Title
A Step Towards a General Framework for Consequential Life Cycle Assessment

Journal
Journal of Industrial Ecology, 21(2)

ISSN
1088-1980

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Publication Date
2017-04-01

DOI
10.1111/jiec.12433

Peer reviewed

A Step Towards a General Framework for Consequential Life Cycle Assessment

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Keywords:

life cycle assessment (LCA)
policy analysis
energy
renewable energy
price effects
industrial ecology

Supporting information is available on the *JIE* Web site

Summary

At the core of consequential life cycle assessment (CLCA) is a model of the economic system of which the activity that motivates the CLCA is a part. While there are several applications of CLCA in the literature, there does not appear to exist a formal, general mathematical framework. To address this gap, this article presents a general multi-market equilibrium framework, which could be adapted to an arbitrary level of complexity depending on the context and data availability. A general expression for total pollution (of a given type) is derived, which highlights different factors that determine the impact on emissions. It is then illustrated how microeconomic theory can help predict the direction of price and quantity changes for each commodity within the modeled system simply based on an activity's relationship to the ultimate activity or service, which motivates the CLCA. The steps involved in converting the multi-market framework to general equilibrium are also discussed.

Introduction

The application of life cycle assessment (LCA) as an aid for public decision-making seems to support a categorization of LCAs into – attributional LCAs and consequential LCAs (Delucchi 2004; Ekvall and Weidema 2004; Finnveden et al. 2009; Guinee et al. 2010; Earles and Halog 2011; Plevin et al. 2014). These two types of LCA differ in their goal, scope, system boundary, methodology, data, and intended use, with the result that the two approaches are more complementary than substitutable (Rajagopal 2014; Suh and Yang 2014; Dale and Kim 2014; Anex and Lifset 2014). At the root of these differences is the fact that ALCA's strength is in describing the present or past state of a system while CLCA's purpose is to predict the possible future states of a system under different scenarios. This is not to say that ALCA cannot be utilized for analyses of possible future scenarios. ALCA is in fact often used for understanding the potential benefits of replacing one activity or service with a substitute. But implicit in such exercises are assumptions such as that one good simply replaces an equivalent quantity of another and that there are no spillover effects on the production and consumption of other goods in the economy. Spillover effects are variously referred to as unintended or market-mediated or indirect effects in the literature, terms I use interchangeably here. CLCA on the other hand aims to make predictions using a different set of assumptions, while accounting for the major, if not all, indirect effects. For instance, predicting with ALCA requires direct assumptions about the changes in the quantity of different goods consumed. An alternative approach involves making assumptions about how individuals will adjust to price changes and then impute how prices and consumption change using a model of economic behavior. A second distinction is that whereas ALCA is product-specific, the economic and policy context is equally if not more important than the attributes of a product's own life cycle to a CLCA (Lemoine et al. 2010; Bento et al. 2015). Lastly, ALCA is a relatively mature approach to formal

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DOI: 10.1111/jiec.12433

Editor managing review: Sangwon Suh

Volume 00, Number 0

guidelines, which are codified in the form of ISO 14000 standards for environmental management. There is now a large literature on CLCA for different product systems. Examples of applications include electric power systems (Mattsson et al. 2003), lead-free solders (Ekvall and Andrae 2006), waste management (Ekvall et al. 2007), fuel cell vehicles (Sandén and Karlström 2007), milk production (Thomassen et al. 2008), the European energy sector (Dandres et al. 2011), and bioenergy and biofuels (Tonini et al. 2012; Earles et al. 2013), to name a few. However, there does not exist a formal, general economic and mathematical framework that could be adapted to any specific application. This article is a step towards addressing this gap. The topic of linking or reconciling potentially different insights one might derive from an ALCA and a CLCA, while it is a related and open research question, is beyond the scope of this article.

The rest of the article is organized as follows. In the next section I discuss the role for and limitations of market equilibrium-based economic models for CLCA. In the subsequent section I illustrate our approach to CLCA using a simplified example and show how the micro-economic principles of supply and demand can be used to predict the direction of price effects on different primary inputs that are responsible for emissions. I then generalize the multi-market model to a system with an arbitrary number of inputs, outputs, and intermediate goods. The next section describes the steps involved in converting the multi-market framework to general equilibrium. The final section summarizes the article.

CLCA and Market Equilibrium Models

The basic motivation underlying CLCA is clear. It is concerned with predicting the environmental impacts of a new technology (or policy) shock taking into consideration how markets will adjust to such a shock. By market adjustment it is meant the change in price and quantity of goods produced (or consumed) and how those changes affect environmental outcome in ways not generally accounted for under an ALCA. Since market adjustment depends on the policy regime in place, a CLCA is also policy-specific. CLCA, therefore, requires an economic model containing the markets for the goods that are affected by the technology or policy in question. This statement raises two further questions. One is what is meant by an economic model, for there are several to choose from. Partial equilibrium (PE) models, computable general equilibrium (CGE) models, and econometric models, are examples of a few different types of economic models that are used for different purposes, and each one can vary in the level of detail and complexity. For a comparison of these approaches see work by Khanna and Zilberman (2012) and Rajagopal and Zilberman (2013). The second question is how one determines which are the goods whose markets are to be included in a CLCA. There do not exist any obvious criteria for selecting or leaving out specific goods, or at least such have not been formally laid out in the literature. This is akin to the problem of system boundary definition under ALCA. This article covers the first of these two questions only.

Irrespective of the type of economic model used, the basic approach to predicting the impact of a shock to a system involves a comparison of outcomes under two different states of the system–one without the shock and another with the shock. Each state is typically assumed to be an equilibrium in which all markets clear, a sort of steady state to which the system settles with or without the shock. For new technologies or policies supporting such technologies, past experience is likely to be of limited use for predicting future impacts. This is one factor that limits the usefulness of econometric or statistically-based models and is a reason why numerical simulation of theoretical models of market equilibrium is the common approach for ex ante assessments. The modeling framework I present in this article falls under this category as well.

Market equilibrium models can range in complexity from a single sector PE model to multi-sector multi-region global CGE models (Sadoulet and De Janvry 1995; Francois and Reinert 1997). I present a general multi-market single region partial equilibrium framework, henceforth simply “multi-market model” which could be adapted to span the gamut of market equilibrium models. I build on work by Rajagopal (2014), which laid out a conceptual approach to CLCA but did not develop a formal mathematical model for CLCA. Specifically, this article does the following. I illustrate how one could assemble a multi-market model for a given set of commodities and also describe how this can be transformed into a CGE model. I also derive a general expression for the impact on emissions, which highlights the role of different factors–some of which are exogenous and others endogenous to the model–that are central to the calculation of change in emissions. I then show how microeconomic theory can predict the direction of price and quantity changes for each good in the system simply based on the good’s relationship to the main good for which the CLCA is being performed.

The limitations and criticisms applicable to the different types of economic equilibrium models extend to the modeling framework I lay out here. For instance, see work by Ackerman (2005) for limitations of CGE models for trade policy analysis, and Böhringer and Rutherford (2008) for limitations of CGE and PE modeling of energy-economy-environment interactions. Despite their shortcomings, there is a long history of use of equilibrium models for predicting the economic effects of technical shocks, and energy, environmental, agricultural, and trade policy shocks (Bhattacharyya 1996; Kretschmer and Peterson 2010). In these applications, equilibrium models are used to inform policy makers of both the net economic benefits and the distribution of economic gains and losses to different economic groups under alternative policies and future scenarios. Because equilibrium models are highly simplified representations of reality, and because of gaps and uncertainty in the data required, these models are not relied upon for their absolute measures of impact. Instead these models are useful for qualitative insights and at best order of magnitude estimates regarding

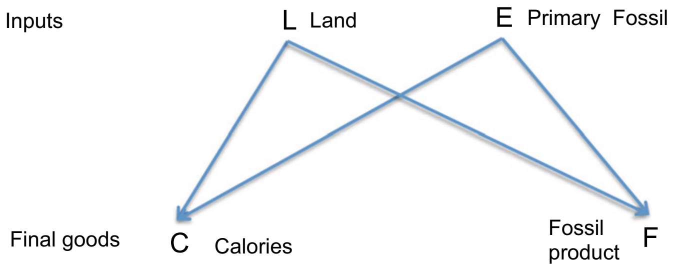

Figure 1 Schematic of a simple subsystem of inter-connected markets used for illustrating a mathematical framework for CLCA.

the direct and unintended effects, given plausible assumptions about model inputs and under different scenarios. This, however, is not the case with the use of such models in the context of regulations such as the US Renewable Fuel Standard and the California Low Carbon Fuel Standard (LCFS), which regulate fuel products based on their life cycle greenhouse gas (GHG) emissions intensity. In these two specific applications, PE and CGE models are being used to guide the selection of a point estimate of the life cycle emission intensity of a fuel or a specific fuel production pathway (EPA 2009; CARB 2009). Under the LCFS, a predicted average indirect emissions intensity of a given type of biofuel, which is derived from the CGE model, is added to a firm-specific emission intensity derived from a traditional LCA to determine the total emissions intensity for that specific firm's biofuel. As to whether this is the appropriate use of the two types of LCA and what might be the best way to incorporate the estimates from a traditional LCA to those from market-equilibrium models, this is beyond the scope of this article. The mathematical framework I lay out here is aimed simply at highlighting the different activities linked to the life cycle of a product or service in question, which could be a source of unintended negative consequences.

A Multi-Market Framework for CLCA

An Illustrative Example

Imagine a simple economic system (shown schematically in figure 1) that relies on two primary natural resources (henceforth, primary inputs) – land (L) and energy (E), which is in the form of a primary fossil resource. These inputs are used to produce two final goods – calories (C) for food consumption and a fossil fuel product (F) for deriving energy services.

To this system, let us now introduce a policy shock that mandates a given minimum level of production of biofuel (B), which is also derived by combining the two primary inputs. For simplicity of exposition and without loss of generality, let us say this policy forces the consumption of a blend or mixture (M) of B and F as the final consumer fuel instead of pure F, which was the case before the policy. B and F are now intermediate goods that are simply used to produce the final consumer good M. See figure 2 for the modified schematic. The system of equations that describe the equilibrium, assuming that the policy is binding, that is, the level of biofuel produced is equal to the mandated minimum level, is shown below. For the sake of brevity, the set of mathematical equations describing the equilibrium for the system before the policy is introduced has been moved to the supporting information available on the Journal's website.

Let P_i and Q_i denote the price and quantity of good i, Q_i^j denote the quantity of good i used in production of good j, α_i^j denote the fixed proportion relationship between input i and output j such that $\alpha_i^j Q_i^j = Q_j$.

Supply functions:

$$\text{Land: } Q_L = S_L(P_L) \tag{1a}$$

$$\text{Primary Fossil: } Q_E = S_E(P_E) \tag{1b}$$

Demand functions:

$$\text{Calories: } Q_C = D_C(P_C) \tag{1c}$$

$$\text{Finished Fuel: } Q_M = D_M(P_M) \tag{1d}$$

Identities for quantities used:

$$\text{Finished Fuel: } Q_M = Q_B + Q_F \tag{1e}$$

$$\text{Land: } Q_L = Q_L^C + Q_L^B + Q_L^F \tag{1f}$$

$$\text{Primary Fossil: } Q_E = Q_E^C + Q_E^B + Q_E^F \tag{1g}$$

Fixed Proportion Production relationships:

$$\text{Land and Calories: } \alpha_L^C Q_L^C = Q_C \tag{1h}$$

$$\text{Primary Fossil and Calories: } \alpha_E^C Q_E^C = Q_C \tag{1i}$$

$$\text{Land and Biofuel: } \alpha_L^B Q_L^B = Q_B \tag{1j}$$

$$\text{Primary Fossil and Biofuel: } \alpha_E^B Q_E^B = Q_B \tag{1k}$$

$$\text{Land and Fossil product: } \alpha_L^F Q_L^F = Q_F \tag{1l}$$

$$\text{Primary Fossil and Fossil product: } \alpha_E^F Q_E^F = Q_F \tag{1m}$$

Zero Economic Profit conditions:

$$\text{Calories production: } P_C Q_C = P_E Q_E^C + P_L Q_L^C \tag{1n}$$

$$\text{Biofuel production: } P_B Q_B = P_E Q_E^B + P_L Q_L^B \tag{1o}$$

$$\text{Fossil fuel production: } P_F Q_F = P_E Q_E^F + P_L Q_L^F \tag{1p}$$

$$\text{Blending process: } P_M Q_M = P_B Q_B + P_F Q_F \tag{1q}$$

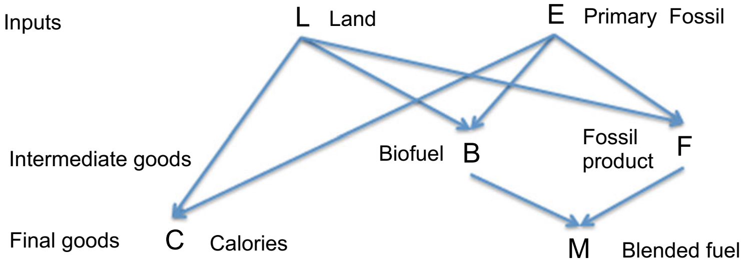

Figure 2 Schematic of the system in figure 1 modified to depict the effect of introduction of biofuel (B) that is blended with fossil fuel (F) to produce a mixed or blend fuel (M).

The above system consists of the following different sets of equations:

(i) A supply function per each primary input–equations (1a)–(1b).
(ii) A demand function for each final good–equations (1c)–(1d).
(iii) An identity for each good that is simply mixed or blended together–equation (1e).
(iv) An identity for the total quantity of each good that is consumed in different processes–equations (1f)–(1g). The identities are akin to the conservation of mass (or energy) conditions
(v) A production relationship for each input used in each process. A fixed proportion (FP) production process, the one depicted above, is one for which the ratio of output per unit of each input is fixed–equations (1h)–(1m).
(vi) A zero economic profit condition for each production or mixing process–equations (1n)–(1p). This condition merits further elaboration, which I do next.

For a commercial activity, if the total revenues are less than total cost, this means that the producers are incurring a loss, so they will not sustain this activity for long. This situation cannot be an equilibrium. If total revenues exceed total cost, then there is positive economic profit from selling the clean good. It is worth emphasizing that I refer to *economic* profit, which refers to profit after all inputs have been paid for. This is different from *accounting* profit, which does include cost of inputs such as the firm's owner compensation. The owner of the firm claims the accounting profit. This is a basic requirement for equilibrium, since if there is economic profit, this will attract further expansion of capacity or new entry into this activity. If there is positive economic profit it means that the producers would voluntarily be willing to supply more of the clean good even without the policy. In other words, the policy is not binding. While this is a plausible and interesting scenario to analyze, our focus here is in illustrating the effect of a binding policy shock. This requires that in equilibrium the total revenues from sales of a blended product just equal the total cost of producing the blend.

The system consists of 17 equations in 18 variables ($P_B, P_C, P_E, P_F, P_L, P_M, Q_B, Q_C, Q_E, Q_F, Q_L, Q_M, Q_L^C, Q_E^C, Q_L^B, Q_E^B, Q_L^F, Q_E^F$). If I fix one of the unknowns, say, Q_B, then I can solve the system. Alternatively, I could fix the ratio of two variables, say, $\frac{Q_B}{Q_M} = 10\%$, in which case I would have 18 equations in 18 unknowns. As an illustration, I solve for a fixed Q_B. Fixing the quantity of B at $\overline{Q}_B$, and after some manipulation, the above system can be reduced to a smaller system of equations involving the price variables in the model–P_B, P_C, P_E, P_F, P_L, and P_M.

$$S_L(P_L) = \frac{D_C(P_C)}{\alpha_L^C} + \frac{\overline{Q}_B}{\alpha_L^B} + \frac{D_M(P_M) - \overline{Q}_B}{\alpha_L^F} \tag{2a}$$

$$S_E(P_E) = \frac{D_C(P_C)}{\alpha_E^C} + \frac{\overline{Q}_B}{\alpha_E^B} + \frac{D_M(P_M) - \overline{Q}_B}{\alpha_E^F} \tag{2b}$$

$$P_C = \frac{P_E}{\alpha_E^C} + \frac{P_L}{\alpha_L^C} \tag{2c}$$

$$P_B = \frac{P_E}{\alpha_E^B} + \frac{P_L}{\alpha_L^B} \tag{2d}$$

$$P_F = \frac{P_E}{\alpha_E^F} + \frac{P_L}{\alpha_L^F} \tag{2e}$$

$$P_M \cdot D_M(P_M) = P_B\overline{Q}_B + P_F[D_M(P_M) - \overline{Q}_B] \tag{2f}$$

For supply and demand functions that are linear in price, the above system is a quadratic system whose explicit solution can be represented as:

$$P_k = P_k(\overline{Q}_B, \vec{\beta}) \tag{3}$$

$$Q_k = Q_k(\overline{Q}_B, \vec{\beta})(\text{using } P_k'\text{s and equations}(1a){-}(1e)) \tag{4}$$

$$Q_i^j = Q_i^j(\overline{Q}_B, \vec{\beta})(\text{using } Q_k'\text{s and equations } (1h){-}(1m)) \tag{5}$$

where, $k \in \{B, C, E, F, L, M\}, i \in \{E, L\}, j \in \{B, C, F\}$, and $\vec{\beta}$ is a vector of exogenous parameters, which includes the constants representing the fixed proportion production relationships (α's) and the parameters that specify the supply and demand functions. Pollution ultimately arises from the consumption of the primary inputs. If $\gamma_i(Q_i)$ represents the average pollution intensity for output level Q, then total emissions can be written as:

$$Z_T = \sum_{i=1}^{I} \gamma_i(Q_i) \cdot Q_i \quad \text{where } I = \{E, L\} \tag{6}$$

where Q_i denotes the total quantity of primary polluting input i consumed in the entire system. Differentiating equation (6), with respect to $\overline{Q}_B$,

$$\frac{\partial Z_T}{\partial \overline{Q}_B} \approx \sum_{i=1}^{I} \left[\frac{\partial \gamma_i}{\partial Q_i} \frac{\partial Q_i}{\partial \overline{Q}_B} Q_i + \gamma_i \frac{\partial Q_i}{\partial \overline{Q}_B} \right] \tag{7}$$

I can write $\frac{\partial Q_i}{\partial \overline{Q}_B} = \frac{\partial Q_i}{\partial P_i} \frac{\partial P_i}{\partial \overline{Q}_B} = \frac{\partial S_i}{\partial P_i} \frac{\partial P_i}{\partial \overline{Q}_B}$ (using the supply functions for the primary inputs, and equations (1a)–(1b) to substitute $\partial Q_i / \partial P_i$). Therefore,

$$\Delta Z_T \approx \left[\sum_{i=1}^{I} \left(\frac{\partial \gamma_i}{\partial Q_i} Q_i + \gamma_i \right) \frac{\partial S_i}{\partial P_i} \frac{\partial P_i}{\partial \overline{Q}_B} \right] \Delta \overline{Q}_B \tag{8}$$

Let $\eta_i^s = \frac{\partial S_i}{S_i} / \frac{\partial P_i}{P_i}$ denote the elasticity of supply of good i with respect to its price of i. Substituting for $\frac{\partial S_i}{\partial P_i}$ and using $Q_i = S_i(P_i)$ in the equation above, I get

$$\Delta Z_T \approx \left[\sum_{i=1}^{I} \left(\frac{\partial \gamma_i}{\partial Q_i} Q_i + \gamma_i \right) \eta_i^s \frac{Q_i}{P_i} \frac{\partial P_i}{\partial \overline{Q}_B} \right] \Delta \overline{Q}_B \tag{9}$$

Key Model Parameters and Variables Driving the Change in Emissions

Equation (9) highlights the role of different factors, some of which are exogenous and the rest endogenous to the model, that appear to drive the change in emissions.

(1) Rate of change of average emission intensity of each primary input ($\frac{\partial \gamma_i}{\partial Q_i}$): The rate at which the average emission intensity changes with total output is positively correlated with change in emissions. The well-known and controversial issue of land use change emissions caused by biofuel expansion is fundamentally due to the fact that farming marginal land is more emission intensive than farming infra-marginal land (Khanna and Crago 2012).

(2) Elasticity of supply of primary inputs (η_i^s): Primary inputs whose supply is inelastic (in the context of the analysis), will have no effect on emissions as their supply will be unchanged due to the policy shock. If each of the primary inputs is supplied inelastically, then aggregate emissions are unchanged. Therefore, in modeling the environmental impact of a policy or a technology, it is essential to trace the effects all the way up the supply chain to the primary sources of pollution.

(3) The price effect ($\frac{\partial P_i}{\partial \overline{Q}_B}$): The change in emissions depends on the effect of the policy shock on the price of primary inputs, which depends not only on the elasticity of the supply of the primary inputs but also on the elasticity of demand for the final products. For linear supply and demand functions, although an analytical solution exists for the system of equations (1a)–(1q), once I make one of the variables exogenous the analytical expression is too unwieldy to perform comparative static analysis to infer the direction of change for any of the variables as a function of the exogenous parameters. One can imagine that this only gets more unwieldy for a larger system of equations. However, I can use economic theory to conceptually analyze the direction of price changes.

(4) The size of the shock ($\Delta \overline{Q}_B$): Last but not least, the size of the shock, not surprisingly, has a direct influence on the change in emissions. However, it is not clear that the relationship between the size of the shock and the change in emissions is monotonic (Rajagopal 2014).

Using Microeconomic Theory to Identify Direction of Price Effects

I solved this system of equations using the symbolic toolbox feature in MATLAB software. The explicit solution is an algebraic expression that is simply too unwieldy to derive general qualitative insights from. However, I can use basic microeconomic intuition to predict the direction of impact on the price (and the quantity) of different goods in the system under any given policy shock, holding all else fixed. A policy that causes consumption of B to increase will lead to less consumption of its substitute, F. Therefore, the price of B will increase while that of F will decrease. The price (quantity) of the blended or mixed product, M, could however either be larger (smaller) or smaller (larger) relative to the price (quantity) of F in the counter-factual state of the world, that is, the state without the policy shock. When the reduction in demand for F causes the marginal cost (which is also its market price under competitive equilibrium) of producing F to fall such that it more than compensates for the higher cost of blending a costlier biofuel, then the price of M could be lower than price of F in the counter-factual state. That the biofuel is costlier is essential for the policy to be binding, otherwise B's consumption would be higher even without the policy shock. Moving up the production chain, the increased production of B represents an additional source of demand for the primary inputs E and L. However, demand for each of these primary inputs for producing fossil products declines. Because of these opposing effects from biofuel production and fossil processing, the net effect on both the price and the quantity supplied of each of the primary inputs, land, and energy, is ambiguous. Because the impact on land and energy prices is ambiguous, such will be the case for the price of calories and hence the production and consumption of calories. The direction of change in prices discussed above is marked in figure 3a by upward (price increase), downward (price decrease) or bi-directional arrows (ambiguous effect).

In reality, the amount of land allocated to fossil fuel processing is minuscule compared to the extent of its use for food production. Likewise, the amount of energy resources consumed in food production is relatively small compared to its use for deriving energy services. Let us therefore drop these two links in the system. The simplified subsystem is shown in figure 3b. With land now used only for production of calories and biofuel, the increase in demand for biofuel increases the demand for land. Since the supply of land is unchanged, this leads to an increase in the equilibrium price of land. An increase in the price of

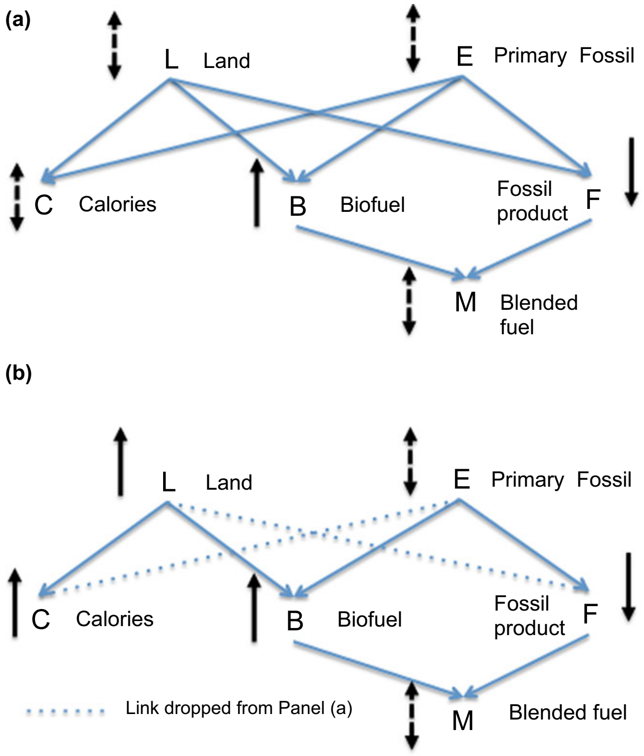

Figure 3 Direction of price effects on various goods due to the biofuel shock. a) Full system: Upward (downward) pointing arrows represent a price increase (decrease) relative to a counter-factual scenario without the biofuel shock; b) Abridged system: The dotted lines represent the links that have been dropped from panel (a) to demonstrate the effect on the price of calories and land, which are no longer ambiguous as they were in panel (a).

land (an input to calorie production) while holding demand fixed leads to an increase in the equilibrium price of calories. Despite energy no longer being an input to calorie production, the opposing effects of increasing demand from biofuel production and reduction in the derived demand of fossil products, mean that the effect on the price of the primary energy resource is still ambiguous.

I can now make general statements about the direction of change in the price and quantity of the primary inputs in response to a given shock to the system. I focus on primary inputs, as these are the ultimate sources of pollution in our model. Specifically, let us consider the case of a positive demand shock to one specific good in the system, which I will refer to as G. A primary input that is associated with the production of G, either directly or indirectly because of its use in the production of an intermediate good that is used in production of G but is used neither in the production of any substitute good to G nor any intermediate good used in the production of any substitute good to G, will experience a positive demand shock as well and hence cause its price and quantity supplied to increase. Likewise, a primary input that is associated with the production of a substitute good to G or any intermediate good used in the production of any substitute good to G, but is neither used in the production of G itself nor used in the production of an intermediate good that is used in the production of G, will experience a negative demand shock and therefore experience a decrease in its price and quantity supplied. Finally, if a primary input, which is associated with the production of G, is also an input in the production of any substitute good to G or any intermediate good used in the production of any substitute good to G, then the net effect on the demand for such a primary input is ambiguous, and such is the case for the net effect on its price and quantity supplied.

Generalizing the Multi-Market Partial Equilibrium Model

I generalize the above example to an arbitrary number of primary inputs, M, and an arbitrary number of produced goods, N, produced using N distinct processes. Let a number R out of the N produced goods ($R \leq N$) be final consumer goods with the remaining $N - R$ being pure intermediate goods. The sets of equations for this system are described in table 1 (with table 2 showing the number of variables in the model). The top portion of the table 1 describes the system of equations and the bottom portion describes the variables. The table shows that there are as many equations as the number of unknowns.

To this system, let us now add a policy shock. Again, for illustrative purposes only, consider a mandate for the use of a new cleaner substitute to an existing dirty good. There is now one at least additional good in the system, the clean good, and the production of this good introduces an additional process. I, therefore, now have a total of M+N+1 produced goods and (M+N+1)*(N+1) production relationships. Further, let the policy require that the producers of the dirtier substitute sell the cleaner good by blending it with the their dirtier product. This introduces yet another an additional product, which is the blended product. It also introduces an additional identity equating the total quantity of the blended product to the sum of the quantity of inputs to the blending process. I, therefore, now have a total of N+2 zero profit conditions–one condition for each of the N original production process and for the two additional process, production of the clean good and the blending process. The equations describing this system are shown in table 3 (with table 4 showing the number of variables in the model). Again there are as many equations as the number of unknowns. When this is linear in the unknown variables, I can derive an analytical solution for the prices and output levels of the various goods. With the similar but appropriate modifications, one would be able to simulate the effect of other types of policy shocks such as a subsidy for a specific technology or set of technologies, tax on any given type of pollution, performance standards, or simply, an innovation that alters the technical relationship between inputs and outputs. This framework could be used to analyze policies in a multi-region framework involving trade in goods between the different regions.

The framework outlined above is not restricted to fixed-proportion in production relationships. More flexible

Table 1 Description of the mathematical model describing a system before the policy shock. This table shows the number of equations or conditions. Table 2 shows the number of variables in the model

Type of equation	*Count*	*Description*
Supply function	M	One supply function per each primary input
Demand function	R	One demand function for each final consumer good
Identity for primary input	M	For each primary input, there will be an identity representing consumption of each input summed across all the production processes.
Identity for intermediate inputs	N-R	Since only R out of the N produced goods are final goods and so there are N-R intermediate goods, which are fully used as input in other processes. There will be one identity for the total quantity consumed of each such good.
Production relationships	(M+N)*N	Each of the M+N different goods could be used in each process. There will be one equilibrium condition describing how much of each of the M+N is used in each of the N processes.
Zero economic profit condition for each good produced	N	One condition for each production process
Total number of conditions	2*M + 2*N + (M+N)*N	

Table 2 Description of the mathematical model describing a system before the policy shock. This table shows that the number of variables in the model equals the number of equations in the system shown in table 1

Variables	*Count*	*Description*
Prices	M+N	One for each of the distinct goods in the system
Quantity	M+N+(M+N)*N	The production of each good for M+N goods, and the use of each of the M+N goods in the N processes
Total number of variables	2*M + 2*N + (M+N)*N	

relationships in the transformation from inputs to outputs, such as a constant elasticity of substitution production function, could also be used. Microeconomic theory could be used to derive an expression for the quantity of each input per unit of output as a function of the prices of the various goods in equilibrium.

The general framework can also be extended to handle multi-output production processes. One approach is to treat each distinct jointly-produced output as though it is produced from a dedicated process. In this case, one would need to simply know the technical relationship between the various inputs and each different jointly-produced output. Similar to the approaches for burden allocation across co-products when performing ALCA (Curran 2006), one among the proportion of mass, volume, energy, or economic value embodied by each different joint-product could be the basis for allocation of each different input and each different type of pollution.

The general framework allows for a higher level of technical richness in any given sector(s) of interest. For instance, any given industry or sector, such as electricity production, might be characterized by a number of different broad categories of technologies for power generation such as coal, gas, and hydro power; a number of different sub-categories within each category, such as integrated gasification combined cycle or combined heat and power generation using coal; and heterogeneity with each sub-category, such as newer more efficient plants and older less efficiency plants. One approach to accommodate such a range of variation within any sector would be to derive a production relationship for the average output of the sector by suitably aggregating across the different individual production relationships. This approach might be unsatisfactory when the primary objective is to analyze the effect of a policy or technical shock on the electricity mix. One alternative is to treat electricity produced from each distinct source as a separate commodity with its own production function. However, if there is but one common price for the output derived in different ways, one would need to introduce additional conditions to resolve indeterminacy. Such conditions could take the form of capacity constraints on a specific type of technology. There is a literature on incorporating technical richness into economic equilibrium models. For instance, see work by Böhringer (1998) and Böhringer and Rutherford (2008) and the various articles published in a 2006 special issue of *The Energy Journal* entitled "Hybrid Modeling of Energy-Environment Policies: Reconciling Bottom-up and Top-down" (Yatchew 2006).

Finally, the sector and regional breadth of the actual model and the technical richness in which each sector is represented will be application-specific and contingent on data availability. For market equilibrium models, the data requirements increase with the level of detail (such as the number of different economic sectors and the number of different regions) and the complexity of the functional forms that represent the technical and the behavioral phenomena that are being modeled.

Table 3 Description of mathematical model describing the system post policy shock. This table shows the number of equations or conditions. Table 4 shows the number of variables in the model

Type of equation	*Count*	*Description*
Supply function	M	One supply function per each primary input
Identity for primary input	M	For each primary input identity representing total consumption of each M across the the N production processes
Demand function	R	One demand function for each final good. This is same as before as the new
Identity for intermediate inputs	N-R*	R out of the N produced goods are final goods and so there are N-R intermediate goods, which are fully used as input in other processes. There will be one identity for the total quantity consumed of each such good
Production relationships	(M+N+1)*(N+1)	Each of the M+N+1 different goods could be used in each process. There will be one equilibrium condition describing how much of each of the M+N+1 is used in each of the *N* processes
Zero economic profit condition	N+2	One condition for each original production process, N, and the two addition process – production of clean good and blending process
Blending or Mixing condition	1	The total quantity of the blended product is equal to sum of the quantity of inputs to blending
Policy conditions	1	In our case, the policy specifies the output of the clean good
Total # of conditions	2*M + 2*N + (M+N+1)*(N+1) +4	

Note: Although the cleaner substitute is not consumed directly but as a blend, it is however not treated here as an intermediate. Since it is a substitute to an existing final consumer good, I therefore also do not have an additional demand function for this product. This is why the blending or mixing identity exists.

Table 4 Description of mathematical model describing the system post policy shock. This table shows that the number of variables in the model equals the number of equations in the system shown in table 3

Variables	*Number*	*Description*
Prices	M+N+2	M+N original goods plus two addition prices – one each for cleaner good and the blended product
Quantity	M+N+2+ (M+N+1)*(N+1)	The production of each good for M+N+2 goods, and the use of each of the M+N+1 goods in the N processes. For simplicity, it is assumed, that the blended product is not used as an intermediate input to any production process.
Total # of variables	2*M + 2*N + (M+N+1)*(N+1) + 4	

Note: Although the cleaner substitute is not consumed directly but as a blend, it is however not treated here as an intermediate. Since it is a substitute to an existing final consumer good, I therefore also do not have an additional demand function for this product. This is why the blending or mixing identity exists.

Extending the Multi-market Framework to a General Equilibrium Framework

I describe briefly the modifications to the system of multi-market model equations that would transform it into a model of general equilibrium (GE). A detailed description of the conditions for general equilibrium can be found in any standard textbook discussion of general equilibrium theory. For instance, refer to work by Sadoulet and De Janvry (1995) or Dixon and Parmenter (1996). To the system described in table 1, one would first replace the supply functions for the primary inputs with a fixed endowment of each primary input. This would eliminate *M* supply equations but it would also eliminate M unknowns, the quantity variables that are now exogenous. Then, one would also replace the *R* demand functions for the final consumer goods with *R* new conditions derived from the theory of consumer choice applied to a representative household. According to this theory, a consumer maximizes utility from consuming the *R* final goods subject to an income constraint. Solving this maximization problem yields *R* first order conditions, each of which involves equating marginal utility from consuming each final good to its price. This process introduces two additional variables into the calculus, which are the wealth of the representative household and the marginal utility of wealth, also known as the shadow price of wealth constraint. It also introduces two additional conditions. One is that the total consumer expenditure across all final goods equal his/her wealth. In the GE framework, since the consumer is also the owner of the primary inputs, the returns to these inputs are the source of his/her wealth. The second condition then is that the total value of the primary inputs, which is simply the product of the price of each input and its quantity summed across all inputs, also equal his/her wealth. However, it so happens that the zero profit conditions collectively will render this equation redundant. So, effectively, I only have one additional condition and so I will have one variable more than the number of equilibrium conditions. It is for this reason that when solving the general equilibrium model, one of the variables in the model is made exogenous. Typically, one of the price variables is set to unity, which is referred to as the numeraire good, and other prices are computed relative to the numeraire.

Quoting from Gohin and Moschini (2006):

> [The] GE model can be viewed as a consistent sum of PE models, with the explicit structural representation of all good and factor markets, as well as the specification of macroeconomic equilibrium conditions. Ceteris paribus, therefore, a GE approach is bound to be more general and the results are more appealing on theoretical grounds. But often there is a meaningful trade-off between a GE and a PE approach. Generally speaking, PE models can provide a detailed analysis of some sectors, while ignoring interactions with other sectors of the economy. In contrast, GE models can take these interactions into account, often at the cost of relying on a more aggregated level of analysis.

Therefore, a multi-market model with a richer representation of the technical and market conditions for a small group of commodities with strongly interlinked supply and demand might represent a satisfactory alternative to a theoretically complete and more expansive CGE with much greater data needs. I refer again to the literature on hybrid modeling mentioned a little earlier. The challenge however is in determining precisely which are those commodities or markets from the entire set of commodities within the economy that need to be included in a multi-market model. This, as mentioned earlier, is similar to the problem of system boundary definition in ALCA. This is a related yet different problem that is beyond the scope of this article.

Conclusion

LCA is one among several decision support tools for environmental policy-making (Höjer et al. 2008). In this, ALCA and CLCA are not substitutable but complement each other (Rajagopal 2014; Suh and Yang 2014). CLCA has a particularly useful role to play in analyzing whether the effectiveness of a new technology or policy could be undermined by unintended effects elsewhere in the economy. Currently, any LCA that takes into account some consideration of price effects using any economic modeling framework could be considered as a CLCA. This article is a step towards addressing the current situation that despite a large literature on CLCA, there does not appear to exist a general mathematical framework that both abstracts from and could be adapted to specific applications. At the core of CLCA is a model of the economic system of which the product that motivates the CLCA is a part. The function of the economic model is to capture the linkages of the main product's supply chain with the rest of the economy so that one could predict the ripple effects of a technical or policy shock to the main product on the rest of the system. Although there exist different potential modeling approaches for this purpose, numerical simulation of theoretical models of market equilibrium is particularly advantageous for visualizing alternative future technology and policy scenarios. However, a market equilibrium model could range in complexity from a single sector PE model to a multi-sector, multi-region global CGE model. I have presented a general multi-market PE framework that can accommodate an arbitrary number of goods and regions and, which with some modification, can be transformed in to a GE framework. Recent advances allow such models to accommodate varying levels of technical richness across the different economic activities represented in the model. The approach I laid out could be used to simulate different types of policy shocks such as mandates, subsidies, taxes, and emission performance standards, under different parametric assumptions in order to identify potential unintended pollution impacts, which is one motivation underlying CLCA. I present our framework as simply one approach to CLCA that rests on and extends the same microeconomic foundation that is used for predicting the direct and indirect effects of economic and environmental policies. The limitations and criticisms that apply to economic simulation models therefore extend to CLCA applications of such models as well. Built and simulated thoughtfully, and the predictions interpreted properly, market equilibrium-based CLCA models could yield useful insights about the unintended environmental consequences of one policy relative to another, which, I contend, is the salient contribution of CLCA to decision making. For any given product or policy, developing a procedure for identifying the commodities and the markets that need to be included and those that could be excluded from a CLCA, as well as the level of technical richness to which each economic sector is represented within the model, are important next steps in this line of research. Last but not least, another important area for further research is exploring how one might combine potentially different insights that might emerge from an ALCA and a CLCA in decision-making.

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Additional Supporting Information may be found in the online version of this article at the publisher's web site:

Supporting Information S1: This supporting information describes some additional systems of mathematical equations that were excluded from the main article due to space constraints.