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# Improved Single Sector Supersymmetry Breaking

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## Abstract

Building on recent work by N. Arkani-Hamed and the present authors, we construct realistic models that break supersymmetry dynamically and give rise to composite quarks and leptons, all in a single strongly-coupled sector. The most important improvement compared to earlier models is that the second-generation composite states correspond to dimension-2 "meson" operators in the ultraviolet. This leads to a higher scale for flavor physics, and gives a completely natural suppression of flavor-changing neutral currents. We also construct models in which the hierarchy of Yukawa couplings is explained by the dimensionality of composite states. These models provide an interesting and viable alternative to gravity- and gauge-mediated models. The generic signatures are unification of scalar masses with different quantum numbers at the compositeness scale, and lighter gaugino, Higgsino, and third-generation squark and slepton masses. We also analyze large classes of models that give rise to both compositeness and supersymmetry breaking, based on gauge theories with confining, fixed-point, or free-magnetic dynamics.

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# 1  Introduction

One of the most exciting results of the recent progress in understanding non-perturbative effects in supersymmetric gauge theories [1] is that it allows the exploration of new possibilities for the realization of supersymmetry (SUSY) in nature. The most important example is the use of dynamical SUSY breaking to explain the origin of the SUSY breaking scale. In recent years, large classes of SUSY gauge theories have been discovered that exhibit dynamical SUSY breaking through a variety of different mechanisms [2], and realistic models have been built using these theories as building blocks in both (super)gravity-mediated [3] and gauge-mediated frameworks [4].

In these conventional approaches to SUSY model building, SUSY breaking arises in a separate sector consisting of fields that are neutral under the standard model gauge group, and SUSY breaking is communicated to the observable fields by messenger (gauge or gravitational) interactions. It is clearly important to know whether such a 'modular' structure is required in order for SUSY to be the solution of the hierarchy problem, or if simpler models without sectors are possible. In Ref. [5], realistic models were constructed that do not require a separate SUSY breaking sector. In these models, SUSY is broken dynamically by fields that are charged under the standard model gauge group, giving rise to composite fields with the quantum numbers of quarks and leptons. (Compositeness avoids the 'no-go' theorem of Dimopoulos and Georgi on SUSY breaking by charged fields [6].) The scalar components of the composite quarks and leptons have SUSY breaking masses induced directly by the strong dynamics, while the masses of the fermions are protected by unbroken chiral symmetries. The masses of the gauginos (which are elementary in this class of models) arise at 1-loop order, are therefore smaller than the composite scalar masses, which must therefore be in the range of 1–10 TeV. Any elementary sfermions in the model also obtain their mass from gauge mediation from the composite scalars. There are non-trivial constraints on this scenario arising from the requirement that the spectrum is phenomenologically acceptable, and these will be discussed below.

If we make the simplest assumption that the first two generations are composite while the third is elementary, we automatically gain a partial understanding of the observed hierarchy of fermion masses. The reason is that all Yukawa couplings involving composite states must arise from higher-dimension operators in the fundamental theory, and are necessarily suppressed, while the top Yukawa coupling can be order one.[1] A highly non-trivial feature of this scenario is that it does not lead to excessive

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[1]This is true as long as there are no trilinear Yukawa couplings generated by the strong dynamics, for a model with such dynamical couplings, but no SUSY breaking see [7]. For other composite SUSY

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flavor-changing neutral currents (FCNC's) from squark non-degeneracy even if the flavor sector has no flavor symmetry. This is because the strong composite dynamics is flavor-blind, and so the composite scalar masses are degenerate to high accuracy, with small corrections due to perturbative flavor-breaking couplings.[2]

We can also consider 'dimensional hierarchy' models in which the first and second generations are composites with different dimensionality.[3] In these models, there is no symmetry enforcing degeneracy of the squark masses, and we must assume that squark masses are in the 10 TeV range to suppress FCNC's.

These scenarios for single sector SUSY breaking have several interesting generic phenomenological implications. First, as mentioned above, gaugino and stop masses will be much smaller than the masses of composite squarks and sleptons. Second, the composite scalar masses unify at the compositeness scale. (In models where the first two generations are composite, *all* the scalars of the first generations unify; in the dimensional hierarchy models the scalars in the first and second generation unify separately.) Third, if we assume that the Yukawa interactions are generated by new physics at a flavor scale above the compositeness scale without special flavor symmetries, predictions for flavor-changing processes such as $\mu \to e\gamma$ are plausibly within experimental reach.

In this paper, we extend the work of Ref. [5] in two important ways. First, we construct models in which the composite quarks and leptons correspond to dimension 2 'meson' operators in the fundamental theory (rather than dimension 3 as in Ref. [5]). This means that the scale of flavor physics is higher in the new models, leading to a completely natural suppression of FCNC effects, including $\epsilon_K$. (In fact, the desirability of models with dimension-2 composites was emphasized in Ref. [5].) Second, we construct explicit dimensional hierarchy models that give a framework for understanding the fermion mass hierarchy that is closely linked to the mechanism of SUSY breaking. Finally, we construct and analyze large classes of supersymmetric gauge theories with non-perturbative dynamics of the type required for this kind of model-building. At low energies the models either confine (like the models of Ref. [5]), have conformal fixed points, or are magnetically free. This shows that the combination of compositeness and SUSY breaking is not uncommon, and suggests that further exploration of the connection between these phenomena is worthwhile.

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models, see Refs. [8, 9].

[2]This mechanism is similar to the one that operates in QCD to give rise to an approximate flavor symmetry for hadrons made of light quarks.

[3]Interesting models with this feature were constructed in Ref. [10]. However, these models do not incorporate SUSY breaking in the manner envisioned here.

This paper is organized as follows. In Section 2, we describe the origin of mass scales and the generic phenomenology of the models that we are describing. In Section 3, we analyze some specific models, and Section 4 contains our conclusions. The detailed analysis of specific gauge theories is relegated to an appendix.

## 2 Mass Scales and Phenomenology

In this Section, we describe the most important qualitative features of the models constructed in this paper. Much of this material appears already in Ref. [5], but we present it here specialized to the two new types of models we will construct: 'meson' models where the first two generations correspond to dimension 2 operators, and 'dimensional hierarchy' models in which the first generation corresponds to dimension 3, the second to dimension 2, and the third generation is elementary (dimension 1). We want to emphasize the fact that the phenomenology is very rich, and is largely independent of the details of specific models. More detail can be found in Ref. [5] and in the appendix to this paper.

### 2.1 SUSY Breaking and Compositeness

We first explain the mechanism that gives rise to SUSY breaking and compositeness. The models we describe have a strong gauge group of the form $G_{\mathrm{comp}} \times G_{\mathrm{lift}}$, where both groups are asymptotically free[4] and $\Lambda_{\mathrm{comp}} \gg \Lambda_{\mathrm{lift}}$. The scale $\Lambda_{\mathrm{comp}}$ is the compositeness scale, in the sense that the degrees of freedom that correspond to the quarks and leptons at low energies are strongly interacting at the scale $\Lambda_{\mathrm{comp}}$. Direct bounds on the compositeness scale imply that $\Lambda_{\mathrm{comp}} \gtrsim 2$ TeV. The role of the gauge group $G_{\mathrm{lift}}$ is to generate a dynamical superpotential that lifts the vacuum degeneracy and gives rise to a local SUSY breaking minimum.

The models contain the following fields[5]

| | $G_{\mathrm{lift}}$ | $G_{\mathrm{comp}}$ | $G_{\mathrm{global}}$ |
|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $\mathbf{1}$ |
| $L$ | $\bar{\square}$ | $\mathbf{1}$ | $\bar{\square}$ |
| $\bar{U}$ | $\mathbf{1}$ | $\bar{\square}$ | $\square$ |
| $P$ | $\mathbf{1}$ | $R$ | $\mathbf{1}$ |

[4]Actually $G_{\mathrm{lift}}$ need not be asymptotically free [5], but we will ignore that possibility here for simplicity.

[5]The names of the fields originate from the fact that these models are all distant cousins of the venerable '3–2' model of dynamical SUSY breaking [11, 12].

where the representation $R$ may be highly reducible (implying additional global symmetries). In addition, the model has a tree-level superpotential

$$W = \lambda Q L \bar{U}. \tag{2.1}$$

There are additional requirements on the model in order for this model to have a local SUSY breaking minimum. We choose $G_{\mathrm{global}}$ such that classically there is a flat direction with $\bar{U} \neq 0$ where $Q$ and $L$ are massive and $G_{\mathrm{comp}}$ is completely broken. Nonperturbative $G_{\mathrm{lift}}$ dynamics lift this flat direction via a dynamical superpotential of the form

$$W_{\mathrm{dyn}} \sim \Lambda_{\mathrm{lift}}^{3-r} \bar{U}^r. \tag{2.2}$$

Whether this superpotential forces $\bar{U}$ to large or small values depends on the value of $r$, but it also depends on the effective Kähler potential for $\bar{U}$. For large $\bar{U}$, $G_{\mathrm{comp}}$ is completely perturbative and the Kähler potential is smooth in $\bar{U}$, so the potential slopes toward the origin for $r > 1$.[6] For $\bar{U} \ll \Lambda_{\mathrm{comp}}$ the $G_{\mathrm{comp}}$ dynamics changes the Kähler potential for $\bar{U}$. For example, if the $G_{\mathrm{comp}}$ dynamics is confining, the Kähler potential will be smooth in terms of a 'composite' field $B = (\bar{U}^n)$. The superpotential can then be written

$$W_{\mathrm{dyn}} \sim B^{r/n}, \tag{2.3}$$

which corresponds to a potential that slopes *away* from the origin if $r/n < 1$. Therefore, for $1 < r < n$ there is no SUSY minimum for any value of $\bar{U}$, and there is a SUSY breaking minimum near the border between the region of validity of the confined and Higgs descriptions. This occurs for

$$\langle \bar{U} \rangle \sim \frac{\sqrt{N} \Lambda_{\mathrm{comp}}}{4\pi}, \tag{2.4}$$

where $N$ is the number of 'colors' of $G_{\mathrm{comp}}$. For an explanation of the factors of $4\pi$ and $N$ see Refs. [5, 14]. We keep track of powers of $N$ in our estimates because we will see that $N \sim 10$ for realistic models.

This mechanism also occurs in the case where the $G_{\mathrm{comp}}$ dynamics gives rise to a conformal fixed point (in the limit where we turn off $G_{\mathrm{lift}}$), provided that the $\bar{U}$ anomalous dimensions are sufficiently large. As long as $\bar{U} \ll \Lambda_{\mathrm{comp}}$ the $G_{\mathrm{comp}}$ dynamics is controlled by the infrared fixed point. Recall that we are assuming that

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[6]The case $r = 1$ is marginal; perturbative interactions determine whether the potential slopes toward or away from the origin [5, 13].

$G_{\mathrm{lift}}$ is weak at the scale $\Lambda_{\mathrm{comp}}$, so the non-perturbative superpotential can be viewed as a perturbation. The 1PI potential for $\bar{U}$ is therefore

$$V_{\mathrm{1PI}} \simeq (K_{\mathrm{1PI}}^{-1})_{\bar{U}^\dagger \bar{U}} \left| \frac{\partial W_{\mathrm{dyn}}}{\partial \bar{U}} \right|^2, \tag{2.5}$$

where $K_{\mathrm{1PI}}$ is the 1PI Kähler metric evaluated at the conformal fixed point. The scaling dimension of the Kähler metric $(K_{\mathrm{1PI}})_{\bar{U}^\dagger \bar{U}}$ is $2 - 2d_{\bar{U}}$, where $d_{\bar{U}}$ is the scaling dimension of $\bar{U}$. Therefore,

$$(K_{\mathrm{1PI}}^{-1})_{\bar{U}^\dagger \bar{U}} \sim \bar{U}^{2(d_{\bar{U}} - 1)/d_{\bar{U}}}. \tag{2.6}$$

This forces the potential to slope away from the origin for

$$\frac{1 - d_{\bar{U}}}{d_{\bar{U}}} > r - 1. \tag{2.7}$$

One might worry that this argument relies on a 'Higgs' description in terms of the elementary field $\bar{U}$ in a regime where the theory is strongly coupled. In many cases there is an alternate description in terms of a weakly-coupled dual theory. For example, if $G_{\mathrm{comp}} = SU(N)$ with $F$ 'flavors' $U, \bar{U}$, the theory has an infrared fixed point for $\frac{3}{2}N < F < 3N$ [15]. There is a dual description in terms of a theory with gauge group $SU(F - N)$ in which the 'baryon' operator $\bar{U}^N$ in the original theory is mapped to an operator $\bar{u}^{F-N}$ in the dual. For $F$ near $\frac{3}{2}N$ the dual description is weakly coupled, and the considerations of the previous paragraph can be made rigorous.One finds that the behavior of the Kähler potential agrees precisely with Eq. (2.5). This equivalence between the 'Higgs' and 'dual' descriptions can be viewed as a generalization of the usual 'complementarity' [16] for theories with scalars in the fundamental representation, and gives us additional confidence in the considerations above.

We see that there is a general mechanism that can stabilize the field $\bar{U}$ and give rise to a SUSY-breaking vacuum. In the models we construct, the above discussion holds only on one branch of the moduli space, and there are other branches with SUSY minima. However, the mechanism still gives rise to a metastable *local* SUSY breaking minimum, which is sufficient.

In these models, SUSY is broken by

$$\langle F_{\bar{U}} \rangle \sim \left\langle \frac{\partial W_{\mathrm{dyn}}}{\partial \bar{U}} \right\rangle \sim \frac{\Lambda_{\mathrm{comp}}^2}{4\pi} (\lambda \sqrt{N})^{r-1} \left( \frac{\Lambda_{\mathrm{lift}}}{\Lambda_{\mathrm{comp}}} \right)^{3-r}, \tag{2.8}$$

where we used Eq. (2.4). Since $r < 3$ (otherwise the dynamical superpotential Eq. (2.2) does not have a good limit $\Lambda_{\mathrm{lift}} \to 0$ when $G_{\mathrm{lift}}$ is asymptotically free),

we have $\langle F_{\bar{U}} \rangle \ll \Lambda^2_{\mathrm{comp}}$. The scalar components of $\bar{U}$ get a SUSY-breaking mass of order

$$m^2_{\bar{U}} \sim \left\langle \frac{\partial^2}{\partial \bar{U}^2} \left| \frac{\partial W_{\mathrm{dyn}}}{\partial \bar{U}} \right|^2 \right\rangle \sim \frac{F^2_{\bar{U}}}{\langle \bar{U} \rangle^2} \equiv m^2_{\mathrm{comp}}. \tag{2.9}$$

The 'preon' fields $P$ charged under $G_{\mathrm{comp}}$ get SUSY-breaking masses from effects such as

$$\Gamma_{\mathrm{1PI}} \sim \int d^2\theta d^2\bar{\theta} \frac{16\pi^2}{\Lambda^2_{\mathrm{comp}}} \bar{U}^\dagger \bar{U} P^\dagger P \sim m^2_{\mathrm{comp}} P^\dagger P. \tag{2.10}$$

The fermion components of $P$ can remain massless because of unbroken chiral symmetries [17], and these can be identified with quarks and leptons.

With this discussion of the mass scales, we have enough information to analyze the main features of the phenomenology of these models. Masses for standard-model gauginos and elementary charged scalars are generated by gauge mediation from the composite scalars, so that

$$m_{\lambda,\mathrm{SM}} \sim N \frac{g^2_{\mathrm{SM}}}{16\pi^2} m_{\mathrm{comp}}, \qquad m^2_{\phi,\mathrm{elem}} \sim N \left( \frac{g^2_{\mathrm{SM}}}{16\pi^2} m_{\mathrm{comp}} \right)^2. \tag{2.11}$$

Note that the multiplicity factor $N$ enhances gaugino masses compared to elementary scalar masses.

In the models we construct, some or all of the quarks and leptons from the first two generations are composite, while the third generation is elementary. The reason for this is that in our models the Yukawa couplings for composite quarks and leptons arise from higher-dimension operators in the fundamental theory, and are naturally small compared to one. It is difficult to accommodate the order-one top Yukawa coupling in this framework unless the top quark is elementary. Another reason for the third generation to be elementary is that stop masses of order $m_{\mathrm{comp}} \sim 1$–$10$ TeV (needed to get sufficiently large gaugino masses) necessitate a large amount of fine-tuning in electroweak symmetry breaking. These arguments do not forbid the possibility that the right-handed bottom quark or third-generation leptons are composite, but we will not take advantage of these loopholes. In order to obtain a third generation scalar mass $m_3 \gtrsim 100$ GeV we therefore require

$$m_{\mathrm{comp}} \gtrsim \frac{10 \text{ TeV}}{\sqrt{N}}. \tag{2.12}$$

We see that this class of models naturally has a superpartner spectrum similar to the 'more minimal' framework [18]. In models of this kind, there is a dangerous

negative contribution to the third-generation squark masses from the heavy scalars [19], given by

$$\mu \frac{dm_3^2}{d\mu} = \frac{8g^2}{16\pi^2} C_2 \left[ \frac{3g^2}{16\pi^2} m_{\text{comp}}^2 - m_\lambda^2 \right], \qquad (2.13)$$

where we have assumed that a single gauge group dominates and specialized to the case of two full composite generations. One way to avoid this problem is to have the compositeness scale close to 10 TeV, so that the negative contribution above does not dominate. If the compositeness scale is high, one can avoid problems if the gaugino contribution is important. From Eq. (2.13), we see that $m_\lambda \gtrsim m_{\text{comp}}/10$ is sufficient. (This estimate is confirmed by the detailed analysis of Ref. [19].) This condition is plausibly satisfied if $N \gtrsim 10$. In addition, we will see below that the sector that breaks flavor symmetries and generates Yukawa couplings can plausibly give large positive contributions to the third-generation scalar masses large enough to eliminate this problem.

Most of the models we construct have of order $N$ 'preonic' generations above the compositeness scale, and for $N \gtrsim 10$ the standard-model gauge groups are far from asymptotically free. This is compatible with perturbative unification if the compositeness scale is above (or near) the GUT scale $10^{16}$ GeV. (Note that for such large compositeness scales, the composite dynamics need not conserve baryon number.)

In this framework, we automatically gain a partial understanding of the fermion mass hierarchy: the Yukawa couplings of the first two generations are naturally suppressed because they arise from higher-dimension operators in the fundamental theory. (Since the compositeness scale can be above the GUT scale, an intriguing possibility is that the flavor scale is the Planck scale.) If all composite states correspond to operators of the same dimension, there are further hierarchies in the fermion masses that must be explained; on the other hand, we will see that a high degree of squark degeneracy can be guaranteed by the strong dynamics in this case. We can also consider models in which the composite states of different generations correspond to operators of different dimension; in this case, there is no squark degeneracy, and FCNC's must be suppressed by large squark masses. We will consider both types of models in what follows.

### 2.2 'Meson' Models

We first discuss 'meson' models where all quarks and leptons of the first two generations correspond to dimension-2 operators $P\bar{U}$ in the fundamental theory. This

means that Yukawa couplings involving the first two generations can be generated by adding the following terms to the tree-level superpotential:

$$\Delta W = \frac{1}{M^2} H (P\bar{U})(P\bar{U}) + \frac{1}{M} H \Phi_3 (P\bar{U}). \tag{2.14}$$

Here $H$ is a Higgs field and $\Phi_3$ is an elementary third-generation quark or lepton field. which gives a Yukawa matrix of the form

$$y \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}, \qquad \epsilon \sim \frac{\langle \bar{U} \rangle}{M}, \tag{2.15}$$

where $M$ is the scale of new physics where flavor symmetries are broken. Additional structure is clearly needed to construct fully realistic Yukawa matrices, but for $\epsilon$ in the range $10^{-1}$–$10^{-2}$ this is a good starting point.

In the above we have assumed that the $G_{\text{comp}}$ gauge coupling is perturbative just above the scale $\Lambda_{\text{comp}}$ (as in QCD). This may not be true in models where the $G_{\text{comp}}$ interactions by themselves have an strongly-coupled infrared fixed point. In such theories the large anomalous dimension of $\bar{U}$ can persist up to momentum scales significantly above the strong dynamics scale. If this is the case then the operators in Eq. (2.14) are enhanced (just as in 'walking technicolor' theories [20]), and the flavor scale can be put even higher.

We will make the conservative assumption that the new physics at the scale $M$ does not have any approximate flavor symmetries that can suppress FCNC's. In particular, this means that the Yukawa couplings $\lambda$ in Eq. (2.1) do not conserve flavor. It is highly non-trivial that the strong dynamics in this theory nevertheless gives rise to an approximate flavor symmetry at low energies that enforces the near degeneracy of the composite scalars. The underlying reason for this is the fact that all of the composites $(P\bar{U})$ are part of a single multiplet from the point of view of the strong interactions.

Let us first consider the $\lambda$-dependent effects. The superpotential Eq. (2.2) depends on $\lambda$ only through $\det(\lambda)$, which is flavor independent. There is nontrivial $\lambda$ dependence in the effective Kähler potential, but it is proportional to $\lambda^2/(16\pi^2) \lesssim 10^{-2}$. We now consider the effects of general higher-dimension operators suppressed by the flavor scale $M$. The largest effects come from terms in the effective Lagrangian of the form

$$\Delta \mathcal{L}_{\text{eff}} \sim \int d^2\theta d^2\bar{\theta} \frac{1}{M^2} (P\bar{U})^\dagger (P\bar{U}), \tag{2.16}$$

which give rise to mixing between the composite generations. This translates to mixing masses between the composite generations of order

$$\frac{\Delta m_{jk}^2}{m_{\text{comp}}^2} \sim \left(\frac{\langle \bar{U} \rangle}{M}\right)^2 \sim y_{jk}. \tag{2.17}$$

The most stringent bounds on squark mixing come from $K^0$–$\bar{K}^0$ mixing, and can be summarized as

$$\text{Re}\left(\frac{\Delta m_{\tilde{d}\tilde{s}}^2}{m_{\text{comp}}^2}\right) \lesssim 10^{-1}\frac{m_{\text{comp}}}{10 \text{ TeV}}, \qquad \text{Im}\left(\frac{\Delta m_{\tilde{d}\tilde{s}}^2}{m_{\text{comp}}^2}\right) \lesssim 10^{-2}\frac{m_{\text{comp}}}{10 \text{ TeV}}. \tag{2.18}$$

Since $y_{ds} \sim 3 \times 10^{-4}$, this is easily satisfied even if we assume that $CP$ violation in the flavor sector is maximal.

Note that the flavor sector generically introduces additional contributions to the third generation scalar masses from operators of the form

$$\Delta \mathcal{L} \sim \int d^2\theta d^2\bar{\theta}\, \frac{1}{M^2} \bar{U}^\dagger \bar{U} \Phi_3^\dagger \Phi_3, \tag{2.19}$$

which gives

$$\Delta m_3^2 \sim \frac{N}{M^2} F_{\bar{U}}^2 \sim N \frac{\langle \bar{U} \rangle^2}{M^2} m_{\text{comp}}^2. \tag{2.20}$$

This can reasonably give contributions to $m_3$ as large as $\sim 1$ TeV, large enough to cancel the negative 2-loop contribution discussed above.

A striking signature of these models is that *all* scalars of the first two generations unify at the scale $\Lambda_{\text{comp}}$ (which need not be close to the GUT scale). The unification holds up to effects suppressed by a loop factor, and is therefore expected to hold to 1%. This striking pattern is difficult to obtain naturally in other models.

## 2.3 'Dimensional Hierarchy' Models

We next discuss 'dimensional hierarchy' models that explain the observed fermion mass hierarchy in terms of a hierarchy of dimensions of operators. Specifically, we assume that the first-generation quarks and leptons correspond to dimension 3 operators of the form $(P\bar{U}\bar{U})$, second-generation quarks and leptons correspond to dimension 2 operators $(P\bar{U})$, and third generation quarks and leptons are elementary (dimension 1). In this case, Yukawa couplings involving the composite states arise from terms in the tree-level superpotential of the form

$$\Delta W = \frac{1}{M^4} H(P\bar{U}\bar{U})(P\bar{U}\bar{U}) + \frac{1}{M^3} H(P\bar{U}\bar{U})(P\bar{U}) + \frac{1}{M^2} H\Phi_3(P\bar{U}\bar{U})$$
$$+ \frac{1}{M^2} H(P\bar{U})(P\bar{U}) + \frac{1}{M} H\Phi_3(P\bar{U}), \tag{2.21}$$

giving rise to a Yukawa matrix of the form

$$y \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \qquad \epsilon \sim \frac{\langle \bar{U} \rangle}{M}. \tag{2.22}$$

This structure reproduces the main features of the observed fermion mass hierarchy for $\epsilon \sim 10^{-1}$.

In this scenario there is no approximate flavor symmetry at low energies because the first- and second-generation fields belong to different strong-interaction multiplets. We therefore have

$$\frac{\Delta m_{\tilde{d}\tilde{s}}^2}{m_{\text{comp}}^2} \sim \sin \theta_{\text{c}} \sim 10^{-1}. \tag{2.23}$$

Comparing with the bounds from the $K^0$–$\bar{K}^0$ system Eq. (2.18), we see that for $m_{\text{comp}} \sim 10$ TeV we require either a 10% fine-tuning or a 10% suppression of $CP$-violating effects in the squark masses.

A striking signature of these models is that the first- and second-generation scalars unify in two multiplets at the scale $\Lambda_{\text{comp}}$.

As in the 'meson' models considered above, operators of the form Eq. (2.19) can give additional positive contributions to the third-generation scalar masses in this class of models of order 1 TeV.

## 3  Model Building

### 3.1  A Two Generation 'Meson' Model

We construct a model with two complete generations of quarks and leptons corresponding to dimension-2 composite operators. The model has $G_{\text{comp}} = SU(15)$, $G_{\text{lift}} = SU(13)$, and is based on the 'fundamentals only' model analyzed in the Appendix. The matter content is

|  | $SU(13)$ | $SU(15)$ | $SU(15)$ | $SU(3)$ |
|---|---|---|---|---|
| $Q$ | □ | □ | **1** | **1** |
| $L$ | □̄ | **1** | □̄ | **1** |
| $\bar{U}$ | **1** | □̄ | □ | **1** |
| $\bar{D}$ | **1** | □̄ | **1** | **1** |
| $S$ | **1** | □ | **1** | □ |

10

The standard model gauge group $SU(5)_{\text{SM}}$ is embedded in the fundamental of $SU(15)$ as[7]

$$\square \to \mathbf{10} \oplus \bar{\mathbf{5}}. \tag{3.1}$$

The theory has the tree-level superpotential

$$W = \lambda QL\bar{U} + m\bar{D}S. \tag{3.2}$$

We assume that $\Lambda_{15} \gg \Lambda_{13}, m$.

The composite spectrum below the scale $\Lambda_{15}$ is

| Higgs | composite | $SU(15)$ | $SU(3)$ |
|---|---|---|---|
| $\bar{D}$ | $\bar{D}\bar{U}^{N-1}$ | $\bar{\square}$ | $\mathbf{1}$ |
| $S$ | $S\bar{U}$ | $\square$ | $\square$ |

The operator corresponding to the composite states is given in both the 'Higgs' and 'composite' description, along with their quantum numbers under the global symmetries (see the Appendix for more details). The mass term breaks the global symmetry $SU(3) \to SU(2)$, and gives the 'baryon' composites and one of the 'meson' composites masses of order $m$. It may appear unattractive to have an explicit mass term in the model, but the low-energy behavior of the model is independent of the value of $m$ as long as 100 GeV $\lesssim m \ll \Lambda_{15}$.[8] In a more fundamental theory, the mass term may arise dynamically, or from the VEV of a singlet field.

Below the scale $m$, the composite spectrum is therefore

| Higgs | composite | $SU(15)$ | $SU(2)$ |
|---|---|---|---|
| $S$ | $S\bar{U}$ | $\square$ | $\square$ |

The 'horizontal' $SU(2)$ symmetry is broken by the Yukawa couplings $\lambda$ and by higher-dimension terms required to give fermion masses, but the approximate $SU(2)$ symmetry is sufficient to suppress FCNC's, as discussed in Section 2 above. In this model, all the squarks and sleptons of the first two generations unify together at the scale $\Lambda_{15}$.

This model has 29 generations of 'preons' above the compositeness scale (plus Higgs fields), and so the $SU(5)_{\text{SM}}$ couplings have a Landau pole within a decade of the compositeness scale.

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[7]Here $SU(5)_{\text{SM}}$ is only a shorthand for the standard embedding of $SU(3)_{\text{C}} \times SU(2)_{\text{W}} \times U(1)_{\text{Y}}$.

[8]Remarkably, the low-energy dynamics is insensitive to the relative size of $m$ and $\Lambda_{13}$; see the Appendix.

11

This is still compatible with perturbative unification if the compositeness scale is *above* the GUT scale. The Landau pole may have a physical interpretation in terms of a 'dual' model. (In fact, since the model contains only fundamentals of all the gauge group factors, it is straightforward to construct a dual for any of the gauge group factors individually.) However, it is certainly unattractive that this model requires new physics so close to the compositeness scale, and it is our hope that more economical models with smaller matter content can be found.

### 3.2  A Composite $\bar{\mathbf{5}}$ 'Meson' Model

A simple way to avoid Landau poles near the compositeness scale using the model-building technology developed in this paper is to have fewer states composite. For example, we can take $G_{\mathrm{comp}} = SU(5)$, $G_{\mathrm{lift}} = SU(3)$, with the following matter content:

|  | $SU(3)$ | $SU(5)$ | $SU(5)_{\mathrm{SM}}$ | $SU(3)$ |
|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L$ | $\bar{\square}$ | $\mathbf{1}$ | $\bar{\square}$ | $\mathbf{1}$ |
| $\bar{U}$ | $\mathbf{1}$ | $\square$ | $\bar{\square}$ | $\mathbf{1}$ |
| $\bar{D}$ | $\mathbf{1}$ | $\bar{\square}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $S$ | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ | $\square$ |

with a tree-level superpotential

$$W = \lambda QL\bar{U} + m\bar{D}S. \tag{3.3}$$

We take $\Lambda_5 \gg \Lambda_3, m$. The mass term breaks the global $SU(3)$ symmetry down to $SU(2)$, and the light composite spectrum below the scale $m$ consists of two $\bar{\mathbf{5}}$ of the $SU(5)_{\mathrm{SM}}$.

Above the compositeness scale $SU(5)_{\mathrm{SM}}$ has $6 \times \bar{\mathbf{5}} \oplus 3 \times \mathbf{5}$ (rather than $3 \times \mathbf{5}$), and so there is no problem with a Landau pole near the compositeness scale.

In this model, the masses of the scalars $\tilde{d}_L$, $\tilde{s}_L$, $\tilde{e}_L$, and $\tilde{\nu}_L$ unify at the compositeness scale. Of course, it is possible to make similar models where a small number of states (such as the first two generation $\mathbf{10}$'s) are composite, and their masses will unify.

### 3.3  An 'Efficient' but Un-unified Model

We now consider a model that gives two generations of composite quarks without generating a Landau pole near the compositeness scale. However, the composite

states in this model do not arise in complete $SU(5)$ multiplets, and so the model cannot be naturally embedded in a grand-unified model.

The model is again based on the 'fundamentals only' model analyzed in the Appendix. The field content is

|        | $SU(2)$ | $SU(N)$ | $SU(N-2)_L$ | $SU(2)$ | $SU(N-2)_R$ |
|--------|---------|---------|-------------|---------|-------------|
| $Q$    | $\square$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L$    | $\square$ | $\mathbf{1}$ | $\bar{\square}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L'$   | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ |
| $\bar{U}$ | $\mathbf{1}$ | $\bar{\square}$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\bar{U}'$ | $\mathbf{1}$ | $\bar{\square}$ | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ |
| $\bar{D}$ | $\mathbf{1}$ | $\bar{\square}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $S$    | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ | $\square$ |
| $S'$   | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $X$    | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ |

with tree-level superpotential

$$W = \lambda QL\bar{U} + \lambda' QL'\bar{U}' + yXS\bar{U}' + m\bar{D}S'. \tag{3.4}$$

The mass term gives mass of order $m$ to all composite states containing $S'$, while the Yukawa coupling proportional to $y$ gives rise to a mass of order $y\Lambda_N$ to all composite states containing $\bar{U}'$. The remaining composite states are

| Higgs | composite | $SU(N-2)_L$ | $SU(N-2)_R$ |
|-------|-----------|-------------|-------------|
| $S$   | $S\bar{U}$ | $\square$  | $\square$   |

In the Higgs description, $S$ refers to only the $N-2$ 'colors' that are orthogonal to $\bar{U}'$.

We can use this to construct a model with two composite generations of quarks using the embedding of Ref. [8]. We take $N = 9$ and embed the standard-model gauge $\times$ flavor group

$$SU(3)_C \times SU(2)_W \times U(1)_Y \times [SU(2)_q \times SU(2)_u \times SU(2)_d \times U(1)_B]$$
$$\subset SU(7)_L \times SU(7)_R \tag{3.5}$$

where $SU(2)_{q,u,d}$ are flavor symmetry groups, and $U(1)_B$ is baryon number (which has no anomalies under the strong groups). The embedding is

$$SU(7)_L \; : \; \square \to (\mathbf{1}, \square)_0 \times (\square, \mathbf{1}, \mathbf{1})_{\frac{1}{7}}$$
$$\oplus (\bar{\square}, \mathbf{1})_{-\frac{1}{3}} \times (\mathbf{1}, \mathbf{1}, \mathbf{1})_{-\frac{4}{21}}, \tag{3.6}$$

$$SU(7)_R \ : \ \square \to (\mathbf{1},\mathbf{1})_{-1} \times (\mathbf{1},\overline{\square},\mathbf{1})_{-\frac{1}{7}}$$
$$\oplus (\mathbf{1},\mathbf{1})_{1} \times (\mathbf{1},\mathbf{1},\overline{\square})_{-\frac{1}{7}}$$
$$\oplus (\square,\mathbf{1})_{\frac{1}{3}} \times (\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{4}{21}}. \tag{3.7}$$

This gives rise to two composite generations of quarks, with additional fields transforming under the group Eq. (3.5) as

$$\Phi_u \sim (\mathbf{1},\square)_{-1} \times (\square,\overline{\square},\mathbf{1})_{0},$$
$$\Phi_d \sim (\mathbf{1},\square)_{1} \times (\square,\mathbf{1},\overline{\square})_{0},$$
$$A \sim (\mathbf{8},\mathbf{1})_{0} \times (\mathbf{1},\mathbf{1},\mathbf{1})_{0},$$
$$B \sim (\mathbf{1},\mathbf{1})_{0} \times (\mathbf{1},\mathbf{1},\mathbf{1})_{0}. \tag{3.8}$$

The fields $\Phi_{u,d}$ are 'flavored' Higgs fields that may play a role in flavor physics. Alternatively, all of the extra fields above can be given masses of order $\Lambda_9$ by adding extra fields $X$ with conjugate quantum numbers and adding Yukawa couplings of the form $\Delta W = XS\bar{U}$. The composite $\mathbf{8}$ can be eliminated by a higher-dimension operator of the form $\Delta W = (S\bar{U})^2$.

Above the composite scale, this model has 12 extra $SU(2)_W$ doublets, and 5 extra $SU(3)_C$ flavors, so the Landau pole is not close to the compositeness scale.

### 3.4  A 'Dimensional Hierarchy' Model

We next consider a model in which the hierarchy of Yukawa couplings is explained by the different dimensionalities of composite states in the different generations. The model we consider is based on the 'antisymmetric tensor' model analyzed in the Appendix. The matter content is

|         | $SU(2)$ | $SU(N)$ | $SU(N)$ | $SU(2)$ |
|---------|---------|---------|---------|---------|
| $Q$     | $\square$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L$     | $\overline{\square}$ | $\mathbf{1}$ | $\overline{\square}$ | $\mathbf{1}$ |
| $\bar{U}$ | $\mathbf{1}$ | $\overline{\square}$ | $\square$ | $\mathbf{1}$ |
| $A$     | $\mathbf{1}$ | $\boxminus$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $S$     | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ | $\square$ |

where we have left $N$ arbitrary for the moment. The tree-level superpotential is

$$W = \lambda QL\bar{U}. \tag{3.9}$$

14

As shown in the Appendix, the composite spectrum of this model below the scale $\Lambda_N$ is

| Higgs | composite | $SU(N)$ |
|:---:|:---:|:---:|
| $A$ | $A\bar{U}^2$ | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ |
| $S$ | $S\bar{U}$ | $\square$ |

We embed the standard model into the global $SU(N)$ by taking $N = 15 + n$ with

$$\square \to \mathbf{10} \oplus \mathbf{\bar{5}} \oplus (n \times \mathbf{1}). \tag{3.10}$$

Then the composite $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ state decomposes as

$$\begin{smallmatrix}\square\\\square\end{smallmatrix} \to (n \times \mathbf{10}) \oplus (n \times \mathbf{\bar{5}}) \oplus \left[\mathbf{45} \oplus \mathbf{\overline{45}} \oplus \mathbf{\overline{10}} \oplus \mathbf{5} \oplus \left(\tfrac{1}{2}n(n-1) \times \mathbf{1}\right)\right]. \tag{3.11}$$

Allowing all possible superpotential terms among the $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ composites gives mass to all of the states in brackets above, leaving $n-1$ composite generations [5]. The largest mass term for the mirror family ($\mathbf{\overline{10}} \oplus \mathbf{5}$) is with one linear combination of the dimension two families, since this is the lowest dimension operator. If we take $n = 1$ ($N = 16$), we obtain one composite generation from the dimension-2 $\square$ composite and one from the dimension-3 $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ composite.

As discussed in Section 2, there is no symmetry relating the the first and second generation squarks, so the composite scalars must have masses of order 10 TeV to suppress FCNC's.

This model has 18 generations of 'preons' above the compositeness scale (plus Higgs fields), and the Landau pole for the standard-model couplings is approximately a factor of $10^2$ above the compositeness scale.

### 3.5  An 'Efficient' but Speculative Model

To obtain more elegant models we would like find examples with smaller gauge groups and matter content so that there are no Landau poles close to the compositeness scale. We now present a model with an efficient group-theoretic embedding, but whose dynamics we do not know how to analyze completely. If we make a reasonable dynamical assumption, this model gives rise to compositeness and SUSY breaking by the mechanism discussed in Section 2. The particle content is:

| | $SU(k)$ | $SO(10)$ | $SU(10)$ | $SU(2)$ |
|:---:|:---:|:---:|:---:|:---:|
| $Q$ | $\square$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L$ | $\bar{\square}$ | $\mathbf{1}$ | $\square$ | $\mathbf{1}$ |
| $\bar{U}$ | $\mathbf{1}$ | $\square$ | $\bar{\square}$ | $\mathbf{1}$ |
| $S$ | $\mathbf{1}$ | $\mathbf{16}$ | $\mathbf{1}$ | $\square$ |

15

with the usual tree-level superpotential

$$W = \lambda Q L \bar{U}. \tag{3.12}$$

For $\langle \bar{U} \rangle \gg \Lambda_{10}$, $SO(10) \times SU(10)$ is broken to the diagonal $SO(10)$ subgroup and $SU(k)$ gaugino condensation gives rise to a dynamical superpotential

$$W_{\mathrm{dyn}} \sim \bar{U}^{10/k}. \tag{3.13}$$

The potential therefore slopes toward $\bar{U} \to 0$ for $k < 10$.

The dynamics for small values of $\langle \bar{U} \rangle$ involves the strong-coupling behavior of the $SO(10)$ gauge theory with spinors, which is presently not well understood. The $SO(10)$ gauge theory has a dual description in terms of an $SU(2) \times SU(7+k)$ gauge theory with a complicated set of matter representations, including a symmetric tensor of $SU(7+k)$ [21]. This dual is not weakly coupled in the infrared, so we cannot use it to determine the behavior of the Kähler potential for $\bar{U}$ in any simple way. Based on analogies with similar duals, one expects this theory to be at a fixed point in the infrared [21, 22].

If we assume that the anomalous dimension of $\bar{U}$ is sufficiently large, then there is a local SUSY-breaking minimum with $\langle \bar{U} \rangle \sim \Lambda_{10}$. The fermions from the **16**'s are exactly massless far from the origin, and because there can be no phase transitions as a function of moduli they are massless at the local minimum as well. This model therefore contains two composite fermionic **16**'s which can be identified with two standard-model generations (with right-handed neutrinos) if we embed the standard model into $SU(10)$ via the standard GUT embedding

$$SU(10) \to SO(10)_{\mathrm{SM}}. \tag{3.14}$$

FCNC's are suppressed by the approximate global $SU(2)$ symmetry of the strong dynamics. Above the compositeness scale $\Lambda_{10}$, this model has $3 + k/2$ additional 'preonic' generations. Note that for *e.g.* $k = 9$, only a 10% change of the $\bar{U}$ scaling dimension is required to obtain a local minimum, and the preonic theory only has 7.5 extra generations. It is therefore very reasonable to assume that this model works and gives a highly 'efficient' composite model.

Yukawa couplings for the composite generations can be induced if we include a Higgs field, $H$ embedded in the $\square$ of the global $SU(10)$ by operators of the form

$$\Delta W = \frac{1}{M} S S H \bar{U} \tag{3.15}$$

This gives Yukawa couplings $y \sim \langle \bar{U} \rangle / M$ for the composite quarks and leptons. (Comparing to our previous expressions, this corresponds to the composite operators being

dimension-$\frac{3}{2}$ operators.) Thus the flavor scale $M$ can be pushed up even higher in this model, and FCNC's are even more suppressed than in our 'meson' models. Mixing between the composite generations and the fundamental third generation $\Phi_3$ is more difficult to obtain. It may arise from operators such as

$$\Delta W = \frac{1}{M^2} \Phi_3 SSH\bar{U} \tag{3.16}$$

provided the right-handed sneutrino components of $S$ get VEVs. Presumably these VEVs must occur below the scale $\langle \bar{U} \rangle$. If they are an order of magnitude below this scale then we get an adequate suppression of mixings with the third generation. It would be interesting to extend the model to include the generation of neutrino masses through a seesaw with masses for the composite right-handed neutrinos, but we will not pursue this subject here.

In the above discussion, we have used a 'Higgs' description where the $SU(10)$ gauge dynamics is spontaneously broken. This model has no 'confined' description in the naive sense, since we cannot write a composite chiral operator transforming as a **16** under the unbroken $SO(10)$ global symmetry. This model therefore does not exhibit 'complementarity' [16], and a deeper understanding of its strong-coupling behavior would be very desirable.

## 4  Conclusions

We have shown that there is a wide class of realistic models which dynamically break SUSY and produce composite quarks and leptons, all in a single strongly-coupled sector. These models are remarkably simple. For example, the complete model presented in Section 3.1 has particle content

|          | $SU(13)$ | $SU(15)$ | $SU(15)$ | $SU(3)$ |
|----------|----------|----------|----------|---------|
| $Q$      | □        | □        | **1**    | **1**   |
| $L$      | □̄        | **1**    | □̄        | **1**   |
| $\bar{U}$| **1**    | □̄        | □        | **1**   |
| $\bar{D}$| **1**    | □̄        | **1**    | **1**   |
| $S$      | **1**    | □        | **1**    | □       |

where the first two groups are strong gauge groups, and the remaining groups are global symmetries of the strong interactions. The standard model gauge group $SU(5)_{\mathrm{SM}}$ is embedded in the $SU(15)$ symmetry via $\square \rightarrow \boxminus \oplus \bar{\square}$. The theory has a tree-level superpotential with the most general couplings of the form

$$W = \lambda Q L \bar{U} + m \bar{D} S + \frac{1}{M^2} H (S\bar{U})^2 + \frac{1}{M} H \Phi S\bar{U} + H\Phi^2 \tag{4.1}$$

where $H$ is a Higgs field and $\Phi$ denotes an elementary (third-generation) quark or lepton field. This model gives rise to two full generations of composite quarks and leptons; breaks SUSY; and gives rise to Yukawa couplings of the hierarchical form

$$y \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}, \qquad \epsilon \sim \frac{\Lambda_{15}}{M}. \tag{4.2}$$

We see that many of the fermion mass hierarchies are automatic consequences of approximate symmetries in this model. Even if we assume that there is no GIM mechanism in the higher-dimension operators that give rise to the Yukawa couplings, approximate flavor symmetries of the strong interactions guarantee the natural suppression of flavor-changing neutral currents (including $\epsilon_K$) with no fine-tuning. The model is compatible with perturbative unification of gauge couplings if the compositeness scale is at or above the GUT scale.

We have presented other models that are similarly simple, including a model that generates hierarchical Yukawa couplings of the form

$$y \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \qquad \epsilon \sim \frac{\Lambda_{16}}{M}. \tag{4.3}$$

All of these models have a very distinctive phenomenology: the composite sfermions of the first two generation are heavier than the gauginos, Higgsinos, and third-generation sfermions; and the composite sfermions unify at the compositeness scale (given by $\Lambda_{15}$ in the first model and $\Lambda_{16}$ in the second).

The main unattractive feature of these models is the large number of 'preon' fields charged under the standard model group, resulting in a Landau pole for the standard-model interactions. This can be avoided with the model-building technology presented here in less ambitious models with fewer composite states. However, another result of this paper is that the dynamics that gives rise to simultaneous compositeness and SUSY breaking occurs in a large class of models. This includes models whose low-energy dynamics is governed by confinement, non-trivial infrared fixed points, or free-magnetic phases. We believe that it is quite likely that further progress in understanding the dynamics of SUSY gauge theories will lead to the discovery of many additional models that display the dynamics illustrated here.

As an example, we presented in Section 3.5 a model that gives rise to two composite generations, without Landau poles near the compositeness scale. This model cannot be completely analyzed, but it works in complete analogy with models that we can analyze provided that a plausible inequality on an anomalous dimension is satisfied.

Our final conclusion is that the connection between compositeness and SUSY breaking is worth further exploration on both the theoretical and phenomenological fronts.

## 5  Acknowledgments

## Appendix A: Analysis of Models

In this Appendix, we present the detailed analysis of gauge theories of the type described in the main text.

### A.2  $SU \times SU$ Fundamentals Only

This model has gauge group $SU(k) \times SU(N)$, and matter content given by

|  | $SU(k)$ | $SU(N)$ | $SU(N)$ | $SU(F)$ | $SU(N-k+F)$ |
|---|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $1$ | $1$ | $1$ |
| $L$ | $\bar{\square}$ | $1$ | $\bar{\square}$ | $1$ | $1$ |
| $\bar{U}$ | $1$ | $\bar{\square}$ | $\square$ | $1$ | $1$ |
| $\bar{D}$ | $1$ | $\bar{\square}$ | $1$ | $\square$ | $1$ |
| $S$ | $1$ | $\square$ | $1$ | $1$ | $\square$ |

In addition, the theory has the usual tree-level superpotential

$$W = \lambda Q L \bar{U}. \tag{A.1}$$

This model is interesting because it shows that s-confinement [23] is not necessary for our mechanism to work. The result of the analysis is that for

$$F < k < N, \qquad F > 0, \tag{A.2}$$

this model has a local SUSY-breaking minimum with $\langle \det \bar{U} \rangle \sim (\Lambda_N)^N$. (For $F = 0$, the theory has a deformed moduli space and we cannot determine whether there is

a local SUSY-breaking minimum.) At the local minimum, the theory has composite states given by

| Higgs | composite | $SU(N)$ | $SU(F)$ | $SU(N-k+F)$ |
|---|---|---|---|---|
| $\bar{D}$ | $\bar{D}\bar{U}^{N-1}$ | $\overline{\square}$ | $\square$ | $\mathbf{1}$ |
| $S$ | $S\bar{U}$ | $\square$ | $\mathbf{1}$ | $\square$ |

The description of the composite states is given in both the 'Higgs' and 'composite' description, along with their quantum numbers under the unbroken global symmetries.

We now give the details of the analysis of this model. The classical moduli space can be labeled by the gauge invariant operators

| | $SU(N)$ | $SU(F)$ | $SU(N-k+F)$ |
|---|---|---|---|
| $\bar{U}S$ | $\square$ | $\mathbf{1}$ | $\square$ |
| $\bar{D}S$ | $\mathbf{1}$ | $\square$ | $\square$ |
| $L^k$ | $\overline{[k]}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $Q^k S^{N-k}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\overline{[F]}$ |
| $\bar{U}^N$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\bar{U}^{N-1}\bar{D}$ | $\overline{\square}$ | $\square$ | $\mathbf{1}$ |
| $\bar{U}^{N-2}\bar{D}^2$ | $\overline{\square\square}$ (vert) | $\square\square$ (vert) | $\mathbf{1}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $\bar{U}^{N-F}\bar{D}^F$ | $\overline{[F]}$ | $[F]$ | $\mathbf{1}$ |

Here $[n]$ denotes the $n$-index antisymmetric tensor. The moduli space has three branches, depending on which baryon operators are nonzero:

$$L \text{ branch:} \quad L^k \neq 0$$
$$Q \text{ branch:} \quad Q^k S^{N-k} \neq 0 \tag{A.3}$$
$$\bar{U} \text{ branch:} \quad \bar{U}^N, \bar{U}^{N-1}\bar{D}, \ldots, \bar{U}^{N-F}\bar{D}^F \neq 0.$$

(This means that *e.g.* if $L^k \neq 0$, all other baryon operators vanish.) We will be interested in the $\bar{U}$ branch. There are classical constraints on the operators on this branch that we will not write.

We assume $\Lambda_N \gg \Lambda_k$, and take arbitrary VEV's on the $\bar{U}$ branch of the moduli space:

$$\langle \bar{U} \rangle = \begin{pmatrix} a_1 & & \\ & \ddots & \\ & & a_N \end{pmatrix}. \tag{A.4}$$

First consider the case when $a_1, \ldots, a_N \gg \Lambda_N$, then all components of $Q$ and $L$ get massive, and $SU(k)$ gaugino condensation gives rise to a dynamical superpotential that pushes $\det \bar{U}$ to zero:

$$W_{\text{dyn}} \propto (\det \bar{U})^{1/k} \sim \bar{U}^{N/k}. \tag{A.5}$$

We next consider the case

$$\Lambda_k \ll a_1, \ldots, a_n \ll \Lambda_N, \qquad a_{n+1}, \ldots, a_N \gtrsim \Lambda_N. \tag{A.6}$$

By taking all VEV's large compared to $\Lambda_k$, we ensure that the non-perturbative superpotential generated by the $SU(k)$ dynamics is less important than the $SU(N)$ $D$-term potential, and therefore it makes sense to restrict attention to the $\bar{U}$ branch of the classical moduli space. We will now show that the small VEV's $a_1, \ldots, a_n$ are driven to larger values by the non-perturbative $SU(k)$ dynamics. We do this by constructing the effective theory below the large VEV's $a_{n+1}, \ldots, a_N$ classically and then analyzing the non-perturbative dynamics in the resulting low-energy theory. In general, other VEV's must be large in order to solve the classical $D$-flat constraints, and we must analyze *all* possibilities and show that $\det \bar{U}$ is driven away from zero in all cases.

The analysis divides into two cases, depending on the VEV's for $\bar{D}$. First, suppose that the $D$-flat conditions for the large VEV's in $\bar{U}$ are satisfied by having $n$ large VEV's for $D$, so that the baryon operator $\bar{U}^{N-n} \bar{D}^n$ is large. This requires $n \leq F$. In this case, the $SU(N)$ gauge group is completely broken, and the only strong dynamics is in the unbroken $SU(k)$ gauge group. The fields $\bar{D}$ and $S$ may have additional large VEV's, but these do not affect the analysis. The large VEV's in $\bar{U}$ leave $n$ flavors of $SU(k)$ massless, denoted by $q, \ell$. The theory has a superpotential

$$W_{\text{eff}} = \lambda q \ell \bar{u}, \tag{A.7}$$

where $\bar{u}$ contains the excitations of the small VEV's of $\bar{U}$. Gaugino condensation for $SU(k)$ leads to a dynamical superpotential

$$W_{\text{dyn}} \propto \bar{u}^{n/k} \tag{A.8}$$

Thus the non-perturbative $SU(k)$ dynamics pushes $\bar{u}$ away from the origin if $k > F$, since $n \leq F < k$.

The remaining case is that the $D$-flat conditions for the large VEV's in $\bar{U}$ are satisfied by having (at least) $N-n$ components of $S$ large, so that the meson operator $\bar{U}S$ is large. This requires $n \geq k-F$. (We need not consider the possibility that some

21

components of $Q$ are large, since this corresponds to a different branch of the moduli space.) We first integrate out the fields that are massive due to the $N - n$ large $S$ VEV's to obtain a theory with gauge group $SU(n) \times SU(k)$, and matter content given by

|       | $SU(k)$ | $SU(n)$ | $SU(n)$ | $SU(F)$ | $SU(n-k+F)$ |
|-------|---------|---------|---------|---------|-------------|
| $q$   | $\square$ | $\square$ | 1 | 1 | 1 |
| $\ell$ | $\bar\square$ | 1 | $\bar\square$ | 1 | 1 |
| $\bar u$ | 1 | $\bar\square$ | $\square$ | 1 | 1 |
| $\bar d$ | 1 | $\bar\square$ | 1 | $\square$ | 1 |
| $s$ | 1 | $\square$ | 1 | 1 | $\square$ |

with some singlets not shown. The superpotential is

$$W_{\text{eff}} = \lambda q \ell \bar u. \tag{A.9}$$

This is just the original model with $N$ replaced by $n$. Some of the components of $\bar d$ and $s$ may also be large, and this can be analyzed in the effective theory above.[9]

Consider first the case where all the components of $\bar d$ and $s$ have small VEV's. When $F \geq 2n$ the effective theory is infrared-free, so we see that gaugino condensation for $SU(k)$ leads to a dynamical superpotential

$$W_{\text{dyn}} \propto \bar u^{n/k} \tag{A.10}$$

which forces $\bar u$ to run away for $k > n$ which is always satisfied provided that $k > \frac{1}{2}F$ since $F \geq 2n$.

For $2 \leq F \leq 2n$, the $SU(N)$ dynamics has a dual description, and the effective theory has gauge group $SU(k) \times SU(F)$, with matter content given by

|       | $SU(k)$ | $SU(F)$ | $SU(n)$ | $SU(F)$ | $SU(n-k+F)$ |
|-------|---------|---------|---------|---------|-------------|
| $(\bar d q)$ | $\square$ | 1 | 1 | $\square$ | 1 |
| $(\bar u s)$ | 1 | 1 | $\square$ | 1 | $\square$ |
| $(\bar d s)$ | 1 | 1 | 1 | $\square$ | $\square$ |
| $\tilde q$ | $\bar\square$ | $\bar\square$ | 1 | 1 | 1 |
| $\tilde u$ | 1 | $\square$ | $\bar\square$ | 1 | 1 |
| $\tilde{\bar d}$ | 1 | $\square$ | 1 | $\bar\square$ | 1 |
| $\tilde s$ | 1 | $\bar\square$ | 1 | 1 | $\bar\square$ |

[9]A strict application of effective field theory ideology would require us to integrate out all states with large mass at the same time. However, the result is the same because the heavy modes can be integrated out at tree level, ignoring the breaking of SUSY.

The theory has a dynamical superpotential

$$W_{\text{dyn}} = \frac{1}{\Lambda_n} \left[ (\bar{d}q)\tilde{q}\tilde{\bar{d}} + (\bar{u}s)\tilde{s}\tilde{\bar{u}} + (\bar{d}s)\tilde{s}\tilde{\bar{d}} \right]. \tag{A.11}$$

In this theory the $SU(F)$ gauge dynamics is infrared free provided $F \leq \frac{1}{2}n$. In this case, the non-perturbative $SU(k)$ dynamics dominates, generating a dynamical superpotential

$$W_{\text{dyn}} \sim \tilde{\bar{d}}^{F/k} \tag{A.12}$$

which pushes $\tilde{\bar{d}}$ away from the origin for $k > F$. Since the duality operator mapping is $\bar{u}^N \leftrightarrow \tilde{\bar{d}}^F$, this means that $\bar{u}$ is forced away from the origin.

The condition $F \leq \frac{1}{2}n$ arose from demanding that the dual theory is infrared free, but this is actually too restrictive since the field $\bar{u}$ can be pushed away from the origin even if the dual theory has an infrared fixed point (see Section 2.1). In the conformal window for the dual ($\frac{1}{2}F < n < 2F$) the scaling dimension of the operator $\tilde{\bar{d}}^F$ is $\frac{3}{2}nF/(n + F)$. The field $\bar{u}$ is therefore pushed away from the origin for $k > \frac{3}{2}nF/(n + F)$. (As $n \to 2F$, the dual becomes infrared free and we recover our previous condition $k > F$, while when $n \to \frac{1}{2}F$, the 'electric' description becomes infrared free and we recover our previous condition $k > \frac{1}{2}F$.) Since this must be satisfied for all $k - F \leq n \leq N$, we have

$$k > \frac{3NF}{2(N + F)}. \tag{A.13}$$

Note that as we approach the end of the conformal window ($N \to \frac{1}{2}F$) this requires $k \to N$, which is the marginal case of inverted hierarchy models. In the conformal window, the bound (A.13) is superseded[10] by the result $k > F$ obtained by studying the case with large VEV's for the baryons $\bar{U}^{N-n}D^n$.

We have been considering the case where there are no additional large VEV's for $\bar{d}$ and $s$ in the effective theory given above Eq. (A.11). These VEV's reduce the number of colors and flavors of the $SU(n)$ group in the effective theory by the same amount. For $F \geq 2n$ this gives the effective theory a larger positive $\beta$ function, so the theory is still infrared-free, and the same analysis applies. For $2 \leq F \leq 2n$, the $SU(n)$ dynamics again has a dual description in terms of an $SU(F)$ gauge group, but now with fewer flavors. It is easily checked that the analysis above still applies and that the $SU(k)$ gauge dynamics forces $\bar{u}$ away from the origin for sufficiently large $k$.

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[10]The bound Eq. (A.13) will be important in the next section where the baryons are removed from the low energy theory.

23

Now go back to the effective theory described above Eq. (A.11) for the case $F = 1$, assuming that the fields $s$ and $\bar{d}$ do not have large VEV's. The $SU(N)$ theory is then s-confining, and the low-energy dynamics can be described by a theory with $SU(k)$ gauge group and matter content given by

| | $SU(k)$ | $SU(n)$ | $SU(n-k+1)$ |
|---|---|---|---|
| $(\bar{d}q)$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(\bar{u}s)$ | $\mathbf{1}$ | $\square$ | $\square$ |
| $(\bar{d}s)$ | $\mathbf{1}$ | $\mathbf{1}$ | $\square$ |
| $(\bar{u}^n)$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(\bar{u}^{n-1}\bar{d})$ | $\mathbf{1}$ | $\bar{\square}$ | $\square$ |
| $(q^k s^{n-k})$ | $\mathbf{1}$ | $\mathbf{1}$ | $\bar{\square}$ |
| $(q^{k-1}s^{n-k+1})$ | $\bar{\square}$ | $\mathbf{1}$ | $\mathbf{1}$ |

The fields are indicated by composite operators with the quantum numbers in parentheses. The theory has a dynamical superpotential

$$W_{\text{dyn}} = \frac{1}{\Lambda_n^{2n-1}} \left[ (\bar{u}^n)(\bar{d}q)(q^{k-1}s^{n-k+1}) + (\bar{u}^n)(\bar{d}s)(q^k s^{n-k}) \right.$$
$$\left. + (\bar{u}^{n-1}\bar{d})(\bar{u}s)(q^k s^{n-k}) \right]. \tag{A.14}$$

The $SU(k)$ gauge group has one flavor with a trilinear coupling that pushes $(\bar{u}^n)$ away from the origin. We must now consider the possibility that $\bar{d}$ and/or $s$ have large VEV's. As in the dual case analyzed above, these VEV's change the number of the $SU(n)$ colors and flavors by the same amount, and therefore again lead to s-confinement [23], and the $SU(k)$ dynamics then pushes $\bar{u}$ away from the origin.

Finally, we consider the case $F = 0$, where the $SU(n)$ theory has a deformed moduli space and the argument for a local SUSY breaking minimum fails. In this case, the effective theory has gauge group $SU(k)$ with matter content given by

| | $SU(k)$ | $SU(n)$ | $SU(n-k)$ |
|---|---|---|---|
| $(\bar{u}s)$ | $\mathbf{1}$ | $\square$ | $\square$ |
| $(\bar{u}^n)$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(q^k s^{n-k})$ | $\mathbf{1}$ | $\mathbf{1}$ | $\bar{\square}$ |

with quantum constraint

$$(\bar{u}^n)(q^k s^{n-k}) = \Lambda_n^{2n}. \tag{A.15}$$

The $SU(k)$ dynamics gives rise to a dynamical superpotential

$$W_{\text{dyn}} \sim (\bar{u}^n)^{1/k}, \tag{A.16}$$

on this moduli space. The criterion for a supersymmetric vacua is that the gradient of $W_{\text{dyn}}$ is proportional to the gradient of the constraint:

$$\frac{\partial W_{\text{dyn}}}{\partial (\bar{u}^n)} = \frac{1}{k} (\bar{u}^n)^{1/k-1} \propto (q^k s^{n-k}),$$

$$\frac{\partial W_{\text{dyn}}}{\partial (q^k s^{n-k})} = 0 \propto (\bar{u}^n). \tag{A.17}$$

There are solutions $(\bar{u}^n) \to 0$, $(q^k s^{n-k}) \to \infty$ as well as $(\bar{u}^n) \to \infty$, $(q^k s^{n-k}) \to 0$ (where the constant of proportionality vanishes). However, we cannot control the Kähler potential sufficiently in this case to know whether $\bar{u}$ is pushed away from the origin.

### A.3  $SU \times SU$ without 'Baryon' Composites

The existence of composite fermions with the quantum numbers of high-dimension 'baryon' operators in the previous model is inconvenient for the type of model-building we are interested in. A simple way to eliminate the unwanted composite states in this model is to add a mass term for all $\bar{D}$ fields to the tree-level superpotential:

$$W = \lambda QL\bar{U} + m\bar{D}S. \tag{A.18}$$

This breaks the global symmetry $SU(N - k + F) \to SU(N - k)$. For $m \gg \Lambda_N$, we can analyze the non-perturbative dynamics by first integrating out $\bar{D}$ and $S$. We then obtain the $F = 0$ model, for which we cannot establish the existence of a SUSY-breaking minimum. However, for $m \ll \Lambda_N$, we will show that the 'baryon' composite fields acquire a mass of order $m$, and the composite spectrum below the scale $m$ is as follows:

| Higgs | composite | $SU(N)$ | $SU(N - k)$ |
|:-----:|:---------:|:-------:|:-----------:|
| $S$ | $S\bar{U}$ | □ | □ |

Remarkably, this result does not depend on the relative sizes of $m$ and $\Lambda_k$. We find that for

$$F < k < N, \qquad 0 < F \le \frac{N}{2},$$

$$\frac{3NF}{2(N + F)} < k < N, \qquad \frac{N}{2} < F \le 2N, \tag{A.19}$$

this model has a local SUSY-breaking minimum.

We first consider the possibility that $\bar{D}$ has large VEV's (in the sense of Eq. (A.6)). In this case, the $SU(N)$ gauge symmetry is completely broken, and the $SU(k)$ dynamics generates a dynamical superpotential that is independent of $\bar{D}$ (see Eq. (A.8)). Therefore, the potential for for $\bar{U}$ slopes away from the origin, while the potential for $\bar{D}$ slopes toward zero.

The remaining cases have $N - n$ components of $S$ large. The effective theory is given above Eq. (A.9), with the superpotential modified to

$$W_{\text{eff}} = \lambda q\ell\bar{u} + m\bar{d}s. \tag{A.20}$$

We now analyze the effective theory for various values of $F$. For $F \geq 2n$ the effective theory is infrared-free, so we see that gaugino condensation for $SU(k)$ leads to a superpotential

$$W_{\text{dyn}} \propto \bar{u}^{n/k} + m\bar{d}s \tag{A.21}$$

which forces $\bar{u}$ to run away for $k > n$ which is always satisfied provided that $k > \frac{1}{2}F$ since $F \geq 2n$.

For $2 \leq F \leq 2n$ the theory has a dual description given near Eq. (A.11), with the dynamical superpotential modified to

$$W_{\text{dyn}} = \frac{1}{\Lambda_n} \left[ (\bar{d}q)\tilde{q}\tilde{\bar{d}} + (\bar{u}s)\tilde{s}\tilde{\bar{u}} + (\bar{d}s)\tilde{s}\tilde{\bar{d}} \right] + m(\bar{d}s). \tag{A.22}$$

The presence of the linear term in $(\bar{d}s)$ does not prevent the theory from forcing $\tilde{\bar{d}}$ away from the origin. Consider giving a VEV for $\tilde{\bar{d}}$ satisfying

$$\Lambda_k, (m\Lambda_n)^{1/2} \ll \tilde{\bar{d}} \ll \Lambda_N. \tag{A.23}$$

In this case, the fields $(\bar{d}q)$, $\tilde{q}$ and $(\bar{d}s)$, $\tilde{s}$ get masses of order $\tilde{\bar{d}}$. The linear term implies $\tilde{s} \sim \Lambda_n m/\tilde{\bar{d}} \ll \tilde{\bar{d}}$, which can be treated as a small perturbation. Below the scale $\tilde{\bar{d}}$, the low-energy effective theory is again $SU(k)$ super Yang–Mills theory. $SU(k)$ gaugino condensation then gives rise to a dynamical superpotential that pushes $\tilde{\bar{d}}$ away from the origin as before.

Note that for $\tilde{\bar{d}} \sim \Lambda_n$, the $(\bar{d}s)$ equation of motion implies that $\tilde{s} \sim m$, which gives a mass of order $m$ to $\tilde{\bar{u}}$. This is how the 'baryon' composites get mass in the dual description.

For $F = 1$, the story is very similar. In this case, the theory s-confines [23], and the effective theory is given near Eq. (A.14), with the dynamical superpotential

modified to

$$W_{\mathrm{dyn}} = \frac{1}{\Lambda_n^{2n-1}} \Big[ (\bar{u}^n)(\bar{d}q)(q^{k-1}s^{n-k+1}) + (\bar{u}^n)(\bar{d}s)(q^k s^{n-k})$$

$$+ (\bar{u}^{n-1}\bar{d})(\bar{u}s)(q^k s^{n-k}) \Big] + m(\bar{d}s). \tag{A.24}$$

Give a VEV to $(\bar{u}^n)$ satisfying

$$\Lambda_k^n, (\Lambda_n^{2n-1}m)^{1/2} \ll (u^n) \ll \Lambda_n^n. \tag{A.25}$$

This VEV gives $(\bar{d}q), (q^{k-1}s^{n-k+1})$ and $(\bar{d}s), (q^k s^{n-k})$ a mass of order $\bar{u}$; the presence of the linear term gives the field $(q^k s^{n-k})$ a small VEV. Below the scale $\bar{u}$, the effective theory is again $SU(k)$ super Yang–Mills, and the dynamical superpotential pushes $\bar{u}$ away from the origin.

For $(\bar{u}^n) \sim \Lambda_n^n$, the $(\bar{d}s)$ equation of motion implies that $(\bar{u}^{n-1}\bar{d})$ gets a mass of order $m$. This is how the 'baryon' composites get mass in the confined description.

Putting together the inequalities for these various cases to work, we arrive at Eq. (A.19). Note that the fact that the VEV $\bar{D}$ is always pushed toward the origin allows this model to work for a wider range of parameters than the massless model considered above.

## A.4  Antisymmetric Tensors

We now turn to a model with dimension-2 and dimension-3 composites.

|        | $SU(k)$ | $SU(N)$ | $SU(N)$ | $SU(F-k)$ | $SU(F-4)$ |
|--------|---------|---------|---------|-----------|-----------|
| $Q$    | $\square$ | $\square$ | $1$ | $1$ | $1$ |
| $L$    | $\bar\square$ | $1$ | $\bar\square$ | $1$ | $1$ |
| $\bar{U}$ | $1$ | $\bar\square$ | $\square$ | $1$ | $1$ |
| $\bar{D}$ | $1$ | $\bar\square$ | $1$ | $1$ | $\square$ |
| $A$    | $1$ | $\boxminus$ | $1$ | $1$ | $1$ |
| $S$    | $1$ | $\square$ | $1$ | $\square$ | $1$ |

The model has the usual superpotential

$$W = \lambda QL\bar{U}. \tag{A.26}$$

For $F = k = 4$, this is the model analyzed in Ref. [5]. The composite fermion spectrum is

| Higgs | composite | $SU(N)$ | $SU(F-k)$ | $SU(F-4)$ |
|-------|-----------|---------|-----------|-----------|
| $\bar{D}$ | $\bar{D}\bar{U}^{N-1}$ | $\bar\square$ | $1$ | $\square$ |
| $A$ | $A\bar{U}^2$ | $\boxminus$ | $1$ | $1$ |
| $S$ | $S\bar{U}$ | $\square$ | $\square$ | $1$ |

We now briefly analyze this model. For $\det \bar{U} \gg (\Lambda_N)^N$, $SU(k)$ gaugino condensation gives rise to (an exact) dynamical superpotential

$$W \propto (\det \bar{U})^{1/k} \sim \bar{U}^{N/k}. \tag{A.27}$$

For $F = 4$ the $SU(N)$ interactions s-confine [23, 24], and the analysis follows that of Ref. [5]. For odd $N$ ($N = 2n+1$) and $k = 2$ the moduli space is parameterized by

| | $SU(N)$ | $SU(2)$ |
|---|---|---|
| $S\bar{U}$ | $\square$ | $\square$ |
| $SA^n$ | $\mathbf{1}$ | $\square$ |
| $SQ^2 A^{n-1}$ | $\mathbf{1}$ | $\square$ |
| $A\bar{U}^2$ | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | $\mathbf{1}$ |
| $\bar{U}^N$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L^2$ | $\bar{\square}$ | $\mathbf{1}$ |

The degrees of freedom below the scale $\Lambda_N$ are

| | $SU(2)$ | $SU(N)$ | $SU(2)$ |
|---|---|---|---|
| $(Q\bar{U})$ | $\square$ | $\square$ | $\mathbf{1}$ |
| $(S\bar{U})$ | $\mathbf{1}$ | $\square$ | $\square$ |
| $(QA^n)$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(SA^n)$ | $\mathbf{1}$ | $\mathbf{1}$ | $\square$ |
| $(SQ^2 A^{n-1})$ | $\mathbf{1}$ | $\mathbf{1}$ | $\square$ |
| $(S^2 QA^{n-1})$ | $\square$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(A\bar{U}^2)$ | $\mathbf{1}$ | $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ | $\mathbf{1}$ |
| $(\bar{U}^N)$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L$ | $\square$ | $\bar{\square}$ | $\mathbf{1}$ |

The effective superpotential is given by the sum of the tree superpotential and a dynamical superpotential [23, 24]:

$$
\begin{aligned}
W_{\text{eff}} = \frac{1}{\Lambda_N^{2N-1}} &\Big\{ \left[ (QA^n)(Q\bar{U})^3(S\bar{U})^2 + (SA^n)(S\bar{U})(Q\bar{U})^2 \right] (A\bar{U}^2)^{n-1} \\
&+ (SQ^2 A^{n-1})(S\bar{U})(A\bar{U}^2)^n + (S^2 QA^{n-1})(Q\bar{U})(A\bar{U}^2)^n \\
&+ (\bar{U}^N)(SA^n)(SQ^2 A^{n-1}) + (\bar{U}^N)(QA^n)(S^2 QA^{n-1}) \Big\} \\
&+ \lambda L (Q\bar{U}).
\end{aligned}
\tag{A.28}
$$

28

Integrating out $L$ and $(Q\bar{U})$ leaves $SU(2)$ with one flavor $(QA^n)$ and $(S^2QA^{n-1})$ as well as some singlets with a superpotential

$$W_{\text{eff}} = \frac{1}{\Lambda_N^{2N-1}}\left[(SQ^2A^{n-1})(S\bar{U})(A\bar{U}^2)^n + (\bar{U}^N)(SA^n)(SQ^2A^{n-1})\right.$$
$$\left. + (\bar{U}^N)(QA^n)(S^2QA^{n-1})\right]. \tag{A.29}$$

The last two terms in this superpotential are mass terms when the baryon $(\bar{U}^N)$ has a VEV, so on this branch of the moduli space we find a dynamical superpotential:

$$W_{\text{dyn}} \sim (\bar{U}^N)^{1/2}, \tag{A.30}$$

which forces the baryon to run away for any $N$.

For odd $N$ ($N = 2n + 1$) and $k = 3$ The moduli space is parameterized by

|  | $SU(N)$ |
|---|---|
| $S\bar{U}$ | $\square$ |
| $SA^n$ | $\mathbf{1}$ |
| $Q^3A^{n-1}$ | $\mathbf{1}$ |
| $A\bar{U}^2$ | $\boxminus$ |
| $\bar{U}^N$ | $\mathbf{1}$ |
| $L^3$ | $\boxminus$ |

The low energy (below $\Lambda_N$) degrees of freedom are

|  | $SU(3)$ | $SU(N)$ |
|---|---|---|
| $(Q\bar{U})$ | $\square$ | $\square$ |
| $(S\bar{U})$ | $\mathbf{1}$ | $\square$ |
| $(QA^n)$ | $\square$ | $\mathbf{1}$ |
| $(SA^n)$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(Q^3A^{n-1})$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(SQ^2A^{n-1})$ | $\bar{\square}$ | $\mathbf{1}$ |
| $(A\bar{U}^2)$ | $\mathbf{1}$ | $\boxminus$ |
| $(\bar{U}^N)$ | $\mathbf{1}$ | $\mathbf{1}$ |
| L | $\bar{\square}$ | $\bar{\square}$ |

The effective superpotential is given by the sum of the tree superpotential and a

29

dynamical superpotential.

$$W_{\text{eff}} = \frac{1}{\Lambda_N^{2N-1}}\left\{\left[(QA^n)(Q\bar{U})^3(S\bar{U})^2 + (SA^n)(S\bar{U})(Q\bar{U})^2\right](A\bar{U}^2)^{n-1}\right.$$
$$+ (Q^3A^{n-1})(S\bar{U})(A\bar{U}^2)^n + (SQ^2A^{n-1})(Q\bar{U})(A\bar{U}^2)^n$$
$$\left.+ (\bar{U}^N)(SA^n)(Q^3A^{n-1}) + (\bar{U}^N)(QA^n)(SQ^2A^{n-1})\right\} \tag{A.31}$$

$$+ \lambda L(Q\bar{U}).$$

Integrating out $L$ and $(Q\bar{U})$ leaves $SU(3)$ with one flavor $(QA^n)$ and $(SQ^2A^{n-1})$ as well as some singlets with a superpotential

$$W_{\text{eff}} = \frac{1}{\Lambda_N^{2N-1}}\left[(Q^3A^{n-1})(S\bar{U})(A\bar{U}^2)^n + (\bar{U}^N)(SA^n)(Q^3A^{n-1})\right.$$
$$\left.+ (\bar{U}^N)(QA^n)(SQ^2A^{n-1})\right]. \tag{A.32}$$

The last two terms in this superpotential are mass terms when the baryon $(\bar{U}^N)$ has a VEV, so on this branch of the moduli space we find a dynamical superpotential:

$$W_{\text{dyn}} \sim (\bar{U}^N)^{1/3} \tag{A.33}$$

Which forces the baryon to run away for any $N$.

Giving $N - m$ large VEVs to $\bar{U}$ and $S$ breaks $SU(N)$ down to $SU(m)$, and the low energy theory is the original theory with $N$ replaced by $m$, so the theory is still s-confining and the analysis goes through as above.

For even $N$ ($N = 2n$) and $k = 2$ the low energy degrees of freedom are

|  | $SU(2)$ | $SU(N)$ | $SU(2)$ |
|---|---|---|---|
| $(Q\bar{U})$ | □ | □ | 1 |
| $(S\bar{U})$ | 1 | □ | □ |
| $(A^n)$ | 1 | 1 | 1 |
| $(Q^2A^{n-1})$ | 1 | 1 | 1 |
| $(SQA^{n-1})$ | □ | 1 | □ |
| $(S^2A^{n-1})$ | 1 | 1 | 1 |
| $(S^2Q^2A^{n-2})$ | 1 | 1 | 1 |
| $(A\bar{U}^2)$ | 1 | 目 | 1 |
| $(\bar{U}^N)$ | 1 | 1 | 1 |
| $L$ | $\bar{\square}$ | $\bar{\square}$ | 1 |

30

The superpotential is

$$W_{\text{eff}} \sim \frac{1}{\Lambda_N^{2N-1}}\Big\{(A^n)(Q\bar{U})^2(S\bar{U})^2(A\bar{U}^2)^{n-2} + (S^2A^{n-1})(Q\bar{U})^2(A\bar{U}^2)^{n-1}$$

$$+ (Q^2A^{n-1})(S\bar{U})^2(A\bar{U}^2)^{n-1} + (S^2Q^2A^{n-2})(A\bar{U}^2)^n$$

$$+ (SQA^{n-1})(Q\bar{U})(S\bar{U})(A\bar{U}^2)^{n-1}$$

$$+ (\bar{U}^N)\Big[(A^n)(S^2Q^2A^{n-2}) + (Q^2A^{n-1})^2 \tag{A.34}$$

$$+(SQA^{n-1})(SQA^{n-1}) + (S^2A^{n-1})^2\Big]\Big\}$$

$$+ \lambda L(Q\bar{U}).$$

Integrating out $L$ and $(Q\bar{U})$ leaves $SU(2)$ with one flavor $(SQA^{n-1})$ and a superpotential

$$W_{\text{eff}} \sim \frac{1}{\Lambda_N^{2N-1}}\Big\{(Q^2A^{n-1})(S\bar{U})^2(A\bar{U}^2)^{n-1} + (S^2Q^2A^{n-2})(A\bar{U}^2)^n$$

$$+ (\bar{U}^N)\Big[(A^n)(S^2Q^2A^{n-2}) + (Q^2A^{n-1})^2 \tag{A.35}$$

$$+(SQA^{n-1})(SQA^{n-1}) + (S^2A^{n-1})^2\Big]\Big\}.$$

The last four terms are mass terms on the branch of moduli space where $(\bar{U}^N)$ has a VEV, so gaugino condensation results in the dynamical superpotential

$$W_{\text{dyn}} \sim (\bar{U}^N)^{1/2} , \tag{A.36}$$

which forces the baryon to run away for any $N$.

For even $N$ ($N = 2n$) and $k = 3$ the low energy degrees of freedom are

| | $SU(3)$ | $SU(N)$ |
|---|---|---|
| $(Q\bar{U})$ | $\square$ | $\square$ |
| $(S\bar{U})$ | $\mathbf{1}$ | $\square$ |
| $(A^n)$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(Q^2A^{n-1})$ | $\bar{\square}$ | $\mathbf{1}$ |
| $(SQA^{n-1})$ | $\square$ | $\mathbf{1}$ |
| $(SQ^3A^{n-2})$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $(A\bar{U}^2)$ | $\mathbf{1}$ | $\boxminus$ |
| $(\bar{U}^N)$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $L$ | $\bar{\square}$ | $\bar{\square}$ |

The superpotential is

$$W_{\text{eff}} = \frac{1}{\Lambda_N^{2N-1}} \Big\{ (A^n)(Q\bar{U})^3(S\bar{U})(A\bar{U}^2)^{n-2} + (Q^2 A^{n-1})(S\bar{U})^2(A\bar{U}^2)^{n-1}$$

$$+ (SQA^{n-1})(Q\bar{U})(S\bar{U})(A\bar{U}^2)^{n-1} + (SQ^3 A^{n-2})(A\bar{U}^2)^n$$

$$+ (\bar{U}^N)\big[(A^n)(SQ^3 A^{n-2}) + (Q^2 A^{n-1})^2 \tag{A.37}$$

$$+(SQA^{n-1})(SQA^{n-1})\big]\Big\}$$

$$+ \lambda L(Q\bar{U}).$$

Integrating out $L$ and $(Q\bar{U})$ leaves $SU(3)$ with one flavor $(Q^2 A^{n-1})$, $(SQA^{n-1})$ some singlets and a superpotential

$$W_{\text{eff}} = \frac{1}{\Lambda_N^{2N-1}} \Big\{ (Q^2 A^{n-1})(S\bar{U})^2(A\bar{U}^2)^{n-1} + (SQ^3 A^{n-2})(A\bar{U}^2)^n$$

$$+ (\bar{U}^N)\big[(A^n)(SQ^3 A^{n-2}) + (Q^2 A^{n-1})^2 \tag{A.38}$$

$$+(SQA^{n-1})(SQA^{n-1})\big]\Big\}.$$

The last three terms are mass terms on the branch of moduli space where $(\bar{U}^N)$ has a VEV, so gaugino condensation results in the dynamical superpotential

$$W_{\text{dyn}} \sim (\bar{U}^N)^{1/3} , \tag{A.39}$$

which forces the baryon to run away for any $N$.

For $F = 5$ and $N$ sufficiently large, the $SU(N)$ theory has an infrared fixed point with a dual gauge group $SU(2) \times SU(2)$ [22]. However for even $N$ it is known that the baryon operator maps to a product of fields with scaling dimension less than 3, so for $k \geq 3$ the baryon should again be forced to have a non-zero SUSY breaking VEV. For larger values of $F$ we expect that the $SU(N)$ dynamics have an infrared fixed point up to the point where asymptotic freedom is lost ($F = 2N + 3$); unfortunately, little or nothing is know about the dual descriptions of these theories.

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