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Circular polarization of the CMB: Foregrounds and detection prospects

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The cosmic microwave background (CMB) is one of the finest probes of cosmology. Its all-sky temperature and linear polarization fluctuations have been measured precisely at a level of \(\delta T/T_{CMB} \sim 10^{-6}\). In contrast, circular polarization (CP) of the CMB has not been precisely explored. The current upper limit on the CP of the CMB is at a level of \(\delta V/T_{CMB} \sim 10^{-4}\) and is limited on large scales. Some of the cosmologically important sources which can induce a CP in the CMB include early Universe symmetry breaking, a primordial magnetic field, galaxy clusters, and Pop III stars (also known as the first stars). Among these sources, Pop III stars are expected to induce the strongest signal with levels strongly dependent on the frequency of observation and on the number, \(N_p\), of the Pop III stars per halo. Optimistically, a CP signal in the CMB resulting from the Pop III stars could be at a level of \(\delta V/T_{CMB} \sim 2 \times 10^{-7}\) in scales of 1° at 10 GHz, which is much smaller than the currently existing upper limits on the CP measurements. Primary foregrounds in the cosmological CP detection will come from the galactic synchrotron emission, which is naturally (intrinsically) circularly polarized. We use data-driven models of the galactic magnetic field, thermal electron density, and relativistic electron density to simulate all-sky maps of the galactic CP. This work also points out that the galactic CP levels are important below 50 GHz and is an important factor for telescopes aiming to detect primordial B modes using CP as a systematic rejection channel. In this paper, we focus on a SNR evaluation for the detectability of the Pop III induced CP signal in the CMB. We find that a SNR higher than unity is achievable, for example, with a 10 m telescope and an observation time of 20 months at 10 GHz, if \(N_p \geq 100\). We also find that, if frequency of observation and resolution of the beam is appropriately chosen, a SNR higher than unity is possible with \(N_p \geq 10\) and resolution per pixel \(\sim 1 \mu\)K at an observation time of 60 months.

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I. INTRODUCTION

The cosmic microwave background (CMB) has been the finest probe of cosmology. A complete characterization of the CMB is made by quantifying the four Stokes parameters \([1]\) associated with its unpolarized intensity, linear polarization (LP) and circular polarization (CP). Temperature and polarization anisotropies of the CMB have provided invaluable insights into the Universe that we live in. The venture into the CMB studies began with its discovery in 1965 [2,3], which showed the CMB radiation to be a black body with a mean temperature of \(T \sim 2.7255\) K. Subsequently, the first detection of angular variation of the CMB mean temperature was measured with the COBE satellite [4], and today the temperature fluctuations have been measured with an unprecedented precision [5–9]. The CMB is also linearly polarized due to Thomson scattering at the surface of last scattering. This polarization level is at \(\sim 5\)% of the temperature anisotropies. Polarization fluctuations have been measured with a high precision, and many future experiments are aimed at increasing the precision of polarization measurements [10–14]. The Planck experiment reached a sensitivity level, \(\delta T/T_{CMB} \sim 10^{-6}\), for temperature and polarization, a much higher level of sensitivity compared to the WMAP satellite. Together, the CMB temperature and the LP data overwhelmingly support the cosmological standard model or the \(\Lambda\) cold dark matter [15] model. It is only natural to wonder about what kinds of information the CP of the CMB stores. The CP of the CMB has not been explored extensively and the current level of CMB CP measurement stands at a level \(\Delta V/T_{CMB} \sim 10^{-4}\) in the scales of 8° and 24°[16]. This upper limit is much higher than the level of the CP signal expected from any cosmologically relevant sources. Previously, Ref. [17] made one of the first efforts to measure the CP of the CMB. Recently, CLASS [18] and PIPER [19] experiments proposed measuring the LP and CP of the CMB; however, the experiment is designed to focus on primarily the measurements of the LP of the CMB. These experiments aim to detect primordial B modes using variable-delay polarization (VPM) instruments where the observing strategy relies on the expectation of CP of the sky to be null to constrain systematic uncertainties. In this paper, we provide galactic CP maps for frequencies relevant for both CLASS at 40 GHz and PIPER at 220 GHz, showing that the galactic CP effects are important for frequencies below 50 GHz.

There are various cosmologically important sources which may induce a CP in the CMB via different
mechanisms. Some of these CP production channels are intrinsic to the emission from a certain type of source, for example, synchrotron emitting radio sources [20,21]. Some are due to the effects of external magnetic fields [22] or other birefringent effects, and, finally, some mechanisms propose CP generation in the CMB by models that stand on departures from the standard model of particle physics [23–28].

Among the sources which can induce a CP in the CMB due to the presence of a magnetic field are the so-called Pop III stars, also known as the first stars. See [29,30] for a review of the Pop III stars. Reference [31] describes how a CP in the CMB can be induced by the remnants of Pop III stars that went supernovae (SNe). These stars, residing in dark matter minihalos, provide a window into the early structure formation which ended the cosmological dark ages and began the reionization along with metal enrichment of the intergalactic medium. Pop III stars are expected to exist based on the numerical simulations of primordial star formation and the fossil abundance of SNe. However, there are no definite constraints on the properties of these stars [32,33]. These stars are generally not expected to be directly detected by the most advanced future space telescopes like the WFIRST and the JWST [34], except under certain conditions when these Pop III stars explode into pair instability SNe [35] or hypernova [36], releasing energies of around $10^{53}$ erg. These stars and their properties are speculated on in numerical simulations [37,38]; however, they are far from being verified by observations. CP of the CMB provides an indirect and very economical way of exploring these Pop III stars.

In this paper we will primarily focus on a SNR determination of the CP signal in the CMB resulting from the Pop III stars. However, there are other cosmologically important sources which induce CP in the CMB. These sources include the primordial magnetic field (PMF) [23,24], different modifications, and symmetry breaking mechanisms [25–27] beyond the standard model particle physics and the galaxy clusters [39]. Most of these sources induce a lower signal level in CP of the CMB when compared to the level of CP induced by the Pop III stars. However, these sources could certainly be explored via the CP in the CMB once the instrumental sensitivity improves.

In addition to the cosmologically important sources of CP, the Milky Way (MW) Galaxy produces synchrotron radiation which is intrinsically circularly polarized. Circularly polarized synchrotron emission from the MW Galaxy acts as a foreground towards the detection of the cosmological CP in the CMB. Currently, there is not enough observational data to accurately shed light on the level of CP from the galactic synchrotron emission (GSE). In this paper, we generate numerical simulations of the galactic CP due to synchrotron emission using data-driven models of the galactic magnetic field (GMF) and cosmic ray electron energy distribution.

CP in the CMB could potentially detect the existence of the Pop III stars, symmetry breaking in the early Universe, or the existence of a primordial magnetic field. Implementing direct detection of these sources will require a revolution on the instrumentation front, involving a long time scale and a very high cost. Exploring some of these highly interesting sources indirectly via the CP in the CMB is possible within the current reach of instrumentation, achievable at a moderate time line and cost.

In this paper, we will discuss the sources, foregrounds, and detection prospects of the cosmological CP of the CMB. Section II discusses the theoretical framework needed for the description of CP, while Sec. III presents an overview of various sources and mechanisms which induce CP in the CMB. Sections IV–VI discuss the galactic foregrounds in CP and simulations. Section VIII presents the results on detection prospects. Finally, we discuss the future directions and implications of this work in Sec. IX.

II. POLARIZATION TRANSFER EQUATION

A complete description of polarization of an electromagnetic (EM) wave is described by four Stokes parameters, intensity $I$, LPs $Q$ and $U$, and $CP$ $V$. Together, $(I, Q, U, V)$ represent the Stokes vector associated with the EM wave. The evolution of different Stokes vector components of an EM wave propagating through a plasma is governed by the following polarization transfer equation [1]:

$$
\begin{pmatrix}
\frac{dI}{dz} \\
\frac{dQ}{dz} \\
\frac{dU}{dz} \\
\frac{dV}{dz}
\end{pmatrix} =
\begin{pmatrix}
\eta_I \\
\eta_Q \\
\eta_U \\
\eta_V
\end{pmatrix} +
\begin{pmatrix}
-\kappa_I & -\kappa_Q & -\kappa_U & -\kappa_V \\
-\kappa_Q & -\kappa_I & -\kappa_F & -h_Q \\
-\kappa_U & \kappa_F & -\kappa_I & -\kappa_C \\
h_Q & \kappa_C & -\kappa_I & -\kappa_V
\end{pmatrix}
\begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix},
$$

(1)

where the spatial derivatives on the left side of the equation indicate the change in Stokes vector along the line of sight, taken to be inclined along the $z$ axis. The coefficients $\eta_{I,Q,U,V}$ indicate emissivity and $\kappa_{I,Q,U,V}$ indicate absorption coefficients corresponding to the Stokes vectors $I$, $Q$, $U$, and $V$. Under an isotropic distribution of unperturbed particles in a plasma, the conversion coefficient, $h_Q$ between $Q$ and $V$, vanishes due to dielectric symmetries [1,40]. Equation (1) uses a coordinate frame where the sky-projected magnetic field component is aligned along the $y$ axis. Under this geometry, the Stokes component $+U$ is
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FIG. 1. Schematic view of the generation of CP via the FC. The top frame shows the overall geometry of the problem. The bottom frame shows exactly how the FC mechanism generates CP when a linearly polarized CMB passes through the magnetized plasma of the Pop III remnants. The phase difference, $\Delta \phi$, between the mutually orthogonal E-field components is zero and represents only a linearly polarized CMB prior to its passage through the Pop III remnant. The magnetic field in the remnant creates an asymmetry due to the difference in Lorentz forces between the charged particles moving along $\hat{y}$ and $\hat{x}$. This now results in a nonzero phase difference between the mutually orthogonal E-field components of the outgoing wave and hence creates a circularly polarized wave.

A detailed description of the generalized transfer equation in the context of synchrotron emission can be found in [20,40]. In this paper we adopt the symbols and conventions from [40] to describe the polarization relevant quantities.

III. OVERVIEW OF DIFFERENT CP SOURCES

A. Pop III stars as CP sources in the CMB

Pop III stars mark the transition of a simple homogeneous Universe of H-He gas into a complex and structured Universe after the cosmic dark ages. Pop III stars are therefore also known as the first stars. A simple interpretation of optical depth observed by the WMAP and the Planck Collaboration suggests star formation activity at $z \geq 11$ [44–46]. Supernovae explosions of the Pop III stars were responsible for the metal enrichment of the intergalactic medium [47]. These Pop III stars are predicted to form in dark matter minihalos of mass $\sim 10^6-7M_\odot$ around $z \sim 20–30$ [32]. A low-mass halo is needed to virial temperature below the threshold of $T_{\text{vir}} = 10^4$ K to allow efficient atomic hydrogen cooling, which is necessary for collapse. The Pop III stars are yet unobserved and there is a significant lack of certainty of their properties, most of which are predicted from numerical simulations and, therefore, are highly model dependent.

Significance of understanding of the Pop III stars is enormous. Many of the implications drawn from the Pop III stars depend on the mass of these primordial stars. If the Pop III stars are massive ($\geq 100M_\odot$), they can be connected to several effects which can be tested in the distant future, for example, Sunyaev-Zeldovich effects of...
the CMB [48,49] and gravity waves from black holes formed from Pop III remnants [50,51]. One of the most certain and significant effects of the Pop III stars is the cosmological heavy element production and the cosmic reionization [52–55].

In [31], Pop III stars are established as a source of appreciable CP in the CMB. Intrinsic CP of such sources is small, however, the CMB acts as a backlight in this scenario. As the CMB photons pass through the relativistic plasma of the Pop III remnants, a fraction of the CMB linear polarization is converted into CP via the FC mechanism. Please refer to Sec. II for a schematic explanation of the FC mechanism under which \( \kappa_C \) in Eq. (1) describes the transfer of the Stokes \( U \) into the Stokes \( V \) component in the CMB.

Using a simple analytical model of a SN remnant of a Pop III star, [31] evaluates \( \kappa_C \) described in Eq. (1) as

\[
\kappa_C \sim 20 \text{ pc}^{-1} \left( \frac{t_{\text{age}}}{10^6 \text{ yr}} \right)^{1/2} \left( \frac{E_{\text{SN}}}{10^{53} \text{ ergs}} \right)^{1/4} \left( \frac{1 + z}{10} \right)^{1/2} \left( \frac{f_{\text{mag}}}{0.1} \right) \left( \frac{f_{\text{rel}}}{0.1} \right) \left( \frac{\nu}{1 \text{ GHz}} \right)^{-3},
\]

(2)

where \( t_{\text{age}} \) is the age of the SN remnant, \( E_{\text{SN}} \) is the energy of the explosion, and \( f_{\text{mag}} \) and \( f_{\text{rel}} \) are the fractions of the explosion energy, respectively, into the relativistic electron energy and the magnetic fields in a SN remnant. \( \nu \) is the CMB observation frequency. Following the so-called halo model, the angular power spectrum of the CP due to the Pop III stars is evaluated. The angular power spectrum is effectively the square of the rms fluctuation in \( V \), or \( \ell (\ell + 1) C_{\ell}^{\text{CP}} / (2\pi) \sim (\delta V)^2 \). In [31], Pop III stars are assumed to only exist in halos with a virial temperature \( T_{\text{vir}} > 10^4 \text{ K} \), where atomic hydrogen cooling is effective for the collapse. Since the signal of the CP due to the FC mechanism falls off with frequency, we set our normalization frequency to be 10 GHz in future equations in this paper. A frequency much lower than 10 GHz calls for a full solution of the transfer equation, which will be addressed in future work.

Around \( \ell \sim 100 \), a simple formula corresponding to the brightness temperature associated with the fluctuation \( \delta V \) can be expressed as follows:

\[
\delta V_{\text{PopIII}}(\nu)|_{t_{\text{age}}=10^6 \text{ yr}, \ell \sim 100}
\sim 7 \times 10^{-2} \left( \frac{\nu}{10 \text{ GHz}} \right)^{-3} \left( \frac{N_p}{100} \right) \left( \frac{E_{\text{SN}}^{16+2\rho_{\text{PopIII}}}/20}{10^{53} \text{ ergs}} \right) \mu \text{K},
\]

\[
\delta V_{\text{PopIII}}(\nu)|_{t_{\text{age}}=10^4 \text{ yr}, \ell \sim 1000}
\sim 8 \times 10^{-1} \left( \frac{\nu}{10 \text{ GHz}} \right)^{-3} \left( \frac{N_p}{100} \right) \left( \frac{E_{\text{SN}}^{16+2\rho_{\text{PopIII}}}/20}{10^{53} \text{ ergs}} \right) \mu \text{K},
\]

(3)

where \( N_p \) is the number of Pop III stars per halo, \( \rho_{\text{PopIII}} \) is the spectral index of the electron energy distribution around the Pop III remnant, and \( \rho_{\text{PopIII}} \sim 2 \) [56].

In Fig. 2 different Pop III associated CP signals are shown. Note that \( C_{\ell}^{\text{CP}} \) falls off very sharply with the
the local magnetic field, \( B \), is a remnant of the Pop III explosion, with increasing age, \( t \sim 10^6 \) yr, of Pop III stars in a halo is uncertain and could be up to \( \sim 10^6 \) yr. The number of Pop III stars in the CMB has been naturally generated in the CMB in the presence of Thomson scattering, a primordial magnetic field, spatial curvature, and adiabatic fluctuations. This CP generation is intrinsic and does not require any preexisting linear polarization, unlike FC. In Fig. 2, \( C_{\ell}^{V,V,MW} \), due to the presence of a primordial magnetic field of an equivalent field strength, \( B_{\text{PMB}} = 1 \) nG, is shown, following [23]:

\[
\delta V_{\text{galaxy cluster}} \sim 3 \times 10^{-3} \left( \frac{\nu}{10 \, \text{GHz}} \right)^{-3} \mu \text{K}. \tag{5}
\]

Note that the CP signal due to galaxy clusters is much smaller than that due to the Pop III stars.

C. Primordial magnetic field as a source of CP in the CMB

PMF is postulated as a source of the magnetic field in the Universe \([58, 59]\). Current limits on the PMF is \( \sim \) a few nanogauss today \([60]\). Reference \([23]\) showed that CP is naturally generated in the CMB in the presence of Thomson scattering, a primordial magnetic field, spatial curvature, and adiabatic fluctuations. This \( CP \) generation is intrinsic and does not require any preexisting linear polarization, unlike FC. In Fig. 2, \( C_{\ell}^{V,V,MW} \), due to the presence of a primordial magnetic field of an equivalent field strength, \( B_{\text{PMB}} = 1 \) nG, is shown, following [23]:

\[
\delta V_{\text{prim}} \sim 6 \times 10^{-7} \left( \frac{\nu}{10 \, \text{GHz}} \right)^{-3} \mu \text{K}. \tag{6}
\]

For all cases in Fig. 2, \( \nu \) is set at 10 GHz.

D. Summary of other \( CP \) sources

Under the realms of the standard model, the CMB does not have a significant intrinsic Stokes \( V \) component. However, an early Universe symmetry breaking, the presence of new physics, or scattering processes could induce an intrinsic Stokes \( V \) component in the CMB. Many of these possibilities are summarized in Table I, as well as in the references. One of the most concerning sources of non-cosmological \( CP \) is synchrotron emission in our Galaxy. GSE is intrinsically circularly polarized and therefore poses as a foreground to cosmologically important \( CP \) sources in the CMB. The GSE is produced by the cosmic ray electrons in the presence of the galactic magnetic field. We will discuss the GSE in much more detail in Secs. IVA and IVB.
IV. MODELING OF FOREGROUNDS TO THE CMB CP

A. Intrinsic CP of the GSE

Total brightness of the galactic radio sky is dominated by the diffused synchrotron emission due to the cosmic ray electrons and positrons gyrating in the GMF [61]. This emission is significant between frequencies of a few tens of megahertz to a few tens of gigahertz. The primary sources of cosmic ray electrons are supernova remnants, pulsars in the Galaxy. The energy range of the cosmic ray electrons are between a few hundred MeV to tens of GeV. The GMF strength ~ a few microgauss. The ordered components of the GMF is coherent on kiloparsec scales; however, the GMF also has a random component which varies in scales of a few hundred parsecs [62].

Synchrotron emission due to the MW Galaxy happens to be the strongest source of foreground to the CMB observations at low frequencies [63,64]. Synchrotron emission is also naturally expected to have a rather high level of LP, and there have not been any significant measurements of its CP. Synchrotron emission coming from relativistic cosmic rays is expected to be elliptically polarized, even with an isotropic distribution of the electron velocity [21]. Emissivity, $\eta_V$, or the intrinsic CP of the GSE under a power-law electron energy distribution is given by

$$\begin{align*}
  dV_{\text{sync}} &= F_1 \cot \theta \left( \frac{\nu}{\nu_{B_1}} \right)^{-1/2} \eta_V^{\text{sync}} dz,
\end{align*}$$  

(7)

where $dz$ is the elemental length along the line of sight. $\eta_V$ is the emissivity associated with the unpolarized intensity of the synchrotron emission. $\nu_{B_1}$ is the gyrofrequency of the magnetic field component perpendicular to the line of sight. $F_1$ is a function of the spectral index of the electron energy distribution, and its detailed form is found in [21]. In this paper, for simplicity, we set $F_1 = 1.37$ [see Eqs. (31)–(35) of [21]], which is obtained using reasonable parameters for the electron energy distribution obtained from the observation of the Crab Nebula [65,66] and low frequency observations from the Planck satellite [67]. Typically, $F_1$ lies between 1 and 2, if $1 < \alpha_{\text{sync}} < 3$, with $\alpha_{\text{sync}}$ being the spectral index of the cosmic ray electron energy distribution.

B. CP generation via FC in the MW Galaxy

GSE is a crucial and significant source of foreground towards cosmologically important sources of CP in the CMB. In this paper, we only focus on the intrinsic emission of the galactic synchrotron as the most significant foreground towards the cosmic CP. Circularly polarized synchrotron emission in the MW Galaxy is described by the term $\eta_V$ in Eq. (1), and its precise form is given in Eq. (7).

Another possible mechanism for galactic CP generation is the FC in the Galaxy resulting from relativistic cosmic ray electrons gyrating in the GMF. In this section, using simple analytic models, we will estimate the magnitude of FC induced galactic CP. We begin with the following cosmic ray electron density distribution [68,69]:

$$N_{\text{cre}} d\gamma = C_{\text{cre}} \exp(-r/h_\gamma) \text{sech}^2(-h/h_\gamma) \gamma^{-p} d\gamma,$$

(8)

where $r$ is the galactic radius and $z$ is the height. The synchrotron spectral index is $p \sim 3$; the Lorentz factor, $\gamma$, lies between 100 and 300 for the Galaxy; and $C_{\text{cre}} = 4 \times 10^{-3} \text{cm}^{-3}$. The radial scale and the disk height are, respectively, set by $h_\gamma = 5$ kpc and $h_z = 1$ kpc. We use $r \sim 0.5$ kpc and $h_z \sim 0.5$ kpc at $\nu = 10$ GHz in Eq. (8) to obtain the relativistic electron density in the MW Galaxy. We then use Appendix D of [40] to obtain the FC coefficient in the Galaxy, $k_{\text{CP}}^{\text{galaxy}} \sim 10^{-15}$ kpc$^{-1}$.

The amount of CP induced via the FC effect in the Galaxy is $\sim k_{\text{CP}} U_\nu$, where $U_\nu$ is the total incoming LP due to the CMB and the GSE at a given frequency $\nu$. $U_{\text{CMB}} \sim 10^{-6}$ K, and it therefore induces a CP due to its passage through the Galaxy as $\delta V_{\text{galactic FC}} \sim k_{\text{CP}} U_\nu \sim 10^{-21}$ kpc$^{-1}$ K, which is much smaller than any other cosmologically important source of CP in the CMB.

GSE also has a significant level of intrinsic LP [21] and therefore is subjected to FC effects in the Galaxy. In Sec. VI B, we show the expected level of LP in the GSE (see Fig. 5) at $\nu = 30$ GHz. We note that the highest level of intrinsic LP of the GSE is $\sim$ few tenths of a kelvin. This level does not change significantly with our frequencies of interest ($\nu > 5$ GHz). Therefore, CP induced in the Galaxy due to FC of the GSE is $\sim k_{\text{CP}} U_\nu \sim 10^{-16}$ kpc$^{-1}$ K. By contrast, intrinsic CP of the GSE at $\nu = 10$ GHz is $\sim 10^{-6}$ kpc$^{-1}$ K, obtained using the equations in Appendix D of [40] with $B \sim 10 \mu G$. This level is also supported by Fig. 6. Therefore, FC induced CP of the GSE is negligible compared to its intrinsic CP.

The CP level in the Galaxy is a function of the frequency, and the relative importance of different channels to generate CP depends on the frequency. The frequency dependence of the ratio of intrinsically generated CP vs FC induced CP in the Galaxy is given by

$$\eta_V \alpha \nu^2 \cot \theta \log \left( \frac{\nu}{\nu_{B_{\min}}^2} \right)^{-1} U_{\nu,\text{GSE}},$$

(9)

where $U_\nu$ is the total incoming LP in the Galaxy, composed of contributions from both the CMB and the GSE. $\theta$ is the angle between the magnetic field and the line of sight, $\nu_{\min}$ is the minimum Lorentz factor for the relativistic electrons in the Galaxy, and $\nu_B$ is the cyclotron frequency given by $\nu_B = 2.8(B/1 \mu G)$. The ratio $\eta_V / k_{\text{CP}} U_\nu$ is $\sim 10^2 / U_\nu(K)$ at $\nu = 10$ GHz, $B = 10 \mu G$, $\nu_{\min} = 100$, and $\theta = \pi/4$. LP in
the CMB is not a function of frequency and is given by \( U_{\nu}^{\text{CMB}} \sim 10^{-6} \) K. The level of LP in the galactic synchrotron emission is significant compared to the unpolarized intensity. \( U_{\nu}^{\text{syn}} < 0.2 \) K at \( \nu = 10 \) GHz and eventually falls off with higher frequency. Therefore, along a given line of sight, \( \eta_{\nu}/\kappa_{\nu} U_{\nu} \) is a monotonically increasing function of frequency. Therefore, in our frequencies of interest (5 GHz < \( \nu < 30 \) GHz) for the CMB CP measurement, the FC induced CP of the CMB or the GSE is not an important effect.

For every emission there is an associated absorption. This applies to both the unpolarized Stokes intensity \( I \) and the circular polarization Stokes intensity \( V \). The intrinsic emission in Stokes \( V \) will be extinct if the emission and absorption were perfectly balanced, or \( (\eta_{\nu} - \kappa_{\nu} I) \sim 0 \). Therefore, it is also important to consider the absorption of the circularly polarized emission in the Galaxy. The absorption is given by \( \kappa_{\nu} I_{\nu} \). Following Appendix D of [40], we obtain

\[
\frac{\eta_{\nu}}{\kappa_{\nu} I_{\nu}} = m_{\text{e}} e^{2} \left( \frac{\nu}{\nu_{B_{0}}} \right)^{1/2} \psi(p)/I_{\nu},
\]

where \( \psi(p) \) is a function of the spectral index, \( p \), of the relativistic electron energy distribution. For the Galaxy, we use \( p \rightarrow 3 \). For the GSE, \( I_{\nu} \sim \nu^{-(p-1)/2} \). The ratio \( \eta_{\nu}/(\kappa_{\nu} I_{\nu}) \) is \( \sim 10^{14} \) at \( \nu = 10 \) GHz, \( B \sim 10 \mu G \), \( I_{\nu} \sim 10^{-2} \) K and is an increasing function of frequency. This implies that absorption of the CP emission in CMB observation frequencies (1 GHz or above) is not significant. Absorption of the synchrotron emission component is, however, significant in so-called self-absorbed synchrotron sources where \( (\eta_{\nu} - \kappa_{\nu} I) \rightarrow 0 \). This scenario is realized at much lower frequencies, \( \nu_{\text{self}} \leq 10 \) MHz [70]. Therefore, synchrotron self-absorption of its circularly polarized emission is not a concern in our case.

Synchrotron self-absorption of the unpolarized intensity is also not important for the Galaxy in the frequencies of interest (\( \nu > 1 \) GHz). In the case of very low frequencies \(< 1\) GHz, some extragalactic sources could become self-absorbed or optically thick. In this low frequency regime, synchrotron flux from the sources decreases with decreasing frequency. On the contrary, at higher frequencies, flux emitted by the synchrotron sources decreases with increasing frequency. This turnover in the flux-frequency relation pollutes the smooth synchrotron frequency dependence, altering the spectral index of the synchrotron brightness temperature. Spectral smoothness is important for successfully removing the foregrounds. Synchrotron flux from the Galaxy is still high at frequencies \( \geq 1 \) GHz. However, because of the smooth dependence of the synchrotron flux on the frequency, foreground removal via a polynomial fit is easier. This is especially relevant where the signal of interest (for example, the CMB CP resulting from galaxy clusters) is lower than the foregrounds. Unless the number of such sources is small enough, it is wiser to confine the search for the cosmic CP at frequencies \( \geq 5 \) GHz. This is also the motivation for us to confine the CMB CP observation frequencies to between 5 and 30 GHz.

Below, we summarize the conclusions from the current section.

(i) CP induced in the CMB resulting from the MW Galaxy (via the FC mechanism) is much smaller than the levels of CP induced in the CMB resulting from cosmologically important sources (see Table I).

(ii) CP induced in the GSE via the FC mechanism is much smaller than the intrinsic emission of circularly polarized synchrotron radiation in the Galaxy.

(iii) CP induced intrinsically [via the \( \eta_{\nu} \) term in Eq. (1)] is higher than the FC induced CP in the Galaxy at all frequencies of interest.

(iv) Self-absorption of the circularly polarized GSE is not important in the frequencies of interest.

(v) Self-absorption of the unpolarized intensity of the GSE may pollute the smoothness of the synchrotron spectra at frequencies \(< 1\) GHz.

V. NONSYNCHROTRON FOREGROUND SOURCES OF CIRCULAR POLARIZATION

The MW Galaxy is bright in free-free emission in our frequencies of interest, 5–30 GHz. The free-free signal is not attributed to a polarized sky. Some spinning dust signal is linear polarized [67]; however, linear polarization in these frequencies resulting from spinning dust is much smaller than LP from the synchrotron. An intrinsic CP resulting from spinning dust is not expected. There is also little chance of FC of the LP (due to dust), as we have seen, in Sec. IV B, that the FC coefficient due to the MW Galaxy is very small. Therefore, we do not expect any appreciable CP due to the free-free and the spinning dust emission in the Galaxy, in the frequencies between 5 and 30 GHz.

Reference [71] discusses recent observations of mesospheric oxygen induced CP at large angular scales. Temperature equivalent CP due to the atmospheric source is sensitive to the height of the atmospheric sources. Typically, this signal is described by a dipolelike large scale structure with \( V \sim (15–100) \times 10^{-3} \) \( \mu K \). We will neglect this atmospheric contribution to the noise estimate. Oxygen related effects are only important in the largest scales for the balloon based experiments. They are, however, more serious for the ground based experiments that are aiming to detect a circularly polarized component in the CMB.

Generally, brightness in free-free emission or dust emission will not affect the CP measurement unless there is leakage, which causes mixing between different types of signal, such as a polarized signal and a nonpolarized signal. In this paper, we neglect leakage of any kind.
VI. CONSTRUCTION OF THE STOKES V MAPS DUE TO THE GSE

A. The HAMMURABI code: Implementation of CP due to the GSE

We use the HAMMURABI code [72] to create maps of Stokes parameters I, Q, U, and V due to the GSE. Calculation of Stokes I, Q, and U parameters are part of the original implementation of HAMMURABI and are clearly described in [72]. These simulations use an input magnetic field, free electron density, and relativistic electron density models to output the Stokes vectors into Healpix formatted maps [73] at a given frequency and spatial resolution. Below, we summarize the precise inputs used for the HAMMURABI code to generate the Stokes V simulated maps.

(i) A 3D GMF model composed of both a large scale field and a turbulent component. The large scale field is coherent over scales ~ kiloparsec and is described in [74]. The turbulent component is described in [75] and is coherent over scales of a few hundred parsecs.

(ii) A 3D model of cosmic ray electron density given in [74].

(iii) A 3D thermal electron density model described in [76].

Please note that a Stokes V calculation is not part of the original HAMMURABI implementation. We use Eq. (7) to implement the construction of a Stokes V field due to the intrinsic synchrotron emission of the MW Galaxy into HAMMURABI. All of the Stokes vector outputs are expressed in temperature units of kelvin and Healpix formatted maps of a user specified resolution. A synchrotron spectral index of $p = 2.8$ was used in all HAMMURABI simulations unless specified otherwise.

B. Comparison between HAMMURABI simulations and observations

The main goal of this section is to justify the use of the HAMMURABI code in galactic CP power spectrum calculation. In order to do so, we use the component separated synchrotron maps of both intensity and polarization provided on the Planck Collaboration Web site.

The data-driven model of GSE in the Planck Web site is based on a few data sets. They are

(i) 408 MHz synchrotron emission map [77],

(ii) WMAP low frequency observations [6] (with resolution of ~1°),

(ii) Planck low frequency observations [67].

Following the methods described in [78] and using the above data sets, a 408 MHz map of the GSE is generated [79]. This map has a resolution of ~1°. Following similar methods and data sets, a 30 GHz map of polarized synchrotron emission [67,80] was generated with a resolution of ~40 arc min.

Next question to ask is, which quantities must one compare between the HAMMURABI simulations and the observed data sets to validate the HAMMURABI code generated Stokes V map? To answer this question, we consult Eq. (7), which describes our implementation of the intrinsic CP of the GSE. Equation (7) indicates that CP depends on the $I_{\text{sync}}$ at a given point in the sky, and ratio of the line-of-sight component and the perpendicular component of the GMF. Generally, synchrotron emission (in polarized and total intensity) are proportional to the $B_{\perp}$ component of the GMF. On the other hand, the galactic rotation measure (RM) is proportional to $B_{\parallel}$ (defined along the line of sight). The magnitude of CP along a line of sight is proportional to $I_{\text{sync}}$ along that line of sight but is also sensitive to $B_{\perp}/B_{\parallel}$, which is more difficult to determine.

We make comparisons between HAMMURABI and the Planck, WMAP, and Haslam joint data set. We do this both in real space and in terms of the angular power spectra. Let us define an all-sky intensity field $I = I_{\text{sync}}(\Omega)$ and use Healpy, a python implementation of the original Healpix, to find

$$\tilde{a}_{LM} = \int d\Omega W(\Omega) I_{\text{sync}}(\Omega) Y_{LM}^*(\Omega),$$

where $W(\Omega)$ is the mask which is 0 if a pixel is masked and 1 if it is not. We then evaluate

$$C_{\ell}^{I,\text{MW}} = \frac{1}{f_{\text{sky}}(2\ell + 1)} \sum_{m=-\ell}^{m=\ell} \tilde{a}_{\ell m} \tilde{a}_{\ell m},$$

where $f_{\text{sky}}$ is the sky fraction. $C_{\ell}^{I,\text{MW}}$ represents the angular power spectrum of the unpolarized intensity of the GSE.

In Fig. 3, we present the 408 MHz Haslam [77,78] data set derived galactic synchrotron intensity in the right column. In the left column, we present the 408 MHz map of the GSE created by the HAMMURABI simulation. The spectral index chosen for this simulation was $p = 2.8$. Other inputs chosen for the simulation are described in Sec. VI A. Both maps are smoothed at a resolution of 1°.

In Fig. 4, we present the angular power spectra of the 408 MHz Haslam data-driven synchrotron power spectrum, and that given by the HAMMURABI simulation. We follow Eq. (12) to derive the power spectra. In each case, an identical mask was used to remove the high-foreground galactic disk from the sky. A 20° symmetric cut around the equator along with the WMAP K-band mask was used. More on the specifics of other masks and their effect on the power spectra is described in Sec. VII A. Figure 4 shows that the shapes of the power spectra are similar and the ratio of power at each angular scale fluctuates around unity. The HAMMURABI map was scaled with the Haslam 408 MHz map at $\ell = 100$. 
In Fig. 5, we present the 30 GHz synchrotron polarized map (left panel) derived from the Haslam, Planck, and WMAP data [6,67,78,80] and the HAMMURABI [72] simulations (right panel). The total polarization from synchrotron emission from the Galaxy is plotted. The maps are presented in Rayleigh-Jeans temperature equivalent units. The maps represent the total polarization given by $P = \sqrt{Q^2 + U^2}$ in the units of $\mu K$. The left panel represents the 30 GHz polarized map of the GSE obtained with an uniform synchrotron spectral index of $p = 2.8$. The right panel represents the data-driven synchrotron polarization map. Each map produces a similar morphology, although there are many mismatches in details. The more detailed and accurate maps could be produced by using more accurate GMF models, which are not available at the moment.

From Figs. 3–5, we draw the following conclusions.

(i) The power spectra of synchrotron intensity between the HAMMURABI simulation and the observed data at 408 MHz match at each scale with their fluctuation within unity.

(ii) The polarization of the GSE between the HAMMURABI simulation and the observed data at 30 GHz match in overall morphology and order of magnitude estimates for the polarization.

(iii) There are many finer details of morphological mismatch between the polarization maps between the HAMMURABI simulation and the observed data. This mismatch arises from inadequate GMF models and models of the cosmic ray electron density, which can only be improved with more data in the future.

(iv) The galactic disk is the highest source of synchrotron intensity and polarization. The disk-removed angular power spectra of the synchrotron intensity between HAMMURABI and the observed data agree reasonably well. Therefore, the SNR derived using GSE angular power spectrum estimates is expected to be reliable [see Eq. (15)].
Currently, there are no reliable observed data sets for all-sky galactic \( CP \). We will describe a HAMMURABI code generated simulation of galactic \( CP \) in the next section.

C. All-sky maps of galactic \( CP \) simulated by HAMMURABI

Below, we will describe maps of both unpolarized and polarized synchrotron emission due to the MW Galaxy, evaluated at 10 and 30 GHz. The resolutions of the maps were managed using the Healpix specified parameter NSIDE, where the corresponding angular resolution of the map is given by \( \delta \theta \sim 3600'/12 \text{ NSIDE}^2 \). Each map was generated with NSIDE = 256.

As we already discussed in Sec. IV B, FC effects due to the MW Galaxy are insignificant compared to the \( CP \) or the Stokes \( V \) induced in the CMB due to the primordial effects or the intrinsic \( V \) of the synchrotron emission of the MW Galaxy itself. Therefore, only the intrinsic generation of \( CP \) due to the galactic synchrotron emission or \( \eta_V \) [in Eq. (1)] term was considered for the galactic Stokes \( V \) map calculation.

In the left column of Fig. 6, we display synchrotron radiation from the MW Galaxy in Stokes \( I, V, \) and \( V_f \). The left panel represents \( \nu = 10 \) GHz and the right panel corresponds to \( \nu = 30 \) GHz. Each map was smoothed at a resolution of 1°.

In Fig. 7, we add two sets of maps at 40 and 220 GHz which are more relevant to the upcoming CMB telescopes, CLASS and PIPER, with capabilities of measuring Stokes \( V \). Please note that, for the frequency relevant for the PIPER telescope, galactic \( CP \) is lower by a factor of \( (40/220)^3 = 6 \times 10^{-3} \) compared to its levels at 40 GHz, which is relevant for the CLASS telescope.

We make the following observations from the maps shown in Figs. 6 and 7.

(i) The magnetic field strengths are highest around the disk of the Galaxy causing the highest synchrotron signal in Stokes \( I \) along the disk.

(ii) Along a given line of sight, synchrotron intensity falls off as a power law with increasing frequency as \( \sim \nu^{-2-\eta_{\text{sync}}/2} \). Note that this frequency dependence is different from the \( \nu^{-3} \) dependence in the case of FC generated \( CP \).

(iii) It follows from Eq. (7) that the emission in Stokes \( V_{\text{sync}} \) is proportional to the synchrotron intensity, \( I_{\text{sync}} \), and also depends on the ratio \( B_\perp/B_\parallel \), where \( B_\perp \) is the sky-projected magnetic field and \( B_\parallel \) is the line-of-sight magnetic field. Following a magnetic field configuration that is symmetric around the galactic disk, a high correlation between \( I \) and \( V \) along the disk is expected. The level of Stokes \( V \) agrees well with analytic calculations using simple equations for the coefficients of the transfer equation in Appendix D of [40].

(iv) Along a given line of sight, \( CP \) of the GSE falls off as a power law with increasing frequency.

(v) The ratio of \( V/V_f \) along a given line of sight decreases with increasing frequency [see Eq. (7)].

VII. ANGULAR POWER SPECTRA OF \( CP \) DUE TO THE GSE

A. Construction of the galactic mask

From Fig. 6, it is clear that the galactic disk is the highest source of foreground emission in Stokes \( V \). Therefore, we create a mask to block these parts of the sky in order to evaluate the SNR in Sec. VIII. The mask used in this paper is a superposition of a WMAP K-band mask [6] and a symmetric 20° cut around the galactic plane. The galactic plane was cut out to avoid the highest source of foreground in Stokes \( V \). The WMAP mask was used to remove additional point sources and a generally high synchrotron source since it is expected from Eq. (7) that \( \eta_V \) increases with \( I_{\text{sync}} \). The effective sky fraction, \( f_{\text{sky}} \), using this particular mask is 0.65.
In this section, we will discuss the angular power spectrum of \( CP \) due to the synchrotron emission of the MW Galaxy. The only source of \( CP \) is the intrinsic emission term \( \eta V \) described in Eq. (1). In order to construct the angular power spectra, we first generate the galactic circular polarization field \( V = V_{\text{sync}}(\Omega) \) using HAMMURABI as a function of the solid angle \( \Omega \). To calculate the power spectra, \( C_{l}^{V,\text{MW}} \), of the \( CP \) due to the GSE, we use the following equation:

\[
C_{l}^{V,\text{MW}} = \frac{1}{2\pi} \int_{0}^{2\pi} \int_{0}^{\pi} \frac{P_{l}^{\text{MW}}(\Omega)}{\sigma_{\text{Gal}}^{2}} \sin \theta \, d\Omega \, d\theta
\]
where $f_{\text{sky}}$ is the sky fraction. Please see Eq. (11) for the definition of $\tilde{a}_{lm}$. The angular power spectra of the Stokes $I$ and $V$ components due to the GSE are shown in Fig. 8. We compute $C^{V,\text{MW}}_\ell$ using different masks, each labeled in the plot. Each mask is a superposition of both the WMAP K-band mask and a symmetric galactic cut. The synchrotron $I$ and $V$ fields were smoothed with a Gaussian beam of FWHM = 1°. We make the following observations in the power spectra of $C_P$ and the unpolarized intensity resulting from the GSE.

(i) A sawtooth feature in the power spectra of both Stokes $I$ and $V$ at larger scale, which is due to symmetry between the northern and southern galactic sky and invariance with a shift in longitude \cite{82}. A perfectly symmetric map around the galactic plane will only have angular power with a nonzero $C_\ell$ if $\ell$ is even. However, if the symmetry is partially broken, or not perfect, then the $C_\ell$ for odd values of $\ell$ will also get populated. This is supported by an increasingly sawtooth nature of the spectra in both $I$ and $V$ as we remove the galactic plane using a mask and tend to a smoother and more symmetric synchrotron sky.

(ii) The sawtooth behavior of $C^{V,\text{MW}}_\ell$ in large scale is the opposite of $C_\ell$ due to the extra factor of $\sin \theta$ in evaluation of $\eta V$ following Eq. (7). This implies, for a perfectly symmetric sky in $I_{\text{sync}}$, that the $C_P$ sky will be perfectly antisymmetric. This results in $C^{V,\text{MW}}_\ell$ being nonzero for only odd values of $\ell$. In the case of a broken or imperfect antisymmetry, $C^{V,\text{MW}}_\ell$ for even values of $\ell$ also gets populated.

(iii) On a smaller scale, the power primarily comes from turbulence and the $C_\ell$’s for both $I$ and $CP$ are relatively smooth. $C_\ell$ falls off as $\sim \ell^{-\frac{11}{3}}$ beyond $\ell \sim 50$ \cite{83}. The scale dependence of $C^{V,\text{MW}}_\ell$ is also similar to the $C_\ell$ of the unpolarized synchrotron because $\eta V$ is proportional to $I_{\text{sync}}$. In the observed spectrum of synchrotron emission at 408 MHz \cite{77} from the MW Galaxy, there is more power in small scales which are mostly contributed by point sources. Since we have removed the point sources, power in a small scale drops \cite{82}.
and future CMB experiments. Let \( C^\ell_{\nu,\text{SOI}} \) be the signal of interest, composed of signals from the Pop III stars, galaxy clusters, or primordial sources. Therefore,

\[
C^\ell_{\nu,\text{SOI}} = C^\ell_{\nu,\text{PopIII}} + C^\ell_{\nu,\text{galaxycluster}} + C^\ell_{\nu,\text{prim}},
\]

where \( C^\ell_{\nu,\text{PopIII}} \) is, by far, the highest signal among the scenarios that are reviewed in this paper. See Fig. 2 for a quick comparison. \( C^\ell_{\nu,\text{SOI}} \) is then dominated by the contribution from \( C^\ell_{\nu,\text{PopIII}} \). Detection prospects of the cosmological signal of interest can then be simply evaluated as

\[
\left( \frac{S}{N} \right)^2 = \sum_{\ell=2}^{\ell_{\text{max}}} \frac{(2\ell + 1)}{2} f_{\text{sky}} \left( \frac{C^{\ell_{\nu,\text{SOI}}}}{C^\ell_{\nu}} \right)^2,
\]

where

\[
\tilde{C}^\ell_{\nu} = C^\ell_{\nu,\text{SOI}} + C^\ell_{\nu,\text{MW}} + C^\ell_{\nu,\text{EG}} + I^\ell_{\nu}.
\]  

In Eq. (15), we use Eq. (7) of [31] to evaluate \( C^\ell_{\nu,\text{PopIII}} \). The galactic foreground contribution, \( C^\ell_{\nu,\text{MW}} \), is primarily due to intrinsically circularly polarized GSE. The calculation for \( C^\ell_{\nu,\text{MW}} \) is described in Sec. VII B and Secs. IV A and IV B. The galactic signal may also include the mesospheric oxygen signal described in Sec. V. However, we have ignored the mesospheric oxygen signal (of CP) due to its limitation to only the largest scales. We have also set angular power, \( C^\ell_{\nu,\text{EG}} \), coming from the extragalactic sources [that are not included in the signal of interest in Eq. (15)] to be zero. The angular power related to the instrumental noise, \( I^\ell_{\nu} \), is given by the following:

\[
I^\ell_{\nu} = A^2 \exp \left( \ell^2 \frac{\Theta^2_{\text{FWHM}}}{8 \ln 2} \right),
\]

where \( A^2 = \Delta^2_\rho \) (in \( \mu \text{K/K} \))\( \Theta_{\text{FWHM}} \) (in radians)\( T_{\text{CMB}} \). An important quantity, the resolution per pixel, \( \Delta_\rho \), is defined using the \( \Delta_\rho \) given in Eq. (17), such that \( \Delta_\rho = \Delta_\rho T_{\text{CMB}} \).

The full width at half maximum of the Gaussian beam is denoted by \( \Theta_{\text{FWHM}} \). Resolution per pixel is related to the detector noise-equivalent temperature, \( s \), and total observation time, \( t_{\text{obs}} \), in the following manner:

\[
(A^2) = \frac{4 \pi f_{\text{sky}}s^2}{t_{\text{obs}}},
\]

where \( s \) is in units of \( \mu \text{K(see)}^{1/2} \) and \( t_{\text{obs}} \) is in seconds. Equation (18) follows from the following. The area covered by each pixel is \( \sim \Theta^2_{\text{FWHM}} \). The time required to get a resolution per pixel of \( \Delta_\rho \) with a detector noise-equivalent temperature of \( s \) is \( \sim (s/\Delta_\rho)^2 \). Therefore, within a given observing time of \( t_{\text{obs}} \), the number of pixels covered will be \( N_{\text{pix}} = t_{\text{obs}} (\Delta_\rho/s)^2 \). Therefore, the fraction of sky area covered by the pixels is \( f_{\text{sky}} = N_{\text{pix}} \Theta^2_{\text{FWHM}}/(4\pi) \).

Goal spatial resolution depends on the type of the telescope used. Generally, \( \Theta_{\text{FWHM}} \) in arc minutes is given by

FIG. 8. Angular power spectrum of the unpolarized synchrotron intensity in terms of Stokes I (the left column) and its intrinsic Stokes \( \nu \text{GHz} \). Pseudo-\( C_\ell \) values, following Eqs. (13) and (12), are plotted using different masks. Each map was smoothed using a Gaussian beam of FWHM = 1°. \( C^\ell_{\nu,\text{MW}} \) and \( C^\ell_{\nu,\text{EG}} \) both have sawtooth behavior in a large scale and smooth nature due to turbulence in smaller scales. \( C^\ell_{\nu} \) for unpolarized synchrotron intensity has peaks for even values of \( \ell \). Sawtooth behavior of the power spectra simply follows from a near-symmetric northern and southern galactic sky with a very weak dependence on longitude.

VIII. DETECTION PROSPECTS

In this section, we attempt to forecast detection prospects of a cosmological signal of Stokes \( I \) in the CMB via current and future CMB experiments. Let \( C^\ell_{\nu,\text{SOI}} \) be the signal of interest, composed of signals from the Pop III stars, galaxy clusters, or primordial sources. Therefore,

\[
C^\ell_{\nu,\text{SOI}} = C^\ell_{\nu,\text{PopIII}} + C^\ell_{\nu,\text{galaxycluster}} + C^\ell_{\nu,\text{prim}},
\]
\[ \Theta_{\text{FWHM}} = \frac{1800}{D(m)\nu(\text{GHz})}, \]  

(19)

where \( D(m) \) is the diameter of the telescope in meters and \( \nu \) is the frequency of the CMB observation in gigahertz. One can therefore use a 10 m telescope at 10 GHz to obtain a resolution of \( \sim 18 \) arc min. It is easier to find dedicated observing time in smaller telescopes than in larger ones.

In Fig. 9, we plot different competing factors, such as the signal, noise, and foregrounds for two different beam resolutions, 18 arc min (left panel) and 1° (right panel). The resolution per pixel in each case is considered to be at three different values, 0.1, 1, and 10, in units of microkelvin.

We plot a comparison between both frequency dependent (FR related) and frequency independent (lensing and primordial gravitational waves) B modes, along with FC generated CP and CP due to the GSE in Fig. 10. In this case, the results were presented at \( \nu = 40 \) GHz due to its immediate relevance to the CLASS telescope. Please note that the angular power in galactic CP will be down by a factor of \( \left( \frac{40}{220} \right)^6 \sim 5 \times 10^{-5} \) at 220 GHz. It is important to note that the galactic CP will be an important factor to consider while probing B modes for lower values of the tensor-to-scalar ratio, \( r \). In probing B modes, galactic CP is a serious effect to consider over large scales, while in smaller scales other effects such as the FR due to the Galaxy and cosmological CP could be more important. However, detectability in smaller scales is limited by the thermal noise of the detector for both cosmological CP and B modes.
In Figs. 11 and 12, we present SNR estimates for the detection of the Pop III stars as the frequency and the beam resolution are varied. We consider various values for the number of Pop III stars per halo: $N_p = 1–1000$. The main observations from the SNR estimates are as follows.

(i) SNR is significantly higher than unity for $N_p \geq 100$.

(ii) SNR increases with a decreasing frequency and beamwidth.

(iii) If $N_p = 1$, a SNR that is higher than unity is generally not expected at any beam resolution within the frequency range of 5–30 GHz.

(iv) If $N_p \geq 10$, a SNR higher than unity is expected with an appropriate choice of frequency and beam resolution.

(v) If $N_p \geq 100$, a SNR significantly higher than unity is expected, with an observing time of 20 months or less. In this case, the choice of frequency and FWHM of the beam is more relaxed. For example, if $N_p = 1000$, a SNR higher than unity can be achieved with FWHM up to 40 arcminutes and a frequency of up to 50 GHz.

Some typical scenarios for observing CP involve the following. If $N_p \geq 100$, a SNR significantly higher than unity is achievable using a 10 m telescope at 10 GHz at 40 months of observing time. If $N_p \geq 10$, a SNR higher than unity is achievable using a 10 m telescope at ~10 GHz at 60 months of observing time.

Note that the signal of interest in our case is composed of the primordial, Pop III star related, and galaxy cluster related CP signals. However, the Pop III related CP signal is much higher (see Fig. 2) and dominates other sources of CP signals of interest. For example, the SNR for solely
observing the \( CP \) signal from the galaxy clusters is much less than unity with the optimal beam resolution and a low frequency. Therefore, a SNR higher than unity will most certainly imply the presence of a \( CP \) signal induced by the Pop III stars.

In Fig. 13, improvements in SNR due to the partial removal of the Galaxy is considered. Partial removal of the Galaxy is achieved using a factor, \( f_{DG} \), whose angular power due to the galactic \( CP \) in Eq. (15) is modified as \( C_{\ell}^{\text{V.MW}} \rightarrow f_{DG} C_{\ell}^{\text{V.MW}} \). Partial removal of the galactic effects extends the detectability prospects to higher frequencies, especially for lower values of \( N_p \). Finally, the detectability remains limited by the thermal noise of the detector.
IX. DISCUSSION

In this paper we have evaluated SNR for the detection of cosmologically important CP signals in the CMB. The frequency range of observation was chosen to be 5–30 GHz. The lower limit of 5 GHz in the frequency range of interest is chosen to avoid irregularities in the synchrotron spectra due to extragalactic self-absorbing sources. Additionally, the treatment of CP in lower frequencies requires a full solution of the polarization transfer equation in many scenarios [84]. An upper limit of ∼30 GHz is chosen due to the sharp falloff of the CP signal with increasing frequency.

Our work follows from [31], which showed that Pop III stars could induce a strong CP signal in the CMB. In addition to the Pop III signal, the CP signal induced in the CMB due to the galaxy clusters and other primordial sources, such as the primordial magnetic field and symmetry breaking mechanisms, were also considered. Of all the cosmologically important sources of CP discussed in this paper, CP due to the Pop III stars dominates, with a significantly higher signal in the frequency range of interest. A detailed description of the mechanisms that produce CP in various sources is provided in Secs. III and II. CP in the CMB is produced by the Pop III stars via the FC mechanism which transforms an incoming linear polarization into a circular polarization in the presence of an external magnetic field.

An important foreground to the CP observation is the galactic CP due to the GSE, which is naturally (intrinsically) circularly polarized. We evaluated the signal level of the CP generated from the GSE using numerical simulations generated by HAMMURABI code where we have implemented the calculation of Stokes V (the component corresponding to CP) following Eq. (7).

The goal frequency of observation is in the 5–30 GHz range, which is chosen for the reasons described earlier in this section. There is not much known about the mass and other properties of the Pop III stars. If the number of Pop III stars per halo is as high as 1000, then observations at higher frequencies (up to 50 GHz) could lead to a SNR that is higher than unity.

The final result of our work is summarized in Figs. 11–13. A SNR significantly higher than unity is achievable if \( N_p \geq 100 \). Generally, a SNR higher than unity is accessible if \( N_p \geq 10 \), with an appropriately chosen frequency of observation and beam resolution. Under the most optimistic scenario (for example, \( N_p = 1000 \)), a SNR higher than unity is accessible at \( \nu = 50 \) GHz, with a beam resolution of up to 40′. Under the least optimistic scenario, where \( N_p = 1 \), we do not expect a SNR higher than unity.

Limitations of our results come from the current status of the GMF models. Our results for the galactic foreground in CP is based on numerical simulations which are partially driven by galactic synchrotron and low frequency CMB data. The GMF models considered in this paper are compared with Haslam data (a 408 MHz map), as well as low frequency CMB data from the Planck and WMAP satellites. However, the resolution of these data sets is, at best, \( 40′ \ldots 1° \), limiting our knowledge of the GMF at smaller scales. The maps of Stokes V presented in this paper depend on the GMF models we have adopted, which need to be improved to produce the accurate maps of the GSE which match the observations more closely. This will also improve the Stokes V maps. Finally, there are no observed maps of the galactic CP. An all-sky map of the observed galactic CP will reveal the nature of the GMF in greater detail, verify the theoretical predictions on CP from the polarization transfer equations, and yield foreground levels to the measurements of the cosmic CP.

A low frequency measurement of the cosmic CP in the CMB can reveal information about the Pop III stars, early Universe symmetry breaking or new physics, galaxy clusters, or even the primordial magnetic field. The signals from the Pop III stars are particularly interesting due to their expected high signal level and, consequently, their significantly high SNR. Observing CP in the CMB will be an indirect probe of the Pop III stars, which are highly significant as the first formed structures since the cosmic dark ages and as the seeds of reionization of the Universe. Not much is known about the Pop III stars currently. Direct observation of these high redshift objects (\( z \geq 15 \)) are generally beyond the reach of future telescopes like the JWST. Observing the CP induced in the CMB provides a very economical way of learning about the existence and the nature of the Pop III stars, which may be realized in the immediate future. With the current status of instrumentation, observing the CP of the CMB offers the greatest promise in probing the Pop III stars. If instrumentation improves in the near future, new physics related signals can also be explored, a possibility which currently remains far out of reach for the extremely expensive modern day accelerators.

Finally, an immediate practical significance of this work is towards the cosmic B-mode exploration. This applies to the telescopes that are now being built and that propose to explore primordial B modes using VPM techniques. The observing strategy relies on using CP as a systematic rejection channel. This work points out that the galactic CP effects are important at large scales, especially for frequencies below 50 GHz.

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[81] The maps may be obtained from sde@ucdavis.edu. See http://somade.faculty.ucdavis.edu for details.