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UNIVERSITY OF CALIFORNIA,
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**Measurement of Anomalous Heat Transfer in
Sapphire**

THESIS

submitted in partial satisfaction of the requirements
for the degree of

Doctor of Philosophy
in Physics

by

Robert Thomas Joachim

Thesis Committee:

Professor Peter Taborek, Chair

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Professor Jing Xia

2016

Dedicated To
My family, friends, and mentors,
who have been vital to my success.

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Abstract of the Thesis

Measurement of Anomalous Heat Transfer in Sapphire

by

Robert Thomas Joachim

Doctor of Philosophy in Physics

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Professor Peter Taborek, Chair

On nearly every length scale radiated heat transfer between two objects can be described using the exact same theory. However, this theory, which is invariant with respect to the distances separating the objects, breaks down over distances of a few microns or less. Radiated heat flux between objects on these scales experiences an enhancement that grows as the distance between the objects shrinks. This enhancement, known as near field radiated heat transfer (NFRT) arises from phonon polartion resonances on the surfaces of the materials.

Not all materials experience these resonances and only those that have resonances in the near IR will experience this phenomenon. One material with such resonances is sapphire. Expanding and improving upon previous work we created a calorimeter capable of probing this effect in sapphire. Our new methods remove many of the previous issues associated with thermal drift and electrostatic forces. This novel system combined with

new probing techniques has allowed us to measure fluxes as small as 30 nW at separation distances as small as a few tens of nanometers. Using this system we were able to observe anomalous radiated heat flow between sapphire near room temperature. Furthermore, the system was robust enough to function at low temperatures allowing us to confirm the temperature dependence of this effect down to temperatures as low as 100K.

Chapter 1

Introduction

Anomalous radiated heat transfer has been the subject of increasing interest over the last decade. It has long been theorized that over distances less than the thermal wavelength a distance dependent enhancement to radiated heat will occur in certain materials. However, while the underlying theory had been thoroughly established [1] and experimental hints of the effect gathered [2],[3], it was not until recently that this phenomenon was conclusively measured[4]. This first measurement pioneered many of the techniques used in our study such as the use of specialized AFM cantilevers as calorimeters [5] and the reliance on the Derjaguin approximation [6] to predict the theoretical outcome. This study lead the way for similar experiments involving different materials and designs [7, 8, 9, 10]. However, this initial experiment also failed to address many issues that have become apparent with these calorimetry measurements. One of the main problems is thermal drift which became obvious in later experiments and had to be systematically addressed[11].

Our newly designed calorimetry apparatus and experimental technique allows us to build on these previous works. We have been able to minimize the effects of thermal drift allowing us to confirm the heat flux sensitivity in the AFM cantilevers theorized in [5]. We have also identified and resolved issues with electrostatic forces that remain in spite

of the AFM pendulum configuration. These and other improvements on the work in [12] have allowed us to make accurate measurements of anomalous radiated heat transfer. We have been able to make more accurate measurements at room temperature and for separation distances of less than 100 nm.

After confirming the setup's abilities by verifying the room temperature theory, the experiment was moved to a cryogenic system. Using this system we were able to probe the behavior of anomalous radiated heat transfer at temperatures substantially below room. Those results are also presented here.

Chapter 2

Background

This chapter will begin by discussing the limitations of classical radiative heat transfer theory for objects separated by less than a few microns. It will then discuss the fluctuation dissipation theorem as a more accurate method of predicting heat transfer on these scales. This theorem will then be used to predict heat transfer between two dielectrics on a nanometer scale. It will also discuss the physical basis for this effect. We will conclude by discussing some of the possible applications for this phenomenon

2.1 Anomalous Near Field Radiation

Over nearly all distances the radiated heat flux between two objects can be described using a simple ballistic model. In this model heat moves between two objects via an exchange of photons. This classical model can be condensed into the simple equation

$$\Phi = \epsilon\sigma A(T^4 - T_0^4) \tag{2.1}$$

Dependent only on the temperatures of the two objects T and T_0 , incident surface area A , and emissivity ϵ . For the purposes of this experiment we will consider the flow of heat between a semi-infinite plain and a sphere. In this geometry the heat flux equation

becomes

$$\Phi = 8\pi R^2 \epsilon \sigma T^3 \Delta T \quad (2.2)$$

Where R is the radius of the sphere. This relationship is relatively simple in that it depends only on temperature and the incident surface area, while most notably having no dependence on separation distance. However, this simple model breaks down at separation distances of a few microns. At these scales the classical ballistic model is no longer complete. This is because this model focuses only on propagating waves while ignoring the contributions of evanescent waves.

Whereas traditional heat transfer is carried out by propagating waves which travel normal to the surface without loss of energy, evanescent waves die off rapidly in the normal direction. In terms of the wave vector $\vec{k}$, propagating waves have k_z real while evanescent waves have k_z imaginary. The nature of evanescent waves is such that they only transfer heat when meeting with another evanescent wave traveling in the opposite direction. As a result their contributions are only relevant over short distances and between parallel surfaces. When these conditions are met it is possible for these evanescent waves to contribute to heat transfer in a way that is not only dependent on distance but also capable of dwarfing the contributions from propagating waves. A concise theory for describing these additional contributions can be described using fluctuation dissipation theorem.

2.2 Fluctuation Dissipation Theorem

In the regime where the classical heat transfer model fails fluctuation dissipation theorem (FDT) can be used to accurately calculate radiated heat transfer. This theory incorporates the contributions of both evanescent and propagating waves. It does so by considering the random thermal fluctuations present in all materials. These random

thermal fluctuations create fluctuating currents, the mean value of which are zero. However, these fluctuations also create fluctuating electromagnetic fields outside the material. These fluctuating fields can dissipate energy in other media via the Joule Effect.

While it is not possible to completely calculate these random fluctuations, it is possible to calculate the correlation function for these fluctuations using FDT. Rigorous derivations carried out in [13] and [1] yield the z component of the Poynting vector as

$$\Phi(\omega) = \langle S_z(\mathbf{r}, \omega) \rangle = \int d\omega' \epsilon'_\omega(\theta) I_\omega^0(T) \cos \theta d\Omega \quad (2.3)$$

Where $I_\omega^0(T)$ is the radiation intensity of a body at a temperature T, which written explicitly is

$$I_\omega^0(T) = \frac{\hbar\omega^3}{4\pi^3c^2} \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} \quad (2.4)$$

In the case of a dielectric media 1 and 2 separated by a vacuum, media 3, this can be expanded into evanescent and propagating flux terms

$$\Phi^{\text{evan}}(\omega) = \int d\omega \int_{\omega/c}^{\infty} \frac{2K dK}{k_0^2} e^{-2\text{Im}(\gamma_3)d} \left[\frac{\text{Im}(r_{31}^p) \text{Im}(r_{32}^p)}{|1 - r_{31}^p r_{32}^p e^{2i\gamma_3 d}|^2} \right] |I_\omega^0(T_1) - I_\omega^0(T_2)| \quad (2.5)$$

$$\Phi^{\text{prop}}(\omega) = \sum_{q=s,p} \int d\omega d\Omega \cos \theta \left[\frac{(1 - |r_{31}^q|^2)(1 - |r_{32}^q|^2)}{|1 - r_{31}^q r_{32}^q e^{2i\gamma_3 d}|^2} \right] |I_\omega^0(T_1) - I_\omega^0(T_2)| \quad (2.6)$$

Where r is the amplitude reflection coefficient and for the evanescent term the sum is dropped as these waves are only p polarized. This complete theory covers the contributions of both propagating and evanescent waves in these two separate terms. For the purposes of our experiment we will consider a radiative heat transfer coefficient $h^R(\omega)$ defined as [13]

$$h^R(\omega) = \lim_{(T_1 - T_2) \rightarrow 0} \frac{\Phi(\omega)}{T_1 - T_2} \quad (2.7)$$

Applying this limit will give us

$$h^R(\omega) = \frac{1}{\pi^2 d^2} \frac{\text{Im}(\epsilon_1) \text{Im}(\epsilon_2)}{|1 + \epsilon_1|^2 |1 + \epsilon_2|^2} k_B \left(\frac{\hbar\omega}{k_B T} \right)^2 \frac{e^{\frac{\hbar\omega}{k_B T}}}{(e^{\frac{\hbar\omega}{k_B T}} - 1)^2} \quad (2.8)$$

Here ϵ is the frequency dependent dielectric function for each of the media. This term has two notable peculiarities. Firstly, unlike propagating waves, the evanescent term has a dependence on d , the separation distance between the two objects. The leading d^{-2} term makes the near-field contribution negligible at larger distances while for smaller distances it can exceed the magnitude of far field contributions. Secondly these waves are decidedly monochromatic as a result of the $|1 + \epsilon|^2$ terms in the denominator which prioritize the contributions of wavelengths for which $\epsilon \rightarrow -1$. While standard black body radiation is distributed across a broad swathe of wavelengths, near-field radiation is usually dependent on one or two wavelengths in the near IR. This unusual feature arises from surface phonon polariton resonances at these frequencies. As a result the effect is greatest when identical materials are used since they will experience resonance in the same wavelength. This is important because not all materials exhibit resonances in the near IR. Only materials with resonances in the frequency range relevant for thermal transport will experience NFRT. One common material which supports these resonances is sapphire the theory for which is worked out below.

We can use equation 2.8 to calculate the expected heat flux in the near field for the sphere plane scenario in our experiment by performing the integral

$$\int_{\omega_i}^{\omega_f} \int_0^R h[\tilde{d}(r), T] 2\pi r dr \quad (2.9)$$

Where we integrate over the relevant wavelengths in the near IR as well as the radius of the sphere R and $\tilde{d} = d + R - \sqrt{R^2 - r^2}$. This relation for $\tilde{d}$ arises from the sphere plane geometry as shown in figure 2.1

This will leave us with an equation for the evanescent heat flux in W/K which is

$$G_{\text{evan}}[d] = 2H\pi\left(\frac{R}{d} + \log[d] - \log[R + d]\right) \quad (2.10)$$

Where H is some numerical constant which changes based on the materials used. We can combine the evanescent contributions with the more familiar near field contributions

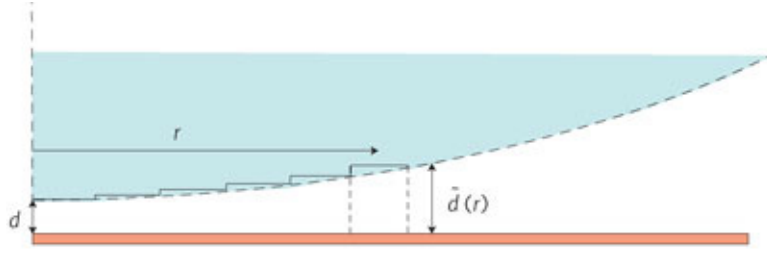


Figure 2.1: For the purposes of calculating h we must consider $\tilde{d}$ the distance between the plate and each layer of the sphere.

to yield an equation for the heat flux per unit K expected at all separation distances

$$G_{\text{total}}[d] = 2H\pi\left(\frac{R}{d} + \log[d] - \log[R + d]\right) + 8\pi R^2\sigma\epsilon T^3 \quad (2.11)$$

This final equation is used to create the theory curves in this thesis.

2.3 Surface Phonon-Polaritons

So far we have discussed the mathematical theory behind these IR resonances without much reference to their physical source. They arise from surface phonon-polariton oscillations. These oscillations occur when transverse optical photons couple with longitudinal phonons in a material. A quantum of this coupled transverse wave field is called a polariton. When the frequencies and wave vectors are nearly equal, this field can be driven to resonance [14]. When such resonances are in the near IR they can be thermally excited. This is the case with sapphire, glass, silicon carbide, and other select insulators. The exact frequency of these resonances varies and is determined by the dielectric function of the material.

This is similar to the phenomenon responsible for surface enhanced Raman spectroscopy (SERS), except in that phenomenon surface plasmon polaritons are the resonators and they tend to resonate at visible or near IR frequencies.

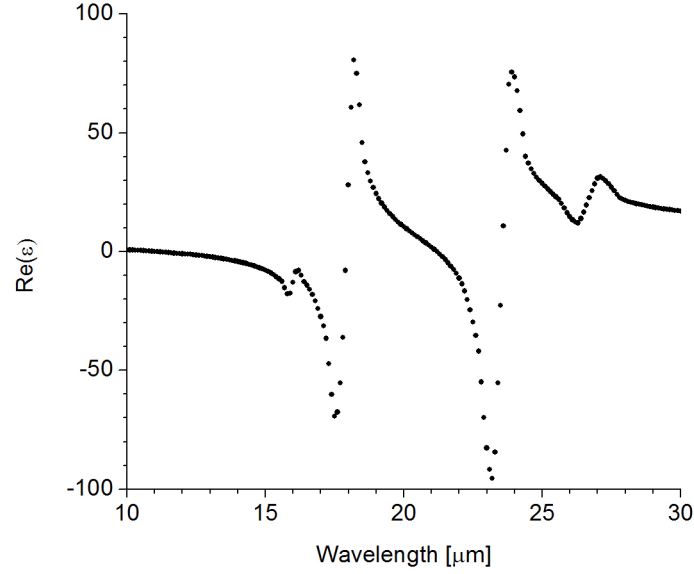


Figure 2.2: Real part of the dielectric constant over the wavelengths relevant for radiated heat. We can expect resonances to occur when this function equals -1.

2.4 Sapphire Theory

As with previously studied materials such as glass and silicon carbide, sapphire exhibits surface phonon polariton resonances in the IR. However, these resonances occur at different wavelengths and greater magnitudes than in those materials. For any material it is possible to calculate the expected resonances using optical data to determine the frequency dependent dielectric constant. For the purposes of this project a program was created to determine the plasmon resonances using provided optical data found in [15]. This program can be found in appendix 5.1.1. The real part of the frequency dependent dielectric function for sapphire over the relevant wavelengths is shown in figure 2.2

For sapphire these resonances were found to occur at wavelengths of about 12 μm and 21 μm . Figure 2.3 compares these with the resonances in glass.

As evident in figure 2.3 the near-field heat transfer coefficient of sapphire is much

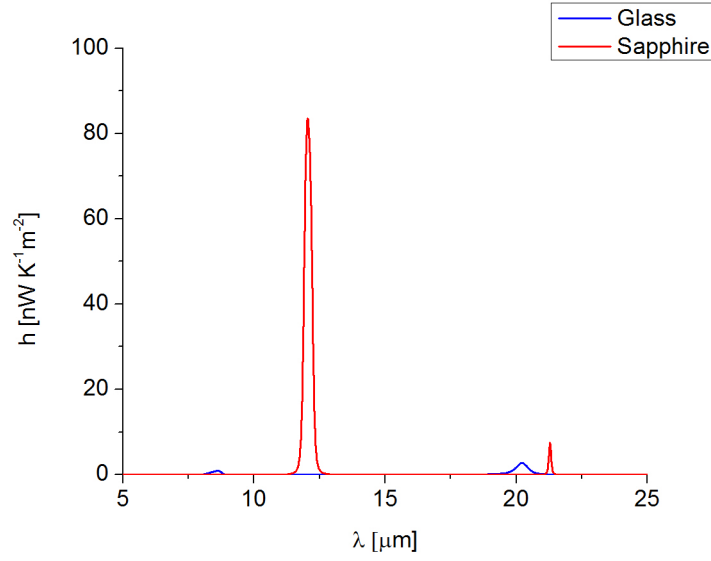


Figure 2.3: A comparison of the heat transfer coefficient in sapphire (red) to that of glass (blue) for surfaces separated by a distance of 10 nm and at a temperature of 300K. As expected the radiation becomes decidedly monochromatic at such a small separation with the different materials experiencing resonances at different frequencies. Interestingly the near field effect in sapphire dwarfs that of glass.

greater than that of glass. This translates into an even greater heat flux between sapphire objects over the relevant length scales. A program was created to determine the heat flux between a plane and sphere as a function of distance using provided optical data 5.1.2. This program was used to create figure 2.4 which shows the heat flux per K between a semi infinite substrate and a $100\text{ }\mu\text{m}$ sphere as a function of distance. The sapphire greatly outmatches the glass causing a stronger effect and therefore a more easily measured phenomenon.

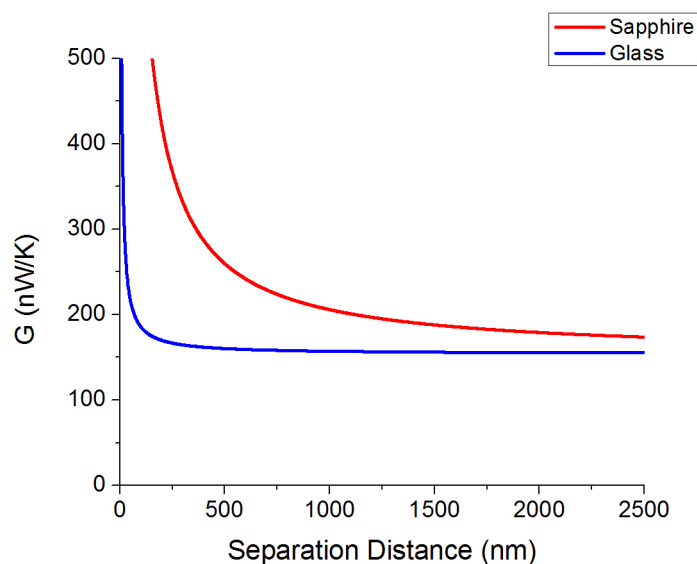


Figure 2.4: Heat flux between a $100\text{ }\mu\text{m}$ diameter sphere and a plane as a function of distance. The near field contributions become increasingly relevant at smaller distances. The effect in sapphire (red) is noticeably greater than the effect in glass (blue).

2.5 Calorimetry Theory

Measuring such small heat fluxes over such small distances required a specialized calorimeter. The crux of this whole experiment was a $22\text{ }\mu\text{m}$ by $300\text{ }\mu\text{m}$ silicon nitride

AFM cantilever with a gold coating. This cantilever was able to function as a very sensitive calorimeter as a result of bimaterial bending. The theoretical sensitivity H in nanometers of bending per nanowatt of heat flux of this calorimeter is fully derived in [5] and given by the equation

$$H = (\gamma_1 - \gamma_2) \frac{t_1 + t_2}{t_2^2 K} \frac{l^3}{(\lambda_1 t_1 + \lambda_2 t_2) w} \quad (2.12)$$

with

$$K = 4 + 6 \left(\frac{t_1}{t_2} \right) + 4 \left(\frac{t_1}{t_2} \right)^2 + \frac{E_1}{E_2} \left(\frac{t_1}{t_2} \right)^3 + \frac{E_2}{E_1} \left(\frac{t_2}{t_1} \right) \quad (2.13)$$

This equation is specifically for a cantilever absorbing power at the tip. The sensitivity is dependent on both the geometry of the cantilever in the length l , width w , and material thicknesses t_1 and t_2 as well as the properties of the materials in the Young's modulus E , thermal expansion coefficient γ and thermal conductivity λ .

This theory gave us an estimate of the cantilever response but in practice each cantilever had slightly different physical dimensions. Furthermore, as we began to experiment at low temperatures some of these properties would change as well. As a result it was necessary to determine H for each cantilever individually and for each temperature. This was done using methods outlined below.

After determining H it was straightforward to translate the cantilever response δ into a heat flux via the relation

$$G_{exp} = \frac{H}{\Delta T} \delta[d] \quad (2.14)$$

For the purposes of this experiment a large dielectric sphere is glued to the tip of the cantilever. This sphere serves two purposes. Firstly it drastically enhances the amount of surface area to absorb incident radiation. Secondly it allows us to approximate two parallel surfaces without the experimental difficulties of aligning two plains.

2.6 Derjaguin Approximation

While the use of two parallel plates in this experiment would simplify the theoretical aspects, it would greatly complicate the physical setup. Instead we compared the heat transfer between a large sphere and a substrate. In order to modify the theory appropriately an approximation is needed. To this end we employ the Derjaguin Approximation [16]. Also known as the proximity force approximation this method was originally designed to approximate forces between two spheres as interactions between two planes. It can be suitably altered for the case of a plate and a sphere and is frequently used in Casimir force measurements [17]. This approximation is most valid when the separation distance $d \ll R$ the radius of the sphere.

For our purposes we integrate the heat transfer coefficient $h[\tilde{d}(r), T]$ over the entire sphere

$$G_{\text{theo}}(d, T) = \int_0^R h[\tilde{d}(r), T] 2\pi r dr \quad (2.15)$$

where R is the sphere radius and $\tilde{d}(r) = d + R - \sqrt{R^2 - r^2}$ is the local distance between the plane and the sphere surface.

2.7 Applications

While our full understanding of how to control near field heat transfer is still developing, many exciting applications have already been proposed [18]. A few of the most interesting applications are shown in figure 2.5a. One widely discussed use is in thermophotovoltaics or TPV. TPVs are similar to traditional photovoltaic cells in that they transform radiation into energy but they differ in that their bandgaps tend to be in the near-IR rather than the visible wavelength. Research into TPVs is popular as the hope is to one day use them to generate electricity from waste heat. During operation they

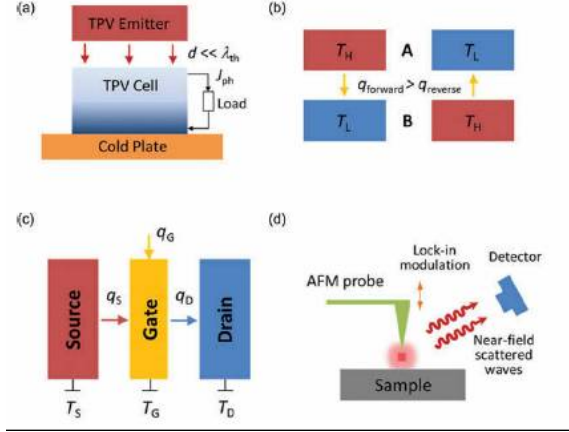


Figure 2.5: Adapted from [18]. A diagram depicting some of the possible applications for NFRT. a) Enhancing the efficiency of TPVs. b&c) Thermal management and creation of thermal circuits. d) Imaging beyond the diffraction limit

are put in close proximity to a heated substrate which acts as an emitter of thermal photons. The hope is that using materials which experience near field transfer as the emitter will greatly increase the number of incident photons and thus raise the currently poor efficiency of such devices. Much work remains though as the effect of NFRT is strongest between matching materials. NFRT materials tend to emit photons at wavelengths in the mid-IR while the materials in TPVs often have bandgaps in the near-IR.

Another possible future use for this effect may lie in thermal management as this enhanced flow of heat allows us to overcome the limits set by the Stefan-Boltzman Law. The ability to preferentially control the flow of heat may one day allow us to construct thermal rectifiers, thermal diodes, or even thermal transistors (figure 2.5b&c). Any manner of heat manipulation relying on NFRT would have the advantage of relying on photons rather than phonons for thermal transport.

Yet another proposed use is in the field of imaging and spectroscopy. It is possible to use an AFM tip to overcome the diffraction limit inherent to imaging. By bringing the

tip to within a distances of less than one wavelength of the substrate, it is possible to image the surface by probing the evanescent field.

These diverse applications may one day be realized but much fundamental work remains to be done. Below we outline the construction of an apparatus and experimental techniques which can be applied to studying NFRT in various materials and at various temperatures.

Chapter 3

Experimental Setup

This chapter will focus on improvements to the experimental apparatus and data analysis over the experiment outlined in [12].

3.1 Physical Setup

3.1.1 Cryogenics System

As the primary focus of the original experiment was to verify the room temperature theory, it was carried out in a simple vacuum can on an air table. The preliminary low temperature data was taken by placing the experiment in a closed cycle refrigerator capable of reaching temperatures around 40K. This setup soon proved problematic however. The vibrations caused by the cooling unit were difficult to counteract and deactivating the unit during data acquisition led to an unacceptable thermal drift.

The solution was to move the experiment into a wet cryogenic system. The experimental system was moved to a new cell which could be submerged in liquid nitrogen for extended periods. This allowed for a lower limit of 77K while higher temperatures were achieved using heaters on the experiment. The thermal and mechanical stability of this setup proved to be much greater than with the closed cycle refrigeration.

As before the vacuum was maintained using a turbo pump backed by a rotary vein roughing pump. Additional support came from the cryopumping induced by the low temperatures. In all we were able to reach pressures <1 mTorr, well below the 8 mTorr threshold needed to ignore heat conduction from air.

3.1.2 Experimental Apparatus

A diagram of the experimental setup is shown in figure 3.1. A substrate heated to temperature T_1 was slowly brought into contact with a sphere held at a lower temperature T_2 . This approach was made using piezo stacks provided by Physik Instrumente as Thor Labs piezo stacks tended to fail after a few cycles at low temperatures. Heat absorbed by the sphere would cause bending in the cantilever as explained in section 3.1.3. This bending was measured using a fiber interferometer system aimed at the back of the cantilever. The fiber optics ran from a 1550 nm diode laser. This wavelength was chosen due to the relatively low absorption expected from the cantilever materials, minimizing heat flux from the laser. Even so the laser was run at the minimum possible power. Each aspect of this apparatus saw improvements over those used in our previous work [12]. These improvements greatly enhanced our experimental capabilities and are outlined below.

3.1.3 Calorimetry

As in the previous work a bimaterial AFM colloid cantilever was used to measure radiated heat flux. The exact details of the operation are outlined elsewhere [12] but, simply put, the sphere on the cantilever absorbs radiation heating the cantilever. The cantilever then bends as a result of the uneven expansion of the two materials (in this case gold and silicon nitride). While such specialized AFM cantilevers can be purchased

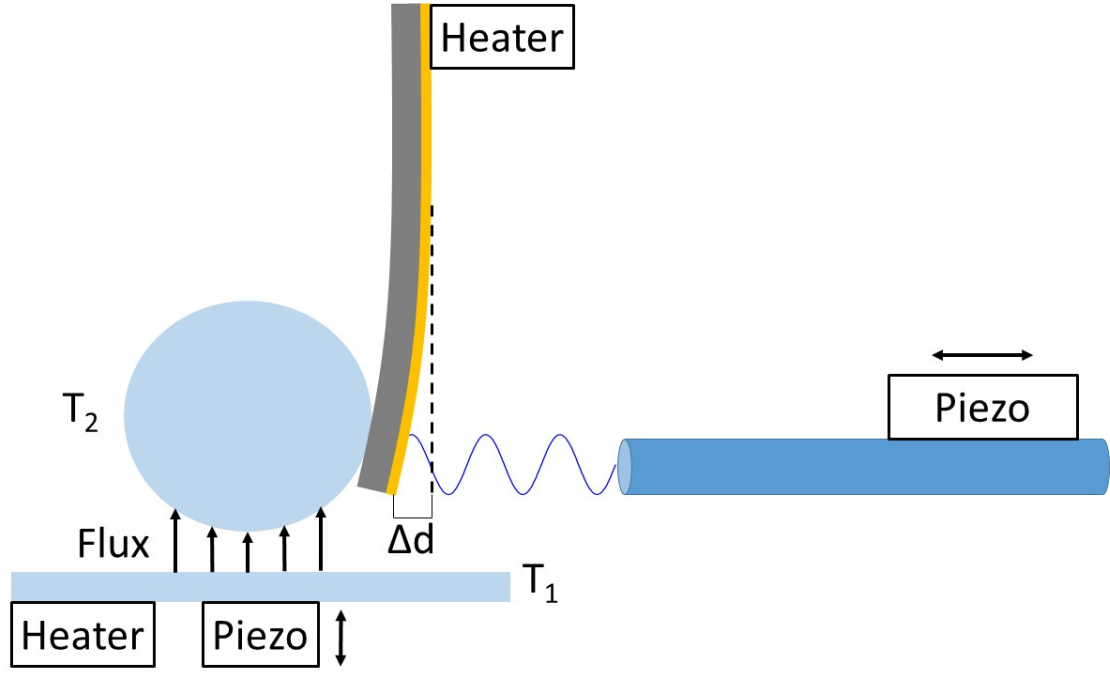


Figure 3.1: A diagram of the apparatus used in this experiment. The cantilever is held at temperature T_2 and approached by the substrate at temperature T_1 where $T_1 > T_2$. The heat flux into the sphere causes the the cantilever to bend a distance Δd which is measured by the interferometer. This configuration, in which the cantilever is perpendicular to the surface, is referred to as the pendulum configuration. It is used to minimize the influence of surface forces

commercially, they are quite costly and of limited selection. To this end it became pertinent to manufacture our own cantilevers.

The cantilevers used were model MLCT-O10 purchased from Bruker AFM Probes. These silicon nitride cantilevers with a gold coating on the reflex side had a length of $310\text{ }\mu\text{m}$ and a width of $20\text{ }\mu\text{m}$. These cantilevers were entirely tipless but the materials and dimensions made the cantilever more sensitive to heat flux than those available commercially with a sphere already attached. A response of about 0.03 nm/nW was theoretically expected [5] and experimentally verified as shown below.

Spheres of various sizes, between $40\text{ }\mu\text{m}$ and $250\text{ }\mu\text{m}$, and various materials including sapphire and borosilicate glass were attached with a conductive silver epoxy. The actual application took place under an optical microscope using the micrometer stage to place the spheres. The spheres were held by a $125\text{ }\mu\text{m}$ optical fiber using static electricity. While keeping the spheres stationary the AFM cantilever, with a dab of epoxy previously applied, was raised into the sphere. Figure 3.2 shows a glass sphere adhered to the optical fiber next to the cantilever for comparison. For the purposes of our experiment we used only cantilever C. It's dimensions made it the most sensitive and insured that it would be the first cantilever to make contact with the substrate.

The introduction of this technique greatly reduced the material costs and gave us more experimental freedom. Specifically when purchasing through a commercial supplier one is limited to glass spheres of radius $10\text{ }\mu\text{m}$ or smaller. Since the absorbed heat flux of the cantilever is $\propto R^2$, the ability to use larger spheres improves the sensitivity of our probe. We were also able to attach sapphire spheres which are not available with commercial cantilevers.

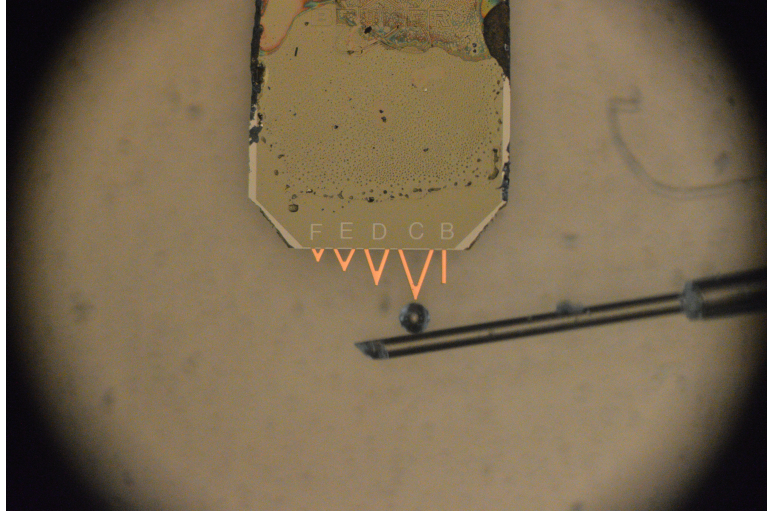


Figure 3.2: A photograph taken through the microscope used to construct the cantilevers. This photo is just for demonstration. In practice the sphere and cantilever would, by necessity, be much cleaner.

3.1.4 Substrate

Earlier work on this subject was only able to yield data at separations greater than 100 nm. This is a key failing as the effects of near field transfer begin to seriously dwarf far field transfer below these separation distances. It was our belief that this failing was due to the surface roughness of the both the substrate and sphere. The roughness of the surfaces was confirmed using an SEM. As shown in figure 3.3 the untreated 200 μm diameter spheres had surface features on the order of 1 μm . To address this issue both the spheres and substrates were cleaned using a method adapted from [19]. They were first soaked in a 10% solution of H_2SO_4 for three hours then in a 10M NaOH solution for 30 minutes. The timing of each treatment was determined experimentally as too much time in NaOH tended to create pits in the glass spheres. This treatment removed the near micron size aberrations found on the untreated spheres as seen in figure 3.3. This

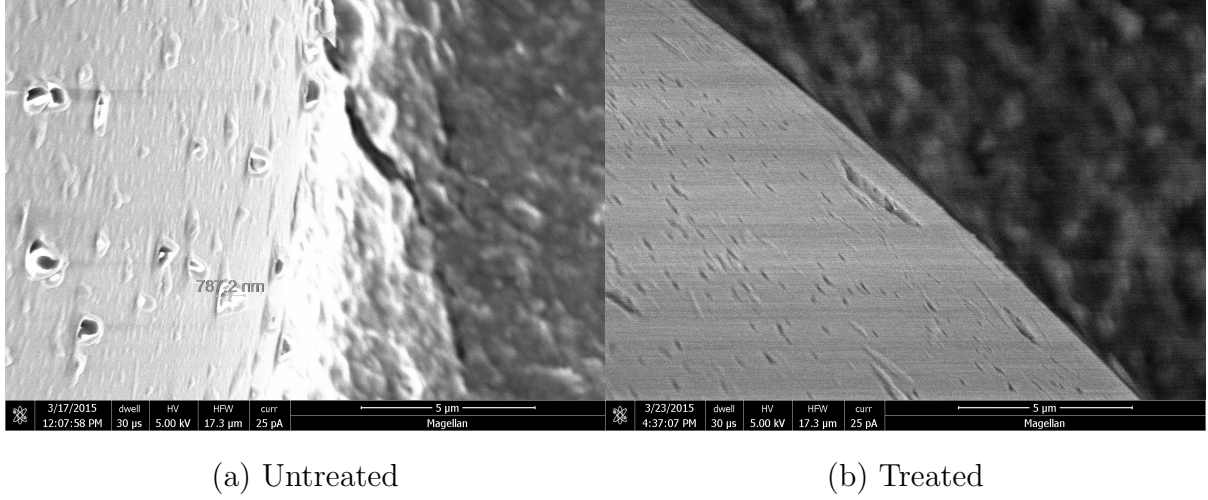


Figure 3.3: Comparison of glass spheres before and after soaking in NaOH and H₂SO₄

allowed for data to be taken at separation distances of less than 100 nm.

3.2 Difficulties and Improvements

3.2.1 Sensitivity Improvements

As the heat flux is calculated directly from the cantilever displacement, the heat sensitivity of our apparatus is limited by the sensitivity of our interferometer system. For this experiment we were able to measure displacements of a nanometer by making several improvements over our previous work. Two major sources of noise were focused on, vibrational noise and electronic noise.

Vibrational noise is a constant problem in many low temperature physics experiments. Moving from a closed cycle cryostat to a wet cryogenic system reduced the vibration associated with the pulse tubes but other vibrations remained. The vibrations from the pump line were dampened by sending it through sand as shown in figure 3.4. The experiment itself was further decoupled from vibrations by suspending on springs. Helium



Figure 3.4: Sending the pump line through a large amount of sand was enough to dampen out the vibrations from the pumps.

was used as an exchange gas to cool the experiment without the need for a physical thermal link that might couple in vibrations.

Other steps were taken to reduce the electronic noise. A new Arroyo Instruments laser driver was used which proved more stable than the Newport Instruments driver, improving the signal. A second photoreceiver was introduced allowing us to measure the overall intensity of the laser diode output. This let us to remove the effects of drift in laser intensity. Efforts were made to temperature control the laser but the results were unimpressive. Perhaps the most effective step taken was to modulate the laser using a noise signal. This modulation greatly improved the quality of the signal as shown in figure 3.5.

A common problem with fiber interferometers arises from reflections of the signal at interfaces other than the object who's displacement is being measured. These "ghost interferometers" contribute to the noise in the system. To reduce such reflections index matching fluid from Thor labs was added between the fiber connections to remove any possible air pockets. This fluid has an index of refraction of 1.4378 almost exactly

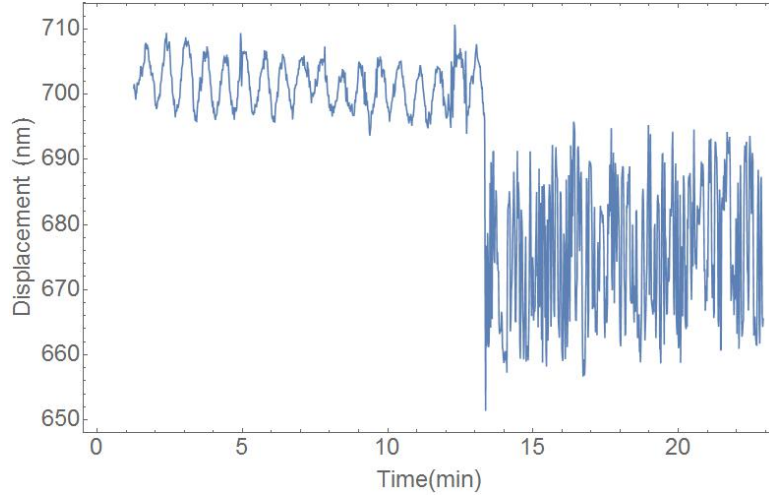


Figure 3.5: Comparison of the signal with noise modulation (left) and without noise modulation (right). With the cantilever held at 305K the substrate temperature was oscillated between 318K and 320K causing the cantilever to oscillate. Without noise modulation the cantilever oscillations are completely obscured.

matching the fiber's which is 1.4465 thus substantially reducing spurious reflections.

While these steps greatly reduced the noise it was still helpful to average the interferometer signal which removed most of the grass-like noise. This left drift on the scale of a minute or more as the main source of noise.

3.2.2 Thermal Considerations

As this experiment required precise displacement measurements and involved various metals, ceramics, and glasses, thermal drift posed a serious problem. Any change in temperature of the system would cause various expansions or contractions giving the illusion of the cantilever bending. Drifts in ambient temperature were avoided by holding the experiment at temperatures either higher or lower than the room temperature with a PID system. This system usually sufficed to keep the temperature of the cantilever

holder within a few thousands of a Kelvin. However, this step alone could not eliminate all thermal drift.

The real problems arose when the substrate temperature was changed substantially (several Kelvin) over a long period of time (i.e. several minutes). These problems were particularly noticeable when attempting to calibrate the response of the cantilever using a known heat flux. Originally this was done by as in [4] by raising the heater power in steps and waiting for the temperature to level off. However problems with this method became obvious when the experimental calibration began yielding sensitivities multiple orders of magnitude greater than those predicted by the theory in [5]. As shown in figure 3.7 this method can produce the appearance of bending when none should be suspected at all. This issue might be partly responsible for the cantilever response of 0.435 nm/nW reported in [4] well in excess of the theoretically predicted value of 0.029 nm/nW.

Several steps were taken to address the problem of thermal drift. The first was to, whenever possible, use Invar in the construction of the experiment. This easily obtainable and machinable metal has a coefficient of linear expansion of $1.2 \times 10^{-6} K^{-1}$ at room temperature, nearly ten times smaller than steel. However while it was convenient to construct the cantilever holder out of Invar other portions of the experiment could not be so easily replaced so further steps were taken. The substrate size was shrunk substantially from about 300 mm² to about 25 mm² and the heater changed from a 10W cartridge heater to a 1/8W thick film resistor. Figure 3.6 gives a size comparison of the two heaters. This smaller substrate required a smaller heater allowing us to heat the substrate while limiting the amount of excess heat flowing into the experiment.

The final, and most effective, step to avoid thermal drift involved altering our experimental methods. Instead of holding the cantilever at one temperature and the substrate at another, higher temperature the substrate temperature was varied in a rapid sinusoidal

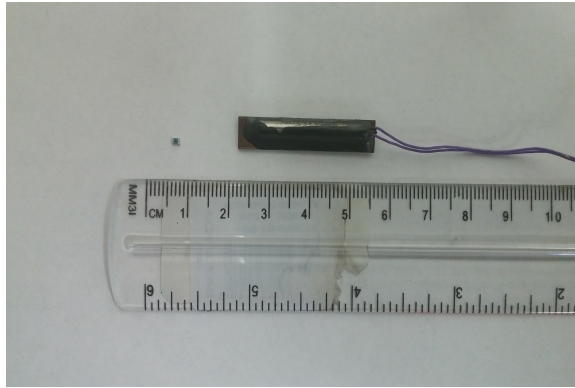


Figure 3.6: The current heater (left) and the old heater (right). Shrinking the substrate size allowed for a much smaller heater to be used without loss of functionality.

manner. The cantilever system had a much smaller thermal inertia than the surrounding materials meaning that only it was responsive enough to follow rapid oscillations while the bulk material was too sluggish. Without such rapid oscillations, it is possible to achieve the illusion of thermal bending rather easily as mentioned above. Figure 3.7 shows a series of tests for a bimaterial cantilever *without* a sphere attached.

The lack of a sphere reduces the incident surface area of the cantilever by three orders of magnitude and as such the amount of absorbed heat to a negligible level. Despite this change the cantilever still appears to bend substantially with increasing temperature. This aberration is most likely due to thermal expansion of the holder and/or thermal heating of the cantilever via conduction from the holder. However, by rapidly varying the temperature over the same range, it was possible to minimize this spurious displacement. Increasingly rapid oscillations lead to a decrease in this spurious bending. Thus for the purposes of our experiment the temperature was oscillated at a rate of 40 mHz.

As an example of the benefits of this method we consider the calibration of our cantilever. Throughout the experiment it was necessary to frequently calibrate the cantilever's response to a known flux in terms of nm of bending per nW of heat absorbed.

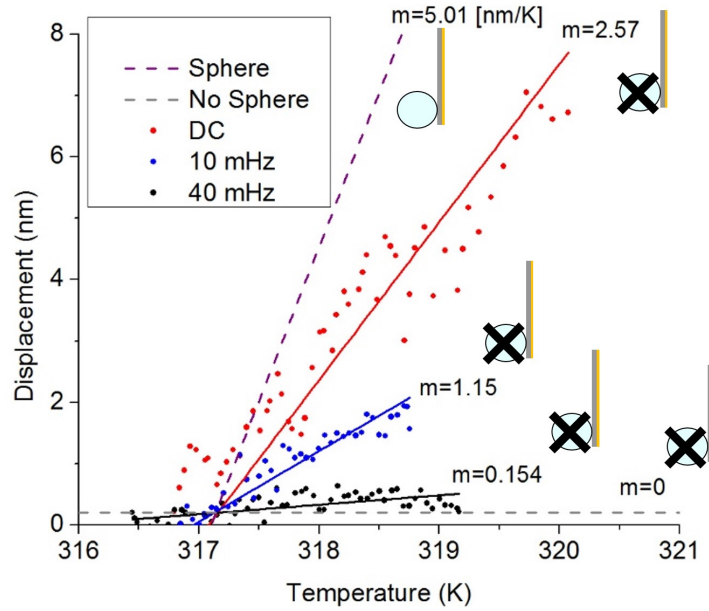


Figure 3.7: Even a cantilever without a sphere can appear to bend. Note that for convenience the data is shown as nm vs K rather than nm vs nW. For a cantilever with a sphere we would expect a bending of about 5 nm/K for these conditions as shown by the dashed line. For a cantilever without a sphere we would expect no bending but that is not the case. The effect is worst for a single discrete temperature change (red) of 3K which causes a response nearly equal to that theoretically predicted for a cantilever with the sphere. Increasing the rate of the temperature oscillations reduces this false response with 10 mHz (blue) giving 1 nm/K and 40 mHz giving about 1 Å/K which is below the sensitivity of our apparatus.

This was necessary for several reasons. First each cantilever differed slightly in its physical dimensions altering its sensitivity to heat. Second the exact alignment of the fiber often changed so whereas the theoretical sensitivity describes a bending of the cantilever tip, our system often read the displacement some distance from the tip. Finally, during the low temperature portion of the experiment we entered a temperature regime where the material properties responsible for the bimaterial bending began to change. Thus while the theoretical value is a good guideline we should not expect our measured sensitivity

to match that value.

The calibration shown in figure 3.8 took place at room temperature with the cantilever being held at 300K and the substrate oscillated between 312.3K and 314.3K at a rate of 40 mHz. Fitting this data finds a slope of 0.021 nm/nW which is in good agreement the theoretical value given the circumstances mentioned above and much closer than the 0.435 nm/nW reported elsewhere [4].

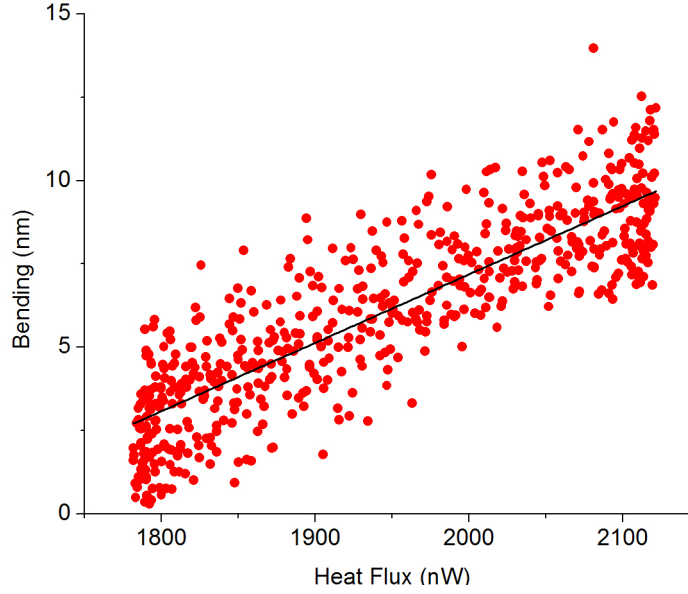


Figure 3.8: A plot showing the cantilever response to many temperature oscillations at 40 mHz. Applying a linear fit to this data yields an H of 0.021 nm/nW which is in good range of the expected value.

This oscillation method had an additional advantage, it allowed us avoid the electrostatic issues that became apparent in earlier measurements.

3.2.3 Electrostatic Issues

Unlike most experiments involving AFM cantilevers our experiment approaches the substrate in such a way that the cantilever is perpendicular to the surface (see fig 3.1)

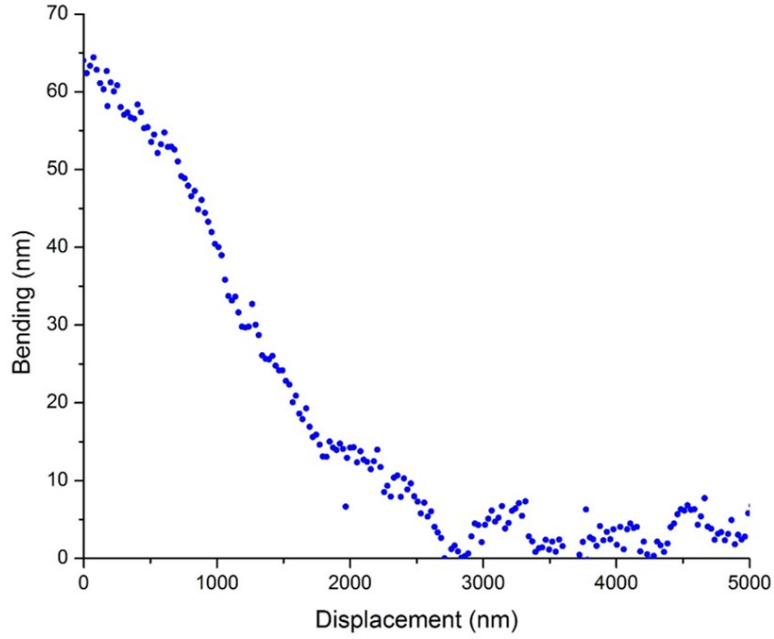


Figure 3.9: Even with the sphere and substrate held at equal temperatures bending could still be observed as a result of surface forces. This plot is the average of many approaches.

rather than parallel to the surface. The goal of this "pendulum configuration" is to remove the possibility of surface forces attracting the sphere and causing bending in the cantilever, thus obscuring the bending due heat flux. However even in this arrangement these forces continued to be an issue. This was most evident when approaches between the sphere and substrate yielded a data curve even when at equal temperatures when none should be expected. Such a test is demonstrated by figure 3.9. The larger $200\text{ }\mu\text{m}$ spheres exacerbated the problem as did some attempts to clean the substrate with Kim wipes. For particularly high charging the sphere could be seen through the microscope snapping to contact with the surface.

A number of smaller fixes were tried at first. The two surfaces were better grounded together removing some of the more glaring discrepancies. However, it was not possible to truly ground the insulating sphere and substrate causing some issues to remain. Some of

the more serious charging resulted from cleaning with Kim wipes so the cleaning methods were modified. The substrates were instead kept submerged in DI water and dried with a nitrogen gun. This too failed to remove the issues completely. The final step was to introduce a polonium-210 disc in an attempt to use the alpha emission to discharge the insulators. Again this was unable to completely dispel bending induced by surface forces. A different approach had to be taken to completely remove the influences of static forces.

3.2.4 Oscillation Method

Due to failings in the traditional experimental method discussed in section 3.2.3 a new experimental approach was pioneered for this thesis. Rather than holding the substrate at constant temperature, we oscillated the temperature in the same manner as the far field calibration experiment described in section 3.2.2. This in turn would cause the cantilever to oscillate with an amplitude corresponding to the difference in flux between the two temperatures. This allowed us to measure the amplitude of the oscillations rather than the overall bending of the cantilever which might be due to factors other than heating. The theoretical value for this amplitude could be found by determining the expected curve for the maximum and minimum temperature using the program 5.1.2 and subtracting the two curves as shown in figure 3.10.

As seen in figure 3.10 the amplitude, while well within our sensitivity range at a micron or more, only really becomes large within a few hundred nanometers. This was actually beneficial because it prevented the measurement system from becoming over saturated, allowing us to confirm this phenomenon at smaller distances than previous works with sapphire [10].

An example data run is shown in figure 3.11. This data was taken by approaching the substrate in discrete 50nm steps while the substrate temperature was oscillated by 2K

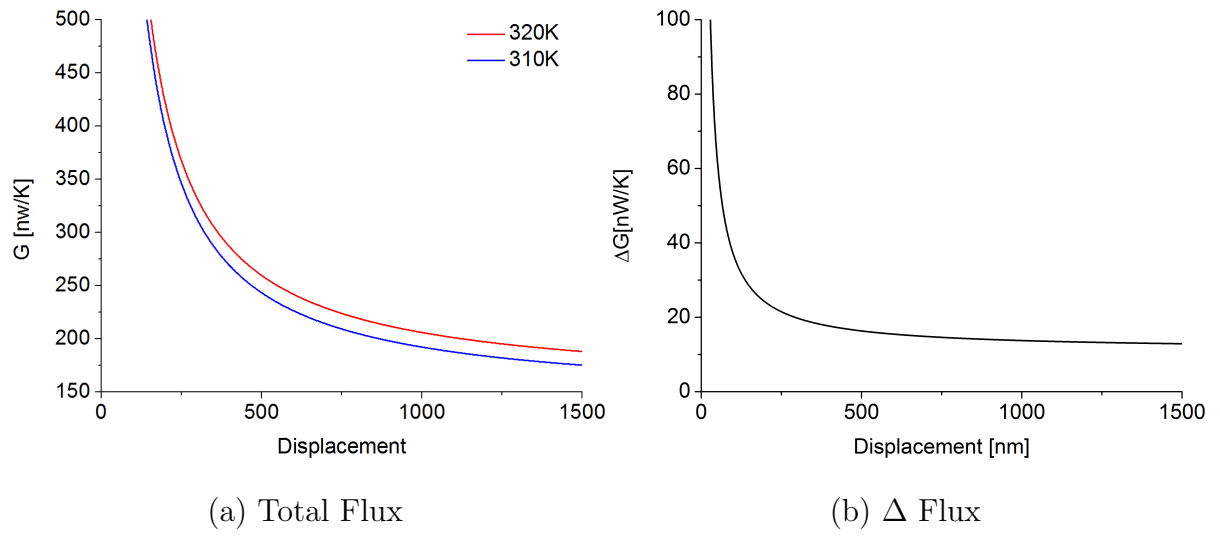


Figure 3.10: An example of the theoretical calculations for a cantilever held at 300K and a substrate oscillated between 310K and 320K. In order to determine the theoretical oscillations of the cantilever two flux curves were found. One for the maximum temperature and one for the minimum temperature. These are shown together in (a). The expected amplitude is just the difference between these two curves as shown in (b).

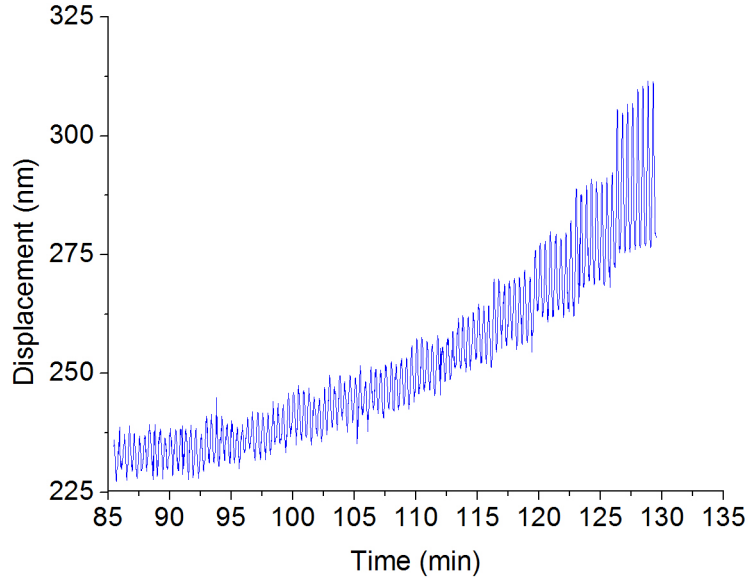


Figure 3.11: Data for an approach with the distance decreasing with time in 50 nm steps. The steps become more noticeable near contact. Note that while the size of the oscillations increases as expected, the overall signal is rising denoting a bending of the cantilever due to both thermal and surface force effects.

at a frequency of 40 mHz. The system sat at each step long enough to complete several oscillations. So while the cantilever might bend with each step, it remained stationary on each step. The multiple oscillations could then be fitted with a trigonometric curve to determine the average amplitude.

One possible issue with this method would be the change in separation distance due to thermal expansion and contraction. The expansion can be easily derived from the equation

$$\Delta L = L\alpha\Delta T \quad (3.1)$$

The sapphire substrate has a thickness of 0.4 mm and a linear expansion coefficient α of $5.3 \times 10^{-6} K^{-1}$ while the stainless steel substrate holder has a thickness of 0.5 mm

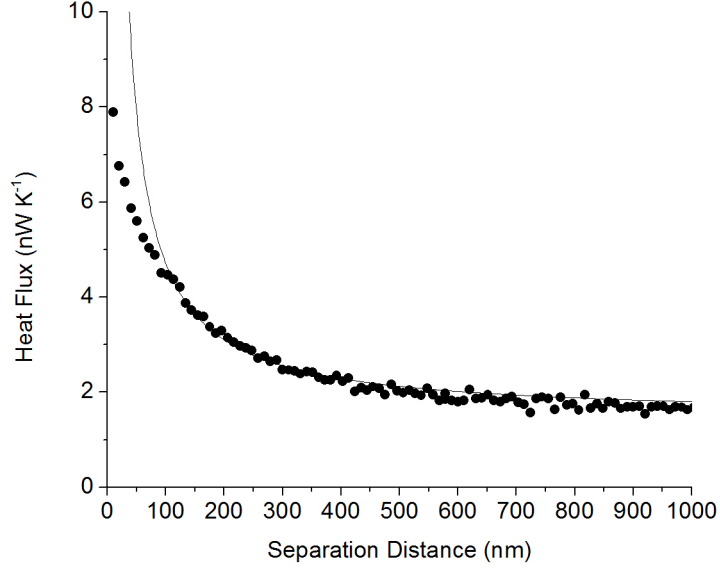


Figure 3.12: Data taken using the oscillation method compared to theory (solid line)

and an α of $17.3 \times 10^{-6} K^{-1}$. This combines to yield a thermal expansion of 9.6 nm/K at room temperature. This contributes a small, but manageable error, to our measurements.

3.3 Results

3.3.1 Oscillation Results

In order to validate the accuracy of this new method it was first used at room temperature using a 200 μm sapphire sphere and a sapphire substrate. The cantilever was held at 300K while the substrate temperature was oscillated between 312K and 314K at a rate of 40 mHz. The approach was preformed in 10 nm steps with each step held for 2 minutes, allowing for about 8 oscillations at each step. The resulting data is compared to theory in figure 3.12.

The results were very encouraging. This is the first time a cantilever calorimetry

system was used to measure the effect of near field heat transfer in sapphire. Furthermore, unlike the measurements taken with sapphire in [8] which stopped at around $2\text{ }\mu\text{m}$ we were able to probe much smaller separation distances where the effect becomes even more pronounced, even to the point of potentially exceeding the Stefan-Boltzmann limit.

We were able to gather data at much smaller separations than our previous experiments due in large parts to improvements in surface roughness. The data matches theory well to within 100 nm. The deviations at smaller displacements could have a few sources. Issues with surface roughness seem unlikely as we would expect them to shift the entire data set and moreover SEM imaging does not support this theory. It is possible that at these distances the sphere begins to be drawn closer by surface forces causing our calculations to underestimate the distance. The slight changes in the separation distance due to thermal expansion may also become relevant in this regime. Another possibility is that at these distances the Derjaguin approximation becomes invalid. This was the conclusion of earlier works [20, 21].

3.3.2 Low Temperature Setup

For the low temperature measurements the system was transferred to a wet cryogenic system. This was simply a vacuum can that could be lowered into a dewar full of liquid nitrogen. A few changes needed to be made to the room temperature setup to insure functionality in this environment. Most changes were simple. For instance the thermistors began to fail around 180K and needed to be replaced with platinum thermometers. However a larger problem arose with the cantilever. Its large deformation in response to temperature made it a good calorimeter but it also caused it to bend at a large angle over the temperature ranges used in the low temperature experiment. Over a great enough temperature range, the bending angle became too great and the interferometer failed as

most of the signal was reflected in such a way that it did not return to the fiber. To compensate for this the cantilever was positioned at a 10° angle relative to the parallel. This was done such that the deformation from cooling returned the cantilever to parallel. This caused some difficulties as the angle was too great to allow for alignment at room temperature. However, if carefully positioned at the outset, the interferometer would become functional around 230K and remain functional down to at least 100K.

3.3.3 Far Field Results

To prove the basic capabilities of the system we will first consider just far field radiation. At distances greater than a few microns the near-field effect is negligible and the radiated heat between the sphere and substrate is governed by the equation $8\pi R^2 \sigma \epsilon T^3 \Delta T$. For this experiment the cantilever was moved at least $10 \mu\text{m}$ from the substrate. The substrate was then oscillated between 43.5K and 56.5K above the cantilever temperature. Such large temperature differences were necessary to keep within the sensitivity of the apparatus at the lower temperatures. These oscillations were performed for four different temperatures as shown in table 3.1. As expected the ratio of the corresponding amplitudes matched closely with the ratio of the average substrate temperature cubed. These are shown in table 3.2. Figure 3.13 compares the amplitude for the highest base temperature and lowest base temperature.

In general our apparatus obeys the Stefan-Boltzmann law in the far field. The amplitudes of oscillation decrease as a function of temperature even though the size of the temperature oscillations was held constant at 13K.

Our comparisons are all in very good agreement. The largest differences occur when comparing ratios for the highest temperature T_4 . This is not surprising since the cantilever had to be rearranged between the low temperature and room temperature tests.

	Avg. Substrate Temperature	Amplitude (nm)
T_1	225K	12.7
T_2	250K	17.6
T_3	275K	24
T_4	355K	61.2

Table 3.1: The experimental oscillation amplitudes for four different cantilever temperatures. The temperature of the substrate was oscillated about a temperature 50K above the cantilever temperature.

	T_{Low}^3/T_{High}^3	A_{Low}/A_{High}
T_1/T_2	0.73	0.72
T_1/T_3	0.55	0.53
T_2/T_3	0.75	0.73
T_1/T_4	0.25	0.21
T_2/T_4	0.35	0.29
T_3/T_4	0.46	0.39

Table 3.2: Ratio of cantilever oscillation amplitudes at the four temperatures compared to the ratio of the temperature cubed. As expected from the equation for far-field radiation the two ratios scale the same.

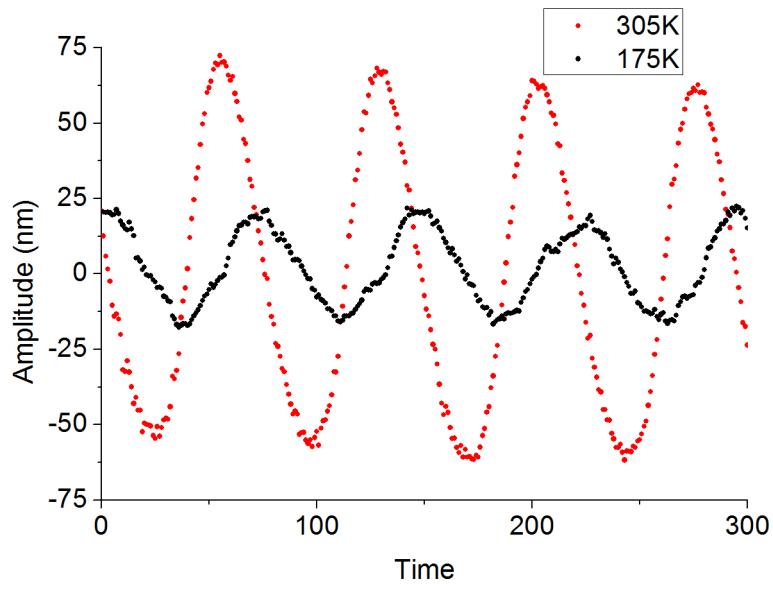


Figure 3.13: A comparison of the room temperature response (red) and the 175K response (black). For the same size temperature fluctuations but different cantilever temperature the response scales like T^3

This rearrangement will cause a slight difference in the cantilever's response coefficient H [nm/nW] which in turn will effect its oscillation amplitude.

3.3.4 Low Temperature Results

Having confirmed the efficacy of our apparatus and methods we probed the behavior of this phenomenon at increasingly low temperatures. In general we looked to confirm two major trends. Firstly we would expect both near field and far field transfer to decrease with decreasing temperature. Secondly we would expect the range of far-field transfer to be reduced as well. This may sound like a mistake as for the room temperature effect the relevant range is quoted in terms of the thermal wavelength, which is $\propto T^{-1}$. However while this comparison is convenient at room temperature it fails at lower temperatures in part because NFRT is dependent on specific wavelengths unrelated to the black body spectrum. A plot of the theoretical behavior changes that should accompany a drop in temperature are shown in figure 3.14.

In order to explore these trends the cantilever was held at temperatures of 150K, 180K, and 200K while the substrate was oscillated between a temperature 30K and 36K above the cantilever temperature. The data is shown separately in figure 3.15. It adheres well to the theory but is noisier than the room temperature data. In general it confirms the expected trends.

3.3.5 Preliminary YBCO Results

Having established a system that can measure small heat fluxes at low temperatures opens the door so some novel applications. One particular area of interest involves the change in emissivity of a high T_c super conductor at the transition temperature. As our device is a very accurate flux meter and the flux between a sphere and a plate in the

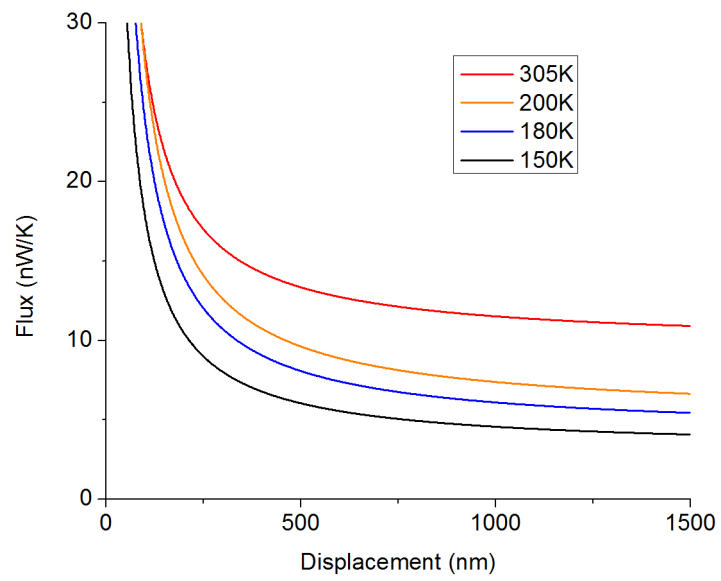


Figure 3.14: As the temperature drops the far-field flux will decrease due to its dependence on T^3 . The strength of the near field transfer will also decrease as will the range over which it is relevant.

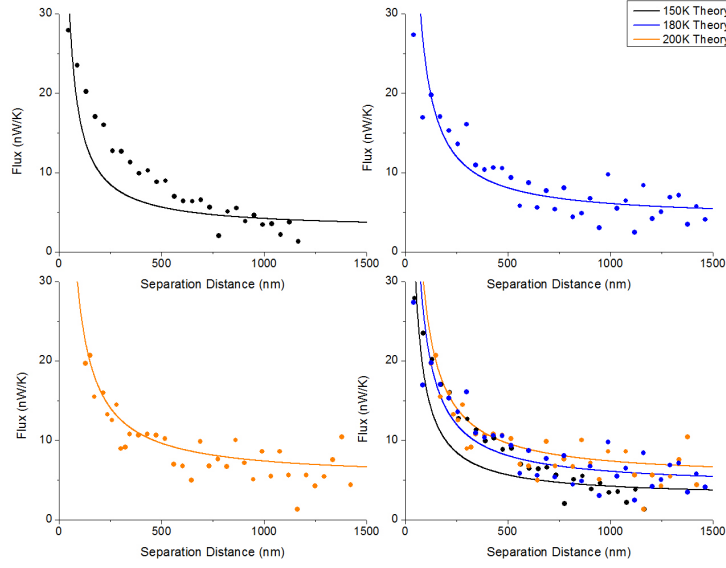


Figure 3.15: Shown together the noise becomes a bit of an issue but trends are still visible. There is a decrease in both the strength of the effect and the relevant range.

far field goes as $\sim \epsilon T^3 \Delta T$ we can monitor changes in the emissivity ϵ . Our first tests focused on the superconductor yttrium barium copper oxide (YBCO) which has a critical temperature of about 93K. This temperature is convenient as it is easily achievable with our current wet cryogenic system. It is also potentially troublesome as is the start of the region where many material properties begin to change. Material properties such as the coefficients of thermal expansion which are critical to the function of our bimaterial beam calorimeter.

The preliminary tests were encouraging. The device appeared to be operating properly. Taking data at temperatures of 80K we were able to measure bending amplitudes of only a few nanometers corresponding to temperature changes of just a few K. This is shown in figure 3.16

However, this response, while only a few nanometers, appears to be much too large.

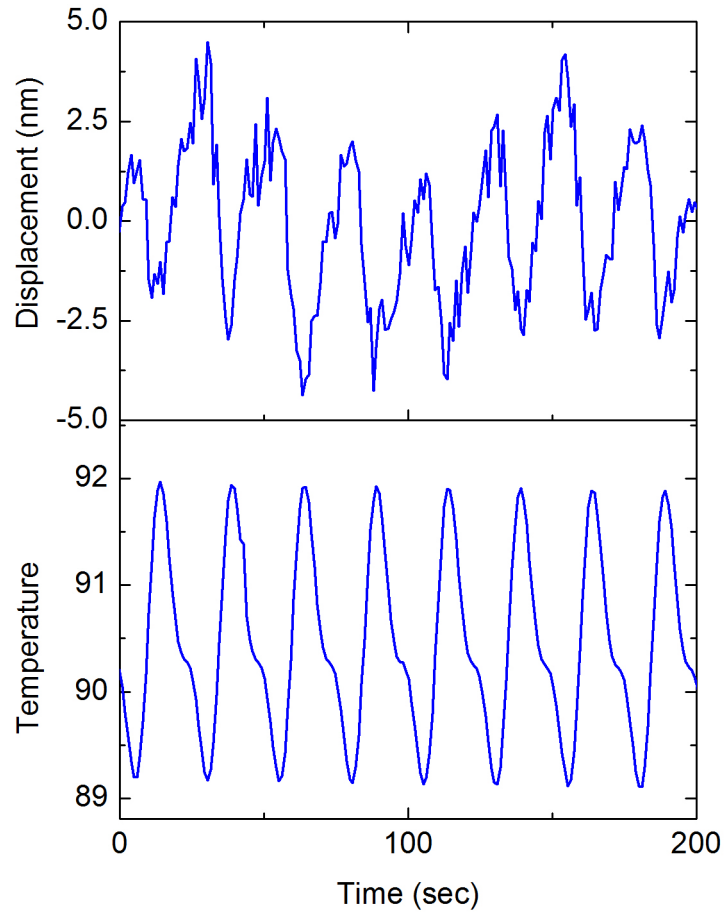


Figure 3.16: The preliminary results were encouraging. Oscillating the superconductor by a temperature of just a few K induced a bending oscillation of a few nanometers in the cantilever. All of which the system was able to measure.

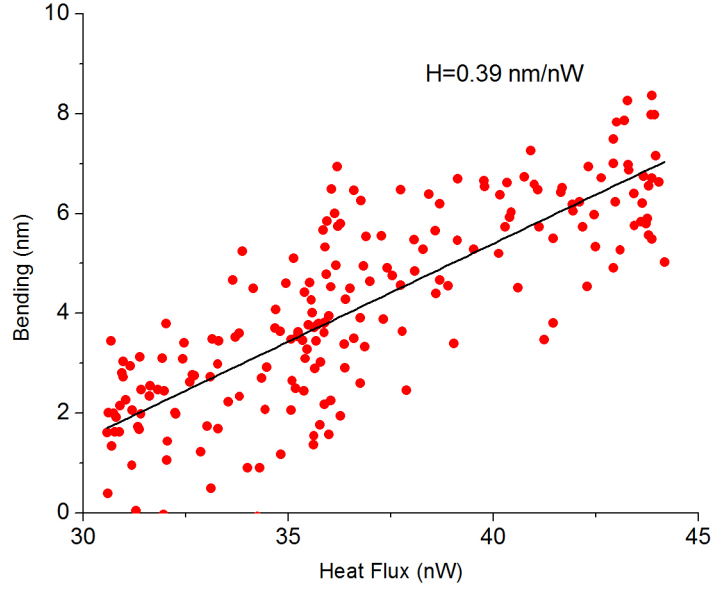


Figure 3.17: As with some of our earlier measurements the sensitivity appears to be too high. Though in this case it is by a factor of 10 rather than 100.

Our theoretical calculations give a guideline of about 0.03 nm/nW while the calibration tests as shown in figure 3.17 give a value of 0.33 nm/nW. The source of this discrepancy is unclear as we have cleared up many of the previous issues. One possibility is that the change in material properties at these temperatures renders the bimaterial bending of the cantilever more sensitive. Readings on this topic do not seem to bear this out though. The other possibility we are pursuing is that there is a small cold leak. So small as to keep the pressure at $< 1 \times 10^{-4}$ torr with the pumps running when submerged in LN but large enough to allow some heat transfer via conduction or convection. This looks to be the most likely source of the problem. A thorough cleaning of the indium seal area seems to have solved the issue.

It remains now to try running the test again using a cantilever without a sphere. As with our earlier tests we would expect this cantilever not to react at all to changes in radiated heat. Following a conclusive test with our dummy cantilever we can resume

using our cantilevers with spheres and hopefully complete the emissivity measurement.

Chapter 4

Conclusion

This project had two distinct goals, the first was to create a reliable apparatus for microcalorimetry and the second was to use this apparatus to probe anomalous heat transfer at the nano scale in a low temperature environment. It was successful in both these goals. Specifically a microcalorimetry apparatus was constructed with a heat sensitivity of 0.029 nm/nW monitored by an interferometer capable of measuring a displacement of ~ 1 nm. During implementation of this apparatus we identified and overcame numerous obstacles. These included various noise sources which both obscured and mimicked the real data. Through a number of improvements to the apparatus and changes to our experimental technique we were able run several successful tests.

We began by confirming the heat sensitivity of the cantilever as predicted by [5]. Next we probed heat flux between a sapphire sphere and a sapphire plate over distances of tens of nanometers, much smaller than distances previously achieved with this material. The room temperature data matched very nicely with theory over most distances but deviated for small separations, an effect seen in previous studies. These measurements represent a number of firsts including the first measurement of this effect in sapphire using an AFM system and the first measurements for sapphire at distances less than $1\text{ }\mu\text{m}$. Finally we were able to make the first conclusive measurements of NFRT at low temperatures. In

doing so we witnessed two broad trends, a general decrease in the strength of the effect as a function of temperature as well as a decrease in the range of the effect.

4.1 Future Work

Having confirmed the efficacy of our apparatus at both room temperature and lower temperatures the door is now open to many intriguing studies. We can continue to pursue near field heat transfer as a growing body of work both theoretical and experimental continues to probe this effect in new materials and structures. Our apparatus is useful in that the materials used can be easily changed and the sphere plane geometry avoids the experimental difficulties of maintaining two parallel planes. We can also examine the effect at different temperature extremes. Much work remains to be done in this field if we are to realize any of the applications described in [refApplications](#). Alternatively, our device also functions as a precise measure of emissivity. This could be put to interesting use in probing the transition of a high T_c superconductor such as YBCO.

Chapter 5

Appendix

5.1 Computer Programs

This section includes programs created for the purpose of carrying out this experiment. They may be of use in similar future experiments.

5.1.1 Plasmon Resonance Program

This is actually two separate but interdependent programs. The first program Find ϵ Data converts optical data for n and k of a material into a frequency dependent dielectric function for the included wavelengths. The second program, Resonance, plots the heat transfer coefficient h versus the wavelength in μm . These two programs are presented with the examples of glass and sapphire

Plasmon Resonances Program

```

FindeDat[FlPthDat_, unit1_] :=
(*Input should be File path for the optical
data followed by the unit of the frequency either Hz,
eV, or wavelength in meters. The data should be arranged as
(frequency,n,k). This program outputs a table ordered as
( $\omega$ ,  $\epsilon_{\text{real}}$ ,  $\epsilon_{\text{imaginary}}$ ). This will be necessary to find the
resonances in  $\hbar\omega^R$  using the program plasmon resonance plots*)Module[
{datSph = DeleteCases[Import[FlPthDat, "Table"], {_String, _String, _String}],
(*This assumes the data is in 3 columns and drops any column labels*)
er, eim,  $\epsilon_{\text{Re1}}$ ,  $\epsilon_{\text{Im1}}$ ,  $c = 3 \times 10^8$  (*m s-1*),  $\hbar = 6.582 \times 10^{-16}$  (*eV s*),
kb =  $8.617 \times 10^{-5}$  (*eV K-1*), kb1 =  $1.38 \times 10^{-23}$  (*J K-1*),
 $\sigma = 5.67 \times 10^{-8}$ ,  $\epsilon$ , conv1,  $\omega1$ , x, Mtr1Re, Mtr1Im,  $\omega_{\text{max1}}$ ,  $\omega_{\text{min1}}$ },

er[n_, k_] :=  $n^2 - k^2$ ; (* $\epsilon$  real*)
eim[n_, k_] :=  $2 n * k$ ; (* $\epsilon$  imaginary *)
x = unit1;

conv1[w_] := Which[x == "eV", w * 1, x == "m",  $\frac{\hbar 2 \pi c}{w}$ , x == "s-1",  $\hbar * w$ ];

(*This line along with the line below convert the frequencies to m*)
 $\omega1$  = Table[conv1[datSph[[n, 1]]], {n, 1, datSph // Length}];
 $\epsilon$  = Table[{conv1[datSph[[n, 1]]], er[datSph[[n, 2]]], datSph[[n, 3]]],
eim[datSph[[n, 2]], datSph[[n, 3]]]}, {n, 1, datSph // Length}];
 $\epsilon$ ]

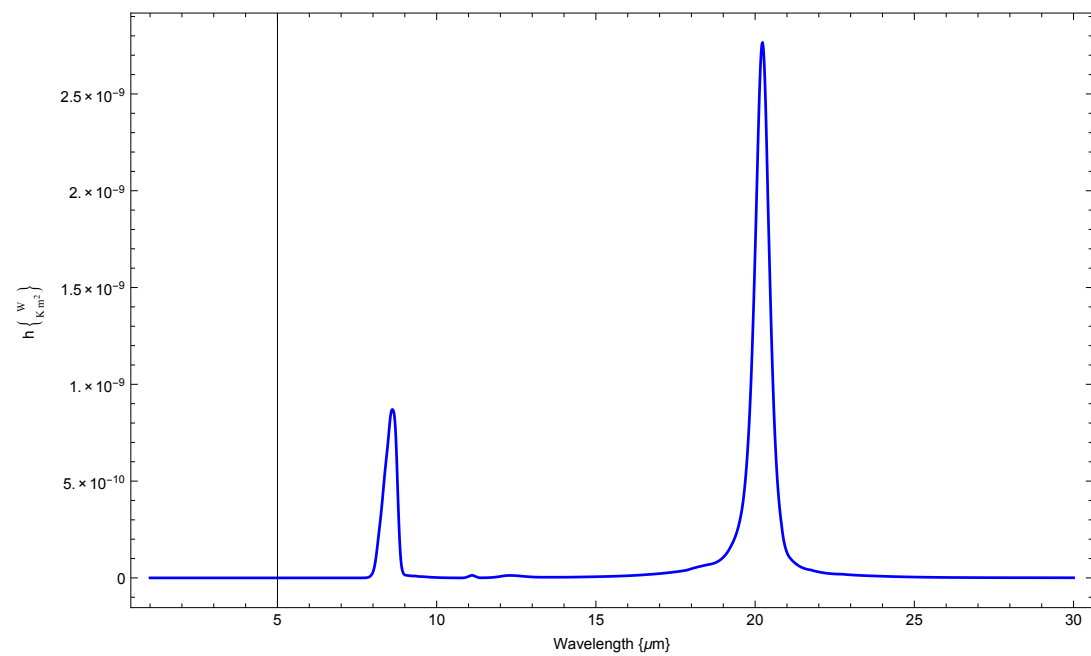
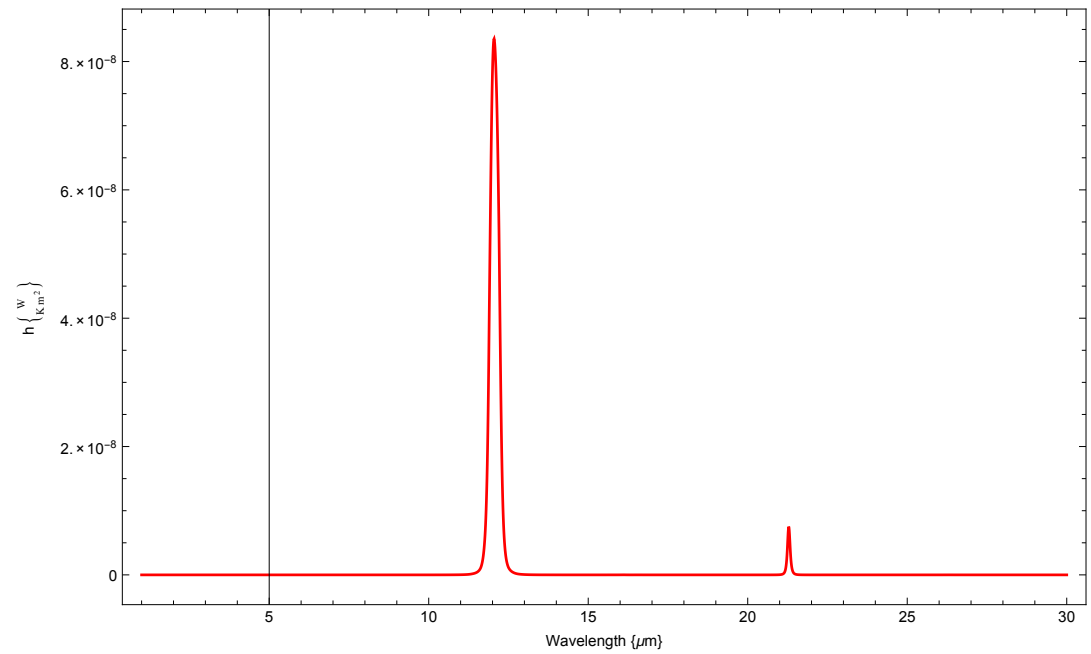
(*As an example the data for sapphire and glass *)

sapphire = FindeDat[
"C:\\Users\\Administrator\\Dropbox\\Bobby's Docs\\AFM Pendulum\\Optical
Data\\sapphire data m n k.txt", "m"];

glass = FindeDat[
"C:\\Users\\Administrator\\Dropbox\\Bobby's Docs\\AFM Pendulum\\Optical
Data\\silica data 3 rows Complete2.txt", "m"];

```

```
sa = Resonance[sapphire, Red]
gl = Resonance[glass, Blue]
```



```

(*This next step requires: input the data from the above program,
the separation distance d, and the color of the plot*)
Resonance[dat_, d_, color_] :=
Module[{Re, Im, Recut, Imcut, fRe, fIm, hExp,  $\omega$ , kb =  $8.617 \times 10^{-5}$  (*eV K-1*) ,
  kb1 =  $1.38 \times 10^{-23}$  (*J K-1*) , T = 300 (*T can be changed accordingly *) ,
  c =  $3 \times 10^8$  (*m s-1*) ,  $\hbar$  =  $6.582 \times 10^{-16}$  (*eV s*) } ,
  Re = Table[{dat[[n, 1]], dat[[n, 2]]}, {n, 1, dat // Length}];
  Im = Table[{dat[[n, 1]], dat[[n, 3]]}, {n, 1, dat // Length}];

  (*gets rid of duplicates so the interpolation will work *)
  Recut = DeleteDuplicatesBy[Re, First];
  Imcut = DeleteDuplicatesBy[Im, First];

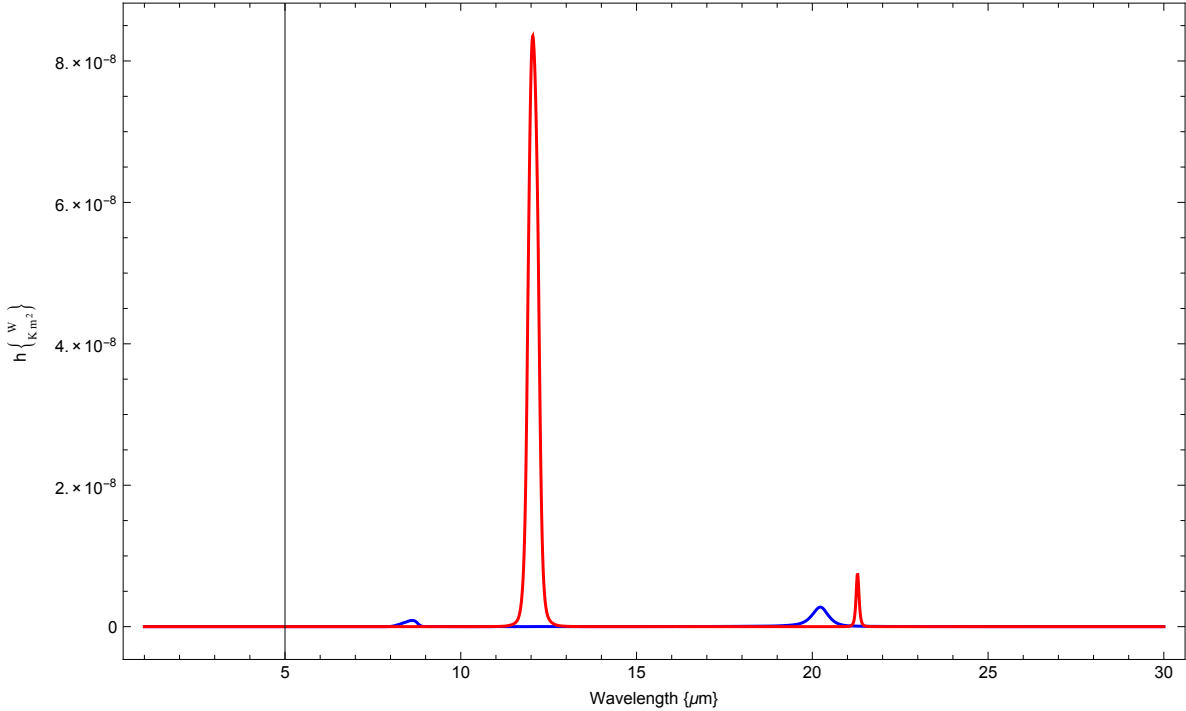
  (*interpolates the data to create functions of  $\epsilon$  in terms of  $\omega$ *)
  fRe = Interpolation[Recut];
  fIm = Interpolation[Imcut];

  (*gives the near field radiated heat transfer coefficient *)
  hExp[ $\omega$ _] :=  $\frac{1}{d^2} \frac{1}{\pi^2} \frac{fIm[\omega] fIm[\omega]}{(Abs[1 + fRe[\omega] + fIm[\omega] i]^2)^2} kb1 \left( \frac{\omega}{kb T} \right)^2 \frac{Exp\left[\frac{\omega}{kb T}\right]}{(Exp\left[\frac{\omega}{kb T}\right] - 1)^2}$ ;

  (*plots the coefficient h vs wavelength in microns*)
  Plot[ $\frac{1}{2} hExp\left[\frac{\hbar 2 \pi c}{x 10^{-6}}\right]$ , {x, 1, 30}, PlotRange → All, PlotStyle → color,
    Frame → True, FrameLabel → {"Wavelength { $\mu$ m}", "h" Text[ $\left\{\frac{W}{m^2 "K"}\right\}$ ]}]]

```

Show[g1, sa]



5.1.2 Theoretical Conductance Curve Generator

This program creates the data needed to graph a theoretical curve for the conductance between a semi-infinite material and a sphere. As input it takes the frequency dependent dielectric of the materials in the form of a three column data set with a column for frequency, one for n , and one for κ . n and κ are the real and imaginary parts of the complex index of refraction respectively. As for frequency it can be in terms of either Hz, eV, or wavelength. The program will convert them to eV for convenience. The units must be indicated in the input for this to work right. The temperature of the substrate along with the emissivity of the substrate must also be input. Also included is a sample input and output for the program.


```

ThrGdat[F1PthSph_, F1PthBlk_, unit1_, unit2_, T_, R_, em_, Xlim_, b_, Hco_, dT_] :=
(*Input should be File path for sphere data, file path for bulk data,
 $\omega$  units for sphere, units for bulk, T of substrate in K, R of colloid in meters,
the emissitivity of the substrate, the upper x limit for the graph,
and the upper y limit for the graph. b is the offset of the theoretical curve
to account for the experimetnal error in establistig the distance*)Module[
{datSph = DeleteCases[Import[F1PthSph, "Table"], {_String, _String, _String}],
datBlk = DeleteCases[Import[F1PthBlk, "Table"], {_String, _String, _String}],
(*This assumes the data is in 3 columns and drops any column labels*)
er, eim, eRe1, eIm1, eRe2, eIm2, wmax1, wmax2, wmin1, wmin2, c =  $3 \times 10^8$ 
(*m s-1*),  $\hbar = 6.582 \times 10^{-16}$  (*eV s*), kb =  $8.617 \times 10^{-5}$  (*eV K-1*),
kb1 =  $1.38 \times 10^{-23}$  (*J K-1*),  $\sigma = 5.67 \times 10^{-8}$  (*W m-2 K-4*),  $\epsilon$ , conv1,
conv2, x, y, Mtr1Re, Mtr2Re, Mtr1Im, Mtr2Im, hExp,  $\omega_1$ ,  $\omega_2$ , H, G, Gff},
 $\epsilon = em$ ;
Gff[t_] :=  $2 \pi R^2 4 \sigma \epsilon t^3$ ; (*Far Field heat contribution*)
er[n_, k_] :=  $n^2 - k^2$ ; (* $\epsilon$  real*)
eim[n_, k_] :=  $2 n k$ ; (* $\epsilon$  imaginary *)
x = unit1;
y = unit2;

conv1[w_] := Which[x == "eV",  $1 * w$ , x == "m",  $\frac{\hbar 2 \pi c}{w}$ , x == "s-1",  $\hbar * w$ ];
(*This line along with the line below convert the frequencies to eV*)
conv2[w_] := Which[y == "eV",  $1 * w$ , y == "m",  $\frac{\hbar 2 \pi c}{w}$ , y == "s-1",  $\hbar * w$ ];

 $\omega_1$  = Table[conv1[datSph[[n, 1]]], {n, 1, datSph // Length}];
 $\omega_2$  = Table[conv2[datBlk[[n, 1]]], {n, 1, datBlk // Length}];
eRe1 = Table[{conv1[datSph[[n, 1]]], er[datSph[[n, 2]], datSph[[n, 3]]]},
{n, 1, datSph // Length}];
eRe2 = Table[{conv2[datBlk[[n, 1]]], er[datBlk[[n, 2]], datBlk[[n, 3]]]},
{n, 1, datBlk // Length}];
eIm1 = Table[{conv1[datSph[[n, 1]]], eim[datSph[[n, 2]], datSph[[n, 3]]]},
{n, 1, datSph // Length}];
eIm2 = Table[{conv2[datBlk[[n, 1]]], eim[datBlk[[n, 2]], datBlk[[n, 3]]]},
{n, 1, datBlk // Length}];

Mtr1Re = Interpolation[DeleteDuplicates[eRe1, #1[[1]] == #2[[1]] &]];
Mtr2Re = Interpolation[DeleteDuplicates[eRe2, #1[[1]] == #2[[1]] &]];
Mtr1Im = Interpolation[DeleteDuplicates[eIm1, #1[[1]] == #2[[1]] &]];
Mtr2Im = Interpolation[DeleteDuplicates[eIm2, #1[[1]] == #2[[1]] &]];

wmax1 = Max[ $\omega_1$ ];
wmax2 = Max[ $\omega_2$ ];
wmin1 = Min[ $\omega_1$ ];
wmin2 = Min[ $\omega_2$ ];

```

```

hExp[ω_] :=  $\frac{1}{\pi^2} \left( \left( \text{Mtr1Im}[\omega] \text{Mtr2Im}[\omega] \right) / \left( \left( \text{Abs}[1 + \text{Mtr1Re}[\omega] + \text{Mtr1Im}[\omega] \text{i}] \right)^2 \right) \right.$ 
 $\left. \text{Abs}[1 + \text{Mtr2Re}[\omega] + \text{Mtr2Im}[\omega] \text{i}]^2 \right) \text{kb1} \left( \frac{\omega}{\text{kb T}} \right)^2 \frac{\text{Exp}\left[\frac{\omega}{\text{kb T}}\right]}{\left(\text{Exp}\left[\frac{\omega}{\text{kb T}}\right] - 1\right)^2};$ 

(*the equation for the heat transfer coefficient sans the  $\frac{1}{d^2}$  component*)
H =  $\frac{1}{\hbar}$  NIntegrate[hExp[i], {i, Max[wmin1, wmin2], Min[wmax1, wmax2]}};

G[x_] := 2 H π  $\left( \frac{R}{x} + \text{Log}[x] - \text{Log}[R + x] \right);$ 

(*This is a result of the integral  $\int_0^R h[\tilde{d}, T] 2\pi r \, dr$  with  $\tilde{d} = d + R - \sqrt{R^2 - r^2}$  *)
Table[{x,  $\left( G\left[\frac{x+b}{10^9}\right] 10^9 + \text{Gff}[T] 10^9 \right)$ }, {x, 0, Xlim, 1}]

(*Plots the theoretical G vs d curve which includes Gff*)
]

```

5.2 Math

5.2.1 Classical Heat Transfer

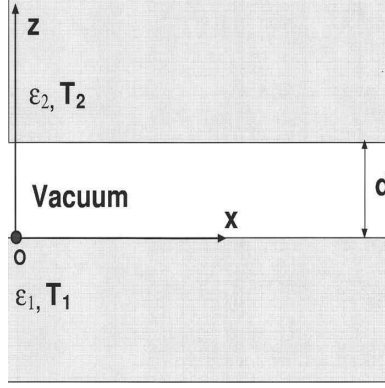


Figure 5.1

It is first useful to consider the classical treatment of two semi-infinite dielectric planes separated by a vacuum as shown in figure 5.1. At pressures < 0.8 mTorr radiative heat transfer dominates over the other two forms of heat transfer. In order to calculate the heat transfer between the two materials we must take the difference of the heat flux between medium 1 and medium 2 ($\phi_{1 \rightarrow 2}$) and the heat flux between medium 2 and medium 1 ($\phi_{2 \rightarrow 1}$). Since the intervening media is transparent, a ray light approach can be used to calculate the heat flux between the media. The elementary energy flux $d\Omega_{1 \rightarrow 2}$ from 1 to 2 taken through a solid angle $d\phi = 2\pi \sin \theta d\theta$ about the θ direction at frequency ω is the sum of the reflected rays to wit

$$d\phi_{1 \rightarrow 2}(\theta) = [\epsilon'_{1\omega} \epsilon'_{2\omega} + \epsilon'_{1\omega} \rho'_{1\omega} \rho'_{2\omega} \epsilon'_{2\omega} + \epsilon'_{1\omega} \rho'_{1\omega} \rho'_{2\omega} \rho'_{1\omega} \rho'_{2\omega} \epsilon'_{2\omega} + \dots] I_{\omega}^0(T_1) \cos \theta d\Omega d\omega \quad (5.1)$$

Where $\epsilon'_{1\omega}$ and $\epsilon'_{2\omega}$ are the directional emissivities at the frequency ω of media 1 and media 2, $\rho'_{1\omega}$ and $\rho'_{2\omega}$ are the specular reflectivities, and $I_{\omega}^0(T_1)$ is the intensity of equilibrium

radiation at temperature T_1 :

$$I_{\omega}^0(T_1) = \frac{\hbar\omega^3}{4\pi^3c^2} \frac{1}{\exp(\hbar\omega/k_B T_1) - 1} \quad (5.2)$$

Equation 5.1 can be written as a sum of a geometric series in order to find the total flux from 1 to 2

$$\phi_{1 \rightarrow 2} = \int_0^{2\pi} \cos \theta d\Omega \int_0^{\infty} d\omega \frac{\epsilon'_{1\omega} \epsilon'_{2\omega}}{1 - \rho'_{1\omega} \rho'_{2\omega}} I_{\omega}^0(T_1) \quad (5.3)$$

The equation for $\phi_{2 \rightarrow 1}$ can be written similarly thus the classical heat exchange is given by

$$q(\omega) = \phi_{1 \rightarrow 2} - \phi_{2 \rightarrow 1} = \int_0^{2\pi} \cos \theta d\Omega \int_0^{\infty} d\omega \frac{\epsilon'_{1\omega} \epsilon'_{2\omega}}{1 - \rho'_{1\omega} \rho'_{2\omega}} [I_{\omega}^0(T_1) - I_{\omega}^0(T_2)] \quad (5.4)$$

Performing this integral will yield

$$q(\omega) = \frac{\epsilon'_{1\omega} \epsilon'_{2\omega}}{1 - \rho'_{1\omega} \rho'_{2\omega}} \sigma [T_1^4 - T_2^4] \quad (5.5)$$

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