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# STOCHASTIC GROWTH IN SCHUMPETERIAN DYNAMICS

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## ABSTRACT

This paper discusses three key elements of stochastic growth in the Schumpeterian dynamics. These elements comprise the new entry of firms in an industry, the displacement of the old technology by the new and the nonlinear impact of learning by doing on the growth of innovating firms. Each of these elements has important implications for the new theory of endogenous growth.

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# I. INTRODUCTION

Two key variables have been strongly emphasized in the Schumpeterian view of growth: innovations in new technology and the expected growth of knowledge in the form of R&D . The first often referred to as 'the technology push hypothesis', e.g., Kamien and Schwartz (1982) generates competition in new technology: its adoption, development and continual improvement through the process of 'creative destruction'. The second emphasizes the externalities and scale economies effect of R&D capital. Recently Thompson (1996) has developed this aspect of the connection of technological opportunity and the growth of knowledge in Schumpeterian model, which termed this economic framework as *trustified capitalism* in which innovations in big units tend to be carried out by the R&D specialists and the firms' market shares respond to innovations by themselves and by others.

Since these two key variables play a central role in new growth theory it is important to analyze their implications in a generalized Schumpeterian framework. Two types of generalizations are attempted here. One is the stochastic aspect of the innovation process in new technology and the other the substitution process underlying the growth of knowledge capital and its diffusion.

One may mention two motivations for this study. One is the overriding stochastic nature of the innovation process in its inception, development, diffusion and survival or extinction. In modern endogenous growth many researchers have stressed the inherent uncertainties of R&D investment and the fact that producers using new technologies rarely achieve commercial viability until after they experience a prolonged period of learning by doing. Secondly, this view of

Schumpeterian growth allows the possibility of self sustained growth in per capita output, just like the new growth theory but there are other possibilities also due to the stochastic nature of the evolutionary process. As a matter of fact we show that the stochastic nature of the nonlinear growth processes such that the cost of output fluctuations measured by variance may tend to retard the mean process of growth, unless the intermediate inputs are forever being improved at a steady rate with a very low rate of variance.

The paper outline is as follows. Section I discusses the innovation flow process in an industry as a demographic process of *births* and *deaths*, where birth may refer to entry of new firms, and death the exit of old firms. Instead of firms one may refer to the technology used by the firms and the new technology competes with the old in the process of 'creative destruction'. This is followed in Section II by a Markov process model with transition probabilities, where the growth of knowledge is affected by past history and future expectations. This section also discusses the economic implications of nonlinearity in the growth process. Section III analyzes the implications of the output process, resulting from the continual improvement of intermediate goods thereby raising productivity in the assembly of final output. Section IV analyses the impact of stochastic innovations on output and discusses the sources of nonlinear fluctuations in the output process. The cost of such fluctuations may sometimes affect the mean output levels adversely, particularly for countries with high growth rates.

## III. A MODEL OF ENTRY AND EXIT

Consider technological innovations as a flow $x_t$, where its expected change during a discrete interval of time t to t+1 can be modelled in terms of the deterministic model

$$E(x_{t+1}) = x_t + B_t - D_t \qquad (1)$$

where $B_t$ and $D_t$ are expected births and deaths during the interval. Denoting expected birth rates (or energy rates) and death rates (or exit rates) by $b_t$ and $d_t$ and assuming a simple birth and death process model of population growth, the mean and variance of the innovation input may be easily written as follows: see, e.g., Sengupta (1999a)

$$E(x_{t+1}) = x_t \; e^{r_t} \, , \; r_t = b_t - d_t \tag{2}$$

$$\mathrm{var}(x_{t+1}) = (b_t - d_t)^{-1} (b_t + d_t) \, [ e^{r_t} - 1] \; e^{r_t} \; x_t$$

As in demographic models the birth and death rates may be considered as shocks to the production process characterized by $y_t = f(x_t)$.

Two economic interpretations of the $x_t$ process are in order. One is the entry and exit interpretation of birth and death rates. The other is the notion that $x_t$ represents the pool of research knowledge in an industry, where $b_t$ is new contributions and $d_t$ is the obsolescence or depletion. Recently Lansbury and Mayes (1996) have empirically analyzed the time variations in the average entry ($b_t$) and exit ($d_t$) rates in the different industrial sectors of UK manufacturing industry over the period 1980 to 1990. They found that exits have remained a relatively stable share of the number of businesses over the whole period. The range has been only 1½ percentage points between 5¼ and 6¾ percent. Entries however have varied more cyclically with a range nearly four times as large, from 3½ to 9 percent. Furthermore, the productivity of most new entrants is higher than that of the sample as a whole in all years and most exits had a lower productivity than the sample as a whole.

The idea of a pool of unexploited research knowledge providing the domestic and foreign sources of innovation as engines of growth has been empirically explored by Kortum and Eaton (1995) and also Helpman (1997). In this model each country (or firm) undertaking research makes a potential contribution to the pool, while any country (or firm) adopting any of the generated ideas depletes the pool in proportion to its size. Their empirical model provided a high degree of explanation of the differences in manufacturing productivity in the major industrial countries over the last four decades 1950-1990. International technological diffusion appears to be the major source of productivity growth.

Consider the mean variance process (2). Clearly if $b_t > d_t$, then the expected innovation process is unbounded and hence there can be persistence in output growth as in new growth theory. In Schumpeterian dynamics the most favorable context for innovation is oligopolistic competition with large firms, and the most important competitive pressure comes from technical change that erodes any monopoly profits of a firm which fails to remain in the forefront of technical change and knowledge frontier.

Given the stochastic innovation process (1) one may now consider the case where births and deaths are endogenously determined as follows:

$$B_t = b_0 + b_1 x_t + b_2 x_{t-1};\ D_t = d_0 + d_1 x_t \qquad (3)$$

As in Lansbury and Mayes (1996) the deaths or exits are proportional to $x_t$, whereas births or entries depend on both $x_t$ and $x_{t-1}$. We obtain then the reduced form equation

$$x_t = \beta_0 - \beta_1 x_{t-1} + \beta_2 E(x_{t+1}) \qquad (4)$$

where $\beta_0 = (d_0 - b_0)\,\beta_2,\ \beta_1 = b_2\beta_2$

$\beta_2 = (1 + b_1 - d_1)^{-1}$

In the more general case $D_t = d_0 + d_1 x_t + d_2 x_{t-1}$ and this yields the same equation as (4) except that $\beta_1$ has to be redefined as $\beta_1 = \beta_2(b_2 - d_2)$.

Several important implications follow from this fundamental equation (4) of the innovation flow in Schumpeterian framework. First of all, this equation captures the impact of the future by

$x_{t+1}$ and the immediate past by $x_{t-1}$. The future expectations may involve demand expectations, whereas the past may reflect various costs due to risk aversion and input fluctuations. Clearly we have

$$\partial x_t/\partial x_{t-1} < 0 \text{ and } \partial x_t/\partial E(x_{t+1}) > 0 \qquad (5)$$

and since $\beta_2$ exceeds $b_2\beta_2$, entry would tend to dominate exit thus implying positive growth in innovation rates. One notes that the impact of past history and the future expectations may also be more generalized in terms of more than one period, e.g.,

$$x_t = \beta_0 - \sum_{i=1}^{m} \beta_{1i} x_{t-1} + \sum_{i=1}^{m} \beta_{2i} E(x_{t+1})$$

Secondly, one may consider (4) as an optimal decision rule for the innovating firm, where $x_{t+1}$ represents the forward looking view of the future, while $x_{t-1}$ is the backward looking view. This is a class of forecast-based rules which feed back from expected values of future innovations (or demands) to the optimal decision rules. These forecast-based rules widely used in rational expectations theory have a number of desirable features, which mean they may approximate the optimal feedback rule of linear quadratic optimal control theory, see e.g., Taylor (1993). For instance the second order difference equation (4) can be easily formulated as the dynamic optimal adjustment behavior of a rational innovating firm. This behavior involves an optimizing decision by the producer, who finds that his current factor uses are not consistent with the long run equilibrium path $(x_t^*, y_t^*)$ as implied by the stochastic model (4). These values $x_t^*, y_t^*$ of inputs and output may also be interpreted as target levels implied by the current input prices and their expected changes in the future. Sengupta (1996) has recently applied a quadratic adjustment cost model to explore the optimal time path of demand for capital and labor inputs in the growth process of Japan over the period 1965-90. By following this model we postulate that the innovating firm minimizes the expected present value of a quadratic loss function as follows

$$\text{Min } E_t L$$

where $$L = \sum_{t=0}^{\infty} \rho^t \left[ (\tilde{X}_t - \tilde{X}_t^*)' A (\tilde{X}_t - \tilde{X}_t^*) + (\tilde{X}_t - \tilde{X}_{t-1})' B (\tilde{X}_t - \tilde{X}_{t-1}) \right] \qquad (6)$$

where $E_t(\cdot)$ is expectation as of time t, $\rho$ an exogenous discount rate, prime is transpose, A and B are diagonal matrices with positive weights and $\tilde{X}_t, \tilde{X}_t^*$ are the vectors of input level and their targets. Here the first component of the loss function is disequilibrium cost due to deviations from either the desired level or steady state equilibrium and the second component characterizes the producer's aversion to input fluctuations under market uncertainty. Kennan (1979) and more recently Callen et al. (1990) have applied this formulation to derive optimal input demand equations which incorporate the producer's response to market fluctuations. On carrying out the minimization in (6) for the research input $\tilde{x}_{1t}$ say, one may easily derive the optimal adjustment behavior as

$$\tilde{x}_{1t} = b_0 + b_1 \tilde{x}_{1t-1} + b_2 \tilde{x}_{1t+1} \qquad (7)$$

where $$b_0 = (a_1 + a_2 + a_2\rho)^{-1} (a_1 \tilde{x}_{1t}^*)$$

$$b_1 = (a_1 + a_2 + a_2\rho)^{-1} a_2$$

$$b_2 = (a_1 + a_2 + a_2\rho)^{-1} (\rho a_2)$$

Similar equations for other inputs can be derived. On comparing (4) and (7) one could easily draw some interesting implications. First of all, one could estimate this model (7) and if it turns out that $\hat{b}_2 > \hat{b}_1 > 0$, or $\hat{b}_2 > 0$ with $\hat{b}_1 < 0$, then the future expectations play a more dominant role than the past history. This is the normal response in a Schumpeterian growth process involving innovating firms and for countries with rapid growth episodes. This may be empirically tested. Secondly, on using the nonexplosive characteristic root $\mu_1$ of the second order difference equation (7), the optimal adjustment equation can be written as

$$\Delta\tilde{x}_{1t} = \tilde{x}_{1t} - \tilde{x}_{1t-1} = c_1 - g_1\tilde{x}_{1t-1} + h_1 d_{1t} \qquad (8)$$

where $d_{1t} = (1 - \mu_1) \sum_{s=0}^{\infty} \mu_1^s \tilde{x}_{1,t+s}^*$

Both Kennan (1979) and Gregory et al. (1993) have discussed two stage methods of estimating this linear decision rule in a statistically consistent manner by using other instrument variables.

Finally, the gap between $x_{1t}$ and $\tilde{x}_{1t}$, i.e., $x_{1t} = \tilde{x}_{1t} + \varepsilon_{1t}$ may be evaluated over time to test if the planned inputs converge to the expected trend following from the stochastic process model. Indirectly it would provide an empirical test of the rational expectations hypothesis which postulates a perfect foresight conditon in the sense that the $\{\varepsilon_{1t}\}$ process is purely white noise.

A third interpretation of model (1) arises when we consider a differential equation form of the model

$$E(x_{t+1}) = (1 + ax_t)^{-1} \lambda x_t; \lambda = 1+b-d$$

$$B_t = bx_t; D_t = dx_t$$

as $$\dot{x}_t / x_t = (r - ax_t); r = b-d \qquad (6)$$

where the solution is

$$x_t = (1 + x_0 e^{-rt})^{-1} (r/a)$$

with r/a as the upper asymptote of the logistic solution, $x_0$ is the initial state of the system and a is a positive constant denoting the rate of decline of ($\dot{x}_t / x_t$) in respect of the level of $x_t$. Now consider the stochastic variations in parameter r, which may be due to the entry and/or exit process being random. For the knowledge input this may be interpreted as firms which allocate resources to R&D buying themselves a *chance* at developing some new targeted product. Assume that r varies randomly as $\alpha + \gamma W(t)$, where W(t) is a Gaussian white noise process with zero mean, $\alpha$, $\gamma$ being positive constants. Then one can derive the infinitesimal mean $\mu(x)$ and variance $\sigma^2(x)$ of the stochastic process $\{x_t\}$ as follows: see, e.g., Karlin and Taylor (1981)

$$\mu(x) = x(\alpha - ax) \qquad (7)$$

$$\sigma^2(x) = \gamma^2 x^2$$

On substitution the mean $\mu(x)$ can be expressed as a function of variance $\sigma^2(x)$

$$\mu(x) = (\alpha\sigma(x)/\gamma) - a\sigma^2(x)/\gamma^2$$

It follows that

$$\frac{\partial\mu(x)}{\partial\sigma^2(x)} < 0, \text{ if } \sigma^2(x) > (\alpha\gamma/(2a))^2 \qquad (8)$$

Otherwise $\partial\mu(x)/\partial\sigma^2(x) > 0$.

If output is proportional to innovation flow, then the negative relationship in (8) would imply that greater volatility leads to lower mean output. Recently Ramey and Ramey (1991) and Binder and Pesaran (1996) have empirically found for real GNP time series data in US a negative and persistent relationship between mean and variance of output. The major source of fluctuation here is the randomness in productivity and demand shocks. In Schumpeterian dynamics this negative relationship holds only at higher levels of volatility when the second inequality in (8) holds; otherwise both mean and variance may rise over time. The asymmetry in the nonlinear process of dynamics is of some importance here.

## III. MARKET SELECTION PROCESS

A key element in Schumpeterian dynamics is the competition from new products. Competition is very broadly defined so as to include innovative ways of market penetration and displacement of the old products using old technologies. Two types of characterization of this market selection process may be easily made. One is the market share dynamics of newly entering firms using newer technologies. If $z_i(t)$ denotes the market share of this group of firms, then this selection process may be modeled as

$$\Delta z_i(t) = k z_i(t-1) \, [C_i(t-1) - 1] \tag{9}$$

where $z_i(t) = y_i(t)/Y(t)$, $y_i(t)$ and $Y(t)$ being output of firm group i and $Y(t)$ the total industry output and $C_i(t-1)$ is a measure of relative competitiveness of firm i relative to the technological level of other firms operating in the market. Recently Perez (1997) has used this type of logistic model of the market selection process to analyze rapid growth episodes of newly industrializing countries (NICs) in Southeast Asia, which actively promote importation of modern technology and the expansion of multinational enterprises. In this framework the process of displacement of the old technology by the new is strictly path dependent, since the absorptive capacity of the old firms depends on their past levels of technological accomplishment and their ability to adopt newer technologies through creative destruction. Clearly this process involves transition probabilities reflecting the knowledge spillover process.

A second type of characterization is in terms of overall output growth $\Delta y(t)/y(t)$, where output growth is proportional to the profitability of new technology.

$$\dot{y}(t)/y(t) = b(D(p,\gamma) - y(t)) \tag{10}$$

Here b is the adoptive coefficient assumed to be constant, dot is time derivative and $D(p,\gamma)$ is the long run demand curve for the new commodity with $p = p(t)$ as price and $\gamma$ is a shift parameter indicating demand shocks. If capacity growth is in equilibrium with demand growth and price is proportional to marginal cost, i.e., p = kc(x), one obtains a balanced diffusion path as a logistic model:

$$\dot{y}(t)/y(t) = (\alpha - \beta y(t)) \quad (11)$$

where $\alpha = b(d_0 - c_0 d_1 k)$, $\beta = 1 - c_1 d_1 k$

$c(x) = c_0 - c_1 x$, $D(p,\gamma) = d_0 - d_1 p(\gamma)$

This type of model has been explored in evolutionary economies by Metcalfe (1988), Bruckner (1996) and others, who emphasized the stochastic nature of the balanced diffusion path, where the parameters $\alpha$, $\beta$ admit of random variations due to uncertain entry and exit. Note that this type of model (11) assumes a flexible competitive price process $p(t,\gamma)$ which varies so as to equilibrate the growth of capacity and long run demand. Here there exist three sources of output growth. First is the diffusion parameter which is affected by learning by doing and the scale economies in the learning curve. The higher the diffusion rate of new technology, the greater the output growth. Secondly, if demand rises over time due, e.g., to lowering of prices and the substitution of new for the old and the innovator has a forward looking view of market growth, it stimulates capacity growth. Finally, the marginal cost tends to decline due to knowledge spillover across different firms and industries. For example Norsworthy and Jang (1992) who estimated the effect of technological change on productivity in three technology-intensive industries in US, e.g., microelectronics, computers and manufacturing, found for the period 1960-80 that the high rate of technical change due to learning by doing led to steady rates of price decline and high rates of obsolescence of capital in the computer and microelectronics industries.

The nonlinear growth dynamics in (11) above exhibits however two different forces at work. One is the positive growth-enhancing effect captured by the parameters $\alpha$ and b. This is the forward looking view of the innovating firms expecting demand growth. Then there is the growth retarding effect due to the increase in the parameter $\beta$ resulting from competitive pressures and demand fluctuations. To capture these two effects in a linear model one may consider the dynamic model of adjustment mentioned before in (7) and (8). In terms of the output variable y(t) this adjustment may be written as

$$y(t) = b_0 + b_1\, y(t-1) + b_2\, y(t+1) \tag{12}$$

where it is assumed that the rational expectations hypothesis holds employing the $E_t\, y(t+1) = y(t+1)$, i.e., expected future output equals the observed level y(t+1). Clearly one could estimate this model in a statistically consistent way by a two-stage method, i.e., estimating $E_t\, y(t+1)$ by the instrumental variable method in the first stage and then estimating the linear equation (12) with y(t-1) and $\hat{y}(t+1)$ as the independent variables, see e.g., Gregory et al. (1993). If it turns out that $\hat{b}_2 > \hat{b}_1 > 0$, or $\hat{b}_2 > 0$ and $\hat{b}_1 \leq 0$, then one could conclude that the future expectations represented by $E_t\, y(t+1)$ play a more dominant role than the past history represented by y(t-1). The future expectations arise due to optimistic demand forecasts and productivity growth by the innovating firms, while the past history reflects the cost structure of the old technology. Clearly when future expectations dominate the growth process, it would exhibit a process of creative destruction in the Schumpeterian sense. This would however be different from the creative destruction model put forward by Aghion and Howitt (1998), which postulates that a successful innovator drives out the previous incumbent by undercutting his process and creating a local monopoly, e.g., through patents until driven out by the next innovator. In our approach here the innovation follows a stochastic process, where the evolutionary growth in the leading edge technology occurs through the market selection process.

We now present a market selection process model in terms of Markovian transition probabilities for the market share variable $z_i(t) = y_i(t)/Y(t)$, where for simplicity we assume that there are three levels of technology i=1,2,3, i.e., low, medium and high. Following Bruckner et al. (1996) we model the transition of firms as a process of stochastic substitution, i.e., one plant or firm substitutes the new technology for the old one, e.g., $y_i \rightarrow y_{i+1}$ and over time $y_i(t) \rightarrow y_i(t+1)$. By the Markovian property only one step is admissible as a transition. The complete stochastic substitution model may then be specified as follows:

$$z_i(t) = \sum_{j=1}^{3} p_{ji} z_j(t-1) + \sum_{j=1}^{3} q_{qi} E_t [z_j(t+1)] \tag{13}$$

where $\sum_{i=1}^{3} p_{ji} = \sum_{i=1}^{3} q_{qi} = 1$ for all j=1,2,3

$p_{ji} \geq 0$; $q_{ji} \geq 0$, all i,j=1,2,3

Under the rational expectations hypothesis it holds that $E_t[z_j(t+1)] = z_j(t+1)$ and if residual errors $u_i(t)$ are admitted in (13) to account for the difference between the actual and the estimated occurrence of $z_i(t)$, then the sample observations may be assumed to be generated by the following stochastic relation in vector matrix form:

$$z(t) = P'z(t-1) + Q'z(t+1) + u(t)$$

$$P, Q \geq 0, \ \sum_i p_{ji} = 1 = \sum_i q_{ji} \tag{14}$$

where $Eu(t) = 0$ and $E(u(t)\, u'(t)) = D$, D being a positive definite diaponal matrix with constant positive elements in the diagonal.

This type of linear Markov probability model with transition probabilities P and Q has been frequently applied in marketing for brand loyalty studies and in labor economies for labor mobility studies, see e.g., Lee, Judge and Zellner (1977). There is one difference however due to the presence of the expected future variable $z_j(t+1)$, which allows future (demand) expectations to influence the current market share of innovating firms. This variable captures the intensity of the creative destruction process more directly. For example, assume that we have two estimators of P and Q as

$$P^{(1)} = \begin{bmatrix} p_{11} & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{bmatrix}, \qquad Q^{(1)} = \begin{bmatrix} q_{11} & q_{12} & 0 \\ 0 & q_{22} & q_{23} \\ 0 & 0 & q_{33} \end{bmatrix}$$

$$P^{(2)} = \begin{bmatrix} p_{11} & 0 & 0 \\ 0 & p_{22} & 0 \\ 0 & 0 & p_{33} \end{bmatrix}, \qquad Q^{(2)} = \begin{bmatrix} q_{11} & 0 & 0 \\ 0 & q_{22} & 0 \\ 0 & 0 & q_{33} \end{bmatrix}$$

such that $q_{12} > p_{12}$, $q_{23} > p_{23}$ and $q_{ii} > p_{ii}$ then the future expected demand growth plays a more dominant role than the past history. The diagonal dominance in each row would indicate a low degree of technological progress.

If we also consider the possibility of transition to a lower level of technology due to obsolescence and lack of updating, then we may have situations where

$$p_{21} > p_{22},\ q_{21} > q_{22} \text{ and } p_{32} > p_{33},\ q_{32} > q_{33}$$

or,

$$p_{21} > p_{12},\ q_{21} > q_{12} \text{ and } p_{32} > p_{23},\ q_{32} > q_{23}$$

implying a state of technological retroregression. Empirical estimates of transition probabilities would thus provide the direction of shift of the technological frontier.

This transition probability matrix formulation is closely connected with the innovative-interaction matrix (IIM) empirically estimated by DeBresson (1996) and his associates. Like

Leontief input output table this IIM measures the interaction between sectors which are suppliers of innovative activity and the other sectors which are users. Empirical applications have been made in recent times of this matrix for countries such as UK, France, Italy, Greece, Canada and China. Two types of hypotheses have been put forward in this context. One is the Schumpeterian hypothesis that the innovations tend to be concentrated in certain sectors due to substantial economies of scale rather than evenly distributed over a large number of sectors. DeBresson finds substantial evidence for this hypothesis of *innovative clusters* in UK, Greece, Italy and other countries. A second hypothesis postulates a close positive correlation between the innovative activity and the two sectoral linkages, forward (through users' demand) and backward (through input needs). This is also borne out by DeBresson's empirical studies.

## IV. TECHNICAL PROGRESS AND GROWTH

Recent advances in the economics of innovation and new technology have shown that the initial development and final adoption of this technology is a lengthy complex process of evolutionary adaptation. Recently Antonelli (1995) has captured this adaptivity aspect in terms of a modified neoclassical production function

$$Yt = A(t)\, K^{a}(t)\, L^{b}(t)\; I_K^{c}(t) \qquad (13)$$

$$A(t) = f(I_K(t))$$

where Y(t) is output, K(t) and L(t) are the usual capital and labor inputs and $I_K(t)$ is the stock of information capital. Due to the dependence of A(t) on the stock of information capital, significant amounts of externalities or spillover effects may be generated. Diffusion of technology embodied in information capital is assumed to follow a logistic process

$$\dot{I}_K(t)/I_K(t) = b(h - I_K(t)) \qquad (14)$$

where b is the positive rate of diffusion and h is the ceiling level of information capital. It is clear in this formulation that the general efficiency of each Cobb-Douglas production function (13) shifts towards the right, as the overall level of information capital increases. To empirically implement this specification Antonelli (1995) estimated the regression function of the average rate of growth of labor productivity on five sets of variables such as GDP per capita, average investment to GDP ratio, ratio of total US patents, diffusion of information and communication technologies (DICT), and a catching-up variable for 29 representative countries over the period 1980-88. His estimates found the DICT variable to be highly significant in a statistical sense (t-value 2.116) and his overall results confirm the finding that the diffusion rates of key technologies in the communications and related fields have generated significant externalities through knowledge spillover effects all throughout the economic system.

Two points are to be noted however in this formulation. First of all, one needs separate data on information capital $I_K(t)$. The DICT variable and the number of patents are only proxy variables. Secondly, the cumulative experience embodied in the learning by doing models is not directly introduced in this framework, although numerous recent studies have established the learning curve effects of new innovation technology in such modern industries as microelectronics, communications engineering and semiconductors, see, e.g., Norsworthy and Jang (1992). Hence we reformulate the system (13) as

$$Y(t) = A(T)\, K^a(T)\, L^b(t) \qquad (15)$$

$$A(t) = Z^{\theta}, \quad 0 < \theta < 1$$

where $Z(t) = \int_0^t Y(\tau)\, d\tau$ is cumulative output representing the embodied form of all knowledge capital and cumulative experience. Clearly technological progress here is endogenous and the externality effect of output growth is captured by the parameter $\theta$. On assuming a fixed saving ratio s and $\dot{L}/L = n$ one could derive from (15) an equilibrium growth equation in terms of the variable $u = \dot{Z}(t)/Z(t)$ which is consistent with the saving-investment equilibrium:

$$\dot{u}(t)/u(t) = r(m - u(t)) \qquad (16)$$

where $r = 1-\theta-a$, $m = (1-\theta-a)^{-1}$ (nb)

Alternatively this can be written as

$$\dot{u}(t)/u(t) = (1-\theta)\,[B - u(t)] \qquad (17)$$

where $B = (1 - \theta)^{-1}$ $(nb + as\beta)$, $\beta = Y(t)/K(t)$

Clearly both equations (16) and (17) display a logistic evolution path, which incorporates both the eternality parameter θ and the capital coefficient a in the production function. Two important types of stochasticity may arise in the models (16) and (17). One is through the random variations in parameter m due to shocks affecting the learning parameter θ and the capital coefficient a. We have discussed this case before. The second arises when we rewrite the evolutionary equation (16) in the framework of a stochastic differential equation as follows:

$$du = [(gu - hu^2)\,dt] + d\varepsilon \qquad (18)$$

where the first term under square bracket on the right hand side represents the systematic part and the second term dε is an error term with a mean zero and variance $(gu - hu^2)$ dt. Note that $g = rm$ and $h = r$ here represent the learning and experience effects as before.

For the second case denote by μ the asymptotic mean of the u(t) process and let $X(t) = u(t) - \mu$. Then

$$X(t + \Delta t) - X(t) = [gu(t) - hu^2(t)]\,dt + d\varepsilon(t)$$

On taking expectations of both sides and letting $\Delta t \to 0$ one obtains

$$(g/h)\,\mu - \mu^2 = \sigma_u^2$$

This shows that $\partial\mu/\partial\sigma_u^2 < 0$ if $2\mu > g/h$. Thus as the mean income level increases above the level set by g/2h, higher variance of u leads to a lower mean. But otherwise, the correlation between $\mu$ and $\sigma_u^2$ is expected to be strongly positive. The empirical studies by Ramey and Ramey (1991) and Binder and Pesaran (1996) have found strong support for the phase when variance has a negative impact on the mean output levels. This derivation shows however that there exists another phase when explosive or chaotic instability may occur. Sengupta (1999b) has discussed elsewhere some empirical tests of this Schumpeterian dynamics over the two high growth economies of Korea and Japan.

## V. CONCLUDING REMARKS

Stochastic sources of growth in Schumpeterian dynamics are discussed here in relation to the new theory of endogenous growth. Three key elements of dynamics are developed here and discussed in relation to the growth of technology-intensive sectors of a modern economy. These elements comprise the process of entry and exit of new products and new innovations, the process of substitution of the old technology by the new and the impact of learning by doing in the technology diffusion process.

Two major hypotheses in the Schumpeterian framework are developed here. One involves the impact of fluctuations on the innovation induced output process and the other the dominance of future expectations over the past history as sources of rapid growth. These hypotheses are amenable to easy empirical testing. Hence it provides an analytical framework for a deeper analysis of Schumpeterian dynamics which can provide new lights on the new growth theory.

# REFERENCES


Aghion, P. and Howitt, P. (1998). *Endogenous Growth Theory*, Cambridge, Massachusetts, MIT Press.

Antonelli, C. (1995). 'The diffusion of new information technologies and productivity growth', *Journal of Evolutionary Economics*, Vol. 5, pp. 1-17.

Binder, M. and Pesaran, M.H. (1996), 'Stochastic Growth', Working Paper No. 96-18, University of Maryland Department of Economics.

Bruckner, E., Ebeling, W., Montano, J. and Scharnhorst, A. (1996). 'Nonlinear stochastic effects of substitution – an evolutionary approach', *Journal of Evolutionary Economics*, Vol. 6, pp. 1-30.

Callen, T., Hall, R. and Henry, S. (1990). 'Manufacturing stocks, expectations risk and cointegration', *Economic Journal*, Vol. 100, pp. 756-772.

DeBresson, C. (1996). *Economic Interdependence and Innovative Activity: An Input Output Analysis*, Cheltenham, UK, Edward Elgar.

Gregory, A.W., Pagan, A.R. and Smith, G.W. (1993). 'Estimating linear quadratic models with integrated processes', in PCB Phillips (ed.), *Models, Methods and Applications in Econometrics*, Blackwell Publishers, Oxford.

Grossman, G.M. and Helpman, E. (1994). 'Endogenous innovation in the theory of growth', *Journal of Economic Perspectives*, Vol. 8, pp. 23-44.

Helpman, E. (1997). 'R&D and productivity', Discussion Paper Series No. 1798, Harvard Institute of Economic Research.

Kamien, M.I. and Schwartz, N.L. (1982). *Market Structure and Innovation*, Cambridge, Cambridge University Press.

Karlin, S. and Taylor, H.M. (1981). *A Second Course in Stochastic Processes*, New York, Academic Press.

Kennan, J. (1979). 'The estimation of partial adjustment models with rational expectations', *Econometrica*, Vol. 47, pp. 1441-1457.

Kortum, S. and Eaton, J. (1995). 'Engines of growth: domestic and foreign sources of innovation', Finance and Economics Discussion Series No. 95-35, Federal Reserve Board, Washington, D.C.

Lansbury, M. and Mayes, D. (1996). 'Entry exit, ownership and the growth of productivity', in D. Mayes (ed.), *Sources of Productivity Growth*, Cambridge University Press, Cambridge.

Lee, T.C., Judge, G.G. and Zellner, A. (1977). *Estimating the Parameters of the Markov Probability Model from Aggregate Time Series Data*, Amsterdam, North Holland.

Metcalfe, J.S. (1988), 'The diffusion of innovation: an interpretive survey', in G. Dosi et al. (eds.), *Technical Change and Economic Theory*, Pinter Publishers, London.

Norsworthy, J.R. and Jang, S.L. (1992), *Empirical Measurement and Analysis of Productivity and Technological Change*, Amsterdam: North Holland.

Ramey, R. and Ramey, V. (1991), 'Technology commitment and the cost of economic fluctuations', NBER Working Paper No. 3755.

Sengupta, J.K. (1996). 'Economic fluctuations in a model of input and output growth in Japan (1965-90)', *Economic Modelling*, Vol. 15, pp. 135-149.

Sengupta, J.K. (1999a). *New Growth Theory: An Applied Perspective*, Cheltenham, UK, Edward Elgar.

Sengupta, J.K. (1999b). 'A stochastic model of Schumpeterian innovations', paper sent for publication.

Taylor, J.B. (1993). 'Discretion versus policy rules in practice', Carnegie-Rochester Conference series on Public Policy, Vol. 39, pp. 195-214.

Thompson, P. (1996). 'Technological opportunity and the growth of knowledge: A Schumpeterian approach to measurement', *Journal of Evolutionary Economics*, Vol. 6, pp. 77-97.