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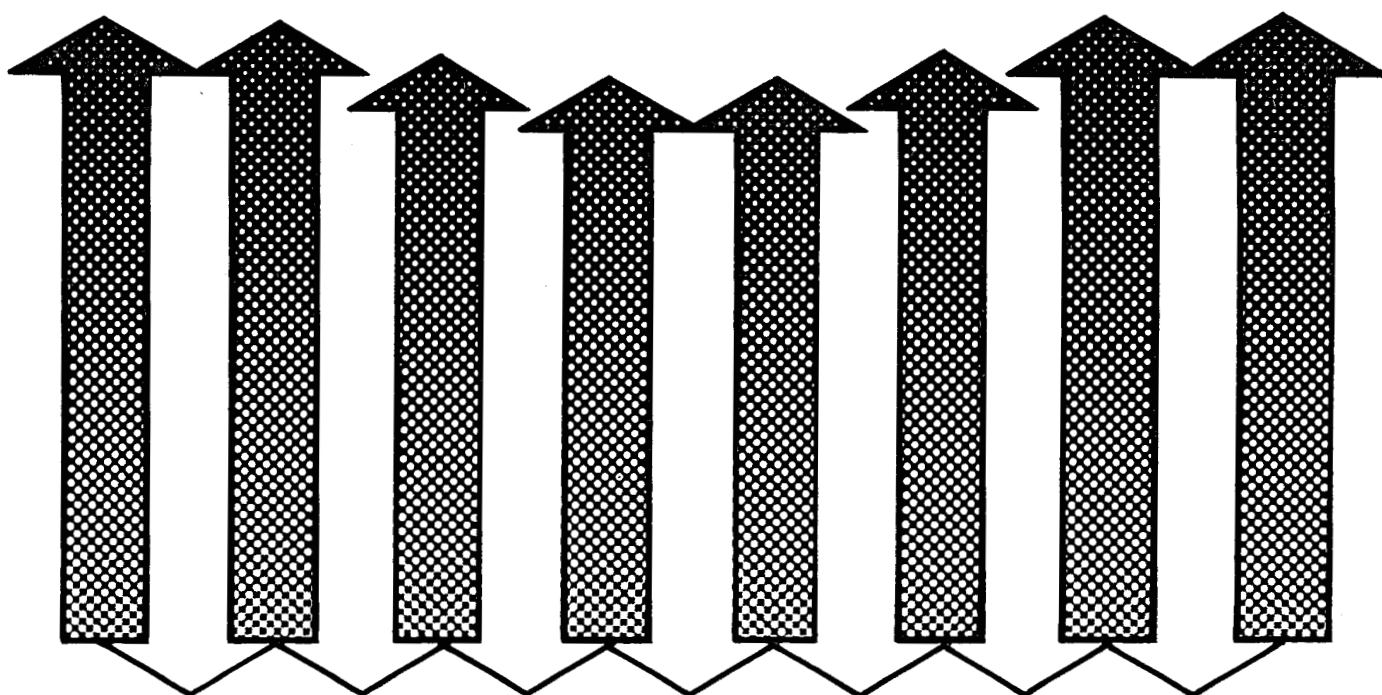
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Golder Associates, Inc.

March 1980

Geothermal Subsidence Research Management Program

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PHYSICAL PROCESSES OF COMPACTION

COMPANION REPORT 1

TO

SIMULATION OF GEOTHERMAL SUBSIDENCE

(LBL 10571)

Prepared For

LAWRENCE BERKELEY LABORATORY

by

GOLDER ASSOCIATES, INC.

UNDER SUBCONTRACT 3730302

Prepared by: I. Miller, W. Dershowitz, K. Jones,
L. Myer, K. Roman, and M. Schauer

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March 1980

This report is the first in a series of three companion reports presenting the results of an investigation into the use of mathematical models for predicting subsidence caused by geothermal fluid extraction. The results of the investigation are summarized in the report, "Simulation of Geothermal Subsidence" (LBL 10571). The titles of the other companion reports are listed below.

Report No.	Title
2	Detailed Report on Tested Models LBL-10837
3	Case Study Data Base LBL-10839

An additional report on the subject of reservoir models was generated as part of this project. The report was produced in 1979 by Dr. George F. Pinder under subcontract to Golder Associates and is titled "State-of-the-Art Review of Geothermal Reservoir Modelling" (LBL 9093).

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ABSTRACT

There are a variety of theories, techniques, and parameters
in the subsidence literature. Biot's theory, Terzaghi's theory,
and the theory of interacting continua (TINC) are used to explain
solid-fluid interaction; stress-strain theories range from linear
elastic to e-log p to plasticity and pore-collapse theories.
Parameters are numerous: void ratio, permeabilility, compaction
coefficient, pore compressibility, Young's modulus, bulk modulus,
shear modulus, Poisson's ratio, Lame coefficients, coefficient of
consolidation, and storage coefficient. This report reviews the
physical processes which govern compaction and deformation in
geothermal systems. The review is an attempt to provide a
reasonably coherent general structure for the theories and para-
meters which were referred to above. The material we present is
a compendium of existing published work.

Processes governing flow and the behavior of the pore fluid
are not discussed. The reader is referred to the George F.
Pinder report (LBL 9093) for a discussion of geothermal reservoir
flow models.

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1.0 INTRODUCTION

There are a variety of theories, techniques, and parameters in the subsidence literature. Biot's theory, Terzaghi's theory, and the theory of interacting continua (TINC) are used to explain solid-fluid interaction; stress-strain theories range from linear elastic to e-log p to plasticity and pore-collapse theories. Parameters are numerous: void ratio, permeabililility, compaction coefficient, pore compressibility, Young's modulus, bulk modulus, shear modulus, Poisson's ratio, Lame coefficients, coefficient of consolidation, and storage coefficient.

The dominant variables affecting the deformation of geological systems are widely accepted to be stress, temperature, pore fluid pressure, and time. In Section 2.0 we will present a discussion of theories describing the effect of these variables.

2.0   <u>BASIC EQUATIONS</u>

The basic equation which follows is based on that by Lewis and Schrefler (1978), with the inclusion of thermal terms. Lewis and Schrefler's work is essentially a generalization to nonlinear problems of the linear theory of Biot (e.g., Biot and Willis 1957) and also uses an approach developed by Bishop (1973). The theory of interacting continua (TINC) (e.g., Garg and Nur 1973), provides identical results to Biot when high-order terms are neglected.

The following sign conventions will be followed in this report.

<u>Stresses</u> will have <u>tensions</u> positive, shear stresses on positive faces acting in positive directions. (See Figure 1.)
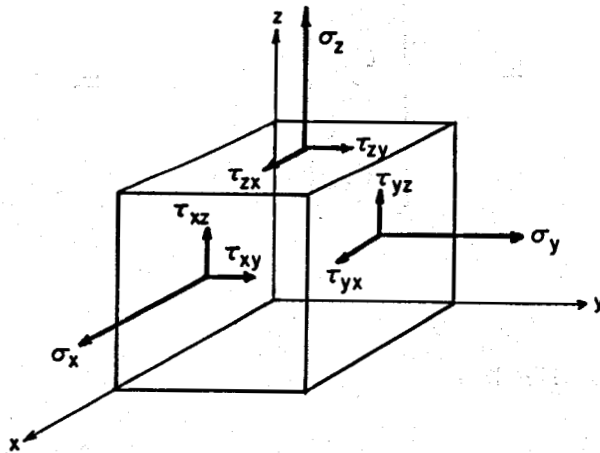
FIGURE 1
SIGN-AND-AXIS CONVENTIONS
FOR STRESS AND STRAIN

Pore Pressure has compression positive.

Strains. Engineering strains are used:

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad , \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad , \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad \gamma_{yz} = \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \quad etc.$$

where (u,v,w) are deformations in (x,y,z), respectively. Only small strains are covered by the theory. Strains are positive in tension.

Notation. Stress-and-strain tensors are in a vectorial representation and are identified by a single underscore. Matrices are represented by a double underscore.

$$\underline{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \qquad \underline{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

2.1 EFFECTIVE STRESS

The method by which Lewis and Schrefler (1978) extend the linear Biot theory to incorporate nonlinear stress-strain laws is similar to that of Nur and Byerlee (1976).

"Effective stress" is defined in the traditional form:

$$[1] \quad \underline{\sigma}' = \underline{\sigma} + P\underline{m} \qquad \underline{m} = \left\{ \begin{array}{c} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} \right\}$$

p = pore fluid pressure.

Terzaghi originally defined effective stress as that portion of the stress tensor which causes strain (i.e., $\delta\underline{\varepsilon} = \underline{\underline{D}}_T^{-1} \delta\underline{\sigma}'$). However, when the above definition of effective stress is used, there is in fact a small strain induced when the fluid pressure is changed while holding the effective stress constant. Imagine a small porous sample submerged in a container of fluid where the fluid pressure is increased by δP. Although the effective stress is constant (and equal to zero), the sample will contract slightly as the solid grains respond to the fluid pressure. In the Lewis and Schrefler (1978) presentation, the strain due to this contraction is in addition to that caused by any effective stress changes:

$$\delta\underline{\varepsilon}^e = \delta\underline{\varepsilon} - \delta\underline{\varepsilon}^{vp} = \underline{\underline{D}}_T^{-1} \delta\underline{\sigma}' - \frac{\delta P}{3K_s} \underline{m}$$

where: $\delta\underline{\varepsilon}^e$ is the elastic change in strain.

$\delta\underline{\varepsilon}^{vp}$ is the nonrecoverable strain due to viscous or plastic deformations. It is often ignored.

$\underline{\underline{D}}_T$ is the tangent stress-strain matrix for the elastic strains. $\underline{\underline{D}}_T$ is discussed in Section 2.4.

$\delta\underline{\sigma}'$ is the change in effective stress.

$$\delta\underline{\sigma}' = \delta\underline{\sigma} + \delta P\underline{m}$$

K_s is the bulk modulus of the matrix material and is assumed constant (linear).   ($K_s \approx 1 - 2 \times 10^9$ psf for geological materials.)

The first term on the right-hand side is the strain due to the effective stress. This strain is computed exactly as though the material were dry. The second term on the right-hand side is the additional strain due to compaction of the matrix material.

For the reader who is unfamiliar with the nature of the stress-strain matrix $\underline{D}_T$, a discussion is presented in Section 2.4. One commonly used variant of $\underline{D}_T$ is that for a linear elastic isotropic material. The following $\underline{D}_T$ matrix for this material is used several times in the following equations:

$$
\underline{\underline{D}}_T = \begin{bmatrix}
\lambda + 2G & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2G & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2G & 0 & 0 & 0 \\
0 & 0 & 0 & G & 0 & 0 \\
0 & 0 & 0 & 0 & G & 0 \\
0 & 0 & 0 & 0 & 0 & G
\end{bmatrix}
$$

2.2 THERMAL EFFECTS

For geothermal problems, it is necessary to add a third term in order to incorporate the effect of temperature variations, which can also cause strains in the absence of effective stress changes. Assuming that thermal expansion is isotropic, we get:

$$[2] \quad \delta\underline{\varepsilon}^e = \delta\underline{\varepsilon} - \delta\underline{\varepsilon}^{vp} = \underline{D}_T^{-1}\,\delta\underline{\sigma}' - \frac{\delta P}{3K_s}\,\underline{m} + \alpha\,\delta T\,\underline{m} = \underline{D}_T^{-1}\,\delta\underline{\sigma}' + \delta\underline{\varepsilon}_g$$

where: α is the coefficient of linear isotropic thermal
expansion ($\alpha \approx 10^{-5}$ °C^{-1} for geological materials).

δT is the change in temperature.

$\delta \underline{\varepsilon} g$ is the strain caused by pure volumetric expansion of
the matrix material due to thermal effects and
pore-pressure effects.

$$\delta \underline{\varepsilon}_g = \left(\frac{-\delta P}{3K_s} + \alpha \, \delta T \right) \underline{m}$$

2.3 GENERAL EQUATION

Multiplying the above expression by $\underline{\underline{D}}_T$ and rearranging
gives:

$$[3] \quad \delta \underline{\sigma}' = \underline{\underline{D}}_T \left(\delta \varepsilon + \frac{\delta P}{3K_s} \underline{m} - \delta \underline{\varepsilon}^{VP} - \alpha \, \delta T \underline{m} \right) = \underline{\underline{D}}_T \left(\delta \underline{\varepsilon} - \delta \underline{\varepsilon}^{VP} - \delta \underline{\varepsilon}_g \right)$$

The above expression is the key equation for the calculation
of deformation in fluid-solid interaction problems. For linear
poroelastic problems, $\underline{\underline{D}}_T$ can be replaced by $\underline{\underline{D}}$.

In the following sections we will derive a number of results
for specific problems, but all will be based on equation [3].
The viscoplastic term $\delta \varepsilon^{VP}$ will be ignored in most of the fol-
lowing derivations, which are therefore valid only for elastic
materials. Nonlinear elasticity of the form commonly observed in
hydrostatic compression tests is within the scope of the simpli-
fied equations.

2.4 PORE VOLUME CHANGES

In order to couple the deformation relationship in equation [3] with the flow equations for the pore fluid, it is necessary to keep track of the amount of fluid stored in the matrix. This is achieved by computing the pore volume. The pore volume is defined by either the porosity, n, or the void ratio, e:

[4] $\quad n = \dfrac{V_P}{V} = \dfrac{e}{1+e}$

[5] $\quad e = \dfrac{V_P}{V_s} = \dfrac{n}{1+n}$

where: Vp is the volume of voids or pores, Vp = V–Vs

V is the total volume

Vs is the volume of solid matrix material.

We will compute the change in the volume of voids as the change in total volume less the change in the volume of solids:

[6] $\quad \delta V_P = \delta V - \delta V_s$

For small deformations, the change in the total volume is:

[7] $\quad \delta V = \delta \underline{\varepsilon}^T \underline{m} = \delta \varepsilon_x + \delta \varepsilon_y + \delta \varepsilon_z$

The change in the volume of solids is computed as the sum of (a) the change due to changing both σ and P by δP, holding $\underline{\sigma}'$ constant, plus (b) the change due to changing $\underline{\sigma}'$ by $\delta \underline{\sigma}' = \delta \underline{\sigma} - \delta P \underline{m}$, holding P constant, and (c) the change due to the change in temperature. During (a) the volume of solid matrix material decreases by $(1-n) \dfrac{\delta P}{K_s}$. During (b) the mean normal (average)

direct stress in the matrix increases by $\frac{1}{3(1-n)}$ $\delta\underline{\sigma}^{T'}\underline{m}$, so the

volumetric strain in the matrix material is $\frac{1}{3K_s(1-n)}$ $\delta\sigma^{T'}\underline{m}$.

Since the matrix material has volume 1-n, the change in its volume is $\frac{1}{3K_s}$ $\delta\sigma^{T'}\underline{m}$.

The volume change due to temperature is $3(1-n)\,\alpha\delta T$. Thus, the total change in the volume of solid matrix material is:

$$[8] \quad \delta V_s = \frac{-(1-n)\delta P}{K_s} + \frac{1}{3K_s}\,\delta\underline{\sigma}'^T\underline{m} + 3(1-n)\propto\delta T$$

$$= \frac{1}{K_s}\left(\frac{1}{3}\,\delta\underline{\sigma}'^T\underline{m} - (1-n)\,\delta P\right) + 3(1-n)\propto\delta T$$

Substituting [7] and [8] into equation [6] gives the change in pore volume:

$$[9] \quad \delta V_P = \delta\underline{\varepsilon}^T\underline{m} - \frac{1}{K_s}\left(\frac{1}{3}\,\delta\underline{\sigma}'^T\underline{m} - (1-n)\,\delta P\right) - 3(1-n)\propto\delta T$$

Substituting for $\delta\underline{\varepsilon}^T\underline{m}$ from equation [3] and simplifying gives, for an isotropic material:

$$[10] \quad \delta V_P = \left(\frac{1}{K_T} - \frac{1}{K_s}\right)\delta\sigma'_m - \frac{n}{K_s}\,\delta P + 3n\propto\delta T$$

where: $\delta\sigma_m' = 1/3\,\delta\sigma^{T'}\underline{m}$ is the change in mean normal effective stress.

$\frac{1}{K_T} - \frac{1}{K_s}$ = bulk compressibility − matrix compressibility.

A term arising in the petroleum literature is the "pore volume compressibility," or "pore compressibility." It is the fractional change in pore volume due to stress or pore-pressure changes. From equation [10], the pore compressibility for an isotropic material is as follows:

$$C_P = \frac{1}{n}\left(\frac{1}{K_T} - \frac{1}{K_S}\right)$$

for changes in mean normal effective stress.

$$C_P = \frac{1}{K_S}$$

for changes in pore pressure with constant effective stress.

For changes in pore pressure with constant total stress $(\delta\sigma_m' = -\delta P)$.

$$C_P = \frac{1}{n}\left(\frac{1+n}{K_S} - \frac{1}{K_T}\right)$$

3.0 THE PORE PRESSURE INTERACTION COEFFICIENT

Equation [3] can be formulated in terms of total stress as:

$$[11] \quad \delta\underline{\sigma} + \delta P\underline{m} = \underline{\underline{D}}_T \left(\delta\underline{\varepsilon} + \frac{\delta P}{3K_S}\underline{m} - \alpha\,\delta T\underline{m} \right)$$

or:
$$\delta\underline{\sigma} + \delta P \left(1 - \frac{1}{3K_S}\underline{\underline{D}}_T\right)\underline{m} = \underline{\underline{D}}_T \left(\delta\underline{\varepsilon} - \alpha\,\delta T\underline{m} \right)$$

If the material is isotropic:

$$\underline{\underline{D}}_T\,\underline{m} = 3K_T\,\underline{m}$$

where K_T is the tangent bulk modulus of the material:

$$K_T = \lambda_T + \frac{2}{3}G_T$$

Substituting into the preceding equation gives:

$$[12] \quad \delta\underline{\sigma} + \delta P\left(1 - \frac{K_T}{K_S}\right)\underline{m} = \underline{\underline{D}}_T \left(\delta\underline{\varepsilon} - \alpha\,\delta T\,\underline{m} \right)$$

or:
$$\delta\underline{\sigma} + \delta P\left(1 - \frac{K_T}{K_S}\right)\underline{m} + \delta T\left(3\alpha K_T\right)\underline{m} = \underline{\underline{D}}_T\,\delta\underline{\varepsilon}$$

where the second form indicates how temperature changes can be treated like pore-pressure changes.

Equation [12] is a form which appears in most discussions of poroelasticity. The term $\left(1 - \frac{K_T}{K_S}\right)$ is the "pore-pressure inter-action coefficient." Terzaghi's theory and conventional soil mechanics practice assume that the skeletal stiffness is much less than the matrix material stiffness, i.e., $K_T \ll K_s$, and thus that the pore pressure interaction factor is unity. For rocks, however, K_T/K_s can become significant and should be considered.

The value of equations [11] and [12] is that they define a reduced stress (the left-hand sides of the equations) which acts as if the material had no pore fluid. $\underline{\underline{D}}_T$ is, however, usually a function of the effective stress.

4.0 HAND CALCULATIONS

4.1 VOLUMETRIC BEHAVIOR

Equation [3] is a general equation for the three-dimensional deformation of fluid-filled porous materials with temperature changes. As such, it can be readily used in finite-element computations. It is, however, too complex for hand calculation except in very simple problems. There are two widely used assumptions which can be used to make equation [3] much simpler: the assumption of purely volumetric behavior of an isotropic material ($\delta\sigma_x = \delta\sigma_y = \delta\sigma_z$, $\delta\varepsilon_x = \delta\varepsilon_y = \delta\varepsilon_z$, $\delta\tau_{xy} = \delta\tau_{yz} = \delta\tau_{zx} = \delta\gamma_{xy} = \delta\gamma_{yz} = \delta\gamma_{zx} = 0$) and the assumption of one-dimensional strains of an isotropic material ($\delta\sigma_x = \delta\sigma_y$, $\delta\varepsilon_x = \delta\varepsilon_y = \delta\gamma_{xy} = \delta\gamma_{yz} = \delta\gamma_{zx} = \delta\tau_{xy} = \delta\tau_{yz} = \delta\tau_{zx} = 0$) that is appropriate for vertical compaction of extensive horizontal layers.

4.1.1 Governing Equations

In the deformation of an isotropic material, the volumetric strain $\delta\varepsilon_v = \delta\varepsilon_x + \delta\varepsilon_y + \delta\varepsilon_z = \delta\underline{\varepsilon}^T\underline{m}$ is a function of only the mean normal effective stress $\delta\sigma_m' = \frac{1}{3}(\delta\sigma_x' + \delta\sigma_y' + \delta\sigma_z') = \frac{1}{3}\delta\underline{\sigma}'^T\underline{m}$ the fluid pressure and the temperature.

For an elastic material, equation [3] becomes:

$$[13] \quad \delta\sigma_m' = K_T\left(\delta\varepsilon_v + \frac{\delta P}{K_s} - 3\alpha\,\delta T\right)$$

where K_t is the tangent bulk modulus, $K_T = \lambda_T + \frac{2}{3}G_T$.

Alternatively, from equation [11]:

$$[14] \quad \delta\sigma_m + \delta P\left(1 - \frac{K_T}{K_s}\right) = K_T\left(\delta\varepsilon_v - 3\alpha\,\delta T\right)$$

Some useful results can be calculated for the case of undrained loading (Bishop 1973). In this case, the change in pore volume has to equal the compression of the pore fluid. For an isotropic material, this case gives the ratio of the generated pore pressure to the mean normal total stress as:

$$\frac{\delta P}{\delta\sigma_m} = -\frac{\frac{1}{K_T} - \frac{1}{K_s}}{n\left(\frac{1}{K_f} - \frac{1}{K_s}\right) + \frac{1}{K_T} - \frac{1}{K_s}}$$

The apparent bulk modulus is:

$$K_T{}^{UD} = \frac{\delta\sigma_m}{\delta\varepsilon_v} = \frac{K_T}{1 - \dfrac{\left(1 - \frac{K_T}{K_s}\right)\left(\frac{1}{K_T} - \frac{1}{K_s}\right)}{\left(\frac{1}{K_f} - \frac{1}{K_s}\right) + \frac{1}{K_T} - \frac{1}{K_s}}}$$

For the case where the matrix material is much stiffer than the bulk material of the fluid, these become:

$$\frac{\delta P}{\delta\sigma_m} = \frac{1}{1 + \dfrac{nK_T}{K_f}}$$

$$K_T{}^{UD} = \frac{\delta\sigma_m}{\delta\varepsilon_v} = K_T + \frac{K_f}{n}$$

4.2 ONE-DIMENSIONAL LINEAR PROBLEMS

4.2.1 Governing Equations

For compacting layers it is commonly assumed that the deformation is one-dimensional, i.e., that there are no lateral strains. If the changes in effective stress are moderate when compared to the initial effective stress, it is also reasonable to treat the material as linear. These two assumptions render equation [3] much simpler.

For one-dimensional strain:

$$\underline{\varepsilon}_1 = \begin{Bmatrix} 0 \\ 0 \\ \varepsilon_z \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

and thus the term $\underset{=T}{D}\underline{\varepsilon}$ in equation [3] becomes:

$$\underline{\underline{D}}\,\underline{\varepsilon}_1 = \varepsilon_z \begin{Bmatrix} \lambda \\ \lambda \\ \lambda + 2G \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

(Dropping the T subscript from $\underline{D}$, since for a linear material $\underline{\underline{D}}$ is constant.)

Similarly, the term $\underline{\underline{D}}_T\underline{m}$ becomes:

$$\underline{\underline{D}}\,\underline{m} = (3\lambda + 2G)\,\underline{m} = 3K\underline{m}$$

Substituting the above expressions into equation [3] gives:

$$[15] \quad \delta \underline{\sigma} + \delta P \left(1 - \frac{K}{K_s}\right) \underline{m} = \delta \varepsilon_z \begin{Bmatrix} \lambda \\ \lambda \\ \lambda + 2G \\ 0 \\ 0 \\ 0 \end{Bmatrix} - 3K\alpha \, \delta T \underline{m}$$

Expanding the individual rows in the above equations gives:

$$[15a] \quad \delta \sigma_x = \delta \sigma_y = -\delta P \left(1 - \frac{K}{K_s}\right) + \lambda \delta \varepsilon_z - 3K\alpha \delta T$$

$$[15b] \quad \delta \sigma_z = -\delta P \left(1 - \frac{K}{K_s}\right) + \left(\lambda + 2G\right) \delta \varepsilon_z - 3K\alpha \delta T$$

$$[15c] \quad \delta \tau_{xy} = \delta \tau_{yz} = \delta_{zx} = 0$$

Solving equation [15b] for $\delta \varepsilon_z$ gives:

$$[16] \quad \delta \varepsilon_z = \frac{\delta \sigma_z + \delta P \left(1 - \frac{K}{K_s}\right) + 3K\alpha \delta T}{\lambda + 2G}$$

Substituting into [15a] gives:

$$[17] \quad \delta \sigma_x = \delta \sigma_y = -\delta P \left(1 - \frac{K}{K_s}\right) + \lambda \left\{ \frac{\delta \sigma_z + \delta P \left(1 - \frac{K}{K_s}\right) + 3K\alpha \delta T}{\lambda + 2G} \right\} - 3K\alpha \delta T$$

4.2.2   The Compaction Coefficient

The compaction coefficient C_m is the compaction per unit fluid pressure drop; thus, if the above assumptions about linearity, isotropy, and constant total vertical stress are made, then:

$$[18] \quad C_m = \frac{\delta \varepsilon_z}{\delta P} = \frac{1 - \frac{K}{K_s}}{\lambda + 2G} = \underline{\text{pore-pressure interaction coefficient}}$$
$$\underline{\text{the constrained (1-D) modulus}}$$

4.2.3 Pore Volume Change

A number of useful relationships can be derived relating the volume or mass of fluid drained to the pore-pressure change. In order to develop them, we will first need to determine the change in pore volume due to loads or fluid pressure changes.

From equation [9] we find:

$$\delta V_P = \delta \varepsilon_z - \frac{1}{K_s} \left(\frac{1}{3} \delta \underline{\sigma}'^T \underline{m} - (1-n)\,\delta P \right) - 3\,(1-n)\,\alpha\,\delta T$$

Now, $\delta \underline{\sigma}'^T \underline{m} = \delta \sigma_x' + \delta \sigma_y' + \delta \sigma_z' = \delta \sigma_x + \delta \sigma_y + \delta_z + 3\,\delta P$

$$[19] \quad \delta V_P = \delta \varepsilon_z - \frac{\delta \sigma_x + \delta \sigma_y + \delta \sigma_z + 3n\,\delta P}{3K_s} - 3\,(1-n)\,\alpha\,\delta T$$

where $\delta \varepsilon_z$ is defined by equation [16], $\delta \sigma_x$ and $\delta \sigma_y$ are defined by equation [17], and $\delta \sigma_z$, δP, and δT are assumed known.

As discussed in Section 2.1.4, the "pore compressibility" is the fractional change in pore volume due to stress or pore volume changes. From equation [19], the pore compressibility for an elastic isotropic material in one-dimensional compression is as follows:

$$[20a] \quad C_P = \frac{1}{n} \left(\frac{1}{\lambda + 2G} - \frac{1}{3K_s} \left(1 + \frac{2\lambda}{\lambda + 2G} \right) \right)$$

for changes in the vertical stress, and

$$[20b] \quad C_P = \frac{1}{n}\left(1-\frac{K}{K_s}\right)\left(\frac{1}{\lambda+2G}\right)\left(1+\frac{4G}{3K_s}\right)-\frac{1}{K_s}$$

for changes in pore pressure with constant vertical stress.

4.2.4 Storage Coefficient

Specific storativity, s, is used in groundwater hydrology to denote the volume of water squeezed out of a unit volume of formation due to a unit drop in head.

$$s = \left(\frac{1}{V}\frac{\partial V_P}{\partial P} + \frac{n}{\rho_f}\frac{\partial \rho_f}{\partial P}\right)\rho_f g$$

where: V_P is the pore volume

   V is the volume of material

   n is the porosity

   ρ_f is the fluid density

   g is the gravitational constant.

$$[21] \quad s = \left(n C_P + \frac{n}{K_f}\right)\rho_f g$$

where: C_P is the pore compressibility from equation [20b]

   K_f is the bulk modulus of the fluid

For the case where K_S is much larger than K or G, the above equation simplifies to:

$$[22] \quad s = \left(\frac{1}{\lambda+2G} + \frac{n}{K_f}\right)\rho_f g$$

4.2.5 Ratio of Volume of Fluid Drained
to Volume of Compaction

For liquid-dominated geothermal reservoirs, it is possible
to directly estimate the amount of compaction without computing
the reservoir pressure drop. This is because water is only
slightly compressible. The following expressions neglect thermal
effects, although they can readily be incorporated into noniso-
thermal problems.

If the total vertical stress σz is constant, then the
relationship between the pressure drop and the volume of water
removed from the reservoir is:

$$\delta V_w = \frac{s}{\rho_f g} \; \delta P$$

where: δV_w is the ratio of the volume of water removed to the
          volume of formation.
       s is the storage coefficient.

Substituting the value of s from equation [21] and re-
arranging gives:

$$\delta P = \frac{\delta V_w \, \rho_f g}{s} = \frac{\delta V_w}{\left(n C_p + \frac{n}{K_f} \right)}$$

[22]

The compaction coefficient relates the pressure to the
vertical strain (equation [18]). Solving for the vertical
strain:

$$\delta \varepsilon_z = C_m \, \delta P = \left(\frac{1 - \frac{K}{K_s}}{\lambda + 2G} \right) \left(\frac{\delta V_w}{n C_p + \frac{n}{K_f}} \right)$$

The ratio of the volume of compaction to the volume of water removed is:

$$[23] \quad \frac{\delta \mathcal{E}_z}{\delta V_w} = \left(\frac{1 - \frac{K}{K_s}}{\lambda + 2G} \right) \left(\frac{1}{n C_P + \frac{n}{K_f}} \right)$$

For the case where Ks is much larger than K or ($\lambda + 2G$), this simplifies to:

$$[24] \quad \frac{\delta \mathcal{E}_z}{\delta V_w} = \frac{1}{1 + \frac{n}{K_f} (\lambda + 2G)}$$

4.2.6 Undrained Loading

For rapidly-applied external loads $\delta \sigma_z$, there is no time for any flow to occur. In this case, the decrease in pore volume equals the amount of fluid compression:

$$- C_P^{\sigma} \, \delta \sigma_z - C_P^{P} \, \delta P = \frac{\delta P}{K_f}$$

where: C_P^{σ} is the pore compressibility with respect to the vertical stress, as defined in equation [20a] and

C_P^{P} is the pore compressibility with respect to the with respect to the pore pressure, as defined in equation [20b].

$$[25] \quad \delta P = \left(\frac{- C_P^{\sigma}}{\frac{1}{K_f} + C_P^{P}} \right) \delta \sigma_z$$

Again, where K_S is much larger than K or $\lambda + 2G$, the equation becomes much simpler:

$$[26] \quad \delta P = \frac{-\delta\sigma_z}{\left(1 + \dfrac{n(\lambda+2G)}{K_f}\right)}$$

Changes in atmospheric pressure cause undrained loading in confined aquifers. Hydrologists use the "barometric efficiency" B to represent the ratio of the pore pressure change to the atmospheric pressure change, i.e., $B = -\dfrac{\delta P}{\delta\sigma_z}$. For the assumption of large K_S, this gives:

$$[27] \quad B = \frac{-\delta P}{\delta\sigma_z} = \left(\frac{1}{1 + \dfrac{n(\lambda+2G)}{K_f}}\right)$$

5.0   STRESS-STRAIN RELATIONSHIPS

In the preceding sections the general form of the formation stress-strain relationship was presented by the use of a tangent stress-strain matrix $\underline{D}_T$:

$$\delta\underline{\sigma}' = \underline{D}_T \left(\delta\underline{\varepsilon} + \frac{\delta P}{3K_s}\, \underline{m} - \delta\underline{\varepsilon}^{VP} - \alpha\, \delta T \underline{m} \right) = \underline{D}_T\, \delta\underline{\varepsilon}^e$$

where $\delta\underline{\varepsilon}^e$ is the "elastic" component of the strain.

This form is quite general, though restricted to cases where strains are small. The $\underline{D}_T$ matrix is normally considered to be a function of the effective stress only and thus in principle can be derived equally well from lab tests on dry or saturated specimens. In the following sections, we will discuss a number of common forms of the stress-strain relationship.

5.1   LINEAR ELASTIC ISOTROPIC MATERIAL

While it is true that no geological material is linear over a large range of stresses, the assumption of linearity is still common. There are several reasons for this, including:

o   The assumption of linearity makes problems much easier to analyze

o   Nonlinear material models require parameters that can be very difficult to determine

o   For modest effective stress changes (say, no less than 50 percent of the initial value) the effects of nonlinearity are usually not large.

These reasons justify the assumption of linearity for a large number of cases.

The assumption of isotropy is purely for convenience, as anisotropic effects are equally significant for small and large stress changes.

For a linear elastic isotropic material the $\underline{D}_T$ matrix is constant and thus the subscript "T" (for tangential) can be dropped. The matrix is a function of just two elastic coefficients. There is a wide choice as to which two to use; in this report, we use the Lamé coefficients λ and G wherever possible. In terms of λ and G, the $\underline{D}_T$ matrix is:

$$
\underline{D}_T = \begin{bmatrix}
\lambda + 2G & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2G & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2G & 0 & 0 & 0 \\
0 & 0 & 0 & G & 0 & 0 \\
0 & 0 & 0 & 0 & G & 0 \\
0 & 0 & 0 & 0 & 0 & G
\end{bmatrix}
$$

For convenience, the following table sets out some common conversions between elastic constants. The constants used are:

λ = Lamé coefficient

G = Second Lamé coefficient (the shear modulus):

$\dfrac{\text{shear stress}}{\text{shear strain}}$

K = Bulk modulus: $\dfrac{\text{volumetric stress}}{\text{volumetric strain}}$; $\dfrac{1}{\text{compressibility}}$

E = Young's modulus: $\dfrac{\text{tensile stress}}{\text{elongation}}$

..

ν = Poisson's ratio: $\dfrac{\text{lateral shrinkage}}{\text{elongation}}$

TABLE 1
RELATIONSHIPS BETWEEN ELASTIC CONSTANTS

	λ,G	k,G	$G,$	$E,$	E,G
$\lambda =$	λ	$1 - \dfrac{2G}{3}$	$\dfrac{2G}{1-2}$	$\dfrac{E}{(1+\ )(1-2\ )}$	$\dfrac{G(E-2G)}{(3G-E)}$
$G =$	G	G	G	$\dfrac{E}{2(1+\ )}$	G
$k =$	$\lambda + \dfrac{2G}{3}$	k	$\dfrac{2G(1+\ )}{3(1-2\ )}$	$\dfrac{E}{3(1-2\ )}$	$\dfrac{EG}{3(3G-E)}$
$E =$	$\dfrac{G(3\lambda+2G)}{\lambda+G}$	$\dfrac{9kG}{3k+G}$	$2G(1+\ )$	E	E
$v =$	$\dfrac{\lambda}{2(\lambda+G)}$	$\dfrac{3k-2G}{6k+2G}$			$\dfrac{E}{2G}-1$

For anisotropic linear elastic materials the $\underline{D}_T$ matrix becomes more complex, though it is still symmetric. In particular, off-diagonal terms can appear which cause shearing stresses to produce direct strains (dilatancy) and direct stresses to produce shear strains. A relatively common anisotropic formulation used for geological materials is isotropic within one plane but has different properties perpendicular to the plane. This material is defined by five elastic coefficients. A discussion of anisotropic materials can be found in Jaeger and Cook (1969).

5.2 NONLINEAR ELASTIC MATERIAL

The implication of assuming nonlinear elasticity is that an unloading stress-strain path will duplicate the loading path.

Use of the tangent $\underline{\underline{D}}_T$ is based on a curve-fitting description of the bulk modulus. It is possible for hysteretic materials which unload along a different path than they loaded to have an "unloading" stress-strain curve which is different from the loading curve. This technique represents an alternative to the use of plasticity theories (see Section 5.3).

There is generally a middle portion of the volumetric stress-strain curve for rocks which can be represented by a nonlinear elastic model. Hertz's theory of the contact of elastic spheres suggests that the bulk modulus should vary with the cube root of the effective stress. A discussion of this concept can be found in Geertsma (1957) or Van der Knaap (1959).

5.3 PLASTIC MATERIAL

Plasticity theories represent a sophisticated phenomenological description of the complete stress-strain behavior of a material. In general, conventional plasticity theories assume that the $\underline{\underline{D}}_T$ matrix is linear and isotropic and treat all non-linear behavior as plastic (irreversible)--an assumption that is not particularly sound for volumetric behavior of rocks. In addition, they assume that plastic behavior is isotropic--an assumption that is not proven for shearing behavior.

Nevertheless, plasticity theories have wide applicability. A discussion of plasticity theories for soil mechanics can be found in Zienkiewicz et al. (1975). Plasticity theories for rock behavior are described in Sandler et al. (1974) and Cheney et al. (1979).

5.4 MECHANISTIC MODELS

These are models where the stress-strain behavior is based on a model of the behavior of the component parts of the material. Hertz's theory (see Geertsma 1957; Van der Knaap 1959) of the behavior of an aggregate of elastic spheres falls in this category, as does Rowe's stress-dilitancy theory (Rowe 1962). A review of mechanistic models can be found in Schatz (1976). In their most sophisticated form, mechanistic models consist of elasto-visco-plastic matrices containing both cracks and spherical pores. These models can be computationally very difficult, but provide a more complete description of the full range of stress-strain behavior than do simpler models.

5.5 TERZAGHI'S E-LOG P RELATIONSHIP

Terzaghi (1943) presented a theory in which solid deformation is described by an empirical relation between pore volume change and stress. The Terzaghi equations are basically a one-dimensional theory, and it is not clear that the one-dimensional equations can be correctly generalized to three dimensions (Jaeger and Cook 1969). For one-dimensional problems, however, the theory has been widely accepted as an adequate representation of the deformation of fine-grained soils. The applicability to fine-grained, low porosity rocks has remained largely untested.

The one-dimensional consolidation theory assumes that there are no lateral deformations and that the soil grains are incompressible. Terzaghi (1943) presented the theory in terms of the void ratio, e = volume of voids divided by the volume of solids, and the effective vertical compressive stress p (p = $-\sigma_z'$). The relationship he derived assumed that void ratio varied with the logarithm of the effective stress:

$$e = e_o - C_c \log_{10} \left(\frac{P_o + \delta P}{P_o} \right)$$

where: C_c is known as the compression index

e_o, P_o are the starting values of e and p.

For unloading situations where p is decreasing (due to decreasing overburden stress or increasing pore pressure), C_c is replaced by C_S, the swelling coefficient. The swelling coefficient is also used for reloading, while p is less than its previous highest value (P_p, the preconsolidation pressure). Two other coefficients are commonly used with Terzaghi's theory: a_V and m_V. Their definitions are:

$$a_V = \frac{\partial e}{\partial P} = \frac{-.434 \; C_c}{P}$$

$$m_V = \frac{-\partial \varepsilon_z}{\partial P} = \frac{1}{1+e} \frac{\partial e}{\partial P} = \frac{-.434 \; C_c}{P(1+e)}$$

Note that m_V is equivalent to the compaction coefficient C_m (equation [9]).

5.6   TIME-DEPENDENT BEHAVIOR

Little experimental work has been performed to assess the time-dependency of deformations. In general, it is assumed that most apparent time-dependency is merely due to the time required for excess pore pressures to be dissipated. However, there is evidence that at least for shales, substantial creep strains can follow the instantaneous elastic strains.

5.7 TEMPERATURE EFFECTS

Temperature effects are fairly well understood. The assumptions that the coefficient of thermal expansion is both isotropic and independent of temperature and stress level are not necessary to the theory set out in Section 2.1. We know of no data which indicate that these are not adequate approximations.

There is evidence that the stress-strain relationship is temperature-dependent. Van Gonten and Choudhary (1969) found that compressibility increased by about 20 percent as the temperature was raised from room temperature to 400°F. In addition, the rate of plastic flow is temperature-dependent in most materials.

6.0   CHEMICAL EFFECTS

All of the preceding material has focused on the effects on
deformation of stresses, temperatures, pore-fluid pressures, and
time. The only other physical processes that appear able to
significantly affect deformation are chemical processes. These
include such effects as leaching and precipitation of minerals
and chemical reactions induced by changing temperature, pressure,
or pore fluid chemistry. We have found no documented evidence of
chemical processes (other than massive dissolution of carbonates)
seriously affecting deformations.

7.0   PARAMETER ESTIMATION

Due to the great difficulty of obtaining representative
parameter values from lab tests on core samples, there may in
some cases be advantages to measuring subsidence parameters
indirectly by field measurements of storage coefficients (from
pump tests) and barometric efficiencies.

Several of the parameters used in the above equations can be
determined relatively easily. Thermal expansion coefficients
vary little between geologic materials and are readily measured
in the laboratory. The matrix material bulk modulus K_S is
also relatively easily measured in the laboratory. Poisson's
ratio is not highly variable for different materials and thus can
often be estimated with sufficient accuracy. Combining these
parameters with indirect measurements of performance such as
storage coefficients and barometric efficiencies should in many
cases allow reasonably accurate deduction of material behavior
models.

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LIST OF SYMBOLS

Symbol	Dimensions	Description
a_v	$ML^{-1}T^{-2}$	Coefficient of compressibility $= \partial e/\partial P$
B	—	Barometric efficiency = pore pressure change/atmospheric pressure change
C_c	—	Compression index
C_m	$M^{-1}LT^2$	Compaction coefficient $= \partial \varepsilon_z /\partial P$
C_P	$M^{-1}LT^2$	Pore compressibility $= \frac{1}{V_P} \frac{\partial V_P}{\partial P}$
C_s	—	Compression index for unloading, Terzaghi theory. Swelling coefficient.
$\underline{\underline{D}}$	$ML^{-1}T^{-2}$	Stress-strain matrix. $\underline{\underline{D}} \, \delta \underline{\varepsilon} = \delta \underline{\sigma}$ for no pore pressure or temperature.
e	—	Void ratio $= V_v/V_s$
E	$ML^{-1}T^{-2}$	Young's modulus
g	LT^{-2}	Acceleration of gravity
G	$ML^{-1}T^{-2}$	Shear modulus
K	$ML^{-1}T^{-2}$	Bulk modulus
K_f	$ML^{-1}T^{-2}$	Bulk modulus of pore fluid
K_s	$ML^{-1}T^{-2}$	Bulk modulus of solid matrix material
$\underline{m}$	—	Special vector: $\underline{m}^t = \langle 1\ 1\ 1\ 0\ 0\ 0 \rangle$
m_v	$M^{-1}LT^2$	Coefficient of volume compressiblity $= \partial \varepsilon_z /\partial P$
n	—	Porosity, V_v/V
P	$ML^{-1}T^{-2}$	Vertical effective stress in Terzaghi theory
P	$ML^{-1}T^{-2}$	Pore-fluid pressure

Symbol	Dimensions	Description
s	L^{-1}	Storage coefficient, specific storativity
T	—	Subscript indicates tangential modulii for nonlinear materials
u,v,w	L	Displacements in x,y,z directions
V	L^3	Overall volume of sample
V_p	L^3	Volume of pores
V_s	L^3	Volume of solid matrix material
V_w	L^3	Volume of water squeezed out of sample
x,y,z	L	Coordinate directions. z = elevation
α	$temp.^{-1}$	Coefficient of isotropic linear thermal expansion
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	—	Shear strains
δ	—	Indicates a small variation in a parameter
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	—	Direct strains.
$\underline{\varepsilon}$	—	Total strain vector $\underline{\varepsilon}^T = \langle \varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \gamma_{xy} \ \gamma_{yz} \ \gamma_{zx} \rangle$
$\underline{\varepsilon}^e$	—	Elastic strain vector
ε_g	—	Volumetric strain in matrix material due to pore-pressure and temperature
$\underline{\varepsilon}^{vp}$	—	Viscous and/or plastic strains vector
ε_v	—	Volumetric strain = $\varepsilon_x + \varepsilon_y + \varepsilon_z$
$\sigma_x, \sigma_y, \sigma_z$	$ML^{-1}T^{-2}$	Stresses in x,y,z directions
$\underline{\sigma}$	$ML^{-1}T^{-2}$	Total stress vector $\underline{\sigma}^T = \langle \sigma_x \ \sigma_y \ \sigma_z \ \tau_{xy} \ \tau_{yz} \ \tau_{zx} \rangle$

Symbol	Dimensions	Description
$\underline{\sigma}'$	$ML^{-1}T^{-2}$	Effective stress vector $\underline{\sigma}' = \underline{\sigma} + P\underline{m}$
σ_m	$ML^{-1}T^{-2}$	Mean normal stress $\sigma_m = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right)$
σ_m'	$ML^{-1}T^{-2}$	Mean normal effective stress $\sigma_m' = \sigma_m + P$
λ	$ML^{-1}T^{-2}$	Lame elasticity coefficient
ρ_f	ML^{-3}	Density of pore fluid
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	$ML^{-1}T^{-2}$	Shear stresses. τ_{xy} = shear stress on x-plane in y-direction
ν	—	Poisson's ratio