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ESTIMATION OF THE ASYMPTOTIC PSEUDOSCALAR-MESON—BARYON TOTAL CROSS SECTION FROM THE ABFST MULTIPERIPHERAL MODEL*

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ABSTRACT

The high-energy forward scattering of a pseudoscalar meson (μ) and a baryon (B) is discussed on the basis of the ABFST multiperipheral model. Assuming that the low-energy μB amplitude is dominated by the baryon pole plus the first elastic resonance, and using the approximate solution to the $\mu\mu$ integral equation proposed by Abarbanel, Chew, Goldberger, and Saunders, we derive an expression for the high energy limit of $\sigma_{\mu B}^{tot}$. A numerical estimate based on SU(3) symmetry is given.

I. INTRODUCTION

Our purpose here is to study, in a crude way, the forward scattering between a pseudoscalar meson (μ) and a baryon (B) at very high energy on the basis of the ABFST model,[1-3] with an internal symmetry SU(n). We shall represent the input low-energy μB kernel by a B pole (which controls the production of even numbers of final μ's), and a prominent elastic (μB) resonance (which controls the production of odd numbers of final μ's). This approach is motivated by known facts about $\sigma_{\mu B}^{e\ell}$, like the curves shown in Fig. 1.

For the μ-B-B vertex in this kernel, it is necessary to introduce off-shell continuation,[4] otherwise this contribution to $\sigma_{\mu B}^{tot}$ is negative. This is in contrast to the usual assumption (of course not a necessity) that we can keep all the low-energy vertices on-shell. With regard to the possible modification of this vertex and/or other vertices, a plausible rule is to be guided by experimental distributions in singly-peripheral reactions. We study this question briefly in our Appendix A. The off-shell dependence of the vertices assumed in the main portion of the paper is partially justified by this material in the Appendix. Otherwise we shall use the solution to the $\mu\mu$ integral equation suggested by Abarbanel, Chew, Goldberger, and Saunders.[5] Even though such a solution is only approximate--both mathematically[6] and physically,[7] because of its appealing simplicity we can elucidate the various factors that control the magnitudes of asymptotic total cross sections.

Based on these assumptions and approximations, and making use of the (assumed) smallness of the μ mass in comparison with all other relevant masses, we shall derive an expression for $\sigma_{\mu B}^{tot}$ as $s \to \infty$. We make a numerical estimate assuming SU(3) symmetry and in the last section some discussion is given of the results.

II. THE FORWARD ABFST EQUATION FOR $\mu\mu$ SCATTERING AND THE APPROXIMATE SOLUTION

It is natural to begin with consideration of the $\mu\mu$ scattering equation.[8] Let us briefly review the approximate solution from Ref. 5, for completeness as well as for introducing notations and conventions.

The imaginary part of the forward elastic amplitude for $\mu\mu$ high-energy scattering satisfies the equation (c.f. Fig. 2):

$$\mathrm{Im}\, T_{\mu\mu}{}^{I}\left((p - p')^2, p^2, p'^2\right) = \int ds_0\, \delta^{+}[(p' - p'')^2 - s_0]$$

$$\times\ \mathrm{Im}\, \tilde{T}_{\mu\mu}{}^{I}(s_0) + \left(\tfrac{2}{\pi}\right)\int d^4p''\ \mathrm{Im}\, T_{\mu\mu}{}^{I}\left((p - p'')^2, p^2, p''^2\right)$$

$$\times \left[\frac{1}{16\pi^3}\,\frac{1}{(p''^2 - \mu^2)^2}\int ds_0\ \delta^{+}\left((p' - p'')^2 - s_0\right)\mathrm{Im}\,\tilde{T}_{\mu\mu}{}^{I}(s_0)\right], \tag{II-1}$$

where in particular we consider only the amplitude which corresponds to the identity representation of the internal symmetry in the crossed channel (the superscript "I" signifies this), and $\mathrm{Im}\,\tilde{T}_{\mu\mu}{}^{I}(s_0)$, being used as input, is related to the low-energy $\mu\mu$ elastic cross section by

$$\mathrm{Im}\,\tilde{T}_{\mu\mu}{}^{I}(s_0) = \Lambda(s_0,\mu^2,\mu^2)\sum_{y} X^{Iy}\,\sigma_{\mu\mu}^{e\ell,y}(s_0), \tag{II-2}$$

where $\Lambda(x,y,z) = [x^2 + y^2 + z^2 - 2(xy + yz + zx)]^{\frac{1}{2}}$, X^{Iy} is the t-s internal symmetry crossing matrix, and μ^2 is the pseudoscalar-meson mass squared. We have neglected the off-shell dependence of $\mathrm{Im}\,\tilde{T}_{\mu\mu}{}^{I}$.

Now Eq. (II-1) can be diagonalized by projecting it onto the unitary irreducible representation of SO(1,3):

$$\mathrm{Im}\, T^{I\lambda}_{\mu\mu}(\tau,\tau') = \mathrm{Im}\, \tilde{T}^{I\lambda}_{\mu\mu}(\tau,\tau') + \int_{-\infty}^{0} d\tau'' \,\mathrm{Im}\, T^{I\lambda}_{\mu\mu}(\tau,\tau'')\, K^{I\lambda}(\tau'',\tau'), \tag{II-3}$$

where $\tau = p^2$ etc.,

$$K^{I\lambda}(\tau'',\tau') = \frac{1}{16\pi^3}\frac{1}{(\tau''-\mu^2)^2}\,\mathrm{Im}\,\tilde{T}^{I\lambda}_{\mu\mu}(\tau'',\tau'), \tag{II-4}$$

$$\mathrm{Im}\,\tilde{T}^{I\lambda}_{\mu\mu}(\tau,\tau') = \int_{4\mu^2}^{\infty} ds_0 \frac{1}{(\lambda+1)} e^{-(\lambda+1)\eta(s_0,\tau,\tau')}\,\mathrm{Im}\,\tilde{T}_{\mu\mu}{}^{I}(s_0), \tag{II-5}$$

$$\cosh\eta(s_0,\tau,\tau') = \frac{s_0-\tau-\tau'}{(-2\tau)^{\frac{1}{2}}(-2\tau')^{\frac{1}{2}}}, \tag{II-6}$$

and the "full" $\mathrm{Im}\, T_{\mu\mu}{}^{I}$ is related to the "partial wave" $\mathrm{Im}\, T^{I\lambda}_{\mu\mu}$ by

$$\mathrm{Im}\, T_{\mu\mu}{}^{I}(\hat{s},\tau,\tau') = \frac{1}{2\pi i}\int_c d\lambda(\lambda+1)\,\frac{e^{+(\lambda+1)\eta(\hat{s},\tau,\tau')}}{\sinh\eta(\hat{s},\tau,\tau')}$$

$$\times\ \frac{1}{(-2\tau)^{\frac{1}{2}}(-2\tau')^{\frac{1}{2}}}\,\mathrm{Im}\, T^{I\lambda}_{\mu\mu}(\tau,\tau'), \tag{II-7}$$

where c is a contour of integration in the λ plane running from $-i\infty$ to $+i\infty$, passing to the right of all λ singularities of $\mathrm{Im}\, T^{I\lambda}_{\mu\mu}$.

The approximate solution to Eq. (3),[5] in the neighborhood of the leading singularity of $\mathrm{Im}\, T^{I\lambda}_{\mu\mu}$ in the λ plane, takes the simple form

$$\mathrm{Im}\, T^{I\lambda}_{\mu\mu}(\tau,\tau') \sim \frac{\mathrm{Im}\,\tilde{T}^{I\lambda}_{\mu\mu}(\tau,\tau')}{1-\mathrm{Tr}\, K^{I\lambda}}, \tag{II-8}$$

where

$$\mathrm{Tr}\, K^{I\lambda} = \frac{1}{16\pi^3}\int_{-\infty}^{0} d\tau \frac{1}{(\tau-\mu^2)^2}\int_{4\mu^2}^{\infty} ds_0 \frac{1}{(\lambda+1)} e^{-(\lambda+1)\eta(s_0,\tau,\tau)}$$

$$\times\ \mathrm{Im}\,\tilde{T}^{I\lambda}_{\mu\mu}(s_0). \tag{II-9}$$

This solution would be exact if the kernel $K^{I\lambda}(\tau'',\tau')$ were factorizable in τ'' and τ'. Next we approximate the low-energy forward elastic $\mu\mu$ amplitude by a pole corresponding to a resonance. Thus we put

$$\mathrm{Im}\,\tilde{T}_{\mu\mu}{}^{I}(s_0) = \Lambda(s_0,\mu^2,\mu^2)[X\,\pi\,mx\,\Gamma\,\sigma^{e\ell}_{\mu\mu}(\max)]_c\,\delta(s_0-m_c{}^2)$$

$$\equiv m_c{}^2 R_c{}^I\,\delta(s_0-m_c{}^2), \tag{II-10}$$

where $m_c{}^2$ is the squared mass and x_c is the elasticity of the resonance. Then from Eq. (II-9), we get[9]

$$\mathrm{Tr}\, K^{I\lambda} \approx \frac{1}{16\pi^3}\frac{2R_c{}^I}{\lambda(\lambda+1)(\lambda+2)} + \frac{1}{16\pi^3}$$

$$\times\left[-\frac{\pi R_c{}^I}{\sin\pi\lambda}\left(\frac{\mu^2}{m_c{}^2}\right)^{\lambda} + \frac{24R_c{}^I}{\lambda(\lambda+1)(\lambda+2)(\lambda+3)(1-\lambda)}\left(\frac{\mu^2}{m_c{}^2}\right)\right]. \tag{II-11}$$

Thus, from Eqs. (II-8) to (II-11), we obtain the leading contribution to $\mathrm{Im}\, T_{\mu\mu}{}^{I}$:

$$\mathrm{Im}\, T_{\mu\mu}{}^{I}(\hat{s},\tau,\tau') \sim \frac{16\pi^3 m_c{}^2 J(\alpha)}{(-2\tau)^{\frac{1}{2}}(-2\tau')^{\frac{1}{2}}}\,\frac{e^{(\alpha+1)[\eta(\hat{s},\tau,\tau')-\eta(m_c{}^2,\tau,\tau')]}}{\sinh\eta(\hat{s},\tau,\tau')}, \tag{II-12}$$

where α is the rightmost singularity of $\mathrm{Im}\, T^{I\lambda}_{\mu\mu}$ in the λ plane, and

$$J(\alpha) = \left[- \frac{\partial}{\partial\lambda} \frac{16\pi^3}{R_c^{\,I}} \, \mathrm{Tr}\, K^{I\lambda} \right]^{-1}_{\lambda=\alpha} . \qquad \text{(II-13)}$$

III. THE μB FORWARD ELASTIC AMPLITUDE AND TOTAL CROSS SECTION AT ASYMPTOTIC ENERGY

We now consider μB scattering. We assume that the production amplitude is given by Fig. 3(a) for even numbers of final μ's, and by Fig. 3(b) for odd numbers of final μ's. The physical absorptive part of the forward elastic μB amplitude at high energy is given by

$$\mathrm{Im}\, T_{\mu B}{}^{I}(s,\mu^2,M^2) = \left(\frac{2}{\pi}\right) \int d^4p' \, \frac{\delta^+[(p_B - p')^2 - s_B]}{16\pi^3(p'^2 - \mu^2)^2} \, \mathrm{Im}\, T_{\mu\mu}{}^{I}(\hat{s},\mu^2,p'^2)$$

$$\times \int ds_B \, \mathrm{Im}\, \tilde{T}_{\mu B}{}^{I}(s_B,p'^2),$$

$$= \frac{1}{16\pi^3} \int_{4\mu^2}^{s} \frac{d\hat{s}}{s} \int_{u^*}^{\infty} \frac{du}{(u+\mu^2)^2} \, \mathrm{Im}\, T_{\mu\mu}{}^{I}(\hat{s},\mu^2,-u)$$

$$\times \int ds_B \, \mathrm{Im}\, \tilde{T}_{\mu B}{}^{I}(s_B,-u), \qquad \text{(III-1)}$$

where $u = -\tau' = -p'^2$,

$$u^* \approx \frac{\hat{s}}{s} \left(\frac{s_B}{1 - \frac{\hat{s}}{s}} - M^2 \right) , \qquad \text{(III-2)}$$

M^2 is the baryon mass squared, and

$$\mathrm{Im}\, \tilde{T}_{\mu B}{}^{I}(s_B,-u) = \mathrm{Im}\, \tilde{T}_{\mu B}{}^{I}(s_B,-u)\big|_{B\text{-pole}}$$

$$+ \Lambda(s_B,\mu^2,M^2) \sum_{y} X^{Iy} \, \sigma^{e\ell,y}_{\mu B}(s_B) . \qquad \text{(III-3)}$$

A discussion of expression (III-3) will be given shortly. In Eq. (III-1), we shall use Eq. (II-12) for $\mathrm{Im}\, T_{\mu\mu}{}^{I}$ and find an explicit integrated form for the whole expression. In doing so we employ following simplifications:

(1) We shall neglect μ^2 in comparison with m_c^2, as well as with $\hat{s}$. In the integrand, we shall put the factor $1/(u+\mu^2)^2 \approx 1/u^2$. This simplification introduces little error as long as the integral is finite and $\mathrm{Re}\,\alpha > \frac{1}{2}$. Correspondingly we neglect the second term (square bracket) in Eq. (II-11), giving

$$J(\alpha) = \left(\frac{18}{11}\right) \quad \text{for} \quad \alpha = 1, \quad \text{independent of } R_c^{\,I}. \tag{III-4}$$

(2) From Eq. (II-6) we have

$$e^{\pm\eta(\chi,\mu^2,-u)} = \frac{1}{2}\,\frac{1}{(-\mu^2)^{\frac{1}{2}}}\left\{\left(\frac{\chi-\mu^2+u}{u^{\frac{1}{2}}}\right) \pm \left[\left(\frac{\chi-\mu^2+u}{u^{\frac{1}{2}}}\right)^2 - 4(-\mu^2)\right]^{\frac{1}{2}}\right\}. \tag{III-5}$$

Observe that the expression $(\chi-\mu^2+u)/u^{\frac{1}{2}}$ diverges both for $u \to 0$ and $u \to \infty$ whenever $(\chi-\mu^2) \neq 0$ and has a minimum value $2(\chi-\mu^2)^{\frac{1}{2}}$ at $u = (\chi-\mu^2)$. So, for $\chi = \hat{s}$, $m_c^2 \gg \mu^2$, we can expand the square root term in Eq. (III-5) in terms of $4(-\mu^2)/[(\chi-\mu^2+u)/u^{\frac{1}{2}}]^2$ for all u in the integration range, and from the leading term obtain

$$e^{\pm\eta} \approx \left[\frac{1}{(-\mu^2)^{\frac{1}{2}}}\left(\frac{\chi+u}{u^{\frac{1}{2}}}\right)\right]^{\pm 1}, \tag{III-6}$$

therefore, Eq. (III-1) with Eq. (II-12), becomes

$$\mathrm{Im}\,T_{\mu B}^{\,I}(s,\mu^2,M^2) \sim \int_{4\mu^2}^{s}\frac{d\hat{s}}{s}\int_{u^*}^{\infty}\frac{du}{u^2}\,\frac{m_c^2 J(\alpha)(\hat{s}+u)^{\alpha}}{(m_c^2+u)^{\alpha+1}}\int ds_B\,\mathrm{Im}\,\tilde{T}_{\mu B}^{\,I}(s_B,-u) \tag{III-7}$$

$$\sim s^{\alpha}\int_0^{\infty}\frac{du}{u^2}\,\frac{m_c^2}{(u+m_c^2)^{\alpha+1}}\,\frac{J(\alpha)}{(\alpha+1)}\left(\frac{1}{-2M^2}\right)^{\alpha+1}$$

$$\times \left\{(s_B - M^2 + u) - \left[(s_B - M^2 + u)^2 + 4M^2u\right]^{\frac{1}{2}}\right\}^{\alpha+1}\int ds_B$$

$$\times\ \mathrm{Im}\,\tilde{T}_{\mu B}^{\,I}(s_B,-u). \tag{III-8}$$

Next we assume[5] that the value of $R_c^{\,I}$ gives $\alpha = 1$ for the Pomeranchuk pole, thus

$$\mathrm{Im}\,T_{\mu B}^{\,I}(s,\mu^2,M^2) \sim s\int_0^{\infty}\frac{du}{u^2(u+m_c^2)^2}\,\frac{J(1)m_c^2}{(-2M^2)^2}$$

$$\left\{\left[u^2 + (\hat{\beta}+\hat{\gamma})u + \hat{\beta}^2\right] - (\hat{\beta}+u)(u^2 + 2\hat{\gamma}u + \hat{\beta}^2)^{\frac{1}{2}}\right\}$$

$$\times \int ds_B\,\mathrm{Im}\,\tilde{T}_{\mu B}^{\,I}(s_B,-u), \tag{III-9}$$

where $\hat{\beta} = s_B - M^2$, $\hat{\gamma} = s_B + M^2$.

Let us now consider the input Eq. (III-3):

1. For the baryon pole contribution, suppose for the moment we have only one type of coupling for the internal symmetry under consideration. Then the most general form for the μ-B-B transition amplitude is

$$\tilde{T}_{\mu B}(q;\,p,p') = i\,G(q^2,p^2,p'^2)\,\bar{u}(q)\,\gamma_5\,\tau_i u(p), \tag{III-10}$$

where G is a real function of the invariants, and in our case $q^2 = p^2 = M^2$; $\frac{1}{2}\tau_i$ is one of the generators of the infinitesimal transformation of the internal symmetry group and i denotes the quantum number of μ; when μ = pion, B = nucleon, in our normalization

$G^2(M^2,M^2,\mu^2)$ is just the π-N-N coupling constant $g^2 \approx 14.4 \times 4\pi$. Thus, from Fig. 4 (cf. Appendix B for our conventions),

$$\mathrm{Im}\ \tilde{T}_{\mu B}\Big|_{B\text{-pole}} = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3\ 2\ E_q}\ (2\pi)^4\ \delta^4(p - p' - q)$$

$$\times\ \tilde{T}^*_{\mu B}(q;\ p,p')\ \tilde{T}_{\mu B}(q;\ p,p'), \qquad \text{(III-11)}$$

a result which, when evaluated with Eq. (III-10) becomes (after spin averaged),

$$\mathrm{Im}\ \tilde{T}_{\mu B}\Big|_{B\text{-pole}} = -(\tau_f\tau_i)_{f=i}\ \pi\ G^2(M^2,M^2,p'^2)\ p'^2\ \delta(s_B - M^2). \qquad \text{(III-12)}$$

Notice that an "on-shell treatment" of $\mathrm{Im}\ \tilde{T}_{\mu B}\big|_{B\text{-pole}}$, setting $p'^2 = \mu^2$, would give the absurd result of a negative cross section for production of any even number of μ's. So we must make some off-shell continuation here. In order to agree with singly peripheral experiments, we shall keep the factor p'^2 and use a Dürr-Pilkuhn form factor[21] for the vertex function G^2:

$$G^2(M^2,M^2,p'^2) \approx g^2\ \frac{r^{-2}}{r^{-2} + u}\ , \qquad \text{(III-13)}$$

where $u = -p'^2$ as before, and r may be described as the "radius" of the μ-B-B vertex. This "radius" is expected to be roughly equal to the Compton wavelength of the lightest particle that μ and B can exchange. Therefore we have

$$\mathrm{Im}\ \tilde{T}_{\mu B}{}^{I}\Big|_{B\text{-pole}} = +\sum_y X^{Iy}\ z_y\ \pi\ g^2\ \frac{r^{-2}u}{r^{-2} + u}\ \delta(s_B - M^2), \qquad \text{(III-14)}$$

where z_y measures the contribution of the factor $(\tau_f\tau_i)$ to the amplitude with internal quantum number y in the direct channel.[10] Notice that $\mathrm{Im}\ \tilde{T}_{\mu B}{}^{I}\big|_{B\text{-pole}}$ becomes independent of u as $u \to \infty$.

2. The (μB) resonance contribution to Eq. (III-3) we approximate by taking only one prominent resonance:

$$\mathrm{Im}\ \tilde{T}_{\mu B}{}^{I}\Big|_{R} = \Lambda(s_B,\mu^2,M^2)\ X^{IR}\ \pi m_R\ x_R\ \Gamma_R\ \sigma^{e\ell,R}_{\mu B}(\max)\ \delta(s_B - m_R^2), \qquad \text{(III-15)}$$

where both $\mathrm{Im}\ \tilde{T}_{\mu B}{}^{I}\big|_R$ and $\sigma^{e\ell,R}_{\mu B}(\max)$ are referred to spin averaged quantities. This (μB) resonance contribution we evaluate on-shell, in the same sense as the $(\mu\mu)$ elastic resonance contribution given by Eq. (II-10).

Putting Eq. (III-14) and Eq. (III-15) into Eq. (III-9), we can calculate $\mathrm{Im}\ T_{\mu B}{}^{I}(s,\mu^2,M^2)$ explicitly. Now, recall that, as far as internal symmetry SU(n) is concerned, any elastic amplitude in the direct channel at high energy and at or near the forward direction is related to the crossed channel amplitude which corresponds to the identity representation by[11,5]

$$T_{\mu B} \underset{s\to\infty}{\sim} \frac{1}{N_\mu^{\frac{1}{2}}\ N_B^{\frac{1}{2}}}\ T_{\mu B}{}^{I}\ , \qquad \text{(III-16)}$$

when $T_{\mu B}{}^{I}$ dominates other amplitudes in the crossed channel. The factor $N_\mu^{-\frac{1}{2}}\ N_B^{-\frac{1}{2}}$ comes from s-t internal symmetry crossing matrix and N_μ (N_B) is the dimension of the subspace to which μ (B) belongs. Combining all these ingredients, we obtain the expression for the spin-averaged total cross section

$$\sigma_{\mu B}^{tot} = \frac{1}{\Lambda(s,\mu^2,M^2)} \operatorname{Im} T_{\mu B}$$

$$\underset{s\to\infty}{\sim} \frac{1}{N_\mu^{\frac{1}{2}} N_B^{\frac{1}{2}}} \sum_y X^{Iy} z_y \pi g^2 \frac{I_B^{\,1}}{m_c^{\,2}}$$

$$+ \frac{1}{N_\mu^{\frac{1}{2}} N_B^{\frac{1}{2}}} X^{IR} x_R \pi m_R \Gamma_R \sigma_{\mu B}^{R}(\max) \frac{I_R^{\,1}}{m_c^{\,2}} , \qquad \text{(III-17)}$$

where the dimensionless quantities

$$I_B^{\,1} = \frac{J(1) m_c^{\,4}}{(-2M^2)^2} \int du \frac{1}{u^2(u+m_c^{\,2})^2} \frac{u\, r^{-2}}{(u+r^{-2})} \{[u^2 + \hat{\gamma} u] - u(u^2 + 2\hat{\gamma}u)^{\frac{1}{2}}\}, \qquad \text{(III-18)}$$

$$= \frac{J(1)}{\gamma^2} \left\{ \frac{a^2}{(a^2-1)^2} \left[(\gamma - a^2) \log \frac{\gamma}{2a^2} + \frac{2\gamma a^2 - a^4}{(2\gamma a^2 - a^4)^{\frac{1}{2}}} \left(\sin^{-1}\left(1 - \frac{a^2}{\gamma}\right) + \sin^{-1}(1) \right) \right] \right.$$

$$- \frac{a^2}{(a^2-1)^2} \left[(\gamma - 1) \log \frac{\gamma}{2} + \frac{2\gamma - 1}{(2\gamma - 1)^{\frac{1}{2}}} \left(\sin^{-1}\left(1 - \frac{1}{\gamma}\right) + \sin^{-1}(1) \right) \right]$$

$$\left. + \frac{a^2}{(a^2-1)} \left[\gamma + \log\frac{\gamma}{2} - \frac{\gamma - 1}{(2\gamma-1)^{\frac{1}{2}}} \left(\sin^{-1}\left(1 - \frac{1}{\gamma}\right) + \sin^{-1}(1) \right) \right] \right\}, \qquad \text{(III-19)}$$

$$\underset{r^{-2}=m_c^{\,2}}{=} \frac{J(1)}{\gamma^2} \left\{ \frac{\gamma^2}{2\gamma - 1} - \frac{\gamma^2}{2(2\gamma-1)^{3/2}} \left(\sin^{-1}\left(1 - \frac{1}{\gamma}\right) + \sin^{-1}(1) \right) \right\}, \qquad \text{(III-20)}$$

with

$$\gamma \equiv \frac{\hat{\gamma}}{m_c^{\,2}} = \frac{2M^2}{m_c^{\,2}}, \qquad a \equiv \frac{r^{-2}}{m_c^{\,2}}, \quad \text{and}$$

$$I_R^{\,1} = \frac{J(1) m_c^{\,4}}{(-2M^2)^2} \int du \frac{\hat{\beta}}{u^2(u+m_c^{\,2})^2}$$

$$\times \{[u^2 + (\hat{\beta} + \hat{\gamma})u + \hat{\beta}^2] - (\hat{\beta} + u)(u^2 + 2\hat{\gamma}u + \hat{\beta}^2)^{\frac{1}{2}}\}, \qquad \text{(III-21)}$$

$$= \frac{J(1)\beta}{\gamma^2} \left\{ 2(\beta - \gamma) + (\beta + \gamma - 2\beta^2) \log \frac{\beta + \gamma}{2\beta^2} \right.$$

$$\left. + \frac{(-\beta - \gamma + 3\gamma\beta + \beta^2 - 2\beta^3)}{(2\gamma - 1 - \beta^2)^{\frac{1}{2}}} \left[\sin^{-1} \frac{\gamma - 1}{(\gamma^2 - \beta^2)^{\frac{1}{2}}} + \sin^{-1} \frac{\gamma - \beta^2}{(\gamma^2 - \beta^2)^{\frac{1}{2}}} \right] \right\} \qquad \text{(III-22)}$$

with

$$\beta = \frac{\hat{\beta}}{m_c^{\,2}} = \frac{m_R^{\,2}}{m_c^{\,2}} - \frac{M^2}{m_c^{\,2}}, \qquad \gamma = \frac{\hat{\gamma}}{m_c^{\,2}} = \frac{m_R^{\,2}}{m_c^{\,2}} + \frac{M^2}{m_c^{\,2}} .$$

It is noteworthy that the behaviors of the integrands in Eq. (III-18) and Eq. (III-21) are similar. In the two limits of integration:

$$\text{Integrand of Eq. (III-18)} \underset{u\to 0}{\longrightarrow} \frac{\hat{\gamma}}{m_c^{\,4}}, \quad \underset{u\to\infty}{\longrightarrow} \frac{\frac{1}{2} r^{-2} \hat{\gamma}^2}{u^4}; \quad \hat{\gamma} = 2M^2, \qquad \text{(III-23)}$$

$$\text{Integrand of Eq. (III-21)} \underset{u\to 0}{\longrightarrow} \frac{\frac{1}{2}\hat{\beta}\left(\frac{\hat{\gamma} - \hat{\beta}}{\hat{\beta}}\right)^2}{m_c^{\,4}}, \quad \underset{u\to\infty}{\longrightarrow} \frac{\frac{1}{2}\hat{\beta}(\hat{\gamma}^2 + \hat{\beta}^2)}{u^4};$$

$$\hat{\beta} = m_R^{\,2} - M^2, \qquad \hat{\gamma} = m_R^{\,2} + M^2. \qquad \text{(III-24)}$$

They both decrease monotonically from a finite value to zero, falling off asymptotically $\propto u^{-4}$. Thus B and R contribute to the last link with the same degree of "peripherality." Without the help of the form factor (III-13), the integrand of Eq. (III-18) would fall off like u^{-3} and thus make the B contribution much less peripheral than the R contribution.

IV. NUMERICAL VALUE OF THE ASYMPTOTIC $\sigma_{\mu B}^{tot}$ IN A SU(3) SYMMETRIC MODEL

We now evaluate the numerical value of $\sigma_{\mu B}^{tot}$ given by Eq. (III-17) when the internal symmetry group is SU(3). Obviously, besides the fact that this symmetry is rather badly broken, the continuing neglect of the μ mass in comparison with the other relevant masses seems not too sound. Nevertheless, as an illustrative example, we shall adopt the following as our "best input" for this hypothetical exact symmetry limit.

Let us consider μ, M, m_R to be the $0^-\underset{\sim}{8}(K,\pi,\eta,\bar{K})$, $\frac{1}{2}^+\underset{\sim}{8}(N,\Sigma,\Lambda,\Xi)$, and $\frac{3}{2}^+\underset{\approx}{10}(\Delta,\Sigma,\Xi,\Omega)$ respectively, and take m_c to be $m_V 1^-\underset{\sim}{8}(K^*,\rho,\phi,\bar{K}^*)$ for definiteness.[12] We ignore the mixing of $\underset{\sim}{1}$ and $\underset{\sim}{8}$ here. For the μ-B-B vertex, we have now,[13] instead of Eq. (III-10),

$$\tilde{T}_{\mu B}(q;\, p,p') = i\alpha G\bar{u}(q)\gamma_5 2D_i u(p) + i(1-\alpha)G\bar{u}(q)\gamma_5 2F_i u(p) \,. \quad \text{(IV-1)}$$

Thus

$$\operatorname{Im}\tilde{T}_{\mu B}{}^{I}\Big|_{B\text{-pole}} = \pi\sum_{y} X^{Iy}\{4[\alpha^2 D_f D_i + (1-\alpha)^2 F_f F_i$$

$$+ \alpha(1-\alpha)(D_f F_i + F_f D_i)]_{f=i}\}_y\, G^2 u\delta(s_B - M^2) \,, \qquad \text{(IV-2)}$$

where $y = 8_{ss}, 8_{aa}, 8_{sa}$, and 8_{as}, and $D_f D_i = \frac{5}{3}(\mathcal{P}_{8_{ss}})_{fi}$, $F_f F_i = 3(\mathcal{P}_{8_{aa}})_{fi}$ etc. (the $\mathcal{P}$'s are projection operators for the corresponding subspaces).

We use the D/F ratio $\alpha = 2/3$ and $g^2 = 14.4 \times 4\pi$. For the mass in each multiplet we use the first term in the corresponding Gell-Mann-Okubo mass formula.[14] Thus M = 1.15 GeV, m_R = 1.385 GeV, and $m_c = m_V = 0.852$ GeV. We furthermore choose $r^{-2} = m_c^2$.[15] For the width Γ_R of R, we use the calculated value from the SU(3) Chew-Low static model[16]

$$\Gamma_R = 2 \times \frac{2}{3} \times \frac{8}{3} \alpha(3 - 2\alpha) \frac{g^2}{4\pi} \frac{1}{4M^2} p_{CM}^3 \approx 0.107 \text{ GeV},$$[17]

and finally we put $x_R = 1$. Recall also that $N_\mu = 8$, $N_B = 8$, $X^{I8}{}_{ss} = X^{I8}{}_{aa} = 1$, $X^{I8}{}_{sa} = X^{8}{}_{as} = 0$ (thus $\Sigma_y z_y = \frac{116}{27}$), $X^{I10} = \frac{5}{4}$, and remember Eq. (III-4). Feeding all these ingredients into the right-hand side of Eq. (III-17), we obtain

$$\sigma_{\mu B}^{tot} \underset{s\to\infty}{\sim} 22.6 \text{ mb} + 4.1 \text{ mb} = 26.7 \text{ mb}. \qquad \text{(IV-3)}$$

The large B to R ratio mainly comes from the factors z_y which represent the crossing complication associated with internal quantum numbers.

Two further points deserve mention:

1. The B and R contributions are about equally peripheral. We can calculate the average u values in the integrals Eqs. (III-18) and (III-21). It is found that

$$\langle u\rangle_B = 0.54\, m_c^2 \text{ for the B contribution}, \qquad \text{(IV-4)}$$

$$\langle u\rangle_R = 0.41\, m_c^2 \text{ for the R contribution}, \qquad \text{(IV-5)}$$

where $m_c^2 = 0.729$ GeV^2.

2. The value of the integral Eq. (III-18) is not sensitive to our particular choice of $r^{-2} = m_c^2$, since the form factor $(r^{-2})/(r^{-2} + u)$ is close to unity in the small u region where the integrand contributes most to the integral, the tail of the integrand (with the factor

$\sim \frac{r^{-2}}{u}$) not contributing much. Indeed it can be shown that if we change r^{-2} by $\pm 50\%$ relative to m_c^2, then the value of the integrals changes only $\pm 10\%$. For example, choosing $r^{-2} = \frac{1}{2} m_c^2$, we find $\sigma_{\mu B}^{tot} \underset{s\to\infty}{\sim} (18.4 + 4.1)$mb, while $\langle u\rangle_B = 0.44\, m_c^2$ for the B contribution. On the other hand, had we used no form factor at all, we would have $\langle u\rangle_B = 3.3\, m_c^2$, evidently unacceptable for a model based on the assumed peripheral nature of high-energy reactions.

V. DISCUSSION AND CONCLUSION

We have treated the bound state pole and the resonance pole in the μB input kernel in an effectively parallel way, as depicted by Eq. (III-14) and Eq. (III-15). That these two contributions enjoy equivalent status in the contribution to $\sigma_{\mu B}^{tot}$ is shown in Eq. (III-17).

Our numerical result based on the SU(3) symmetric model of the last section is reasonably close to the values $\sigma_{\pi N}^{tot} \sim 21.0$ mb and $\sigma_{KN}^{tot} \sim 17.2$ mb projected by Barger and Cline.[18] This result, of course, crucially depends on the factor $N_\mu^{-\frac{1}{2}} N_B^{-\frac{1}{2}}$, i.e., on our knowledge of the multiplet structure of the μ and the B. An SU(2) symmetric model, ignoring the existence of strange mesons and baryons, would give a result for σ^{tot} almost an order of magnitude greater, in addition to suffering from the well-known difficulty that the $\pi\pi$ kernel is too weak to build up $\alpha_P(0) \approx 1$.[2,3]

In conclusion, although the model employed here is an over-simplification, we have again demonstrated, with multiperipheral dynamics, that it is conceivable, for the properties of a few low-lying states to control decisively the magnitude of total cross sections at very high energy.

ACKNOWLEDGMENTS

It is a pleasure to thank Professor G. F. Chew for invaluable advice and encouragement, and to Professor J. D. Jackson for conversations about the OPE model. The discussions with S. S. Shei and C. Sorensen were very helpful.

APPENDIX A. SINGLY-PERIPHERAL MODEL AT LOW ENERGY: SOME EXAMPLES

We discuss here briefly the off-shell problem in the singly-peripheral model. This question, by the method of construction, relates closely to the ABFST multiperipheral model. We will attempt to give support to the input μB kernel we used in the text.

The differential cross section for a two-body quasi-elastic reaction $ab \to 12$ via one pion exchange (OPE) is given by (cf. Fig. A-1)

$$\frac{d^3\sigma}{d\Delta^2\, ds_1\, ds_2} = \frac{1}{16\pi^3\, \Lambda^2(s,m_a{}^2,m_b{}^2)} \frac{V_1\, V_2}{(\Delta^2+\mu^2)^2}, \qquad \text{(A-1)}$$

where $\Delta^2 = -t$, the momentum transfer; Λ is the triangle function; and $V_1 = \mathrm{Im}\,\tilde{T}_{a\pi}(s_1,\Delta^2)$, the imaginary part of the forward elastic amplitude for the scattering of a particle (with mass $m_a{}^2$) and a pion (with mass $-\Delta^2$) at their CM energy square, s_1. A similar relation for V_2 is implied.

Knowing the spin (j_1) and parity of system number 1, one can calculate V_1 from lowest-order perturbation theory (the Born term model: BTM). In case particle a is spinless, for example, BTM gives

$$V_1{}^B = \left(\frac{\Lambda_{off}}{\Lambda_{on}}\right)^{2j_1} V_1{}^{on}, \qquad \text{(A-2)}$$

where $\Lambda_{off} = \Lambda(s_1,\mu^2,-\Delta^2)$, $\Lambda_{on} = \Lambda(s_1,\mu^2,\mu^2)$, and $V_1{}^{on}$ is the vertex if the exchanged pion were on-shell. It is well known that such a primitive OPE model is not sufficiently peripheral to cope with experimental data when $j_1 = 1$. Improvements can be made, e.g., by

introducing absorption corrections,[19] or Dürr-Pilkuhn form factors.[20] Here we shall consider the latter type only.

The Dürr-Pilkuhn (DP) method is to introduce in each vertex a form factor which depends on an additional parameter r (the "radius" of the vertex), e.g.,

$$V_1{}^{DP} = F_1(s_1,\mu^2,-\Delta^2;\, r)\left(\frac{\Lambda_{off}}{\Lambda_{on}}\right)^{2j_1} V_1{}^{on} \qquad \text{(A-3)}$$

such that $F_1 \xrightarrow[\Delta^2\to -\mu^2]{} 1$ and gives adequate damping as $\Delta^2 \to \infty$. We remark that the precise way to construct such form factors is not unambiguous.[20] In fact, more sophisticated form factors which decrease faster than those of DP for high values of momentum transfer were provided by Benecke and Dürr[21] (BD). G. Wolf[22] demonstrated the success of those form factors in a variety of reactions.

On the other hand, it seems empirically true that in many cases one can also achieve a fairly satisfactory description of the data by keeping the vertices on the mass shell of the exchanged pion. Such a simple recipe, already advocated by ABFST, may be used if we do not demand a high degree of accuracy and want to avoid the introduction of additional parameters. There are, however, exceptional situations. For example, when the emitted particle is p-wave and lies below the threshold of the pion and the incoming particle, the on-shell vertex is negative, e.g., the π-N-N vertex.[4] Then some form of off-shell continuation is inevitable.

We shall not go into this dilemma in detail but consider two experimental examples:

(1) $\pi^+ p \to \rho^0 \Delta^{++}$ at 4 GeV/c from Ref. 23.

Let us keep the vertices at ρ^0 and Δ^{++} on-shell and use the narrow width approximation

$$V_1 = \Lambda(s_1,\mu^2,\mu^2)\ \sigma(\rho^0 \to \pi^-\pi^+)$$
$$= \Lambda(m_\rho^2,\mu^2,\mu^2)\ \pi\ m_\rho\ \Gamma_\rho\ \frac{1}{2}\ \sigma^\rho(\max)\ \delta(s_1 - m_\rho^2), \qquad \text{(A-4)}$$

$$V_2 = \Lambda(s_2,\mu^2,M^2)\ \sigma(\Delta^{++} \to \pi^+ p)$$
$$= \Lambda(m_\Delta^2,\mu^2,\mu^2)\ \pi\ m_\Delta\ \Gamma_\Delta\ \sigma^\Delta(\max)\ \delta(s_2 - m_\Delta^2)\ , \qquad \text{(A-5)}$$

where $\sigma^\rho(\max) = 231$ mb, the unitarity limit on the isospin-1 amplitude. So our prediction for the differential cross section is

$$\frac{d\sigma}{d\Delta^2} = \frac{0.0855}{(\Delta^2 + 0.02)^2}\ \text{mb/GeV}^2, \qquad \text{(A-6)}$$

where we use $\mu^2 = 0.02$ GeV^2. The curve is shown in Fig. A-2, lying somewhat but not grossly higher than the data. Using a DP form factor for the ρ^0 vertex amounts to multiplying Eq. (A-6) by a factor

$$\left(\frac{4m_\rho^2\ r_\rho^{-2} + \Lambda_{on}^2}{4m_\rho^2\ r_\rho^{-2} + \Lambda_{off}^2}\right)\left(\frac{\Lambda_{off}}{\Lambda_{on}}\right)^2 . \qquad \text{(A-7)}$$

Then we will obtain even a larger $(d\sigma)/(d\Delta^2)$, since Eq. (A-7) is always larger than 1 in the physical region, no matter what the magnitude of the parameter r_ρ^{-2}. A similar argument holds if we use a DP form factor for the Δ^{++} vertex. If we use more strongly convergent

form factors, like those BD form factors employed by G. Wolf,[22] we can fit the experimental distribution somewhat better [cf. Fig. 6(c) of Ref. 22].

(2) $\pi^- p \to \rho^- p$ at 4 GeV/c from Ref. 24.

For $V_1(\rho^- \to \pi^0\pi^-)$, we use the expression in Eq. (A-4). For $V_2(p \to \pi^0 p)$, we notice that $V_2^B = \pi g^2 \Delta^2 \delta(s_2 - M^2)$ is not adequate. So we adopt a DP form factor for this vertex,

$$V_2 = \pi g^2 \Delta^2 \left(\frac{r_N^{-2} - \mu^2}{r_N^{-2} + \Delta^2}\right)\ \delta(s_2 - M^2). \qquad \text{(A-8)}$$

Choosing $r_N^{-2} = 10\ \mu^2 = 0.2$ GeV^2 (as they did in Ref. 20), and with $g^2 = 14.4 \times 4\pi$, the prediction for the differential cross section is

$$\frac{d\sigma}{d\Delta^2} = \frac{0.068\ \Delta^2}{(\Delta^2 + 0.2)(\Delta^2 + 0.02)^2}\ \text{mb/GeV}. \qquad \text{(A-9)}$$

This curve is shown as line I in Fig. A-3.

Thus the simple prescription employed in this paper gives satisfactory description of the magnitudes and shapes of the data in these two examples. Indeed a rationale for keeping vertices on-shell (whenever possible) has been given by the OPE-δ prescription of Williams.[26] We refer the reader to Williams' paper for details.

A more consistent approach to the singly-peripheral model and thus for the input of the multiperipheral model would seem to be to use a DP or BD form factor in each vertex so that every emitted particle is treated on the same footing. In practice, however, any two prescriptions for these vertices which both give adequate descriptions of singly-peripheral data will give essentially the same input for the multiperipheral model. Encouraged by the success shown above, and in view of the crudity of other aspects of our model, we believe that our hybrid approach to the vertices, motivated by simplicity, is justified.

APPENDIX B. THE CONVENTIONS WE USED AND THE EQUATIONS FOR THE INVARIANT AMPLITUDES

We have

$$S = 1 + i(2\pi)^4 \delta^4(p_f - p_i)T. \tag{B-1}$$

The phase space factor $(d^3p)/[(2\pi)^3 2E_p]$ is used for both bosons and fermions. Thus $\sigma^{tot} = (1/\Lambda)\mathrm{Im}\,T$ for all cases. For μB scattering

$$T_{\mu B} = \bar{u}(A + \gamma\cdot QB)u; \quad \text{with} \quad (\gamma\cdot p - M)u(p) = 0, \text{ and } \bar{u}u = 2M , \tag{B-2}$$

$$\mathrm{Im}\, T_{\mu B}(s,0) \underset{s\to\infty}{\sim} [2M\, \mathrm{Im}\, A(s,0) + s\, \mathrm{Im}\, B(s,0)]. \tag{B-3}$$

Instead of Eq. (III-1), we have two separate equations for the invariant amplitudes. At $t = 0$, we have

$$\mathrm{Im}\, A^I(s,0) \sim s^\alpha \int_0^1 dx\, x^\alpha\, \theta(x - x^*) \int_0^1 \frac{du\, J(\alpha)\, m_c^2}{u^2(u + m_c^2)^{\alpha+1}} \times \int ds_B[\mathrm{Im}\, \tilde{A}^I - Mx\, \mathrm{Im}\, \tilde{B}^I], \tag{B-4}$$

$$\mathrm{Im}\, B^I(s,0) \sim s^{\alpha-1} \int_0^1 dx\, x^\alpha\, \theta(x - x^*) \int_0^1 \frac{du\, J(\alpha)\, m_c^2}{u^2(u + m_c^2)^{\alpha+1}} \times \int ds_B[(\hat{\beta} + u + 2M^2x)\mathrm{Im}\, \tilde{B}^I], \tag{B-5}$$

where

$$x^* = \frac{(\hat{\beta} + u) - (\hat{\beta}^2 + 2\hat{\gamma}u + u^2)^{\frac{1}{2}}}{(-2M^2)}, \quad x = \frac{\hat{s}}{s}, \quad \hat{\beta} = s_B - M^2,$$

$$\hat{\gamma} = s_B + M^2 . \tag{B-6}$$

1. For the B-pole contribution:

$$\mathrm{Im}\,\tilde{A}^{I} = 0\,,$$
$$\mathrm{Im}\,\tilde{B}^{I} = \sum_{y} X^{Iy} z_y\, \pi\, G^2\, \delta(s_B - M^2). \tag{B-7}$$

2. For the (μB) resonance, R contribution,[27]

$$\mathrm{Im}\,\tilde{A}^{I} = X^{IR}\left\{4\pi\left[\frac{3(W+M)}{(E+M)} + \frac{(W-M)}{(E-M)}\right] \mathrm{Im}\, f_R\right\}, \tag{B-8}$$

$$\mathrm{Im}\,\tilde{B}^{I} = X^{IR}\left\{4\pi\left[\frac{3}{(E+M)} - \frac{1}{(E-M)}\right] \mathrm{Im}\, f_R\right\}, \tag{B-9}$$

where $W = s_B^{\frac{1}{2}}$ and E is the baryon energy in the CM system of the μ (here off-shell) and the baryon. We have used the fact that R is a $\mathcal{J}^P = 3/2^+$ resonance, and, in the narrow resonance approximation we have

$$\mathrm{Im}\, f_R = \frac{p_{CM}}{4\pi} \frac{\pi\, m_R\, \Gamma_R\, \sigma^{e\ell,R}(\max)}{2} \delta(s_B - m_R^2). \tag{B-10}$$

Substituting Eqs. (B-7) - (B-10) into Eq. (B-4) and Eq. (B-5), we can calculate $\mathrm{Im}\, A^I$ and $\mathrm{Im}\, B^I$ separately at high energy.

On the other hand, we can recapture our previous formula Eq. (III-7) by using relation (B-3). Notice, however, that we have to put $(W \pm M)^2 = 2W(E \pm M) - u$ in Eqs. (B-8) and (B-9) [or, alternatively, $2M\,\mathrm{Im}\,\tilde{A} + (\hat{\beta} + u)\,\mathrm{Im}\,\tilde{B} = \mathrm{Im}\,\tilde{T}_{\mu B}$], in order get exact agreement.

FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U. S. Atomic Energy Commission.

1. L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1962); D. Amati, A. Stanghellini, and S. Fubini, ibid. 26, 6 (1962).
2. G. F. Chew, T. Rogers, and D. Snider, Phys. Rev. D2, 765 (1970).
3. D. M. Tow, Phys. Rev. D2, 154 (1970).
4. See, for example, the Chew-Low extrapolation formula: G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).
5. H. D. I. Abarbanel, G. F. Chew, M. L. Goldberger, and L. M. Saunders, Phys. Rev. Letters 25, 1735 (1970).
6. The one-side off-shell dependence of this solution is essentially the same as that of the "big u" solution obtained by ABFST. We can see this, for example, in Eq. (III-7). The great improvements achieved in this new work are, of course, the determination of the overall scale from the inhomogeneous term, and the inclusion of the crossing symmetry factor.
7. In comparison with the Schizophrenic Pomeron Model of G. F. Chew and D. Snider, Phys. Rev. D3, 420 (1971).
8. We use the modern version of this model formulated in Ref. 2. But we shall neglect the high-energy parts of the inputs in this "first approximation."
9. The terms within the bracket are due to S. S. Shei, Lawrence Radiation Laboratory Report UCRL-20203 (to be published in Phys. Rev.).

10. For example, $z_{\frac{1}{2}} = 3$ in a SU(2) symmetric theory.

11. D. Amati, L. L. Foldy, A. Stanghellini, and L. Van Hove, Nuovo Cimento 32, 1686 (1964).

12. One can interpret Eq. (II-10) as actually representing a series of $(\mu\mu)$ resonances. For example, if we put $\mathrm{Im}\,\tilde{T}_{\mu\mu}{}^{I} = \sum_i m_i{}^2 R_i{}^I \delta(s_0 - m_i{}^2)$, then for the $\mu\mu$ amplitude when $\tau = \tau' = \mu^2$, we have, from Eq. (II-12), the relation $(\Sigma R_i{}^I)/(m_c{}^2) = \sum_i (R_i{}^I)/(m_i{}^2)$, i.e., we replace "all" those $(\mu\mu)$ resonances with a "center of $(\mu\mu)$ resonances" at position m_c with gross strength $R_c{}^I = \sum_i R_i{}^I$ as depicted by the relation above. This "resonance" certainly has a larger strength and likely a higher mass than m_V. The input seems then more realistic and the output cross sections become smaller. But there are drawbacks: (i) Obviously we need in general another value of $m_c{}^2$ for the μB problem since the weighting relation is different from that given above, (ii) the indeterminacy of these values of m_c's and (iii) the use of a higher value of $m_c{}^2$ will increase the mean invariant momentum transfer in each link.

13. Our conventions and crossing matrix etc., can be found e.g., in P. Carruthers, Introduction to Unitary Symmetry (Interscience Publishers, New York, 1969).

14. For example, $M = M_0 + M_1 Y + M_2[2 - 2I(I+1) + \frac{1}{2} Y^2]$ for B $\underset{\sim}{8}$. Such assignment of M_0 as the mass in the "exact symmetry limit" is not unambiguous.

15. A value of r^{-2} ranging from $5\,\mu^2$ to $20\,\mu^2$ is used in most data fittings. See further discussion about this choice below.

16. In the sense of R. E. Cutkosky, Ann. Phys. 23, 415 (1963); R. H. Capps, Nuovo Cimento 27, 1208 (1963); A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963), Nuovo Cimento 31, 1324 (1964). The static model is, of course, consistent with our neglect of the μ mass here. On the other hand, if we take the mass formulas seriously, then $\mu = 0.412$ GeV and thus R lies below the (μB) threshold in the "exact symmetry limit." This was noted by R. J. Oakes and C. N. Yang, Phys. Rev. Letters 11, 174 (1963). In any case, we may defend the plausibility of the width Γ_R calculated here as corresponding to the most commonly observed magnitude of (μB) resonance widths. Furthermore the value of the factor $\pi\, m_R\, \Gamma_R\, \sigma^R(\max)$ computed in our way turns out to be nearly the same as that computed from the observed values of the corresponding quantities for $\Delta(1236)$. Therefore it seems likely that we have not underestimated this factor in the (μB) resonance contribution.

17. From our choice of the values of α and g^2, and using the observed masses and phase space, the calculated width of $\Delta(1236)$ is $\Gamma_{\Delta \to N\pi} \approx 0.1$ GeV. The decay channel $\Delta \to \Sigma K$, which represents half of the total decay probability with SU(3) symmetry, is closed.

18. V. Barger et al., Nucl. Phys. B5, 411 (1968), or Barger and Cline, Phenomenological Theories of High Energy Scattering (W. A. Benjamin, Inc. Publishers, New York, 1969).

19. See, for example, J. D. Jackson, Rev. Mod. Phys. 37, 484 (1965).

20. H. P. Dürr and H. Pilkuhn, Nuovo Cimento 40, 899 (1965).

21. J. Benecke and H. P. Dürr, Nuovo Cimento 56, 269 (1968).

22. G. Wolf, Phys. Rev. 182, 1538 (1969).

23. Aachen-Berlin-Birmingham-Bonn-Hamburg-London-München Collaboration, Nuovo Cimento 35, 659 (1965).

24. Aachen-Berlin-Birmingham-Bonn-Hamburg-London-München Collaboration, Nuovo Cimento 31, 729 (1964).

25. J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. 139, B428 (1965).

26. P. K. Williams, Phys. Rev. 181, 1963 (1969). The author gave an on-shell prediction, not for the differential cross section, but for the density matrix element of the ρ in the reaction $\pi^-p \to \rho^-p$ at 3 GeV/c.

27. In our convention, the relations between the invariant amplitudes and the conventional amplitudes f_1, f_2 remain the same as that in G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1337 (1957).

FIGURE CAPTIONS

Fig. 1. Schematic representation of $\sigma_{\pi N}$'s.

Fig. 2. Production amplitude of $\mu\mu$ scattering.

Fig. 3. Production amplitude of μB scattering of

(a) even numbers of final μ's, and

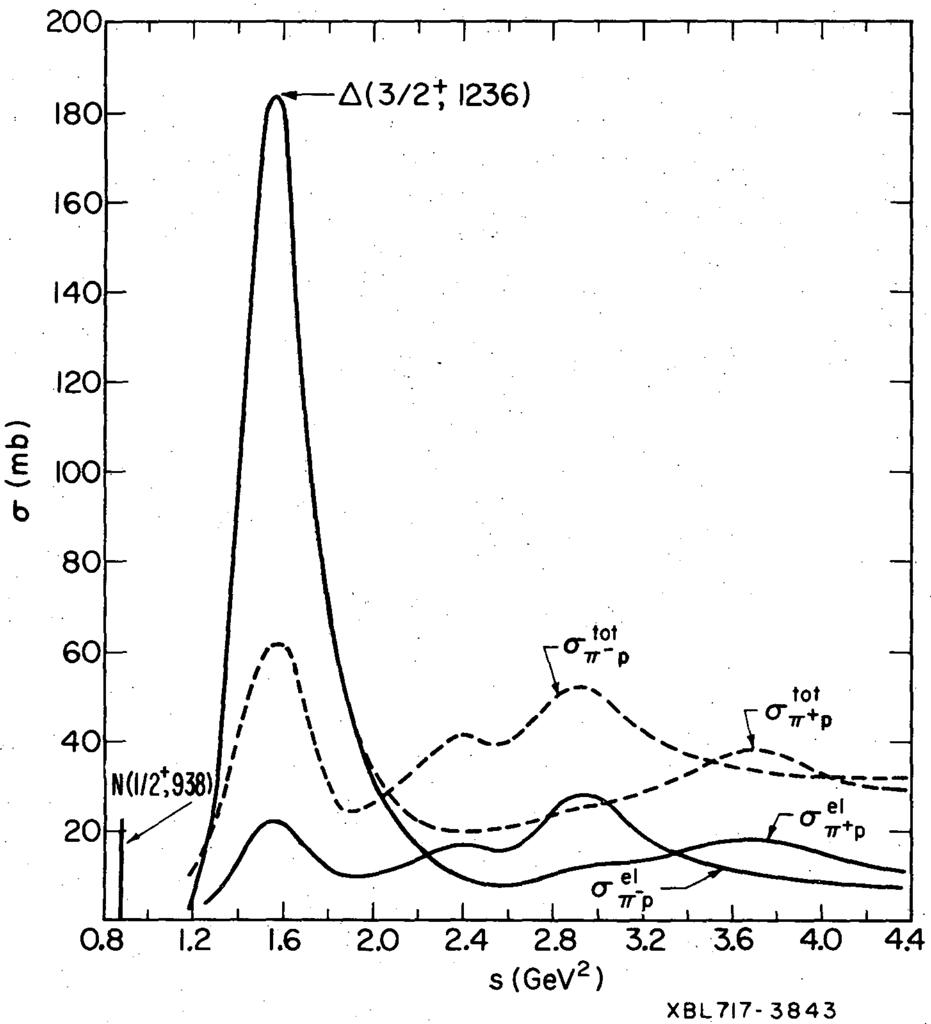
(b) odd numbers of final μ's.

Fig. 4. B-pole contribution to the unitarity sum.

Fig. A-1. Single peripheral model for two-body quasi-elastic scattering.

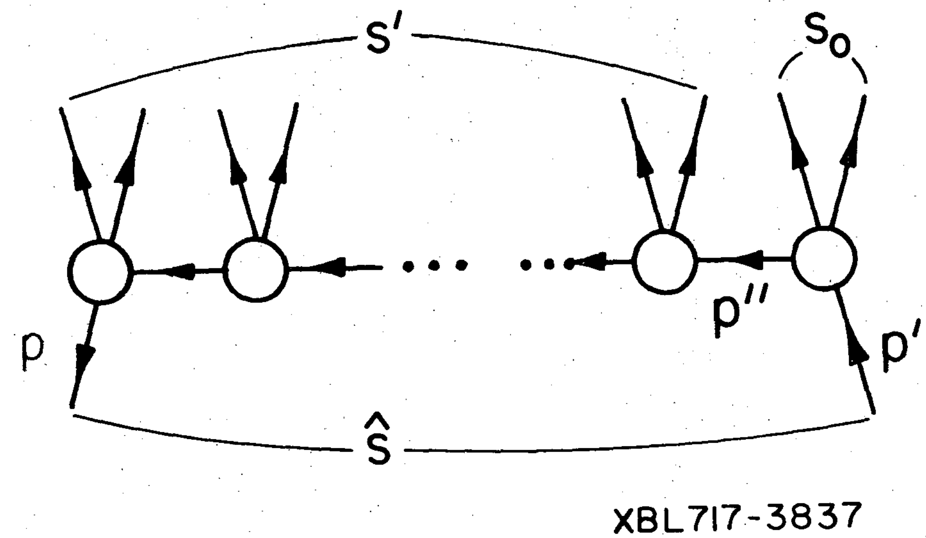
Fig. A-2. Differential cross section of $\pi^+p \to \rho^0\Delta^{++}$ at 4 GeV/c, taken from Ref. 23. The sharp theoretical lower limit to Δ^2, not present in experimental data, is due to our use of a sharp mass for each of the resonances. Notice that this figure should not be compared with Fig. 4 of Ref. 20 or Fig. 13 of Ref. 19, since those figures are fits to a distribution with arbitrary normalization.

Fig. A-3. Differential cross section of $\pi^-p \to \rho^-p$ at 4 GeV/c. The data are taken from Ref. 24. The figure is adapted from Ref. 25. Curve I is from Eq. (A-9). Curve II is the prediction from a full DP model, i.e., in addition to Eq. (A-8), a form factor Eq. (A-7) is also used in the ρ^- vertex with $r_\rho^{-2} = 10\ \mu^2$. Curve III is the prediction from OPE with absorption correction from Ref. 25. Curve IV is the prediction from the unmodified Born term model.



200
180
160
140
120
100
80
60
40
20
σ (mb)
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4.0
4.4
s (GeV²)
Δ(3/2⁺, 1236)
N(1/2⁺, 938)
σ tot π⁻p
σ tot π⁺p
σ el π⁺p
σ el π⁻p
XBL717-3843

Fig. 1



s′
s₀
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Fig. 2

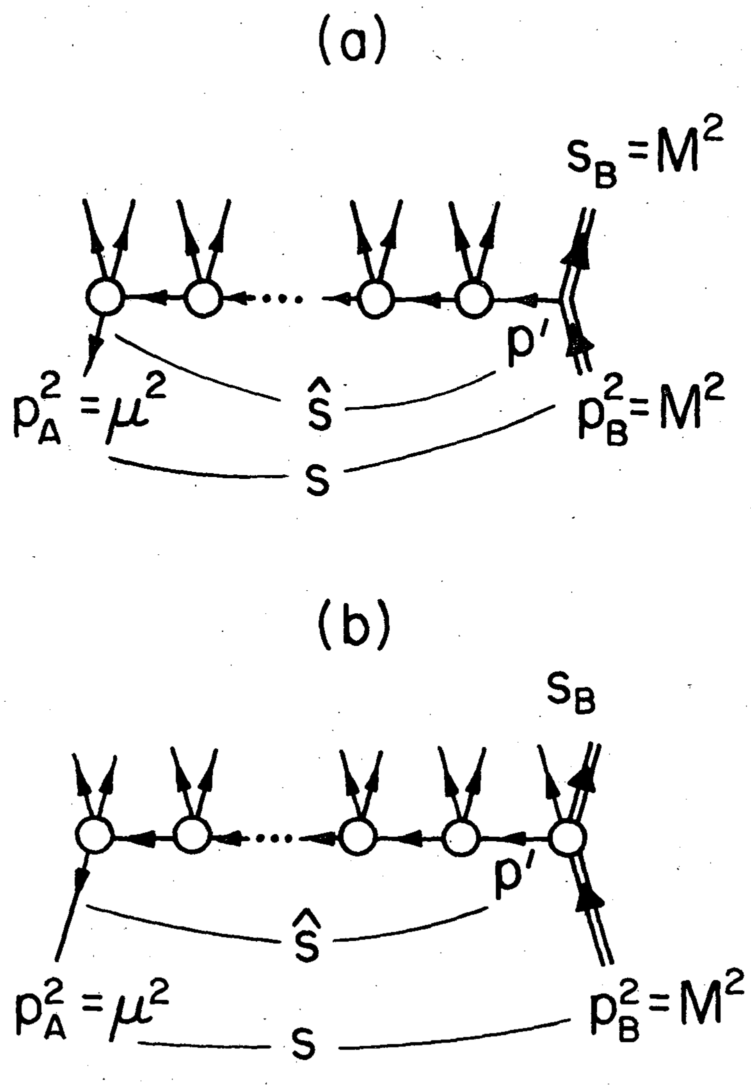

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Fig. 3

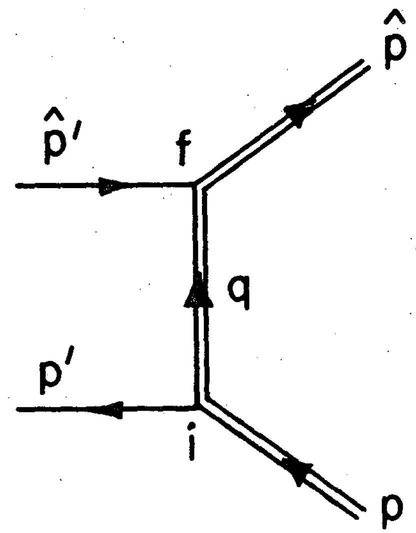

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Fig. 4

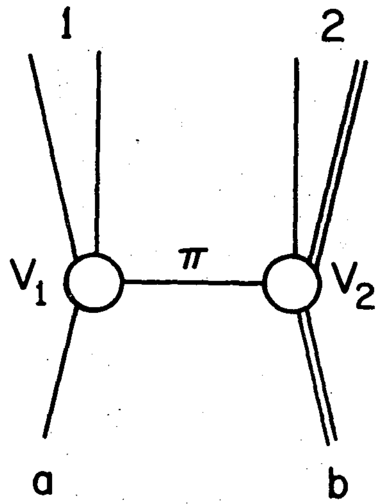

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Fig. A-1

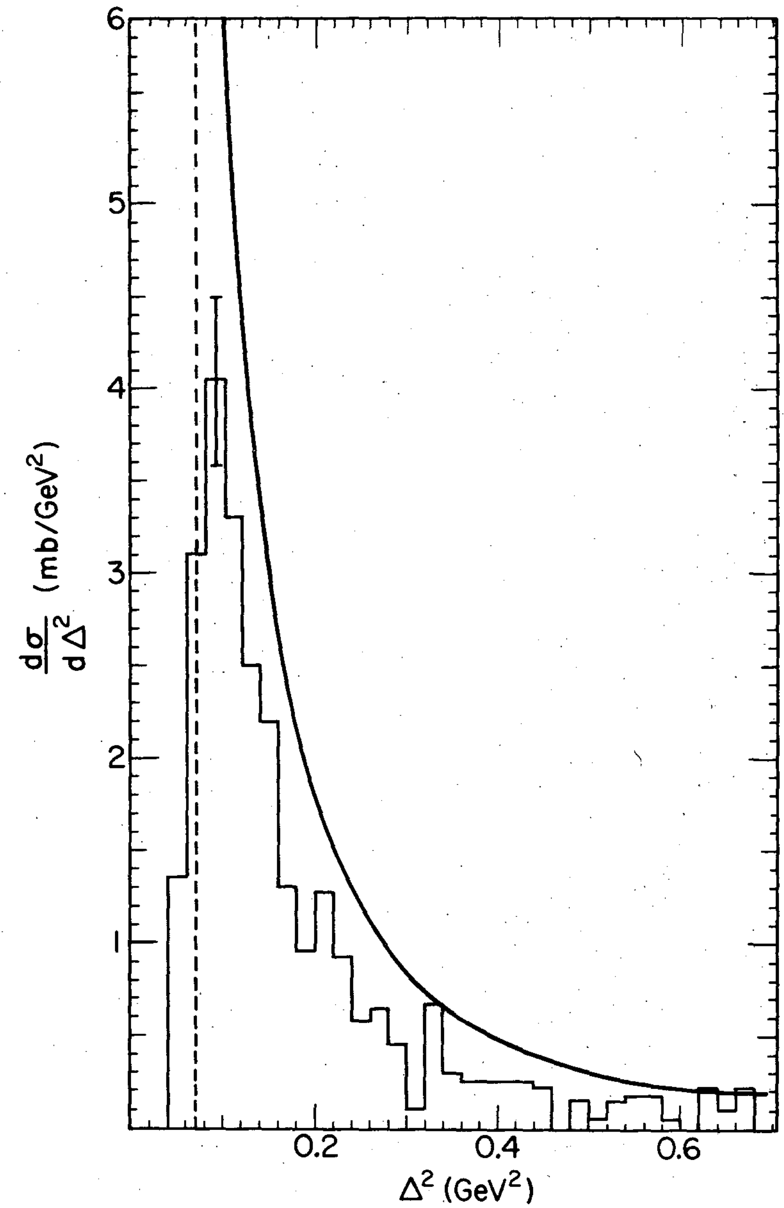

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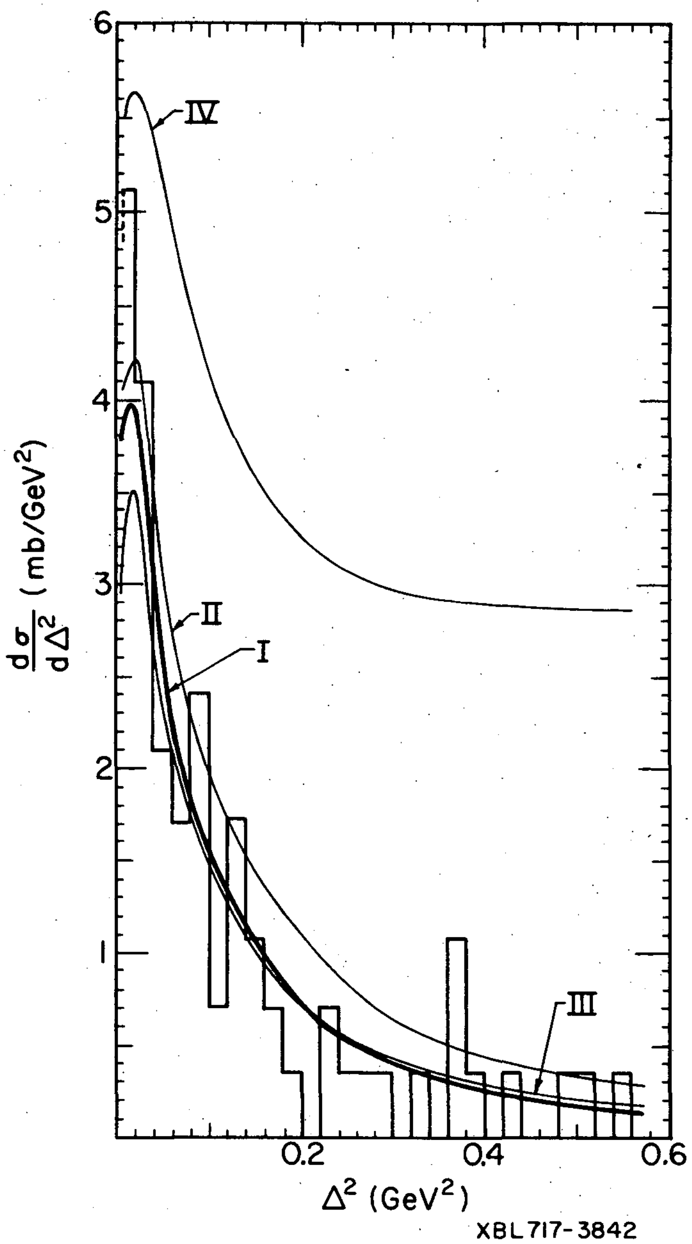
Fig. A-2



6
5
4
3
2
1
0.2
0.4
0.6
IV
II
I
III
$\frac{d\sigma}{d\Delta^2}$ (mb/GeV^2)
Δ^2 (GeV^2)
XBL717-3842

Fig. A-3

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