Title
E.E. REVIEW COURSE - LECTURE VI.

Authors
Martinelli, E.
Gordon, H.
Hernandez, P.

Publication Date
1952-04-07

UNIVERISITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

UCRL 1802

ELECTRICAL ENGINEERING REVIEW COURSE

LECTURE VI
April 7, 1952
E. Martinelli

(Notes by: H. Gordon, P. Hernandez)

STEADY ELECTRIC CURRENT

I. GENERAL

The vectors and equations which are valid for Electrostatic and Electromagnetic processes are:

Electric Field Vector $\vec{E}$ volts/meter

Electric Displacement Vector $\vec{D} = K\vec{E}$ where:

$\text{Div}\,\vec{D} = 4\pi\rho$ ρ = Electric Charge Density

$u = \frac{1}{8\pi}\vec{E}\vec{D}$ u = Electrical Energy Density

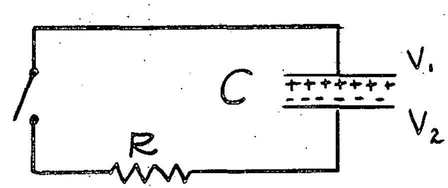
Relations belonging only to Electrostatics:

$\vec{E} = -\text{grad}\,V = -\nabla V$ irrotational field

Potential V is constant in conductor

II. OHM'S LAW

Consider a charged condenser with potentials V_1 and V_2 on the plates and the switch open. This is the Electrostatic case and the potential V is constant all along the conductor.



When the switch is closed the potential is no longer constant in the conductor; it now has values V_1 and V_2 at the ends, and we have given up one of the Electrostatic relations (potential V is constant in a conductor). The charges + e and -e neutralize each other by passage of current in the conductor of strength:

$I = -\frac{de}{dt}$ Equilibrium is re-established when the charges have neutralized each other; current has stopped flowing; and the electrical fields have completely collapsed.

DISCLAIMER

It is not possible to produce a perfectly steady current by electrostatic means; but it can be approximated by taking C and R very large. A large value of C provides a great number of charges, and a large value of R retards the flow of current, then $\frac{de}{dt}$ approximates a steady current, and the field of E remains approximately irrotational if the current changes very slowly, that is, we assume that the field is not changing with time.

Ohm's Law is established by measuring the rate of discharge $\left(-\frac{de}{dt}\right)$

$$\vec{I} = \frac{V_2 - V_1}{R}$$

This is the integral form and applies to steady currents only. This form cannot be used in field theory since only results which refer to a single point can be used.

R is the resistance of the wire and depends only on its dimensions and material.

$$R = \frac{1}{\sigma}\frac{\ell}{s} = \rho\frac{\ell}{s}$$

Where:

R = resistance, ohms

σ = specific conduction, mho/unit length.

ℓ = length

s = cross section area of conductor.

ρ = specific resistance; and is equal to the resistance of a 1 cm cube, units ohm-cm.

$$\rho = \frac{R\ (\text{ohms})\ S\ (\text{cm}^2)}{\ell\ (\text{cm})} = \text{ohm} - \text{cm}$$

The differential form of Ohm's Law can be applied to field theory since it refers to small general elements.

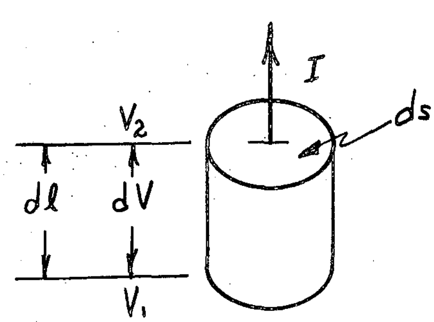
Assume that the same relation as has been found in the first instance for the wire as a whole now holds for any arbitrary element of its volume.

$$I = -\frac{V_2 - V_1}{R}$$

$$= -\frac{dV}{R}$$

$$R = \frac{1}{\sigma}\frac{d\ell}{ds}$$

$$I = -\frac{\sigma\, dV ds}{d\ell}$$



$$\frac{I}{ds} = -\sigma \frac{dV}{d\ell}$$

$$= -\sigma \text{ grad } V = -\sigma \nabla V$$

$$= \sigma \vec{E}$$

$$\vec{i} = \sigma E \text{ (differential form)}$$

$\text{grad } V = \frac{dV}{d\ell} = \nabla V$

Field component along conductor. Vectors I and E are parallel. $\vec{i} = \frac{I}{ds}$ is the current density

This form is suitable for process varying with time. Holds only for isotropic substances, that is, those that do not depend on direction.

III. JOULE'S LAW

Joule's law defines the quantity of heat developed in a wire transversed by a current, and all energy disappearing is lost in form of heat.

In the short circuited condenser case:

$$Q = -(V_2 - V_1) \frac{d\,e}{d\,t}$$

$$= (V_2 - V_1)\, I$$

$$= I^2R \text{ integral form}$$

$I = \frac{V_2 - V_1}{R}$

$V_2 - V_1 = I\,R$

Translate into the different form:

$$R = \frac{1}{\sigma} \frac{d\ell}{d\,s}$$

$\frac{I}{ds} = \vec{i}$ current density

$I = \vec{i}ds = \sigma \vec{E}ds$

$dV = d\ell\, ds$

Joule heat per unit volume

$$Q = I^2R = \sigma^2 E^2 ds^2 \left[\frac{1}{\sigma} \frac{d\ell}{ds}\right] = \sigma \vec{E}^2 dV$$

$$\frac{Q}{dV} = \sigma \vec{E}^2 = \frac{\vec{i}}{\vec{E}} \vec{E}^2 = \vec{i} \circ \vec{E}$$

$\vec{i} = \sigma \vec{E}$

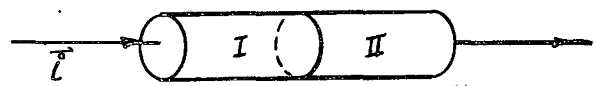
$\sigma = \frac{\vec{i}}{\vec{E}}$

It can also be shown that $\vec{i} \circ \vec{E}$ is also the heat developed per unit time.

IV. CONDUCTION, DISPLACEMENT, AND POLARIZATION CURRENT

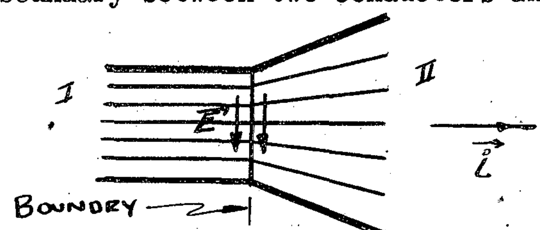
A) Conduction current, $\vec{i}$

Consider a steady current in conductor:



Within the conductor the div $\vec{i} = \nabla \cdot \vec{i} = 0$, which says that charge neither ends nor begins in a unit volume (conservation of charge). It behaves like an incompressible fluid in a pipe.

$\vec{i}$ is continuous at the boundary between two conductors and is the true current actually measured.



At the boundary of two conductors the normal component of $\vec{i} = \sigma \vec{E}$ is continuous which means all current crosses the boundary. Also the tangential component of $\vec{E}$ is continuous which says that the voltage on both sides of the boundary at any point is identical.

B) Displacement current, $\vec{D}$

Consider a non-steady current in a media. This has a time rate of change of charge density, ρ.

By Gauss's Theorum

$$\text{div } \vec{i} = \nabla \cdot \vec{i} = -\frac{\partial \sigma}{\partial t}$$ rate at which current is created in unit volume.

and $\rho = \nabla \cdot \vec{D}$

$$\nabla \cdot \vec{i} = -\frac{\partial \rho}{\partial t} = -\frac{\partial \nabla \cdot \vec{D}}{\partial t} = -\nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \left(\vec{i} + \frac{\partial \vec{D}}{\partial t} \right) = 0$$

The displacement current is made up of

(1) The displacement current in Vacuo, $\frac{\partial \vec{E}}{\partial t}$, and

(2) The polarization current, $\frac{\partial \vec{P}}{\partial t}$.

Then

$$\nabla \cdot \left(\vec{i} + \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} \right) = 0$$

Where: $\vec{i} + \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$ is the solenoidal total current which is continuous across a boundary.

There is one more current due to magnetic induction which has not yet been covered in this course.

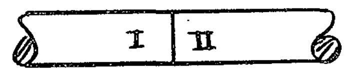
		$\vec{i}$	$\frac{\partial \vec{E}}{\partial t}$	$\frac{\partial \vec{P}}{\partial t}$
Steady Current	Conductor	High	0	0
	Insulator	Low	0	0
Time Varying Current 60 Mc	Conductor	High	Low	Low
	Insulator	Low	High	High
Time Varying Current Light	Conductor	High	Low	Low
	Insulator	Low	High	High
Time Varying Current 200 KV X-Ray	Conductor	Low	High	Low
	Insulator	Low	High	Low

Displacement current in Vacuo is the current that flows in the electric field of a vacuum condenser as the charge builds up on the plates; or as the condenser discharges and the field collapses.

Polarization current is the current that appears to flow when dipoles in a dielectric rotate and orient themselves when a condenser with a dielectric is being charged or discharged.

V. IMPRESSED FORCES AND ELECTROMOTIVE FORCE

For non-homogeneous conductors and in the transition layer between one conductor and another $\vec{i} \neq \sigma \vec{E}$

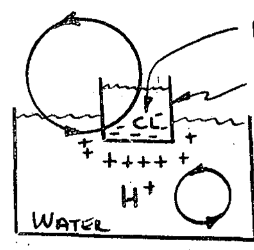


An example of the electric field built up at the transition layer of two dissimilar metals is a thermocouple which builds up a potential V_1 and V_2 on the conductors. The electric field is present without current flow.

There are electric fields in disconnected batteries but no current flow.

To account for this force Ohm's Law can be extended to $\vec{i} = \sigma (\vec{E} + \vec{E}^e)$ where $\vec{E}^e$ is the impressed or applied force and in a battery is equal and opposite to $\vec{E}$.

Another example of impressed force is an electrolytic cell where the conductor is the electrolyte in the cell.



The electrolyte dissociates into H^+ and CL^- ions which diffuse independently of each other; form a difference in concentration on each side of the cup; and cause a current to flow.

The diffusion current causes the dilute parts of the solution to positively charge and the concentrated parts negatively.

The electric field checks the diffusion of H^+ ions and accelerates the CL^- ions.

When equilibrium is reached the two kinds of ions are compensated by the field and current has stopped flowing.

Then $\vec{E} + \vec{E}^e = 0$ and $\vec{E} = -\vec{E}^e$

This is the case of the open circuited battery.

The impressed field $\vec{E}^e$ is present only in the interior of the electrolyte.

Outside $\vec{E}$ is determined by the principles of electrostatics; that is, continuity of the tangential component of $\vec{E}$ and the normal component of $\vec{I}$ at the boundary; hence it is irrotational.

$$\oint \vec{E}^e \, ds = 0$$

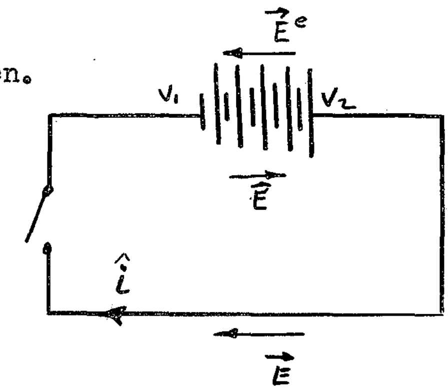
Under the conditions:

(1) Impressed forces are so distributed that they can be compensated within the conductor by an electrostatic field.

(2) Path of integration lies wholly within the electrolyte.

VI. VOLTAIC CIRCUIT

$$V_{1-2} = V_2 - V_1 = \int_1^2 \vec{E}^e \, ds \text{ with switch open.}$$



$$\oint \vec{E}^e \, ds = \int_1^2 \vec{E}^e \, ds = \vec{E}^e_{12} \quad \text{switch closed.}$$

$\vec{E}^e_{12}$ cannot be counterbalanced by an electric field, therefore current $\vec{i} = \sigma \, (\vec{E} + \vec{E}^e)$ must flow to develop steady conditions.

The characteristic property of a steady current is that it is everywhere solenoidal, that is, $\nabla \circ \vec{i} = 0$. For the field $\vec{E}$ then $\nabla \times \vec{E} = \text{curl } \vec{E} = 0$. When σ and $\vec{E}^e$ are given, $\vec{i}$ and $\vec{E}$ can be calculated.

Integrate: $\vec{i} = \sigma \, (\vec{E} + \vec{E}^e)$

$$\frac{\vec{i}}{\sigma} = (\vec{E} + \vec{E}^e)$$

$$\frac{\vec{I}}{s\sigma} = (\vec{E} + \vec{E}^e)$$

$$\vec{I} \oint \frac{ds}{s\sigma} = \oint (\vec{E} + \vec{E}^e) \, ds$$

$$= \oint \vec{E}^e \, ds = E^e_{12}$$

Let s = cross section area of conductor.

$$\vec{i} = \frac{\vec{I}}{s}$$

R = Total resistance

Since the line integral of $\vec{E}$ must vanish, since $\vec{E}^e$ cannot be counterbalanced by an electrostatic field, then:

$$\oint \frac{ds}{\sigma s} = R \qquad \text{and } \vec{I} R = \vec{E}^e_{12}$$

Thus: The product of the current strength and the resistance of the whole ring-shaped closed circuit is equal to the line integral of the impressed electric force.

E^e_{12} is called the electromotive force, EMF.

When $\vec{I}$ has been found, the $\vec{i}$ and $\vec{E}$ can be found.

$$\vec{E} = \frac{\vec{i}}{\sigma} - \vec{E}^e$$

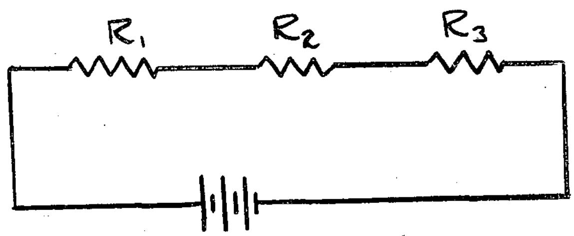
VII. KIRCHHOFF'S LAWS:

FIRST LAW: $\sum i = 0$ for any junction of wires in a circuit.

SECOND LAW: $\sum i_i R_i = E$ for any circuit (or branch)

For series resistors:

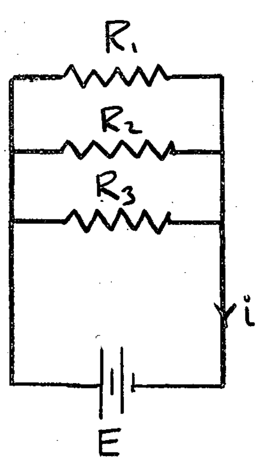
$$i\,(R_1 + R_2 + R_3) = E$$



or $iR_e = E$

where: $R_e = R_1 + R_2 + R_3$ = Equivalent resistance.

For parallel resistors:



$$i = i_1 + i_2 + i_3$$

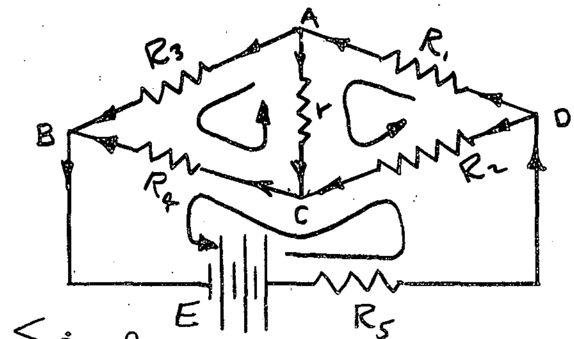
$$i_1R_1 = i_2R_2 = i_3R_3 = E = i\,R_{eff.}$$

$$i_1 = i\,\frac{R_{eff.}}{R_1};\ i_2 = i\,\frac{R_{eff.}}{R_2};\ i_3 = i\,\frac{R_{eff.}}{R_3}$$

or $$i = i\,R_{eff.}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\therefore \frac{1}{R_{eff.}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

WHEATSTONE BRIDGE:



Using the first and second laws in conventional manner, the conventions for assumed current flow and the order for taking the voltage drops around the circuit meshes are set up as shown in the diagram and used as follows:

$\sum i = 0$

At A: $i_1 - i_r - i_3 = 0$ (1)

B: $i_3 + i_4 - i_5 = 0$ (2)

C: $i_2 + i_r - i_4 = 0$ (3)

D: $i_5 - i_1 - i_2 = 0$ (4)

$\sum i R = 0$

For ACD: $i_1 R_1 + i_r r - i_2 R_2 = 0$ (5)

ABC: $i_3 R_3 - i_4 R_4 - i_r r = 0$ (6)

EDCB: $i_2 R_2 + i_4 R_4 + i_5 R_5 - E = 0$ (7)

EDAB: $i_1 R_1 + i_3 R_3 + i_5 R_5 - E = 0$ (8)

NOTE: All variables appear once in six of the eight equations, therefore two equations may be neglected.

For the bridge in balance:

$i_r = 0$

Thence:

From Equation (1)

$i_1 = i_3$ (1a)

From Equation (3)

$$i_2 = i_4 \tag{3a}$$

From Equations (2) and (4)

$$i_5 = i_1 + i_2 = i_3 + i_4 \tag{2a}$$

From Equation (5)

$$i_1 R_1 = i_2 R_2 \tag{5a}$$

From Equation (6)

$$i_3 R_3 = i_4 R_4 \tag{6a}$$

Equation (7) may be written:

$$E = i_2 R_2 + i_4 R_4 + i_5 R_5 \tag{7a}$$

And substituting (3a), (2a), and (5a) into (7a):

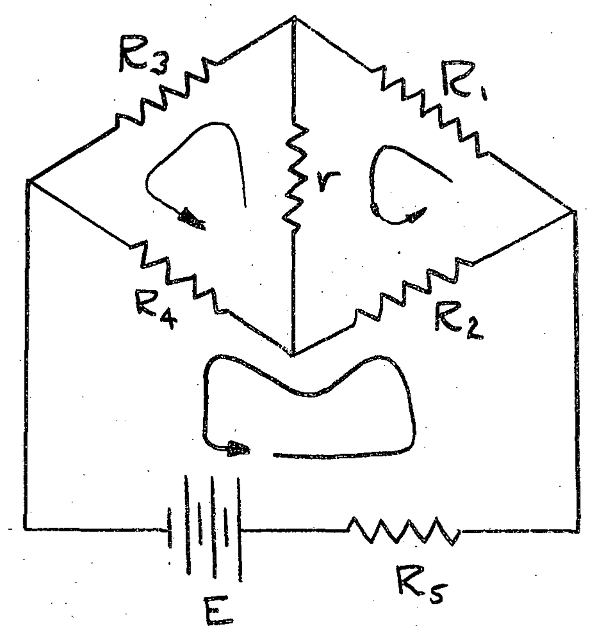
$$E = i_2 R_2 + i_2 R_4 + i_2 \frac{R_2}{R_1} R_5 + i_2 R_5 \tag{8}$$

or, the desired solutions become:

$$i_2 = E \Big/ \left[R_2 + R_4 + R_5 \left(1 + \frac{R_2}{R_1} \right) \right] = i_4 \tag{9}$$

and, $$i_1 = \frac{R_2}{R_1} E \Big/ \left[R_2 + R_4 + R_5 \left(1 + \frac{R_2}{R_1} \right) \right] = i_3 \tag{10}$$

and, $$i_5 = E \left(1 + \frac{R_2}{R_1} \right) \Big/ \left[R_2 + R_4 + R_5 \left(1 + \frac{R_2}{R_1} \right) \right] \tag{11}$$



Using Maxwell's Method:

Let i_1, i_2, and i_3 be the currents in each mesh.

Then current in ;

$R_1 = i_1$

$R_2 = (i_3 - i_1)$

$R_3 = i_2$

$R_4 = (i_3 - i_2)$

$r = (i_1 - i_2)$

$R_5 = i_3$

AROUND THE CIRCUITS:

$$i_1R_1 + (i_1 - i_2)r + (i_3 - i_1)R_2 = 0 \qquad (1)$$

$$i_2R_3 + (i_2 - i_3)R_4 + (i_2 - i_1)r = 0 \qquad (2)$$

$$i_3R_5 + i_1R_1 + i_2R_3 = E \qquad (3)$$

For the bridge in balance $i_1 - i_2 = 0$

And equations (1) and (2) become

$$i_1R_1 + (i_3 - i_1)R_2 = 0 \qquad (1a)$$

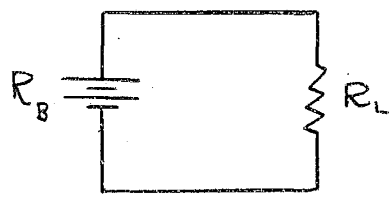
$$i_2R_3 + (i_2 - i_3)R_4 = 0 \qquad (2a)$$

Equations (1a) and (2a) may then be solved for i_2 and i_3 which are then substituted into Equation (3) and solved in turn for i_1 to yield:

$$i_1 = E \Big/ \left[R_1 - R_5\left(\frac{R_1 - R_2}{R_2}\right) - R_3\left(\frac{R_1 - R_2}{R_2}\right)\left(\frac{R_4}{R_3 + R_4}\right)\right] \qquad (4)$$

Then i_2 and i_3 are found and the currents through each resistance obtained from the original conventions assumed for the mesh circuits.

POWER TRANSFER



Power dissipated in load

$$P = i^2 R_L$$

But $i = \dfrac{E}{R_L + R_B}$

or $$P = \left(\frac{E}{R_L + R_B}\right)^2 R_L$$

Then, for maximum

$$\frac{d\,P}{d\,R_L} = \frac{E^2}{(R_L + R_B)^2} - \frac{2\,E^2\,R_L}{(R_L + R_B)^3} = 0$$

or

$$2\,R_L = (R_L + R_B)$$

and

$$R_L = R_B$$

then

$$P = \frac{E^2}{4\,R_B}$$

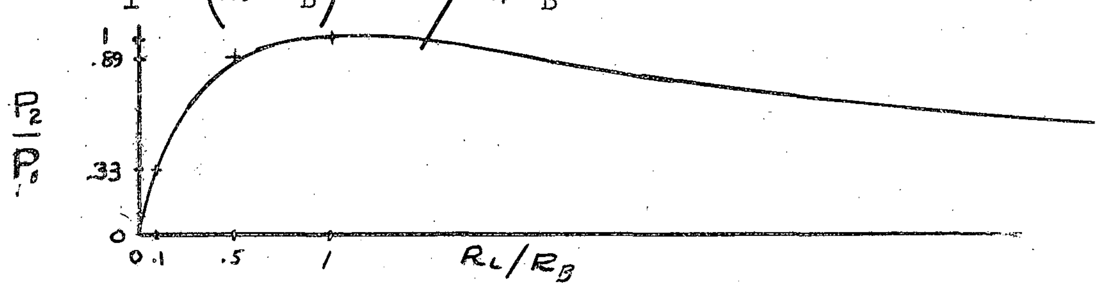
However, power delivered to load is not a particularly sensitive function of the load.

If $R_L = 1/2\ R_B$

$$\frac{P_2}{P_1} = \left(\frac{E}{3/2\ R_B}\right)^2 \frac{R_B}{2} \Bigg/ \frac{E^2}{4\,R_B}$$

$$= 8/9 \quad (\text{also if } R_L = 2\,R_B)$$

If $R_L = 0.1\ R_B$; $\dfrac{P_2}{P_1} = \left(\dfrac{E}{1.1\ R_B}\right)^2 0.1\ R_B \Bigg/ \dfrac{E^2}{4\ R_B} = 0.33$ (also if $R_L = 10\ R_B$)



plc: 5/13/52