Title
Essays on Bimatrix Games in the Laboratory

Author
Zhao, Shuchen

Publication Date
2021

UNIVERSITY OF CALIFORNIA
SANTA CRUZ

ESSAYS ON BIMATRIX GAMES IN THE LABORATORY

A dissertation submitted in partial satisfaction of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

Shuchen Zhao

June 2021

The Dissertation of Shuchen Zhao
is approved:

Professor Daniel Friedman, Chair

Professor Natalia Lazzati

Professor Kristian López Vargas

Quentin Williams
Interim Vice Provost and Dean of Graduate Studies

Table of Contents

List of Figures **v**

List of Tables **vii**

Abstract **ix**

Dedication **xii**

Acknowledgments **xiii**

1 When Are Mixed Equilibria Relevant? **1**
- 1.1 Introduction . . . 1
- 1.2 Previous literature . . . 4
- 1.3 Theoretical Considerations . . . 7
- 1.4 Laboratory Implementation . . . 14
  - 1.4.1 Treatment variables . . . 14
  - 1.4.2 Design . . . 16
  - 1.4.3 Testable Hypotheses . . . 17
- 1.5 Results . . . 20
  - 1.5.1 Point Predictions . . . 22
  - 1.5.2 Qualitative Dynamics . . . 25
  - 1.5.3 Fitted Dynamic Model . . . 29
- 1.6 Discussion . . . 31

2 Coordination Games in Continuous Time **35**
- 2.1 Introduction . . . 35
- 2.2 Literature Review . . . 38
  - 2.2.1 Experiments in Continuous Time . . . 38
  - 2.2.2 Coordination Failure and Repeated Coordination . . . 40
- 2.3 Theoretical Predictions . . . 43
  - 2.3.1 Nash Equilibria for One-Shot Games . . . 43
  - 2.3.2 Comparing Continuous and Discrete Time Repeated Interactions 45

2.4 Experimental Design . . . 48
2.5 Results in Stag Hunt Games . . . 50
2.5.1 Treatment Effects . . . 53
2.5.2 Classification at Pair Level . . . 54
2.6 Results in Battle of the Sexes Games . . . 58
2.6.1 Treatment Effects . . . 60
2.6.2 Classification at Pair Level . . . 62
2.7 Conclusions . . . 65

3 Taking Turns in Continuous Time **69**
3.1 Introduction . . . 69
3.2 Related Literature . . . 73
3.3 Experimental Design . . . 77
3.3.1 Time Environments and User Interface . . . 78
3.3.2 Payoff Matrices . . . 80
3.3.3 Hypotheses . . . 82
3.3.4 Sessions . . . 85
3.4 Results . . . 86
3.4.1 Treatment Effects . . . 86
3.4.2 Pair Classification and Learning . . . 88
3.4.3 Reaction to Changes . . . 94
3.4.4 Transition Probability Matrix and Alternating Pattern . . . 97
3.5 Conclusions . . . 102

A Supplement to Chapter One **105**
A.1 Computation of NE, Maximin and QRE for AMPa games . . . 105
A.2 Numerical proof of the limit cycle . . . 106
A.3 Results in details . . . 108

B Supplement to Chapter Two **120**
B.1 Survey Data . . . 120
B.2 Further Investigation of Dynamics in BOS . . . 121
B.2.1 Transition Probabilities Between Corner Profiles . . . 121
B.2.2 Length of Equilibrium Play and Corner Mismatches in BOS . . . 125
B.3 Explaining the Treatment Effects . . . 126
B.3.1 Continuous vs Discrete Time . . . 126
B.3.2 Mixed vs Pure Action Sets . . . 129

C Supplement to Chapter Three **134**
C.1 Payoff Inequality . . . 134
C.2 Transition Probability Matrices with Diagonal Cells . . . 135
C.3 Logistic Regressions for Statistical Significance . . . 136
C.4 Transition Probabilities at the Pair Level . . . 137
C.5 Figures with All Combinations of Treatments . . . 139

List of Figures

1.1 Heatmap for AMPa row player. 8
1.2 Classifying directional changes in AMPa Matching Pennies. 11
1.3 Main features of oTRw screen for MCrp AMPa game. 16
1.4 Sample time paths of action profiles. 21
1.5 Classification over time in random pairwise matching sessions. 26
1.6 Classification over time in mean matching sessions. 27
1.7 Simulation of (1.5) using fitted parameters from continuous time mixed strategy AMPa games. 31

2.1 Subjects interface in practice periods for games in the pure action sets and discrete time treatment. 50
2.2 Subjects interface in practice periods for games in the mixed action sets and continuous time treatment. 51
2.3 Classification over time in stag hunt games. 55
2.4 Classification at pair level in SH. 58
2.5 Classification over time in battle of the sexes games. 62
2.6 Classification at pair level in BOS. 65

3.1 Sample user interface in the games in continuous time. 79
3.2 Three possible transition dynamics. 85
3.3 Examples of alternating dynamics. 89
3.4 Scatter plot showing the classification of all 504 supergames colored by the time environments under the 80%/2 periods criteria. 91
3.5 Coordination rate within supergames. 93
3.6 CDF of the time at which the pairs first reach either pure Nash equilibrium. 95
3.7 CDF of the duration during which the subjects stay at mismatches. . . 96
3.8 CDF of the duration during which the subjects stay at Nash equilibria. 100
3.9 Average duration at Nash over time in the supergames. 101

A.1 The figure shows the distance between observations and Nash equilibrium over time with 50 simulations and 3000 iterations in each simulation. . . 107
A.2 Time average profiles. 116

A.3 Data summary of population time averages in 3 games colored by treatments. 117
A.4 The fraction of time pairs play each cycle classifications sorted by the fraction of time each pair plays clockwise cycles. 118
A.5 Simulation result in AMPa games mixed strategy treatments. 119

C.1 Difference in the cumulative payoff between two players within supergames. 135
C.2 Scatter plot showing the probabilities of transitioning between Nash and Aggressive and between Nash and Accommodating colored by the time environments. 138
C.3 Coordination rate within supergames. 139
C.4 The empirical cumulative distribution function of the timing at which the pairs first reach a pure Nash equilibrium. 140
C.5 The empirical cumulative distribution function of the duration during which the subjects stay at mismatches. 140
C.6 The empirical cumulative distribution function of the duration during which the subjects stay at Nash equilibria. 141

List of Tables

1.1 Payoff bimatrices and equilibrium mixtures. 7
1.2 Experiment Design. 18
1.3 Mean distance to predictions of time average profiles. 23
1.4 Coefficients ($\pm$ standard errors) for OLS regressions with clustering at session level. 24
1.5 Directional Learning Model (1.5) coefficient estimates ($\pm$ standard error) for Row and Col(umn) player actions in Continuous and Discrete time. . 30

2.1 Payoff bimatrices of stag hunt games and battle of the sexes games. . . 43
2.2 Design table of block 1 for stag hunt sessions and battle of the sexes sessions. 51
2.3 Summary table for classified strategy profiles in stag hunt games. 52
2.4 One vs all logistic regression of classification over time in SH clustered at pair level. 56
2.5 One vs all logistic regression of classification at pair level in SH. 59
2.6 Summary table for classified strategy profiles in battle of the sexes games. 60
2.7 One vs all logistic regression of classification over time in BOS clustered at pair level. 63
2.8 One vs all logistic regression of classification at pair level in BOS. 66

3.1 Payoff matrices in the experiment. 81
3.2 Session Information. 86
3.3 Coordination rates by treatments and order effect. 88
3.4 Classification at the pair level. 92
3.5 Coordination rates between Games 1-6 (first half of the session) and Games 7-12 (second half of the session). 94
3.6 Transition probability matrices. t' represents t+500 milliseconds in continuous time and t+1 period in discrete time. 98
3.7 The probability of each type of transitional dynamics being played in the two time environments. 99

A.1 Multinomial logistic regressions of cycle classifications on treatment dummy variables. 108
A.2 Mean of the mean observations of pairs with mean Distance to predictions. 109
A.3 Median of the mean observations of pairs with median Distance to predictions. 110
A.4 Regressions of each classified cycle type on treatment dummies collapsed at pair level. 111
A.5 BR learning regression for eq (1.3). 112
A.6 Pure directional learning regression for eq (1.4). 113
A.7 Directional learning for eq (1.5) with discrete regret classified into discrete values. 114
A.8 Directional learning for eq (1.2) with 5 lagged regret terms. 115
A.9 Directional learning for eq (1.2) with 1 lagged regret term. 115

B.1 Results for the post experimental survey. 121
B.2 Number of times pairs move into and move out of any pure Nash equilibria. 123
B.3 Transition probability matrix of BOS pure continuous treatments. . . . 124
B.4 Transition probability matrix of BOS pure discrete treatments. 124
B.5 Transition probability matrix of BOS mixed continuous treatments. . . 124
B.6 Transition probability matrix of BOS mixed discrete treatments. 124
B.7 Average length of equilibrium plays and two corner mismatches in all BOS treatments. 125
B.8 Continuous vs Discrete in starting strategy, convergence speed and efficiency after convergence in SH. 128
B.9 Continuous vs Discrete in starting strategy, convergence speed, alternations and efficiency after convergence in BOS. 130
B.10 Distributional frequencies of mismatches in each treatment of SH. 132
B.11 Distributional frequencies of mismatches in each treatment of BOS. . . . 133

C.1 Transition probability matrices with diagonal terms. 136
C.2 Logistic regression used to assess significance under a robust environment. 137

Abstract

Essays on Bimatrix Games in the Laboratory

by

Shuchen Zhao

The dissertation presents three experimental studies with an emphasis on dynamics of some repeated bimatrix games. The first chapter studies the equilibrium convergence and cyclical dynamics in asymmetric matching pennies games. The second and third chapters focus on coordination and dynamic patterns in two continuous time coordination games, especially the turn taking dynamics in continuous time battle of the sexes games.

The first chapter focuses on a matching pennies game with a unique mixed Nash equilibrium under various environments. Mixed Nash equilibrium (NE) is a cornerstone of game theory, but its empirical relevance has always been questionable. The chapter studies in the laboratory two games whose unique NE is in completely mixed strategies; other treatments include the matching protocol (pairwise random vs population mean matching), whether time is discrete or continuous, and whether players can specify mixtures explicitly or only pure strategy realizations. NE mixes predict observed behavior better than maximin in all treatments, but uniform mixes are better predictors than any equilibrium mixture in many treatments. By contrast, in a control game with a unique NE in pure strategies, the best point prediction is NE. Mixed equilibrium predictions are more useful in population mean matching than in standard

pairwise matching. Regret-based sign preserving dynamics capture regularities across all treatments.

The second chapter switches to study the impact of continuous time interaction on two iconic coordination games: stag hunt and battle of the sexes in a laboratory environment and compare results to possible theoretical explanations. Experimental results show that subjects consistently coordinate better in continuous time than in discrete time under various treatment environments. In continuous time, they are also more likely to converge to payoff-dominant equilibrium in stag hunt games and alternating dynamics in battle of the sexes games. Furthermore, the coordination rate is affected by complexity of action sets and weakly influenced by payoff matrices. The chapter also explores some stylized facts that result in the treatment effects.

The third chapter follows the results of the second chapter and goes deeper into the alternating dynamics in continuous time battle of the sexes with laboratory experiments. In discrete time battle of the sexes games, players often interact with alternating strategies and switch between two pure Nash equilibria, which is difficult to coordinate in continuous time because the players must determine both the order of alternations and how long to remain at each pure equilibrium. Overall, laboratory subjects behave differently in the two time environments. Although the continuous time interactions accelerate the initial convergence to one of the pure strategy equilibria and divergence from mismatches, they undermine the subjects' subsequent ability to coordinate moves from one equilibrium to the other. Compared to the alternating pattern in discrete time, the pattern in continuous time is unstable and diverse, and the transitions are mainly motivated by the disadvantaged subjects. The difficulty in alternation also slows the learning process in continuous time.

To myself,

"Two things fill me with constantly increasing admiration and awe,the longer and more earnestly I reflect on them: the starry heavens without and the moral law within." It is a great honor to devote my life to pursuing the mystery of human's soul.

Acknowledgments

My journey at UCSC is enjoyable and full of unforgettable memories. In the last five years, countless memories have been created, together with many people. Here I wish to express my gratitude towards those who have supported me in the pursuit of my Ph.D.

First, I would like to thank my advisor, Professor Daniel Friedman, for his invaluable advice, continual encouragement, and excellent guidance in my journey of exploring game theory and experimental economics. Dan is not only my mentor in research but also my role model in academia and my friend in real life. I learned a lot from him and this journey would have been much more difficult without Dan's guidance. It is my great honor to be one of Dan's students and to start my academic career with Dan as my advisor.

I would also like to extend my gratitude to Professor Natalia Lazzati and Professor Kristian López Vargas for their honest, constructive, and timely feedback on my research. Professor Lazzati and Professor López Vargas are generous in sharing their expertise in economics throughout my Ph.D. studies. Whenever I have a question, I know their doors are always open and I can always rely on their valuable inputs. I am also grateful to Professor Donald Wittman and Professor John Musacchio for their support and suggestions in my oral exam.

Moreover, I have received so much constructive feedback from researchers outside the department and the dissertation cannot be completed without their help. I would like to express my gratitude to Professor Simin He, Professor Dongming Zhu, Professor Ailin Leng, Professor Todd Kaplan, and Professor Vincent Crawford for their feedback on the second and the third chapter of the dissertation, and to Professor Ciril Bosch, Professor Tim Cason, Professor Ed Hopkins, Professor Steffen Huck, Professor Heinrich Nax, Professor Julian Romero, Professor Daniel Stephenson, and Professor John Wooders for their feedback on the first chapter of the dissertation. There are also many other participants in seminars and referees from journals who contribute their thoughts to the dissertation and I would love to express my gratitude to them, too.

Furthermore, I would like to thank my friends at UCSC, including my cohorts, the members of the experimental workshop, and my friends in the bay area. The Ph.D. journey is difficult and happiness is not something I can often expect in my daily life. But life is indeed more colorful when we were discussing problems, playing games, hanging out, and going through so many challenges together. It is my fortune to have so many life-long connections and thank you all for making the Ph.D. journey enjoyable.

Last but not least, I would like to thank my family for their support throughout the last five years. I would love to thank my parents for their patience and encouragement. I understand that pursuing a Ph.D. is never what they want me to do, but I really appreciate that they respect my decision and support me with all they can. I am especially grateful to my wife, who stays by my side for the last eight years. I have her support and understanding in the darkest time of my Ph.D. journey, and I always know for sure that we can rely on each other as a family. It is my great fortune to marry her and I am definitely not able to complete my Ph.D. degree without her.

Chapter 1

When Are Mixed Equilibria Relevant? [1]

"There he goes," said Holmes, as we watched the [special train] carriage swing and rock over the point. "There are limits, you see, to our friend's intelligence. It would have been a *coup-de-maître* had he deduced what I would deduce and acted accordingly."
— Arthur Conan Doyle (1893)

1.1 Introduction

Generalized matching pennies games capture the essence of strategic situations (e.g., in hunting, warfare, and sports) where success comes from outguessing opponents. For example, in the epigraph above, Sherlock Holmes gloats that his own level-3 strategy of exiting at Canterbury bested his archenemy Moriarty's level-2 strategy of engaging a special train to Dover,[2] but Holmes recognizes that higher levels are possible. Since level-$(k+1)$ beats level-k for every positive k in generalized matching pennies, these games suffer from infinite regress, a Gordian knot that blocked progress in game theory

[1] The first chapter is a joint work with Daniel Friedman.

[2] Aficionados of A.C. Doyle's story and of level k reasoning might regard random attacks by Moriarty as level 0, and regard Holmes taking the train to Dover as level 1.

for centuries. Neumann (1928) finally cut that knot with the idea of mixed strategy equilibrium.

Although mixed strategy equilibrium remains a cornerstone of game theory, it continues to provoke theoretical and empirical controversy, as noted in the next section. We will see that different theoretical models differ sharply on the predictive power of mixed Nash equilibrium. Applied economists typically focus on a pure strategy Nash equilibrium if any exist, but turn to mixed NE in games (such as generalized matching pennies) with no pure NE. We will see that field and laboratory evidence supporting those mixed equilibria is, however, itself mixed at best.

The present paper is motivated by the following research questions. First, under what conditions (if any) does mixed Nash equilibrium do a good job of predicting behavior in generalized matching pennies games? Second, is there a better point prediction — perhaps maximin, as von Neumann proposed, or quantal response equilibrium? Third, can qualitative or quantitative dynamic models explain behavior when it departs from point predictions?

These questions are important to applied social scientists and biologists as well as to theorists. Cyber-attacks and -defenses are naturally modeled as generalized matching pennies games, as are military exercises and actual conflicts. Standard biological models of predator-prey interactions can be regarded as dynamic versions of matching pennies games. Answers to our research questions should help guide work in such applications.

We use a laboratory experiment to address those questions. The experiment deploys a variety of treatment combinations intended to reveal circumstances under which each of the competing equilibrium concepts has predictive power. The treatments include two different simple matching pennies games as well as a control game that has only a pure Nash equilibrium; two matching protocols that contrast population games (as in biology and some economic applications such as national cybersecurity policy) to standard pairwise matching; do-it-yourself randomization versus automated randomization with chosen mixing weights; and simultaneous moves in discrete time versus asynchronous moves in continuous time.

The results are instructive. We find some circumstances under which Nash equilibrium predicts human subject behavior well, but find no circumstances under which maximin does better. There is a surprisingly wide range of circumstances under which level-0 uniform random mixing predicts better than Nash equilibrium. For the generalized matching pennies games we consider, logit quantal response equilibrium generally does not predict better than its edge cases, Nash equilibrium, and uniform mixing. The upshot is that mixed equilibria are empirically relevant over a narrower range of circumstances than one might have supposed.

After reviewing some previous literature in Section 2 and some established theory in Section 3, we present our experimental design in Section 4. We implement our 2x2 bimatrix games using a new graphical screen display for players. Section 4 concludes with lists of testable hypotheses about competing point predictions, about treatment effects on mean choices and on dispersion, and about adaptive learning dynamics.

Section 5 collects results. Some point predictions do better than others in some circumstances, but overall none of them predicts especially well. The data are generally consistent with a qualitative directional learning model, and a quantitative regret-based version of directional learning captures important regularities. A concluding discussion in Section 6 summarizes our findings and suggests implications for game theory and for applied research. Appendices include supplementary data analysis, and instructions to subjects.

1.2 Previous literature

The early game theory emphasized two-player zero-sum bimatrix games, where Nash equilibrium (NE) and maximin (MM) mixed strategies coincide, but recognized that NE and MM mixes differ in asymmetric matching pennies games (e.g., Solan et al. (2013)). Early theoretical work on fictitious play dynamics (Robinson (1951); Brown (1951)) established that time-average play converges to equilibrium in the zero-sum case, but Shapley (1964) devised a non-zero-sum game with a unique NE in mixed strategies for which fictitious play dynamics converge to a limit cycle and not to the NE.

Subsequent theory does not yield clear predictions on dynamic stability. Stahl II (1988), Crawford (1985) and others showed that convergence to equilibrium in asymmetric matching pennies games generally fails for their favored dynamics. On the other hand, such games Fudenberg and Kreps (1993) show that stochastic fictitious play converges, and the noise inherent in the Erev and Roth (1998) reinforcement learning model produces a similar result. Hopkins (2002) shows that that for both of those stochastic models, convergence is to QRE and thus to NE in the limit as noise amplitude vanishes. Convergence fails for replicator dynamics (Taylor and Jonker (1978)): the matching pennies NE is neutrally stable. Hofbauer and Hopkins (2005) find that for a broad class of dynamics, stable mixed equilibria are vanishingly rare in games with more than 2 pure strategies. Of course, unless the step size decreases to zero, discrete dynamics in a finite population typically can't converge to an interior equilibrium but at best will bounce around in its neighborhood.

These heterogeneous theoretical results on point predictions and dynamics underline the need for empirical work, but so far the empirical results are also quite heterogeneous. Rapoport and Orwant (1962) surveyed early laboratory experiments, and found that average play typically was closer to a uniform mix (e.g., .50-.50) than to a NE or MM mix. O'Neill (1987) found that overall time-average play is surprisingly close to the NE mix in a particular zero-sum 4x4 game, but Brown and Rosenthal (1990) noted that this does not imply that individual players employ NE strategies, and they indeed found substantial departures from the specified iid mixes. Subsequent empirical contributions such as Walker and Wooders (2001), Chiappori et al. (2002) and Palacios-Huerta (2003) left many readers with the impression that professionals closely approximate equilibrium mixed strategies but the usual undergrad lab subjects cannot. A closer reading suggests that, outside their familiar environments, professionals are no more successful than the usual subjects (Wooders (2010); Levitt et al. (2010)), but that populations of the usual subjects can collectively, if not individually, implement equilibrium mixtures (e.g., Friedman (1996); Binmore et al. (2001)).

The empirical literature pertaining directly to our second research question is limited and inconclusive. To our knowledge, only one previous empirical paper compares maximin mixtures to NE mixtures. Ochs (1995) considers several treatments (including one that uses a set of 9 explicit mixtures) in asymmetric matching pennies games, but finds that neither Nash equilibrium nor maximin tracks the observed changes in average play when game parameters change. Goeree et al. (2003) find that quantal response equilibrium with one free parameter (for logit precision) also fails to track such changes, but adding a second parameter (for risk aversion) improves performance.

There is also empirical literature on adaptive dynamics in matching pennies games. Mookherjee and Sopher (1994) find that belief learning (responsive to payoffs that would have been earned by strategies not employed) beats reinforcement learning. On the other hand, Erev and Roth (1998) find that their three parameter reinforcement learning model compares favorably to other learning models in predicting behavior in 12 laboratory studies of games with unique interior mixed NE. Camerer and Hua Ho (1999)'s EWA model includes an extra parameter to hybridize belief learning (a la Cheung and Friedman (1997)) with reinforcement learning; the authors show that it is able to fit a variety of games, including some matching pennies games. Tang (2001) presents two 3x3 bimatrix games with different predicted dynamic stability properties; the data favor the Selten (1991) anticipatory dynamics model over the Crawford (1985) model. Stephenson (2019) reports an experimental test of evolutionary models in coordinated attacker-defender games that include own-population effects (Friedman (1991)) not considered in our generalized matching pennies games. His results are consistent with a class of adaptive dynamic models. Taken together, these papers suggest that there may be a useful empirical role for dynamic models of some sort, but it is not yet clear which sorts are best nor when they will predict convergence or non-convergence to a point prediction.

In sum, despite important prior work by leading game theorists and experimentalists, all three motivating research questions remain open.

1.3 Theoretical Considerations

Table 1.1: Payoff bimatrices and equilibrium mixtures.

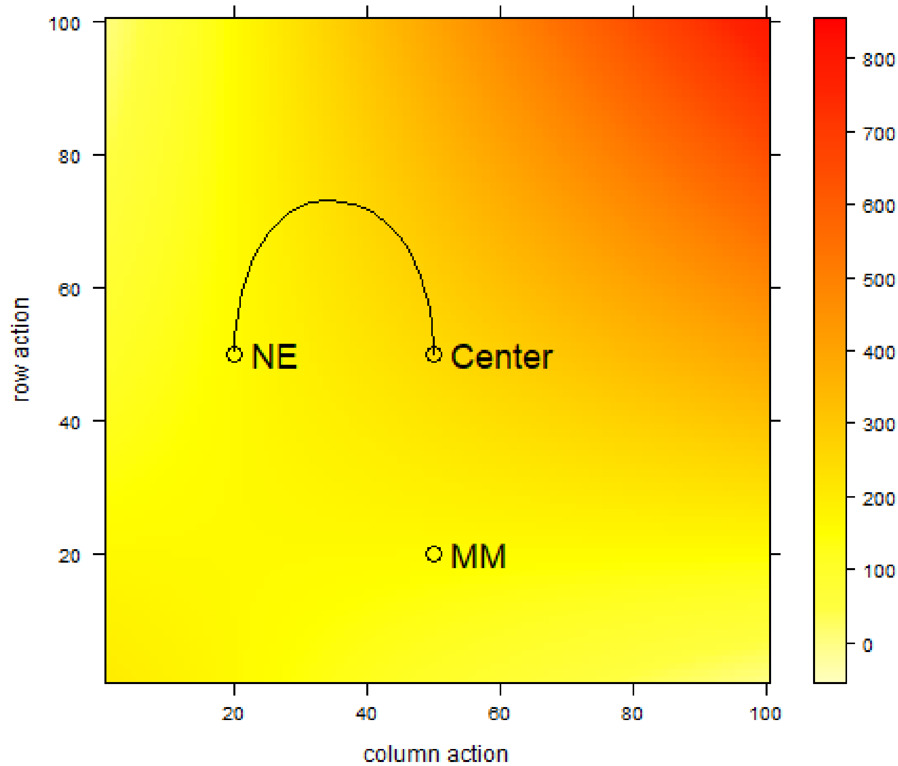
Name	AMPa	AMPb	IDDS
Bimatrix	$\begin{pmatrix} 800,0 & 0,200 \\ 0,200 & 200,0 \end{pmatrix}$	$\begin{pmatrix} 300,100 & 100,300 \\ 100,200 & 700,100 \end{pmatrix}$	$\begin{pmatrix} 200,500 & 0,600 \\ 400,300 & 200,100 \end{pmatrix}$
NE	(0.5, 0.2)	(0.33, 0.75)	(0, 1)
Maximin	(0.2, 0.5)	(0.75, 0.67)	(0, 1)

The notation (a, b) refers to row mixture aTop$\oplus(1-a)$Bottom and column mixture bLeft$\oplus(1-b)$Right.

Point predictions. Table 1.1 shows the specific bimatrix games that we will study. The first two are asymmetric matching pennies games. The Appendix includes the straightforward computation of the unique Nash equilibrium (NE) and maximin strategies; these games were chosen in part to create separation between those mixtures. The third game, named IDDS because it is dominance solvable, is intended as a control; its unique NE is in pure strategies.

Figure 1.1 graphically displays the mixed extension of the AMPa game from

Figure 1.1: Heatmap for AMPa row player.



The color at coordinates (x, y) indicates, via scaled thermometer at right, the row player's expected payoff at mixed strategy profile $100(b, a)$. NE, MM, Center respectively mark the coordinates of Nash equilibrium, maximin and Center profiles. The arc connecting Center $(x, y) = (50, 50)$ to NE includes all Logit quantal response equilibrium (QRE) profiles. The Figure uses a limited palette to better display overlaid text; the actual heatmap palette seen by subjects is shown in Figure 1.3.

the Row player's perspective: at mixed strategy profile $(a, b) \in [0, 1]^2$ her payoff is

$$\begin{aligned} f_R(a,b) &= (a, 1-a) \begin{pmatrix} 800 & 0 \\ 0 & 200 \end{pmatrix} \begin{pmatrix} b \\ 1-b \end{pmatrix} = 800ab + 200(1-a)(1-b) \\ &= 1000ab - 200a - 200b + 200. \end{aligned} \tag{1.1}$$

These payoffs are displayed as colors in a "heat map;" the thermometer bar on the right side shows the corresponding numerical values. These range bi-linearly from 0 at the corners $(a, b) = (0, 1), (1, 0)$ to 200 at (0,0) and to 800 at (1,1). Superimposed on the heatmap are alternative point predictions of empirical average mixtures: Nash equilibrium (NE), Maximin (MM), Center (i.e., the point (0.50, 0.50)), and the arc of Logit quantal response equilibria (QRE) as the precision parameter ranges from 0 (at Center) to ∞ (at NE).

At least since Nash (1951), game theorists have recognized two different interpretations of equilibrium in 2-player games. In the first interpretation, two highly rational individuals, fully aware of each other's circumstances, make choices (possibly mixtures) that they have no incentive to change. In the second, members of a large row player population match anonymously with members of a large column player population, and an equilibrium distribution of action profiles remains unchanged as individual players adapt. Binmore et al. (2001), among others, claim that the appropriate dynamic model of how players adapt their choices, and thus the stability of an equilibrium profile, may depend on whether the game is played by pairs of individuals or by populations. That claim motivates the mean-matching vs random-pairwise protocols presented in Section 4.

Of course, the adaptation process, and hence the stability of mixed equilibrium, may also depend on whether individual players can choose mixture weights explicitly (as for example, a political strategist deciding the fraction of positive vs negative ads to run) or can choose only the frequency of pure actions. The adaptation process also depends on whether players' choices are made simultaneously in discrete time (as in most laboratory experiments) or asynchronously in continuous time (as in online pricing of airline tickets, for example). Such considerations motivate the action space and time treatments presented in Section 4.

Sign preserving dynamics. In games where each player has only two pure strategies, there is a broad class of adaptive dynamics that applies to both the individualistic and the population interpretations (Friedman (1991); Friedman and Fung (1996); Weibull (1997)). The idea is simply that players (individually or collectively) should increase the weight on the pure strategy with the currently higher payoff.

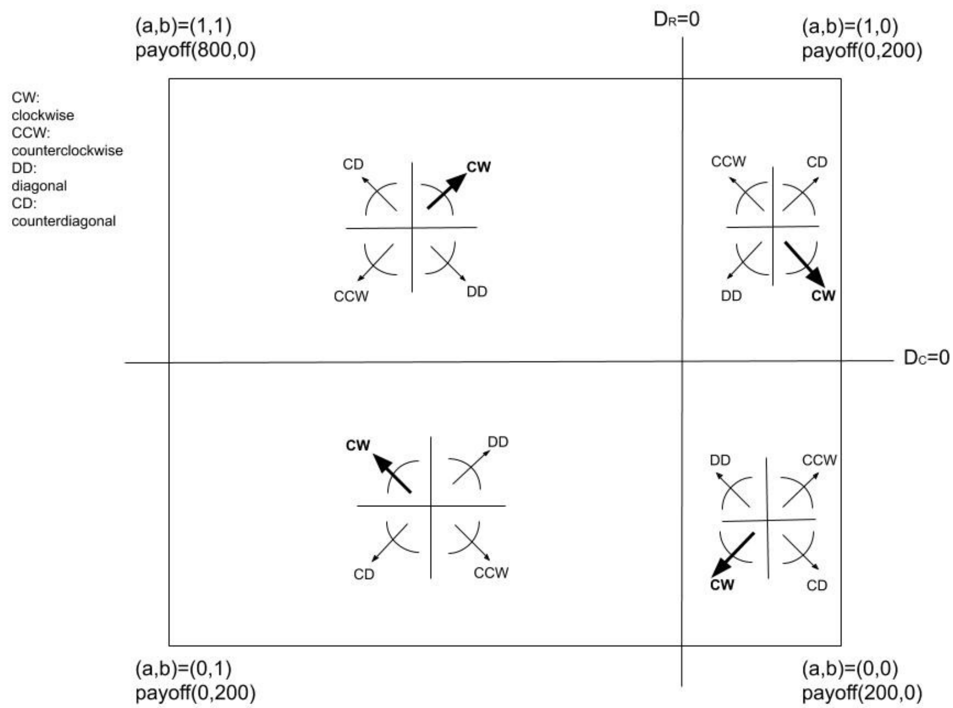
To formalize, let the time t mixed strategy profile be $(a(t), b(t))$ for a bimatrix game $M = (M^R; M^C)$. For example, in the AMPa game, $M^R = \begin{pmatrix} 800 & 0 \\ 0 & 200 \end{pmatrix}$. For smooth play in continuous time, $(\dot{a}(t), \dot{b}(t))$ denotes time rate of change. The payoff difference between pure strategies is denoted $D_R(t) = (1, -1)M^R \cdot (b(t), 1 - b(t))$ for the row player(s) and $D_C(t) = (1, -1)M^C \cdot (a(t), 1 - a(t))$ for the column player(s).

The dynamic process is *sign preserving* if, at all interior profiles $(a(t), b(t)) \in (0, 1)^2$, we have $\dot{a}(t)D_R(t) > 0$ unless $D_R(t) = 0$, and $\dot{b}(t)D_C(t) > 0$ unless $D_C(t) = 0$. That is, for both row and column players, the weight $a(t)$ or $b(t)$ on the first pure strategy strictly increases (resp. decreases) whenever it has a strictly higher (resp. lower) payoff than the alternative strategy. This is a fundamental property of learning and evolution, satisfied (at least approximately)[3] by all standard versions of adaptive dynamics. To

[3]The reason for this qualification is that a lower payoff strategy's share when sufficiently small may be increased by a noise term (e.g., in stochastic best response dynamics), as noted e.g. by Stephenson (2019).

see the implications, suppose that dynamics are continuous and sign preserving. Draw the isoclines $D_R = 0$ and $D_C = 0$, i.e., the lines for which, respectively, row players and column players are indifferent between their pure strategies. These isoclines divide the state space in $(a(t), b(t)) \in [0, 1]^2$ into regions, each with its own implied direction of change.

Figure 1.2: Classifying directional changes in AMPa Matching Pennies.



The horizontal (column player's) axis is reversed to be consistent with the heatmap in Figure 1.1. Arcs and arrows in each rectangle indicate the four possible directions defined in the text.

Figure 1.2 illustrates for the AMPa game. From equation (1.1), $D_R = f(1, b) - f(0, b) = 800b - (200 - 200b) = 1000b - 200$, so the isocline $D_R = 0$ is the vertical line

$b = 0.2$. Similar calculations show that $D_C = 200 - 400a$ so the isocline $D_C = 0$ is the horizontal line $a = 0.5$. These isoclines necessarily intersect at the NE point, and they chop the state space into four rectangles. For example, in the Northeast rectangle $a > 0.5, b < 0.2$, we have $D_R < 0, D_C < 0$, so sign preserving dynamics imply a trajectory with $\dot{a} < 0, \dot{b} < 0$, that is, moving clockwise towards the Southeast rectangle $a < 0.5, b < 0.2$. Similarly, in that Southeast rectangle, sign preserving dynamics imply $\dot{a} < 0, \dot{b} > 0$, moving clockwise towards the Southwest rectangle. Indeed, straightforward calculations show that sign preserving dynamics for the AMPb game as well as the AMPa game imply clockwise moves from each rectangle to the next.

Of course, human subject behavior is noisy, so the prediction from this theory is only that clockwise (CW) will be the most common direction of change. Figure 1.2 depicts the other three possible directions: counterclockwise (CCW), diagonal (DD) towards the Nash equilibrium mix, and counterdiagonal (CD) towards the corner (pure strategy profile) contained in that rectangle. These directions are all defined by the signs of $\dot{a}$ and $\dot{b}$ or, in empirical practice, by the signs of first differences in successive observations of $a(t)$ and $b(t)$.

In do-it-yourself randomization treatments, pairs of individual subjects must satisfy $a(t), b(t) \in \{0, 1\}$, so sign preserving dynamics will jump clockwise from one corner to the next. If the time between jumps is roughly constant, the average strategy profile will approximate the Center, $(.5, .5)$. On the other hand, for protocols that either allow automated mixing or that involve population matching, the trajectory typically lies in the interior of the state space. Clockwise trajectories may converge to NE via damped cycles, or may cycle endlessly with constant average amplitude, or may spiral away from the intersection (at NE) of the zero isoclines. These possibilities are all consistent with sign preserving dynamics.

Directional learning model. We now construct a more quantitative model of sign preserving dynamics called regret-based directional learning. Let $s_{it} \in [0, 1]$ denote player (or player population) i's mixture at time (subperiod or tick) t, and let $f_i(s_{it}, s_{-it})$ be the corresponding payoff. For example, for a row player i in the AMPa game, $f_i(s_{it}, s_{-it}) = f_R(a(t), b(t))$. Regret is defined as the normalized shortfall from maximal payoff,[4] $R_{it} = \frac{f_i(\hat{s}_{it}, s_{-it}) - f_i(s_{it}, s_{-it})}{\max_{0 \le x,y \le 1} f_i(x,y)} \ge 0$ for $\hat{s}_{it} \in argmax_x f_i(x, s_{-it})$. The model predicts the change in mixture $\triangle s_{it} = s_{i,t+1} - s_{it}$ as a sign-preserving linear function of regret,

$$\triangle s_{it} = \beta_1 R_{it} sign(\hat{s}_{it} - s_{it}) + \epsilon_{it}. \tag{1.2}$$

The sign function is $sign(\hat{s}_{it} - s_{it}) = +1$ if $\hat{s}_{it} - s_{it} > 0$; $= 0$ if $\hat{s}_{it} - s_{it} = 0$; and $= -1$ if $\hat{s}_{it} - s_{it} < 0$. When $argmax_x f_i(x, s_{-it})$ includes some values larger than s_{it} and other values smaller than s_{it}, then the convention is that $sign\{\hat{s}_{it} - s_{it}\} = 0$.

As long as $\beta_1 > 0$, equation (1.2) implies sign preserving dynamics in which the adaptation speed is proportional to regret, that is, to the potential advantage. Cruder variants (whose estimation is reported in the Appendix) include best response learning

$$\triangle s_{it} = \beta_1 (\hat{s}_{it} - s_{it}) + \epsilon_{it} \tag{1.3}$$

[4] The normalization assumes that maximal payoff is positive, as is the case in the bimatrix games used in the present paper. Otherwise the denominator could be replaced by $\max_{0 \le x,y \le 1} f_i(x, y) - \min_{0 \le x,y \le 1} f_i(x, y)$.

and pure directional learning

$$\triangle s_{it} = \beta_1 sign(\hat{s}_{it} - s_{it}) + \epsilon_{it} \ . \tag{1.4}$$

1.4 Laboratory Implementation

1.4.1 Treatment variables

Our experiment has four treatments. The first is the payoff bimatrix: as noted earlier, we consider two generalized matching pennies games, denoted AMPa and AMPb, as well as a dominance solvable game denoted IDDS.

The second treatment is the action set. In condition P (pure strategy), subjects use radio buttons to select one of two bimatrix rows. The display highlights the cell with payoffs for that choice, given the column chosen by the matched player.[5] In condition M (mixed strategy), subjects use a vertical slider to adjust an explicit mixture of the two strategies, as illustrated in Figure 1.3. The heatmap display indicates the payoff resulting from the player's chosen mix and the matched players' mix.

The third treatment concerns time. In our standard discrete time (D) condition, subjects' choices are updated simultaneously at regular time intervals, here 6000 ms (6 seconds). In the continuous time (C) condition, subjects update choices asynchronously in real time, with an imperceptible latency of around 50 ms, and data are recorded every 500 ms. Previous literature (e.g., Oprea et al. (2011) and Friedman et al. (2015)) argues that continuous time is more realistic in many applications in

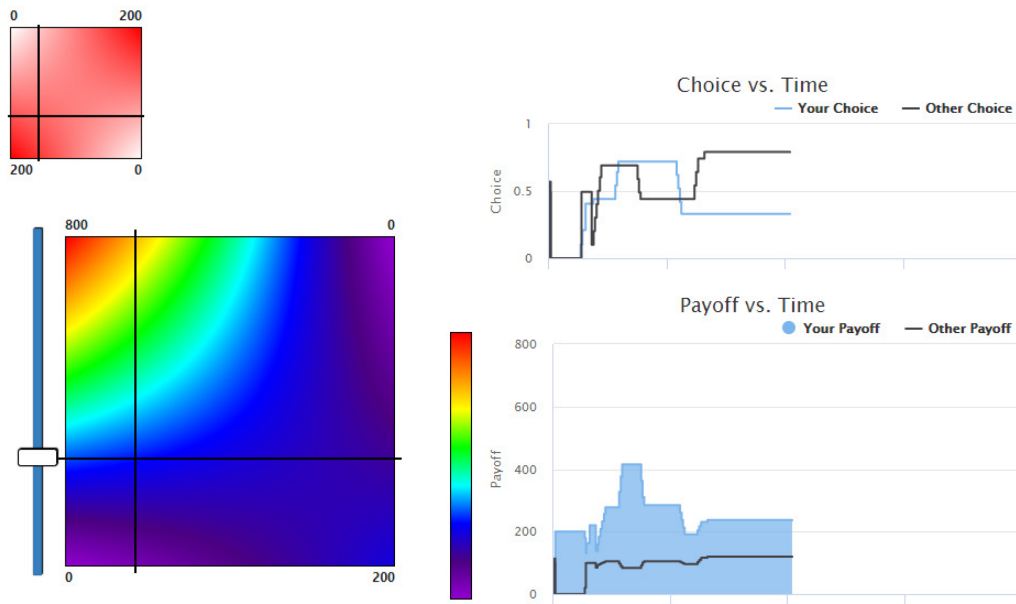
[5]To maintain consistency from the subjects' perspective, every player's screen shows her own choice as between rows, even for subjects labelled in this paper as column players.

sports, e-commerce, and elsewhere, and that it can facilitate cooperation and speed convergence. To our knowledge, there is no previous report of the impact of continuous time treatments on games such as generalized matching pennies where cooperation is infeasible.

Payoffs are flows accumulated over time in condition C, as illustrated in the lower right graph in Figure 1.3. In condition D, the blue area representing payoffs is of adjoining rectangles of width 6 seconds and of height given by the payoff at the chosen profile.

The remaining treatment is the matching protocol. As noted earlier, the dynamic adjustment process and the stability of an equilibrium profile may depend on whether the game is played out by pairs of individuals or at the population level, and each interpretation has real-world applications. In our experiment, there are always two distinct populations: row players match only with column players and vice-versa. In the standard random pairwise (rp) protocol, each subject interacts directly with only one matched opponent, and subjects are randomly rematched at the beginning of each new period. In the mean matching (mm) protocol, each subject plays against the average choice of all subjects in the other population or, equivalently for bimatrix games, gets the mean payoff over matches with all subjects in the other population. In terms of notation introduced earlier, s_{-it} is the time-t action of a particular randomly assigned opponent in rp, while in mm it is the time-t mean action of all possible opponents.

Figure 1.3: Main features of oTRw screen for MCrp AMPa game.



The subject uses slider at left to adjust her mixture (horizontal line); vertical line shows matched player's current mix. Heatmap color at intersection of these lines codes her current flow payoff; thermometer sets scale. Graph at lower right shows how her flow payoffs accumulate (blue area); black line is matched player's (average) flow payoff. Graph in upper right shows evolution of own and matched player's mixtures. Small red heatmap in upper left shows matched player's payoff function.

1.4.2 Design

The data analyzed below come from the 8 sessions specified in Table 1.2. Each session is equally divided into 5 blocks, each with a given payoff bimatrix; the sequence across blocks is AMPa, AMPb, IDDS, AMPa, and AMPb. The other three treatments

— action set, matching, and time — are constant within each session but vary across sessions according to the classic full factorial design; this enables the entire data set to be used to test the direct impact of each of those treatments. In each block, subjects play several periods as detailed in Table 1.2: in rp sessions, the periods last 90 seconds, with random rematching to start each new period, while they last 150 seconds in mm sessions. In D sessions, each period is divided into numerous 6-second subperiods and subjects update choices simultaneously, while in C sessions the subjects update freely in real time.

The oTRw software for conducting the experiment is a hybrid of oTree (Chen et al. (2016)) and LEEPS lab's Redwood suite, illustrated for the most distinctive treatments in Figure 1.3. Inexperienced subjects were recruited from the LEEPS lab subject pool using a local implementation of ORSEE (Greiner (2015)). Each session lasted for around 90 minutes, with a 20-minute instruction/practice stage, 60-minute game play stage and 10-minute payment/closing stage. Payments ranged from US \$14 to \$24, and averaged about \$17 per subject.

1.4.3 Testable Hypotheses

Our design transforms the original research questions into a series of testable hypotheses. Hypotheses H1-H3 below concern first two research questions on point predictions, while H4-H5 concern the third research question on dynamics.

H1: The time-average observed profile will not differ significantly from: (H1a) Nash equilibrium, or (H1b) Maximin, or (H1c) Center (.5, .5) , or (H1d) logit quantal response

Table 1.2: Experiment Design.

Date	Treatment	Block Size	# Subjects
4/4/2019	PCrp	6 x 90s periods	8
4/4/2019	MDrp	6 x 90s periods	10
4/5/2019	PDrp	6 x 90s periods	12
4/5/2019	MCrp	6 x 90s periods	8
4/11/2019	PDmm	4 x 150s periods	8
4/11/2019	MCmm	4 x 150s periods	10
4/16/2019	PCmm	4 x 150s periods	8
4/16/2019	MDmm	4 x 150s periods	12

P=pure, M=mixed action set; C=continuous, D=discrete time; rp=random pairwise, mm=mean matching protocol. For these three treatments, the 2x2x2 full-factorial design allows us to split the data into two 4-session groups to test the treatment effect. The 5 blocks of each session use the following sequence of bimatrices: AMPa, AMPb, IDDS, AMPa, and AMPb. Block Size reports the number and the length of periods in each block.

equilibrium for some positive precision parameter.

That is, we shall test the predictive power of NE mixture in our overall data. We shall do the same for the alternative equilibrium predictions MM and QRE, as well as for the non-equilibrium uniform mix Center.

H2 and H3 below summarize our initial conjectures, based on folk wisdom and fragmentary existing evidence, on the circumstances most conducive to convergence to equilibrium.

H2: The time-average observed profile will be closer to Nash equilibrium (NE): (H2a) under mean matching (mm) than under random pairwise (rp) matching protocol; (H2b) with automated mixes (M) than with only pure actions (P); and (H2c) in continuous time interaction (C) than in discrete time (D). For other versions of H2, replace NE by an alternative point prediction such as Maximin (MM).

H3: There will be less dispersion around the time average observed profile: (H3a) under mm than under rp matching protocol; (H3b) with M (automated mixes) than with P (only pure actions); and (H3c) in C (continuous time) than in D (discrete time).

We operationalize dispersion as the geometric mean interquartile range. That is, for d_R = 75th percentile - 25th percentile of Row mixes $a(t)$ in the sample, and d_C similarly defined for Column mixes $b(t)$, dispersion is defined as $d_G = d_R^{0.5} d_C^{0.5}$. Alternative dispersion measures explored in the Appendix include the harmonic mean, $d_H = (d_R^{-1}/2 + d_C^{-1}/2)^{-1} = \frac{2d_R d_C}{d_R + d_C}$ and the arithmetic mean $\frac{d_R + d_C}{2}$. All measures explored give qualitatively similar results.

The remaining hypotheses deal with our qualitative and quantitative models of dynamics. H4 comes directly from discussion of sign-preserving dynamics in Section 3, and from noting that both players can easily switch pure actions in a 6 second subperiod. H5 is the basic hypothesis of regret-based adjustment together with our conjecture on conducive treatments.

H4: In all treatments in generalized matching pennies games, the most frequently observed direction of change will be clockwise (CW). In pure strategy discrete time (PD) treatments, diagonal (DD) will also be frequently observed.

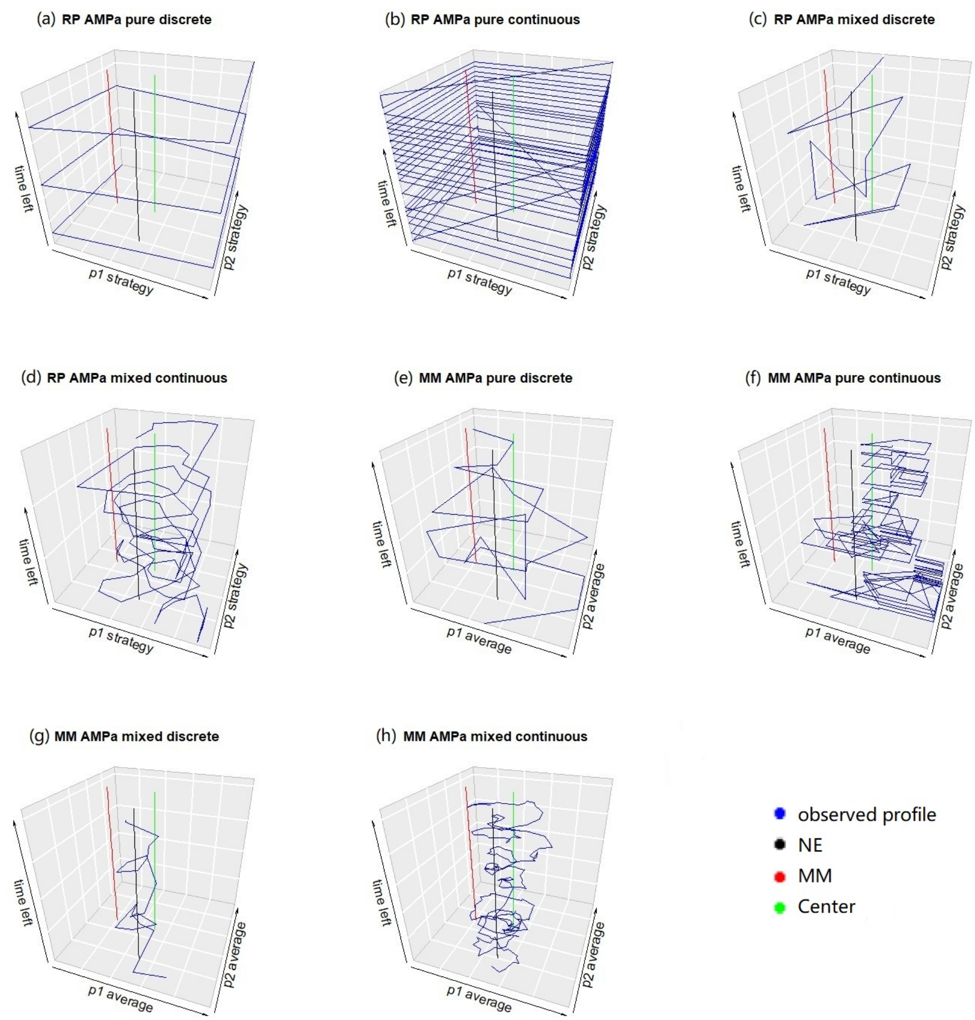
H5: Estimates of the rate coefficient β_1 in learning model (1.2) will be significantly positive in all treatments. Estimates will be more positive for mm than for rp matching, and more positive for P than for M action treatments.

1.5 Results

To gain perspective before reporting hypothesis tests, we examine a few examples of raw data. Each panel in Figure 1.4 displays the time path of action profiles for one instance of each treatment combination in the AMPa bimatrix game. Panel a shows the pure strategy choices of a pair of players in discrete time, the treatment combination most common in previous lab studies. In this instance, the players always best respond to the previous period profile, resulting in a clockwise tour of the four corners of state space. Thus average play is close to the Center, and dispersion is maximal. In Panel b, time is continuous and the (pure) strategy profile is recorded twice per second. The short vertical segments of the time path indicate episodes where both players stayed with their previous strategies, but again the most common change is a clockwise motion to the next corner of the state space. In a handful of episodes, both players switch strategies in the same half-second interval and so make a diagonal (DD) move. In panel c, the players can choose explicit mixtures in discrete time, and there is far less dispersion, but it is unclear whether the average play is closer to the Center or to NE in this instance. The player pair in Panel d usually moves clockwise, sometimes wandering around NE and sometimes wandering away. The next four panels of Figure 1.4 come from mean-matching (population game) sessions. They all have less dispersion than their random-pairwise counterparts. In particular, the mixed strategy discrete time profile path shown in Panel g is usually in the vicinity of NE.

In testing point predictions, it is appropriate to focus on settled behavior,

Figure 1.4: Sample time paths of action profiles.



The horizontal plane is the state space, the (a, b) square. The vertical axis is time remaining, so the time paths begin at the top and spiral downward, and reach the bottom plane at the end of the period. Point predictions are time-invariant and therefore appear as vertical lines.

so subsequent analysis drops the first period in each block, and first 18 seconds (or 3 subperiods) of each period. For the remaining part of each remaining period in a given matching, we collapse the time path to its time average profile, and look at the distribution over all instances for a given treatment combination. Figure A.2 and A.3 in the Appendix respectively display the mean and standard deviation, and the median and interquartile of these time averages.

1.5.1 Point Predictions

Table 1.3 summarizes tests of Hypotheses 1 and Table 1.4 summarizes tests of Hypotheses 2 and 3; robustness checks using both mean and median profiles can be found in the Appendix. For each instance (matching and period) τ, we compute the time average profile (a_τ, b_τ) and its Euclidean distance $[(a_\tau - a_p)^2 + (b_\tau - b_p)^2)]^{0.5}$ from a given point prediction (a_p, b_p). For example, the first line of Table 1.3 shows that for mean matching instances in the AMPb game (pooling over Continuous and Discrete time, and over Pure and Mixing action sets), the mean distance between the time-average profile and the NE prediction is just 0.157. According to a two sample t-test, this is significantly ($p < 0.05$) less than 0.224, the mean distance between those same time average profiles and the Center point. The rest of that line shows that the mean distance to the maximin prediction, 0.398, is significantly larger.

Thus the first lines of Panels A and B in Table 1.3 support Hypothesis H1a, that NE is the best point prediction, for mean-matching treatments in our matching pennies games. Consistent with Hypothesis H1c, Center is best in all other treatments in

Table 1.3: Mean distance to predictions of time average profiles.

	Distance to NE		Distance to Center		Distance to MM	Num of Pairs
			Panel A: AMPb games			
mm	0.157	$<_{**}$	0.224	$<_{***}$	0.398	24
rp	0.313	$>_{***}$	0.124	$<_{***}$	0.295	40
Mixed	0.242	$>_{***}$	0.135	$<_{***}$	0.339	32
Pure	0.268	$>_{***}$	0.188	$<_{***}$	0.328	32
Continuous	0.301	$>_{***}$	0.181	$<_{***}$	0.330	32
Discrete	0.208	$>_{***}$	0.142	$<_{***}$	0.337	32
			Panel B: AMPa games			
mm	0.150	$<_{***}$	0.255	$<_{***}$	0.463	24
rp	0.247	$>_{***}$	0.115	$<_{***}$	0.321	40
Mixed	0.211	$>_{**}$	0.159	$<_{***}$	0.352	32
Pure	0.210	$>$	0.176	$<_{***}$	0.397	32
Continuous	0.263	$>_{***}$	0.156	$<_{***}$	0.375	32
Discrete	0.159	$<$	0.179	$<_{***}$	0.374	32
			Panel C: IDDS games			
mm	0.219	$<_{***}$	0.521	$>_{***}$	0.219	12
rp	0.283	$<_{***}$	0.479	$>_{***}$	0.283	20
Mixed	0.241	$<_{***}$	0.517	$>_{***}$	0.241	16
Pure	0.277	$<_{***}$	0.472	$>_{***}$	0.277	16
Continuous	0.244	$<_{***}$	0.496	$>_{***}$	0.244	16
Discrete	0.274	$<_{***}$	0.494	$>_{***}$	0.274	16

Subscripted asterisks indicate p-values of .10(*), .05(**) and .01(***) for one-sided t tests assuming unequal variance between adjacent columns.

these games, with the possible exception of discrete AMPa, where NE is insignificantly better. In Panels A and B, there is no support for the maximin hypothesis H1b. Panel C confirms that the pure strategy NE (which coincides here with maximin) is a better point prediction than Center in all treatments for the dominance solvable game.

Testing Hypothesis H1d is potentially more complicated, since there is a whole arc of QRE that connects NE to Center, not just a single point prediction. However, as shown in the Appendix and Figure 1.1, that arc usually bends away from the mean profiles. In all treatments, the closest point on the arc turns out to be at or very near to either NE or Center. We, therefore, conclude that our data do not support Hypothesis H1d.

Table 1.4: Coefficients (± standard errors) for OLS regressions with clustering at session level.

continuous	0.10±0.018***	-0.04±0.031	0.02±0.017	-0.06±0.004***
pure	-0.06±0.018**	-0.06±0.025*	0.05±0.016**	0.18±0.005***
mm	-0.18±0.023***	0.02±0.021	0.11±0.024***	-0.00±0.007
AMPa	-0.06±0.023**	-0.01±0.022	0.04±0.016**	0.02±0.006***
continuous_pure	0.06±0.012***	0.01±0.028	0.04±0.016*	0.04±0.008***
continuous_mm	-0.10±0.013***	0.06±0.031*	0.02±0.018	0.01±0.009
continuous_AMPa	0.01±0.035	0.01±0.033	-0.06±0.023**	0.00±0.007
pure_mm	0.15±0.013***	0.11±0.031**	-0.04±0.018*	-0.02±0.009*
pure_AMPa	-0.03±0.035	0.05±0.033	-0.04±0.023	-0.02±0.007**
mm_AMPa	0.06±0.035	0.04±0.036	0.04±0.023	-0.01±0.007
Constant	0.28±0.013***	0.34±0.017***	0.08±0.014***	0.31±0.003***
Observations	128	128	128	128
R-squared	0.630	0.538	0.611	0.954

Dependent variable in last column is geometric mean dispersion, and in other columns is Euclidean distance from mean observed profile to predicted profile. Independent variables are treatment indicators and interactions; the baseline (omitted) indicators are for AMPb, mixed strategy, discrete time, and random pairwise matching. Observations is total number of periods in all sessions. Significance levels 1, 5, and 10% respectively denoted ***,** ,*.

For the same empirical time average profiles, Table 1.4 shows the results of regressing treatment dummies and their interactions on prediction error and on dispersion. Hypothesis H2 asserts that relevant predictions are more accurate for certain treatments. The first column of Table 1.4 supports Hypothesis H2a, that NE is more accurate under mean matching than under random pairwise matching, since the mm coefficient is significantly negative. Conclusions regarding H2b and H2c are more nuanced due to significant interactions: the direct effect of pure strategies and of mean matching both reduce NE prediction error while continuous time increases prediction error, but these are largely offset by the interactions of mean matching with continuous and pure. The upshot is that NE predicts especially well in mixed mean-matching treatments, confirming the impression from the previous table. The entries in the second column confirm that maximin prediction errors are large in all treatments. Many treatments and interactions have opposite signs in the first and third columns, suggesting that they shift the observed behavior away from NE and towards Center, or the reverse.

Hypothesis 3 concerns dispersion. The last column of Table 1.4 reports the geometric mean of row dispersion (standard deviation) and column dispersion (standard deviation) as defined in the previous section. The second line of the Table supports H3b, that dispersion is less with mixed strategies. The significantly negative coefficient in line 8 offers limited support for H3a: mean matching reduces dispersion in pure strategy treatments but perhaps not in general. The Table also supports H3c: dispersion seems to be less for Continuous than for Discrete time treatments.

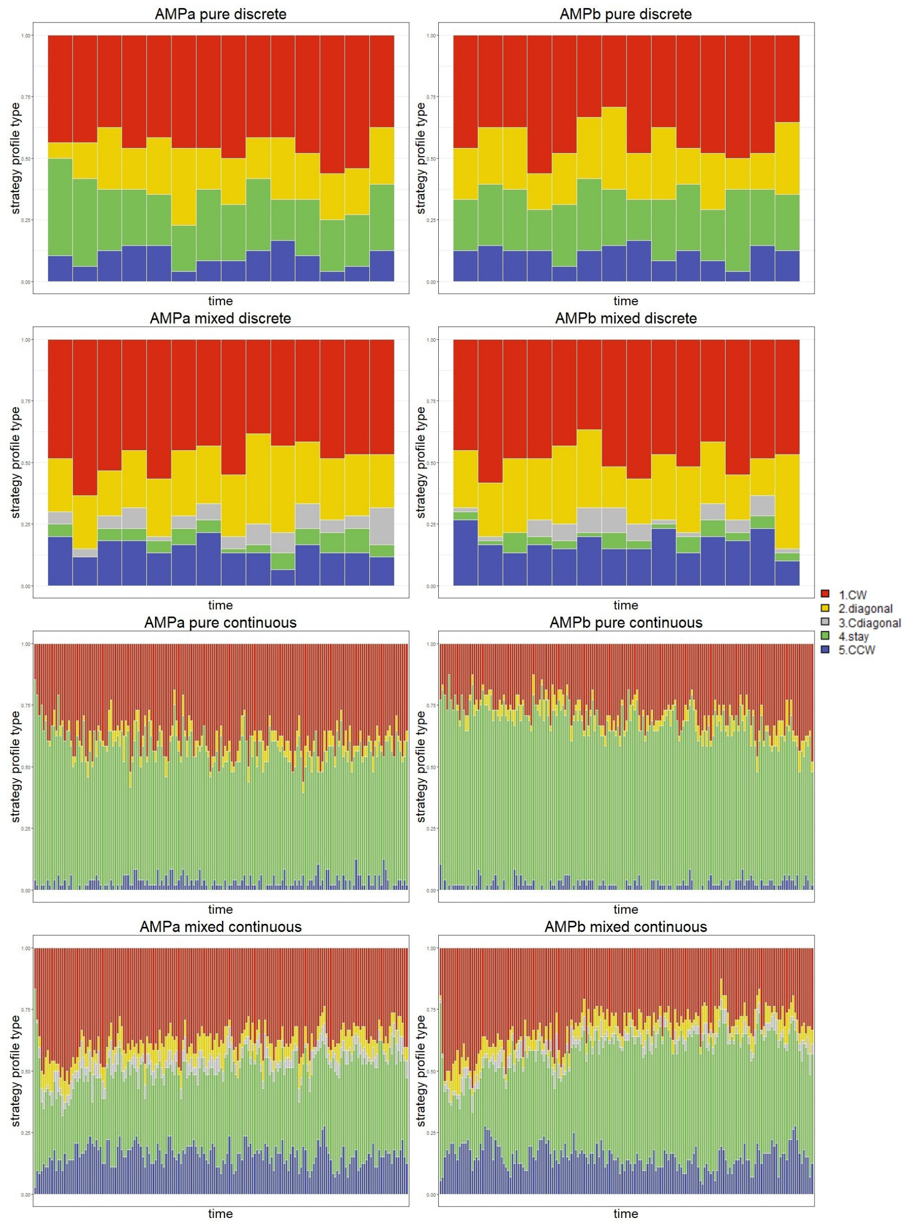
1.5.2 Qualitative Dynamics

The large constant term in the last column of Table 1.4 suggests that behavior typically does not settle down to a behavioral equilibrium. Does that mean that players wander aimlessly, or is there some regularity such as clockwise cycles?

To investigate, recall how Figure 1.2 classified profile moves $\Delta s_t = (\Delta s_{Rt}, \Delta s_{Ct})$ $= (s_{Rt+1} - s_{Rt}, s_{Ct+1} - s_{Ct}) \neq 0$ as clockwise (CW), diagonal (DD), counterclockwise (CCW), or counter diagonal (CD).

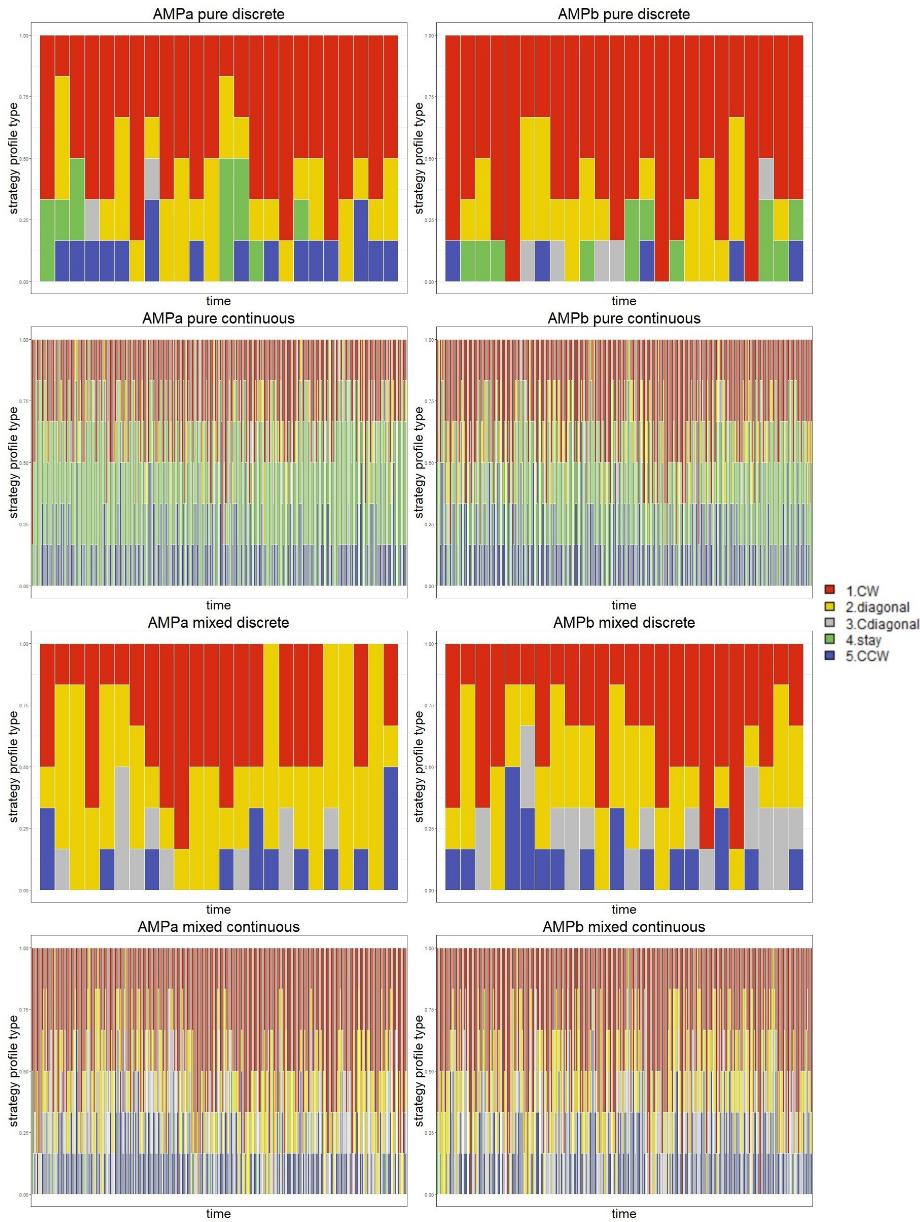
Figure 1.5 shows how the classifications change over time in random pairwise matching sessions. For example, in the top panels we see that in the 15 six-second Discrete subperiods (14 moves since the cyclical behavior is determined by two consec-

Figure 1.5: Classification over time in random pairwise matching sessions.



clockwise (red), diagonal (yellow), counter-diagonal (grey), stay (green) and counter-clockwise (blue). AMPa on left, and AMPb on right, by subperiod or tick within period and averaged across periods.

Figure 1.6: Classification over time in mean matching sessions.



clockwise (red), diagonal (yellow), counter-diagonal (grey), stay (green) and counter-clockwise (blue). AMPa on left, and AMPb on right, by subperiod or tick within period and averaged across periods.

utive subperiods), there is a preponderance of CW moves (in red), a fair number of DD moves, no CD moves (impossible in Pure treatments), rather few CCW moves, and perhaps 10 - 30% Stay ($\Delta s_t = 0$). Indeed, in all treatments CW is more common than other moves, as predicted by sign preserving dynamics. It is no surprise that Stay is far more common and DD is relatively rare in Continuous treatments, since sampling there is twice a second. CCW is especially rare in Pure Continuous sessions. CD is rare even in mixed treatments. DD is not uncommon in discrete time treatments, where it may indicate anticipatory behavior in the sense of Selten (1991). There seem to be no strong trends in behavior within periods, nor major differences between AMPa and AMPb games. There is, however, considerable heterogeneity across matched pairs, as can be seen from the by-pair breakdown in Appendix Figure A.4.

Figure 1.6 presents similar evidence for mean matching sessions, where Δs_t represents population profile moves rather than individual pair moves. Not surprisingly, with mean matching we see fewer Stay and more CW moves in most treatments. DD becomes more common while CD and CCW remain rare.

The data shown here (and in the Appendix, e.g., Tables A.1 and A.4) thus support Hypothesis H4. Overall, CW moves are indeed the most prevalent, representing up to half of the total observations. DD often ranks second, and other directional moves are relatively rare. Move types have similar distributions in the two bimatrices and (if we ignore Stay) in continuous time and discrete time. The distributions also seem roughly similar in pure and mixed strategy conditions and in mean matching and random pairwise.

1.5.3 Fitted Dynamic Model

To test the more quantitative dynamic hypothesis H5, we fit the regret-based learning model (1.2), allowing for fixed effects and using indicator variables D_k to capture treatment-specific response to regret,

$$\triangle s_{it} = (\beta_1 + \sum_k \beta_k D_k) R_{it} sign\{\hat{s}_{it} - s_{it}\} + b_i + c_t + \epsilon_{it}. \tag{1.5}$$

Table 1.5 collects the results. The first row clearly supports H5: the baseline response to regret β_1 is very significantly positive. Remaining rows show that this support is not reversed by any treatment or interaction considered. H5 also predicts that response is stronger in pure strategy treatments (since moves there must be to corners, not just incremental); the continuous time data clearly support this prediction, but the impact is insignificant in discrete time. The remaining part of H5 predicts stronger response in mean matching than in random pairwise matching. This prediction is supported for Row players in continuous time sessions, but elsewhere the impact is insignificant.

Other entries in the Table mostly seem reasonable upon reflection. Since continuous time data are sampled twelve times as frequently as discrete time data, it is natural for the continuous time coefficients to be much smaller in absolute value. Column players adjust more slowly in AMPa, perhaps because of the greater asymmetry in that game than in the AMPb baseline. Adjustment is faster in IDDS in discrete time with pure strategies, perhaps due to the strategic clarity of that treatment combination. The Appendix reports regressions for related specifications (1.3) and (1.4), with results

Table 1.5: Directional Learning Model (1.5) coefficient estimates (± standard error) for Row and Col(umn) player actions in Continuous and Discrete time.

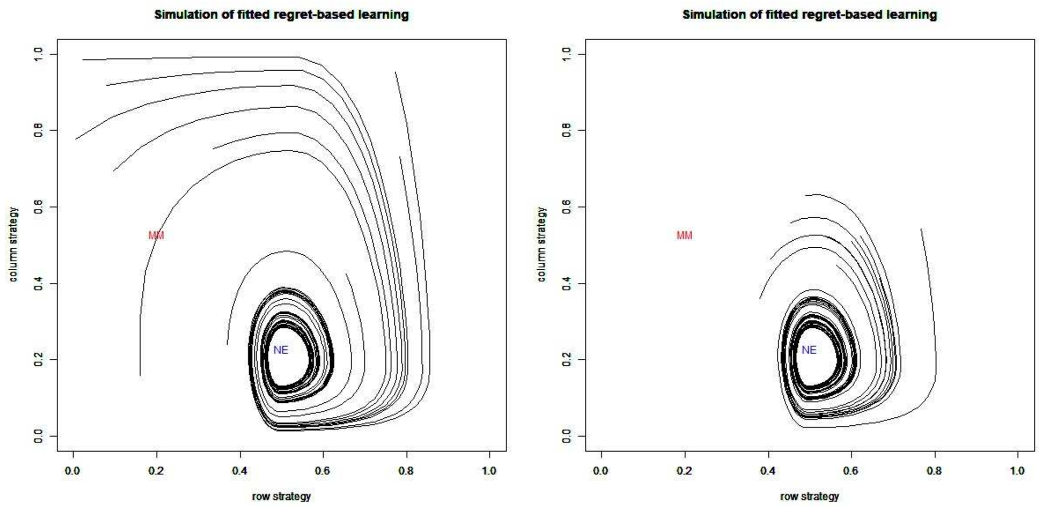
	Row-Continuous	Row-Discrete	Col-Continuous	Col-Discrete
β_1	0.12±0.021***	1.14±0.099***	0.26±0.028***	1.06±0.111***
pure	0.47±0.067***	-0.14±0.116	0.55±0.078***	-0.02±0.131
mm	0.24±0.050***	-0.16±0.218	-0.04±0.051	0.33±0.267
AMPa	-0.03±0.025	-0.22±0.134	-0.17±0.032***	-0.62±0.120***
IDDS	0.04±0.057	0.07±0.244	-0.24±0.031***	0.28±0.319
pure_mm	-0.03±0.153	0.75±0.292**	-0.18±0.151	0.35±0.345
pure_AMPa	0.10±0.090	0.22±0.155	-0.14±0.087	0.17±0.147
pure_IDDS	-0.21±0.123*	0.98±0.335***	-0.20±0.123	1.12±0.406***
mm_AMPa	-0.03±0.066	0.00±0.316	0.06±0.061	0.17±0.303
mm_IDDS	-0.13±0.099	-0.12±0.368	0.06±0.055	-0.35±0.450
pure_mm_AMPa	-0.34±0.195*	0.24±0.390	-0.10±0.164	-0.75±0.384*
pure_mm_IDDS	0.53±0.314*	-0.36±0.472	0.36±0.217*	-0.26±0.626
Observations	79,145	4,995	79,145	4,995
R-squared	0.213	0.337	0.251	0.253
Number of Pairs	415	345	415	345

Observations show number of time ticks in each regression. Least squares with pair and tick fixed effects. the baseline (omitted) indicators are for AMPb, mixed strategy, discrete time, and random pairwise matching. Nominal significance levels 1, 5, and 10% denoted ***,** ,* .

generally consistent with those of Table 1.5.

To complement the hypothesis tests, we ran simulations of equation (1.5) using the coefficient estimates reported in Table 1.5 with error terms set to zero. Figure 1.7 shows that, according to two of the fitted models (the others are qualitatively similar), players (or populations) move in clockwise cycles that converge to a limit cycle surrounding the Nash equilibrium. Thus the data suggest that, practically speaking, there will never be convergence to any point prediction, but rather that (a) cycles will persist for a very long time, and (b) Nash equilibrium is a crude approximation of the long-run time-average profile. See the Appendix for a more careful analysis using a Poincarè map.

Figure 1.7: Simulation of (1.5) using fitted parameters from continuous time mixed strategy AMPa games.



panel (a) is for random pairwise matching and (b) is for mean matching. Simulations are each run 300 periods from 10 different initial profiles.

1.6 Discussion

Our results speak to the broader questions behind the testable hypotheses, and suggest the following practical advice for applied researchers.

First, we find that mixed Nash equilibrium is a reasonably good predictor of behavior in population games. For example, consider an application to the market for an intermediate good where upstream firms deal with many downstream firms, and conversely. When the market model has no pure NE, our results suggest that mixed Nash equilibrium may be a good candidate for describing time-average behavior.

Second, for most other circumstances in generalized matching pennies games, we find that uniform random behavior outpredicts mixed Nash equilibrium. In an otherwise similar application where a given upstream firm deals mainly with a single downstream firm and conversely, the mixed NE may not be the best predictor of time average behavior.

Third, for our asymmetric matching pennies games, we found no circumstances in which alternative equilibrium concepts such as maximin and Quantal Response Equilibrium improved on both Nash equilibrium and uniform mixes. This may simplify work for applied economists.

Applied economists more interested in short run dynamics than in long run time average behavior should find our experiment particularly instructive. We found persistent dispersion and systematic directional change in our data. A regret-based directional learning model had significant explanatory power, and its coefficients were fairly consistent across treatments. Applied economists may be able to deploy models that similarly aim to capture directional adaptations that are sensitive to regret (i.e., to the magnitude of potential gains).

Our main message for theorists is that mixed equilibrium (Nash or otherwise) is relevant over a narrower range of circumstances than might have been supposed. We find reliable convergence to pure Nash equilibrium in a (control) dominance solvable game, and to a lesser extent we find convergence to mixed NE in matching pennies population games. However, in traditional pairwise matching (especially with traditional pure strategy choice) for matching pennies, we find persistent dispersion that centers more on the uniform random mix than on an equilibrium mix.

A more positive message is that there is order beneath the dispersion. We find a clear tendency for players to cycle in generalized matching pennies games. Typically one player (or player population) has a stronger incentive to switch strategies, and doing so strengthens that incentive in the other player or population, creating (with our sign conventions) clockwise cycles. In discrete time treatments we saw some evidence that players tried to anticipate and exploit these regularities, but they nevertheless persist, especially in continuous time and in population games. We encourage game theorists to further develop models of adaptive dynamics that are sensitive to circumstances of the sort we have explored.

Our experiment also suggests possible follow-up work in the laboratory. Our subjects explicitly choose their mixes in M treatments and receive the expected payoff, and so do not need to randomize actions dynamically. In contrast, Romero and Rosokha (2018) elicit subjects' history-dependent actions in a repeated prisoners' dilemma. It might be interesting to try similar elicitation procedures in repeated matching pennies games. It may also be worthwhile to seek new treatments that facilitate convergence to Nash equilibrium or other point predictions. In some pilot sessions, we tried displays of best and worst possible payoffs, and tried slowing adjustment speed in continuous time, but found little impact. Finally, it seems worthwhile to design experiments that can better distinguish among different models of adaptive learning.

We hope especially that our results encourage applied researchers to work in a more nuanced fashion with mixed strategy equilibrium and adjustment dynamics. Biologists since Lotka and Volterra (Lotka (1926)) have recognized that dynamics are crucial to understanding generalized matching pennies interactions such as between predators and prey. Social scientists may benefit from similar thinking. For example, 'hot spot' dispatch of law enforcement resources (e.g., Lazzati and Menichini (2016)) is a generalized matching pennies population game, and our work suggests how adaptive dynamics could supplement equilibrium analysis.

Chapter 2

Coordination Games in Continuous Time

2.1 Introduction

Coordination failure has been studied for decades both in laboratory environments and in real world scenarios where people fail to coordinate their choices at certain strategy profiles. To illustrate coordination failure, game theorists commonly point to two iconic games. In the pareto-ranked games such as stag hunt games, there exist two pure Nash equilibria with one's payoff strictly higher than the other. However, the one with a lower payoff comes with an insurance when players mismatch, which creates tension between two equilibria and players tend to choose the risk-dominant strategy instead of payoff-dominant strategy. In the mixed-motives games such as battle of the sexes, players prefer different pure Nash equilibria and find it difficult to coordinate their conflicts to achieve both efficiency and equality. Lack of coordination reduces the efficiency in both games.

Previous studies of coordination failure have focused on efficiency improvement and optimal learning in one-shot games and discrete time repeated interactions (e.g. Devetag and Ortmann (2007)). However, continuous time interactions may also enhance human cooperation, as is shown in general by both theoretical work (Simon and Stinchcombe (1989); Bergin and MacLeod (1993)) and by laboratory results (e.g. Friedman and Oprea (2012)). In continuous time, decision makers can choose the reaction timing almost freely and make quick responses to observable changes, thus creating a richer set of strategies than that in the discrete time where actions can only be chosen at fixed points in time. Based on these studies, the continuous time environment provides an opportunity for researchers to better understand and solve coordination failure as it allows players to interact at high frequency and move asynchronously and thus reduces the strategic uncertainty people face and motivates their cooperative behaviors.

The empirical study of the continuous time interaction can provide results applicable to real-world contexts. With the development of technologies, the world is becoming faster than ever before and many decisions nowadays can be considered as continuous time processes, especially with the introduction of the 5th generation network. For instance, the price in online marketplaces can be adjusted at any time with biddings and transactions updated in seconds. Understanding the impact of such technological growth on human coordination becomes necessary for firms and policy makers, especially when they face such choices in the high-tech industries.

This paper introduces coordination games in the continuous time to the laboratory and compares possible theoretical predictions to the laboratory results in a systematic environment. As the first paper that studies stag hunt games and battle of the sexes games in the continuous time environment, this paper contributes to the literature in both fields of repeated coordination and continuous time interaction by showing that continuous time interaction facilitates coordination and increases the efficiency, which provides another method of solving coordination failure and enhances the general impression that continuous time interaction improves human cooperation.

The experimental results show how continuous time environments improve the efficiency in both games from different perspectives. In the continuous time environments, lab subjects interact in milliseconds, while the discrete time environment is the same as conventional repeated games. This study demonstrates that, in both types of games, lab subjects coordinate better in continuous time than in discrete time. In stag hunt games, subjects tend to switch from risk-dominant equilibrium to payoff-dominant equilibrium. In battle of the sexes games, subjects are more likely to alternate between equilibria in battle of the sexes games.

In addition to the comparison between continuous and discrete time environments in two types of coordination games, the experiment is also designed to test the difference in a systematic environment. Subjects experience three payoff matrices in each game that motivates them towards different equilibrium. They also experience action sets with two levels of complexity. In pure action sets treatment, subjects' action sets are binary, while in mixed action sets treatment they face action sets with continuous values. Although both treatments share the same pure Nash equilibrium as focal points, mixed action sets treatments are visually more complicated than pure action sets treatments and could cause less coordinate behavior. The results confirm the consistency of the main treatment effect of continuous time interaction. Furthermore, the differences between the two action set treatments are statistically significant but there is little difference among payoff matrices.

The paper begins with reviewing some previous theoretical and experimental work related to our investigation in Section 2. Sections 3 and 4 layout our theoretical foundation and experimental design, respectively. Experimental results of stag hunt games are discussed in Section 5, which in general show that continuous time interaction enhances coordination. Similar results are found for battle of the sexes games in Section 6 with a unique turn taking pattern. Section 7 concludes the main findings and points out unsolved questions for future studies.

2.2 Literature Review

2.2.1 Experiments in Continuous Time

Although most economic laboratory experiments employed a discrete time framework, continuous time theory has been developed for quite a long time. Models that work in discrete time cannot be directly copied to the continuous time environment because the concept of "period" is not well-defined in continuous time. Previous theories have been focused on modelling traditional games in continuous time environments and show new equilibria, where the most well-known findings show that both perfect continuous time and inertia continuous time with reaction lags support cooperation in games similar to prisoners' dilemma (Simon and Stinchcombe (1989); Bergin and MacLeod (1993); Park (2014); Alós-Ferrer and Kern (2015); Calford and Oprea (2017)). With the development of laboratory techniques, experimental economists began to study continuous time games in recent years in the lab environment where subjects interact in milliseconds. Although subjects cannot really respond in milliseconds, the high frequency interaction allows them to react much faster than they react in discrete time environments and many experimental findings support theoretical predictions. In prisoner's dilemma games, the cooperation rate between players tends to be higher in continuous time than in discrete time (Friedman and Oprea (2012)), while a follow-up study found that termination rule and time horizon affect the coordination rate (Bigoni et al. (2015)). Similar cooperative behaviors have also been observed in the continuous time public good games with communication (Oprea et al. (2014)).

Some lab studies in continuous time environments focus on other types of behavioral difference between continuous and discrete time environments. In the continuous time evolutionary hawk and dove game, symmetric mixed equilibrium is more likely to be selected in the one-population game, whereas separation equilibrium is stronger in the two-population game (Oprea et al. (2011)). This finding has been further developed under the uniparametric model (Benndorf et al. (2016)) and the perturbed best response dynamics (Benndorf and Martinez-Martinez (2017)). Furthermore, continuous time treatments affect subjects' behaviors in market competition games. Subjects are more likely to converge to Nash equilibrium in the 4-player Hotelling location competition in continuous time (Kephart and Friedman (2015)). The effect of continuous time in oligopoly competition requires further exploration: there are findings showing that tacit collusion exists in long term Cournot game (Friedman et al. (2015)) but other evidence shows that collusion rate is higher in discrete time than in continuous time, which contrasts the general idea that continuous time treatments tend to facilitate co-operation (Horstmann et al. (2016)). Besides these, continuous time treatments have also been adopted in minimum effort games (Deck and Nikiforakis (2012); Leng et al. (2018)) and network formation (Berninghaus et al. (2006)).

2.2.2 Coordination Failure and Repeated Coordination

Although coordination games have been popular over past decades, coordination games in continuous time have received little attention in the previous literature. In early studies, coordination failure is considered to be a common phenomenon in the laboratory (e.g. Van Huyck et al. (1990), Van Huyck et al. (1991); Cooper et al. (1990), Cooper et al. (1992); See Devetag and Ortmann (2007) for an overview). Coordination failure describes either failure to coordinate on any one of the multiple equilibria or failure to coordinate on the payoff dominant equilibrium. Researchers have studied coordination failure in the past decades for robustness tests and potential improvements using games such as stag hunt, battle of the sexes and order statistics. In discrete time treatments, what researchers have learned is that efficiency can be enhanced through lower attractiveness of the secure action (Battalio et al. (2001); Dubois et al. (2012)), lower cost of experimentation (Van Huyck et al. (2007)), less stringent coordination requirements (Van Huyck et al. (2007)), fixed matching protocols (Clark and Sefton (2001)), full information feedback (Berninghaus and Ehrhart (2001)), communication (Blume and Ortmann (2007)) and social interaction (Bolton et al. (2016)). Furthermore, asymmetric information also affects coordination in global games but the effect remains ambiguous (Cabrales et al. (2007); Van Huyck et al. (2018)).

Theorists have studied coordination problems and focus on repeated interaction since Crawford and Haller (1990). They built a model where players apply randomized mixed strategies to facilitate coordination, optimize intertemporal payoffs and maintain coordination thereafter. Their model indicates that maintaining coordination requires players to stay at either payoff-dominant or risk-dominant equilibrium in stag hunt games, while players can also take turns between two pure Nash equilibria in battle of the sexes games. Lau and Mui (2008) and Lau and Mui (2012) developed the idea of randomized coordination as "turn taking with independent randomization" and applied it to explain alternating dynamics. Strategies such as individual evolutionary learning (Arifovic and Ledyard (2018)) also involve a randomized experimentation stage before players converge to any equilibrium. Romero and Zhang (2018) compares equilibrium with various normative principles and finds that equal payoff and equal opportunities are the most frequently played guideline for alternating dynamics. Correlated equilibrium and third-party signals have also been proved as a method to solve coordination problems in battle of the sexes games (Duffy et al. (2017); Anbarci et al. (2017)). Some other papers discuss alternating dynamics with pre-defined punishment strategy and randomized cost structure (Leo (2017)) and comparison between alternations and cut-off strategy (Kaplan and Ruffle (2011)).

Another flow of research was focused on players' learning and convergence in coordination games with both multiple-period random matching and repeated interactions. In early literature (Crawford (1995)), subjects update their belief based on the observed actions from the other player in previous periods and form the best response in the current period. Cheung and Friedman (1997) tested games with a three-parameter belief learning model and found considerable heterogeneity among individual players. Hyndman et al. (2009) added a forward-looking component to the model , which better fits the data in coordination games. Strategic teaching is a method of efficient coordination where experienced players sacrifice short-term payoff for long-term coordination. In stag hunt games, for example, experienced players may intentionally stay at payoff-dominant strategy and wait for the other player to catch up (Hyndman et al. (2009)). In battle of the sexes games, experienced players can also demonstrate alterations to the other player with their observable choices (Cason et al. (2013b)). Subjective principled-player learning is another approach to study interactions in long-run coordination games where players form subjective ideas of their opponents' behavior instead of quantified belief of actions (Sandroni (2000)). One of the recent breakthroughs is from Ioannou and Romero (2014b) and Ioannou and Romero (2014a). Instead of using action-based learning, they built a strategy-based learning model from a finite strategy space, a new mapping from history to strategies and an asynchronous belief updating process. Although it is difficult to define the strategic space and history in continuous time, their idea provides a new direction for understanding repeated interactions. These learning models provide a solid reference of how subjects interact in repeated coordination games though none of them is tested under continuous time treatments.

Studying coordination games in the continuous time contributes new findings to the literature in both fields of repeated coordination and continuous time interaction. On one hand, it could provide another method of solving coordination failure, which has not been tested before. On the other hand, it enriches our understanding of how continuous time interaction impacts human coordination and cooperation.

2.3 Theoretical Predictions

2.3.1 Nash Equilibria for One-Shot Games

The paper focuses on two standard coordination games: stag hunt (SH) and battle of the sexes (BOS). In each game, three payoff matrices are implemented to test the consistency of the treatment effects. Table 2.1 shows the payoff matrices.

Table 2.1: Payoff bimatrices of stag hunt games and battle of the sexes games.

SH			BOS		
	a	b		a	b
A	(x,x)	(0,y)	A	(d,c)	(0,0)
B	(y,0)	(z,z)	B	(0,0)	(c,d)

Row players choose between A (top) and B (bottom). Column players choose between a (left) and b (right).

To simplify the notation, let (p, q) denotes players mixture $(pA+(1-p)B, qa+(1-q)b)$. Three payoff matrices of stag hunt games are: SH0.6R ((x,y,z)=(450,420,120)), SH1R ((x,y,z)=(450,400,200)) and SH2R ((x,y,z)=(450,350,400)) (Battalio et al., 2001). On one hand, all three payoff matrices have the same pure Nash equilibria (1,1) and (0,0) ((A,A) and (B,B)) and mixed Nash equilibrium ((0.8,0.8)) ((0.8A+0.2B, 0.8A+0.2B)). On the other hand, they are different in the optimization premium (Battalio et al., 2001, defined as the expected payoff difference between the two strategies, which shows the steepness of the payoff functions near the equilibrium. For row players, given column players' mixture q, $OP(q) = \pi(A, q) - \pi(B, q) = \delta(q - q^*)$, where $\delta = x - y + z$ and q^* refers to the column player's mixed Nash strategy) and relative riskiness (Dubois et al., 2012, defined as the ratio of the expected payoff ranges between safe and risky strategies. For row player, $RR = \frac{|y-z|}{x}$). SH0.6R has the lowest optimization premium and the highest relative riskiness so players are predicted to be more likely to play the payoff-dominant strategy. On the contrary, SH2R has the highest optimization premium and the lowest relative riskiness, which makes the risk-dominant strategy more attractive. SH1R is between these two payoff matrices.

Three payoff matrices of battle of the sexes games are: BOSha ((d,c)=(400,40)), BOSma ((d,c)=(400,160)) and BOSla ((d,c)=(400,280)) (Anbarci et al., 2017). All three payoff matrices have the same pure Nash equilibria ((1,1) and (0,0)) but different mixed Nash equilibrium ((0.91,0.09) for BOSha, (0.71,0.29) for BOSma, (0.59,0.41) for BOSla). The difference between payoff matrices is mainly about how asymmetric the payoff is in its two pure Nash equilibria (Lau and Mui (2008)). BOSla apparently has the lowest level of asymmetry while the payoffs are highly asymmetric in BOSha. BOSma is between these two payoff matrices. Players are less likely to coordinate when the level of asymmetry is higher.

Both games have the same two pure Nash equilibria and one distinct mixed Nash equilibrium. The Nash equilibria provide a reference of potential convergence in the laboratory experiments. In most cases, the literature focuses on two pure Nash equilibrium as they are most frequently played. There are two other normative reasons that explain the importance of pure Nash equilibria. In the evolutionary context, although there exists a saddle path that leads to mixed strategy equilibrium, the population mixture converges to either of the two pure Nash equilibria. Mixed strategy equilibrium is also less efficient than any of the pure Nash equilibria. Given the instability of mixed strategy Nash equilibrium, there should be little difference between pure action sets and mixed action sets, as the pure Nash equilibria are the same between two action sets treatments.

2.3.2 Comparing Continuous and Discrete Time Repeated Interactions

Literature in repeated coordination games explains the repeated interactions mainly through strategic play and belief updating. However, as discussed in literature on continuous time theory, models that work in discrete time cannot be directly copied to the continuous time environment because the concept of "period" is not well-defined in continuous time. History-dependent strategies thus require a new mapping between continuous time history to strategies. Similarly, the strategies that describe the future by "period" such as turn taking strategies can no longer be implemented. Although the basic ideas of learning and coordination in both SH and BOS do not vary between continuous time and discrete time environments, strategies need to be redefined to meet the definitions in continuous time environments.

However, if we focus on the difference between the time treatments, some qualitative hypotheses can be summarized easily. Although human beings cannot react and respond in milliseconds, the reaction lag is usually much lower than the length of "period" in repeated games. Players interact with their counterpart, update their belief and adjust their strategies much more frequently in continuous time than in discrete time, which speed up the convergence to equilibrium in all types of repeated coordination models. The nature of continuous time interaction also causes asynchronous play and further facilitates coordination.

In models with randomized experimentation, players can experiment and adjust strategies faster than they do in discrete time. For instance, the mixed strategy randomization in battle of the sexes games in discrete time with homogeneous reaction lag can be easily transformed to a continuous time version. At every time tick where players can adjust their strategy, they pick one of two possible actions with a mixed strategy p^* that maximizes the intertemporal payoff. If they observe a mismatch at the current time tick t, they continue to play the mixed strategy. If they observe coordination at t, they stop the mixed strategy and stay at the current equilibrium (or alternate between two equilibria). The game becomes even simpler if players share heterogeneous reaction lags: fast players immediately converge to the other players' location and the coordination is then satisfied. In the learning models, the total time consumed in the learning stage can be greatly reduced with any learning mechanisms given the high frequency interactions. The continuous time environment also provides a richer set of observable data per unit of time than that in the discrete time. Both speed up convergence to Nash equilibria. Another benefit from continuous time treatment is that players have a higher expected utility of playing the risky actions in continuous time than in discrete time. Players can also easily switch back to safe actions if the other player does not cooperate. Both benefits reduce strategic uncertainty and make them more willing to try risky actions and perform strategic teaching during the game.

As discussed above, continuous time interaction is expected to enhance coordination and cooperation in both SH and BOS. In perfect continuous time, the limit of players' reaction lag goes to 0 and players have the ability to reach instantaneous coordination, especially in SH. Due to positive reaction lag, subjects in the experiment are expected to coordinate at a slower speed compared to that in perfect continuous time but still faster than the convergence in discrete time.

However, is the efficiency always increased with the continuous time environment? The answer to the question could be ambiguous in BOS. On one hand, shortening the learning stage apparently raises efficiency. On the other hand, players suffer from an efficiency loss in the coordination stage each time they alternate, as players have reaction lags and the player who follows the alternation needs time to switch. The loss becomes larger when the game is longer. In discrete time, however, once the alternation is formed, players alternate without loss of efficiency. The idea will be further explored in the appendices of this paper.

2.4 Experimental Design

The data is collected from 10 sessions run between October 26th to November 15th in 2018. 5 sessions applied stag hunt games and the others applied battle of the sexes games. In each session, the experiment used a balanced 3 x 2 x 2 full-factorial within-subjects design with 3 types of payoff matrices, 2 time treatments (continuous and discrete) and 2 action sets treatments (pure and mixed). Overall, the paper implements a balanced 2 x 3 x 2 x 2 full-factorial design, where the selection between SH and BOS is between-subjects and other treatments are within-subjects.

80 subjects (42 in stag hunt sessions and 38 in battle of the sexes sessions) were recruited on UCSC LEEPS online platform ORSEE. In each session, subjects played 36 periods of 60-second bi-matrix games. Each subject was randomly paired anonymously each period with a counterpart so the experiments use the random matching protocol between periods and fixed matching protocol within each period. The experiment uses the continuous bi-matrix program on software called oTree/Redwood. The average payment is 19 US dollars and the length of the sessions is about 1 hour and 20 minutes, with a 20-minute instruction, a 50-minute gameplay, and a 10-minute closing.

In each session, all 36 60-second periods are equally divided into 3 blocks. Table 2.2 shows the design for both stag hunt sessions and battle of the sexes sessions in block 1, respectively. Subjects experienced all 12 combinations of treatments in each block. The design in block 2 is similar to that in block 1 but in reverse order. Block 3 and block 1 share the same design. Figure 2.1 and 2.2 show two samples of user interface subjects faced in the experiment. In each period, subjects had access to the previous payoffs and actions of both players in the current period, which are displayed on the right side of the screen. Before the blocks start, subjects play a 2-period practice with the same treatments but a different payoff matrix.

In continuous time treatments, payoffs and actions move in real time, are calculated and exchanged between players every 50 milliseconds and are recorded every 500 milliseconds.

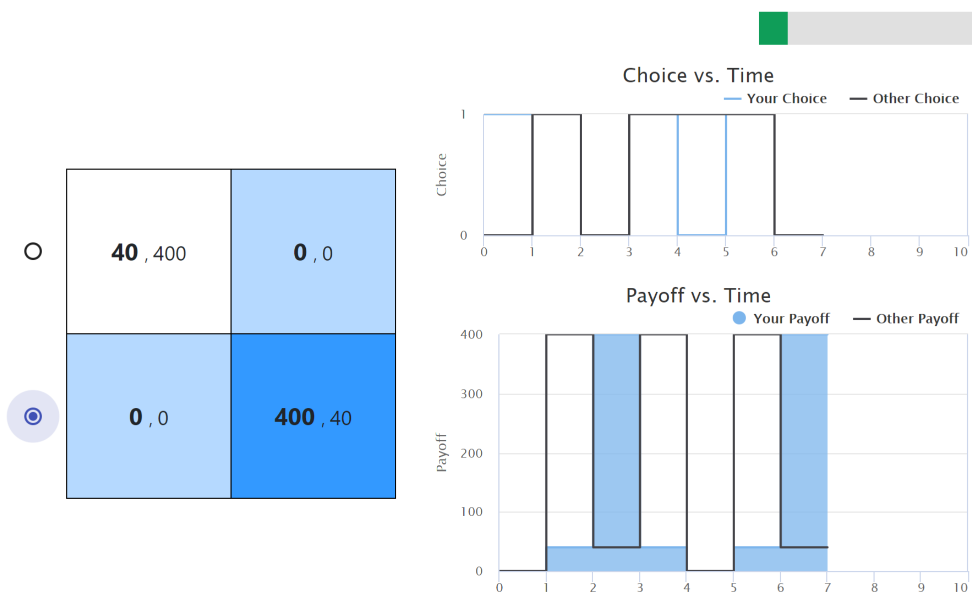
In discrete time treatments, each period is equally divided into 10 subperiods, each lasting for 6 seconds. The green fill bar at the top shows the time remaining of the current subperiod. Subjects' subperiod payoff depends only on the last choices both players make before the end of the subperiod. The data is recorded every subperiod.

In pure action sets treatments, subjects can only choose two options A and B. Subjects can freely click back and forth between the rows using the radio buttons and the current chosen strategies will be shown in the blue shade on the payoff matrix.

In mixed action sets treatments, subjects make their choices in an action set with a continuous number of actions by clicking the slider next to the heat map. The action sets are constructed by adding all possible mixtures between A and B. Although the mixed action sets treatment does not necessarily allow subjects to play mixed strategy, both action sets treatments have the same corner Nash equilibria and mixed action sets treatment is visually more complicated than pure action sets treatment. The heat map shows the payoff of all possible profiles, where the horizontal line shows the subject's current chosen action and the vertical line shows the counterpart's current chosen one. The counterpart's payoff heat map is displayed on the top left of the screen.

At the end of each session, subjects were asked to complete a post experimental survey about how they choose their actions in the experiment. The results of the survey are considered as a supplement material to study subjects' behavior and are explained in Appendix A.

Figure 2.1: Subjects interface in practice periods for games in the pure action sets and discrete time treatment.



2.5 Results in Stag Hunt Games

Data from block 1, the first 2 subperiods in discrete time treatments and equivalently the first 12 seconds in continuous time treatments are dropped to eliminate learning during the experiment and unintentional mismatch caused by the transition between periods. In this case, 8 observations in discrete time treatments and 96 obser-

Figure 2.2: Subjects interface in practice periods for games in the mixed action sets and continuous time treatment.

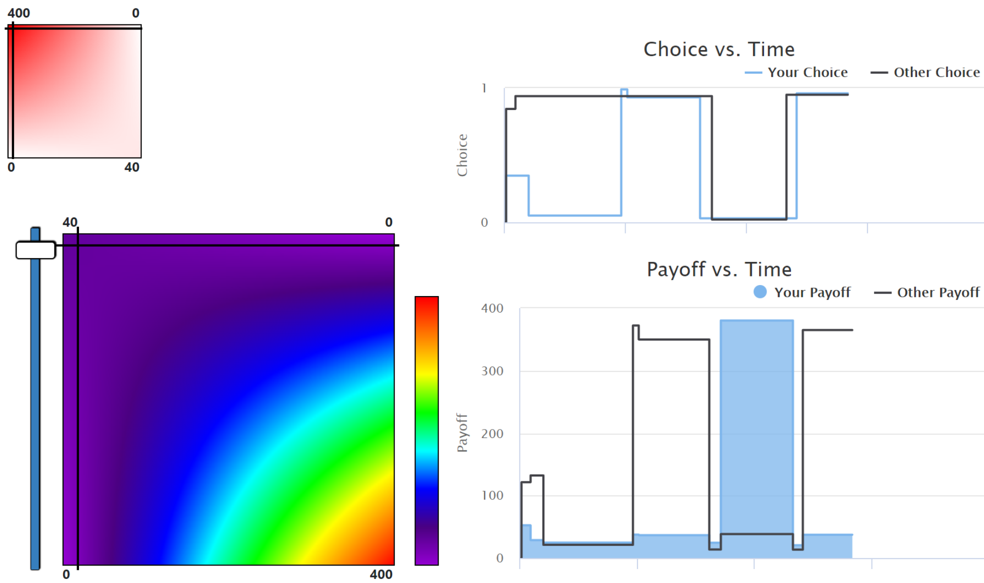


Table 2.2: Design table of block 1 for stag hunt sessions and battle of the sexes sessions.

Period	Matrix in SH	Matrix in BOS	Time	Action sets
1	SH2R	BOSla	Discrete	Pure
2	SH1R	BOSma	Discrete	Pure
3	SH0.6R	BOSha	Discrete	Pure
4	SH0.6R	BOSha	Discrete	Mixed
5	SH1R	BOSma	Discrete	Mixed
6	SH2R	BOSla	Discrete	Mixed
7	SH2R	BOSla	Continuous	Pure
8	SH1R	BOSma	Continuous	Pure
9	SH0.6R	BOSha	Continuous	Pure
10	SH0.6R	BOSha	Continuous	Mixed
11	SH1R	BOSma	Continuous	Mixed
12	SH2R	BOSla	Continuous	Mixed

The order of time treatments and action sets treatments are the same in both sessions and the only difference in design is the order of payoff matrices. In each session, all 36 periods are equally divided into 3 blocks. Block 2 is similar to block 1 but in reverse order and block 3 is the same as block 1.

vations in continuous time treatments are collected for each pair of subjects in blocks 2 and 3 and there are 504 pairs left in SH.

To study the behavior at pair level instead of at individual level, strategy profiles are classified into different types. In stag hunt games, strategy profiles can be classified into 4 types: "payoff dominant" ($(p, q \geq 0.9)$, "risk dominant" ($p, q \leq 0.1$), "mixed NE" ($p, q \in (0.7, 0.9)$) and "mismatch" (the rest of the profiles). Table 2.3 provides a summary of classified strategy profiles while focusing on 4 corner strategy profiles based on the classification. Overall speaking, subjects play more payoff dominant equilibrium and less mismatch in continuous time treatments than in discrete time treatments and in pure action sets treatments than in mixed action sets treatments. They also play more risk dominant equilibrium and less payoff dominant equilibrium when the payoff matrix favors risk dominance. Mixed NE is rarely played.

Table 2.3: Summary table for classified strategy profiles in stag hunt games.

	payoff dominant	risk dominant	mixed NE	mismatch	observations
SH0.6R,PD	0.63	0.21	-	0.15	336
SH0.6R,PC	0.81	0.11	-	0.08	4032
SH0.6R,MD	0.43	0.16	0.03	0.38	336
SH0.6R,MC	0.64	0.17	0.01	0.17	4032
SH1R,PD	0.56	0.28	-	0.16	336
SH1R,PC	0.80	0.16	-	0.05	4032
SH1R,MD	0.40	0.23	0.01	0.36	336
SH1R,MC	0.58	0.23	0.00	0.18	4032
SH2R,PD	0.43	0.47	-	0.10	336
SH2R,PC	0.67	0.27	-	0.06	4032
SH2R,MD	0.37	0.30	0.01	0.32	336
SH2R,MC	0.59	0.27	0.01	0.13	4032

First four columns show the frequency of each type of strategy profile. Last column shows the total number of observations of each treatment.

2.5.1 Treatment Effects

The summary tables show an overview of subjects' choices but little about the dynamics, which makes it necessary to check the dynamics by the classified strategy profiles we defined in the previous subsection and show how the fraction of each type changes over time. Figure 2.3 shows how the fraction of each type being played changes over time in all treatments in stag hunt games. Under the same payoff matrix, there exists an obvious difference between the two time treatments and the two action sets treatments. Pairs are more likely to play payoff-dominant equilibrium in continuous time than in discrete time and in pure action sets treatments than in mixed action sets treatments. The fraction of mismatches is low in pure action sets treatments and is decreasing in mixed action sets treatments. Furthermore, mixed Nash equilibrium is rarely played in all mixed action sets treatments. Between payoff matrices, pairs are less likely to play payoff-dominant equilibrium and are more likely to switch to risk-dominant equilibrium when the optimization premium is getting higher.

Table 2.4 shows some statistical evidence. As "mixed NE" is rarely played, I only create three "type dummy variables" for type "payoff dominant", "risk dominant" and "mismatch". Another type "either NE" combines the first two types and denotes observations that stay at either pure NE. To focus more on the treatment effects instead of relations between types, I run "one vs all" logistic regressions as follows:

$$P(Y=1|X=x) = \frac{e^{x'\beta}}{1+e^{x'\beta}} \tag{2.1}$$

$$x'\beta = \beta_0 + \beta_1 * treatments. \tag{2.2}$$

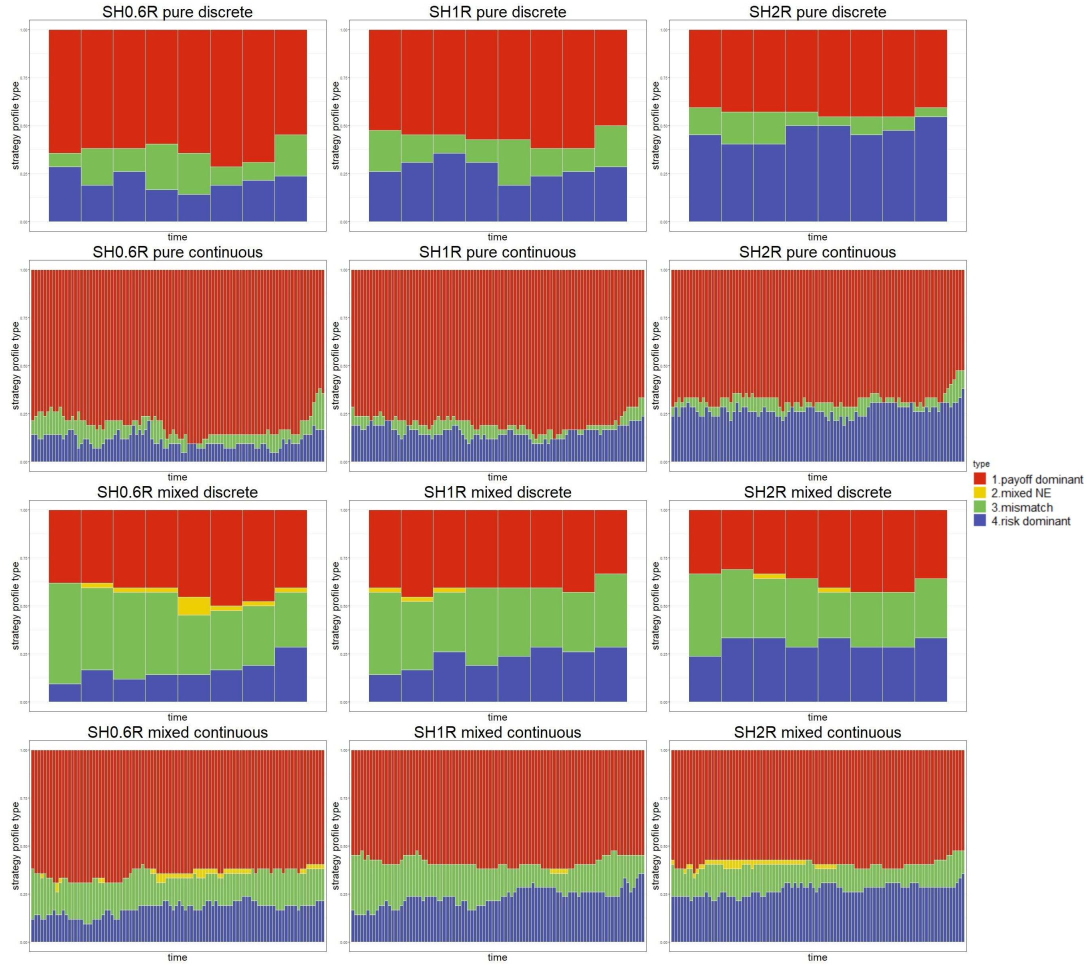
Here Y represents either of the four type dummies. Treatments include all basic treatments and their intersections. As this is an experiment with a 3 x 2 x 2 design, there exist 11 independent treatment terms and "mixed action sets, discrete time, SH1R" is served as the baseline. There is also a dummy variable for the second half of each period and another dummy for block 3 to study learning within each period and between blocks.

Results from Table 2.4 reports all βs in (2) and confirms impressions from Figure 2.3. Both continuous time treatment and the pure action sets treatment affect the probability of playing pure NE and "mismatch" just as what we have seen in Figure 2.3. But if we consider two pure NE separately, only continuous time treatments have a significant effect on "payoff-dominant" equilibrium. Furthermore, the coefficient of within period learning shows significant learning effects: pairs learn to coordinate from the first half to the second half of the period. Another finding is that optimization premium weakly affects pairs' probability of playing risk-dominant equilibrium and the direction is consistent with previous papers: the coefficient of SH2R and SH0.6R in column 2 is large but insignificant.

2.5.2 Classification at Pair Level

Showing how each fraction of classified types moves over time provides an overview of the dynamics within each period. However, as the data is aggregated, the real dynamics of each pair is lost, which makes it necessary to check the dynamics and classification problem at the pair level. Here pairs are classified into various types and

Figure 2.3: Classification over time in stag hunt games.



payoff dominant(red), risk dominant(blue), mixed NE(yellow) and mismatch(green). The figures show how each fraction of dynamics types changes in the period.

show how pairs' types change between treatments.

In stag hunt games, pairs can be classified into 4 types: "payoff dominant" (pairs play payoff-dominant equilibrium for at least 75% of the time), "risk dominant" (pairs play risk-dominant equilibrium for at least 75% of the time), "mixed NE" (pairs play mixed Nash equilibrium for at least 75% of the time) and "mismatch" (the rest of

Table 2.4: One vs all logistic regression of classification over time in SH clustered at pair level.

Treatments	payoff	risk	either NE	mismatch
SH2R	-0.15(0.424)	0.39(0.401)	0.17(0.365)	-0.16(0.361)
SH0.6R	0.09(0.412)	-0.42(0.400)	-0.19(0.340)	0.10(0.327)
continuous	0.73(0.424)*	0.03(0.419)	0.97(0.396)**	-0.94(0.391)**
pure	0.64(0.401)	0.25(0.398)	1.11(0.329)***	-1.07(0.325)***
continuous_pure	0.40(0.602)	-0.76(0.621)	0.41(0.547)	-0.44(0.544)
continuous_SH2R	0.16(0.597)	-0.18(0.596)	0.12(0.589)	-0.24(0.588)
continuous_SH0.6R	0.15(0.591)	0.04(0.619)	0.15(0.545)	-0.14(0.529)
pure_SH2R	-0.38(0.583)	0.45(0.555)	0.36(0.493)	-0.37(0.490)
pure_SH0.6R	0.21(0.566)	0.06(0.577)	0.23(0.477)	-0.15(0.469)
continuous_pure_SH2R	-0.28(0.844)	0.04(0.849)	-0.86(0.774)	0.98(0.773)
continuous_pure_SH0.6R	-0.36(0.845)	-0.08(0.903)	-0.76(0.766)	0.76(0.755)
second half	0.05(0.056)	0.13(0.069)*	0.30(0.102)***	-0.30(0.106)***
block 3	0.35(0.231)	-0.34(0.249)	0.20(0.242)	-0.20(0.236)
Constant	-0.60(0.327)*	-1.13(0.310)***	0.27(0.283)	-0.30(0.275)
Observations	26,208	26,208	26,208	26,208

Dependent variables are dummy variables of the type of strategy profiles. Payoff and risk refer to payoff dominant type and risk dominant type, respectively. Either NE refers to the observations that play either pure NE. Standard deviation shown in the parentheses. Significance level: *** 0.01 ** 0.05 * 0.1.

the pairs that can not be classified into any of the types above). The classification of pairs is completely based on the classification of observations in section 5.2 and the data is still based on the modified dataset where the first 20% of observations are dropped. As a result, 75% of the time refers to 6 subperiods in discrete time and 36 seconds in continuous time

Figure 2.4 shows the frequency and percentage of each type in all treatments and the results are similar to those in section 5.2. Under the same payoff matrix, pairs are more likely to stay in payoff-dominant equilibrium and less likely to stay mismatching in continuous time than in discrete time and in pure action sets treatments than in mixed action sets treatments. Between payoff matrices, pairs are more likely to play payoff-dominant equilibrium instead of risk-dominant equilibrium when the optimization premium of the payoff matrix is higher. Not a single pair stays in mixed Nash equilibrium in mixed action sets treatments.

Table 2.5 shows some statistical evidence. Four type dummies are created for type "payoff dominant", "risk dominant", "mismatch" and "either EQ" that combines the first two types. The results supported what has been shown in Figure 2.4 about the treatment effects of continuous time treatments on payoff-dominant equilibrium and mismatch. The coefficient of pure action sets treatments becomes significant if we combine two types of coordination. Although the coefficients are not significant, the effect of payoff matrices still affects the probability that a pair stays at risk-dominant equilibrium given the large magnitude of the coefficients.

The results in stag hunt games can be summarized as follows.

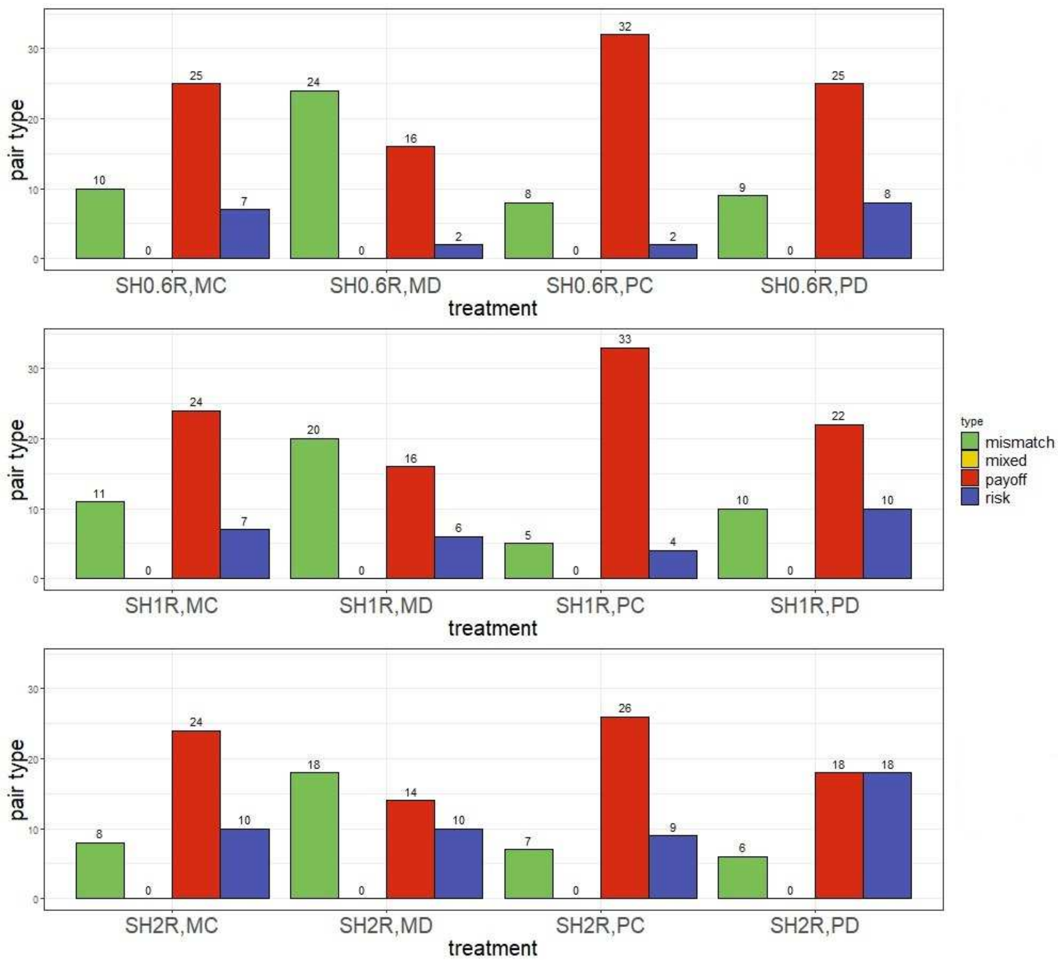
Result 1:

(a) *In stag hunt games, subjects are more likely to coordinate at pure Nash equilibria in continuous time treatments and pure action sets treatments.*

(b) *Subjects are more likely to converge to payoff-dominant equilibrium in stag hunt games in continuous time than in discrete time.*

(c) *Subjects also learn to coordinate within each period.*

Figure 2.4: Classification at pair level in SH.



Numbers show the number of pairs that coordinate at certain equilibrium for at least 75 percent of the time in a certain period.

2.6 Results in Battle of the Sexes Games

Data is dropped in BOS in the same way as it is dropped in SH. In BOS, 456 pairs remain in the dataset. In battle of the sexes games, strategy profiles can be classified into 6 types: “coordinate(A,A)” ($p, q \geq 0.9$), “coordinate(B,B)” ($p, q \leq 0.1$),

Table 2.5: One vs all logistic regression of classification at pair level in SH.

Treatments	payoff	risk	either EQ	mismatch
SH2R	-0.21(0.457)	0.63(0.571)	0.19(0.439)	-0.19(0.439)
SH0.6R	-0.00(0.451)	-1.21(0.849)	-0.38(0.439)	0.38(0.439)
continuous	0.78(0.446)*	0.18(0.606)	0.94(0.468)**	-0.94(0.468)**
pure	0.58(0.444)	0.63(0.571)	1.07(0.477)**	-1.07(0.477)**
continuous_pure	0.43(0.661)	-1.27(0.880)	-0.10(0.760)	0.10(0.760)
continuous_SH2R	0.21(0.636)	-0.18(0.794)	0.22(0.686)	-0.22(0.686)
continuous_SH0.6R	0.10(0.632)	1.21(1.031)	0.51(0.669)	-0.51(0.669)
pure_SH2R	-0.18(0.635)	0.25(0.745)	0.44(0.721)	-0.44(0.721)
pure_SH0.6R	0.29(0.631)	0.92(1.003)	0.52(0.683)	-0.52(0.683)
continuous_pure_SH2R	-0.64(0.917)	0.26(1.130)	-1.24(1.094)	1.24(1.094)
continuous_pure_SH0.6R	-0.53(0.932)	-1.67(1.467)	-1.20(1.050)	1.20(1.050)
block 3	0.30(0.187)	-0.25(0.238)	0.18(0.211)	-0.18(0.211)
Constant	-0.64(0.333)*	-1.67(0.455)***	0.01(0.327)	-0.01(0.327)
Observations	504	504	504	504

Dependent variables are dummy variables of the type of pairs. Payoff and risk refer to payoff dominant pairs and risk dominant pairs, respectively. Either EQ combines the first two types of pairs. Standard deviation shown in the parentheses. Significance level: *** 0.01 ** 0.05 * 0.1.

"mismatch ohenry" ($p \leq 0.1, q \geq 0.9$, showing they choose strategies that are benefical to the other player, the type is named after the novel called "The Gift of the Magi"), "mismatch selfish" ($p \geq 0.9, q \leq 0.1$, showing they choose strategies that are benefical to themselves), "mixed NE" (difference between subject's mixture and mixed Nash equilibrium is less than 0.05) and "mismatch" (the rest of the profiles). Table 2.6 shows the summary table of battle of the sexes games. The frequency of playing two pure Nash equilibria is almost equally likely in all treatments. However, mismatches are more common in battle of the sexes games than in stag hunt games, especially the mismatches when both subjects prefer their advantageous strategy. The comparison of coordination and mismatch between the two time treatments and the two action sets treatments is similar to the results in stag hunt games. Though the result comes with no statistical testing and does not consider the heterogeneity among pairs, it provides a first impression of the data.

Table 2.6: Summary table for classified strategy profiles in battle of the sexes games.

	(A,A)	(B,B)	mixed NE	mismatch selfish	mismatch ohenry	mismatch	observations
BOSla,PD	0.27	0.33	-	0.30	0.11	-	304
BOSla,PC	0.38	0.44	-	0.15	0.03	-	3648
BOSla,MD	0.15	0.14	0.00	0.11	0.04	0.55	304
BOSla,MC	0.23	0.25	0.00	0.13	0.01	0.38	3648
BOSma,PD	0.33	0.27	-	0.31	0.09	-	304
BOSma,PC	0.35	0.43	-	0.19	0.02	-	3648
BOSma,MD	0.16	0.14	0.01	0.15	0.02	0.51	304
BOSma,MC	0.24	0.25	0.01	0.14	0.02	0.34	3648
BOSha,PD	0.22	0.31	-	0.38	0.10	-	304
BOSha,PC	0.34	0.37	-	0.27	0.03	-	3648
BOSha,MD	0.07	0.09	0.03	0.18	0.02	0.62	304
BOSha,MC	0.17	0.18	0.01	0.14	0.01	0.49	3648

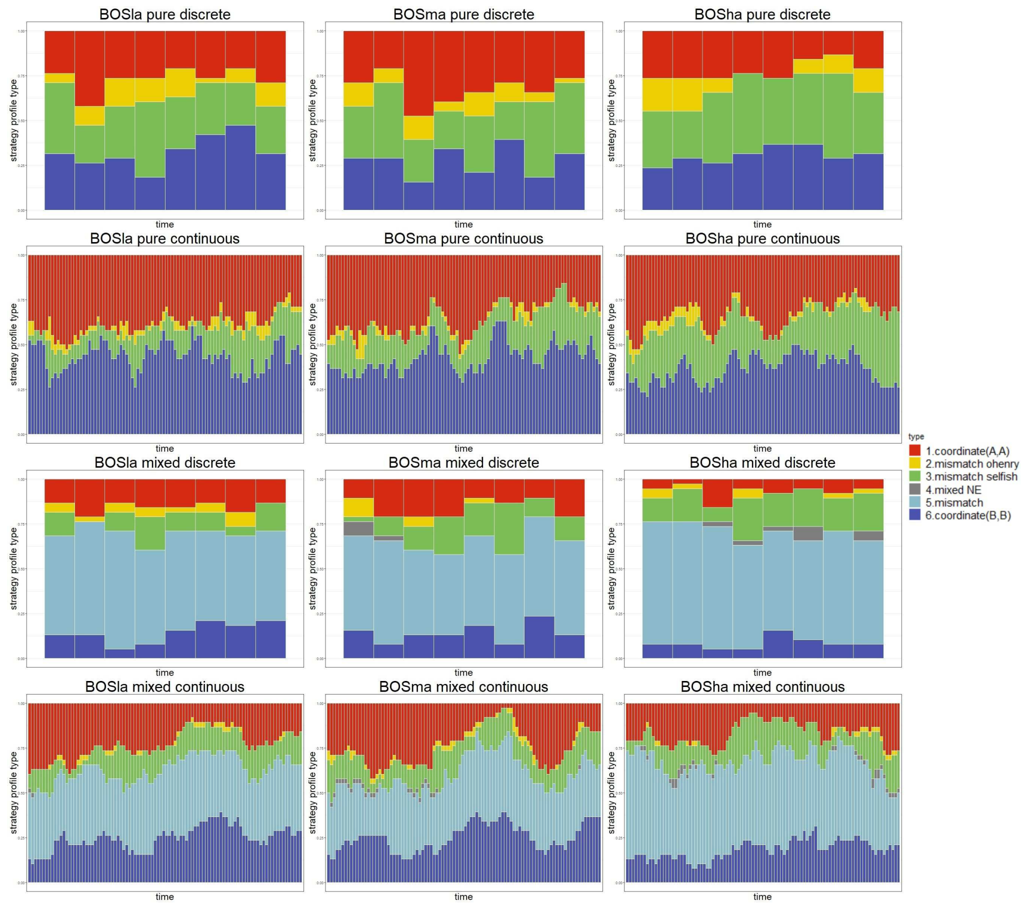
First four columns show the frequency of each type of strategy profile. Last column shows the total number of observations of each treatment.

2.6.1 Treatment Effects

Figure 2.5 shows how the fraction of each type being played changes over time in all treatments in battle of the sexes games. Under each payoff matrix, it seems that pairs are more likely to coordinate in continuous time than in discrete time and in pure action sets treatments than in mixed action sets treatments. What is more, neither of the pure Nash equilibria dominates the other one. Comparing two kinds of mismatches at corners, the fraction of pairs playing selfishly is always higher than the fraction of pairs playing altruistically. However, there exists a large area of general mismatches in mixed action sets treatments and the fraction seems consistent over time, which indicates that the coordination problem might be worse in battle of the sexes games than that in stag hunt games. Similar to previous results, mixed Nash equilibrium is rarely played. Between payoff matrices, there is little difference in pure action sets treatments but pairs are less likely to coordinate in mixed action sets treatments when the level of payoff asymmetry is higher.

Table 2.7 shows some statistical evidence. Four type dummies are created for type “coordinate at (A,A)”, “coordinate at (B,B)”, “mismatch” and “either NE” which combines the first two types. “mismatch” includes all 3 kinds of mismatches in the previous classification. “mixed action sets, discrete time, BOSma” is served as the baseline. The results show that both continuous time treatments and pure action sets treatments increase the probability of playing pure Nash equilibria and lower the probability of mismatches, which is consistent with Figure 2.5. Learning effects also exist within each period and between blocks. However, in the second half of the period pairs are less likely to play (A,A) but more likely to play (B,B). One possible reason could be that for alternating dynamics, pairs tend to start with (A,A), which causes them to play more (B,B) in the second half of the period. It is also worth noticing that the probability of mismatch increases in the second half of the game and the probability of coordination decreases, which is counter-intuitive and possibly results from the fact that pairs systematically mismatch when switching between two pure Nash equilibria in continuous time. What is more, a high level of payoff asymmetry increases the probability of mismatches.

Figure 2.5: Classification over time in battle of the sexes games.



(A,A)(red), (B,B)(blue), mismatch ohenry(yellow), mismatch self(green), mixed NE(grey) and other mismatches(light blue). The figures show how each fraction of dynamics types changes in the period.

2.6.2 Classification at Pair Level

In battle of the sexes games, pairs can be classified into 5 types: "alternating" (pairs play either pure Nash equilibria for at least 75% of the time and both Nash equilibria exist at least 37.5% of time), "one NE" (pairs play either pure Nash equilibria

Table 2.7: One vs all logistic regression of classification over time in BOS clustered at pair level.

Treatments	(A,A)	(B,B)	either NE	mismatch
BOSla	-0.05(0.329)	0.03(0.352)	-0.02(0.310)	0.08(0.303)
BOSha	-0.96(0.388)**	-0.57(0.346)	-0.87(0.343)**	0.74(0.320)**
continuous	0.52(0.284)*	0.73(0.311)**	0.83(0.307)***	-0.80(0.296)***
pure	0.93(0.270)***	0.83(0.252)***	1.25(0.274)***	-1.19(0.266)***
continuous_pure	-0.39(0.342)	-0.02(0.356)	0.07(0.398)	-0.10(0.389)
continuous_BOSla	-0.02(0.417)	-0.05(0.449)	-0.05(0.433)	0.02(0.425)
continuous_BOSha	0.51(0.466)	0.16(0.467)	0.28(0.463)	-0.17(0.439)
pure_BOSla	-0.22(0.387)	0.23(0.398)	0.00(0.390)	-0.06(0.384)
pure_BOSha	0.43(0.441)	0.73(0.381)*	0.59(0.414)	-0.46(0.395)
continuous_pure_BOSla	0.41(0.497)	-0.18(0.520)	0.28(0.564)	-0.25(0.558)
continuous_pure_BOSha	-0.06(0.539)	-0.59(0.527)	-0.43(0.582)	0.32(0.562)
second half	-0.47(0.105)***	0.33(0.092)***	-0.13(0.075)*	0.15(0.076)*
block 3	0.20(0.116)*	0.30(0.126)**	0.47(0.161)***	-0.45(0.158)***
Constant	-1.47(0.249)***	-2.18(0.252)***	-1.00(0.249)***	0.92(0.240)***
Observations	23,712	23,712	23,712	23,712

Dependent variables are dummy variables of the type of strategy profiles. (A,A) and (B,B) refer to types that coordinate at (A,A) and (B,B), respectively. Either NE refers to the observations that play either pure NE. Standard deviation shown in the parentheses. Significance level: *** 0.01 ** 0.05 * 0.1.

for at least 75% of the time but the pair does not play alternation), "mismatch selfish" (pairs play the selfish type of mismatch for at least 75% of the time), "mixed NE" (pairs play mixed Nash equilibrium for at least 75% of the time) and "mismatch" (the rest of the pairs that can not be classified into any of the types above). Figure 2.6 shows the frequency and percentage of each type in all treatments and the results are similar to those in section 5.2. Under the same payoff matrix, pairs are more likely to coordinate and less likely to mismatch in continuous time than in discrete time and in pure action sets treatments than in mixed action sets treatments. Pairs also play alternating dynamics more frequently in continuous time or in the pure action sets treatments. Between payoff matrices, pairs are less likely to coordinate when the level of asymmetry of the payoff matrix is higher. No pair stays in mixed Nash equilibrium in mixed action sets treatments but there are a few pairs that consistently play selfishly and do not compromise.

Table 2.8 shows some statistical evidence. Four type dummies are created for type "alternating", "oneNE", "mismatch" and "either EQ" that combines the first two types. "mismatch" includes all 3 types of mismatches in the previous classification. Pairs are more likely to coordinate and less likely to mismatch in continuous time than in discrete time and in pure action sets than in mixed action sets. The treatment effects on "alternating" types are large but not significant. The payoff matrices also have an insignificant effect on the coordination rate.

The results in battle of the sexes games can be summarized as follows.

Result 2:

(a) *In battle of the sexes games, subjects are more likely to coordinate at pure Nash equilibria in continuous time treatments and pure action sets treatments.*

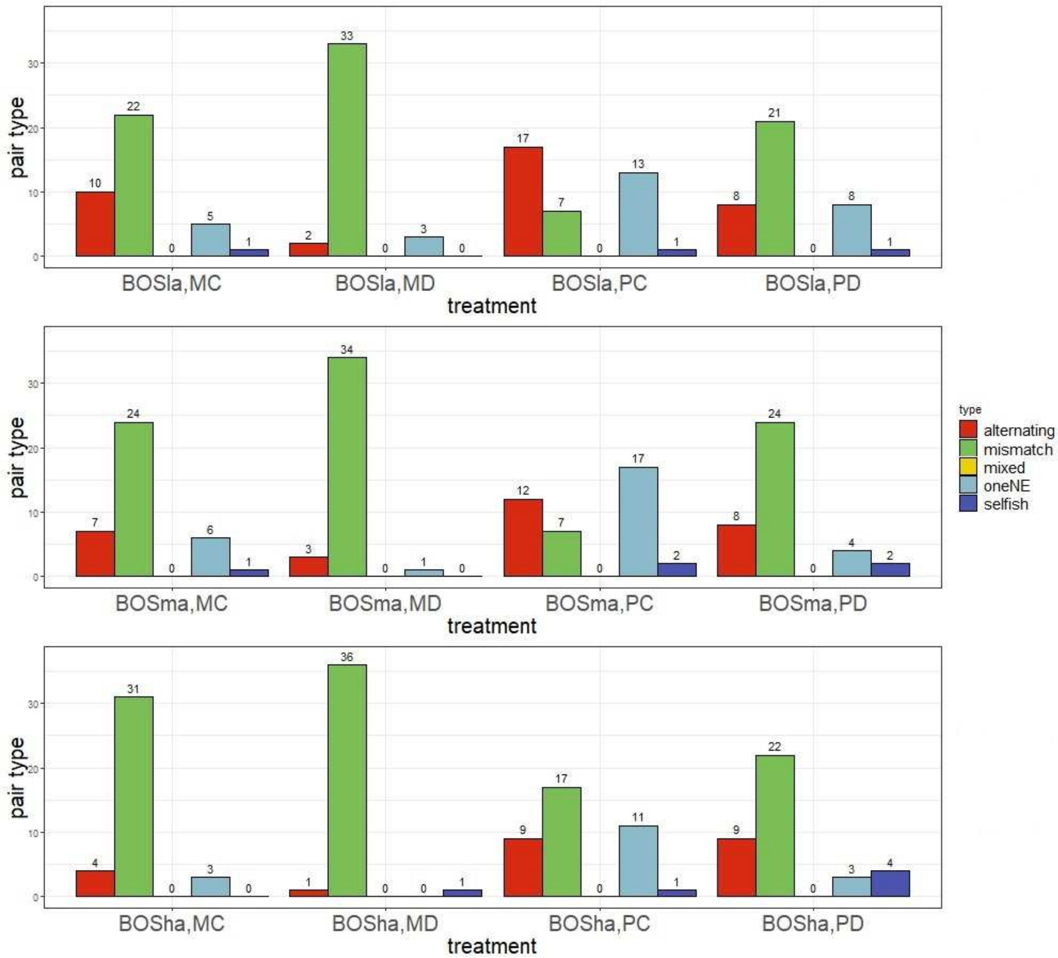
(b) *Subjects are more likely to converge to alternating dynamics in battle of the sexes games in continuous time than in discrete time. The treatment effects are large but not significant.*

(c) *High level of payoff asymmetry reduces the coordination rate.*

(d) *Similar to stag hunt games, learning exists between blocks and within each period.*

(e) *Coordination problem in BOS is worse than that in SH.*

Figure 2.6: Classification at pair level in BOS.



Numbers show the number of pairs that coordinate at certain equilibrium for at least 75 percent of the time in a certain period.

2.7 Conclusions

As has been studied both in theory and in laboratory experiments, continuous time interaction is believed to be an effective method to improve human cooperation (e.g. Friedman and Oprea (2012)). This paper applies continuous time environments to

Table 2.8: One vs all logistic regression of classification at pair level in BOS.

Treatments	alternating	one NE	either EQ	mismatch
BOSla	-0.43(0.944)	1.15(1.178)	0.25(0.714)	-0.25(0.714)
BOSha	-1.16(1.179)	-13.70(932.832)	-1.47(1.143)	1.47(1.143)
continuous	0.97(0.734)	1.94(1.107)*	1.49(0.630)**	-1.49(0.630)**
pure	1.14(0.722)	1.47(1.143)	1.37(0.634)**	-1.37(0.634)**
continuous_pure	-0.42(0.905)	-0.01(1.269)	0.46(0.815)	-0.46(0.815)
continuous_BOSla	0.89(1.097)	-1.37(1.348)	-0.03(0.859)	0.03(0.859)
continuous_BOSha	0.50(1.359)	12.92(932.833)	0.64(1.265)	-0.64(1.265)
pure_BOSla	0.43(1.099)	-0.34(1.351)	0.20(0.861)	-0.20(0.861)
pure_BOSha	1.31(1.302)	13.39(932.833)	1.47(1.245)	-1.47(1.245)
continuous_pure_BOSla	-0.33(1.323)	0.11(1.574)	-0.28(1.128)	0.28(1.128)
continuous_pure_BOSha	-1.05(1.556)	-13.29(932.833)	-1.70(1.448)	1.70(1.448)
block 3	0.30(0.247)	-0.07(0.271)	0.20(0.223)	-0.20(0.223)
Constant	-2.62(0.618)***	-3.57(1.022)***	-2.24(0.542)***	2.24(0.542)***
Observations	456	456	456	456

Dependent variables are dummy variables of the type of pairs. "alternating" and "one NE" refer to types that play alternating dynamics and stay at one pure Nash equilibrium, respectively. Either EQ combines the first two types of pairs. Standard deviation shown in the parentheses. Significance level: *** 0.01 ** 0.05 * 0.1.

the classic problem of coordination failure using stag hunt games and battle of the sexes games with a balanced 2 (the battle of the sexes and the stag hunt) x 2 (continuous and discrete time treatments) x 2 (pure and mixed action sets treatments) x 3 (three payoff matrices for each game) full-factorial experimental design.

The experimental results show how continuous time environments improve the efficiency in both games from different perspectives. Subjects consistently coordinate better in continuous time than in discrete time. In stag hunt games, subjects are more likely to reach the payoff-dominant equilibrium in continuous time than in discrete time. In battle of the sexes game, subjects prefer to alternate between two pure Nash equilibria when the time treatment is continuous. Apart from the two time environments, payoff matrices and the complexity of the action sets also affect subjects' coordinative behavior, though payoff matrices only show a weak influence. Comparing two coordinate games, stag hunt games receive a higher coordination rate in all treatments, while subjects are less cooperative in battle of the sexes games. This is suggested by the nature of two coordination games as people have common interests in stag hunt games but mixed motivations in battle of the sexes games. The results match our qualitative hypotheses in Section 3 that continuous time environments enhance coordination.

This paper provides instructive results to the literature by showing that continuous time interaction facilitates coordination and improves efficiency. The findings provide a method of solving coordination failure and enhance the general impression that continuous time interaction improves human cooperation. However, people may lose efficiency if they need to alternate among multiple focal points as there are mismatches between alternations. Furthermore, people can coordinate better when the problem is easier and when the parametrized incentives are stronger. Another investigation of battle of the sexes games in the appendices shows that subjects' switches between equilibrium status and mismatches are mainly caused by selfish motivations. The positive impact of continuous time interaction encourages firms and policy makers to adopt new technologies that enable high frequency of interaction such as high speed networks. The new technologies can potentially mitigate the coordination failure problem.

Some quantitative questions are still unsolved. First, as the concept of "period" does not exist in continuous time, what is the optimal frequency of alternation that players should follow in battle of the sexes games? Second, the learning algorithm in continuous time remains a puzzle and needs further study. Although the difference between learning in the two time environments seems obvious, how players learn and update belief in continuous time requires a quantitative modelling approach. If we follow Ioannou and Romero (2014b)'s discrete-time strategic learning, it would also be a fruitful topic to transform the history-dependent finite automation from discrete time to continuous time environment. Third, stylized facts in the experiment show that subjects also tend to compete for higher payoff instead of coordination, which is different from traditional theory prediction. To address this problem, future theoretical researchers can study similar games with turn taking dynamics under such a competitive framework.

Chapter 3

Taking Turns in Continuous Time

3.1 Introduction

In game theoretical models with repeated interactions, players usually make decisions in "discrete time"; the game is divided into a finite or infinite number of periods, and the players interact in one period after another. The theoretical work by Simon and Stinchcombe (1989) and Bergin and MacLeod (1993) introduced another time environment called "continuous time" in which the game moves in real time, and the players can adjust their strategies and affect the outcomes at almost any time during the game. Their most notable finding is that the continuous time environment supports full cooperation in finitely repeated social dilemma games[1], which is supported by recent laboratory experiments by Friedman and Oprea (2012). Subsequently, the continuous time environment has been applied to other game theoretical models, which have been

[1]The equilibrium is proved in "perfect continuous time" and "inertia continuous time" with ϵ-equilibria.

shown to support cooperation and improve equilibrium convergence (e.g., Oprea et al. (2014); Calford and Oprea (2017); Leng et al. (2018)).

However, whether the benefit of continuous time interactions can be extended to all games remains unclear. We consider 2-player battle of the sexes games in which continuous time interactions may harm social welfare. As a type of mixed-motive coordination game, battle of the sexes games are quite common in real-world scenarios (e.g., cooperative teamwork, resource allocation, and congestion problems). The players generally prefer distinct pure strategy Nash equilibria and, thus, may fail to coordinate. With the development of technology, people are currently facing such problems in online environments. For instance, parties may face online congestion problems when they need to access resources on multiple servers as follows: the parties may prefer a single server but accessing this server together substantially slows the connection speed, causing a low payoff for everyone. Since the problem occurs in an online environment, coordination may take place under online tracking systems, where both actions and performance can be adjusted, tracked and updated in real time. Would it be better to allow people to make decisions in discrete intervals or real time? An understanding of how the continuous time environment affects coordination could have vital empirical value and indicate whether people should accelerate or slow interactions with such problems.

What is the critical feature of battle of the sexes games that renders coordination difficult in continuous time? The recent literature concerning continuous time games mostly considers social dilemma games (e.g., the prisoner's dilemma) and games in which the efficient Nash equilibrium is difficult to reach (e.g., minimum effort games).

A common feature of these games in continuous time is that the players proactively make only a single decision and best respond to others' strategy in the equilibrium. However, the previous literature investigating battle of the sexes games in discrete time has found that players tend to take turns and play their preferred pure Nash equilibria when the game is played repeatedly. The equilibrium path is modeled as "alternating dynamics" (or "turn-taking dynamics") and is supported by many theories and laboratory experiments (e.g., Lau and Mui (2008); Ioannou and Romero (2014b); Arifovic and Ledyard (2018)). Although alternating dynamics achieve both efficiency and fairness between players, directly transferring such dynamics to a continuous time environment is difficult because a "turn" is not well-defined in continuous time. The players must determine how to coordinate both the order of alternations and how long they remain at each pure Nash equilibrium. Furthermore, the players must make decisions at multiple history-dependent timings and decide both whether to switch and how long to stay at each time. In contrast, in a discrete time setting, the players need to coordinate only the order of alternations and take turns. Even though a player can react quickly and asynchronously (Alós-Ferrer and Kern (2015)) in continuous time, it is still difficult to replicate alternating dynamics and achieve both efficiency and fairness.

This paper compares subjects' interactions in battle of the sexes games under continuous time with their interactions under discrete time to address the following three major research questions. First, will a continuous time environment facilitate coordination in battle of the sexes games? Second, what may cause the difference between the two time environments? Third, how do subjects alternate between Nash equilibria in continuous time? If the players choose another dynamics path, what other types of dynamics will they choose? In addition to the major comparison between the two time environments, our experiment applies three payoff matrices as a robustness check.[2]

This paper contributes to the literature from both empirical and theoretical perspectives. First, this paper offers practical suggestions for people who face coordinative tasks in online environments. Second, this paper provides evidence advancing the general theory of continuous time games because the alternating dynamics in battle of the sexes games are quite different from the equilibrium pattern in the current literature. Third, as people can move asynchronously in continuous time, the order of alternations and the duration of the Nash equilibrium reveal what motivates players to take turns from a new perspective.

Our experimental results show that subjects behave quite differently in the two time environments. Although continuous time interactions accelerate initial convergence to one of the pure strategy equilibria and divergence from mismatches, they undermines players' subsequent ability to coordinate moves from one equilibrium to another. Laboratory subjects are substantially less likely to play alternating dynamics in continuous time than discrete time. Compared to the strong alternating pattern observed in discrete time, the dynamics in continuous time are diverse and unstable, further slowing the learning process in continuous time. Ultimately, the two significant

[2] The matrices are adopted directly from Anbarci et al. (2017), who introduce the inequality aversion model (Fehr and Schmidt (1999)) to battle of the sexes games. The three payoff matrices differ in the level of payoff asymmetry between the players at pure Nash equilibria and, thus, show how fairness concerns affect coordination.

forces offset each other and render the treatment effect insignificant. Moreover, the difference among the three payoff matrices is minimal.

We also find the following general patterns of how subjects interact in continuous time: the disadvantaged players who earn a low payoff tend to move first, and the advantaged players follow. The transitions between pure Nash equilibria and mismatches in which both players choose what they prefer also represent the major motivation of transitions in continuous time. In continuous time, it appears that the dynamics motivated by the disadvantaged players are more common than the traditional dynamics (e.g., strategic teaching). Furthermore, the subjects' willingness of staying at their opponents' preferred Nash equilibrium is squeezed out over time, especially in the last 30 seconds of the game.

The remainder of this paper is organized as follows. Section 2 reviews the literature concerning both continuous time games and battle of sexes games. Section 3 introduces the experimental design and hypotheses of this study. Section 4 presents the experimental results and discusses the differences between the two time environments. Finally, Section 5 concludes the main findings and notes unsolved questions to be addressed in future studies.

3.2 Related Literature

Although most economic laboratory experiments have employed a discrete time framework, continuous time games have been developed in theory for quite a long time.

Models that work in discrete time cannot be directly transferred to continuous time environments because the concept of "period" is not well-defined in continuous time. Previous theories focusing on modeling traditional games in continuous time have identified new equilibria, and the most notable finding shows that continuous time interactions support full cooperation in social dilemma games, such as the prisoner's dilemma, and timing games in finite time horizons, which fundamentally differs from predictions in discrete time and improves social welfare (Simon and Stinchcombe (1989); Bergin and MacLeod (1993)). Recent continuous time theories focus on the effect of natural reaction lags (also called inertia) in social dilemma games (Park (2014); Calford and Oprea (2017)) and the asynchronous nature of continuous time games (Alós-Ferrer and Kern (2015)).

With the development of laboratory techniques, experimental economists have recently begun to study continuous time games in laboratory environments in which subjects interact in milliseconds. Although subjects cannot respond in milliseconds (except for using the "time freezing" technique described by Calford and Oprea (2017)), the high frequency interactions allow them to react much faster than they could in discrete time, and many experimental findings support the theoretical predictions. In prisoner's dilemma games, the cooperation rate between players in continuous time tends to be higher than that in discrete time (Friedman and Oprea (2012)). A follow-up study finds that the termination rules and time horizon also affect the cooperation rate (Bigoni et al. (2015)). Similar cooperative behavioral patterns have been observed unconditionally in long-term Cournot games (Friedman et al. (2015)), conditionally on communication in public good games (Oprea et al. (2014)), and conditionally on the information structure in minimum effort games (Deck and Nikiforakis (2012); Leng et al. (2018)). Some other laboratory studies in continuous time focus on convergence to Nash equilibria. In the continuous time evolutionary hawk and dove game, a symmetric mixed equilibrium is more likely to be selected in the one-population game. In contrast, the separation equilibrium is stronger in the two-population game (Oprea et al. (2011)). This finding has been further developed under the uniparametric model (Benndorf et al. (2016)) and the perturbed best response dynamics (Benndorf and Martinez-Martinez (2017)). Continuous time interactions have also been found to support Nash equilibria in the 4-player Hotelling competition (Kephart and Friedman (2015)) and network formation games (Berninghaus et al. (2006)).

However, the games used in theories and experiments thus far do not require a complex equilibrium path, such as alternation between Nash equilibria. Research investigating continuous time battle of the sexes games expands our ideas of how players generally interact in continuous time when they need to make multiple decisions at multiple timings. Furthermore, to the best of our knowledge, no theory or experiment provided evidence that continuous time can potentially harm social welfare, but battle of the sexes games may show different results.

This paper is also related to the literature concerning battle of the sexes games, especially the "alternating dynamics" (or "turn-taking dynamics") in repeated interactions. Since the first discussion of alternating dynamics by Luce and Raiffa (1989), such dynamics have been widely discussed and applied to a wide range of empirical topics, including the management of common pool resources (Janssen and Ostrom (2006)), repeated traffic route choices (Helbing et al. (2005)), entry into markets of natural monopoly (Dixit and Shapiro (1984)), and network externalities (Besen and Farrell (1994)). The first experimental evidence of alternating dynamics in battle of the sexes games in laboratory studies was derived from the fixed-matching protocol by Prisbrey (1992), and some subsequent experiments found similar patterns (e.g., Bhaskar (2000); Sonsino and Sirota (2003); Kuzmics et al. (2014)). For example, in Kuzmics et al. (2014), alternating dynamics are used in more than 80% of the repeated battle of the sexes games.

Many theories and related experiments support alternating dynamics in discrete time. Lau and Mui (2008) and Lau and Mui (2012) develop the idea by Crawford and Haller (1990) as "turn taking with independent randomization", where players choose mixed strategies before they reach the equilibrium path and play Nash equilibria strategies after they determine the alternating dynamics. Arifovic and Ledyard (2018) introduce their individual evolutionary learning theory to battle of the sexes games and prove that this theory can support the alternating dynamics if one player knows the dynamics at the beginning of the game. Romero and Zhang (2018) compare equilibrium with various normative principles and find that equal payoff and equal opportunities are the most frequently played guidelines for alternating dynamics. Experienced players can also teach the other player to play alternating dynamics during the game (Cason et al. (2013b)). The other mechanisms that support alternating dynamics include correlated equilibrium (Duffy et al. (2017); Anbarci et al. (2017)), action-based belief learning

(Cheung and Friedman (1997)), subjective principle learning (Sandroni (2000)), and strategy-based belief learning (Ioannou and Romero (2014b)). Some other papers also discuss alternating dynamics with a predefined punishment strategy and random cost structure (Leo (2017)), comparison between alternations and cut-off strategies (Kaplan and Ruffle (2011)), and how coordination is affected by preplay communication (Cooper et al. (1989)) and a third compromise option (He and Wu (2020)).

However, these theories cannot be directly transferred to the continuous time environment as the critical concept of a "period" is not well-defined in real numbers. Although we do not propose a theory in this paper, we hope that our experimental results can shed light on the continuous time battle of the sexes theory for theorists. Furthermore, the comparison of subjects' behaviors in the two time environments has an empirical impact. Currently, people face traditional coordinate problems in online environments, such as online teamwork assignments, the allocation of online resources that require cooperation, and the coordination of internet traffic. Research investigating continuous time battle of the sexes games could indicate whether people should accelerate or slow interactions with such problems.

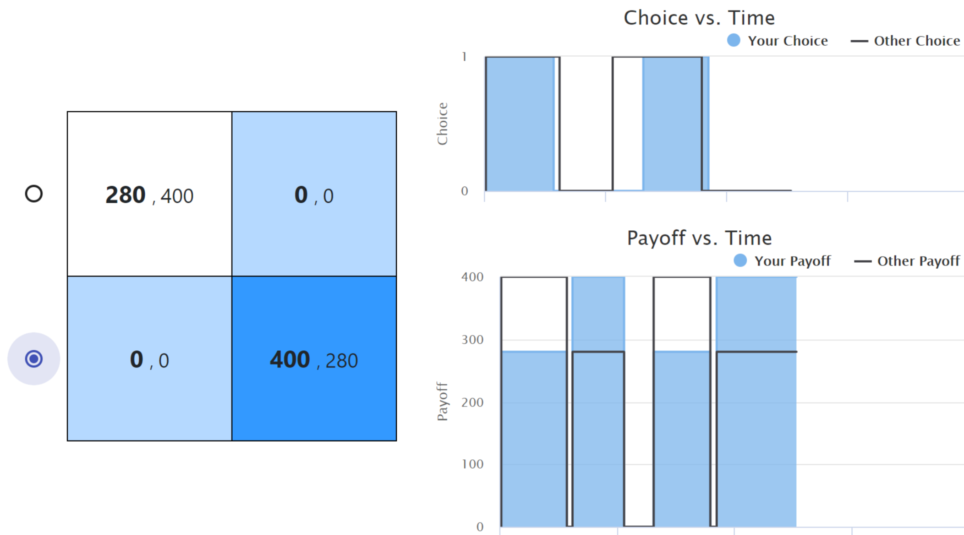
3.3 Experimental Design

This paper applies a 2x3 full-factorial design with two time environments (continuous time and discrete time) and three payoff matrices. The experiments use a between-subjects design for the two time treatments and a within-subject design for the three payoff matrices. In each experimental session, the subjects play 12 *supergames* after 3 practice games, each lasting for 2 minutes. The subjects are randomly paid based on their earned payoff in one of the supergames. Each subject is randomly and anonymously paired with another subject in each supergame; thus, the experiments use a random matching protocol between supergames and a fixed matching protocol within each supergame. The remainder of this section discusses the treatments, hypotheses, and session information.

3.3.1 Time Environments and User Interface

Each supergame lasts for 2 minutes. In the continuous time scenario, the payoffs and choices move in real time, are calculated and exchanged between players every 50 milliseconds and are recorded twice each second. Each supergame is equally divided into 20 periods in discrete time, and each period lasts for 6 seconds. The data are recorded every period. The length of the periods is similar to that in Friedman and Oprea (2012)'s "grid-8" treatment. More data are collected during sessions with a short period length than sessions with a longer period. Based on Friedman and Oprea (2012), there are significant differences between the continuous time treatment and "grid-8" treatments; thus, we could expect the subjects to play the 6-second discrete time games differently from the continuous time games. In both time environments, the subjects' initial choices are randomly determined. In the data sections, we remove the data from the practice games and the first 2 seconds of the continuous time scenarios to avoid a reaction lag to the initial random choices.

Figure 3.1: Sample user interface in the games in continuous time.



Since we adapt a between-subjects design in the two time environments, the subjects are presented with different user interfaces in different sessions based on the time environment. Figure 3.1 shows a sample user interface presented to the subjects on their screen in the continuous time setting. The payoff matrix on the left side of the screen shows the battle of the sexes game subjects playing in the current supergame. The subjects are randomly divided into two roles (row player and column player) in each game. The subjects can freely click back and forth between the rows using the radio buttons, and the current chosen choice is shaded in blue on the payoff matrix. The other subject controls the columns. We transpose the payoff matrix for the column players such that all subjects see themselves as the row player on their screen. As the game is played in continuous time, their decisions affect their payoffs in real time. The history of the current supergame is shown on the right side of the screen. The subjects have access to the previous payoffs and choices of both players in the current supergame. In the continuous time scenario, both charts move in real time and are updated every 50 milliseconds, and the subjects can monitor their counterpart's choices in real time. In the discrete time scenario, each supergame is divided into 20 periods, and only the last choice in each period is used to determine the payoff for that period. Additionally, the subjects cannot observe their counterparts' current decision. The subjects see a green bar on the top right showing the remaining time of the current period, and both choices and payoff charts are updated period-by-period.

3.3.2 Payoff Matrices

Table 3.1 shows the three payoff matrices used in the experiment, which are variants adapted from Anbarci et al. (2017). The major difference among the payoff matrices is the level of payoff asymmetry. The name of the matrix "BoSX" is based on the level of payoff asymmetry when players coordinate at either pure Nash equilibria. "X" is the ratio of the advantaged player's payoff over the disadvantaged player's payoff at either pure Nash equilibria and represents the level of payoff asymmetry. All three payoff matrices have the same pure Nash equilibria but different mixed Nash equilibrium. In this study, we focus on two pure strategy Nash equilibria and two mismatches. To make it understandable, we rename the four cells of the payoff matrix as follows: (A,a) as "Row player preferred Nash equilibrium (*RPNE*)", (B,b) as "Column player preferred Nash equilibrium (*CPNE*)", (A,b) as "Aggressive mismatch (*Aggressive*)", and (B,a) as "Accommodating mismatch (*Accommodating*)". In battle of the sexes games, players reach the efficient status when they choose either pure Nash equilibrium. In the repeated interactions, fairness is another concern and players achieve complete fairness when they alternate equally between two pure Nash equilibria.

Since we adopt a within-subject design for the payoff matrices, we design two between-subjects game sequences to assess the order effect. The following two sequences show the order of the payoff matrices in the practice games and the 12 supergames. In Sequence 1, the subjects start with the matrix that is the easiest to coordinate among the three payoff matrices and play in the order of (1*BoS1.4, 1*BoS2.5, 1*BoS10, 2*BoS10, 2*BoS2.5, 2*BoS1.4, 2*BoS1.4, 2*BoS2.5, 2*BoS10). In Sequence 2, the subjects start with the matrix that is the most difficult to coordinate among the three payoff matrices and play in the order of (1*BoS10, 1*BoS2.5, 1*BoS1.4, 2*BoS1.4, 2*BoS2.5, 2*BoS10, 2*BoS10, 2*BoS2.5, 2*BoS1.4). In both game sequences, the sessions are divided into 3-game practice and two 6-game blocks, and the subjects play each payoff matrix twice in each block. Due to technical constraints, we do not fully randomize the order of the games; however, the two game sequences show two opposite cases and can fully represent how the order effect affects the subjects' coordination.

Table 3.1: Payoff matrices in the experiment.

BoS1.4			BoS2.5			BoS10		
	a	b		a	b		a	b
A	(400,280)	(0,0)	A	(400,160)	(0,0)	A	(400,40)	(0,0)
B	(0,0)	(280,400)	B	(0,0)	(160,400)	B	(0,0)	(40,400)

The row players choose between A (top) and B (bottom). The column players choose between a (left) and b (right).

3.3.3 Hypotheses

Although no directly relevant theory explains how players interact in continuous time battle of the sexes games, we can still develop some hypotheses to guide our experimental study. The hypotheses proposed in this section are based on a discussion regarding how the continuous time environment affects behavior in general based on the continuous time games literature, how people learn and alternate based on the battle of the sexes games literature, and intuitive thinking.

The most important research question in this paper is related to a comparison of coordination rates between continuous time and discrete time. The coordination rate is defined as the average fraction of time that pairs of subjects stay at either pure Nash equilibria (RPNE or CPNE). As discussed in the introduction, there are considerable differences between the two time environments, including the difficulty in playing alternating strategies and how fast the subjects can respond. We first hypothesize that the coordination rate in continuous time is higher than that in discrete time based on general conclusions in the previous literature.

Hypothesis 1 *Define the coordination rate as the average fraction of time that pairs of subjects stay at either pure Nash equilibria (RPNE or CPNE). The coordination rate in continuous time is higher than that in discrete time.*

Another dimension of the treatments is the payoff matrices adapted from Anbarci et al. (2017), who introduce inequality aversion from Fehr and Schmidt (1999) to battle of the sexes games. Based on their theoretical predictions and experimental results, the subjects' coordination rate decreases from BoS1.4 to BoS10. With two between-subjects game sequences, the subjects may also learn to coordinate in general more slowly in Sequence 2 than Sequence 1. We propose the following hypothesis based on their research findings:

Hypothesis 2 *The subjects' coordination rate increases when the level of payoff asymmetry decreases from BoS10 to BoS1.4. The average coordination rate in the sessions with Sequence 1 is higher than that in the sessions with Sequence 2.*

The following assumptions more closely examine the differences between the two time environments. As discussed in the introduction, it is more difficult for subjects to play alternating dynamics in continuous time than discrete time mainly because the subjects must coordinate both the order and the duration of alternations in continuous time. In contrast, the subjects only need to coordinate the order of alternations and take turns in discrete time.

Hypothesis 3 *Define alternating dynamics as pairs who maintain both efficiency (fraction of time at pure Nash) and fairness (difference in time at either pure Nash). The subjects are less likely to play alternating dynamics in continuous time than discrete time.*

The difficulty in playing alternating strategies may also affect the learning process in continuous time. The subjects learn with their counterpart during each supergame. Although the subjects switch counterparts between supergames, they can bring their experience to new supergames and, thus, improve coordination between supergames. We hypothesize that learning occurs in both time environments, but we investigate the learning pattern based on the data.

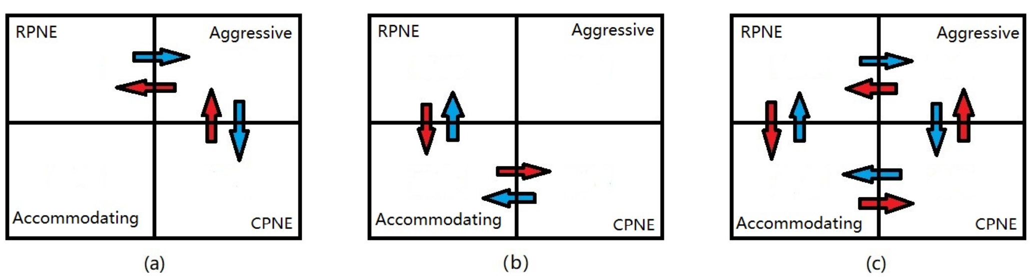
Hypothesis 4 *The subjects learn to coordinate. The coordination rate increases over time between supergames and within each supergame.*

Although the players have difficulty playing alternating dynamics, the previous literature shows that players can interact quickly and asynchronously in continuous time. The subjects do not have to wait for the next period and can immediately follow what the other subject is playing. As a result, the initial convergence to pure Nash equilibria in continuous time is faster than that in discrete time and subjects also deviate from mismatches more quickly in continuous time than discrete time.

Hypothesis 5 *The subjects spend less time reaching their first pure Nash equilibria in the supergames and spend less time remaining at mismatches in continuous time than discrete time.*

Regarding the second research question, we are interested in how subjects alternate between pure Nash equilibria, especially in continuous time. To construct the dynamics pattern, we investigate what motivates the subjects to transition between action profiles and the duration of time the subjects remain at either pure Nash equilibria. First, there are three possible transition patterns. If the subjects follow strategic teaching, the transitions should be motivated by a single player in a cyclical pattern, and we should observe that the transition probabilities between two mismatches and pure Nash equilibria are almost equal. If the dynamics are motivated by either the disadvantaged players or advantaged players, they should be motivated by two players in turn. In disadvantaged-player dynamics, the transitions between Aggressive and Nash equilibria should be more frequent than those between Accommodating and Nash equilibria. In advantaged-player dynamics, the transition frequencies should be the opposite. A showcase of all three dynamics are shown in Figure 3.2. Second, given the difficulty in alternation in continuous time, we expect the duration of time the subjects remain at pure Nash equilibria to be more diverse in continuous time than discrete time.

Figure 3.2: Three possible transition dynamics.



(a) disadvantaged-player dynamics; (b) advantaged-player dynamics; (c) strategic teaching. The red arrows show the transitions motivated by row players and the blue arrows show the transitions motivated by column players.

3.3.4 Sessions

The data are collected during 8 sessions conducted between August 26th and October 5th of 2020, and in total, 84 subjects were recruited from the UCSC Leeps Online platform ORSEE (Greiner (2015)). The session information is shown in Table 3.2. The experiment uses a continuous bimatrix program using the software oTree/Redwood (Pettit et al. (2014); Chen et al. (2016)). The subjects earn points based on the results of one randomly selected nonpractice game, and the conversion rate between points and US dollars is 20:1, with a show-up payment of 4 US dollars. The average payment is approximately 15 US dollars, and the length of a session is approximately 1 hour, including 20 minutes of instruction and 40 minutes of gameplay. All experiments are conducted in online environments following the UCSC online experimental protocol.

Table 3.2: Session Information.

Time	Sequence	Num of subjects	Num of sessions
Continuous	1	22	2
Continuous	2	18	2
Discrete	1	22	2
Discrete	2	22	2

The "Time" column shows the time environment implemented during the session. The "Sequence" column shows the order of the payoff matrices, which is explained in the main text.

3.4 Results

3.4.1 Treatment Effects

The data are collected during 504 supergames after removing the data from the practice games and the first 2 seconds in the continuous time scenario. In each supergame, the subjects are grouped in pairs, and we use *pairs* to represent two subjects in the same supergame. Each of the three continuous time environments consists of

18880 observations, and each discrete time environment has 1760 observations. Starting from Hypotheses 1-2, panel A in Table 3.3 shows a summary of the coordination rate (the fraction of time the pairs stay at either pure Nash equilibria) in each treatment. The subjects are approximately 3% less likely to coordinate in continuous time than discrete time and are more likely to coordinate when the level of payoff asymmetry is lower. Although the sign of the treatment effects follows the hypotheses, no effect is statistically significant, and neither Hypothesis 1 nor Hypothesis 2 appears to be supported by the data.

One possible reason for the failure to find support for Hypothesis 2 is the within-subject design. Treatment effects are difficult to observe when subjects learn all three payoff matrices as a whole. To assess the treatment effect of the payoff matrices from another perspective, we design two game sequences and vary the sequences between sessions. How does the order effect in the two game sequences influence the coordination rate? Notably, the subjects start with a low level of payoff asymmetry (BoS1.4) in Sequence 1 and a high level of payoff asymmetry (BoS10) in Sequence 2. Panel B in Table 3.3 compares the coordination rates between the sessions that apply Sequence 1 and the sessions that use Sequence 2. Although the subjects indeed have a higher chance to coordinate in Sequence 1 than Sequence 2, the effect is insignificant. Overall, it is difficult to confirm that the game sequence affects the coordination rate.

Column (1) in Table C.2 in the Appendix also shows the logistic regressions and confirms our findings in this section. The treatment effects can be summarized as follows.

Table 3.3: Coordination rates by treatments and order effect.

Panel A: coordination rates by treatment

	Continuous	Discrete	C-D
BoS1.4	0.68	0.71	-0.03
BoS1.4-BoS2.5	0.01	0.02	
BoS2.5	0.66	0.69	-0.03
BoS2.5-BoS10	0.03	0.00	
BoS10	0.63	0.69	-0.05

Panel B: coordination rates by sequence

	Sequence 1	Sequence 2	S1-S2
Continuous	0.67	0.64	0.04
Discrete	0.70	0.69	0.02

Subscripted asterisks indicate p-values of .10(*), .05(**) and .01(***) in two-sided t-tests assuming unequal variance between adjacent columns. The data are clustered at the supergame level.

Result 1 *The treatment effects of both the time environments and payoff matrices are weak.*

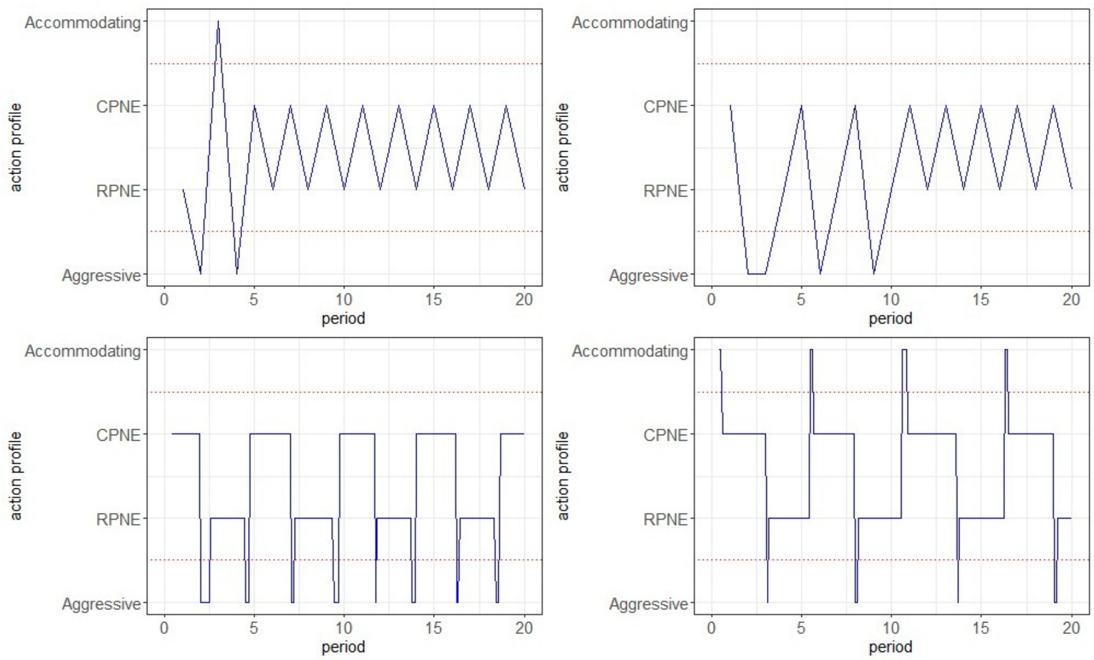
(1) The coordination rate in continuous time is insignificantly lower than that in discrete time under all payoff matrices.

(2) The payoff matrices only slightly affect the coordination rates, and the order effect is weak.

3.4.2 Pair Classification and Learning

Based on the observed insignificant treatment effects, can we conclude that the subjects behave similarly in the two time environments? Is there any difference between the two time environments beneath the coordination rate? We raise some possible mechanisms in the introduction and summarize them in the hypotheses section with Hypotheses 3-5. We explore Hypotheses 3 and 4 in this section.

Figure 3.3: Examples of alternating dynamics.



The upper row shows two examples in discrete time and the lower row shows two examples in continuous time.

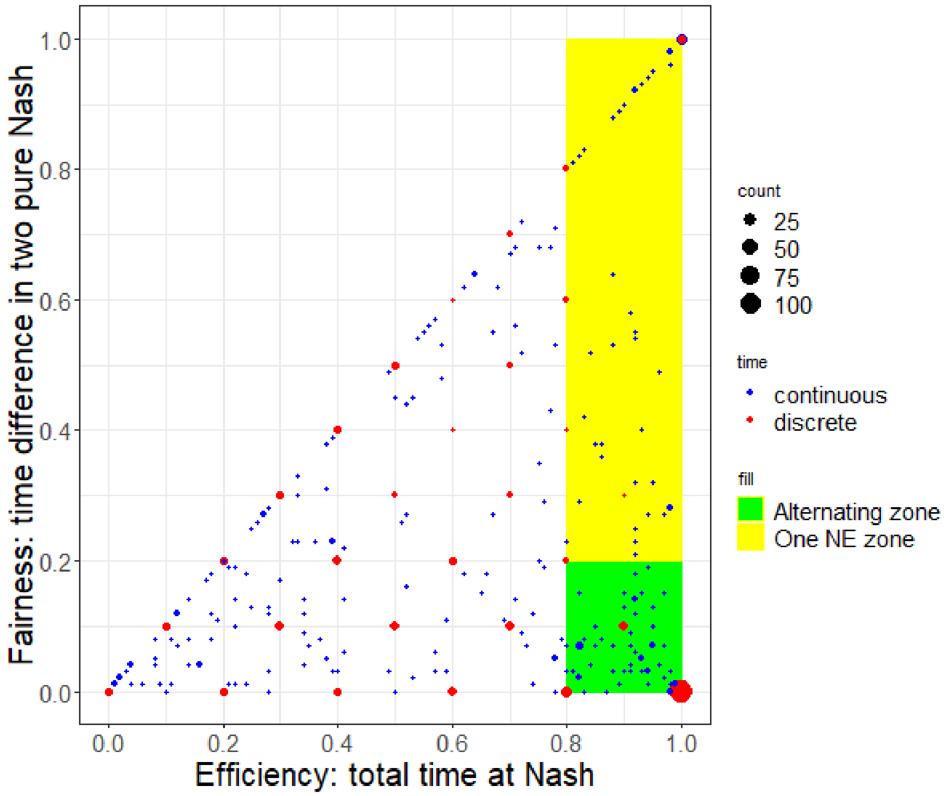
In Hypothesis 3, we state that it is more challenging to play alternating dynamics in continuous time than discrete time as the players need to coordinate both the order and the duration of alternations in continuous time but need to coordinate only the order of alternations and take turns in discrete time. The alternating dynamics in the two time environments can also be quite different and Figure 3.3 shows some examples of alternating dynamics in the two time environments. The pairs may also remain at one pure Nash equilibrium as it is easier to reach than reaching alternating dynamics in continuous time. To compare the dynamics in the two time environments, we first classify each pair of subjects by the dynamics path they play. Since we are interested in stabilized equilibrium paths, we first use the data from the second half of each supergame when classifying the types (last 10 periods in discrete time and last 60 seconds in continuous time).

To classify the pairs, we consider both efficiency and fairness. We classify the pairs as "Alternating" pairs if they coordinate for no less than 80% of the time (8 periods in discrete time and 48 seconds in continuous time) and the difference between the time they remain at the two pure Nash equilibria is no more than 20% of the time (2 periods in discrete time and 12 seconds in continuous time). If the pairs coordinate for more than 80% of the time but do not satisfy the second criteria, they are classified as "One NE". Since we focus on two subsets of equilibrium types, we classify all other pairs as "Other".[3] For example, suppose a pair of subjects play 5 RPNE, 3 CPNE, and 2 mismatches in the last 10 periods in the discrete time scenario. In this case, the pair is classified as playing alternating dynamics regardless of the exact alternating pattern. If the pairs play 6 RPNE, 2 CPNE, and 2 mismatches, they are classified as preferring one pure Nash equilibrium to the other. If the pairs mismatch more than 3 times, they are classified as "Other". Figure 3.4 shows a scatter plot showcasing the classification of all 504 supergames under the current criteria.

Panel A in Table 3.4 shows the results. The subjects are significantly more likely to play alternating dynamics and are less likely to prefer one Nash equilibrium

[3]More pairs are classified as belonging to the two coordinate types if the criteria decrease, and fewer pairs become coordinative types if the criteria increase. We change the criterion of coordination from 60% to 90% and the criterion of alternation from 3 periods (18 seconds) to 0 periods (0 seconds). The treatment effect remains robust.

Figure 3.4: Scatter plot showing the classification of all 504 supergames colored by the time environments under the 80%/2 periods criteria.



The size of the points indicates the number of supergames overlapping at the coordinate.

in discrete time than continuous time. Since the subjects might alternate based on the entire supergame in continuous time, will the treatment effects differ if we use the whole supergame to classify the pairs? Panel B in Table 3.4 shows the classification under the same criteria based on the whole dataset, and the results are similar. In both panels, approximately 50% of the pairs in discrete time and 25% of the pairs in continuous time are classified as "Alternating" types. Staying at one pure Nash equilibrium is almost as popular as alternating dynamics in continuous time but quite uncommon in discrete time. Columns (2)-(5) in Table C.2 in the Appendix confirm the statistical significance with a logistic regression.

Table 3.4: Classification at the pair level.

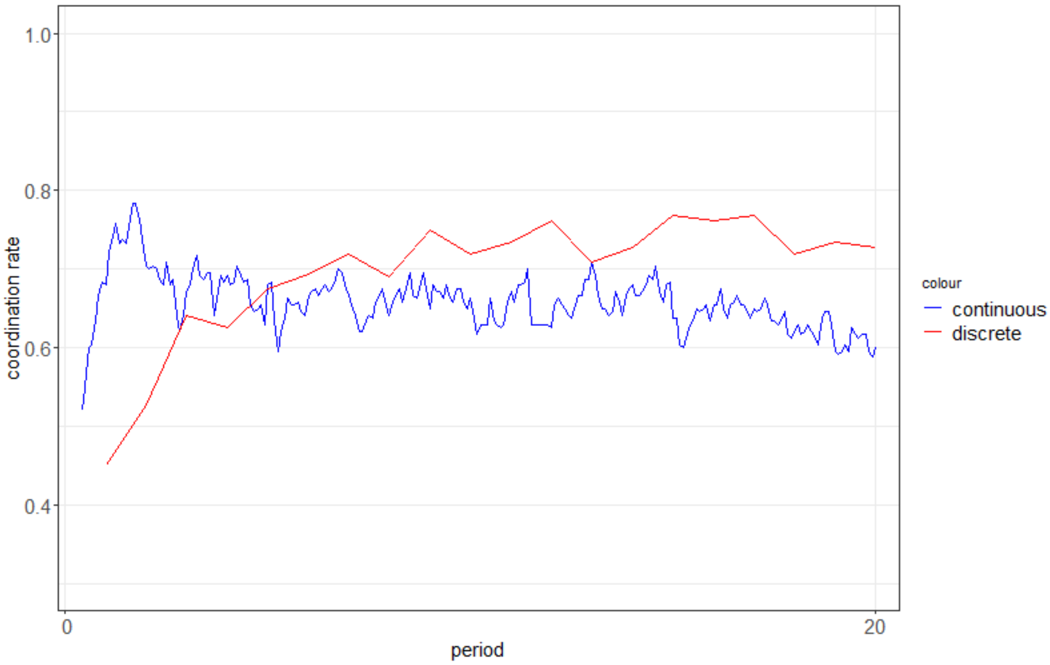
	Alternating	One NE
Panel A: second half		
Continuous	0.24	0.23
Discrete	0.53	0.06
C-D	-0.30***	0.18***
Panel B: full dataset		
Continuous	0.25	0.17
Discrete	0.45	0.02
C-D	-0.20***	0.15***

Subscripted asterisks indicate p-values of .10(*), .05(**) and .01(***) in two-sided t-tests assuming unequal variance between adjacent columns.

Does the difficulty of playing alternating strategies affect the learning process? On the basis of Hypothesis 4, we assess the learning effect and determine whether the coordination rate is improved between supergames and within each supergame. Figure 3.5 shows the average coordination rate within supergames. We find strong within-game learning in discrete time as follows: the coordination rates in discrete time are lower than those in continuous time at the beginning of the supergames, but the rates catch up approximately one-fourth of the way through the game. In the second half of the supergames, coordination in discrete time becomes better than coordination in continuous time. Meanwhile, we do not observe any learning in continuous time. Figure C.3 in the Appendix further separates the two curves by payoff matrices and shows the same trend.

To determine whether the subjects carry what they learn between supergames, we compare the average coordination rates in block 1 (Game 1-6) and block 2 (Game

Figure 3.5: Coordination rate within supergames.



7-12). Table 3.5 shows the coordination rates in the two blocks of the session. Similar to the observations in Figure 3.5, the coordination rate in block 2 is significantly higher than that in block 1 under discrete time but not under continuous time. This result is further supported in column (1) in Table C.2 in the Appendix. In general, both between-supergame learning and within-supergame learning support Hypothesis 4 in discrete time but not continuous time.

Although there is no significant treatment effect between the two time environments, the subjects interact and learn quite differently. The results in this section confirm Hypothesis 3 and part of Hypothesis 4, illustrating the following advantages of discrete time: it is easier to learn to play alternating strategies and learn to coordinate in discrete time than continuous time. In the Appendix (Figure C.1), we also

Table 3.5: Coordination rates between Games 1-6 (first half of the session) and Games 7-12 (second half of the session).

	Game1-6	Game7-12	(G7-12)-(G1-6)
Continuous	0.67	0.64	-0.03
Discrete	0.66	0.73	0.08**

Subscripted asterisks indicate p-values of .10(*), .05(**) and .01(***) in two-sided t-tests assuming unequal variance between adjacent columns. The data are clustered at the supergame level.

show that the payoff in continuous time is more unequal than that in discrete time as a consequence of the lack of alternation.

Result 2 *The discrete time environment has the following advantages:*

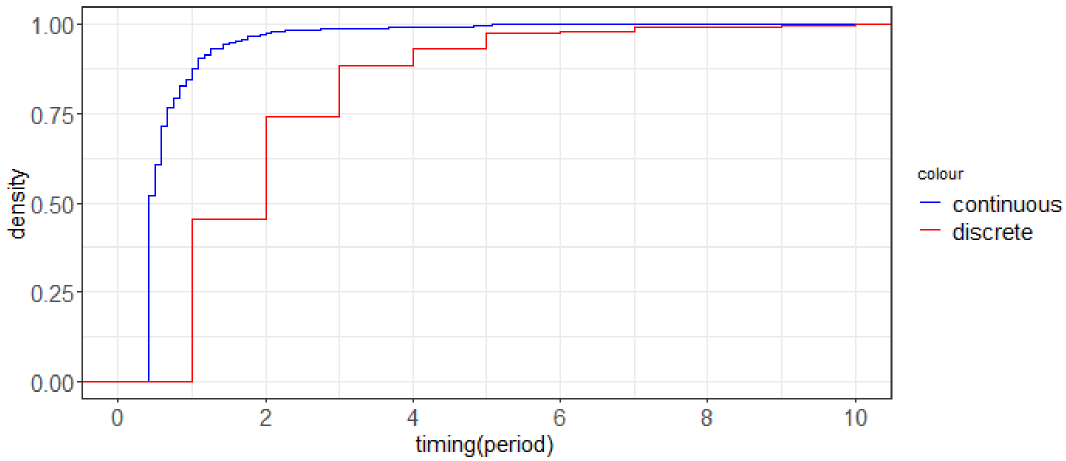
(1) The subjects are more likely to stay at one Nash equilibrium and less likely to alternate between Nash equilibria in continuous time than discrete time.

(2) The subjects learn between and within supergames in discrete time; however, the learning pattern in continuous time is weaker than that in discrete time.

3.4.3 Reaction to Changes

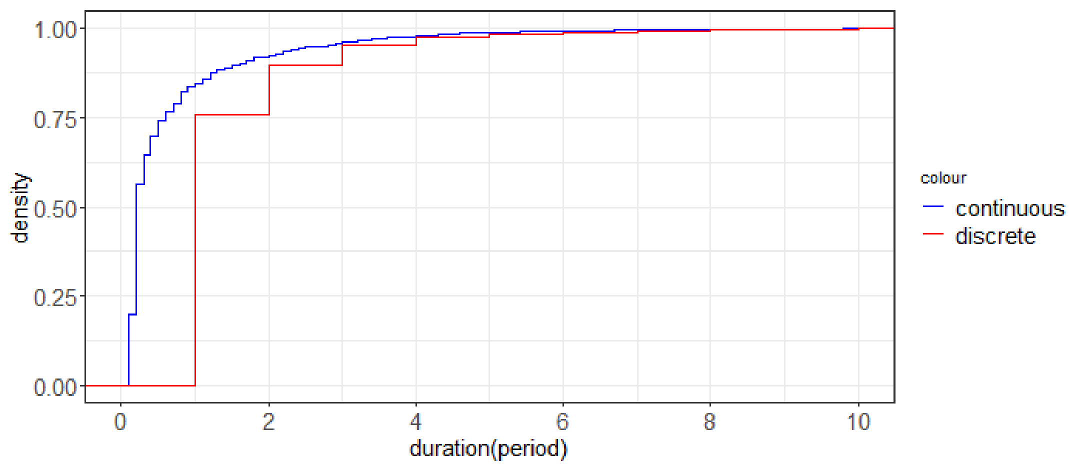
The previous literature has shown that players react faster to changes in continuous time than discrete time. Given the findings in the literature, the subjects should deviate from mismatches and reach Nash equilibria more quickly in continuous time than discrete time. Does this expectation hold in battle of the sexes games? Alternatively, is the continuous time environment dominated by the discrete time environment? Figure 3.6 shows the empirical cumulative distribution functions of the time at which the subjects first reach either Nash equilibrium in each supergame and supports the first statement of Hypothesis 5. The subjects spend less time reaching their first Nash equilibria in continuous time than discrete time. This result is also consistent with the initial high coordination rate in continuous time. Figure C.4 in the Appendix further separates the two curves by payoff matrices and shows the same trend.

Figure 3.6: CDF of the time at which the pairs first reach either pure Nash equilibrium.



To determine how long subjects stay at mismatches, we first combine the adjacent observations in which the subjects make the same decisions and reorganize the dataset into multiple "events" (notably, the data are collected in each period in discrete time and every 500 milliseconds in continuous time). Each event shows the subjects' choices and how long they maintain their choices. For instance, if the subjects play RPNE for 5 periods, switch to CPNE for 10 periods, and return to RPNE in the last 5 periods, there are three "events" in this pair of subjects. Here, we focus only on the events involving mismatches (Aggressive and Accommodating). Figure 3.7 shows the empirical cumulative distribution functions of the duration during which the subjects remain at either of the two mismatches (length of mismatch events) and supports the second statement of Hypothesis 5. Clearly, the subjects adjust their strategies to move away from mismatches in continuous time more quickly than they do in discrete time. Indeed, the minimum reaction time in discrete time is one period and the subjects can respond much faster in continuous time by design. However, the curve shows that the subjects use the advantages of the continuous time environment and do not remain at mismatches for long. Figure C.5 in the Appendix further separates the two curves by payoff matrices and shows the same trend.

Figure 3.7: CDF of the duration during which the subjects stay at mismatches.



Figures 3.6 and 3.7 show the relative advantage of the continuous time environment discussed in the hypotheses in which the subjects can quickly change decisions and move to Nash equilibria given the other player's choices. Both time environments appear to have advantages, and the two forces point in opposite directions, which could explain the insignificance of the treatment effect in the continuous time environment. However, undoubtedly, the subjects play differently in the two time environments. Hy-

pothesis 5 is supported in this section, and the results can be summarized as follows.

Result 3 *The continuous time environment has the following advantages:*

(1) The subjects spend less time reaching their first pure Nash equilibrium in the supergames under continuous time than those under discrete time.

(2) The subjects deviate faster from mismatches in continuous time than discrete time.

3.4.4 Transition Probability Matrix and Alternating Pattern

We observed that the two time environments are different in the difficulty of alternation, within- and between-supergame learning, and response time. However, what exactly is the dynamics pattern in continuous time? We focus on the following two critical elements of the subjects' interactions: the probability and direction of moving from one action profile to another and the duration of time the subjects stay at pure Nash equilibria.

Table 3.6 focuses on the first element and shows the transition probability matrices in both the continuous and discrete time environments. The last column indicates that the subjects are almost equally likely to switch from either pure Nash equilibria RPNE and CPNE. Regarding mismatches, the players are more likely to switch from Aggressive, where they hold their own preferred positions, than from Accommodating, where they benefit the other player. In both tables, we can observe how the subjects move into and move away from pure Nash equilibria from the lower left cells and upper right cells as follows: the subjects are approximately four to five times more likely to move away from pure Nash equilibria to Aggressive than to Accommodating. Considering the large number of observations from Aggressive, the subjects are approximately three to four times more likely to switch to any pure Nash equilibria from Aggressive than from Accommodating. The transitions between the matrix cells appear to be mainly motivated by the disadvantaged subjects, followed by the advantaged subjects, especially in continuous time. Based on the findings, the disadvantaged-player dynamics appear to be the most popular transitions among the three transition dynamics we raise in our hypotheses section. As both dynamics require the subjects to frequently switch between Accommodating and Nash equilibria, strategic teaching and advantaged-player dynamics are unlikely to be applied in continuous time. In addition to the above findings, panel B shows a clear pattern of turn-taking between RPNE and CPNE with a conditional probability of approximately 80% in discrete time. Table C.1 in the Appendix reports the transition probability matrices with the diagonal terms, and we can observe the probabilities of staying at each position. The result is similar to that found in this section. Figure C.2 in the Appendix confirms the results at the pair level.

Table 3.6: Transition probability matrices. t' represents t+500 milliseconds in continuous time and t+1 period in discrete time.

Panel A: continuous time.

	Aggressive at t'	Accommodating at t'	RPNE at t'	CPNE at t'	num of transitions
Aggressive at t	-	0.02	0.46	0.51	1870
Accommodating at t	0.07	-	0.46	0.47	459
RPNE at t	0.79	0.19	-	0.02	1112
CPNE at t	0.82	0.15	0.03	-	1182

Panel B: discrete time.

	Aggressive at t'	Accommodating at t'	RPNE at t'	CPNE at t'	num of transitions
Aggressive at t	-	0.22	0.38	0.41	686
Accommodating at t	0.41	-	0.28	0.31	325
RPNE at t	0.17	0.04	-	0.78	1520
CPNE at t	0.17	0.05	0.79	-	1528

To remove the diagonal terms, where the pairs stay at the current profile, we consider only the transitions in which the pairs choose different profiles at t and t'.

However, the results above are only sufficient but not necessary to prove the dynamics subjects play in the games. The central problem is that all the three types of dynamics involve two correlated transitions among two or three cells of the matrix but Table 3.6 only demonstrates independent transitions directly between two cells. Table 3.7 focuses on the correlated transitions by considering transitions between two Nash equilibria. Subjects can transit directly between two Nash equilibria, or through either Aggressive or Accommodating. It is also possible that they fail at transitioning between Nash equilibria and Table 3.7 reveals the probabilities of all circumstances. Each correlated transition can be classified into four types: "Disadvantaged" and "Advantaged" refer to the transitions that can represent disadvantaged-player dynamics and advantaged-player dynamics, where subjects move between Nash equilibria through either Aggressive or Accommodating. "Direct" refers to the direct transitions between two Nash equilibria and "Fail" refers to the transitions that start from one Nash but fail to reach the other Nash. The results confirm what we have found: Disadvantaged-player dynamics are much more popular than other dynamics in continuous time. In discrete time, there are more direct transitions between two Nash equilibria. The comparison of the probability of failure between the two time environments also verify difference in their coordination rates.

Table 3.7: The probability of each type of transitional dynamics being played in the two time environments.

	Disadvantaged	Advantaged	Direct	Fail
Continuous	0.61	0.14	0.00	0.25
Discrete	0.10	0.02	0.82	0.05

Figure 3.8 focuses on the second element and shows the distribution of the duration during which the subjects stay at the same Nash equilibrium. Here, we continue to use the "events" defined in Section 4.3 but focus on the events involving Nash equilibria. Figure 3.8 shows the density of the duration during which the subjects stay at the two pure Nash equilibria (length of Nash events). In discrete time, we observe a peak at 1-period duration, providing strong evidence for 1-period alternating dynamics. The density in continuous time is much flatter than that in discrete time, indicating that the subjects do not have a clear dynamics pattern. Figure C.6 in the Appendix further separates the two curves by payoff matrices and shows the same trend. Furthermore, are the durations at Nash remain consistent over time? Figure 3.9 shows the average durations at Nash over time in the supergames and the data reveal an obvious end-game effect only in continuous time: the subjects are not willing to stay at their opponents' preferred Nash equilibrium in the last 30 seconds of the supergame. As a comparison, the durations at Nash in discrete time are quite consistent over time.

Figure 3.8: CDF of the duration during which the subjects stay at Nash equilibria.

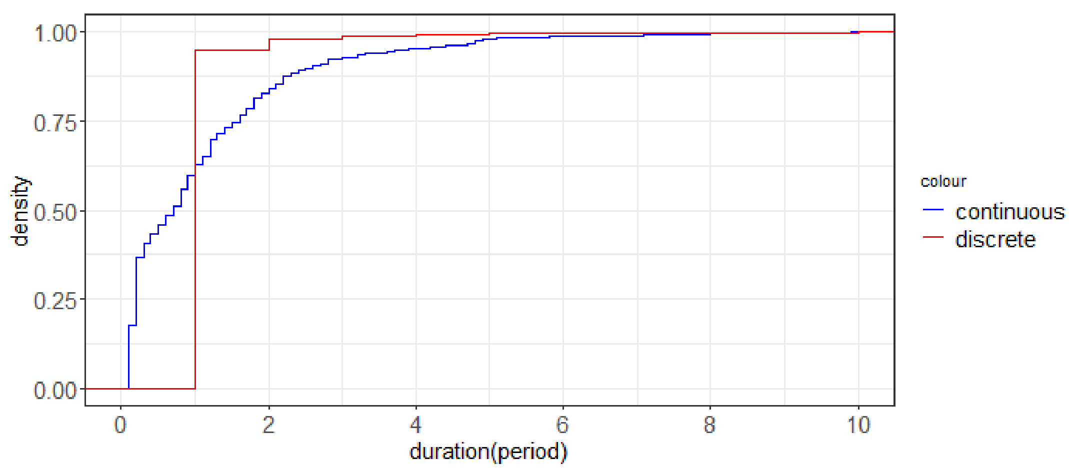
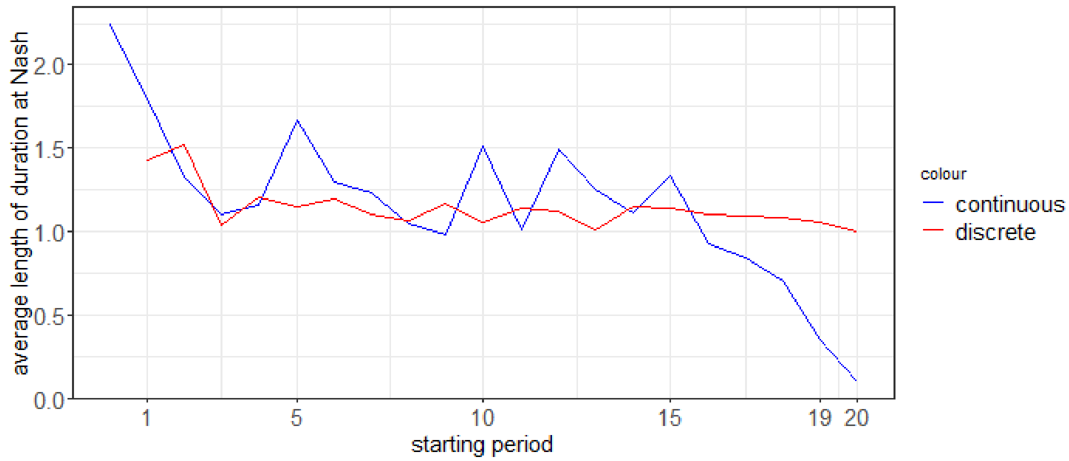


Figure 3.9: Average duration at Nash over time in the supergames.



Results 2-4 can be combined to produce some basic dynamics patterns in continuous time. The subjects interact and switch decisions much more frequently in continuous time than they do in discrete time due to the nature of the continuous time interactions. However, because of the difficulty in learning both the order and the duration of alternations, the subjects are less likely to follow alternating dynamics, and their behavioral patterns are more diverse in continuous time than discrete time. Moreover, the players who earn less at Nash equilibria tend to switch to their preferred positions first, and the dynamics are unlikely to be driven by a single player. The disadvantaged-player dynamics are rarely observed in discrete time and we often observe direct transitions between two pure Nash equilibria. The dynamics also reveal the motivation for turn taking from a new perspective as follows: the disadvantaged players tend to move to their preferred actions, and the advantaged players predict this move and follow simultaneously. Last but not least, the durations at Nash are squeezed out in the last 30 seconds of the supergames in continuous time, but not in discrete time.

Result 4 *(1) Both the transition probability matrices and the distribution of the duration during which the subjects stay at Nash equilibria reveal a clear alternating pattern in discrete time, but the alternating pattern is diverse and unstable in continuous time.*
(2) The most frequent transitions are between Aggressive and the Nash equilibria, supporting the disadvantaged-player dynamics, where the disadvantaged subjects move first and the advantaged subjects follow.
(3) The low frequency of the transitions between Accommodating and the Nash equilibria shows that the subjects are unlikely to play either strategic teaching or advantaged-player dynamics.
(4) The durations at Nash are squeezed out in the last 30 seconds of the supergames in continuous time, but not in discrete time.

3.5 Conclusions

Do continuous time interactions always improve social welfare and lead to a Nash equilibrium? This paper discusses a scenario in which continuous time interactions may harm social welfare by studying battle of the sexes games in continuous time. With the development of Internet technology, people are facing coordination problems, such as online teamwork, the allocation of online resources, and internet congestion. Whether interacting in continuous time is efficiency-improving is empirically important and could guide our behavior in the real world. From the previous literature and the nature of continuous time interactions, we know that it is more difficult to coordinate in battle of the sexes games in continuous time than discrete time.

Our experimental results show that the subjects behave quite differently in the two time environments as follows: the subjects are more likely to stay at one pure Nash equilibrium and are less likely to play alternating strategies in continuous time than discrete time. The difficulty in alternations also slows the learning process in continuous time. However, the subjects reach Nash equilibrium and deviate from mismatches more quickly in continuous time than discrete time, which is consistent with the advantages of continuous time interactions in the previous literature. The two forces lead the game in opposite directions, offset each other, and eventually render the treatment effect insignificant. In addition to the treatment effect, we find some patterns of how the subjects interact in continuous time as follows: the disadvantaged players tend to move to their preferred profiles first, and the advantaged players follow. The transition between the mismatch when both players choose their preferred actions and Nash equilibria is the major force that motivates the dynamics.

As shown in the data, a good way to solve the coordination problem is to search for the first equilibrium under continuous time and then switch to a discrete time environment for turn taking; this approach fully uses the advantages of both time environments. We can also expand our experimental findings to a general context as follows: continuous time interactions may harm efficiency when the players must switch between multiple Nash equilibria in the equilibrium path.

However, some questions remain unsolved. No theory explaining how the players interact in continuous time games that require subjects to move between Nash equilibria exists, especially the disadvantaged-player dynamics observed in our experiment. It is necessary to develop an approach that can explain such an equilibrium path; we hope that our results shed light on this research topic. This study also leads to reviewing other efficiency-improving treatments in continuous time battle of the sexes games. Will preplay communication help people coordinate alternations in continuous time? Will the continuous time environment become more efficient than the discrete time environment with the help of various tools? These questions could also be fruitful topics for future research.

Appendix A

Supplement to Chapter One

A.1 Computation of NE, Maximin and QRE for AMPa games

In this section we use AMPa games as an example to show the computation of NE, Maximin and QRE curve in Table 1.1 and Figure 1.1.

To calculate Nash equilibrium, recall from sign preserving dynamics that we calculated $D_R(t)$ and $D_C(t)$, which show the payoff difference between pure strategies for row and column players, respectively. By definition, the unique mixed Nash equilibrium can be solved by

$$D_R(t) = 1000b - 200 = 0 \tag{A.1}$$

$$D_C(t) = 200 - 400a = 0 \tag{A.2}$$

Hence $(a_{NE}, b_{NE}) = (0.5, 0.2)$.

The Maximin problem for row players is:

$$\max_a \min\{f_R(a,1), f_R(a,0)\}, \tag{A.3}$$

where $f_R(a,b) = 1000ab - 200a - 200b + 200$ from equation (1.1). Since $f_R(a,1)$ (resp. $f_R(a,0)$ increases (resp. decreases) linearly in a, the max must occur where $f_R(a,1) = f_R(a,0)$, yielding $a_{MM} = 0.2$. The analogous problem for column players yields $b_{MM} = 0.5$.

To calculate the QRE(λ) curve for both players, recall that the logit payoff function in AMPa games is implicitly defined by the fixed point equations

$$a = \frac{\exp(\lambda f_R(1,b))}{\exp(\lambda f_R(1,b)) + \exp(\lambda f_R(0,b))} \tag{A.4}$$

$$b = \frac{\exp(\lambda f_C(a,1))}{\exp(\lambda f_C(a,1)) + \exp(\lambda f_C(a,0))}. \tag{A.5}$$

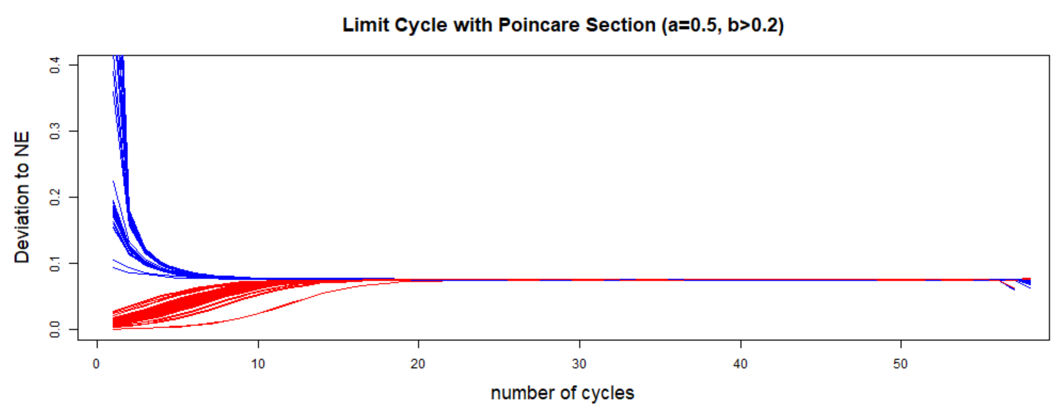
When $\lambda \to 0$, we have $(a,b) = (.5,.5)$. When $\lambda \to \infty$, we have $(a,b) = (a_{NE}, b_{NE}) = (.5,.2)$. The arc curve between two extreme cases in shown in Figure 1.1.

A.2 Numerical proof of the limit cycle

Figure A.5 shows the limit cycle with a 3-d plot and provides a more vivid image than Figure 1.5. In this section we numerically prove the limit cycle by collecting distance between observation and Nash equilibrium over time when the trajectory crosses the Poincaré section from the simulation data in Section 5.3.

Given strategy profile (a_t, b_t), the Poincaré section we select is the isocline $(a = 0.5, b > 0.2)$. The trajectory (a_t, b_t) at time t is considered to cross the isocline when $b_t > 0.2$ and $(a_t - 0.5)(a_{t+1} - 0.5) \leq 0$ are satisfied. Whenever the trajectory crosses the isocline, the distance between the observation and Nash Equilibrium ($d = \sqrt{(a_t - 0.5)^2 + (b_t - 0.2)^2}$) is recorded and graphed. Figure A.1 overlaps such distance over time with 50 simulations and 3000 iterations in each simulation. The blue (red) lines shows the distance over time when trajectory crosses the isocline and the initial position is outside (inside) the limit cycle. As shown in the figure, both blue and red lines converge to the limit cycle where the radius of the cycle is about 0.09, which proves that instead of converging to the Nash equilibrium, the dynamics converge to a limit cycle that is affected by the payoff matrices and the speed of adjustment. With 3000 iterations, the average number of cycles is around 60. The result is robust for other 3 isocline between Nash Equilibrium and the boundary of the action space.

Figure A.1: The figure shows the distance between observations and Nash equilibrium over time with 50 simulations and 3000 iterations in each simulation.



The blue (red) lines shows the distance over time when trajectory crosses the isocline and the initial position is outside (inside) the limit cycle.

A.3 Results in details

Table A.1: Multinomial logistic regressions of cycle classifications on treatment dummy variables.

Dependent	CW	Diagonal	CCW	CD
continuous	-2.94***	-4.06***	-2.61***	-2.84***
	(0.319)	(0.381)	(0.322)	(0.385)
pure	-2.16***	-2.21***	-2.40***	-22.28***
	(0.307)	(0.348)	(0.329)	(0.370)
AMPa	-0.54	-0.60	-0.61	-0.20
	(0.462)	(0.496)	(0.468)	(0.534)
continuous_pure	1.49***	1.24**	0.09	2.27***
	(0.384)	(0.523)	(0.425)	(0.459)
continuous_AMPa	0.89*	0.93	0.84	0.76
	(0.506)	(0.584)	(0.511)	(0.609)
pure_AMPa	0.66	0.60	0.50	0.32
	(0.497)	(0.552)	(0.531)	(0.593)
continuous_pure_AMPa	-0.39	-0.54	-0.15	-0.57
	(0.587)	(0.753)	(0.646)	(0.708)
second_half	-0.16**	-0.06	-0.17**	-0.16
	(0.066)	(0.083)	(0.080)	(0.103)
block_2	-0.58***	-0.57***	-0.62***	-0.75***
	(0.150)	(0.218)	(0.156)	(0.249)
Constant	3.04***	2.37***	1.99***	0.76**
	(0.301)	(0.347)	(0.307)	(0.359)
Observations	37,920	37,920	37,920	37,920

Dependent variables are dummy variables of classified types of dynamics. "AMPb, mixed strategy, discrete time" and type "Stay" are served as the baseline comparison group. Significance level: *** 0.01, ** 0.05, * 0.1.

Table A.2: Mean of the mean observations of pairs with mean Distance to predictions.

Mean of by-period Mean Summary Table

Treatments	row median	column median	To NE	p-value	To Center	p-value	To MM	Harmonic Disp	Geometric Disp
				Panel A: AMPb games					
mm	0.371	0.632	0.157	0.011	0.224	0.000	0.398	0.187	0.376
rp	0.523	0.519	0.313	0.000	0.124	0.000	0.295	0.192	0.386
p-value	0.000	0.000	0.000	-	0.000	-	0.000	0.720	0.721
Mixed	0.461	0.558	0.242	0.000	0.135	0.000	0.339	0.140	0.284
Pure	0.472	0.565	0.268	0.006	0.188	0.000	0.328	0.240	0.480
p-value	0.705	0.813	0.433	-	0.009	-	0.674	0.000	0.000
Continuous	0.498	0.529	0.301	0.000	0.181	0.000	0.330	0.181	0.364
Discrete	0.434	0.593	0.208	0.001	0.142	0.000	0.337	0.200	0.400
p-value	0.027	0.034	0.005	-	0.052	-	0.763	0.148	0.155
				Panel B: AMPa games					
mm	0.605	0.289	0.150	0.000	0.255	0.000	0.463	0.189	0.380
rp	0.501	0.440	0.247	0.000	0.115	0.000	0.321	0.200	0.401
p-value	0.000	0.000	0.000	-	0.000	-	0.000	0.374	0.378
Mixed	0.512	0.396	0.211	0.057	0.159	0.000	0.352	0.152	0.305
Pure	0.568	0.371	0.210	0.121	0.176	0.000	0.397	0.240	0.481
p-value	0.014	0.380	0.953	-	0.460	-	0.069	0.000	0.000
Continuous	0.550	0.429	0.263	0.000	0.156	0.000	0.375	0.188	0.377
Discrete	0.531	0.337	0.159	0.259	0.179	0.000	0.374	0.204	0.409
p-value	0.429	0.001	0.000	-	0.314	-	0.988	0.161	0.170
				Panel C: IDDS games					
mm	0.083	0.804	0.219	0.000	0.521	0.000	0.219	0.112	0.240
rp	0.106	0.752	0.283	0.000	0.479	0.000	0.283	0.132	0.278
p-value	0.409	0.178	0.119	-	0.238	-	0.119	0.327	0.318
Mixed	0.078	0.782	0.241	0.000	0.517	0.000	0.241	0.084	0.188
Pure	0.116	0.761	0.277	0.000	0.472	0.000	0.277	0.165	0.340
p-value	0.187	0.577	0.351	-	0.167	-	0.351	0.000	0.000
Continuous	0.122	0.805	0.244	0.000	0.496	0.000	0.244	0.123	0.258
Discrete	0.073	0.738	0.274	0.000	0.494	0.000	0.274	0.126	0.270
p-value	0.084	0.072	0.429	-	0.950	-	0.429	0.888	0.747

Harmonic and geometric distances are calculated by standard deviation of both players. p-value in column 5 and 7 shows p-value for t test of by-period mean data for given treatments between distance to predictions.

Table A.3: Median of the mean observations of pairs with median Distance to predictions.

Median of by-period Mean Summary Table

Treatments	row median	column median	To NE	p-value	To Center	p-value	To MM	Harmonic Disp	Geometric Disp
Panel A: AMPb games									
mm	0.376	0.664	0.134	0.007	0.216	0.000	0.391	0.255	0.516
rp	0.516	0.529	0.302	0.000	0.107	0.000	0.314	0.261	0.540
p-value	0.000	0.000	0.000	-	0.000	-	0.000	0.579	0.530
Mixed	0.465	0.544	0.241	0.002	0.114	0.000	0.346	0.219	0.452
Pure	0.500	0.582	0.240	0.009	0.176	0.000	0.317	0.500	1.000
p-value	0.692	0.702	0.524	-	0.007	-	0.653	0.000	0.000
Continuous	0.529	0.544	0.341	0.001	0.198	0.000	0.352	0.219	0.456
Discrete	0.450	0.604	0.212	0.023	0.128	0.000	0.326	0.276	0.558
p-value	0.017	0.092	0.011	-	0.045	-	0.909	0.055	0.066
Panel B: AMPa games									
mm	0.584	0.284	0.107	0.000	0.243	0.000	0.443	0.253	0.511
rp	0.500	0.432	0.238	0.000	0.110	0.000	0.320	0.440	0.881
p-value	0.000	0.000	0.000	-	0.000	-	0.000	0.111	0.101
Mixed	0.510	0.411	0.218	0.102	0.162	0.000	0.352	0.243	0.488
Pure	0.542	0.359	0.198	0.207	0.167	0.000	0.371	0.500	1.000
p-value	0.041	0.702	0.732	-	0.577	-	0.161	0.000	0.000
Continuous	0.521	0.431	0.265	0.002	0.138	0.000	0.352	0.276	0.573
Discrete	0.527	0.333	0.154	0.449	0.173	0.000	0.371	0.376	0.753
p-value	0.914	0.003	0.000	-	0.219	-	0.541	0.386	0.356
Panel C: IDDS games									
mm	0.055	0.807	0.227	0.001	0.536	0.001	0.227	0.013	0.000
rp	0.084	0.737	0.278	0.000	0.487	0.000	0.278	0.002	0.000
p-value	0.360	0.155	0.116	-	0.224	-	0.116	0.423	0.983
Mixed	0.075	0.747	0.262	0.000	0.516	0.000	0.262	0.029	0.096
Pure	0.078	0.775	0.244	0.004	0.490	0.004	0.244	0.000	0.000
p-value	0.509	0.611	0.402	-	0.270	-	0.402	0.055	0.001
Continuous	0.102	0.799	0.230	0.002	0.500	0.002	0.230	0.007	0.000
Discrete	0.061	0.738	0.272	0.000	0.490	0.000	0.272	0.017	0.000
p-value	0.147	0.147	0.515	-	0.669	-	0.515	0.821	0.916

Harmonic and geometric distances are calculated by IQR of both players. p-value in column 5 and 7 shows p-value for Wilcoxon signed-rank test of by-period mean data for given treatments between distance to predictions.

Table A.4: Regressions of each classified cycle type on treatment dummies collapsed at pair level.

Dependent	CW	CCW	diagonal	stay
continuous	-0.16***	-0.01	-0.21***	0.40***
	(0.036)	(0.018)	(0.024)	(0.041)
pure	-0.07*	-0.06***	-0.05*	0.22***
	(0.040)	(0.020)	(0.027)	(0.046)
AMPa	-0.01	-0.01	-0.02	0.02
	(0.038)	(0.019)	(0.025)	(0.043)
continuous_pure	0.01	-0.07***	0.03	0.02
	(0.056)	(0.027)	(0.037)	(0.063)
continuous_AMPa	0.06	0.02	0.03	-0.10*
	(0.051)	(0.025)	(0.034)	(0.058)
pure_AMPa	0.04	-0.00	0.01	-0.03
	(0.057)	(0.028)	(0.038)	(0.065)
continuous_pure_AMPa	0.03	0.01	-0.01	-0.03
	(0.079)	(0.039)	(0.052)	(0.090)
Constant	0.49***	0.17***	0.26***	0.04
	(0.027)	(0.013)	(0.018)	(0.030)
Observations	380	380	380	380
R-squared	0.125	0.234	0.359	0.475

Dependent variables count fraction of time pairs play each type of classified cycles. "AMPb, mixed strategy, discrete time" is served as the baseline comparison group. Significance level: *** 0.01, ** 0.05, * 0.1.

Table A.5: BR learning regression for eq (1.3).

	(1) Row	(2) Row	(3) Col	(4) Col
β_1	0.05***	0.45***	0.05***	0.40***
	(0.007)	(0.042)	(0.006)	(0.036)
pure	0.36***	0.15***	0.39***	0.18***
	(0.043)	(0.053)	(0.040)	(0.050)
mm	-0.01*	-0.21***	-0.02***	-0.15***
	(0.009)	(0.060)	(0.008)	(0.055)
AMPa	-0.01	-0.01	-0.01	-0.04
	(0.009)	(0.061)	(0.008)	(0.046)
IDDS	0.03	0.15	-0.04***	-0.03
	(0.027)	(0.119)	(0.008)	(0.100)
pure_mm	-0.14**	0.35***	-0.27***	0.11
	(0.056)	(0.097)	(0.050)	(0.080)
pure_AMPa	0.07	0.09	0.06	0.05
	(0.059)	(0.077)	(0.053)	(0.075)
pure_IDDS	-0.23***	0.27	-0.32***	0.11
	(0.067)	(0.166)	(0.046)	(0.136)
mm_AMPa	0.01	-0.06	0.00	0.08
	(0.011)	(0.089)	(0.011)	(0.075)
mm_IDDS	0.07	0.08	0.03***	0.10
	(0.043)	(0.160)	(0.010)	(0.122)
pure_mm_AMPa	-0.16**	-0.09	-0.03	-0.08
	(0.072)	(0.127)	(0.067)	(0.123)
pure_mm_IDDS	0.39***	-0.15	0.35***	0.07
	(0.148)	(0.209)	(0.066)	(0.187)
Observations	79,145	4,995	79,145	4,995
R-squared	0.263	0.371	0.262	0.274
Number of Pairs	415	345	415	345

Columns (1)(3) use continuous time data and columns (2)(4) use discrete time data. Significance level flags are *** 0.01, ** 0.05, * 0.1.

Table A.6: Pure directional learning regression for eq (1.4).

	(1) Row	(2) Row	(3) Col	(4) Col
β_1	0.03***	0.20***	0.03***	0.16***
	(0.004)	(0.025)	(0.004)	(0.026)
pure	0.39***	0.40***	0.42***	0.43***
	(0.042)	(0.041)	(0.040)	(0.044)
mm	-0.01	-0.10***	-0.01***	-0.05
	(0.005)	(0.034)	(0.005)	(0.038)
AMPa	-0.01	0.03	-0.00	-0.00
	(0.005)	(0.043)	(0.005)	(0.033)
IDDS	-0.00	-0.01	-0.03***	-0.13*
	(0.007)	(0.059)	(0.005)	(0.077)
pure_mm	-0.15***	0.23***	-0.28***	0.00
	(0.055)	(0.083)	(0.050)	(0.069)
pure_AMPa	0.07	0.05	0.06	0.01
	(0.058)	(0.063)	(0.052)	(0.068)
pure_IDDS	-0.20***	0.43***	-0.33***	0.21*
	(0.062)	(0.129)	(0.045)	(0.120)
mm_AMPa	0.00	-0.09*	0.00	0.02
	(0.006)	(0.053)	(0.006)	(0.047)
mm_IDDS	-0.01	0.03	0.02***	0.14
	(0.008)	(0.071)	(0.006)	(0.099)
pure_mm_AMPa	-0.16**	-0.07	-0.03	-0.03
	(0.071)	(0.105)	(0.067)	(0.109)
pure_mm_IDDS	0.47***	-0.10	0.36***	0.04
	(0.142)	(0.152)	(0.065)	(0.173)
Observations	79,145	4,995	79,145	4,995
R-squared	0.263	0.339	0.262	0.240
Number of Pairs	415	345	415	345

Column (1)(3) use continuous time data and column (2)(4) use discrete time data. Significance level flags are *** 0.01, ** 0.05, * 0.1.

Table A.7: Directional learning for eq (1.5) with discrete regret classified into discrete values.

	(1) Row	(2) Row	(3) Col	(4) Col
β_1	0.02***	0.19***	0.03***	0.14***
	(0.004)	(0.020)	(0.004)	(0.022)
pure	0.15***	0.10***	0.19***	0.15***
	(0.019)	(0.027)	(0.020)	(0.028)
mm	-0.00	-0.09***	-0.01***	-0.02
	(0.004)	(0.030)	(0.004)	(0.034)
AMPa	-0.01	0.00	-0.01**	-0.03
	(0.004)	(0.031)	(0.005)	(0.025)
IDDS	0.00	0.02	-0.03***	0.01
	(0.007)	(0.052)	(0.004)	(0.062)
pure_mm	0.05	0.26***	-0.11***	0.08
	(0.035)	(0.063)	(0.029)	(0.051)
pure_AMPa	0.05**	0.03	-0.04*	-0.06*
	(0.027)	(0.039)	(0.023)	(0.034)
pure_IDDS	-0.07**	0.20***	-0.13***	0.11
	(0.029)	(0.078)	(0.025)	(0.076)
mm_AMPa	0.00	-0.06	0.01	0.04
	(0.006)	(0.043)	(0.006)	(0.040)
mm_IDDS	-0.02**	0.01	0.02***	-0.01
	(0.008)	(0.064)	(0.005)	(0.087)
pure_mm_AMPa	-0.15***	-0.00	0.02	-0.08
	(0.043)	(0.075)	(0.034)	(0.063)
pure_mm_IDDS	0.12*	-0.15	0.14***	-0.03
	(0.074)	(0.102)	(0.040)	(0.114)
Observations	79,145	4,995	79,145	4,995
R-squared	0.231	0.337	0.259	0.247
Number of Pairs	415	345	415	345

Values: 1 ($R_{it} \leq 0.33$), 2 ($R_{it} \in (0.33, 0.67)$) and 3 ($R_{it} \geq 0.67$). Column (1)(3) use continuous time data and column (2)(4) use discrete time data. Significance level flags are *** 0.01, ** 0.05, * 0.1.

Table A.8: Directional learning for eq (1.2) with 5 lagged regret terms.

	(1) Row	(2) Row	(3) Col	(4) Col
β_1	1.16***	0.62***	0.78***	0.50***
	(0.046)	(0.032)	(0.048)	(0.029)
β_1 L1	0.08*	0.01	-0.04	-0.02
	(0.042)	(0.022)	(0.036)	(0.020)
β_1 L2	0.08*	-0.18***	0.07*	-0.12***
	(0.046)	(0.026)	(0.043)	(0.017)
β_1 L3	0.17***	-0.08***	0.10**	-0.01
	(0.046)	(0.027)	(0.042)	(0.020)
β_1 L4	0.13***	0.01	-0.01	-0.01
	(0.043)	(0.016)	(0.041)	(0.015)
β_1 L5	0.24***	-0.06***	0.09**	-0.04***
	(0.044)	(0.019)	(0.036)	(0.013)
Observations	3,270	77,070	3,270	77,070
R-squared	0.284	0.220	0.187	0.231
Number of Pairs	345	415	345	415

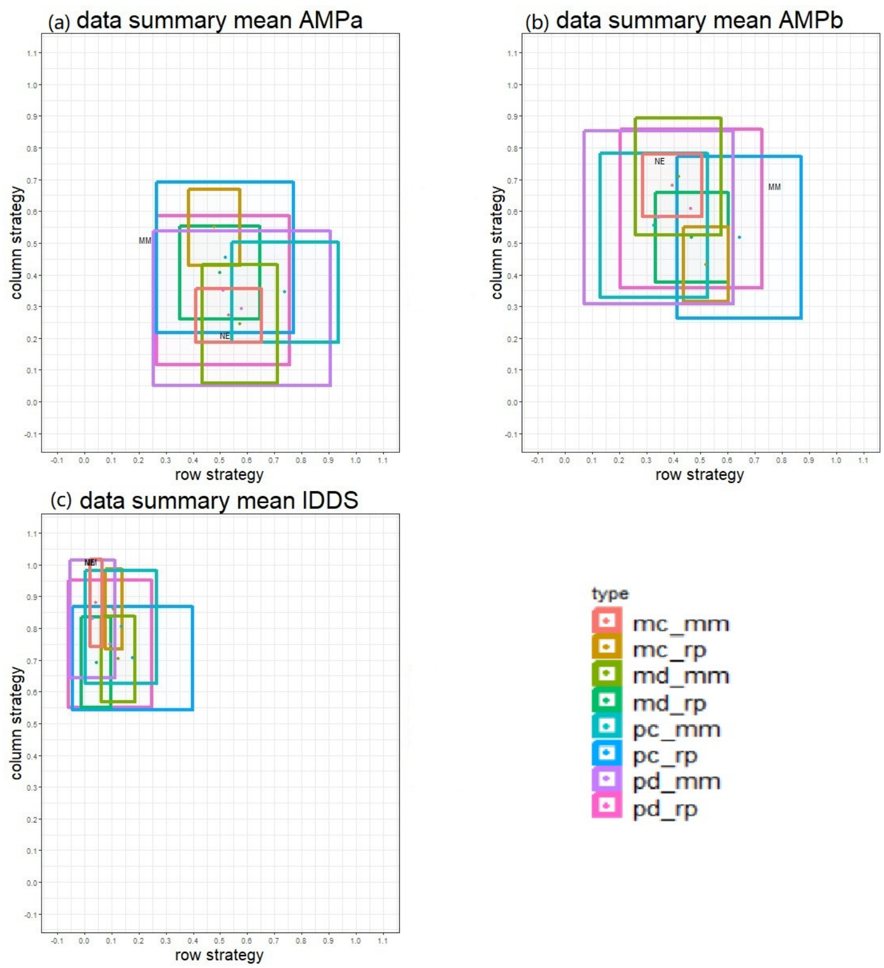
Column (2)(4) use continuous time data and column (1)(3) use discrete time data. Significance level flags are *** 0.01, ** 0.05, * 0.1.

Table A.9: Directional learning for eq (1.2) with 1 lagged regret term.

	(1) Row	(2) Row	(3) Col	(4) Col
β_1	1.12***	0.61***	0.74***	0.51***
	(0.038)	(0.032)	(0.037)	(0.030)
β_1 L1	0.01	-0.12***	-0.08***	-0.11***
	(0.033)	(0.023)	(0.025)	(0.024)
Observations	4,650	78,730	4,650	78,730
R-squared	0.308	0.190	0.199	0.218
Number of Pairs	345	415	345	415

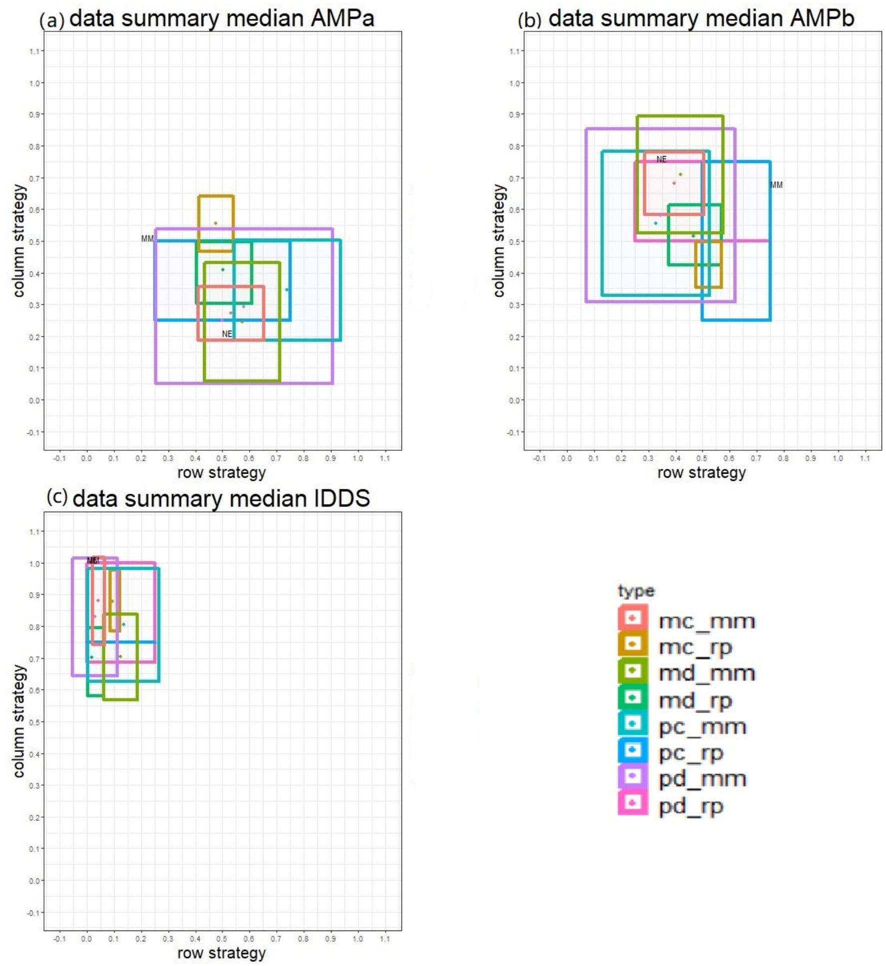
Column (2)(4) use continuous time data and column (1)(3) use discrete time data. Significance level flags are *** 0.01, ** 0.05, * 0.1.

Figure A.2: Time average profiles.



Treatments are p=pure, m=mixed strategy choice; c=continuous, d=discrete time; rp=random pairwise, mm=mean matching protocol. Dots inidcate mean of the time averages by treatment, and rectangles enclose Row x Column mean +/- standard deviation. The figure suggests that maximin is seldom the best predictor of central tendency, while NE predicts well in some cases. More often Center seems best, especially in treatment combinations that have large boxes, indicating considerable heterogeneity across instances. The IDDS data under all treatments cluster much closer to the NE=MM point (0,1) than to the Center point (.5, .5).

Figure A.3: Data summary of population time averages in 3 games colored by treatments.



p=pure, m=mixed strategy choice; c=continuous, d=discrete time; rp=random pairwise, mm=mean matching protocol. Dots are median of the time averages by treatments. Rectangles are shaped by median and 1st/3rd quantiles.

Figure A.4: The fraction of time pairs play each cycle classifications sorted by the fraction of time each pair plays clockwise cycles.

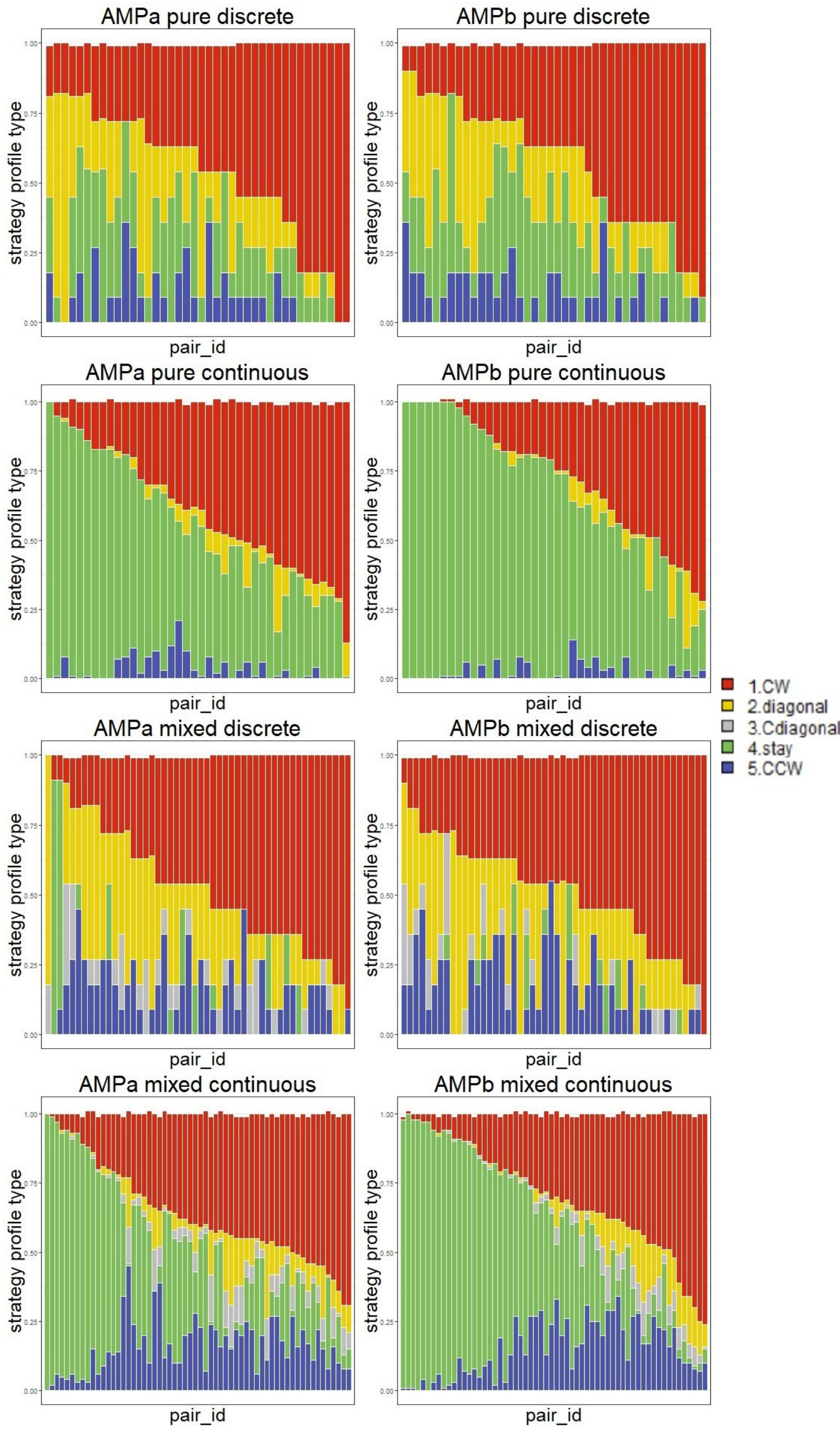
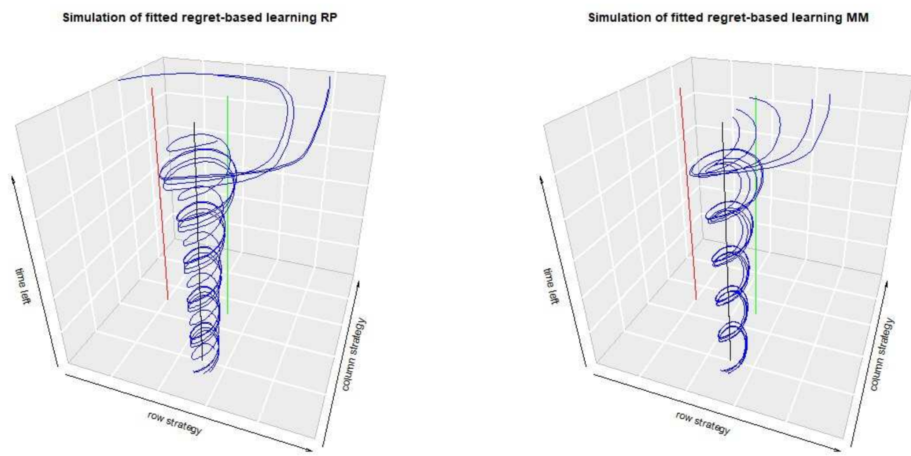


Figure A.5: Simulation result in AMPa games mixed strategy treatments.



Simulation under random pairwise matching is on the left and simulation under mean matching is on the right.

Appendix B

Supplement to Chapter Two

B.1 Survey Data

At the end of the session, subjects are asked to complete a post-experimental-open-ended survey to explain their behavioral pattern and the reason that they choose those strategies. Several important key words are extracted from subjects' answers and the frequency of each key word is counted in Table B.1.

In SH, 30 out of 42 subjects claim to realize the efficiency of payoff-dominant strategy. 5 subjects mention strategic teaching and 19 discussed adaptive learning. It shows that most subjects behavioral thinking is consistent with previous findings and learning theories.

In BOS, 28 out of 38 subjects mentioned alternating between two pure Nash equilibrium. 9 of them discuss equalizing the payoff and a few others mention retaliation and adjusting strategy. There also exist some selfish subjects. The answers here indicate that subjects realize the necessity of alternation. Although it might be difficult to alternate between Nash equilibria under some treatments, the problem should be the way they alternate, not whether they realize it or not.

Table B.1: Results for the post experimental survey.

Survey data results			
Panel A: stag hunt sessions			
payoff dominant	30	follower	2
PD educator	5	selfish	3
PD adapter	19	unclear	7
Panel B: battle of the sexes sessions			
alternate	28	adjust	3
equalizer	9	selfish	4
retaliate	6	unclear	2
betrayal	1	random	1

Numbers show the frequency of each type of answers about how subjects play the game.

B.2 Further Investigation of Dynamics in BOS

B.2.1 Transition Probabilities Between Corner Profiles

As can be observed from previous literature and results so far, the coordination problem in stag hunt games follows a straightforward comparison between payoff dominant and risk dominant equilibrium, while the dynamics in battle of the sexes games is complicated. Though the experimental data shows some dynamics patterns, the results are harmed by the large fraction of mismatch and the question requires further investigation.

Table B.2 studies the cases when subjects move from mismatch to any pure Nash equilibria and move away from Nash equilibrium to any mismatches. Equilibrium plays that last for at least 1 second in continuous time and 1 subperiod in discrete time are recorded. The total number of equilibrium plays in each treatment is listed in the last column of Table B.2. The first 3 columns ("before_10", "before_01", "before_mis") represent number of cases when subjects move to an equilibrium status from selfish mismatch, ohenry mismatch and other mismatches, respectively. Column 4 to 6 ("after_10", "after_01", "after_mis") represent number of cases when subjects move away from equilibrium status to selfish mismatch, ohenry mismatch and other mismatches, respectively.

As can be learned from previous result sections, people are more likely to coordinate in the continuous time treatments and pure action sets treatments, which is reflected as those treatments have higher numbers of equilibrium plays than their counterparts. More importantly, if we compare column 1 to column 2, we can find that subjects are more likely to walk into the equilibrium from selfish mismatch than from ohenry mismatch. Similarly, column 4 and 5 show that subjects are more likely to leave the equilibrium for selfish reasons than altruistic reasons.

Another fact comes from the transition probability matrices shown in Table B.3-B.6. Data in discrete time treatments is recorded by subperiods and is recorded twice every second in continuous time. Given the nature of continuous time interaction, subjects in continuous time treatments have a higher probability of staying at the previous position compared to the probability in discrete time. Except for the difference between time treatments, some other results can be drawn from the tables. First, pairs have similar transition probabilities from mismatch to either (A, A) or (B, B) in most of

Table B.2: Number of times pairs move into and move out of any pure Nash equilibria.

	before_10	before_01	before_mis	after_10	after_01	after_mis	# of equilibrium
BOSla,PD	35	11	0	30	14	0	90
BOSla,PC	139	41	0	152	34	0	366
BOSla,MD	12	4	18	14	5	12	65
BOSla,MC	37	9	32	39	10	30	157
BOSma,PD	47	13	0	49	11	0	120
BOSma,PC	149	36	0	167	25	0	377
BOSma,MD	16	1	21	17	1	17	73
BOSma,MC	50	17	32	47	18	29	193
BOSha,PD	47	10	0	46	10	0	113
BOSha,PC	176	31	0	194	23	0	424
BOSha,MD	15	1	10	10	2	13	51
BOSha,MC	29	8	44	37	8	30	156

The first 3 columns ("before_10", "before_01", "before_mis") represent number of cases when subjects move to an equilibrium status from selfish mismatch, ohenry mismatch and other mismatches, respectively. Column 4 to 6 ("after_10", "after_01", "after_mis") represent number of cases when subjects move away from equilibrium status to selfish mismatch, ohenry mismatch and other mismatches, respectively.

the treatments. Second, pairs are more likely to move away from (A, A) or (B, B) to selfish mismatch than ohenry mismatch. Third, it is obviously easier for subjects to move between two pure Nash equilibria in discrete time than in continuous time. Last but not least, mismatches in mixed action sets treatments are more serious than mismatches in pure action sets treatments, mostly driven by mismatch outside the corners.

Table B.2-B.6 show some important stylized facts in battle of the sexes games that might indicate some behavior pattern of subjects. From the tables, it seems that selfish motivation is the major force of switching into and away from equilibrium status in battle of the sexes games. Although few pairs reach alternating dynamics in battle of the sexes games as shown in Section 6, the switching between equilibria, and between equilibria and mismatches are quite frequent. The data also indicates a potential

Table B.3: Transition probability matrix of BOS pure continuous treatments.

	selfish at t+1	ohenry at t+1	AA at t+1	BB at t+1	observations
selfish at t	0.75	0.01	0.11	0.14	2176
ohenry at t	0.03	0.57	0.18	0.22	326
AA at t	0.06	0.02	0.92	0.01	3870
BB at t	0.07	0.01	0.00	0.91	4458

Table B.4: Transition probability matrix of BOS pure discrete treatments.

	selfish at t+1	ohenry at t+1	AA at t+1	BB at t+1	observations
selfish at t	0.41	0.10	0.18	0.32	262
ohenry at t	0.38	0.18	0.27	0.17	77
AA at t	0.27	0.07	0.37	0.30	220
BB at t	0.28	0.08	0.30	0.34	239

Table B.5: Transition probability matrix of BOS mixed continuous treatments.

	selfish at t+1	ohenry at t+1	AA at t+1	BB at t+1	mis at t+1	observations
selfish at t	0.84	0.00	0.04	0.04	0.08	1477
ohenry at t	0.01	0.62	0.11	0.19	0.07	115
AA at t	0.03	0.01	0.94	0.00	0.02	2324
BB at t	0.02	0.01	0.00	0.94	0.02	2473
mis at t	0.03	0.00	0.01	0.02	0.95	4361

Table B.6: Transition probability matrix of BOS mixed discrete treatments.

	selfish at t+1	ohenry at t+1	AA at t+1	BB at t+1	mis at t+1	observations
selfish at t	0.21	0.02	0.16	0.23	0.39	111
ohenry at t	0.23	0.00	0.08	0.15	0.54	26
AA at t	0.19	0.06	0.47	0.11	0.18	102
BB at t	0.23	0.02	0.11	0.39	0.25	96
mis at t	0.11	0.02	0.06	0.05	0.76	447

pattern of alternating dynamics in continuous time: disadvantageous player switches to her preferred position and advantageous player follows. Both players maintain the equilibrium for a period of time and repeat the switching, which causes a short period of natural mismatch between equilibrium plays.

B.2.2 Length of Equilibrium Play and Corner Mismatches in BOS

Another comparison between two time treatments in battle of the sexes games is the length of time subjects stay at each action profile. Three profiles are interesting to us: the pure Nash equilibria, the selfish mismatch, and the ohenry mismatch. Table B.7 shows the average time and its standard deviation pairs stay at either pure Nash equilibria, the selfish mismatch and the ohenry mismatch in battle of the sexes games. The length of stay in continuous time is normalized to be consistent with "subperiod" in discrete time (data is divided by 12 as 12 time ticks in continuous time equal to 1 subperiod in discrete time). For instance, Column 1 Row 4 shows that pairs on average stay at pure Nash equilibria for about 1.43 subperiods (approximately 8.58 seconds) in "BOSla, Mixed Continuous" treatments.

Table B.7: Average length of equilibrium plays and two corner mismatches in all BOS treatments.

	NE_mean	NE_sd	selfish_mean	selfish_sd	ohenry_mean	ohenry_sd
BOSla,PD	1.68	1.18	1.63	1.18	1.20	1.18
BOSla,PC	1.00	1.01	0.25	1.01	0.18	1.01
BOSla,MD	1.93	1.48	1.15	1.48	1.00	1.48
BOSla,MC	1.43	1.06	0.55	1.06	0.24	1.06
BOSma,PD	1.35	0.81	1.36	0.81	1.19	0.81
BOSma,PC	0.90	1.08	0.32	1.08	0.16	1.08
BOSma,MD	1.47	0.90	1.23	0.90	1.00	0.90
BOSma,MC	1.20	1.01	0.44	1.01	0.21	1.01
BOSha,PD	1.34	0.79	1.72	0.79	1.23	0.79
BOSha,PC	0.78	0.71	0.38	0.71	0.25	0.71
BOSha,MD	1.24	0.50	1.27	0.50	1.00	0.50
BOSha,MC	1.05	1.07	0.51	1.07	0.26	1.07

Though turn taking can be considered as a social norm in discrete time, the fact that the average time of subjects stays on either pure Nash equilibria is higher than 1 results from the noise in the data. Caused by the same reason, the average also comes with a higher standard deviation. Still, we can draw some results from the table. On one hand, the average length of stay at either pure Nash equilibria in continuous time treatments is similar to the length in discrete time with a shorter average and a similar standard deviation. The result indicates how subjects might choose the frequency of switching when they perform alternating dynamics: the average switching frequency is about 5-8 seconds which is close to a "one period" alternation in discrete time. On the other hand, the average length of stay at both mismatches in continuous time is much shorter than the length in discrete time, showing the benefit of fast response in the continuous time environment. The fast response and quick adjustment indicate one possible reason that explains the efficiency increase in the learning stage by continuous time treatments. Furthermore, subjects switch faster at ohenry mismatch than at selfish mismatch, which is consistent with the inequality between two mismatches discussed multiple times in the paper. The slow switch at selfish mismatch indicates a potential competitive framework in battle of the sexes games where both subjects prefer to stay at their advantageous positions and wait for the other subject to switch.

B.3 Explaining the Treatment Effects

B.3.1 Continuous vs Discrete Time

In section 3, some possible mechanisms that cause the differences in subjects' behavior between continuous time and discrete time treatments are discussed. In stag hunt games, as subjects can respond quickly and switch actions freely, they have a high probability to try to play payoff-dominant action at the start of the game. They are also more likely to accelerate the convergence process and shorten the “learning stage”. In battle of the sexes games, subjects are more likely to play altruistic behavior at the beginning and speed convergence. Table B.8 and B.9 show some findings that support this idea.

Table B.8 compares two time treatments in starting strategy, convergence speed and after-convergence efficiency (defined by compare subjects’ payoff to the optimal payoff they can get e.g. In SH, subjects’ efficiency equal to 1 when they are at payoff-dominant equilibrium) in stag hunt games. “row start” and “col start” denote the fraction of row and column subjects that start with payoff-dominant strategy. As can be observed from the table, subjects have a higher probability of choosing payoff-dominant strategy in continuous time than in discrete time. However, the difference is not significant. “Converge AA” and “Converge NE” denote the average timing when pairs stabilize at payoff-dominant equilibrium or either pure Nash equilibria, respectively. Stabilization is defined when the cumulative probability (from current time tick to the end of the period) of pairs playing a certain equilibrium is higher than 90%. The timing can also be considered as the length of “learning stage”. Timing in continuous time treatment is compressed into subperiods as that in discrete time. Pairs that do not converge are numbered as 10 (total number of subperiods). Both columns show that pairs spend less time converging to payoff dominant equilibrium or either pure Nash equilibria in continuous time than in discrete time. “convergence efficiency” shows the average efficiency when pairs successfully converge. There should be little difference between two time treatments in this column and the results meet the expectation.

Table B.8: Continuous vs Discrete in starting strategy, convergence speed and efficiency after convergence in SH.

Treatments	row start	col start	converge AA	converge NE	convergence efficiency
SH0.6R,PD	0.74	0.76	5.76	4.86	0.81
SH0.6R,PC	0.79	0.81	4.32	3.83	0.90
SH0.6R,MD	0.52	0.55	7.40	6.90	0.77
SH0.6R,MC	0.67	0.69	4.35	3.12	0.82
SH1R,PD	0.64	0.69	6.67	5.43	0.80
SH1R,PC	0.81	0.74	3.88	1.89	0.87
SH1R,MD	0.50	0.67	7.05	6.00	0.74
SH1R,MC	0.62	0.60	4.91	3.28	0.83
SH2R,PD	0.52	0.48	6.57	3.69	0.94
SH2R,PC	0.67	0.67	5.18	3.17	0.94
SH2R,MD	0.40	0.48	7.33	5.71	0.94
SH2R,MC	0.64	0.62	4.98	2.82	0.94

"row start" and "col start" denote the fraction of row and column subjects that start with payoff-dominant strategy. "Converge AA" and "Converge NE" denote the average time tick when pairs stabilize at payoff-dominant equilibrium or either pure Nash equilibrium, respectively. Pairs which do not converge are numbered as 10 (total number of subperiods). "convergence efficiency" shows the average efficiency when pairs successfully converge.

Table B.9 compares two time treatments in starting strategy, convergence speed, alternations and after-convergence efficiency in battle of the sexes games. "row start" and "col start" denote the fraction of row and column subjects that start with altruistic strategy. Column 1 of Table B.9 does not show a consistent pattern, although subjects behave more altruistically at the beginning in continuous time than in discrete time in "BOSla,Pure" and "BOSha, Pure" treatments. "Converge NE" denotes the average time tick when pairs stabilize at pure Nash equilibria. As there are too many mismatches, pairs which do not converge are numbered as "NA" and are removed when computing the mean (As a comparison, the number in the parentheses shows the averages when no-convergence is numbered as 10). Column 3 shows that pairs spend less time on learning to coordinate in continuous time than in discrete time. The difference seems larger when the level of payoff asymmetry is higher. “alternate index” denotes the average difference between the frequency of pairs playing two pure Nash equilibria when successfully converging. Pairs seem to alternate worse when the level of payoff asymmetry is low but overall speaking they seem to alternate pretty well if they successfully converge. “convergence efficiency” shows the average efficiency when pairs successfully converge. In continuous time, pairs suffer from switching cost because of inertia, as one subject follows the other subject from one pure NE to the other pure NE with a reaction delay. So the efficiency is expected to be a bit lower in continuous time than the efficiency in discrete time. The last column of Table B.9 confirms the expectation.

B.3.2 Mixed vs Pure Action Sets

There should be no difference between pure action sets and mixed action sets as two treatments in fact use the similar payoff matrices and the same pure Nash equilibria. However, there are significant treatment effects from action sets treatments. In this subsection, one potential mechanism that is inferred from the data by taking a deeper look at the mismatches is discussed.

Table B.10 and B.11 show the main findings. Two reference lines in the following tables, 0.1 and 0.9, as well as the definition of “mismatch”, are directly from the classification of observations in section 5.2. Table B.10 shows the frequency of each type

Table B.9: Continuous vs Discrete in starting strategy, convergence speed, alternations and efficiency after convergence in BOS.

Treatments	row start	col start	converge NE	alternate index	convergence efficiency
BOSla,PD	0.37	0.39	6.68(8.34)	2.58	0.99
BOSla,PC	0.53	0.50	4.84(7.28)	1.74	0.93
BOSla,MD	0.21	0.34	7.00(9.21)	3.50	0.98
BOSla,MC	0.16	0.45	6.29(8.63)	2.04	0.95
BOSma,PD	0.53	0.34	7.00(9.05)	1.58	0.99
BOSma,PC	0.45	0.42	4.01(7.32)	1.91	0.93
BOSma,MD	0.16	0.34	7.22(9.34)	1.78	0.97
BOSma,MC	0.21	0.29	5.36(8.66)	1.99	0.92
BOSha,PD	0.29	0.47	5.64(8.74)	1.73	1.00
BOSha,PC	0.45	0.55	2.72(8.28)	1.05	0.93
BOSha,MD	0.24	0.18	8.00(9.84)	1.67	0.99
BOSha,MC	0.21	0.21	4.06(8.59)	2.15	0.94

"row start" and "col start" denote the fraction of row and column subjects that start with altruistic strategy. "Converge NE" denotes the average time tick when pairs stabilize at pure Nash equilibria. Pairs which do not converge are numbered as NA and are removed from computation (the number in the parentheses shows the averages when no-convergence is numbered as 10). "alternate index" and "convergence efficiency" denote the average difference between the frequency of pairs playing two pure Nash equilibria and the average efficiency after pairs successfully converge, respectively.

of mismatches in stag hunt games. Mismatches are further classified into four types: "corner" denotes mismatches when either subject's mixture is above 0.9 and the other's below 0.1. "one at payoff(risk)" denotes the case when one subject stay at payoff(risk)-dominant strategy (mixture greater than or equal to 0.9 for payoff dominant strategy and lower than or equal to 0.1 for risk dominant strategy) while the other subject's mixture is between 0.1 and 0.9. "middle" represents the rest of the mismatches while p, q are between 0.1 and 0.9. Rows of pure action sets treatments are less interesting since there is only one possibility of mismatches. As discussed in previous sections, there are more mismatches in mixed action sets treatments than in pure action sets treatments. Those additional mismatches are mostly from three types: "corner", "one at risk" and "one at payoff", which contribute to about 80% of the total mismatches in mixed action sets treatments. All types indicate insufficient strategic teaching or insufficient following in the experiment, describing the situation when one subject sends out some insufficient signals to the other subject who plays risk-dominant strategy or the situation when one subject wanders in the middle while the other subject has already reached payoff-dominant strategy. Both situations come with low efficiency and are mainly the result from the fact that subjects can freely try out the "middle" mixtures so they are less likely to directly go to the corner equilibria, which is in fact less efficient compared to corner equilibria. Furthermore, less pairs mismatch at corners in mixed action sets treatments as expected and there is a consistent but small fraction of mismatches in the middle of the action space.

Table B.11 shows the frequency of each type of mismatches in battle of the sexes games. Similarly, mismatches are classified into five types: "ohenry(selfish) corner" is defined as $p(q) \leq 0.1, q(p) \geq 0.9$. "one at ohenry(selfish)" denotes the case when one subject play altruistically(selfishly) (mixture within 10% range of the other's (own) preferred strategy) and the other subject's mixture is between 0.1 and 0.9. "middle" represents the rest of the mismatches while p, q are between 0.1 and 0.9. Despite the large difference in the number of mismatches, it seems that subjects behave consistently in pure and mixed action sets treatments: most of the mismatches come from selfish behavior from one ("one at selfish type") or both ("selfish corner" type) sides, which

Table B.10: Distributional frequencies of mismatches in each treatment of SH.

Treatments	corner	one at risk	one at payoff	middle	# of mismatch
SH0.6R,PD	52	0	0	0	52
SH0.6R,MD	33	36	31	28	128
SH0.6R,PC	315	0	0	0	315
SH0.6R,MC	78	134	321	164	697
SH1R,PD	54	0	0	0	54
SH1R,MD	26	33	27	34	120
SH1R,PC	185	0	0	0	185
SH1R,MC	96	255	213	157	721
SH2R,PD	34	0	0	0	34
SH2R,MD	23	38	19	0	108
SH2R,PC	225	0	0	28	225
SH2R,MC	58	268	116	70	512

"corner" denotes mismatches when either $p, q \geq 0.9$ and the other ≤ 0.1. "one at payoff(risk)" denotes the case when one subject stay at payoff(risk)-dominant equilibrium while the other subject's mixture is between 0.1 and 0.9. "middle" represents the rest of the mismatches while p, q are between 0.1 and 0.9.

describes the situations when either both subjects play selfishly at their preferred actions or one of them play selfishly but the other subject send out insufficient coordinative signal at the middle. This description also works for the "ohenry" type that describes the situation when both subjects altruistically mismatch or when one subject plays altruistically but the other subject does not sufficiently follow. Similar to SH, less pairs mismatch at corners in the mixed action sets treatments and the small fraction of mismatches in the middle is still consistent.

The difference between subjects' behavior in two action sets treatments can be described as a problem of insufficient signalling and insufficient coordination. With large action sets, subjects are more likely to try out the "middle" mixtures to insufficiently lead or follow their counterpart. However, the middle mixtures tend to lower the efficiency compared to corner equilibria. In SH, choosing the middle mixtures can neither switch subjects' best response from risk-dominant to payoff-dominant nor help the "payoff-dominant" leaders to stay at payoff-dominant strategy. In BOS, middle mixtures cannot attract the subjects who play selfish strategies to altruistic strategies and are also insufficient as a follower strategy.

Table B.11: Distributional frequencies of mismatches in each treatment of BOS.

Treatments	ohenry corner	one at ohenry	selfish corner	one at selfish	middle	# of mismatch
BOSla,PD	32	0	91	0	0	123
BOSla,MD	13	43	32	86	39	213
BOSla,PC	126	0	537	0	0	663
BOSla,MC	33	274	479	694	415	1895
BOSma,PD	27	0	95	0	0	122
BOSma,MD	7	25	45	88	43	208
BOSma,PC	91	0	698	0	0	789
BOSma,MC	57	247	502	681	323	1810
BOSha,PD	29	0	114	0	0	143
BOSha,MD	7	40	54	94	54	249
BOSha,PC	111	0	976	0	0	1087
BOSha,MC	25	200	518	1070	505	2318

"ohenry(selfish) corner" is defined as $p(q) \leq 0.1, q(p) \geq 0.9$. "one at ohenry(selfish)" denotes the case when one subject play altruistically(selfishly) and the other subject's mixture is between 0.1 and 0.9. "middle" represents the rest of the mismatches while p, q are between 0.1 and 0.9.

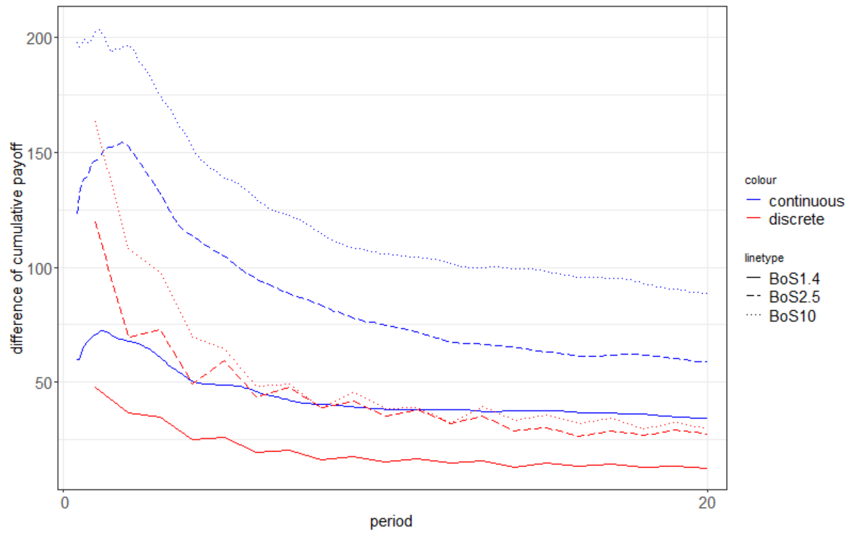
Appendix C

Supplement to Chapter Three

C.1 Payoff Inequality

If subjects prefer staying at one Nash equilibrium to alternating between Nash equilibria, the payoff between subjects could become less equal and it raises fairness concern. Figure C.1 supports this idea. We observe large payoff differences between the two time environments. Notably, we use the average cumulative payoff of the subjects in each supergame rather than their payoff at any time. The payoff difference could result from the fact that the subjects are more likely to prefer to stay at one Nash equilibrium to alternating between two Nash equilibria in continuous time and discrete time. "Equal payoff" is considered an important normative motivation for players to take turns in the literature. According to our experimental results, the subjects do not manage payoff equality well in continuous time. Thus, equal payoff is unlikely to be the major motivation guiding the subjects' interactions in our experiments.

Figure C.1: Difference in the cumulative payoff between two players within supergames.



The two time environments are represented by colors, and the three payoff matrices are represented by line types.

C.2 Transition Probability Matrices with Diagonal Cells

Table C.1 reports the transition probability matrices with the diagonal terms; thus, we can observe the probability of the pairs staying at each action profile. Despite the high probability of staying at the current position in continuous time, the results are similar to those shown in Table 3.6. Given that pairs are more likely to choose Aggressive than Accommodating, they are actually more likely to switch to a pure Nash equilibrium from Accommodating than from Aggressive. We can also confirm the one-period turn-taking pattern in discrete time. Furthermore, competition exists in discrete time as follows: the subjects are very likely to stay at Aggressive, with a conditional probability of 41%.

Table C.1: Transition probability matrices with diagonal terms.

Panel A: continuous time.

	Aggressive at t'	Accommodating at t'	RPNE at t'	CPNE at t'	num of transitions
Aggressive at t	0.89	0.00	0.05	0.05	17697
Accommodating at t	0.02	0.71	0.13	0.14	1583
RPNE at t	0.05	0.01	0.94	0.00	18338
CPNE at t	0.05	0.01	0.00	0.94	18782

Panel B: discrete time.

	Aggressive at t'	Accommodating at t'	RPNE at t'	CPNE at t'	num of transitions
Aggressive at t	0.41	0.13	0.22	0.24	1167
Accommodating at t	0.36	0.12	0.24	0.27	371
RPNE at t	0.15	0.04	0.15	0.67	1779
CPNE at t	0.15	0.04	0.71	0.10	1699

t' represents t+500 milliseconds in continuous time and t+1 period in discrete time.

C.3 Logistic Regressions for Statistical Significance

Table C.2 uses a logistic regression to assess the significance of all tables in the paper. In all five regressions, the dependent variables are coordination indicators or type indicators, and the independent variables are treatment dummy variables. Column (1) shows the treatment effects and the order effect with a random effect model. Columns (2)-(5) show the difference in playing "Alternating" and "One NE" among the treatments as follows: columns (2) and (3) use the second half of each supergame, and columns (4) and (5) use the full dataset. The results support the observations reported in the paper.

Table C.2: Logistic regression used to assess significance under a robust environment.

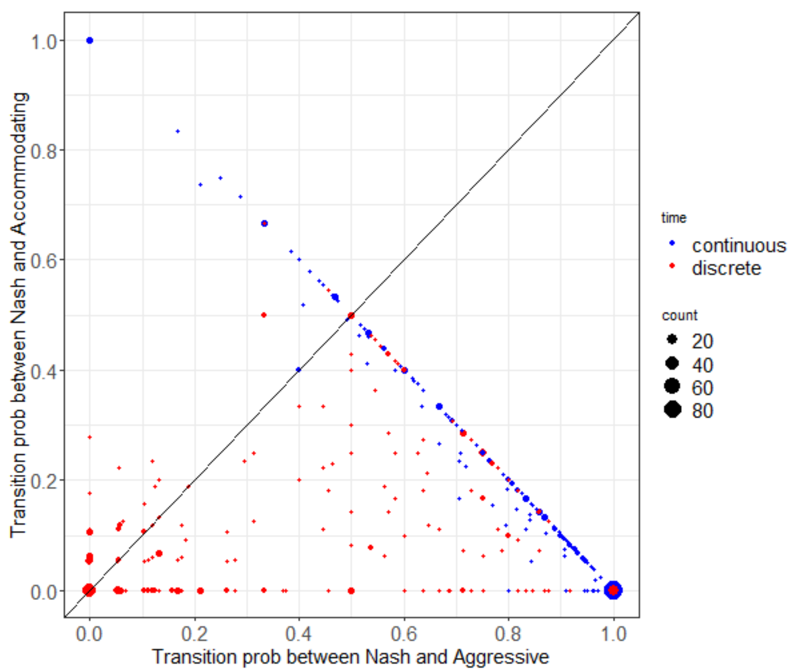
	(1)	(2)	(3)	(4)	(5)
Dependent	Coordinate	Alternating	One NE	Alternating	One NE
continuous	0.233	-1.106***	1.306**	-0.813**	1.808***
	(0.337)	(0.338)	(0.509)	(0.347)	(0.671)
BoS1.4	0.041	0.138	-0.000	0.235	-0.420
	(0.250)	(0.303)	(0.601)	(0.307)	(0.926)
BoS10	-0.087	0.277	-0.735	0.328	-1.127
	(0.252)	(0.302)	(0.722)	(0.304)	(1.165)
continuous*BoS1.4	0.031	-0.206	0.218	0.027	0.420
	(0.370)	(0.480)	(0.715)	(0.478)	(1.021)
continuous*BoS10	-0.040	-0.417	0.883	-0.328	1.127
	(0.362)	(0.482)	(0.816)	(0.485)	(1.239)
sequence2	-0.025	0.069	-0.492*	-0.054	-0.406
	(0.202)	(0.193)	(0.275)	(0.192)	(0.325)
sequence2*continuous	-0.114				
	(0.300)				
block2	0.609***	0.406**	-0.687**	0.529***	-0.933***
	(0.204)	(0.194)	(0.274)	(0.193)	(0.337)
block2*continuous	-0.852***				
	(0.300)				
Constant	0.944***	-0.237	-2.101***	-0.612**	-2.792***
	(0.238)	(0.251)	(0.454)	(0.257)	(0.623)
Observations	61,920	504	504	504	504

Subscripted asterisks indicate p-values of .10(*), .05(**) and .01(***) in two-sided t-tests assuming unequal variance between adjacent columns. The regression in column (1) is clustered at the supergame level.

C.4 Transition Probabilities at the Pair Level

The transition probability matrices in Section 4.4 support the disadvantaged-player dynamics at the aggregate level, but does this result holds at the pair level? In this section we look into the transitions for each pair of subjects. To reduce the dimensions of comparison, we reduce the transitions to three types: transitions directly between Nash equilibria, transitions between Nash and Aggressive, and transitions between Nash and Accommodating. Figure C.2 shows the scatter plot where pairs are located on the map by the conditional probabilities of the transitions. If a pair of subjects is located on the top left of the graph, the transitions between Nash and other profiles are likely to be between Nash and Accommodating, which supports the advantaged-player dynamics. If a pair is located near the bottom right of the graph, they are more likely to be motivated by the disadvantaged-player dynamics. If a pair is close to the 45 degree line, it can be either strategic teaching or a hybrid of advantaged-player and disadvantaged-player dynamics. As is shown in the figure, for the majority of the pairs, the transitions between Nash and Aggressive occur more often than the transitions between Nash and Accommodating.

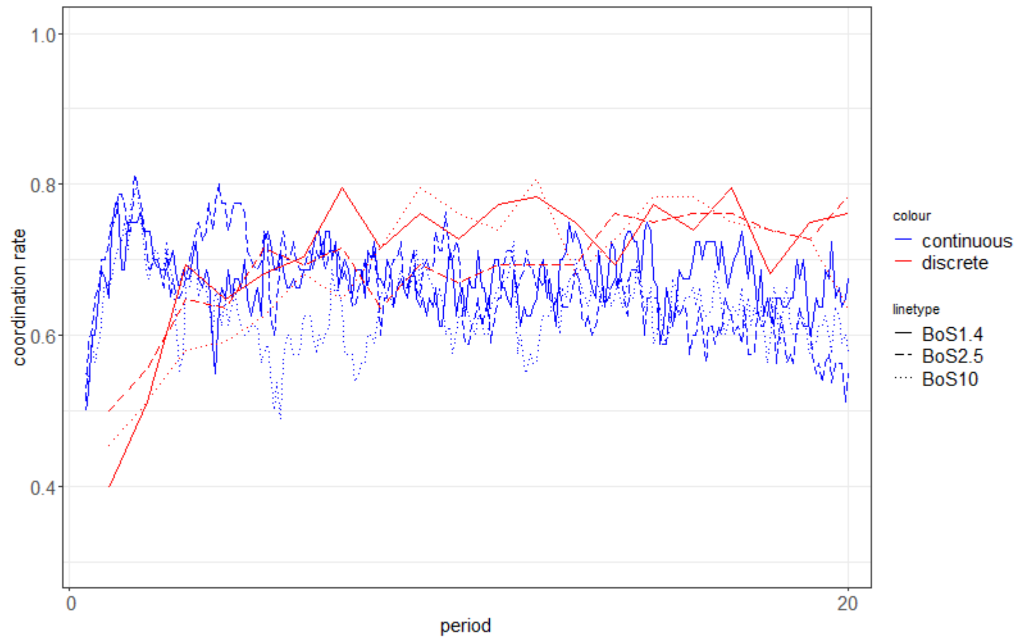
Figure C.2: Scatter plot showing the probabilities of transitioning between Nash and Aggressive and between Nash and Accommodating colored by the time environments.



The size of the points indicates the number of supergames overlapping at the coordinate.

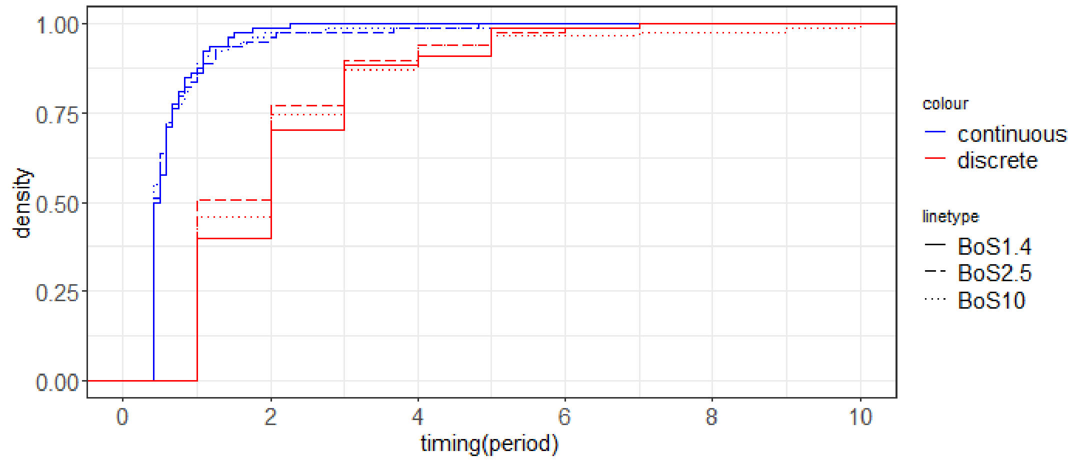
C.5 Figures with All Combinations of Treatments

Figure C.3: Coordination rate within supergames.



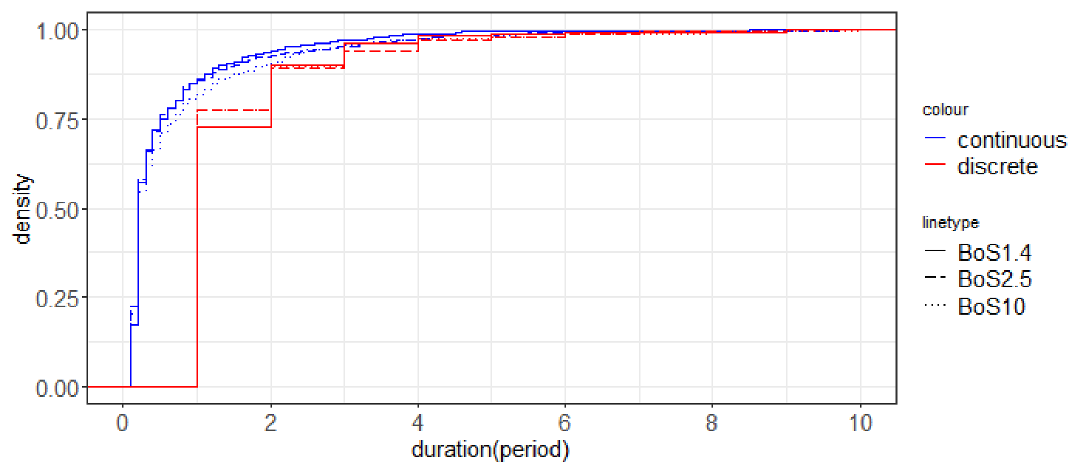
The two time environments are represented by colors, and the three payoff matrices are represented by line types.

Figure C.4: The empirical cumulative distribution function of the timing at which the pairs first reach a pure Nash equilibrium.



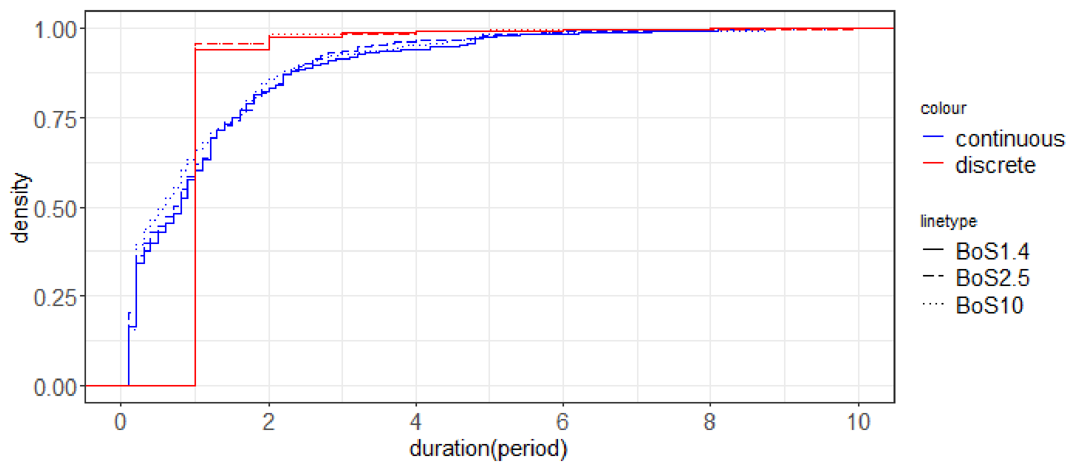
The two time environments are represented by colors, and the three payoff matrices are represented by line types. All pairs but one reach a Nash equilibrium before period 10.

Figure C.5: The empirical cumulative distribution function of the duration during which the subjects stay at mismatches.



The two time environments are represented by colors, and the three payoff matrices are represented by line types. The curves are flat and close to 1 when the duration is greater than 10 periods.

Figure C.6: The empirical cumulative distribution function of the duration during which the subjects stay at Nash equilibria.



The two time environments are represented by colors, and the three payoff matrices are represented by line types. The curves are flat and close to 1 when the duration is greater than 10 periods.

Bibliography

Al-Ubaydli, O., Jones, G., and Weel, J. (2013). Patience, cognitive skill, and coordination in the repeated stag hunt. *Journal of Neuroscience, Psychology, and Economics*, 6(2):71.

Alós-Ferrer, C. and Kern, J. (2015). Repeated games in continuous time as extensive form games. *Journal of Mathematical Economics*, 61:34–57.

Anbarci, N., Feltovich, N., and Y.Gurdal, M. (2017). Payoff inequity reduces the effectiveness of correlated-equilibrium recommendations. *Journal of Economic Literature*.

Anderson, A., Rosen, J., Rust, J., and Wong, P. (2020). Disequilibrium play in tennis.

Arifovic, J. and Ledyard, J. (2018). Learning to alternate. *Experimental Economics*, 21(3):692–721.

Battalio, R., Samuelson, L., and Van Huyck, J. (2001). Optimization incentives and coordination failure in laboratory stag hunt games. *Econometrica*, 69(3):749–764.

Benndorf, V. and Martinez-Martinez, I. (2017). Perturbed best response dynamics in a hawk–dove game. *Economics Letters*, 153:61–64.

Benndorf, V., Martinez-Martinez, I., and Normann, H.-T. (2016). Equilibrium selection with coupled populations in hawk–dove games: Theory and experiment in continuous time. *Journal of Economic Theory*, 165:472–486.

Bergin, J. and MacLeod, W. B. (1993). Continuous time repeated games. *International Economic Review*, pages 21–37.

Berninghaus, S. K. and Ehrhart, K.-M. (2001). Coordination and information: Recent experimental evidence. *Economics Letters*, 73(3):345–351.

Berninghaus, S. K., Ehrhart, K.-M., and Ott, M. (2006). A network experiment in continuous time: The influence of link costs. *Experimental Economics*, 9(3):237–251.

Besen, S. M. and Farrell, J. (1994). Choosing how to compete: Strategies and tactics in standardization. *Journal of economic perspectives*, 8(2):117–131.

Bhaskar, V. (2000). Egalitarianism and efficiency in repeated symmetric games. *Games and Economic Behavior*, 32(2):247–262.

Bigoni, M., Casari, M., Skrzypacz, A., and Spagnolo, G. (2015). Time horizon and cooperation in continuous time. *Econometrica*, 83(2):587–616.

Binmore, K., Swierzbinski, J., and Proulx, C. (2001). Does minimax work? an experimental study. *Economic Journal*, pages 445–464.

Blume, A. and Ortmann, A. (2007). The effects of costless pre-play communication: Experimental evidence from games with pareto-ranked equilibria. *Journal of Economic theory*, 132(1):274–290.

Bolton, G. E., Feldhaus, C., and Ockenfels, A. (2016). Social interaction promotes risk taking in a stag hunt game. *German Economic Review*, 17(3):409–423.

Brandts, J. and Cooper, D. J. (2006). A change would do you good.... an experimental study on how to overcome coordination failure in organizations. *American Economic Review*, 96(3):669–693.

Brindisi, F., Çelen, B., and Hyndman, K. (2014). The effect of endogenous timing on coordination under asymmetric information: An experimental study. *Games and Economic Behavior*, 86:264–281.

Brown, G. W. (1951). Iterative solution of games by fictitious play. *Activity analysis of production and allocation*, 13(1):374–376.

Brown, J. N. and Rosenthal, R. W. (1990). Testing the minimax hypothesis: a re-examination of o'neill's game experiment. *Econometrica (1986-1998)*, 58(5):1065.

Cabrales, A., Nagel, R., and Armenter, R. (2007). Equilibrium selection through incomplete information in coordination games: an experimental study. *Experimental Economics*, 10(3):221–234.

Calford, E. and Oprea, R. (2017). Continuity, inertia, and strategic uncertainty: A test of the theory of continuous time games. *Econometrica*, 85(3):915–935.

Camerer, C. and Hua Ho, T. (1999). Experience-weighted attraction learning in normal form games. *Econometrica*, 67(4):827–874.

Cason, T. N., Friedman, D., and Hopkins, E. (2013a). Cycles and instability in a rock–paper–scissors population game: A continuous time experiment. *Review of Economic Studies*, 81(1):112–136.

Cason, T. N., Lau, S.-H. P., and Mui, V.-L. (2013b). Learning, teaching, and turn taking in the repeated assignment game. *Economic Theory*, 54(2):335–357.

Cason, T. N., Savikhin, A. C., and Sheremeta, R. M. (2012). Behavioral spillovers in coordination games. *European Economic Review*, 56(2):233–245.

Chen, D. L., Schonger, M., and Wickens, C. (2016). otree—an open-source platform for laboratory, online, and field experiments. *Journal of Behavioral and Experimental Finance*, 9:88–97.

Cheung, Y.-W. and Friedman, D. (1997). Individual learning in normal form games: Some laboratory results. *Games and economic behavior*, 19(1):46–76.

Chiappori, P.-A., Levitt, S., and Groseclose, T. (2002). Testing mixed-strategy equilibria when players are heterogeneous: The case of penalty kicks in soccer. *American Economic Review*, 92(4):1138–1151.

Clark, K., Kay, S., and Sefton, M. (2001). When are nash equilibria self-enforcing? an experimental analysis. *International Journal of Game Theory*, 29(4):495–515.

Clark, K. and Sefton, M. (2001). Repetition and signalling: experimental evidence from games with efficient equilibria. *Economics Letters*, 70(3):357–362.

Cooper, R., DeJong, D. V., Forsythe, R., and Ross, T. W. (1989). Communication in the battle of the sexes game: some experimental results. *The RAND Journal of Economics*, pages 568–587.

Cooper, R., DeJong, D. V., Forsythe, R., and Ross, T. W. (1992). Communication in coordination games. *The Quarterly Journal of Economics*, 107(2):739–771.

Cooper, R. W., DeJong, D. V., Forsythe, R., and Ross, T. W. (1990). Selection criteria in coordination games: Some experimental results. *The American Economic Review*, 80(1):218–233.

Crawford, V. P. (1985). Learning behavior and mixed-strategy nash equilibria. *Journal of Economic Behavior & Organization*, 6(1):69–78.

Crawford, V. P. (1995). Adaptive dynamics in coordination games. *Econometrica: Journal of the Econometric Society*, pages 103–143.

Crawford, V. P. and Haller, H. (1990). Learning how to cooperate: Optimal play in repeated coordination games. *Econometrica: Journal of the Econometric Society*, pages 571–595.

Deck, C. and Nikiforakis, N. (2012). Perfect and imperfect real-time monitoring in a minimum-effort game. *Experimental Economics*, 15(1):71–88.

Devetag, G. and Ortmann, A. (2007). When and why? a critical survey on coordination failure in the laboratory. *Experimental economics*, 10(3):331–344.

Dixit, A. K. and Shapiro, C. (1984). *Entry dynamics with mixed strategies.* Woodrow Wilson School, Princeton University.

Doyle, A. C. (2018 (originally 1893)). *The final problem.* Johan Brown.

Dubois, D., Willinger, M., and Van Nguyen, P. (2012). Optimization incentive and relative riskiness in experimental stag-hunt games. *International Journal of Game Theory*, 41(2):369–380.

Duffy, J., Lai, E. K., and Lim, W. (2017). Coordination via correlation: an experimental study. *Economic Theory*, 64(2):265–304.

Duffy, S., Naddeo, J., Owens, D., and Smith, J. (2016). Cognitive load and mixed strategies: On brains and minimax.

Erev, I. and Roth, A. E. (1998). Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria. *American economic review*, pages 848–881.

Fehr, E. and Schmidt, K. M. (1999). A theory of fairness, competition, and cooperation. *The quarterly journal of economics*, 114(3):817–868.

Friedman, D. (1991). Evolutionary games in economics. *Econometrica*, 59(3):637–666.

Friedman, D. (1996). Equilibrium in evolutionary games: Some experimental results. *Economic Journal*, 106(434):1–25.

Friedman, D. and Fung, K. (1996). International trade and the internal organization of firms: An evolutionary approach. *Journal of International Economics*, 41(1-2):113–137.

Friedman, D., Huck, S., Oprea, R., and Weidenholzer, S. (2015). From imitation to collusion: Long-run learning in a low-information environment. *Journal of Economic Theory*, 155:185–205.

Friedman, D. and Oprea, R. (2012). A continuous dilemma. *American Economic Review*, 102(1):337–63.

Fudenberg, D. and Kreps, D. M. (1993). Learning mixed equilibria. *Games and economic behavior*, 5(3):320–367.

Gauriot, R., Page, L., and Wooders, J. (2016). Nash at wimbledon: evidence from half a million serves. *Available at SSRN 2850919*.

Goeree, J. K., Holt, C. A., and Palfrey, T. R. (2003). Risk averse behavior in generalized matching pennies games. *Games and Economic Behavior*, 45(1):97–113.

Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with orsee. *Journal of the Economic Science Association*, 1(1):114–125.

He, S. and Wu, J. (2020). Compromise and coordination: An experimental study. *Games and Economic Behavior*, 119:216–233.

Helbing, D., Schönhof, M., Stark, H.-U., and Hołyst, J. A. (2005). How individuals learn to take turns: Emergence of alternating cooperation in a congestion game and the prisoner's dilemma. *Advances in Complex Systems*, 8(01):87–116.

Hirsch, M. W., Smale, S., and Devaney, R. L. (2012). *Differential equations, dynamical systems, and an introduction to chaos.* Academic press.

Hofbauer, J. and Hopkins, E. (2005). Learning in perturbed asymmetric games. *Games and Economic Behavior*, 52(1):133–152.

Hopkins, E. (2002). Two competing models of how people learn in games. *Econometrica*, 70(6):2141–2166.

Horstmann, N., Kraemer, J., and Schnurr, D. (2016). Oligopoly competition in continuous time. *Available at SSRN 2630664*.

Hyndman, K., Terracol, A., and Vaksmann, J. (2009). Learning and sophistication in coordination games. *Experimental Economics*, 12(4):450–472.

Ioannou, C. A. and Romero, J. (2014a). A generalized approach to belief learning in repeated games. *Games and Economic Behavior*, 87:178–203.

Ioannou, C. A. and Romero, J. (2014b). Learning with repeated-game strategies. *Frontiers in neuroscience*, 8:212.

Janssen, M. A. and Ostrom, E. (2006). Governing social-ecological systems. *Handbook of computational economics*, 2:1465–1509.

Kaplan, T. R. and Ruffle, B. J. (2011). Which way to cooperate. *The Economic Journal*, 122(563):1042–1068.

Kaplan, T. R., Ruffle, B. J., and Shtudiner, Z. (2018). Cooperation through coordination in two stages. *Journal of Economic Behavior & Organization*, 154:206–219.

Kephart, C. and Friedman, D. (2015). Hotelling revisits the lab: equilibration in continuous and discrete time. *Journal of the Economic Science Association*, 1(2):132–145.

Kuroda, K. and Kameda, T. (2019). You watch my back, i'll watch yours: Emergence of collective risk monitoring through tacit coordination in human social foraging. *Evolution and Human Behavior*.

Kuzmics, C., Palfrey, T., and Rogers, B. W. (2014). Symmetric play in repeated allocation games. *Journal of Economic Theory*, 154:25–67.

Lau, S.-H. P. and Mui, V.-L. (2008). Using turn taking to mitigate coordination and conflict problems in the repeated battle of the sexes game. *Theory and Decision*, 65(2):153–183.

Lau, S.-H. P. and Mui, V.-L. (2012). Using turn taking to achieve intertemporal cooperation and symmetry in infinitely repeated $2\times$ 2 games. *Theory and Decision*, 72(2):167–188.

Lazzati, N. and Menichini, A. A. (2016). Hot spot policing: A study of place-based strategies for crime prevention. *Southern Economic Journal*, 82(3):893–913.

Leng, A., Friesen, L., Kalayci, K., and Man, P. (2018). A minimum effort coordination game experiment in continuous time. *Experimental Economics*, 21(3):549–572.

Leo, G. (2017). Taking turns. *Games and Economic Behavior*, 102:525–547.

Levitt, S. D., List, J. A., and Reiley, D. H. (2010). What happens in the field stays in the field: Exploring whether professionals play minimax in laboratory experiments. *Econometrica*, 78(4):1413–1434.

Lotka, A. J. (1926). Elements of physical biology. *Science Progress in the Twentieth Century (1919-1933)*, 21(82):341–343.

Luce, R. D. and Raiffa, H. (1989). *Games and decisions: Introduction and critical survey.* Courier Corporation.

Masiliūnas, A. (2017). Overcoming coordination failure in a critical mass game: strategic motives and action disclosure. *Journal of Economic Behavior & Organization*, 139:214–251.

Mookherjee, D. and Sopher, B. (1994). Learning behavior in an experimental matching pennies game. *Games and Economic Behavior*, 7(1):62–91.

Mookherjee, D. and Sopher, B. (1997). Learning and decision costs in experimental constant sum games. *Games and Economic Behavior*, 19(1):97–132.

Nash, J. (1951). Non-cooperative games. *Annals of mathematics*, pages 286–295.

Neumann, J. v. (1928). Zur theorie der gesellschaftsspiele. *Mathematische annalen*, 100(1):295–320.

Ochs, J. (1995). Games with unique, mixed strategy equilibria: An experimental study. *Games and Economic Behavior*, 10(1):202–217.

O'Neill, B. (1987). Nonmetric test of the minimax theory of two-person zerosum games. *Proceedings of the national academy of sciences*, 84(7):2106–2109.

Oprea, R., Charness, G., and Friedman, D. (2014). Continuous time and communication in a public-goods experiment. *Journal of Economic Behavior & Organization*, 108:212–223.

Oprea, R., Henwood, K., and Friedman, D. (2011). Separating the hawks from the doves: Evidence from continuous time laboratory games. *Journal of Economic Theory*, 146(6):2206–2225.

Palacios-Huerta, I. (2003). Professionals play minimax. *The Review of Economic Studies*, 70(2):395–415.

Park, I.-U. (2014). Cooperation in continuous dilemma and uncertain reaction lag. Technical report, Working Paper, 1–29.

Pettit, J., Friedman, D., Kephart, C., and Oprea, R. (2014). Software for continuous game experiments. *Experimental Economics*, 17(4):631–648.

Prisbrey, J. E. (1992). An experimental analysis of two-person reciprocity games.

Pruzhansky, V. (2013). Maximin play in completely mixed strategic games. *Theory and decision*, 75(4):543–561.

Rankin, F. W., Van Huyck, J. B., and Battalio, R. C. (2000). Strategic similarity and emergent conventions: Evidence from similar stag hunt games. *Games and Economic Behavior*, 32(2):315–337.

Rapoport, A. and Orwant, C. (1962). Experimental games: A review. *Behavioral Science*, 7(1):1–37.

Robinson, J. (1951). An iterative method of solving a game. *Annals of mathematics*, pages 296–301.

Romero, J. and Rosokha, Y. (2018). Constructing strategies in the indefinitely repeated prisoner's dilemma game. *European Economic Review*, 104:185–219.

Romero, J. and Zhang, H. (2018). Normative conflict and history dependence in repeated coordination games.

Sandroni, A. (2000). Reciprocity and cooperation in repeated coordination games: the principled-player approach. *Games and Economic Behavior*, 32(2):157–182.

Schmidt, D., Shupp, R., Walker, J. M., and Ostrom, E. (2003). Playing safe in coordination games:: the roles of risk dominance, payoff dominance, and history of play. *Games and Economic Behavior*, 42(2):281–299.

Selten, R. (1991). Anticipatory learning in two-person games. In *Game equilibrium models I*, pages 98–154. Springer.

Shapley, L. (1964). Some topics in two-person games. *Advances in game theory*, 52:1–29.

Simon, L. K. and Stinchcombe, M. B. (1989). Extensive form games in continuous time: Pure strategies. *Econometrica: Journal of the Econometric Society*, pages 1171–1214.

Solan, E., Maschler, M., and Zamir, S. (2013). *Game Theory*. Cambridge U Press.

Sonsino, D. and Sirota, J. (2003). Strategic pattern recognition—experimental evidence. *Games and Economic Behavior*, 44(2):390–411.

Stahl II, D. O. (1988). On the instability of mixed-strategy nash equilibria. *Journal of Economic Behavior & Organization*, 9(1):59–69.

Stephenson, D. (2019). Coordination and evolutionary dynamics: When are evolutionary models reliable? *Games and Economic Behavior*, 113:381–395.

Tang, F.-F. (2001). Anticipatory learning in two-person games: some experimental results. *Journal of economic behavior & organization*, 44(2):221–232.

Taylor, P. D. and Jonker, L. B. (1978). Evolutionary stable strategies and game dynamics. *Mathematical biosciences*, 40(1-2):145–156.

Van Huyck, J., Viriyavipart, A., and Brown, A. L. (2018). When less information is good enough: experiments with global stag hunt games. *Experimental Economics*, pages 1–22.

Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1990). Tacit coordination games, strategic uncertainty, and coordination failure. *The American Economic Review*, 80(1):234–248.

Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1991). Strategic uncertainty, equilibrium selection, and coordination failure in average opinion games. *The Quarterly Journal of Economics*, 106(3):885–910.

Van Huyck, J. B., Battalio, R. C., and Rankin, F. W. (2007). Evidence on learning in coordination games. *Experimental Economics*, 10(3):205–220.

Walker, M. and Wooders, J. (2001). Minimax play at wimbledon. *American Economic Review*, 91(5):1521–1538.

Weibull, J. W. (1997). *Evolutionary game theory*. MIT press.

Wooders, J. (2010). Does experience teach? professionals and minimax play in the lab. *Econometrica*, 78(3):1143–1154.