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Generalized Pencil-of-Function Method for Extracting Poles of an EM System from Its Transient Response

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Abstract—A generalized pencil-of-function (GPOF) method for extracting the poles of an EM system from its transient response is developed. The GPOF method needs the solution of a generalized eigenvalue problem to find the poles. This is in contrast to the conventional Prony and pencil-of-function methods which yield the solution in two steps, namely, the solution of an ill-conditioned matrix equation and finding the roots of a polynomial. Subspace decomposition is also used to optimize the performance of the GPOF method. The GPOF method has advantages over the Prony method in both computation and noise sensitivity, and approaches the Cramer–Rao bound when the signal-to-noise ratio (SNR) is above threshold. An application of the GPOF method to a thin-wire target is also presented.

I. INTRODUCTION

IT IS KNOWN [1]–[8] that in target identification extracting various features of the target from its transient response is desired. The target poles that contain the information of the decay factors and the resonant frequencies should be estimated with high accuracy while the residues can be computed by solving a linear least squares problem [1] after the poles are obtained.

The Prony method [1]–[3] has been a very popular technique for pole retrieval. There are also many versions of the Prony method, which include the least square (LS) Prony method, the total least square (TLS) Prony method, and the SVD Prony method. An alternative method is the pencil-of-function (POF) method [4], [9]. Very recently, the idea of the POF method has been explored along with ESPRIT [10], and this has resulted in improved and generalized versions [11]–[14]. This paper, which is a result of this exploration, presents a generalized pencil-of-function (GPOF) method. The GPOF method finds poles by solving a generalized eigenvalue problem instead of the conventional two-step process where the first step involves the solution of a matrix equation, and the second step entails finding the roots of a polynomial, as is required by the Prony method. We develop the GPOF method and discuss its computational aspects in Section II. The noise sensitivity of the GPOF method is addressed in Section III. In Section IV, an application of the GPOF method to a thin-wire target is presented.

II. GENERALIZED PENCIL-OF-FUNCTION METHOD

It is known that an EM transient signal can be described by

$$y_k = \sum_{i=1,M} b_i \exp{(s_i \delta t k)} \tag{1}$$

where $k = 0, 1, \cdots, N - 1$, b_i are the complex residues, s_i are the complex poles, and δt is the sampling interval. For notational brevity, we can let $z_i = \exp{(s_i \delta t)}$ which are the poles in the Z-plane. It is clear that b_i and s_i should, respectively, be in complex conjugate pairs for real valued y_k.

Following the idea of the pencil-of-function method, we consider the following set of "information" [9] vectors:

$$\mathbf{y}_0, \mathbf{y}_1, \cdots, \mathbf{y}_L$$

where

$$\mathbf{y}_i = [y_i, y_{i+1}, \cdots, y_{i+N-L-1}]^T. \tag{2}$$

The superscript T denotes transpose of a matrix. Based on these vectors, we define the matrices Y_1 and Y_2 as

$$Y_1 = [\mathbf{y}_0, \mathbf{y}_1, \cdots, \mathbf{y}_{L-1}] \tag{3}$$

$$Y_2 = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_L]. \tag{4}$$

To look into the underlying structure of the two matrices, one can write

$$Y_1 = Z_1 B Z_2 \tag{5}$$

$$Y_2 = Z_1 B Z_0 Z_2 \tag{6}$$

where

$$Z_1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \cdot & \cdot & \cdots & \cdot \\ z_1^{N-L-1} & z_2^{N-L-1} & \cdots & z_M^{N-L-1} \end{bmatrix} \tag{7}$$

$$Z_2 = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{L-1} \\ & & \cdots & \\ 1 & z_M & \cdots & z_M^{L-1} \end{bmatrix} \tag{8}$$

$$Z_0 = \text{diag } [z_1, z_2, \cdots, z_M] \tag{9}$$

$$B = \text{diag } [b_1, b_2, \cdots, b_M]. \tag{10}$$

Based on the above decomposition of Y_1 and Y_2, one can show

that if $M \leq L \leq N - M$ the poles $\{z_i; i = 1, \cdots, M\}$ are the generalized eigenvalues [10], [17] of the matrix pencil $Y_2 - zY_1$. Namely, if $M \leq L \leq N - M$, $z = z_i$ is a rank-reducing number of $Y_2 - zY_1$. If $L = M$, this method is the same as the basic POF method as in [12] and equivalent to a version of Ibrahim time-domain (ITD) method [16]. However, in the GPOF method, we are more interested in different values of L for $M < L < N - M$. The significance of this will be shown later.

To develop and illustrate the use of an algorithm for computing the generalized eigenvalues of the matrix pencil problem we can write

$$Y_1^+ Y_2 = Z_2^+ B^{-1} Z_1^+ Z_1 B Z_0 Z_2$$
$$= Z_2^+ Z_0 Z_2 \qquad (11)$$

where the superscript $+$ denotes the (Moore–Penrose) pseudo-inverse [17], whereas we use "-1" for the (regular) inverse. It can be seen from (11) that there exist vectors $\{\mathbf{p}_i; i = 1, \cdots, M\}$ such that

$$Y_1^+ Y_1 \mathbf{p}_i = \mathbf{p}_i, \qquad (12)$$

and

$$Y_1^+ Y_2 \mathbf{p}_i = z_i \mathbf{p}_i. \qquad (13)$$

The $\mathbf{p}_i$ are called the generalized eigenvectors of $Y_1 - zY_2$. To compute the pseudo-inverse Y_1^+, one can use the singular value decomposition (SVD) [17] of Y_1 as follows:

$$Y_1 = \sum_{i=1,M} \sigma_i \mathbf{u}_i \mathbf{v}_i^H$$
$$= U D V^H \qquad (14)$$

$$Y_1^+ = V D^{-1} U^H \qquad (15)$$

where $U = [\mathbf{u}_1, \cdots, \mathbf{u}_M]$, $V = [\mathbf{v}_1, \cdots, \mathbf{v}_M]$, and $D = \text{diag} [\sigma_1, \cdots, \sigma_M]$. The superscript H denotes the conjugate transpose of a matrix. U and V are matrices of left and right singular vectors, respectively. Note that for noisy data y_k one should choose $\sigma_1, \cdots, \sigma_M$ to be the M largest singular values of Y_1, and the resulting Y_1^+ is called the truncated pseudo-inverse of Y_1. Since $Y_1^+ Y_1 = VV^H$ and $V^H V = I$, substituting (15) into (13) and left multiplying (13) by V^H yields

$$(Z - z_i I) \mathbf{z}_i = 0 \qquad (16)$$

where $i = 1, \cdots, M$, and

$$Z = D^{-1} U^H Y_2 V, \qquad (17)$$

and

$$\mathbf{z}_i = V^H \mathbf{p}_i. \qquad (18)$$

Note that Z is an $M \times M$ matrix, and z_i and $\mathbf{z}_i$ are, respectively, eigenvalues and eigenvectors of Z. Now we have completed the description of an algorithm of the GPOF method.

It is important to mention that the number of poles, M, can be estimated from the singular values, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_M$

$\geq \cdots \geq \sigma_{\min(N-L,L)}$, since $\sigma_{M+1} = \cdots = \sigma_{\min(M-L,L)} = 0$ for noiseless data.

If $L = M$, the SVD of the Y_1 is not required, and $\{z_1; i = 1, \cdots, M\}$ are the eigenvalues of the $M \times M$ matrix $(Y_1^H Y_1)^{-1} Y_1^H Y_2$ which is obtained by substituting $Y_1^+ = (Y_1^H Y_1)^{-1} Y_1^H$ into (13) for $L = M$. Furthermore, one can verify that with or without noise,

$$(Y_1^H Y_1)^{-1} Y_1^H Y_2 = \begin{bmatrix} 0 & \cdots & 0 & -c_M \\ 1 & & 0 & -c_{M-1} \\ & \ddots & & \vdots \\ & & 1 & -c_1 \end{bmatrix} \qquad (19)$$

which is the companion matrix of the polynomial

$$1 + \sum_{i=1,M} c_i z^{-i} = 0, \qquad (20)$$

where

$$\begin{bmatrix} c_M \\ \vdots \\ c_1 \end{bmatrix} = -(Y_1^H Y_1)^{-1} Y_1^H \mathbf{y}_M \qquad (21)$$

which is the solution of the least square Prony method. So for $L = M$ the GPOF method is equivalent to the LS Prony method.

The total least square Prony method is to compute the polynomial coefficients as follows:

$$\mathbf{c} = \arg \min \{\mathbf{c}^H Y^H Y \mathbf{c}; \|\mathbf{c}\| = 1\} \qquad (22)$$

where $Y = [\mathbf{y}_0, \mathbf{y}_1, \cdots, \mathbf{y}_M]$ and $\mathbf{c} = [c_M, \cdots, c_1, c_0]^T$. A perturbation analysis [12] has shown that the LS Prony method and the TLS Prony method are equivalent to the first-order perturbation approximation.

The SVD Prony method (for $L > M$) is to compute $\mathbf{c}_L = [c_{L-1}, \cdots, c_0]^T$ by

$$\mathbf{c}_L = -(Y_2^H Y_2)^+ Y_2^H \mathbf{y}_0 = -Y_2^+ \mathbf{y}_0 \qquad (23)$$

and detect the M signal roots, i.e., $\{z_i^{-1}; i = 1, \cdots, M\}$, from the L roots of the L-degree polynomial $1 + \Sigma_{i=i,L} c_{L-i} z^{-1}$. But the detection is guaranteed (without noise) to be successful only when all $z_i (z_i^{-1})$ are inside (outside) or on the unit circle [21]. Computationally, solving for the roots of an L-degree polynomial is also a disadvantage of the SVD Prony method, compared to solving M eigenvalues of an $M \times M$ matrix for the GPOF method.

III. NOISE SENSITIVITY OF GPOF METHOD

In this section, we illustrate the noise sensitivity of the GPOF method through some numerical examples. Specifically, we let

$$y_k = \sum_{i=1,J} A_i \sin (w_i k + \phi_i) \exp (-\alpha_i k) \qquad (24)$$

where $k = 0, 1, \cdots, N - 1$, $N = 30$, $J = 2$ ($M = 2, J = 4$) $A_1 = A_2 = 1$, $w_1 = 0.2\pi$, $w_2 = 0.35\pi$, $\phi_1 = \phi_2 = 0$, $\alpha_1 = 0.02\pi$, and $\alpha_2 = 0.035\pi$. Note that $z_1 = \exp (s_i \delta t) = \exp (-\alpha_i + jw_i)$, for $i = 1$ and 2, and $z_i = z_{i-2}^*$, for $i = 3$ and 4. The superscript asterisk denotes the complex conjugation. It is
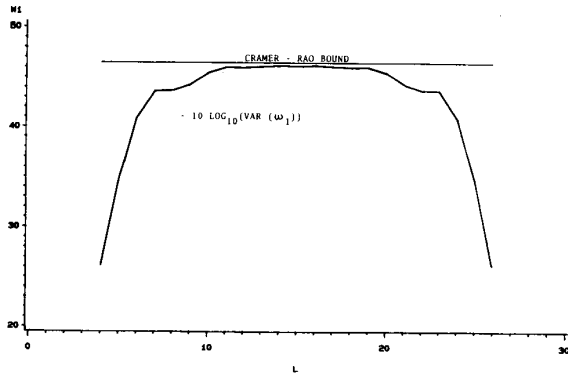
Fig. 1. Inverted perturbation variances in dB of the estimated w_1 versus the pencil parameter L.
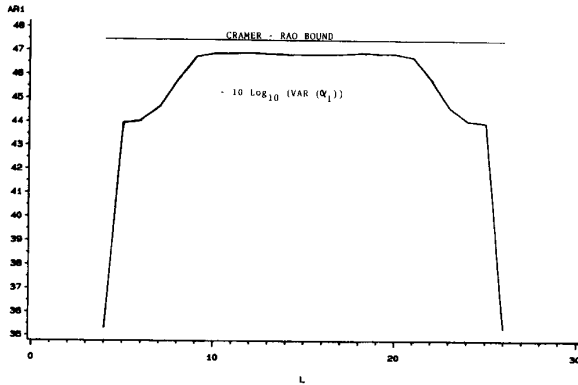

Fig. 2. Inverted perturbation variances in dB of the estimated α_1 versus the pencil parameter L.

Fig. 3. Simulation results (denoted by +) of the inverted sample variance in dB of w_1 estimated by the GPOF method and the LS Prony method. The straight lines are obtained from the perturbation analysis.

Fig. 4. Simulation results (denoted by +) of the inverted sample variance in dB of α_1 estimated by the GPOF method and the LS Prony method. The straight lines are obtained from the perturbation analysis.

also important to note that α_i and w_i are, respectively, damping factors and resonant frequencies normalized by the sampling frequency $f_s = 1/\delta t$. (There is no need to specify the sampling frequency for our numerical simulations. The sequence as in (24) is the only sampled data sequence used in this section for illustration purposes.) The first-order perturbation analysis of the GPOF method is outlined in the Appendix. For the analysis, it is assumed that the additive noise in y_k is white and sufficiently weak so that the first-order approximation can be carried out through the derivation. Fig. 1 shows the inverted perturbation variance in dB of w_1 (imaginary part of the pole) versus the pencil parameter L. The Cramer-Rao bound provides the "absolute" best result that any technique can achieve in the present "noisy" environment. The GPOF method approximately reached the Cramer-Rao bound. This implies that "no" other theoretical technique can do any better! Fig. 2 shows the same thing for α_1 (real part of the pole). The plots for w_2 and α_2 are similar to the above two figures, and hence are omitted from this paper. As one observes, the optimal choice of L is around $L = N/2$. Intuitively, this phenomenon can be explained as follows.

The noiseless Y_1 (or Y_2) has a column subspace of dimension M. (Note that the GPOF method requires $L \geq M$ and $N - L \geq M$.) This subspace is called the signal subspace, denoted by S_S. But the noisy Y_1 (or Y_2), which consists of
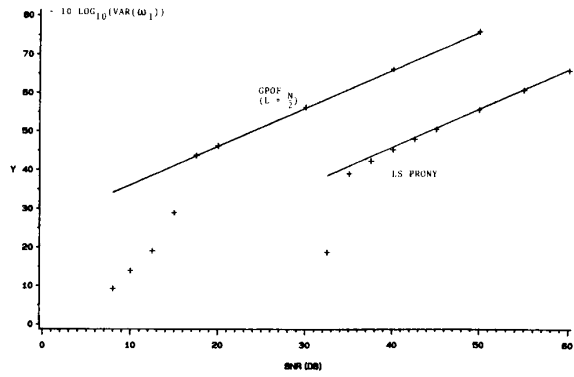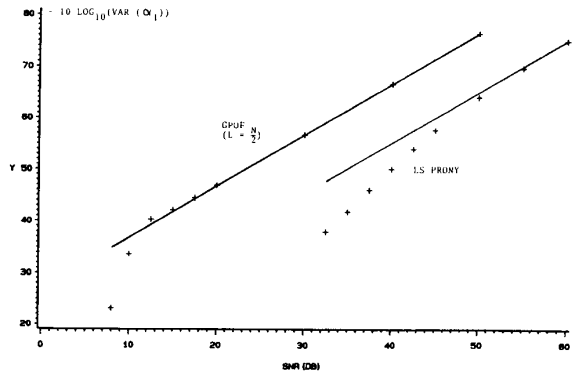
signal and noise, spans a column subspace, denoted by S_{S+N}, of dimension equal to min $(N - L, L)$. It is clear that we can write $S_{S+N} = S_S + S_N$, where S_S contains all signal components and the noise components projected onto the signal subspace, and S_N contains noise only. As is seen in the GPOF method, we perform subspace decomposition of Y_1 as in (14) and throw away the noise component in S_N which is spanned by u_i for $i > M$. It seems, therefore, that the larger the noise subspace S_N, the more noise can be filtered out by the GPOF method. The dimension of S_N is the largest when $N - L = L$, i.e., $L = N/2$. This is consistent with our perturbation analysis. Note that around $L = N/2$ the performance of the GPOF method is very close to the optimal bound, i.e., the Cramer-Rao bound [14], [19]. A different interpretation of a similar phenomenon with the ITD method was made in [16].

With the choice $L = N/2 = 15$, some simulation results for the GPOF method are shown in Figs. 3 and 4. The (Monte-Carlo) simulation was conducted with 200 runs. During each run, we computed the estimated α_i and w_i from the data contaminated by (pseudo) white Gaussian noise. (Note that $\alpha_i = -\text{Re}\{\log[z_i]\}$ and $w_i = \text{Im}\{\log[z_i]\}$.) The noise used in each run is independent of that used in others. Figs. 3 and 4 show the inverted sample variances (denoted by the plus signs) in dB of the estimated w_1 and α_1 versus SNR which is defined
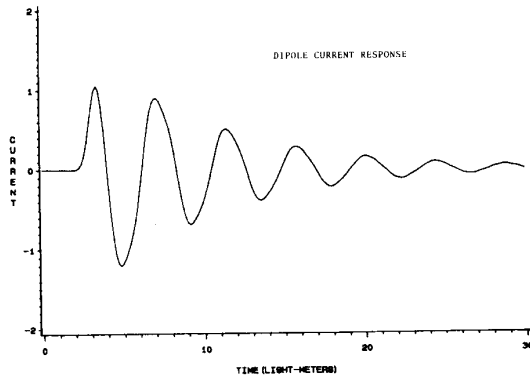
Fig. 5. Current response at the center of the 2-m dipole illuminated by a short pulse.

Fig. 6. FFT amplitude spectrum of the current.

by

$$SNR = \sum_{k=0,N-1} |y_k|^2/N\sigma^2 \qquad (25)$$

and

$$SNR \text{ (dB)} = 10 \log_{10} (SNR). \qquad (26)$$

σ^2 is the noise variance. The straight lines are obtained from the perturbation analysis. As one observes, for high SNR, the simulation results agree with the analytical results. As a reference, the simulation results of the LS Prony method are also shown in the plots. The detailed perturbation analysis of the Prony method is available in [14], [15]. The noisy data used for the LS Prony method were the same as those used for the GPOF method. We should mention that the SVD Prony method as represented by (21) performs better than the LS Prony method and the TLS Prony method [21]. In fact, the SVD Prony method performs almost as well as the GPOF method for this particular example. However, it can be shown, in general, [12]-[14] that the GPOF method is less sensitive to noise than the SVD Prony method.

IV. An Application

Consider that a 2-m dipole antenna of radius 0.001m is illuminated by a short EM pulse of the form

$$E^{inc} = \eta/(\pi^{1/2}c\sigma) \exp\left[-(t-t_0)^2/\sigma^2\right]$$

from broadside where $\eta = 377$ ohms, $c = 3 \times 10^8$ m/s, $\sigma = 0.5/c$, and $t_0 = 6\sigma$. The current response at the center point of the dipole was computed by the approach in [20] with sampling time $\delta t = 0.5$ ns, and is shown in Fig. 5. To get the intrinsic poles of the dipole itself, we considered a segment of the current for $t = 8.4$ through 25.5 lightmeters, which consists of 114 samples. Note that for $t > 8.4$ lightmeters the EM pulse E^{inc} is almost zero. Fig. 6 shows the fast Fourier transform (FFT) amplitude spectrum of the current. From the spectrum, four resonant components are detected at 0.0684, 0.203, 0.391, and 0.414 GHz. The spectrum was computed with resolution equal to $f_s/1024 = 0.00195$ GHz.

Applying the GPOF method with $L = N/2 = 57$ to the 114 sampled data, we observed that the ten largest singular values of the data matrix Y_1 are 9.3, 7.7, 0.45, 0.42, 0.057, 0.056,
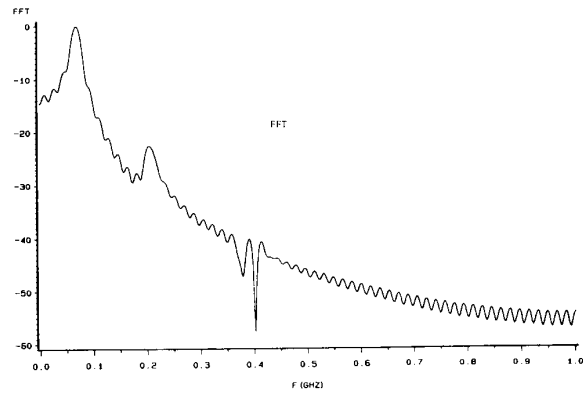
0.039, 0.0388, 0.0052, and 0.0049. With $M = 8$ (since there is a large drop from the eighth singular value to the ninth), the GPOF method yielded the following poles (i.e., $s_i\delta t = \log [z_i] = -\alpha_i + jw_i$):

$$-0.0204 \pm j0.218 \quad -0.0266 \pm j0.642$$
$$-0.0046 \pm j1.17 \quad 0.0039 \pm j1.27.$$

By least squares fitting [1], the corresponding residues (absolute values) were computed to be

$$0.3835 \times 10^0 \quad 0.2456 \times 10^{-1} \quad 0.9379 \times 10^{-3} \quad 0.7506 \times 10^{-3}$$

From the first six stable poles, three estimated resonant frequencies ($f_1 = w_1/2\pi\delta t$) are obtained to be 0.0694, 0.204, and 0.372 GHz. The unstable estimated poles with the frequency 0.404 GHz appear to correspond to the "fourth" resonant component as is shown in the FFT spectrum. Applying the LS Prony method to the same data samples and with the assumption $M = 8$, we found the following poles:

$$-0.0203 \pm j0.218 \quad -0.0261 \pm j0.645$$
$$0.3 \times 10^{-3} \pm j1.251 \quad 0.611 \pm j2.244.$$

It is seen that the first two pairs of poles are close to the corresponding pairs obtained by the GPOF method while the next two pairs differ from those obtained by the GPOF method. In the following table, the resonant frequencies estimated by different approaches are compared for the identical data records.

Frequency (GHz)	f_1	f_2	f_3	f_4
By $f_1 = c(2i - 1)/2L$	0.0750	0.225	0.375	*
By GPOF	0.0694	0.204	0.372	0.404
By FFT	0.0684	0.203	0.391	0.414
By LS Prony	0.0694	0.205	0.398	0.714

L is the length of the dipole and c is the light velocity. It is clear that for the same data, the GPOF method provides a stable solution.

V. Conclusion

We have presented the GPOF method which solves a generalized eigenvalue problem to estimate poles of EM

system. In this new approach, one finds the poles directly by solving a generalized eigenvalue problem. This is in contrast to the contemporary methods like Prony and pencil-of-function methods, which are generally two step processes. In the first step one solves a matrix equation followed by finding the roots of a polynomial. Compared to the SVD Prony method, the GPOF method is less restricted because it does not require that all system poles must be either stable or nonstable, and computationally more efficient because it does not solve an Lth-degree polynomial. Compared to the LS or TLS Prony method, the GPOF method is more robust to noise as has been shown. An application of the GPOF method to a synthesized dipole has been shown, with comparison to the FFT and the LS Prony method.

APPENDIX

In this section, we give an outline of the first order perturbation analysis of the GPOF method. For detailed discussion, see [14].

First, it can be verified that the eigenvalues of Z are same as the nonzero eigenvalues of $Y_1^+ Y_2$ whether or not the data are noisy. Then, it is known [18] that the first-order variation in the poles is given by

$$\delta z_i = \mathbf{q}_i \delta(Y_1^+ Y_2) \mathbf{p}_i / \mathbf{q}_i^H \mathbf{p}_i \tag{27}$$

where δ is the first-order differential operator and $\mathbf{p}_i$ and $\mathbf{q}_i$ are, respectively, the right and the left eigenvectors of the noiseless $Y_1^+ Y_2$. So $\mathbf{p}_i$ (see (11)) is the ith column of Z_2^+, and $\mathbf{q}_i$ is the ith column of Z_2^H. Therefore, $\mathbf{q}_i^H \mathbf{p}_i = 1$.

Secondly, it can be shown [13]-[14] that

$$\mathbf{q}_i^H \delta Y_1^+ \mathbf{p}_i = -\mathbf{q}_i^H Y_1^+ \delta Y_1 Y_1^+ \mathbf{p}_i. \tag{28}$$

Note that Y_1^+ is a truncated pseudoinverse of Y_1 in the noisy data case. From (27) and (28), one can obtain the following after some algebraic manipulations:

$$\delta z_i = (1/b_i) \mathbf{r}_i^H (\delta Y_2 - z_i \delta Y_1) \mathbf{p}_i \tag{29}$$

where $\mathbf{r}_i^H$ is the ith row of Z_1^+, and

$$\delta Y_1 = \begin{bmatrix} n_0 & n_1 & \cdots & n_{L-1} \\ & & \cdots & \\ n_{N-L-1} & n_{N-L} & \cdots & n_{N-2} \end{bmatrix} \tag{30}$$

$$\delta Y_2 = \begin{bmatrix} n_1 & n_2 & \cdots & n_L \\ & & \cdots & \\ n_{N-L} & n_{N-L+1} & \cdots & n_{N-1} \end{bmatrix}. \tag{31}$$

Finally, it can be shown from (29)-(31) that if $\{n_i; i = 0, \cdots, N - 1\}$ are white with variance equal to σ^2, then the first-order perturbation variances are

$$E\{|\delta z_i|^2\} = [\sigma^2/|b_i|^2] \sum_{k=1,N} |C_{i,k} - z_i C_{i,k+1}|^2 \tag{32}$$

$$E\{|\delta\alpha_i|^2\} = \sigma^2 \sum_{k=1,N} \text{Re}^2 \{(C_{i,k} - z_i C_{i,k+1})/b_i z_i\} \tag{33}$$

$$E\{|\delta w_i|^2\} = \sigma^2 \sum_{k=1,N} \text{Im}^2 \{(C_{i,k} - z_i C_{i,k+1})/b_i z_i\} \tag{34}$$

where

$$C_{i,k} = \begin{bmatrix} \sum_{m=1,k-1} f_{i,k,m}, & 2 \le k \le \min(L, N-L) \\ \sum_{m=k-L,k-1} f_{i,k,m}, & L \le N-L \text{ and} \\ & L+1 \le k \le N-L+1 \\ \sum_{m=1,N-L} f_{i,k,m}, & N-L \le L \text{ and} \\ & N-L+1 \le k \le L+1 \\ \sum_{m=k-L,N-L} f_{i,k,m}, & \max(L, N-L)+2 \le k \le N \end{bmatrix} \tag{35}$$

$$f_{i,k,m} = r_{i,m}^* p_{i,k-m} \tag{36}$$

in which $r_{i,m}^*$ is the mth element of the row vector $\mathbf{r}_i^H$ and $p_{i,m}$ is the mth element of the column vector $\mathbf{p}_i$.

REFERENCES

[1] M. L. Van Blaricum and R. Mittra, "A technique for extracting the poles and residues of a system directly from its transient response," IEEE Trans. Antennas Propagat., vol. AP-23, no. 6, pp. 777-781, Nov. 1975.

[2] A. J. Poggio et al., "Evaluation of a processing technique for transient data," IEEE Trans. Antennas Propagat., vol. AP-26, no. 1, pp. 165-173, Jan. 1978.

[3] M. L. Van Blaricum and R. Mittra, "Problem and solutions associated with Prony's method for processing transient data," IEEE Trans. Antennas Propagat., vol. AP-26, no. 1, pp. 174-182, Jan. 1978.

[4] T. K. Sarkar et al., "Sub-optimal approximation/identification of transient waveforms from electromagnetic system by pencil-of-function method," IEEE Trans. Antennas Propagat., vol. AP-28, no. 6, pp. 928-933, Nov. 1980.

[5] F. I. Tseng and T. K. Sarkar, "Experimental determination of resonant frequencies by transient scattering from conducting spheres and cylinders," IEEE Trans. Antennas Propagat., vol. AP-32, no. 9, pp. 914-918, Sept. 1984.

[6] ——, "Deconvolution of the impulse response of a conducting sphere by the conjugate gradient method," IEEE Trans. Antennas Propagat., vol. AP-35, no. 1, pp. 105-109, Jan. 1987.

[7] E. M. Kennaugh and D. L. Moffatt, "Transient and impulse response approximations," Proc. IEEE, vol. 53, pp. 893-901, Aug. 1965.

[8] D. L. Moffatt and R. K. Mains, "Detection and discrimination of radar targets," IEEE Trans. Antennas Propagat., vol. AP-23, no. 3, pp. 358-367, May 1975.

[9] V. K. Jain, T. K. Sarkar, and D. D. Weiner, "Rational modeling by pencil-of-function method," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-31, no. 3, pp. 564-573, June 1983.

[10] A. Paulraj, R. Roy, and T. Kailath, "Estimation of signal parameters via rotational invariance techniques-ESPRIT," in Proc. 19th Asilomar Conf. Circuit, Syst. Compat., Asilomar, CA, Nov. 1985.

[11] A. J. Mackay and A. McCowen, "An improved pencil-of-function method and comparisons with traditional methods of pole extraction," IEEE Trans. Antennas Propagat., vol. AP-35, no. 4, pp. 435-441, Apr. 1987.

[12] Y. Hua and T. K. Sarkar, "Further analysis of three modern techniques for pole retrieval from data sequence," in Proc. 30th Midwest Symp. Circuits Syst., Syracuse, NY, Aug. 1987.

[13] ——, "Matrix pencil method and its performance," in Proc. ICASSP-88, New York, NY, Apr. 1988.

[14] Y. Hua, "On techniques for estimating parameters of exponentially damped/undamped sinusoids in noise," Ph.D. dissertation, Syracuse Univ., 1988.

[15] Y. Hua and T. K. Sarkar, "Perturbation analysis of TK method for harmonic retrieval problem," IEEE Trans. Acoust. Speech Signal Processing, vol. ASSP-36, no. 2, pp. 228-240, Feb. 1988.

[16] S. R. Ibrahim and R. S. Pappa, "Large model survey testing using the Ibrahim time domain identification technique," J. Spacecraft, vol. 19, no. 5, pp. 459-465, Sept.-Oct. 1982.

[17] G. H. Golub and C. F. Van Loan, Matrix Computations. Baltimore, MD: Johns Hopkins Univ., 1983.

[18] J. H. Wilkinson, *The Algebraic Eigenvalue Problem*. Oxford, U.K.: Clarendon, 1965.

[19] C. R. Rao, *Linear Statistical Inference and Its Applications*, 2nd ed. New York, Wiley, 1973.

[20] S. M. Rao, T. K. Sarkar, and S. A. Dianat, "The application of the conjugate gradient method to the solution of transient electromagnetic scattering from thin wires," *Radio Sci.*, vol. 19, no. 5, pp. 1319–1326, Sept.-Oct. 1984.

[21] R. Kumaresan, "Estimating the parameters of exponentially damped or undamped sinusoidal signals in noise," Ph.D. dissertation, Univ. Rhode Island, 1982.
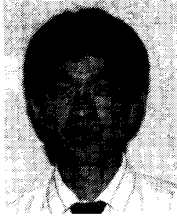
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