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THE BARBAROUS ω PIERCES THE ρ^{o} TO REVEAL A π - B EXCHANGE DEGENERACY*

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ABSTRACT

We show that ρ^{o} - ω interference effects in the $\pi^{+}\pi^{-}$ decay mode can be predicted without any essential ambiguity. The data permit the determination of the relative phase of the production amplitudes, in agreement with the prediction of a Regge model with exchange degenerate trajectories. The model explains the anomalously small width of the rho measured in colliding beams experiments.

Experimental attempts to positively identify $\rho^0 - \omega$ interference have for several years resembled a search for a needle in a haystack. The recent observation by G. Goldhaber *et al.*[1] of a sizeable dip at the ω mass in the dipion mass distribution in $\pi^+ p \to \pi^+\pi^-\Delta^{++}$ is rather dramatic evidence for the existence and magnitude of the effect. We first show how one can calculate the $\omega \to \pi^+\pi^-$ amplitude. This has no essential ambiguities once one knows the effect to be large. Then we calculate the interference phenomena by means of a model for the relative phase of the ρ^0 and ω production amplitudes. Our predictions for hadronic and photoproduction experiments and for $e^+e^- \to \pi^+\pi^-$ agree well with present data and are summarized in both Table I and Fig. 4. The experimental sign and magnitude of the $\omega \to \pi^+\pi^-$ amplitude agree, within a factor of two, with the symmetry breaking theory of Coleman and Glashow.[2]

A convenient formalism to describe the 2π decays of the ρ^0 and ω is the mass mixing theory which has been described by many authors.[3] Any 2π production amplitude may be written

$$S(\pi^+\pi^-) = \begin{bmatrix} A(\rho) & A(\omega) \end{bmatrix} \underset{\sim}{P}(m) \begin{bmatrix} T(\rho^0 \to \pi^+\pi^-) \\ T(\omega \to \pi^+\pi^-) \end{bmatrix} \qquad (1)$$

where the matrix propagator is given by

$$\left[\underset{\sim}{P}(m)\right]^{-1} = m \underset{\sim}{1} - \underset{\sim}{M} \qquad \underset{\sim}{M} = \begin{bmatrix} m_\rho - i\Gamma_\rho/2 & -\delta \\ -\delta & m_\omega - i\Gamma_\omega/2 \end{bmatrix} ; \qquad (2)$$

$A(\rho)$ and $A(\omega)$ are the respective vector meson production amplitudes; the T's are decay amplitudes in the absence of mass mixing, and m is the invariant mass of the dipion system.

A priori, $\delta = -\langle\rho^0|M|\omega\rangle$ where M is the electromagnetic (EM) mass operator, is a free parameter. However Coleman and Glashow[2] have related such matrix elements to the medium strong, SU(3) breaking, mass splitting of the well-known baryon and meson states. This theory gives quite reasonable results for the EM mass shifts of the baryon octet and decuplet and pseudoscalar meson nonet. They predict

$$\delta = -\langle\rho^0|M|\omega\rangle \simeq \frac{(m_{K^*}^2 - m_\rho^2)}{\sqrt{3}\, m_\rho} \langle\pi'\rangle/\langle\eta'\rangle$$

$$\simeq 2.5 \text{ MeV}$$

for the value $\langle\pi'\rangle/\langle\eta'\rangle = .018$ they determined from the baryon mass splittings. They also predict $m_{K^{*o}} - m_{K^{*+}} \simeq 2.5$ MeV which difference is experimentally measured[4] to be 6.3 ± 4.1 MeV. This equality between δ and the K^* EM mass splitting follows in the more flexible theory[5] which assumes both that M transforms like an octet under SU(3) and that it satisfies the quark model type relation $\langle\rho|M|\phi\rangle = 0$.[6]

Actually this type of theory is only intended to calculate the tadpole or subtraction terms in the matrix elements of M. One must add by hand to the previous results any contributions from low lying intermediate states. For the K^* mass difference, the γK^* intermediate state will lower the prediction by an MeV or so. For δ such a term is absent as neither ρ^0 nor ω is blessed with a charge or magnetic moment. One may however couple the ρ^0 and ω directly to a photon leading to the rather thin diagram of Fig. 1.[7] This gives a contribution[8]

$$\delta' = \frac{e^2 m_\omega}{8 \gamma_\rho \gamma_\omega} \approx 0.7 \text{ MeV}$$

The presence of real intermediate states $(\pi\gamma, \pi\pi\gamma)$ will lead to an imaginary part for δ. One may use unitarity and the experimental bounds[9] on the ρ and ω decay rates to the these states, to find

$$|\text{Im } \delta| \lesssim 0.5 \text{ MeV}.$$

More accurately this is a bound for the full $\omega \to 2\pi$ amplitude including the intrinsic $T(\omega \to \pi^+\pi^-)$ contribution. This is all to the good for it is this full amplitude $\propto \delta_{eff} = \delta + (m_\rho - m_\omega - i\Gamma_\rho/2)\, T(\omega)/T(\rho)$ which occurs in the expression for $S(\pi^+\pi^-)$, the amplitude to produce two final state pions. The fact that δ_{eff}, if large, must be real is simply Watson's theorem that if the $\pi\pi$ intermediate state dominates then $\omega \to 2\pi$ must have the same phase as $\pi\pi$ elastic scattering.

We will not distinguish between δ_{eff} and δ as the $T(\omega)$ term is so small. So we have for the production amplitude

$$S(\pi^+\pi^-) = \frac{A(\rho)\, T(\rho \to \pi^+\pi^-)}{m_\rho - m - i\Gamma_\rho/2} \left\{ 1 + \frac{A(\omega)}{A(\rho)} \frac{1}{m_\omega - m - i\Gamma_\omega/2}\, \delta \right\} \tag{3}$$

We now turn to the problem of estimating the phase of $A(\omega)/A(\rho)$. As we shall indicate shortly, a simple Regge model with some experimental support implies

$$\frac{A(\omega)}{A(\rho)} = \pm i \text{ (Real, positive number)} \quad (4)$$

for the reactions $\pi^{\pm}N \to (\rho^{o}, \omega)N^{(*)}$. This sign is theoretically determined to be plus for incident π^{+} and minus for incident π^{-}. Consequently we expect a dip (rise), of maximum size at the ω mass, coming from the factor

$$\left(1 \mp \frac{2\delta}{\Gamma_{\omega}} \left| \frac{A(\omega)}{A(\rho)} \right| \right)^2 ,$$

for incident $\pi^{\pm}$. Such a dip and peak have been seen in two different high statistics experiments:[1,10]

$$\pi^{+}p \to \pi^{+}\pi^{-}\Delta^{++} \qquad \text{at 3.7 GeV/c; dip} \qquad \text{(I)}$$

$$\pi^{-}p \to \pi^{+}\pi^{-}n \qquad \text{at 3 to 5 GeV/c; peak} \qquad \text{(II)}$$

The present best estimate of $|A(\omega)/A(\rho)|$ for reaction (I) gives $\delta \approx 4$ MeV, in fair agreement with our crude estimate. These data confirm the sign predictions of both exchange degeneracy [see Eqn. (4)] and the Coleman-Glashow mass-mixing theory.

A reaction with relatively real production amplitudes is $e^{+}e^{-} \to (\rho^{o}, \omega)$, where universal vector meson-photon coupling suggests

$$A(\omega)/A(\rho) \approx + \frac{1}{3} .$$

We therefore deduce a modulating function in the ρ^o- peak,

$$\left| 1 + \frac{\delta/3}{m_\omega - m - i\Gamma_{\omega/2}} \right|^2 \quad (5)$$

which gives a shoulder or kink on the high side of the dipion ρ peak. We understand[11] that such a shape is seen in the preliminary data from Orsay. We note in passing that the processes $\pi N \to \ell^+ \ell^- N^{(*)}$ are also amenable to detailed analysis[12] in our picture.

Our claim for the relative phases of the ρ^o and ω production amplitudes is justified by a simple exchange degeneracy[13] argument. Since the production reactions primarily populate the density matrix element ρ_{oo}, we expect $\rho^o(\omega)$ production to be dominated by $\pi(B)$ exchange near the forward direction.[14] The mechanism is indicated in Fig. 2.a). The meson spectrum suggests [Fig. 2.b)] that the π and B may lie on the same trajectory. To apply exchange degeneracy, we consider the SU(3) related reaction,

$$K^+ p \to K^{*o} \Delta^{++} ,$$

which also proceeds by π and B exchange. From the absence of s-channel resonances, we infer that the imaginary parts of the π and B amplitudes [the phases of which are $(1 \pm e^{-i\pi\alpha})$, respectively] must cancel. Consequently the π and B must lie on the same trajectory, and must have the same residue functions. The SU(3) rotations back to the reactions of interest then give the result, $A(\omega)/A(\rho) \propto \mp i$.[15]

From the prediction for reaction (I), we obtain by isospin rotations five other results for pion-nucleon reactions. All of our results are summarized in Table I. Quantitative predictions may be obtained from eqns. (3) and (5), and either the theoretical or experimental estimates of the mixing parameter δ. In Fig. 3, we show typical predictions for $\pi N \to \pi\pi N^{(*)}$ and for $e^+e^- \to \pi^+\pi^-$. We have modulated a pure Breit-Wigner for simplicity. In our example, the unadorned Breit-Wigner has $\Gamma_\rho = 100$. The distribution for the $e^+e^- \to \pi^+\pi^-$ reaction displays i) a rho peak shifted toward the omega mass and ii) a full-width at half-maximum of 80 MeV. Both these features are seen in the preliminary data from Orsay.[11] Thus we predict that colliding beam experiments should see an anomalously small rho width. The same shape is predicted for diffractive photoproduction of rho and omega.

For reactions with kaons incident, we can also apply exchange degeneracy to obtain the relative production phase,[16]

$A(\omega)/A(\rho) = \pm 1.$

The high energy kaon data are consistent with the prediction of equal omega and rho production amplitudes. However it is not sufficiently precise to resolve the rho-omega interference in the two pion decay mode and so test the relative phase of the amplitudes. The low energy data[17] contain substantial background ($\approx 75\%$) under the rho, and a clean test is impossible.

Figure 4 summarizes the wide range of possible phases of $A(\omega)/A(\rho)$ obtainable in different experiments. In the theoretical dream of an experiment with infinite statistics we could hope to use ρ-ω interference to investigate the t-dependence of the phase of $A(\omega)/A(\rho)$. Similarly one can examine its value in different amplitudes by looking at various moments of the decaying 2π system.

While our results do not embody any revolutionary theoretical concepts, they do illustrate a pleasing union of some simple and appealing ideas. First we have the mass mixing theory of the $\omega \to \pi^+\pi^-$ amplitude itself. This kind of model was previously familiar from the superweak theory[18] of the CP-violating $K_2^o \to \pi^+\pi^-$. The hypothesis of octet dominance for the EM mass splitting relates the mass mixing parameter (and thus the $\omega \to \pi^+\pi^-$ amplitude) to the $K^{*o} - K^{*+}$ mass difference. Secondly, we find exchange degenerate Regge trajectories. This notion has received much attention lately in both πN and KN scattering,[19] and appears, by virtue of our results, to be valid in vector meson production reactions as well. Thirdly, the relative sign prediction of the universal (ρ^o,ω) - photon couplings [see eqn. (5)] is tested in $e^+e^- \to \pi^+\pi^-$.

It is a pleasure to thank Professor J. D. Jackson and Dr. E. L. Berger for wisdom and encouragement.

We also wish to thank our experimental colleagues for their generosity with new and still preliminary data. In particular, we thank Professor G. Goldhaber, Dr. A. R. Clark, Dr. D. G. Coyne, and Dr. W. A. Wenzel for discussions.

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TABLE I. Predictions of the Interference In Various Reactions

Reaction	Exchange	Phase (ω/ρ)	Prediction
$\pi^+ p \to \pi^+\pi^-\Delta^{++}$	π,B	+i	dip
$\pi^+ n \to \pi^+\pi^-\Delta^+$	π,B	+i	dip
$\to \pi^+\pi^- p$	π,B	+i	dip
$\pi^- p \to \pi^+\pi^- n$	π,B	-i	peak
$\to \pi^+\pi^-\Delta^0$	π,B	-i	peak
$\pi^- n \to \pi^+\pi^-\Delta^-$	π,B	-i	peak
$K^- p \to \pi^+\pi^-(\Lambda,\Sigma^0)$	K, K*(1320)	+1	constructive below ω, destructive above
$K^- n \to \pi^+\pi^-\Sigma^-$	or	+1	
$\overline{K^0} p \to \pi^+\pi^-\Sigma^+$	K*(890), K*(1420)	-1	destructive below ω, constructive above
$\overline{K^0} n \to \pi^+\pi^-(\Lambda,\Sigma^0)$		-1	
$e^+e^- \to \pi^+\pi^-$	γ	+1	constructive below ω, destructive above
$\gamma A \to \pi^+\pi^- A$	(π), P	+1	

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* This work was supported in part by the U. S. Atomic Energy Commission.

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6. If, as is done in Ref. 5, we identify the tadpole with the contribution of the A_2 trajectory, we predict $\langle\rho|M|\phi\rangle = 0$.

7. This diagram was first suggested to us by Prof. J. D. Jackson. The same diagram has been employed by M. Gourdin (preprint, 1969) to estimate the electromagnetic mixing of rho and omega. Our results indicate that this will underestimate the effect.

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11. J. E. Augustin et al., Lund Conference, 1969. The same features are seen in the DESY photoproduction data (E. Lohrmann, Proc. 1967 International Symposium on Electron and Photon Interactions at High Energies); and in the preliminary photoproduction data of the SLAC--LRL--U. C. Berkeley--Tufts Collaboration. We thank K. Moffeit for bringing these data to our attention.

12. J. T. Donohue, University of Illinois Thesis, 1967 (unpublished). See also L. Durand and Y. T. Chiu, Phys. Rev. Letters 14 (1965) 1039. The explicit results are unfortunately incorrect, because the authors do not consider the important issue of the phase of the production amplitudes.

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14. This is confirmed by detailed fits. See. G. C. Fox and L. Sertorio, Phys. Rev. 176 (1968) 1739.

15. We realize that the simple π - B Regge pole model does not fit the data quantitatively. (See the contortions performed in Ref. 14.) However, exchange degeneracy is such a general phenomenon that we can reasonably hope our phase prediction will survive complications, such as cuts. Further, our predictions only hold in the intermediate energy range (as in Ref. 1, for example) where the B exchange contribution dominates omega production.

16. The exchange degeneracy prediction is not uniquely specified for these kaon reactions. One must also assume that both members of the exchange degenerate pair have the same F/D ratio at the baryon-antibaryon vertex. This can be established by using factorization plus exchange degeneracy in baryon-baryon reaction. (See R. H. Capps, Purdue preprint, 1969, "Exchange Degeneracy and SU(3)".)

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FIGURE CAPTIONS

Fig.1. Simplest diagram for $\omega \to \rho^0$, from which the intrinsic mixing is estimated.

Fig.2. a) Regge exchange diagram for the production of ρ^0 or ω in πN collisions. b) Chew-Frautschi plot of the postulated $\pi - B$ trajectory. We assume the $\pi_A(1640)$ to be the first recurrence of the pion.

Fig.3. a) Typical predictions for $\pi N \to \pi^+\pi^- N^{(*)}$: $\delta |A(\omega)/A(\rho)| = -2, 0, +2$ for constructive, no, and destructive interference. b.) Results for $e^+e^- \to \pi^+\pi^-$: $\delta = +4$.

Fig.4. Summary of the relative phases of ω and ρ^0 production amplitudes.

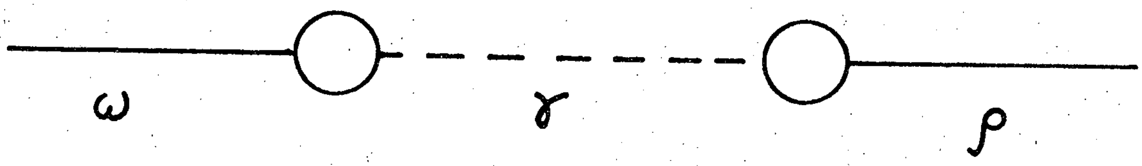

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Fig. 1

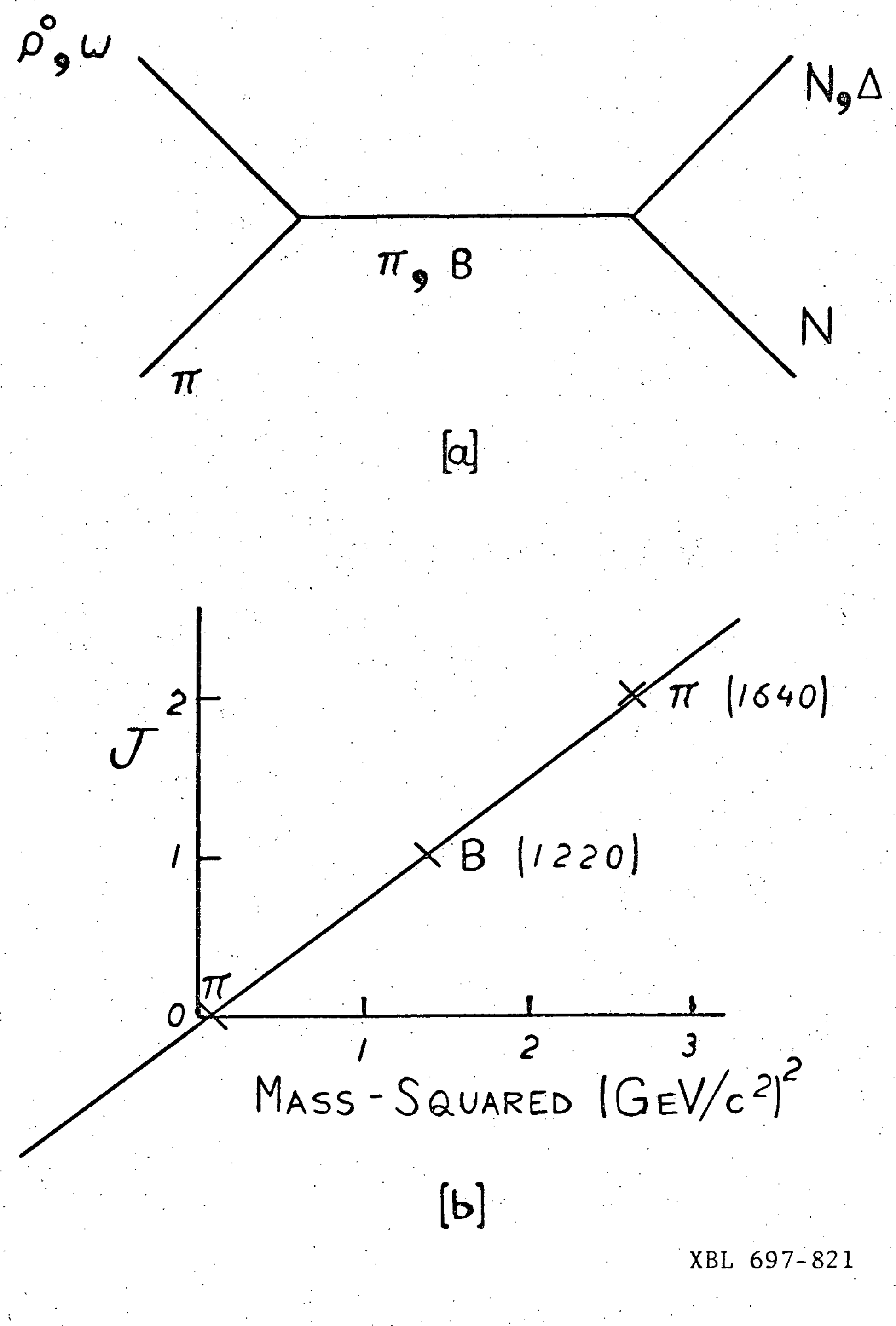

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Fig. 2

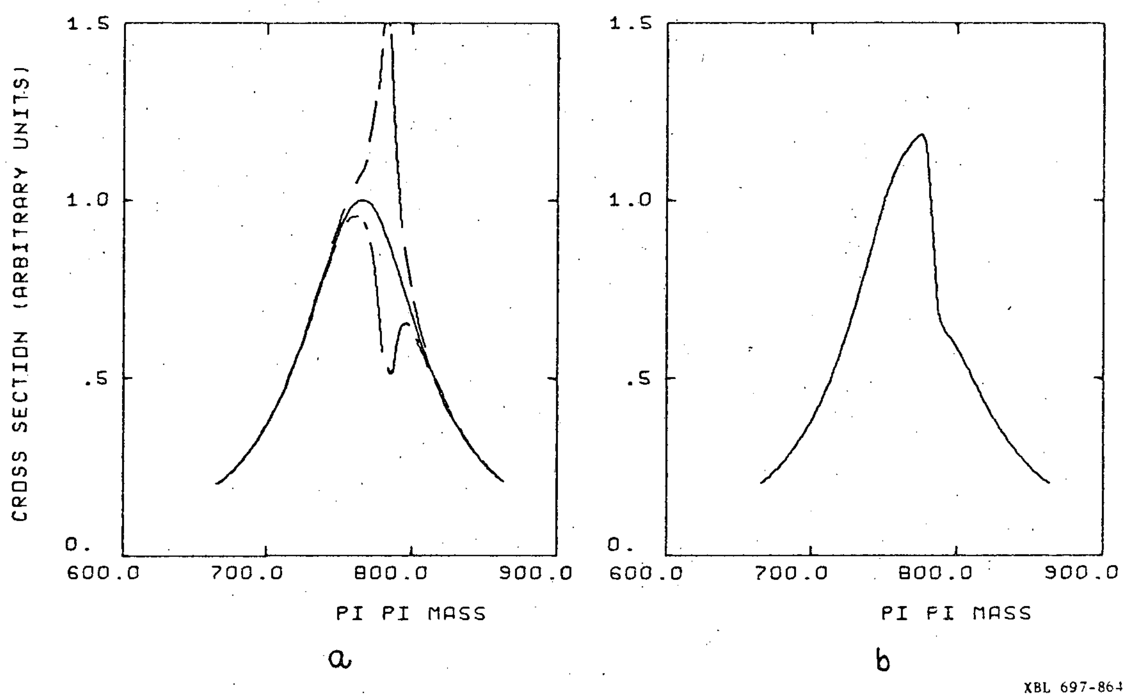
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Fig. 3

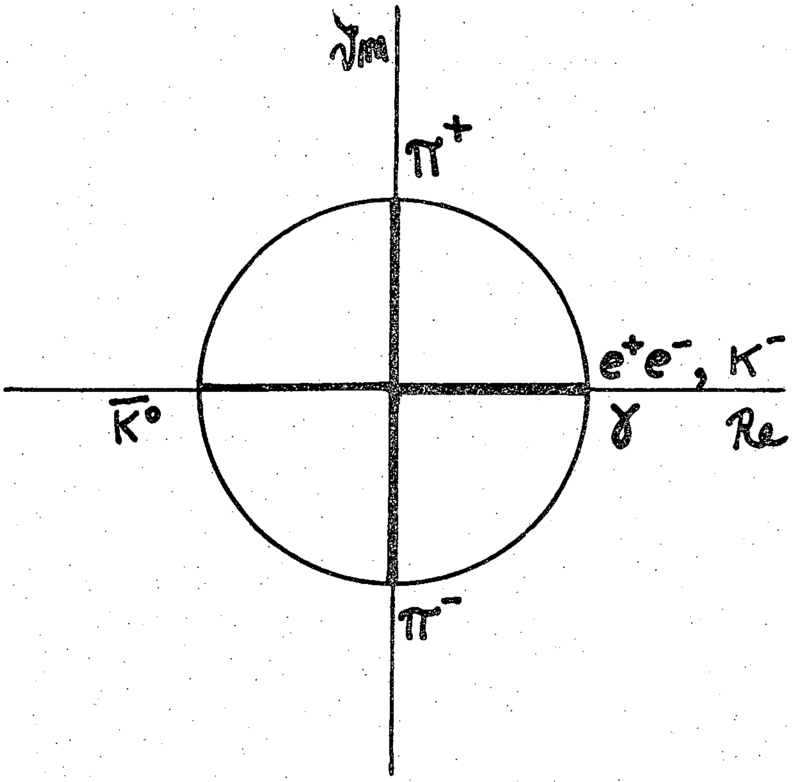

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Fig. 4

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