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Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 42(0)

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Publication Date
2020

Peer reviewed

Children use one-to-one correspondence to establish equality after learning to count

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Abstract

Humans make frequent and powerful use of external symbols to express number exactly, leading some to question whether exact number concepts are *only* available through the acquisition of symbolic number systems. Although prior work has addressed this longstanding debate on the relationship between language and thought in innumerate populations and semi-numerate children, it has frequently produced conflicting results, leaving the origin of exact number concepts unclear. Here, we return to this question by replicating methods previously used to assess exact number knowledge in innumerate groups, such as the Pirahã, with a large sample of semi-numerate US toddlers. We replicate previous findings from both innumerate cultures and developmental studies showing that numeracy is linked to the concept of exact number. However, we also find evidence that this knowledge is surprisingly fragile even amongst numerate children, suggesting that numeracy alone does not guarantee a full understanding of exactness.

Keywords: Number; language; cognitive development; conceptual development

Introduction

Human numerical abilities are built upon a foundation of core cognitive mechanisms shared with nonhuman animals; humans, however, enjoy a concept of number that is both *exact* and *unbounded*, and far exceeds the limits of what these foundational systems afford (Carey & Barner, 2019). Humans are distinct from other animals in another critical sense, however, in that they regularly use symbols to externalize these exact number representations. This relationship between the uniquely human capacities of symbolic expression and exact number representation has prompted an enduring debate about whether representations of exactness are dependent upon knowledge of a symbolic number system.

There is generally agreement that, even without access to exact number language, humans possess two numerical representation mechanisms: the Parallel Individuation (PI) system, which can represent small sets (3-4), and the Approximate Number System (ANS), which offers imprecise representations of large quantities (Feigenson, Dehaene, & Spelke, 2004). While both mechanisms are available early in life (Izard, Sann, Spelke, & Streri, 2009; Wynn, 1992a) and are refined over development, neither is capable of supporting large exact number representations. Although the PI system furnishes precise representations, it is limited to quantities of 3 or 4, and while the ANS supports large number representations, they are imprecise. In particular, a key failing of the ANS in capturing integer properties is that it operates as a function of the ratio between two quantities (i.e., Weber's law). Thus, the ANS cannot detect quantity differences when ratios are sufficiently small (e.g., 9:10).

There is some evidence from innumerate cultures that, without access to linguistic number, human numerical representations are limited to these two systems and do not permit representations of large exact numerosities. Gordon (2004) investigated how the Pirahã, an indigenous Amazonian group with no exact number language, perform on a task requiring them to create a set of objects matching an experimenter's. In the simplest version of this task, the experimenter placed a row of objects (e.g., batteries) in front of the participant, and then asked them to copy the set with another collection of objects. The logic of this task as a test of exact number stems from Hume's Principle – that one-to-one correspondence between sets ensures exact equality. Therefore, if a participant understands this, then they should use one-to-one correspondence to generate matches for *all* numerosities. On the other hand, if no such knowledge exists, then their matches for large quantities should show the ratio-dependent signatures of the ANS.

Using this diagnostic, Gordon (2004), and later Everett & Madora (2012), found that the Pirahã succeeded for items within the PI range (e.g., up to 3 or 4), but approximated for larger quantities, even when one-to-one correspondence could be easily established. While these results seem to suggest the concept of exact equality is linked to symbolic number language, they were contradicted by Frank, Everett, Fedorenko, & Gibson (2008), who found that the Pirahã *could* deploy one-to-one correspondence for all numerosities, although they were less likely to do so when this correspondence was more difficult to establish (e.g., if the experimenter's set was hidden after a brief presentation). However, Everett & Madora (2012) contended that the participants who succeeded in Frank et al. (2008) had been exposed to exact number language and training on one-to-one procedures, leaving open the possibility that these participants may have failed the set-matching task without such training or symbolic number language, consistent with Gordon (2004). Due to the challenges associated with testing remote populations such as the Pirahã, however, these discrepant findings are difficult to adjudicate and resolve.

The origin of exact number concepts has also been explored in another semi-numerate population: young children. Although most children in industrialized cultures hear number language early in life, they do not begin to acquire the meanings of number words until around 2.5 years of age (Wynn, 1992b), and even then do not achieve adult-like knowledge of number words for at least another 2-3 years. Children begin the learning process by acquiring the meanings of small number words – e.g., *one, two, three, four* – one at a time, in sequential order, over a period of many months, and do not seem to understand their relationship to the count list. Around 4 years of age, however, children progress beyond this "subset knower" stage, and acquire some form of the "Cardinal Principle" (CP). These "CP-knowers" understand how to use the count routine to generate sets for larger number words (Wynn, 1992b), and they seem to possess a qualitatively different understanding of number relative to subset knowers (Sarnecka & Carey, 2008). The developmental trajectory associated with children's number acquisition is remarkably consistent (Mollica & Piantadosi, n.d.), and can be reliably assessed in the lab, offering a compelling case study in which to explore the origin of exact number concepts and their relationship to symbolic number.

Recent investigations of whether children can reason about large exact number prior to CP acquisition have produced mixed results. For instance, Izard, Streri, & Spelke (2014) found that subset knowers could track equality between 6 puppets placed on 6 tree branches, although they failed to do so if the perceptual identity of the puppets changed. Similarly, Sarnecka & Wright (2013) found that subset knowers could use one-to-one correspondence to determine whether two sets of 6 were "just the same." Finally, Jara-Ettinger, Piantadosi, Spelke, Levy, & Gibson (2017) found that subset knowers could track changes to equality between two large sets, indicating a fairly robust understanding of exact equality. In contrast, other work has shown that subset knowers fail to spontaneously use one-to-one correspondence to establish equality, even for sets within their PI range. For example, Negen & Sarnecka (2009) found that children's ability to match sets <5 cumulatively increased as a function of known number words. Additionally, Mix (1999) and Mix, Huttenlocher, & Levine (1996) found that subset knowers' ability to match sets of 2-4 was significantly affected by their perceptual similarity.

While the developmental literature suggests that subset knowers may have some partial understanding of Hume's Principle, this work is limited in several ways. First, much of the current literature focuses on sets within the PI range, and does not compare performance between small and large numerosities. Second, it leaves open whether subset knowers' successes for larger quantities stems from understanding the relationship between one-to-one correspondence and exact number, or because this relationship was highlighted within these paradigms. Finally, comparisons of nonsymbolic one-to-one knowledge between subset and CP-knowers are limited, leaving open the question of whether this knowledge is affected by learning the significance of the count routine.

Here, we address these outstanding questions in the developmental literature along with a set of contested findings from innumerate cultures. We adapt methods previously used in work with the Pirahã (Everett & Madora, 2012; Frank et al., 2008; Gordon, 2004) to investigate exact number knowledge in a large group of 3- to 5-year-old children. Specifically, we use the set-matching task to explore whether young children recognize the nonsymbolic relationship between one-to-one correspondence and exact equality, and how this knowledge is related to their acquisition of symbolic number. In Experiment 1, we replicate findings from innumerate cultures and some of the developmental literature that numeracy is significantly related to performance on a set-matching task. In Experiment 2, we rule out one alternative hypothesis for CP-knowers' increased accuracy in comparison to subset knowers. Surprisingly, we find that while CP-knowers outperformed subset knowers in both experiments, their performance was far from ceiling. Together, our findings suggest that the relationship between one-to-one correspondence and exact number becomes more salient to children after they have acquired the CP, and that this knowledge may continue to develop for some time after children achieve this level of numeracy.

Experiment 1

Method

This study was pre-registered on OSF (`https://osf.io/3wta2`), and all methodological and analytical choices were as preregistered, unless stated otherwise in-text.

Participants Our final analyzable sample included 144 children (M_{age} = 3.94 years, SD_{age} = 0.52 years, range = 3.01 - 5.07 years) recruited from local preschools and the surrounding community in San Diego, California, USA and Comox Valley, British Columbia, Canada. In this sample, 70 were identified as CP-knowers, while the remaining 74 were classified as subset knowers.

Procedure

Set-matching This task, modeled on Gordon (2004), was framed as a "matching game." Children were presented with a 6"x30" blue cardboard rectangle and a container with 15 fish. The experimenter introduced the game by saying, "Let's play a matching game. Do you know what matching is? Matching is when you make things look the same. So, in this game, you're going to make things look like each other." The experimenter explained that the child could put their fish in their pond. Next, the experimenter then placed another blue board with one plastic fish glued to the center directly above the child's and said, "Using your fish, can you make your pond look like my pond?"

In an effort to replicate the methods of Gordon (2004) as closely as possible, and to obtain a measure of children's unprompted attention to exactness, the experimenter did not explicitly direct children to attend to number when giving either task instructions or feedback. One deviation from Gordon (2004) was the inclusion of two training trials with 1 and 2 fish (on the experimenter's board), during which the children received non-numerical feedback to ensure that they understood the purpose of the task (e.g., "Look! These ponds match, because there is a fish here, and a fish here!").

Boards were presented to children either in a Parallel or Orthogonal orientation, based on Gordon (2004). In the Parallel orientation the experimenter's board was was placed directly above the child's, such that one-to-one correspondence was a readily available strategy for solving the task. In the Orthogonal orientation the experimenter's board was placed perpendicularly to the right of the child's, requiring a spatial transformation of one-to-one correspondence. All children received tasks in both orientations, with Parallel trials presented before Orthogonal trials.

To test whether children's ability to use one-to-one correspondence was affected by the identity of sets (Mix, 1999; Mix et al., 1996), we manipulated the similarity of the fish relative to the experimenter's between subjects. Half of the children were randomly assigned to the Identical condition, in which fish were the same for both the experimenter and the child, and half were assigned to the Non-identical condition, in which fish where were matched on relative size, but were different varieties.

Training trials were presented for both the Parallel and Orthogonal orientations in a fixed order (1, then 2). After passing these training trials, children received 5 test trials in both board orientations with small (3, 4) and large (6, 8, and 10) quantities with neutral feedback. Trial order was fixed for the Parallel (3, 4, 10, 8, and 6) and Orthogonal (4, 3, 8, 10, and 6) orientations. Fish on the experimenter's boards were always approximately 1" apart, regardless of set size; although the set of 10 was spread across the majority of the board, the maximum number of fish (15) could still be placed on the board with approximately .25" separation.

Children who attempted to count were immediately stopped and told "This isn't a counting game - this is just a matching game!" Counting attempts were relatively rare: In both conditions, CP-knowers attempted counts on 64/700 trials, while subset knowers attempted counts 28/740 trials.

Give-N Children's CP knowledge was assessed using an abbreviated version of a titrated Give-N (Wynn, 1992b). The experimenter gave the child a plate and 10 plastic objects (e.g., bears, apples, buttons), and asked the child to place some number on the plate. After children placed some number of objects on the plate and indicated that they were finished, the experimenter asked, "Is that *N*? Can you count and make sure?" If the child answered in the negative, they were permitted to fix the set. If children successfully generated a given *N*, they were asked for *N*+1 on the next trial; otherwise, they were asked for *N*-1. Children were considered CP-knowers if they were able to generate sets of 6 (the maximum number tested) at least 2 out of 3 times when requested. Children were classified as subset knowers if they gave another *N* correctly at least two of three times, and did not give that *N* more than once for another number, as in Wynn (1992b).

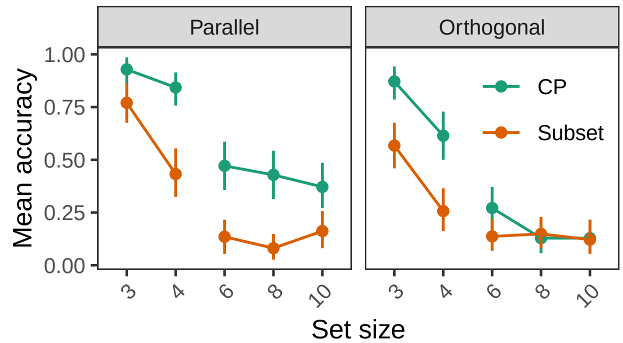

Figure 1: Mean accuracy on the set-matching tasks by CP-knower level. Error bars represent 95% confidence intervals computer by nonparametric bootstrap.

Results and Discussion

Our primary question was whether CP-knowers were more likely than subset knowers to generate exact matches for both large and small set sizes. To test this, we built a generalized linear mixed effects model (GLMM) predicting exact matches from CP-knower status, set size, orientation (Parallel/Orthogonal), and age, with a random effect of subject.[1] This model indicated that CP-knowers generated exact matches significantly more often than subset knowers overall ($\beta = 1.09$, $p < .0001$; Figure 1), even when controlling for age ($\beta = 0.3$, $p = .01$). This final model also revealed decreased accuracy with increasing set sizes ($\beta = -1.24$, $p < .0001$), and for Orthogonal trials ($\beta = -0.86$, $p < .0001$). Follow-up analyses found no interaction between set size and orientation ($\chi^2 = 0.99$, $p = 0.32$); however, there was a significant 3-way interaction between set size, orientation, and CP-knower status ($\chi^2 = 12.5$, $p = 0.006$), such that the difference in performance between subset and CP-knowers for increasing set sizes was greatest in Parallel, as opposed to Orthogonal, orientations ($\beta = 0.98$, $p = .002$).

We next tested whether, consistent with prior work (Mix, 1999), CP-knowers were more likely than subset knowers to ignore perceptual dissimilarities in this task by constructing another GLMM predicting an exact match from an interaction between CP-knower status and identity condition (Identical/Non-identical), orientation (Parallel/Orthogonal), set size, and age, with a random effect of subject. This model indicated a significant interaction between set identity and CP knowledge, with CP-knowers significantly more accurate than subset knowers in the Non-identical condition ($\beta = 1.13$,

[1] All mixed effects models were fit in `R` using the `lme4` package. The final model specification was: `Correct ~ CP-knower status + Set size + Orientation + Age + ( 1 | subject)`. Although we pre-registered a model containing a CP-knower status x Set size interaction, a Likelihood Ratio Test indicated that this interaction did not improve the fit of the main effects model ($\chi^2 = 1.2$, $p = .21$).

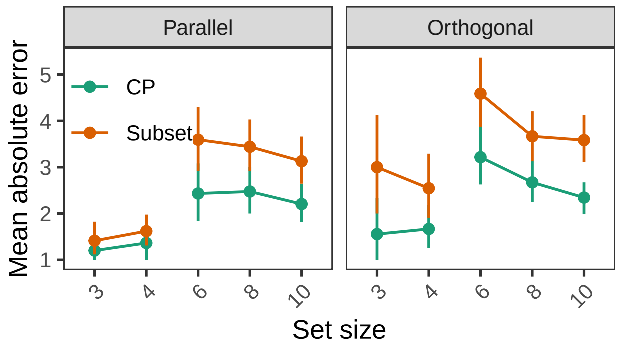

Figure 2: Mean absolute error for incorrect trials on the set-matching tasks by CP-knower level. Error bars represent 95% confidence intervals computer by nonparametric bootstrap.

p = .002), which was again significant when controlling for age (β = 0.31, *p* = .007). In addition to these effects, this model again indicated decreased accuracy when boards were presented in the Orthogonal orientation (β = -0.86, $p < .0001$) and with increasing set size (β = -1.24, $p < .0001$).

These analyses indicate that, when one-to-one correspondence was a readily available strategy for matching sets, CP-knowers were more likely than subset knowers to capitalize on this relationship in order to generate numerically equal matches, even when the two sets were perceptually dissimilar. When the boards were presented in an Orthogonal orientation, and one-to-one correspondence was less easily implemented, however, CP-knowers' performance for large numerosities was not appreciably different from subset knowers'. However, despite CP-knowers' increased accuracy for boards in the Parallel orientation, their knowledge of one-to-one's significance was strikingly limited, with overall accuracy for large set sizes only 42% and far below adult-like levels (Frank et al., 2008). This surprisingly variable performance (Figure 3) suggests that acquisition of the CP alone may not guarantee an understanding of exact equality and its relationship to one-to-one correspondence.

Next, we also investigated errors in children's matching as a less conservative signal for whether they were attempting a one-to-one match, even if they were not perfectly accurate. We reasoned that exploring children's errors as a complement to our accuracy analyses may yield more information about the matching strategy that children are deploying: If children are attempting a one-to-one match, we should find that their responses are closer to the target set, whereas if they are not attempting a one-to-one match their errors should be farther from the target set. For these analyses, we specifically investigated differences between subset and CP-knowers' absolute error (|Target set - Response|) on incorrect trials, as well as their Coefficient of Variation (CoV).

We first analyzed differences between subset and CP-knowers' absolute error on incorrect trials with a linear mixed effects model predicting absolute error from CP-knower status, set size, orientation, and age, with a random effect of subject.[2] Likelihood Ratio Tests indicated a main effect of CP knowledge ($\chi^2(1) = 9.76$, *p* = .004), with lower absolute error for CP-knowers' in comparison to subset knowers (β = -0.79, *p* = .002), even when controlling for age (β = -0.21, *p* = .10; Figure 2). Absolute error increased with set size (β = 0.39, $p < .0001$), and in the Orthogonal orientation (β = 0.57, $p < .0001$).

Once again, we found evidence that CP-knowers are less affected by perceptual dissimilarities when establishing numerical equality; a second linear mixed effects model predicting absolute error from an interaction of CP knowledge and Identity condition, set size, orientation, and age, with a random effect of participant showed a significant interaction between CP knowledge and Identity ($\chi^2(1) = 10.3$, *p* = .002), such that CP-knowers had significantly lower error on Non-identical trials in comparison to subset knowers (β = -1.39, *p* = .001).

Finally, we also used CoV, which captures noise in participants' responses to a given set size[3] as an additional measure of error in this task. Mirroring our accuracy and error analyses above, CoVs were significantly lower for CP-knowers in comparison to subset knowers for trials in both the Parallel ($t(142) = -5.73$, $p < .0001$) and Orthogonal orientations ($t(141) = -4.99$, $p < .0001$). While the slope of CoV relative to target quantity has been used to identify whether participants are deploying a one-to-one or approximation strategy in previous set-matching work (Everett & Madora, 2012; Frank, Fedorenko, Lai, Saxe, & Gibson, 2012), this analysis is not appropriate for our dataset for several reasons. First, both the number of items that participants could use to match sets, as well the area in which they could be matched, were bounded, which truncated the full distribution of responses for larger sets and affected CoV approximations. Prior work has shown that such bounds preclude assessments of the kind of scalar variability associated with ANS (Wagner, Chu, & Barner, 2019). Second, accuracy in our task was much lower than in previous work, and more children seemed to default to error-prone heuristics to solve this task (e.g., many subset knowers gave the maximum number of fish for large sets), yielding noisier CoV estimates. Due to these considerations, we cannot use CoV to make inferences about whether children were using one-to-one correspondence or approximation to solve this task.

Together, the results of our accuracy and error analyses broadly replicate the finding that numeracy is significantly related to the availability of exact number concepts. Similar to the Pirahã, and consistent with developmental work

[2]The final model specification was: `Absolute error ~ CP-knower status + Set size + Orientation + Age + ( 1 | subject)`. Although we again pre-registered a model containing a CP-knower status x Set size interaction, we pruned this interaction after finding it did not improve the fit of the model ($\chi^2 = 2.74$, *p* = .10).

[3]CoV was approximated as in Frank et al. (2012), with the formula $\sqrt{(t_i - r_i)^2}/t_i$, where *t* is the target quantity, and *r* is the child's response.

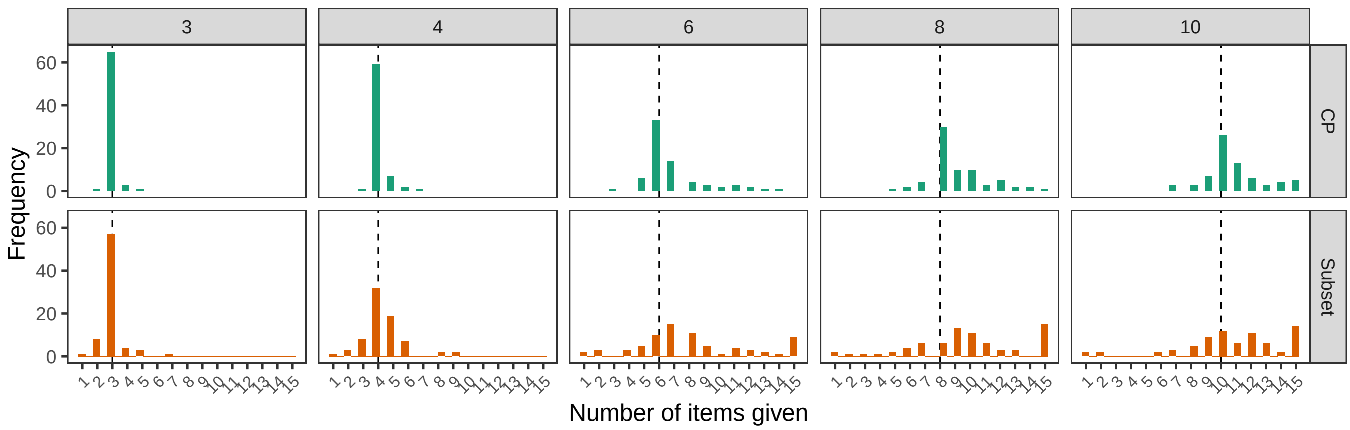

Figure 3: Frequency of set-size response (x-axis) for each target size in the Parallel condition, grouped by CP knowledge.

showing set-matching failures prior to CP acquisition, we found that although subset knowers were generally accurate for sets within their PI range, they were less likely to generate an exact match for large numerosities. In contrast, CP-knowers were significantly more accurate in comparison to subset knowers for these larger numerosities, with their pattern of errors suggesting that a majority of children in this group were potentially attempting to deploy one-to-one correspondence. CP-knowers' performance was far from ceiling, however, and reflected some striking limitations. We return to this novel finding in the General Discussion.

Experiment 2

In Experiment 1, we found that CP-knowers were much more likely than subset knowers to establish exact equality between two large quantities. One possible reason for CP-knowers outperforming subset knowers in this task, however, is that they might have subvocally counted the experimenter's set and used it to generate a match. Experiment 2 was designed to test for this possibility.

Method

This study was pre-registered on OSF (`https://osf.io/pj4zy`), and all methodological and analytical choices were as preregistered, unless stated otherwise in-text.

Participants Our current sample includes 28 children out of a planned sample of 40 (M_{age} = 4.38 years, SD_{age} = 0.51 years, range = 3.25 - 5.03 years) recruited in preschools and the surrounding community in San Diego, California, USA. All children were classified as CP-knowers by the Give-N task.

Procedures

Procedures and tasks were identical to Experiment 1 with two exceptions in the set-matching task. First, because the results of Experiment 1 indicated that children were unable to use one-to-one correspondence with orthogonally oriented sets, boards were only presented in a Parallel orientation, with trial order for larger sets counterbalanced across participants. Second, to test whether CP-knowers' performance could be explained by subvocal counting, after the last trial of set-matching (8 or 10 fish) the experimenter covered both boards and asked the child, "How many fish are in my pond?" If the child did not know, they were prompted to guess. The logic of this follow-up question was that, if the child had succeeded by counting, then they should provide an accurate answer when asked to report the target set's cardinality. To control for differences in working memory, and to test whether children were capable of remembering a recently counted set, the experimenter then let the child count the board, covered it again, and then asked, "How many fish are in my pond?"

Results and Discussion

The majority of children (93%) were able to remember the cardinality of a recently counted set. Of the 28 children currently included in this dataset, 15 first responded "I don't know." Of the 13 children who first offered a numeric answer, only 2 gave a correct response. Children who were able to provide a numeric response gave verbal estimates that were, on average, about 4 numbers off from the correct response (*Min* = 1, *Max* = 18, *SD* = 5.5). Additionally, these children did not show evidence of having counted as they were generating their sets, as their verbal responses were almost 5 numbers off from the size of the set they had generated (*M* = 4.92, *SD* = 3.7). Finally, children who gave a numeric response did not have higher overall mean performance in comparison to children who were unable to provide a numeric response ($t(26) = 0.37$, $p = 0.72$). Together, these results suggest that the greater accuracy CP-knowers demonstrated in Experiment 1 was likely not due to subvocal counting.

Additionally, we found that CP-knowers' set-matching performance closely matched Experiment 1, with 93% accuracy for small numerosities (compared with 89% in Experiment 1), and 40% accuracy for large numerosities (compared with 42% in Experiment 1).

General Discussion

Although concepts associated with linguistic expressions such as "*eleven*" are unequivocally tied to symbolic number, it is unknown whether the idea that number *can* be exact is only available through exact number language. Previous work exploring this question has produced conflicting results, with exact number representations sometimes hinging on symbolic number (Everett & Madora, 2012; Gordon, 2004; Negen & Sarnecka, 2009); and sometimes being available in its absence (Frank et al., 2008; Jara-Ettinger et al., 2017). In the current work, we returned to this question with a large sample of children to explore whether one diagnostic of exact number – the ability to use one-to-one correspondence to generate numerically equal matches – changed as a function of symbolic number acquisition.

By adapting a method previously used in innumerate cultures, we provide a broad and flexible test of children's exact number knowledge at varying stages of their symbolic number acquisition. Our findings adjudicate between previous studies by providing a large and robust set of findings, replicated across two experiments, that show a link between large exact number representations and knowledge of symbolic number language. We replicate previous findings with the Pirahã and young children that numeracy is related to exact number concepts (Everett & Madora, 2012; Gordon, 2004; Negen & Sarnecka, 2009): While both subset and CP-knowers were generally accurate on the set-matching task for quantities within the PI range, only CP knowers were more likely to generate an exact (as opposed to approximate) match for larger quantities. Subset knowers, on the other hand, were unlikely to spontaneously deploy a set-matching strategy that would guarantee exact, rather than approximate, equality. These findings are compatible with Gordon's (2004) claim that large exact number concepts are related to numeracy, but sharpen this claim to show that (1) simply knowing some exact number words is not sufficient to acquire a full understanding of one-to-one correspondence, and (2) even learning to count does not ensure perfect performance on this task.

While CP-knowers were more accurate than subset knowers, their performance was surprisingly variable and well below adult levels (Frank et al., 2008). This indicates that while acquisition of the CP may make one-to-one correspondence more accessible, knowledge of the CP alone may not be sufficient to furnish a complete understanding of how exact number and one-to-one correspondence are related. One possible reason for this pattern of performance may be that children learn the significance of one-to-one correspondence not through number language, but through its associated procedures. Specifically, as CP-knowers learn how to deploy one-to-one correspondence in the count routine to create a partition between counted and uncounted sets, they may notice that one-to-one procedures can also be deployed nonsymbolically to the same end, as in the set-matching task. Thus, it is possible that CP-knowers' increased accuracy on the set-matching task may not reflect new conceptual knowledge stemming solely from exact number language, but rather their abstraction of a general principle from learning the procedures of the count routine. Through gaining more experience with deploying the count routine and recognizing how it is coextensive with nonsymbolic one-to-one procedures, CP-knowers may discover the integral role of one-to-one correspondence in exact number.

This hypothesis may account for CP-knowers' variable performance in the current work; CP-knowers are an unbounded group, and demonstrate a high degree of heterogeneity in both their understanding of counting and the integers. For example, many young CP-knowers may have only a surface-level understanding of counting, and blindly deploy it to generate cardinalities without necessarily grasping its deeper logical entailments and numerical meaning (Barner, 2017). Prior work has shown that many children discover other properties of the integers, such as the successor function (Cheung, Rubenson, & Barner, 2017), well after acquiring the CP, suggesting that as children progress from a procedural to numerical understanding of the count list, their understanding of exact number similarly grows more robust. Children's performance here raises the possibility that a full understanding of the role of one-to-one correspondence in establishing equinumerosity may emerge some time after acquiring the CP. Future work should investigate the trajectory of this understanding, and its implications for the development of other numerical knowledge (Carey & Barner, 2019).

There are two important limitations of this work. First, because we wished to provide a measure of children's unprompted attention to exactness, our ambiguous instructions to "Make your pond look like mine" may have created too large a hypothesis space, prompting some children to generate matches on the basis of length, density, or some other set feature. That subset knowers' matching behavior was affected by set identity is consistent with this alternative, and with prior work showing that perceptual identity is more salient than numerical equality for subset knowers (Izard et al., 2014; Mix, 1999), and even some CP-knowers (Chan & Mazzocco, 2017). Second, the set-matching paradigm does not fully rule out an approximation strategy. Children's ANS becomes more precise after they acquire the CP (Shusterman, Slusser, Halberda, & Odic, 2016), leaving open the possibility that CP-knowers' lower rates of error and higher accuracy may reflect some mix of both one-to-one and approximation strategies. These limitations provide direction for future work testing the effects of directing children's attention to number, and also disambiguating between different set-matching strategies.

Together, this work provides key data on the previously unclear role of language in the development of exact number concepts. We find that, consistent with the hypothesis that the availability of exact number concepts is linked to exact number language (Núñez, 2017), children with limited symbolic number knowledge struggled on a nonsymbolic test of exact equality. While children with greater symbolic number knowledge had higher accuracy, their performance on even the simplest version of this task was surprisingly low, suggesting that exact number language alone may not be sufficient to fully grasp Hume's Principle, and that the numerical significance of one-to-one correspondence may be discovered through counting experience. Future work should explore the development of this knowledge in numerate children, and the process through which children might acquire a more complete understanding of the relationship between one-to-one correspondence and exact number.

Acknowledgements

We thank the Research Assistants of the UCSD Language and Development Lab for their assistance with data collection. We also thank the parents and families of our participants for their support. This work was supported by an NSF GRFP to RMS.

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