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Gradation-Dependent Thermal Conductivity of Sands

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Abstract: Although thermal conductivity is a widely-applied parameter in geotechnical engineering, the effect of soil gradation on the thermal conductivity is not well understood. Thermal needle tests were performed to analyze the influence of gradation on the thermal conductivity of carbonate sands. The thermal conductivity of carbonate sand specimens having different gradations prepared to three void ratios was observed to increase with uniformity coefficient or fractal dimension. Although an increase in void ratio leads to a decrease in thermal conductivity, the percent difference in thermal conductivity was independent of the initial void ratio. A maximum increase in thermal conductivity of 13.9% was observed for uniformity coefficients ranging from 2 to 20. Empirical equations employing the uniformity coefficient are proposed to quantify the gradation-dependent thermal conductivity of carbonate sand for use in the design of insulating layers.

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Introduction

The thermal conductivity of soil is a basic physical property pertaining to the transfer of heat by conduction (Bristow 2002). The thermal conductivity is a key parameter in simulations of a variety of problems, including granular insulation layers for pavements or landfills in cold regions (e.g., Humphrey and Eaton 1995; Benson et al. 1996), ground source heat exchange systems and energy piles (e.g., McCartney et al. 2016), radioactive waste disposal systems (e.g., Cui et al. 2011), and oil-carrying pipelines (e.g., Lee et al. 2010). As soil is a multi-phase system consisting of solid particles, water, and air, each with disparate thermal conductivity values, the relative amounts and distributions of each of the phases can have a significant effect on the soil thermal conductivity. For example, the thermal conductivity of solid particles ranges from 2.0 to 7.7 W/m/K depending on the mineralogy, the thermal conductivity of water (0.61 W/m/K) is slightly smaller than that of the solid particles, while the thermal conductivity of air (0.026 W/m/K) is nearly two orders of magnitude smaller (Tarnawski and Leong 2016).

According to previous studies (Dong et al. 2015; Zhang and Wang 2017), factors affecting the soil thermal conductivity could be divided into two categories: (1) ***internal factors***, such as soil fabric (i.e., mineral composition, particle shape, particle size, etc.) and soil structure (i.e., porosity, gradation, pore size distribution, etc.); (2) ***external factors***, such as water content, temperature, etc. The thermal conductivity of soils is greatly affected by internal factors (i.e., mineral composition, packing density) due to the differences in thermal conductivity of solid particles, water and air (Dong et al. 2015). Moreover, the thermal conductivity of unsaturated soils increases with increasing degree of saturation (an external factor), due to the formation of water films between the particles that can increase the contact area for heat transfer. An important internal factor that has not been fully investigated is the effect of gradation, which is a relevant topic in the design of granular blankets for insulation layers in pavements, landfills, or other geothermal heat exchange applications.

In the current study, a series of laboratory tests were performed to investigate the influence of the gradation on the thermal conductivity of sand at constant void ratios. The uniformity coefficient and fractal dimension are used to establish empirical models for the gradation-dependent thermal conductivity. The fundamental mechanisms behind the change of thermal conductivity with gradation are also discussed.

Carbonate Sand and Testing Protocols

Characteristics of the Materials Investigated

For convenience, a dry carbonate sand was selected in this study because it could be crushed by different amounts to result in specimens with different gradations but the same mineralogy. This is critical in isolating the effects of the particle size gradation. Further, carbonate sand is widely present in coastal engineering applications, and may be encountered in energy pile applications that involve stress levels sufficient to cause particle breakage and changes in gradation that may alter heat transfer processes. Carbonate sand from Yongxing Island of the Xisha archipelago located in the South China Sea is used in the current study, which is mainly composed of shell fragments and coral debris. The specific gravity of this sand is 2.79.

The gradations of carbonate sands with a wide range of gradations in the percentage finer versus logarithm of particle size plane are shown in **Fig. 1(a)** and in the logarithm of percentage finer versus logarithm of particle size plane are shown in **Fig. 1(b)**. The linear fitting curves in **Fig. 1(b)** indicate that gradations of these carbonate sands can described by a fractal function (Tyler and Wheatcraft 1992):

$$F(d)=\left(\frac{d}{d_M}\right)^{3-f_d} \tag{1}$$

where F is percentage finer; d is particle diameter; d_M is the maximum particle diameter; f_d is fractal dimension. The fractal dimension f_d obtained from best fitting curve ranges from 0.4 to 2.4. Scanning electron micrograph (SEM) images of carbonate sand specimens in **Fig. 1(c)** reveal the components of carbonate sand specimens with different uniformity coefficients ($\mathrm{C}_u=\frac{d_{60}}{d_{30}}$, where d_{60} and d_{30} are the particle diameters corresponding to the 60% and 30% finer fractions, respectively) and specification (an image of a 60-fold magnification of single particle). The lower-right image in **Fig. 1(c)** indicates that inter-particle voids may also be encountered in carbonate sand particles that may make them prone to crushing (Xiao et al. 2016).

Experimental Details and Testing Procedures

The thermal conductivity values of dry carbonate sand specimens were measured using a single-needle probe TR-1 along with the KD2 Pro Thermal Properties Analyzer obtained from Decagon Devices of Pullman, WA. This device uses the transient thermal probe method with an analysis that is provided in the user manual. The stainless-steel mold used in this study has an inner diameter of 50 mm and a height of 137 mm. As recommended by the user manual, the TR-1 probe having a diameter of 2.4 mm and length of 100 mm is suitable for measuring the thermal conductivity of dry soils. The specimen preparation and experimental procedures are as follows:

(1) Sands with different gradations were obtained from post-impact tests on carbonate sands reported by Xiao et al. (2016). The crushed sand was heated in an oven at 105°C for 24 hours then was permitted to cool before sieving to reach different gradations. As listed in **Table 1**, maximum and minimum void ratios of the sands with the different gradations shown in **Fig. 1** were measured based on ASTM D4253 (ASTM 2016a), ASTM D4254 (ASTM 2016b), to ensure that the selected target void ratio evaluated in this study were possible for the different gradations.

(2) A group of five specimens with a given void ratio (0.80, 0.85, and 0.90 in **Table 2**) were prepared according to the five gradations in **Fig. 1**. The specimens were prepared in eight equal lifts. The under-compaction method proposed by Ladd (1978) was used to obtain uniform specimens. Specifically, each lift was poured into the mold to form a layer with its compacted dry density slightly greater (about 1%) than that of the substratum layer. Compaction was not observed to further alter the gradations.

(3) The TR-1 probe was inserted into the center position of the specimen using a guide to ensure verticality. The thermal conductivity of the sand species was measured by KD2 Pro Thermal Properties Analyzer. The thermal conductivity for each testing condition (i.e., combination of void ratio and gradation) was measured five times. Although a slight densification of the sand is possible during needle insertion, measurements of the five specimens were within a ±2% range of the

average value of measured thermal conductivities that are reported in **Table 2**, which indicates that this approach is repeatable. The thermal conductivity was measured under controlled room temperature conditions (25 °C) to minimize the influence of the ambient temperature.

Experimental Thermal Conductivity Results

The results in **Fig. 2** show that the thermal conductivity λ at a given void ratio increases with increasing the uniformity coefficient (C_u) when $2.0 < C_u \leq 4.7$, while λ increases gradually when $C_u > 4.7$. It is also interesting to note that the relationships between λ and C_u for different void ratios are parallel, indicating that the gradation effect on the thermal conductivity is attributed to the same intrinsic mechanism that is independent of the void ratio. The change in thermal conductivity with the gradation was analyzed using the percent difference calculated as follows:

$$\Delta\lambda = \frac{100(\lambda - \lambda_0)}{\lambda_0} \tag{2}$$

where λ_0 is the thermal conductivity for sand with C_u=2.0 (or f_d=0.4). The value of λ_0, as shown in **Fig. 2**, is dependent on void ratio e with a decrease in thermal conductivity with increasing void ratio. However, the percent difference was found to not be sensitive to the void ratio. The maximum percent difference in the thermal conductivity is about 13.9% for the sand with C_u=19.8, which is relatively large from a practical engineering point of view. **Fig. 3** shows that the void ratio was observed to not have a significant effect on the percent difference in thermal conductivity, which can be fitted by the following relationship:

$$\Delta\lambda = \beta_0\left[\exp(-2\chi_0) - \exp(-\chi_0 C_\mathrm{u})\right] \tag{3}$$

where β_0 (=26.038) and χ_0 (=0.357) are empirical fitting parameters that are specific to this carbonate sand. The form of **Eq. (3)** shows a good fit to the data, reflected by the coefficient of determination R^2 close to 1.0. This empirical equation indicates that the thermal conductivity does not change significantly for uniformity coefficient greater than 10. The ultimate value of the fitted relationship for $\Delta\lambda$ from **Eq. (3)** is 12.7%, which is slightly lower than the experimental value of 13.9%. Because a linear relation between λ and e was observed in **Fig. 2**, the following fitting equation can be defined:

$$\lambda = \lambda_{c0} - \alpha e - \beta \exp\left(\chi C_u\right) \tag{4}$$

where λ_{c0} (=0.425), α (=0.218), β (=0.051) and χ (=0.332) are fitting parameters representative of carbonate sands. The fitting surface in **Fig. 4** obtained from **Eq. (4)** captures well the variations of thermal conductivity of the tested sands, based on a maximum error of -0.003 and R^2=0.986.

The fractal dimension is an important parameter for describing the characteristics of particle size distribution. For specimens with C_u =19.8, the corresponding fractal dimension f_d is approximately 2.4, which is close to the ultimate fractal dimension of carbonate sands as observed in one-dimensional compression tests (Zhang and Baudet 2013) and also in impact tests (Xiao et al. 2016). The fractal dimension f_d of carbonate sands in this study ranges from 0.4 (C_u=2.0) to 2.4 (C_u=19.8). Effect of gradation with the fractal dimension on the thermal conductivity is shown in **Fig. 5**. The variation of thermal conductivity with gradation and void ratio in **Fig. 5** is similar to that in **Fig. 2**. The relationship between the percent difference in thermal conductivity $\Delta\lambda$

defined in **Eq. (2)** and f_d for different void ratios, as shown in **Fig. 6**, can be uniformly described by the following exponential equation:

$$\Delta\lambda = m_0\left[\exp\left(n_0 f_d\right) - \exp\left(0.4 n_0\right)\right] \tag{5}$$

where m_0 (=6.369) and n_0 (=0.505) are fitting parameters. The fitting results by **Eq. (5)** agree well with the test data (R^2=0.990). Based on the approximately linear relationship between λ and e observed from **Fig. 5** and the formulation in **Eq. (5)**, a nonlinear multiple fitting equation can be defined as follows:

$$\lambda = \lambda_{f0} - le + m\exp\left(nf_d\right) \tag{6}$$

where k_{f0} (=0.386), l (=0.218), m (=0.0092), n (=0.61) are fitting parameters. It is observed that the fitting result from **Eq. (6)** is slightly better than that by **Eq. (4)** based on the values of the maximum relative error (-0.002) and coefficient of determination R^2 (0.993). This indicates that the relationships between λ, f_d and e may lead to more accurate predictions than the relationships between λ, C_u and e.

Discussion

The variations of thermal conductivity with gradation are mainly attributed to the changes in inter-particle contact area and coordinate number (i.e., the number of contacts per particle) with gradation. For a given void ratio, as the uniformity coefficient (or the fractal dimension) increases, the inter-particle contact area and coordinate number increase (McDowell et al. 1996), leading to an increase in the effective conduction among particles for heat transfer. However, the contact area has minimal effect on heat transfer when the uniformity coefficient comes to a critical value (C_u

=10). Likewise, for a given sand gradation, increasing the void ratio will result in a decrease of inter-particle contact area and contact points, leading to a decrease in the effective conduction among particles. These are important conclusions that can have an impact on the design and performance of granular insulation blankets used in pavements or landfill systems.

It should be noted that the parameters in the empirical equations are representative of the dry carbonate sand under investigated in this study, but it is expected that the trends with uniformity coefficient and fractal dimension will be similar when changing the gradation of sands with different mineralogy (e.g., quartz, mica). The experimental methodology used in this study can be used to calibrate parameters in the empirical equations for these other sands, although it is recommended to perform tests on at least four different gradations spanning the uniformity coefficients investigated in this study due to the nonlinearity observed in the empirical relationships. The gradation effect observed in this study could also be combined in the future with other factors, such as particle shape and mineralogy, as they may also play a major role in the number of particle contacts and packing relationships that could affect the thermal conductivity. Further, the impact of the gradation on the soil-water retention curve may lead to different trends in the thermal conductivity with the degree of saturation (Dong et al. 2015).

Conclusions

The influence of gradation on the thermal conductivity is investigated through a series of laboratory tests on dry carbonate sand specimens with different gradations. For a given void ratio, the thermal conductivity of the sand sample increases with increasing the uniformity coefficient or the fractal dimension. Although an increase in void ratio leads to a decrease in thermal conductivity, the percent difference in thermal conductivity was independent of the initial void ratio. The maximum percent difference in thermal conductivity for different gradations and void ratios investigated in this study is 13.9%, which indicates the importance of considering this variable in the design of granular insulation layers or in energy geostructures involving sands prone to crushing. Two empirical equations obtained from nonlinear surface fitting are proposed for the thermal conductivity in relation to the uniformity coefficient and void ratio as well as in relation to the fractal dimension and void ratio. The correlation between the thermal conductivity and fractal dimension has a slightly better than that with the uniformity coefficient and may be more useful in particle mechanics models.

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References

ASTM (2016a). "Standard test method for maximum index density and unit weight of soils using vibratory table." *D4253-16*, ASTM International, West Conshohocken, PA.

ASTM (2016b). "Standard test method for minimum index density and unit weight of soils and calculation of relative density." *D4254-16*, ASTM International, West Conshohocken, PA.

Benson, C., Olson, M., and Bergstrom, W. (1996). "Temperatures of insulated landfill liner." *Proc., 75th Annual Meeting*, Transportation Research Board, Washington, D.C., 24-31.

Bristow, K. L. (2002). "Thermal conductivity." *Methods of Soil Analysis: Part 4 Physical Methods*, J. H. Dane, and C. G. Topp, eds., Soil Science Society of America, Madison, WI, 1209-1226.

Cui, Y.-J., Tang, A.-M., Qian, L.-X., Ye, W.-M., and Chen, B. (2011). "Thermal-mechanical behavior of compacted GMZ bentonite." *Soils Found.*, 51(6), 1065-1074.

Dong, Y., McCartney, J. S., and Lu, N. (2015). "Critical review of thermal conductivity models for unsaturated Soils." *Geotech. Geol. Eng.*, 33(2), 207-221.

Humphrey, D. N., and Eaton, R. A. (1995). "Field performance of tire chips as subgrade insulation for rural roads." *Proc., 6th International Conference on Low-Volume Roads*, Transportation Research Board, Washington, DC, 77-86.

Ladd, R. S. (1978). "Preparing test specimens using undercompaction." *Geotech. Test. J.*, 1(1), 16-23.

Lee, J., Kim, J.-T., Chung, I.-M., and Kim, N. W. (2010). "Analytical solution for heat and moisture diffusion in layered materials." *Can. Geotech. J.*, 47(6), 595-608.

McCartney, J. S., Sanchez, M., and Tomac, I. (2016). "Energy geotechnics: Advances in subsurface energy recovery, storage, exchange, and waste management." *Comput. Geotech.*, 75, 244-256.

McDowell, G. R., Bolton, M. D., and Robertson, D. (1996). "The fractal crushing of granular materials." *J. Mech. Phys. Solids.*, 44(12), 2079-2102.

Tarnawski, V. R., and Leong, W. H. (2016). "Advanced geometric mean model for predicting thermal conductivity of unsaturated soils." *Int. J. Thermophys.*, 37(2), 1-42.

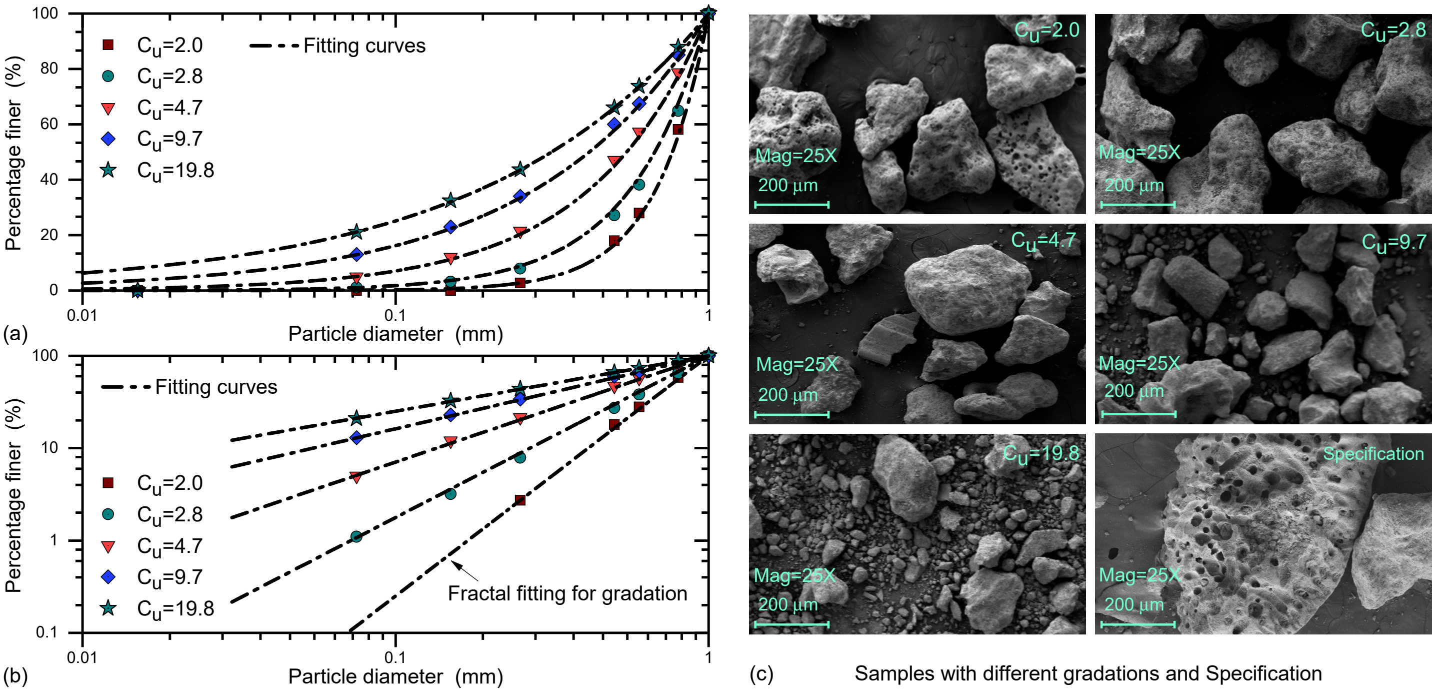
Tyler, S. W., and Wheatcraft, S. W. (1992). "Fractal scaling of soil particle-size distributions: Analysis and limitations." *Soil Sci. Soc. Am. J.*, 56(2), 362-369.

Xiao, Y., Liu, H., Xiao, P., and Xiang, J. (2016). "Fractal crushing of carbonate sands under impact loading." *Geotechnique letters*, 6(3), 199-204.

Zhang, N., and Wang, Z. (2017). "Review of soil thermal conductivity and predictive models." *Int. J. Therm. Sci.*, 117, 172-183.

Zhang, X., and Baudet, B. A. (2013). "Particle breakage in gap-graded soil." *Geotechnique Letters*, 3(2), 72-77.

Fig 1 size: Height = 9.1 cm, Width = 19 cm; 600 dpi



Samples with different gradations and Specification

Fig 2 size: Height = 8.8 cm, Width = 8.8 cm; 600 dpi

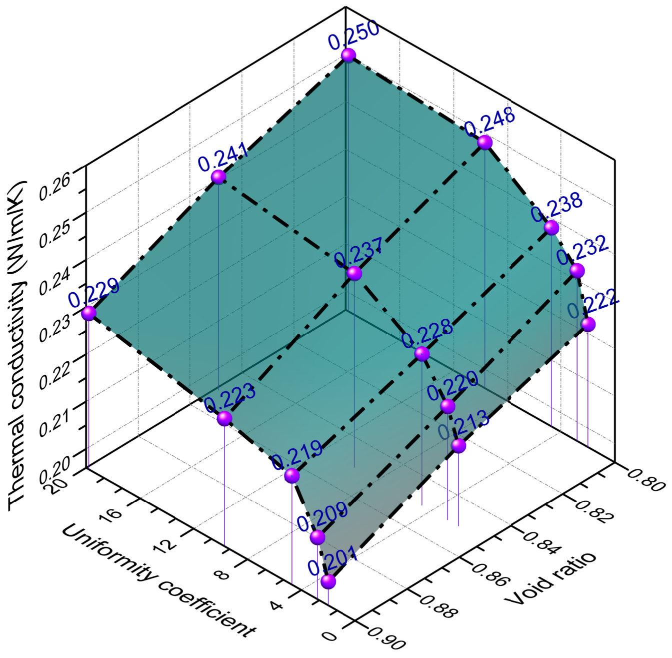


Fig 3 size: Height = 4.6 cm, Width = 8.8 cm; 600 dpi

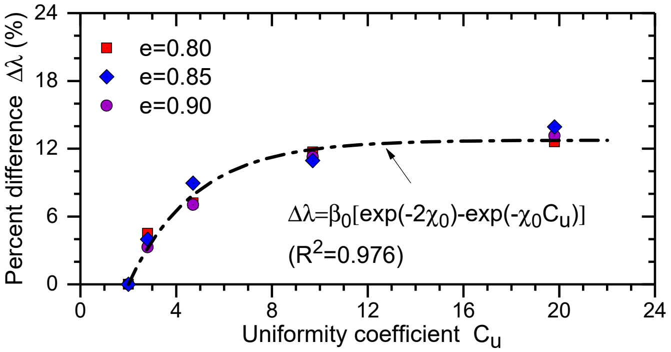


Fig 4 size: Height = 8.8 cm, Width = 8.8 cm; 600 dpi

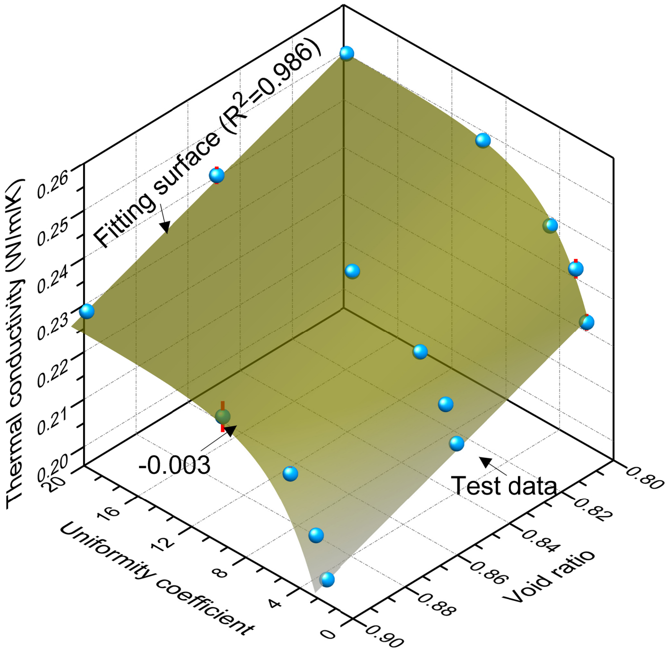


Fig 5 size: Height = 8.8 cm, Width = 8.8 cm; 600 dpi

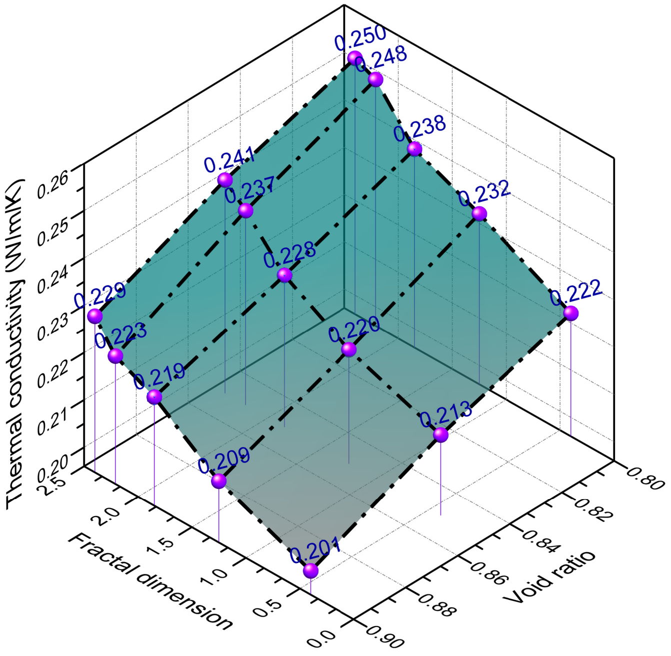


Fig 6 size: Height = 4.6 cm, Width = 8.8 cm; 600 dpi

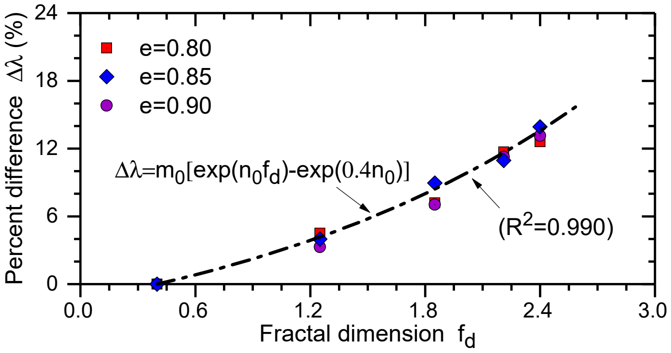


Fig 7 size: Height = 8.8 cm, Width = 8.8 cm; 600 dpi