Computer Program for Cellular Structures of Arbitrary Plan Geometry

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COMPUTER PROGRAM FOR CELLULAR STRUCTURES OF ARBITRARY PLAN GEOMETRY

by

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and

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Report to the Sponsors: Division of Highways, Department of Public Works, State of California, and the Bureau of Public Roads, Federal Highway Administration, United States Department of Transportation.

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Department of Civil Engineering
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COMPUTER PROGRAM FOR CELLULAR STRUCTURES OF ARBITRARY PLAN GEOMETRY

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to

the Division of Highways
Department of Public Works
State of California
Under Research Technical Agreement
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and
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College of Engineering
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ABSTRACT

A computer program is presented for the analysis of cellular structures of constant depth with arbitrary geometry in plan view. The development is based on the finite element method and uses two different element types designed to capture the main behavior of deck and web components. The structure may be subjected to a variety of force and displacement boundary conditions, such as distributed dead and live-loads in addition to concentrated nodal loads and prescribed nodal displacements. The well established direct stiffness method is used for the element assembly. After solving for the unknown nodal displacements and reactions, internal forces are computed at the center and at nodes of deck and web elements selected by the user.

KEY WORDS

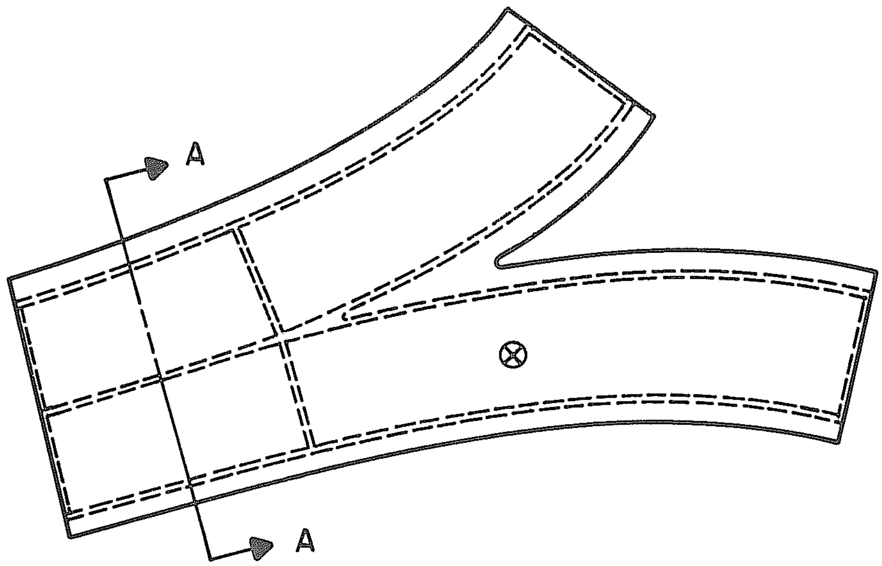
Box girder bridges, multi-cell bridges, skew bridges, interchange structures, orthotropic folded plates, anisotropic folded plates, elastic analysis, structural analysis, structural design, finite elements, direct stiffness method.

TABLE OF CONTENTS

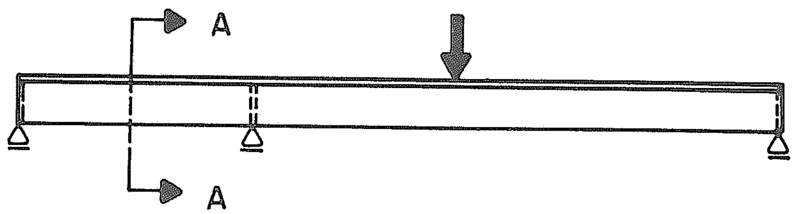
1. INTRODUCTION

Cellular systems are used extensively for various types of civil engineering structures, such as box girder bridges, buildings, aircraft and hydraulic structures. A variety of methods have been presented for the analysis of prismatic box girders [1,2,3,4] but an analytical tool for cellular structures with arbitrary geometric configuration has not been introduced yet except within the context of a general shell analysis.

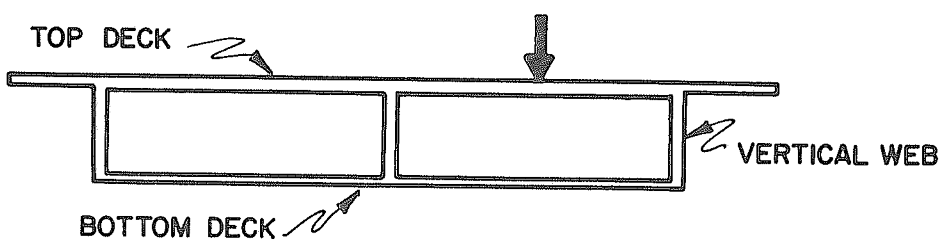
This report describes a finite element computer program for the analysis of cellular structures of constant depth with arbitrary geometry in plan view. The well known finite element method of analysis is ideally suited for computer applications combining versatility and efficiency in an optimum fashion. A variety of special elements have been developed in reference [5] for the analysis of box structures. These elements were assembled in two cellular programs developed to compare efficiency and accuracy. The program using elements without midside nodes, which was identified by the name CELL, has been further developed for general usage as a box girder bridge program and will be presented in this report. Its present version is capable of treating box structures with arbitrary plan view, subjected to any type of loading and boundary conditions. There is still one limitation on the geometric configuration of the cellular structure, it must have vertical web components of constant height. This restriction could be easily removed by changing the input format and by including more general element transformations as all elements can be of either quadrilateral or triangular shape. The structure and input format of program CELL takes advantage of the simplicity of geometry and topology found in most box girder bridges. Cases with completely arbitrary geometry should be analyzed by other less convenient and less efficient shell programs. A typical box structure, which can be analyzed by program CELL is illustrated in Fig. 1. All structural components exhibit in plane and flexural stiffness which are accounted for in the analysis.



PLAN VIEW



ELEVATION



SECTION A – A

FIG. I BOX GIRDER BRIDGE EXAMPLE OF A CELLULAR STRUCTURE

2. METHOD OF ANALYSIS

The finite element method of analysis is summarized very briefly since it has been described extensively in numerous publications.

2.1 Finite Element Idealization

The structure is idealized by means of general quadrilateral or triangular elements which are interconnected at the corner nodes by 5 degrees of freedom, U, V, W, θ_X and θ_Y, omitting the global rotation θ_Z from the general 6 degrees of freedom system used in shell analysis. In reference [5] results are presented for the analysis of single cell box girders which are discretized by a variety of nodal configurations. The 5 degrees of freedom system appeared the most promising tool for the analysis of box structures both in terms of accuracy and efficiency. Different types of displacement models and mixed models are utilized to represent the in plane and plate bending behavior of the deck and web components. Their nodal configurations and formation is briefly described in Chapter 3 while a detailed derivation is given in reference [5].

2.2 Direct Stiffness Method

Compatibility of the kinematic nodal quantities is enforced by the well known direct stiffness method used for the assembly of elements. As only 5 degrees of freedom exist complete continuity of displacements cannot be maintained along the element interfaces. Therefore, a bound of the results cannot be guaranteed from a theoretical point of view but convergence is assured as all elements satisfy the constant energy criterion.

The resulting system of simultaneous equations is solved for the unknown nodal displacements taking advantage of the symmetric, positive definite, and banded character of the structural stiffness matrix. Sparsity is accounted for to increase the efficiency of the direct solution scheme.

Finally, the internal forces are determined from the resulting element displacement field by using the standard stress displacement relationships.

3. DESCRIPTION OF FINITE ELEMENTS

A variety of finite elements are used to account for the different behavior of the two main structural components, the horizontal deck and the vertical webs. They have been described and tested in reference [5]. All elements satisfy the constant energy criterion and maintain full continuity as long as adjacent elements are coplanar.

3.1 Deck Elements

The deck is idealized by quadrilateral or triangular elements having 5 DOF per node. As the local coordinates of the deck elements x,y,z coincide with the global coordinates of the box structure, X,Y,Z, no transformation is required, a rearrangement of the in-plane and plate bending contributions suffices to form the global element stiffness $\underline{K}_D$ (20 X 20).

a) Plane Stress Behavior.

The in-plane action is represented by the mixed model Q8D11 having 8 external DOF and 3 internal ones. This quadrilateral element degenerates to the constant strain triangle by letting the coordinates of the first and fourth node coincide. The nodal configurations of both, the quadrilateral element Q8D11 and the triangular element, CST, are illustrated in Fig. 2.

The mixed model is constructed using separate expansions for the displacement and the strainfield. The variation of the components u and v of the displacement field is approximated by the standard bilinear expansion. The normal strain components ϵ_x and ϵ_y are derived from the displacement field by the linearized strain displacement relationships. The shear-strain variation is assumed to be constant.

It has been shown in reference [5] that this choice for the field variables provides a stiffness matrix which yields more flexible and better results than the associated displacement model. With the help of an extension of the Hu-Washizu Variational Principle the stiffness is formed using a two point Gaussian numerical integration formula. The three internal DOF, the u and v components of the center node and the generalized coordinate associated with the constant shear strain variation are eliminated by an internal static condensation process.

Due to the lack of invariance of the stiffness matrix with regard to coordinate rotations, this element is formed in the convected coordinate system $\bar{x}$, $\bar{y}$, illustrated in Fig. 2, with the $\bar{x}$ axis being defined by nodes 1 and 2. The $\bar{y}$ axis is now fully defined in order to provide a right hand Cartesian coordinate system. The element stiffness is generated in this convected coordinate system and subsequently transformed into the local x, y coordinates.

b) Plate Bending Behavior.

The flexural action is represented by the Q19 quadrilateral two-way bending element having 12 external DOF and 7 internal ones. This quadrilateral displacement model has been described in detail in reference [6]. It degenerates to the LCCT-9 triangular plate bending element if the first and the last node in the nodal array coincide. The nodal configurations of the quadrilateral element Q19 and the triangular element LCCT-9 are illustrated in Fig. 3.

The displacement model Q19 is assembled from four LCCT-11 triangles with 11 bending DOF at 5 nodal points. The 7 internal DOF are eliminated by an internal static condensation process. The linear curvature compatible triangle, denoted by LCCT, uses complete

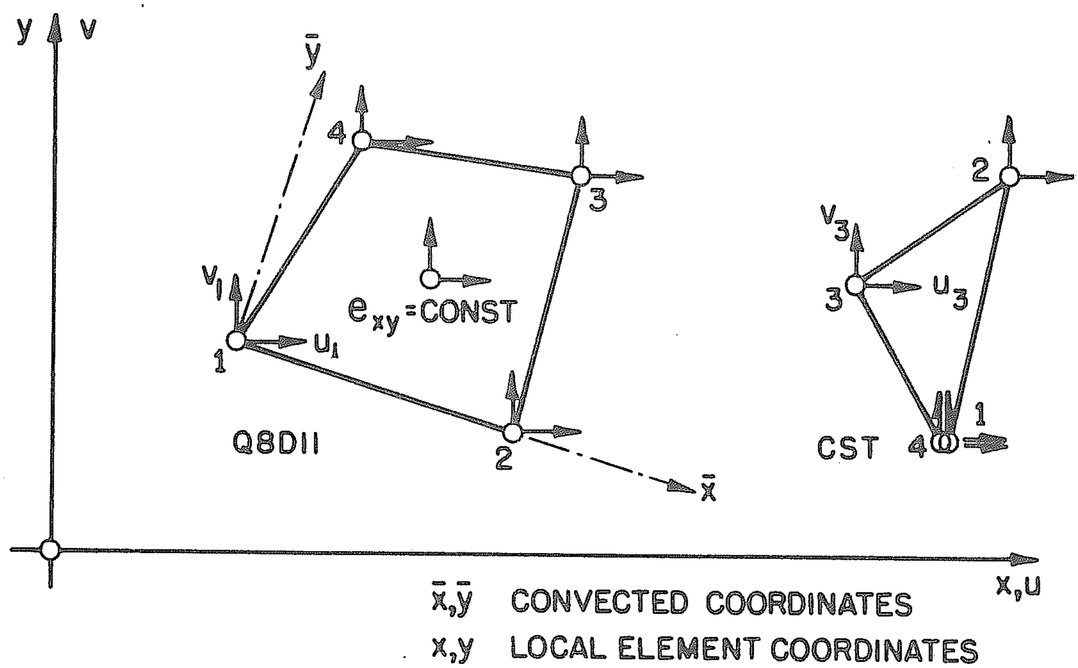


FIG. 2 NODAL CONFIGURATION OF PLANE STRESS ELEMENTS Q8D11 AND CST

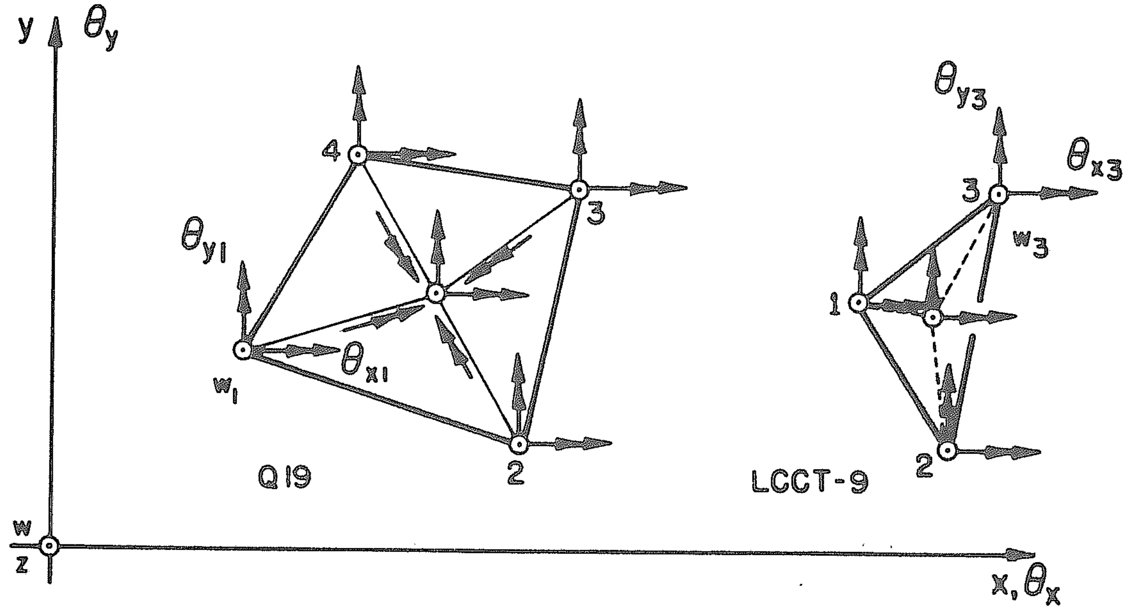


FIG.3 NODAL CONFIGURATION OF PLATE BENDING ELEMENTS Q19 AND LCCT-9

cubic expansions for the transverse displacement field w over each of the triangular subregions. Enforcing continuity between the subregions yields the element LCCT-12 having 12 DOF with a quadratic variation of normal slopes along the element edges. Note that second derivatives are piecewise continuous within each subregion of the triangle. This element degenerates to the LCCT-11, LCCT-10 or LCCT-9 simply by using kinematic constraint conditions to enforce a linear variation of normal slopes along the edges.

3.2 Web Elements

The web is idealized by quadrilateral elements having 5 DOF per node. They are assumed to lie in a vertical plane with arbitrary orientation with respect to the global X-Y coordinates. The transformation involves simple rotations of the local nodal quantitites u, w and $\bar{\theta}_x$, ω into the global DOF U, V and θ_X, θ_Y, and after rearrangement of the in-plane and plate bending contributions the global element stiffness $\underline{K}_w$ (20 X 20) is developed.

a) Plane Stress Behavior.

The in-plane action is represented by the displacement model QUSP12 having 12 fundamental DOF. The nodal configuration of this element which has been described in detail in reference [5] is illustrated in Fig. 4. This displacement model is constructed using different expansion for the u and v components of the displacement field in order to capture the beam behavior of the web components by single elements over the depth. A bilinear expansion for u and v assures that the element contains rigid body modes and constant energy states. Moreover, a cubic variation in the x-direction of the v component can be described introducing the nodal rotations $\omega = \frac{\partial v}{\partial x}$ normal to the plane of the element. This makes it possible to represent the beam behavior by a single plane stress element. Due to the different variation of the displacement components u and v the stiffness depends on the orientation of the coordinate system and lacks invariance. Hence, as a general rule governing the numbering of nodal points the edge 1-2 and 4-3 have to be approximately parallel to the local x-axis. Since the present program is restricted to box structures of constant depth the spar elements remain rectangular. Hence, no problem arises choosing a convected coordinate system to minimize the lack of invariance.

b) Plate Bending Behavior.

The flexural action is represented by the element ONEW, a quadrilateral one-way bending element having 8 fundamental DOF. The nodal configuration of this beam type of element is illustrated in Fig. 5. This element is constructed using cubic beam type expansions to approximate the flexural action by one-way bending in the y-direction between nodes 1-4 and 2-3. Due to the assumption of one-way bending there is no coupling between nodes 1-4 and 2-3. The width B of the equivalent beams is computed as average of the x differences of nodes 1-2 and 3-4, and the spans equal the difference of y-coordinates between nodes 1-4 and 2-3.

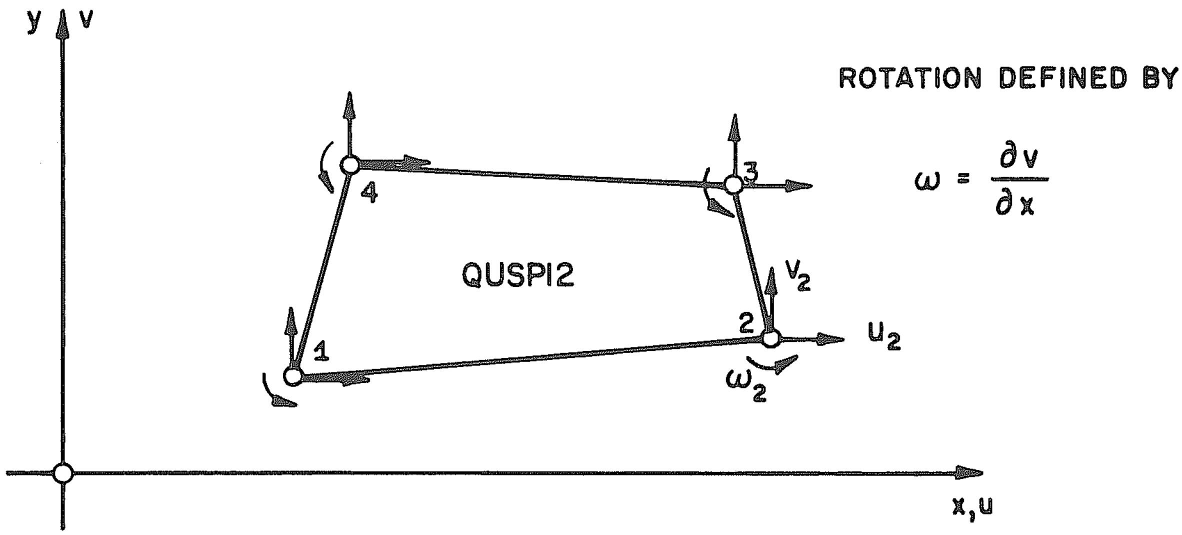


FIG. 4 NODAL CONFIGURATION OF PLANE STRESS "SPAR" ELEMENT QUSPI2

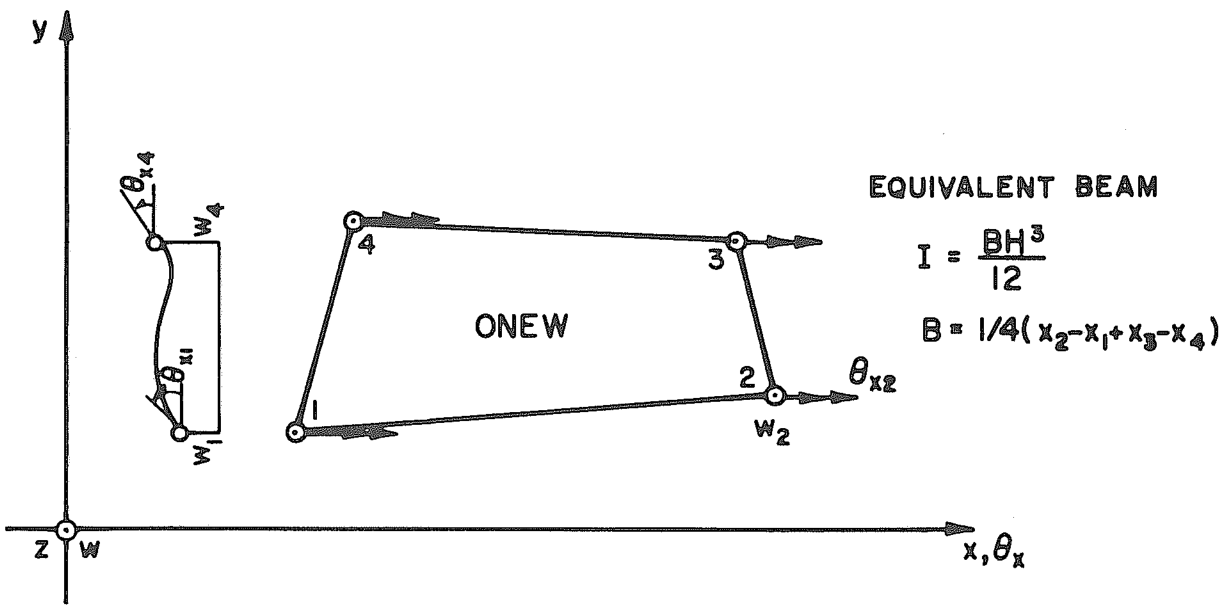


FIG. 5 NODAL CONFIGURATION OF ONE WAY PLATE BENDING ELEMENT ONEW

4. DESCRIPTION OF PROGRAM CELL

4.1 Nature of Program

This program is capable of analyzing cellular structures consisting of top and bottom decks which are interconnected by vertical web elements. At this stage the depth of these box structures is assumed constant. Such a cellular system is illustrated in Fig. 1.

The structure is idealized by different types of finite elements which are interconnected at the nodes by the 5 global degrees of freedom U, V, W, θ_X, θ_Y. Advantage is taken of the special behavior of these box structures by omitting θ_Z from the 6 DOF system commonly used for the finite element analysis of arbitrary shells. All elements exhibit multi-component action possessing both in-plane and flexural stiffness. While the flexural action of deck elements is represented by two-way bending elements that of vertical web elements has to be idealized by one-way bending due to the lack of the θ_Z DOF. The various types of elements which may be of arbitrary quadrilateral or triangular shapes have been described in Chapter 3.

The elastic material constitution of each element can be specified in the form of an orthotropic stress-strain law or equivalent anisotropic laws relating stress resultants and moments to strains and curvatures. This makes is possible, if desired, to eliminate bending contributions or to consider equivalent material laws for stiffened plate structures. The thickness is assumed constant over each element. A more detailed description of the anisotropic elastic analysis is given in Section 4.3.

The structure can be subjected to a variety of loading conditions such as concentrated nodal loads, live loads uniformly distributed over the area of each top deck element and dead loads which are all determined using the tributary area concept. Arbitrary displacement boundary conditions can be imposed along the global DOF of each node and in any skewed direction within the X-Y plane.

The input is so arranged that the problem regarding mesh layout and nodal coordinates is fully defined by describing the top deck. Hence, the data is arranged similarly to a typical plate bending or plane stress program. Additional information is only needed for the position and location of vertical web elements. The output of resulting nodal displacements, nodal forces and internal forces is so arranged that the static quantity of the top deck is output together with its corresponding value in the bottom deck. A variety of output options have been incorporated to provide the user with enough flexibility to print out particular quantities at selected points of interest.

4.2 Coordinate System and Sign Conventions

The cellular structure is referred to a global right-handed Cartesian system X, Y, Z, with X-Y lying in the plane of the top deck, as illustrated in Fig. 6. U, V, W are linear nodal displacements which are positive in the global directions X, Y, Z. θ_X, θ_Y are nodal rotations about the X, Y axes whose positive direction are given by the right hand rule. Each element is formed in a local right handed Cartesian system x, y, z with x, y lying in the plane of the element as illustrated in Fig. 6. For the deck elements only, the global and local systems are identical. The sign convention of the internal forces is referred to these local coordinates and is indicated in Fig. 7.

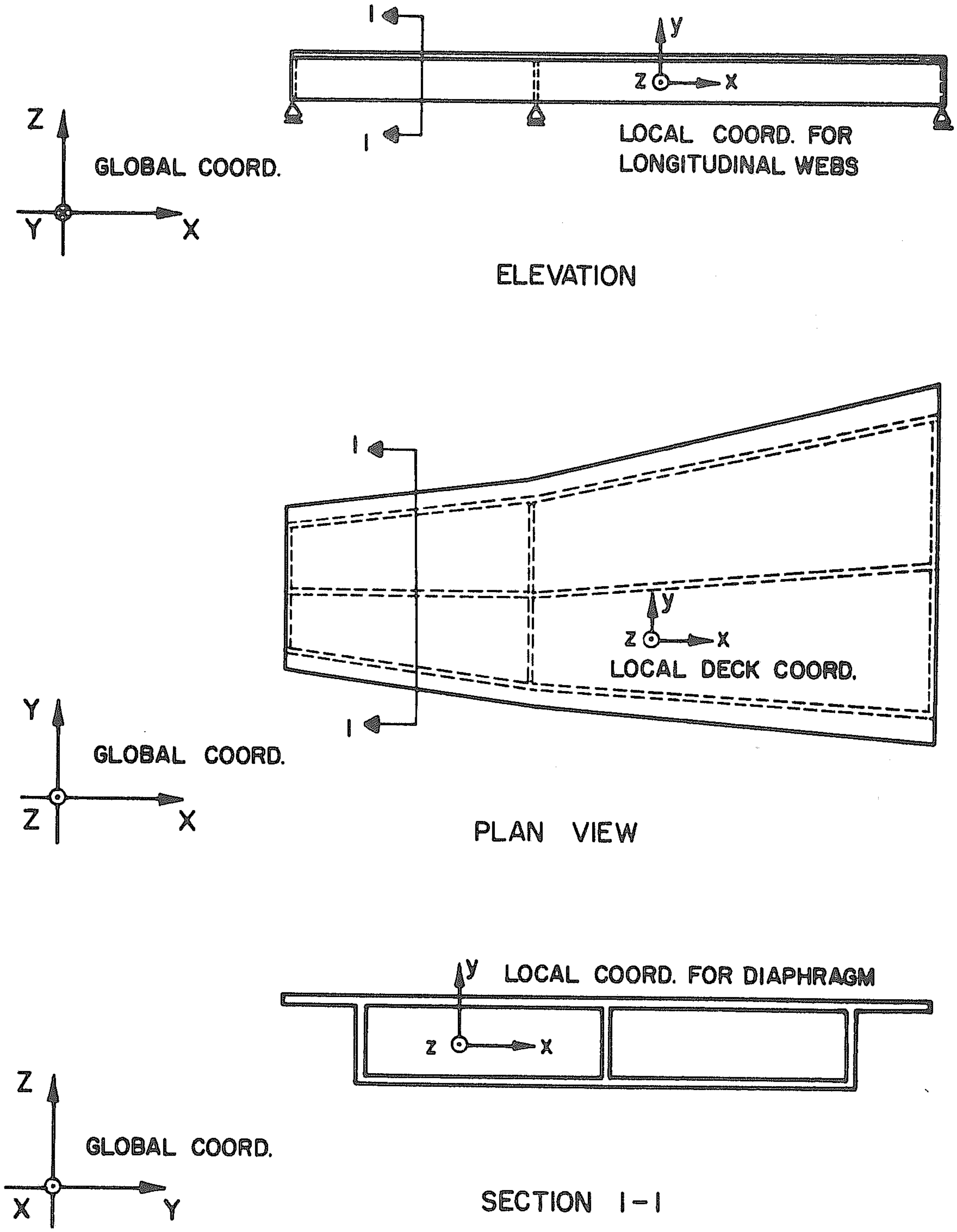
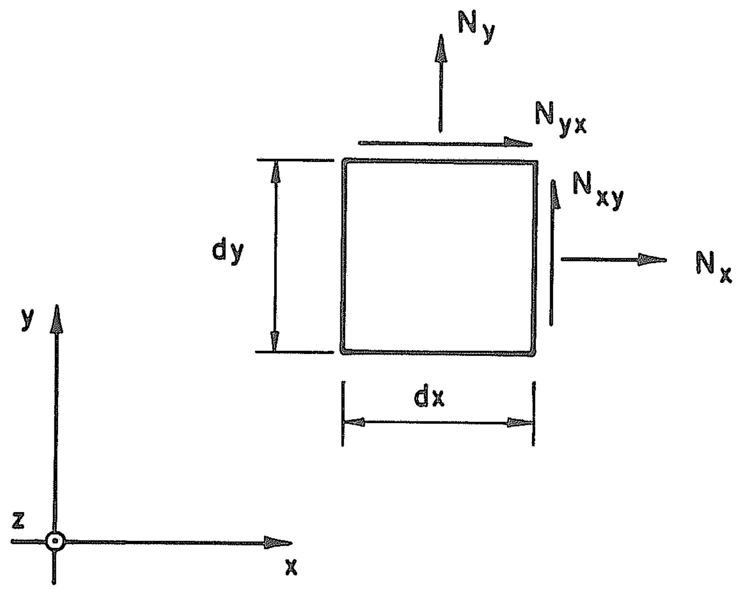
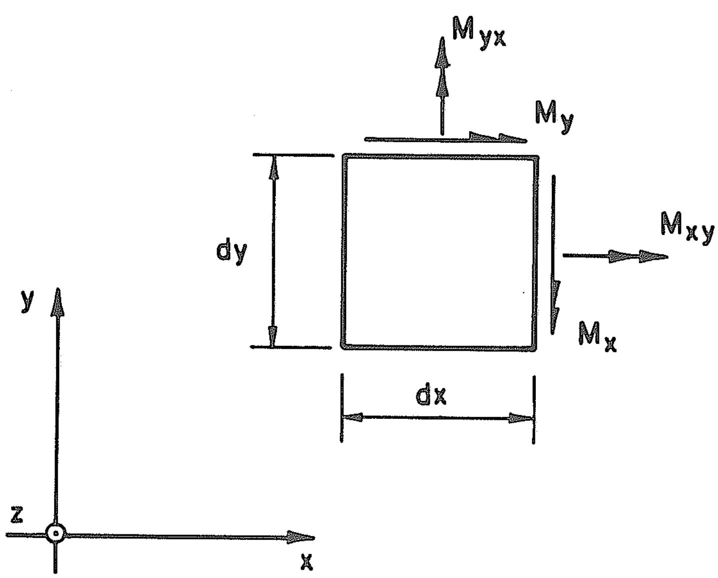


FIG. 6 SIGN CONVENTIONS OF LOCAL AND GLOBAL COORD. SYSTEMS



LOCAL ELEMENT COORD.

IN PLANE STRESS RESULTANTS



LOCAL ELEMENT COORD.

PLATE BENDING MOMENT RESULTANTS

FIG. 7 SIGN CONVENTION OF INTERNAL FORCES

4.3 Elastic Analysis

The material properties can be specified at two levels, a) a plane stress law relating stresses to strains, and b) a law relating internal forces to strains and curvatures.

a) Orthotropic Plane Stress Material.

The principal axes of orthotropy for each element are denoted by $\bar{x}_1, \bar{y}_1, \bar{z}_1$ where $\bar{x}_1$ forms an angle φ with the local element x-axis. The orthotropic stress-strain relation refers to these principal axes and is defined by

$$\begin{Bmatrix} \bar{\tau}_{xx} \\ \bar{\tau}_{yy} \\ \bar{\tau}_{xy} \end{Bmatrix} = \begin{bmatrix} E_1/\chi & E_1 \nu_{12}/\chi & 0 \\ E_2 \nu_{21}/\chi & E_2/\chi & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\gamma}_{xy} \end{Bmatrix}$$

E_1, E_2	Elastic moduli in the $\bar{x}$ and $\bar{y}$ direction.
ν_{12}, ν_{21}	Poisson's ratio with $E_1 \nu_{12} = E_2 \nu_{21}$ for symmetry. ν_{12} is defined as the ratio of the strain in the $\bar{x}$-direction to the strain in the $\bar{y}$-direction due to a uniaxial stress in the $\bar{y}$-direction.
$\chi = 1-\nu_{12} \nu_{21}$	Denominator in material law.
G_{12}	Shear modulus.

The corresponding material law in the x, y coordinates requires a tensor transformation eliminating zero coefficients if $\varphi \neq 0$. For isotropic material the constants degenerate to $E_1 = E_2 = E$, $\nu_{12} = \nu_{21} = \nu$, $G_{12} = G$ and the angle φ is not needed. The in-plane stress resultants strain relation is obtained by multiplying this matrix by h, where h is the plate thickness. The moment-curvature relation is obtained by multiplying this matrix by $\frac{h^3}{12}$.

b) Anisotropic Stress Resultant-Strain and Moment-Curvature Relations.

In order to increase the versatility of the program direct access is available to the material law relating stress resultants to strains and moments to curvatures. This capability makes it possible to take account for instance of stiffeners arranged along two skewed directions or of various levels of flexural action, such as two-way bending, one-way bending or no bending action altogether. In-plane stress resultants are related to the strains by

$$\underline{N} = \underline{DS}\ \underline{\varepsilon}$$

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} DS(1) & DS(4) & DS(5) \\ & DS(2) & DS(6) \\ Sym. & & DS(3) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

Internal moments are related to the curvatures by

$$\underline{M} = \underline{DM}\ \underline{\kappa}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} DM(1,1) & DM(1,2) & DM(1,3) \\ & DM(2,2) & DM(2,3) \\ Sym. & & DM(3,3) \end{bmatrix} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}$$

The coefficients of the material matrices DS and DM can be determined either by integration of the associated plane stress material law over the thickness as it is done in a) or by performing appropriate experiments to evaluate these coefficients.

4.4 Method of Solution

The solution is based on the direct stiffness method which has been described extensively in the literature. The unknown kinematic quantities are the 5 nodal displacements $U, V, W, \theta_X, \theta_Y$. They are matched for all elements meeting at a node to enforce interelement continuity. Full conformity is achieved between all coplanar elements. At junctures of deck and web elements full continuity of deformations cannot be maintained. Due to the different functional variation of in-plane deck displacements and transverse web displacements, conformity of displacement is violated in the X-Y plane. Due to the one-way bending representation of the flexural web action no slope continuity can be enforced except for θ_X at the nodes. It has been shown in reference [5] that this lack of conformity is of minor influence. The numerical results of the proposed finite element analysis tend to remain on the stiff side which implies that the displacements obtained are smaller than the true values in the continuum. But from a theoretical point of view one cannot claim an upper bound to the total energy due to the lack of continuity of displacements and slopes and due to the use of mixed models in the finite element development.

4.5 Capabilities and Restrictions

The program in its present form imposes the following restrictions onto the geometry, material and the discretization of the problem considered.

a) Geometry of Box Structure.

The geometry of the cellular structure is restricted to box structures of constant depth and vertical webs interconnecting the horizontal top and bottom deck. The geometry can be arbitrary in plan view.

b) Material Constitution.

The material properties are specified for each element type. An elastic anisotropic material law describes the constitution within each element. Similar to the constant element thickness the material properties are assumed constant within each element domain.

c) Programming Limitations.

There are a few restrictions on the discretization of the cellular structure by finite elements which originate in the programming.

First, the vertical web is idealized by a single element over the whole depth.

Second, there is a restriction on the largest bandwidth, 130, and hence the connectivity of the element layout. This limitation can be eliminated by incorporating into the direct block solution technique a capability which allows the bandwidth to be larger than the block length.

Third, there is an additional restriction on the largest number of DOF, 200, which may be affected by kinematic constraints.

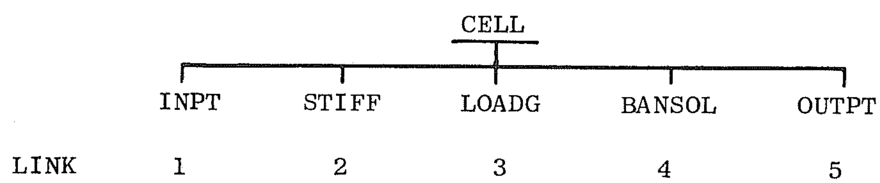
No restrictions on the number of elements, number of nodal points, number of material types etc. are imposed since the program features dynamic storage allocation coupled with an automatic field length reduction. This capability adjusts the field length automatically during execution to the current storage requirements. An explicit formula for the hand calculation of the required field length is given in Section 5.7. There is no limitation on the number of load cases which can be treated without repeated decomposition of the stiffness matrix.

5. PROGRAMMING INFORMATION

The program is written in FORTRAN IV and has been tested on the CDC 6400 at the Berkeley Computer Center.

5.1 Program Structure

The program uses the Overlay feature. The main program CELL is in permanent residence while 5 links of primary level are called consecutively into residence from the main program CELL. Schematically it can be shown as follows:



5.2 Program Decks

The program contains the following decks which need not be in sequence within each link as long as they are contained in the appropriate link.

MAIN PROGRAM

PROGRAM CELL

SUBROUTINE FL (in COMPASS)

CELL monitors the calling sequence of the primary overlays, links 1 to 5. FL has two purposes: If called LWA(N) it retrieves the last word address N of the program during execution, and if called RFL(N) it resets the field length to N. This program is not written in standard FORTRAN IV, but its equivalent should be available at any computer center. Otherwise a fixed amount of storage has to be calculated by hand using formulas given in Section 5.7. This storage has to be reserved in the area of blank COMMON.

LINK 1

PROGRAM INPT

SUBROUTINE INPU

SUBROUTINE FORMC

SUBROUTINE Q8D11

SUBROUTINE QUSP12

SUBROUTINE SPLATE

SUBROUTINE SLCCT

SUBROUTINE SONEW

SUBROUTINE DECK

SUBROUTINE WEB

INPT reads in the input control, computes the required field length for this link and resets it if necessary.

INPU reads in the data, generates the finite element mesh, the nodal coordinates, the material properties and forms the element stiffnesses for both deck and web elements in the global coordinate system.

FORMC determines the anisotropic plane stress material law in the element coordinates from given orthotropic constants which are specified in their principal directions.

Q8D11 computes the in-plane stiffness of the deck elements Q8D11 (8X8) in local coordinates.

QUSP12 computes the in-plane stiffness of the web elements QUSP12 (12X12) in local coordinates.

SPLATE assembles the plate bending stiffness for the quadrilateral deck element Q19 (12X12) in local coordinates.

SLCCT computes the plate bending stiffness of the triangular subelement LCCT-9 (9X9) or LCCT-11 (11X11).

SONEW computes the one-way plate bending stiffness for the web element ONEW (8X8) in local coordinates.

DECK assembles the in-plane and plate bending stiffness into the global deck element (20X20).

WEB assembles the in-plane and plate bending stiffness into the global web element (20X20) after transforming it into global coordinates.

LINK 2

PROGRAM STIFF

SUBROUTINE BIGK

STIFF computes the required field length for this link and resets it if necessary.

BIGK assembles the global stiffness matrices of the deck and web elements into the structural stiffness matrix in block form and imposes displacement boundary conditions on the structural level.

LINK 3

PROGRAM LOADG

SUBROUTINE LOAD

LOADG computes the required field length for this link and resets it if necessary.

LOAD reads in the loading information for the concentrated nodal loads, the live loads and the dead loads. In addition, non-zero displacement boundary conditions may be input in the global directions which are converted into equivalent loads.

LINK 4

PROGRAM BANSOL

SUBROUTINE BAND

BANSOL computes the required field length for this link and resets it if necessary.

BAND has two entries: First, the stiffness matrix is decomposed block by block, a single load vector is reduced with the total vector in core and is overwritten after back substitution by the total solution. Second, with the reduced stiffness available from a previous load case each load vector is reduced separately and overwritten by the corresponding solution after back substitution. Two blocks of the stiffness matrix and the total load vector need to be in core at a time. A block is defined by NBLKL rows and IBANDW columns.

IBANDW equals the bandwidth which depends on the connectivity of the finite element mesh layout and NBLKL is the block length. The block length NBLKL can be specified on the input control card. Otherwise it is calculated from the maximum available core in LINK 2 or LINK 4, the smaller of which determines the length of NBLKL:

LINK 2 NBLKL = (120,000 B - 4*NUMEL - 2*NBEAM)/(2*IBANDW)

LINK 4 NBLKL = (120,000 B - NEQ)/(2*IBANDW + 1)

where B indicates the octal base.

This out of core symmetric band solver takes advantage of variable bandwidth with 45 degrees shadows and accounts for zeros within the band. These small alterations of the standard Gauss algorithm for symmetric matrices without pivoting increase tremendously the efficiency of the algorithm in combination with the use of single subscripted arrays. Note that only two files are needed since the total load vector is in core at a time leading to considerable savings in peripheral processing. The storage requirement remains the same since the buffer requirement for two additional tapes offsets eventual reductions by treating the load vector block by block. The present version imposes the following limitation upon the shape of each block:

$$IBANDW \leq NBLKL$$

This limitation can be removed introducing two additional files for temporary storage and requires a substantial change in the element assembly.

LINK 5

PROGRAM OUTPT

SUBROUTINE OTPT

SUBROUTINE PSD11

SUBROUTINE PLSP12

SUBROUTINE FPLATE

SUBROUTINE FLCCT

SUBROUTINE FONEW

OUTPT reads in the output control, computes the required field length for this link and resets it if necessary.

OTPT outputs first the global displacement components at each node of the top and bottom deck. Then the nodal forces and reactions and the internal forces at the center of deck elements are computed and printed if the appropriate output controls are activated. Furthermore, the internal forces are computed and averaged at NSTO nodes of the deck elements and web elements. Their element contributions to each node and averaged nodal values are output separately for both types of structural components together with their principal values.

PSD11 determines the in-plane stress resultants in local coordinates at the four nodes and at the centroid of the plane stress element Q8D11.

PLSP12 determines the in-plane stress resultants in local coordinates at the four nodes and at the centroid of the spar element QUSP12.

FPLATE determines the moments in local coordinates at the four nodes and at the centroid of the plate bending element Q19.

FLCCT determines the curvatures at the nodes of each triangular plate bending element LCCT.

FONEW determines the transverse moments M_y in local coordinates at the four nodes of the one-way bending element ONEW.

5.3 File Usage

FORTRAN logical units 1, 3, 8 are used as binary tapes for temporary storage. File unit 7 is available for binary punched output.

On-line input-output is done via FORTRAN statements READ n, list and PRINT n, list. On the CDC 6400 these conventions are used instead of the standard IBM commands READ (i,n) list and WRITE (i,n) list.

5.4 Input Specifications

The input data is key punched on cards as specified below. The sequential order of the input cards must be strictly adhered to and consistent units must be used throughout.

INPUT DATA FOR IDEALIZATION

1. Start Card for Problem (A6)

   Col. 1 to 5 - CHECK, each problem has to begin with this card with the word "START" in the first 5 columns which initiates the input of each new problem.

2. Title Card (13A6)

   Col. 1 to 78 - HED (13), alphanumeric information to identify problem.

3. Input Control Card (6I4, F10.2)

   Col. 1 to 4 - NUMEL, number of elements in top deck

   Col. 5 to 8 - NUMNP, number of nodal points in deck

   Col. 9 to 12 - NBEAM, number of vertical web elements

   Col. 13 to 16 - NMAT, number of different material types which include the element thickness.

   Col. 17 to 20 - NUMBC, total number of top and bottom deck nodes with specified kinematic constraints; these must be imposed separately for top and bottom deck nodes.

   Col. 21 to 24 - NBLKL, specified block length of structural stiffness matrix. If left blank the block length is computed internally using the current bandwidth and the maximum available core storage of 140000B (B indicates octal base) to reduce the number of blocks, and hence the tape handling.

Col. 25 to 34 - AH, height of the box structure; this is assumed constant for entire structure.

4. Deck Element Array (10I4)

Col. 1 to 4 - L, element number.

Col. 5 to 20 - NP(4) element nodal array, must be listed counter-clockwise (viewed from above).

Col. 21 to 24 - MA, material number of top deck element.

Col. 25 to 28 - MB, material number of bottom deck element.

One way generation is activated if previous element numbers have been omitted, in which case following further data must then be given.

Col. 29 to 32 - NDIF, difference in corresponding nodal numbers of consecutive elements. If it is left blank NDIF is set to 1 for the one way generation.

If two way generation is desired the following data must be added.

Col. 33 to 36 - MOD, modular difference in the element number of adjacent element strings.

Col. 37 to 40 - NLIM, number up to which two way generation is to be activated.

5. Web Element Array (7I4)

Col. 1 to 4 - L, element number

Col. 5 to 12 - NBP(2), element nodal array, list nodes in increasing numerical order.

Col. 13 to 16 - MC, material number.

One way generation is activated if previous elements are omitted, in which case the following further data must then be given.

Col. 17 to 20 - NDIF, difference in corresponding nodal numbers of consecutive elements. If it is left blank NDIF is set to 1 for the one way generation.

If two way generation is desired the following data must be added.

Col. 21 to 24 - MOD, modular difference in the element number in adjacent element strings.

Col. 25 to 28 - NLIM, element number up to which two way generation is to be activated.

6. Nodal Coordinate Array (I4, 2F10.2, 2I4)

Col. 1 to 4 - N, nodal point

Col. 5 to 14 - XORD(N), global x-coordinate of node N

Col. 15 to 24 - YORD(N), global y-coordinate of node N

One way generation is activated if previous nodes are omitted, in which case NDIF = 1 in all cases.

If two way generation is desired the following data must be added.

Col. 25 to 28 - MOD, modular difference in the nodal numbers of adjacent nodal strings.

Col. 29 to 32 - NLIM, number of node up to which two way generation is activated.

Note, for a detailed description of the generation options with some examples, see Section 5.5.

7. <u>Material Array</u> (I4, 6F10.2)

The element material properties must be input totally as a constitutive law relating stresses to strains or as a law relating in plane stress resultants and moments to strains and curvatures.

a) An elastic orthotropic material law is specified in the local $\bar{x}_1$-$\bar{y}_1$ frame of the principal directions of orthotropy. It relates stresses to strains for each material type of the elements and incorporates the element thickness.

Col. 1 to 4 - N, material number, must be nonzero and ordered sequentially.

Col. 5 to 14 - E1(N), elastic modulus in principal direction of orthotropy $\bar{x}_1$.

Col. 15 to 24 - E2(N), elastic modulus in principal direction of orthotropy $\bar{x}_2$.

Col. 25 to 34 - G12(N), shear modulus

Col. 35 to 44 - PR(N), mean Poisson's ratio $\nu = \sqrt{\nu_{12}\ \nu_{21}}$

Col. 45 to 54 - ANG(N), angle in degrees between local element x-coordinate axis and principal axis of orthotropy $\bar{x}_1$. Use right hand rule for + direction.

Col. 55 to 64 - TH(N), thickness of element assumed to be uniform over the element.

To increase the versatility of the program an alternative input of the material constitution can be specified for each element type.

b) An elastic anisotropic material law relating moments to curvatures and in plane stress resultants to strains is specified for each material type of the elements in the local x,y element coordinate system.

First set of cards describes plate bending action (I4, 6F10.2).

Col. 1 to 4 - N, normally set to material number is now always set to zero to activate input of material properties in the form 7b instead of 7a. Materials must be ordered sequentially.

Col. 5 to 14 - DM(I,1,1), element 1,1 of (3 X 3) material law I.

Col. 15 to 24 - DM(I,2,2), element 2,2 of (3 X 3) material law I.

Col. 25 to 34 - DM(I,3,3), element 3,3 of (3 X 3) material law I.

Col. 35 to 44 - DM(I,1,2), element 1,2 of (3 X 3) material law I.

Col. 45 to 54 - DM(I,1,3), element 1,3 of (3 X 3) material law I.

Col. 55 to 64 - DM(I,2,3), element 2,3 of (3 X 3) material law I.

Second set of cards describing in plane action (I4, 6F10.2).

Col. 1 to 4 - N, material number, must be ordered sequentially.

Col. 5 to 14 - DS(I,1), element 1,1 of (3 X 3) material law I.

Col. 15 to 24 - DS(I,2), element 2,2 of (3 X 3) material law I.

Col. 25 to 34 - DS(I,3), element 3,3 of (3 X 3) material law I.

Col. 35 to 44 - DS(I,4), element 1,2 of (3 X 3) material law I.

Col. 45 to 54 - DS(I,5), element 1,3 of (3 X 3) material law I.

Col. 55 to 64 - DS(I,6), element 2,3 of (3 X 3) material law I.

Note for definition of the above constants, see Section 4.3 in Chapter 4.

8. <u>Displacement Boundary Condition Array</u> (I4, 6L2, 2F10.2)

Kinematic constraints, 3 translations and two rotations can be specified on NUMBC cards for each top and bottom node. A separate card must be used to impose boundary conditions at each of the top and bottom nodes constrained. Note that a sufficient number of kinematic constraints have to be imposed to remove possible rigid body motion of the structure and the singularity of the stiffness matrix.

Col. 1 to 4 - M, nodal point number

Col. 5 to 6 - J1, set true, T, if U-component constrained

Col. 7 to 8 - J2, set true, T, if V-component constrained

Col. 9 to 10 - J3, set true, T, if W-component constrained

Col. 11 to 12 - J4, set true, T, if θ_X component constrained

Col. 13 to 14 - J5, set true, T, if θ_Y component constrained

If above specifications apply to a top node, no further data is needed. For a bottom node the following data need to be added.

Col. 15 to 16 - JB, set true, T, if constraints are applied at the nodes of the bottom deck.

Omit subsequent data if the above kinematic constraints act in the global coordinate direction. However, if a displacement or a rotation component is constrained in a direction which differs from the global X,Y axes then an angle must be specified separately for each of these constraints. The first angle is denoted by ALF for the displacement constraint V and is taken from the + global X axis to the direction along which the node remains free to displace $\bar{U}$. The second angle is denoted by BET for the rotational constraint θ_Y and is taken from the + global X-axis to the axis about which the node remains free to rotate, $\bar{\theta}_X$. Use right hand rule for + direction of both angles. The corresponding orthogonal components $\bar{V}$ and $\bar{\theta}_Y$ are set to zero if the constraint for V or θ_Y was set true, T .

Col. 17 to 26 - ALF, angle of skew for constraint $\bar{V}$ in degrees

Col. 27 to 36 - BET, angle of skew for constraint $\bar{\theta}_Y$ in degrees.

Nonzero displacement boundary conditions can be imposed by applying the corresponding kinematic constraints and by specifying the magnitude of given displacements in the input for nodal loads.

INPUT DATA FOR LOADING AND OUTPUT CONTROLS

Repeat subsequent set of cards for each load case.

9. Start Card for Loading (A6)

   Col. 1 to 4 - CHECK, each load case has to begin with a card having the word "LOAD" in the first 4 columns which initiates the input of the loading and output controls. Any number of load cases can be treated.

10. Loading Control Card (3I4, 2F10.2)

    Col. 1 to 4 - NLD, number of nodes with concentrated forces and moments. These must be applied separately at top and bottom deck nodes.

    Col. 5 to 8 - NLL, number of different live load intensities acting on the top deck elements in the z-direction, leave blank if no live load included.

    Col. 9 to 12 - NDL, set to 1 if dead load is included, otherwise leave blank.

If NLL > 0 the following data must be added.

Col. 13 to 22 - PLL, live load intensity [force/area] for NLL = 1 which is assumed uniform over the area of all top deck elements. If NLL > 1 the remaining live load intensities have to be specified together with the top deck elements over which they act on cards specified in paragraph 12.

If NDL = 1 the following data must be added.

Col. 23 to 32 - PDL, specific weight for dead load [force/volume]. Leave blank if NDL = 0.

11. <u>Concentrated Nodal Loads</u> (I4, 2L2, 2x, 5F10.2)

If NLD = 0 these cards are omitted.

Global nodal forces, 3 linear force components P_X, P_Y, P_Z and two moment components M_X and M_Y, can be specified on NLD cards for each node. A separate card must be used to input nodal loads at the top or bottom deck nodes.

Col. 1 to 4 - M, nodal point number

Col. 5 to 6 - TBOT, set true, T, if nodal force acts at the node of the bottom deck.

Col. 7 to 8 - TDIS, set true, T, if a nonzero displacement boundary condition is prescribed.

Col. 11 to 60 - FORCE(5), nodal load or displacement components in the global directions. They are ordered in the following sequence. First, the linear forces P_X,P_Y,P_Z then the moments M_X, M_Y.

12. <u>Uniform Live Loads and Dead Loads</u>

If NLL $\leq$ 1 these cards are omitted. Uniform live loads can be specified over individual deck elements which differ from the overall live load distribution PLL. For each additional live load intensity NLL > 1 the following information needs to be furnished.

First card (2I4, F10.2)

Col. 1 to 4 - I, live load number

Col. 5 to 8 - NEL, total number of deck elements subjected to this live load

Col. 9 to 12 - PLL, live load intensity [force/area]

Second set of cards (20I4)

Col. 1 to 80 - NOLL(NEL), sequential list of top deck elements subjected to live load number I having intensity PLL .

13. Output Control Card (5I4, F10.2)

Various options can be activated for computing, printing and punching of nodal displacements and nodal forces. In addition to that, internal forces can be determined and output at element center and at the nodes where all individual element contributions are averaged.

Col. 1 to 4 - T1, set to 1 if global nodal forces and reactions should be computed and printed.

Col. 5 to 8 - T2, set to 1 if internal forces at the center of the deck elements should be computed and printed

Col. 9 to 12 - T3, set to 1 if binary punched output desired. For detailed description of the punched output see Section 5.6, paragraph i.

Col. 13 to 16 - NSTO, total number of nodes at which internal forces should be computed, averaged and output together with their principal values. If NSTO = 0, output of nodal quantities is supressed; if NSTO = NUMNP (total number of nodes), these quantities are evaluated and output at all nodes; if 0 < NSTO < NUMNP, these quantities are determined and output at NSTO nodes only which have to be listed on subsequent cards.

Col. 17 to 20 - NDIA, total number of transverse web elements idealizing the diaphragms for which the averaging procedure at the nodes has to remain separate from that of the longitudinal web elements.

Col. 21 to 30 - ALF, angle in degrees from global X-axis to the direction along which the normal nodal stress resultants are to be computed and output. Use right hand rule for + direction of angle. These values override the computation of principal stress resultants at the nodes which are usually determined from the nodal averages.

If 0 < NSTO < NUMNP input on the following set of cards list of nodes at which internal forces should be computed, averaged and output together with their principal values.

14. List of Nodes for Internal Forces (20I4)

Col. 1 to 80 - NPS(NSTO), list of nodes where internal forces desired.

Omit these cards if NSTO = 0 or NSTO = NUMNP.

RUNNING SEQUENCE

Repeat for each subsequent load case cards specified in paragraphs 9 - 14 beginning with the start card LOAD.

Repeat for each subsequent problem cards specified in paragraphs 1 - 14 beginning with the start card START.

The program stops when a card with the word STOP in the first 4 columns is encountered.

5.5 Commentary on Generation Options

It is always possible to input all components of the deck and web element arrays and to specify all nodal coordinates. For mesh layouts with a certain degree of regularity, it is convenient to make use of mesh and coordinate generation options to reduce the required input data.

a) Deck Element Generation (see paragraph 4 of input specifications in Section 5.4).

Two types of generations are available for the nodal arrays of deck elements.

1) One Way Generation: If element cards N+1, N+2 -- N+L-1 are omitted and columns 33 to 40 are left blank on element card N+L the missing (L-1) element arrays will be generated by increasing the nodal numbers of the preceding element by NDIF. NDIF is assumed to be 1 if it is not specified in Cols. 29 to 32 of element card (N+L). The material numbers of the top and bottom deck elements are the same as those input for element N.

2) Two Way Generation: This option can be used when two adjacent strings of sequentially numbered elements have been defined previously. Two parameters need to be specified on the card for element N, which is the last element so far numbered:

Col. 33 to 36 - MOD, module ($m > 0$) equalling the difference in the element numbers of corresponding elements in adjacent strings of elements.

Col. 37 to 40 - NLIM, largest element number up to which generation desired ($> N$).

The i = 1,..4 nodal numbers of element N+1, N+2,..NLIM are generated by

$$i_K = i_{K-MOD} + (i_{K-MOD} - i_{K-2MOD})$$

where i_K denotes node i of the K^{th} element with K = N+1, N+2...NLIM. The material type numbers are set equal to their values for element (K-MOD). If NLIM = NUMEL no more element cards are needed. If NLIM < NUMEL the card of node (NLIM + 1) must follow.

b) Web Element Generation (see paragraph 5 of input specifications in Section 5.4).

Two types of generations are available for the nodal arrays of web elements similar to the generation of deck element arrays.

1) One Way Generation: If element cards N+1, N+2...N+L-1 are omitted and columns 21 to 28 are left blank on element card N+L, the missing (L-1) element arrays will be generated by increasing the nodal numbers of the preceding element by NDIF. NDIF is set to 1 if it is not specified in Cols. 17 to 20 of element card N+L.

2) Two Way Generation: This option can be used when two adjacent strings of sequentially numbered elements have been defined previously. Two parameters need to be specified on the card for element N, which is the last element already numbered:

Col. 20 to 24 - MOD, module (m > 0) equalling the difference of corresponding element numbers in adjacent strings.

Col. 25 to 28 - NLIM, largest element number up to which generation desired (> N).

The i = 1,2 nodal numbers of element N+1, N+2,..NLIM are generated by

$$i_K = i_{K-MOD} + (i_{K-MOD} - i_{K-2MOD})$$

where i_K denotes node i of the K^{th} element with K = N+1, N+2,..NLIM. The material type number is the same as for element (K-MOD). If NLIM = NBEAM, no more element cards are needed. If NLIM < NBEAM, the card of element NLIM + 1 must follow.

In order to make averaging of nodal quantities between coplanar web elements possible the following numbering scheme has to be adopted to identify web elements: Number first in increasing order transverse elements idealizing all transverse diaphragms, then continue numbering sequentially longitudinal elements idealizing all longitudinal web components.

c) Nodal Coordinate Generation (see paragraph 6 of input specifications in Section 5.4).

Two types of coordinate generations are available similar to the element array generations.

1) One Way Generation: If (L-1) nodal cards for points N+1, N+2, N+3, ...N+L-1 are omitted and Cols. 25-32 of the N^{th} card are left blank, the missing coordinates will be generated as those of L equally spaced points on a line joining N and (N+L). That is

$$DIV = L$$

$$\left.\begin{aligned} x_{N+K} &= x_{N+K-1} + (x_{N+L} - x_N)/DIV \\ y_{N+K} &= y_{N+K-1} + (y_{N+L} - y_N)/DIV \end{aligned}\right\} \quad \text{for } K = 1, 2, \ldots (L-1)$$

2) Two Way Generation: This option of coordinate generation can be used after two adjacent strings of sequential nodal points have been defined previously. It is activated by specifying on the card for node N.

Col. 25 to 28 - MOD, module (m > 0) equalling the difference of corresponding nodal numbers on the adjacent coordinate lines.

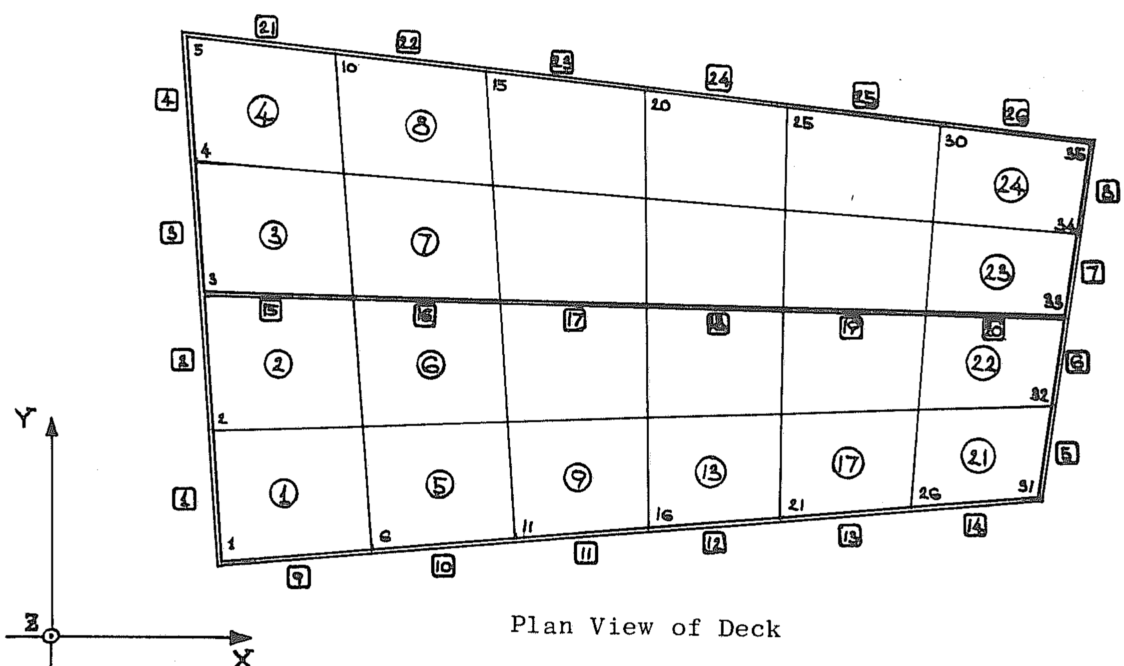
Col. 29 to 32 - NLIM, the largest node up to which the coordinate generation desired (> N).

The x,y coordinates of nodes N+1, N+2,..NLIM is generated from

$$x_K = x_{K-MOD} + (x_{K-MOD} - x_{K-2MOD})$$
$$y_K = y_{K-MOD} + (y_{K-MOD} - y_{K-2MOD})$$
for K = N+1,..NLIM

If NLIM = NUMNP no more nodal cards are needed. If NLIM < NUMNP, the card of node (NLIM + 1) must follow.

d) Example for Mesh Generation



Plan View of Deck

1,2,3 ... Nodal Numbers
① ② ... Deck Element Numbers
[1] [2] ... Web Element Numbers

In above figure each exterior edge has been subdivided equally to form mesh layout.

1) Deck Element Cards: 3 cards are needed

L	NP(1)	NP(2)	NP(3)	NP(4)	MA	MB	NDIF	MOD	NLIM
1	1	6	7	2	1	2			
5	6	11	12	7	1	2			
9	11	16	17	12	1	2		4	24

2) Web Element Cards: 5 cards are needed

L	NBP(1)	NBP(2)	MC	NDIF	MOD	NLIM
1	1	2	3			
5	31	32	3			
9	1	6	4			
15	3	8	4	5		
21	5	10	4	5	6	26

3) Nodal Coordinates: 4 cards are needed

N	X-ORD	Y-ORD	MOD	NLIM
1	x_1	y_1		
5	x_5	y_5		
6	x_6	y_6		
10	x_{10}	y_{10}	5	35

5.6 Output Description

The solution of each problem contains the following output information.

a) Input Echo

A printout of the input data is output with proper description of the data.

b) Nodal Displacements

All global displacements are output at the nodes of the top and bottom deck. The 5 components U, V, W, θ_X and θ_Y are listed sequentially for each node.

c) Nodal Forces

The global nodal forces are computed and printed if output control T1 is set to 1. This information yields the reactions to the loading and gives an insight into the round-off error and magnitude of the residual forces at all nodes.

d) Internal Forces at Center of Deck Elements.

Both stress resultants and moments are computed and printed at the center of the deck elements if output control T2 is set to 1.

e) Internal Forces at Nodes of Deck Elements.

If output control, NSTO > 0 internal force contributions of each element to a node, their averages and associated principal values are computed and output at NSTO specified nodes of both top and bottom deck.

f) Internal Forces of Web Elements

If output control NSTO > 0 the internal force contributions of each element, their averages and corresponding principal values are computed and output at NSTO specified nodes of both, transverse and longitudinal web elements. Averaging takes place if the second node of a web element coincides with the first one of an adjacent web element. Hence, to activate averaging of coplanar web elements only, one has to provide information regarding how many web elements lie in the transverse direction, NDIA, and how many lie in the longitudinal direction, NBEAM - NDIA. Therefore, averaging necessitates a general rule concerning the web element numbering. It is assumed that the transverse web elements in the diaphragms are numbered first increasing with the global X-direction while the longitudinal web elements follow second, increasing with the global Y-direction.

g) Direction of Internal Forces (for deck element only)

To reduce the effort for equilibrium checks in arbitrary directions, the output control ALF is available. ALF is the angle in degrees from the global X-axis to the direction along which the normal stress resultants are desired. Use right hand rule for + direction. The rotated components N_{nn}, N_{ns} and M_{nn}, M_{ns} are output together with the direction cosines instead of the principal values N_{max}, N_{min} and M_{max}, M_{min} which are usually output together with the angle in radians between the global X-axis and the direction of the maximum principal value given in radians. This capability simplifies tremendously equilibrium checks, e.g. for skewed box girders at sections parallel to the skewed support.

h) Log of Problem

The printed output of each problem is terminated by a summary listing execution times in different links and general information such as the number of degrees of freedom, the bandwidth and the block length.

i) Punched Output

With output control T_3 set to 1 the following arrays are punched on cards in binary format. First, the global displacement vector D(NEQ) is always punched out as one record. The nodal components are ordered in the same way as in the stiffness matrix, namely, nodal displacements $U, V, W, \theta_X, \theta_Y$ first at the nodes of the bottom deck and then at those of the top deck. An end of file card (789/1/7) follows. Second, the matrix SNP(NSTO, 12) of averaged internal forces in the deck is punched at NSTO nodes. The internal force components are arranged in the columns of matrix SNP, first the N_{xx}, N_{yy}, N_{xy}, M_{xx}, M_{yy}, M_{xy} components of the top deck and then those of the bottom deck. An end of file card (789/1/7) follows. Third, the matrix YYMO (NSTO, 12) of the M_{yy} contributions of each deck element to the nodal averages is punched at NSTO nodes. For each of the NSTO nodes the columns contain the element contributions to that node. The first 6 columns are reserved for the top deck and the remaining 6 columns contain element contributions to the bottom deck. The number of element contributions to a node coincides with the number of columns used, increasing in order. If there are less than a maximum of 6 element contributions per node, then the remaining columns are initialized and set to zero. An end of file card (789/1/7) follows. Fourth, if the set of NSTO nodes where nodal averages are output contains also nodes of web elements, the same nodal information as in second and third is punched for the longitudinal web elements. Since they exhibit one way bending only the M_{xx} and M_{xy} components are set to zero. As averaging of internal forces takes place between coplanar web elements only a maximum of two element contributions per node is considered.

5.7 General Remarks

The following comments are in place regarding the usage of program CELL.

a) Bandwidth

Number the nodes so as to minimize the bandwidth which depends on the largest difference of nodal numbers for any finite element.

b) Interpretation of Internal Forces

Recall that the finite element method is an approximate method of analysis. It is well known that the stresses derived through the standard stress-displacement relationships from the resulting nodal displacements do not satisfy differential equilibrium. In fact element stresses exhibit usually large discontinuities if simple expansions are used to approximate the field variables within each element. Hence, stresses at the element center are considered more reliable. If nodal stresses are desired, a simple averaging of all element contributions at a node leads to the most meaningful representation. In general, these averages violate considerably the natural boundary conditions at free edges and approximate poorly local stress concentrations. Refinement of the mesh is usually required to capture steep stress gradients within the element domains or stress boundary conditions along the surfaces.

c) Finite Element Types

Considerable computer time during formation of element stiffnesses can be saved if one considers repetitious element types. Two factors define an element type, the element geometry and the material properties. The program compares internally the geometries and material laws of consecutive elements. If both factors agree, the formation of the element stiffness is skipped together with its transformation into global coordinates. Hence, the user is able to reduce considerably the formation time by numbering consecutively elements of the same type.

d) Storage Requirements

The required storage for a given problem is determined during execution and is allocated automatically within the program. The following formulas are useful for computing the field length by hand if it is not possible to retrieve automatically the last word address of the program and to reset the field length during execution. This estimate is based on experience with a CDC 6400 computer using the FUN compiler.

ST = FIX + VAR

describes in general the amount of storage required for a specific problem. FIX is the fixed amount of storage area which is independent of the problem being solved while VAR denotes the variable storage area which is a function of the data being processed. The program passes through its 5 links during execution, each of which requires a minimum storage area for FIX and the associated blank COMMON areas. In most cases link 2 or 4 governs the storage requirement.

Define AA = 6*NUMEL + 5*NBEAM + 2*NUMNP + 15*NMAT

Link 1: FIX = 36267B (where B denotes the octal base)

VAR = AA + 6*NMAT

Link 2: FIX = 17573B

VAR = 4*NUMEL + 2*NBEAM + 2*NBLKL*IBANDW

Link 3: FIX = 21264B

VAR = AA + 2*NUMEL + 2*NEQ

Link 4: FIX = 17655B

VAR = 2*NBLKL*(IBANDW + 1) + NEQ

Link 5: FIX = 30316B

VAR = AA + NEQ + [NEQ + NUMEL + 7*12*NSTO + 2*NSTO]

The expression in brackets should be deleted if no controls for the printed output are activated.

The following variables depend on the size of the problem and are defined by

NUMEL - number of top deck elements

NBEAM - number of web elements

NUMNP - number of nodal points

NMAT - number of different material types

NBLKL - number of equations in a block

IBANDW - half of maximum bandwidth

NEQ - total number of equations

NSTO - number of nodes at which internal forces are output

c) Execution Time

A summary of central processing times is presented below for the solution of different problems by program CELL. The total execution times are given in seconds in addition to the times spent within each link of the overlay system as obtained from the CDC 6400 computer using the FUN compiler. Moreover, parameters such as NUMEL, NBEAM, NEQ, IBAND and NSTO are listed since they govern the computational effort of the problem considered. For the purpose of identification the finite element mesh is described by the notation $i*j/\alpha$, where i stands for the number of deck elements in the transverse direction, j for the number of deck elements in the longitudinal direction and α for the angle of skew.

TABLE 1

EXECUTION TIMES FOR PROGRAM CELL IN SECONDS

PROBLEM MESH	NUMEL	NBEAM	NEQ	IBAND	NSTO	LINK 1	LINK 2	LINK 3	LINK 4	LINK 5	TOTAL
Single Cell Box 1*3/90	3	8	80	40	0	1	0	0	1	0	2
Single Cell Box 2*5/90	10	14	180	50	0	1	1	0	3	0	5
Four Cell Box 6*22/90	132	122	1610	90	8	46	17	1	71	25	162
Example 4 8*20/90	160	72	1890	110	29	14	27	1	137	28	207
Example 5 8*20/45	160	72	1890	110	49	22	23	1	118	30	193
Example 6 5*20/90	93	49	1180	90	25	38	10	1	35	17	101

It is evident that from this limited data no general formula can be constructed for the timing of program CELL. Hence, the purpose of this summary is to present execution times for a few typical problems and to suggest certain relationships between the timing and some parameters. The timing of Link 1 where the global element stiffnesses are generated depends mainly on the number of web elements, NBEAM, but no clear trend can be seen since the type generation of repetitious elements obscures the picture. The timing of Link 2 where the element stiffness is added into the structural stiffness matrix is almost proportional to the number of top deck elements, NUMEL. The timing of Link 3 where the global load vector is formed is negligible in comparison to the other links. The timing of Link 4 where the system of equations is solved is in general proportional to the number of operations, $0.5 \times NEQ \times IBAND^2$, but the variable bandsolver obscures this relationship taking advantage of the sparsity of the matrix. The timing required for the solution of additional load cases amounts to 10% of the time used for decomposition of the structural stiffness matrix. The timing of Link 5 where the solution of the problem is output depends mainly on the total number of elements, NUMEL + NBEAM, and the number of nodes where the results are output, NSTO.

A very rough estimate for the total time can be obtained by assuming that the execution time is proportional to the number of degrees of freedom, NEQ.

6. EXAMPLES

Several examples of gradually increasing complexity have been chosen to illustrate the application of the program CELL. Whenever possible, the results obtained are compared with values from other independent solutions.

Examples 1, 2 and 3 (Figs. 8, 9 and 10) deal with the analysis of isotropic plates of rectangular or parallelogrammic shape subjected to in plane or transverse loadings. The results can be compared in the case of the skewed sheet with solutions using a refined finite element analysis, in the case of the cantilever with results obtained from refined beam theory and in the case of the rhombic plate with analytical solutions using series expansions. These independent solutions may be considered exact for the purpose of comparison.

Example 4 (Fig. 11) deals with the analysis of a non-skew, two cell box girder bridge simply supported at the end diaphragms which are perpendicular to the longitudinal axis. This structure is subjected to a concentrated load acting on the outside girder at the midspan cross-section. The results can be compared directly with those obtained by the folded plate theory using the program MULTPL, which may be considered exact for the purpose of comparison.

Example 5 (Fig. 15) deals with the analysis of the same two cell box girder bridge, but now supported on end diaphragms which are skewed 45° to the longitudinal axis. Since no analytical or experimental solutions are presently available for skew box girder bridges, internal equilibrium checks are performed at sections parallel to the skewed supports providing an insight into the accuracy of the resulting stress distribution.

Example 6 (Fig. 19) is selected to illustrate the capabilities of program CELL. An idealized highway branch is chosen to show one of the complex geometric configurations the program can treat. Again, since no other method of analysis is available, internal equilibrium checks are performed to assess the accuracy of the resulting stress distribution.

6.1 Example 1 - In Plane Analysis of Skewed Sheet

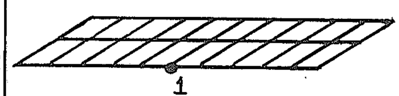
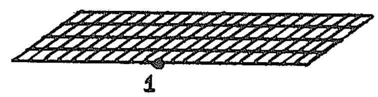
This example has been chosen to demonstrate the accurate representation of the in plane behavior by using the deck element Q8D11. This mixed model is illustrated in Fig. 2 and described in detail in Section 3.1. The parallelogrammic structure with an angle of skew of 30 degrees is shown in Fig. 8. The sheet is subjected to two concentrated in plane loads and is supported along its skewed edges. This plane stress example was used in reference [5] to compare the accuracy of displacements and stresses obtained from various elements of the 8 DOF and the 12 DOF families.

The results from a refined finite element analysis, denoted by LSE, serve as basis of comparison because there is no exact solution available for this complex boundary value problem. A detailed formulation of this linear strain quadrilateral having 16 DOF is given in reference [7]. The LSE analysis maintains a lower bound to the exact displacements as displacement continuity is fully maintained.

Table 2 presents a summary of V-displacements at point 1 to measure the accuracy of the solutions. The results of a coarse and a fine mesh are given for the element Q8D11 using an "exact" 3 X 3 and an "approximate" 2 X 2 Gaussian integration scheme which are indicated by a (3) or (2) after the element designation Q8D11. For purpose of comparison, analogous solutions are obtained with the displacement model Q8D8 which is formed using the standard bilinear expansions for both displacement components and using a 2 X 2 numerical integration scheme.

TABLE 2

EXAMPLE 1 - COMPARISON OF V-DISPLACEMENTS
(ft X 10^{-4}) AT POINT 1

FINITE ELEMENT	COARSE MESH 	FINE MESH 
LSE [7]	-	54.51
Q8D8	11.48	22.16
Q8D11(3)	30.44	42.80
Q8D11(2)	51.49	53.74

First, one observes that all solutions lie below the lower bound obtained from the LSE analysis, hence they maintain a lower bound too.

Second, the results exhibit the tremendous effect of the internal node and the mixed formulation coupled with the relaxation of the integration rule. The combination of all these "improvements" applied to the standard element Q8D8 yields the element Q8D11(2) which performs as well as the higher order quadrilateral LSE having 16 fundamental DOF instead of 8. For these reasons this element was finally chosen to represent the in plane behavior of the deck elements in the program CELL.

6.2 Example 2 - In Plane Analysis of Cantilever

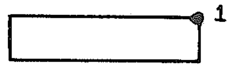
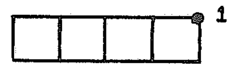
This example has been chosen to demonstrate the accurate representation of the in plane behavior by web element QUSP12. This displacement model is illustrated in Fig. 4 and described in Section 3.2. The structure, shown in Fig. 9, is subjected to two concentrated in plane loads at the tip of the cantilever. This plane stress example was used in reference [5] to compare displacements and stresses of various so called "spar" elements. The theoretical tip deflection is obtained from refined beam theory including shear deformations. As the root section is prevented from warping the theoretical value forms an upper bound to the exact tip deflection, but the error is less than 1/2000 as shown in reference [7]. In the finite element idealization displacement boundary conditions are prescribed to prevent warping at the fixed end section.

Table 3 presents a summary of V-displacements at the tip of the cantilever to measure the accuracy of the solutions. The results are given for a coarse and a fine mesh both idealizing the depth of the cantilever by one element. The solutions are obtained from the analysis using the web element QUSP12 with a 3 X 3 Gaussian integration scheme and the mixed model Q8D11(2) with a 2 X 2 Gaussian integration scheme. For purpose of comparison, an analogous solution is obtained using the displacement model Q8D8.

TABLE 3

EXAMPLE 2 - COMPARISON OF V-DISPLACEMENTS (in)

AT POINT 1

FINITE ELEMENT	COARSE MESH 	FINE MESH 
THEORY [5]	0.3558	
Q8D8	0.0475	-
Q8D11(2)	0.2693	0.3493
QUSP12	0.2533	0.3283

First, one observes that all solutions including that obtained from the mixed model Q8D11(2) with a relaxed integration rule lie below the theoretical results maintaining a lower bound for all practical purposes.

Second, the results exhibit again the tremendous effect of the internal node and the mixed formulation which combined with a relaxed integration rule yields the element Q8D11(2). Note the excellent performance of this element if subjected to in plane bending.

Third, results of the element QUSP12 compare very well with the theoretical solution. This element provides a cubic variation of the v-displacements along the longitudinal edges introducing a rotational $\omega = \frac{\partial v}{\partial x}$ at each node in order to capture the in plane bending of the web. It also maintains continuity with the transverse bending displacements of adjacent deck elements simply by enforcing compatibility of rotations. For these reasons the element QUS12 was finally chosen to represent the in plane behavior of the webs in the program CELL.

6.3 Example 3 - Plate Bending Analysis of Rhombic Plate

This example has been chosen to demonstrate the plate bending representation of the deck by element Q19. This displacement model is illustrated in Fig. 3 and described in detail in Section 3.1. The rhombic plate with an angle of skew of 40 degrees is shown in Fig. 9. This structure is subjected to a uniformly distributed transverse loading normal to the plane of the plate and is simply suppported along its edges.

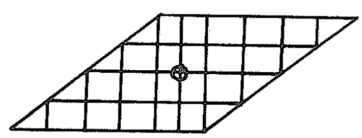
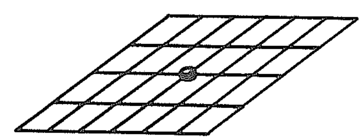
The theoretical results for transverse displacements and principal moments at the center of the plate are obtained from reference [8] which presents analytical solutions using series expansions. As this solution uses collocation to enforce equilibrium and continuity along a diagonal of the plate the results depend on sufficient control of the accuracy of boundary collocation. In general, the results of reference [8] on the flexural analysis of rhombic plates can be considered exact from a practical point of view.

Two finite element idealizations are used to discretize the structure, both having the same number of DOF and bandwidth, hence, posing the same computational problem. The first mesh layout idealizes the structure by triangular and rectangular elements while the second mesh layout uses parallelogrammic elements exclusively. Two types of boundary conditions are enforced at the corners of the idealized rhombic plate. The first clamps the corners by imposing $W = \theta_X = \theta_Y = 0$ at the corner nodes while the second enforces only $W = 0$ with the rotations being free.

Table 4 presents a summary of various solutions for transverse displacements and principal moments at the center of the rhombic plate. The results are given for both types of mesh layouts and boundary conditions described above.

TABLE 4

EXAMPLE 3 - COMPARISON OF VERTICAL DISPLACEMENTS $W[ft \times 10^{-2}]$ AND MOMENTS $M[k\text{-}ft/ft]$ AT CENTER OF RHOMBIC PLATE

BOUND. COND. AT CORNERS	TRIANG.-RECTANG. MESH 			PARALLELOGRAMMIC MESH 		
	W	M_{max}	M_{min}	W	M_{max}	M_{min}
THEORY [8]	9.58	-1.80	-2.81	9.58	-1.80	-2.81
$\theta = 0$	6.95	-1.21	-2.47	6.88	-1.06	-2.29
$\theta \neq 0$	9.98	-1.95	-2.85	9.69	-2.07	-3.16

First, one observes that the solutions obtained by clamping the rotations, $\theta = 0$, lie far below the theoretical results. The parallelogrammic mesh yields stiffer or lower results than the rectangular mesh layout with adjustment for the skewed boundaries by triangles. It is a well known fact that the finite element approximations are too crude to capture the singularities at the obtuse corners. Since convergence of the finite element results is very slow in the case of skewed plates a highly refined mesh layout is required to provide acceptable results.

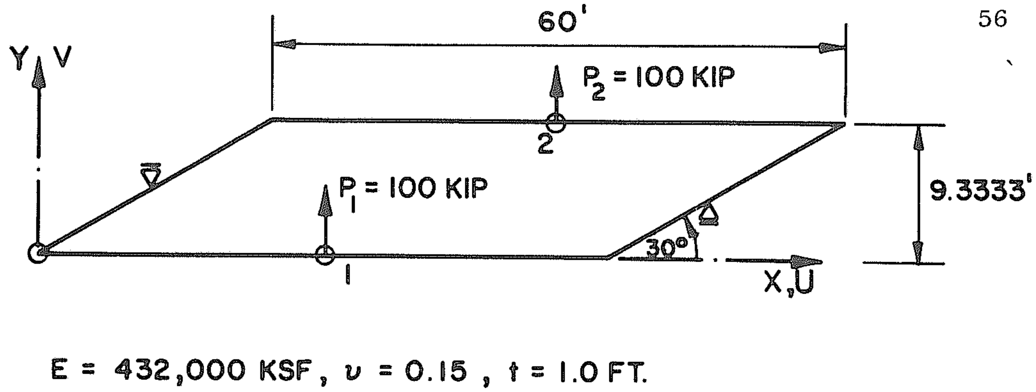


FIG. 8 EXAMPLE 1 - IN PLANE ANALYSIS OF SKEWED SHEET

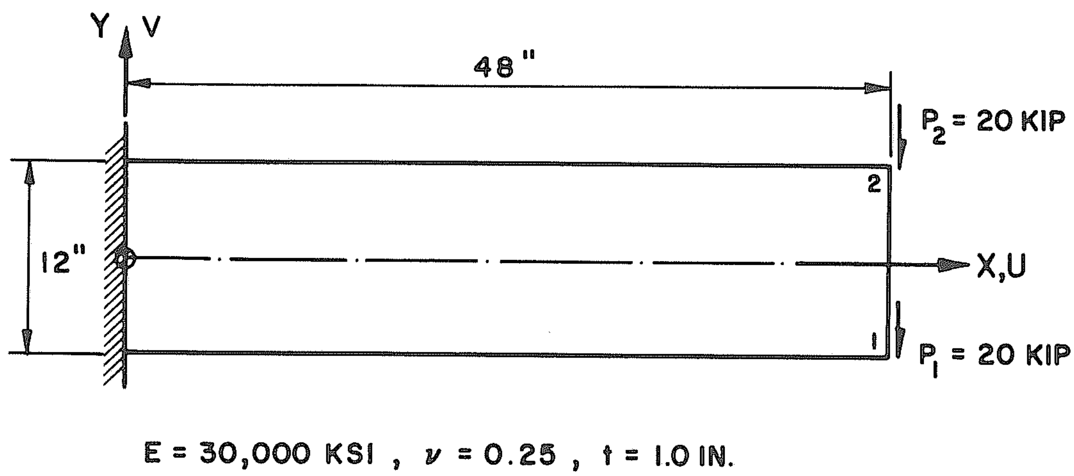


FIG. 9 EXAMPLE 2 - IN PLANE ANALYSIS OF CANTILEVER

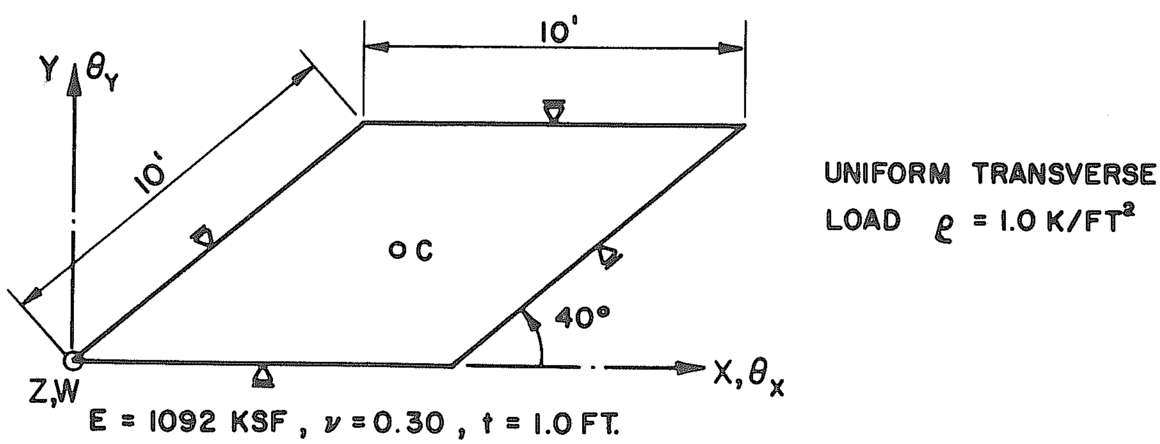


FIG. 10 EXAMPLE 3 - PLATE BENDING ANALYSIS OF RHOMBIC PLATE

Second, to speed up the slow rate of convergence the corner rotations are released from clamping. The elements adjacent to the obtuse corners are now able to approximate the steep gradients of the curvature field. One can observe that this artificial release improves the results tremendously but it destroys boundedness of the solution. Note that the rectangular mesh layout yields again displacements which are larger than those obtained from the parallelogrammic mesh but the moment resultants follow the opposite pattern.

6.4 Example 4 - Two Cell Box Girder Bridge on Right Supports

This example has been chosen to demonstrate the accuracy of the finite element program CELL as applied to the analysis of cellular bridge structures. The geometry and material properties of the non-skew box girder bridge to be analyzed are described in Fig. 11 together with its loading and boundary conditions.

As this box girder is prismatic and simply supported at the end diaphragms, folded plate theory can be applied to the analysis of this structure. Program MULTPL, which was presented in reference [1], is based on folded plate theory. It is applied to the analysis of this two cell box girder and the results may be considered exact for the purpose of comparison.

The box structure is idealized by the finite element mesh layout illustrated in Fig. 12. Due to the simple geometric configuration 3 ft. X 3 ft. square elements can be used throughout with three elements idealizing the top and bottom deck of each cell in the transverse direction and one element the height of each web.

Observe that more realistic boundary conditions are imposed in the finite element layout than those assumed in folded plate theory. In the element idealization the nodes underneath the ends of the longitudinal webs are prevented from vertical movement supporting the flexible end diaphragms while the end diaphragms of folded plate theory are assumed completely rigid within their plane and do not exhibit any stiffness out of their plane. This difference of boundary conditions is negligible from a practical point of view if localized regions near the supports are excluded from the comparison.

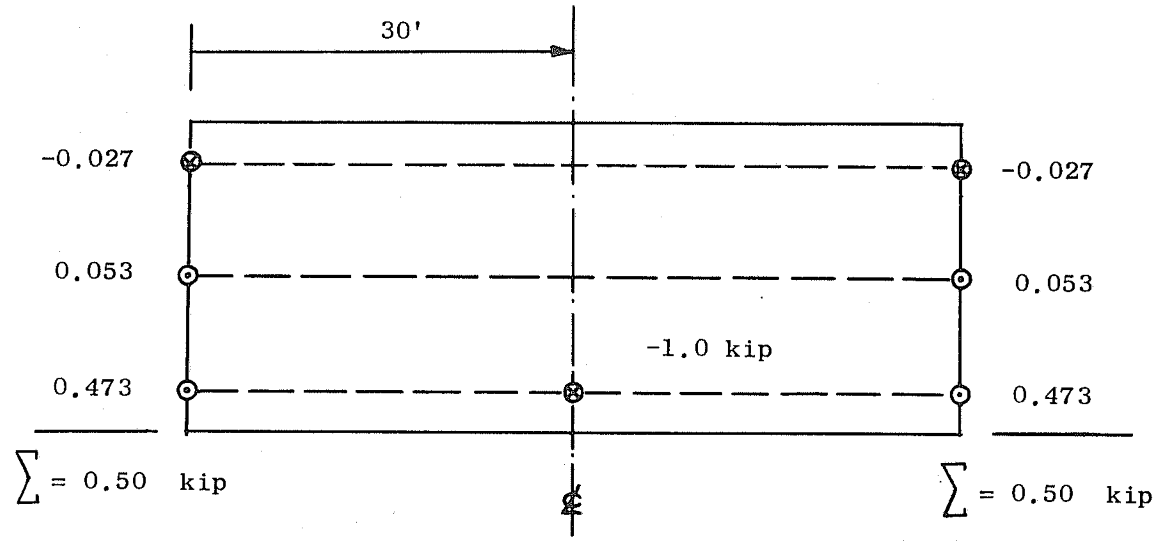
Figure 13 illustrates the distribution of vertical displacements W at the midspan cross-section and along the top of the longitudinal outside girders. Note the excellent agreement of displacements between both solutions, the theoretical results obtained from MULTPL and the approximate finite element results obtained from CELL, maintaining throughout a relative difference below 2%.

In standard finite element analysis stresses are computed for each individual element. Recall that the resulting stress distribution neither satisfies differential equilibrium nor stress boundary conditions in a local sense. Furthermore, as long as simple expansions are used to describe the energy variation within an element, large discontinuities of stresses occur along interelement faces. In order to obtain a representative value for the resulting stress distribution it is common practice to determine nodal stresses simply by averaging all element contributions to a specific node. Averaging has to take place for the deck and web components separately.

Figure 14 illustrates the distribution of the longitudinal stress resultants N_x at the midspan cross-section and along the top of the longitudinal outside girders. The theoretical solutions obtained from MULTPL agree very well with the results from CELL in regions far enough from the singularity underneath the concentrated load. In the vicinity of this point, the chosen finite element mesh layout cannot capture the steep stress gradients. Further mesh refinement is required to improve the values near the load. The longitudinal stress resultant N_x is presented along the top of the outside girders to provide some insight into the load distribution of the two cell box structure. The transverse distribution of N_x yields the information necessary for calculating the total internal moment at midspan. The ratio of this internal moment, M_{int}, determined by integration of the resulting internal stress distribution and the moment, M_{stat}, computed from overall statical considerations at a cross-section will be defined by the coefficient μ.

$$\mu = \frac{M_{int}}{M_{stat}}$$

This coefficient gives some insight into the accuracy of the resulting stress distribution which is particularly important in cases where no theoretical solutions are available. It provides a criterion for establishing confidence in the finite element results which are approximate in nature. As long as one considers only cross-sections parallel to the supports, the magnitude of individual reactions is not required, only their sum need to be known to evaluate M_{stat}. The sum of the reactions at each support are easily computed from statics while the finite element analysis yields their individual magnitudes which look for Example 4 as follows:



Two contributions, M_N and M_M, make up the total internal moment M_{int}. The contribution M_N to the internal moment M_{int} is established by taking moments of those forces in deck and web components which are equivalent to the internal resulting stress distribution about the neutral axis of the cross-section. The statically equivalent forces are simply determined by numerical integration of the normal stress resultants N_n at a cross-section, say by the trapezoidal rule. The factor M_M is determined by numerical integration of the normal slab moments M_n at the same section.

The contribution of internal stress and moment resultants to the gross internal moment at midspan is given by

$$M_{int} = M_N + M_M = 14.08 + 0.19 = 14.27 \text{ k-ft.}$$

The statical moment, M_{stat}, can be found directly from the external loading and reactions and equals in this case $0.500 \times (30) = 15.0$ k-ft.

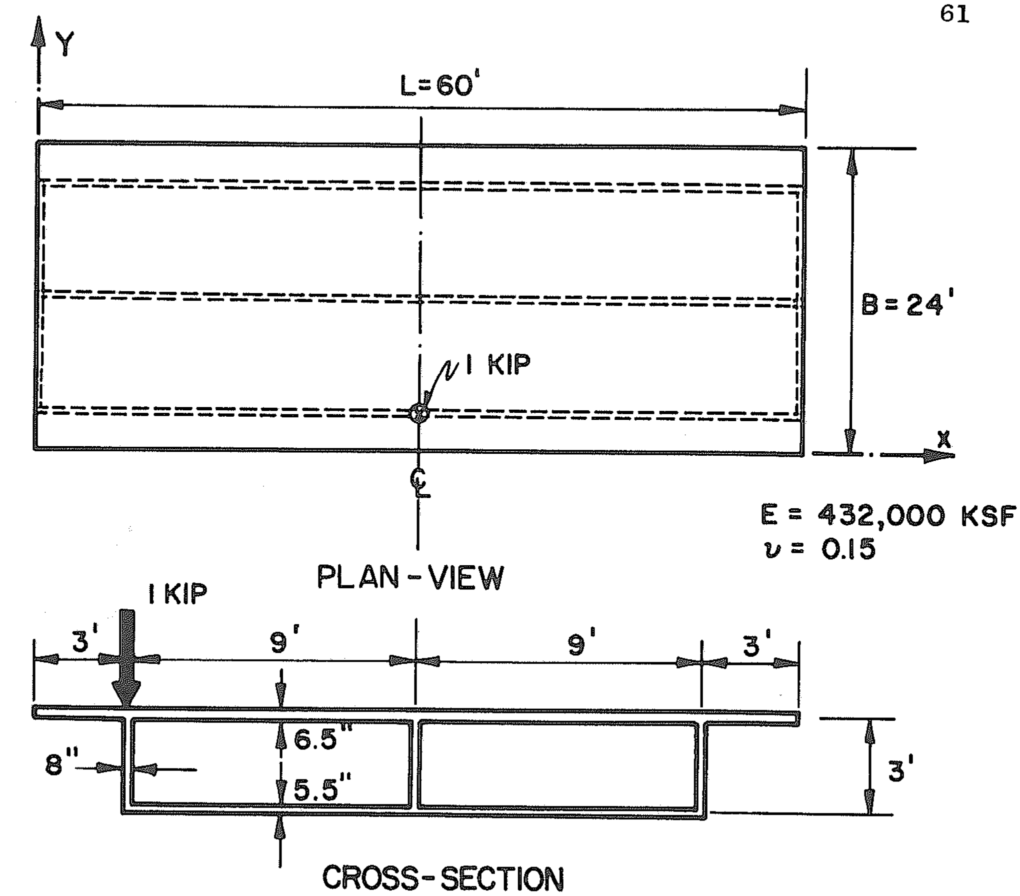


FIG. 11 EXAMPLE 4 - TWO CELL BOX GIRDER BRIDGE ON RIGHT SUPPORTS

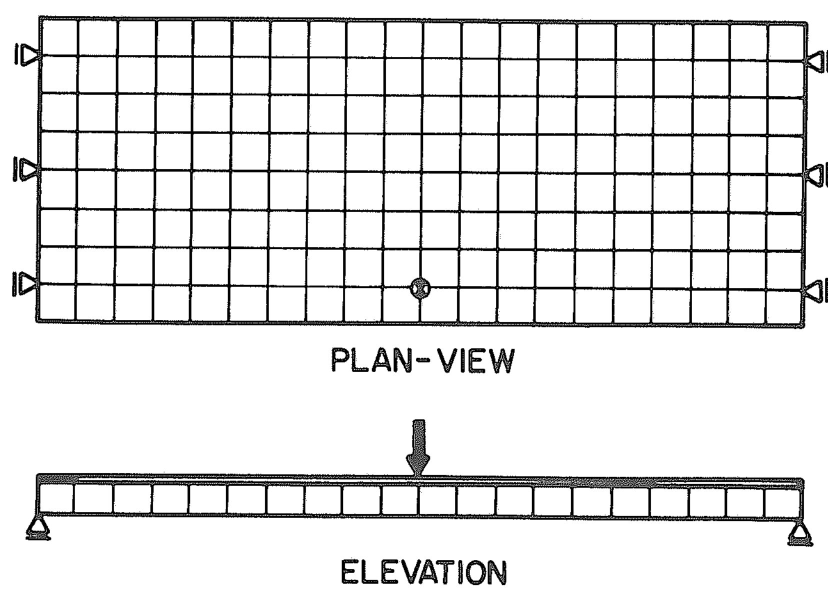
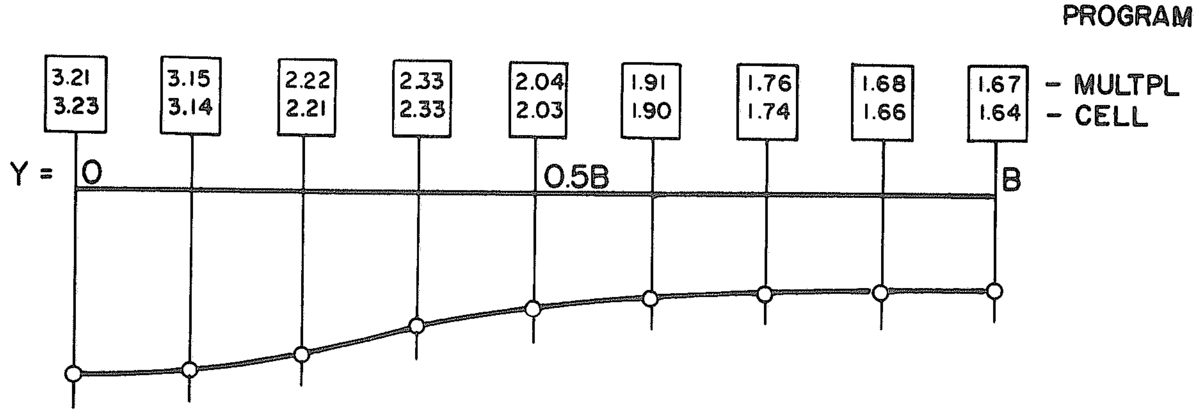
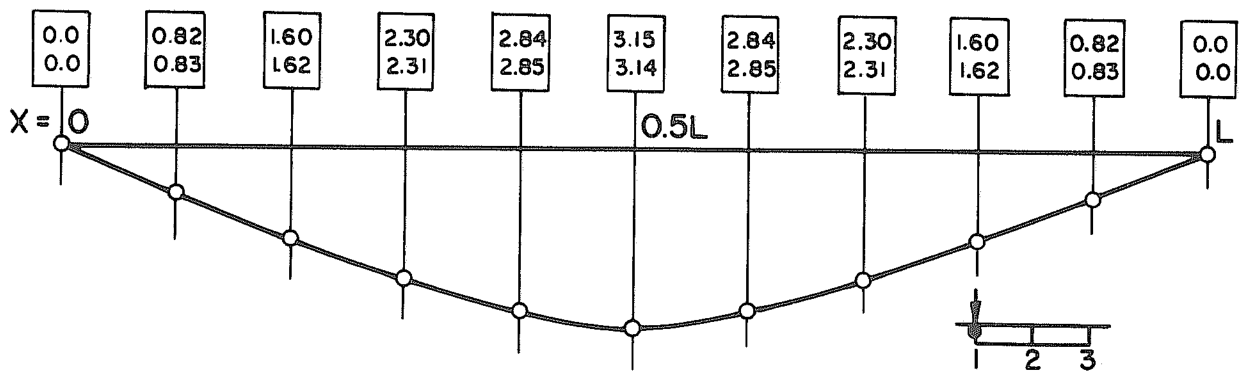


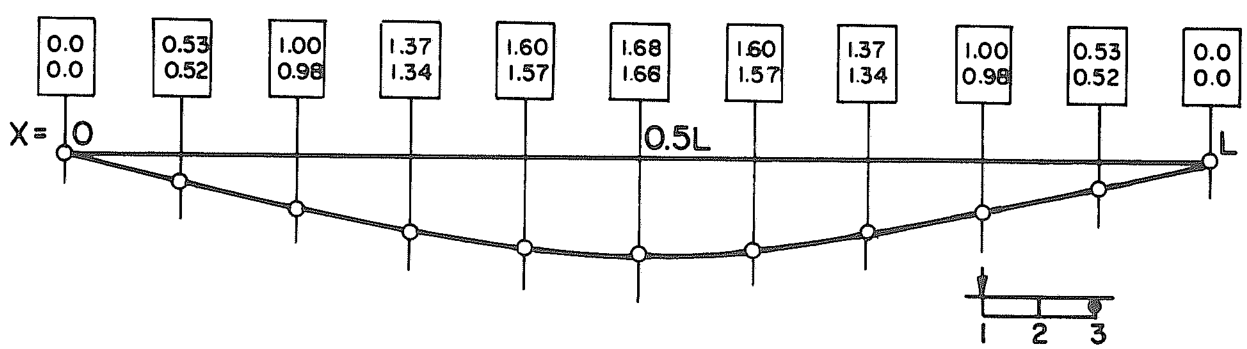
FIG. 12 EXAMPLE 4 - FINITE ELEMENT IDEALIZATION



TRANSVERSE DISTRIBUTION OF W AT TOP DECK OF MIDSPAN CROSS-SECTION



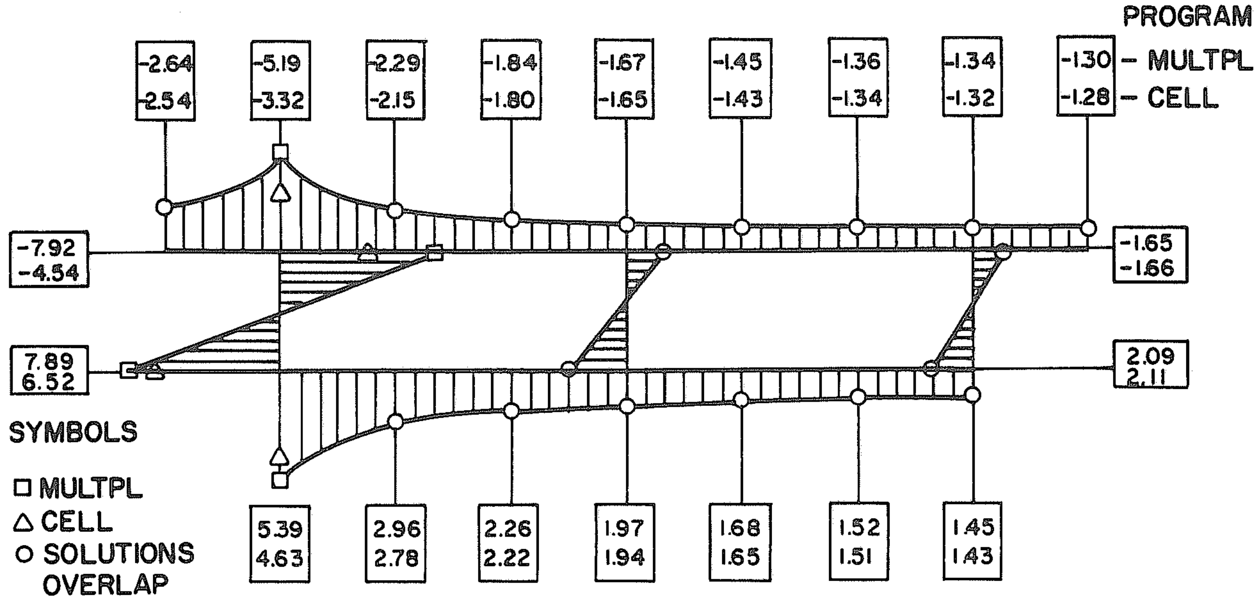
LONGITUDINAL VARIATION OF W ALONG TOP OF GIRDER 1



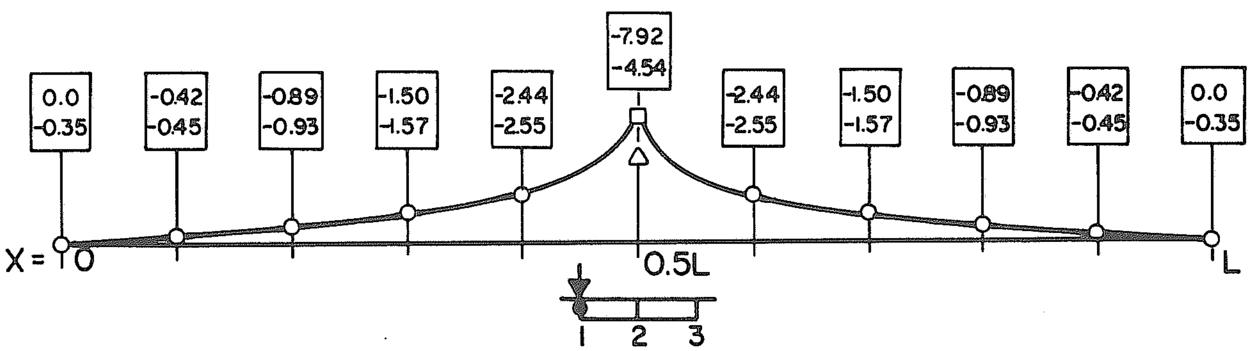
LONGITUDINAL VARIATION OF W ALONG TOP OF GIRDER 3

FIG. 13 EXAMPLE 4 - RIGHT BOX, VERTICAL DISPLACEMENTS

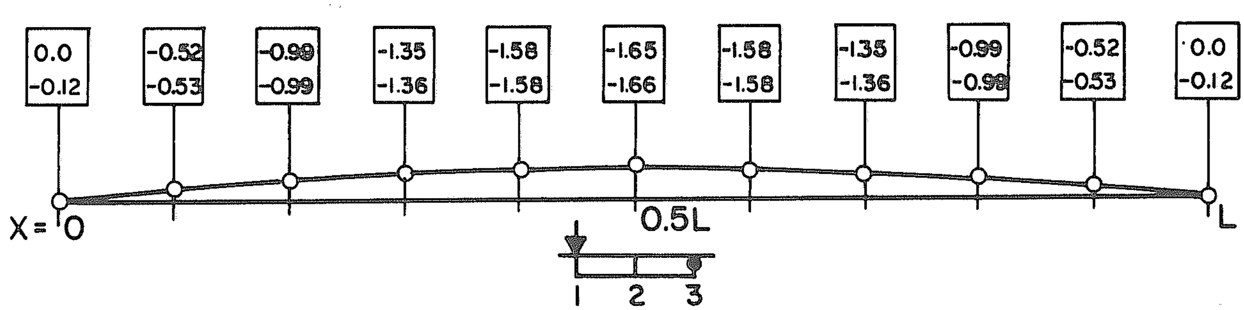
$W(ft \cdot 10^{-4})$



TRANSVERSE DISTRIBUTION OF N_x AT MIDSPAN CROSS-SECTION



LONGITUDINAL VARIATION OF N_x AT TOP OF GIRDER 1



LONGITUDINAL VARIATION OF N_x AT TOP OF GIRDER 3

FIG. 14 EXAMPLE 4 - RIGHT BOX, LONGITUDINAL STRESS RESULTANTS N_x ($K/FT \cdot 10^{-1}$)

The coefficient μ is then computed from

$$\mu = \frac{M_{int}}{M_{stat}} = \frac{14.27}{15.0} = 0.95$$

Observe that the violation of equilibrium by the resulting stress distribution by 5% is negligible considering that the section is taken directly underneath the concentrated loading. This error would decrease appreciably at any other section away from stress concentrations and singularities as shown in reference [5].

The total computer time necessary for execution of this problem amounted to 3 minutes and 27 seconds. A detailed discussion of execution times as obtained on the CDC 6400 computer is presented in Section 5.7

6.5 Example 5 - Two Cell Box Girder Bridge on Skewed Supports

This example has been chosen to demonstrate the capability of the finite element program CELL to analyze box girder bridges on skewed supports. In principle the same structure as in example 4 is used, the only difference being the direction of the supporting end diaphragms which are now inclined at an angle of 45 degrees with the longitudinal axis. The geometry and material properties of the skewed box girder bridge are described in Fig. 15 together with its loading and boundary conditions.

Unfortunately, no other analytical or experimental solutions are presently available to compare these results with. Only internal equilibrium checks in form of the μ coefficients can be performed in order to assess the accuracy of the resulting stress distribution.

The box structure is idealized by the finite element mesh layout illustrated in Fig. 16. Results in reference [5] suggest that rectangular and triangular elements should be used instead of parallelogrammic elements to idealize skewed regions. Hence, the structure is discretized by 3 ft. X 3 ft. square elements while right triangles adjust the mesh to the skewed boundaries. Similar to example 4 three elements idealize the top and bottom deck of each cell in the transverse direction and one element the height of each web.

The listing of the data for this typical example is included in Appendix B for those wishing a check case from Program CELL.

Figure 17 illustrates the distribution of vertical displacements W along the midspan cross-section and along the top of the longitudinal outside girders. The midspan section refers to the skewed section A-A parallel to the supports which passes through the center of the parallelogrammic plan view. In comparison to the associated right box the displacements are considerably smaller, especially those of the unloaded outside girder. This is no surprise since the clear span between the supports reduces from 60 ft. to 42.42 ft. and the effective moment of inertia of the skewed section increases by 1.414. Moreover, due to the continuity between longitudinal webs and diaphragms the corners are effectively restrained from rotation.

Figure 18 illustrates the distribution of the normal stress resultants N_n at the skewed midspan cross-section and of N_x along the top of the longitudinal outside girders. Again, only the averages of all element contributions to a node are given, thus smoothing stress discontinuities along interfaces. A comparison with the values obtained for the associated right box illustrates a vast reduction in the magnitude of the stress resultants due to reduction of effective

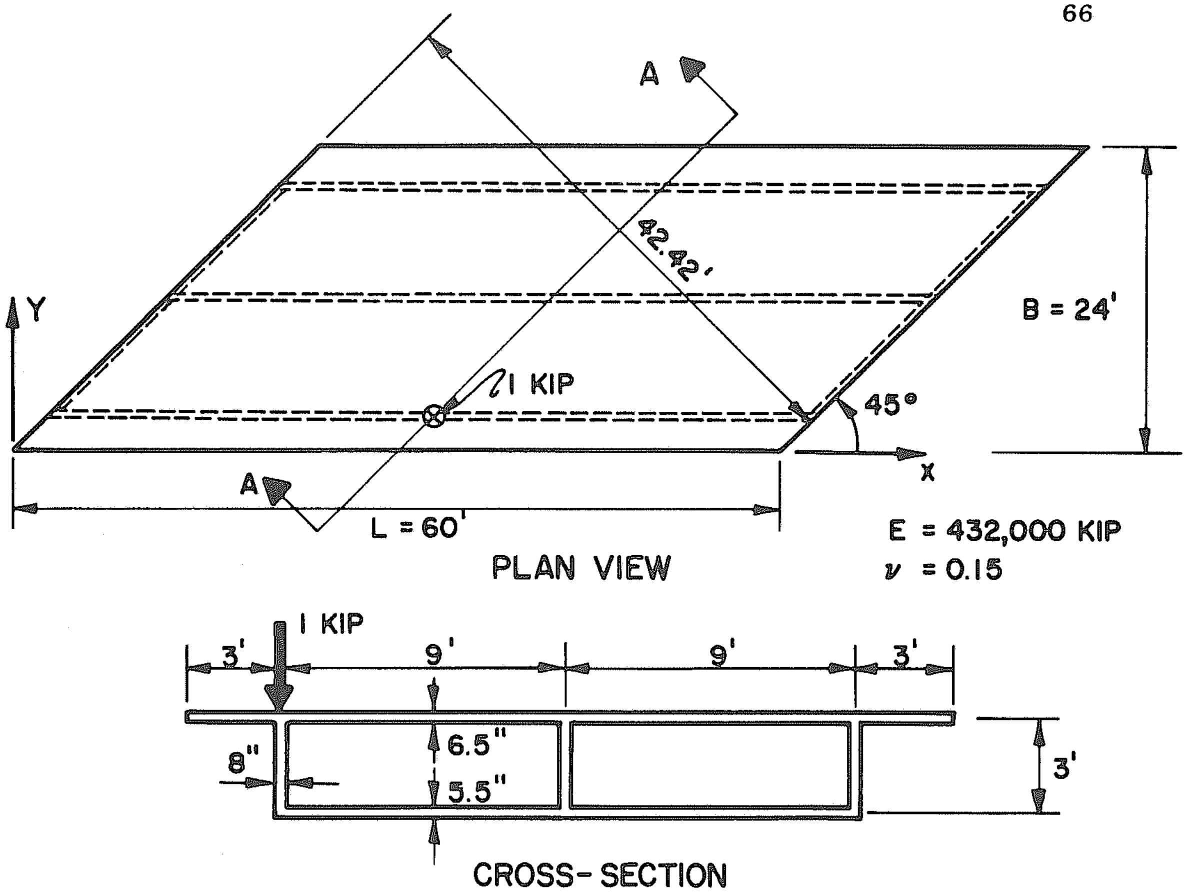


FIG. 15 EXAMPLE 5 - TWO CELL BOX GIRDER BRIDGE ON SKEWED SUPPORTS

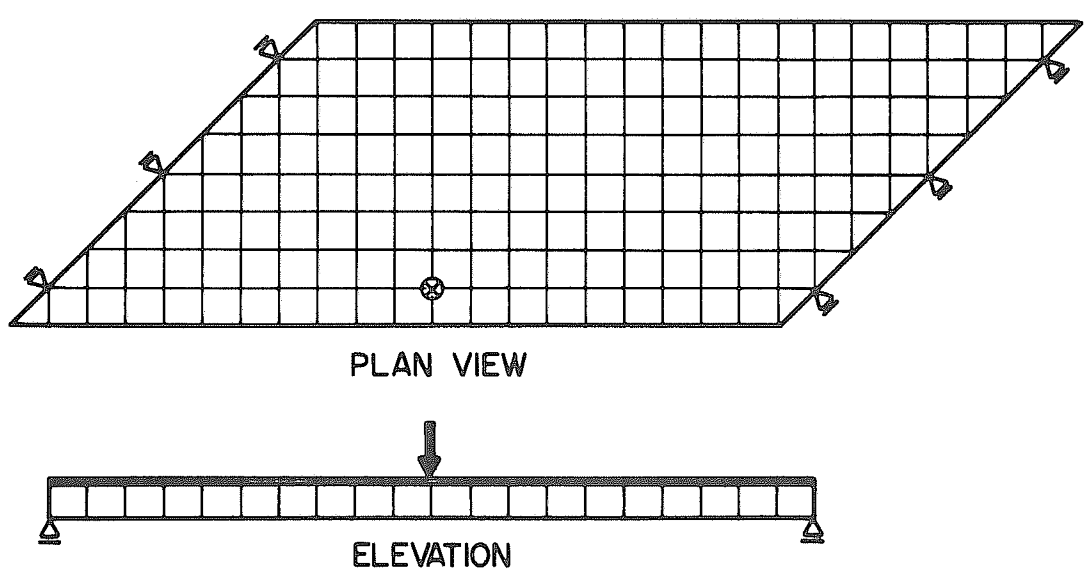
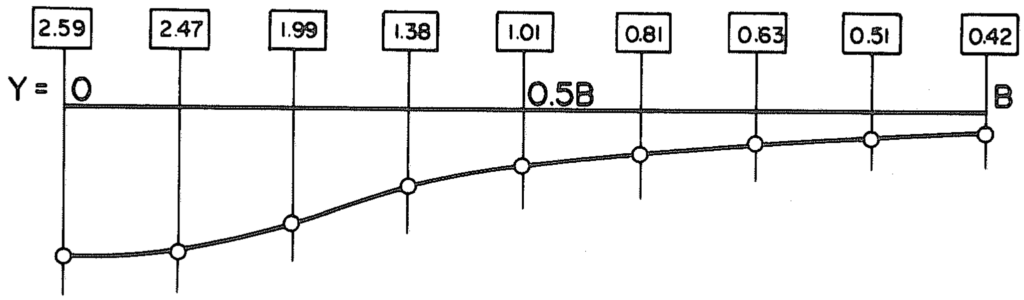
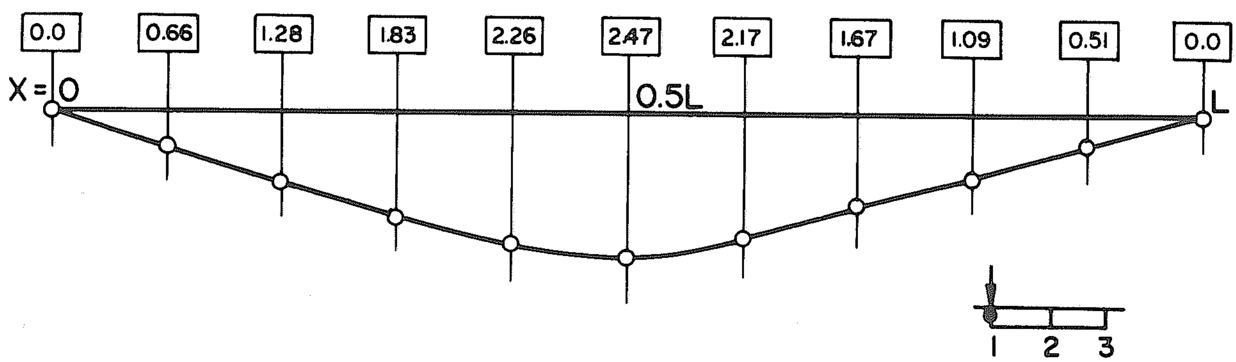


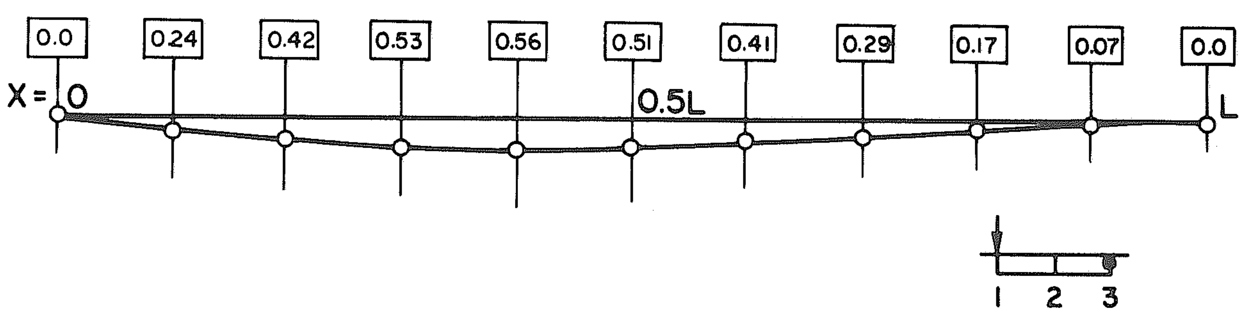
FIG. 16 EXAMPLE 5 - FINITE ELEMENT IDEALIZATION



TRANSVERSE DISTRIBUTION OF W AT TOP DECK OF MIDSPAN CROSS-SECTION

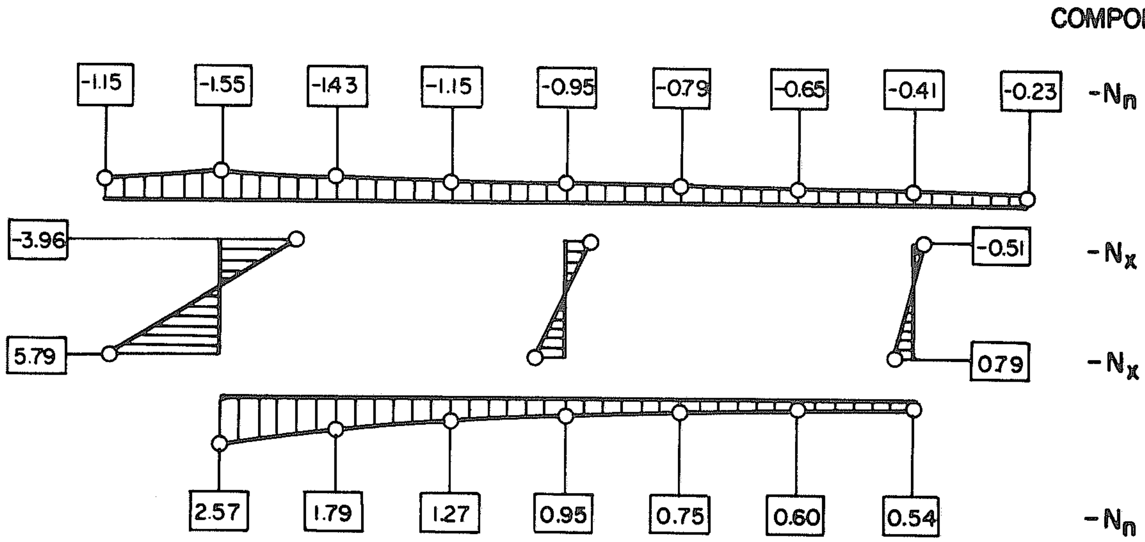


LONGITUDINAL VARIATION OF W ALONG TOP OF GIRDER 1

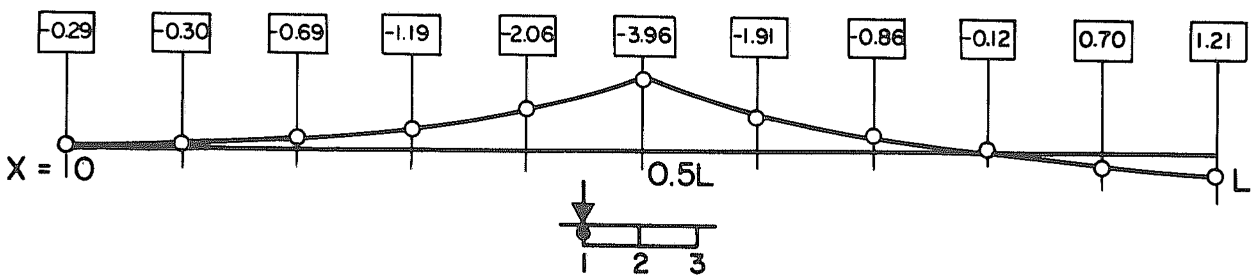


LONGITUDINAL VARIATION OF W ALONG TOP OF GIRDER 3

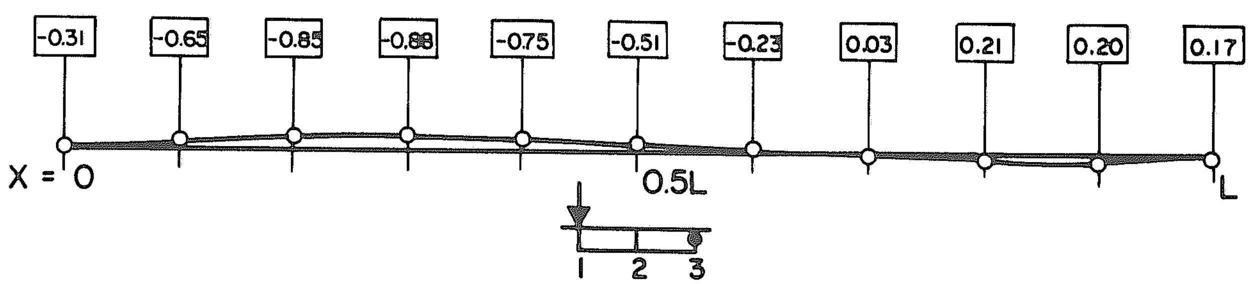
FIG. 17 EXAMPLE 5 – SKEWED BOX, VERTICAL DISPLACEMENTS $W\ (ft \cdot 10^{-4})$



TRANSVERSE DISTRIBUTION OF N_n AND N_x AT MIDSPAN CROSS-SECTION PARALLEL TO SKEWED SUPPORTS



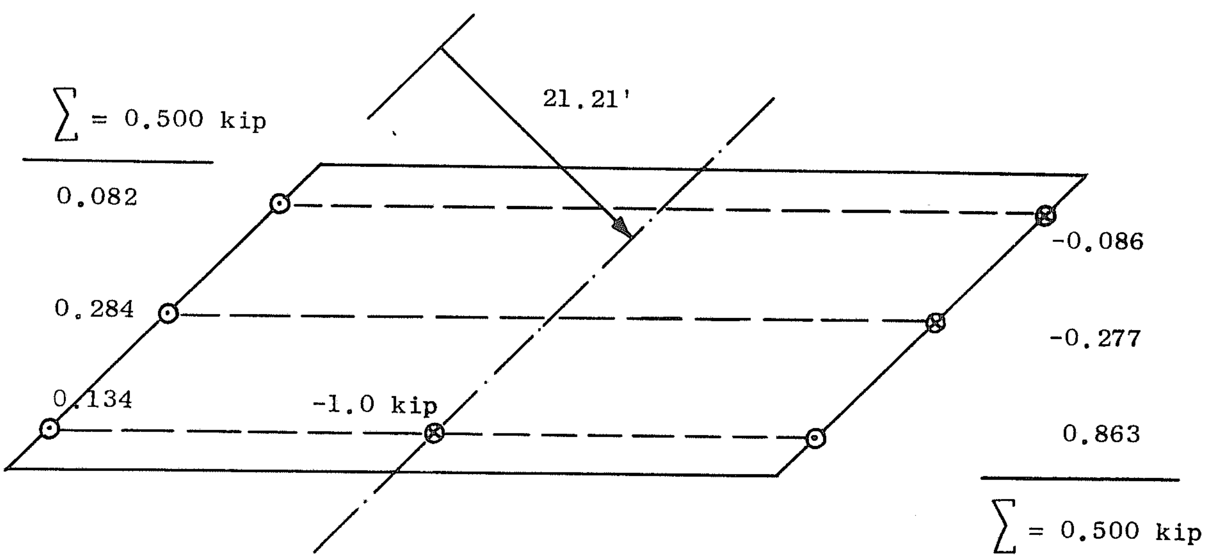
LONGITUDINAL VARIATION OF N_x AT TOP OF GIRDER 1



LONGITUDINAL VARIATION OF N_x AT TOP OF GIRDER 3

FIG. 18 EXAMPLE 5 – SKEWED BOX, LONGITUDINAL STRESS RESULTANTS N_x AND N_n (K/FT · 10^{-1})

span, the increase in effective moment of inertia and the clamping effect at the corners. This damping effect causes a change in sign of the longitudinal N_x stress resultants along the top of the outside girders. This quantity provides some insight into the load distribution of a two cell box girder on skewed supports. The transverse distribution of N_n yields the information necessary for calculating the internal moment at the midspan cross-section. As this section is taken parallel to the skewed supports the total sum of reactions at each support is known from statics and the sum of the reactions obtained from the computer output should equal this value. Hence, the gross statical moment, M_{stat}, can be easily computed for any section parallel to the skewed supports. For completeness, the magnitudes of individual reactions are given below as obtained from the finite element analysis. For this case, $M_{stat} = 0.500X(21.21) = 10.61$ k-ft.



M_N, the contribution of the internal stress resultants normal to the skewed midspan section, is obtained by taking moments of the resulting stress distribution about the neutral axis. M_M, the contribution of the internal moments normal to the skewed midspan section, is determined by numerical integration of the normal moments at this section. The total contribution of internal stress and moment resultants to the gross internal moment at midspan is given by

$$M_{int} = M_N + M_M = 9.92 + 0.33 = 10.25 \text{ k-ft.}$$

while the coefficient μ is computed from

$$\mu = \frac{M_{int}}{M_{stat}} = \frac{10.25}{10.61} = 0.97$$

Observe that the resulting stress distribution at midspan satisfies equilibrium within 3%. This violation of statics is negligible if one considers that the section is taken directly underneath the concentrated loading.

The total computer time necessary for execution of this problem amounted to 3 minutes and 13 seconds. A detailed discussion of execution times as obtained on the CDC 6400 computer is presented in Section 5.7.

6.5 Example 6 - Two Cell Box Girder Highway Branch

This example has been chosen to demonstrate the versatility of the finite element program CELL as applied to the analysis of a cellular highway branch. The geometry and material properties of the box girder bridge to be analyzed are described in Fig. 19 together with its loading and boundary conditions.

As no other analytical tool is presently available for the solution of problems with such complex geometric configurations, no comparison with alternate results can be made. In order to assess the accuracy of the internal stress distribution equilibrium checks are performed as described in the discussion of the previous examples.

The curved box structure is idealized by the finite element mesh layout illustrated in Fig. 20. Due to symmetry of the geometry, only half of the structure need to be considered in the analysis. Two separate cases have to be treated to account for the non-symmetric loading acting on the actual structure. First, half of the structure is analyzed by imposing symmetric boundary conditions in form of $V = \theta_X = 0$ at the nodes in the plane of symmetry, and by applying half of the actual loading. Second, the same half is analyzed by imposing anti-symmetric boundary conditions in form of $W = \theta_Y = U = 0$ at the nodes in the plane of symmetry, and by applying half of the actual loading. The final results for the actual structure are obtained by simply superimposing the solutions of both analyses previously described. Analogous to the previous examples, each cell is idealized by three elements for top and bottom deck and by one element for the webs.

Figure 21 illustrates the distribution of vertical displacements W at the Section of X = 0.4L along which a transverse diaphragm is located and also along the top of the longitudinal outside girders. Observe the effect of this transverse diaphragm upon the transverse distribution of W at the same section enforcing a linear response. A comparison of the displacements with those obtained from the analysis of the non-skew box girder bridge in Example 4 indicates a considerable reduction in the vertical displacements W. It is interesting to note that the displacements in the loaded outside girder agree very well with those of the skew box girder bridge in Example 5 in contrast to the displacements along the unloaded outside girder. This observation is easily explained by the different load distribution in those two structures mainly due to the inclusion of a transverse diaphragm and due to the clamping effects at the corners of the skewed box girder bridge.

Figure 22 illustrates the distribution of the longitudinal stress resultant N_x at the cross-section X = 0.4L and along the top of the outside girders. The distribution along the longitudinal girders provide some insight into the load distribution of this structure. Observe the similarity with the analogous results obtained for the non-skew box girder bridge in Example 4. The improvement in the load distribution becomes apparent, which is caused mainly by the inclusion of a midspan diaphragm. The transverse distribution of N_x yields the information necessary for an internal equilibrium check. The internal moment M_{int} is determined by numerical integration of the internal forces N_x and M_x at the section of the transverse

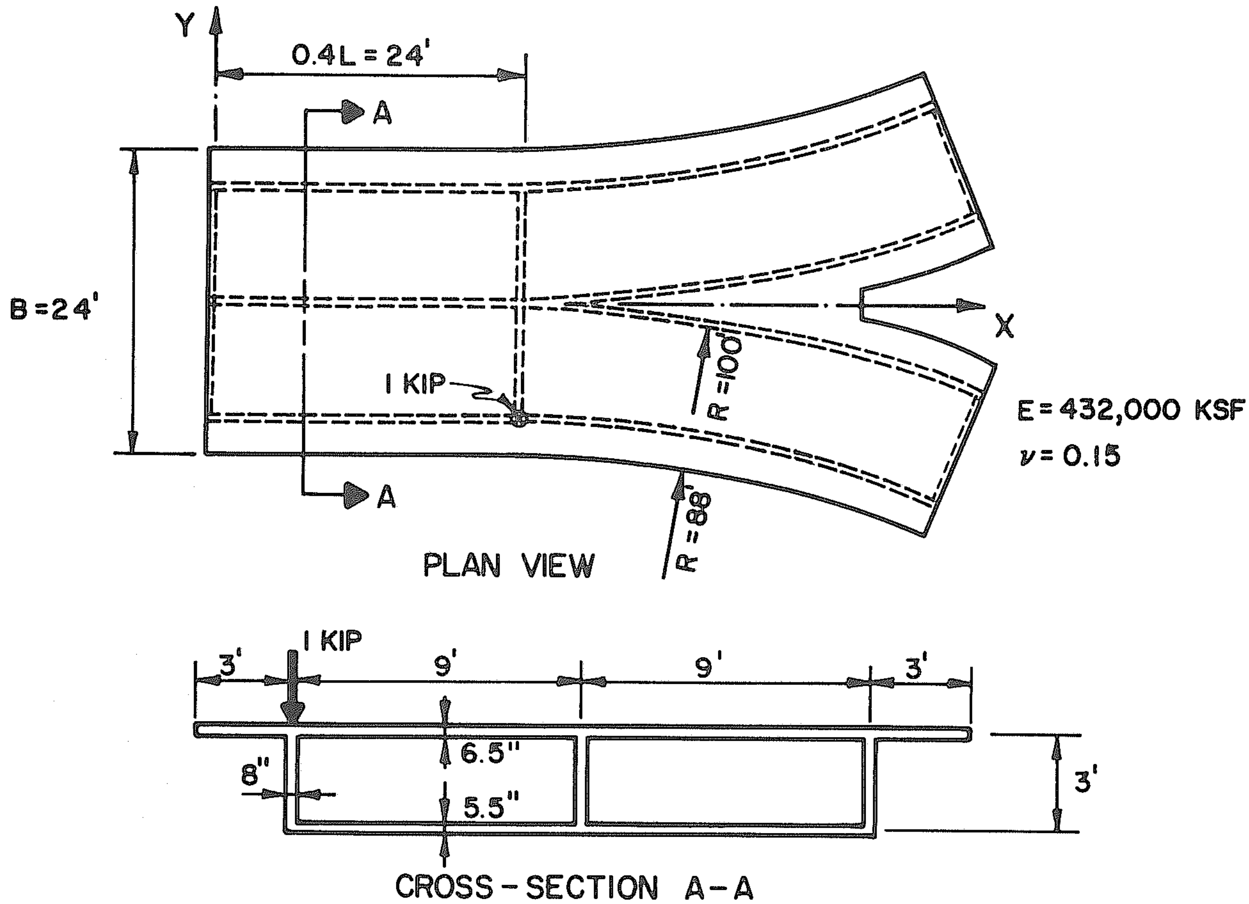


FIG. 19 EXAMPLE 6 - TWO CELL HIGHWAY BOX GIRDER BRANCH

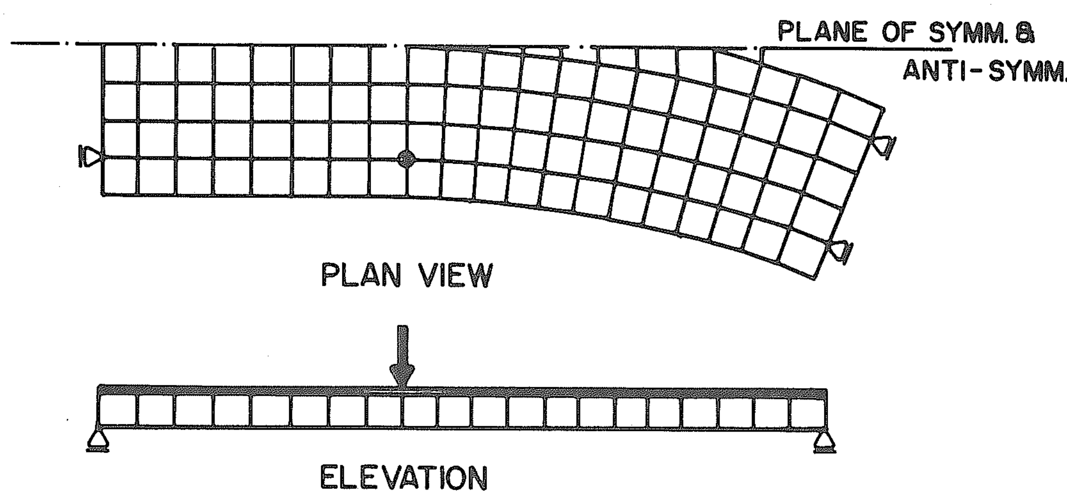
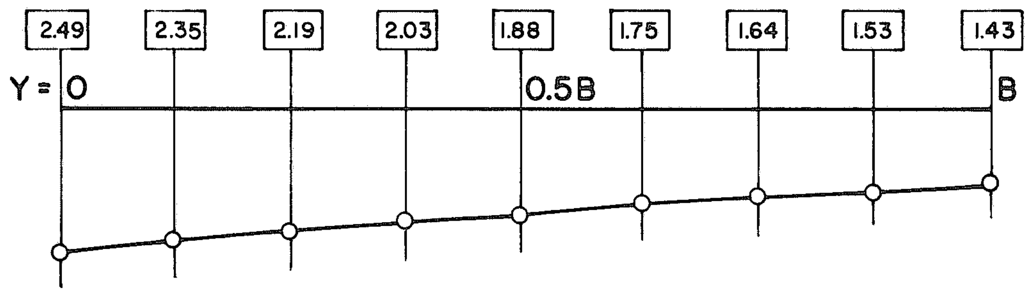
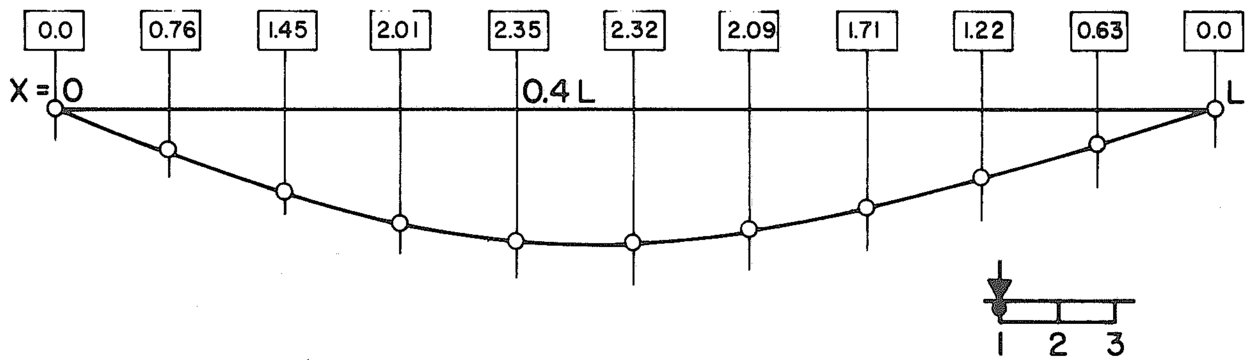


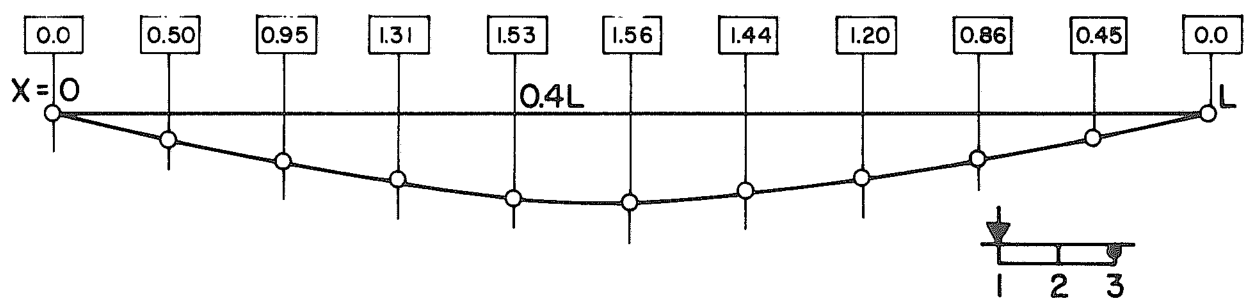
FIG. 20 EXAMPLE 6 - FINITE ELEMENT IDEALIZATION



TRANSVERSE DISTRIBUTION OF W AT THE TOP DECK OF SECTION X = 0.4 L

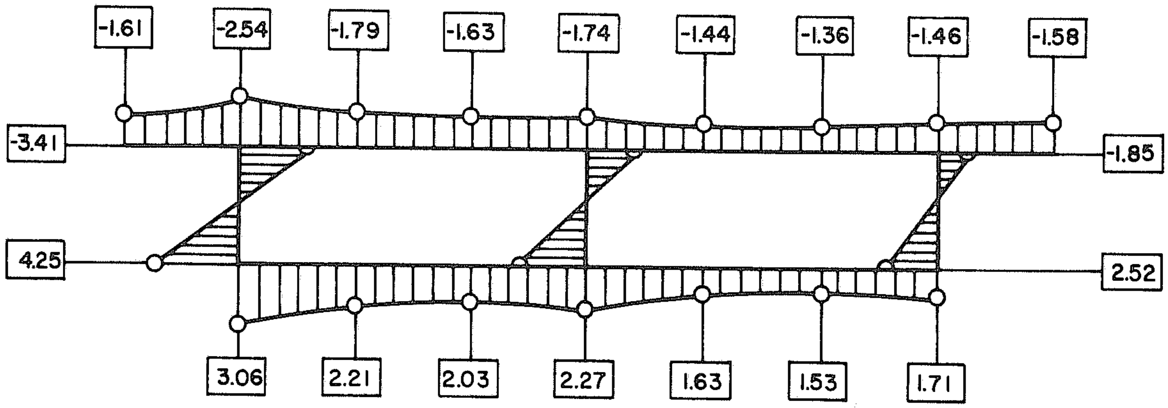


LONGITUDINAL VARIATION OF W ALONG TOP OF GIRDER 1

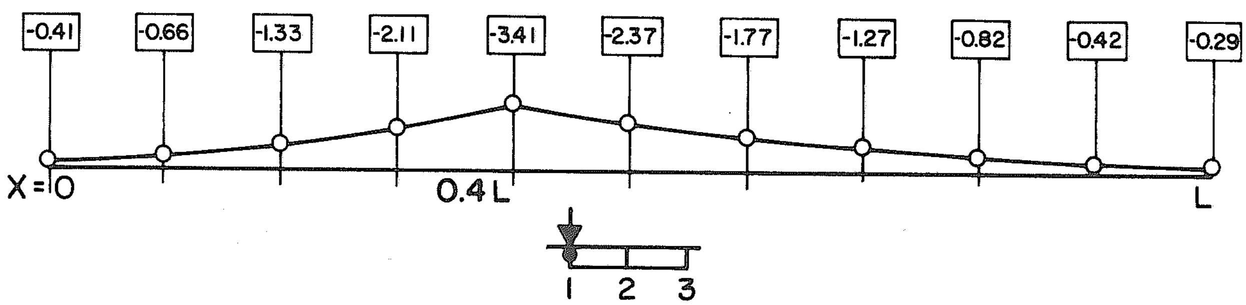


LONGITUDINAL VARIATION OF W ALONG TOP OF GIRDER 3

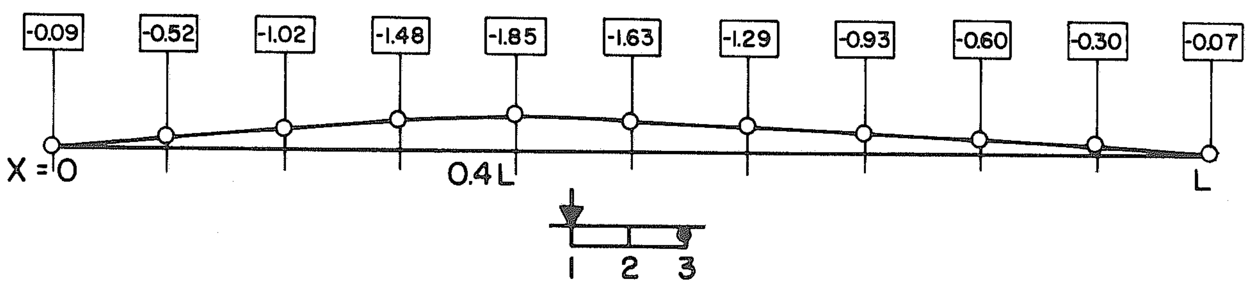
FIG. 21 EXAMPLE 6 - HIGHWAY BRANCH, VERTICAL DISPLACEMENTS W $(FT \cdot 10^{-4})$



TRANSVERSE DISTRIBUTION OF N_x AT SECTION X = 0.4 L UNDERNEATH THE LOADING



LONGITUDINAL VARIATION OF N_x AT TOP OF GIRDER 1



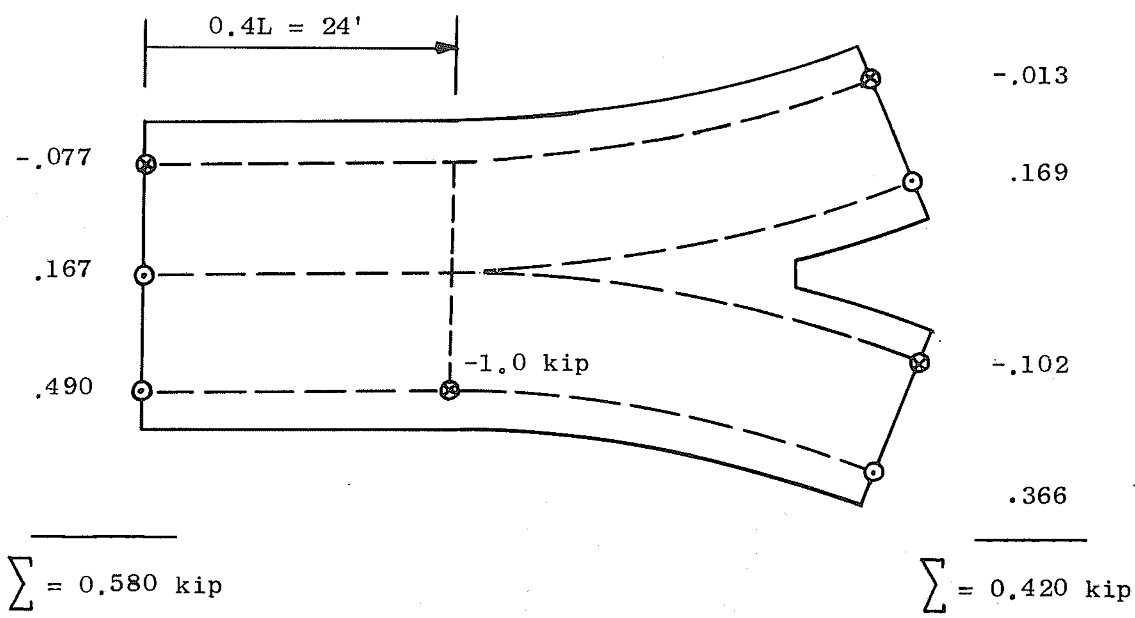
LONGITUDINAL VARIATION OF N_x AT TOP OF GIRDER 3

FIG. 22 EXAMPLE 6 - HIGHWAY BRANCH, LONGITUDINAL STRESS-RESULTANTS N_x ($K/FT \cdot 10^{-1}$)

diaphragm, X = 0.4L, where the load is applied. The gross internal moment is given by

$$M_{int} = M_N + M_M = 12.84 + 0.22 = 13.06 \text{ k-ft.}$$

The gross external moment is computed from overall statical considerations. The finite element analysis yields magnitudes of individual reactions as follows



The total static moment at the cross-section X = 0.4L = 24 ft. is given by $M_{stat} = 0.580 \times (24.0) = 13.92$ k-ft. The coefficient μ can now be calculated to provide some insight into the accuracy of the resulting stress distribution

$$\mu = \frac{M_{int}}{M_{stat}} = \frac{13.06}{13.92} = 0.94$$

Observe that a 6% violation of equilibrium by the internal stress distribution can be neglected considering that equilibrium was checked at a section directly underneath the concentrated loading.

The total computer time necessary for execution of this problem amounted to 1 minute and 41 seconds for each of the two cases analyzed. A detailed discussion of execution times as obtained on the CDC 6400 is presented in Section 5.7.

7. ACKNOWLEDGEMENTS

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APPENDIX A

Source Listing of Computer Program CELL

Considerable time, effort and expense have gone into the development of this computer program. It is obvious that it should be used only under the conditions and assumptions for which it was developed. These are described in the report. Although the program has been extensively tested by the authors, no warranty is made regarding the accuracy and reliability of the program and no responsibility is assumed by the authors or the sponsors of this research project.

```
      OVERLAY (BOX,0,0)
      PROGRAM CELL (INPUT,OUTPUT,PUNCHB,TAPE1,TAPE3,TAPE8,
     *               TAPE7=PUNCHB)
C
C**********************************************************************
C        LINEAR ELASTIC ANALYSIS OF CELLULAR SYSTEMS WITH ARBITRARY
C        GEOMETRY INCLUDING TWO WAY BENDING IN DECK AND ONE WAY BENDING
C        IN WEBELEMENTS, DEPTH IS CONSTANT, 5 DEGREES OF FREEDOM/NODE
C
C        PROGRAMMED ON CDC 6400 BY KASPAR J. WILLAM
C        UNIVERSITY OF CALIFORNIA, BERKELEY          JANUARY 1969
C**********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON / BOCON / NEBC(200),ANGLE(200)
      DIMENSION TIM(10)
      DATA STOPF /4HSTOP/, SFLAG /5HSTART/, PFLAG /4HLOAD/
C
C        ASSUME BLOCKLENGTH FOR PARTITIONING OF STRUCTURAL STIFFNESS
C
  500 READ 400, CHECK
      IF (CHECK.EQ.STOPF) GO TO 600
      IF (CHECK.EQ.SFLAG) GO TO 100
      IF (CHECK.EQ.PFLAG) GO TO 200
      GO TO 500
C
C        SEQUENTIAL CALL OF FIVE PRIMARY OVERLAYS
C
  100 NFLDL = 42000B
      CALL RFL (NFLDL)
C
C        INPUT DATA AND GENERATE ELEMENT STIFFNESSES
C
      NRUN = 1
      NA = 1
      CALL SECOND (TIM(1))
      CALL OVERLAY (3HBOX,1,0,0)
      CALL SECOND (TIM(2))
C
C        ASSEMBLE STRUCTURAL STIFFNESS IN BLOCKFORM
C
      CALL SECOND (TIM(3))
      CALL OVERLAY (3HBOX,2,0,0)
      CALL SECOND (TIM(4))
C
C        DETERMINE LOAD VECTOR IN BLOCKFORM
C
  450 READ 400, CHECK
      IF (CHECK.EQ.STOPF) GO TO 600
      IF (CHECK.EQ.SFLAG) GO TO 100
      IF (CHECK.EQ.PFLAG) GO TO 200
      GO TO 450
  200 CALL SECOND (TIM(5))
      CALL OVERLAY (3HBOX,3,0,0)
      CALL SECOND (TIM(6))
```



```
C
C        DIRECT SOLUTION OF SET OF LINEAR EQUATIONS IN BLOCKFORM
C
      CALL SECOND (TIM(7))
      CALL OVERLAY (3HBOX,4,0,0)
      CALL SECOND (TIM(8))
C
C        OUTPUT OF NODAL DISPLACEMENTS, NODAL FORCES AND INTERNAL FORCES
C
      CALL SECOND (TIM(9))
      CALL OVERLAY (3HBOX,5,0,0)
      CALL SECOND (TIM(10))
      IF (NRUN.GT.1) NA=3
      PRINT 310
      DO 300 I=NA,5
      J = 2*I
      K = J-1
      TIM(I) = TIM(J) - TIM(K)
  300 PRINT 320,I,TIM(I)
      NEQ = 10*NUMNP
      PRINT 330,IBANDW, NEQ
      NRUN = NRUN+1
      GO TO 500
  310 FORMAT (///25H1 TIME USED IN EACH LINK      ///)
  320 FORMAT (6H LINK   I3, F10.3/)
  330 FORMAT (//14H BANDWIDTH      I5/
     *          14H NO OF EQUAT    I5//)
  400 FORMAT (A6)
  600 STOP
      END
```

```
      OVERLAY (BOX,1,0)
      PROGRAM INPT
C
C***********************************************************************
C        THIS PROGRAM READS IN CONTROL INFORMATION FOR DYNAMIC STORAGE
C        ALLOCATION OF ELEMENT AND COORDINATE ARRAYS AND RESETS
C        THE FIELDLENGTH ACCORDINGLY
C***********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON / BCCON / NEBC(200),ANGLE(200)
      COMMON D(1)
      DIMENSION HED(13)
      INTEGER FWORD
C
C        CONTROL INFORMATION FOR INPUT
C
      READ  11, HED
      PRINT 12, HED
   11 FORMAT (13A6)
   12 FORMAT (1H1,13A6)
      READ  15, NUMEL,NUMNP,NBEAM,NMAT,NUMBC,NBLKL,AH
      PRINT 16, NUMEL,NUMNP,NBEAM,NMAT,NUMBC,NBLKL,AH
   15 FORMAT (6I4,F10.2)
   16 FORMAT(//
     * 35H NUMBER OF ELEMENTS............... I6/
     * 35H NUMBER OF NODAL POINTS........... I6/
     * 35H NUMBER OF VERTICAL WEB ELEMENTS.. I6/
     * 35H NUMBER OF MATERIAL TYPES ........ I6/
     * 35H NUMBER OF NODES W/ SPEC. DIS BC.. I6/
     * 35H SPECIFIED BLOCK LENGTH .......... I6/
     * 35H HEIGHT OF BOX STRUCTURE (CONST).. F5.4//)
      MARG = 3000
      CALL LWA (FWORD)
      N1  = 1
      N2  = N1  + 4*NUMEL
      N3  = N2  + 2*NBEAM
      N4  = N3  + NUMEL
      N5  = N4  + NUMEL
      N6  = N5  + NBEAM
      N7  = N6  + NBEAM
      N8  = N7  + NBEAM
      N9  = N8  + NUMNP
      N10 = N9  + NUMNP
      N11 = N10 + 9*NMAT
      N12 = N11 + 6*NMAT
      N13 = N12 + NMAT
      N14 = N13 + NMAT
      N15 = N14 + NMAT
      N16 = N15 + NMAT
      N17 = N16 + NMAT
      N18 = N17 + NMAT
      LWORD = FWORD + N18
      IF (LWORD.GT.(NFLDL-MARG).AND.LWORD.LT.NFLDL) GO TO 100
      NFLDL = LWORD
```



```
      CALL RFL (NFLDL)
  100 CALL INPU (D(N1),D(N2),D(N3),D(N4),D(N5),D(N6),D(N7),D(N8),D(N9),
     *           D(N10),D(N11),D(N12),D(N13),D(N14),D(N15),D(N16),
     *           D(N17),NUMEL,NBEAM,NUMNP,NMAT,NUMBC)
      RETURN
      END
```

```
      SUBROUTINE INPU (NP,NBP,MAT,NAT,MBAT,BSIN,BCOS,XORD,YORD,DM,DS,
     *                 E1,E2,PR,G12,ANG,TH,IE,IB,IP,IM,NUMBC)
C
C*********************************************************************
C        THIS SUBROUTINE READS IN DATA, GENERATES NODAL MESH AND
C        COORDINATES AND EVALUATES THE PLANE STRESS AND PLATE BENDING
C        STIFFNESS MATRICES FOR THE DECK AND SPAR ELEMENTS
C        GEOMETRY  PLAN  ARBITRARY QUADRILATERAL BOX WITH CONST DEPTH
C*********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON / BOCON / NEBC(200),ANGLE(200)
      COMMON / PLSTR / XA(4),YA(4),CS(6),SPD(12,12),RL(12),ESIG(5,3)
      COMMON / PLBDG / XX(5),YY(5),CM(3,3),PP(5),BM(3,5),CV(3,5),
     *                 SPB(19,19),VA(19)
C
      DIMENSION NP(IE,4),NBP(IB,2),MAT(IE),NAT(IE),MBAT(IB),BSIN(IB),
     *          BCOS(IB),XORD(IP),YORD(IP),DM(IM,3,3),DS(IM,6),E1(IM),
     *          E2(IM),PR(IM),G12(IM),ANG(IM),TH(IM)
      DIMENSION ITE(2),IPE(4),NPT(4),XD(4),YC(4),XT(4),YT(4),
     *          ST(20,20),SB(20,20),SBT(19,19),SBB(19,19),S(20,20)
      DATA ITE /1,4/, IPE /2,3,4,1/
      LOGICAL J1, J2, J3, J4, J5, JB
C
C         INITIALIZATION
C
      IFLAG= 0
      IMAT = 0
      MAXBC = 200
      MAXPD = 130
C
C*********************************************************************
C         READ AND ECHO OF INPUT DATAS
C*********************************************************************
C
C
C         INPUT AND GENERATION OF DECK ELEMENT ARRAY
C
      N = 1
   20 READ 18, L,(NPT(I),I=1,4),MA,MB,NDIF,MCD,NLIM
   18 FORMAT (10I4)
      NCT = 0
      IF (L-N) 50,22,30
   22 DO 24 I=1,4
   24 NP(N,I) = NPT(I)
      MAT(L) = MA
      NAT(L) = MB
      GO TO 46
   30 IF (N.LE.1) GC TO 52
      IF (NDIF.EQ.0) NDIF=1
      DO 32 I=1,4
   32 NP(N,I) = NP(N-1,I)+NDIF
      N1 = N-1
      GO TO 40
   42 IF (NCT.EQ.0) PRINT 68, MOD,NLIM
```



```
      NCT = NCT+1
      N1 = N-MOD
      N2 = N1-MOD
      IF (N1.LE.0.OR.N2.LE.0) GO TO 54
      DO 44 I=1,4
   44 NP(N,I) = 2*NP(N1,I) - NP(N2,I)
   40 MAT(N) = MAT(N1)
      NAT(N) = NAT(N1)
   46 N1 = N
      N = N+1
      IF (L-N) 34,22,30
   34 L = N
      IF (MOD.GT.0.AND.N1.LT.NLIM) GC TO 42
      IF (N1-NUMEL) 20,70,56
C
C         ERROR EXITS FOR DECK ELEMENT GENERATICN
C
   50 PRINT 60, L
      IFLAG = 1
      GO TO 70
   52 PRINT 62
      IFLAG = 1
      GO TO 70
   54 PRINT 64
      IFLAG = 1
      GO TO 70
   56 PRINT 66, N1,NUMEL
   60 FORMAT (//17H ELEMENT CARD N    I4,16H NOT IN SEQUENCE  )
   62 FORMAT (//27H FIRST ELEMENT CARD MISSING  )
   64 FORMAT (//50H  INSUFFICIENT INFORMATICN TO GENERATE DECK MESH  )
   66 FORMAT (//17H ELEMENT NUMBER  I4 ,22H EXCEEDS GIVEN NUMEL  I4)
   68 FORMAT (//34H DECK ELEMENT GENERATION WITH MOD= I4,6H NLIM=  I4)
C
C         INPUT AND GENERATION OF WEB ELEMENT ARRAY
C
   70 N = 1
   71 READ 99, L,(NPT(I),I=1,2),MC,NDIF,MOD,NLIM
   99 FORMAT (7I4)
      NCT = 0
      IF (L-N) 90,75,80
   75 DO 72 I=1,2
   72 NBP(N,I) = NPT(I)
      MBAT(L) = MC
      GO TO 73
   80 IF (N.LE.1) GO TO 92
      IF (NDIF.EQ.0) NDIF=1
      DO 76 I=1,2
   76 NBP(N,I) = NBP(N-1,I)+NDIF
      N1 = N-1
      GO TO 74
   82 IF (NCT.EQ.0) PRINT 98, MOD,NLIM
      NCT = NCT+1
      N1 = N-MOD
      N2 =N1-MOD
      IF (N1.LE.0.OR.N2.LE.0) GO TO 94
      DO 78 I=1,2
```

```
78 NBP(N,I) = 2*NBP(N1,I) - NBP(N2,I)
74 MBAT(N) = MBAT(N1)
73 N1 = N
   N = N+1
   IF (L-N) 79,75,80
79 L = N
   IF (MOD.GT.0.AND.N1.LT.NLIM) GO TO 82
   IF (N1-NBEAM) 71,100,96
C
C       ERROR EXITS FOR WEB ELEMENT GENERATION
C
90 PRINT 91, L
   IFLAG = 1
   GO TO 100
92 PRINT 93
   IFLAG = 1
   GO TO 100
94 PRINT 95
   IFLAG = 1
   GO TO 100
96 PRINT 97, N1,NBEAM
91 FORMAT (//13H WEB CARD N  I4,16H NOT IN SEQUENCE)
93 FORMAT (//23H FIRST WEB CARD MISSING  )
95 FORMAT (//50H INSUFFICIENT INFORMATION TO GENERATE WEB MESH      )
97 FORMAT (//20H WEB ELEMENT NUMBER  I4,22H EXCEEDS GIVEN NBEAM    )
98 FORMAT (//34H WEB ELEMENT GENERATION WITH MOD=  I4,6H NLIM= I4)
C
C       INPUT AND GENERATION OF NODAL POINT COORDINATES
C
100 L = 0
110 READ 112, N,XORD(N),YORD(N),MOD,NLIM
    L1 = L+1
    IF (N.LE.L1) GO TO 120
    IF (L.LE.0) GO TO 180
    DIV = N-L
    DX = (XORD(N)-XORD(L))/DIV
    DY = (YORD(N)-YORD(L))/DIV
    DO 140 L=L1,N
    XORD(L) = XORD(L-1) + DX
140 YORD(L) = YORD(L-1) + DY
    GO TO 120
150 L1 = N+1
    PRINT 170, MOD,NLIM
160 N = N+1
    N1 = N-MOD
    N2 = N1-MOD
    IF (N1.LE.0.OR.N2.LE.0) GO TO 182
    XORD(N) = 2*XORD(N1) - XORD(N2)
    YORD(N) = 2*YORD(N1) - YORD(N2)
    IF (N.LT.NLIM) GO TO 160
    MOD = 0
120 L = N
    IF (MOD.GT.0) GO TO 150
    IF (N-NUMNP) 110,200,186
C
C       ERROR EXITS FOR COORDINATE GENERATION
```



```
C
180 PRINT 190
    IFLAG = 1
    GO TO 200
182 PRINT 192, N
    IFLAG = 1
    GO TO 200
186 PRINT 196, N,NUMNP
    IFLAG = 1
112 FORMAT (I4,2F10.2,3I4)
170 FORMAT (//34H COORDINATE GENERATION WITH MOD=   I4,6H NLIM= I4)
190 FORMAT (//31H 1 NODAL POINT CARD MISSING       )
192 FORMAT (//27H COORDINATE CARD FOR NODE =  I4 ,16H NOT IN SEQUENCE)
196 FORMAT (//12H NODE NUMBER  I4,22H EXCEEDS GIVEN NUMNP =  I4)
C
C       INPUT OF ELEMENT MATERIAL PROPERTIES
C
200 READ 210,(N,E1(I),E2(I),G12(I),PR(I),ANG(I),TH(I),I=1,NMAT)
    IF (N.GT.0) GO TO 332
    IMAT = 1
    READ 210, (N,(DS(I,J),J=1,6),I=1,NMAT)
    DO 212 I=1,NMAT
    DM(I,1,1) = E1(I)
    DM(I,2,2) = E2(I)
    DM(I,3,3) = G12(I)
    DM(I,1,2) = PR(I)
    DM(I,1,3) = ANG(I)
    DM(I,2,3) = TH(I)
    DM(I,2,1) = PR(I)
    DM(I,3,1) = ANG(I)
212 DM(I,3,2) = TH(I)
C
C       DETERMINATION OF BANDWIDTH AND DEGREES OF FREEDOM
C
332 NPD = 0
    DO 385 N=1,NUMEL
    DO 385 I=1,4
    K = NP(N,I)
    DO 385 J=1,I
    L = IABS(NP(N,J)-K)
    IF (NPD.LT.L) NPD=L
385 CONTINUE
    IBANDW = 10*(NPD+1)
    NEQ = NUMNP*10
    IF (NBLKL.NE.0) GO TO 330
    NLK2 = (120000B-4*NUMEL-2*NBEAM)/(2*IBANDW)
    NLK4 = (120000B-NEQ)/(2*IBANDW+1)
    NBLKL = MIN0 (NLK2,NLK4)
330 IF (NBLKL.GT.NEQ) NBLKL=NEQ
    IF (IBANDW.GT.NBLKL) GO TO 1010
340 NUMBLK = (NEQ-1)/NBLKL+1
C
C       ECHO OF INPUT INFORMATION
C
    PRINT 363, NEQ,IBANDW,NBLKL,NUMBLK
363 FORMAT (// 20H NUMBER OF EQUATIONS  I5/
```

```
     *              20H BANDWIDTH            I5/
     *              20H BLOCK LENGTH         I5/
     *              20H NUMBER OF BLOCKS     I5)
      PRINT 310
  310 FORMAT (// 30H NODAL POINT COORDINATES        ////
     * 5X, 12H NODAL POINT ,5X, 8H X-COORD, 4X, 8H Y-COORD //)
      PRINT 320,(I,XORD(I), YORD(I), I=1,NUMNP)
  320 FORMAT (I12,5X,2F12.4)
      PRINT 382
      PRINT 384, (N,(NP(N,I),I=1,4),MAT(N),NAT(N),N=1,NUMEL)
      PRINT 392
      PRINT 394, (N,(NBP(N,I),I=1,2),MBAT(N),N=1,NBEAM)
  382 FORMAT(///20H DECK ELEMENT ARRAY   ////
     * 8H ELEMENT , 9X, 15H EXTERNAL NODES,5X, 8H MAT TOP ,5X,8H MAT BOT
     *  ///)
  384 FORMAT (I5,11X,4I4,I10,I13)
  392 FORMAT(///20H WEB ELEMENT ARRAY  ////
     * 8H ELEMENT ,5X, 10H EXT NODES ,5X, 8H MAT     ///)
  394 FORMAT (I5,8X,2I4,I10)
      IF (IMAT.EQ.0) GO TO 222
      PRINT 224, (I,DS(I,1),DS(I,2),DS(I,3),DS(I,4),DS(I,5),DS(I,6),
     * I=1,NMAT)
      PRINT 226, (I,DM(I,1,1),DM(I,2,2),DM(I,3,3),DM(I,1,2),DM(I,1,3),
     *             DM(I,2,3),I=1,NMAT)
  224 FORMAT (/// 48H MATERIAL LAW RELATING STRESS RES TO STRAINS     //
     * 9H MAT. NO., 8X,6H DS(1),14X,6H DS(2),14X,6H DS(3),14X,6H DS(4),
     * 14X,6H DS(5),14X,6H DS(6), //(I5,6E20.5))
  226 FORMAT (/// 48H MATERIAL LAW RELATING MOMENTS TO CURVATURES     //
     * 9H MAT. NO.,7X,8H DM(1,1),12X,8H DM(2,2),12X,8H DM(3,3),12X,
     * 8H DM(1,2),12X,8H DM(1,3),12X,8H DM(2,3)  //(I5,6E20.5))
      GO TO 390
  222 PRINT220,(I,E1(I),E2(I),G12(I),PR(I),ANG(I),TH(I),I=1,NMAT)
  210 FORMAT (I4, 6F10.2)
  220 FORMAT (///20H MATERIAL PROPERTIES//
     * 9H MAT. NO.  ,5X, 15H ELAST MOD  E1   ,5X, 15H ELAST MOD  E2  ,
     * 5X, 15H ELAST MOD G12 , 5X, 15H POISSONS RATIO , 5X, 15H PRINCIPL
     *E DIR , 5X, 15H PL THICKNESS    //
     * (I5,  3E20.5, 3F20.5))
C
C        DETERMINATION OF DIRECTION COSINES FOR WEB ELEMENTS
C
  390 DO 395 N=1,NBEAM
      N1 = NBP(N,1)
      N2 = NBP(N,2)
      DX = XORD(N2) - XORD(N1)
      DY = YORD(N2) - YORD(N1)
      DL = SQRT(DX**2 + DY**2)
      BSIN(N) = DY/DL
  395 BCOS(N) = DX/DL
C
C        INPUT OF DISPLACEMENT BOUNDARY CONDITIONS
C
      DO 283 N=1,MAXBC
  283 ANGLE(N)=0.0
      J=0
      PRINT 230
```



```
      DO 290 N=1,NUMBC
      READ 240, M,J1,J2,J3,J4,J5,JB,ALF,BET
      PRINT250, M,J1,J2,J3,J4,J5,JB,ALF,BET
      K5 = M*10
      K4 = K5-1
      K3 = K4-1
      K2 = K3-1
      K1 = K2-1
      L5 = K5-5
      L4 = K4-5
      L3 = K3-5
      L2 = K2-5
      L1 = K1-5
      IF (.NOT.J1) GO TO 271
      IF (JB) GO TO 275
      J=J+1
      NEBC(J) = K1
      GO TO 271
  275 J = J+1
      NEBC(J) = L1
  271 IF (.NOT.J2) GO TO 272
      IF (JB) GO TO 276
      J=J+1
      NEBC(J) = K2
      ANGLE(J) = ALF/57.29578
      GO TO 272
  276 J = J+1
      NEBC(J) = L2
      ANGLE(J) = ALF/57.29578
  272 IF (.NOT.J3) GO TO 273
      IF (JB) GO TO 277
      J=J+1
      NEBC(J) = K3
      GO TO 273
  277 J = J+1
      NEBC(J) = L3
  273 IF (.NOT.J4) GO TO 274
      IF (JB) GO TO 278
      J=J+1
      NEBC(J) = K4
      GO TO 274
  278 J = J+1
      NEBC(J) = L4
  274 IF (.NOT.J5) GO TO 290
      IF (JB) GO TO 279
      J=J+1
      NEBC(J) = K5
      ANGLE(J) = BET/57.29578
      GO TO 290
  279 J = J+1
      NEBC(J) = L5
      ANGLE(J) = BET/57.29578
  290 CONTINUE
      NEQBC=J
  240 FORMAT (I4,6L2,2F10.2)
  230 FORMAT (20H1BOUNDARY CONDITIONS///// 15X,
```

```
     * 12H NODAL POINT ,3X,7H TAG-U ,3X,7H TAG-V ,3X,7H TAG-W ,3X,
     *  7H TAG-DX,3X,7H TAG-DY,3X,8H TAG-BOT,2X,9H ANGLE-V ,2X,
     * 9H ANGLE-DY  ///)
  250 FORMAT (15X,I7,2X,6L10,2X,2F13.2)
C
C         CHECK IF DATA WITHIN LIMITATIONS, OTHERWISE STOP
C
  262 IF (NEQBC.GT.MAXBC) GO TO 1020
  263 IF (IBANDW.GT.MAXPD)GO TO 1030
  264 IF (IFLAG.EQ.1) STOP
C
C         FORMATION OF ANISOTROPIC MATERIAL LAW
C
      IF (IMAT.EQ.1) GO TO 640
      DO 630 M=1,NMAT
      EA = E1(M)
      EB = E2(M)
      XU = PR(M)
      GA = G12(M)
      AG = ANG(M)
      TS = TH(M)
      TM = TS**3/12.0
      CALL FORMC (EA,EB,XU,GA,AG,CM,CS)
      DO 632 I=1,3
      DO 632 J=1,3
  632 DM(M,I,J) = CM(I,J)*TM
      DO 634 I=1,6
  634 DS(M,I) = CS(I)*TS
  630 CONTINUE
C
C         STORE ELEMENT AND COORDINATE ARRAYS ON TAPE 1
C
  640 REWIND 1
      REWIND 3
      WRITE (1) NP,NBP
      WRITE (1) XORD,YORD,MAT,NAT,MBAT,DM,DS,BSIN,BCOS
C
C*********************************************************************
C         DETERMINATION OF ELEMENT STIFFNESSES
C*********************************************************************
C
C         FORMATION OF GLOBAL DECK ELEMENT STIFFNESS
C
      DO 652 I=1,4
      XT(I) = 0.0
  652 YT(I) = 0.0
      DO 650 N=1,NUMEL
      M = MAT(N)
      NCT = 0
      NTRI = 4
      ITYPG = 0
      ITYPM = 0
      IF (NP(N,1).EQ.NP(N,4)) NTRI=3
  635 DO 660 I=1,4
      L = NP(N,I)
      XX(I) = XORD(L)
```



```
      YY(I) = YCRD(L)
      PP(I) = 1.0
      XA(I) = XORD(L)
      YA(I) = YCRD(L)
  660 CONTINUE
C
C         IF DECK ELEMENT STIFFNESS IS OF THE SAME TYPE AS PREVIOUS
C         ONE ITS STIFFNESS IS NOT RECOMPUTED
C
      DO 654 I=1,4
      J = IPE(I)
      XD(I) = XX(J)-XX(I)
      YD(I) = YY(J)-YY(I)
      IF (XD(I).NE.XT(I)) ITYPG=1
      IF (YD(I).NE.YT(I)) ITYPG=1
      XT(I) = XD(I)
  654 YT(I) = YD(I)
      IF (N.EQ.1) GO TO 670
      IF (M.NE.MAT(N-1)) ITYPM=1
      IF (ITYPG.EQ.0.AND.ITYPM.EQ.0) GO TO 656
      IF (ITYP.EQ.0) GO TO 656
  670 DO 665 I=1,3
      DO 665 J=1,3
  665 CM(I,J) = DM(M,I,J)
      DO 675 I=1,6
  675 CS(I) = DS(M,I)
      CALL Q8D11 (2)
      IF (NTRI.NE.3) GO TO 642
      DO 646 I=1,8
      SPD(I,1) = SPD(I,1) + SPD(I,7)
  646 SPD(I,2) = SPD(I,2) + SPD(I,8)
      DO 648 I=1,8
      SPD(1,I) = SPD(1,I) + SPD(7,I)
  648 SPD(2,I) = SPD(2,I) + SPD(8,I)
      DO 647 I=1,8
      DO 647 J=1,2
      SPD(I,J+6) = 0.0
  647 SPD(J+6,I) = 0.0
  642 CALL SPLATE (NTRI)
      CALL DECK (S)
      IF (NCT.GT.0) GO TO 658
      DO 662 I=1,361
  662 SBT(I) = SPB(I)
      DO 664 I=1,400
  664 ST(I) = S(I)
      GO TO 656
  658 DO 666 I=1,361
  666 SBB(I) = SPB(I)
      DO 668 I=1,400
  668 SB(I) = S(I)
  680 WRITE (1) SB,SBB
      WRITE (3) SB
      GO TO 650
  656 WRITE (1) ST,SBT
      WRITE (3) ST
      M = NAT(N)
```

```
      NCT = NCT + 1
      IF (N.EQ.1) GO TO 670
      IF (M.NE.NAT(N-1)) ITYPM=1
      IF (ITYPG.NE.0.OR.ITYPM.NE.0) GO TO 67C
      GO TO 680
  650 CONTINUE
C
C         FORMATION OF GLOBAL STIFFNESS FOR VERTICAL WEB ELEMENTS
C
      ASI = 0.0
      ACO = 0.0
      AXL = 0.0
      DO 740 N=1,NBEAM
      ITYP = 0
      SI = BSIN(N)
      CO = BCOS(N)
      L  = MBAT(N)
      NA = NBP(N,1)
      NB = NBP(N,2)
      XL = (XORC(NB)-XORD(NA))*CC + (YORD(NB)-YORD(NA))*SI
C
C         IF WEB ELEMENT IS OF THE SAME TYPE AS PREVIOUS ONE, ITS
C         STIFFNESS IS NCT RECCMPLTED
C
      IF (SI.NE.ASI) ITYP=1
      IF (CO.NE.ACO) ITYP=1
      IF (XL.NE.AXL) ITYP=1
      ASI = SI
      ACO = CC
      AXL = XL
      IF (N.EQ.1) GC TO 730
      IF (L.NE.MBAT(N-1)) ITYP=1
      IF (ITYP.EQ.0) GO TO 720
  730 DO 746 I=1,2
      K = I+1
      LB = ITE(I)
      XX(LB) = 0.0
      XX(I+1) = XL
      YY(I) = 0.0
  746 YY(I+2) = AH
      DO 748 I=1,4
      XA(I) = XX(I)
      YA(I) = YY(I)
  748 PP(I) = 1.0
      DO 734 I=1,3
      DO 734 J=1,3
  734 CM(I,J) = DM(L,I,J)
      DO 736 I=1,6
  736 CS(I) = DS(L,I)
      CALL QUSP12 (3)
      CALL SONEW
      CALL WEB (S,SI,CO)
  720 WRITE (1) S
      WRITE (3) S
  740 CONTINUE
      PRINT 1091
```



```
 1091 FORMAT (/// 30H LINK 1 IS COMPLETED                    //)
      GO TO 1009
C
C***********************************************************************
C          PROGRAM EXIT DUE TO INPUT ERRORS
C***********************************************************************
C
 1010 PRINT 1051, IBANDW,NBLKL
 1051 FORMAT (// 10H BANDWIDTH I5,30H IS GREATER THAN BLOCK LENGTH  I3)
      IFLAG = 1
      GO TO 340
 1020 PRINT 1021, NEQBC
 1021 FORMAT (4CH2 MAX. NO. OF BC CONSTRAINTS EXCEEDED      I5//)
      IFLAG=1
      GO TO 263
 1030 PRINT 1031, IBANDW
 1031 FORMAT (4CH2 MAXIMUM BANDWIDTH EXCEEDED               I5//)
      IFLAG=1
      GO TO 264
 1009 RETURN
      END
```

```
      SUBROUTINE FORMC  (E1,E2,XU,C12,ANG,CM,CS)
C
C*************************************************************************
C     THIS SUBROUTINE COMPUTES THE STRESS-STRAIN COEFFICIENTS
C     IN THE GLOBAL X-Y-Z SYSTEM FOR A Z-CYLINDRICALLY ORTHOTROPIC
C     MATERIAL DEFINED BY CONSTANTS REFERRED TO THE PRINCIPAL MATERIAL
C     AXES X1-X2-Z
C*************************************************************************
C
C     INPUT
C
C        E1             ELASTIC MODULUS IN X1 DIRECTION
C        E2             ELASTIC MODULUS IN X2 DIRECTION
C        G12            SHEAR MODULUS
C        XU             MEAN POISSONS RATIO  SQRT(V12*V21)
C        ANG            ANGLE BETWEEN X-AXIS AND PRINCIPAL X1-AXIS
C
C     OUTPUT
C
C        CM(3,3)        PLANE STRESS MATERIAL MATRIX
C        CS(I)          PLANE STRESS MATERIAL VECTOR
C        I=1,6          ELEMENTS 11,22,33,12,13,23
C
      DIMENSION CM(3,3), CS(6)
C
      C = 1.
      S = 0.
      IF (ANG)  120,150,120
  120 PHI = ANG/57.2957795
      S = SIN(PHI)
      C = COS(PHI)
  150 C2 = C*C
      SC = S*C
      S2 = S*S
      C4 = C2*C2
      S2C2 = S2*C2
      S4 = S2*S2
      XUH = 1.-XU**2
      C11 = E1/XUH
      C22 = E2/XUH
      C12 = SQRT(E1*E2)*XU/XUH
      Z = C12+2.*G12
      RZ1 = C11-Z
      RZ2 = Z-C22
      RZ = (RZ1-RZ2)*S2C2
      ZS2C2 = 2.*Z*S2C2
      CM(1,1) = C11*C4+C22*S4+ZS2C2
      CM(2,2) = C22*C4+C11*S4+ZS2C2
      CM(1,2) = C12+RZ
      CM(3,3) = G12+RZ
      CM(1,3) = (RZ1*C2+RZ2*S2)*SC
      CM(2,3) = (RZ2*C2+RZ1*S2)*SC
      CM(2,1) = CM(1,2)
      CM(3,1) = CM(1,3)
      CM(3,2) = CM(2,3)
      CS(1)   = CM(1,1)
```



```
      CS(2)   = CM(2,2)
      CS(3)   = CM(3,3)
      CS(4)   = CM(1,2)
      CS(5)   = CM(1,3)
      CS(6)   = CM(2,3)
      RETURN
      END
```

```
      SUBROUTINE Q8D11(INT)
C
C***********************************************************************
C         STIFFNESS SUBROUTINE FOR  8 OF PLANE STRESS QUADRILATERAL
C         8 FUNDAMENTAL DEGREES OF FREEDOM AND 3 INTERNAL DEGREES FOR
C         AN INTERNAL NODE AND A CONSTANT SHEAR STRAIN VARIATION
C         AN ANISOTROPIC MATERIAL LAW IS USED
C***********************************************************************
C
C      INPUT
C
C        INT*INT         GAUSSIAN INTEGRATION RULE
C        D(I)            CONSTIT LAW RELATING STRESS-RES TO STRAINS
C        I=1,6           COMPONENTS OF D (11,22,33,12,13,23)
C        XA              GLOBAL X-COORDINATES
C        YA              GLOBAL Y-COORDINATES
C
C      OUTPUT
C
C        S(I,J)          8*8 STIFFNESS MATRIX OF PLANE STRESS ELEMENT
C                        IN GLOBAL XA-YA COORDINATES  (FORMATION IN
C                        LOCAL CONVECTED X-Y COORDINATES)
C        I=1,8           NODAL DISPLACEMENTS  (U1,V1,U2,V2,..)
C        I=9,10          CONDENSATION OF CENTER NODE DOF
C        I=11            CONDENSATION OF CONSTANT SHEAR STRAIN DOF
C
      COMMON / PLSTR / XA(4),YA(4),D(6),S(12,12),R1(12),ST(5,3)
C
      DIMENSION P(5,2),DC(4,2),A(2,2),U(5),V(5),ETA(2),IPERM(2),
     *          XK(4,4),WGT(4,4),X(4),Y(4),TEMP(8,4)
C
      DATA XK /      0.,              0.,              0.,              0.,
     * -.577350269189б,  .5773502691896,  0.,              0.,
     * -.7745966692415,  .0000000000000,  .7745966692415,  0.,
     * -.8611363115941,-.3399810435849, .3399810435849, .8611363115941/
      DATA WGT / 2.000,               0.,              0.,              0.,
     * 1.0000000000000,1.0000000000000,  0.,              0.,
     *  .5555555555556, .8888888888889, .5555555555556,  0.,
     *  .3478548451375, .6521451548625, .6521451548625, .3478548451375/
      DATA DC / -1., 1., 1.,-1.,  -1.,-1., 1., 1. /, IPERM / 2, 1/
      EQUIVALENCE (A11,A(1)),(A21,A(2)),(A12,A(3)),(A22,A(4))
C
C         INITIALIZATION
C
      DO 100 I=1,11
      DO 100 J=1,11
  100 S(I,J) = 0.
      NA = 0
      NINT = INT
C
C         TRANSFORMATION OF COORDINATES INTO LOCAL CONVECTED COORDINATES
C
      DX = XA(2) - XA(1)
      DY = YA(2) - YA(1)
      AL = SQRT(DX**2 + DY**2)
      CO = DX/AL
```



```
      SI = DY/AL
      DO 520 I=1,4
      X(I) = (XA(I)-XA(1))*CO + (YA(I)-YA(1))*SI
  520 Y(I) =-(XA(I)-XA(1))*SI + (YA(I)-YA(1))*CO
C
C         LOOP FOR DETERMINING INTEGRANDS AT SAMPLING POINTS OF
C         NUMERICAL INTEGRATION SCHEME
C
  500 DO 300 LX = 1,NINT
      DO 300 LY = 1,NINT
      ETA(1) = XK(LX,NINT)
      ETA(2) = XK(LY,NINT)
C
C         FORMATION OF LOCAL DERIVATIVES
C
      DO 150  I = 1,2
      J = IPERM(I)
      A(I,1) = 0.
      A(I,2) = 0.
      DO 140  L = 1,4
      C=0.250*DC(L,I)*(1.0+DC(L,J)*ETA(J))
      P(L,I)=C
      A(I,1)=A(I,1)+C*X(L)
  140 A(I,2)=A(I,2)+C*Y(L)
  150 P(5,I)=-2.0*ETA(I)*(1.0-ETA(J)**2)
      DET = A11*A22 - A12*A21
      FAC=WGT(LX,NINT)*WGT(LY,NINT)/DET
      IF (DET.LE.0.0) GO TO 1000
C
C         FORMATION OF GLOBAL DERIVATIVES
C
      DO 200 J=1,5
      U(J) =  A22*P(J,1) - A12*P(J,2)
  200 V(J) = -A21*P(J,1) + A11*P(J,2)
C
C         FORMATION OF TRIPLEPRODUCT
C
      DO 360 I=1,5
      K2 = I + I
      K1 = K2 - 1
      IF (NA.EQ.1) GO TO 320
      DO 340 J=1,5
      L2 = J + J
      L1 = L2 - 1
      UU = U(I)*U(J)
      VV = V(I)*V(J)
      UV = U(I)*V(J)
      VU = V(I)*U(J)
      S(K1,L1) = S(K1,L1) + FAC*(D(1)*UU + D(5)*(UV+VU))
      S(K2,L2) = S(K2,L2) + FAC*(D(2)*VV + D(6)*(UV+VU))
      S(K1,L2) = S(K1,L2) + FAC*(D(4)*UV + D(5)*UU+D(6)*VV)
      IF (I.EQ.J) GO TO 340
      S(K2,L1) = S(K2,L1) + FAC*(D(4)*VU + D(5)*UU+D(6)*VV)
  340 CONTINUE
      GO TO 360
  320 S(K1,11) = S(K1,11) + FAC*D(3)*V(I)*DET
```

```
      S(K2,11) = S(K2,11) + FAC*B(3)*B(I)*DET
      IF (I.EQ.1) S(11,11) = S(11,11) - FAC*B(3)*DET**2
  360 CONTINUE
  300 CONTINUE
      NA = NA + 1
      NINT = INT - 1
      IF (NINT.LT.1) NINT = 1
      IF (NA.EQ.1) GC TC 500
C
C        TRANSPOSE STIFFNESS FOR SYMMETRY
C
      DO 400 I=2,11
      K = I - 1
      DO 400  J = 1,K
  400 S(I,J) = S(J,I)
C
C        STATIC CONDENSATION
C
      DO 702 M=1,3
      K=11-M
      L=K+1
      PIVOT=S(L,L)
      IF (PIVOT.EQ.0.0) GO TO 702
      DO 700 I=1,K
      C=S(I,L)/PIVOT
      S(I,L)=C
      DO 700 J=I,K
      S(I,J)=S(I,J)-C*S(L,J)
  700 S(J,I)=S(I,J)
  702 CONTINUE
C
C        TRANSFORM CONVECTED STIFFNESS INTO GLOBAL CARTESIAN COORDINATES
C
      SS = SI*SI
      CC = CO*CO
      SC = SI*CO
      DO 560 K=1,4
      IQ = K + K
      IP = IQ - 1
      DO 550 J=1,8
      IF (J.EQ.IP) GO TO 550
      IF (J.EQ.IQ) GO TO 550
      TEMP(J,K) = S(J,IP)*SI + S(J,IQ)*CO
      S(J,IP) = S(J,IP)*CO - S(J,IQ)*SI
      S(IP,J) = S(J,IP)
      S(IQ,J) = TEMP(J,K)
      S(J,IQ) = TEMP(J,K)
  550 CONTINUE
      TEMP(IQ,K) = S(IP,IP)*SS + S(IQ,IQ)*CC + 2.*S(IP,IQ)*SC
      TEMP(IP,K) =-(S(IQ,IQ)-S(IP,IP))*SC + S(IP,IQ)*(CC-SS)
      S(IP,IP) = S(IP,IP)*CC + S(IQ,IQ)*SS - 2.*S(IP,IQ)*SC
      S(IP,IQ) = TEMP(IP,K)
      S(IQ,IP) = TEMP(IP,K)
      S(IQ,IQ) = TEMP(IQ,K)
  560 CONTINUE
      GO TO 1600
```



```
 1000 PRINT 1200, DET
 1200 FORMAT(// 30H DETERMINANT OF JACOBIAN IS     E15.3//)
      PRINT 1400, (X(I), Y(I), I=1,4)
 1400 FORMAT (5X, 2F10.3)
      STOP
 1600 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE QUSP12 (NN)
C
C*************************************************************************
C        THIS SUBROUTINE ASSEMBLES THE PLANE STRESS STIFFNESS MATRIX
C        S(12,12) FOR A QUADRILATERAL ELEMENT WITH 12 DEGREES OF FR.
C        EMPLOYING CORNER ROTATIONS (DV/DX) NORMAL TO THE PLANE
C        TO PROVIDE A COMPLETE BEAM DISPL. FUNCTION FOR THE DISPL.
C        IN THE V- DIRECT.A CONSTITUTIVE LAW FOR ANISOTROPIC
C        MATERIAL IS USED (IN GLOBAL X,Y COORDINATE SYSTEM)
C*************************************************************************
C
C     INPUT
C
C        NN*NN           GAUSSIAN INTEGRATION RULE
C        D(I)            CONSTIT LAW RELATING STRESS-RES TO STRAINS
C        I=1,6           COMPONENTS OF D (11,22,33,12,13,23)
C        XORD (4)        LOCAL X-COORDINATES
C        YORD (4)        LOCAL Y-COORDINATES
C
C     OUTPUT
C
C        S(I,J)          PLATE STIFFNESS MATRIX
C        I=1,4           U-DISPLACEMENTS
C        I=5,8           V-DISPLACEMENTS
C        I=9,12          ROTATIONS (DV/DX)
C        R=.TRUE.        STATIC CONDENSATION OF THE ROTATIONS
C
      COMMON / PLSTR / XORD(4),YORD(4),D(6),S(12,12),R1(12),ST(5,3)
C
      DIMENSION WT(3,4),ORD(3,4),FX4(4),FY4(4),FX(4),FY(4)
      DIMENSION EXX(4),EYY(8),EXY(4),EYX(8),X1(4),X2(4),Y1(4),Y2(4),
     *          UU(2),UV(2),VL(2),VV(2),VX(8),VY(8),ALFX(2),ALFY(2),
     *          ITE(2),ITA(4),ITI(4),FT1(4),FT2(4)
C
      DATA WT/1.000000000000000, 0.555555555555556, 0.347854845137434,
     *        1.000000000000000, 0.888888888888889, 0.652145154862546,
     *        0.0,               0.555555555555556, 0.652145154862546,
     *        0.0,               0.0,               0.347854845137454/
      DATA ORD/-.577350269189626, -.774596669241483, -.861136311594C53,
     *         .577350269189626, 0.000000000000000, -.339981043584856,
     *        0.0,                .774596669241483,  .339981043584856,
     *        0.0,               0.0,                .861136311594053/
      DATA FY4 /-1.0,-1.0, 1.0, 1.0/, FX4 /-1.0, 1.0, 1.0,-1.0 /
      DATA ITE /1,4/, ITA /1,2,2,1/, ITI /4,3,3,4/
      EQUIVALENCE (FX,X1), (FY,Y1), (EXX,X2), (EXY,Y2)
      LOGICAL R
C
C
C        INITIALIZATION
C
C
      R = .FALSE.
      DO 100 I=1,12
      DO 100 J=1,12
      S(I,J)=0.0
  100 S(J,I)=S(I,J)
```



```
C
C        GEOMETRICAL TRANSFORMATION OF QUADRILATERAL BOUNDARIES
C
      X1(1)=0.5*(XORD(2)-XORD(1))
      X1(3)=0.5*(XORD(3)-XORD(4))
      X1(2)=X1(1)
      X1(4)=X1(3)
      Y1(1)=0.5*(YORD(2)-YORD(1))
      Y1(3)=0.5*(YORD(3)-YORD(4))
      Y1(2)=Y1(1)
      Y1(4)=Y1(3)
      X2(1)=0.5*(XORD(4)-XORD(1))
      X2(2)=0.5*(XORD(3)-XORD(2))
      X2(3)=X2(2)
      X2(4)=X2(1)
      Y2(1)=0.5*(YORD(4)-YORD(1))
      Y2(2)=0.5*(YORD(3)-YORD(2))
      Y2(3)=Y2(2)
      Y2(4)=Y2(1)
      DO 120 I=1,4
      DET =X1(I)*Y2(I)-Y1(I)*X2(I)
      FCT=DET+X2(I)*Y1(I)
      FT1(I)= Y1(I)*X1(I)/2.0/FCT
  120 FT2(I)=X1(I)*(1.0-Y1(I)*X2(I)/FCT)
C
C        LOOP FOR DETERMINING INTEGRANDS AT SAMPLING POINTS OF
C        NUMERICAL INTEGRATION SCHEME
C
      DO 200 II=1,NN
      DO 200 JJ=1,NN
      X=ORD(NN-1,II)
      Y=ORD(NN-1,JJ)
      XA=X*X
      XB=X*XA
C
C        FORMATION OF LOCAL DERIVATIVES
C
      DO 210 I=1,4
      FX(I)=0.25*FX4(I)*(1.0+FY4(I)*Y)
  210 FY(I)=0.25*FY4(I)*(1.0+FX4(I)*X)
C
C        FORMATION OF JACOBIAN TRANSF. MATRIX
C
      X11=0.0
      X22=0.0
      X12=0.0
      X21=0.0
      DO 300 I=1,4
      X11=X11+FX(I)*XORD(I)
      X22=X22+FY(I)*YORD(I)
      X12=X12+FX(I)*YORD(I)
  300 X21=X21+FY(I)*XORD(I)
C
C        CALCULATION OF JACOBIAN DETERMINANT
C
      DET=X11*X22-X12*X21
```

```
      FAC=WT(NN-1,I)*WT(NN-1,JJ)/DET
      IF (DET.LE.0.0) GC TO 1CCC
C
C        FORMATICN OF LCCAL CERIVATIVES
C
      DO 400 I=1,4
      VY(I)=0.125*FY4(I)*(2.0+3.C*FX4(I)*X-FX4(I)*XB)
      VX(I)=0.375*(1.C+FY4(I)*Y)*(FX4(I)-FX4(I)*XA)
      VY(I+4)=0.125*FY4(I)*(-FX4(I)-X+FX4(I)*XA+XB)
  400 VX(I+4)=0.125*(1.C+FY4(I)*Y)*(-1.0+2.C*FX4(I)*X+3.0*XA)
C
C        FORMATICN OF CERIVATIVES IN GLOBAL COORDINATES
C
      DO 420 I=1,4
      EXX(I)= X22*FX(I)-X12*FY(I)
  420 EXY(I)=-X21*FX(I)+X11*FY(I)
      DO 440 I=1,4
      K=ITA(I)
      J=ITI(I)
      AA= VX(I)+FY4(I)*FT1(K)*VX(K+4) + FY4(I)*FT1(J)*VX(J+4)
      BB= VY(I)+FY4(I)*FT1(K)*VY(K+4) + FY4(I)*FT1(J)*VY(J+4)
      EYY(I+4)=-X21*FT2(I)*VX(I+4)+X11*FT2(I)*VY(I+4)
      EYX(I+4)= X22*FT2(I)*VX(I+4)-X12*FT2(I)*VY(I+4)
      EYY(I)=-X21*AA + X11*BB
  440 EYX(I)= X22*AA - X12*BB
C
C        FORMATION CF TRIPLEPRCDUCT  BT * C * B
C
      DO 500 I=1,4
      EA = EXX(I)
      EB = EXY(I)
      DO 510 J=I,4
  510 S(I,J) = S(I,J) + FAC*(EA*(D(1)*EXX(J)+D(5)*EXY(J)) +
     *                      EB*(D(3)*EXY(J)+D(5)*EXX(J)))
      DO 500 J=1,8
  500 S(I,J+4) = S(I,J+4) + FAC*(EA*(D(4)*EYY(J)+D(5)*EYX(J)) +
     *                            EB*(D(3)*EYX(J)+D(6)*EYY(J)))
      DO 540 I=1,8
      EC = EYY(I)
      ED = EYX(I)
      DO 540 J=I,8
  540 S(I+4,J+4) = S(I+4,J+4) + FAC*(EC*(D(2)*EYY(J)+D(6)*EYX(J)) +
     *                                ED*(D(3)*EYX(J)+D(6)*EYY(J)))
  200 CONTINUE
C
C        TRANSPCSE STIFFNESS FCR SYMMETRY
C
      DO 620 I=2,12
      K=I-1
      DO 620 J=1,K
  620 S(I,J)=S(J,I)
      IF (.NOT.R) GC TO 7CO
C
C        STATIC CONCENSATION CF CP RCTATIONS
C
      DO 61C M=1,4
```



```
      K = 12 - M
      L = K + 1
      PIV = S(L,L)
      IF (PIV.EC.0.0) GC TO 61C
      DO 600 I=1,12
      C = S(I,L)/PIV
      S(I,L) = C
      DO 600 J=I,K
      S(I,J) = S(I,J) - C*S(L,J)
  600 S(J,I) = S(I,J)
  610 CONTINUE
      GO TC 160C
 1000 PRINT 120C, DET
 1200 FORMAT(// 30H DETERMINANT CF JACOBIAN IS    E15.3//)
      PRINT 140C, (XCRD(I),YORD(I),I=1,4)
 1400 FORMAT (5X, 2F10.3)
      STOP
 1600 CONTINUE
  700 RETURN
      END
```

```
      SUBROUTINE SPLATE (NTRI)
C
C***********************************************************************
C     THIS SUBROUTINE ASSEMBLES THE STIFFNESS MATRIX AND CONSISTENT
C        LOAD VECTOR OF THE QUADRILATERAL PLATE BENDING ELEMENT Q19
C        FORMED BY FOUR TRIANGLES LCCT-11 (NTRI=4) OR THE TRIANGULAR
C        PLATE BENDING ELEMENT LCCT-9 (NTRI=3)
C***********************************************************************
C
C     INPUT
C        CM(3,3)         CONSTITUTIVE LAW RELATING MOMENTS TO CURVATURES
C        X(4)            GLOBAL X-COORDINATES OF NODES
C        Y(4)            GLOBAL Y-COORDINATES OF NODES
C        P(4)            NODAL LOAD INTENSITIES OF DISTRIBUTED LOADS
C
C     OUTPUT
C
C        S(I,J)          PLATE BENDING STIFFNESS MATRIX (12*12 OR 9*9)
C        F(I)            CONSISTENT LOAD VECTOR
C        I=1,12          W,DX,DY NODAL DISPLACEMENTS
C        I=13,19         CONDENSED INTERNAL DEGREES OF FREEDOM
C
      COMMON / PLBDG / X(5),Y(5),CM(3,3),P(5),BM(3,5),CV(3,5),
     *                 S(19,19),F(19)
      COMMON / TRIAG / B(3),A(3),CMT(3,3),PT(3),BMT(3,3),
     *                 ST(12,12),FT(12)
      DIMENSION LOC(11,4), IPERM(4), NC(3)
      DATA IPERM /2,3,4,1/
      DATA LOC   /1, 2, 3, 4, 5, 6,13,14,15,17,16,
     *            4, 5, 6, 7, 8, 9,13,14,15,18,17,
     *            7, 8, 9,10,11,12,13,14,15,19,18,
     *           10,11,12, 1, 2, 3,13,14,15,16,19/
      LOGICAL TRIG
C
C        INITIALIZATION OF PARAMETERS FOR TRIANGULAR AND
C        QUADRILATERAL PLATE BENDING STIFFNESS
C
C
      TRIG = NTRI.EQ.3
      NTR = 4
      IF (TRIG) NTR = 1
      NBF = 11
      NEF = 12
      NIF = 7
      IF (.NOT.TRIG) GO TO 120
      NBF = 9
      NEF = 9
      NIF = 0
  120 NDF = NBF
      NTF = NEF + NIF
      NC(3) = NBF - 6
      X(5) = 0.25*(X(1)+X(2)+X(3)+X(4))
      Y(5) = 0.25*(Y(1)+Y(2)+Y(3)+Y(4))
      P(5) = 0.25*(P(1)+P(2)+P(3)+P(4))
      DO 150 I=1,380
  150 S(I,1) = 0.
```



```
C
C        INPUT FOR TRIANGULAR ELEMENT STIFFNESS  LCCT
C
      DO 260 N=1,NTR
      M = IPERM(N)
      NC(1) = N
      NC(2) = M
      L = NC(3)
      A(1) = X(L)-X(M)
      A(2) = X(N)-X(L)
      A(3) = X(M)-X(N)
      B(1) = Y(M)-Y(L)
      B(2) = Y(L)-Y(N)
      B(3) = Y(N)-Y(M)
      DO 200  I = 1,3
      L = NC(I)
      PT(I) = P(L)
      DO 200  J = 1,3
  200 CMT(I,J) = CM(I,J)
      CALL SLCCT (NBF)
      IF (TRIG) GO TO 280
C
C        INSERT CONTRIBUTION OF TRIANGULAR STIFFNESS INTO QUADRILATERAL
C
      FT(11) = -FT(11)
      DO 250  I = 1,NDF
      K = LOC(I,N)
      F(K) = F(K) + FT(I)
      ST(I,11) = -ST(I,11)
      ST(11,I) = -ST(11,I)
      DO 250  J = 1,I
      L = LOC(J,N)
      C = S(K,L) + ST(I,J)
      S(K,L) = C
      S(L,K) = C
  250 CONTINUE
  260 CONTINUE
      GO TO 350
C
C        BENDING STIFFNESS FOR TRIANGULAR ELEMENT  LCCT-9 (HCT)
C
  280 DO 300 I=1,NDF
      F(I) = FT(I)
      DO 300 J=1,NDF
  300 S(I,J) = ST(I,J)
      GO TO 500
C
C        STATIC CONDENSATION OF  7  INTERNAL DEGREES OF FREEDOM
C        OF QUADRILATERAL PLATE BENDING STIFFNESS
C
  350 DO 420  M = 1,NIF
      L = NTF - M
      N = L + 1
      PIVOT = S(N,N)
      IF (PIVOT.LE.0.0) GO TO 420
      FN = F(N)
```

```
      F(N) = FN/PIVOT
      DO 400  I = 1,L
      C = S(I,N)/PIVOT
      S(I,N) = C
      F(I) = F(I) - C*FN
      DO 400  J = I,L
      S(I,J) = S(I,J) - C*S(N,J)
  400 S(J,I) = S(I,J)
  420 CONTINUE
  500 RETURN
      END
```



```
      SUBROUTINE SLCCT  (NBF)
C
C**********************************************************************
C     STIFFNESS SUBROUTINE FOR COMPATIBLE TRIANGULAR PLATE ELEMENT
C     WITH 'NBF' BENDING D.F. (NBF=12,11,10,9) - RIGHT-HAND SYSTEM -
C     ORTHOTROPIC ELASTIC MATERIAL, FOR NBF=9  LCCT9=HCT
C**********************************************************************
C
      COMMON / TRIAG / B(3),A(3),CMT(3,3),PT(3),BMT(3,3),
     *                 ST(12,12),FT(12)
      DIMENSION P(21,12),H(21),U(21),Q(3,6),HT(3),IPERM(3),TX(3),TY(3),
     *          NKN(4,3),NSN(4,3)
      EQUIVALENCE (CM11,CMT(1)),(CM12,CMT(2)),(CM13,CMT(3)),
     *            (CM22,CMT(5)), (CM23,CMT(6)), (CM33,CMT(9))
      DATA IPERM/2,3,1/, NKN/2,5,3,6, 8,2,9,3, 5,8,6,9/,
     *     NSN/2,3,5,6, 3,1,6,4, 1,2,4,5/
C
C        INITIALIZATION
C
      NDF = NBF
      AREA = A(3)*B(2)-A(2)*B(3)
      FAC = AREA/72.0
      DO 150  I = 1,3
      J = IPERM(I)
      K = IPERM(J)
      X = A(I)**2+B(I)**2
      U(I) = -(A(I)*A(J)+B(I)*B(J))/X
      X = SQRT(X)
      HT(I) =  4.0*AREA/X
      TY(I) = -0.5*B(I)/X
      TX(I) =  0.5*A(I)/X
      A1 = A(I)/AREA
      A2 = A(J)/AREA
      B1 = B(I)/AREA
      B2 = B(J)/AREA
      Q(1,I)   = B1*B1
      Q(2,I)   = A1*A1
      Q(3,I)   = 2.*A1*B1
      Q(1,I+3) = 2.*B1*B2
      Q(2,I+3) = 2.*A1*A2
  150 Q(3,I+3) = 2.*(A1*B2+A2*B1)
C
C        FORMATION OF CURVATURE MATRIX  P(21,12)

      DO 200  I = 1,3
      J = IPERM(I)
      K = IPERM(J)
      II = 3*I
      JJ = 3*J
      KK = 3*K
      A1 = A(I)
      A2 = A(J)
      A3 = A(K)
      B1 = B(I)
      B2 = B(J)
      B3 = B(K)
```

```
      U1 = U(I)                                                         SLCC  57
      U2 = U(J)                                                         SLCC  58
      U3 = U(K)                                                         SLCC  59
      W1 = 1.-U1                                                        SLCC  60
      W2 = 1.-U2                                                        SLCC  61
      W3 = 1.-U3                                                        SLCC  62
      B1D = 2.*B1                                                       SLCC  63
      B2D = 2.*B2                                                       SLCC  64
      B3D = 2.*B3                                                       SLCC  65
      A1D = 2.*A1                                                       SLCC  66
      A2D = 2.*A2                                                       SLCC  67
      A3D = 2.*A3                                                       SLCC  68
      C21 = B1-B3*U3                                                    SLCC  69
      C22 = -B1D+B2*W2+B3*U3                                            SLCC  70
      C31 = A1-A3*U3                                                    SLCC  71
      C32 = -A1D+A2*W2+A3*U3                                            SLCC  72
      C51 = B3*W3-B2                                                    SLCC  73
      C52 = B2D-B3*W3-B1*U1                                             SLCC  74
      C61 = A3*W3-A2                                                    SLCC  75
      C62 = A2D-A3*W3-A1*U1                                             SLCC  76
      C81 = B3-B2D-B2*U2                                                SLCC  77
      C82 = B1D-B3+B1*W1                                                SLCC  78
      C91 = A3-A2D-A2*U2                                                SLCC  79
      C92 = A1D-A3+A1*W1                                                SLCC  80
      U37 = 7.*U3                                                       SLCC  81
      W27 = 7.*W2                                                       SLCC  82
      U34 = 4.*U3                                                       SLCC  83
      W24 = 4.*W2                                                       SLCC  84
C                                                                       SLCC  85
C        CONSISTENT NODAL LOADS FOR LINEARLY VARYING LATERAL PRESSURE   SLCC  86
C                                                                       SLCC  87
      FT(II-2) = ((90.+U37+W27)*PT(I)+(36.+U37+W24)*PT(J)+(36.+U34+W27) SLCC  88
     1 *PT(K))*AREA/1080.                                               SLCC  89
      FT(II-1) = (((54.+W27)*B2-(54.+U37)*B3)*PT(I)+((15.+W24)*B2-(39.  SLCC  90
     1  +U37)*B3)*PT(J)+((39.+W27)*B2-(15.+U34)*B3)*PT(K))*AREA/6480.   SLCC  91
      FT(II  ) = (((54.+W27)*A2-(54.+U37)*A3)*PT(I)+((15.+W24)*A2-(39.  SLCC  92
     1  +U37)*A3)*PT(J)+((39.+W27)*A2-(15.+U34)*A3)*PT(K))*AREA/6480.   SLCC  93
      FT(K+9)  =  (7.*(PT(I)+PT(J))+4.*PT(K))*AREA*HT(K)/6480.          SLCC  94
      DO 200  N = 1,3                                                   SLCC  95
      L = 6*(I-1) + N                                                   SLCC  96
      Q11 = Q(N,I)                                                      SLCC  97
      Q22 = Q(N,J)                                                      SLCC  98
      Q33 = Q(N,K)                                                      SLCC  99
      Q12 = Q(N,I+3)                                                    SLCC 100
      Q23 = Q(N,J+3)                                                    SLCC 101
      Q31 = Q(N,K+3)                                                    SLCC 102
      Q2333 = Q23-Q33                                                   SLCC 103
      Q3133 = Q31-Q33                                                   SLCC 104
      P(L    ,II-2) = 6.*(-Q11+W2*Q33+U3*Q2333)                         SLCC 105
      P(L    ,II-1) = C21*Q23+C22*Q33-B3D*Q12+B2D*Q31                   SLCC 106
      P(L    ,II  ) = C31*Q23+C32*Q33-A3D*Q12+A2D*Q31                   SLCC 107
      P(L    ,JJ-2) = 6.*(Q22+W3*Q2333)                                 SLCC 108
      P(L    ,JJ-1) = C51*Q2333+B3D*Q22                                 SLCC 109
      P(L    ,JJ  ) = C61*Q2333+A3D*Q22                                 SLCC 110
      P(L    ,KK-2) = 6.*(1.+U2)*Q33                                    SLCC 111
      P(L    ,KK-1) = C81*Q33                                           SLCC 112
```



```
      P(L    ,KK  ) = C91*Q33                                           SLCC 113
      P(L    ,I+9 ) = 0.                                                SLCC 114
      P(L    ,J+9 ) = HT(J)*Q33                                         SLCC 115
      P(L    ,K+9 ) = HT(K)*Q2333                                       SLCC 116
      P(L+3 ,II-2) = 6.*(Q11+U3*Q3133)                                  SLCC 117
      P(L+3 ,II-1) = C21*Q3133-B3D*Q11                                  SLCC 118
      P(L+3 ,II  ) = C31*Q3133-A3D*Q11                                  SLCC 119
      P(L+3 ,JJ-2) = 6.*(-Q22+U1*Q33+W3*Q3133)                          SLCC 120
      P(L+3 ,JJ-1) = C51*Q31+C52*Q33+B3D*Q12-B1D*Q23                    SLCC 121
      P(L+3 ,JJ  ) = C61*Q31+C62*Q33+A3D*Q12-A1D*Q23                    SLCC 122
      P(L+3 ,KK-2) = 6.*(1.+W1)*Q33                                     SLCC 123
      P(L+3 ,KK-1) = C82*Q33                                            SLCC 124
      P(L+3 ,KK  ) = C92*Q33                                            SLCC 125
      P(L+3 ,I+9 ) = HT(I)*Q33                                          SLCC 126
      P(L+3 ,J+9 ) = 0.                                                 SLCC 127
      P(L+3 ,K+9 ) = HT(K)*Q3133                                        SLCC 128
      P(N+18,II-2) = 2.*(Q11+U3*Q12+W2*Q31)                             SLCC 129
      P(N+18,KK-1) = ((B1D-B2D)*Q33+C82*Q23+C81*Q31)/3.                 SLCC 130
      P(N+18,KK  ) = ((A1D-A2D)*Q33+C92*Q23+C91*Q31)/3.                 SLCC 131
  200 P(N+18,K+9 ) = HT(K)*Q12/3.                                       SLCC 132
C                                                                       SLCC 133
C        STATIC CONDENSATION OF MIDSIDE DEGREES OF FREEDOM              SLCC 134
C                                                                       SLCC 135
      NK = 12 - NBF                                                     SLCC 136
      IF (NK.LE.0)  GO TO 300                                           SLCC 137
      DO 220  N = 1,NK                                                  SLCC 138
      K = 13 - N                                                        SLCC 139
      DO 220  L = 1,4                                                   SLCC 140
      J = NKN(L,N)                                                      SLCC 141
      IF (L.LE.2)  C = TX(K-9)                                          SLCC 142
      IF (L.GT.2)  C = TY(K-9)                                          SLCC 143
      FT(J) = FT(J) + C*FT(K)                                           SLCC 144
      DO 220  I = 1,21                                                  SLCC 145
  220 P(I,J) = P(I,J) + C*P(I,K)                                        SLCC 146
C                                                                       SLCC 147
C        FORMATION OF MOMENT VECTOR  U (21)                             SLCC 148
C                                                                       SLCC 149
  300 DO 400  J = 1,NDF                                                 SLCC 150
      DO 340  L = 1,3                                                   SLCC 151
      KK = L + 18                                                       SLCC 152
      II = L - 6                                                        SLCC 153
      P3 = P(KK,J)                                                      SLCC 154
      H(KK) = 0.                                                        SLCC 155
      DO 340  N = 1,3                                                   SLCC 156
      M = IPERM(N)                                                      SLCC 157
      II = II + 6                                                       SLCC 158
      JJ = II + 3                                                       SLCC 159
      P1 = P(II,J)                                                      SLCC 160
      P2 = P(JJ,J)                                                      SLCC 161
      SUM = P1 + P2 + P3                                                SLCC 162
      T1 = SUM + P1                                                     SLCC 163
      T2 = SUM + P2                                                     SLCC 164
      T3 = SUM + P3                                                     SLCC 165
      H(II) = T1                                                        SLCC 166
      H(JJ) = T2                                                        SLCC 167
  340 H(KK) = T3 + H(KK)                                                SLCC 168
```

```
      DO 360  N = 1,19,3
      U(N)   = CM11*H(N) + CM12*H(N+1) + CM13*H(N+2)
      U(N+1) = CM12*H(N) + CM22*H(N+1) + CM23*H(N+2)
  360 U(N+2) = CM13*H(N) + CM23*H(N+1) + CM33*H(N+2)
C
C        FORMATION OF STIFFNESS MATRIX  ST (12,12)
C
      DO 400  I = 1,J
      X = 0.
      DO 380  N = 1,21
  380 X = X + U(N)*P(N,I)
      ST(I,J) = X*FAC
  400 ST(J,I) = ST(I,J)
      RETURN
      END
```



```
      SUBROUTINE SONEW
C
C*********************************************************************
C        THIS SUBROUTINE FORMS A ONE WAY BENDING 8 * 8 STIFFNESS
C        MATRIX (BENDG IN Y-DIR) IN FORM  OF A 12 * 12 MATRIX
C        INPUT AND OUTPUT SAME AS FOR  SPLATE
C*********************************************************************
C
      COMMON / PLBDG / X(5),Y(5),CM(3,3),P(5),BM(3,5),CV(3,5),
     *                 S(19,19),F(19)
      DIMENSION ITE(2,2),IPE(2),AL(2)
      DATA ITE /1,10,4,7/, IPE /0,3/
C
C        INITIALIZATION
C
      DO 100 I=1,12
      DO 100 J=I,12
  100 S(I,J) = C.0
      AL(1) = Y(4) - Y(1)
      AL(2) = Y(3) - Y(2)
      FT = CM(1,1)*(X(2)-X(1)+X(3)-X(4))/2.C
C
C        FORMATION OF STIFFNESS ES OF TWO ELIVALENT BEAMS IN Y-DIRECTION
C
      DO 200 I=1,2
      XL = AL(I)
      SIG = -1.C
      DO 300 J=1,2
      IA = ITE(J,I)
      SIG = -SIG
      S(IA  ,IA  ) = 6.C/(XL**3)*FT
      S(IA+1,IA+1) = 2.C/XL*FT
  300 S(IA  ,IA+1) = 3.C/(XL**2)*SIG*FT
      IA = 1 + IPE(I)
      JA = 10- IPE(I)
      S(IA  ,JA  ) =-6.C/(XL**3)*FT
      S(IA  ,JA+1) = 3.C/(XL**2)*FT
      S(IA+1,JA  ) =-3.C/(XL**2)*FT
  200 S(IA+1,JA+1) = 1.C/XL*FT
C
C        TRANSPOSE STIFFNESS FOR SYMMETRY
C
      DO 400 I=2,12
      K = I - 1
      DO 400 J=1,K
  400 S(I,J) = S(J,I)
      RETURN
      END
```

```
      SUBROUTINE DECK (S)
C
C***********************************************************************
C         THIS SUBROUTINE FORMS THE GLOBAL DECK ELEMENT STIFFNESS
C         ASSEMBLING THE PLATE BENDING STIFFNESS  SPB (12*12) OR (9*9)
C         COMPUTED BY SPLATE AND THE IN PLANE STIFFNESS SPD(8*8) OR (6*6)
C         COMPUTED BY QdC11
C         STRESS STIFFNESS QBD11
C***********************************************************************
C
      COMMON / PLSTR / XA(4),YA(4),CS(6),SPD(12,12),R1(12),ESIG(5,3)
      COMMON / PLBDG / XX(5),YY(5),CM(3,3),PP(5),BM(3,5),CV(3,5),
     *                 SPB(19,19),VA(19)
      DIMENSION S(20,20),IU(4),IV(4),IW(4)
      DATA IU /1,6,11,16/, IV /1,3,5,7/, IW /1,4,7,10/
C
C         INITIALIZATION
C
      DO 100 I=1,400
  100 S(I) = 0.0
C
C         ADD PLANE STRESS AND PLATE BENDING STIFFNESS INTO  S
C
      DO 200 I=1,4
      IA = IU(I)
      IB = IA+1
      IC = IA+2
      ID = IA+3
      IE = IA+4
      I1 = IV(I)
      I2 = I1+1
      I3 = IW(I)
      I4 = I3+1
      I5 = I3+2
      DO 220 J=1,4
      JA = IU(J)
      JB = JA+1
      JC = JA+2
      JD = JA+3
      JE = JA+4
      J1 = IV(J)
      J2 = J1+1
      J3 = IW(J)
      J4 = J3+1
      J5 = J3+2
      S(IA,JB) = SPD(I1,J2)
      S(IA,JA) = SPD(I1,J1)
      S(IB,JB) = SPD(I2,J2)
      S(IC,JD) = SPB(I3,J4)
      S(IC,JE) = SPB(I3,J5)
      S(ID,JE) = SPB(I4,J5)
      S(IC,JC) = SPB(I3,J3)
      S(ID,JD) = SPB(I4,J4)
      S(IE,JE) = SPB(I5,J5)
  220 CONTINUE
  200 CONTINUE
```



```
C
C         TRANSPOSE  S  FOR SYMMETRY
C
      DO 260 I=1,4
      IA = IU(I)
      IB = IA+1
      IC = IA+2
      ID = IA+3
      IE = IA+4
      DO 260 J=1,4
      JA = IU(J)
      JB = JA+1
      JC = JA+2
      JD = JA+3
      JE = JA+4
      S(IB,JA) = S(JA,IB)
      S(ID,JC) = S(JC,ID)
      S(IE,JC) = S(JC,IE)
      S(IE,JD) = S(JD,IE)
  260 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WEB (S,SI,CC)
C
C**********************************************************************
C         THIS SUBROUTINE FORMS THE GLOBAL ELEMENT STIFFNESS MATRIX
C         FOR THE VERTICAL WEB ELEMENTS ASSEMBLING THE ONE WAY BENDING
C         STIFFNESS SPB(12*12) COMPUTED BY SCNEW WITH THE IN PLANE
C         STIFFNESS SPD(12,12) COMPUTED BY CLSP12
C**********************************************************************
C
      COMMON / PLSTR / XA(4),YA(4),CS(6),SPD(12,12),R1(12),ESIG(5,3)
      COMMON / PLBDG / XX(5),YY(5),CM(3,3),PP(5),BM(3,5),CV(3,5),
     *                 SPB(19,19),VA(15)
      DIMENSION S(20,20),IU(4),IW(4)
      DATA IU /1,11,16,6/, IW /1,4,7,10/
C
C         INITIALIZATION
C
      DO 100 I=1,400
  100 S(I) = 0.0
C
C         ADD PLANE STRESS AND PLATE BENDING STIFFNESS INTO  S  AND
C         APPLY TRANSFORMATION INTO GLOBAL COORDINATES
C
      DO 200 I=1,4
      IA = IU(I)
      IB = IA+1
      IC = IA+2
      ID = IA+3
      IE = IA+4
      I1 = I
      I2 = I+4
      I3 = I+8
      I4 = IW(I)
      I5 = I4+1
      DO 220 J=1,4
      JA = IU(J)
      JB = JA+1
      JC = JA+2
      JD = JA+3
      JE = JA+4
      J1 = J
      J2 = J+4
      J3 = J+8
      J4 = IW(J)
      J5 = J4+1
      S(IA,JA) = SPD(I1,J1)*CC**2 + SPB(I4,J4)*SI**2
      S(IB,JB) = SPD(I1,J1)*SI**2 + SPB(I4,J4)*CO**2
      S(IC,JC) = SPD(I2,J2)
      S(ID,JD) = SPB(I5,J5)*CC**2 + SPD(I3,J3)*SI**2
      S(IE,JE) = SPB(I5,J5)*SI**2 + SPD(I3,J3)*CO**2
  240 S(IA,JC) = SPD(I1,J2)*CC
      S(IA,JD) = SPD(I1,J3)*CC*SI
      S(IA,JE) =-SPD(I1,J3)*CC*CC
      S(IB,JC) = SPD(I1,J2)*SI
      S(IB,JD) = SPD(I1,J3)*SI*SI
      S(IB,JE) =-SPD(I1,J3)*SI*CC
```



```
      S(IC,JD) = SPD(I2,J3)*SI
      S(IC,JE) =-SPD(I2,J3)*CC
      S(IB,JD) = S(IB,JD) - SPB(I4,J5)*CO*CC
      S(IB,JE) = S(IB,JE) - SPB(I4,J5)*CO*SI
      S(IA,JD) = S(IA,JD) + SPB(I4,J5)*SI*CC
      S(IA,JE) = S(IA,JE) + SPB(I4,J5)*SI*SI
      S(IA,JB) = (SPD(I1,J1) - SPB(I4,J4))*SI*CC
      S(ID,JE) = (SPB(I5,J5) - SPD(I3,J3))*SI*CO
  220 CONTINUE
  200 CONTINUE
C
C         TRANSPOSE  S  FOR SYMMETRY
C
      DO 250 I=1,4
      IA = IU(I)
      IB = IA +1
      IC = IA + 2
      ID = IA + 3
      IE = IA + 4
      DO 250 J=1,4
      JA = IU(J)
      JB = JA +1
      JC = JA +2
      JD = JA +3
      JE = JA +4
      S(IC,JA) = S(JA,IC)
      S(IC,JB) = S(JB,IC)
      S(ID,JC) = S(JC,ID)
      S(IE,JC) = S(JC,IE)
      S(ID,JB) = S(JB,ID)
      S(IE,JB) = S(JB,IE)
      S(ID,JA) = S(JA,ID)
      S(IE,JA) = S(JA,IE)
      S(IB,JA) = S(JA,IB)
      S(IE,JD) = S(JC,IE)
  250 CONTINUE
      RETURN
      END
```

```
      OVERLAY (BOX,2,0)
      PROGRAM STIFF
C
C***********************************************************************
C        CALCULATE REQUIRED FIELD LENGTH FOR BIGK
C***********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NECBC,MARG,AH
      COMMON / BOCON / NEBC(200),ANGLE(200)
      COMMON D(1)
      INTEGER FWORD
C
      CALL LWA (FWORD)
      ND = NBLKL
      NH = ND+NC
      N1 = 1
      N2 = N1 + 4*NUMEL
      N3 = N2 + 2*NBEAM
      N4 = N3 + NH*IBANDW
      LWORD = FWORD + N4
      IF (LWORD.GT.(NFLDL-MARG).AND.LWORD.LT.NFLDL) GO TO 100
      NFLDL = LWORD
      IF (NFLDL.LT.335000B) NFLDL=335000B
      CALL RFL (NFLDL)
  100 CALL BIGK (D(N1),D(N2),D(N3),NUMEL,NBEAM,NH,IBANDW)
      RETURN
      END
```



```
      SUBROUTINE BIGK (NP,NBP,C,IE,IH,IF,IB)
C
C***********************************************************************
C        THIS SUBROUTINE FORMS THE COMPLETE STRUCTURAL STIFFNESS MATRIX
C        IN BLOCK FORM, IMPOSES BOUNDARY CONDITIONSAND STORES IT ON TAPE
C***********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NECBC,MARG,AH
      COMMON / BOCON / NEBC(200),ANGLE(200)
C
      DIMENSION NP(IE,4),NBP(IH,2),C(IH,IB)
      DIMENSION STQ(20,20),SPB(20,20)
      LOGICAL LC
      EQUIVALENCE (STQ,SPB)
C
C        INITIALIZATION
C
      REWIND 8
      REWIND 1
      READ (1) NP,NBP
      NBLK = 0
      IFLAG = 0
      ND = NBLKL
      NN = IH*IB
      DO 50 N=1,NN
   50 C(N) = 0.0
C
C***********************************************************************
C        FORMATION OF STIFFNESS MATRIX IN BLOCKS
C***********************************************************************
C
   60 NBLK = NBLK+1
      NH=(NBLK+1)*ND
      NZ=NH-ND
      NF=NZ-ND+1
      KSHIFT=NF-1
      REWIND 3
C
C***********************************************************************
C        CONTRIBUTION OF DECK ELEMENTS TO UPPER BLOCK
C***********************************************************************
C
      DO 710 N=1,NUMEL
C
C        TOP DECK ELEMENTS
C
      LK = 5
      LO = .FALSE.
      IF (NP(N,1).LT.0) GO TO 700
      DO 420 I=1,4
      LA = NP(N,I)
      NL4 = LA*10 - 9
      IF (NZ.LT.NL4) GO TO 420
      IF (NF.LE.(NL4+9)) GO TO 440
  420 CONTINUE
```

```
      GO TO 700
  440 READ (3) STQ
      DO 2000 I=1,4
      L=NP(N,I)
      IL = 5*I - 5
      NL = L*10 - KSHIFT - LK
      DO 2010 J=1,4
      M=NP(N,J)
      IF(L.GT.M) GO TO 2010
      JM = 5*J - 5
      NM = M*10 - KSHIFT - LK
      MC=NM-NL+1
 2040 DO 2020 II=1,5
      NR=NL+II
      MC=MC-1
      NC=MC
      IR=IL+II
      DO 2030 JJ=1,5
      NC=NC+1
      IF(NC.LE.0) GO TO 2030
      IC=JM+JJ
      C(NR,NC) = C(NR,NC) + STQ(IR,IC)
 2030 CONTINUE
 2020 CONTINUE
 2010 CONTINUE
 2000 CONTINUE
C
C        BOTTOM DECK ELEMENTS
C
      IF (LO) GO TO 410
      LO = .TRUE.
      LK = 10
      GO TO 440
  410 NP(N,1) =-NP(N,1)
      GO TO 710
  700 READ (3) STQ
      READ (3) STQ
  710 CONTINUE
C
C***********************************************************************
C        CONTRIBUTION OF BEAM ELEMENTS
C***********************************************************************
C
      DO 110 N= 1,NBEAM
      IF (NBP(N,1).LT.0) GO TO 100
      DO 120 I=1,2
      LA = NBP(N,I)
      NL4 = LA*10 - 9
      IF (NZ.LT.NL4) GO TO 120
      IF (NF.LE.(NL4+9)) GO TO 140
  120 CONTINUE
      GO TO 100
  140 READ (3) SPB
      DO 4000 I=1,2
      L = NBP(N,I)
      IL = 10*I - 10
```



```
      NL = L*10 - KSHIFT - 10
      DO 4010 J=1,2
      M = NBP(N,J)
      IF (L.GT.M) GO TO 4010
      JM = 10*J - 10
      NM = M*10 - KSHIFT - 10
      MC = NM - NL + 1
      DO 4020 II=1,10
      NR = NL + II
      MC = MC - 1
      NC = MC
      IR = IL + II
      DO 4020 JJ=1,10
      NC = NC + 1
      IF (NC.LE.0) GO TO 4020
      IC = JM + JJ
      C(NR,NC) = C(NR,NC) + SPB(IR,IC)
 4020 CONTINUE
 4010 CONTINUE
 4000 CONTINUE
      NBP(N,1) =-NBP(N,1)
      GO TO 110
  100 READ (3) SPB
  110 CONTINUE
C
C***********************************************************************
C        EFFECT OF BOUND. COND. ON  BIGK
C***********************************************************************
C
      DO 750 I=1,NEQBC
      IF (NEBC(I).LE.0) GO TO 750
      NL=NEBC(I)
      NL1=NL-1
      PHI=ANGLE(I)
      IF (PHI.EQ.0.0) GO TO 780
C
C        SKEWED BOUNDARY CONDITION IN UPPER BLOCK
C
      IF (NH.LT.NL) GO TO 750
      IF (NF.GT.NL1) GO TO 750
      NR=NL-KSHIFT
      NR1 = NR - 1
      CC = COS(PHI)
      SS = SIN(PHI)
      IF (NZ.LT.NL1) GO TO 760
      C(NR1,1)=C(NR1,1)*CC*CC+2.0*C(NR1,2)*SS*CC+C(NR,1)*SS*SS
      L = NR1
      DO 744 J=3,IBANDW
      C(NR1,J) = C(NR1,J)*CC + C(NR,J-1)*SS
      L = L-1
      IF (L.LE.0) GO TO 744
      C(L,J-1)=C(L,J-1)*CC+C(L,J)*SS
  744 CONTINUE
      DO 746 J=1,IBANDW
      C(NR,J)=0.0
      L=NR-J+1
```

```
      IF (L.LE.0) GO TO 746
      C(L,J)=0.0
  746 CONTINUE
      NEBC(I)=-NEBC(I)
      GO TO 750
C
C         EFFECT OF SKEWED BOUNDARY CONDITIONS IN LOWER BLOCK UNTO
C         EQUATIONS IN UPPRER BLOCK
C
  760 NK=NR-ND
      L=ND+1
      IF (NK.GE.IBANDW) GO TO 750
      IBAND = IBANDW - 1
      DO 762 J=NK,IBAND
      L=L-1
      C(L,J)=C(L,J)*CC+C(L,J+1)*SS
  762 C(L,J+1)=0.0
      GO TO 750
C
C         SIMPLE BOUNDARY CONDITION IN UPPER BLOCK
C
  780 IF (NH.LT.NL) GO TO 750
      IF (NF.GT.NL) GO TO 750
      NR=NL-KSHIFT
      IF (NZ.LT.NL) GO TO 785
      DO 790 J=1,IBANDW
      C(NR,J)=0.0
      L=NR-J+1
      IF (L.LE.0) GO TO 790
      C(L,J)=0.0
  790 CONTINUE
      NEBC(I)=-NEBC(I)
      GO TO 750
C
C         EFFECT OF SIMPLE BOUNDARY CONDITION IN LOWER BLOCK ONTO
C         EUATIONS IN UPPER BLOCK
C
  785 NJ=NR-ND+1
      L=ND+1
      IF (NJ.GT.IBANDW) GO TO 750
      DO 766 J=NJ,IBANDW
      L=L-1
      C(L,J)=0.0
  766 CONTINUE
  750 CONTINUE
C
C***********************************************************************
C         WRITE UPPER BLOCK ON TAPE AND SHIFT LOWER BLOCK UP
C***********************************************************************
C
  530 WRITE (8) ((C(I,J),I=1,ND),J=1,IB)
      DO 360 N=1,ND
      K=N+ND
      DO 360 M=1,IBANDW
      C(N,M)=C(K,M)
  360 C(K,M)=0.0
```



```
C
C         CHECK FOR LAST BLOCK
C
      IF (NZ.LT.NEQ) GO TO 60
C
C***********************************************************************
C        CHECK COMPLETENESS OF STIFFNESS MATRIX PUT ON TAPE
C***********************************************************************
C
      DO 450 N=1,NUMEL
      IF (NP(N,1).LT.0) GO TO 450
      PRINT 416,N
  416 FORMAT (25H DECKELEMENT MISSING         I4)
      IFLAG=1
  450 NP(N,1) =-NP(N,1)
  621 DO 620 N=1,NEQBC
      IF (NEBC(N).LE.0) GO TO 620
      PRINT 626,N
  626 FORMAT (25H BOUND. COND. MISSING        I4)
      IFLAG=1
  620 NEBC(N)=-NEBC(N)
      DO 600 N=1,NBEAM
      IF (NBP(N,1).LT.0) GO TO 600
      PRINT 606,N
  606 FORMAT (25H BEAM MISSING                I4)
      IFLAG = 1
  600 NBP(N,1) =-NBP(N,1)
      IF (NUMBLK.EQ.NBLK) GO TO 650
      PRINT 646, NUMBLK,NBLK
  646 FORMAT (///25H NUMBER OF BLOCKS DIFFER    2I5)
      IFLAG = 1
  650 IF (IFLAG.NE.0) STOP
      PRINT 630
  630 FORMAT (/// 23H LINK NO 2 COMPLETED     ///)
      RETURN
      END
```

```
      OVERLAY (BOX,3,0)
      PROGRAM LOADG
C
C***********************************************************************
C         THIS PROGRAM COMPUTES THE REQIRED FIELDLENGTH AND RESETS IT
C         IF NECESSARY
C***********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON / BOCON / NEBC(200),ANGLE(200)
      COMMON D(1)
      INTEGER FWORD
C
C         COMPUTE AND RESET FIELD LENGTH IF NECESSARY
C
      CALL LWA (FWORD)
      N1  = 1
      N2  = N1  + 4*NUMEL
      N3  = N2  + 2*NBEAM
      N4  = N3  + NUMEL
      N5  = N4  + NUMEL
      N6  = N5  + NBEAM
      N7  = N6  + NBEAM
      N8  = N7  + NBEAM
      N9  = N8  + NUMNP
      N10 = N9  + NUMNP
      N11 = N10 + 9*NMAT
      N12 = N11 + 6*NMAT
      N13 = N12 + NUMEL
      N14 = N13 + NUMEL
      N15 = N14 + NEQ
      N16 = N15 + NEQ
      LWORD = FWORD + N16
      IF (LWORD.LT.NFLDL) GO TO 100
      NFLDL = LWORD
      CALL RFL (NFLDL)
  100 CALL LOAD (D(N1),D(N2),D(N3),D(N4),D(N5),D(N6),D(N7),D(N8),D(N9),
     *           D(N10),D(N11),D(N12),D(N13),D(N14),D(N15),NUMEL,NBEAM,
     *           NUMNP,NMAT,NEQ)
      RETURN
      END
```



```
      SUBROUTINE LOAD (NP,NBP,MAT,NAT,MBAT,BSIN,BCOS,XORD,YORD,DM,DS,
     *                 NOLL,PI,P,D,IE,IB,IP,IM,NAK)
C
C***********************************************************************
C         THIS SUBROUTINE FORMS THE LOADVECTOR FOR EACH LOADCASE
C***********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON / BOCON / NEBC(200),ANGLE(200)
C
      DIMENSION NP(IE,4),NBP(IB,2),MAT(IE),NAT(IE),MBAT(IB),BSIN(IB),
     *          BCOS(IB),XORD(IP),YORD(IP),DM(IM,3,3),DS(IM,6),
     *          NOLL(IE),PI(IE),P(NAK),D(NAK)
      DIMENSION IPE(4),ITE(4),X(5),Y(5),FORCE(5),ST(20,20),SPB(19,19)
      LOGICAL TBCT,TDIS
      DATA IPE /4,1,2,3/, ITE /2,3,4,1/
C
C         INITIALIZATION
C
      REWIND 1
      LDG = 3
      IF (NRUN.GT.1) LDG=8
      REWIND LDG
      READ (1) NP,NBP
      READ (1) XORD,YORD,MAT,NAT,MBAT,DM,DS,BSIN,BCOS
      NN = NBLKL
      DO 130 I=1,NEQ
      D(I) = 0.0
  130 P(I) = 0.0
      DO 140 I=1,NUMEL
  140 PI(I)= 0.0
C
C***********************************************************************
C         LOADINPUT
C***********************************************************************
C
      PRINT 104, NRUN
  104 FORMAT (12H1 LOAD CASE      I4 //)
      READ 106, NLD,NLL,NDL,PLL,PDL
  106 FORMAT (3I4,2F10.2)
      PRINT 107, NLD,NLL,NDL
  107 FORMAT (//40H NUMBER OF NODAL LOADS                        I4/
     *          40H NUMBER OF DIFF LL FOR TOP DECK ELEM        I4/
     *          40H DEAD LOAD INCLUDED (NDL=1)                  I4//)
      IF (NLD.EQ.0) GO TO 300
C
C         INPUT OF CONCENTRATED NODAL LOADS
C
      PRINT 81
      DO 120 N=1,NLD
      READ  80, M,TBCT,TDIS,(FORCE(I),I=1,5)
      PRINT 82, M,TBCT,TDIS,(FORCE(I),I=1,5)
      KB = M*10
      KC = KB-1
      KD = KC-1
```

```
      KE = KD-1
      KF = KE-1
C
C         ADD COCENTRATED NODAL LOADS TO GLOBAL LOAD VECTOR
C
      IF (FORCE(1).EQ.0.0) GO TO 720
      IF (TBOT) GO TO 721
      IF (TDIS) D(KF) = FORCE(1)
      P(KF) = FORCE(1)
      GO TO 720
  721 IF (TDIS) D(KF-5) = FORCE(1)
      P(KF-5) = FORCE(1)
  720 IF (FORCE(2).EQ.0.0) GO TO 722
      IF (TBOT) GO TO 723
      IF (TDIS) D(KE) = FORCE(2)
      P(KE) = FORCE(2)
      GO TO 722
  723 IF (TDIS) D(KE-5) = FORCE(2)
      P(KE-5) = FORCE(2)
  722 IF (FORCE(3).EQ.0.0) GO TO 724
      IF (TBOT) GO TO 725
      IF (TDIS) D(KD) = FORCE(3)
      P(KD) = FORCE(3)
      GO TO 724
  725 IF (TDIS) D(KD-5) = FORCE(3)
      P(KD-5) = FORCE(3)
  724 IF (FORCE(4).EQ.0.0) GO TO 726
      IF (TBOT) GO TO 727
      IF (TDIS) D(KC) = FORCE(4)
      P(KC) = FORCE(4)
      GO TO 726
  727 IF (TDIS) D(KC-5) = FORCE(4)
      P(KC-5) = FORCE(4)
  726 IF (FORCE(5).EQ.0.0) GO TO 120
      IF (TBOT) GO TO 729
      IF (TDIS) D(KB) = FORCE(5)
      P(KB) = FORCE(5)
      GO TO 120
  729 IF (TDIS) D(KB-5) = FORCE(5)
      P(KB-5) = FORCE(5)
  120 CONTINUE
   80 FORMAT (I4,2L2,2X,5F10.2)
   81 FORMAT (//80H NODE   BOTTOM   DIS-BC      U-FORCE    V-FORCE    W-FOR
     *CE    X-MOMNT   Y-MOMNT        //)
   82 FORMAT (I4,2L8,4X,5F10.2)
C
C        INPUT OF LIVE LOAD INTENSITIES FOR DECK ELEMENTS IF NLL.GT.1
C
  300 IF (NLL.LE.0.AND.NDL.LE.0) GO TO 200
      DO 304 I=1,NUMEL
  304 PI(I) = PLL
      IF (NLL.LE.1) GO TO  311
      DO 306 I=2,NLL
      READ 307, I,NEL,PLL
  307 FORMAT (2I4,F10.2)
      READ 305, (NOLL(K),K=1,NEL)
```



```
  305 FORMAT (20I4)
      DO 308 K=1,NEL
      IA = NOLL(K)
  308 PI(IA) = PLL
  306 CONTINUE
  311 PRINT 303
      DO 301 I=1,NUMEL
  301 PRINT 309, I,PI(I),PDL
  303 FORMAT (30H1 UNIFORM LOADS FOR ELEMENT      //
     1 10X, 8H ELEMENT,3X,13H LL INTENSITY,7X,20H UNIT WEIGHT FOR DL //)
  309 FORMAT (10X,I5,8X,F10.3,10X,F10.3)
C
C         DETERMINATION OF DL AND LL CONTRIBUTION OF EACH DECK ELEMENT
C         TO VERTICAL NODAL LOADS  (TRIBUTARY AREA CONCEPT)
C
  310 DO 240 I=1,NUMEL
      M = MAT(I)
      L = NAT(I)
      THU = SQRT(12.0*DM(M,3,3)/DS(M,3))
      THL = SQRT(12.0*DM(L,3,3)/DS(L,3))
      DO 220 J=1,4
      NB = NP(I,J)
      X(J) = XORD(NB)
  220 Y(J) = YORD(NB)
      X(5) = 0.25*(X(1)+X(2)+X(3)+X(4))
      Y(5) = 0.25*(Y(1)+Y(2)+Y(3)+Y(4))
      PLL = PI(I)
      IF (NP(I,1).NE.NP(I,4)) GO TO 235
C
C         FOR TRIANGULAR DECK ELEMENT
C
      A3 = X(2)-X(1)
      A2 = X(1)-X(3)
      B2 = Y(3)-Y(1)
      B3 = Y(1)-Y(2)
      WT = (A3*B2-A2*B3)/6.0
      DO 222 J=1,3
      NA = NP(I,J)
      KD = NA*10 - 2
      LD = KD-5
      P(KD) = P(KD) - WT*THU*PDL
      P(LD) = P(LD) - WT*THL*PDL
      P(KD) = P(KD) + WT*PLL
  222 CONTINUE
      GO TO 240
C
C         FOR QUADRILATERAL DECK ELEMENT
C
  235 DO 230 J=1,4
      L = IPE(J)
      K = ITE(J)
      X41 = 0.5*(X(L) - X(J))
      Y41 = 0.5*(Y(L) - Y(J))
      X21 = 0.5*(X(K) - X(J))
      Y21 = 0.5*(Y(K) - Y(J))
      WT = 0.5*((X(5)-X(J))*(Y41-Y21) + (Y(5) - Y(J))*(X21 - X41))
```

```
      NA = NP(I,J)
      KD = NA*10 - 2
      LD = KD - 5
      P(KD) = P(KD) - WT*THU*PCL
      P(LD) = P(LD) - WT*THL*PCL
      P(KD) = P(KD) + WT*PLL
  230 CONTINUE
  240 CONTINUE
      IF (NCL.NE.1) GO TO 200
C
C        DETERMINATION OF DEADLOAD CONTRIBUTION OF WEB ELEMENTS
C
      DO 260 I=1,NBEAM
      M = MBAT(I)
      THB = SQRT(12.0*CM(M,3,3)/DS(M,3))
      NA = NBP(I,1)
      NB = NBP(I,2)
      DL = (XORD(NB) - XORD(NA))**2 + (YORD(NB) - YORD(NA))**2
      WT = SQRT(DL)*AH*THB*PCL/4.0
      KA = NA*10 - 2
      KB = NB*10 - 2
      P(KA) = P(KA)-WT
      P(KB) = P(KB)-WT
      P(KA-5) = P(KA-5)-WT
  260 P(KB-5) = P(KB-5)-WT
C
C***********************************************************************
C        EFFECT OF BOUNDARY CONDITIONS ON LOAD VECTOR
C***********************************************************************
C
  200 DO 400 I=1,NEQBC
      NR = NEBC(I)
      DIS = D(NR)
      IF (DIS.EQ.0.0) GO TO 400
      NB =(NR-1)/10 + 1
      ICO = NR - (NB-1)*10
      NCA = 0
      IF (ICO.GT.5) NCA = 5
C
C        LOAD CONTRIBUTION FROM DECK ELEMENTS
C
      DO 440 J=1,NUMEL
      MA = 0
      READ (1) ST,SPB
      DO 450 L=1,4
      NA = NP(J,L)
      IF (NA.NE.NB) GO TO 450
      MA = (L-1)*5 + ICO - NCA
      GO TO 442
  450 CONTINUE
      GO TO 460
  442 IF(NCA.EQ.0) GO TO 460
  446 DO 444 KA=1,4
      NK = NP(J,KA)
      NJ = (KA-1)*5
      NL = (NK-1)*10 + NCA
```



```
      DO 448 KB=1,5
  448 P(KB+NL) = P(KB+NL) - ST(KB+NJ,MA)*DIS
  444 CONTINUE
      IF (NCA.EQ.0.AND.MA.NE.0) GO TO 440
  460 READ (1) ST,SPB
      IF (NCA.EQ.0.AND.MA.NE.0) GO TO 446
  440 CONTINUE
C
C        LOAD CONTRIBUTION FROM BEAM ELEMENTS
C
      DO 470 J=1,NBEAM
      READ (1) ST
      DO 480 L=1,2
      NA = NBP(J,L)
      IF (NA.NE.NB) GO TO 480
      MA = (L-1)*10 + ICO
      GO TO 472
  480 CONTINUE
      GO TO 470
  472 DO 474 KA=1,2
      NK = NBP(J,KA)
      NJ = (KA-1)*10
      NL = (NK-1)*10
      DO 476 KB=1,10
  476 P(KB+NL) = P(KB+NL) - ST(KB+NJ,MA)*DIS
  474 CONTINUE
  470 CONTINUE
  400 CONTINUE
      DO 420 I=1,NEQBC
      NR = NEBC(I)
  420 P(NR) = D(NR)
      PRINT 494
      DO 490 I=1,NUMNP
      KA = I*10
  490 PRINT 492, I,P(KA-4),P(KA-3),P(KA-2),P(KA-1),P(KA),
     *                 P(KA-9),P(KA-8),P(KA-7),P(KA-6),P(KA-5)
  494 FORMAT (/// 30H FINAL LOAD VECTOR                    ////
     * 10X,10H NODAL PT., 6X,8H U-FORCE, 8X,8H V-FORCE, 8X,8H W-FORCE,
     * 7X,9H X-MOMENT, 7X,9H Y-MOMENT    /)
  492 FORMAT (10X,I5,8X,1P5E16.5/23X,1P5E16.5//)
C
C***********************************************************************
C        WRITE TOTAL LOAD VECTOR ON TAPE LDG
C***********************************************************************
C
      WRITE (LDG) P
      PRINT 2019
 2019 FORMAT (// 22H LINK NO 3 COMPLETED    //)
      RETURN
      END
```

```
      OVERLAY (BCX,4,0)
      PRCGRAM BANSOL
C
C*********************************************************************
C         COMPUTES AND RESETS FIELDLENGTH IF NECESSARY FOR BAND
C*********************************************************************
C
      COMMCN / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON D(1)
      INTEGER FWORD
C
      CALL LWA (FWORD)
      ND = NBLKL
      NS = ND*IBANDW
      NORG = 8
      NRED = 3
      N1 = 1
      N2 = N1+NS
      N3 = N2+NS
      N4 = N3+NEQ
      N5 = N4+ND
      LWORD = FWORD+N5
      IF (LWORD.GT.(NFLDL-MARG).AND.LWORD.LT.NFLDL) GO TO 100
      NFLDL = LWORD
      IF (NFLDL.LT.335000B) NFLDL=335000B
      CALL RFL (NFLDL)
  100 CALL BAND (D(N1),D(N2),D(N3),D(N4),NEQ,ND,IBANDW,NS,NUMBLK,NRUN,
     *           NORG,NRED)
      RETURN
      END
```



```
      SUBROUTINE BAND (A,AA,B,NBL,NEQ,NEQB,MBAND,NSB,NUMBLK,
     *                 KKK,NORG,NRED)
C
C*********************************************************************
C       OUT OF CORE BAND SOLVER FOR SYMMETRIC, POSITIVE DEFINITE MATRIX
C       RESTRICTICN  BANDWIDTH=MBAND.LE.NECB=THE NUMBER OF EQUATIONS
C       PER BLCCK.  SPARSITY IS ACCOUNTED FCR BY SKIPPING INNERMOST DO-
C       LOOP EITHER IF MULTIPLIER C(I,K)=0 WITHIN BAND OR IF A(I,J)=0
C       AT THE EDGE OF THE BAND BY VARYING THE BAND WIDTH WITH 45 DEGR
C       SHADCWS.  IF PIVOT=C ALGORITHM SKIPS ASSOCIATED EQUATION
C*********************************************************************
C
C          A(NSB)           LEADING BLCCK CF RECTIFIED BANDMATRIX (IN SINGLE
C                           SUBSCRIPYS) - OVERWRITTEN BY REDUCED  A
C          AA(NSB)          TRAILING BLCCK CF RECTIFIED BANDMATRIX(IN SINGLE
C                           SUBSCRIPTS).  PART CF IT IS OVERWRITTEN BY
C                           REDUCTICN OF A
C          B(NEQ)           TOTAL LCADVECTOR, ONE AT A TIME.  IS OVERWRITTEN
C                           BY SCLUTICN VECTOR
C          NBL(NECB)        VARIABLE BAND VECTCR FCR LEADING BLOCK
C          NEQ              TOTAL NUMBER OF EQUATICNS
C          NECB             NUMBER CF EQUATIONS PER BLOCK
C          MBAND            MAXIMUM WIDTH OF HALF BAND
C          NSB=NECB*MBAND   STORAGE PER BLCCK
C          NUMLK=(NEQ-1)/NEQB+1  NUMBER OF BLCCKS
C          KKK.LE.1         REDUCTICN CF A AND B WITH BACKSUBSTITUTION
C          KKK.GT.1         REDUCTION OF B WITH BACKSUBSTITUTION
C       TAPES
C          NORG             FILE FCR ORIGINAL MATRIX A, OVERWRITTEN BY
C                           SOLUTICN VECTOR AND LOAC VECTORS FOR CASES.GT.1
C          NRED             FILE WITH FIRST LOAD VECTOR, OVERWRITTEN BY
C                           REDUCED MATRIX A DURING DECOMPOSITION
C
      DIMENSICN A(NSB),AA(NSB),B(NEC),NBL(NECB)
C
C         INITIALIZATICN
C
      MB = MBANC-1
      INC= NEQB-1
      IF (KKK.GT.1) GO TO 800
C
C         DECOMPCSITICN CF MATRIX A BLCCK BY BLCCK
C
      REWIND NORG
      REWIND NRED
      READ (NCRG) A
      READ (NREC) B
      REWIND NRED
      CALL NOBLCK (NRED)
      DO 700 N=1,NUMBLK
C***
      DO 300 I=1,NECB
      D = A(I)
      IF (D) 120,300,13C
  120 M= NECB*(N-1)+I
      PRINT 1000, M,D
```

```
 1000 FORMAT (// 20H PIVOT IS NEGATIVE   /
     *          26 H DIAGONAL TERM OF EQUATICN  I6,8H EQUALS  1PE12.5/)
C
C        ESTABLISH VARIABLE BANDWIDTH
C
  130 DO 140 J=NEQB,NSB,NEQB
      IF (A(NSB-J+I).NE.0.0) GO TO 150
  140 CONTINUE
  150 NBL(I) = NSB-J+I
C
      JL = I+1
      MAX = NBL(I)
      JH = (MAX-1)/NEQB+I
      II = I
C**
      DO 200 J=JL,JH
      II = II+NEQB
      C = A(II)/D
      IF (C.EQ.0.0) GO TO 200
      IF (J.GT.NEQB) GO TO 220
      KK = J
C*
      DO 100 JJ=II,MAX,NEQB
      A(KK) = A(KK)-C*A(JJ)
  100 KK=KK+NEQB
  220 A(II) = C
  200 CONTINUE
  300 CONTINUE
C
C        STORE REDUCED BLCCK CN TAPE NRED
C
  320 WRITE (NRED) A,NBL
C
C        REDUCTICN OF TRAILING BLOCK
C
      IF (N.EQ.NUMBLK) GO TO 800
      READ (NORG) AA
C***
      NE = NECB-MB
      IL = NSB-MB
      DO 600 I=1,MB
      NE = NE+1
      IL = IL+1
      II = IL
C**
      DO 500 J=NE,NEQB
      C = A(II)*A(J)
      IF (C.EQ.0.0) GO TO 500
      MAX = NBL(J)
      KK = I
C*
      DO 400 JJ=II,MAX,NEQB
      AA(KK) = AA(KK)-C*A(JJ)
  400 KK = KK+NEQB
  500 II = II-INC
  600 CONTINUE
```



```
C
C        REPLACE LEACING BLCCK BY TRAILING BLOCK
C
      DO 650 I=1,NSB
  650 A(I) = AA(I)
  700 CONTINUE
C
C        REDUCTION OF SINGLE LCACS - TOTAL LCACVECTOR IN CORE
C
  800 REWINC NRED
      IF (KKK.LE.1) GO TO 810
      REWIND NORG
      READ (NCRG) B
C
C        REDUCE LCACING BLOCK BY BLCCK
C
  810 DO 820 N=1,NUMBLK
      READ (NREC) A,NBL
      IN = (N-1)*NECB
C***
      DO 840 I=1,NEQB
      IN = IN+1
      IF (A(I).EQ.0.0) GO TO 840
      JL = I+1
      II = I
      JH = (NBL(I)-1)/NEQB+I
      IF (IN-NEQ) 830,850,840
  830 JN = IN
      C  = B(IN)
      IF (C.EQ.0.0) GO TO 840
C**
      DO 860 J=JL,JH
      II = II+NEQB
      JN = JN+1
  860 B(JN) = B(JN)-C*A(II)
  850 B(IN) = B(IN)/A(I)
  840 CONTINUE
  820 CONTINUE
C
C        BACKSUBSTITUTICN - TCTAL SOLUTICN IN CORE
C
      DO 900 N=1,NUMBLK
      BACKSPACE NRED
      READ (NREC) A,NBL
  910 BACKSPACE NRED
      NQB = (NUMBLK-N+1)*NEQB+1
      NE = NEQB+1
C***
      DO 920 I=1,NECB
      JI = NE-I
      IF (A(JI).EQ.0.0) GO TO 920
      IL = JI+NEQB
      MAX = NBL(JI)
      KNI = NQB-I
      JNI = KNI+1
      IF (JNI.GT.NEQ) GC TO 920
```

```
      C = B(KNI)
C**
      DO 930 II=IL,MAX,NEQB
      C = C-A(II)*B(JNI)
  930 JNI = JNI+1
      B(KNI) = C
  920 CONTINUE
  900 CONTINUE
      REWIND NORG
      WRITE (NORG) B
      RETURN
      END
```



```
      OVERLAY (BCX,5,0)
      PROGRAM OUTPT
C
C*********************************************************************
C        THIS PROGRAM READS  IN INFORMATION FCR OUTPUT OPTIONS AND
C        RESETS  THE FIELDLENGTH IF NECESSARY
C*********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON / BCCON / NEBC(2C0),ANGLE(200)
      COMMON D(1)
      INTEGER FWORC,T1,T2,T3
C
C        CONTROL INFORMATION CF CUTPUT CPTICNS
C
      READ 200, T1,T2,T3,NSTC,NDIA,ALF
  200 FORMAT (5I4,F10.2)
      PRINT 220, T1,T2,T3,NSTC,NDIA,ALF
  220 FORMAT (/// 32HICONTROL PARAMETERS FCR OUTPUT       ///
     *            47H GLOBAL NCDAL FORCES CALCLLATED AND PRINTED      I4/
     *            47H STRESS RES. PRINTED AT CENTER OF DECK ELEM      I4/
     *            47H OUTPUT IS PUNCHED ON CARCS                      I4/
     *            47H NC. OF NCDES WHERE STRESS RES. AVER. AND PRINT  I4/
     *            47H NUMBER CF WEB ELEMENTS IN Y-DIRECTION           I4/
     *            40H ANGLE FROM X-AXIS TO DIR OF STRESS RES    F17.5)
C
C        DETERMINE REQUIRED FIELD LENGTH AND RESET IF NECESSARY
C
      NEO = 0
      NPO = 0
      NNO = 12*NSTO
      IF (T1.EQ.1) NPO=NEQ
      IF (T2.EQ.1) NEO=NUMEL
      CALL LWA (FWORD)
      N1  = 1
      N2  = N1  + 4*NUMEL
      N3  = N2  + 2*NBEAM
      N4  = N3  + NUMEL
      N5  = N4  + NUMEL
      N6  = N5  + NBEAM
      N7  = N6  + NBEAM
      N8  = N7  + NBEAM
      N9  = N8  + NUMNP
      N10 = N9  + NUMNP
      N11 = N10 + 9*NMAT
      N12 = N11 + 6*NMAT
      N13 = N12 + NEQ
      N14 = N13 + NPO
      N15 = N14 + NEO*12
      N16 = N15 + NSTO
      N17 = N16 + NSTO
      N18 = N17 + NNO
      N19 = N18 + NNO
      N20 = N19 + NNO
      N21 = N20 + NNO
```

```
      N22 = N21 + NNC
      N23 = N22 + NNO
      N24 = N23 + NNC
      LWORD = FWORD+N24
      IF (LWORD.GT.(NFLDL-MARG).AND.LWORD.LT.NFLDL) GO TO 100
      NFLDL = LWORD
      CALL RFL (NFLDL)
  100 CALL OTPT (D(N1),D(N2),D(N3),D(N4),D(N5),D(N6),D(N7),D(N8),D(N9),
     *           D(N10),D(N11),D(N12),D(N13),D(N14),D(N15),D(N16),
     *           D(N17),D(N18),D(N19),D(N20),D(N21),D(N22),D(N23),
     *           NUMEL,NBEAM,NUMNP,NMAT,NEQ,NSTC,NDIA,T1,T2,T3,NPO,NEO,
     *           ALF)
      RETURN
      END
```



```
      SUBROUTINE OTPT (NP,NBP,MAT,NAT,MBAT,BSIN,BCOS,XORD,YORD,DM,DS,
     *                 D,P,SCT,NPS,NWT,XXNS,YYNS,XYNS,XXMO,YYMO,XYMO,
     *                 SNP,IE,IW,IP,IM,IQ,NSTO,NDIA,T1,T2,T3,NPO,NEO,
     *                 ALF)
C
C***********************************************************************
C         THIS SUBROUTINE CALCULATES AND PRINTS THE GLOBAL NODAL DIS-
C         PLACEMENTS AND FORCES AND DETERMINES CENTER AND NODAL
C         STRESS RESULTANTS TOGETHER WITH THEIR AVERAGES AND PRINCIPAL
C         VALUES FOR BOTH DECK AND WEB ELEMENTS
C***********************************************************************
C
      COMMON / SETUP / NUMNP,NUMEL,NBEAM,NMAT,NRUN,NFLDL,IBANDW,NEQ,
     *                 NBLKL,NUMBLK,NEQBC,MARG,AH
      COMMON / BOCON / NEBC(200),ANGLE(200)
      COMMON / PLBDG / XX(5),YY(5),CM(3,3),PP(5),BM(3,5),CV(3,5),
     *                 SPB(19,19),RA(19)
      COMMON / PLSTR / XA(4),YA(4),CS(6),SPD(12,12),RI(12),ST(5,3)
C
      DIMENSION NP(IE,4),NBP(IW,2),MAT(IE),NAT(IE),MBAT(IW),BSIN(IW),
     *          BCOS(IW),XORD(IP),YORD(IP),DM(IM,3,3),DS(IM,6),D(IQ),
     *          P(NPO),SCT(NEO,12),NPS(NSTC),NWT(NSTO),XXNS(NSTO,12),
     *          YYNS(NSTO,12),XYNS(NSTO,12),XXMO(NSTO,12),YYMO(NSTO,12),
     *          XYMO(NSTO,12),SNP(NSTO,12)
      DIMENSION SK(20,20),ITE(2),ILE(2)
      DATA ITE /1,4/, ILE /9,4/
      INTEGER T1,T2,T3
C
C         INITIALIZATION
C
      REWIND 1
      REWIND 8
      READ (1) NP,NBP
      READ (1) XORD,YORD,MAT,NAT,MBAT,DM,DS,BSIN,BCOS
      READ (8) D
C
C***********************************************************************
C         OUTPUT OF NODAL POINT DISPLACEMENTS
C***********************************************************************
C
      DO 780 I=1,NEQBC
      NR=NEBC(I)
      PHI=ANGLE(I)
      IF (PHI.EQ.0.0) GO TO 780
C
C         TRANSFORM SKEW DISPL. INTO GLOBAL X-Y COORD DISPL
C
      NR1=NR-1
      D(NR) = D(NR1)*SIN(PHI)
      D(NR1)= D(NR1)*COS(PHI)
  780 CONTINUE
      PRINT 900
  900 FORMAT (21H1 DISPLACEMENT VECTOR////
     * 10X,10H NODAL PT., 7X,7H U-DIS.,9X,7H V-DIS.,9X,7H W-DIS.,
     *  8X,8H X-ROTAT,8X,8H Y-ROTAT  //)
      DO 901 I=1,NUMNP
```

```
      K2 = I*10
      K3 = K2 - 1
      K4 = K2 - 2
      K5 = K2 - 3
      K6 = K2 - 4
      PRINT 907, I, D(K6),D(K5),D(K4),D(K3),D(K2),
     * D(K6-5),D(K5-5),D(K4-5),D(K3-5),D(K2-5)
  901 CONTINUE
  907 FORMAT (10X,I5,8X,1P5E15.5/23X,1P5E15.5//)
      IF (T3.NE.1) GO TO 120
      WRITE (7) D
      ENDFILE 7
C
C***********************************************************************
C         DETERMINATION OF GENERALIZED FORCES AND REACTIONS
C***********************************************************************
C
  120 IF (T1.NE.1) GO TO 400
      DO 140 I=1,NEQ
  140 P(I) = 0.0
C
C         CONTRIBUTION FROM DECK ELEMENTS
C
      DO 310 N=1,NUMEL
      NCT = 0
      NKA = 5
  325 READ (1) SK,SPB
      DO 340 L=1,4
      LA = NP(N,L)*10 - NKA
      DO 350 K=1,5
      IA = (L-1)*5 + K
      SUM = 0.0
      DO 320 I=1,4
      KA = NP(N,I)*10 - NKA
      DO 330 J=1,5
      JA = (I-1)*5 + J
  330 SUM = SUM + SK(IA,JA)*D(J+KA)
  320 CONTINUE
      P(K+LA) = P(K+LA) + SUM
  350 CONTINUE
  340 CONTINUE
      NKA = 10
      NCT = NCT + 1
      IF (NCT.EQ.1) GO TO 325
  310 CONTINUE
C
C         CONTRIBUTION FROM BEAM ELEMENTS
C
      DO 360 N=1,NBEAM
      READ (1) SK
      DO 365 L=1,2
      LA = NBP(N,L)*10 - 10
      DO 370 K=1,10
      IA = (L-1)*10 + K
      SUM = 0.0
      DO 375 I=1,2
```



```
      KA = NBP(N,I)*10 - 10
      DO 380 J=1,10
      JA = (I-1)*10 + J
  380 SUM = SUM + SK(IA,JA)*D(J+KA)
  375 CONTINUE
      P(K+LA) = P(K+LA) + SUM
  370 CONTINUE
  365 CONTINUE
  360 CONTINUE
      PRINT 395
      DO 390 J=1,NUMNP
      KA = J*10
  390 PRINT 907, J,P(KA-4),P(KA-3),P(KA-2),P(KA-1),P(KA),
     *                P(KA-9),P(KA-8),P(KA-7),P(KA-6),P(KA-5)
  395 FORMAT (30H1 GENERALIZED FORCE VECTOR          /////
     * 10X,10H NODAL PT., 6X,8H U-FORCE, 8X,8H V-FORCE, 8X,8H W-FORCE,
     * 7X,9H X-MOMENT, 7X,9H Y-MOMENT  ////)
C
C***********************************************************************
C         DETERMINATION OF INTERNAL FORCES AT CENTER AND AT THE NODES
C         OF EACH ELEMENT
C***********************************Q***********************************
C
  400 IF (NSTO.LE.0.AND.T2.NE.1) GO TO 800
      REWIND 1
      READ (1) NP,NBP
      READ (1) XORD,YORD,MAT,NAT,MBAT,DM,DS,BSIN,BCOS
      IF (NSTO.LE.0) GO TO 640
      IF (NSTO.NE.NUMNP) GO TO 454
      DO 452 I=1,NUMNP
  452 NPS(I) = I
      GO TO 460
  454 READ 456, (NPS(I),I=1,NSTO)
  456 FORMAT (20I4)
C
C         INITIALIZATION OF ARRAYS FOR NODAL POINT STRESSES
C
  460 DO 464 J=1,NSTO
      DO 462 L=1,12
      XXNS(J,L) = 0.0
      YYNS(J,L) = 0.0
      XYNS(J,L) = 0.0
      XXMO(J,L) = 0.0
      YYMO(J,L) = 0.0
  462 XYMO(J,L) = 0.0
  464 NWT(J) = 0
      PRINT 457
      PRINT 459, (NPS(I),I=1,NSTO)
  457 FORMAT (// 65H NODES WHERE INTERNAL FORCES COMPUTED, AVERAGED AND
     * OUTPUT         ///)
  459 FORMAT (20I4)
C
C***********************************************************************
C         DETERMINATION OF STRESS RESULTANTS AND MOMENTS FOR DECK ELEMENT
C***********************************************************************
C
```

```
  640 IF (T2.EQ.1) PRINT 421
      DO 450 N=1,NUMEL
      NCT = 0
      M = MAT(N)
      NKA = 4
      NKB = 3
      NTRI = 4
      IF (NP(N,1).EQ.NP(N,4)) NTRI=3
  620 KKK = 0
  434 DO 441 I=1,4
      JA = NP(N,I)
      XA(I) = XORD(JA)
      YA(I) = YORD(JA)
      XX(I) = XORD(JA)
      YY(I) = YORD(JA)
      PP(I) = 1.0
      KA = JA*10 - NKA
      KB = KA + 1
      I1 = 2*I - 1
      I2 = 2*I
      R1(I1) = C(KA)
      R1(I2) = C(KB)
      KC = JA*10 - NKB
      DO 662 JL=1,3
      KKK=KKK+1
      KC = KC + 1
  662 RA(KKK) = D(KC)
  441 CONTINUE
      DO 444 I=1,6
  444 CS(I) = DS(M,I)
      DO 668 J=13,19
  668 RA(J) = 0.0
      DO 674 I=1,3
      DO 674 J=1,3
  674 CM(I,J) = DM(M,I,J)
      READ (1) SK,SPB
      CALL PSD11 (NTRI)
      CALL FPLATE (NTRI)
      NC4 = NCT*6
C
C        OUTPUT OF INTERNAL FORCES AT CENTER OF EACH ELEMENT
C
      IF (T2.NE.1) GO TO 477
      IF (NCT.EQ.0) PRINT 423,N
      DO 429 J=1,3
      I = J+NCT*3
      SCT(N,I) = ST(5,J)
  429 SCT(N,I+3) = BM(J,5)
      PRINT 469, ST(5,1),ST(5,2),ST(5,3),BM(1,5),BM(2,5),BM(3,5)
  423 FORMAT (12H ELEMENT      I5)
  421 FORMAT (50H1 INTERNAL FORCES AT CENTER OF DECK ELEMENTS          ///
     * 25X, 12H    NXX      ,5X,12H     NYY      ,5X,12H     NXY       ,
     * 6X,10H   MXX     ,7X,10H   MYY     ,7X,10H   MXY        ///)
  477 IF (NSTO.LE.0) GO TO 450
C
C        AVERAGE INTERNAL FORCES OF ADJACENT ELEMENTS AT EACH NODE
```



```
C
      DO 455 K=1,4
      IF (K.EQ.4.AND.NTRI.EQ.3) GO TO 455
      N2 = NP(N,K)
      DO 472 J=1,NSTC
      IF (N2.EQ.NPS(J)) GO TO 474
  472 CONTINUE
      GO TO 455
  474 IF (NCT.EQ.1) GO TO 478
      NWT(J) = NWT(J)+1
  478 L = NWT(J)+NC4
      XXNS(J,L) = ST(K,1)
      YYNS(J,L) = ST(K,2)
      XYNS(J,L) = ST(K,3)
      XXMO(J,L) = BM(1,K)
      YYMO(J,L) = BM(2,K)
      XYMO(J,L) = BM(3,K)
  455 CONTINUE
      M = NAT(N)
      NKA = 9
      NKB = 8
      NCT = NCT+1
      IF (NCT.EQ.1) GO TO 620
  450 CONTINUE
C
C        OUTPUT OF AVERAGE INTERNAL FORCES AT THE NODES
C
      IF (NSTO.LE.0) GO TO 800
      ALF = ALF/57.29578
      BSI = SIN(ALF)
      BCO = COS(ALF)
      PRINT 465
      DO 467 I=1,NSTO
      PRINT 468, NPS(I)
      K = NPS(I)
      LL = 0
      KJ = K*10-2
      NN = NWT(I)
      NKA = 1
      NKB = NN
      NCT = 0
  480 SNXX = 0.0
      SNYY = 0.0
      SNXY = 0.0
      SMXX = 0.0
      SMYY = 0.0
      SMXY = 0.0
  475 DO 476 L=NKA,NKB
      PRINT 469, XXNS(I,L),YYNS(I,L),XYNS(I,L),XXMO(I,L),YYMO(I,L),
     *             XYMO(I,L)
      SNXX = SNXX + XXNS(I,L)
      SNYY = SNYY + YYNS(I,L)
      SNXY = SNXY + XYNS(I,L)
      SMXX = SMXX + XXMO(I,L)
      SMYY = SMYY + YYMO(I,L)
      SMXY = SMXY + XYMO(I,L)
```

```
  476 CONTINUE
      SNP(I,LL+1) = SNXX/NN
      SNP(I,LL+2) = SNYY/NN
      SNP(I,LL+3) = SNXY/NN
      SNP(I,LL+4) = SMXX/NN
      SNP(I,LL+5) = SMYY/NN
      SNP(I,LL+6) = SMXY/NN
      PRINT 463, (SNP(I,LL+J),J=1,6)
C
C         DETERMINATICN CF PRINCIPAL VALUES CF INTERNAL FORCES
C
  226 IF (BSI.NE.0.0) GC TO 272
      AB = 0.5*(SMXX+SMYY)/NN
      BA = 0.5*(SMXX-SMYY)/NN
      AA = 0.5*(SNXX+SNYY)/NN
      BB = 0.5*(SNXX-SNYY)/NN
      TAV = SQRT(BA**2+(SMXY/NN)**2)
      TAU = SQRT(BB**2+(SNXY/NN)**2)
      SMMAX = AB+TAV
      SMMIN = AB-TAV
      SNMAX = AA+TAU
      SNMIN = AA-TAU
      IF (BA.EQ.0.0.AND.SMXY.EQ.0.0) GO TO 221
      BAG = 57.29578*ATAN2((SMXY/NN),BA)/2.0
      GO TO 219
  221 BAG = 360.0
  219 IF (BB.EQ.0.0.AND.SNXY.EQ.0.0) GO TO 211
      BANG = 57.29578*ATAN2((SNXY/NN),BB)/2.0
      GO TO 218
  211 BANG = 360.
      GO TO 218
  272 SS = BSI*BSI
      SC = BSI*BCO
      CC = BCO*BCO
      SNMAX = (SNXX*CC+SNYY*SS+2.0*SNXY*SC)/NN
      SMMAX = (SMXX*CC+SMYY*SS+2.0*SMXY*SC)/NN
      SNMIN = ((SNYY-SNXX)*SC+SNXY*(CC-SS))/NN
      SMMIN = ((SMYY-SMXX)*SC +SMXY*(CC-SS))/NN
      BANG = BSI
      BAG = BCO
  218 PRINT 486, SNMAX,SNMIN,BANG,SMMAX,SMMIN,BAG
      IF (NCT.GT.0) GO TO 467
      NKA = 7
      NKB = NWT(I)+6
      LL = 6
      NCT = NCT+1
      GO TO 480
  467 CONTINUE
      IF (T3.NE.1) GO TO 240
      WRITE (7) SNP
      ENDFILE 7
      WRITE (7) YYMO
      ENDFILE 7
C
C************************************************************************
C         DETERMINATION OF INTERNAL FORCES IN THE WEB ELEMENTS
```



```
C************************************************************************
C
  240 IF (NCIA.EQ.0) NCIA=NBEAM
      NTA = 1
      NTB = NCIA
  570 IF (NTB.EQ.NBEAM) PRINT 764
      IF (NTB.LT.NBEAM) PRINT 762
      DO 510 J=1,NSTO
      DO 505 L=1,4
      XXNS(J,L) = 0.0
      YYNS(J,L) = 0.0
      XYNS(J,L) = 0.0
      XXMO(J,L) = 0.0
      YYMO(J,L) = 0.0
  505 XYMO(J,L) = 0.0
  510 NWT(J) = 0
      DO 500 N=NTA,NTB
      M = MBAT(N)
      SI = BSIN(N)
      CO = BCOS(N)
      NA = NBP(N,1)
      NB = NBP(N,2)
      XL = (XORD(NB)-XORD(NA))*CO + (YORD(NB)-YORD(NA))*SI
      DO 520 I=1,2
      LA = ITE(I)
      XX(LA) = 0.0
      XX(I+1)= XL
      YY(I)  = 0.0
      YY(I+2)= AH
  520 CONTINUE
      DO 512 I=1,4
      XA(I) = XX(I)
      YA(I) = YY(I)
  512 PP(I) = 1.0
      DO 506 I=1,3
      DO 506 J=1,3
  506 CM(I,J) = DM(M,I,J)
      DO 508 I=1,6
  508 CS(I) = DS(M,I)
      DO 540 J=1,2
      JA = ITE(J)
      IB = ILE(J)
      I1 = NA*10
      I2 = NB*10
      R1(JA)  = D(I1-IB)*CO + D(I1-IB+1)*SI
      R1(J+1) = D(I2-IB)*CO + D(I2-IB+1)*SI
      R1(JA+4)= D(I1-IB+2)
      R1(J +5)= D(I2-IB+2)
      R1(JA+8)= D(I1-IB+3)*SI - D(I1-IB+4)*CC
      R1(J +9)= D(I2-IB+3)*SI - D(I2-IB+4)*CC
  540 CONTINUE
      CALL PLSP12
      DO 550 J=1,2
      IA = ITE(J)
      IB = ILE(J)
      I1 = NA*10
```

```
      I2 = NB*1C
      I3 = IA*3 - 2
      I4 = (J+1)*3 - 2
      RA(I3)   = D(I1-IB)*SI - D(I1-IB+1)*CC
      RA(I4)   = D(I2-IB)*SI - D(I2-IB+1)*CC
      RA(I3+1) = D(I1-IB+3)*CO + D(I1-IB+4)*SI
      RA(I4+1) = D(I2-IB+3)*CO + D(I2-IB+4)*SI
      RA(I3+2) = 0.0
  550 RA(I4+2) = 0.0
      CALL FONEW
C
C        DETERMINATION AND AVERAGING OF INTARNAL FORCES OF WEB
C        ELEMENTS AT THE NODES
C
      DO 560 K=1,2
      KA = 5 - K
      N2 = NBP(N,K)
      L = K
      DO 544 J=1,NSTO
      IF (N2.EQ.NPS(J)) GO TO 546
  544 CONTINUE
      GO TO 560
  546 XXNS(J,L) = ST(KA,1)
      YYNS(J,L) = ST(KA,2)
      XYNS(J,L) = ST(KA,3)
      XXMO(J,L) = BM(1,KA)
      YYMO(J,L) = BM(2,KA)
      XYMO(J,L) = BM(3,KA)
      XXNS(J,L+2) = ST(K,1)
      YYNS(J,L+2) = ST(K,2)
      XYNS(J,L+2) = ST(K,3)
      XXMO(J,L+2) = BM(1,K)
      YYMO(J,L+2) = BM(2,K)
      XYMO(J,L+2) = BM(3,K)
      NWT(J) = NWT(J)+1
  560 CONTINUE
  500 CONTINUE
C
C        AVERAGING OF INTERNAL FCRCES AT THE NODES OF WEB ELEMENTS
C
      DO 571 N=NTA,NTB
      DO 580 M=1,2
      I = NBP(N,M)
      DO 582 J=1,NSTO
      IF (I.EQ.NPS(J)) GO TO 584
  582 CONTINUE
      GO TO 580
  584 IF (N.EQ.NBEAM) GO TO 586
      IF (NBP(N,1).EQ.NBP(N+1,1)) GO TC 586
      IF (NBP(N,M).EQ.NBP(N+1,1)) GO TO 580
  586 PRINT 468, I
  562 NKA = 1
      NKB = 2
      LL = 0
      NN = NWT(J)
  572 SNXX = 0.C
```



```
      SNYY = 0.C
      SNXY = 0.0
      SMXX = 0.0
      SMYY = 0.0
      SMXY = 0.0
      DO 530 L=NKA,NKB
      PRINT 469, XXNS(J,L),YYNS(J,L),XYNS(J,L),XXMO(J,L),YYMO(J,L),
     *            XYMO(J,L)
      SNXX = SNXX + XXNS(J,L)
      SNYY = SNYY + YYNS(J,L)
      SNXY = SNXY + XYNS(J,L)
      SMXX = SMXX + XXMO(J,L)
      SMYY = SMYY + YYMC(J,L)
      SMXY = SMXY + XYMO(J,L)
  530 CONTINUE
      SNP(J,LL+1) = SNXX/NN
      SNP(J,LL+2) = SNYY/NN
      SNP(J,LL+3) = SNXY/NN
      SNP(J,LL+4) = SMXX/NN
      SNP(J,LL+5) = SMYY/NN
      SNP(J,LL+6) = SMXY/NN
C
C        DETERMINATION OF PRINCIPAL INTERNAL FCRCES IN WEBS
C
  568 AA = 0.5*(SNXX+SNYY)/NN
      BB = 0.5*(SNXX-SNYY)/NN
      AB = 0.5*(SMXX+SMYY)/NN
      BA = 0.5*(SMXX-SMYY)/NN
      TAU = SQRT(BB**2+(SNXY/NN)**2)
      TAV = SQRT(BA**2+(SMXY/NN)**2)
      SNMAX = AA+TAU
      SNMIN = AA-TAU
      SMMAX = AB+TAV
      SMMIN = AB-TAV
      IF (BB.EQ.0.0.AND.SNXY.EQ.0.0) GO TO 574
      BANG = 57.29578*ATAN2((SNXY/NN),BB)/2.0
      GO TO 575
  574 BANG = 360.
  575 IF (BA.EQ.0.0.AND.SMXY.EQ.0.0) GO TO 576
      BAG = 57.29578*ATAN2((SMXY/NN),BA)/2.0
      GO TO 577
  576 BAG = 360.0
  577 PRINT 463, (SNP(J,LL+I),I=1,6)
      PRINT 486, SNMAX,SNMIN,BANG,SMMAX,SMMIN,BAG
      IF (NKA.EQ.3) GO TO 580
      NKA = 3
      NKB = 4
       LL=6
      GO TO 572
  580 CONTINUE
  571 CONTINUE
      IF (NTB.EQ.NBEAM) GO TO 484
      NTA = NTB+1
      NTB = NBEAM
      GO TO 570
  484 IF (T3.NE.1) GO TO 600
```

```
      WRITE (7) SNP
      ENDFILE 7
      WRITE (7) YYMC
      ENDFILE 7
  600 PRINT 2019
C
C         FORMAT STATEMENTS
C
  463 FORMAT (5X,12H NP. AVERAGE,3X,1P6E17.5/)
  486 FORMAT (2X,17H     PRIN NP QUANT,1X,1P6E17.5///)
  469 FORMAT (20X,1P6E17.5)
  468 FORMAT (12H NODAL POINT  I5/)
  465 FORMAT (48H1 INTERNAL FORCES AT NODES CF DECK PLATES            //
     * 25X,12H     NXX      ,5X,12H      NYY      ,5X,12H      NXY      ,
     * 6X,10H   MXX     ,7X,10H   MYY     ,7X,10H   MXY     //)
  762 FORMAT (50H1 INTERNAL FORCES AT NODES CF TRANSV WEB ELEMENTS    //
     * 25X,12H     NXX      ,5X,12H      NYY      ,5X,12H      NXY      ,
     * 6X,10H   MXX     ,7X,10H   MYY     ,7X,10H   MXY     //)
  764 FORMAT (50H1 INTERNAL FORCES AT NODES CF LONGIT WEB ELEMENTS    //
     * 25X,12H     NXX      ,5X,12H      NYY      ,5X,12H      NXY      ,
     * 6X,10H   MXX     ,7X,10H   MYY     ,7X,10H   MXY     //)
 2019 FORMAT (// 23H LINK NO 5 CCMPLETED      //)
  800 RETURN
      END
```



```
      SUBROUTINE PSC11 (NTRI)
C
C**********************************************************************
C        THIS SUBROUTINE COMPUTES THE STRESS RESULTANTS FOR THE PLANE
C        STRESS ELEMENTS  Q8D9  ANC  Q8D11(2)  AT NODES AND AT CENTER
C**********************************************************************
C
C       INPUT
C
C         D(I)           CONSTITUTIVE LAW RELATING STRES-RES TO STRAINS
C         I=1,6          ELEMENTS  11,22,33,12,13,23
C         XA(4)          GLOBAL X-COCRDINATES CF NODES
C         YA(4)          GLOBAL Y-CCC DINATES OF NODES
C         R1(I)          GLOBAL U-V DISPLACEMENTS AT THE NODES
C
C       OUTPUT
C
C         ST(I,J)        INTERNAL STRESS RESULTANTS AT CENTER AND
C                        CCRNER NCDES
C         I=1,4          CCRNER NCDES
C         I=5            CENTER NODE
C         J=1,2,3        NXX,NYY,NXY  STRESS RESULTANTS
C
      COMMCN / PLSTR / XA(4),YA(4),D(6),S(12,12),R1(12),ST(5,3)
C
      DIMENSION P(4,2),CC(4,2),A(2,2),ETA(2),IPERM(2),IT(5),
     *          AX(5),AY(5),X(4),Y(4),DIS(8),U(4),V(4)
      DATA DC / -1., 1., 1.,-1.,  -1.,-1., 1., 1. /
      DATA AX / 0.,-1., 1., 1.,-1. /
      DATA AY / 0.,-1.,-1., 1., 1. /
      DATA IT /5,1,2,3,4/, IPERM /2,1/
      EQUIVALENCE (A11,A(1)), (A21,A(2)), (A12,A(3)), (A22,A(4))
C
C         INITIALIZATION
C
      DO 100 J=1,3
      DO 100 I=1,5
  100 ST(I,J)=0.0
      EXY = 0.0
C
C         TRANSFCRMATION INTO LOCAL CONVECTED CCORDINATES
C
      DX = XA(2)-XA(1)
      DY = YA(2)-YA(1)
      AL = SQRT(DX**2+DY**2)
      CO = DX/AL
      SI = DY/AL
      DO 520 I=1,4
      X(I) = (XA(I)-XA(1))*CO + (YA(I)-YA(1))*SI
  520 Y(I) =-(XA(I)-XA(1))*SI + (YA(I)-YA(1))*CO
      CC = CO*CC
      SS = SI*SI
      SC = SI*CC
      DO 540 I=1,4
      I2 = I+I
      I1 = I2-1
```

```
      DIS(I1) = R1(I1)*CO + R1(I2)*SI
  540 DIS(I2) =-R1(I1)*SI + R1(I2)*CC
C
C         COMPUTE STRESS RESULTANTS AT SAMPLING POINTS
C
      DO 300 II=1,5
      JJ = IT(II)
      IF (II.GT.1.ANC.NTRI.EQ.3) GC TO 70C
      ETA(1)=AX(II)
      ETA(2)=AY(II)
C
C         FORMATICN CF LCCAL DERIVATIVES
C
      DO 150  I = 1,2
      J = IPERM(I)
      A(I,1) = 0.
      A(I,2) = 0.
      DO 140  L = 1,4
      C=0.250*DC(L,I)*(1.0+DC(L,J)*ETA(J))
      P(L,I)=C
      A(I,1)=A(I,1)+C*X(L)
  140 A(I,2)=A(I,2)+C*Y(L)
  150 CONTINUE
      DET = A11*A22 - A12*A21
      IF (CET.GT.0.0) GC TO 250
      PRINT 101C, CET,ETA(1),ETA(2)
      PRINT 1020
      PRINT 103C, (X(I),Y(I),I=1,4)
 1010 FORMAT (//30H DETERMINATE CF JACOBIAN          E15.5/
     *             30H SAMPLING AT LCCATICN  X       E15.5/
     *             30H SAMPLING AT LOCATICN  Y       E15.5)
 1020 FORMAT (//30H ELEMENT CCCRCINATES           /)
 1030 FORMAT (1CX,2E15.5)
C
C         FORMATION OF GLOBAL CERIVATIVES
C
  250 DO 200 J=1,4
      U(J) =  A22*P(J,1) - A12*P(J,2)
  200 V(J) = -A21*P(J,1) + A11*P(J,2)
C
C         DETERMINATICN CF STRESS RESULTANTS AT NODES AND CENTER
C
      XX=0.0
      YY=0.0
      ZZ=0.0
      DO 500 I=1,4
      K = I + I
      J = K - 1
      IF (II.NE.1) GC TC 560
      EXY = EXY + (V(I)*DIS(J)+U(I)*DIS(K))/DET
  560 XX = XX + D(1)*U(I)*DIS(J) + D(4)*V(I)*DIS(K)
      YY = YY + D(4)*U(I)*DIS(J) + D(2)*V(I)*DIS(K)
      ZZ = ZZ + D(5)*U(I)*DIS(J) + D(6)*V(I)*DIS(K)
  500 CONTINUE
      SXX = XX/CET+D(5)*EXY
      SYY = YY/CET+D(6)*EXY
```



```
      SXY = ZZ/CET+D(3)*EXY
C
C         TRANSFCRM STRESS RESULTANTS INTO GLOBAL COORDINATES
C
      ST(JJ,1) = SXX*CC+SYY*SS-2.0*SXY*SC
      ST(JJ,2) = SXX*SS+SYY*CC+2.0*SXY*SC
      ST(JJ,3) = (SXX-SYY)*SC+SXY*(CC-SS)
  300 CONTINUE
      IF (NTRI.NE.3) GC TO 4CC
C
C         DETERMINATICN CF NOCAL STRESSES FCR CCNSTANT STRAIN TRIANGLE
C
  700 DO 420 I=1,4
      DO 420 J=1,3
  420 ST(I,J) = ST(5,J)
  400 RETURN
      END
```

```
      SUBROUTINE PLSP12
C
C***********************************************************************
C         THIS SUBROUTINE DETERMINES STRESS RESULTANTS AT THE NODES AND
C         AT THE CENTER FOR THE PLANE STRESS ELEMENT  QUSP12
C***********************************************************************
C
C     INPUT
C
C        D(I)            CONSTITUTIVE LAW RELATING STRESS RES TO STRAINS
C        I=1,6           ELEMENTS 11,22,33,12,13,23
C        XORD(I)         LOCAL X-COORDINATES OF NODES
C        YORD(I)         LOCAL Y-COORDINATES OF NODES
C        V(I)            DISPLACEMENT VECTOR
C        I=1,4           U DISPL (LB,RB,RT,LT)
C        I=5,8           V DISPL (LB,RB,RT,LT)
C        I=9,12          0 ROTATIONS  DV/DX   (LB,RB,RT,LT)
C        R=.TRUE.        ROTATIONS HAVE BEEN ELEIMINATED BY STAT COND
C
C     OUTPUT
C
C        ST(I,J)         INTERNAL STRESS RESULTANTS AT CORNER AND
C                        CENTER NODES
C        I=1,4           CORNER NODES
C        I=5             CENTER NODE
C        J=1,2,3         NXX, NYY, NXY STRESS RESULTANTS
C
      COMMON / PLSTR / XORD(4),YORD(4),D(6),S(12,12),V(12),ST(5,3)
C
      DIMENSION EXX(4),EYY(8),EXY(4),EYX(8),EXZ(2),EYZ(2),EZZ(2),
     *          UU(2),UV(2),VU(2),VV(2),VX(8),VY(8),ALFX(2),ALFY(2),
     *          X1(4),X2(4),Y1(4),Y2(4),FT1(4),FT2(4),FX4(4),FY4(4),
     *          FX(4),FY(4),AX(5),AY(5),ITE(2),ITA(4),ITI(4)
C
      DATA AX / -1.0, 1.0, 1.0,-1.0, 0.0 /
      DATA AY / -1.0,-1.0, 1.0, 1.0, 0.0 /
      DATA FY4 /-1.0,-1.0, 1.0, 1.0/, FX4 /-1.0, 1.0, 1.0,-1.0 /
      DATA ITE /1,4/, ITA /1,2,2,1/, ITI /4,3,3,4/
      EQUIVALENCE (FX,X1), (FY,Y1), (EXX,X2), (EXY,Y2)
      LOGICAL R
C
C        INITIALIZATION
C
      R = .FALSE.
      DO 100 J=1,3
      DO 100 I=1,5
  100 ST(I,J)=0.0
C
C        GEOMETRICAL TRANSFORMATION OF QUADRILATERAL BOUNDARIES
C
      IF (.NOT.R) GO TO 140
C
C        RECOVER ROTATIONS AT CP
C
      DO 110 L=9,12
      M = L - 1
```



```
      DO 110 K =1,M
  110 V(L) = V(L) - S(K,L)*V(K)
  140 X1(1)=0.5*(XORD(2)-XORD(1))
      X1(3)=0.5*(XORD(3)-XORD(4))
      X1(2)=X1(1)
      X1(4)=X1(3)
      Y1(1)=0.5*(YORD(2)-YORD(1))
      Y1(3)=0.5*(YORD(3)-YORD(4))
      Y1(2)=Y1(1)
      Y1(4)=Y1(3)
      X2(1)=0.5*(XORD(4)-XORD(1))
      X2(2)=0.5*(XORD(3)-XORD(2))
      X2(3)=X2(2)
      X2(4)=X2(1)
      Y2(1)=0.5*(YORD(4)-YORD(1))
      Y2(2)=0.5*(YORD(3)-YORD(2))
      Y2(3)=Y2(2)
      Y2(4)=Y2(1)
      DO 120 I=1,4
      DET =X1(I)*Y2(I)-Y1(I)*X2(I)
      FCT=DET+X2(I)*Y1(I)
      FT1(I)= Y1(I)*X1(I)/2.0/FCT
  120 FT2(I)=X1(I)*(1.0-Y1(I)*X2(I)/FCT)
C
      DO 200 II=1,5
      X=AX(II)
      Y=AY(II)
      XA = X*X
      XB = XA*X
C
C         FORMATION OF LOCAL DERIVATIVES
C
      DO 210 I=1,4
      FX(I)=0.25*FX4(I)*(1.0+FY4(I)*Y)
  210 FY(I)=0.25*FY4(I)*(1.0+FX4(I)*X)
C
C         FORMATION OF JACOBIAN TRANSF. MATRIX
C
      X11=0.0
      X22=0.0
      X12=0.0
      X21=0.0
      DO 300 I=1,4
      X11=X11+FX(I)*XORD(I)
      X22=X22+FY(I)*YORD(I)
      X12=X12+FX(I)*YORD(I)
  300 X21=X21+FY(I)*XORD(I)
C
C         CALCULATION OF JACOBIAN DETERMINANT
C
      DET = (X11*X22-X12*X21)
      IF (DET.GT.0.0) GO TO 450
      PRINT 1010, DET,X,Y
      PRINT 1020
      PRINT 1030, (XORD(I),YORD(I),I=1,4)
 1010 FORMAT (//30H DETERMINANT OF JACOBIAN      E15.5/
```

```
     *          30H OUTPUT AT LOCATION  X          E15.5/
     *          30H OUTPUT AT LOCATION  Y          E15.5//)
 1020 FORMAT (//30H ELEMENT COORDINATES             /)
 1030 FORMAT (10X,2E15.2)
      GO TO 200
C
C         FORMATION OF LOCAL DERIVATIVES
C
  450 DO 400 I=1,4
      VY(I) = 0.125*FY4(I)*(2.0+3.0*FX4(I)*X-FX4(I)*XB)
      VX(I) = 0.375*(1.0+FY4(I)*Y)*(FX4(I)-FX4(I)*XA)
      VY(I+4) = 0.125*FY4(I)*(-FX4(I)-X+FX4(I)*XA+XB)
  400 VX(I+4) = 0.125*(1.0+FY4(I)*Y)*(-1.0+2.0*FX4(I)*X+3.0*XA)
C
C         FORMATION OF DERIVATIVES IN GLOBAL COORDINATES
C
      DO 420 I=1,4
      EXX(I)= X22*FX(I)-X12*FY(I)
  420 EXY(I)=-X21*FX(I)+X11*FY(I)
      DO 440 I=1,4
      K=ITA(I)
      J=ITI(I)
      AA= VX(I)+FY4(I)*FT1(K)*VX(K+4)  + FY4(I)*FT1(J)*VX(J+4)
      BB= VY(I)+FY4(I)*FT1(K)*VY(K+4)  + FY4(I)*FT1(J)*VY(J+4)
      EYY(I+4)=-X21*FT2(I)*VX(I+4)+X11*FT2(I)*VY(I+4)
      EYX(I+4)= X22*FT2(I)*VX(I+4)-X12*FT2(I)*VY(I+4)
      EYY(I)=-X21*AA + X11*BB
  440 EYX(I)= X22*AA - X12*BB
C
C
C         COMPUTATION OF NODAL POINT STRESSES
C
      XX=0.0
      YY=0.0
      ZZ=0.0
      DO 500 I=1,4
      XX=XX+D(1)*EXX(I)*V(I)+D(5)*EXY(I)*V(I)
      YY=YY+D(4)*EXX(I)*V(I)+D(6)*EXY(I)*V(I)
  500 ZZ=ZZ+D(5)*EXX(I)*V(I)+D(3)*EXY(I)*V(I)
      DO 510 I=1,8
      XX=XX+D(4)*EYY(I)*V(I+4)+D(5)*EYX(I)*V(I+4)
      YY=YY+D(2)*EYY(I)*V(I+4)+D(6)*EYX(I)*V(I+4)
  510 ZZ=ZZ+D(6)*EYY(I)*V(I+4)+D(3)*EYX(I)*V(I+4)
      ST(II,1) = XX/DET
      ST(II,2) = YY/DET
      ST(II,3) = ZZ/DET
  200 CONTINUE
      RETURN
      END
```



```
      SUBROUTINE  FPLATE (NTRI)
C
C************************************************************************
C         THIS SUBROUTINE DETERMINES THE MOMENT RESULTANTS FOR THE
C         QUADRILATERAL PLATE BENDING ELEMENT QL19 AND THE TRIANGULAR
C         PLATE BENDING ELEMENT  LCCT-9
C************************************************************************
C
C      INPUT
C         CM(3,3)           CONSTITUTIVE LAW RELATING MOMENTS TO CURVATURES
C         X(4)              GLOBAL X-COORDINATES OF NODES
C         P(4)              NODAL LOAD INTENSITIES OF DISTRIBUTED LOADS
C
C      OUTPUT
C
C         BM(I,J)           INTERNAL MOMENTS AT CORNER AND CENTER NODES
C         J=1,4             CORNER NODES
C         J=5               CENTER NODE
C         I=1,3             MXX,MYY,MXY MOMENT COMPONENTS
C
      COMMON / PLBDG / X(5),Y(5),CM(3,3),P(5),BM(3,5),CV(3,5),S(19,19),
     *                 R1(19)
      COMMON / TRIAG / B(3),A(3),CMT(3,3),PT(3),CVT(3,3),ST(12,12),R(12)
C
      DIMENSION IPERM(4),NC(3),FAC(3)
      DATA IPERM /2,3,4,1/, FAC /.5,.5,.25/
      LOGICAL TRIG
C
C         INITIALIZE PARAMETERS
C
      NTR = 4
      TRIG = NTRI.EQ.3
      IF (TRIG) NTR = 1
      DO 130 I=1,3
      DO 130 J=1,5
  130 CV(I,J) = 0.0
      X(5) = 0.25*(X(1)+X(2)+X(3)+X(4))
      Y(5) = 0.25*(Y(1)+Y(2)+Y(3)+Y(4))
      P(5) = 0.25*(P(1)+P(2)+P(3)+P(4))
      NBF = 11
      L1 = 13
      L2 = 19
      IF (.NOT.TRIG) GO TO 120
      NBF = 9
      GO TO 160
C
C         RECOVER DISPLACEMENTS FOR  DOF  ELIMINATED BY STAT COND
C
  120 DO 140 L=L1,L2
      M = L - 1
      DO 140  K = 1,M
  140 R1(L) = R1(L) - S(K,L)*R1(K)
C
C         PREPARE INPUT FOR MOMENT CALCULATION IN EACH  LCCT TRIANGLE
C
  160 NC(3) = NBF - 6
```

```
      DO 300 N=1,NTR
      M = IPERM(N)
      NC(1) = N
      NC(2) = M
      L = NC(3)
      A(1) = X(L)-X(M)
      A(2) = X(N)-X(L)
      A(3) = X(M)-X(N)
      B(1) = Y(M)-Y(L)
      B(2) = Y(L)-Y(N)
      B(3) = Y(N)-Y(M)
      IF (TRIG) GO TO 330
      DO 180  K = 1,3
      KI = 3*(N-1) + K
      KJ = 3*(M-1) + K
      R(K  ) = R1(KI)
      R(K+3) = R1(KJ)
  180 R(K+6) = R1(K+12)
      R(10) =  R1(M+15)
      R(11) = -R1(N+15)
      GO TO 360
  330 DO 340 I=1,9
  340 R(I) = R1(I)
  360 CALL FLCCT (NBF)
C
C        ACCUMULATE AND AVERAGE CURVATURES AT THE NODES
C
      DO 260  J = 1,3
      L = NC(J)
      C = FAC(J)
      IF(TRIG) C=1.0
      DO 260  I = 1,3
  260 CV(I,L) = CV(I,L) + C*CVT(I,J)
  300 CONTINUE
C
C        DETERMINE INTERNAL MOMENTS AT THE NODES AND AT CENTER
C
      DO 380  K = 1,5
      DO 380  I = 1,3
  380 BM(I,K) = CM(I,1)*CV(1,K) + CM(I,2)*CV(2,K) + CM(I,3)*CV(3,K)
      RETURN
      END
```



```
      SUBROUTINE FLCCT  (NBF)
C
C*********************************************************************
C        THIS SUBROUTINE COMPUTES CURVATURES FOR A  LCCT TRIANGULAR
C     PLATE BENDING ELEMENT WITH  'NBF' BENDING D.F. -RIGHT HAND SYSTEM
C*********************************************************************
C
      COMMON / TRIAG / B(3),A(3),CMT(3,3),PT(3),CVT(3,3),ST(12,12),R(12)
C
      DIMENSION U(3),HT(3),TX(3),TY(3),Q(3,6),IPERM(3),NKN(2,3)
      DATA IPERM/2,3,1/, NKN/2,5, 8,2, 5,8/
C
C        INITIALIZATION
C
      AREA = A(3)*B(2)-A(2)*B(3)
      DO 120  I = 1,3
      J = IPERM(I)
      X = A(I)**2+B(I)**2
      U(I) = -(A(I)*A(J)+B(I)*B(J))/X
      X = SQRT(X)
      TX(I) =  0.5*A(I)/X
      TY(I) = -0.5*B(I)/X
      HT(I) =  4.0*AREA/X
      A1 = A(I)/AREA
      B1 = B(I)/AREA
      A2 = A(J)/AREA
      B2 = B(J)/AREA
      Q(1,I)   = B1*B1
      Q(2,I)   = A1*A1
      Q(3,I)   = 2.*A1*B1
      Q(1,I+3) = 2.*B1*B2
      Q(2,I+3) = 2.*A1*A2
  120 Q(3,I+3) = 2.*(A1*B2+A2*B1)
C
C        RECOVER DISPLACEMENTS FOR  DOF  ELIMINATED BY KINEMAT  COND
C
      M = 12 - NBF
      IF (M.LE.0)  GO TO 160
      DO 140  N = 1,M
      K = 13 - N
      L1 = NKN(1,N)
      L2 = NKN(2,N)
  140 R(K) = (R(L1)+R(L2))*TX(K-9)+(R(L1+1)+R(L2+1))*TY(K-9)
C
C        DETERMINE CURVATURES AT THE NODES
C
  160 DO 300  I = 1,3
      J = IPERM(I)
      K = IPERM(J)
      II = 3*I
      JJ = 3*J
      KK = 3*K
      A2 = A(J)
      A3 = A(K)
      B2 = B(J)
      B3 = B(K)
```

```
      U2 = U(J)
      U3 = U(K)
      W2 = 1.-U2
      W3 = 1.-U3
      C21 = -(2.+W2)*B2-(2.+U3)*B3
      C22 = B2*W2-B3*U3
      C31 = -(2.+W2)*A2-(2.+U3)*A3
      C32 = A2*W2-A3*U3
      C51 = 4.*B3-B2+B3*W3
      C52 = B2-B3*W3
      C61 = 4.*A3-A2+A3*W3
      C62 = A2-A3*W3
      C81 = B3-4.*B2-B2*U2
      C82 = B2*U2-B3
      C91 = A3-4.*A2-A2*U2
      C92 = A2*U2-A3
      CO21 = -B2-(3.+U3)*B3
      CO22 =  B3+(3.+W2)*B2
      CO31 = -A2-(3.+U3)*A3
      CO32 =  A3+(3.+W2)*A2
      DO 200  N = 1,3
      Q11 = Q(N,I)
      Q22 = Q(N,J)
      Q33 = Q(N,K)
      Q12 = Q(N,I+3)
      Q23 = Q(N,J+3)
      Q31 = Q(N,K+3)
      Q1 = Q22-C33
      Q2 = Q22-C23
      Q3 = C33-C23
      Q4 = C23+C1
      Q5 = Q23-C1
  200 CVT(N,I) =  (-6.*C11+3.*((U3-W2)*Q1+(U3+W2)*Q23))*R(II-2)
     1        + (6.*Q22+3.*W3*C4)*R(JJ-2) + (6.*C33+3.*U2*Q5)*R(KK-2)
     2        +((C21*Q1+C22*Q23+4.*(B2*Q31-B3*C12))  *R(II-1)
     3        + (C31*Q1+C32*Q23+4.*(A2*Q31-A3*C12))  *R(II)
     4        + (C51*Q22+C52*C3) *R(JJ-1) + (C61*Q22+C62*Q3) *R(JJ)
     5        + (C81*Q33+C82*Q2) *R(KK-1) + (C91*Q33+C92*Q2) *R(KK)
     6        + HT(K)*Q4*R(K+9) + HT(J)*C5*R(J+9))/2.
  300 CONTINUE
      RETURN
      END
```



```
      SUBROUTINE FONEW
C
C***********************************************************************
C        THIS SUBROUTINE COMPUTES THE MY-MOMENTS AT EACH NODE
C        ASSUMING ONE WAY BENDING  IN THE Y - DIRECTION
C        INPUT AND OUTPUT ARE THE SAME AS FOR FPLATE
C***********************************************************************
C
      COMMON / PLBDG / X(5),Y(5),CM(3,3),P(5),EM(3,5),CV(3,5),S(19,19),
     *                 R(19)
C
      DIMENSION FA(4,2), ITE(4), ILE(4), IPE(2), FB(4), FC(4)
      DATA FA /-3.0,-4.C,3.0,-2.C,  3.C, 2.C,-3.0,4.0 /
      DATA ITE /1,2,10,11/, ILE /4,5,7,8/, IPE /1,4/
C
C        INITIALIZATION
C
      DO 100 I=1,3
      DO 100 J=1,5
  100 BM(I,J) = 0.0
      FCT = CM(1,1)/2.0
      XL =(Y(4) - Y(1))/2.0
      XR =(Y(3) - Y(2))/2.0
      FAC = 0.25*(X(2)-X(1)+X(3)-X(4))
C
C        DETERMINE MY-MOMENTS AT THE NODES ASSUMING BEAM BEHAVIOR
C
      DO 200 I=1,2
      DO 210 J=1,3,2
      FB(J) = FA(J,I)/XL/XL
  210 FC(J) = FA(J,I)/XR/XR
      DO 220 J=2,4,2
      FB(J) = FA(J,I)/XL
  220 FC(J) = FA(J,I)/XR
      IA = IPE(I)
      IB = I + 1
      SUM = 0.0
      SUN = 0.0
      DO 300 J=1,4
      JA = ITE(J)
      JB = ILE(J)
      SUM = SUM + FB(J)*R(JA)
  300 SUN = SUN + FC(J)*R(JB)
      BM(2,IA) = FCT*SUM*FAC
      BM(2,IB) = FCT*SUN*FAC
  200 CONTINUE
      RETURN
      END
```

APPENDIX B

Listing of Data for Example 5 -
Two Cell Box Girder Bridge on
Skewed Supports.

B1

```
START
ANALYSIS OF 45 DEGREE 2 CELL BOX
 168 189  72   4   6 172  3.
   1   1   2   2   1   1   3
   2   2   4   5   3   1   3
   3   3   5   6   3   1   2
   4   4   7   8   5   1   3
   5   5   8   9   6   1   2
   6   6   9  10   6   1   2
   7   7  11  12   8   1   3
   8   8  12  13   9   1   2
  10  10  14  15  10   1   2
  11  11  16  17  12   1   3
  12  12  17  18  13   1   2
  15  15  20  21  15   1   2
  16  16  22  23  17   1   3
  17  17  23  24  18   1   2
  21  21  27  28  21   1   2
  22  22  29  30  23   1   3
  23  23  30  31  24   1   2
  28  28  35  36  28   1   2
  29  29  37  38  30   1   3
  30  30  38  39  31   1   2
  36  36  44  45  36   1   3
  37  37  45  47  38   1   3
  38  38  47  48  39   1   2
  44  44  53  54  45   1   3
  45  46  55  56  47   1   3
  46  47  56  57  48   1   2
  52  53  62  63  54   1   3          8 132
 133 145 154 146 145   1   3
 134 146 154 155 147   1   2
 140 152 160 161 153   1   3
 141 154 162 155 154   1   2
 142 155 162 163 156   1   2
 147 160 167 168 161   1   3
 148 162 169 163 162   1   2
 149 163 169 170 164   1   2
 153 167 173 174 168   1   3
 154 169 175 170 169   1   2
 155 170 175 176 171   1   2
 158 173 178 179 174   1   3
 159 175 180 176 175   1   2
 160 176 180 181 177   1   2
 162 178 182 183 179   1   3
 163 180 184 181 180   1   2
 164 181 184 185 182   1   2
 165 182 185 186 183   1   3
 166 184 187 185 184   1   2
 167 185 187 188 186   1   3
 168 187 189 188 187   1   3
   1   3   6   4
   2   6  10   4
   3  10  15   4
   4  15  21   4
   5  21  29   4
```

B2

```
  6  28  36   4
  7 154 162   4
  8 162 169   4
  9 169 175   4
 10 175 180   4
 11 180 184   4
 12 184 187   4
 13   3   5   4
 14   5   8   4
 15   8  12   4
 16  12  17   4
 17  17  23   4
 18  23  30   4
 19  30  38   4
 20  38  47   4
 32 146 154   4   9
 33  15  20   4
 34  20  26   4
 35  26  33   4
 36  33  41   4
 37  41  50   4
 49 149 157   4   9
 50 157 164   4
 51 164 170   4
 52 170 175   4
 53  36  44   4
 54  44  53   4
 66 152 160   4   9
 67 160 167   4
 68 167 173   4
 69 173 178   4
 70 178 182   4
 71 182 185   4
 72 185 187   4
  1
  23.
  33.        3.
  46.
  66.        6.
  79.
 109.        9.
 1112.
 1512.       12.
 1615.
 2115.       15.
 2218.
 2818.       18.
 2921.
 3621.       21.
 3724.
 4524.       24.
 4627.
 5427.       24.
 5530.
 6330.       24.          9 153
15463.       3.
```

```
 16163.        24.
 16266.        6.
 16866.        24.
 16969.        9.
 17469.        24.
 17572.         12.
 17972.         24.
 18075.        15.
 18375.        24.
 18478.        18.
 186 78.       24.
 18781.        21.
 18881.        24.
 18984.        24.
   1432000.    432000.    187826.087.15                        .542
   2432000.    432000.    187826.087.15                        .458
   3432000.    432000.    187826.087.15
   4432000.    432000.    187826.087.15                        .667
   3 T   T      T
  15     T      T
  36 T T T      T
 154     T      T
 175     T      T
 187     T      T
LOAD
   1
  65                           -1.0
   1           49  12  -45.0
   3   5   8   12  17  23  30  38  47  56  74  83  92 101 110 119 128 137 146 154
  36  44  53   62  71  80  89  98 107 116 134 143 152 160 167 173 178 182 185 187
  55  65  75   85  95 105 115 125 135
```