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EVIDENCE DISCLOSURE AND VERIFIABILITY

BY

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Evidence Disclosure and Verifiability

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Abstract

We explore the conceptual basis of “verifiability” by explicitly modeling the process of evidence production in contractual relationships of complete information. Evidence is represented by documents, on the submission of which an enforcement authority conditions transfers between the contracting parties. Our analysis includes the opportunity for parties to engage in side-dealing and renegotiation during the enforcement phase. The central contracting problem involves determining whether a contract can be designed to induce given transfers as a function of the outcome of productive interaction. We study how this objective is constrained by the need to motivate parties to disclose documents during the enforcement phase. We prove the *Full Disclosure Principle*, justifying constraining attention to equilibria in which all documents are disclosed in every contingency. We also obtain insights on the implications of “positive” and “negative” evidence and we briefly discuss the relevance of our results to the design of legal institutions. *JEL Classification: C70, D74, K10.*

The notion of “verifiability” — that a court can observe a given aspect of a contractual relationship — is at the heart of the contract theory literature. Economic models of contract generally start with a specification of things considered verifiable and then assume that court-enforced contracts can condition transfers arbitrarily on (and only on) these things.¹ In reality, however, court action is a function not of what *can* be observed by the court but what evidence is *actually* presented to the court by the contracting parties. Further, presentation of evidence is subject to the parties’ incentives. Thus, verifiability critically depends on motivating parties to submit evidence.

This paper offers a foundation for the notion of verifiability, by explicitly modeling evidence disclosure and contract enforcement in contractual relationships of complete

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¹Often constraints are added to reflect an opportunity for the parties to renegotiate before court enforcement.

information. In our model, players first agree to a contract, then engage in productive interaction, and afterward can voluntarily submit evidence in a court-administered enforcement phase. Our key methodological step is the representation of evidence as physical documents; importantly, not all documents exist in all contingencies.² We study agents' incentives to disclose documents and we characterize how incentives in the enforcement phase impose limits on the extent to which court-enforced transfers can be conditioned on the outcome of productive interaction. Our results show how verifiability is sensitive to the nature of documented evidence.

We utilize an equilibrium condition that accounts for players' individual and joint incentives in the enforcement phase. Given a contract and productive contingency (the *state*), individuals selectively produce feasible documents to maximize their court-enforced transfers. In addition we assume that coalitions of players can engage in externally-enforced side contracting on the documents each is to disclose. We require in any equilibrium that no coalition of players can gain by deviating from the prescribed disclosure rules to spot contract over their own documents. That is, equilibria of the evidence disclosure phase must be *impervious to side contracting*. This condition applies Bernheim, Peleg, and Whinston's (1987) notion of coalition-proof Nash equilibrium in a setting in which players can write externally-enforced side contracts.

To understand the limits of verifiability revealed by our model, first consider a three player production game where each player can exert either high effort H or low effort L . Suppose that players wish to support productive behavior where all exert high effort. The literature has recognized that, given an adequately rich message space, it is possible to support such behavior using court-enforced "message game" schemes, regardless of whether the court can observe effort or output. This is done with a contract prescribing players to name any deviator who is then punished severely by the court. If the contract prescribes punishing player i when both of i 's opponents declare he has cheated, naming the deviator can be supported as a Nash equilibrium. For example, suppose that when two or more players name i as a deviator, the court forces i to pay 2 units to each of the others. However, such a contract is not impervious to side contracting. In fact, regardless of whether anyone has actually deviated, there always exists a two player coalition having the incentive to state that the third player deviated. For instance, suppose player 1 has deviated, so that all of the players are supposed to name player 1. Prior to evidence disclosure, player 1 and player 2 have the joint incentive to enter into a side deal where they agree to both name player 3. This would result in the same transfer of 2 to player 2, but yields in a transfer of 2 to player 1 instead of -4. The documents in this example are *cheap* because they can be produced in any state. As the example suggests, cheap documents cannot be the basis of verifiability when players can enter into side contracts over which documents to disclose.

²By modeling production of evidence as disclosure of documents, we presume that any testimony, statements, objects, etc. submitted as evidence can also be described in writing.

Our main technical result is the *Full Disclosure Principle*, which establishes that, when evaluating the scope of externally-enforced transfers, one can restrict attention to full disclosure behavioral rules. Full disclosure means that, at the time of contract enforcement, every player always submits all of the documents in his possession. This result simplifies the analysis of evidence disclosure and contract enforcement. It also implies, as the above example suggests, that message game schemes are not effective when players can engage in externally-enforced side deals.

Our analysis allows us to characterize verifiability and limits to court enforcement as corresponding to different types of evidence. The following two player example illustrates the issues we address. Suppose player 2 either expends high effort H with disutility x , or expends low effort L with no disutility. To induce her to select H , she must receive a transfer $t \geq x$ from player 1 when H is played (above whatever she might receive when L is chosen). Further, suppose there is a document d which exists in state H but not in state L (an example would be a completed product). There is a sense in which the disclosure or nondisclosure of d verifies whether H or L occurred. However, arbitrary transfers conditional on player 2's effort are not always possible. For example, suppose player 1 possesses the document. He has no interest in submitting the document unless $t \leq 0$ since only in this case is he rewarded for doing so. We conclude that if only player 1 possesses the document then there is no externally-enforced contract that supports H . On the other hand, H can be induced if player 2 possesses the document, since she has the incentive to submit it when $t \geq 0$, which allows $t = x$. Further, it is possible to induce H if player 1 possesses a document d' that only exists when player 2 *fails* to exert effort H . This can be done by giving player 2 a transfer of x *unless* d' is disclosed by player 1.

The literature typically treats verifiability as implying a partition of the productive action space; one then looks for an externally-enforced contract that is measurable with respect to this partition and which supports a particular productive action profile.³ Our analysis indicates where the standard methodology is appropriate and where contracts are further constrained by incentives in the evidence disclosure phase. We show that when two states cannot be differentiated by the existence of documents, these states must involve the same court-enforced transfers. That is, court-enforced transfers must be measurable with respect to a partition that reflects the distribution of documents over states. In addition, we find additional restrictions on the partition implied by the extent to which *positive* and *negative* evidence are available. In the example in the preceding paragraph, d is positive evidence of H (and negative evidence of L), while d' is positive evidence of L (and negative evidence of H). We also describe evidence environments for which the set of supportable court-enforced transfers is unconstrained by incentives in disclosure.

Our model helps organize one's thinking about actual legal institutions. In particular, the U.S. legal system is sensitive to the distinction between positive and

³Examples of this include Holmstrom (1982), Legros and Matthews (1993), and more recently Bernheim and Whinston (1999).

negative evidence. The theory identifies positive evidence as the strongest form of evidence and, in fact, positive evidence presented to substantiate a claim is always given weight by the legal system. Further, although U.S. courts recognize the significance of negative evidence, they generally treat it as less compelling than positive evidence. Wigmore (1935) suggests that failure by a party to disclose evidence that would be favorable to her claim if it existed is “open to the inference that it does not exist.”⁴ That is, in our example above, it is known that document d exists in state H but does not exist in state L . As suggested, a contract can be structured to give player 2 the incentive to disclose d when this document is available. Thus, under such a contract (and given rational behavior), her failure to disclose d can be taken to mean that d does not exist. However, this conclusion may not be reached if the original contract failed to provide player 2 with the incentive to disclose d . A classic case, *State v. Simons* (1845) provides a good example of a setting where the court’s decision is based upon negative evidence.⁵

The modeling exercise reported herein combines elements of several noteworthy papers in the literature. Our notion of documents existing in different contingencies is much like the “truthful reporting” constraint in Milgrom and Roberts (1986), Okuno-Fujiwara, Postlewaite and Suzumura (1990), and Shin (1994) and the “sending a message cannot be verified” setting studied by Hart and Moore (1988). We adopt the complete-contract perspective.⁶ Following the large body of work on Nash implementation and implementation with renegotiation, we study the case of complete information between contracting parties. See, for example, Holmstrom (1982), Legros and Matthews (1993), Maskin and Moore (1999), and Maskin and Tirole (1999). As noted above, we also utilize a version of Bernheim, Peleg, and Whinston’s (1987) coalition-proofness concept.

The paper is organized as follows. In Section 1 we describe the model of evidence disclosure and discuss the concept of equilibrium that is impervious to side contracting. In Section 2, we characterize supportable court-enforced transfers and prove the *Full Disclosure Principle*. Section 3 analyzes the relationship between the manner by which players can distinguish one state from another and the relevant partition of the state space. In Section 4, we turn our attention to incentives in productive interaction and we characterize, given an induced partition, those productive action profiles that can be supported. Appendix A contains proofs not found elsewhere in the text. Appendices B and C present extensions noted in the text.

⁴Wigmore (1940) notes that uncertainty exists as to the exact “nature of the inference and conditions in which it may legitimately be drawn.”

⁵Simons was accused of selling spirits without a license to do so. The court held that the failure of Simons to present a license, even though State presented no evidence showing lack of a license, was sufficient to conclude that he did not possess such license. The court’s opinion suggests that evidence “need not be of the most direct and positive kind.”

⁶Tirole (1999) contains a thorough discussion of incomplete vs. complete contracts.

1 A Model of Contract, Evidence Disclosure, and External Enforcement

We consider a contractual relationship between n players (also called agents), who interact over four periods of time. In the first period, the players form a contract. This contract has an externally-enforced component m which specifies monetary transfers to be compelled by the court in period 4, conditional on evidence presented to the court in period 3.

In the second period, productive interaction occurs, leading to an outcome a which we call the *state of the relationship*. The state is commonly observed by the players. We let A denote the set of possible states and we assume A is finite. Players receive an immediate payoff given by $u : A \rightarrow \mathbf{R}^n$. Since most of our analysis addresses how behavior later in the game is conditioned on the state, we defer to Section 4 further discussion of the details of the production phase.

External contract enforcement occurs in periods 3 and 4. Specifically, in period 3 the players simultaneously and independently submit documents to the court. Documents represent evidence on which the court conditions transfers. Denote by $D_i(a)$ the set of documents that can be presented by player i in state a . Since not all documents may be available in all states, $D_i(a) \neq D_i(b)$ is generally, but not necessarily, the case. Let $D_i \equiv \bigcup_{a \in A} D_i(a)$ denote the set of documents available to player i over all states. We assume D_i is finite. We also assume D_1, D_2, \dots, D_n are disjoint sets and, defining $N \equiv \{1, 2, \dots, n\}$, we let $D \equiv \bigcup_{i \in N} D_i$ and $D(a) \equiv \bigcup_{i \in N} D_i(a)$ for each a . For any set of documents $E \subset D$, we write E_i as those documents disclosed by player i . The feasible sets of disclosed documents are given by $\mathcal{D} \equiv \{E \mid E \subset D(a), \text{ for some } a \in A\}$. Note that the empty set (no documents disclosed) is an element of \mathcal{D} .

In period 4 the court imposes the transfer m as a function of the documents disclosed by the players.⁷ Formally, $m : \mathcal{D} \rightarrow \mathbf{R}^n$, so for any evidence set $E \subset \mathcal{D}$, $m_i(E)$ is the monetary transfer made to player i . Thus, player i 's total payoff in the contract game is $u_i(a) + m_i(E)$. We assume $\sum_{i \in N} m_i \leq 0$. Remember that m is jointly selected by the players in period 1.

Regarding document disclosure, we apply the term *disclosure rule* to any function $\beta : A \rightarrow \mathcal{D}$, satisfying $\beta(a) \subset D(a)$ for each $a \in A$, which describes how the players behave in period 3, conditional on the state.⁸ For example, in state a the players disclose documents $\beta(a)$. Let $\beta_i(a)$ denote the documents presented by player i in state a and, for any $J \subset N$, define $\beta_J(a) \equiv \bigcup_{i \in J} \beta_i(a)$.⁹

We call any function $g : A \rightarrow \mathbf{R}^n$ a *transfer function*. An externally-enforced

⁷Our model applies equally well to other external enforcement systems.

⁸For simplicity, we focus here on pure disclosure rules. In Appendix B, we show that consideration of mixed rules (where players submit documents with nondegenerate probabilities) adds little to the analysis; our main results are unchanged when mixed disclosure rules are allowed.

⁹Note that an individual player's behavior can easily be deduced from $\beta(a)$ since the players' document spaces are disjoint.

	d_d^1	d_d^2	d_d^3	d_c^1	d_c^2
Paid	X	X	X		X
Did not pay			X	X	X

Figure 1: Description of $D_i(a)$ sets in the debtor/creditor example.

contract m and a disclosure rule β imply the transfer function g defined by $g(a) = m(\beta(a))$ for every $a \in A$. The following terminology will be handy.

Definition 1 *A transfer function g is called **supported by externally-enforced contract m and disclosure rule β** if $g(a) = m(\beta(a))$ for every $a \in A$. Also, g is called **supported by disclosure rule β** if there is an externally-enforced contract m such that m and β support g .*

On a technical level, our main goal is to characterize the degree to which court-compelled transfers can be indirectly conditioned on the state, given that the players behave rationally in the disclosure phase. In other words, we seek to understand whether any given transfer function g can be supported by an externally-enforced contract m and a disclosure rule β that is consistent with rationality.

Interpretation

Our model associates the common notion of “verifiability” with the court’s observation of actual documents disclosed by the players. As an example, consider a relationship between a creditor and a debtor. Suppose there are two states, one representing that the debtor has paid the creditor and the other representing that no payment was made. Three documents can be presented by the debtor. The first, denoted d_d^1 , is a receipt given to the debtor by the creditor certifying payment. The second, d_d^2 , is a canceled check that exists only if payment was made. The third document, d_d^3 , is written testimony of payment made by the debtor’s friend, who is brought forth as a witness. Two documents may be presented by the creditor. The first, d_c^1 , is a bank notice demonstrating that the debtor’s check was returned due to insufficient funds. The second, d_c^2 , is testimony of non-payment made by the creditor’s colleague, who is brought forth as a witness.¹⁰

Figure 1 describes how the availability of these documents depends on the state (that is, the specification of $D_i(a)$ sets) in this example. In the figure, an X indicates that a particular document exists and can feasibly be disclosed in a given state. For example, the debtor can produce a canceled check (document d_d^2) or a receipt

¹⁰One can imagine other documents that might be relevant and could easily be included in this example. In addition, “document” should be interpreted broadly to include items such as videotapes, audio recordings, oral statements made in court, etc.

(document d_d^2) only if she actually paid the creditor. However, it may be that she can find a witness to testify (document d_d^3) regardless of whether she paid. Similarly, the creditor can only produce the bounced check (document d_c^1) if the debtor did not pay, but the creditor can always find a witness to say that payment has not occurred (document d_c^2).

One way of thinking about documents is that players can present any particular document at a *cost* which depends on the state. We assume that, given a , documents in $D(a)$ can be disclosed at zero cost, while documents outside of this set can be produced only at infinite — or sufficiently prohibitive — cost; for example, a player cannot convincingly forge a canceled check.

Our assumption that the D_i sets are disjoint does not limit application of the model. For example, suppose in reality both the creditor and debtor can produce the canceled check in some state. Then, in fact, it will be the case that one of them can produce the actual canceled check while the other can produce *a copy* of the canceled check, with these two items considered as different documents.

Equilibrium Concept

To analyze behavior in period 3, we use the standard Nash equilibrium concept. Given m , we call β an *equilibrium disclosure rule* if β specifies a Nash equilibrium for each a ; that is,

$$m_i(\beta(a)) \geq m_i(E_i \cup \beta_{-i}(a))$$

for each $i \in N$, $E_i \subset D_i(a)$, and $a \in A$. In addition to equilibrium, we add constraints reflecting the players' ability to renegotiate between periods 3 and 4 and to make side deals between periods 2 and 3. On the former, suppose the players can renegotiate the externally-enforced contract m between periods 3 and 4. If their outstanding contract is such that $\sum_{i \in N} m_i(a) < 0$ for some a , then in state a the players would re-specify m before the court compels transfers. This justifies restricting attention to *balanced* externally-enforced contracts, which are functions of the form $m : \mathcal{D} \rightarrow \mathbf{R}_0^n$, where $\mathbf{R}_0^n \equiv \{x \in \mathbf{R}^n \mid \sum_{i \in N} x_i = 0\}$.

Regarding side deals made between periods 2 and 3, we shall impose a version of Bernheim, Peleg, and Whinston's (1987) *coalition-proofness* concept that assesses whether a sub-group of players can benefit from side contracting. We examine whether a coalition can gain from writing a side contract m' to be externally enforced along with the players' original contract m . To illustrate, suppose that a coalition of players $J \subset N$ wishes to write an additional contract m' specifying transfers between members of this coalition as a function of the documents disclosed in period 3. It must be that $m'_i = 0$ for each $i \notin J$ since coalition J cannot force a side contract on players outside the coalition. In addition, we require m' to be balanced due to the specter of renegotiation between periods 3 and 4. The side contract m' is assumed to be enforced by the same court that enforces m ; alternatively, m' may be enforced by some other institution, legal or otherwise. In any case, with the side contract, externally enforced transfers in period 4 are given by $m + m'$.

Players outside the coalition do not observe the side contract until after documents are disclosed. Thus, the point of writing a side contract is to induce members of the coalition to change their behavior in the disclosure phase in a way that benefits the coalition. To evaluate whether this is possible, define $M_J(E) \equiv \sum_{j \in J} m_j(E)$ and let M' be defined analogously. Suppose that in state a the players would coordinate on disclosure of documents $\beta(a)$ in the absence of side contracting. Further suppose that, by side contracting, a coalition J can induce its members to disclose documents E_J . Then the coalition strictly gains from the side deal if and only if

$$M_J(\beta_{-J}(a) \cup E_J) + M'_J(\beta_{-J}(a) \cup E_J) > M_J(\beta(a)) + M'_J(\beta(a)),$$

which is equivalent to $M_J(\beta_{-J}(a) \cup E_J) > M_J(\beta(a))$ since $M'_J = 0$.

Between periods 2 and 3, a J coalition can always find a side contract that induces its members to disclose any particular set of documents, as long as the specified documents exist given the state. This is because the coalition writes a side contract *after players observe the state* — knowing which documents exist. The coalition can write a *forcing* side contract which punishes individuals for not disclosing exactly those documents desired by the coalition (by way of arbitrarily large transfers to the other members of the coalition). The side contract can also implement any desired split of its gains between the coalition members. Thus, between periods 2 and 3, coalitions can effectively spot contract on which documents to disclose. Mathematically, in state a , a coalition J can side contract to force its members to disclose of any set of documents $E_J \subset D_J(a) \equiv \bigcup_{j \in J} D_j(a)$.

A side contract between members of a coalition J may be undermined by a subsequent side contract between members of a sub-coalition $K \subset J$. Following Bernheim, Peleg, and Whinston (1987), we view a side contract as viable only if it is immune to disruption by sub-coalitions (who have to pass the same test). In fact, the issue of sub-coalitions is easily handled in our model, because forcing contracts can always be designed to stifle any further side dealing by sub-coalitions. Specifically, a coalition J can specify m' so that any player $j \in J$ who does not disclose a specified set of documents must pay an amount y to each of the other players in the coalition. Then any sub-coalition $K \subset J$ will lose at least y when one or more of its members deviate from the prescription of m' . The number y can be set large enough so that this loss is greater than any gain the sub-coalition can get by way of the original contract m .

We look for specifications of behavior that are coalition-proof with respect to externally-enforced side contracts. Given the discussion above, it is sufficient to simply evaluate whether coalitions can gain from spot contracting on disclosure of documents. With reference to a state a and a disclosure rule β , we say that $E \in \mathcal{D}$ is a *J-deviation from $\beta(a)$ at a* if $E \subset D(a)$ and $E_i = \beta_i(a)$ for all $i \notin J$. In words, E is a set of documents disclosed when only the players in J deviate from the prescription $\beta(a)$. Coalition-proofness is defined as follows.

Definition 2 *Given an externally enforced contract m , a disclosure rule β is called **impervious to side contracting (ISC)** if $M_J(\beta(a)) \geq M_J(E)$ for each $a \in A$, each coalition $J \subset N$, and each $E \subset D(a)$ that is a J -deviation from $\beta(a)$ at a .*

We use the term *impervious to side contracting* instead of *coalition-proof Nash equilibrium* because the latter is defined for self-enforced contracts (Nash equilibria) of standard non-cooperative games, while we require a version that examines externally-enforced contracts. That is, ISC is Bernheim, Peleg, and Whinston’s (1987) definition applied to externally-enforced contracts. Note that ISC includes the self-enforced component, since for each $a \in A$, $\beta(a)$ must be a Nash equilibrium with payoffs defined by m .¹¹ Considering coalitions of single players, we see that ISC implies Nash equilibrium:

Lemma 1 *Every ISC disclosure rule is an equilibrium disclosure rule. Further, in a two player setting, every equilibrium disclosure rule is an ISC disclosure rule.*

We use the ISC concept to study rational behavior in the evidence disclosure and enforcement phases of the contractual relationship.

Regarding transfer functions, recall the definition of g supported by m and β . In the same vein, we say that g is *supported by an ISC disclosure rule* if there are functions m and β such that (i) g is supported by m and β , and (ii) β is an ISC disclosure rule with respect to m . We shall also use the term “ g supported by ISC disclosure rule β ,” meaning there is an m such that (i) and (ii) are satisfied. Since any externally-enforced contract is required to be balanced, we can restrict attention to transfer functions that are also balanced; that is, $g : A \rightarrow \mathbf{R}_0^n$.

2 Supportable Transfer Functions

In this section, we characterize the set of transfer functions that are supported by ISC disclosure rules. An important part of our analysis concerns whether supporting a transfer function relies on players strategically withholding documents that exist.

Definition 3 *The **full disclosure rule** $\bar{\beta}$ is defined by $\bar{\beta}(a) \equiv D(a)$ for all $a \in A$.*

With full disclosure, each player submits all of the documents in his possession in every state.

Definition 4 *A transfer function g is called **supported by an ISC/full disclosure rule** if and only if there exists an externally enforced contract m such that (i) the full disclosure rule $\bar{\beta}$ is ISC and (ii) g is supported by m and $\bar{\beta}$.*

¹¹In section 4, we further discuss self-enforced components of contract.

Our main technical result is:

Theorem 1 (Full Disclosure Principle) *If a transfer function g is supported by an ISC disclosure rule, then g is supported by an ISC/full disclosure rule.*

Before proceeding to the characterization of supportable transfer functions and the proof of Theorem 1, we comment on the significance of the full disclosure principle. Note that the principle is similar in theme to the classic revelation principle of mechanism design theory.¹² Like the revelation principle, it simplifies the analysis of implementation/evidence production by allowing us to focus on the full disclosure rule. As demonstrated below, this streamlines the analysis of supportable transfer functions.

However, there are several significant ways in which the full disclosure principle differs from the revelation principle. First, the former applies to document disclosure in situations where the existence of documents depends on the state, while the latter concerns settings in which agents can freely submit messages, regardless of the state.¹³ Second, we impose the ISC condition to model the possibility of side deals.¹⁴ Third, full disclosure refers to players submitting all available documents, not necessarily “truthfully reporting” the state. For example, suppose that in both states a and b players are able to write “ a occurred” on a piece of paper and submit this document. Suppose also that in both states players can write “ b occurred” on a piece of paper. Where the revelation principle applies, it allows one to restrict attention to behavioral rules in which the players submit “ a occurred” in state a and “ b occurred” in state b . The full disclosure principle, on the other hand, justifies restricting attention to behavioral rules in which *both* documents are submitted in *both* states. In other words, in both states a and b , the players present “ a occurred” and “ b occurred.” This implies that externally-enforced transfers rely on distinguishing between states via the *existence* of documents (or lack thereof) which is our foundation for the concept of verifiability. Section 3 elaborates on this point and discusses the implication for “message game phenomena.”

Characterization of Supportability Given β

We begin the analysis with a simple characterization of the ISC condition. For each $a \in A$, $E \subset D(a)$, and $E' \in \mathcal{D}$, we define the function $R(a; E, E')$ as follows.

¹²See, for example, Dasgupta, Hammond, and Maskin (1979).

¹³There is an exception to our characterization of the literature. Green and Laffont (1986) study implementation of social choice functions in a principal-agent model where the agent’s message space is state dependent, and they characterize those message spaces for which the revelation principle is valid. Our modeling exercise pursues another line of inquiry marked by the other two differences noted here; in addition, we study multiple agents with complete information and we offer an interpretation of message restrictions in terms of documents. Our model requires different analytical techniques than those employed by Green and Laffont (1986).

¹⁴Laffont and Martimort (2000) consider mechanism design in a public goods environment when agents can enter collusive agreements. See also Baliga and Brusco (2000) and the references therein.

If $E' \not\subset D(a)$, then we let $R(a; E, E') \equiv N$. If $E' \subset D(a)$ then we let $R(a; E, E') \equiv \{i \in N \mid E_i \neq E'_i\}$. That is, $R(a; E, E')$ is the minimum set of players that would be needed to deviate from E in order to achieve E' in state a .

Lemma 2 *Given m , β is an ISC disclosure rule if and only if $m_i(\beta(a)) \leq m_i(E)$, for each $a \in A$, $E \in \mathcal{D}$, and $i \notin R(a; \beta(a), E)$.*

The proof of Lemma 2 is intuitive and straightforward. Suppose for some equilibrium disclosure rule $\beta(a)$ and some state a , there is a set of documents E and a player i such that $i \notin R(a; \beta(a), E)$ and $m_i(\beta(a)) > m_i(E)$. This means that if the players in group $R(a; \beta(a), E)$ achieve disclosure of E by altering what documents they present, player i is strictly worse off. Since m is balanced, this implies that the other players ($-i$) are collectively strictly better off when E is disclosed rather than $\beta(a)$. Further, since $R(a; \beta(a), E) \subset -i$ we know that E is a $-i$ -deviation from $\beta(a)$ at a , which means $\beta(a)$ could not be ISC. In the other direction the condition of the lemma obviously implies that no coalition can strictly gain by deviating from β . *Q.E.D.*

Next we provide a necessary and sufficient condition for a transfer function g to be supported by an ISC disclosure rule, where we focus on a given disclosure rule β . The analysis examines the different ways in which coalitions of players can deviate to induce disclosure of an arbitrary set of documents. For example, consider some set of documents E and states a and b . Disclosure rule β prescribes presentation of documents $\beta(a)$ in state a and $\beta(b)$ in state b . It may be that E is a J -deviation from $\beta(a)$ in state a , while E is a K -deviation from $\beta(b)$ in state b . If there is a function m with respect to which β is ISC, then it must be that m deters the J coalition from deviating to E in state a and also deters the K coalition from deviating to E in state b . We use Lemma 2 to translate this constraint into an inequality defined for E and β .

For any disclosure rule β , each $i \in N$, and $E \in \mathcal{D}$, let

$$B(i; \beta, E) \equiv \{a \in A \mid i \notin R(a; \beta(a), E)\}.$$

This is the set of states at which player i is *not* needed to induce disclosure of E by deviation from the prescription of β . Note that $E \not\subset D(a)$ implies $a \notin B(i; \beta, E)$. For any $a \in B(i; \beta, E)$, using Lemma 2, we know that player i 's transfer when E is disclosed must be at least as great as his transfer when $\beta(a)$ is disclosed; further, the latter transfer is supposed to be $g_i(a)$, given the transfer function g . Examining all states in $B(i; \beta, E)$, we have the following lower bound on player i 's transfer conditional on E .

$$z_i(E; \beta, g) \equiv \begin{cases} \max_{a \in B(i; \beta, E)} g_i(a) & \text{if } B(i; \beta, E) \neq \emptyset \\ -\infty & \text{if } B(i; \beta, E) = \emptyset \end{cases}.$$

Our characterization result is

Theorem 2 (Characterization) *Take as given a disclosure rule β and a transfer function g . There exists an externally enforced contract m such that (i) g is supported by β and m , and (ii) β is an ISC disclosure rule, if and only if*

$$\sum_{i \in N} z_i(E; \beta, g) \leq 0, \text{ for every } E \in \mathcal{D}.$$

To generate intuition, recall that $z_i(E; \beta, g)$ is a lower bound on the transfer to player i when E is the set of documents disclosed. Since the externally-enforced contract m must be balanced, it is possible to meet the lower bounds for all of the players only if the sum of z_i 's is non-positive. Note that $z_i(E; \beta, g)$ equals negative infinity when player i is needed to deviate to E from every state. In this case, deviations to E can be easily deterred by punishing player i severely.

Proof of the Full Disclosure Principle

We use the characterization theorem to prove Theorem 1. Suppose transfer function g is supported by disclosure rule β . We define $\bar{\beta}$ to be the full disclosure rule; that is, $\bar{\beta}(a) = D(a)$ for all $a \in A$. Using Theorem 2, to ascertain whether g is supported by $\bar{\beta}$ we need to check whether

$$\sum_{i \in N} z_i(E'; \bar{\beta}, g) \leq 0, \text{ for all } E' \in \mathcal{D}. \quad (1)$$

To do this, consider any set of disclosed documents E' . If $z_i(E'; \bar{\beta}, g) = -\infty$ for some i , then

$$\sum_{i \in N} z_i(E'; \bar{\beta}, g) \leq 0$$

is assured. Therefore, suppose $z_i(E'; \bar{\beta}, g) \neq -\infty$, for all $i \in N$. Then for each player i , let a^i be a state such that a^i maximizes $g_i(a)$ over all $a \in B(i, \bar{\beta}, E')$. Then we can define E so that $E_i = \beta_i(a^i)$ for all i . This implies that player i is not needed to deviate to E from $\beta(a^i)$ at a^i , unless we have the non-feasibility case where $E \not\subset D(a^i)$. However, we can rule out $E \not\subset D(a^i)$ by the definition of B and since $a^i \in B(i, \bar{\beta}, E')$. This is because $E' \subset D(a^i)$ for every $i \in N$ and, by the definition of E' , it must be that $E'_i = D_i(a^i)$. Thus, $E_i \subset E'_i$ for all $i \in N$, which implies that $E \subset D(a^i)$ for all $i \in N$. We conclude that $a^i \in B(i, \beta, E)$. Thus

$$z_i(E; \beta, g) \geq z_i(E'; \bar{\beta}, g) \text{ for all } i \in N,$$

which implies that

$$\sum_{i \in N} z_i(E'; \bar{\beta}, g) \leq \sum_{i \in N} z_i(E; \beta, g) \leq 0,$$

for all $E' \in \mathcal{D}$. That g is supported by ISC disclosure implies (1). *Q.E.D.*

Note how the full disclosure principle simplifies checking whether a given transfer function can be supported by an ISC disclosure rule. One need only verify the condition of Theorem 2 for the full disclosure rule. That is, defining $\bar{\beta}(a) \equiv D(a)$ for each $a \in A$, one evaluates whether

$$\sum_{i \in N} z_i(E; \bar{\beta}, g) \leq 0, \text{ for every } E \in \mathcal{D}. \quad (2)$$

A transfer function g is supportable by ISC disclosure if and only if condition (2) holds.

3 Evidence and the Relevant Partition

The examples in the Introduction show that, regarding whether a given transfer function can be supported, it matters how the players can distinguish between states by submitting documents. In this section we refine the characterization of supportable transfer functions on the basis of the manner in which players can differentiate between states. We begin by noting that meaningful differentiation between states a and b cannot occur unless players disclose in at least one state a document that *does not exist* in the other state.

Theorem 3 *Suppose g is supported by m and ISC disclosure rule β . If $\beta(a), \beta(b) \subset D(a) \cap D(b)$, then $g(a) = g(b)$.*

The intuition behind Theorem 3 is that, since document sets $\beta(a)$ and $\beta(b)$ are feasible in both states a and b , it is easy for coalitions of players to pretend that one state occurred when in fact the other occurred. To prove the theorem, suppose any player i deviates to disclose $\beta_i(b)$ in state a , when the others are disclosing $\beta_{-i}(a)$. Since i must be deterred from doing this, we have $m_i(\beta(a)) \geq m_i(\beta_i(b) \cup \beta_{-i}(a))$. Next suppose the $-i$ coalition were to deviate to disclose $\beta_{-i}(a)$ in state, b when i discloses $\beta_i(b)$. Then it must be that $M_{-i}(\beta(b)) \geq M_{-i}(\beta_i(b) \cup \beta_{-i}(a))$, which implies $m_i(\beta(b)) \leq m_i(\beta_i(b) \cup \beta_{-i}(a))$. Combining the inequalities yields $m_i(\beta(a)) \geq m_i(\beta(b))$. The same argument can be repeated starting with i deviating in state b , yielding $m_i(\beta(b)) \geq m_i(\beta(a))$. Thus, $m_i(\beta(a)) = m_i(\beta(b))$ for all i . *Q.E.D.*

Theorems 1 and 3 suggest that “cheap talk” is of little use for contract enforcement when players can make side deals. To formalize this idea, let us use the term *cheap document* for a document that can be provided in any state. In the implementation literature, “message game phenomena” refers to a situation in which (i) players distinguish between various states by disclosing different cheap documents, and (ii) different transfers are supported in various states.¹⁵ Theorem 3 establishes

¹⁵The literature takes as fact that message game schemes solve contracting problems when there is complete information and more than two players.

that, under ISC, these cheap documents *cannot* serve to distinguish between states in any instrumental manner. Specifically, an ISC disclosure rule that differentiates between states only on the basis of cheap documents must imply the same transfer values over the states. We summarize this discussion with:

Corollary 1 *Message game phenomena using cheap documents cannot occur under ISC disclosure.*

Clearly, this result is due to the combination of the ISC condition and the zero-sum aspect of externally-enforced transfers (given renegotiation possibilities). The conclusion does not necessarily hold for settings in which some productive actions are taken after documented messages are sent, since then continuation payoffs may not be zero-sum. Yet our result does demonstrate that, in general, the opportunity for players to side contract can significantly constrain message game phenomena.

Partitions and Supportability

Next we focus on how documents can be used to distinguish between states. We show how different forms of evidence imply partitions of the state space and we examine how the partitions relate to the set of supportable transfer functions. Denote by F^D the set of transfer functions supported by ISC disclosure rules; that is, $g \in F^D$ if and only if g satisfies condition (2). Consider an arbitrary partition P of the state space. We use the convention that, for any state a , $P(a)$ denotes the element of the partition containing a ; that is, $b \in P(a)$ if and only if a and b are in the same element of the partition P . Let us denote by $G(P)$ the set of transfer functions that are measurable with respect to P .

When considering how evidence is provided by documents, it is important to differentiate between “positive” and “negative” evidence.¹⁶ Suppose there is a document d that can be presented in state a (that is $d \in D(a)$) but it does not exist in state b ($d \notin D(b)$). Then disclosure of d is considered *positive evidence* of a since a player can present this document in state a but not in some other state. Further, *nondisclosure* of d is considered *negative evidence* of b because a player cannot disclose d in state b but can disclose the document in some other state. For example, consider the canceled check in the debtor/creditor story. When the canceled check is presented, it is positive evidence that payment has been made. However, when the canceled check is not presented, this is negative evidence that payment has not been made.

We say that a player can *distinguish between states a and b* if she has positive evidence of at least one of these states to the exclusion of the other (which is thus supported by negative evidence). That is, player i can distinguish between a and b if and only if $D_i(a) \neq D_i(b)$. In the debtor/creditor example, the debtor can distinguish “paid” from “not paid” and vice versa. Let P^D denote the partition of A induced by the notion of distinguishing between states. Formally P^D is defined so that $a \in P^D(b)$ if and only if $D(a) = D(b)$. Theorem 3 implies:

¹⁶Lipman and Seppi (1995) study similar notions.

Corollary 2 *Every transfer function g that is supported by an ISC disclosure rule has the property that $g(a) = g(b)$ for states a and b that cannot be distinguished. That is, $F^D \subset G(P^D)$.*

This result establishes that measurability with respect to P^D is necessary for supportability. However, it is generally not sufficient.

We say that player i can *positively distinguish a from b* if there exists a document $d \in D_i(a)$ such that $d \notin D_i(b)$. Whether action profiles can be positively distinguished is relevant in considering the transfer functions that can be supported. As the next theorem shows, lack of positive evidence limits enforceable transfers, since players must be given the incentive to disclose documents.

Lemma 3 *Take a subset of the players $K \subset N$. Suppose the K players cannot positively distinguish b from a ($D_K(b) \subset D_K(a)$) and the $N \setminus K$ players cannot positively distinguish a from b ($D_{N \setminus K}(a) \subset D_{N \setminus K}(b)$). If g is supported by an ISC disclosure rule then $\sum_{i \in K} g_i(a) \geq \sum_{i \in K} g_i(b)$.*

This lemma establishes that lack of positive evidence implies $F^D \neq G(P^D)$. Intuitively, if the K players as a whole did worse in state a then they would prefer to behave as though state b had occurred; further, the $N \setminus K$ group is unable to counter this by providing positive evidence of a .

We say that player i can *fully distinguish between a and b* if neither $D_i(a) \subset D_i(b)$ nor $D_i(b) \subset D_i(a)$. To fully distinguish between a and b , a player must have positive evidence of a , to the exclusion of b , as well as positive evidence of b , to the exclusion of a . For any player i , we define the partition \overline{P}^{D_i} as the finest partition satisfying

$$D_i(a) \subset D_i(b) \text{ implies } a \in \overline{P}^{D_i}(b).$$

This partition reflects the notion of player i fully distinguishing between states, which the next result associates with a lower bound on the set of supportable transfer functions.

Theorem 4 *Fix some $i \in N$. If transfer function g satisfies $g(a) = g(b)$ for all states a and b between which player i cannot fully distinguish, then g is supported by an ISC disclosure rule. That is, $G(\overline{P}^{D_i}) \subset F^D$.*

As Theorem 4 shows, if a player can fully distinguish between states then a contract exists that essentially forces her to disclose evidence isolating one state from the others. This is accomplished by severely punishing the player for not disclosing one of these critical documents.

Implications of a Legal Rule

We now consider an implication of our theory for the design of legal institutions, in particular with regard to rules of evidence and discovery. This is not meant to be an exhaustive study of legal institutions, but suggests that further research would be valuable. Recall that we have assumed players have common knowledge of documents. Suppose that rules of discovery allow a player to force another player to provide all documents of a particular type. We term this a setting of *enforced discovery requests*. Since our model assumes disjoint document spaces, we capture the setting of enforced discovery requests by assuming that, essentially, each player has *copies* of the documents in the possession of other players. Mathematically, this means that for $i, j \in N$ and each document $d \in D_i$, there exists a document $d' \in D_j$ such that $d \in D_i(a)$ if and only if $d' \in D_j(a)$. In words, if one player can positively distinguish a from b , then all players can do so.

In such an environment, F^D is not constrained by the failure of positive evidence.

Theorem 5 *In the setting of enforced discovery requests, the difference between positive and negative evidence is not critical, and $G(P^D) = F^D$.*

The intuition behind this result is that no constraint is created by an individual player wanting to suppress a document that lowers her transfer, since then another player will benefit from the document being disclosed and thus disclose it.

We acknowledge that in practice enforced discovery requests may be difficult to implement. This is due to the possibility of players destroying or suppressing evidence, and incentives in the discovery process. Brazil (1978) and Shapiro (1979) suggest that in practice the discovery process does not result in the intended open exchange of information since parties seek to suppress evidence. Cooter and Rubinfeld (1994), in a setting of costly evidence production, characterize an efficient level of discovery requests, while Cooter and Rubinfeld (1995) show that current federal law does not adequately prevent the problem of excessive discovery requests. Although Theorem 5 does not address these issues, it does lend support to those who suggest that more attention should be given to improving the workings of the discovery process. However, we maintain that enforced discovery requests are more plausible in today's society than during earlier periods of legal history. For example, in *State v. Simons* (1845), noted in the Introduction, the state's difficulty in keeping accurate, accessible records made the issue of whether Simons presented the license relevant.

4 Productive Interaction

Thus far, our analysis has been geared toward understanding behavior in periods 3 and 4 of the contractual relationship. In this section, we turn our attention to interaction in periods 1 and 2 and we take a broader perspective on the components of contract. Let us presume a simple specification of productive interaction. Suppose that in

period 2 the players simultaneously and independently select actions. Player i 's action space is denoted A_i ; we define $A \equiv A_1 \times A_2 \times \dots \times A_n$. In other words, in period 2 the agents play a one shot “production” game with action profiles A and payoffs given by u plus the continuation value from period 3. The resulting action profile $a = (a_1, a_2, \dots, a_n)$ represents the state of the relationship at the end of period 2.

In period 1, before playing the production game, the players jointly agree to a contract. The contract has two components: an externally-enforced part m and a self-enforced part which describes the disclosure rule β and behavior in the production phase. As modeled in the preceding sections, we suppose that β is an ISC disclosure rule with respect to m . Thus, interaction in periods 3 and 4 can be summarized by the implied transfer function g . In period 1, the players indirectly select a transfer function g through their selection of m and their coordination on β . This justifies viewing interaction in period 2 as a game with action profiles A and payoffs given by $u+g$, where $g \in F^D$ is selected by the players in period 1. We analyze this contracting game by determining whether there is a transfer function in F^D that supports a given a^* as a Nash equilibrium of period 2 interaction.¹⁷

Given $a^* \in A$, let $F(a^*)$ be the set of (balanced) transfer functions that support a^* as a Nash equilibrium of the production phase. Mathematically, $g \in F(a^*)$ if and only if g is balanced and

$$u_i(a^*) + g_i(a^*) \geq u_i(a_i, a_{-i}^*) + g_i(a_i, a_{-i}^*),$$

for each player i and each $a_i \in A_i$. Clearly, there is a contract that yields action profile a^* if and only if $F(a^*) \cap F^D \neq \emptyset$. Thus, in period 1 players implement a transfer function associated with the solution of

$$\max_{a^* \in A^*} \sum_{i \in N} u_i(a^*), \text{ where } A^* \equiv \{a^* \in A \mid F(a^*) \cap F^D \neq \emptyset\}.$$

This maximization problem yields the highest joint value that can be attained in the contractual relationship, given the incentive constraints in the production and evidence disclosure phases.

As explored in the preceding section, sometimes it is helpful to analyze the set of transfer functions that are measurable with respect to a particular partition of the state space. Given a partition P , we can provide a necessary and sufficient condition for a^* to be supported by a transfer function that is measurable with respect to P . The condition relates to the function

$$w_i(a, a^*) \equiv \begin{cases} -\infty & \text{if } Q^i(a^*) \cap P(a) = \emptyset \\ \max\{u_i(a'_i, a_{-i}^*) \mid (a'_i, a_{-i}^*) \in P(a)\} - u_i(a^*) & \text{if } Q^i(a^*) \cap P(a) \neq \emptyset \end{cases},$$

¹⁷Here we focus on supporting pure action profiles. Appendix C presents analysis of mixed action profiles and moves of nature. One can easily extend the analysis to more complicated period 2 interaction, using the appropriate equilibrium concept. For example, production in period 2 may be modeled as a dynamic game. We simply require that all productive activity takes place in period 2, prior to evidence disclosure.

where $Q^i(a^*) \equiv \cup_{a_i \in A_i} P(a_i, a_{-i}^*)$. The function w_i represents the maximal increase in productive payoff that can be achieved by player i by unilaterally deviating from a^* in a way that yields an action profile in the partition element $P(a)$.

Theorem 6 *Consider any partition P of the action space. Then $F(a^*) \cap G(P) \neq \emptyset$ if and only if*

$$\sum_{i \in N} w_i(a, a^*) \leq 0, \text{ for all } a \in A. \quad (3)$$

Theorem 6 is closely related to Theorem 1 of Legros and Matthews (1993).¹⁸ The intuition for necessity is simple. As transfers must be balanced due to renegotiation, summing each player's Nash equilibrium condition gives the result. In the sufficiency direction, we consider separately each element of the partition of A . If players generally do better by deviating to the element $P(a)$ than by being in $P(a^*)$ (i.e., $\sum_{i \in N} w_i(a, a^*) > 0$), then we cannot expect to support play of a^* as a Nash equilibrium with any transfer function g as we require $\sum_{i \in N} g_i = 0$. That is, there does not exist a transfer function g that sufficiently punishes all players for unilateral deviation. However, when $\sum_{i \in N} w_i(a, a^*) \leq 0$ there does exist a transfer function that is measurable with respect to P and prevents deviation to the element of the partition $P(a)$.

5 Conclusion

We have presented a model of contract enforcement where verifiability depends upon players' disclosure of evidence. Our model treats evidence as documents, the availability of which is state dependant. This differs from the typical treatment in law and in the law and economics literature.¹⁹ However, a possible motivation for our view of evidence is that, in reality, litigants know that witnesses are subject to cross-examination, and this leads to litigants calling only those witnesses whose testimony will stand up to cross-examination.²⁰ Our model allowed us to characterize how incentives in evidence disclosure determine those productive outcomes that can be supported by a court-enforced contract. With the Full Disclosure Principle, we showed that attention can be focused on those disclosure rules that involve disclosure of all

¹⁸By specifying that an individual player appropriates the verifiable partnership output and confining attention to finite actions Legros and Matthew's result is seen as a special case of ours. See Appendix C for analysis of mixed productive actions in our model.

¹⁹Classic legal treatments of evidence are Bentham (1827) and Wigmore (1940). Cooter and Rubinfeld (1989) survey the law and economics literature which views evidence as being costly to produce and increasing a party's chances of success at trial. Bernardo, Talley, and Welch (2000) analyze legal presumptions in a setting where the cost of evidence production depends upon the actual state. Sanchirico (1999) models evidence cost as depending on the actual state. Sobel (1989) studies pretrial bargaining in a model where the court can discern the true state at trial.

²⁰This notion is discussed in Posner (1999).

available documents. We have also characterized evidence environments on the basis of how players can distinguish one state from another.

We have assumed that players disclose evidence simultaneously and have required that equilibrium disclosure rules be impervious to side contracting. A possibility for future research is to consider settings where players disclose evidence sequentially. Further, future research could relax our assumption that players have common knowledge of documents. The analysis of various legal institutions is another possible avenue for future research. We have provided a framework in which many institutional issues can be addressed. Examples of such institutional issues include default damage rules, burden of proof considerations, and evidence admissibility rules. Some of these issues may be best addressed by considering special cases of our model. A generalization of our model could be used to address the issue of whether the court or parties should gather and present evidence, in terms of efficiency.

A Proofs Not in the Text

Proof of Theorem 2

(Necessity) Suppose β is an *ISC* disclosure rule, but

$$\sum_{i \in N} z_i(E; \beta, g) > 0, \text{ for some } E \in \mathcal{D}.$$

ISC disclosure requires $m_i(\beta(a)) \leq m_i(E)$, for every $i \notin R(a; \beta(a), E)$, for any $E \in \mathcal{D}$, for all $a \in A$. But if $\sum_{i \in N} z_i(E; \beta, g) > 0$, for some $E \in \mathcal{D}$, then (by $\sum_{i \in N} m_i = 0$) for some a , β is not an ISC disclosure.

(Sufficiency) Take any $E \in \mathcal{D}$. $\sum_{i \in N} z_i(E; \beta, g) \leq 0$, for any $E \in \mathcal{D}$ implies the existence of an enforced contract $m : \mathcal{D} \rightarrow \mathbf{R}_0^n$ such that $m_i(\beta(a)) \leq m_i(E)$, for every $i \notin R(a; \beta(a), E)$, for any $E \in \mathcal{D}$, for all $a \in A$. Since it must be that $\sum_{i \in N} m_i(E) = 0$ for all $E \in \mathcal{D}$ implies there exists an enforced contract m such that $M_J(\beta(a)) \geq M_J(E)$, for each $J \subset N$, for any $E \in \mathcal{D}$, for every $a \in A$, where E is any J deviation from $\beta(a)$. *Q.E.D.*

Proof of Lemma 3

By the Full Disclosure Principle we need to only consider disclosure rules such that $\bar{\beta}(a) = D(a)$ for all $a \in A$. That g is supported implies there exists an m such that $M_K(\bar{\beta}_K(b) \cup \bar{\beta}_{N \setminus K}(a)) \leq M_K(\bar{\beta}(a))$, and that $M_{N \setminus K}(\bar{\beta}_K(b) \cup \bar{\beta}_{N \setminus K}(a)) \leq M_{N \setminus K}(\bar{\beta}(b))$, which can be written alternatively as $M_K(\beta_K(b) \cup \bar{\beta}_{N \setminus K}(a)) \geq M_K(\bar{\beta}(b))$. Combining these two expressions, we get $M_K(\bar{\beta}(a)) \geq M_K(\bar{\beta}(b))$, or equivalently $\sum_{i \in K} g_i(a) \geq \sum_{i \in K} g_i(b)$. *Q.E.D.*

Proof of Theorem 4

By the Full Disclosure Principle we need to only consider disclosure rules such that $\bar{\beta}(a) = D(a)$ for all $a \in A$. Consider a contract m as follows. If $E_i \in \{E_i \mid E_i = D_i(a) \text{ for some } a \in A\}$, then $m_i(E_i) = g(a)$ for that a . If $E_i \notin \{E_i \mid E_i = D_i(a) \text{ for some } a \in A\}$, then $m_i(E_i) = -x$, where $x < \min_{a \in A} g_i(a)$. As $g(a) = g(b)$ for those states a, b between which i cannot fully distinguish, and $D_i(a') \not\subset D_i(b')$ and $D_i(b') \not\subset D_i(a')$ for any other states a' and b' , i will always fully disclose. *Q.E.D.*

Proof of Theorem 5

By the Full Disclosure Principle, we need only consider disclosure rules which specify $\bar{\beta}(a) = D(a)$ for all $a \in A$. Take any g and $E \in \mathcal{D}$. We consider first the case where E is such that $K(a) \subset N$, $D_{K(a)} = E_{K(a)}$ and $E_{N \setminus K(a)} \subset D_{N \setminus K(a)}(a)$ (strictly) for some $a \in A$. We proceed by finding all a 's such that for some $K(a) \subset N$, $D_{K(a)} = E_{K(a)}$ and $E_{N \setminus K(a)} \subset D_{N \setminus K(a)}(a)$ (strictly). Let \tilde{A} denote the set of such a 's. It can be shown that $\cap_{a \in \tilde{A}} N \setminus K(a) \neq \emptyset$. We show this and then continue the proof. Suppose that for some E , $\cap_{a \in \tilde{A}} N \setminus K(a) = \emptyset$. This implies there exists $i, j \in N$ and $a, b \in \tilde{A}$ such that $E = (\bar{\beta}_i(a) \cup \bar{\beta}_j(b) \cup E_{-i,j})$. So it must be that at a , j is required to deviate, but at b , i is required to deviate. In other words, $\bar{\beta}_i(b) \subset \bar{\beta}_i(a)$ and $\bar{\beta}_j(a) \subset \bar{\beta}_j(b)$. But this contradicts the assumption of full discovery requests. Thus, $\cap_{a \in \tilde{A}} N \setminus K(a) \neq \emptyset$, and the court can always tell that at least one player has deviated at any such E . This implies that for any such E , it is always the case that $\sum_{i \in N} z_i(E; \beta, g) \leq 0$.

We must also rule out all players deviating from $\bar{\beta}(a)$. If all players deviate to some $E' \neq \bar{\beta}(b)$ for any b , we can easily construct an m such that some player i has incentive to disclose $\bar{\beta}_i(a)$. However, suppose that all players deviate from $\bar{\beta}(a)$ to $\bar{\beta}(b)$. If this is possible, it must be that $\bar{\beta}(b) \subset \bar{\beta}(a)$. However, unless $g(a) = g(b)$, $\sum_{i \in N} g_i = 0$ implies there exists some player i such that $g_i(a) > g_i(b)$. Thus player i will gain by disclosing $D_i(a)$. *Q.E.D.*

Proof of Theorem 6

(Necessity) If player i deviates to $P(a)$, he gains $w_i(a, a^*) + g_i(a) - g_i(a^*)$. That a^* is supported implies that $w_i(a, a^*) + g_i(a) - g_i(a^*) \leq 0$, for all $i \in N$, for all $a \in A$. As transfers are balanced, this implies

$$\sum_{i \in N} w_i(a, a^*) \leq 0, \text{ for all } a \in A.$$

(Sufficiency) Suppose (3) holds for some a^* . We need to construct a transfer function such that a^* is a Nash equilibrium. Take $a \notin Q^i(a^*)$ for some i . Specify

$$g_j(a) = -w_j(a, a^*), \text{ for all } a \in P(a), j \text{ such that } a \in Q^j(a^*).$$

Let

$$g_k(a) = \frac{1}{(n-K)} \left[\sum w_j(a, a^*) \mid j \text{ such that } a \in Q^j(a^*) \right],$$

for each k such that $a \notin Q^k(a^*)$, where K is the number with $a \in Q^j(a^*)$. Next take $a \in \cap_{i \in N} Q^i(a^*)$. For players $j = 2, \dots, n$, define $g_j(a) = -w_j(a, a^*)$, for all $a \in P(a)$. For player 1, let $g_1(a) = \sum_{j \in N \setminus 1} w_j(a, a^*)$, for all $a \in P(a)$. Note that by construction, players 2, ..., n have no incentive to deviate from a^* . If player 1 deviates to a , he gains

$$w_1(a, a^*) + g_1(a) = \sum_{i \in N} w_i(a, a^*).$$

However, by assumption

$$\sum_{i \in N} w_i(a, a^*) \leq 0, \text{ for all } a \in A. \text{ Q.E.D.}$$

B Analysis of Random Disclosure Rules

In this section we show that our main results are not changed when mixed disclosure rules are allowed. We denote a random disclosure rule by $\sigma : A \rightarrow \Delta^u \mathcal{D}$, where Δ^u denotes that the randomization is uncorrelated across players. We use $\mathcal{E} [m_i(E) \mid \sigma(a)]$ to denote the expected court-enforced transfer for player i in state a when σ is played. Let θ denote a deviation from $\sigma(a)$. In this setting, the ISC condition is as follows.

$$\mathcal{E} [M_J(E) \mid \sigma(a)] \geq \mathcal{E} [M_J(E) \mid \theta],$$

where θ is a J deviation from $\sigma(a)$ and θ_J is degenerate. We are justified in considering only degenerate deviations because if J cannot gain from a degenerate deviation, then J cannot gain by using a mixture.

We define $\hat{R}(a; \sigma(a), \theta)$ to be the minimum set of players that would be needed to deviate from $\sigma(a)$ in order to achieve θ in state a . analysis proceeds as in the text and we have:

Lemma 2' σ is ISC if and only if $\mathcal{E} [m_i(E) \mid \sigma(a)] \leq \mathcal{E} [m_i(E) \mid \theta]$, for all $i \notin \hat{R}(a; \sigma(a), \theta)$, θ , and a .

The proof is straightforward. Suppose the lemma does not hold. Then it must be that $\mathcal{E} [M_{-i}(E) \mid \sigma(a)] < \mathcal{E} [M_{-i}(E) \mid \theta]$. However, note that $\hat{R}(a; \sigma(a), \theta) \subset -i$. So $-i$ players can reach θ . Further, there must be a θ' such that θ'_{-i} is degenerate and $\theta'_i = \theta_i$, with $\mathcal{E} [M_{-i}(E) \mid \sigma(a)] < \mathcal{E} [M_{-i}(E) \mid \theta']$. But this violates ISC. Q.E.D.

To generalize the Full Disclosure Principle to mixed disclosure rules we first define the following notation. For any disclosure rule σ , each $i \in N$, and θ , let

$$\hat{B}(i, \sigma, \theta) \equiv \{a \in A \mid i \notin \hat{R}(a; \sigma(a), \theta)\}.$$

We define

$$\hat{z}_i(\theta; \sigma, g) \equiv \begin{cases} \max_{a \in \hat{B}(i, \sigma, \theta)} g_i(a) & \text{if } \hat{B}(i, \sigma, \theta) \neq \emptyset \\ -\infty & \text{if } \hat{B}(i, \sigma, \theta) = \emptyset \end{cases}.$$

We now show that if a transfer function g is supported by a mixed ISC disclosure rule, then g is supported by an ISC/full disclosure rule. Take any ISC rule σ , with respect to m , that supports g . Let $\bar{\beta}(a) \equiv D(a)$ for all $a \in A$. Consider any E' . As in the proof for the pure disclosure case, let a^i be a state that maximizes $g_i(a)$ over all $a \in B(i, \bar{\beta}, E')$. Define θ such that $\theta_i \equiv \sigma_i(a^i)$ for all $i \in N$. We have $a^i \in \hat{B}(i, \sigma, \theta)$. This implies that $\hat{z}_i(\theta; \sigma, g) \geq z_i(E'; \bar{\beta}, g)$ for all $i \in N$.

C Analysis of Random Productive Actions

In this section we study mixed strategies in productive interaction. Let A_0 denote the action space of nature, let $A \equiv A_0 \times A_1 \times \dots \times A_n$, and consider mixtures $\Delta^u A$, where Δ^u denotes that randomization is uncorrelated across players. We take nature's mixed action α_0 as given. We say that $\alpha^* \in \Delta^u A$ is supported when there exists a transfer function g such that α^* is a Nash equilibrium of the induced game as in Section 4. Nash equilibrium requires

$$\sum_{a_{-i} \in A_{-i}} [u_i(a'_i, a_{-i}) + g_i(a'_i, a_{-i})] [\Pi_{k \neq i} \alpha_k^*(a_k)] \leq \sum_{a \in A} [u_i(a) + g_i(a)] [\Pi_{j \in N} \alpha_j^*(a_j)].$$

There is no simple extension of Theorem 6 for this setting because the analysis of equilibrium involves a system of inequalities corresponding to the different elements of the partition P that are encountered with positive probability when any given action profile is played. However, we can provide a result indicating conditions under which the analogy of $w_i = -\infty$ holds, in which case some productive actions can be disregarded when checking whether a given profile can be supported. Given an action profile α and an element p of partition P , we say that α *leads to p with positive probability* if there is a pure action profile a such that $a \in p$ and a is in the support of α .

Lemma 4 *Consider whether α^* can be supported by a transfer function that is measurable with respect to P . Suppose that there is an element p of P and a player j such that, for each $a'_j \in A_j$, (a'_j, α_{-j}^*) does not reach p with positive probability. Then if player i 's action a_i is such that (a_i, α_{-i}^*) reaches p with positive probability then a_i can be disregarded when checking whether α^* can be supported.*

This lemma is easily proved. Only by deviation of a player other than player j can p be reached with positive probability. Thus, for each $a \in p$, $g_i(a)$ we specify an arbitrarily large transfer from each of the players in $-j$ to player j (the same thing for all states in p), which ensures that no player has the incentive to play an action that leads to p with positive probability. *Q.E.D.*

Lemma 4 generalizes the main component of Legros and Matthews' (1993) Theorem 2.

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