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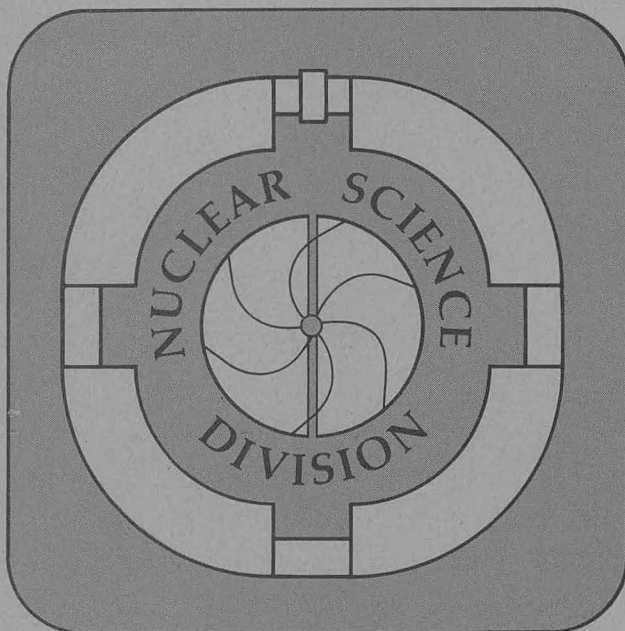
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L.F. Canto, R.J. Donangelo, M.W. Guidry, A.R. Farhan,  
J.O. Rasmussen, P. Ring, and M.A. Stoyer

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BERRY PHASE EFFECTS IN THE TWO-NEUTRON  
TRANSFER CROSS SECTIONS

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ABSTRACT:

This letter presents a new method of calculating rotational populations in two-neutron transfer reactions. In particular, the Berry Phase signature is identified and observed in Pb + Dy reactions. The sharp reduction in transfer probability associated with the Berry's phase or nuclear SQUID effect is calculated to be at significantly lower spins than previous sudden approximation estimates. We find the inclusion of an angular-dependent surface form-factor an essential ingredient in this method. This form-factor is obtained from a Cranked Hartree-Fock-Bogoliubov treatment.

Recently Nikam et al<sup>1)</sup> have predicted that the two-neutron-transfer matrix elements between Cranked Hartree-Fock-Bogoliubov (CHFB) states  $|A, I\rangle$  and  $|A + 2, I\rangle$ :

$$a_{spec}(I) = \langle A + 2, I | (a^\dagger a^\dagger)_{L=0} | A, I \rangle, \quad (1)$$

where  $I$  labels the nuclear angular momentum, have an oscillating behavior for specific deformed nuclei. The zeroes of  $a_{spec}(I)$  arise from diabolic points in the CHFB Hamiltonian connected with a sharp level crossing between the ground-state band and a band of two quasi-particles aligned along the rotational axis, known for many years as the backbending effect. These oscillations have been explained by Nikam and Ring<sup>2)</sup> as a manifestation of Berry's phase<sup>3)</sup> occurring at these diabolical points.

Several attempts have been made to establish the signature of the Berry phase for the two-neutron transfer cross section. The first such attempts<sup>4)</sup> considered  $^{118}\text{Sn}$  as a projectile incident on a  $^{160}\text{Dy}$  target at several collision energies in the Coulomb barrier region. This calculation assumed no excitation of the projectile, was restricted to the head-on collision case, and, most important, neglected the rotation of the target nucleus during the collision (*sudden approximation*). It was found that these effects should manifest themselves as a relatively sudden change in the slope of the cross section as a function of  $I$  in the high angular momentum region. Since in this region the cross section is already small and rapidly decreasing with angular momentum, this signature would have been difficult to observe. A way out of this difficulty would be to consider an even heavier projectile, which would Coulomb excite the target to much higher spins. The sudden approximation is, however, not valid in such a case, and the approach of Ref. 4 cannot be employed.

Price et al.<sup>5)</sup> considered a  $^{208}\text{Pb}$  projectile, also incident on  $^{160}\text{Dy}$ . They also perform their calculations in the sudden limit, which they try to make more realistic by artificially reducing the deformation parameter by 20%. This was done in order to compensate the overestimation of the final angular momentum produced by the sudden approximation. These authors find no evidence of Berry phase effects unless the angular momentum of the diabolic point is set, in an arbitrary way, to a lower value. The higher Coulomb excitation provided by the  $^{208}\text{Pb}$  nucleus is, in this calculation, almost cancelled by the 20% decrease in the rotational coupling. The sudden limit is, however, a rather poor approximation in this case, and the 20% decrease in the deformation parameter may be insufficient to account for the dynamical effects disregarded when the rotation of the target nucleus was neglected.

In the present letter we calculate the signature of the two-neutron transfer on the rotational spectra, improving the methods used so far in three important aspects:

- i) We take into account the adiabaticity of the collision process and show that the consideration of the non-sudden character of the collective excitation provides a signature of the Berry phase already at much lower angular momenta than previously predicted.
- ii) We use much more detailed information on the structure of the intrinsic wavefunction of the target in the diabolic region, by considering angular form-factors.
- iii) We take into account in the calculation of the transfer process not only the diagonal transition from  $I \rightarrow I$  but also all off-diagonal transitions  $I \rightarrow I'$ .

In order to qualitatively illustrate the effects of the finite rotation of the

target during the collision we perform a purely classical trajectory calculation of the excitation process. In Fig. 1 we consider a  $^{208}\text{Pb}$  nucleus at a bombarding energy  $E_{\text{lab}} = 1100$  MeV incident with zero impact parameter on  $^{160}\text{Dy}$ . For the  $^{160}\text{Dy}$  nucleus we take a quadrupole moment  $Q_0 = 7.13$  eb and a constant moment-of-inertia obtained from the energy of its  $2^+$  state. We plot the angular momenta of the Dy nucleus both at the point of closest approach in the collision,  $I_{CA}$ , and its final value,  $I_f$ . The results are presented as a function of the initial angle between the deformation axis of the Dy nucleus and the beam direction,  $\chi_0$ .

We notice that the values of the angular momentum at the point of closest approach peak near  $45^\circ$  and are almost symmetric around this peak, and that the final angular momentum curve is strongly shifted to the left. We remind the reader that in the sudden limit both curves are symmetric around  $45^\circ$  and that, in this case,  $I_f(\chi_0) = 2I_{CA}(\chi_0)$ . A direct consequence of this situation on our problem is that if the transfer process has some anomaly at a given angular momentum, this anomaly will be reflected in two values of the final angular momentum, as schematically illustrated in Fig. 1, and not in only one, as was the case in the sudden limit. In the case where  $^{160}\text{Dy}$  receives a neutron pair the vanishing of the spectroscopic factor  $a_{\text{spec}}$ , leading to diabolical pair transfer, occurs at  $I_{CA} \approx 12 \hbar$ . We note from the figure that in this case one of the final angular momenta takes a relatively low value. This calculation suggests, therefore, that anomalies such as the diabolical points mentioned above could imprint a signature on the transfer process in two different regions of the angular momentum distribution, and that one or both of these regions could lie at lower values than the one predicted in the sudden-limit calculations.

In order to test the validity of this suggestion, we performed semiclassical



calculations employing a modified version of the the Winther-deBoer Coulomb excitation computer code. While our work was in progress, we learned from Pollarolo that he and colleagues were independently making calculations by a similar method of modifying the deBoer-Winther codes<sup>6</sup>). Their assumption of the diabolic transfer matrix  $T_{I'I}$  as purely diagonal and a step function going from +1 to -1 at spin  $12\hbar$  is very different from our treatment of the transfer physics.

The original version of this code<sup>7</sup>) solves the Alder-Winther<sup>8</sup>) semiclassical coupled differential equations for the time evolution of the occupation amplitudes  $A_I(t)$ , where  $I$  is an even number that denotes the rotational angular momentum quantum number of the even-even deformed target nucleus. These equations are solved for initial conditions  $A_I(-\infty) = \delta_{I,0}$ . From the final values of the occupation amplitudes,  $A_I(+\infty)$ , we deduce the Coulomb excitation probabilities  $P(I) = |A_I(+\infty)|^2$ . It is generally agreed that the transfer process is localized in a narrow region of time and separation distance around the point of closest approach. This is a consequence of the sharp exponential drop of the two-neutron tunneling probability with distance. We have thus considered that the transfer process takes place at time  $t = 0$ , which is associated with the minimum distance between projectile and target. This was done by multiplying the occupation amplitudes  $A_I(0)$  by the matrix elements  $T_{I'I}$  obtained by performing the integral

$$T_{I'I} = \int_{-1}^1 P_{I'}(x) a_{tun}(x) a_{abs}(x) F_{I'I}(x) P_I(x) dx, \quad (2)$$

where  $x = \cos \chi$  and  $\chi$  is the angle between the symmetry axis of the target and the projectile at the distance of closest approach. The tunneling and absorption amplitudes  $a_{tun}$  and  $a_{abs}$  are defined similarly to ref. 4 with standard absorptive optical model parameters  $W = 40$  MeV,  $r_0 = 1.2$  fm, and  $a = 0.65$  fm.

$F_{I'I}(x)$  is the value of the form factor

$$F_{I'I}(\mathbf{R}) = \langle A + 2, I' | S^\dagger(\mathbf{R}) | A, I \rangle \quad (3)$$

at the inner turning point  $\mathbf{R}$  in  $\chi$ -direction. The intrinsic states  $| A, I \rangle$  are approximated by CHFB functions with proper average particle number and angular momentum. The operator

$$S^\dagger(\mathbf{R}) = (a^\dagger(\mathbf{R})a^\dagger(\mathbf{R}))_{S=0} \quad (4)$$

creates a pair of neutrons coupled to spin  $S = 0$  at the point  $\mathbf{R}$ . The angular form-factors  $F_{I'I}(\mathbf{R})$  represent in a sense the wavefunction of the Cooper-pair in the rotating nucleus and contain all information on the structure, in particular on Berry's phase.  $F_{I'I}(x)$  therefore accounts for the angular dependence of the transferred pair on the deformed target surface region.  $F_{I'I}(x)$  is proportional to  $| Y_{6m} |^2$  corresponding to a Nilsson orbit in the  $i_{13/2}$  intruder shell at the Fermi surface<sup>9,10</sup>), in a model without pairing, neglecting the intrinsic spin, and at angular momentum of zero. In the calculation of this paper, however, we used CHFB functions in a single  $\nu i_{13/2}$ -shell. The quadrupole deformation  $\beta$ , the pairing-gap parameter  $\Delta$ , and the chemical potential  $\lambda$  were fixed for the different nuclei at angular velocity zero at the values of Kumar and Baranger<sup>11</sup>) and kept constant for the two yrast bands in the initial and the final nucleus. For each angular momentum the cranking frequency  $\omega$  has been determined by the cranking condition

$$\langle J \rangle + \omega \mathcal{J}_c = [I(I + 1)]^{1/2} \quad (5)$$

where  $\langle J \rangle$  is the angular momentum of the valence particles along the rotational axis and  $\mathcal{J}_c = 30 \text{ MeV}^{-1}$  is the moment-of-inertia of the core.

There is no space in this letter to plot these functions  $F_{I'I}(x)$ . Suffice it to say that they are rather smooth functions between low-spin states because of pairing, but they become strongly oscillatory at spins near diabolic. The details will be published elsewhere<sup>12</sup>).

This leads to the amplitudes after transfer

$$A'_{I'}(0) = \sum_I T_{I'I} A_I(0) \quad (6)$$

and after this multiplication, the time evolution is continued with initial conditions  $A'_{I'}(0)$ . The values of the occupation probabilities of the rotational states of the final nucleus are estimated from  $P^{tr}(I) = |A'_{I'}(+\infty)|^2$ .

The results of several calculations with a bombarding energy of  $E_{\text{lab}} = 1100$  MeV are presented in Fig. 2. For the dotted line we used transition matrix elements from Eq. (2) containing no microscopic structure information ( $F_{I'I}(x) = 1$ ). The population pattern is rather flat in this case, showing only the typical enhancement at the rainbow angle. The dashed line shows a calculation using matrix elements from Eq. (2), but containing the CHFB form-factors  $F_{I'I}(x)$ . The microscopic calculation in the (A,I)-plane of Ref. 2 shows that there is a diabolical point at  $I \approx 12 \hbar$  between the nuclei  $^{162}\text{Dy}$  and  $^{160}\text{Dy}$ , which is encircled by the transfer trajectories in this region.<sup>†</sup> The oscillatory functions  $F_{I'I}(x)$  change their sign going through this region, causing a strong reduction already of the population pattern for spins as low as  $10 - 12 \hbar$ . In the sudden approximation<sup>4</sup>) we would have expected this reduction to occur at much higher

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<sup>†</sup> The precise position of the diabolical point depends in a very sensitive way on the parameters of the model, such as deformation, pairing gap, and position of the intruder orbit. We use the model of Kumar and Baranger because it is well established in the literature, however, experiments may determine that the diabolical point is shifted to a different neutron-number. The present theories are not precise enough to predict its exact position.

spin values of  $I \approx 24 \hbar$ . As discussed in Fig. 1, this early reduction is caused by the fact that the trajectory corresponding to the backward root has a final spin slightly lower than at closest approach. We also observe for the dashed line in Fig. 2 an indication of a second dip, which might possibly be connected with the forward root, which produces a much larger final angular momentum.

The oscillatory  $F_{I'I}(x)$  functions in the integral of Eq. (2) produce a  $T_{I'I}$  matrix with substantial off-diagonal values. That is, the transferred pair of neutrons may carry off substantial angular momentum,  $\Delta I = I' - I$ . There are two factors which should act to attenuate these off-diagonal transfer matrix elements connecting widely differing spins. First there is a centrifugal potential which will add to the two-neutron tunneling barrier. Second, there is an effect due to the finite size of the spherical partner in the transfer reaction. In contrast to the (p,t) reaction a ( $^{206}\text{Pb}, ^{208}\text{Pb}$ ) transfer is not a point-like probe of the rapidly oscillating  $F_{I'I}(x)$  functions. Indeed, there will be an angular smearing over the surface, being larger the greater the radius of curvature is. We estimate these two factors by considering a square tunneling barrier between the nuclear surface and the classical turning distance. In this approximation the two factors that modify the transfer matrix elements  $T_{I'I}$  are as follows:

$$g_1(\Delta I) = \exp \left\{ - \left( \left[ 4m_n(B_{cent} + S_{2n}) \right]^{\frac{1}{2}} - \left[ 4m_n S_{2n} \right]^{\frac{1}{2}} \right) \frac{(r_{CA} - R_T - R_P)}{\hbar} \right\} \quad (7)$$

where  $m_n$  is the neutron mass,  $S_{2n}$  is the two-neutron binding energy,  $r_{CA}$  is the distance of closest approach,  $R_T$  and  $R_P$  are the radii of target and projectile respectively, and

$$B_{cent} = \hbar^2 \Delta I(\Delta I + 1) / [4m_n R_T^2]. \quad (8)$$

$$g_2(\Delta I) = \exp \left\{ - \left[ \frac{y\Delta I}{R_T} \right]^2 \right\}, \quad (9)$$

where  $y$ , the characteristic width parameter of the tunneling wave packet is as follows:

$$y = (2\hbar)^{\frac{1}{2}}(4m_n S_{2n})^{-\frac{1}{4}} \left( \frac{1}{R_T} + \frac{1}{R_P} \right)^{-\frac{1}{2}} \quad (10)$$

We carry out calculations both with and without these  $\Delta I$  attenuation factors. The calculations with attenuation simply multiply the matrix elements  $T_{I'I}$  in Eq. (6) by  $g_1$  and  $g_2$ , and the results are shown in the full line in Fig. 2. It changes the details of the population pattern, but not the essential fact of the strong reduction at relatively low angular momenta.

In Fig. 3 we show similar calculations for the transfer from  $^{160}\text{Dy}$  to  $^{158}\text{Dy}$ . The structure calculations show in this case that there is no diabolical point between these two nuclei. The form-factors  $F_{I'I}(x)$  still show an oscillatory behavior in the spin region around  $I \approx 12 \hbar$  because of the close vicinity of the diabolical point, but going through this region they do not change their sign, and apart from a few small oscillations, no strong reduction is observed in the population pattern.

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## REFERENCES:

- 1) R.S. Nikam, P. Ring and L.F. Canto, **Z. Phys. A234** (1986) 241 and **Phys. Lett. 185B** (1987) 269.
- 2) R.S. Nikam and P. Ring, **Phys. Ref. Lett. 58** (1987) 980.
- 3) M.V. Berry, **Proc. Roy. Soc.(London) A392** (1984) 15.
- 4) L.F. Canto, R. Donangelo, R.S. Nikam, and P. Ring, **Phys. Lett. 192B** (1987) 4.
- 5) C. Price, H. Esbensen, and S. Landowne, **Phys. Lett. 197B** (1987) 15.
- 6) J. DeBoer, C.H. Dasso, and G. Pollarolo "Calculations for Diaboloic Pair-Transfer Below the Coulomb Barrier", preprint sub. to **Z. Phys.** (1989).
- 7) A. Winther and J. DeBoer, "A Computer Program for Multiple Coulomb Excitation", Caltech Report, Nov. 18, 1965, ref. 8, p. 303, reprinted in Ref. 8).
- 8) K. Alder and A. Winther, **Coulomb Excitation**, Academic Press, N.Y. (1966).
- 9) M.W. Guidry, T.L. Nichols, R.E. Neese, J.O. Rasmussen, L.F. Oliveira, and R.J. Donangelo, **Nucl. Phys. A391** (1981) 275.
- 10) L.F. Canto, R.J. Donangelo, A.R. Farhan, M.W. Guidry, J.O. Rasmussen, P. Ring, and M.A. Stoyer, **Proc. Symp. on Exotic Nuclear Spectroscopy**, Am. Chem. Soc., Miami Beach, Oct., 1989.
- 11) K. Kumar and M. Baranger, **Nucl.Phys. A110** (1968) 529.
- 12) A.R. Farhan et al., to be published.

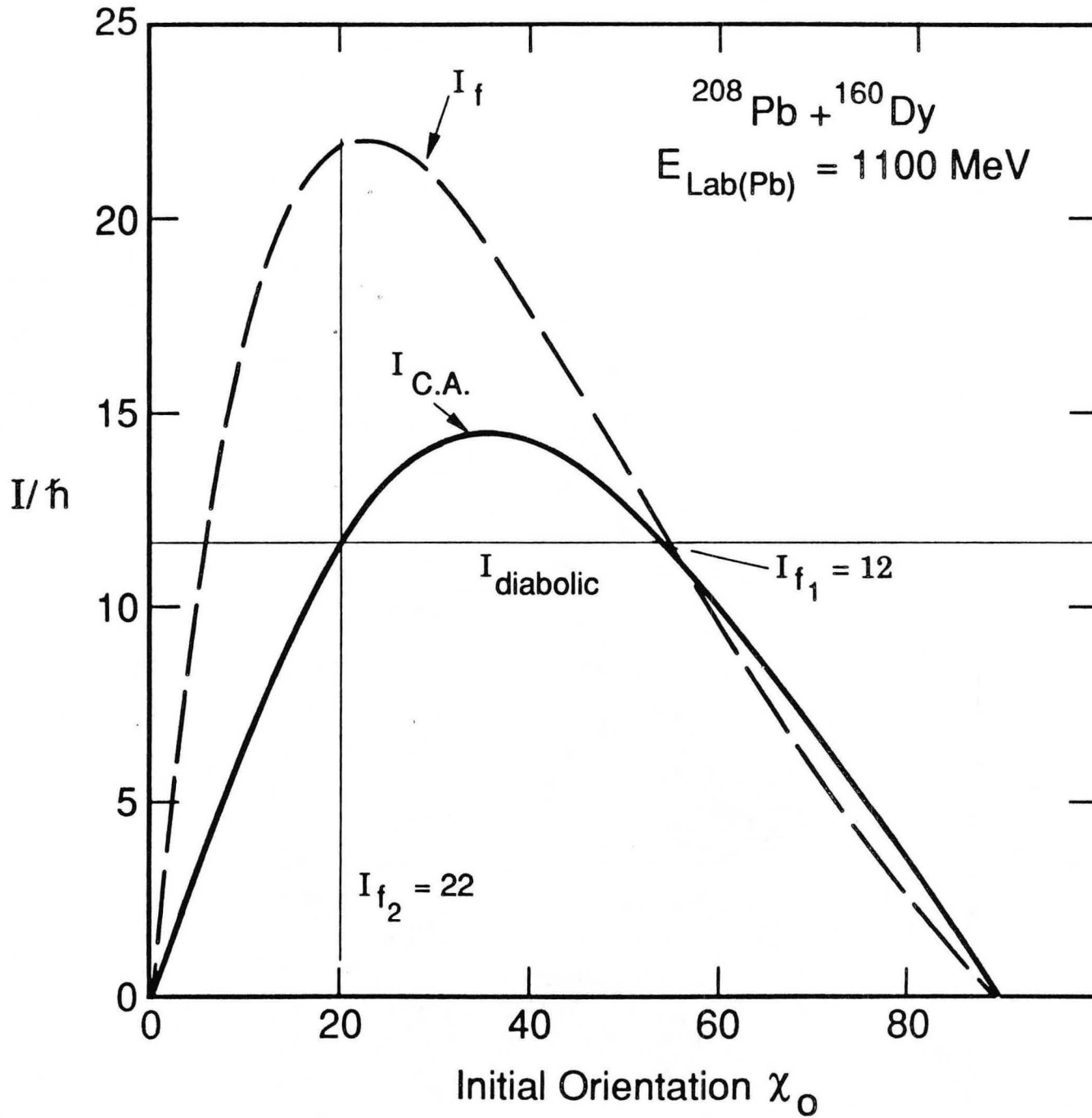
## FIGURE CAPTIONS:

Fig. 1 Classical quantum number functions at closest approach (solid line) and long after the collision (dashed line) for  $^{208}\text{Pb}$  on  $^{160}\text{Dy}$  at  $E_{\text{lab}} = 1100$  MeV. The spin at the diabolic (band-crossing) region is shown by the horizontal line labelled  $I_{\text{diabolic}}$ . The two roots for the diabolic spin at closest approach are indicated by vertical lines. Note that the forward root has a final spin nearly twice that at closest approach (as in the sudden approximation), whereas the back root has a final spin slightly lower than at closest approach, a dramatic consequence of the finite moment-of-inertia and rotation of the nucleus past  $\chi = 90^\circ$  during the time the Coulomb torque acts.

Fig. 2 Rotational transfer population patterns for a case with a diabolical point:  $^{208}\text{Pb}$  on  $^{160}\text{Dy}$ . The full line ( $\bullet$ ) corresponds to calculation with the transfer matrix elements in Eq. 2 taking into account the corrections  $g_1$  and  $g_2$  of Eq. (7) and Eq. (9) due to the presence of a centrifugal barrier and to the finite size of the projectile; in the dashed line ( $\circ$ ) these corrections are neglected and in the dotted line (open boxes) the functions  $F_{I'I}(x)$  in Eq. 2 are not taken into account. The bombarding energy is  $E_{\text{lab}} = 1100$  MeV and the sum of the population probabilities are in all cases normalized to one.

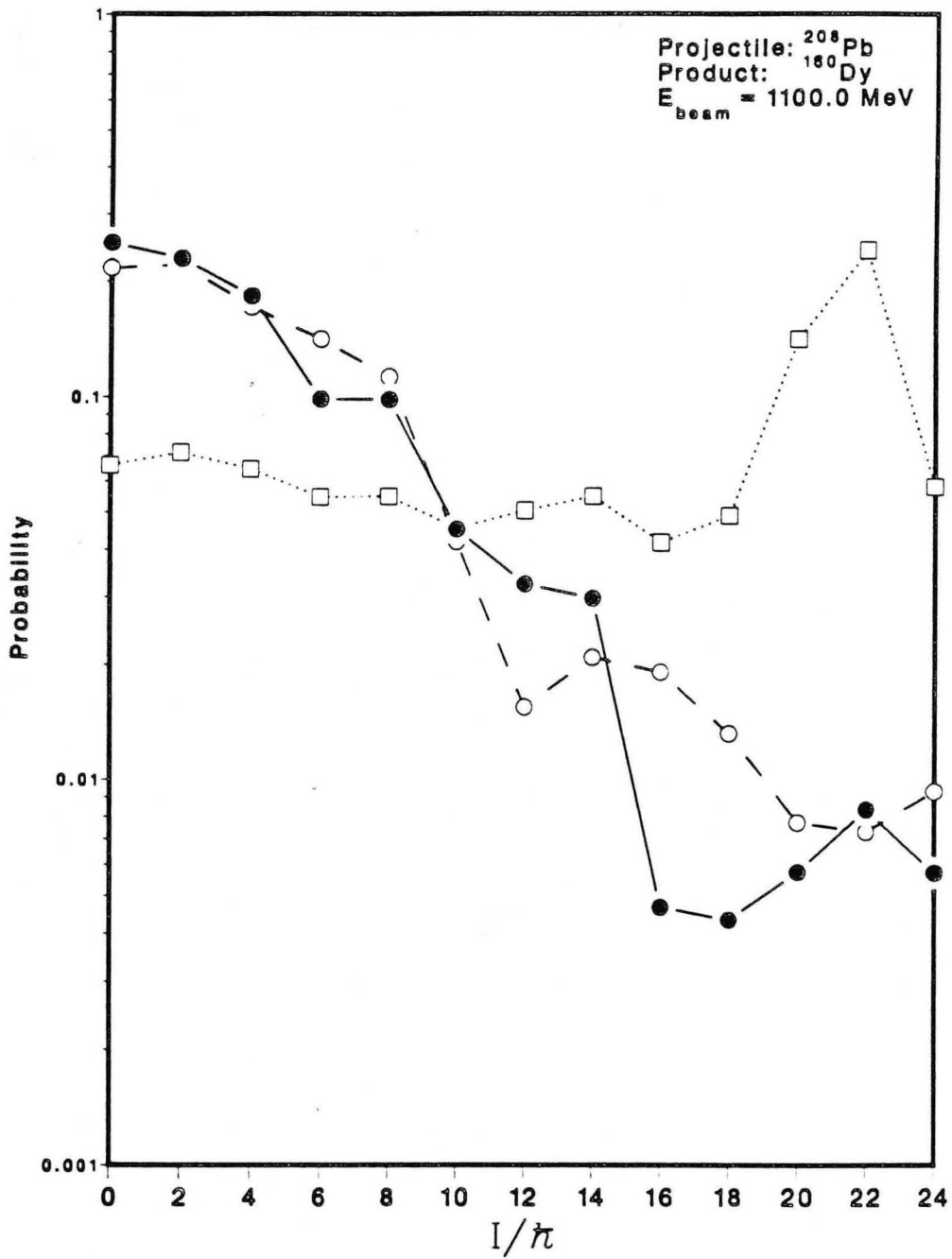
Fig. 3 Rotational transfer population patterns for a case without a diabolical point:  $^{208}\text{Pb}$  on  $^{158}\text{Dy}$  again at  $E_{\text{lab}} = 1100$  MeV. The details are identical to those in Fig. 2.

Figure One  
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# Spin Distributions



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Figure Two

# Spin Distributions

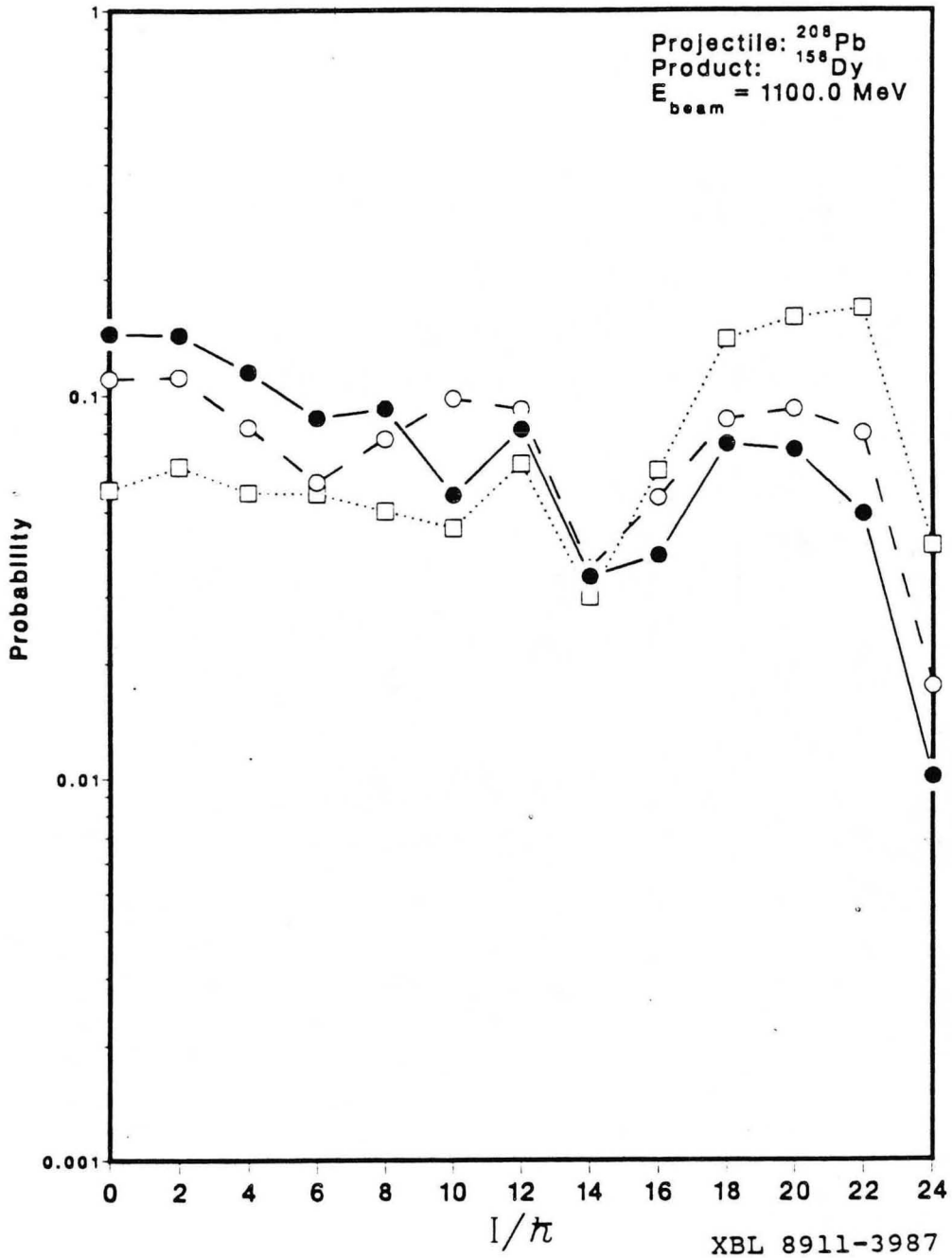


Figure Three  
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