the critical curve that facilitate both physical insight into the parameter dependence of the shadow geometry and practical inference of black hole properties.

6.2.2 Photon subrings

We refer primarily to [\[260\]](#) for the discussion here. The critical curve, as discussed above, partitions photon orbits that escape the black hole to infinity and those that terminate on the event horizon. Photons on the critical curve itself occupy an unstable circular orbit and an infinitesimal outward perturbation to the trajectory will diverge until the photon ultimately escapes. Within a narrow range of impact parameters just outside the critical curve, photons may complete one or more half-orbits around the

black hole before escaping to the observer. These photons produce a series of exponentially nested subrings on the image indexed by their half-orbit winding number n , with successive rings related by a fixed exponential factor in both width and flux. The decreasing width $\delta\rho$ is set by the increasingly narrow range of trajectories whose window is defined by,

$$\frac{\delta\rho_n}{\rho_c} \approx e^{-\gamma n}, \quad (6.3)$$

where n is the number of target half-orbits, γ is the Lyapunov exponent, and ρ_c is the radial extent of the shadow edge. Similarly, we find that the flux density ratio of each successive to its predecessor also scales with the Lyapunov exponent,

$$\frac{F^{n+1}}{F^n} \approx e^{-\gamma}. \quad (6.4)$$

Each successive ring becomes less sensitive to surrounding astrophysical effects and therefore a more direct probe of the black hole's spacetime.

6.2.3 Expected physical scales for M87* and Sgr A*

The angular gravitational radius of M87*, adopting $M \approx 6.5 \times 10^9 M_\odot$ and $D \approx 16.8 \text{ Mpc}$, is $\theta_g = GM/c^2 D \approx 3.8 \mu\text{as}$, yielding an expected shadow diameter of $2 \times 3\sqrt{3}\theta_g \approx 40 \mu\text{as}$ [266]. For Sgr A*, with $M \approx 4.15 \times 10^6 M_\odot$ at $D \approx 8.15 \text{ kpc}$, the same calculation gives $\theta_g \approx 5.0 \mu\text{as}$ and a shadow diameter of $\approx 52 \mu\text{as}$ [295]. The characteristic widths of the photon subrings are governed by the Lyapunov exponent of the unstable circular photon orbit; for a near-Schwarzschild spacetime with $\gamma \approx \pi$, the n th subring has width $w_n \sim w_0 e^{-n\pi}$ relative to the direct emission scale $w_0 \sim \theta_{\text{shadow}}$, giving $w_1 \approx 1.7 \mu\text{as}$ and $w_2 \approx 75 \text{ nas}$ for M87*, and $w_1 \approx 2.2 \mu\text{as}$ and $w_2 \approx 95 \text{ nas}$ for Sgr A*

[260]. The $n = 1$ ring thus lies marginally below the nominal BHEX resolution of $\approx 6 \mu\text{as}$ in spatial width, but remains detectable through its characteristic oscillatory signature in the interferometric visibility amplitude; the $n = 2$ ring, at $\lesssim 100 \text{ nas}$, falls well beyond the reach of any currently proposed instrument in direct imaging. Finally, the well-constrained jet inclination of M87* places the observer at $\sim 17^\circ$ from the spin axis, a nearly pole-on geometry [318]. The spin-induced shadow asymmetry, which reaches $\sim 5 - 10\%$ of the shadow diameter ($\sim 2-4 \mu\text{as}$) at equatorial inclinations, is suppressed by a geometric factor of order $a \sin^2(17^\circ) \approx 0.07$.

6.3 Overview of interferometry

Recent advances in interferometry (to long-baseline and then very-long-baseline interferometry) have made it possible to probe the horizon-scale phenomena discussed in [section 6.1](#) and [section 6.2](#). We review the technical fundamentals of interferometry as needed for the analyses presented in this dissertation. We primarily cite [91] for the conventions used and discussions below.

6.3.1 Basics of interferometry

Radio interferometry exploits the wave nature of light to synthesize an effective telescope aperture far exceeding the physical size of any single dish. When two spatially separated antennas (indexed i and j) receive the same wavefront from a distant source, the signals can be cross-correlated to produce a complex visibility V , a quantity that encodes both the amplitude and phase of the interference fringe formed between the two stations. By the van Cittert-Zernike theorem, the visibility $V(u, v)$ and image

functions $I(x, y)$ are related by the Fourier transform,

$$V_{ij} = \int dx dy I(x, y) e^{2\pi i(ux+vy)}. \quad (6.5)$$

For any given interferometer configuration, the N antennas will form $N(N + 1)/2$ projected baselines as seen by the source, which probe the Fourier transform of the source image at a location corresponding to the direction and magnitude of each individual baseline. A baseline of dimensionless length $\rho = l/\lambda$ (where λ is the operational wavelength of the interferometer and l is the physical extent of the baseline) probes a spatial scale $\approx 1/\rho$. Therefore, an interferometer can be thought of having resolution (by the Rayleigh criterion) equivalent to the inverse of its longest baseline length. The visibility along each baseline is modified by various sources of error, including thermal noise ε_{ij} , phase error ϕ_i , and multiplicative complex gains G_i . These produce the modified visibility,

$$V'_{ij} = G_i G_j e^{i(\phi_i - \phi_j)} (V_{ij} + \varepsilon_{ij}). \quad (6.6)$$

When analyzing data with these sources of error, the corruptions must be incorporated into fit statistics (as with thermal noise) or modeled/fit directly (as with complex gains). However, we may construct derivative data products from the visibility known as *closure quantities* which are insensitive to one or more sources of noise. For example, by multiplying three visibilities together in a triangle, we obtain the complex bispectrum V_B ,

$$V_B = V_{12} V_{23} V_{31}. \quad (6.7)$$

The argument of this quantity is known as the closure phase ϕ_{123} ,

$$\phi_{123} = \arg V_{12} + \arg V_{23} - \arg V_{13}. \quad (6.8)$$

Conveniently, the closure phase is insensitive to complex gains and phase errors. In general, an array with N antennas will support $\approx \binom{n}{3}$ closure triangles and $\approx \binom{n}{4}$ closure quadrangles.

The effective number of baselines may be increased via Earth rotation aperture synthesis, which exploits the rotation of the Earth to translate and rotate the projected baseline from the perspective of the source, sampling the Fourier transform in different locations. A full night of aperture synthesis can lead to significantly denser and more symmetric sampling than a snapshot of the array alone; we demonstrate this concept in [Figure 6.4](#). Generally, the low number of antennas means that, even with aperture synthesis, the Fourier transform of the source is only sampled sparsely, producing an incomplete characterization of the visibility function. As a result, the problem of inverting the Fourier transform to produce an image is ill-defined, or underdetermined.

6.3.2 Interferometric reconstruction

Reconstruction of an image from—and inference using—VLBI data is an active and open problem. We divide this discussion into two separate topics: (i) the problem of interferometric imaging, whereby we attempt to recover the on-sky source from the Fourier transform, and (ii) interferometric modeling, where we assume a model structure and recover parameters by fitting a representation of said model to the visibilities, without first inverting.

Interferometric imaging historically has been performed using variations of the CLEAN algorithm [[319](#); [320](#)], which attempts to directly invert the Fourier transform

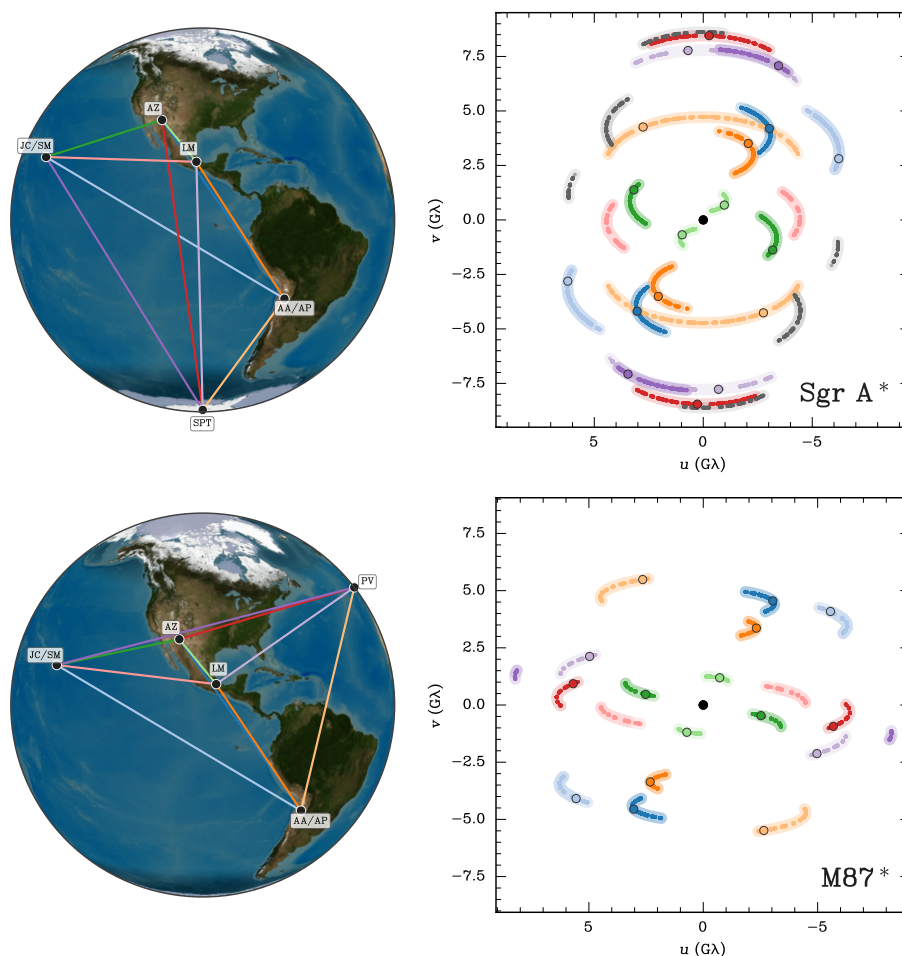


Figure 6.4: Demonstration of Earth rotation aperture synthesis for Sgr A* and M87*. At any given moment, the coverage is determined by the projection of the baselines formed by visible stations along the line-of-sight to the target (left column). At any given moment, these baselines probe only a handful of sparsely separated points in the Fourier plane (solid points, right column). However, as the Earth rotates, the projected baselines change orientation and length, probing new points in the Fourier plane. Over the course of a full night of observation, each baseline sweeps out elliptical trajectories in the Fourier plane (colored bands) that contain significantly more information combined than any given snapshot.

(producing a “dirty image”) and iteratively identify the location of real flux in the map. Deconvolution via CLEAN accounts for gain corruption by performing several iterations of self-calibration to systematically fit the gain terms [e.g., 321]. Once a representative image has been identified via placement of a series of point sources

corresponding to peaks in the dirty image, the image is convolved with the nominal beam of the interferometer to reintroduce the effects of the point-spread function.

Recently, alternatives to CLEAN have been developed, which forward-model an image by maximizing consistency with the visibilities [e.g., 5; 322]. Chief among these are regularized maximum likelihood (RML)-based methods, which maximize a likelihood term in addition to several “regularizers” which encode additional information that can be incorporated into the reconstruction (e.g., knowledge of flux location from a longer-wavelength prior map) [e.g., 323]. RML has several advantages over CLEAN deconvolution procedures, including the flexibility to use arbitrary data products (such as the closure quantities discussed above) and the ability to explore the space of images consistent with the data by sampling around the maximum a posteriori solution, yielding an ensemble of representative reconstructions that can be used to assess the robustness of image-domain features.

Both CLEAN- and RML-based imaging methods require the assumption of a static source, i.e., a source that does not dynamically evolve during aperture synthesis. When this is violated, as in the case of e.g., Sgr A*, image reconstruction algorithms must be modified to support dynamical imaging [324]. The most straightforward way to perform dynamical imaging is to restrict the dataset to short “snapshots” in time over which the static assumption is met. CLEAN algorithms create an average image over the full dataset and restrict identification of peak flux locations for each snapshot to the average emission region. RML similarly forward-models individual snapshots but with an additional regularizer (temporal inertia, $R_{\Delta t}$) that enforces similarity between successive snapshots. More advanced methods, such as StarWarps [325], frame dynamical imaging as exact posterior inference over a probabilistic graphical model, representing each snapshot as a multivariate Gaussian and coupling adjacent frames through transitional probabilities that jointly enforce spatial and temporal regular-

ization. We investigate data processing methods which can greatly enhance these methods in [chapter 8](#).

6.3.3 The Event Horizon Telescope

The Event Horizon Telescope (EHT) is a global very-long-baseline interferometer that, in 2019, achieved a landmark milestone in observational astrophysics: the first direct imaging of black holes [266; 290; 291; 5; 292–294; 326]. Composed of eight geographically distributed stations operating at an observing frequency of 230GHz, the EHT achieved a maximum baseline length of $\approx 1.2 \times 10^4$ km, corresponding to an effective angular resolution of $\lambda/l \approx 20 \mu\text{as}$. This unprecedented resolving power was sufficient to image the shadows cast by the two largest supermassive black holes on the sky: Sgr A*, the SMBH at the centre of the Milky Way, and M87*, the SMBH residing at the core of the giant elliptical galaxy Messier 87. The successful imaging of both sources enabled robust measurements of their masses, providing powerful tests of general relativity in the strong-field regime and placing new constraints on accretion and jet-launching physics.

Ongoing instrumental and logistical upgrades to the EHT have resulted in the development of the next-generation Event Horizon Telescope (ngEHT), a substantially expanded array designed to overcome the current limitations of its predecessor [327; 328]. The ngEHT concept encompasses several key improvements: doubling the number of component stations to increase collecting area and redundancy, strategically optimising station locations to improve coverage of the Fourier (u, v) -plane and thereby reduce imaging artefacts [329], and extending operations to shorter wavelengths of 0.87mm (≈ 345 GHz). Together, these enhancements are expected to yield an approximately 50% improvement in angular resolution relative to the current EHT, and critically, will

provide access to wavelengths at which the surrounding accretion flows are expected to be more optically thin [330; 328; 331]. At 345 GHz, synchrotron self-absorption becomes less severe, allowing emission from regions closer to the event horizon to contribute to the observed flux and enabling a more direct probe of the innermost accretion and jet structure. The combination of improved resolution, (u, v) -coverage, and optical depth conditions will make the ngEHT a transformative instrument for time-domain and multi-frequency imaging of Sgr A* and M87* on event-horizon scales. The ngEHT is under active construction and recently successfully achieved the first VLBI detections at 870 microns, an important developmental milestone [331].

Nevertheless, these specifications approach the practical upper limits of what is achievable using ground-based interferometric arrays. In particular, the remaining atmospheric transmission windows beyond 345 GHz occur at increasingly high frequencies approaching the terahertz regime, presenting a significantly greater technological challenge for the development of receiver and backend instrumentation [see 332, and references therein]. Additionally, the finite size of the Earth fundamentally caps the maximum achievable baseline length and therefore the angular resolution, regardless of the number of stations deployed. Finally, the number of viable high-altitude, low-precipitable-water-vapour sites suitable for sub-millimetre VLBI is limited, constraining the extent to which (u, v) -coverage can be improved through the addition of new ground stations alone. These physical and practical limitations motivate consideration of more radical concepts for extending VLBI capabilities beyond what ground-based infrastructure can offer.

6.3.4 The Black Hole Explorer

A natural solution to the limitations of ground-based VLBI is to extend baselines beyond the Earth entirely, into space. The Black Hole Explorer [BHEX; 299] is a NASA Small Explorer (SMEX) mission concept designed to do precisely this, by placing a 3.5 m antenna in medium Earth orbit and operating it in concert with existing ground-based mm/sub-mm VLBI arrays. By extending baselines well beyond the diameter of the Earth, BHEX will achieve angular resolutions of $\lesssim 6 \mu\text{as}$, approximately three times finer than the current EHT, operating across a dual-band receiver system spanning 80–106 GHz and 240–320 GHz. Similarly, by observing above the atmosphere, BHEX sidesteps atmospheric absorption constraints and removing the practical frequency ceiling imposed by the narrowing of ground-accessible transmission windows at sub-millimetre wavelengths.

The primary science driver of BHEX is the first detection and characterisation of the black hole *photon ring* discussed in subsection 6.2.2 [302]. While the $n = 0$ direct emission has been resolved by the EHT, the higher-order photon ring substructure requires the $\gtrsim 20 \text{ G}\lambda$ baselines only accessible from space. Detection of this feature in Sgr A* and M87* would provide the first direct measurements of black hole spin, offering a meaningfully cleaner test of the Kerr metric in the strong-field regime that is largely free from the astrophysical uncertainties that complicate interpretation of the $n = 0$ ring morphology.

Beyond the photon ring, BHEX will substantially broaden the scope of horizon-scale black hole science. The combination of improved angular and temporal resolution as well as (u, v) -coverage will enable detailed studies of jet launching and collimation on scales approaching the event horizon, where the coupling between the accretion flow and the relativistic outflow remains poorly understood [333]. Furthermore, the

extended baseline coverage provided by the space-ground configuration will facilitate imaging of a significantly larger population of SMBHs beyond Sgr A* and M87*, including AGN jets and systems in low-luminosity accretion states, opening a new window on the demographics of black hole masses and spins across cosmic time [299].

Several chapters of this dissertation focus on testing the capabilities of the future BHEX spacecraft and array to measure angular momentum of science targets. We present a deep-learning-based approach to spin inference from the $n = 1$ photon ring in [chapter 9](#). In [chapter 10](#), we extend the results of [chapter 9](#) to a Fourier analog which can be performed in the visibility domain alone.

Chapter 7

Polar approximations of the black hole critical curve in image domain

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[NATURE] IS AN INFINITE SPHERE, THE CENTRE OF WHICH IS EVERYWHERE, THE
CIRCUMFERENCE NOWHERE.

— Blaise Pascal, son of Etienne Pascal, *Pensées*, 1670



A black hole embedded within a bright, optically thin emitting region imprints a nearly circular “shadow” on its image, corresponding to the observer’s line-of-sight into the black hole. The shadow boundary depends on the black hole’s mass and spin, providing an observable signature of both properties via high resolution images. However, standard expressions for the shadow boundary are most naturally parametrized by Boyer-Lindquist radii rather than by image coordinates. In this chap-

ter, we explore simple, approximate parametrizations for the shadow boundary using ellipses and a family of curves known as *limaçons*. We demonstrate that these curves provide excellent and efficient approximations for all black hole spins and inclinations. In particular, we show that the two parameters of the limaçon naturally account for the three primary shadow deformations resulting from mass and spin: size, displacement, and asymmetry. These curves are convenient for parametric model fitting directly to interferometric data, they reveal the degeneracies expected when estimating black hole properties from images with practical measurement limitations, and they provide a natural framework for parametric tests of the Kerr metric using black hole images.

7.1 Permissions and Attributions

1. The content of [chapter 7](#) is the result of a collaboration with Dr. Dom Pesce, Prof. Michael Johnson, and Dr. Lindy Blackburn, and has previously appeared in the *Astrophysical Journal* as Joseph R. Farah *et al.* (2020). It is reproduced here with the permission of the *Astrophysical Journal*: <https://iopscience.iop.org/article/10.3847/1538-4357/aba59a>.
2. We acknowledge Jonathan Delgado, Chi-Kwan Chan, Paul Tiede, and Roman Gold for helpful discussions.

7.2 Introduction

When surrounded by optically thin emitting material, a black hole produces a nearly circular “shadow” on the image seen by a distant observer, corresponding to the observer’s line-of-sight into the black hole [see, e.g., [285](#); [334](#); [335](#); [283](#); [336–338](#)]. The size and shape of the shadow encode information about the black hole’s angular

momentum, inclination relative to the observer, and mass. Moreover, the shadow boundary is achromatic and independent of the emission and astrophysical details of a source. However, the angular diameter of the shadow is only $\approx 10GM/(c^2D) \approx 10 \mu\text{as} \left(\frac{M}{10^9 M_\odot} \right) \left(\frac{D}{10 \text{ Mpc}} \right)^{-1}$ in size, where G is the gravitational constant, M is the mass of the black hole, c is the speed of light, and D is the distance from the observer [339; 340; 285; 317]. Thus, even for nearby supermassive black hole candidates, very high angular resolution is required to study the shadow.

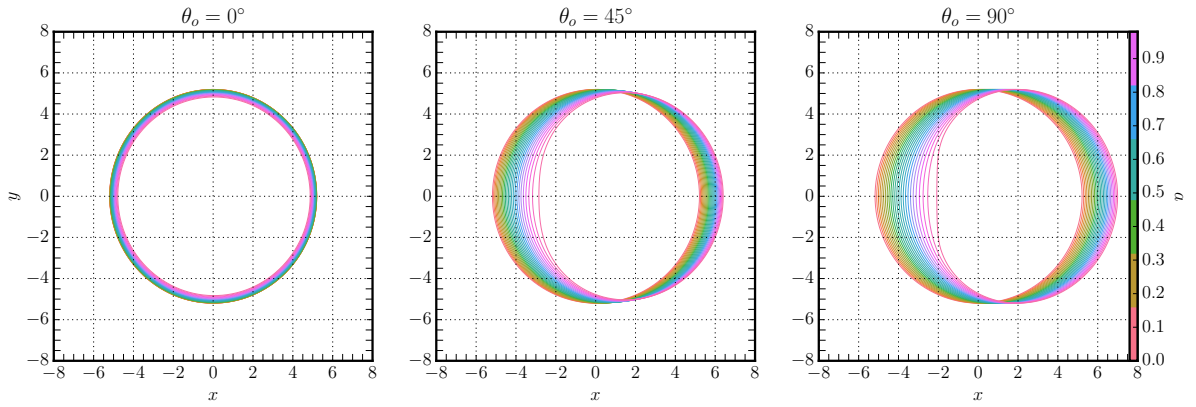


Figure 7.1: Black hole shadow boundary as a function of spin and inclination. The dimensionless spin magnitude a is sampled uniformly from 0 to 1 in steps of 0.02, while the three panels correspond to inclinations $\theta_o = 0^\circ$ (left), $\theta_o = 45^\circ$ (center), and $\theta_o = 90^\circ$ (right).

This resolution was recently achieved by the Event Horizon Telescope, using Very Long Baseline Interferometry (VLBI) at 1.3mm wavelength to image the supermassive black hole in the galaxy M87 [266; 290; 291; 5; 292; 293, hereafter, EHT1-6]. The EHT observations revealed a bright, asymmetric ring, corresponding to a black hole with $M = (6.5 \pm 0.7) \times 10^9 M_\odot$ at a distance $D = (16.8 \pm 0.8) \text{ Mpc}$. While the EHT does not yet provide significant constraints on the shadow shape or asymmetry, planned extensions to shorter wavelengths and longer baselines will achieve even finer resolution and a greater reduction of the astrophysical foreground. Thus, it is essential to develop tools and techniques that are capable of more detailed model fitting and inference and to

assess what can be learned about a black hole from higher resolution observations of its shadow.

To this end, many authors have quantified the shadow of a Kerr black hole using simple diagnostics such as mean diameter, asymmetry, and displacement from the origin [e.g., 317; 341–344]. Other authors have moved beyond these diagnostics, presenting general approximation frameworks to model arbitrary shadows using a Legendre expansion [345] or a basis of principal components [346]. Several authors have explored simpler parametric approximations for the shadow, either using approximations for the full relativistic calculation of the shadow [337] or using families of simple curves [347]. Here, we expand this latter approach, developing approximations for the shadow using simple polar curves. We show that these curves provide excellent representations for the exact shadow shape, while offering a variety of concrete advantages. Relative to simple shadow diagnostics, these curves have the advantage of completely characterizing the information in the shadow, thereby highlighting potential parameter degeneracies that are expected to persist even for high resolution images. They also provide tools for model fitting directly to interferometric data, and they can be used as a parametric framework to assess the validity of the Kerr shadow assumption.

We begin, in [section 7.3](#), with a brief introduction to the shadow and its properties. Next, we explore approximations to the shadow using a shifted ellipse ([section 7.4](#)), a limaçon ([section 7.5](#)), and the convex hull of a shifted limaçon ([section 7.6](#)). In [section 7.7](#), we compare these results and discuss their implications for the inference of black hole parameters with high resolution images. We summarize our results in [section 7.8](#).

7.3 The Shadow of a Kerr Black Hole

The Kerr metric [278] describes the vacuum spacetime around a rotating, non-charged black hole of mass M and dimensionless spin $a = Jc/M^2$, where J is the angular momentum of the black hole and $0 \leq a \leq 1$. A crucial feature of this metric is that photons near the black hole can orbit on spheres with constant Boyer-Lindquist radius, r . For a Schwarzschild black hole, all spherical photon orbits have $r = 3GM/c^2$, while a spinning black hole has $r_+ \leq r \leq r_-$, where the radii of the prograde (+) and retrograde (−) equatorial photon orbits are

$$r_{\pm} = 2 \frac{GM}{c^2} \left\{ 1 + \cos \left[\frac{2}{3} \arccos (\mp |a|) \right] \right\}. \quad (7.1)$$

For simplicity, we will set $G = c = M = 1$ for the remainder of our discussion and will express later quantities in units of $r_g \equiv GM/c^2$.

Null geodesics in the Kerr metric are fully characterized by two constants of motion: the energy-rescaled angular momentum about the spin axis $\ell = L/E$, and the energy-rescaled Carter constant $\chi = Q/E^2$. For a spherical photon orbit at radius r , these are given by [see, e.g., 285; 348; 337]

$$\begin{aligned} \ell &= -\frac{r^3 + a^2 r + a^2 - 3r^2}{a(r-1)}, \\ \chi &= \frac{r^2}{r^2 - a^2} (3r^2 + a^2 - \ell^2). \end{aligned} \quad (7.2)$$

A slight perturbation from a spherical orbit will exponentially diverge away from the photon orbit. Slightly tighter orbits plunge into the black hole, while slightly larger orbits escape to infinity. This instability produces a brightness enhancement at the boundary of the shadow if the region $r_+ \leq r \leq r_-$ is populated by emitting material

and is optically thin [see, e.g., 287; 286; 260]. A distant observer viewing the black hole at inclination θ_o relative to the spin axis will then see this asymptotic brightness enhancement at Cartesian screen coordinates given by [285]:

$$\begin{aligned}
 x(r) &= -\ell / \sin \theta_o \\
 &= \frac{r^3 + a^2 r + a^2 - 3r^2}{a(r-1) \sin \theta_o}, \\
 y(r) &= \pm \sqrt{\chi + a^2 \cos^2 \theta_o - \ell^2 / \tan^2 \theta_o} \\
 &= \pm \frac{1}{a(r-1)} \left\{ a^4 (r-1)^2 \cos^2 \theta_o - [a^2 (r+1) \right. \\
 &\quad \left. + (r-3)r^2]^2 \cot^2 \theta_o - r^3 [r(r-3)^2 - 4a^2] \right\}^{1/2}.
 \end{aligned} \tag{7.3}$$

Here, the projected spin direction on the observer's screen lies along the y -direction. The interior of this curve corresponds to the observer's line of sight into the black hole.

Figure 7.1 shows example shadows at three inclinations and all spins. For a Schwarzschild black hole ($a = 0$), the shadow is circular and has a radius on the image of $\sqrt{x(r)^2 + y(r)^2} = \sqrt{27}r_g$ [339; 340]. As a increases, the shadow is displaced horizontally and flattens on the side approaching the origin. This effect becomes most pronounced in the limit $a \rightarrow 1$ and $\theta_o \rightarrow \pi/2$ (a maximally spinning black hole viewed edge-on).

While Equation 7.3 defines the exact boundary of the black hole shadow, it is inconveniently parametrized by the corresponding (Boyer-Lindquist) *emission radius* rather than by a natural image coordinate such as the polar angle. Next, we will explore simple polar approximations for the shadow.

7.4 Ellipse approximation

7.4.1 Definitions

Our first polar curve to describe the black hole shadow is an ellipse, including a displacement parameter. In addition to its simple parametric form, the ellipse has convenient Fourier relationships to a circle, making it particularly effective as a tool for model fitting directly to interferometric data. Specifically, by the Fourier scaling theorem, any circular image with a known Fourier transform can be stretched by a factor f to create an ellipse, with a Fourier transform given by stretching baselines and rescaling visibility amplitudes of the circular image by the factor f^{-1} along the same direction. For instance, any circular ring model used to fit the EHT observations in [293] could be trivially adapted to allow elliptical shapes [see also 349; 350].

We parametrize the ellipse as $(x(\tau), y(\tau))$, where

$$x(\tau) = r_0(\tau) \cos(\tau) + \delta, \quad (7.4)$$

$$y(\tau) = r_0(\tau) \sin(\tau), \quad (7.5)$$

$$r_0(\tau) = \frac{r_{\parallel} r_{\perp}}{\sqrt{r_{\parallel}^2 \cos^2 \tau + r_{\perp}^2 \sin^2 \tau}}, \quad (7.6)$$

Here, δ is a shift in the positive x -direction, $r_{\parallel}$ is the ellipse radius along the spin axis, $r_{\perp}$ is the ellipse radius orthogonal to the spin axis, and $\tau \in [0, 2\pi]$ parametrizes the curve. If $\delta = 0$, then τ is equal to the polar angle φ .

7.4.2 Fidelity of Shadow Fits

For every shadow given by a pair of black hole parameters a and θ_o , we determined the best-fitting ellipse parameters by minimizing the root-mean-squared (RMS) radial residual, weighted by arc length $d\ell(\varphi) = \sqrt{r(\varphi)^2 + r'(\varphi)^2}d\varphi$. Namely, we minimized

$$\sigma(r_{\perp}, r_{\parallel}, \delta) \equiv \sqrt{\frac{\int_0^{2\pi} d\varphi [\Delta x(\varphi)^2 + \Delta y(\varphi)^2] \sqrt{r(\varphi)^2 + r'(\varphi)^2}}{\int_0^{2\pi} d\varphi \sqrt{r(\varphi)^2 + r'(\varphi)^2}}}. \quad (7.7)$$

Weighting by arc length gives results that are independent of how the curve is parametrized, including the assumed centroid. Other weighting choices, such as $d\tau$, $d\varphi$, or the line element dr resulting from parametrization by Boyer-Lindquist radius (Equation 7.3) can heavily upweight particular segments of the shadow curve.

Figure 7.2 shows the best-fitting ellipse parameters and RMS radial residual at four inclinations and all spins. In general, the parameters vary smoothly and monotonically with spin except at very high spin. Note that the shadow radius orthogonal to the spin axis monotonically decreases with spin, while the shadow axis parallel to the spin axis decreases at low inclination but increases at high inclination.

At all non-zero inclinations, the RMS residual increases with spin. As shown in Figure 7.1, the left shadow edge flattens as spin increases, which the ellipse cannot reproduce. Nevertheless, the RMS residual is $\sigma \lesssim 0.1r_g$ for all cases, corresponding to a fractional radial error of approximately 2% or less. For a black hole viewed at low inclination or a black hole with low spin, the accuracy of the ellipse is excellent. For the supermassive black hole in M87, which has $\theta_o \approx 17^\circ$ [351] and $r_g/D \approx 3.8 \mu\text{as}$ [EHT6], the ellipse has $\sigma \lesssim 0.005r_g$ ($\sigma/D \lesssim 0.02 \mu\text{as}$) or a fractional radial error of approximately 0.1%.

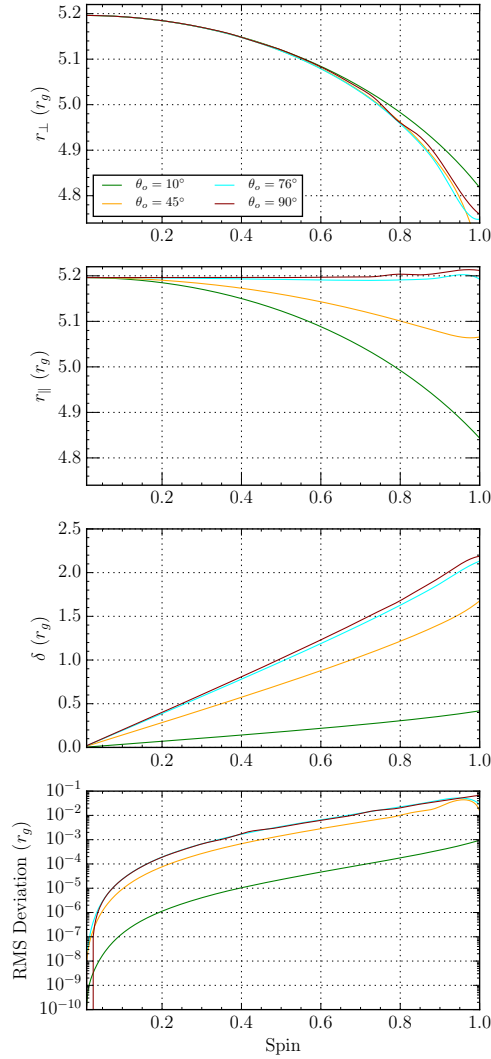


Figure 7.2: Ellipse parameters that best fit the shadow boundary as a function of black hole spin and inclination. From top to bottom, panels show the ellipse radius perpendicular to the projected spin axis ($r_\perp$), the ellipse radius parallel to the projected spin axis ($r_\parallel$), the ellipse displacement (δ), and the RMS error of the fitted curve weighted by arc length ($\sigma = \sqrt{\langle \delta r(\varphi)^2 \rangle_\ell}$).

7.5 Limaçon approximation

7.5.1 Definitions

Our second polar curve to describe the black hole shadow is a limaçon, formed by tracing a fixed point on a circle as it rotates around another circle of equal radius

displaced by one diameter. The limaçon has a simple polar representation that depends on only two parameters:

$$r(\varphi) \equiv \lambda_1(1 + \lambda_2 \cos \varphi). \quad (7.8)$$

Here, φ is the polar angle, $\lambda_1 > 0$ is a scale parameter, while λ_2 contributes both asymmetry and displacement. Note that the limaçon is convex when $|\lambda_2| \leq 1/2$. For $|\lambda_2| > 1$, the limaçon intersects itself at the origin for $\varphi = \pm \cos^{-1}(-\lambda_2^{-1})$. For $\lambda_2 = 1$, the limaçon is exactly a cardioid, with a cusp at $\varphi = \pi$. Because changing the sign of λ_2 simply gives an image reflection, we will assume that $\lambda_2 > 0$ for the remainder of this chapter.

The limaçon exhibits similar behavior to the shadow, shifting itself over from the origin by a similar amount and producing an edge on the left side as it progresses further from the origin. The similarity between the shadow and the limaçon is demonstrated in [Figure 7.3](#) (and is quantified in [subsection 7.5.3](#)). Hence, unlike the ellipse, we do *not* include a shift parameter for fits to the pure limaçon. [\[347\]](#) proposed the limaçon as an approximation to the shadow of a Kerr-Newman black hole and provided several example fits, demonstrating excellent agreement. The quality of these fits declined with increasing spin. In [Section 7.6](#), we will generalize the limaçon by including a shift parameter and an additional operation to ensure convexity; these modifications substantially improve the quality of the shadow fits.

7.5.2 Characteristics

We now compute some simple derived quantities for the limaçon, to clarify how the parameters λ_1 and λ_2 affect the curve and relate to shadow properties.

The simplest characteristics of the limaçon are quantities weighted uniformly by

angle with respect to the origin of the limaçon coordinates. The mean radius is $\langle r \rangle_\varphi = \lambda_1$, while the standard deviation of the radius is $\sqrt{\langle \Delta r^2 \rangle_\varphi} = \lambda_1 \lambda_2 / \sqrt{2}$. The horizontal displacement is $\Delta \equiv \frac{x_{\max} + x_{\min}}{2} = \lambda_1 \lambda_2$.

However, for comparisons with other families of curves (including the Kerr shadow), it is convenient to instead compute quantities with respect to coordinates that are centered on the curve, according to the horizontal displacement, and weighted by arc length. For example,

$$\langle r_\Delta \rangle_\ell \approx \lambda_1 \left(1 + \frac{\lambda_2^2}{4} - \frac{\lambda_2^4}{64} + O(\lambda_2^6) \right). \quad (7.9)$$

It is also useful to evaluate the curve radii along directions orthogonal ($r_\perp$) and parallel ($r_\parallel$) to the vertical axis (i.e., the projected spin axis). These radii are independent of the assumed centroid or curve weighting and have been discussed extensively for the specific case of a black hole's shadow [e.g., 343; 344]. For the limaçon,

$$\begin{aligned} r_\perp &\equiv \frac{1}{2} (\max_\varphi [r(\varphi) \cos(\varphi)] - \min_\varphi [r(\varphi) \cos(\varphi)]) \\ &= \lambda_1 \times \begin{cases} 1 & \lambda_2 \leq 1/2 \\ \frac{(1+2\lambda_2)^2}{8\lambda_2} & \lambda_2 > 1/2, \end{cases} \end{aligned} \quad (7.10)$$

and

$$\begin{aligned} r_\parallel &\equiv \max_\varphi [r(\varphi) \sin(\varphi)] \\ &= \lambda_1 \frac{\left(3 + \sqrt{1 + 8\lambda_2^2} \right) \sqrt{\sqrt{1 + 8\lambda_2^2} + 4\lambda_2^2 - 1}}{8\sqrt{2}\lambda_2} \\ &\approx \lambda_1 \left(1 + \frac{1}{2}\lambda_2^2 - \frac{5}{8}\lambda_2^4 + \dots \right). \end{aligned} \quad (7.11)$$

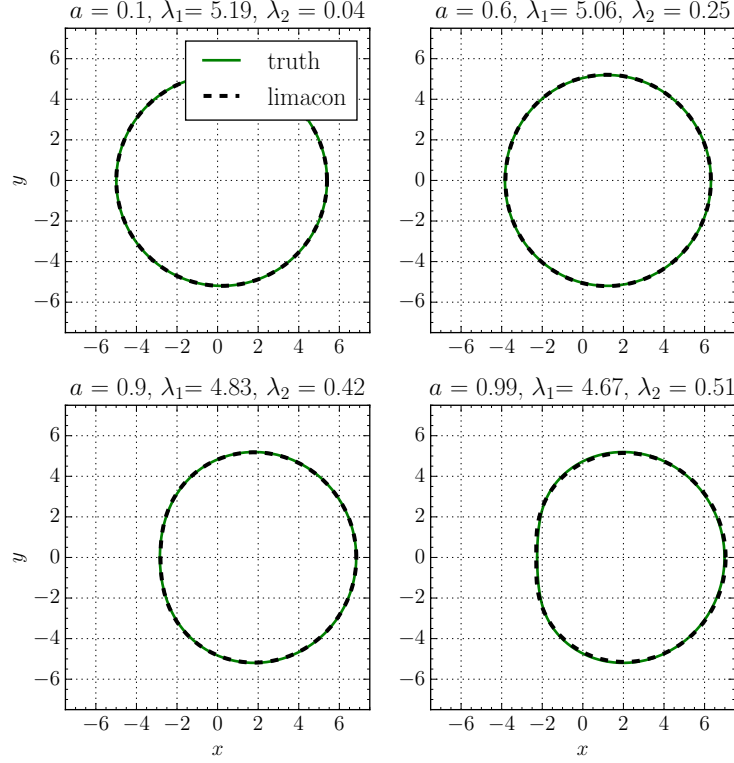


Figure 7.3: Comparison between exact shadows (green, solid) and their best-fitting limaçons (black, dashed) for $\varphi = 90^\circ$ (i.e., edge-on viewing). The limaçon parameter λ_2 increases with spin, producing both the asymmetry and centroid displacement seen in the exact shadow.

7.5.3 Fidelity of Shadow Fits

Figure 7.4 shows the best-fitting limaçon parameters at four inclinations and for all spins, obtained by minimizing the RMS radial residual weighted by arc length as described in Section 7.4.2. The limaçon parameters are monotonic in spin and inclination, and the limaçon generally outperforms the ellipse in approximating the shadow despite requiring one fewer parameter. For $a < 0.95$, limaçons fit the shadow with $\sigma \lesssim 10^{-2} r_g$, or a fractional radial error of less than 0.2%.

Figure 7.5 shows the best-fitting limaçon parameters as a function of black hole spin and inclination. Importantly, the mapping is one-to-one, so the limaçon parameters uniquely determine the black hole spin and inclination if the mass-to-distance ratio of the black hole is known. Because the Kerr metric occupies only a small part of the (λ_1, λ_2) parameter space, a limaçon fit can also be used to assess the validity of the Kerr assumption for the shadow and to quantify departures from Kerr.

7.6 Convex hull limaçon approximation

7.6.1 Definitions

As shown in Figure 7.4, the limaçon provides an excellent approximation to the shadow but is least accurate at high spin and inclinations approaching edge-on. Remarkably, the shape of the shadow in the limit of maximal spin and edge-on inclination is precisely a limaçon with $\lambda_1 = 4r_g$ and $\lambda_2 = 1$, with two modifications. First, it is horizontally displaced from the origin (see Figure 7.6). Namely, it takes the form,

$$r(\varphi; \delta = -1) = 4(1 + \cos \varphi), \quad (7.12)$$

where $\delta = -1$ indicates that r and φ are polar coordinates defined relative to an origin that is horizontally displaced by $\delta = -1$. Second, the shadow for these asymptotic parameters has a hard, flat edge joining the discontinuous portion in the region $\varphi \in [2\pi/3, 4\pi/3]$, which deviates from the signature cusp of the $\lambda_2 = 1$ limaçon. This behavior can be reproduced by taking the convex hull of the shifted limaçon. Equation 7.10 then gives that the shadow of an extremal black hole viewed edge-on has $r_\perp = \frac{9}{2}r_g$ and $r_\parallel = 3\sqrt{3}r_g$. This result was derived by [343], who provide a general

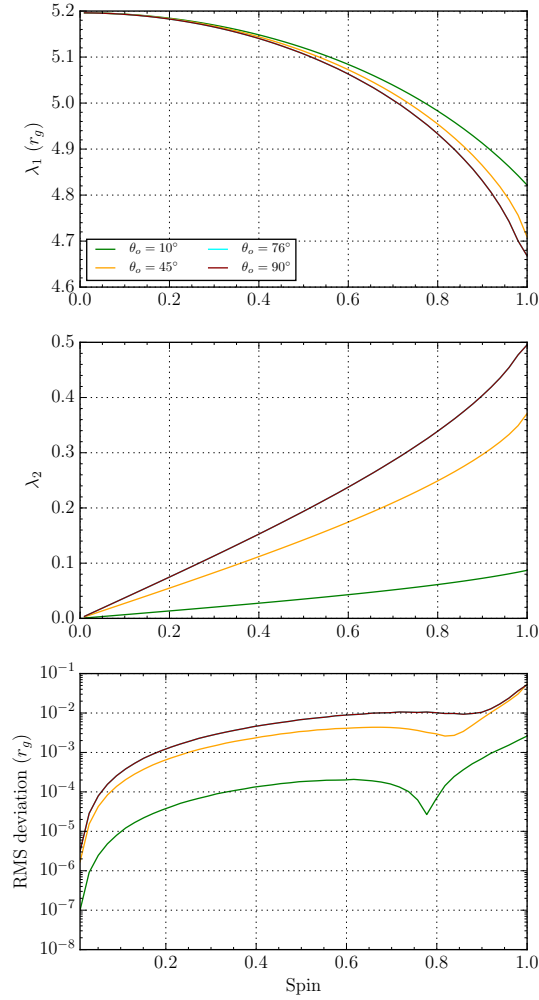


Figure 7.4: Limaçon parameters that best fit the shadow boundary as a function of black hole spin and inclination. From top to bottom, panels show the limaçon scale parameter λ_1 , the limaçon asymmetry parameter λ_2 , and the RMS error of the fitted curve $\sigma = \sqrt{\langle \delta r(\varphi)^2 \rangle_\ell}$. Unlike the ellipse, both limaçon parameters vary monotonically with spin and inclination (although the RMS error is not monotonic in spin).

expression for $r_\perp$ and $r_\parallel$ for a black hole of any spin viewed edge-on.

Motivated by the exact form of the shadow in this limit, our third family of parametric curves is the convex hull of a shifted limaçon. This curve adds a single parameter, the shift δ ; the convex hull is only relevant for $|\lambda_2| > 1/2$, where a flat edge is intro-

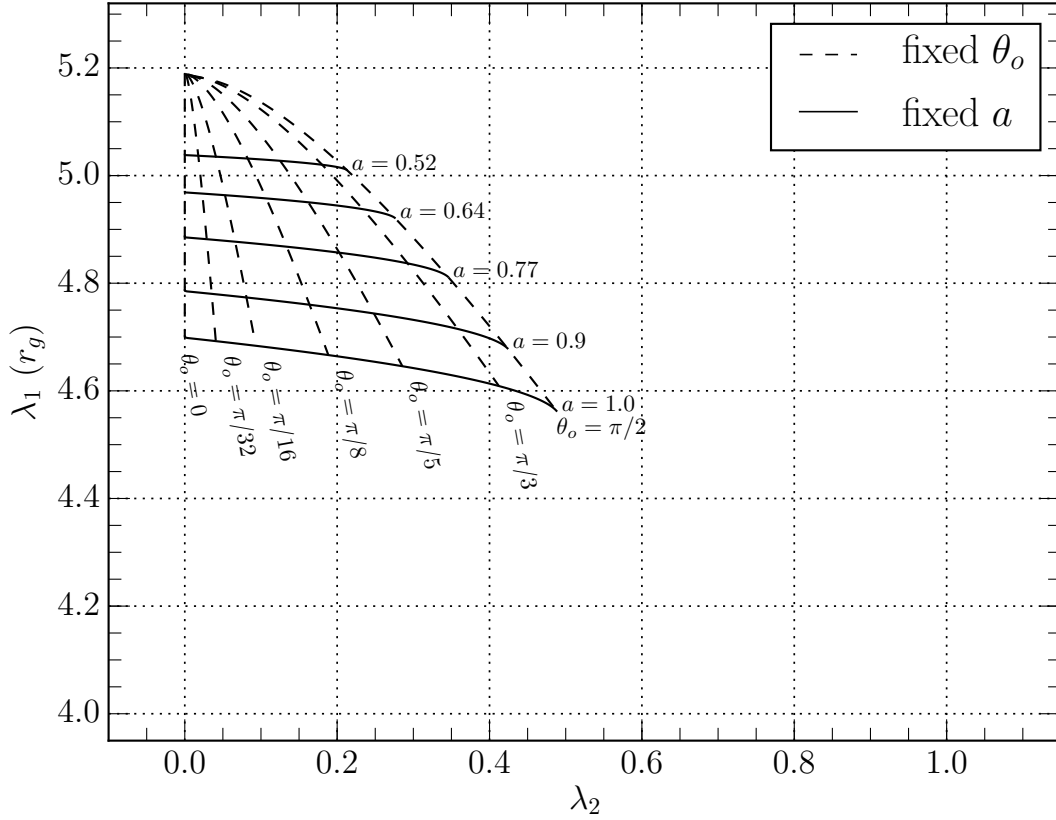


Figure 7.5: Best-fitting limaçon parameters (λ_1, λ_2) for all spins and inclinations.

duced at $\varphi = \pm \arccos(-1/2\lambda_2)$ or at $x_{\min} = \delta - \lambda_1/4\lambda_2$. In the Kerr parameter space, the convex hull is only needed for $a \gtrsim 0.95$.

7.6.2 Fidelity of Shadow Fits

Using the methodology described in [subsection 7.4.2](#), limaçons were fit to shadow shapes at four inclinations for all spin. The results are shown in [Figure 7.7](#).

The convex hull limaçon offers fits that can be an order of magnitude better than those of the unshifted limaçon or the ellipse. Size and asymmetry parameter variation are monotonic, but the curves quantifying the horizontal shift of the approximation

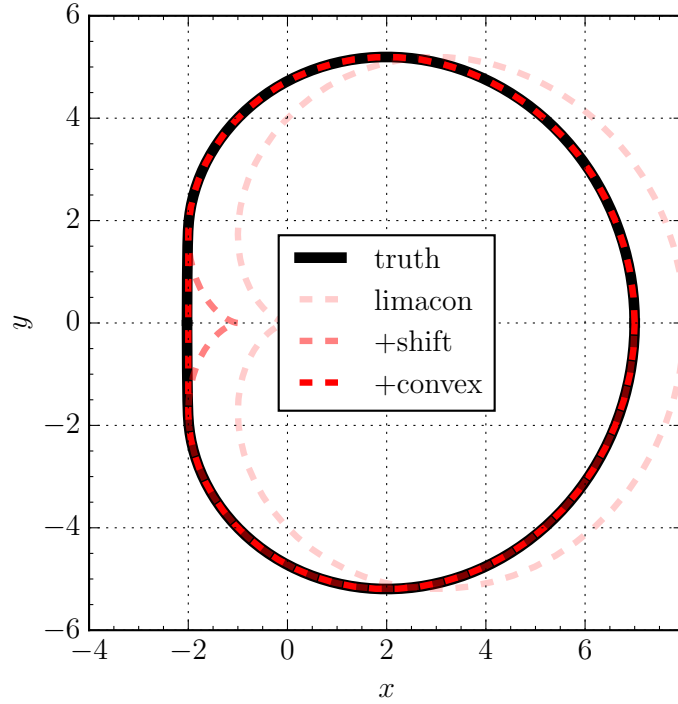


Figure 7.6: In the maximal-spin, edge-on limit, the shadow boundary (black) is the convex hull of a shifted limaçon with $\lambda_1 = 4$, $\lambda_2 = 1$, and $\delta = -1$. Because $\lambda_2 > 1/2$, the pure limaçon curls into a cardioid. Hence, both the displacement and convex hull are necessary to exactly match the shadow.

increase in concavity while shifting downwards as inclination approaches $\theta_o = \pi/2$. All inclinations and spins have a point at around $a = 0.8$ where $\delta = 0$; here, the best-fitting convex hull limaçons match the normal, unshifted limaçon provided that $\lambda_2 < 1/2$. Indeed, this agreement can be seen in the intersections with the lighter, dotted curves in the bottom panel of [Figure 7.7](#) representing the resulting accuracy of the limaçon fits from [Figure 7.4](#). The fit fidelity generally worsens as the asymmetry of the true shadow increases, but it improves sharply near maximal spin.

While the diagnostic quantities of the convex hull limaçon are not as trivial to derive as those of the ordinary limaçon, the convex hull limaçon similarly constrains the Kerr metric tightly in the (λ_1, λ_2) parameter space, as shown in [Figure 7.8](#).

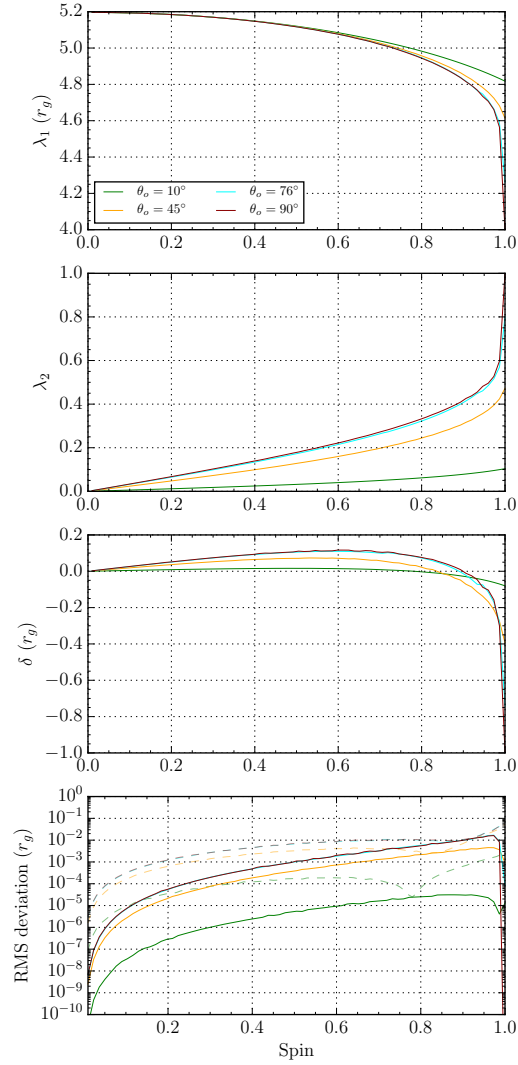


Figure 7.7: Convex hull limaçon parameters that best fit the shadow boundary as a function of black hole spin and inclination. From top to bottom, panels show the scale parameter λ_1 , the asymmetry parameter λ_2 , the displacement δ , and the RMS radial residual $\sqrt{\langle \delta r(\varphi)^2 \rangle_\ell}$. For comparison, the RMS error of the limaçon is also shown as dashed lines (see Figure 7.4). All inclinations and spins have a point near $a = 0.8$ where $\delta = 0$; at this location the limaçon and convex hull limaçon fit equally well unless $\lambda_2 > 1/2$ because the only improvement in the convex hull limaçon is its additional shift parameter.

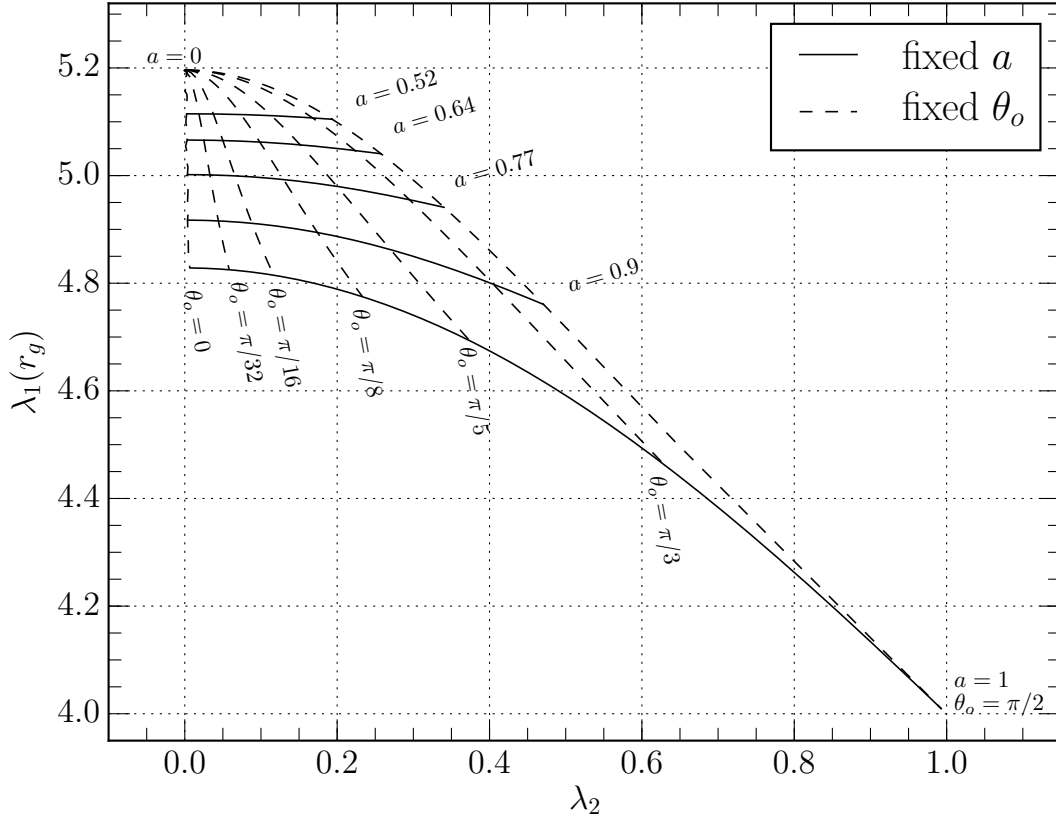


Figure 7.8: Best-fitting convex hull limaçon parameters (λ_1, λ_2) for all spins and inclinations (a displacement parameter δ is fitted but not shown). As for the limaçon, the mapping uniquely determines the black hole spin and inclination if the mass-to-distance ratio (i.e., r_g) is known. Moreover, the Kerr metric occupies only a small part of the (λ_1, λ_2) parameter space, so a convex hull limaçon fit can be used to assess the validity of a Kerr assumption for the shadow and to quantify departures from Kerr.

7.7 Discussion

7.7.1 Comparison of Polar Approximations

Figure 7.9 compares the accuracy for each of our parametric shadow approximations as a function of spin and inclination. For low spin, $a \lesssim 0.5$, the black hole shadow is

nearly circular and all approximations are excellent (even a circle fits reasonably well). At high spin, the circle ceases to be a reasonable fit except at very low inclination, and it has a fractional radial RMS of approximately 4% at high inclination. The limaçon, which can produce a flat left edge, outperforms the ellipse at high spin and inclination, while the convex hull limaçon significantly outperforms both the ellipse and limaçon at all inclinations for maximal spin.

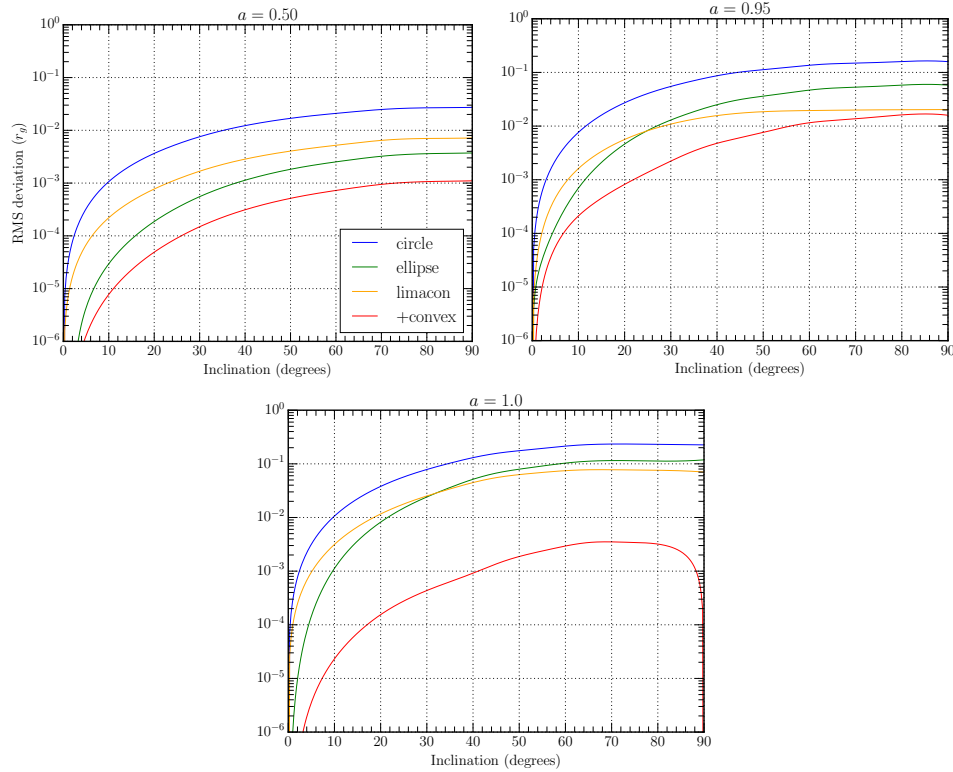


Figure 7.9: Accuracy of best-fitting circle (blue), ellipse (green), limaçon (orange), and convex hull limaçon (red) approximations as a function of inclination for $a = 0.5$ (left), $a = 0.95$ (center), and $a = 1$ (right).

7.7.2 Implications of Accurate Polar Approximations

The accuracy of simple curves such as the limaçon in describing the shadow has subtle implications for the general shadow geometry. For example, the limaçon is defined

by only two parameters, implying that the displacement is determined entirely by the mean radius and asymmetry. This is analogous to the exact shadow, in which the shape is determined by the two parameters of black hole mass and spin. The limaçon relationship can be expressed using “observable” quantities such as the perpendicular radius $r_{\perp}$ and the area $A = \pi\lambda_1^2 (1 + \lambda_2^2/2)$:

$$\Delta = \sqrt{2}r_{\perp} \sqrt{\frac{A}{\pi r_{\perp}^2} - 1}. \quad (7.13)$$

This relationship is exact for a limaçon with $\lambda_2 < 1/2$ and is approximately true for the black hole shadow (Δ is correct to within $0.4r_g$ and has a fractional error smaller than $\approx 20\%$ for all values of spin and inclination). It shows that a shadow displacement only occurs when there is asymmetry and that the two effects are intimately linked. This relationship also provides a simple way to estimate the black hole’s location on an image from measurements of the shadow size and shape.

7.7.3 Degeneracies of Inferred Black Hole Parameters

Another useful application of our geometrical shadow approximations is to assess the degeneracies in spin and inclination. Apart from the trivial inclination degeneracy at zero spin, the shadow has a single additional discrete degeneracy for supplementary inclination angles: $\theta_o \rightarrow \pi - \theta_o$ [see, e.g., 352]. However, if the mass-to-distance ratio of the black hole is unknown, then shadows that differ by a constant rescaling are indistinguishable (e.g., all face-on observers see a perfectly circular shadow, regardless of spin). Moreover, identification of the coordinate origin is impractical, so shadows that differ only by a displacement will likely be indistinguishable. Finally, shadows that may not be formally degenerate may still have differences that are so small as to

be indistinguishable for all practical purposes. Our framework allows us to assess all these cases.

These various classes of degeneracies are evident in Figures 7.5 and 7.8. For example, the convex hull limaçon fits make no assumptions about knowledge of the coordinate centroid. For both the limaçon and the convex hull limaçon, a priori knowledge of the mass-to-distance ratio is necessary to place a measurement of λ_1 in units of r_g , while λ_2 is dimensionless and requires no a priori knowledge.

For instance, because of the a priori estimates of r_g for Sgr A* using stellar orbits, measuring the (convex hull) limaçon parameters λ_1 and λ_2 for Sgr A* would provide unambiguous estimates for both the spin and inclination of the black hole (up to the single discrete degeneracy for supplementary inclination angles). Indeed, a measurement of λ_1 alone is sufficient to narrowly constrain the spin and (if $\lambda_1 < 4.83r_g$) to give a lower bound on the inclination.

Likewise, while the supermassive black hole in M87 does not have a strong prior on r_g and, thus, only λ_2 can be measured directly, it does have a tightly constrained jet inclination. A measurement of λ_2 would then determine the spin magnitude a under the assumption that the spin axis is aligned with the jet.

For sources with no prior constraints on r_g or θ_o , a measurement of λ_2 provides a lower limit on both spin and inclination but does not absolutely determine either. Larger values of spin and inclination would correspond to smaller values of λ_1/r_g and, thus, larger values of the black hole mass. Because the convex hull limaçon is an excellent approximation to the true shadow, these degeneracies are unlikely to be resolved even with highly accurate measurements of the shadow shape, although they could be broken with complementary measurements (e.g., temporal signatures, modeling of the complete system, or relationships between sequential photon subrings).

7.8 Summary

We have shown that simple polar curves can provide excellent approximations to the exact boundary of the shadow from a Kerr black hole ([Equation 7.3](#)). A suitable polar curve must account for three geometric properties of the black hole shadow: diameter, asymmetry, and horizontal displacement. For a Kerr black hole, the shadow diameter and displacement are directly proportional to the black hole mass, while all three properties are affected by the spin magnitude and inclination.

We first explored fitting the shadow with a displaced ellipse, which provides a marked improvement in fit quality over a circle (see [Figure 7.9](#)) and offers convenient analytic expressions for directly fitting interferometric quantities. The RMS error in radius, weighted by arc length, is less than $0.1r_g$ for all spins and inclinations, corresponding to a fractional error of only 2%. For a source viewed at low inclination, as is expected for the supermassive black hole in M87, the fractional RMS error is less than 0.1% (corresponding to $0.02 \mu\text{as}$ for M87). Hence, we expect the ellipse to provide an excellent shadow approximation when fitting parametric models to EHT observations of M87, with sufficient complexity to provide estimates of the shadow asymmetry and, hence, the black hole inclination and spin. A set of ellipse parameters uniquely determines the black hole spin if the mass-to-distance ratio or inclination is known a priori (as is relevant for Sgr A* and M87, respectively).

Next, we explored fitting the shadow with a family of curves known as limaçons, which are two-parameter polar curves, as originally proposed and studied by [\[347\]](#). We showed that a limaçon often fits the shadow better than an ellipse despite requiring one fewer parameter. In addition, limaçons have an inherent relationship between displacement and asymmetry just as Kerr black holes do. Hence, they provide convenient expressions for this relationship for the exact black hole shadow (e.g., [Equation 7.13](#)).

Finally, we explored fitting the shadow with a generalized limaçon, adding a displacement parameter and taking a convex hull. Especially at high spin, this convex hull limaçon provides a significantly better fit than the unmodified limaçon or ellipse. The best-fitting convex hull limaçon has a fractional RMS radial residual of less than 0.3% for all black hole spins and inclinations. It also gives an exact description of the shadow in the limit of a maximally spinning black hole viewed edge-on.

These polar curves offer many utilities including parametric model fitting to interferometric data, simple diagnostics of the shadow properties, and a low-dimensional geometrical representation of the full shadow shape (just as the exact shadow is defined solely by the mass and spin of the black hole). They also reveal degeneracies that are expected when inferring a black hole's parameters from images of its shadow. For instance, because a displacement cannot be directly observed and the convex hull limaçon provides an excellent fit to the shadow shape, an accurately measured shadow only gives a bounded range for the black hole mass as well as lower limits on the spin and inclination (see [Figure 7.8](#)). However, an a priori measurement of the black hole mass, spin, or inclination is sufficient to break this degeneracy, yielding the other two parameters from the measured shadow. Finally, our curves provide a flexible parametric framework to assess whether a particular shadow is compatible with the Kerr metric. While simple circular models were suitable for the initial EHT observations of M87, these curves offer a pathway to interpret the increasingly sensitive observations expected in the coming decades, with correspondingly more stringent demands for model fidelity.

Chapter 8

Selective dynamical interferometric imaging of horizon-scale accretion flows



THERE IS A TIME FOR EVERYTHING,
AND A SEASON FOR EVERY ACTIVITY UNDER THE HEAVENS.

—Ecclesiastes 3:1



Recent developments in very-long-baseline interferometry (VLBI) have made it possible for the Event Horizon Telescope (EHT) to resolve the innermost accretion flows of the largest supermassive black holes on the sky. The sparse nature of the EHT's (u, v) coverage presents a challenge when attempting to resolve highly time-variable sources. We demonstrate that the changing (u, v) coverage of the EHT can contain regions of time over the course of a single observation that facilitate dynamical imaging. These optimal time regions typically have projected baseline distributions that are

approximately angularly isotropic and radially homogeneous. We derive a metric of coverage quality based on baseline isotropy and density that is capable of ranking array configurations by their ability to produce accurate dynamical reconstructions. We compare this metric to existing metrics in the literature, and investigate their utility by performing dynamical reconstructions on synthetic data from simulated EHT observations of sources with simple orbital variability. We then use these results to make recommendations for imaging the 2017 EHT Sgr A* dataset.

8.1 Permissions and Attributions

1. The content of [chapter 8](#) is the result of a collaboration with the Event Horizon Telescope Collaboration, and has previously appeared in the *Astrophysical Journal Letters* as Joseph R. Farah *et al.* (2022). It is reproduced here with the permission of the *Astrophysical Journal*: <https://iopscience.iop.org/article/10.3847/2041-8213/ac6615>. Full acknowledgements are available on the journal page.

8.2 Introduction

Interferometric astronomical observations offer much larger resolving power than do single telescopes, with the interferometric resolution depending on the distance between the elements rather than the diameters of the individual apertures. Since an interferometer probes the Fourier transform of an on-sky source (and not the source image itself), the placement, selection and availability of baselines to maximize coverage of the (u, v) plane is an important and open optimization problem inherent to the interferometric image synthesis. For very-long-baseline interferometry (VLBI), which represents some of the most extreme interferometric observations, the arrays are gen-

erally very sparse, and optimized placement or selection of baselines becomes critical to recovering reliable information about the source.

For any ground-based interferometric observations, including VLBI, Earth rotation causes each individual baseline to trace an elliptical path in the (u, v) plane. Such “Earth rotation aperture synthesis” can be used to improve the (u, v) coverage of sparse arrays. Standard VLBI imaging uses this additional coverage under the assumption that the target source structure remains unchanged for the duration of the observation. For the cases of sources that vary on shorter timescales (e.g., the Galactic Center, x-ray binaries, microquasars), “snapshot images” can be produced using only time intervals over which the source can be considered static [e.g., 353; 354; 272]. In the most extreme scenarios, only instantaneous coverage may be suitable for use in static image reconstructions, severely limiting the (u, v) coverage and resulting image quality. Such a case is the main concern of this chapter. Moreover, this limitation will generally result in portions of an observation that are better than others for imaging because the snapshot coverage is time-dependent.

In particular, directionally-biased (u, v) coverage results in a non-isotropic resolution that complicates interpretation of the reconstructed source geometry. The (u, v) coverage of a given array varies with the declination of the source, due to its dependence on the projected baseline lengths and orientations. The impact of source declination on the (u, v) coverage varies throughout a night of observation as the target rises and sets. A network of baselines which produces an evenly distributed (u, v) coverage of an equatorial source will be East-West biased when targeting a northern source. Observations where the source morphology and orientation of the source are unknown cannot exploit *a priori* knowledge to determine whether a directionally-biased (u, v) configuration will be able to reproduce source structure reliably in an image. Therefore, for cases where the source morphology is unknown, the optimal

(u, v) coverage for producing high-quality image reconstructions will be those that are approximately isotropic in angular distribution.

Periods of sustained isotropic (u, v) coverage may allow multiple high-quality image reconstructions on short timescales to be strung together, resulting in “dynamical” reconstructions. For instance, the most basic dynamical reconstruction can be produced by joining a series of snapshot images. The quality of such a reconstruction will be time-dependent due to the evolving snapshot coverage, and it may be necessary to exclude periods with poor (u, v) coverage. In periods of time where the (u, v) coverage is optimal, the coverage used to produce each snapshot of a dynamical reconstruction will be sufficiently isotropic to reproduce source features on snapshot timescales, allowing for the production of reconstructions that adequately recover the evolution of highly time-variable sources. By contrast, periods of sub-optimal (u, v) coverage may be unable to provide high-quality reconstructions on snapshot timescales, making the algorithmic detection and subsequent flagging of these periods important for the production of meaningful dynamical reconstructions.

The challenges of observing a variable source with a sparse interferometer can be exacerbated by poor coverage geometry. In particular, given the shortage of available facilities supporting high frequency radioastronomical observations, corresponding VLBI arrays exhibit substantial variance in coverage quality in time.

The Event Horizon Telescope (EHT) is a unique VLBI network of telescopes that exploits the full diameter of the Earth and the performance at the challenging 1.3 mm (230 GHz) wavelength to achieve the required angular resolution for horizon-scale tests of general relativity for the largest black holes on the sky (see e.g., [341; 279; 280]). The EHT operates in millimeter wavelengths, the optimal range which enables the resolution of the Sgr A* black hole shadow and reduces the impact of the interstellar medium scattering effects dominant for longer wavelengths, while still

being observable and manageable in the radio interferometric framework. Operating at 230 GHz, the EHT achieved a resolution of $\sim 20\text{--}25\ \mu\text{as}$, which the EHT Collaboration (EHTC) used to produce the first images of a supermassive black hole [266; 290; 291; 5; 292; 293]. These data and the resulting images were used to estimate a mass of $M \approx 6.5 \times 10^9 M_\odot$ for the supermassive black hole in M87 [293], based on an observed angular shadow diameter of $\sim 42\ \mu\text{as}$ [266; 5; 293].

During the 2017 campaign, the EHT also observed the radio source Sgr A* in the Galactic Center [355–360], associated with a supermassive black hole with $M \approx 4.1 \times 10^6 M_\odot$ [361; 362; 360]. The expected mass-to-distance ratio (M/D) of Sgr A* yields a predicted angular shadow diameter of $\sim 50\ \mu\text{as}$ [363], and a minimum variability timescale (light crossing time) of $GM/c^3 \approx 20$ seconds. The corresponding timescale for M87 is ~ 1600 times longer due to the larger mass. Indeed, structural variability of the M87 shadow has been reported on timescales from ~ 1 week [5] to several years [364]. The rapid minimum variability timescale of Sgr A* combined with the extreme sparsity of the EHT presents an urgent and unique need to characterize the effects of time-dependent instantaneous (u, v) coverage.

In this chapter, we develop a procedure for selective imaging of highly variable sources. In section 8.3 and section 8.4, we summarize the synthetic data generation and imaging methods used herein. In section 8.5, we show the limitations of imaging in sparse and uneven coverage. In section 8.6, we survey several metrics capable of ranking (u, v) coverage quality. Additionally, we derive a novel isotropy-based metric which addresses the limitations described in section 8.5. In section 8.7, we apply these metrics to the 2017 EHT coverage of Sgr A*, validate their ability to predict reconstruction quality from (u, v) coverage geometry, and make recommendations for selective dynamical imaging of the 2017 EHT Sgr A* dataset. In section 8.8, we briefly discuss the utility of coverage metrics in ranking and selecting between different

available observing periods. Finally, in [section 8.9](#), we summarize our results.

8.3 Model definition and synthetic data generation

In order to test the ability of various EHT array configurations to recover source variability in different observation periods, we designed and generated synthetic data for three different models. The models are chosen due to their structural similarity (in image and visibility domain) to expected images of Sgr A*. Similarity between the model and data was characterized as either displaying time variability or producing a Bessel-function Fourier representation with nulls between 2 and 4 $G\lambda$ and between 6 and 9 $G\lambda$. The models are described in [subsection 8.3.1](#). The synthetic data generation is expanded upon in [subsection 8.3.2](#).

8.3.1 Models

Here, we describe the suite of models used to test the effects of coverage on reconstructions. Examples of each model with CLEAN beam convolution can be seen in the first row of [Figure 8.1](#).

Rotating elliptical Gaussian

The rotating elliptical Gaussian model is generated using a bivariate exponential with major axis full-width-half-maximum (FWHM) Γ_a , minor axis FWHM Γ_b , and overall flux density A :

$$I(x, y) = \frac{4A \ln 2}{\pi \Gamma_a \Gamma_b} \exp \left(-4 \ln 2 \left[\left(\frac{x}{\Gamma_a} \right)^2 + \left(\frac{y}{\Gamma_b} \right)^2 \right] \right). \quad (8.1)$$

The image is padded and rotated by polar angle φ after the model is generated. By default, the overall flux density A was set to 1.0 Jy. Periods for the rotation ranged between 30 and 1000 minutes (longer than one night of observation).

Ring and orbiting hotspot

The ring model is generated by the subtraction of two concentric uniform-brightness disks, equivalent to the crescent model described in [349], with the parameters $a = b = 0$. The positive disk has a radius of $25 \mu\text{as}$ and the subtracted disk has a radius of $18 \mu\text{as}$. All ring models in this chapter use these parameters with the exception of the ring model in Figure 8.4, which has a subtracted disk radius of $20 \mu\text{as}$. The ring model is used with two synthetic data tests: the static ring (with no hotspot) and the dynamic ring (a static ring plus an orbiting hotspot, referred to as ring+hs). The underlying ring has a diameter of $50 \mu\text{as}$ and a flux density of 1.0 Jy. A hotspot total flux density of 0.25 Jy and a FWHM of $10 \mu\text{as}$ is added to the image, centered on the ring. After construction, the total flux density of each image is normalized to 1.0 Jy. In the dynamic ring model, the orbiting hotspot is centered on the ring and circularly orbiting at a distance of $21.5 \mu\text{as}$ with periods of 30 and 270 minutes. The static ring model is a special case of the ring+hotspot model with the flux density of the hotspot set to zero.

8.3.2 Synthetic data generation

Synthetic data were generated based on April 7 of the 2017 EHT coverage using the `eht-imaging` library [92]. A simulated observation corresponded to approximately 11.5 hours of observing on all available baselines. Snapshot images from the simulated sources and the resulting amplitudes and closure phases can be seen in Figure 8.1 and

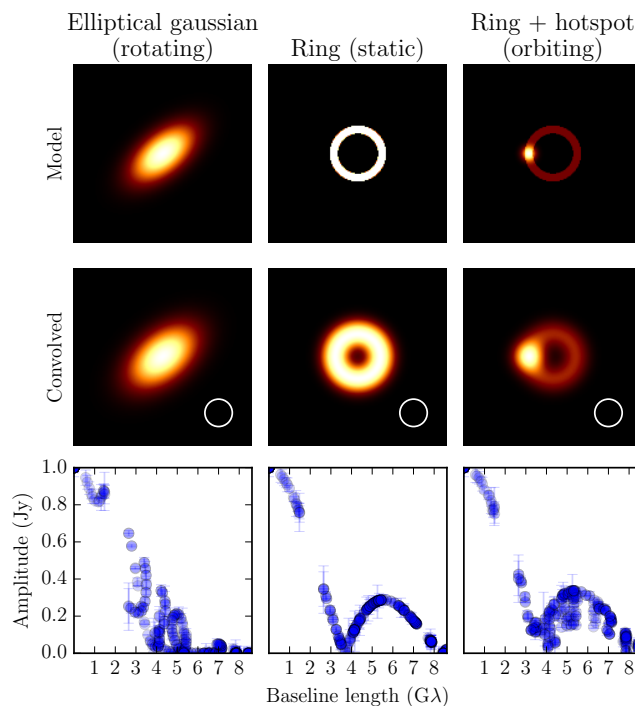


Figure 8.1: The three synthetic models detailed in [section 8.3](#) are displayed. The first row shows each model as seen on-sky at 6 UT, just after the observation begins. The second row shows the model convolved with an $18 \mu\text{as}$ diameter CLEAN beam. The white circle in the lower right shows the size of the beam. The third row shows the measured visibility amplitudes as a function of baseline length for the entire observation. A static imaging routine would fit to the full set of these amplitudes; however, a dynamical imaging routine only attempts to fit to small chunks of the full dataset at any one time.

[section B.1.](#)

Observation parameters (e.g., right ascension and declination of the source, observing frequency, bandwidth) were duplicated from the 2017 EHT survey of Sgr A*. The stations included in the simulated observations were the Atacama Large Millimeter Array (ALMA), the Atacama Pathfinder Experiment (APEX), the Large Millimeter Telescope (LMT), the James Clerk Maxwell Telescope (JCMT), the Submillimeter Array (SMA), the IRAM 30-meter telescope on Pico Veleta (PV), and the South Pole Telescope (SPT). Weather complications were ignored and the simulated observations assumed

all stations were observing the source for the chosen night.

All data were generated with thermal noise only. Realistic values for the thermal noise power were based on estimates from the real 2017 EHT data. No other noise, scattering, leakage, or simulated gain errors were applied to the simulated visibilities.

8.4 Imaging approaches

Since the interferometric measurements are often incomplete in the Fourier domain, the inverse problem of reconstructing an image from the observed data set is usually under-determined. Consequently, the image reconstruction requires prior information, assumptions, or constraints to derive a reasonable image from the infinite number of possibilities that can explain the measurements.

The two most popular categories of imaging methodologies are inverse modeling (e.g., CLEAN) and forward modeling (e.g., regularized maximum likelihood). See [5] for a general overview of the two methods. For time-variable sources, both approaches may allow for more effective reconstructions of dynamic structures than snapshot imaging by including assumptions or constraints on temporal variations of the source structure in addition to the spatial properties regularized in static imaging [e.g. 5]. One such way imposes a temporal similarity constraint between images at different times, and between each time snapshot and the time-averaged structure. In the following Subsections, we briefly describe each dynamical reconstruction method used in this chapter. See [357] for more details.

8.4.1 Inverse-modeling approaches

Imaging of radio interferometric data is traditionally carried out through CLEAN deconvolution algorithms [e.g., 319; 320]. These inverse modeling approaches iteratively

deconvolve the effects associated with the limited sampling of the (u, v) plane, corresponding to the interferometer’s point source response (the so-called “dirty beam”) to the inverse Fourier transform of the measured visibilities, commonly referred to as the “dirty image”. The source brightness distribution is modelled as a collection of point sources, which are extracted at the location of the peaks in the dirty image through an iterative process until some specified stopping criteria is reached. In observations with limited (u, v) sampling, such as those obtained with the EHT, it is important to guide the CLEAN deconvolution process through the inclusion of the so-called “cleaning windows”, restricting the sky areas within which the point components are localized.

Mitigation of the *a priori* calibration uncertainties is commonly carried out through multiple rounds of CLEAN deconvolution followed by self-calibration, which solves for the station gains that maximize consistency between the current model and the measured visibilities [e.g., 321; 365–367]. Amplitude self-calibrations are necessarily limited to intervals of time larger than the expected variability in order to retain information about source variability. The final image is obtained by convolving the model components with a Gaussian CLEAN beam that approximates the central lobe of the point-spread function of the interferometer, with the addition of the last residual image, which represents some unconvolved additional structure and noise. In this chapter we use the `Di fmap` software [e.g., 368; 369] for CLEAN imaging.

Once the imaging procedure converges based on a specified stopping criterion into an average static image, CLEAN dynamic imaging is performed by first dividing the data set into smaller portions with a time duration similar to that of the expected source time scale variability (i.e., “snapshots”). Under the assumption of small structural changes over time, the model corresponding to the static image is used as an initial model, upon which we look for structural changes by cleaning the residual map corresponding to each data snapshot. To guide the deconvolution with such a limited

(u, v) coverage, we limit the extra cleaning to the imaging regions in which we have emission in the averaged image by placing tight cleaning windows. In addition, further self-calibrations in phase and amplitude are performed to refine antenna gain corrections.

The CLEAN algorithms do not enforce similarity between snapshots, other than the use of common initial image priors, which facilitates tracking of rapid source structural changes at arbitrarily separated spatial locations. However, these image changes are restricted to occur within the tight cleaning windows established around the emission found in the averaged static image.

8.4.2 Forward-modeling approaches

Unlike the inverse-modeling methods, which solve for a sparse image on the image domain from the dirty map transformed from the measurement sets, the forward modeling methods solve for an image by evaluating the data likelihood derived from the consistency between actual measurements and the model dataset forward-transformed from the image. It offers flexibility to the imaging through robust data products (e.g., closure quantities that are not affected by station-based calibration uncertainties) and incorporates various observing effects into the observational equation used in the forward transform.

Regularized Maximum Likelihood (RML) methods [see 5, for an overview] optimize a cost function composed of χ^2 terms (proportional to log-likelihood terms) of visibility components and regularization terms that describe the prior assumptions for images. Each regularization term is described by a product of its relative weight (i.e. hyperparameter) and regularization functions. These regularization functions include, e.g., maximum entropy [e.g. 323; 370], total variation and its variants [e.g.

371; 372], and sparsity priors [e.g. 373]. The cost function can be interpreted as a maximum-a-posteriori (MAP) estimation by considering the regularization terms as log prior distribution of the image, although regularization functions do not always have a probabilistic interpretation. The final reconstruction is convolved with the CLEAN beam of the interferometer to remove the effects of methodology-specific super-resolution.

The RML approach can be extended to dynamic reconstruction (henceforth RML dynamic imaging) in a conceptually simple way [324]. The likelihood term can be formulated by forward-transforming snapshots of a video, instead of a single image, to data. One can add temporal regularization terms, that penalize temporal variations of the source structure by defining a metric for the “distance” between adjacent frames. A popular choice is a sum of squared pixel differences between two adjacent snapshots, assuming that snapshot-to-snapshot transition of the source brightness is piecewise smooth (e.g. the $R_{\Delta t}$ regularizer in [324]). Another widely-used choice is a sum of squared differences between the time-averaged image and each snapshot, based on an assumption conceptually similar to dynamic CLEAN imaging (subsection 8.4.1) that the deviations of each snapshot from the mean image is small and sparse (e.g. $R_{\Delta I}$ regularizer in [324]). The temporal regularization term necessarily suppresses intrinsic source variability if weighted too high; however, what constitutes “too high” varies depending on the source structure and variability timescale. Popular image distance metrics include the Euclidean norm or a relative entropy such as Kullback–Leibler divergence [374].

StarWarps [325] is another forward-modeling method for dynamical imaging adopted in this work. StarWarps, based on a probabilistic graphical model, solves for snapshots of a video by solving its posterior probability distribution defined as a product of three terms: data likelihood, multi-variate Gaussian distributions for each snapshot,

and transitional probabilities between adjacent snapshots effectively working as spatial and temporal regularizations, respectively. StarWarps allows for the exact inference of the video by computing a mean and covariance of the image, which provides a complete description under Gaussian approximation. By contrast, the RML dynamic reconstruction derives only a MAP estimation. StarWarps requires an initial static image, which can either be data-driven (e.g., a best-fitting static image of the entire dataset being dynamically reconstructed) or prior-driven (e.g., a synthetic image of a ring).

In this chapter, we use the RML dynamic imaging algorithms implemented in `eht-imaging` (also referred to as `ehtim`) and SMILI (Sparse Modeling Imaging Library for Interferometry), and the StarWarps algorithm in `eht-imaging`. See [357] for more details of regularization functions and other imaging parameters used in the reconstructions.

8.5 Limitations of sparse (u, v) coverage

In this section, we explore limitations of imaging with limited and directionally-biased (u, v) coverage. In particular, we will show that sparse (u, v) coverage results in predictable limitations (i.e., deviations from the true source morphology), which are determined by the geometry of the (u, v) coverage.

Figure 8.2 shows how imaging using directionally-biased (u, v) coverage (column A) fails to properly recover the orientation of an intrinsically non-circularly-symmetric source when the (u, v) coverage does not sufficiently sample the source structure in the relevant direction. By contrast, the same baselines—oriented in a more isotropic way (column B)—are capable of recovering the source profile in all directions. In addition, reconstructions of circularly symmetric sources by directionally-biased (u, v) coverage

(column A of [Figure 8.3](#)) can introduce a lack of circular symmetry that is not present in the underlying source or in reconstructions performed on non-directionally-biased coverage (column B of [Figure 8.3](#)). Due to the minimization of the artifacts introduced into a reconstruction via incomplete (u, v) coverage, imaging algorithms work better when applied to isotropic (u, v) coverages.

In addition to angular inhomogeneity, radially inhomogeneous coverage also leads to ambiguous image reconstruction. The Fourier transforms of various source types (e.g., rings, crescents, Gaussians, etc.) are approximately degenerate when observed using an interferometer that only marginally resolves the image [e.g., [91](#); [375](#)]. More complex approximate degeneracies exist for radially inhomogeneous interferometers which only probe short and long but not intermediate baselines [e.g., [289](#)]. For the EHT observing a $\sim 50 \mu\text{as}$ diameter ring, as expected for Sgr A*, short baselines correspond to those with length $< 2 G\lambda$ and long baselines correspond to those with length $> 6 G\lambda$. [Figure 8.4](#) demonstrates that a Gaussian model describes the simulated EHT observation of a $50 \mu\text{as}$ static ring nearly as well as the static ring model itself if only particular subsets of the data are fit. Even an infinitesimally thin ring model, when only fit to medium and long baselines, can provide a high-quality fit while misrepresenting the total intensity of the source. Without sufficient radial homogeneity (i.e., coverage of short, medium, and long baselines, as seen in column C of [Figure 8.2](#)), fitting and interpreting a model confidently can be difficult.

Periods of (u, v) coverage where these limitations are more likely to occur can be identified by constructing a metric that scores directional bias and radial homogeneity (i.e., coverage of short, medium, and long baselines). The prevalence and severity of reconstruction artifacts that result from coverage limitations form a continuum that can be used to rank different (u, v) configurations. A metric based on these limitations could be applied to a full observation to distinguish different observing

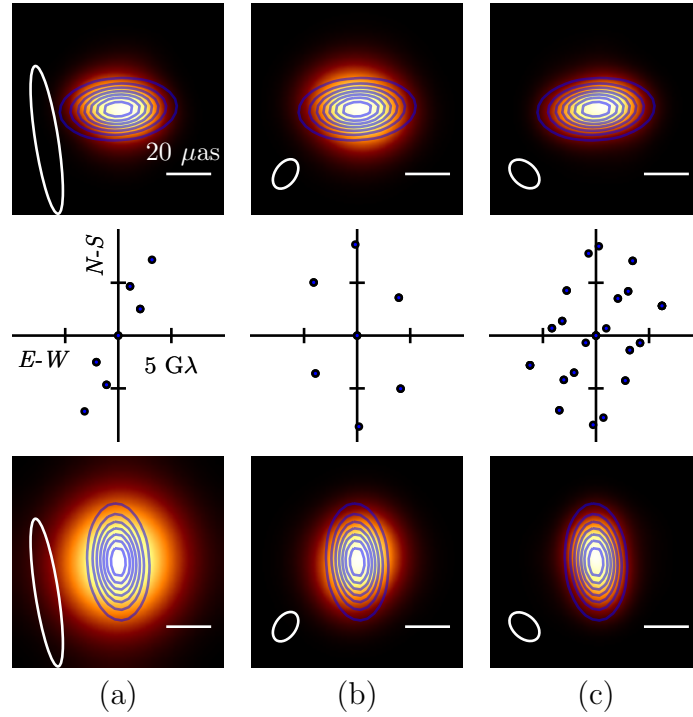


Figure 8.2: Images from a snapshot RML reconstruction of a simulated 2017 EHT observation of a rotating elliptical Gaussian. The reconstructions, shown in orange, are compared to the model images, shown in blue contours corresponding to 5%, 20%, 40%, 60%, and 80% of the model maximum brightness. The top row of panels are reconstructions of an approximately horizontally-oriented elliptical Gaussian, while the bottom row of panels are reconstructions of an approximately vertically-oriented Gaussian. The bottom left ellipse in each image subfigure shows the elliptical CLEAN beam. The middle row of panels show the snapshot coverages used to produce the reconstructions shown. The coverage shown in (a) is produced by the simultaneous observing of ALMA, APEX, SPT, and LMT; in (b), ALMA, APEX, SPT, and PV; in (c), all sites except for PV. Snapshots (a) and (b) have an identical number of baselines in their (u, v) coverage configuration but different angular isotropy. The (u, v) coverage in (b) is relatively capable of resolving the source along the North-South and East-West baselines, and orients the reconstruction properly. By comparison, the (u, v) coverage in (a) can only constrain flux in the direction of the collimated coverage. As a result, it incorrectly recovers the orientation of the Gaussian. The (u, v) coverage in column (c) is a maximally- (u, v) -filling case for the 2017 array configuration, with baselines relatively evenly distributed both radially and angularly. The result is an accurate recovery of the model behavior.

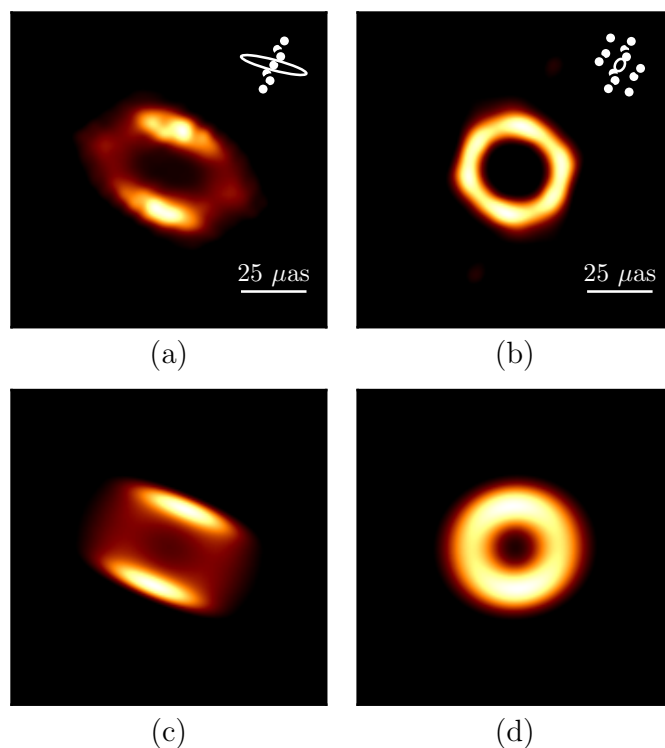


Figure 8.3: Images from a snapshot reconstruction of a simulated 2017 EHT observation of an axisymmetric static ring. Panel (a) is chosen to represent a portion of the observation where the (u, v) coverage was heavily anisotropic, and panel (b) is chosen to represent a portion of the observation where the (u, v) coverage was approximately isotropic and radially dense. The reconstruction is demonstrably and predictably affected by the angular homogeneity of the instantaneous (u, v) coverage, shown as a set of white dots. The white ellipse corresponds to the CLEAN beam and is directly linked to the (u, v) coverage. The predictability of the image artifacts resulting from (u, v) coverage is exemplified by the bottom panels, which show the model ring convolved with the CLEAN beams shown in the top row. Simple convolution with the CLEAN beam is enough to reproduce the salient artifacts.

periods (composed of many evolving (u, v) configurations) by their ability to produce high-quality reconstructions.

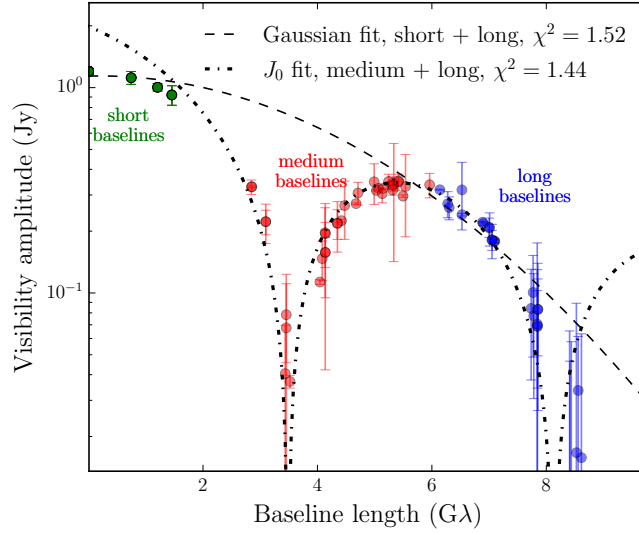


Figure 8.4: Visibility amplitudes of a simulated 2017 EHT observation of a ring ($25 \mu\text{as}$ radius, $5 \mu\text{as}$ width, convolved with $10 \mu\text{as}$ circular Gaussian) are shown as a function of radial baseline length ρ . Short ($\leq 2 \text{ G}\lambda$), medium ($2 \text{ G}\lambda < \rho < 6 \text{ G}\lambda$), and long ($\geq 6 \text{ G}\lambda$) baselines are displayed in green, red and blue, respectively. Continuous fits of different (but equally well-fitting) models are overlaid. Fitting a ring model with infinitesimal thickness (denoted by J_0 , representing a zeroth-order Bessel function of the first kind) to only medium and long baselines accurately represents the source shape and size, but poorly constrains the total intensity of 1 Jy. In addition, an equally good fit can be obtained with a simple Gaussian fit to only short and long baselines. Data from all three baseline types are required to correctly constrain key properties of the source. This result highlights how model misspecification can lead to severe systematic errors, especially when working with limited baseline coverage.

8.6 Coverage metrics

Multiple (u, v) coverage metrics with different underlying considerations exist in the literature. Here, we summarize several metrics and compare the way they score a given observation. In addition, we develop a novel “isotropy metric” that has been tailored to the specific vulnerabilities detailed in [section 8.5](#).

Our “selective dynamical imaging” approach uses such a metric to identify intervals during an observation where the coverage is optimally configured for imaging.

Importantly, these intervals are chosen before image reconstruction is attempted. In contrast, methods such as “lucky imaging” (e.g., [376]) identify particularly useful images (e.g., with minimal distortion) after and on the basis of the reconstruction. A single score computed directly from (u, v) coverage known *a priori* is preferable to an empirical approach (e.g., performing a simulated observation of a synthetic model and comparing reconstructions with the model) as it provides source-structure-agnostic assessments with a substantially lower performance cost.

Any metric capable of scoring different periods of (u, v) coverage would have demonstrable limitations. Within a single observation, a comparison of (u, v) coverage at two different points in time is a reliable way of determining which time region of the observation will produce superior image reconstructions. However, certain reconstruction-impacting data quantities can vary independently of the (u, v) coverage. Sensitivity, calibration, and systematic uncertainty can also be important factors, but are not probed by coverage metrics.

8.6.1 Normalized cross-correlation

The normalized cross-correlation between two images is a measure of their similarity. By performing a dynamical reconstruction on a simulated observation and comparing each image of the dynamical reconstruction to the model, we can heuristically identify which portions of the observation produced the best reconstruction (i.e., the portions of the reconstruction with the greatest similarity to the model). We define the normalized cross-correlation $\rho_{\text{NX}}(X, Y)$ of two images X and Y in an identical fashion to [5],

$$\rho_{\text{NX}}(X, Y) = \frac{1}{N} \sum_i \frac{(X_i - \langle X \rangle)(Y_i - \langle Y \rangle)}{\sigma_X \sigma_Y}. \quad (8.2)$$

To compute the normalized cross-correlations used in [Figure 8.6](#), we reconstruct a

static ring (one image per snapshot of the observation) and compute the metric value between the model image and the reconstruction.

A normalized cross-correlation between a model and the associated reconstruction is the most straightforward way to identify trustworthy periods of an observation, assuming that the reconstruction on synthetic data will behave in a similar fashion to a reconstruction on real data. This assumption is only upheld if care is taken to ensure that the qualities of the synthetic data match those of the real data. In addition, heuristic tests such as the normalized cross-correlation can be biased depending on the structure and inherent variability of the model chosen. If the source randomly aligns with collimated (u, v) coverage at some point in the observation, it can result in a misleadingly high normalized cross-correlation that cannot be replicated for a different source model.

8.6.2 (u, v) filling fraction

[377] proposes a geometric scoring procedure for the (u, v) coverage based on the specification of a desired array resolution θ_{res} and imaging field of view θ_{FOV} . θ_{res} sets an outer boundary with radius $1/\theta_{\text{res}}$ in the (u, v) plane within which a “filling fraction” is computed, and is typically taken to be the nominal array resolution set by the longest baseline in an observation. θ_{FOV} determines a convolution radius of $0.71/\theta_{\text{FOV}}$ corresponding to the scale in the (u, v) plane over which the Fourier response to a filled disc on the sky of diameter θ_{FOV} would decay to half of its maximum amplitude; we use $\theta_{\text{FOV}} = 100\mu\text{as}$ for the filling fraction computation in [Figure 8.6](#). Intuitively, the largest image feature considered in the optimization of coverage sets the smallest scale of interest in the (u, v) plane; thus, convolving a proposed set of (u, v) points by $0.71/\theta_{\text{FOV}}$ yields a measure of what region in the (u, v) plane is sampled by

measured visibilities. The fraction of the bounding circle sampled by the convolved coverage is the filling fraction.

Increasing the specified resolution (perhaps by increasing observation frequency) extends the bounding circle, decreasing the filling fraction unless θ_{FOV} is correspondingly decreased. In this way, the filling fraction captures some features of the “spatial dynamic range” discussed in [378]. As shown in figure 7 of [377], the filling fraction metric is a successful and nearly linear predictor of image fidelity until the filling fraction reaches values near 0.9, at which point imaging techniques are limited by methodology-specific super-resolving scales, which for many imaging algorithms is at approximately half of the diffraction-limited CLEAN beam width in the case of the EHT [5].

8.6.3 Largest coverage gap

An alternative metric probing the coverage isotropy is based on identifying the largest gap in the (u, v) coverage, hence we refer to it as the largest coverage gap (LCG) metric [379]. In this approach we consider the coverage as a set of sampled (u, v) plane locations, to find the largest circle that can be drawn within the limits of the coverage that does not contain a coverage point in itself. Such a largest circular gap can be efficiently calculated with Delaunay triangulation of the coverage set [380]. Then the diameter of the gap d_{max} can be turned into a metric coefficient with

$$m_{\text{LCG}} = 1 - d_{\text{max}}/\rho_{\text{max}} , \quad (8.3)$$

where ρ_{max} is the longest projected baseline length. If we demand that the (u, v) distance corresponding to the center of the circle is less than ρ_{max} , then we have $0 \leq m_{\text{LCG}} \leq 1$, with $m_{\text{LCG}} = 1$ corresponding to the limit of a complete continuous

coverage. A coverage consisting of a single detection would correspond to $m_{\text{LCG}} = 0$. Unlike the filling fraction metric, the LCG metric is independent of the assumed field of view.

8.6.4 Isotropy and radial homogeneity

We propose a novel metric of (u, v) coverage isotropy and radial homogeneity (hereafter referred to as the “isotropy metric”) based on the limitations described in [section 8.5](#). Similarly to the LCG metric, the isotropy metric penalizes anisotropy of the coverage, although the two approaches differ appreciably. In this approach, we treat the distribution of baselines in the Fourier plane as a mass distribution and quantify the radial and angular homogeneity using the second moments of inertia. We define the isotropy metric coverage parameter C for a given snapshot as

$$C = I \left(1 - \frac{\mathcal{K}}{\mathcal{K}_{\text{max}}} \right), \quad (8.4)$$

with

$$I = 1 - \frac{\sqrt{(\langle u^2 \rangle - \langle v^2 \rangle)^2 + 4\langle uv \rangle^2}}{\langle u^2 \rangle + \langle v^2 \rangle}, \quad (8.5)$$

where $\langle u^2 \rangle$, $\langle v^2 \rangle$ and $\langle uv \rangle^2$ are the second moments of the baseline distribution, $\mathcal{K}$ is the Kolmogorov-Smirnov (KS) distance of the radial distribution of baseline lengths from uniform, and $\mathcal{K}_{\text{max}}$ is the maximum value of $\mathcal{K}$ at any point in time during the observation (or an arbitrary value for the purpose of cross-observation comparisons). The isotropy metric has the benefit of being fully analytic and automatically normalized between 0 and 1. A full derivation of the isotropy metric is presented in [section B.2](#).

8.6.5 Discussion of metrics

Despite differences in methodology and implementation, the metrics we examined found similar fluctuations in (u, v) coverage quality and identify similar candidate time regions for high-quality imaging. A comparison of the metrics detailed in Subsections 8.6.1-8.6.4 applied to the 2017 EHT (u, v) coverage of Sgr A* is shown in Figure 8.6. In general, the second half of the observation has superior (u, v) coverage as indicated by the metrics, and the period from ~01:00 GMST to ~03:30 GMST maximizes the various metrics.

The (u, v) filling fraction and LCG metric generally produce results similar to the isotropy metric. Disagreements between the metrics can be seen especially at the beginning of the observation (e.g., 17–19 GMST) and in the middle (e.g., 21–23 GMST). These are periods where the (u, v) coverage is extremely sparse and not suitable for imaging, though the degree to which these periods are determined to be unsuitable varies depending on the specific considerations of the individual metrics.

The normalized cross-correlation metric, while the most direct measurement of time-varying reconstruction quality, lacks source-structure agnosticism. If the source structure is known, the normalized cross-correlation metric can be a useful method of determining what periods of an observation are most advantageous for imaging that particular source structure. However, if the source structure is unknown, then a wide suite of representative source models must be tested to mitigate possible biases. Additionally, the normalized cross-correlation method will be tied to the particular imaging algorithm and hyperparameters used, making this metric less robust than the others considered.

The particular constructions of the metrics can lead directly to unintuitive or undesirable behavior. One example of undesirable behavior is a metric punishing a

coverage for adding data points. Intuitively, more baselines lead to better coverage of the Fourier plane and therefore more information about the source. However, if these additional baselines are placed strategically, they can result in an unintuitive score assignment. A trivial example of this can be generated for the LCG metric. Consider a coverage with maximum baseline length less than ρ that achieves $m_{\text{LCG}} \approx 1$. By placing a single baseline of length L far outside the initial coverage (i.e., $L \gg \rho$), $\rho_{\text{max}} = L$ and d_{max} goes as $\sim L - \rho$. This drives m_{LCG} to zero and seems to indicate the coverage has become demonstrably worse, when in reality the coverage quality has largely stayed the same, with the improvement of a single ultra-long-baseline. This type of array pathology does not occur in the 2017 EHT coverage but may present an issue if the EHT goes to ultra-long space baselines.

The isotropy metric exhibits similar misbehavior, as demonstrated in [Figure 8.5](#). Given an isotropic coverage with a low number of baselines, adding just two baselines strategically can decrease the isotropy metric value by a substantial amount. With so few baselines, the addition of new baselines would intuitively be considered an improvement. However, the metric detects a decrease in isotropy and reports accordingly. This problem is only present for arrays with small numbers of baselines—it is difficult or impossible to significantly alter the isotropy of an array configuration for larger arrays using only a few baselines without resorting to ultra-long-baseline placement as in the LCG example.

An additional limitation that any metric based purely on coverage possesses results from unusual source structure. The metrics described above attempt to predict reconstruction quality by analyzing the coverage available, but this prediction is performed under the assumption of “reasonable” source structure (i.e., source structure with smooth, continuous Fourier representation). However, we can construct simple examples that would render these metrics unhelpful by violating the assumption of

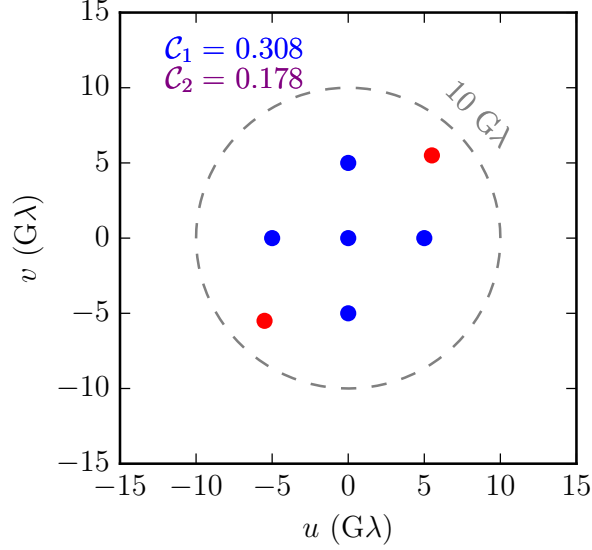


Figure 8.5: A pathological case for the isotropy metric which demonstrates an undesirable behavior. The isotropic coverage shown (blue) results in an isotropy metric value of ≈ 0.3 (C_1 , shown in blue). Adding two data points (red) strategically (i.e., in an anisotropic configuration) decreases the overall isotropy of the array and lowers the metric score by a factor of $\approx 1/2$ (C_2 , shown in red+blue=purple). This change makes sense given the considerations of the metric—the new array is more anisotropic and therefore has a lower score. However, this behavior is undesirable since, intuitively, we expect that an array with more baselines will perform better than an array with fewer baselines. Note: in order to compute the isotropy metric as defined in [subsection 8.6.4](#), the example coverages shown above are assumed to be part of the Apr 7, 2017 EHT coverage of Sgr A* and the corresponding value of $\mathcal{K}_{\max}$ is adopted (see [section B.2](#)).

reasonable source structure. Consider a source whose Fourier transform has zero flux density everywhere an array has (u, v) coverage, and non-zero flux density everywhere else. Regardless of how good the coverage itself is (and therefore how well a given metric may score the coverage), there is no way to produce an accurate reconstruction of the source. This limitation is not likely to be an issue, as the restriction that source emission be non-negative induces sufficient correlation in the Fourier domain that the possibility for arbitrary pathologies to “hide” in coverage-deficient swathes of the plane is severely limited.

The isotropy metric offers substantial performance (i.e., time and overhead cost) benefits over the other metrics considered, while reporting similar results. We test the performance of each metric by computing the per-snapshot score for a full 12 hour observation five times, resulting in $\sim 10\text{k}$ snapshots available for scoring. We run the performance assessments on a i5-1038NG7 10th-generation Intel x86_64 processor with 16 GB of RAM. The normalized cross-correlation, which must reconstruct hundreds of images and compare them to model images, takes several hours to complete. The isotropy metric performs $\sim 10^5$ times faster than the full normalized cross-correlation, ~ 20 times faster than the (u, v) filling fraction and ~ 10 times faster than the LCG metric, while producing similar assessments of coverage. The substantial performance differences between the metrics will become more pronounced with larger arrays, such as the next-generation EHT (ngEHT) coverage. The low overhead generated by the isotropy metric in comparison to the other metrics examined makes the isotropy metric a more optimized method of scoring (u, v) coverages in important contexts, such as real-time track selection and long-term ngEHT site placement [329], which is expanded upon in [section 8.8](#).

8.7 Application of metric to 2017 EHT array

We apply the coverage metrics discussed in [section 8.6](#) to the EHT (u, v) coverage of Sgr A* corresponding to April 7, 2017. The instantaneous metric values for each snapshot of the observation are shown in [Figure 8.6](#). This method of scoring the observation clearly identifies distinct periods of varying coverage quality. The time region from $\sim 01:30$ Greenwich Mean Sidereal Time (GMST) to $\sim 03:10$ GMST (denoted as “Region II” in [Figure 8.6](#)) has the highest overall isotropy and baseline density of the observation. We select this period as a candidate time region for high-quality

	Start time		End time	
	UT	GMST	UT	GMST
Observation	4.0458	17.087	15.598	4.6721 Apr 8
Region I	6.7766	19.825	7.9763	21.028
Region II	12.618	1.6834 Apr 8	14.046	3.1155 Apr 8
LMT drop	13.435	2.5045 Apr 8	13.737	2.8058 Apr 8

Table 8.1: Timestamps for the observation and beginning and end of the time regions of interest in the Apr 7, 2017 EHT coverage of Sgr A*. Region I and Region II correspond to the time regions identified in red and blue (respectively) in Figure 8.6. The LMT dropout corresponds to the sudden loss of coverage that occurs partway through Region II. All timestamps correspond to Apr 7, 2017 unless otherwise noted. In UT, the observation begins and ends on Apr 7; however, when converted to GMST, Region I lies on Apr 7, 2017 while Region II lies on Apr 8, 2017.

imaging (a “good” time region, i.e., one where the typical metric score is high). In contrast, the time region from ~19:45 GMST to ~21:00 GMST (denoted as “Region I” in Figure 8.6), while relatively stable, displays substantially lower coverage quality. We select this time region to examine the behavior of reconstructions in periods of ambiguous coverage quality. Exact timestamps for these time regions are given in Table 8.1.

By performing reconstructions in these time periods, we can validate the capability of the (u, v) coverage metrics to predict reconstruction quality based on coverage alone. We reconstruct four configurations of the ring+hs toy model detailed in section 8.3: a 270 minute orbital period clockwise (CW) and counterclockwise (CCW), and a 30 minute orbital period clockwise and counterclockwise. The reconstructions in each time region are produced according to the RML and CLEAN imaging methods in section 8.4, and we perform feature extraction on each image using the REX module of eht-imaging ([5]) to recover the position angle of the hotspot at each moment in time. The extracted hotspot angles and the model orbits for Regions I and II are shown in Figures 8.7 and 8.8, respectively. Images sampled from dynamical reconstructions gen-

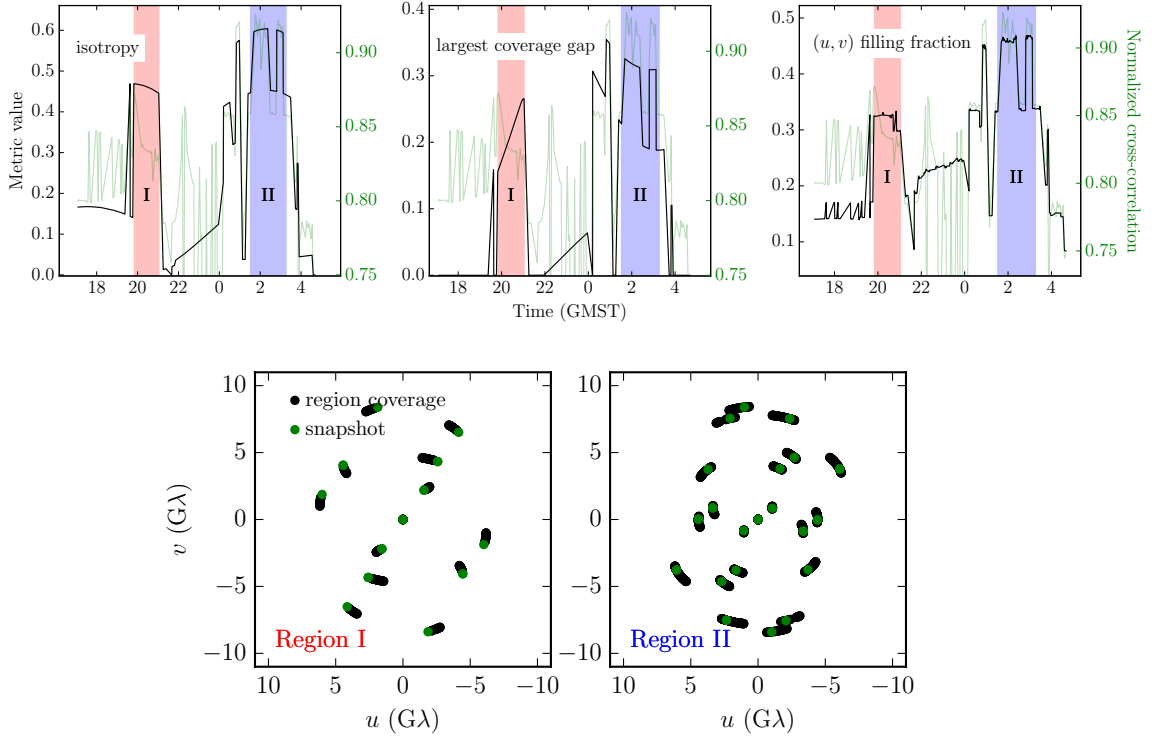


Figure 8.6: The application of all metrics (top left: isotropy; top middle: largest coverage gap; top right: (u, v) filling fraction) described in [section 8.6](#) to the 2017 EHT coverage of Sgr A*. The observation begins on April 7, and the 00:00 GMST indicates the day change to April 8. The red time region corresponds to Region I, and the blue time region to Region II, as described in [section 8.7](#). The normalized cross-correlation (green, representing the comparison between a snapshot reconstruction of a static ring and the model) is shown as a reference for all metrics as it is the only direct measure of instantaneous reconstruction quality. All three (u, v) -coverage-based metrics show high-quality coverage in the time region from $\sim 01:30$ GMST to $\sim 03:10$ GMST (Region II). The bottom row of the figure shows the coverage of Region I (bottom right) and Region II (bottom left) along with a representative snapshot.

erated by each method are displayed in [section B.3](#). The “success” of a reconstruction is determined by the successful extraction of the hotspot position angle.

We find that Region II produces reconstructions that facilitate accurate recovery of dynamical variability. The reconstructions show a ring of approximately $50 \mu\text{as}$ in diameter with a distinct hotspot. The recovered hotspot orientations are shown in

Figure 8.8, along with a comparison to the model values. To compare the N recovered position angles φ_γ^r with the model angle φ_γ^m , we use a phase-adjusted rms $\mathcal{R}$, defined as

$$\mathcal{R} = \left[\frac{1}{N} \sum_{\gamma} \begin{cases} (\varphi_\gamma^r - \varphi_\gamma^m)^2, & \text{if } |\varphi_\gamma^r - \varphi_\gamma^m| \leq \pi \\ (2\pi - \varphi_\gamma^r + \varphi_\gamma^m)^2, & \text{if } |\varphi_\gamma^r - \varphi_\gamma^m| > \pi \end{cases} \right]^{1/2}$$

Overall, the Region II reconstructions successfully recover the dynamical variability in the model, with rms of the recovered angles and model varying between 0.16 and 0.20 radians. Decreases in coverage quality as measured by the metrics (indicated point-wise in Figure 8.8 by increases in transparency, using the isotropy metric as an example) correlate with lower-quality recovery of the hotspot position angle. These low-quality angle recoveries are most obvious in the RML reconstructions of the counterclockwise $T = 30$ minute case. The CLEAN algorithm appears to be more resistant to the sudden loss of coverage, and maintains reconstruction quality even through drops in metric score. Excluding these lapses in coverage quality, Region II clearly facilitates the recovery of source structure and at least one kind of dynamical variability, covering a wide range of periods and directions.

By contrast, a comparatively low-scoring time region (Region I) does not produce reconstructions capable of accurately recovering dynamical variability. Dynamically reconstructed images show a ring-like feature, but the time variation of the brightness asymmetry does not match the model. Overall, the Region I reconstructions fail to recover the dynamical variability in the model, with rms of the recovered angles and model varying between 1.21 and 1.73 radians. The rms on the recovery in Region I is between 7 and 10 times higher than in Region II. For all tests, the scatter around the model is substantially larger than the scatter in Region II, rendering the accurate extraction of a period difficult or impossible. Reconstructions in periods outside of

Region I and Region II cannot recover even basic source structure without significant *a priori* information.

Based on these results, we expect that reconstructions on real data will produce the most accurate and robust recoveries of orbital dynamical variability in Region II of April 7 of the 2017 EHT observations of Sgr A*. Reasonable imaging and feature extraction procedures failed to produce meaningful results in Region I. This ranking is consistent with the predictions of the coverage metrics described in [section 8.6](#) and [section B.2](#). A wide array of factors impact reconstructions, and the tests presented here do not comprise a realistic synthetic data suite for assessing whether or not Region II can accurately recover dynamical variability for Sgr A*. The tests provided here solely demonstrate that Region II is the best time region for performing dynamical reconstructions based on coverage considerations. Additional testing on more complex source types with realistic data corruptions can be found in [\[357\]](#).

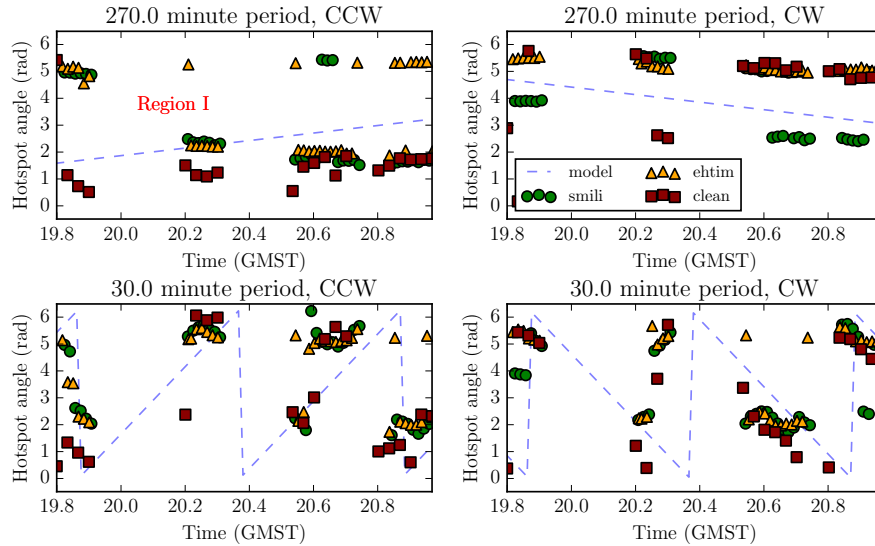


Figure 8.7: SMILI (green circles), ehtim (yellow triangles) and CLEAN (red squares) position-angle reconstructions in Region I of a ring+hs model for orbits of various periodicities and directions. Comparing the resulting hotspot position angle to the model angle (blue dashed line) clearly demonstrates that Region I provides a poor reconstruction of time variability.

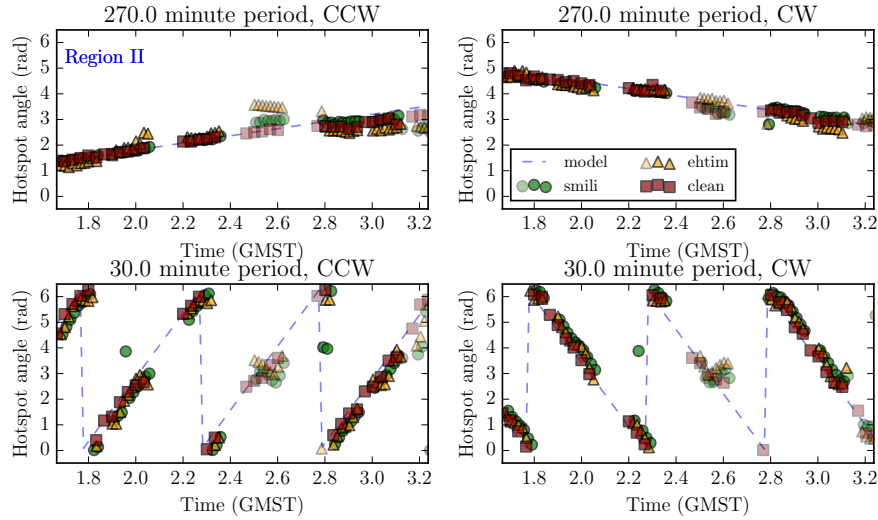


Figure 8.8: SMILI (green circles), ehtim (yellow triangles) and CLEAN (red squares) position-angle reconstructions in Region II of a ring+hs model for orbits of various periodicities and directions. The transparency of each data point corresponds to the isotropy of the snapshot observation used to reconstruct it. Comparing the resulting hotspot position angle to the model angle (blue dashed line) clearly demonstrates that Region II provides excellent conditions and coverage for meaningful recovery of periodicity given reasonable imaging assumptions.

8.8 Metric-based array comparisons

Go/no-go decisions about whether to proceed with an observing run are often made with limited information about the readiness or weather conditions at particular sites. Simulating (and scoring) array configurations with different dropouts— and characterizing changes in the size and quality of the identified candidate temporal regions—can facilitate those go/no-go choices while incorporating uncertainties about station status. One observation configuration can be considered “better” than another if it provides a candidate time region with a higher metric score, or a similar metric score for a longer duration. [Figure 8.9](#) shows an example of this kind of analysis performed with a hypothetical ngEHT array. The top panel shows the isotropy metric (see [subsection 8.6.4](#)) score per snapshot for the full array during a night of observation in which

every station is observing, which represents the ideal scenario. The middle and bottom panels reproduce the metric score per snapshot assuming 4 sites are unable to observe. By identifying a candidate time region and characterizing its quality (based on, e.g., average metric score in the time region) and duration (denoted as $\Delta\tau$ in [Figure 8.9](#)), we can track these characterizations through different combinations of site dropouts and estimate how critical the sites are to the array. Based on the computations in [Figure 8.9](#), ALMA, JCMT, LMT, and SMA dropping out would be catastrophic to the array performance, as the optimal time region for dynamical imaging reduces in duration by $\approx 80\%$ and reduces in quality by $\approx 50\%$. By comparison, the combined dropout of PV, PDB, CARMA, and LMT does not substantially change the duration or quality of the identified candidate time region.

A metric-based comparison additionally provides a natural and quantitative ranking for identifying which day of an observation campaign produced the most optimal coverage for dynamical imaging. We can rank observations on separate days in a similar fashion to the dropout scenarios visualized in [Figure 8.9](#) by comparing the duration and quality of the identified candidate time regions for each day. An example of such a comparison between the Apr 7 and Apr 10 runs of the 2017 EHT observation campaign is shown in [Figure 8.10](#). Instantaneous metric scores are computed for each scan of both days and candidate time regions for dynamical imaging (green) are identified. While both candidate time regions are of approximately the same duration, the candidate time region associated with Apr 7 displays substantially better coverage, making Apr 7 a better choice for dynamical imaging than Apr 10.

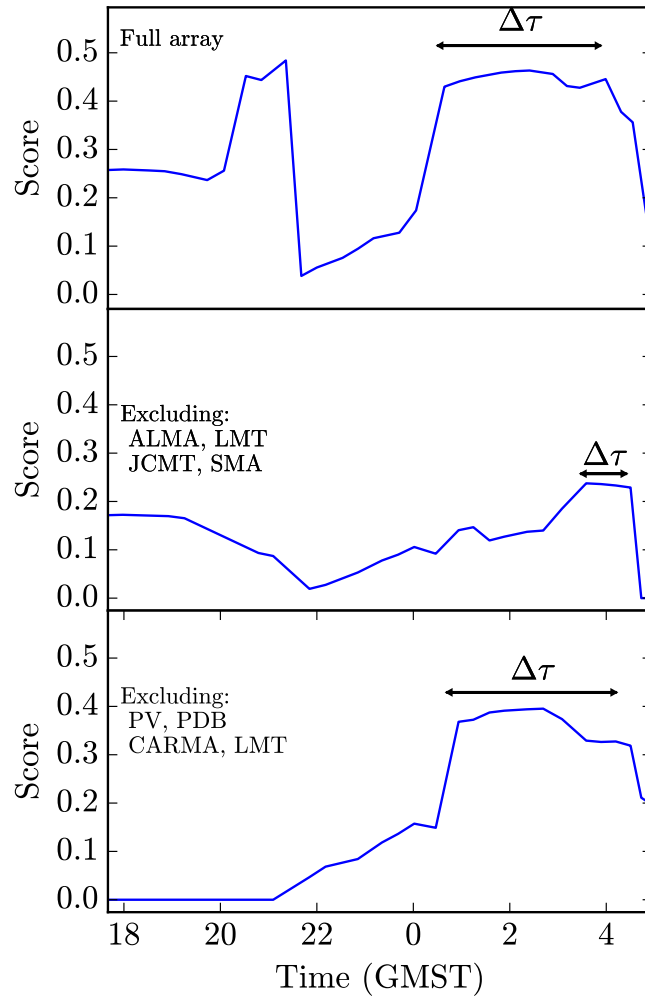


Figure 8.9: Isotropy metric scores for a hypothetical ngEHT array with different dropout scenarios. The full array has all of the sites in the 2017 EHT array (see [section 8.3](#)) with the addition of the Northern Extended Millimeter Array (PDB), the Combined Array for Research in Millimeter-wave Astronomy (CARMA), and the Kitt Peak National Observatory (KP). A universal value of $\mathcal{K}_{\max} \approx 0.513$ was used to perform the computation (see [section B.2](#)). Generally, dropouts will impact the maximum coverage score achieved throughout the observation and the duration (shown as $\Delta\tau$) of the most optimal time region. Characterizing a site’s importance to the observation is a useful way of informing a go/no-go decision, which takes into account station readiness and probability of dropout. For the above observational scenario, the loss of e.g., ALMA, JCMT and SMA would likely motivate a “no-go” decision; whereas the loss of e.g., CARMA and PV would not motivate cancelling the night of observation.

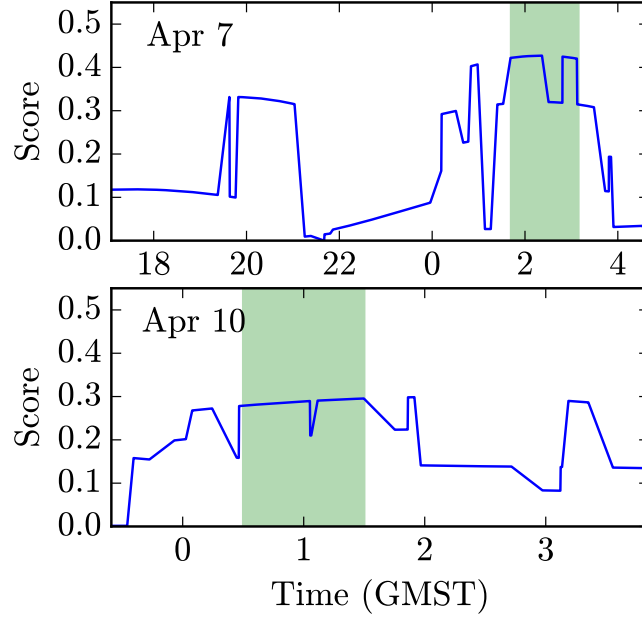


Figure 8.10: Isotropy metric scores for two days of the 2017 EHT observation of Sgr A*. Candidate time regions for dynamical imaging are highlighted in green. A universal value of $\mathcal{K}_{\max} \approx 0.513$ was used to perform the computation (see [section B.2](#)). Dropouts can significantly change the available coverage, resulting in candidate time regions of different duration and quality. By scoring the coverage for the entire observation and directly comparing the predicted quality of candidate time regions, we can identify the best day to perform dynamical imaging. Although the candidate time regions identified on April 7 and April 10 are similar in duration, the candidate time region on April 7 presents $\approx 40\%$ better coverage.

8.9 Conclusions

We have demonstrated that limitations of imaging associated with sparse baseline distribution can be inferred from the specific geometric properties of the (u, v) coverage. Highly anisotropic coverage produces artifacts in reconstructions that distort the image in a direction consistent with stripes in the dirty beam. Additionally, an uneven radial distribution of the Fourier plane (u, v) coverage tends to result in ambiguous image reconstruction. These limitations can be partially avoided by imaging in radially and angularly homogeneous coverage. The reconstruction issues associated with sparse

coverage are exacerbated by rapid short-timescale variability, as seen in a wide array of astrophysical sources, including Sgr A* and the precessing jets of X-ray binaries.

Next, we surveyed and compared existing geometric measures of (u, v) coverage, in addition to deriving a novel isotropy-based metric that addressed the specific limitations demonstrated in [section 8.5](#). The examined metrics included the normalized cross-correlation ([Equation 8.2](#)), the (u, v) filling fraction (see [\[377\]](#)) and the largest coverage gap metric ([Equation 8.3](#), see [\[379\]](#)). The isotropy metric treats the distribution of baselines as a mass distribution and examines the second moment to rank coverages by homogeneity in the Fourier plane. The isotropy metric gives similar results to other (u, v) -coverage-based metrics while being more computationally efficient.

These metrics were applied to the April 7 data of the 2017 EHT coverage of Sgr A* [\[355–360\]](#) and used to select candidate time regions for high-quality dynamical imaging. All metrics identify a period from $\sim 01:30$ GMST to $\sim 03:10$ GMST (“Region II”) that minimizes the coverage limitations each metric addresses. We also select a period from $\sim 19:45$ GMST to $\sim 21:00$ GMST (“Region I”) with reconstruction capability. Reconstructions of time-variable sources allowed successful recovery of the characteristic source variability in Region II. In contrast, reconstructions in Region I were unable to recover the characteristic motion. The ranking determined by the suite of reconstructions performed on synthetic data verify the predictions made by the examined coverage metrics. We expect that attempts to recover variability in real EHT observations of the Galactic Center will produce the most robust and accurate recoveries in Region II of the April 7, 2017 dataset, and therefore recommend performing dynamical imaging procedures in that time region.

Coverage metrics have additional utility for ranking inter-observation comparisons based on their ability to recover dynamical variability, which has a variety of applications to the broader field of interferometry. These metrics provide the ability to make

select observation time-slots based on the capability of available antennas to recover particular dynamical evolution in the target. Such a scored assessment of coverage could well prove of use to other VLBI arrays both as they make go/no-go decisions about whether to observe on a particular night, and then when identifying the periods of best coverage to perform static and dynamic imaging.

Chapter 9

Machine- and deep-learning-driven angular momentum inference from BHEX observations of the $n = 1$ photon ring

“

THERE IS A CRACK, A CRACK IN EVERYTHING

THAT’S HOW THE LIGHT GETS IN.

— Leonard Cohen, “Anthem”, 1992



The $n = 1$ photon ring is an important probe of black hole (BH) properties and will be resolved by the Black Hole Explorer (BHEX) for the first time. However, extraction of black hole parameters from observations of the $n = 1$ subring is not trivial. Developing this capability can be achieved by building a sample of $n = 1$ subring simulations, as

well as by performing feature extraction on this high-volume sample to track changes in the geometry, which presents significant computational challenges. Here, we present a framework for the study of $n = 1$ photon ring behavior and BH property measurement from BHEX images. We use KerrBAM to generate a grid of $\gtrsim 10^6$ images of $n = 1$ photon rings spanning the entire space of Kerr BH spins and inclinations. Intensity profiles are extracted from images using a novel feature extraction method developed specifically for BHEX. This novel method is highly optimized and outperforms existing EHT methods by a factor of ~ 3000 . Additionally, we propose a novel, minimal set of geometric measurables for characterizing the behavior of the $n = 1$ subring geometry. We apply these measurables to our simulation grid and test spin recovery on simulated images using: (i) gradient boosting, a machine learning algorithm; and (ii) an extension of Deep Horizon, a deep learning framework. We find $\gtrsim 90\%$ correct recovery of BH properties using the machine/deep learning approaches, and characterize the space of resolution-dependent geometric degeneracies. Finally, we test both approaches on GRMHD simulations of black hole accretion flows, and report accurate recovery of spin at the expected inclination of M87*.

9.1 Permissions and Attributions

1. The content of [chapter 9](#) is the result of a collaboration with Dr. Jordy Davelaar, Dr. Daniel Palumbo, Prof. Michael Johnson, and Jonathan Delgado, and has previously appeared in the *Astrophysical Journal* as Joseph R. Farah *et al.* (2025). It is reproduced here with the permission of the *Astrophysical Journal*: <https://iopscience.iop.org/article/10.3847/1538-4357/adbbe3>.
2. We acknowledge Yaxuan Shen for helpful discussions regarding regularized

gradient boosting.

9.2 Introduction

In April 2019, the Event Horizon Telescope (EHT) Collaboration published the first image of a black hole [266; 5], representing a significant step forward in resolving and imaging astrophysical phenomena via very-long-baseline interferometry (VLBI). The unprecedented resolution of the image ($\approx 20 \mu\text{as}$) uniquely probed event-horizon scale phenomena and directly facilitated the strongest tests of general relativity to-date [e.g., 293; 294; 281; 280, etc.]. However, while this remarkable resolution was sufficient to probe the shadows of the M87* and Sgr A* black holes [295; 296], it was still inadequate to probe the more complex features of event-horizon scale emission predicted by theory, such as the photon ring. Recently, a space-baseline extension to the EHT (the Black Hole Explorer; BHEX) has been proposed to overcome these limitations and help resolve open questions about general relativity and other fundamental physics [299; 301].

In particular, the higher resolution of BHEX will enable direct observation and study of a black hole’s photon ring [302], a shell of unstable photon orbits which, in aggregate, produce a sharp (width $\approx 10\%$ of the photon ring diameter, or $\approx 1M$) emission feature on the gravitationally lensed image of the black hole seen by a distant observer. The photons completing n half-orbits around the black hole produce n distinct and nested subrings [260]. Of these subrings, the $n = 0$ direct image is primarily determined by the fluid flow and only partially determined by the spacetime. By contrast, the morphology of the $n \geq 1$ subrings (the “photon rings”) is only dictated by unstable photon orbits and are therefore mostly dependent on the properties of the black hole spacetime (e.g., spin, inclination relative to the observer, etc.) [e.g., 303].

At the expected imaging resolution of BHEX ($\approx 6\mu\text{as}$), black hole targets must have a gravitational radius which subtends at least $\sim$ a few microarcseconds in order to have resolvable photon ring substructure. The resolution of BHEX, combined with time-averaging to remove sharp but transient features, is expected to be sufficient to probe at least the $n = 1$ subring of M87* and Sgr A*, thus enabling direct and precise measurements of black hole properties, particularly spin [260].

The behavior of the $n = 1$ photon ring as modified by spin is poorly understood, presenting a potential hurdle for precision measurements via BHEX observations [see e.g., 381, for a review]. Unlike the critical curve, which shows substantial deviation from circularity at edge-on inclination and spins approaching unity [e.g., 285; 347; 337; 382; 288, etc], the equivalent modification to the $n = 1$ subring is more complex and more subtle. Previous studies have characterized the overall size and ellipticity [303; 383; 288], which varies with the angular momentum of the black hole. The azimuthal brightness modulation of the $n = 1$ subring, which can be modeled by a shape known as an m -ring [260], also varies with spin. The precise relationship between these quantities and the properties of the black hole is still unknown; however, characterizing these relationships is essential for measurements with detections of the first subring.

The limited ability to observe such objects in nature results in a heavy reliance on simulations [e.g., 292; 293; 297; 298], which are computationally challenging to perform with existing tools. In particular, in order to study the $n = 1$ subring properties, we rely on computational methods such as general relativistic magnetohydrodynamic (GRMHD) simulations, which are computationally expensive to perform for the full parameter space in e.g. spin. In addition, image feature extraction tools developed for EHT applications involving a small number of images are robust but have generally poor scalability, requiring weeks or even months to analyze the large ($\gtrsim 10^5$) number

of images needed for machine-learning-enabled analyses.

In this paper, we present a novel framework to enable large-scale studies of the $n = 1$ photon subring. We exploit the fast semi-analytic simulation software **KerrBAM** [384] to simulate the first subring and develop bespoke tools to perform analyses on feasible timescales. In [section 9.3](#), we describe the efficient generation of a suite of $n = 1$ photon rings spanning all possible spins and inclinations using **KerrBAM**, a recently-developed semi-analytic accretion modeling code. In [section 9.4](#), we introduce and evaluate “recursive brightest-point extraction”, a novel feature extraction approach designed specifically for BHEX and built to be thousands of times more computationally efficient than existing tools. In [section 9.5](#), we introduce a set of non-redundant geometric observables which can characterize the shape and emission characteristics of the various subrings in our simulation grid. We use these geometric observables to provide the first numerical analysis of the behavior of the $n = 1$ subring across the Kerr parameter space of spins and inclinations. In [section 9.6](#) we demonstrate the ability of these observables as an unambiguous measurement tool by training machine- and deep-learning algorithms to estimate black hole properties from both numerical observables as well as full images of subrings simulated with **KerrBAM**. Finally, in [section 9.7](#) we directly apply our methods to GRMHD-simulated black holes. We summarize our conclusions and outlook for BHEX measurements in [section 9.8](#).

9.3 Simulation generation via **KerrBAM**

The inaccessibility of the $n = 1$ photon ring given current observational limitations results in our understanding being theoretical and largely simulation-based [e.g., [292](#); [293](#); [297](#); [298](#)]. Machine learning algorithms in particular benefit from large datasets to improve performance, particularly when generalizing training features to real-world

behavior or attempting to recover numerical quantities in a continuous fashion [385]. Full GRMHD simulations, while frequently used and closer to reality than semi-analytic simulations, are also computationally expensive to generate, presenting an early bottleneck in attempts to characterize subtle feature behavior. Fortunately, recent years has seen the development of new models and techniques for analytic or semi-analytic computation and/or approximation of black hole accretion flows, which have come with a substantial increase in performance [386; 384; 387].

Investigation of the $n \geq 1$ photon ring has revealed that snapshots of the black hole accretion flow may have flares or fluctuations which will contaminate the image of and obscure the photon ring. However, averaging on timescales significantly longer than the light-crossing time of the black hole $t_c \sim M$ exploits the persistent nature of the photon ring and effectively suppresses contaminating transient phenomena [260; 388; 386; 387]. The planned BHEX mission will provide sufficient coverage of target sources to allow for time-averaging on timescales $\gg t_c$, presenting a clear view of the $n = 1$ photon ring. Indeed, simulations indicate that the bright and narrow $n = 1$ subring will dominate BHEX images, while $n > 2$ subrings will be largely below the BHEX resolution, motivating examinations of the spin-distinguishing power of this subring [299; 301].

We generated a simulated dataset of 10^6 images to use for training. This is a substantial (10x) increase in dataset size as compared to previous similar analyses [e.g., Deep Horizon: 389]. The dataset size is motivated by the subtlety of the subring behavior we hope to capture; the spin and inclination parameter space has steep gradients (particularly towards maximal spin or edge-on inclination) which require dense sampling to characterize. The simulated images are generated with the `KerrBAM` simulator, a semi-analytic ray tracing engine capable of producing images of all $n \geq 0$ subrings [384]. We fixed all accretion flow parameters to standard values following [384]. For

Parameter	FOV ^a	M^b	D^c	PA ^d
Value	$80 \mu\text{as}$	$6.5 \times 10^9 M_\odot$	$5.1 \times 10^{23} \text{ m}$	0°

^aThe field-of-view of the output image.

^bThe mass of the simulated black hole, set to the M87* mass.

^cThe distance between the observer and the simulated black hole, set to the M87* distance.

^dThe position angle of the spin axis projected in the observer plane, corresponding to a rotation of the image.

Table 9.1: KerrBAM simulation parameters controlling the black hole spacetime. We cite [5] where needed for relevant values.

the parameters controlling the black hole spacetime, we fixed all parameters to values consistent with known values for M87*, with the exception of angular momentum (spin) a , inclination θ_0 , and emission radius. The fixed values are summarized in [Table 9.1](#). Spin was varied between almost minimal ($|a| = 0.04$, in units of Jc/M^2 , where J is the angular momentum of the black hole) and almost maximal ($|a| = 0.94$) for both negative (spin axis oriented away from observer) and positive (spin axis oriented towards observer) spin. Inclination was varied between face-on ($\theta_0 = 0^\circ$, parallel to the spin axis) and near-edge-on ($\theta_0 = 85^\circ$, almost perpendicular to the spin axis). The emission radius was varied between $3 M$ (where M is equivalent to the Schwarzschild radius of the black hole via $M \sim r_s c^2 / 2G$) and $7 M$. Images were generated with adaptive ray tracing enabled and resized to a field-of-view (FOV) of $80 \mu\text{as}$ and resolution 128×128 pixels using a linear spline interpolation.

A sample set of images from the dataset is shown in [Figure 9.1](#). These images show the behavior of the $n = 1$ photon ring for varying black hole parameters. At edge-on inclination, the brightness and circular asymmetry becomes more pronounced at higher spin. However, at face-on inclinations, this behavior is significantly less prominent. The variation in spin at these low inclinations is more apparent as a change in the brightness modulation around the ring.

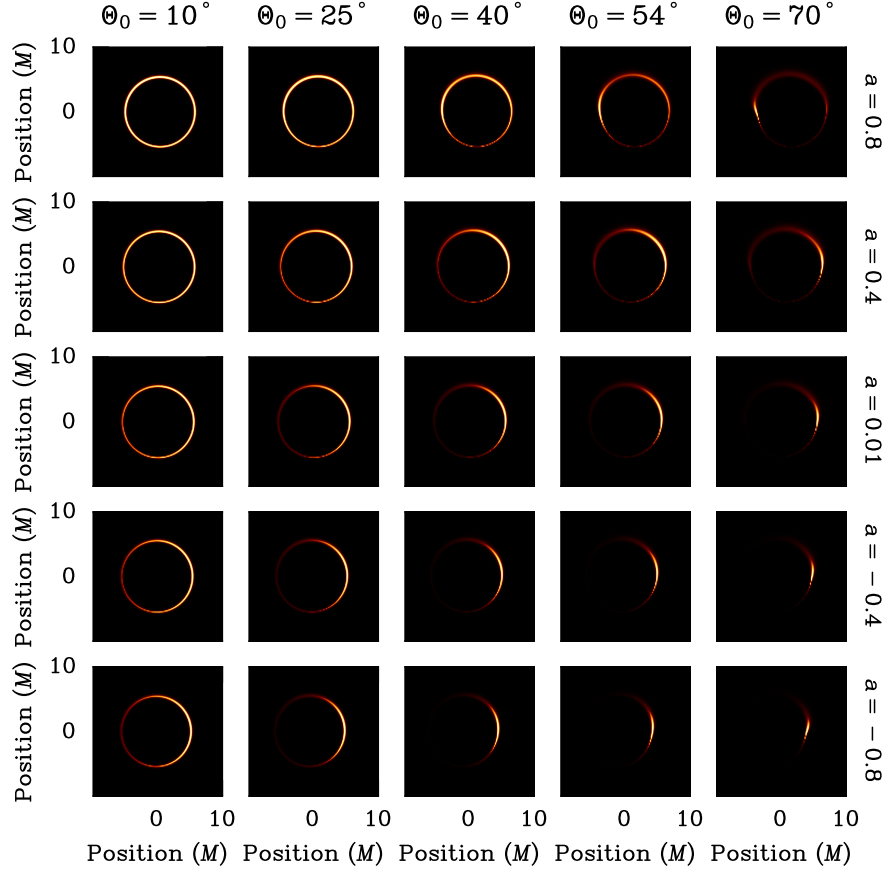


Figure 9.1: Sampled $n = 1$ photon rings generated with KerrBAM. Our dataset consists of $\gtrsim 10^6$ images spanning almost the entire Kerr parameter space.

9.4 Recursive brightest-point extraction

For this analysis, we seek to quantify the emission profile and shape of the $n = 1$ subring, in order to assess how it varies with spin. Image-domain, ring-based feature extraction has been developed for previous VLBI imaging analyses and has enabled, among other things, size and mass measurements of the M87* black hole based on images of the shadow [390; 322]. Once a series of points corresponding to the brightness profile has been measured, Bayesian inference methods can be used to quantify the geometry of the ring shape. By quantifying the modification of the shape with spin

and inclination, we can make assessments about the feasibility of using the subring shape to recover black hole parameters in data taken by BHEX.

The most common method for extracting a ring-like brightness profile involves identifying peak points of flux along rays emanating in all directions from an assumed center. This method was used to make measurements of the black hole shadows of M87* and Sgr A* [5; 296]. In some implementations, the assumed center is fixed and is not recalculated; however, in others, the assumed center is recalculated afterwards to minimize the average peak distance from the center. We use the ray-based feature extraction implementation in `eht-imaging`, specifically the `REX` module, to test the performance and efficiency of ray-based feature extraction [322]. We find in practice that ray-based feature extraction has several limitations that present potential obstacles to our analysis. First, due to the need to scan along dozens of rays and perform several objective function minimizations, this method of feature extraction is slow, with peak flux extractions ranging from ≈ 1 –10 seconds. This time range would result in a required time budget of ≈ 10 –100 days to fully analyze our 10^6 simulated images. Additionally, the method is somewhat dependent on the assumption of circularity, which is not consistently met by images of the $n = 1$ subring. Particularly at high spin, the shape may be significantly asymmetrical in both brightness distribution and ellipticity, resulting in incorrect or incomplete extractions.

To address these limitations, we have developed a novel and bespoke method of feature extraction designed for maximum robustness and performance specifically on BHEX subring images. The new algorithm distills the process of extracting peak fluxes into two vectorized steps which are then applied recursively. Given a BHEX image $I(x, y)$ of a photon subring with emission following some arbitrary parameterized curve $C(t)$, a characteristic angular radius $\mathcal{R}$, an intensity threshold τ , and an initially empty list of emission profile points $\{\mathcal{L}\}$, the algorithm is as follows:

1. Identify the brightest point (x_B, y_B) in the image based on intensity $I(x, y)$.
 - (a) IF $I(x_B, y_B) \leq \tau$: this is considered noise. QUIT and RETURN the list of points $\{\mathcal{L}\}$ corresponding to the emission profile.
 - (b) IF $I > \tau$: The coordinate (x_B, y_B) is considered a peak flux location; ADD to the list $\{\mathcal{L}\}$ corresponding to the emission profile of the image.
2. Mask out a circular region of radius $\mathcal{R}$ and center (x_B, y_B) in the image.
3. RETURN the masked image, the updated list of points $\{\mathcal{L}\}$, the radius $\mathcal{R}$, and the threshold τ to the function recursively.

A visualization of this algorithm is shown in [Figure 9.2](#). The masking radius $\mathcal{R}$ is chosen based on the estimated width α of the emission profile; a choice of $\mathcal{R} < \alpha$ may result in an incorrect recovery. When $\mathcal{R} \geq \alpha$, $\mathcal{R}$ controls the density of the points recovered along the ring. For an emission profile total integrated length $\ell = \int_C dt$, the number of points N produced with a chosen $\mathcal{R} \geq \alpha$ will be, on average, $N \sim \ell/\mathcal{R}$. The choice of τ is more complex but simultaneously less impactful; an optimal choice would be slightly lower than the dimmest point on the desired emission curve, but this is difficult to ascertain prior to calculating the emission profile. Another natural choice is simply an estimate of the average background intensity, which is straightforward to calculate quickly. As the emission profile is repeatedly masked out by step (2) in the algorithm, leaving only the background, the average background intensity served as an effective stopping criterion in our testing.

We implement our novel recursive brightest-point (RBP) extraction algorithm in `python` using vectorized tools in the `scipy` scientific computing package and compare performance to REX. The short, recursive nature of the RBP algorithm, combined with the full vectorization, results in a speed increase of $\approx 1\text{--}3 \times 10^3\times$ over ray-based

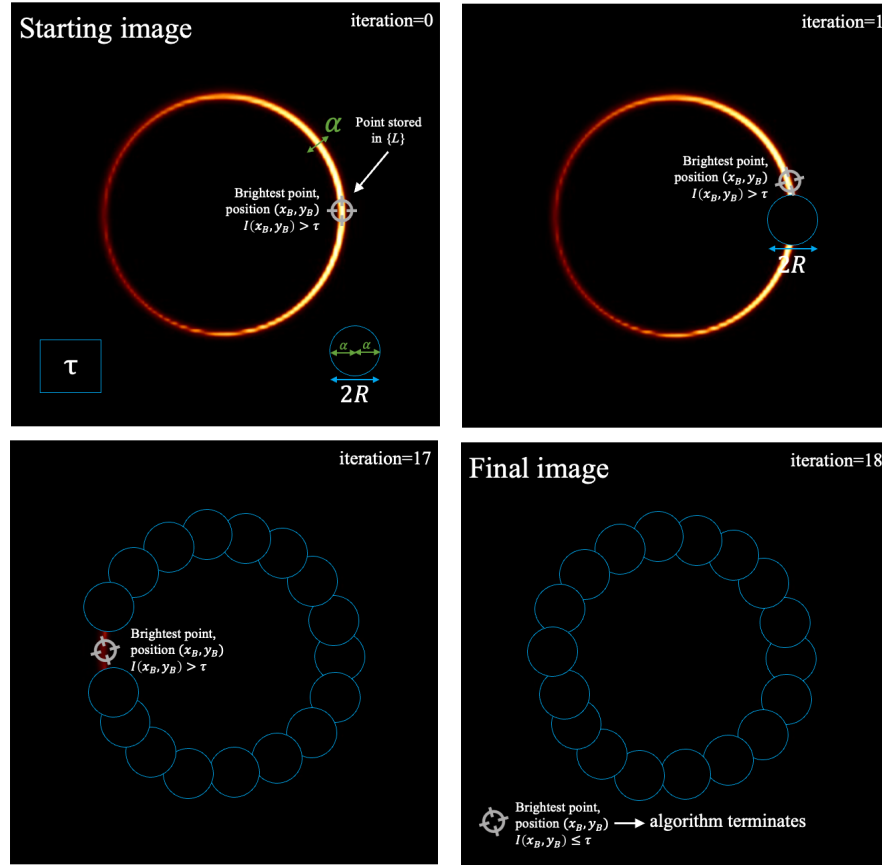


Figure 9.2: Visualization of the recursive brightest-point (RBP) extraction algorithm. The starting image configuration (far left) is used to estimate acceptable values for the algorithm parameters. The masking radius $\mathcal{R}$ is set to some value greater than the average radial emission profile width α . The noise stopping criterion τ is estimated based on a 5σ upper limit on the background flux distribution, sampled from within the blue square region. For the zeroeth iteration, the brightest point, at position (x_B, y_B) is found and added to $\{\mathcal{L}\}$. A region of radius $\mathcal{R}$ is masked out centered on (x_B, y_B) , and the masked image is recursively passed to the algorithm. For the next iteration, the next brightest point is found and added to $\{\mathcal{L}\}$, and the masking is repeated. This process is repeated for 17 iterations (middle right), during which the final point on the ring is masked out. On the final iteration (far right), all meaningful emission from the $n = 1$ photon ring has been masked out and the only flux left is noise. The brightest point in the image now has an intensity less than our stopping criterion τ , so the algorithm terminates.

feature extraction. On average, the RBP algorithm takes $\sim 10^{-3}$ seconds per extraction, compared to ≈ 1 second for REX. The spread in this extraction time for RBP is also small, as the performance of RBP is completely unaffected by the geometry of the image ($\sim 1 \times 10^{-4}$). By contrast, REX tends to perform worse (both in terms of time complexity and extraction success) on images with higher spins and inclinations, as the image becomes less circular and the azimuthal modulation of the brightness becomes more asymmetric.

However, RBP is strongly sensitive to the assumption that the brightest points in the image lie on the $n = 1$ subring. Other emission present—for example, in an aggregate image containing all subrings—will cause incorrect extractions using RBP. By contrast, REX scans for the brightest point along a particular ray, which the $n = 1$ subring emission will frequently satisfy. In practice, RBP is useful for developing large training sets on clean images where speed is a priority, while REX is better suited for more complex individual cases (e.g., real image reconstructions), where the time budget is greater but greater robustness is desired.

9.5 Geometric measurables

In order to quantify the transformation of the $n = 1$ subring shape with spin and inclination, we next choose and calculate geometry-characterizing statistics (“geometric measurables”) on the extracted peak flux locations. For the purposes of training a machine learning algorithm, we seek to identify the smallest set of geometric measurables that can successfully distinguish between subrings at different spins. Ideally, these measurables will be computable in $O(n)$ (linear) or $O(n \log n)$ (logarithmic) time complexity, to minimize the time required to generate training data. Additionally, these measurables cannot be constructed in reference to any absolute coordinate system, as

an absolute phase reference will likely be unavailable to BHEX.

The shape of the $n = 1$ subring is well-approximated by an ellipse for a large majority of the a - θ_0 parameter space. An ellipse fit encodes useful information, but the iterative nature of any objective function minimization routine makes a fit sub-optimal for performance. As an alternative, we use the *smallest enclosing ellipse* (SEE), the ellipse of smallest area which encloses all recovered points. The SEE can be calculated recursively in $O(n)$ time [391]. Additionally, the SEE has a much higher degree of variation with spin than a corresponding ellipse fit, making it a more effective diagnostic. This higher degree of variation is caused by the SEE encoding information about the brightness modulation around the ring, but also makes the SEE diagnostic more sensitive to the dynamic range of the interferometer. To construct our geometric measurables, we define the following quantities: r_a , the semimajor axis of the SEE; r_b , the semiminor axis of the SEE; φ_S , the principal angle of the SEE; and $\mathbf{C}_S = (x_{ab}, y_{ab})$, the centroid of the SEE. We additionally construct the center-of-mass $\mathbf{C}_m$ of the peak flux locations $\{\mathcal{L}\}_{i=0}^N$ with

$$\mathbf{C}_m = \begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} \frac{1}{N} \sum_i L_i^x \\ \frac{1}{N} \sum_i L_i^y \end{bmatrix}, \quad (9.1)$$

where L_i^x denotes the x component of a point identified by the RBP algorithm. We propose the following geometric measurables to comply with the constraints discussed above:

1. ε : an eccentricity parameter, constructed via

$$\varepsilon \equiv \frac{1}{r_a} \sqrt{r_a^2 - r_b^2},$$

2. $|\mathbf{C}_m - \mathbf{C}_S|/r_a$: Magnitude of vector between SEE centroid and center-of-mass of points, and
3. $\varphi_S - \arg(\mathbf{C}_m - \mathbf{C}_S)$: Angle of SEE relative to vector between centroid and center-of-mass of points.

These geometric measurables can be computed with linear time complexity, and are generally non-redundant, making them ideal for both generating and fitting our 10^6 element training set. Critically, these measurables are also scaled to a proxy of the overall size of the subring image (r_a), making them insensitive to the M/D ratio or the emission radius of the target black hole. In addition to these measurables, we also fit an ellipse with semimajor axis a_e , semiminor axis b_e , and principal angle φ_e to the peak flux locations $\{\mathcal{L}\}$ for the purpose of characterizing the $n = 1$ subring shape independent of the brightness modulation. These quantities (a_e, b_e, φ_e) are not used for training, and are computed for a smaller ($\sim 3 \times 10^4$) grid of images spanning the a - θ_0 parameter space for comparison to the SEE measurables.

9.5.1 Numerical variation with black hole properties

We compute each of the quantities described above for the images in our dataset and plot a heatmap of each result in [Figure 9.3](#). Broadly, all geometric measurables show relatively smooth and simple behavior modification with spin and inclination. At inclinations below $\theta_0 \approx 40^\circ$, much of the geometric behavior of the rings is highly degenerate, indicating likely significant challenges distinguishing spin at low inclination (such as that of M87*). Most parameters tend to an extreme value in the limit of high spin and edge-on inclination, except for the vertical component of $|\mathbf{C}_m - \mathbf{C}_S|$, which becomes most extreme towards low spin and high inclination. After the $\theta_0 \gtrsim 40^\circ$ boundary, variation in the parameters is fairly linear up to maximum spin and edge-

on inclination. In general, all parameters varied substantially more for negative spins than for positive spins.

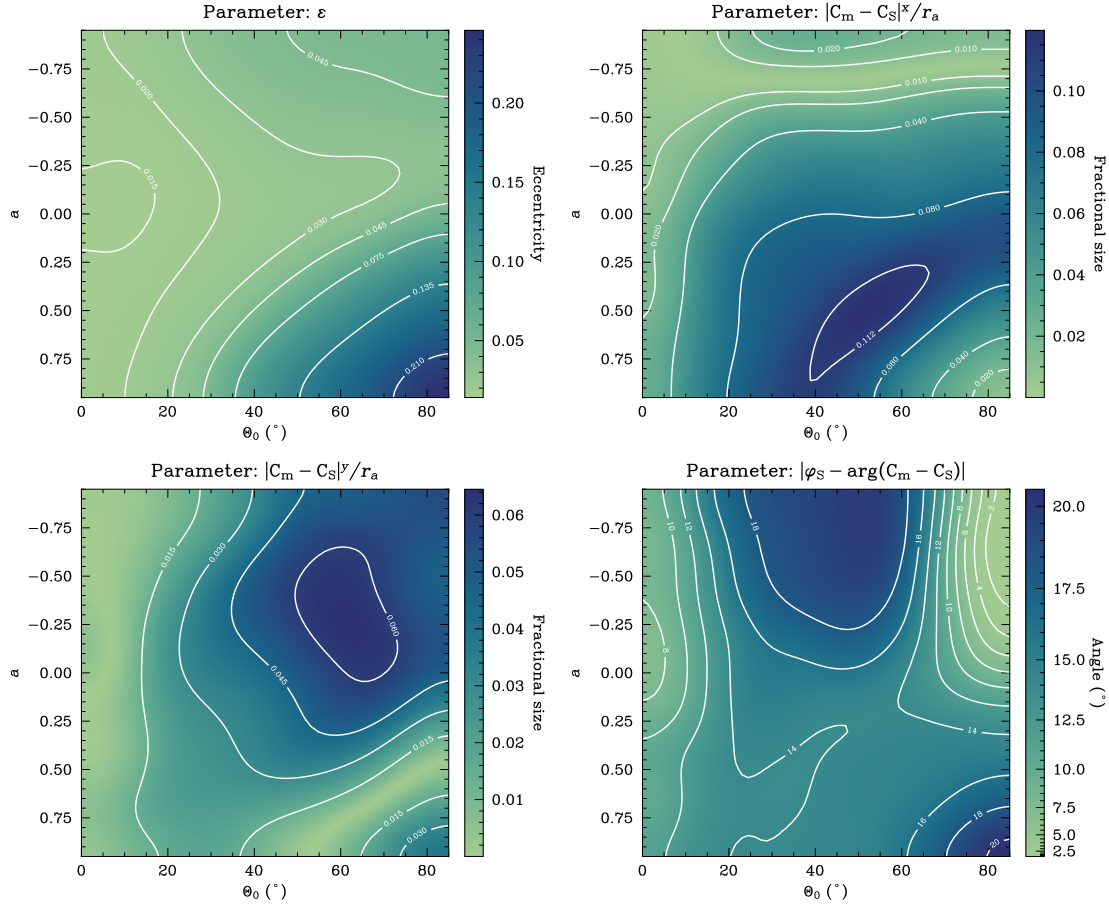


Figure 9.3: Heatmaps of the SEE quantities described in [section 9.5](#).

The ellipse semimajor and semiminor axes as well as the orientation show substantially less variation across the Kerr parameter space. In contrast to the SEE parameters, the size parameters of the ellipse fit vary $\lesssim 6\%$ as spin and inclination are modified. In general, the lower variation makes spin predictions based solely on ellipse fits significantly less discriminatory than predictions based on the SEE model, or a similar model which incorporates brightness distribution.

The variation for all SEE parameters is more easily characterized by considering

only positive or negative spin. To this end, we present approximate formulae for the variation of the geometric measurables with spin and inclination. We fit a third-order bivariate polynomial (BV) Q in a and θ_0 to an arbitrary quantity $Q(a, \theta_0)$ using the following expression:

$$Q = \sum_{i,j=0}^3 A_{i,j} a^i \theta_0^j, \quad (9.2)$$

where $\{A_{i,j}\}$ are the fit coefficients. These fits are performed separately for positive and negative spins. We perform a least-squares minimization over the heatmap distributions in [Figure 9.3](#) using this function and use the best-fit bivariate polynomial to approximate the behavior of the $n = 1$ photon ring morphology. An example visualization of the fit to the r_b parameter of the SEE model is shown in [Figure 9.4](#). We present BV parameters for all SEE model parameters in [Table 9.2](#) and [Table 9.3](#). On average, we find an average residual of $\lesssim 2\%$ across all spins and inclinations, which indicates excellent fits for all parameters.

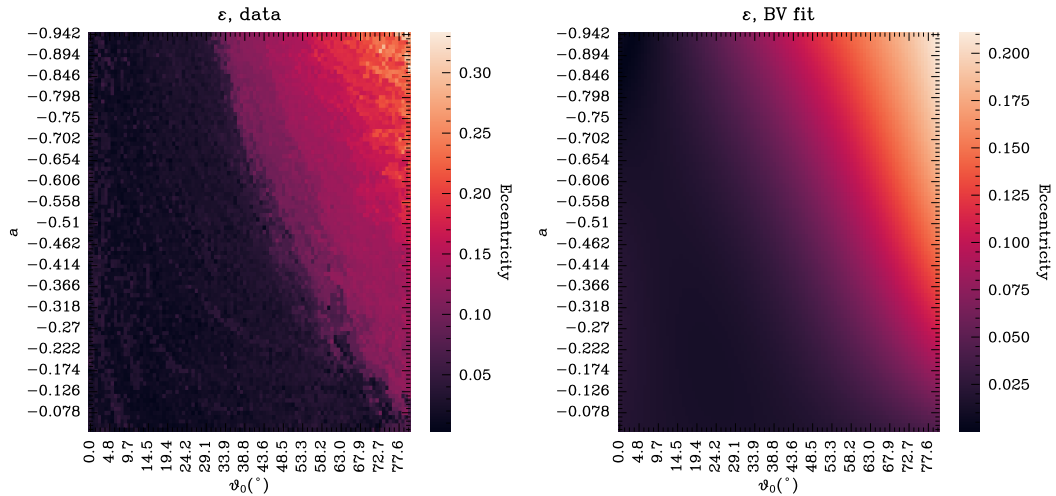


Figure 9.4: Example visualization of the fits to the SEE and ellipse parameters using a third-order bivariate polynomial (BV). (*left*) The best-fit values of r_b for every combination of spin and inclination in the dataset. (*right*) A description of the r_b parameter using a third-order BV fit. The fit has an average residual of $\lesssim 2\%$, indicating a good fit.

$$a < 0$$

Parameter	A_{00}	A_{01}	A_{02}	A_{10}	A_{11}	A_{12}	A_{20}	A_{21}	A_{22}
ε	0.206	-0.008	1.0×10^{-4}	-0.002	0.023	-7.6×10^{-4}	-0.412	0.065	-0.001
$ \mathbf{C}_m - \mathbf{C}_S ^x/r_a$	-0.062	0.006	-9.3×10^{-5}	-0.338	0.031	-3.5×10^{-4}	-0.248	0.019	-1.8×10^{-4}
$ \mathbf{C}_m - \mathbf{C}_S ^y/r_a$	-0.008	5.5×10^{-4}	-2.0×10^{-5}	-0.081	0.010	-1.5×10^{-4}	-0.080	0.010	-1.2×10^{-4}
$\varphi_S - \arg(\mathbf{C}_m - \mathbf{C}_S)$	0.796	-3.400	0.036	150.000	-8.830	0.123	133.000	-8.790	0.123

Table 9.2: BV fit values for the A_{ij} matrix components for negative spin ($a < 0$), computed for each SEE model parameter, as constructed in [section 9.5](#).

$$a > 0$$

Parameter	A_{00}	A_{01}	A_{02}	A_{10}	A_{11}	A_{12}	A_{20}	A_{21}	A_{22}
ε	0.011	2.5×10^{-4}	-8.1×10^{-7}	-0.002	2.8×10^{-4}	1.0×10^{-6}	0.006	5.4×10^{-4}	-7.1×10^{-6}
shfitx	8.0×10^{-4}	-0.003	3.2×10^{-5}	0.004	0.002	-4.3×10^{-5}	-0.019	0.004	-1.6×10^{-5}
shifty	0.014	8.0×10^{-4}	-9.2×10^{-7}	-0.055	0.007	-9.3×10^{-5}	0.059	-0.008	1.1×10^{-4}
alpha	-23.700	-3.610	0.044	24.100	-6.140	0.097	-10.200	3.660	-0.061

Table 9.3: BV fit values for the A_{ij} matrix components for positive spin ($a > 0$), computed for each SEE model parameter, as constructed in [section 9.5](#).

9.6 Machine learning application

Recent advancements in machine learning have revolutionized various scientific domains, including astrophysics, by enabling the development of sophisticated data analysis techniques. Among these, gradient boosting and convolutional neural networks (CNNs) have emerged as particularly impactful. Gradient boosting [392; 393] has proven effective for structured data analysis, offering robust predictive performance through the ensemble of weak learners. Concurrently, CNNs [394; 395] have significantly advanced computer vision tasks, providing powerful tools for image recognition and classification. Here, we apply these techniques to the study of the $n = 1$ photon ring. In subsection 9.6.5, we implement a convolutional neural network following the Bayesian neural network in [389] to estimate spin a directly from images of the $n = 1$ photon ring. In addition, we attempt to approximate the anticipated performance of such a neural network by training a gradient boosting regression model on the geometric measurables presented in section 9.5.

9.6.1 Initial algorithm selection

Performance of machine-learning algorithms can vary in non-trivial ways depending on the nature and characteristics of the training data. Here, we briefly explore several machine learning algorithms (linear regression, k -neighbors regression, random forest regression, and gradient-boosted regression tree; all implemented in `scikit-learn`, see subsection 9.6.2) and compare their performance on our dataset. For all models except linear regression, we varied a primary hyperparameter and chose the value corresponding to the start of a plateau in R^2 , to minimize the effect of hyperparameter tuning/overfitting. The performance of the models was as follows:

Linear regression: this model performed poorly ($R^2 \ll 0.5$) on both training and validation datasets.

k-neighbors regression (KNN): this model performed significantly better than simple linear regression ($R^2 \sim 0.8$). We varied $n_{\text{neighbors}}$ to find an optimal hyperparameter choice of $n_{\text{neighbors}} = 75$.

Random forest regression (RFR): this model was moderately superior to KNN ($R^2 \sim 0.88$). Performance was exceptional on training data but poorer on validation data; indeed, this model had the widest spread between scores on training vs. validation data of any of the models besides linear regression. We vary the depth hyperparameter and note a plateau around depth= 17.

Gradient-boosted regression tree (GBRT): this model performed comparably to RFR ($R^2 \sim 0.9$). Among the models, the difference between the performance on training vs. validation data for GBRT was the most minimal ($\sim 0.1\%$). We varied the $n_{\text{estimators}}$ hyperparameter and identified a plateau around $n_{\text{estimators}} = 250$.

Given the similar performance of the GBRT and RFR, we select the GBRT for its greater consistency between training and validation. Additionally, our dataset is noise-free and has subtle features, which favors the iterative correction of GBRT; by contrast, we may favor the robustness of a RFR-like model when working on noisier training data. We expand in significantly more detail on the setup and training of the GBRT model below.

9.6.2 Setup and training

We implement a GBRT in `scikit-learn` following the notation of [396] and the references therein in order to estimate spin from the geometric measurables presented in

[section 9.5](#). We briefly summarize the techniques used here. Gradient boosting is a machine learning technique that incrementally builds an ensemble of weak predictive models, typically decision trees, to optimize performance by minimizing a loss function through iterative improvements. A GBRT regressor constructs a prediction $\hat{y}_i$ for a given input x_i by adding the predictions of a series of M weak “learners” $h_m x_i$; i.e.,

$$\hat{y}_i = \sum_{m=1}^M h_m(x_i). \quad (9.3)$$

Each estimator h_m is a decision tree regressor of fixed size, constructed to minimize the losses of the summed previous $m - 1$ estimators. The initial estimator h_0 is arbitrary and, for a least-squares loss function, is typically chosen as the mean of the response variables. The parameters of the trees are fit via a functional space gradient descent.

We implement the `GradientBoostingRegressor` in `scikit-learn` with a least-squares loss function. For training the GBRT, our 10^6 image dataset is broken up as follows:

1. **Testing data:** we set aside a fixed 10% of our dataset for universal validation. This is a representative section of the dataset which no network sees during training; we use it to gauge model performance after training is completed.
2. **Training data:** the remaining 90% of the data is used for training the models. The split between “training” subset and “validation” subset discussed below is randomized for each run.
 - (a) **Training subset:** a subset of the training dataset is used for training the models. The size of this subset corresponds to 90% of the training data (which itself is 90% of the full 10^6 images).
 - (b) **Validation subset:** a subset of the training dataset is used for validating the

models *during* training. This is separate from the testing set. The size of this subset corresponds to 10% of the training data (which itself is 90% of the full 10^6 images).

We use the universal validation testing data to optimize hyperparameters (namely M) and estimate performance of the network. As M increases, the performance of the network asymptotically approaches some maximum accuracy, and can even decrease as overfitting effects increase. We choose the number of learners M such that the accuracy achieves $\approx 99\%$ of the maximum, to maximize performance while avoiding overfitting. We train on the input quantities $(r_a, r_b, |\mathbf{C}_m - \mathbf{C}_s|, \varphi_s - \arg(\mathbf{C}_m - \mathbf{C}_s))$ described in [section 9.5](#) and response variable spin (a). For testing purposes, we also attempted training with 95% and 80% of the training datas (with a 5% and 20% validation subset, respectively) and did not notice a significant variation ($\lesssim 1\%$) in the performance of the algorithm. The raw prediction of the algorithm $\hat{y}_i$ is treated as the spin estimate with no modifications.

We next estimate an uncertainty on our measurement. Broadly, there are two types of estimate uncertainties: *epistemic* uncertainties, capturing the insufficiency of the model; and *aleatoric* uncertainties, which capture the intrinsic uncertainty present in the data. We discuss both below.

To estimate an epistemic uncertainty, we implement cross-validation similar to the stochastic gradient boosting technique [397]. In addition to the main prediction algorithm, we perform 10 leave-one-out bootstrapping steps, which we refer to as “bootstrap samples”. Each training set of these samples is randomly selected to be 40% of the whole dataset, with the remaining 10% used as the validation set. After an estimate of spin is made by using the whole dataset for a prediction, the samples are similarly queried to produce a range of predictions $\{\hat{y}_k\}_{k=0}^{10}$ based on their limited

sample of the dataset. The standard deviation of this range $\sigma_{\hat{y},k}$ is used as an estimate of the epistemic uncertainty. The purpose of this uncertainty is to estimate how sensitive our prediction is to the particular construction of the dataset and choice of input images.

To estimate an aleatoric uncertainty, we consider the largest source of uncertainty in an estimate of spin: namely, the degeneracy of the photon ring shape as the inclination varies. At low inclination, the geometric shape of the photon ring is virtually invariant to spin. The degeneracies are visible in [Figure 9.3](#), where most parameters see little-to-no variation at inclinations $\theta_0 \lesssim 40^\circ$. To quantify the uncertainty due to this degeneracy, for each spin a we consider a range of inclinations $[\theta_0, \theta_0 + \delta\theta_0]$. Within this range, we adopt the standard deviation σ_{θ_0} of the estimates of spin for each (a, θ_0) combination as an estimate of the aleatoric uncertainty resulting from the inclination degeneracy. We choose $\delta\theta_0 = 19.75^\circ$, corresponding to 25% of the inclination range we generated for the dataset. Each bin of inclination would therefore contain $\approx 2.5 \times 10^3$ spin estimates at that inclination range, making it an effective approximation of the estimate variance due to the inclination degeneracy.

We add together our aleatoric and epistemic uncertainties together in quadrature to construct a global uncertainty estimate. In general, we found that $\sigma_{\hat{y},k} \ll \sigma_{\theta_0}$, making the inclination degeneracy the dominant source of uncertainty in our estimates.

9.6.3 Overall performance of the GBRT

We consider the overall performance of the GBRT described in [subsection 9.6.2](#) with and without convolution, and with retraining. The maximum nominal BHEX beam is expected to be $\approx 9 \mu\text{as}$, corresponding to operations at 230 GHz, and resulting in an imaging resolution of $\approx 4.5 \mu\text{as}$. At 345 GHz, the BHEX beam is expected to be $\approx 6 \mu\text{as}$,

corresponding to an imaging resolution of $\approx 3 \mu\text{as}$. Thus, to conservatively simulate BHEX imaging conditions, we test the performance of the GBRT under a convolution of $6 \mu\text{as}$, with and without retraining. The distribution of residuals between the predicted and estimated spins is shown in [Figure 9.5](#). We find an average residual of ≈ 0 ($r^2 \approx 0.95$) with a spread of $\lesssim \pm 0.2$ on the unconvolved dataset. This corresponds to $\lesssim 20\%$ of the full range of spins and is convincing evidence that, in the absence of noise and other artifacts, the photon ring geometry and brightness distribution is sufficient to distinguish between different spins. The performance is significantly worsened after convolution with a $6 \mu\text{as}$ beam. At this convolution, the average residual is still ≈ 0 ; however, the spread of deviations is 2-3x larger than the spread of deviations with no convolution. Additionally, there is a skew towards negative values of $a_{\text{estimated}} - a_{\text{truth}}$, indicating that the effect of convolution on our pipeline is an underestimate of spin for a significant portion of the dataset. By contrast, the global (i.e., across all inclinations) performance of the model when trained on convolved images is approximately comparable to the performance of the model when trained on non-convolved images.

Both conditions represent insight into the utility of the $n = 1$ photon ring to measure spin. However, neither truly represents the model's performance in real observational conditions. The performance under non-convolution represents a proof-of-concept—an effective demonstration that under perfect conditions, the $n = 1$ photon ring has characteristics which can be used to measure spin. While this is of course a hyperideal scenario, it is still an important demonstration, as it sets an upper limit on how precise a measurement of spin we can achieve using this method. The retraining under $6 \mu\text{as}$ convolution is a better (but still idealized) analog for real observational conditions, as convolution is the dominant artifact in interferometric images. The excellent performance is a strong indicator that the spin-distinguishing variations of the $n = 1$ photon ring will persist even in convolved images.

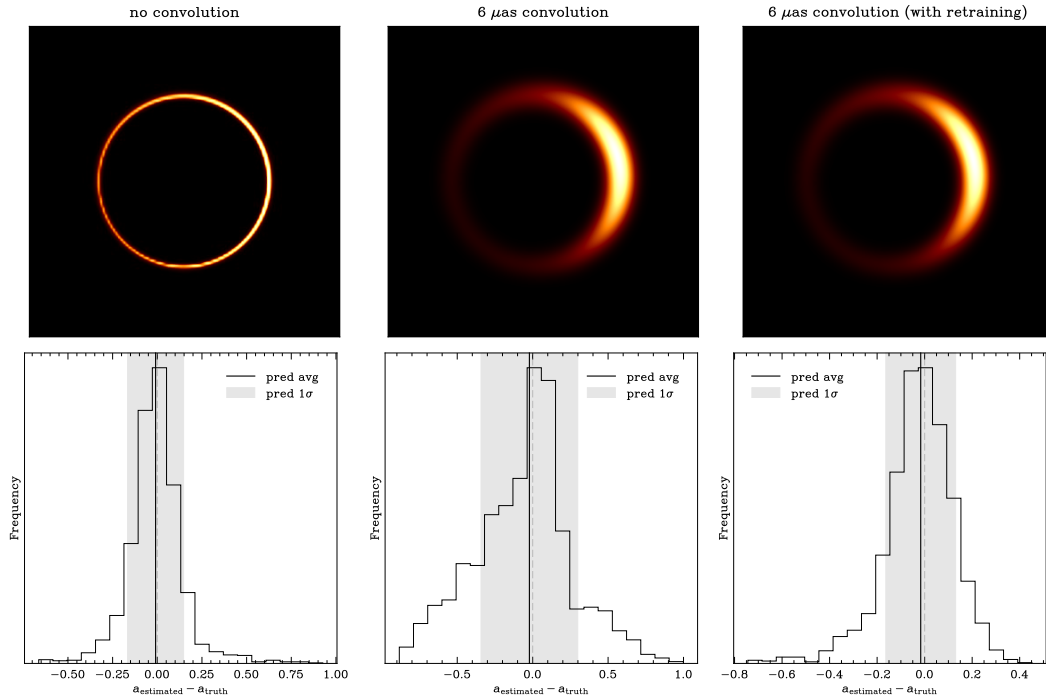


Figure 9.5: Performance at varying convolution for the GBRT described in [subsection 9.6.2](#). (top) Representative appearances of the $n = 1$ photon ring at varying convolutions, ranging from no convolution (left) to the anticipated BHEX beam at 345 GHz (center and right). (bottom) Performance of the GBRT with the convolution level shown in the top row. With no convolution, or convolution with retraining, recovery is highly accurate. As the size of the convolving beam is increased, the performance of the network without retraining predictably deteriorates.

9.6.4 Intra-inclination performance of the GBRT

We now consider the performance of the GBRT at different inclinations. Due to the inclination degeneracy, we expect variable performance of the neural network as inclination is varied. In [Figure 9.6](#), we visualize the performance of the GBRT on four inclination ranges: face-on (0° to 20°), low (20° to 40°), high (40° to 60°), and edge-on (60° to 90°). We perform this visualization for the GBRT trained on non-convolved data operating on non-convolved data, as well as the equivalent analysis for the GBRT operating on data convolved with a $6 \mu\text{as}$ beam. Unexpectedly, we find that

GBRT performance at low and face-on inclinations in the non-convolved case is similar to the performance at high and edge-on inclinations. Recovery at all inclinations is consistent (within the aleatoric uncertainty) with perfect recovery, although there is a systematic under-estimation of the spin magnitude of $\lesssim 5\%$ (of the full spin range) at spins with $a < -0.8$ for $\theta_0 \in [40^\circ, 60^\circ]$. The other inclination ranges show comparable performance. The excellent performance even at low spins and inclinations indicates that, in the absence of imaging convolution and artifacts, the $n = 1$ photon ring can be a distinguishing proxy for spin.

The network performance is significantly worsened when realistic BHEX convolution is introduced in images. When images are convolved with a $6 \mu\text{as}$ beam (no retraining), negative spins are recovered accurately (though with a significantly larger spread in predictions). However, positive spins are largely under-estimated, with most estimates being consistent with $a = 0$. When the network is retrained on images convolved with the $6 \mu\text{as}$ BHEX beam, the performance is comparable to the $0 \mu\text{as}$ convolution test at inclinations $\theta_0 > 20^\circ$. At inclinations $\theta_0 < 20^\circ$, estimates are still consistent with the true values, but there is a greater tendency to underestimate the spin magnitude.

The lower performance with convolution indicates that an image feature-extraction approach may be inadequate to perform spin extraction using the $n = 1$ photon ring unless estimation tools are trained specifically at the anticipated BHEX resolution. However, the excellent performance with no convolution indicates that the $n = 1$ photon ring can be a distinguishing proxy for spin in the absence of convolution or imaging artifacts. Performing spin estimates using Fourier analogs of the geometric measurables may sidestep confounding effects from imaging; we investigate this method of spin inference in [316].

9.6.5 Comparison to Deep Horizon

Convolutional neural networks (CNNs) and the cousin Bayesian neural networks (BNNs) have been used in past analyses to demonstrate recovery of simulation parameters from GRMHD simulations of black hole accretion flows [389]. The primary benefit of a CNN is the ability to train on and input entire images, without needing to perform feature extraction or calculate geometric measurables. This feature of CNNs practically guarantees that they will outperform alternative machine learning algorithms such as GBRT. However, CNNs are also slow to train as a result of running on full, often high-resolution images. By implementing a CNN and comparing the performance to our GBRT, we can assess how effectively our choice of geometric measurables is utilizing the information present in the images.

We reproduce the methodology of [389] with minor modifications. We implement a CNN in Tensorflow version 2.14 [398] leveraging the keras [399] API version 2.14. We choose and implement a 10-layer CNN architecture. Following [389], we use a rectified linear unit activation function [400] for all layers except the output layer. We use the same testing-training-validation data split described in subsection 9.6.2 to train and evaluate the CNN. The network is trained on the input data using the legacy implementation of the Adam optimizer [401]. The network is initialized with the mean squared error as the primary loss function, with the mean absolute error as an evaluation metric. The learning rate and number of training epochs was determined by a grid search to optimize performance on the testing data. We use a fixed learning rate of 0.001 and 50 training epochs for all implementations and runs.

We apply the network architecture and training pipeline to both non-convolved images and images convolved with the BHEX beam. The results are shown in Figure 9.7 and Figure 9.8. The performance on non-convolved images is almost identical to the

performance for the corresponding GBRT, featuring a mean residual of ≈ 0 and a $\lesssim 20\%$ improvement in the residual spread. We follow [389] and also visualize the performance via a scatter plot of spin recoveries, color-coded by standard deviations from perfect recovery. The network reports consistently accurate recovery of spin at all spin and inclination combinations, with marginally worse performance below $a \lesssim 0.2$. The similarity of the performance between the CNN and the GBRT is a strong validation in the choice of geometric measurables being sufficient to capture the salient and spin-distinguishing features of the $n = 1$ photon ring. However, the CNN drastically outperforms the GBRT on images convolved with the BHEX beam, indicating the GBRT may not be fully suitable for direct application to BHEX images.

Performance on the CNN—particularly in the non-convolved case—is excellent despite the relatively simple network architecture. The success of the model is likely due to the simplicity of the dataset itself, which features no noise or complications which may require a more complex network to interpret. However, real image reconstructions from BHEX will be significantly more contaminated with multiple noise sources and reconstruction artifacts beyond convolution. For application to real data, exploiting the latest advancements in neural network architecture (e.g., skip connections and attention mechanisms) may not only be desirable but essential to achieve performance sufficient to conduct spin measurements.

9.7 Application to GRMHD simulations and BHEX

The analysis previously discussed in this work was based on the semi-analytic simulation engine KerrBAM. Here, we apply the methods developed using KerrBAM to GRMHD simulations of black hole accretion flows. The purpose of this analysis is to provide validation that methods developed using semi-analytic simulation engines can

be applied to complex simulations more analogous to a real black hole environment. Additionally, full numerical GRMHD simulations were not included in the training for either the GBRT or CNN networks, making a test on GRMHD simulations a more challenging evaluation of network performance.

9.7.1 Preparation of synthetic data

We analyze the decomposed sub-images from the model set ray-traced in [402]. These simulations have a mass-to-distance ratio corresponding to the EHT estimate for M87*. The fluid models were simulated with `iharm3D` [403; 404] and ray traced with `ipole` [405], using the $R_{\text{high}} = 40$ electron heating parameter described in [406]. More details about the GRMHD and General Relativistic Ray Tracing (GRRT) pipeline can be found in [407]. The resulting images are time-averaged to produce smooth images of typical accretion flow structures, including the photon ring. The time-averaged GRMHD represent an accurate if noise and artifact-free representation of the appearance of sources to BHEX. We generate three GRMHD simulations at $a = 0.0, -0.5$ and -0.94 . The inclinations are generated at the expected inclination angle for M87* ($\theta_0 \approx 17^\circ$).

9.7.2 Application to $n = 1$ GRMHD simulations

We apply the two algorithms discussed previously (GBRT and CNN) to the GRMHD simulations constructed [subsection 9.7.1](#). For the GBRT, we measure the geometric observables constructed in [section 9.5](#) and pass them into the GBRT, which returns an estimate of the spin. For the CNN, we simply resize the image to the 128x128px input size and pass it into the CNN, which returns an estimate of the spin. The results are shown in [Figure 9.9](#). Performance between the two approaches is very similar, despite the difference in method. The similarities in performance on GRMHD are an additional

validation that the geometric measurables constructed in [section 9.5](#) capture most, if not all, of the salient features distinguishing spin. Both methods recovery spin values consistent with the ground truth simulation at $|a| \gtrsim 0.5$, but overestimate the low spin simulations in a manner consistent with complications from the spin-inclination degeneracy problem. Performance could be improved by breaking this degeneracy; future methods may incorporate information from e.g., polarization, which is modified by frame-dragging but is also heavily influenced by properties of the plasma flow in the vicinity [[408](#); [294](#); [402](#)]; or constraints on the diameter measured in Fourier space, known to vary tightly with spin [[260](#); [302](#); [316](#)].

The accuracy of the spin recovery on both the KerrBAM and GRMHD images is a strong preliminary indication that the $n = 1$ photon ring has utility as a proxy to measure spin, with the essential caveat that these tests have been performed on noiseless images without the contribution of the $n = 0$ and $n > 1$ subring emission. The presence of additional subrings in BHEX images—particularly the $n = 0$ direct emission, which is dominated by surrounding astrophysical effects and not the black hole properties—will likely overwhelm the $n = 1$ photon ring emission, making the image-domain approach described here potentially infeasible for precise spin measurements.

9.7.3 Observational limitations

Our analysis demonstrates that the $n = 1$ photon subring can, in idealized conditions, be used to distinguish spin at the resolution of the upcoming BHEX instrument. However, the methods shown represent an upper bound of measurement performance using the $n = 1$ subring, as BHEX image reconstructions will feature contamination which we do not include in our training. These contaminations may include:

Presence of other subrings: BHEX observations of target sources will not exclusively

observe the $n = 1$ subring; instead, they will observe a superposition of emission from the black hole environment [260]. The additional emission will complicate extraction of the $n = 1$ subring shape by the RBP extraction method presented in [section 9.4](#). Incorrect extraction of the subring will directly worsen performance of the GBRT model, as the computation of the geometric measurables will likely be affected. This complication may be addressed by slower but more robust feature extraction methods (e.g., REX, see [section 9.4](#)) which are not optimized to run on large datasets but may be better-suited to extract the $n = 1$ photon subring from BHEX reconstructions.

Thermal (and other) noise: BHEX images will be reconstructed from sparse Fourier domain measurements which will have at minimum thermal noise, as well as systematic error and propagation effects [e.g., 291; 299]. Significant noise (regardless of source) can dominate small-scale features, such as the photon ring substructure, and may introduce false structures in the reconstructed image. Extraction and measurement of these false features will naturally lead to unreliable estimates of spin.

Image snapshots: the methods applied here are designed to be applied to observations averaged over many dynamical timescales, thus ensuring transient features smooth out and leaving only the photon ring [260; 388; 386; 387]. BHEX is expected to produce such time-averaged images [299; 301]. However, the methods discussed in this analysis are not robust to artifacts in non-time-averaged (i.e., snapshot) reconstructions. In such images, sharp features may be present that are unrelated to the photon ring substructure. Extraction or measurement may mistake these sharp features for the photon ring and lead to erroneous estimates of spin.

9.8 Conclusions

The upcoming BHEX orbiting observation platform will enable an unprecedented view of the photon rings of the largest black holes on the sky. In particular, BHEX will probe the $n = 1$ subring of M87* and Sgr A*, which is largely invariant to surrounding astrophysical phenomena and primarily affected by the black hole’s spin and inclination. As a result, the $n = 1$ photon ring has the potential to serve as a proxy for precisely measuring the spin of the black hole. However, the behavior of the $n = 1$ subring as spin is varied is not completely understood, presenting a potential obstacle for tools hoping to exploit it to measure angular momentum.

We present an end-to-end pipeline for characterizing the variation of black hole subrings as the properties of the black hole is modified. We take advantage of the fast semi-analytic simulation engine KerrBAM to generate a large volume ($\gtrsim 10^6$) of $n = 1$ subrings, varying spin and inclination in small increments. We develop a novel feature extraction algorithm (recursive brightest-point extraction) that is capable of extracting the non-circular brightness profiles of the subrings from these images on a short timescale, representing a $\sim 10^3\times$ performance increase over existing EHT-based tools. Next, we construct a series of geometric measurables which characterize the shape, orientation, and brightness distribution of the emission profile, and construct a large training set from these quantities.

Our large dataset of $n = 1$ subrings, combined with the minimal set of descriptive observables, provides an opportunity to precisely characterize the behavior of the $n = 1$ subring. We quantify the behavior by fitting a bivariate cubic polynomial to each quantity and provide a mapping between combinations of (a, θ_0) and each geometric measurable. We then develop and apply a gradient boosted regression tree (GBRT) to the dataset of geometric measurables to broadly test whether these measurables are

sufficient to constrain spin. We additionally test the performance of the GBRT against a convolutional neural network (CNN), which theoretically represents the maximum performance achievable on the images in our dataset (as the image necessarily contains more data than any set of geometric measurables). We find that, under no convolution, the GBRT and CNN are both able to measure spin from images with a high accuracy ($< 10\%$ deviation from truth spin for all spins and inclinations). However, when convolution up to the BHEX beam ($\approx 10\mu\text{as}$) is introduced, both algorithms suffer at low spin and particularly low inclination, where the photon rings appear highly degenerate. Overall, the GBRT performs remarkably similarly to the CNN, indicating that the geometric measurables we have chosen successfully capture the salient features of the images.

To more fully test the ability of the spin extraction methods described above, we test recovery on GRMHD simulations of black holes. These simulations are typically more accurate to reality but also more computationally expensive to generate. We chose simulations at the M87* inclination of $\theta_0 = 17^\circ$, which was particularly challenging for our algorithms during validation on the KerrBAM data. We begin by testing the GBRT and CNN algorithms on GRMHD simulations of the $n = 1$ emission, to probe the utility of training on faster but potentially less accurate semi-analytic models. Remarkably, we find recovery of spin from the GRMHD images to be comparable in accuracy to the KerrBAM counterpart tests. Both the CNN and GBRT perform better at high spin and over-estimate the truth values at low spin.

We have presented a comprehensive pipeline for the study and analysis of black hole photon subrings. Analysis of the $n = 1$ subring has illuminated aspects of how its geometry is modified with spin and inclination across the Kerr parameter space. Additionally, we have developed and trained several tools for extracting spin from semi-analytic and GRMHD simulations. Future research can expand upon any of

these approaches in order to maximize utility to BHEX. The machine learning algorithms would benefit from an increased training set over a wider range of parameter combinations (e.g., electron heating) to both build more robust spin estimates and estimate parameters beyond just spin and inclination. To ensure that these procedures are effective, it is essential to adapt our methods to the specific observational constraints of BHEX. These tools are the basis of viable procedures to measure spin from future, sparse BHEX measurements of target black holes.

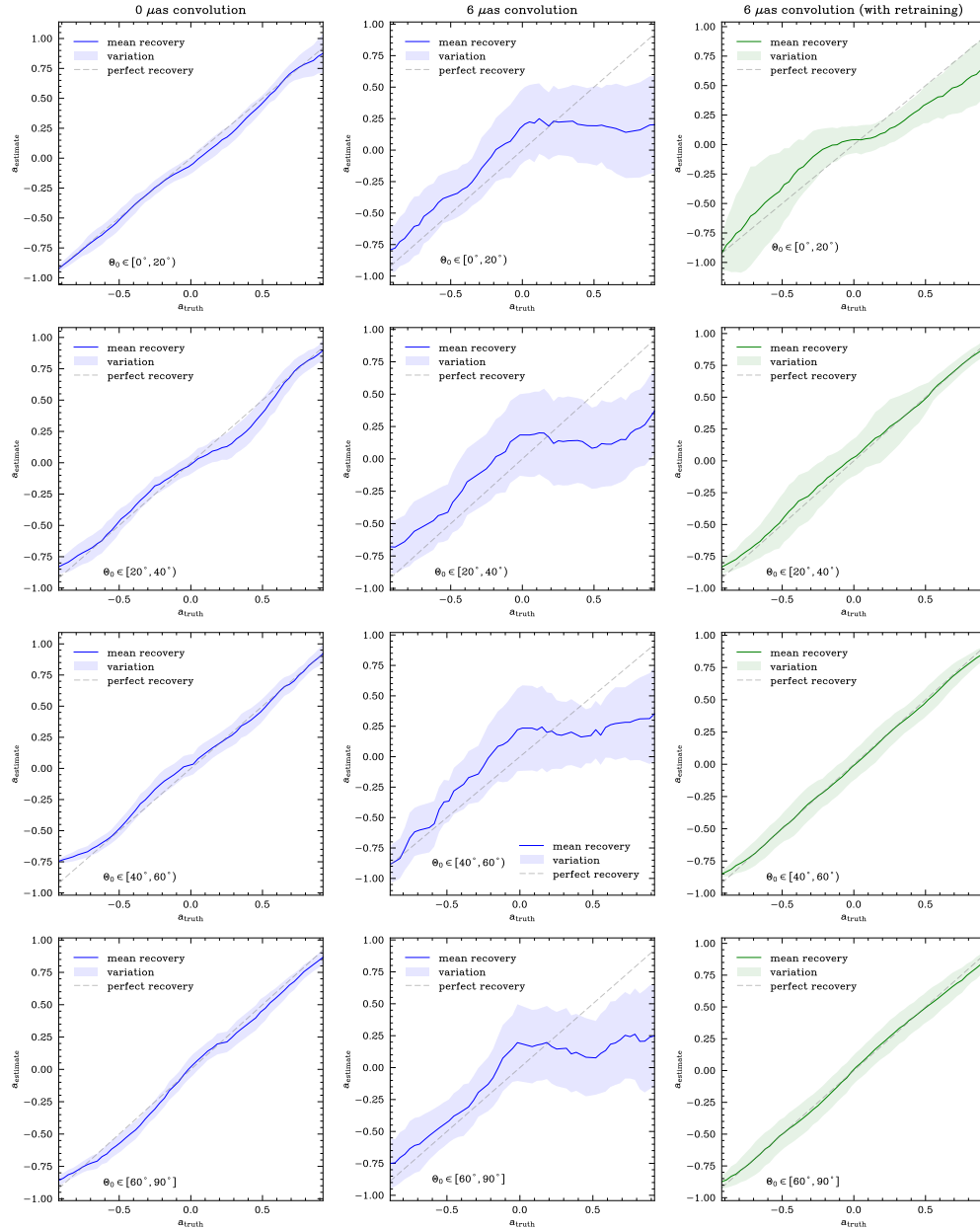


Figure 9.6: Intra-inclination performance of the GBRT on four inclination ranges: face-on (0° to 20°), low (20° to 40°), high (40° to 60°), and edge-on (60° to 90°). The left column (blue) corresponds to the GBRT trained on non-convolved data operating on non-convolved data; the right column (green) is the equivalent analysis for the GBRT operating on data convolved with the $6 \mu\text{as}$ beam. The center column corresponds to the GBRT trained on non-convolved data operating on data convolved with a $6 \mu\text{as}$ BHEX beam. The shaded region in all plots corresponds to the aleatoric uncertainty σ_{θ_0} computed in [subsection 9.6.2](#).

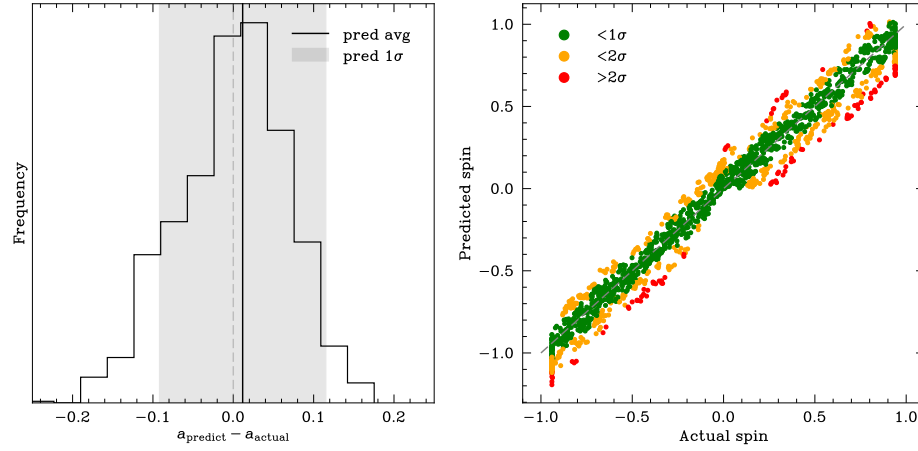


Figure 9.7: Performance of a CNN trained on non-convolved images. We find recovery performance approximately comparable to the performance of the GBRT. The left histogram is an aggregation of recovery residuals, showing an average residual close to zero with a narrow spread of $\lesssim 0.1$. The scatterplot on the right shows sample recoveries at all spins, color-coded by the deviation from the mean recovery at a particular spin.

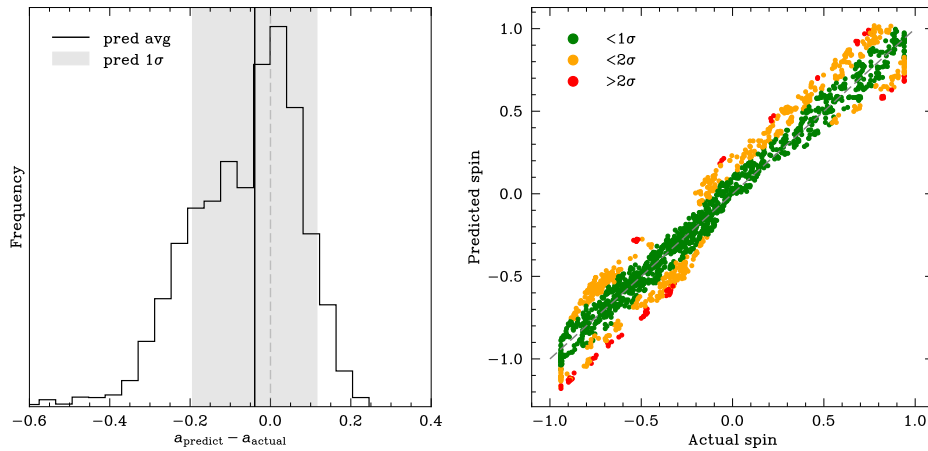


Figure 9.8: Same as Figure 9.7, but for a CNN operating on images convolved with the nominal BHEX beam ($6\mu\text{as}$, bottom), without retraining. We find recovery performance approximately comparable to the performance of the GBRT and the non-convolved network.

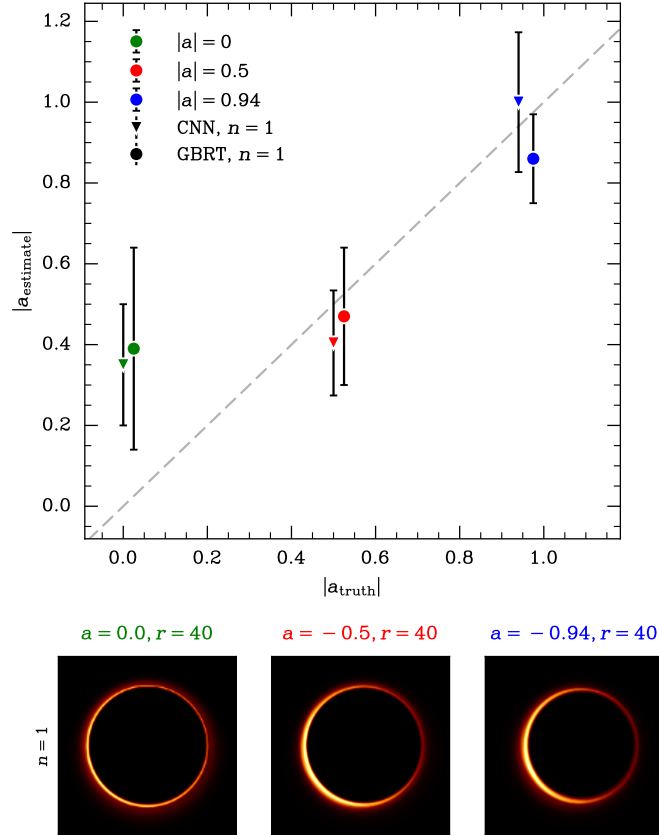


Figure 9.9: (top) Performance of the GRBT (circle points) and CNN (triangle points) on GRMHD images generated following the description in [subsection 9.7.1](#). The two approaches report very similar performance on GRMHD simulations. Recovery of spin is excellent, despite the low inclination and despite neither using GRMHD images in their training data. Perfect recovery is shown with the gray dashed line. Note: the $|a_{\text{truth}}|$ values are offset by ± 0.025 to improve clarity; the true spin values were either 0.0, -0.5 , or -0.94 . (bottom) The simulated images $n = 1$ passed into the GBRT and CNN.

Chapter 10

Interferometric inference of black hole spin from photon ring size and brightness



OH ABBONDANTE GRAZIA OND'IO PRESUNSI FICCAR LO VISO PER LA LUCE ETTERNA, TANTO

CHE LA VEDUTA VI CONSUNSI!

NEL SUO PROFONDO VIDI CHE S'INTERNA LEGATO CON AMORE IN UN VOLUME, CIÒ CHE PER

L'UNIVERSO SI SQUADERNA:

SUSTANZE E ACCIDENTI E LOR COSTUME QUASI CONFLATI INSIEME, PER TAL MODO CHE CIÒ

CH'I' DICO È UN SEMPLICE LUME.

—Dante, *Paradiso*, 33.82–90



The $n = 1$ photon ring is a full image of the astrophysical source around a black hole, produced by photons that execute $n \approx 1$ half-orbit around the event horizon on their way to an observer. The Black Hole Explorer (BHEX) is a proposed extension of the Event Horizon Telescope to space that will target the $n = 1$ photon rings of the supermassive black holes M87* and Sgr A*. In this chapter, we introduce a new interferometric observable that will be directly measurable on BHEX baselines and which admits a clear image-domain interpretation in terms of the photon ring brightness profile. Across a wide range of semi-analytic equatorial emission models, we find that the azimuthal intensity profile of the ring can change depending on the astrophysics of the source, but its width w_b is weakly sensitive to these details—much like the ring shape, which has previously been identified as a probe of the spacetime geometry. Our survey suggests that interferometric measurements of the photon ring diameter and w_b can place constraints (to $\lesssim 20\%$) on the spin and inclination of a black hole with a known mass-to-distance ratio, such as Sgr A*. State-of-the-art numerical simulations support this finding, paving the way to a precise photon-ring-based spin measurement for Sgr A* with BHEX.

10.1 Permissions and Attributions

1. The content of [chapter 10](#) is the result of a collaboration with Prof. Alexandru Lupasca, Prof. Eliot Quataert, Prof. Michael Johnson, and has previously appeared in the *Astrophysical Journal Letters* as Joseph R. Farah *et al.* (2025). It is reproduced here with the permission of the *Astrophysical Journal Letters*: <https://arxiv.org/pdf/2509.23628>.
2. We acknowledge Prof. Charles Gammie and his students for helpful discussions

and for providing the GRMHD simulations used in this chapter.

10.2 Introduction

The Event Horizon Telescope (EHT) has used an array of ground radio stations to observe the black holes M87* and Sgr A* via very-long-baseline interferometry (VLBI) [266; 295]. The Black Hole Explorer (BHEX) is a mission under development to extend terrestrial (sub)millimeter VLBI to space by launching a satellite into Earth orbit at $\gtrsim 20,000$ km altitude [299; 300]. With an effective aperture three times larger than the Earth, BHEX will take the sharpest images in the history of astronomy, enabling it to measure the first ($n = 1$) photon rings around the supermassive black hole targets M87* and Sgr A* [302], which so far remain unresolved [409; 410].

The $n = 1$ photon ring is a full image of the astrophysical source around a black hole, produced by photons that execute $n \approx 1$ half-orbit around the horizon on their way to their observer [284; 287; 260; 286]. Since these photons probe the strong gravity of the spacetime just outside the horizon, the ring image that they produce carries information about the geometry of the black hole, including its mass, inclination and spin. In particular, inferring the spins of the black holes M87* and Sgr A* from measurements of their $n = 1$ photon rings is a primary goal for BHEX [302].

The interferometric response to M87* or Sgr A* on the baselines sampled by an Earth-orbiting satellite is dominated by the $n = 1$ ring. This nearly circular ring has a typical diameter of $d \approx 10M$ and a narrow width $w \lesssim M \ll d$, and its interferometric signature on baselines $1/d \ll u \ll 1/w$ sampled by BHEX is essentially *universal*: that is, the visibility is almost completely fixed by the ring size, shape, and brightness asymmetry, with scant dependence on its (unresolved) radial profile. The ring produces a characteristic ringing signature whose periodicity and amplitude encode

its angle-dependent diameter and brightness asymmetry, respectively. The brightness asymmetry visibly varies with the rotation of the black hole and is clearly worth investigating as a tool for spin inference.

Here, we consider two visibility-domain observables that are directly measurable on the BHEX baselines where the $n = 1$ subring dominates ($u \sim 20\text{--}40\text{ G}\lambda$) and that together encode the geometry of the ring as well as its azimuthal brightness profile. While this profile can rotate and change in ways that do depend on the astrophysics of the source, we find that its *width* (illustrated in Fig. 10.1) is weakly sensitive to these details—much like the diameter itself, which has previously been identified as a probe of the spacetime geometry [411].

A naïve measure of the brightness asymmetry of the photon ring based on its visibility-amplitude envelope—e.g., $(V_{\max} + V_{\min}) / (V_{\max} - V_{\min})$ as a function of baseline angle φ —is attractive but proves sensitive to astrophysical details and is difficult to estimate with low signal-to-noise ratio (SNR). Instead, we integrate the squared visibilities over the $n = 1$ -dominated annulus in the (u, v) plane to obtain an angular profile $b_\varphi \equiv \int_{u_-}^{u_+} |V(u, \varphi)|^2 du$, whose full width at half maximum we denote by w_b (see Fig. 10.1). Physically, b_φ tracks the azimuthal distribution of subring brightness, and w_b provides a robust, directly interferometric proxy for the width of that distribution.

We pair w_b with a geometric observable d_+ , the maximum of the angle-dependent diameter d_φ of the $n = 1$ subring, inferred from the periodicity of the Bessel-like ringing in the visibility amplitude—see Eq. (10.1). Across semi-analytic equatorial emission models, we find that w_b is primarily controlled by inclination and is only weakly affected by nuisance emission parameters, whereas d_+ approximately correlates with spin at low inclinations and exhibits different (and largely orthogonal) degeneracies. A joint measurement of (d_+, w_b) therefore constrains spin and inclination simultaneously.

This spin-inference method is typically subject to uncertainties at the $\lesssim 20\%$ level

for sources with known mass-to-distance ratio such as Sgr A*, consistent with our survey and with tests against state-of-the-art simulations using general-relativistic magnetohydrodynamics (GRMHD; [403]). A major advantage of this technique is that it can be carried out entirely within the visibility domain (without requiring reconstruction of an image from its sparsely sampled Fourier transform) while still relying on data with a clear geometric interpretation.

Our approach complements and extends prior image-domain and geometric strategies. Image-fitting and “photogrammetry” frameworks can recover spin in controlled settings [412], and equatorial semi-analytic models such as KerrBAM reproduce many MAD features (though not SANE) and constrain inclination but find spin difficult to pin down [384]. Measuring multiple subring diameters can in principle break shape degeneracies and yield $\sim 10\%$ spin constraints, albeit with strong mass degeneracy [303]. Machine-learning analyses of photon-ring images also indicate $\lesssim 10\%$ spin precision in favorable cases [413]. Additional channels—e.g., linear polarization of the ring to break mass-spin degeneracies at low inclination [402] and tests based on detailed ring-shape systematics [383]—provide valuable cross-checks. Most closely related to our goals, recent geometric attempts using $n = 1$ diameter and asymmetry alone reported limited constraining power in the BHEX regime, especially at moderate spins [414]; by contrast, our visibility-integrated w_b is both easier to measure at low SNR and more robust to astrophysical variations, particularly for $\theta_o \lesssim 50^\circ$.

Finally, we outline assumptions and practicalities. Our semi-analytic survey and GRMHD validation focus on primarily equatorial emission (appropriate to KerrBAM and many MAD simulations) and on baselines where the $n = 1$ subring dominates; off-equatorial or highly turbulent emission can broaden the (u, v) support and introduce biases that we quantify below. For Sgr A*, where M/D is known, (d_+, w_b) alone can yield meaningful spin constraints. For M87*, combining w_b with a mass-

sensitive observable—such as a diameter ratio (e.g., d_1/d_0) or independent polarization constraints—can break residual degeneracies and potentially enable joint mass-spin inference with high precision.

In this chapter, we investigate two interferometric observables of the $n = 1$ subring which, when measured jointly, can constrain spin to $\lesssim 20\%$ or better at low inclinations. We begin in [section 10.3](#) by briefly reviewing the response of an interferometer to a thin ring on the sky. In [section 10.4](#), we motivate and construct d_+ and w_b (a measure of maximum diameter and of brightness-profile width) and chart their behavior across spins and inclinations. In [section 10.5](#), we validate their performance on a suite of GRMHD simulations. We conclude with limitations and prospects in [section 10.6](#).

10.3 Interferometric signature of a thin ring

Here, we review the interferometric response to a thin ring. Following Gralla [\[288\]](#), we do not assume that the ring is circular, but rather allow its shape to describe any closed convex curve. Thus, we consider an arbitrary convex ring of angle-dependent diameter $d_\varphi \sim d$ and width $w \ll d_\varphi$, with local transversely-integrated intensity I . Then, on baselines u satisfying $1/d \ll u \ll 1/w$, the visibility amplitude $|V|$ can be approximated as [\[288\]](#)

$$|V| \approx \frac{1}{\sqrt{u}} \sqrt{\left(\alpha_\varphi^{\text{L}}\right)^2 + \left(\alpha_\varphi^{\text{R}}\right)^2 + 2\alpha_\varphi^{\text{L}}\alpha_\varphi^{\text{R}} \sin(2\pi d_\varphi u)}, \quad (10.1)$$

where the angle-dependent functions

$$\alpha_\varphi^{(\text{L/R})} = I \sqrt{\mathcal{R}} \Big|_{\text{L/R}} \quad (10.2)$$

encode the integrated intensity $\mathcal{I}$ and radius of curvature $\mathcal{R}$ around the ring. A primary feature of this interferometric signature is the periodicity $1/d_\varphi$ of the sinusoid, which can be measured and used to infer the diameter of the ring. Additionally, the visibility amplitude oscillates within an envelope given by

$$V_{\max} = \frac{\alpha_\varphi^{\text{L}} + \alpha_\varphi^{\text{R}}}{\sqrt{u}}, \quad V_{\min} = \frac{|\alpha_\varphi^{\text{L}} - \alpha_\varphi^{\text{R}}|}{\sqrt{u}}. \quad (10.3)$$

Thus, measurements of the damped oscillation can also yield information about the source integrated intensity $\mathcal{I}$, particularly for rings of near-constant curvature $\mathcal{R}$.

10.4 Construction of interferometric observables

Recent works [288; 305; 413] have shown that the azimuthal profile and maximum diameter of the $n = 1$ subring correlate with spin in both the image and visibility domains. Forming images with limited coverage can be challenging, so here we construct and investigate Fourier analogs of these spin-distinguishing geometric observables.

[260] showed that emission from the $n = 1$ subring with size d and width α is expected to dominate the Fourier transform over a finite range of baseline lengths $1/d \ll u \ll 1/\alpha$ probeable by BHEX for M87* and Sgr A*. We can exploit this phenomenon to make inferences about the $n = 1$ subring while ignoring complications from surrounding astrophysical environment by constructing our observables in this (u, v) region of dominance. A demonstration of the behavior of the subring Fourier transform in this region of dominance is shown in Figure 10.1.

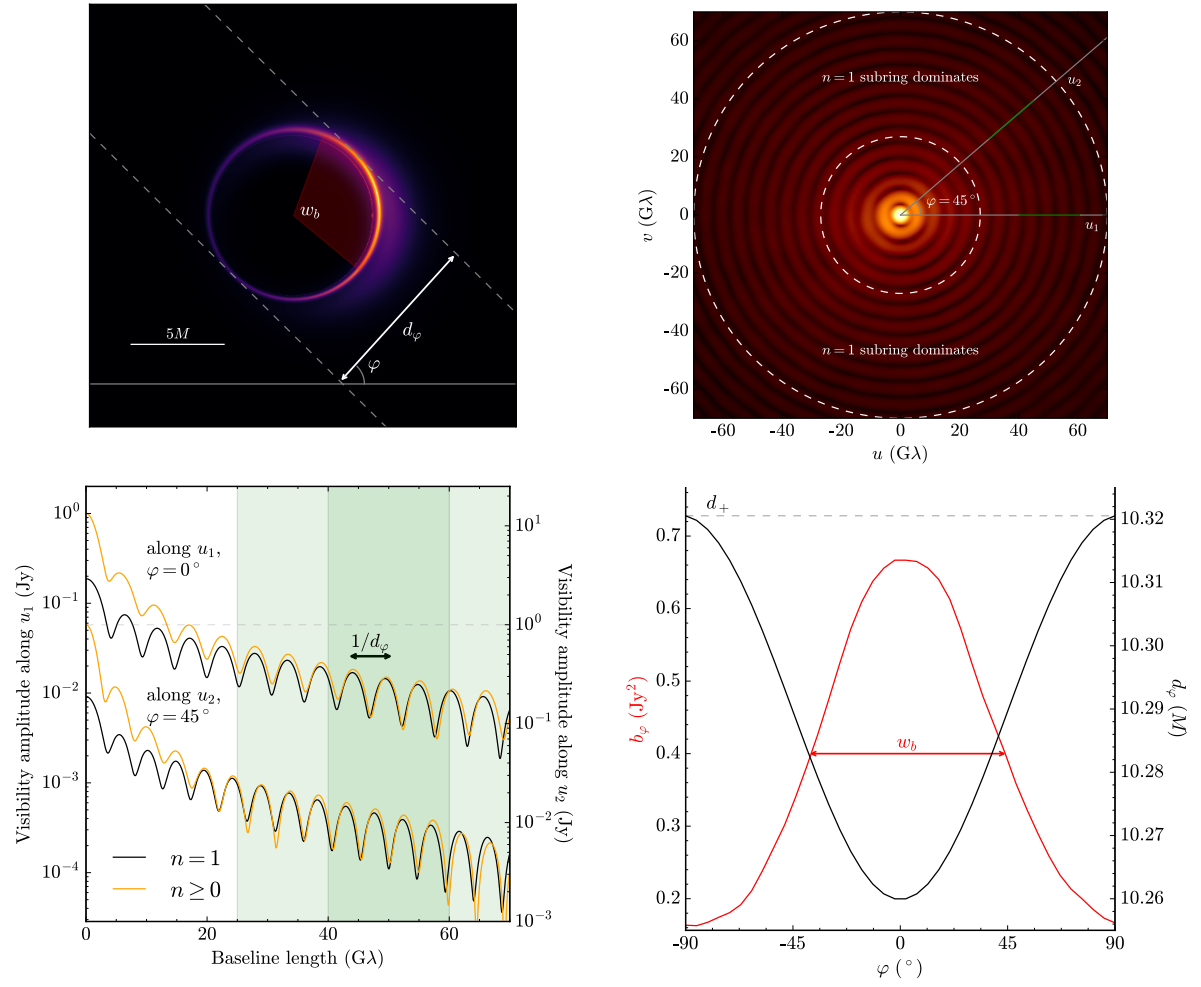


Figure 10.1: Demonstration of how the region of dominance of the $n = 1$ subring arises in the Fourier transform of an image containing multiple subrings. (*top left*) An image containing a simulated black hole. In this image, at least 3 subrings are clearly visible: (i) the $n = 0$ subring, visible as broad diffuse flux particularly on the right side of the image; (ii) the $n = 1$ subring of size d_φ at angle φ , a sharp ring of light and the most dominant image feature, and (iii) the $n = 2$ subring, concentrically nested within the first subring and visible as a much thinner and dimmer ring of light. (*top right*) the absolute value of the Fourier transform of the image in the first panel. Using properties derived in [260], we expect the $n = 1$ subring to dominate the Fourier transform in the region bounded by the dashed white lines. The gray rays correspond to u_1 and u_2 , which will be examined in the following panel. (*bottom left*) Cross-sections of the Fourier transform along the rays u_1 and u_2 , showing the characteristic Bessel function behavior. If we compute the $n = 1$ subring visibility function and compare it to the full Fourier transform, we see it dominates in the light green region, with the most significant domination (i.e., almost all of the emission is generated by the $n = 1$ photon ring) occurring in the darker green region. The spacing of the peaks in this region scales as $1/d_\varphi$ and therefore is a direct measurement of d_φ of the $n = 1$ subring. We can also integrate the visibilities in this region to compute b_φ as described in section 10.4. (*bottom right*) construction of the diameter d_φ and b_φ for all distinct angles in the Fourier plane, computed over the dark green shaded region in the bottom left panel. Both functions look Gaussian- or sinusoid-like. We compute the maximum diameter d_+ (dashed line) and the full-width half-maximum (FWHM) of b_φ as our interferometric observables.

We begin by constructing the observable of maximum diameter, d_+ . Following the treatment of [288] and [260], the diameter of the $n = 1$ subring in a particular direction can be inferred by measuring the frequency of the visibility amplitude oscillation (see Equation 10.1) over the region of dominance. We estimate the diameter d_φ in every direction and construct our first interferometric observable by taking the maximum inferred diameter d_+ .

We next examine the azimuthal brightness profile. [413] heuristically found that variation of this brightness profile tracked the inclination of the black hole, particularly at low spins and inclinations. When normalized to remove overall offsets or scalings in intensity, the width (i.e., full width half maximum) of the azimuthal brightness profile

appears to scale directly with inclination, motivating its use as an observable.

A natural measure of brightness asymmetry $\mathcal{A}$ in Fourier space is, following [288],

$$\mathcal{A} = \frac{V_{\max} + V_{\min}}{V_{\max} - V_{\min}}. \quad (10.4)$$

However, in testing on simple semi-analytic toy models, we find that in the range of BHEX space baselines ($\approx 20\text{-}35 \text{ G}\lambda$) this quantity varies more with the astrophysics of the source than with the properties of the black hole spacetime. The measure in particular is somewhat sensitive to the choice of baseline region covered by BHEX. The consequence of this sensitivity is that the fluid properties that impact the $n = 0 \rightarrow n = 1$ transition point could directly affect the performance of this quantity.

Therefore, we propose an alternative analogous observable: the integrated sum of the squared visibility amplitudes on the region of $n = 1$ subring dominance. The following derivation is described in greater detail in [section C.2](#). Using Plancherel's theorem and the projection-slice theorem, we can relate the interferometric response to the $n = 1$ photon ring to the second moment of the total intensity of a projection $\mathcal{P}_\varphi$ of the image $I(r, \varphi)$, i.e.,

$$\int_{-\infty}^{\infty} du |V(u, \varphi)|^2 = \int_{-\infty}^{\infty} dr |\mathcal{P}_\varphi I(r, \varphi)|^2 \equiv B_\varphi^{(2)}, \quad (10.5)$$

where $V(u, \varphi)$ is the visibility of the source measured by an interferometer. However, in order to directly probe the $n = 1$ subring, we may only measure on a range of frequencies $u_- < u < u_+$ where the $n = 1$ subring dominates. We therefore define a related new quantity b_φ which only probes this range of baselines,

$$b_\varphi \equiv \int_{u_-}^{u_+} du |V(u, \varphi)|^2. \quad (10.6)$$

We next assume we are probing baseline lengths far from the origin, and use the approximation derived in Eq. (5) of [288], reproduced in [Equation 10.1](#). Under these conditions,

$$b_\varphi \propto (\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2 \propto (I_\varphi^L)^2 + (I_\varphi^R)^2, \quad (10.7)$$

where in the last step we have exploited the fact that the $n = 1$ subring has an approximately constant radius of curvature $\mathcal{R}$. The quantity b_φ can be measured relatively easily, and has an approximately Gaussian-like profile. We characterize this profile with our second interferometric observable w_b , equal to the full-width half-maximum (FWHM) of b_φ .

We next seek to characterize the behavior of d_+ and w_b for the $n = 1$ subring at a range of spins and inclinations. Additionally, we expect that characteristics of the azimuthal brightness distribution will vary with properties of the accretion and fluid flow, which may complicate spin inference. Following [413], we use KerrBAM [384] to construct a simulation grid of black holes at combinations of 50 spins ($-0.99 < a < 0.99$) and inclinations ($0 < \theta_0^\circ < 75^\circ$). Additionally, for each combination of spin and inclination, we vary: $0.05 < \beta < 0.95$, the fluid velocity in the zero angular momentum observer (ZAMO) frame; $\pi/5 < \iota < \pi/2$, the vertical angle of the magnetic field vector; $-\pi/2 < \eta < \pi/2$, the equatorial angle of the magnetic field vector; and $-\pi/2 > \chi > -3\pi/2$, the equatorial fluid velocity angle. At each spin magnitude and inclination, we calculate values of d_+ and w_b over the range $25 \text{ G}\lambda < u < 35 \text{ G}\lambda$ for each combination of the fluid parameters, forming a distribution of values from which we compute a discrete median and standard deviation. We expand on the details of observable measurement in [section C.1](#).

The resulting distributions of w_b and d_+ are shown in [Figure 10.2](#) and [Figure 10.3](#), respectively. Across all tested spins and inclinations, we find that w_b approximately

correlates with inclination, and the modification of the parameter due to the surrounding astrophysical environment (as controlled by β and ι) is less than a 10% effect on average. By contrast, d_+ is largely orthogonal to w_b at inclinations $\theta_o \lesssim 50^\circ$, and approximately correlates with spin. Above inclinations of $\sim 50^\circ$, d_+ tracks almost directly with inclination. In the region of the Kerr space where d_+ maps to spin, we find variation significantly less than 1% with changes to β and ι .

We also investigate the impact of additional parameters and choices on the resulting distributions of d_+ and w_b . First, we note that d_+ and w_b display significant symmetry in spin (i.e., the values do not vary significantly with the sign of the spin). The variation with sign is a $\lesssim 1\%$ effect, which we incorporate into the fractional spread (Figure 10.3), and consider only the magnitude of the spin $|a|$. Second, we remark that more parameters than the ones varied here ($a, \theta_o, \beta, \eta, \iota$, and χ) are expected to have an impact on the d_+ and w_b quantities. For example, we find that varying the characteristic radius of the emission profile (R in [384]) can significantly modify the measured value of d_+ . Additionally, the synchrotron cross product index (α_ξ in [384]) and spectral index (α_ν) moderately modify the azimuthal brightness profile beyond what is probed in our simulation library. Finally, our analysis of d_+ and w_b is additionally limited in that we assume equatorial emission (by nature of the KerrBAM toy model) and dominance of the $n = 1$ subring.

10.5 Validation on GRMHD simulations

We next validate the effectiveness of our spin characterization method using general relativistic magnetohydrodynamic (GRMHD) simulations. Semi-analytic engines such as KerrBAM are powerful tools for studying the subring behavior in idealized conditions, but the underlying toy models are too simple to form the basis for a meaningful

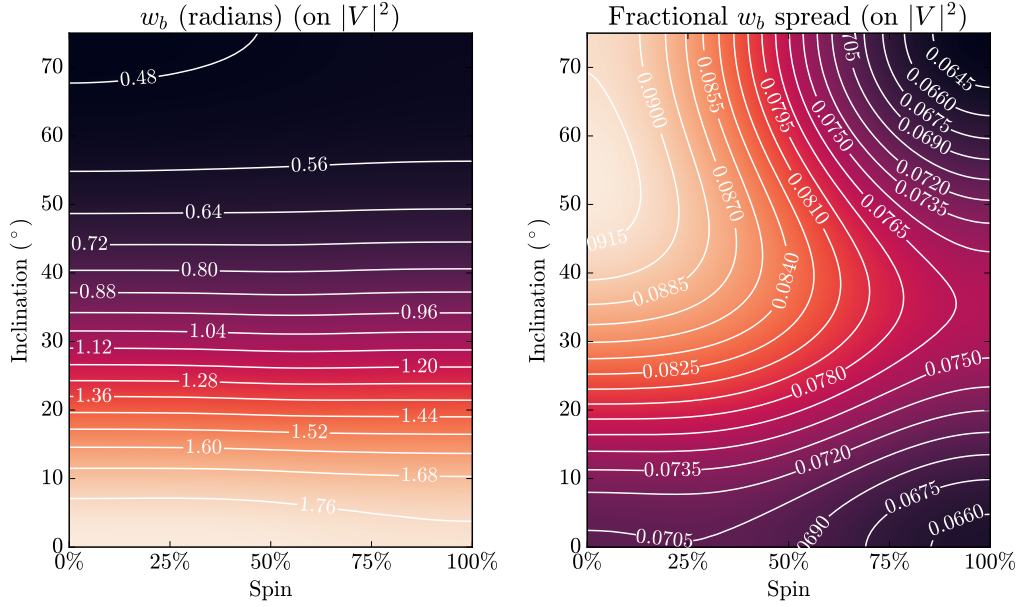


Figure 10.2: Median (*left*) and spread (*right*) of values of w_b computed from the simulation grid described in [section 10.4](#). By eye, w_b approximately correlates with inclination for the entire Kerr parameter space. The spread in values generated by varying properties of the fluid flow is generally $\approx 8\%$ of the value of w_b at inclinations $\theta_o < 50^\circ$.

procedure validation. By contrast, GRMHD simulations are a best-guess at what a real horizon-scale environment may look like. Full numerical GRMHD simulations are far more complex and realistic than semi-analytic simulation engines, and incorporate many features likely to cause complications in a real observation (e.g., time variability). Performing inference on time-variable data is still an open challenge with more required to solve. However, these features can be time-averaged over long timescales to reduce their presence in the image. In this way, testing on GRMHD simulations can provide an accurate estimation of how a spin measurement procedure may perform on a real observation.

To construct a realistic test suite, we generate GRMHD simulations at inclinations of 30° and 10° , typically a challenging range for performing spin estimations, and aligned with inclination estimates for M87* ($10\text{--}20^\circ$). In the construction of our test

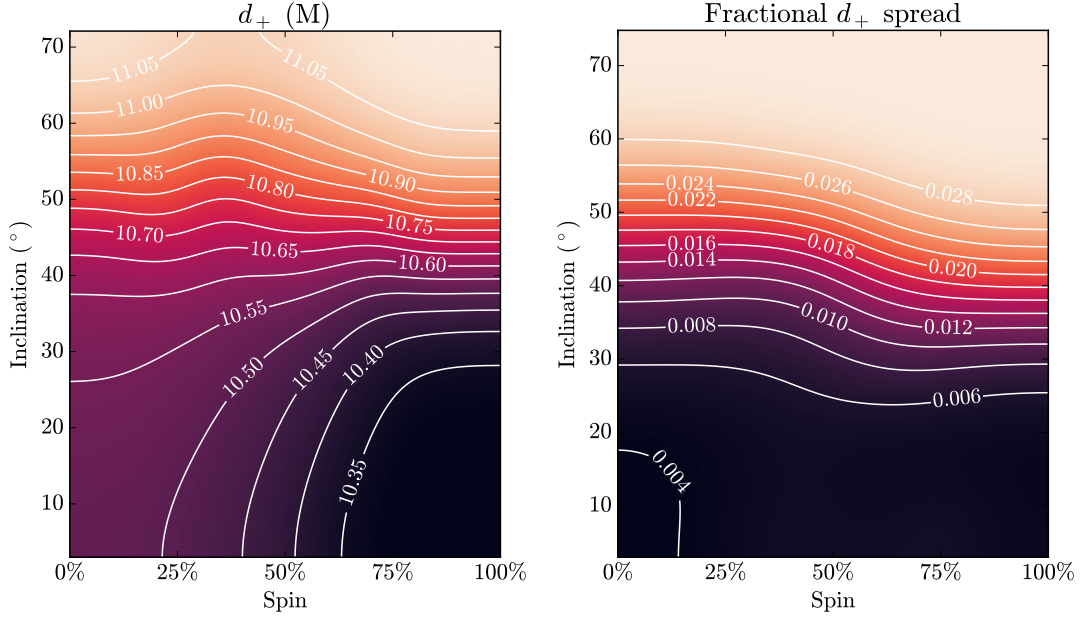


Figure 10.3: Median (*left*) and spread (*right*) of values of d_+ computed from the simulation grid described in [section 10.4](#). By eye, d_+ approximately correlates with spin at inclinations $\lesssim 40^\circ$. The spread in values generated by varying properties of the fluid flow at these inclinations is generally $\lesssim 1\%$ of the value of d_+ .

suite, we varied (in addition to the spin of the black hole): $1 \leq R_{\text{high}} \leq 160$, the electron heating parameter in the weakly magnetized disk; $1 \leq R_{\text{low}} \leq 10$, the electron heating parameter in the highly magnetized jet; and the poloidal magnetic field strength, varied between a fully magnetically-arrested disk (MAD) or standard and normal evolution (SANE). The GRMHD simulations were generated for spins with $|a| = 0.25, 0.50$ and 0.93 . The simulations are then time-averaged over $\sim 1.5 \times 10^4$ gravitational timescales $t_g \equiv GM/c^3$ (for M87* this is $t_g \sim 9$ hours, for Sgr A* this is $t_g \sim 20$ seconds).

We perform measurements of w_b and d_+ in a similar fashion to the application on the KerrBAM simulation grid, with some minor modifications due to the increased complexity of the GRMHD ([section C.1](#)). We time-average each GRMHD simulation and obtain a single representative image of the black hole and the surrounding environment. We then Fourier transform this image and construct w_b and d_+ in the

region of dominance of the $n = 1$ subring, based on the properties of the simulation (demonstrated in [Figure 10.4](#), elaborated on in [section C.1](#)).

Given a measured pair (d_+, w_b) we evaluate, at each grid point (a, θ_o) , the likelihood that this pair was drawn from the KerrBAM distributions characterized by the corresponding means and spreads. Evaluating this likelihood over the full grid yields a two-dimensional pseudo-posterior in (a, θ_o) . Parameter estimates are obtained from the medians of the marginalized pseudo-posteriors in a and θ_o ; these are shown as black points in [Figures 10.5](#) and [10.6](#).

The distribution of values in each (a, θ_o) bin in the KerrBAM grid can be characterized by a mean μ_{a, θ_o} and a standard deviation σ_{a, θ_o} . The colored bands in [Figures 10.5](#) and [10.6](#) visualize the KerrBAM standard deviations in each (a, θ_o) bin (or “spread”). For a given observable (e.g., d_+), the region between $\mu_{a, \theta_o} \pm \sigma_{a, \theta_o}$ maps to a band bounded by two isocontours in the parameter space. Overlap of the corresponding d_+ and w_b bands defines the KerrBAM 1σ region.

We first broadly quantify performance by measuring what fraction of GRMHD realizations in our sample fall within this region. Our finding of $\lesssim 67\%$ indicates that the KerrBAM underestimates the full GRMHD variability, as expected for a semi-analytic framework. As a direct assessment of spin-inference accuracy, we compare inferred spins to the GRMHD truth values and compute a residual magnitude $|a_{\text{infer}} - a_{\text{GRMHD}}|$. The median residual is $\approx 0.1 \pm 0.07$. We find that $\approx 37\%$ of realizations have residuals ≤ 0.05 , $\approx 50\%$ have residuals ≤ 0.1 , and $\approx 91\%$ have residuals ≤ 0.2 , corresponding to an overall 1σ systematic uncertainty for the method of $\approx 10\text{--}20\%$ of the dimensionless spin range.

Next, we examine the performance more specifically. We find that the KerrBAM-based framework is able to constrain the GRMHD parameters to $\pm 20\%$ in spin and $\pm 20^\circ$ in inclination for most of the simulations. For low inclination ($\theta_o = 10^\circ$), the recovery

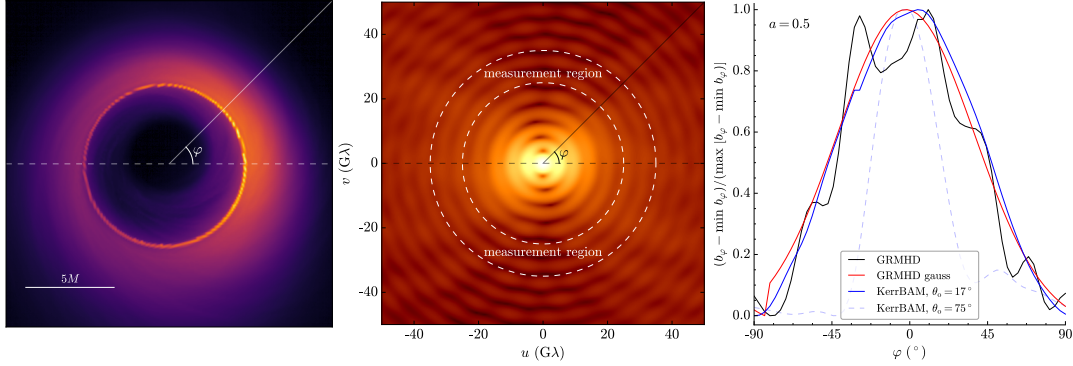


Figure 10.4: Demonstration of the extraction of w_b as performed on GRMHD simulations. (left) a time-averaged GRMHD simulation of a black hole with $a = 0.5$ and $\theta_o = 17^\circ$. (center) Visibility amplitudes of the same simulation. The BHEX instrument is expected to achieve a maximum baseline length of $\sim 35G\lambda$, so we perform our measurement from $25G\lambda$ (where the $n = 1$ subring is expected to begin dominating) out to the maximum BHEX baseline length. (right) We compute the b_ϕ quantity for the simulated image (solid black line) following the procedure in section 10.4. The function is well-approximated by a Gaussian (solid red line). When compared to the b_ϕ quantity computed on the corresponding KerrBAM image, the spreads (analogous to w_b) are consistent. Importantly, the spread in b_ϕ at 17° is also clearly distinct from the spread when computed on a much higher inclination KerrBAM image (light blue dashed line), making the w_b quantity an effective proxy of inclination.

of inclination is excellent, consistent up to $\approx 1\sigma$ with the KerrBAM spread. Spin is well-constrained at $a = 0.25$ and $a = 0.93$ but more poorly constrained at moderate spins ($a = 0.5$). We observe a “pile-up” effect, particularly around $\theta_o \approx 5^\circ$, reflecting cases where the posterior distribution of some estimates is constrained by its lower bound. We interpret this behavior to indicate the GRMHD w_b values extend beyond the range predicted by the KerrBAM model and confirm this is the case for several of the tests.

At $\theta_o = 30^\circ$, spin recovery is comparable to $\theta_o = 10^\circ$ but inclination recovery is substantially worse. The GRMHD simulations present a significantly larger spread in the measured w_b parameter than predicted by the KerrBAM grid. We attribute this behavior to the significantly more complex structure present in the simulations at moderate inclinations, leading to greater contamination of the 25-35 $G\lambda$ baseline region

with emission not originating from the $n = 1$ subring. In particular, SANE simulations with large R_{high} and small R_{low} produce off-equatorial emission, deviating from a key underlying assumption of the KerrBAM model grid. The performance of the framework on GRMHD highlights the challenge of translating patterns identified on semi-analytic or toy-model simulations to more realistic scenarios. However, we note that spin and inclination were somewhat constrained, as the framework could distinguish between high/low spin, and moderate/low inclination, using only the d_+ and w_b parameters. The introduction of additional parameters and a more accurate training grid may improve performance to the level required for a 10% spin constraint with BHEX.

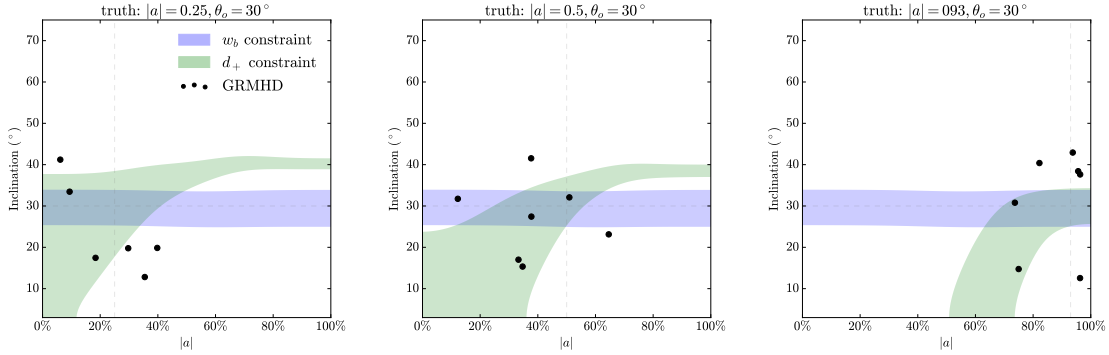


Figure 10.5: Ability of d_+ and w_b to constrain spin and inclination when applied to GRMHD simulations, for $\theta_o = 30^\circ$. The bands correspond to average value and spread of the w_b and d_+ parameters as computed from the KerrBAM grid. Black dots represent an estimate of spin and inclination based on measurements of d_+ and w_b on the GRMHD simulation.

10.6 Discussion and conclusions

In this chapter, we explore a novel method of measuring black hole spin via horizon-scale images from the upcoming Black Hole Explorer mission. Given that the brightness asymmetry in black hole photon rings exhibits clear dependence on spin and inclination, we propose that direct measurement of this asymmetry in Fourier space

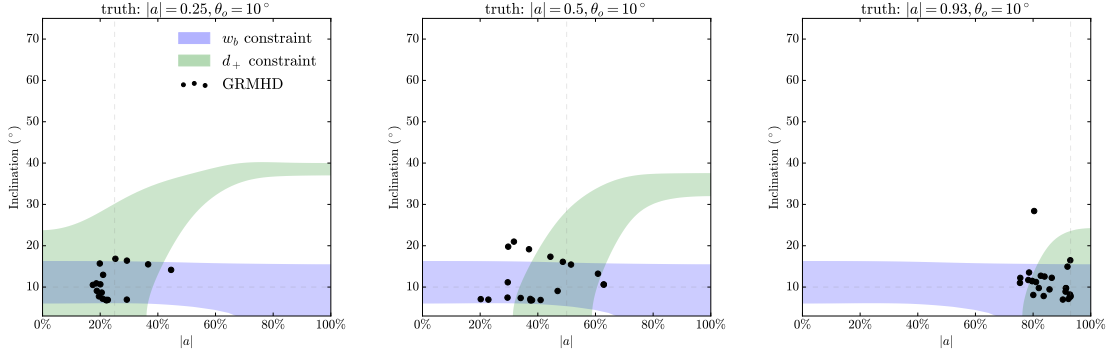


Figure 10.6: Ability of d_+ and w_b to constrain spin and inclination when applied to GRMHD simulations, for $\theta_o = 10^\circ$. The bands correspond to average value and spread of the w_b and d_+ parameters as computed from the KerrBAM grid. Black dots represent an estimate of spin and inclination based on measurements of d_+ and w_b on the GRMHD simulation. While not centered on the truth values (dashed gray lines), the extractions on GRMHD simulations are consistent with the semi-analytic KerrBAM distributions to $\approx 1\sigma$.

combined with measurements of spin-sensitive quantities (e.g., the maximum diameter d_+ of the photon ring) may provide a path to robust constraints on black hole spin.

We introduce a natural visibility-amplitude-based asymmetry metric, defined by $(V_{\max} + V_{\min})/(V_{\max} - V_{\min})$ evaluated on the $n = 1$ subring region of dominance, measured as a function of baseline angle φ . Investigations using simplified semi-analytic models indicate that this particular measure, while intuitive, is sensitive to astrophysical parameters, limiting its utility for measurement.

As an alternative, we propose direct integration of the visibility amplitudes on the $n = 1$ subring region of dominance, which is less sensitive to the parameters of the fluid dynamics. We define a new quantity w_b constructed by taking the width of the angular profile found by integrating the visibility amplitudes over the region of dominance. Our investigation of this quantity on semi-analytic simulations indicates that w_b correlates with inclination across a range of black hole spins and fluid physics

parameters. Preliminary validation against general relativistic magnetohydrodynamic (GRMHD) simulations confirms that w_b and d_+ can somewhat constrain spin and inclination to $\approx 20\%$.

With these encouraging preliminary findings, the next step is to characterize the space of systematic uncertainties. Our current analysis assumes primarily equatorial emission, a condition valid for KerrBAM and MAD GRMHD scenarios. Additional physical factors governing the fluid flow may complicate the accurate recovery of spin and inclination using the integrated quantity w_b . Understanding how deviations from the equatorial emission assumption influence measurement outcomes is therefore essential. Although these uncertainties present challenges, incorporating supplementary observables (e.g., minimum or average diameter, quantities from the $n = 0$ subring, etc.) could mitigate degeneracies, thereby enhancing the robustness and accuracy of spin and inclination measurements.

Part III

Appendix

Appendix A

Additional material for chapter 4

A.1 Details of the reflection and occlusion by the disk in the magnetar+disk system

A precessing disk with misalignment angle β may periodically intersect the line of sight between the observer and magnetar. Depending on the angle between the observer and the magnetar spin axis θ_{obs} , three distinct cases are possible: there will be 0, 1, or 2 occlusions per precession period. In general, observers near the spin axis of the magnetar will experience zero occlusions, while one occlusion per period is seen only by observers in a narrow range of θ_{obs} near $\pi/2 - \beta$. Those near the equatorial plane will experience two occlusions from opposite sides of the disk. The length of the occlusions relative to the disk precession period can be quantified as a function of β , θ_{obs} , and disk aspect ratio H/R ,

$$\frac{T_{\text{occlusion}}}{T_{\text{prec}}} = \begin{cases} 0 & \theta_{\text{obs}} < \pi/2 - \beta - \chi, \\ \frac{1}{\pi} \arccos[\cot(\beta + \chi) \cot \theta_{\text{obs}}] & \pi/2 - \beta - \chi < \theta_{\text{obs}} < \pi/2 - \beta + \chi, \\ \frac{1}{\pi} \{ \arccos[\cot(\beta + \chi) \cot \theta_{\text{obs}}] - \arccos[\cot(\beta - \chi) \cot \theta_{\text{obs}}] \} & \pi/2 - \beta + \chi < \theta_{\text{obs}}. \end{cases} \quad (\text{A.1})$$

Here we have defined $\chi \equiv \arcsin(H/2R)$ for brevity. Additionally, in the case of 2 occlusions per period, the delay between those occlusions is given by

$$\frac{T_{\text{delay}}}{T_{\text{prec}}} = \frac{1}{\pi} \arccos[\cot(\beta - \chi) \cot \theta_{\text{obs}}]. \quad (\text{A.2})$$

These quantities are shown in Figure [A.2](#) (panel a) as a function of viewing angle, where we see that the occlusion period peaks at $\sim 20\%$ of the precession period. For an observer at a random location on the sky, the probabilities P_n for the number of occlusions n depend only on β and H/R :

$$P_0 = 1 - \sin[\beta + \chi], \quad P_1 = \sin[\beta + \chi] - \sin[\beta - \chi], \quad P_2 = \sin[\beta - \chi]. \quad (\text{A.3})$$

Figure [A.2](#) (panels b and c) illustrates the dependence of P_1 and P_2 on β and H/R , showing that both are significant over some region of the parameter space. As an example, for $\beta = \pi/6$ and $H/R = 0.1$ we obtain $P_0 = 45.7\%$, $P_1 = 8.66\%$, and $P_2 = 45.6\%$. The fact that the 1-occlusion case is the least likely but tends to produce the longest occlusion times may explain why this mechanism is not observed in the majority of SLSNe.

We assume that the disk reflects some fraction of the magnetar luminosity, and that the amount of this reflected luminosity directed toward the observer is proportional

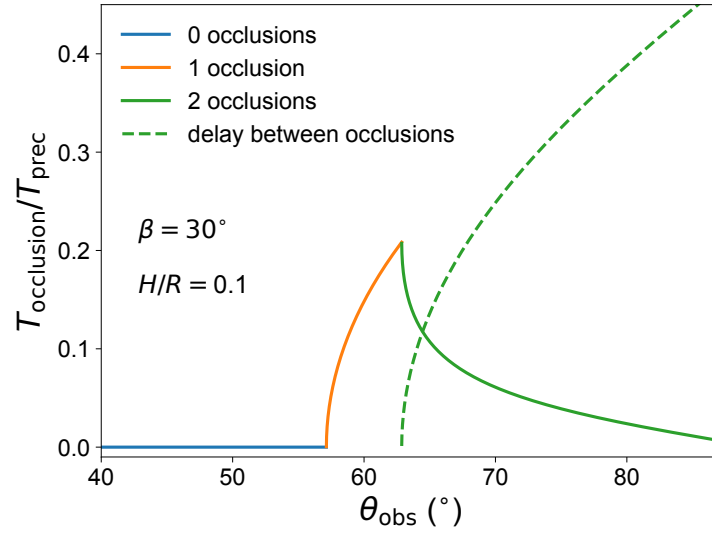


Figure A.1: | **Occlusion periods.** Fraction of the precession period during which the magnetar is occluded by the disk. The dotted line shows the delay between occlusions in the case of two occlusions per period.

to the disk area projected along the line of sight,

$$L_{\text{reflected}} \propto \cos \beta \cos \theta_{\text{obs}} - \sin \beta \sin \theta_{\text{obs}} \cos \Omega_{\text{prec}} t. \quad (\text{A.4})$$

This is in addition to the (constant) luminosity of the magnetar itself along a line of sight L_{magnetar} . The total luminosity along a line of sight is then

$$L = L_{\text{magnetar}}[1 + f(\cos \beta \cos \theta_{\text{obs}} - \sin \beta \sin \theta_{\text{obs}} \cos \Omega_{\text{prec}} t)], \quad (\text{A.5})$$

where f is a free parameter. A caveat to Equation (A.5) is that L is assumed to be zero during an occlusion, as there is a direct line of sight from neither the magnetar nor the surface of the disk to the observer. This assumption is pedagogically simple but somewhat unrealistic since some energy from the magnetar will still reach the observer through the non-shadowed portion of the ejecta; however, we find that we are not particularly sensitive to the energy level during occlusion, as a nonzero occlusion luminosity can be compensated by decreasing the fractional reflectance f while producing a similar signal. To translate this into the observed light curve, we assume 1 occlusion per period, and take as input Equation (A.5) with an occlusion of 20% of the precession period centered on the minima. We then apply the diffusion model of [19] (following [71]) to obtain the light curve shown in [Figure A.3](#). We see that the observed light curve after accounting for photon diffusion through the ejecta is exceptionally well-reproduced by a cosine of the input signal phase function, with a constant phase offset to approximate the diffusion delay. Indeed, our reflection-occlusion model explains two interesting attributes.

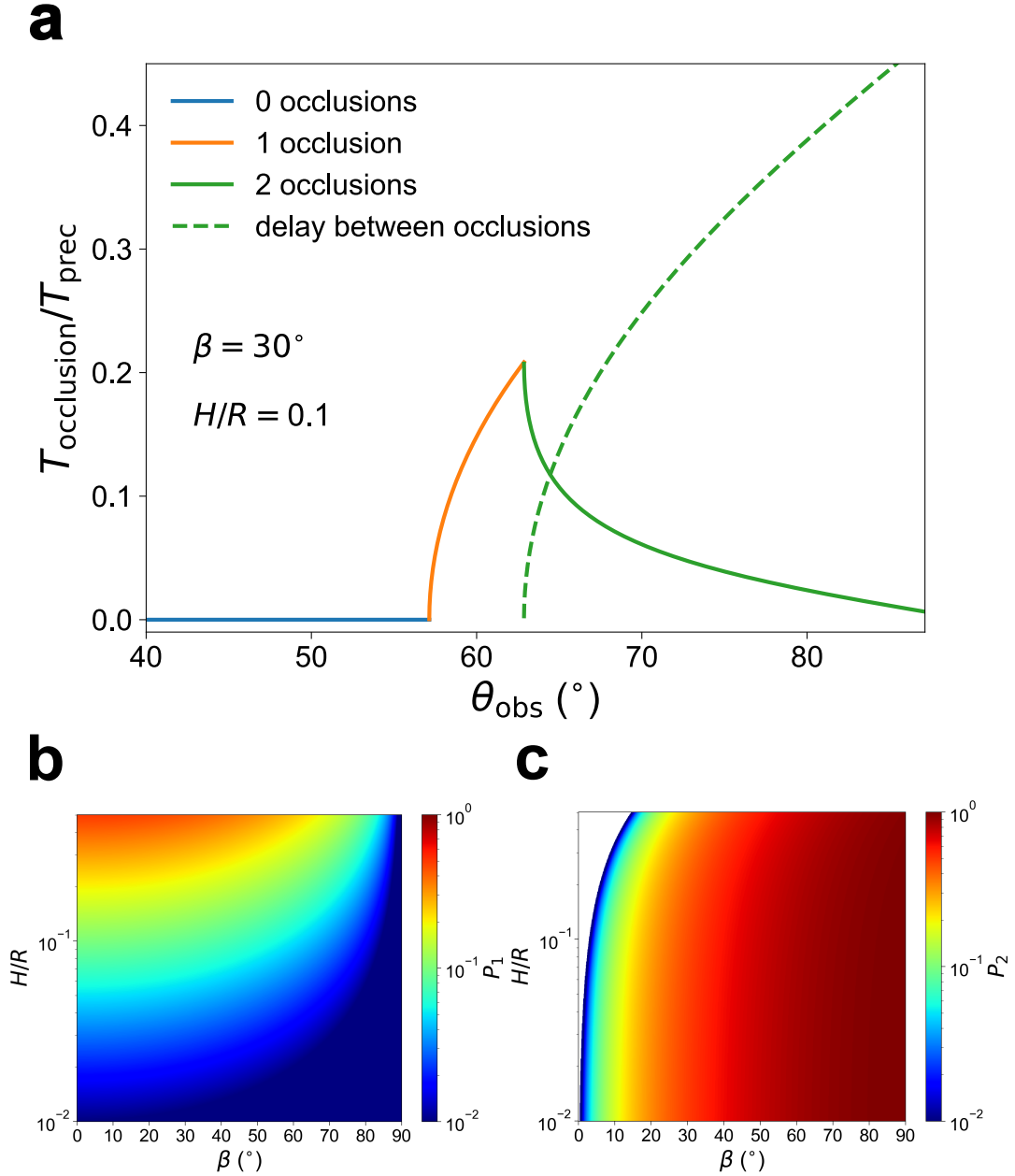


Figure A.2: | **Occlusion periods and probability heatmaps.** (a) Fraction of the precession period during which the magnetar is occluded by the disk. The dotted line shows the delay between occlusions in the case of two occlusions per period. (b) and (c) Heatmaps of the probabilities for 1 and 2 occlusions (P_1 and P_2 , respectively) versus β and H/R . We find that both are reasonably probable over a significant region of the parameter space.

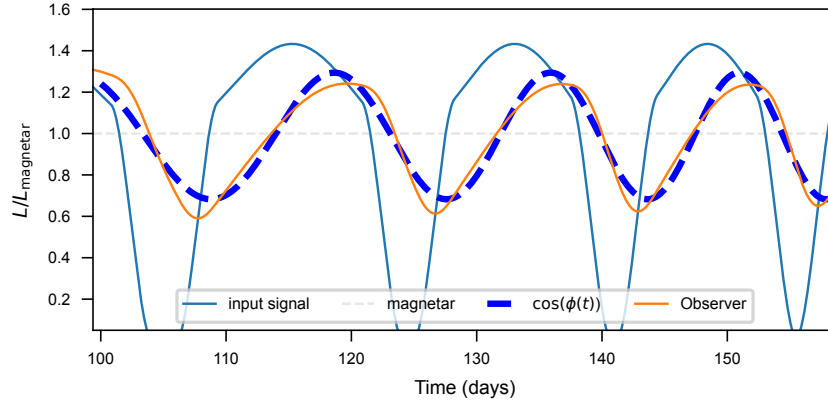


Figure A.3: | **Ocluded light curve.** Simulated reflection and occlusion for an accretion disk with $f = 0.5$, $\beta = 30^\circ$, $\theta_{\text{obs}} = 60^\circ$, and Ω_{prec} tuned to produce a chirp. We compute the input signal based on the precessing accretion-disk model (Equation A.5) combined with the periodic occlusions (blue). We then evolve this input energy through an expanding ejecta with the same properties as SN 2024afav (orange). The resulting light curve is approximately sinusoidal and symmetric around $L/L_{\text{magnetar}} \approx 1$, as we see in the data. The blue curve is the cosine of a phase function which includes a constant phase offset parameter (i.e., $\phi(t) = \omega(t)t + \phi_0$), which motivates the use of the model we proposed in the paper.

1. **Symmetry around the magnetar model:** our residual analysis suggests that the modulations are roughly symmetrical around the magnetar light curve. Figure A.3 shows that while the reflections increase the average input energy above the magnetar model, the occlusion significantly dampens the average input energy and the diffusion process smooths out the modulations to be approximately symmetric around the magnetar light curve, as we see in the data.
2. **Sinusoidal modulation structure:** the data display a very sinusoidal structure, despite our model predicting large nonsinusoidal occlusions. Figure A.3 reveals that the nonsinusoidal occlusions are smoothed out by diffusion through the ejecta and produce modulations which are well modeled by sinusoidal behavior.

Appendix B

Additional material for chapter 8

B.1 Sample synthetic data products

Here, we present representative samples of the visibility amplitudes and closure phases that are generated from the synthetic data described in [subsection 8.3.1](#).

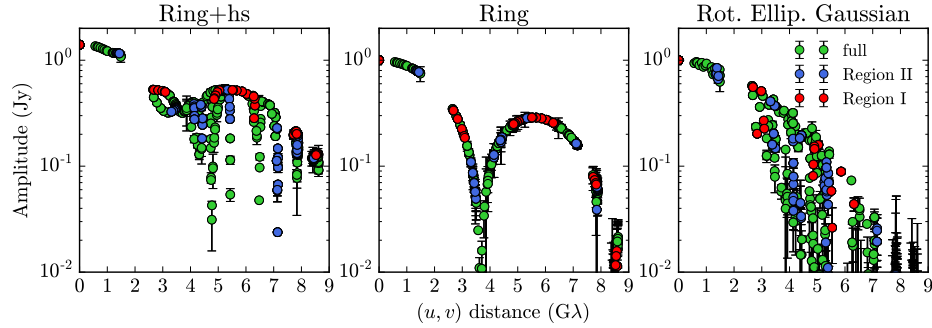


Figure B.1: Example amplitudes for the three main synthetic data types described in [subsection 8.3.1](#). The time-variable models have periods of 270 minutes. The amplitudes recorded during Region I and Region II (see [Table 8.1](#)) are shown in red and blue, respectively. Region II has higher radial homogeneity in (u, v) distance, which contributes to its higher metric score and increased dynamical imaging capability. Errorbars show 1σ thermal noise.

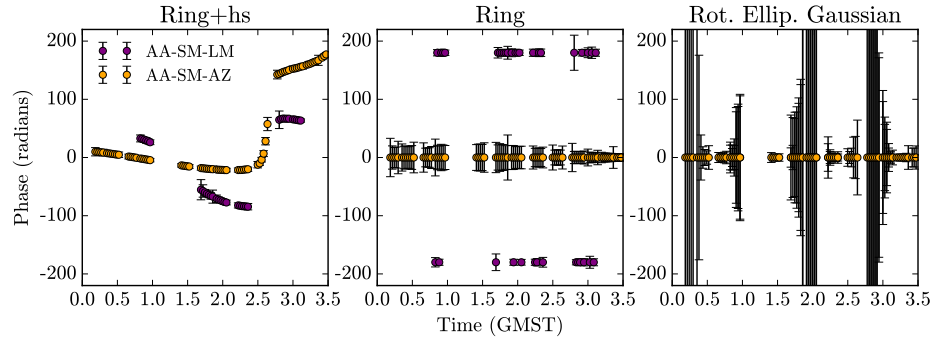


Figure B.2: Example closure phases on two triangles for the three main synthetic data types described in [subsection 8.3.1](#). The time-variable models have periods of 270 minutes. Closure phases are useful for identifying and constraining asymmetry and time variability in the source.

B.2 Derivation of isotropy-based coverage metric

Section [8.5](#) demonstrated that quantifying the isotropy of a (u, v) coverage configuration can indicate whether it is suitable for producing accurate reconstructions of a

dynamic source. We adopt a coverage metric of the form

$$C = I(\{\mathbf{u}_i\})R(\{\mathbf{u}_i\}), \quad (\text{B.1})$$

where $\{\mathbf{u}_i\} = \{(u_i, v_i)\}$ is the set of $2N$ baselines (including their Hermitian conjugates), I is a measure of the isotropy of the coverage, and R is a measure of the radial homogeneity of the coverage.

To estimate the radial homogeneity, we compare the cumulative distribution function (CDF) of the distribution of baseline lengths against the uniform CDF via the Kolmogorov-Smirnov (KS) test. The uniform distribution examined for this test ranges from 0 $G\lambda$ to the maximum baseline length achieved in the observation. This test returns a “distance” $\mathcal{K}$ between the distributions, which increases as the distribution becomes less radially homogeneous, making use of the test in this context a measure of radial inhomogeneity. To convert the result of the KS test into a measure of radial homogeneity, we select an upper bound $\mathcal{K}_{\max}$ corresponding to the maximum distance from uniform any individual baseline distribution obtains throughout the observation. We then subtract the result of the KS test from this maximum, i.e.,

$$\mathcal{K}' = \frac{\mathcal{K}_{\max} - \mathcal{K}}{\mathcal{K}_{\max}} = 1 - \frac{\mathcal{K}}{\mathcal{K}_{\max}}, \quad (\text{B.2})$$

where $R(\{\mathbf{u}_i\}) = \mathcal{K}'$ is our new metric of homogeneity. This metric is conveniently bounded between 0 and 1 . To make this metric absolute, a fixed value of $\mathcal{K}_{\max}$ can be chosen arbitrarily and applied to multiple observations. For the absolute comparisons in this paper (see e.g., [section 8.8](#)) a value of $\mathcal{K}_{\max} = 0.513338437261774 \approx 0.513$ is adopted. This value of $\mathcal{K}_{\max}$ is chosen to be the maximum value of $\mathcal{K}$ achieved during the April 7 observation.

In order to measure the isotropy of the coverage, we examine the second moment (moment of inertia) of the distribution of baselines. As a spatial configuration of points with uniform weighting, the (u, v) coverage can be treated as a mass distribution. For a two-dimensional mass distribution, a disk is considered isotropic, and a rod is considered anisotropic, and the spectrum between the two cases can be probed using the moment of inertia tensor. Given $2N$ baselines and conjugate baselines with coordinates $\{\mathbf{u}_i\} = \{(u_i, v_i)\}$, we can compute the second moments of the distribution

as

$$\begin{aligned}\langle u^2 \rangle &= \frac{1}{2N} \sum_i^{2N} u_i^2, \\ \langle v^2 \rangle &= \frac{1}{2N} \sum_i^{2N} v_i^2, \\ \langle uv \rangle &= \frac{1}{2N} \sum_i^{2N} u_i v_i.\end{aligned}\tag{B.3}$$

The moment of inertia tensor $\mathcal{M}$ is

$$\mathcal{M} = \begin{bmatrix} \langle u^2 \rangle & \langle uv \rangle \\ \langle uv \rangle & \langle v^2 \rangle \end{bmatrix}.\tag{B.4}$$

The principal moments of inertia can be computed from the eigenvalues λ_1 and λ_2 of $\mathcal{M}$. From these, we can derive the following orientation-independent measure of isotropy

$$\tilde{I}(\{\mathbf{u}_i\}) = 1 - \frac{|\lambda_1 - \lambda_2|}{\lambda_1 + \lambda_2} = 1 - \frac{\sqrt{(\langle u^2 \rangle - \langle v^2 \rangle)^2 + 4\langle uv \rangle^2}}{\langle u^2 \rangle + \langle v^2 \rangle}.\tag{B.5}$$

Note that $\tilde{I}(\{\mathbf{u}_i\})$ can also be written in terms of the FWHM of the dirty beam,

$$\tilde{I}(\{\mathbf{u}_i\}) = 1 - \frac{1/\theta_{\min}^2 - 1/\theta_{\text{maj}}^2}{1/\theta_{\min}^2 + 1/\theta_{\text{maj}}^2}\tag{B.6}$$

$$= \frac{2\theta_{\min}^2}{\theta_{\text{maj}}^2 + \theta_{\min}^2}.\tag{B.7}$$

This measure of isotropy is naturally normalized between 0 and 1. Substituting this expression and [Equation B.2](#) into [Equation B.1](#) gives the following expression for a coverage quality metric:

$$\begin{aligned}C &= I(\{\mathbf{u}_i\})R(\{\mathbf{u}_i\}) \\ &= \left(1 - \frac{\sqrt{(\langle u^2 \rangle - \langle v^2 \rangle)^2 + 4\langle uv \rangle^2}}{\langle u^2 \rangle + \langle v^2 \rangle}\right) \left(1 - \frac{\mathcal{K}}{\mathcal{K}_{\max}}\right)\end{aligned}\tag{B.8}$$

This measure of coverage quality can be applied to partition an arbitrary VLBI observation into time regions ranked by their ability to accurately reconstruct dynamical sources.

B.3 Sampled results from synthetic data reconstructions

In [section 8.7](#), we test the coverage metrics examined in [section 8.6](#) by performing reconstructions of synthetic observations in selected time regions of the 2017 EHT observational coverage of Sgr A*. Here, we provide representative snapshot images from each of the time regions. Sampled snapshot images from Region I are shown in [Figure B.3](#) and sampled snapshot images from Region II are shown in [Figure B.4](#).

Reconstructions in both Region I and Region II demonstrate clear recovery of a ring-like feature of approximately $50 \mu\text{as}$ in diameter, as both sufficiently probe the radial distribution of the source Fourier transform to constrain the overall size of the source. However, the directionally-biased coverage of Region I produces incorrect reconstructions of hotspot location. By contrast, reconstructions in Region II repeatedly recover the correct hotspot location across all periods, directions, and imaging algorithms.

Though both time regions recover a ring-like feature of the approximately correct size, the Region II reconstructions provide a more accurate ring-to-central-depression flux density ratio. The increased accuracy is present in both the CLEAN and RML reconstructions. By contrast, reconstructions in Region I fail to consistently provide a visually distinctive depression and misrepresent the angular brightness profile of the source.

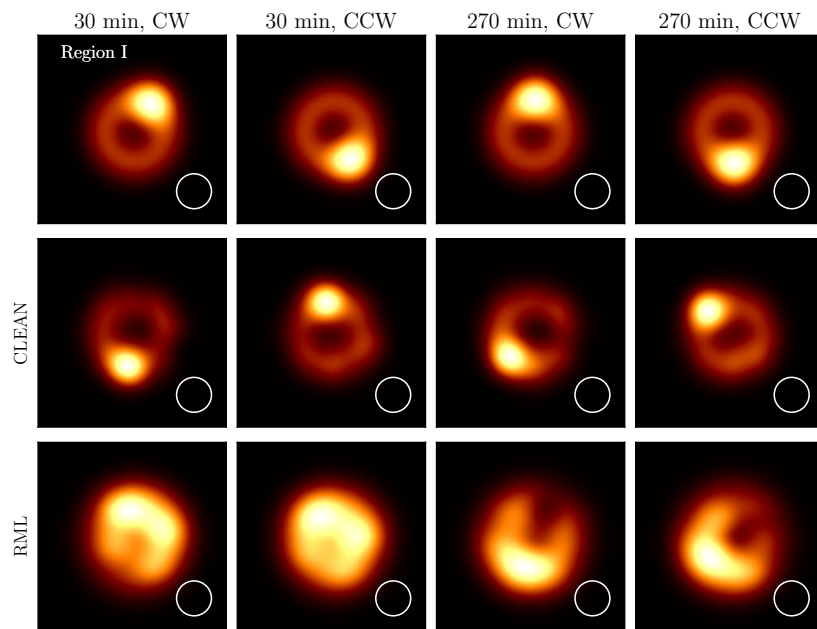


Figure B.3: Sampled reconstructions from Region I. The top row shows the true model image for each configuration. All panels show model images and reconstructions at $\sim 21:00$ GMST on April 7. The white circle in the bottom right of each panel corresponds to an $18 \mu\text{as}$ diameter CLEAN beam. Even with substantial prior assumptions that facilitate ring reconstruction, the hotspot is frequently placed incorrectly, rendering this time region unsuitable for recovery of orbital angular variability.

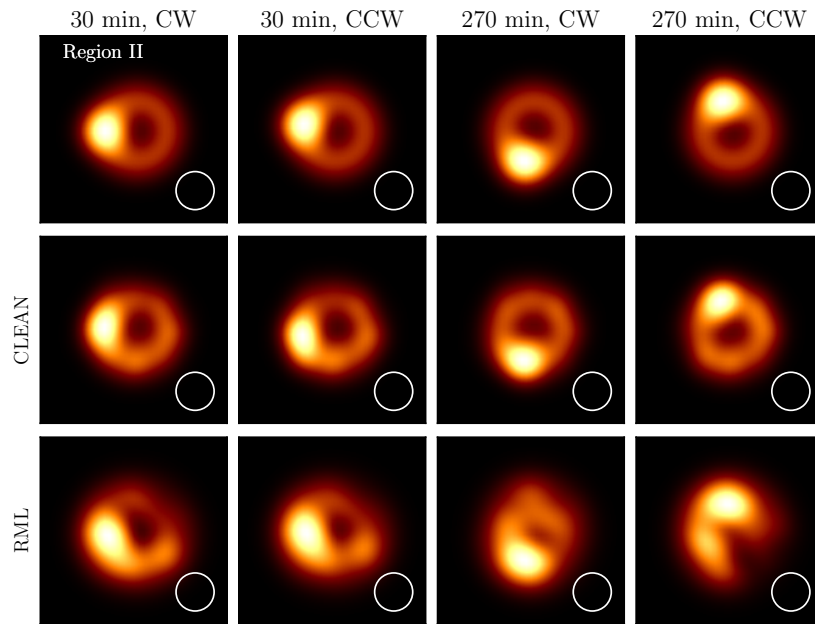


Figure B.4: Sampled reconstructions from Region II. The top row shows the true model image for each configuration. The white circle in the bottom right of each panel corresponds to an $18 \mu\text{as}$ diameter CLEAN beam. All panels show model images and reconstructions and reconstructions at $\sim 1:50$ GMST on April 8.

Appendix C

Additional material for chapter 10

C.1 Details of w_b and d_+ observable measurement

Here, we review the measurement of the d_+ and w_b observables in more detail.

C.1.1 Construction of d_+

The d_+ observable is a measurement of the maximum angular diameter of the $n = 1$ subring as a function of azimuthal angle. This observable is directly inferred from the periodicity of oscillations in the visibility function. [304] and [260] both showed that the visibility function on baselines corresponding to length scales much larger than the subring width can be approximated with a sinusoid whose wavelength varies with $1/d_\varphi$. We directly estimate the location of peaks and nulls in the visibility function and compute an average wavelength Λ_V . The estimated wavelength is then converted to a diameter measurement via

$$d_\varphi \approx 10 M \left(\frac{\Lambda_V}{5.7 G\lambda} \right)^{-1}, \quad (\text{C.1})$$

where $5.7 G\lambda$ corresponds to the $\approx 10 M$ photon ring diameter at the distance of M87*. This procedure is sensitive to the resolution of the original image. We test for convergence of the measured diameter and find that convergence is achieved at a resolution $>200 \times 200$ pixels, which we use for all simulations generated with KerrBAM. In addition, when numerically calculating the Fourier transform on the measurement region $25 G\lambda < u < 35 G\lambda$, the d_+ quantity is sensitive to the resolution in frequency. To balance performance and accuracy, we sample $N_{\text{res}} = 100$ evenly spaced points on the measurement region. This results in a $\approx 0.1 G\lambda$ systematic error, corresponding to $\approx 0.4\%$ of the characteristic diameter. To reduce this systematic error even further, we sample several points around the peaks and nulls in the visibility function and compute an expectation value for each peak location, which is then used to compute the wavelength. In our testing, we found that this reduces the resolution systematic error to $\ll 0.2\%$ and is equivalent to performing the wavelength estimation with $N_{\text{res}} \gtrsim 300$ ($\lesssim 0.033 G\lambda$ spacing).

The increased complexity of the GRMHD simulations combined with their lower resolution made it more challenging to consistently extract diameters. Given the low number of GRMHD tests in our study, we individually checked the accuracy and performance of each fit and attempted multiple methods of diameter measurement. These methods included (i) using the frequency of a best-fit sinusoid plus linear offset, and (ii) using the peak frequency of a Lomb-Scargle periodogram (see e.g., [415] for a review). Consistent measurement of the photon ring diameter in a complex and noisy astrophysical environment is still an open problem, which we do not attempt to present a robust pipeline for here.

C.1.2 Construction of w_b

A recent analysis [413] has suggested that the azimuthal brightness distribution of the $n = 1$ subring varies with the spin and inclination of the black hole. A convenient proxy to this quantity is the definite integral over the squared visibility amplitudes in the region $u_- < u < u_+$ of $n = 1$ domination:

$$b_\varphi \equiv \int_{u_-}^{u_+} du |V(u, \varphi)|^2. \quad (\text{C.2})$$

We compute this quantity for 20 angles $0 < \varphi < \pi$ in the Fourier plane. We find that the distribution of b_φ is Gaussian-like with Fourier angle φ . For the purposes of this analysis, we characterize this distribution based on its FWHM. Similarly to the d_+ quantity, we compute b_φ and fit the resulting distribution to a Gaussian, which will have an associated FWHM, using which we set w_b . In contrast to the diameter measurement, measurement of w_b was of comparable difficulty in both the KerrBAM and GRMHD cases, and the same pipeline was used on both simulation classes.

C.1.3 Construction of parameter heatmaps

For each combination of spin magnitude and inclination, there are several simulations that must be considered. These simulations feature each combination of the fluid parameters, and positive/negative spin. We calculate d_+ and w_b for each simulation, creating a distribution for both for each spin/inclination combination. We perform a single outlier-rejection step, rejecting any values which lie $> 5\sigma$ away from the average of the bin. We then calculate the median and standard deviation of the remaining values, which form the heatmaps shown in [Figure 10.3](#) and [Figure 10.2](#). For display purposes, these heatmaps feature a small amount of global Gaussian smoothing and interpolation to remove minor artifacts that are unlikely to correspond to real behavior.

C.2 Derivation of b_φ

We seek to explore a novel quantity heuristically identified to be a potential inclination discriminant for black hole images. Specifically, recent work has identified the approximately-Gaussian azimuthal brightness profile of the $n = 1$ photon ring as varying almost directly with the inclination of the black hole relative to the observer. We seek to find a Fourier representation of this quantity which can be measured in the $n = 1$ region of dominance.

We begin with an image-domain representation of the $n = 1$ subring $I(r, \varphi)$. We can calculate a projection of this image function in an arbitrary direction φ , $\mathcal{P}_\varphi$. This projection can be performed via e.g., the Radon transform. Once operating on projections, we can exploit the *projection slice theorem*. We define a new quantity $B_\varphi^{(k)}$ which

tracks moments of the total intensity of a projection $\mathcal{P}_\varphi$,

$$B_\varphi^{(k)} = \int_{-\infty}^{\infty} ds [\mathcal{P}_\varphi I(s)]^k, \quad (\text{C.3})$$

where s parameterizes the one-dimensional slice. We can relate this definition to the total intensity of the image. The zero frequency component of the Fourier transform of $I(r, \varphi)$, denoted $V(0)$, necessarily is the total intensity of the image I_{tot} , as no spatial scale can contain more flux than the entire image. This property is direction independent, since $V(0, \varphi) = V(0)$; i.e., the total intensity of the image I_{tot} is independent of direction. Therefore, since the projection of an image along any axis will have the same total intensity as the full image,

$$B_\varphi^{(1)} = \int_{-\infty}^{\infty} ds [\mathcal{P}_\varphi I(s)] = \int d^2\mathbf{x} I(\mathbf{x}) = I_{\text{tot}} = V(0). \quad (\text{C.4})$$

We next make use of *Plancherel's theorem* to investigate a special feature of $B_\varphi^{(k)}$. Plancherel's theorem naturally motivates examining the $B_\varphi^{(2)}$ quantity. We consider a slice of the two-dimensional Fourier transform of our image function $I(r, \varphi)$, which we define to be our visibility function:

$$V(u, \varphi) \equiv \mathcal{F}_{2,r \rightarrow u} I(r, \varphi). \quad (\text{C.5})$$

By taking the squared amplitude of this quantity (which can be measured by an interferometer), we can immediately invoke both Plancherel's theorem and the projection slice theorem to state that

$$\int_{-\infty}^{\infty} du |V(u, \varphi)|^2 = \int_{-\infty}^{\infty} dr |\mathcal{P}_\varphi I(r, \varphi)|^2 = B_\varphi^{(2)}. \quad (\text{C.6})$$

This proves that integrating the Fourier transform over the entire range of frequencies for a particular angle φ will yield information about $B_\varphi^{(2)}$ and therefore the azimuthal brightness distribution of the subring. However, an interferometer will not have access to the full range of frequencies. Even more constraining, the $n = 1$ subring will only dominate on a range of frequencies $u_- < u < u_+$. Therefore, we must demonstrate that this integral still scales with the total value of $B_\varphi^{(2)}$ even when integrated on a small, finite range.

For a closed convex curve with diameter d_φ (e.g., the $n = 1$ subring), we define a new quantity b_φ , for which

$$B_\varphi^{(2)} \propto b_\varphi \equiv \int_{u_-}^{u_+} du |V(u, \varphi)|^2, \quad (\text{C.7})$$

where $u_- < u < u_+$ is the $n = 1$ subring region of dominance, corresponding to

baselines much longer than $1/d_\varphi$.

Proof. It has been shown that the visibility function of a closed convex curve can be approximated as

$$|V(u, \varphi)| \approx \frac{1}{\sqrt{u}} \sqrt{(\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2 + 2\alpha_\varphi^L \alpha_\varphi^R \sin(2\pi d_\varphi u)}, \quad (\text{C.8})$$

where α_φ^L [α_φ^R] corresponds to the product of the integrated intensity I_φ and square root of the radius of curvature $\sqrt{\mathcal{R}}$ on the left [right] edge of the closed convex curve, and d_φ corresponds to the diameter of the curve in the same direction.

Let us now use this approximation to calculate b_φ :

$$\begin{aligned} b_\varphi &= \int_{u_-}^{u_+} du |V(u, \varphi)|^2 \\ &= \int_{u_-}^{u_+} du \left| \frac{1}{\sqrt{u}} \sqrt{(\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2 + 2\alpha_\varphi^L \alpha_\varphi^R \sin(2\pi d_\varphi u)} \right|^2 \\ &= \int_{u_-}^{u_+} du \frac{1}{u} \left[(\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2 + 2\alpha_\varphi^L \alpha_\varphi^R \sin(2\pi d_\varphi u) \right] \\ &= \int_{u_-}^{u_+} du \frac{(\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2}{u} + \int_{u_-}^{u_+} du \frac{2\alpha_\varphi^L \alpha_\varphi^R \sin(2\pi d_\varphi u)}{u} \\ &= \left[(\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2 \right] \log \left(\frac{u_+}{u_-} \right) \\ &\quad + 2\alpha_\varphi^L \alpha_\varphi^R \left[\text{Si}(2\pi d_\varphi u_+) - \text{Si}(2\pi d_\varphi u_-) \right]. \end{aligned} \quad (\text{C.9})$$

For large values (and we are at very long baselines), the sine integral function $\text{Si}(x)$ tends to a constant value, which allows us to drop the second term in our expression and simply write

$$b_\varphi \approx \left[(\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2 \right] \log \left(\frac{u_+}{u_-} \right). \quad (\text{C.10})$$

In the limit of integrating over all frequencies, we have shown (Equation 10.5) that

$$B_\varphi^{(2)} = \lim_{u_\pm \rightarrow \pm\infty} b_\varphi, \quad (\text{C.11})$$

and since Equation C.10 shows that $b_\varphi \propto \lim_{\substack{u_+ \rightarrow \infty \\ u_- \rightarrow 0}} b_\varphi$ (scaling precisely by $\log[u_+/u_-]$), we have therefore shown that

$$B_\varphi^{(2)} \propto b_\varphi. \quad (\text{C.12})$$

Thus, we can make measurements of b_φ on the $n = 1$ subring region of dominance with BHEX, and in fact probe the azimuthal brightness distribution of the $n = 1$ subring,

which varies directly with the inclination of the black hole.

The construction of b_φ can also help us intuitively understand why this quantity seems to directly track the azimuthal brightness distribution of the $n = 1$ subring and not simply the more abstract B_φ quantity.

Earlier, we showed that

$$b_\varphi \propto (\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2. \quad (\text{C.13})$$

The explicit definition of the α_φ quantity is

$$\alpha_\varphi^{L,R} = I_\varphi^{L,R} \sqrt{\mathcal{R}}, \quad (\text{C.14})$$

where $\mathcal{R}$ is the radius of curvature of the closed convex curve. However, the $n = 1$ subring has a very low ellipticity ($\lesssim 6\%$ in the most extreme case), meaning that the radius of curvature is largely constant (certainly significantly less variable than the intensity I_φ) and approximately simply the radius of the subring, $\mathcal{R} \approx R$ for all φ . This lets us treat $\mathcal{R}$ as constant, and write

$$b_\varphi \propto (\alpha_\varphi^L)^2 + (\alpha_\varphi^R)^2 \propto (I_\varphi^L)^2 + (I_\varphi^R)^2. \quad (\text{C.15})$$

From this expression, it becomes clear that the quantity b_φ is not just analogous but in fact a *direct* probe of the azimuthal brightness distribution.

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