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ESSAYS ON EFFICACY OF MACROECONOMIC POLICIES

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Abstract

Essays on Efficacy of Macroeconomic Policies

by

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This research analyzes efficacy of the macroeconomic policies and the role of policymakers to deal with a recessionary case. In particular, I focus on the instruments policymakers have in hand, to stimulate the economic activity. It does the quantitative multiplier analysis for economies with various forms of financial market imperfections to provide a greater degree of realism into macroeconomic modeling. The research is inspired by the claim that if individual incentives for decisions, regarding risk taking and providing liquidity, differ from what is socially optimal, then there is a 'prima facie' case for a policy intervention. This difference between interests primarily comes from the fact that 'asset prices' and 'liquidity' are usually considered as public goods. The papers focus on interaction of the zero-lower-bound constraint on nominal interest rates with some form of financial friction such as a credit friction or household heterogeneity, or an overlapping generations setup. This thesis mostly focuses on the short-term period in a recessionary case. In a deflationary case such as the 2008 crisis, output is demand determined and demand is not sufficient; so policies should focus on tools to stimulate the demand that is needed. The first two chapters are directly related to this idea, while the latter one has a longer focus.

The first chapter analyzes efficacy of a fiscal policy tool, a tax cut in particular, in a liquidity trap scenario where monetary expansion is ineffective. It basically answers a question, as to: when the zero-lower-bound is binding and the conventional monetary policy is not working, whether the discretionary fiscal policy is really ineffective as has recently been argued. The second chapter focuses on unconventional monetary policy in a closed economy and researches a question as to whether certain assumptions regarding constraints and rigidities amplify or mitigate the macroeconomic or real effects of unconventional monetary policy. The third chapter examines the macroeconomic effects of a social security reform. It analyzes contributions from different forms of changes; quantifies the macroeconomic implications of various reforms. Scenario analysis reveal positive effects for labor supply, capital stock and output to the reform implemented.

To my family,

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Chapter 1

Introduction

In a deflationary period such as the Great Recession of 2008, where the zero-lower-bound (ZLB) was hit, policymakers and in particular a central-bank (CB) has an important constraint against its fight with the recession or deflation. However, this does not mean there is really nothing policymakers (e.g. the central bank) can do. One solution to avoid output losses (real costs) is targeting a higher expected inflation such that the natural rate of interest (real interest rate) goes to negative at the zero nominal interest rate (in case of a shock). Krugman (1998) offers an inflation target of like 4 percent to decrease real interest rate and increase the demand and spending. However, such a high inflation causes welfare losses (causes distortions in the real economic activity and therefore is inefficient). Eggertsson and Woodford (2003), for instance, offers commitment to future monetary policy to increase inflation expectations. This paper analyzes efficacy of a number fiscal, unconventional monetary policy and structural reforms. I use a model based on RBC theory, and as in the RBC literature, I use the method of calibration to work out the detailed numerical example of the theory. The chapters of the thesis provide micro foundations, as in RBC model, for macro models.

This research analyzes efficacy of the macroeconomic policies and the role of policymakers to deal with a recessionary case. In particular, I focus on the tools and policies they have in hand, and how effective those policies are to stimulate the economic activity. It does the quantitative multiplier analysis for economies with various forms of financial market imperfections to provide a greater degree of realism into macroeconomic modeling. In that sense, the very basic and standard financial markets are perfect assumption is relaxed with the introduction of some form of financial frictions.

The research is inspired from Rajan's (2005) claim that if individual incentives (for decisions regarding risk taking and providing liquidity) differ from what is socially optimal, then there is a 'prima facie' case for a policy intervention. This difference between interests primarily comes from the fact that 'asset prices' and 'liquidity' are usually considered as public goods.

The papers focus on interaction of the zero-lower-bound constraint on nominal interest rates with some form of financial friction such as a credit friction or household heterogeneity, or an overlapping generations setup. This thesis mostly focuses on the short-term period in a recessionary case. In a deflationary case such as the 2008 crisis, output is demand determined and demand is not sufficient; so policies should focus on tools to stimulate the demand that is needed. The first two chapters are directly related to this idea, while the latter one has a longer focus.

Rather than a relatively less interventionist solution, such as managing expectations or incentives (e.g. asking managers to invest part of their capital in assets they are managing) - which is also difficult and takes time to build such a reputation, chapters of this dissertation focus on discretionary policies, e.g. the tools policymakers are able to use to intervene in the market directly.

All the three chapters of my research focus on different policy instruments at the discretion of policymakers to deal with (certain issue such as) deflationary situations such as the 2008-10 financial crisis (in line with the great depression, the Japanese recession, or the most recent European recession). In particular, I focus on efficacy of these policies and check which of the policy instruments are more effective to stimulate the economic activity.

Part I

Efficacy of Fiscal Policy

Chapter 2

Efficacy of a Fiscal Policy Change

Executive Summary

This paper aims to provide a better understanding of the efficacy of fiscal policy and distortionary-tax cuts in a zero interest rate environment. The paper uses a standard New-Keynesian model, but allows for heterogeneity in consumption behavior by including Keynesian (rule-of-thumb) households that consume their current after tax income. The paper studies how the fraction of the Keynesian households interacting with nominal rigidities, in an economy with distortionary taxes, changes the effectiveness of countercyclical fiscal policy. As a starting point, the model employs labor-income tax cuts to analyze the effectiveness of tax cuts for recovery. Further, the model employs a range of other distortionary taxes (such as income tax and sales tax changes then, as has been offered by many economists during the 2008 crisis) for a richer fiscal policy setup, and the automatic stabilizers analysis as well as the financing method. I look

if the estimated effects change in a more realistic taxation and household set-up where distortionary taxes interact with fraction of the Keynesian agents.

"Fiscal policy ... seems to have been an unsuccessful recovery device in the thirties - not because it did not work, but because it was not tried."

Cary Brown (1956)

2.1 Introduction

Following the outbreak of the Great Recession, the newly elected Obama administration in the US initially announced a three-year fiscal stimulus package, in 2008, to stimulate the aggregate demand; though it was mostly deemed insufficient. This paper aims to better quantify the real effects of similar fiscal stimulus packages. It analyzes whether the fiscal policy itself is able to reverse the output collapse in a liquidity trap. I try to measure the quantitative effects of fiscal shocks.[1] The paper provides a critical analysis on the empirical evidence on the effectiveness of fiscal policy shocks. In particular, I ask whether the evidence based on post-WWII data (that the FP ineffective) is relevant for a liquidity trap case such as the 2008 crisis. All these questions are more relevant today than they have ever been; in particularly, after the policies implemented post the Great Recession, such as the American Recovery Act that was passed in January 2009.

Although the full tax-smoothing prescriptions, due originally to Barro (1979), have mostly been found relevant in most of the developed countries (and for the U.S.

[1]The *Fiscal Multiplier* (or just the multiplier) is a measure that shows by how much GDP (or any other measure of output) responds to a tax change or a change in government spending (a fiscal variable or a fiscal policy shock), a one percent change as a fraction of GDP. For example: How much, in dollars, does GDP change if government expenditure increases by one dollar. $multiplier = \frac{\Delta GDP}{\Delta G}$.

federal tax rates), which Talvi and Vegh (2004) calls irresponsive fiscal policy over business cycles, particularly after the 2008 financial crisis, severe discussions on tax-cuts has reemerged. This paper discusses the efficacy of distortionary tax cuts in a more realistic model with constrained agents and nominal rigidities. The economy has a group of agents that do not use (or do not have access to) the financial and capital markets. As will be discussed below, this is important to get a positive demand effect after a tax cut. The paper studies how the fiscal multipliers found in the literature on the liquidity trap vary by existence of the Keynesian households.[2]

This paper aims to provide a better understanding of the efficacy of changes in fiscal policy in a recessionary environment that represents the 2008 financial crisis. It considers a banking shock to put the economy into a recession. Output is demand determined in a liquidity trap (as in sticky price models, output adjusts to the demand in the economy) and demand is usually not adequate. This is the main problem and policies aimed at increasing production capacity or the potential output level as in the Neoclassical theory, would not be directly relevant in a zero interest rate case. The paper is mainly concerned with the question of whether fiscal policy can reverse an output collapse in a recession such as 2008. Labor-income tax cuts are used as an example of a distortionary tax cut (starting point). I then analyze some other controversial tax cuts that have recently been proposed, such as capital income taxes and consumption taxes.

This paper uses a standard New-Keynesian dynamic stochastic general equi-

[2]A liquidity trap is a case where conventional monetary policy is ineffective in increasing demand or dealing with deflation because the standard tool, short-term nominal interest rate (i.e. the federal funds rate or the overnight nominal interest rate), is down against the zero lower bound constraint.

librium (DSGE) model, but extends it by adding households with heterogenous consumption behavior to study the effectiveness of the fiscal stabilizers in a liquidity trap.[3] I assume there are two kinds of households: a Keynesian (rule-of-thumb) household that consumes his current after tax income - due to Campbell and Mankiw (1989) - and a Ricardian household whose consumption decisions follow the permanent income hypothesis - due to Friedman (1957).[4] This model set-up is motivated by significant empirical and theoretical evidence, see e.g. Mankiw (2000) and Gali et al. (2007) among others, for heterogenous consumption-saving decisions and varying effects of fiscal policy in a liquidity trap.[5] The paper studies how the fraction of Keynesian households interacting with nominal rigidities, in an economy with distortionary taxes, changes the effectiveness of countercyclical fiscal policy. This model considers the fiscal policy effects from automatic stabilizers and those from labor-income tax cut that has been debated widely among leading economists, including Mankiw (2008), Hall and Woodward (2008), Barro (2009) and Feldstein (2009) among other.

Given a government expenditure shock or tax cut, the Ricardian agents (inter-temporally optimizing) increase their labor supply and decrease their consumption considering future taxes (needed to satisfy the inter-temporal GBC). The Keynesian agents, on the other hand, are not much concerned with future taxes. Including sticky prices (and wages), real wages (or real income) goes up (or at least a smaller decline is ob-

[3]The NK DSGE models have recently (after 2000) been very common, particularly in policy analysis, in policy institutions and the academic world to find the best policy and offer (the new normative macroeconomics in Taylor 2000, and Eggertsson (2010)). They allow showing policies explicitly.

[4]I use the term 'Keynesian' because consumption of these agents is proportional to current income in the Keynesian models, while in the traditional classical DSGE models it is proportional to the wealth (hence the need to use the inter-temporal budget constraint).

[5]As argued by, among others, Feldstein (2009), Erceg and Linde (2010), Eggertsson (2010).

served), as is discussed in section 1.2, and labor income increases. Increasing labor income, raises consumption and therefore demand in the economy. Therefore, including the Keynesian agents eliminates some of the negative wealth and substitution effect (of fiscal expansion) observed in the traditional Neoclassical models. This is why Gali et al. (2007) argue the combination of the existence of hand-to-mouth (Keynesian) agents, nominal rigidity and deficit financing is required for a positive demand effect following a fiscal shock. Having Keynesian agents causes larger multiplier effects as in the traditional Keynesian models, because in particular, the Keynesian agents' consumption goes up as it is depended upon the current income. Thus the aggregate consumption will not be crowded out.

Theoretically, adding the Keynesian agents affects the two demand equations in my model. The fraction of the Keynesian households shows up in the aggregate households' consumption Euler equation (showing off direct demand effect) and the firms' investment Euler equation (by change in labor supply and marginal product of labor). By including nominal rigidities and hand-to-mouth agents (via the direct demand effect) into the model, I primarily focus on the positive effect from this countercyclical discretionary fiscal policy that has been controversial in recent studies. As a mater of fact, given these changes, and compared to the benchmark Eggertsson (2010) model, the paper finds significant effects for consumption and labor-income taxes.

2.1.1 The case for tax cuts

Temporary one-time tax-cuts (tax rebates) have been offered by many economists recently. They have also been used, both by the classical supply side Reagan and Bush and the demand side leader president Obama; and have the advantage of being implemented instantaneously. One problem is, they could be saved instead of being spent. Yet they are mostly found to increase the after-tax income of households and result in high demand increase, see e.g. Gali et al. (2007) for a discussion of findings in Parker (1999) and Johnson et al. (2004) and support for another tax-cut.[6] In an empirical analysis of the European economy, Forni et al. (2009) use a similar model set up and quarterly data from the Euro area and find significant effects from labor-income and consumption tax cut on consumption and output. Capital income tax cuts, on the other hand, increase investment and output in a longer period (medium-run). His consumption taxes are VAT, though, unlike sales taxes in the U.S.[7]

Feldstein (2002 and 2009) and Barro (2009), for instance, offer tax cuts on capital and labor income and taxes on firm's profits in order to stimulate the economic activity. Mankiw (2008) and Robert Hall and Susan Woodward, on the other hand, called on for labor-income tax cuts by the end of 2008. Meanwhile, Christiano et al. (2009), Eggertsson (2010), and Erceg and Linde (2010) argue that the efficacy of

[6]It should be noted that, although some, as Feldstein (2009), argue the May-June tax rebate in 2008, had a much lower marginal propensity to consume (MPC) compared to the others in the post-1980 period, that tax cut was permanent. President Obama promised a permanent 500 dollar tax-cut per worker per annum (total of 70 billion).

[7]Uhlig and Drautzburg (2011), on the other hand, compare effectiveness of multiplier effects of distortionary taxes and lump-sum taxes. They find similar short-run multipliers for both, while in the long-run multipliers from distortionary taxes decrease substantially (to almost -1 compared to over-1 for lump-sum transfers).

fiscal policy changes is changing substantially in the zero nominal interest rate case.[8] However, classicals such as Barro (2009) would say this is only an excuse for the use of old Keynesian prescriptions. Taylor (2000), on the other hand, claims discretionary fiscal policy could also be appropriate for long-term issues. For example, reducing the marginal tax rates is helpful for long-term growth and economic efficiency. This goes back to the same point stated by Barro (2009) and Feldstein (2009). For instance, permanent capital tax cuts could increase investment and capital stock and hence output level in the steady state (under normal circumstances).

In Japan, the newly elected Abe government increased its consumption taxes from %5 to %8 in April 2014.[9] Another increase is also planned in the future (maybe in Spring of 2017, to about 10 percent). In 1997, when it was raised, it was reported to negatively affect the economy. But this time, they plan to accompany this increase with a fiscal stimulus package worth \$70bn. Additionally, as Krugman (1998) and Eggertsson (2010) discuss, in Japan, traditional government expenditure increases that were used since 1992 have been ineffective and the focus has shifted to tax cut offers.

On the other hand, Romer and Romer (2007), and Mountford and Uhlig (2009) find much higher multipliers for exogenous distortionary tax cuts than the multipliers for government expenditures. Burnside, Eichenbaum and Fisher (2003) and Mountford and Uhlig (2009) find that private consumption does not change much in response to

[8]Indeed, as discussed in the monetary chapter, Del Negro et al. (2010) show even nonstandard monetary policy actions has large effects in a zero short-term nominal interest rate case with nominal rigidities in both price and wage.

[9]In October 2013, Abe decided to impose a massive tax hike on consumers (doubling of Japans consumption tax) beginning in April 2014.

government expenditure increases. All findings in these papers, regarding response of C, I and real wage to the government expenditure changes, are not consistent with what the standard Keynesian theory would suggest.

Moreover, in explaining management of expectations regarding future policy, Eggertsson and Woodford (2003) argue, when zero bound binds, one policy that can help managing expectations is cutting taxes and financing it by issuing nominal debt. Another option is, cutting taxes and financing it by printing money (considering inflation is a tax)[10].

Bils and Klenow (2008) discuss labor tax cuts as a stabilizer in a recessionary case. As in Gali et al. (2007) and Bils and Klenow (2008), business cycle accounting theorem in Chari et al. (2007) explains the process for a decrease in employment with increase in distortions between intra-temporal consumption-leisure MRS and MPL, i.e. the Hall residual. See discussion in nominal rigidities below. Chari, Kehoe and McGrattan (2007) find dominance of labor wedges (from labor tax or sticky wages or prices, i.e Hall residual), together with the efficiency/productivity wedges, in causing most of the fluctuations in real activity. This model is therefore consistent with findings and suggestions in Chari et all. (2007). Uhlig and Drautzburg (2011), using a similar model setup with constrained agents, evaluate fiscal multipliers for the 2009 fiscal stimulus in the US (ARRA). They find fiscal multipliers around 0.52 for the short-run and -0.42 in the long-run. If the Keynesians have a very low discount rate, as in my model, the fiscal stimulus transfers some of the wealth to the Keynesians (negative welfare effects

[10] Walsh (2010) chapter 4 and Wickens (2008) chapter 5.

for the Ricardians).

I focus on income tax and sales tax changes, as an example of distortionary tax cuts. Cutting labor income taxes has been widely discussed recently (in the context of 2008 crisis) and offered by many economists including Barro (2009), Feldstein (2002 and 2009), Hall and Woodward (2008) and Mankiw (2008). Most of these discussions were in their blogs and were limited to policy discussions without a concrete model. The theoretical studies were missing. This paper is an attempt to understand its efficacy theoretically as there is still a limited number of papers on the issue, particularly for the case of a liquidity trap. This tax-cuts has been found contractionary in Eggertsson (2010) again for a liquidity trap environment.[11] In contrast to my paper, the labor tax in Eggertsson (2010) is a payroll-tax paid by firms. It acts more like a VAT (value-added-tax). It has been analyzed and offered by Eggertsson and Woodford (2004) and Feldstein (2002) for European countries and Japan, respectively.

2.1.2 Why do we need nominal rigidities?

The existence of the Keynesian agents, itself, is not enough to capture the positive demand effect according to Gali et al. (2007). This is briefly explained in the log-linearized equation below that shows the relationship between the (aggregate) marginal product of labor (MPL) and the marginal rate of substitution (MRS), that

[11] In Eggertsson's (2010) model, a reduction in labor tax stimulates deflationary pressures through its effect on firms' marginal cost. That is, people start working more, which decreases real wages and therefore the marginal cost of production for firms. With the decreasing marginal cost, firms start producing more and prices go down. Deflationary expectations increase the real interest rate, but the Fed is not able to respond since the federal funds rate is already at the zero bound. A higher real interest rate decreases demand in the economy.

would be equal to each other, absent nominal rigidity and perfect competition as in RBC models.[12]

$MPL = \frac{U_L}{U_C}$

where L is leisure and C is private consumption. The idea is that it is not possible to explain simultaneous changes (drops) in consumption and employment, in the above equation, during a recession by movements in productivity of labor (or wage movements) due originally to Mankiw, Rotemberg and Summers (1985).

$M\hat{P}L_s = \hat{\mu}_s + M\hat{R}S_s,$

with

$M\hat{R}S_s = \sigma_u \hat{C}_s + \eta \hat{N}_s$, and $\sigma_u > 0$, $\eta > 0$

where in (log-linear form) $M\hat{P}L_s$ is the marginal product of labor, $\hat{\mu}_s$ is the wedge between the marginal product of labor (MPL) and the marginal rate of substitution (MRS) that comes from monopolistic competition, and wage and price rigidities (if constant, frictionless, then only from monopolistic competition), $M\hat{R}S_s$ is the marginal rate of substitution between labor and consumption, $\hat{C}_s$ is aggregate consumption and $\hat{N}_s$ is labor supply.

In the case of a labor tax cut (as in an increase in G), the nominal wage from work goes up and that increases labor supply (theory and evidence supports, both SVAR and NE models above, Gali et al. (2007) and Eggertsson (2010)) and the latter is followed by a marginal product of labor (MPL) fall.[13] If we had a constant wedge

[12] As I mentioned above, consumption and labor income taxes also distort this relationship and therefore have the same function as this markup coming from monopolistic competition and sticky wage and prices.

[13] Why does labor demand increase? Here, I assume, as in the standard NK models, firms are commit-

(or zero as in standard RBC models) then consumption would have to go down to have equality in the above equation. However, by assuming imperfect competition and nominal rigidities in both goods and labor markets I allow the wedge to go down, such that consumption does not have to fall. This means nominal rigidities are necessary in this case.

It is crucial to get a positive co-movement of consumption and real wages, due originally to a 1992 paper by Rotemberg and Woodford, in a theoretical model. This is because high real wages are an empirical reality. Although some papers using the standard RBC models and some empirical papers using the narrative approach find decreasing real wages as a response to a fiscal expansion (as in Ramey and Shapiro (1998)), most of the empirical papers using the SVAR method find an increasing real wage. Examples include Fatas and Mihov (2001) and Gali et al. (2007).

2.1.3 Heterogeneity in consumption behavior

As discussed above, since standard models with inter-temporally optimizing agents alone are not able to capture the positive response of consumption to a fiscal shock, there is clearly a gap between the empirical evidence and the (NK or RBC) literature (especially for C and real wage responses).[14] Hence, heterogeneity in consumption behavior is needed (is necessary along with nominal rigidities) in the standard New-Keynesian models. This paper allows for existence of constrained agents, households in

ted to supply any amount of good demanded at the price they set. They have to increase their demand for labor, thus, in order to increase their production. They set a price of their goods and supply any amount that is demanded at that price. Households are also supplying any labor that is demanded since real wage will always be higher than the MRS as I will assume later.

[14] See Gali et al. (2007) and references therein.

particular, in addition to the basic setup in a standard New-Keynesian model.

The problem with a basic set-up that includes only the inter-temporally optimizing agents is that we are ignoring a significant positive direct effect on demand and making a pretty strong assumption (simplification) about the consumption behavior of the agents. This, as was discussed earlier, causes discrepancies between forecasts of standard DSGE models (that are widely used in policy analysis) and findings in empirical evidence (Gali et al., 2007). Meanwhile, because of the decreased participation in the financial markets after the crisis, if the argument of Gali et al. (2007) and Bilbiie et al. (2005) about the declining fiscal multipliers in empirical studies caused by the rising fraction of Ricardians is true, then the financial crisis in 2008 may be of particular importance for analyzing the effect of the fraction of Keynesian agents that might have increased. What I mean is that the fraction of people having access to the financial markets to smooth their consumption most probably have fallen. Ilzetzki et al. (2010) consider this change more in terms of monetary policy change, where my argument is that fraction of the Keynesian agents might also be changing.

The idea of the rule-of-thumb consumers is from a 1989 paper by Campbell and Mankiw, Campbell and Mankiw (1989). They employ some households that follow the permanent income hypothesis of Friedman (1957) and some others that consume their current disposable income only (which they call the rule-of-thumb of consumption) and find that half of the income goes to the rule-of-thumb consumers. They provide evidence for the importance of the Keynesian (rule-of-thumb) households and heterogeneity in consumption-saving decisions in major economies. As for why they behave in the Keynesian fashion: Gali et al. (2007) and Campbell and Mankiw (1989) claim a fraction of agents do not smooth consumption in response to labor-income changes or do inter-temporal substitution for interest rate movements; while Uhlig and Drautzburg (2011) argue either because their discount factor is very small such that they do not want to smooth consumption by capital or bond accumulation or because they are not able to borrow due to high risks of default.

By adding the rule-of-thumb agents, I look at the direct effects from the income and substitution effects of the fiscal policy changes on spending in addition to the indirect effects revealed in the Eggertsson (2010) model. For instance, the Keynesian agents eliminate some of the negative wealth and substitution effects from future lump-sum taxes. The case for direct effects of tax-cuts on aggregate demand is motivated by Gali et al. (2007) who show that when Keynesian households are added to the model, consumption and therefore demand (key issue in the short-run for a deflationary situation) increase in response to fiscal shocks.

Eggertsson and Krugman (2010), on the other hand, use a model where some agents are debt constrained (as the Keynesians in my model) and a deleveraging shock to the economy, and show that they both results in depression (fall in aggregate demand) since agents are not able to consume due to high debt payments. They show that making some agents debt constrained is very helpful in understanding most of the disputed propositions from mostly the Keynesian economics including effective expansionary fiscal policy and very high multipliers for fiscal shocks. They find the fiscal multipliers positively related to share of the debtor agents.[15] Consumption of debtors in Eggertsson and Krugman (2010) model is also depended on current income, at the margin. This causes larger multiplier effects as in the traditional Keynesian models.

As rightly pointed out by Stiglitz (2002) the fact that individuals don't behave rationally is well known in practice and even in some theory (experimental, imperfect information). The rational expectations theory (which assumes all agents have the same information, act rationally, markets are perfectly efficient, unemployment never exists, credit rationing (crunch) never happens) is not applicable anymore (or in practice). Adding the Keynesian aspect with the constrained households in Uhlig and Drautzburg (2011) shows that; very high negative long-run multipliers in Uhlig (2010b), due to distortionary tax increases in the long-run to finance short-run debt financed expenditures, are going down to slightly negative numbers.

2.1.4 Relation to the literature

This paper analyzes the effective role of the countercyclical fiscal policy argued in the standard Keynesian models in the special case that conventional monetary policy is not effective. The paper deviates from the ad-hoc nature of lump-sum taxes and focuses on distortionary tax cuts that have been popular in policy discussions lately. The model includes a variety of taxes that distort choices of households with heterogenous consumption behavior. It considers the income and substitution effects of fiscal changes for different households. I expect to have better estimates for the effective role of the

[15] For a horizontal SR-AS curve: With a share of 1/3, they find multiplier equal to 1.5; and with share of debt-constrained agents being 1/2, they get a multiplier equal to 2.

countercyclical fiscal policy, particularly for the case that policy-makers have only the fiscal policy instruments.

This paper studies a liquidity trap case where the conventional monetary policy is not effective, demand is low and the economy experiences deflation. I study how effective fiscal policy is in a heterogenous agents model with distortionary taxes in a zero-interest rate environment. The paper aims to contribute to the existing literature in several ways. First, it is related to Eggertsson and Krugman (2010), Forni et al. (2009), Gali et al. (2007), Carroll (1997), Mankiw (2000) and Campbell and Mankiw (1989) in that it considers heterogenous households (including some NonRicardian and some other Ricardian households) with different consumption-saving behaviors. I use this household setup to particularly consider distortionary taxes and study a liquidity trap case to look at the efficacy of discretionary fiscal policy. This is why I include heterogeneity in consumption behavior and allow for the existence of Keynesian agents with the direct demand effect (direct spending effect from tax-cuts).

Secondly, this paper is related to literature on the effectiveness of fiscal policy and the quantitative measures of this effect. Christiano (2004), Christiano et al. (2009), Eggertsson (2010), and Erceg and Linde (2010), as well as Romer and Bernstein (2009) argue that the multipliers are changing substantially in the zero nominal interest rate case, there are many problems related to use of public spending that causes inefficiencies (as has been discussed at the beginning). Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2011) Woodford (2011) Carlstrom, Fuerst, and Paustian (2012) all find large fiscal multipliers, and multipliers that increase with the duration of fiscal expansion, in new-Keynesian zero-bound models. However, Barro (2009) says this is only an excuse for use of the old Keynesian prescriptions. Barro (2009) and Feldstein (2002 and 2009) are in favor of stimulating the economic activity via private sector support, by some tax changes for instance. This paper argues that instead of inefficient government expenditure increases, the government should use its tax policy to substitute for the interest rate instrument. It is consistent with the idea of Barro (2009), Feldstein (2002), Hall and Woodward (2008) and Correia et al. (2010) in that sense.

Third, the paper is related to a literature studying distortionary taxes. Correia et al. (2010) and Feldstein (2009) argue that tax policy is very flexible in a recession, such as 2008, due to the need for the use of fiscal tools. Tax cuts on labor income and capital are found to be leading to further deterioration (contraction) according to some NK analysis, such as Eggertsson (2010). Feldstein (2002), Feldstein (2009) and Barro (2009) offered capital income tax cuts for the U.S. economy and Japan economy. Mankiw (2008) and Woodward and Hall (2008) offered labor tax cuts again for the U.S. Adding the Keynesian households, as in Mankiw (2000) and Gali et al (2007), the tax cuts such as those on wage or capital may have positive direct effects on spending and aggregate demand. Moreover, most of the papers discussed above were policy discussions more than theoretical analysis. This paper is an attempt to see the theoretical validity of these offers.

Lastly, this paper is related to a line of papers that show the economy is in a liquidity trap such that open-market-economies are irrelevant. Wallace (1981), Krugman (1998), Curdia and Woodford (2011) and Eggertsson and Woodford (2003)

- if expectations are not changed. This paper is most closely related to Eggertsson and Woodford (2003 and 2004), Christiano (2004), Eggertsson (2010), Christiano et al. (2009), Erceg and Linde (2010), Feldstein (2002), and Romer and Bernstein (2009) who analyze effective policy in a zero-interest rate case. In contrast to my paper, all of these papers consider only the Ricardian households. They find an increasing multiplier effect for mainly government expenditure shocks. I use a range of distortionary and lump-sum taxes.

The New-Keynesian (NK) dynamic stochastic general equilibrium (DSGE) model is built on Chari et al. (2000), Christiano (2004), Woodford (2003), Smets and Wouters (2007), Gali et al. (2007) and Eggertsson (2010). The paper puts the heterogeneity idea of Gali et al. (2007) into the zero-interest rate framework of Eggertsson and Woodford (2003).

2.2 The Model

I use a real business cyclye (RBC) model with a Dixit-Stiglitz (1977) monopolistic competition framework among firms and workers in the goods and labor markets respectively, and Calvo (1983) type nominal rigidities in firms' price setting and fixed wages[16]. Given these nominal frictions and markups, (actual) labor and output will be demand determined. In other words, firms and workers commit to supply any amount of goods and labor demanded at the prices set by the corresponding agents. All agents in the model, except the CB that follows the Taylor rule, are optimizing. Up to this point, this is a standard New-Keynesian DSGE model. However, I add some Keynesian households to the model. The economy has two types of households therefore: Keynesian (rule-of-thumb) households and Ricardian (inter-temporally optimizing) households. The households are accompanied by a continuum of intermediate good producers and a representative final good producer. Additionally, there is a central bank conducting monetary policy and a government as the fiscal authority. Time is discrete and the only uncertainty comes from an aggregate banking shock (ξ). I assume a complete asset market and the economy is cashless (as is common in the NK literature. Therefore, I ignore the costs of inflation associated with the inflation tax resulting from deviations from the Friedman rule).

[16]I make these changes to a RBC model because, as Gali et al. (2007) show, the standard RBC models are not able to capture the positive response of private consumption to a fiscal shock that exists in the empirical analysis.

2.2.1 Households' problem

I assume a continuum of households, indexed by $j \in [0,1]$, who are monopolistically competitive in their labor supply as in Erceg et al. (2000). The growth rate of population is zero and the population is normalized to one. Each household is infinitely-lived and provides a differentiated labor service $l_t(j)$ to the (single, economy-wide) factor market.[17]. $'f'$ fraction of households are Keynesian and do not have access to the capital and financial markets.[18] They consume only their current after tax (disposable) income. The remaining '$1-f$' fraction are Ricardian. They buy and sell assets and use capital to smooth their consumption inter-temporally. The two type of households differ in their rate of time preference (most basically with $\beta^r >> \beta^k$). The Keynesian and the Ricardian households are uniformly distributed across labor types.[19] I assume real wages are always higher than the mrs, therefore agents always provide any amount of labor demanded by firms, as in Gali et al. (2007).

Following Erceg et al. (2000) and Forni et al. (2009), I assume an (zero-profit) employment agency (labor aggregator agency) combines all the imperfectly substitutable labor supply provided by different household in accordance with firms' demand and creates homogenous labor inputs for firms.[20] The employment agency's demand for

[17]Whereas in Woodford (2003), Christiano (2004) and alike - Eggertsson (2010) etc, each household provides every type of labor.

[18]See footnote 5 for the reason.

[19]Or both Ricardian and Keynesian households supply labor of any type j and demand for a differentiated labor type j is uniformly distributed among these households, R and K.

[20]This is in contrast with Gali et al. (2007), and Chari, Kehoe and McGrattan (2007) that assume a continuum of labor unions (as a continuum of hhs and represented by j) that are monopolistically competitive (consistent with labor markets where labor-unions are powerful - as in Europe). Each of these unions set their own wages and each of them represents all households who supply a specific type of labor. Gali et al. (2007) and Forni at al. (2009) consider an alternative labor market structure for robustness check. In this setup the Keynesians do not necessarily change their labor supply in the same

each particular labor type will be equal to the total demand for that labor type by all firms. The aggregate labor index has the Dixit-Stiglitz (1977) form as in equation (1).

$$L_s = \left[\int_0^1 L_s(j)^{\theta_l} dj\right]^{\frac{1}{\theta_l}}, \tag{2.1}$$

where $0 < \theta_l < 1$ and $L_s(j)$ is labor supply of type j and unit cost of labor demand is W_s. I implicitly assume that firms allocate their labor demand uniformly across the continuum of labor types.

The employment agency takes $W_s(j)$ - the nominal wage chosen by the Ricardian households (consistent with labor market in the US) - and W_s as given and maximizes profit (or minimizes the cost of producing the aggregate labor index) in the same way as firms demanding labor, below, subject to L_s equation above and with respect $L_s(j)$, as in Forni et al. (2009) or the labor packers in Smets and Wouters (2007). In other words, it minimizes the cost of producing the aggregate-labor-index L_s demanded by firms. The Aggregator's problem is below.

$\max_{L_s(j)} \Pi_s = W_s L_s - \int_0^1 W_s(j) L_s(j) dj = W_s \int_0^1 l_s(i) di - \int_0^1 W_s(j) L_s(j) dj$

The cost minimization (or profit maximization) problem for the labor aggregator agency

way as the Ricardians and a union representing both the Ricardians and the Keynesians sets wages in a monopolistically competitive labor market, by maximizing weighted average of utility of the two type of households. This means a common wage and labor supply for both types. However, the differences in their results are insignificant. It therefore makes sense to stick to the this structure and assume the same labor supply and same average wages. Moreover, Gali et al. (2007) shows in the log-linearized form both labor market structures give even the same linear equation (Appendix A of Gali et al. (2007), under the assumption of the same SS consumptions).
The aggregator's labor demand in my model, L_t, will be equal to the firms' labor demand, N_t.
See also Uhlig and Drautzburg (2011) that also assume Calvo type differentiated sticky wages set by unions. They assume wages are set by maximizing utility of the unconstrained households, however, under the assumption that they represent the majority in unions. Firms hire labor from both types randomly again and labor supply will be the same for both types in the equilibrium.

results in the following overall demand, across all firms, for household j's labor (the employment agency's demand for that particular labor type $L_s(j)$).

$$L_s(j) = \left[\frac{W_s(j)}{W_s}\right]^{\frac{-1}{1-\theta_l}} L_s \tag{2.2}$$

The wage-aggregator for the labor index L_s, by using equation (2) in (1) is below.

$$W_s = \left[\int_0^1 W_s(j)^{\frac{\theta_l}{(\theta_l-1)}} dj\right]^{\frac{(\theta_l-1)}{\theta_l}} \tag{2.3}$$

Households' decision problem is two stages. The first step is cost minimization and the second step is utility maximization. The final-good producer, we will see later, deals with the first step by minimizing the cost of producing a composite consumption good $C_s^h(j)$, $h = r,\ k$ ('k' for Keynesian and 'r' for Ricardians). I show that part below for illustration only. This means households face the following cost minimization problem (given that $c_s(i)$ is consumption of goods of type i).

$\min_{c_s(i)} \int_0^1 P_s(i)c_s(i)di$

subject to achieving a Dixit-Stiglitz aggregate consumption level C_s^h, where

$$\int_0^1 N_s(j)C_s^h(j)dj = \left[\int_0^1 (c_s(i))^\theta di\right]^{\frac{1}{\theta}}, \tag{2.4}$$

where 0< θ <1. And their total consumption, C, provided by all of the firms, is a Dixit-Stiglitz composite consumption index.

P$_s = \left[\int_0^1 (P_s(i))^{\frac{\theta}{(\theta-1)}} di\right]^{\frac{(\theta-1)}{\theta}}$

is the corresponding Dixit-Stiglitz composite price index (the same as equation 22 below).

A fraction, $'1-f'$, of households hold bonds and own a share of firms. They buy composite consumption goods, and supply labor to the single economy-wide labor market. All of the households earn an after-tax labor income $'(1-\tau_s^w)W_s(j)L_s(j)'$. The agents decide $W_s(j)$ - the nominal wage - endogenously. I assume all capital is owned and accumulated by firms and therefore the Ricardian agents in particular. Hence, capital does not show up in the household problem.[21] Firms are owned by the Ricardian households, hence, all the after-tax profits goes to these households as $(1-\tau_s^P)Z_s(j)$. The utility maximization problem of a Ricardian agent j is below. They maximize the expected present discounted value of their inter-temporal utility with respect to (wrt) $C_s^r(j)$, $B_{s+1}(j)$ (and $l_s^r(j)$, if needed).

$$\max_{\{C_s^r(j),L_s^r(j),B_{s+1}(j)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \xi_s \left[U(C_s^r(j), G_s^{not}, l_s^r(j)) \right]^{22} \tag{2.5}$$

I assume a separable period utility function (in C, L and G) for agents. This is both for simplicity and to account for importance of the fraction of rule-of-tumb behavior as in Campbell and Mankiw (1989) - as they use separable utility. As discussed by Gali et al. (2007), a non-separable utility function might have some other implications that do not guarantee a significant fraction of Keynesian behavior in consumption (See Basu and Kimball (2002) in their references). The utility function is a common labor-leisure decision utility function consistent with stylized facts (balanced growth path). All agents have the following identical period utility function.

$\mathrm{U}(\mathrm{C}_s^r(j), G_s, l_s^r(j)) = \frac{(C_s^r(j))^{1-\sigma_u}}{1-\sigma_u} + b_1 \frac{(G_s)^{1-b}}{1-b} - \frac{(l_s^r(j))^{1+\eta}}{1+\eta}$[23], [24]

[21] I follow Christiano et al. (2005) - footnote 8 - and assume that it does not matter whether capital is endogenously accumulated by households or firms.

[22] The preference shock functions as in Christiano et al. (2009), Eggertsson (2010) and Correia et al. (2011). It affects the consumption Euler equation, but not the MRS between C_s and L_s.

[23] Or simply: u is increasing and concave in C and G; v is increasing and convex in L.

[24] The exogenous government expenditure is separable from private consumption to make sure it does not distort inter-temporal decision of households, as in Eggertsson and Krugman (2010) and Eggerts-

The Ricardian agents maximize the utility function (5) subject to the inter-temporal version of the following period budget constraint for each household,

$$(1+\tau_s^c)P_sC_s^r(j)+R_s^{-1}B_{s+1}^r(j) \tag{2.6}$$

$$=(1\text{-}\tau_s^a)B_s(j)+(1-\tau_s^p)Z_s(j)+(1-\tau_s^w)W_s(j)l_s^r(j)-P_sT_s^r(j)$$

where $R_s = (1 + i_s)$ is the gross nominal interest rate (R_s^{-1} is price of the riskless nominal bond at time s). Households take the tax rates, prices, transfers (or taxes) from government and all the aggregates as given and maximize utility subject to the budget constraint (and the demand function for their labor, if needed). In other words, they maximize inter-temporal utility function (5), subject to equations (6) (the period budget constraint) - and (2) (demand for its labor supply). Each household has monopoly power over his/her nominal wage $'W_t(j)'$ such that he/she resets his wage at the end of the contract periods (which have random durations). This process is analogous to the price setting process for firms for their output, which we will see in the next section. $0 < \beta < 1$ is the discount factor, and $L_s(j)$ is amount of household-specific labor supplied. Agents get wage $W_s(j)$ for the labor supplied. ξ_s is a preference shock representing the banking crisis.[25] B_s, the beginning of period bond holding, is a one period risk-less bond issued by the government. $Z_s(j)$ ($\int_0^1 Z_s(i)di = \int_0^1 Z_s(j)dj$) is lump-sum profit distributed between households (households have the same share of firms).

son (2010). I assume, all the government expenditure consists of expenditures non-substitutable with private consumption (such as infrastructure and military spending), i.e. $G_s = G_s^n$. It's a Dixit-Stiglitz aggregator analogous to C_s: $G_s = \left[\int_0^1 (g_s(i))^\theta di\right]^{\frac{1}{\theta}}$. Government expenditure perfectly substitutable with private consumption, G_s^p, has proven to have no effect on equilibrium outcomes, See, e.g., discussion in Eggertsson (2010).

[25] If it was a simple taste shock (basic demand shock), it would only show up with C_t (Erceg and Linde (2010), Walsh (2010)); here it is a (general) preference shock.

T_s^h, for $h = k, r$ are lump-sum taxes (or transfers if negative) from the government. The distortionary taxes will be: a sales tax τ_s^c, a labor-income (paid-by household) tax τ_s^w, financial asset (savings) tax τ_s^a, investment tax credit τ_s^I and profit tax τ_s^p. Because I am particularly interested in how tax cuts affect households' behavior in the two models, I use the same differentiation between capital (that affects households' consumption/saving behavior) and profit taxes (that affects firms' investment/hiring and pricing behavior) as in Eggertsson (2010). Capital taxes, τ_s^a, in my model follow the similar set up in Eggertsson (2010).[26] E_s is conditional expectation based on information available at time s.

The first-order necessary conditions for the Ricardian households' optimality (for the rational expectations equilibrium) are derived below by maximizing the utility function subject to the households inter-temporal budget constraint with respect to their choice variables $C_s^r(j)$, $l_s^r(j)$, B_{s+1}, and $Z_s(j)$. Note that I assume wages are a markup over the mrs due to the monopolistic competition.

An Euler equation (EE) for inter-temporal consumption allocation for the Ricardian household (the Keynesian households don't have inter-temporal consumption/saving decisions), which links the marginal cost of consumption today to the ex-

[26]Eggertsson (2010) uses a tax on the stock of savings instead of taxing nominal capital income, which we observe in practice. So, τ_s^a is a tax on the capital/financial stock of households in his model. This is because the tax on nominal capital income, τ_s^k, is zero in a zero-interest rate environment. He rescales τ_s^a such that 1 percent variation in τ_s^a equals to a change in tax equivalent to a 1 percent variation in tax on capital income in steady state. In other words, a tax cut that is equal to a 1 percent fall in capital income tax in steady state.

pected marginal benefit of consumption in the future period, is below.[27]

$$U_{c,s} = R_s(1-\tau^A_{s+1})\beta E_t U_{c,s+1}\frac{\xi_{s+1}}{\xi_s}\frac{P_s}{P_{s+1}}\frac{1+\tau^c_s}{1+\tau^c_{s+1}} \tag{2.7}$$

(Assuming wages are not set by households, there is also) an optimality condition that sets marginal rate of substitution between leisure and consumption to real wage with taxes (an intra-temporal EE for labor):

$$\frac{1-\tau^w_s}{1+\tau^c_s}\frac{W_s(j)}{P_s} = \mu_L\frac{U_{l,s}}{U_{c,s}}{}^{28} \tag{2.8}$$

which corresponds to the $\gamma_l \to 0$ case that will be shown below, in the Calvo type nominal wage setting. More broadly, it should be as:

$$\frac{1-\tau^w_s}{1+\tau^c_s}\frac{W_s(j)}{P_s} = H(C_s, N_s) > \frac{U_{l,s}}{U_{c,s}} \tag{2.9}$$

and the Transversality condition (TVC) - no-ponzi condition (that hhs use all of their inter-temporal BC).

$$\lim_{s\to\infty} E_t\frac{B_{s+1}}{P_s(1+\tau^c_s)}U_{c,s} = 0\left(\lim_{s\to\infty} E_s\lambda_s b_{s+1} = 0\right) \tag{2.10}$$

I assume, as in Gali et al. (2007), that μ_l over mrs is sufficiently high and fluctuations in μ_{ls} (due to stickiness) are small enough such that real wage is always higher than mrs ($\frac{U_{l,s}}{U_{c,s}}$). This makes sure both types of households are always willing to (they promise) supply any amount of labor demanded by firms, at this high wage set by the union. Meanwhile, balanced-growth path requires that $H(C_s, N_s)$ could be written

[27]Sacrificing one unit of consumption, c_t, today to buy $\frac{1}{p_t}$ units of bond/money and $\frac{1}{q_t}$ units equity, in order to consume one unit of consumption good at time 't+1', c_{t+1}.

[28]Where the markup $\mu_L = \frac{1}{\theta_l}$.

as $C_s h(N_s)$. We assumed wage is set by households (or by market, with a markup), and labor supply is determined by demand of firms (given the wage, employment is demand-determined as output is demand determined) and households are supplying any amount of labor demanded by assuming wage is always above mrs. Demand for a differentiated labor type j is uniformly distributed among these households, r and k. Therefore, $N_s^r = N_s^k$, $\forall$ s.

If we assume households are setting their wage, then labor is not a choice variable any more. An alternative labor setting is that I assume wages are set at the beginning of period, before government cuts taxes. And hence fixed for one period.[29] Then, we have the above intra-temporal Euler equation for labor consumption decisions. Assumption of fixed wages for one period has the advantage of not causing any inefficiency or distortions, since Calvo type nominal wage setting causes dispersions in wages across households. This means distortion in the employment allocation - (since demand for each labor is depended on its price).

A fraction $'f'$ of the households are assumed to be Keynesian ($\beta = 0$ or very low) in their consumption behavior. While the Ricardian households have no limits on borrowing against all their future income, the Keynesian are constrained fully. Keynesian households have the same labor supply $'L_s^r(j) = L_s^k(j)'$ as the Ricardians (they face the same labor demand), the same wage and the same utility function. The only difference is that they don't have inter-temporal decisions, and capital and assets are removed from their budget constraints. As pointed out by Forni et al. (2009), this also

[29] Another sticky wage model such as a one period model, as suggested by Prof. Walsh and Prof. Aizenman, might also be considered.

means labor supply of both agents responds to a tax cuts or other fiscal changes in the same way. The Keynesian households do not have access to the capital and financial market and only consume their current after tax income. I assume (since they cannot inter-temporally optimize to set an optimal wage) they set their wage as the average of the optimizing/Ricardian households (W_s) (as in Gali et al. (2007), Forni et al. (2009) and Erceg et al. (2005)). Their utility function and the budget constraint will be as below.

$$\max_{\{C_s^r(j), L_s^r(j)\}} \left[U(C_s^k, G_s, l_s^k(j)) \right]$$

$$(1+\tau_s^c) P_s C_s^k = (1-\tau_s^w) W_s l_s^k - P_s T_s^k(j) \tag{2.11}$$

consumption in real terms is as follows.

$\mathrm{C}_s^k = \frac{(1-\tau_s^w) w_s l_s - T_s^k(j)}{(1+\tau_s^c)}$

And they have an optimality condition that sets marginal rate of substitution between leisure and consumption to the marginal product of labor (MPL) - an intra-temporal EE for labor, as in the Ricardians case.

$$\frac{1-\tau_s^w}{1+\tau_s^c} \frac{W_s(j)}{P_s} = \frac{U_{l^k,s}}{U_{c^k,s}} \mu_L \tag{2.12}$$

I first assume $f = 1/2$ of population are Keynesian households. Then, I change it to see the effect of the fraction of Keynesian households on fiscal multipliers.[30]

Aggregation among the Keynesian and the Ricardian households is as below. 'f' is the fraction of the aggregate consumption coming from the Keynesian households,

[30] Consistent with estimates in Campbell and Mankiw (1989). Erceg and Linde (2010) take it as 1/3 (1/3 of consumption goes to Keynesians). Gali et al. (2007) start with 1/2 as well.

while '1-f' is the fraction of the Ricardian households.

$$C_s = fC_s^k + (1-f)C_s^r \tag{2.13}$$

where $C_s^r = \int_0^1 C_s^r(j)dj$ and $C_s^k = \int_0^1 C_s^k(j)dj$.

In the labor market, on the other hand, we have the following aggregation.

$$L_s = fl_s^k + (1-f)l_s^r \text{[31]} \tag{2.14}$$

In the goods market,

$$Y_s = C_s + I_s + G_s \tag{2.15}$$

2.2.1.1 Calvo type nominal rigidity in wage setting

Assuming each household has monopoly power over his/her nominal wage $'W_t(j)'$ such that he/she resets his wage at the end of the contract periods (which have random durations). This process is analogous to the price setting process for firms for their output, which we will see in the next section. Households take the tax rates, prices, transfers (or taxes) from government and all the aggregates as given and maximize utility subject to the budget constraint. In other words, households maximize inter-temporal utility function (5), subject to equations (6) (the period budget constraint) and (2) (demand for its labor supply). Each period a fraction of agents (from the law of large numbers) are able to reset their wages. It could also be read as the

[31] $L_s^k = L_s^r = L_s$, for all t, in aggregate. See, for example, Appendix A of Gali et al. (2007). Forni et al. (2007): Given that Keynesians set their wage at average of the Ricardians and since all agents face the same labor demand, their labor supply and wages should be (and will be) the same. Ricardian and Keynesian's labor supply responds to a fiscal shock in the same way (in case of a G change).

probability of each household resetting its wage in each period is the same. I will call this probability as $'1-\gamma_l'$.[32]

Given all the information available, when an agent is allowed to set the optimal wage W_s^*, he/she chooses the wage $W_s(j)$ that maximizes his utility - equation (5) - subject to the period budget condition and demand for his demand, considering that he might not even have another chance ever to reoptimize. Households take L_s and W_s as given.[33] E_s is again a conditional expectation based on information available at time 's'. I assume wages are not changing at all when a household type is not allowed to reset its wage for a period.[34] First order conditions (FOC) of the above utility maximization problem (inter-temporal decision/problem), subject to the the two conditions and, with respect to W_s^*, then, are as follows ($\max_{W_s^*}$).

Here, we will need to assume that, since all the households face the same problem, we are looking for a 'symmetric equilibrium' where all the households choose the same wage $W_s^*(j) = W_s^*$.

$$E_t\left\{\sum_{s=t}^{\infty}(\gamma_l\beta)^{s-t}\left[\frac{W_t^*}{P_s}L_s(\frac{W_t^*}{W_s})^{-\theta_l}U_{C,s}-\frac{1}{\theta_l}(-U_{L_s(j)})L_s(\frac{W_t^*}{W_s})^{-\theta_l}\right]\right\}=0 \tag{2.16}$$

or $\mathrm{E}_t\left\{\sum_{s=t}^{\infty}(\gamma_l\beta)^{s-t}\left[\frac{W_t^*}{P_s}L_s(\frac{W_t^*}{W_s})^{-\theta_l}+\frac{1}{\theta_l}\frac{U_{Ls(j)}}{U_{C,s}}L_s(\frac{W_t^*}{W_s})^{-\theta_l}\right]\right\}=0$ [35]

Rearranging this FOC, I get the following simpler form:

$$W_t^* = \frac{1}{\theta_l}\frac{E_t\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}ln(C_s)MC_s(i)(\frac{p_t^*}{P_s})^{\frac{1}{1-\theta}}}{E_t\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}ln(C_s)(\frac{p_t^*}{P_s})^{\frac{\theta}{1-\theta}}}. \tag{2.17}$$

[32] γ_l fraction of agents are <u>not able</u> to reset their wages.
[33] Christiano et al.(2005)
[34] Christiano et al. (2005) and Erceg and Linde (2010)
[35] Where $MRS_s = -\frac{U_{L_s}}{U_{C_s}}$ and the markup $\mu_L = \frac{1}{\theta_l}$.

If wages are sticky here (with the monopoly power over labor) and set by households or unions, then households promise to supply any labor that is demanded at that wage. Therefore there is no FOC wrt labor ($L_s(j) = L_s^d(j)$). See the discussion in the footnote 32 above about findings in Forni et al. (2009) and Gali et al. (2007).[36]

In other words, wages are set such that the expected discounted marginal benefits are equal to the expected discounted marginal disutility from working.

If I assume a perfectly flexible case, then $\gamma_l \to 0$ (meaning there is no agent that is not allowed to reoptimize) and the above equation reduces to,

$$W_t^* = \frac{1}{\theta_l} MRS_s P_s{}^{37} \tag{2.18}$$

By taking γ_l as the fraction of agents that keep their wages fixed each period, I get the following wage index.

$$W_s = [\gamma_l W_{s-1}^{1-\theta_l} + (1-\gamma_l)(W_s^*)^{1-\theta_l}]^{\frac{1}{1-\theta_l}} \tag{2.19}$$

Wage inflation π^w depends on the real marginal cost, which is equal to the gap between real wage and marginal rate of substitution between consumption and leisure.

Given sticky wages, in the steady state, we get the following equality that I used above a lot.

$\frac{1-\tau^w}{1+\tau^c}\frac{W}{P} = \frac{1}{\theta_l}\frac{U_l}{U_c}$

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[36] If we have a competitive labor market, then given the wage set by market, households choose amount of labor supplied. $w = mrs$.

[37] This is (real) marginal rate of substitution.

2.2.2 Firms' problem

I assume there is only one final good produced by a representative final-good-firm and a continuum of intermediate goods, indexed by $i \in [0,1]$, produced by a continuum of firms indexed by the good they produce. Final-good firm buys intermediate goods, assembles them in the same proportions as consumers (households, firms and the government) demand and sells it to the private sector and the government in a competitive market.[38] They do not use any labor, and therefore pay only for the intermediate good they use. Following Chari, Kehoe and McGrattan (2000), I use the following Dixit-Stiglitz form production function for the final good produced by a perfectly competitive firm.

$$Y_s = \left[\int_0^1 Y_s^d(i)^\theta di \right]^{\frac{1}{\theta}}, \tag{2.20}$$

with 0< θ <1. Final-good firms choose Y_s (given) and $Y_s^d(i)$ (demand for good 'i') to maximize profit (or minimize cost) below:

$\max_{Y_s(i)} \Pi_s = P_s Y_s - \int_0^1 P_s(i) Y_s(i) di$

subject to equation (20), where $Y_s(i)$ is an intermediate good produced by firm i.[39] Final good producers face a perfectly competitive market for both their output and the intermediate goods they need for production. They take the prices of final good P_s and intermediate good $P_s(i)$ as given, and choose final good Y_s and intermediate good $Y_s(i)$

[38]Following findings in Eggertsson (2010), I assume all government expenditures are imperfectly substitutable with private consumption. These include military expenditures and infrastructure spendings. Eggerstsson (2010) show that the perfectly substitutable government expenditures (with C_t) are not changing output or inflation.

[39]Constant elasticity of substitution (CES) between the intermediate goods is $\frac{1}{1-\theta}$ - this is also price elasticity. As $\theta \to 0$, ES goes to 1 (unit elastic). Constant markup over the marginal cost for monopolistically competitive firms is $\frac{1}{\theta}$.

for production. Profit maximization of the final good producers results in the following demand function for each intermediate good. This is the sum of the demand for a particular intermediate good, by consumers in the economy, since final-good producers assemble intermediate goods according to the demand in the market.[40]

$$Y_s^d(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta}} Y_s \tag{2.21}$$

The zero-profit condition (due to perfect competition in final goods market) for the final good producers gives the price of final good (or the aggregate price index), which is equal to the marginal cost of production.

$$P_s = \left[\int_0^1 P_s(i)^{\frac{\theta}{(\theta-1)}} di\right]^{\frac{(\theta-1)}{\theta}} \tag{2.22}$$

The intermediate goods, on the other hand, are produced by monopolistically competitive firms. Each intermediate-good firm uses firm-specific capital and (composite) labor rent from labor aggregator agency (thus households), and produces a differentiated good $Y_s(i)$. All producers hire the same kind of labor (homogenous labor input) and face the same wages, W_s. In a way, all types of labor are used in producing a differentiated good. They face the demand function in equation (21) for their output (they all have the same constant demand elasticity). All of the firms are owned by the Ricardian households (each household own an equal share of all firms and capital stock as in Erceg at al. (2000)). Therefore, all the profit goes to households as the dividend payment. The production function for intermediate goods has a usual CRS Cobb-Douglass form. All

[40]Government expenditure and therefore its demand for each differentiated good is analogous to that of households and firms. The same aggregation as consumption and investment.

intermediate-good firms have the same following production function.[41]

$$Y_s(i) = F(K_s(i), N_s(i)) = K_s(i)^{\alpha} N_s(i)^{1-\alpha} \tag{2.23}$$

where $N_s(i)$ and $K_s(i)$ are labor and firm specific capital inputs for production of the intermediate goods. Capital is assumed to accumulate endogenously by firms. The total factor productivity (TFP) is normalized to 1. And the intermediate-good firms maximize profit (or minimize cost) below.

$$\max_{\{N_s(i),K_s(i)\}} \Pi_s(i) = P_s(i)Y_s(i) - W_sN_s(i) - (1+\tau_s^I)(1+\tau_s^c)P_sI_s(i)^{42}$$

The monopolistically competitive intermediate-goods firms face a perfectly competitive factor market in contrast to the imperfect goods market for their output. They take the purchasing price for the investment P_s, all taxes and the aggregate wage W_s as given and choose $Y_s(i)$ (*function given), $N_s(i)$ and $I_s(i)$ to maximize profit function w.r.t. $P_s(i)$, $K_{s+1}(i)$ subject to the demand function (21). They set a price of their goods and supply any amount that is demanded at that price. They have to hire labor to produce goods demanded.

Firms need to consider their capital accumulation process. I include endogenous capital variations into the model because access to the capital markets by a fraction of households is a key point of my paper. I assume, in order to increase the capital stock

[41]Since the production function is constant returns to scale, $Y_s(i) = F(K_s(i), N_s(i)) = F_k(K_s(i), N_s(i))K_s(i) + Fn(K_s(i), N_s(i))N_s(i)$ and $\frac{K_s(i)}{N_s(i)}$ is the same across firms, $\frac{K_s(i)}{N_s(i)} = \frac{K_s}{N_s}$.

[42]Where $Y_s(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta}} Y_s$. The union sells units of labor index L_s, $L_s(i)$, at the cost W_s to the intermediate-goods sector. $w_s = \frac{W_s}{P_s}$ is real wage.

from $K_s(i)$ to $K_{s+1}(i)$, a firm must invest (one period in advance) according to the rule at (21).[43] I first don't include a preference shock in the cost of adjustment function $I(.)$, as in Christiano (2004), and then add it as in Eggertsson (2010) to compare the two cases. ξ_t is a banking crisis shock which increases the default risk in a crisis period and hence cost of loans. It raises cost of borrowing for firms as well as consumers. Therefore, I include the shock into the cost of adjustment function of investment for the firms.

$$I_s(i) = I(\frac{K_{s+1}(i)}{K_s(i)}, \xi_s)K_s(i) \tag{2.24}$$

where $I_s(i)$ is again a Dixit-Stiglitz composite analogous to C_s^h or G_s. The function $I(.)$, in the steady state, satisfies $I(1,\xi) = \zeta$ (the depreciation rate of capital), $I_{I\xi}(1,\xi) \neq 0$ and $I_\xi(1,\xi) = 0$ (to make it comparable with Eggertsson (2010)), $I_I(1,\xi) = 1$, and $I_{II}(1,\xi) = \epsilon_x$ (the degree of adjustment cost in log-linear approximation or curvature on the investment adjustment cost function[44]) with the following conditions for parameters: $0 < \zeta < 1$ and $\epsilon_x > 0$. I_s in the resource constraint is the sum over all the firms' investment.

$$\int_0^1 I_s(i)di = I_s \tag{2.25}$$

Firms pay wages for the labor they hire from households and buy investment goods from the final good producers. They also pay consumption tax τ_s^c for the investment good they buy, a profit tax τ_s^P and an investment tax credit τ_s^I. The profit maximization

[43] The investment adjustment cost is as in Woodford (2003), Christiano (2004) and Eggertsson (2010).

[44] When ϵ_x is large (the more concave), capital stock is constant. When it is small, then investment is elastic, or changing (Christiano (2004)).

problem of firm i is below.

$$\max_{\{P_s^*, K_{s+1}(i)\}} \Pi_s(i) = \Big[P_s(i)Y_s(i) - W_sN_s(i) - (1+\tau_s^I)(1+\tau_s^c)P_sI_s(i)\Big] \tag{2.26}$$

and adding the profit taxes,

$\max_{\{P_s^*, K_{s+1}(i)\}} E_t \left\{\sum_{s=t}^{\infty} (\beta)^{s-t} Q_s (1-\tau_s^P)\Big[P_s(i)Y_s(i) - W_sN_s(i) - (1+\tau_s^I)(1+\tau_s^c)P_sI_s(i)\Big]\right\}$

where as in Correia et al. (2011) and Eggertsson and Woodford (2003) the following variables (excluding the β) will be used, $Q_{s+1} = \beta\frac{\lambda_{s+1}}{\lambda_s} = \frac{R_s^{-1}}{(1-\tau_{s+1}^A)} = \frac{1}{(1+i_s)(1-\tau_{s+1}^A)} = \frac{\beta U_{c,s+j}\xi_{s+j}}{U_{c,s}\xi_s}\frac{(1+\tau_s^c)P_s}{(1+\tau_{s+1}^c)P_{s+j}}$ is the nominal price at time s of a unit of money at a state in period $s+1$ (I use real Q_{s+1} though: $Q_{s+1}^{real} = Q_{s+1}\frac{P_{t+1}}{P_t}$) - stochastic discount factor for Ricardians (share owners of firms) and $\lambda_s = \frac{U_{c,s}\xi_s}{(1+\tau_s^c)P_s}$ is the Lagrange multiplier on the households' BC. It is the shadow value of a dollar to the hh. The FOC with respect to the capital stock chosen for time $'s+1'$, $K_{s+1}(i)$, is below.

$$I'(I_s^N(i), \xi_s)(1+\tau_s^c)(1+\tau_s^I)(1-\tau_s^P) = \tag{2.27}$$

$E_sQ_{s+1}\Pi_{s+1}(1-\tau_{s+1}^P)\Big[r_{s+1}^k(i) + I'(I_{s+1}^N(i), \xi_{s+1})I_{s+1}^N(i)(1+\tau_{s+1}^c)(1+\tau_{s+1}^I)$

- $\mathrm{I}(\mathrm{I}_{s+1}^N(i), \xi_{s+1})(1+\tau_{s+1}^c)(1+\tau_{s+1}^I)$

where

$$I_s^N(i) = \frac{K_{s+1}(i)}{K_s(i)} \tag{2.28}$$

is the net increase in the capital stock per period, $\Pi_{s+1} = (1+E_s\pi_{s+1})$ and

$$R_s^k(i) = \frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)}W_s(j) = \frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)}MRS_sP_s\frac{(1+\tau_s^c)}{(1-\tau_s^w)}.^{45} \tag{2.29}$$

[45] where $\frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)} = \frac{MPK_s}{MPL_s}$.

$\frac{R^k_s}{P_s} = r^k_s$ is the real shadow value of a marginal unit of additional capital (functions as the 'rental cost of capital' in models where capital is rent from hhs).[46] From the production function, it is possible to derive the following equality (firm choices must satisfy this according to Correia et al. (2011) and Chari, Kehoe and McGrattan (2007)).[47]

$\frac{F_N(K_s(i),N_s(i))}{F_K(K_s(i),N_s(i))} = \frac{(1-\alpha)K_s(i)}{\alpha N_s(i)} = \frac{W_s}{R^k_s} = \frac{w_s}{r^k_s}(= \frac{F_N(K_s,N_s)}{F_K(K_s,N_s)} = \frac{W_s(i)}{R^k_s(i)})$

The nominal marginal cost for all firms (common, because of the CRS property of the prodution function - also average cost) and per output is below.MC$_s$[48] $= \frac{w_s}{MPL_s} = \frac{r^k_s}{MPK_s}$

This implies

$MC_s(i) = \frac{r^k_s(i)}{MPK_s(i)} = (\frac{K_s(i)}{\alpha Y_s(i)})\frac{\alpha}{1-\alpha}\frac{N_s(i)}{K_s(i)}W_s(i) = \frac{N_s(i)}{(1-\alpha)Y_s(i)}\frac{1+\tau^c_s}{1-\tau^w_s}MRS_s\frac{1}{\theta_l}$

$MC_s = \frac{N_s(i)}{(1-\alpha)Y_s(i)}MRS_s\frac{1+\tau^c_s}{1-\tau^w_s}\frac{1}{\theta_l} = \frac{N_s(i)}{(1-\alpha)Y_s(i)}\frac{(l_s)^\eta}{(C_s)^{-\sigma u}}\frac{1+\tau^c_s}{1-\tau^w_s}\frac{1}{\theta_l}$

where

$MPK_s = \alpha K_s^{\alpha-1}N_s^{1-\alpha} = \alpha Y_s/K_s$

with K_t and L_t are aggregate capital and labor stocks.

Firms also consider profit maximizing price setting P^*_t in addition to their capital accumulation process. Intermediate-good producers have a monopoly power over the price of the differentiated good they produce. Prices for intermediate goods are set in a staggered fashion (at random durations) *à* la Calvo (1983). Each period a fraction $('1-\gamma'$, since we have a continuum of intermediate good firms) of firms are

[46]Steady state analysis show that $r^k = (\beta^{-1} - 1 + \zeta)(1 + \bar{\tau}^c)(1 + \bar{\tau}^I)$ and $\gamma_k = \frac{K}{Y} = \frac{\alpha}{r^k}\frac{1}{\theta}$ where $\frac{1}{\theta} = \mu_{ss}$ is the pricing markup.

[47]Firms choose capital, investment and labor supply to maximize profit function. Given production function and capital accumulation law.

[48]MC_s is real marginal cost and w_s (small) is real wage.

able to reset their prices and all firms that reset their prices set the same price for their goods in equilibrium. It could also be read as the probability of each firm resetting its price in each period is the same. I will call this probability, $'1-\gamma'$.[49] Intermediate-good firms set their price and then decide the amount of labor they need for production, as they are obliged to supply any amount that is demanded at that price. All the after-tax profit of intermediate-good firms is going to the households, since they own all firms.

The profit maximization problem for a typical intermediate-good firm resetting its price $p_t^* = p_t(i)$ is below. When a firm is allowed to set the optimal price, given all the information available, it chooses the price that would maximize its profit even if it never has another chance to re-optimize. E_t is conditional expectation based on the information available at time t.

$\max_{\{P_t^*\}} E_t \left\{ \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} Q_{s+1} (1-\tau_s^P) \left[(\frac{p_t^*}{P_s}) Y_s(i) - (\frac{MC_s(i)}{P_s}) Y_s(i) \right] \right\}$[50]

subject to demand determined output $Y_s(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta}} Y_s$. This equals to the following,

$$\max_{\{P_t^*\}} E_t \left\{ \sum_{s=t}^{\infty} (\gamma\beta)^{s-t} Q_{s+1} (1-\tau_s^P) \left[(\frac{p_t^*}{P_s})^{\frac{\theta}{\theta-1}} - (\frac{MC_s(i)}{P_s})(\frac{p_t^*}{P_s})^{\frac{-1}{1-\theta}} \right] Y_s \right\} \tag{2.30}$$

[49] γ fraction of firms are not able to reset their prices.

[50] Where $\lambda_s = \frac{U_{c,s}\xi_s}{(1+\tau_s^c)P_s}$ And, as in Forni et al. (2009) and Correia et al. (2011), $Q_{s+1} = \frac{\beta U_{c,s+j}\xi_{s+j}}{U_{c,s}\xi_s} \frac{(1+\tau_s^c)P_s}{(1+\tau_{s+j}^c)P_{s+j}}$ (nominal price at time s of a unit of money at a state in period $s+j$ - stochastic discount factor for Ricardians (share owners of firms)). Eggertsson and Krugman (2010): Q or λ does not play any role in log-linear economy.

The first order condition (FOC) of the above profit maximization problem wrt to p_t^*, is then

$$E_t\left\{\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}Q_{s+1}(1-\tau_s^P)\left[\frac{\theta}{\theta-1}(\frac{p_t^*}{P_s})^{\frac{1}{\theta-1}}(\frac{1}{P_s})+\frac{1}{1-\theta}(\frac{MC_s(i)}{P_s})(\frac{1}{P_s})(\frac{p_t^*}{P_s})^{\frac{\theta-2}{1-\theta}}\right]Y_s\right\}=0 \tag{2.31}$$

which equals

$E_t\left\{\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}Q_{s+1}(1-\tau_s^P)\left[\frac{\theta}{\theta-1}(\frac{p_t^*}{P_s})+\frac{1}{1-\theta}(\frac{MC_s(i)}{P_s})\right](\frac{1}{P_t^*})(\frac{p_t^*}{P_s})^{\frac{1}{\theta-1}}Y_s\right\}=0.$

Rearranging this FOC and substituting $Q_{s+1}=\frac{\beta\lambda_{s+1}}{\lambda_s}$ we get the following simpler form: and Q_{s+1} is the real (that is why we multiply it by inflation - $(1+E_s\pi_{s+1})$- to get stochastic discount factor $Q_{s+1}\Pi_{s+1}$) stochastic discount factor ($\frac{\beta\lambda_{s+1}}{\lambda_s}$ for some lagrange multiplier $\lambda_s=\frac{U_{c,s}}{(1+\tau_s^c)P_s}$)

$$P_t^*=\frac{1}{\theta}\frac{E_t\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}ln(C_s)MC_s(i)(\frac{p_t^*}{P_s})^{\frac{1}{1-\theta}}}{E_t\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}ln(C_s)(\frac{p_t^*}{P_s})^{\frac{\theta}{1-\theta}}}. \tag{2.32}$$

If I assume a perfectly flexible case, then $\gamma\to 0$ (meaning there is no firm that is not allowed to reoptimize) and the above equation reduces to,

$$P_t^*=\frac{1}{\theta}MC_s{}^{51} \tag{2.33}$$

By taking γ as the fraction of firms that keep their prices fixed each period. We will get the following price index. I assume prices are not changing at all when a firm is not allowed to reset its price. [52]

$$P_t=[\gamma P_{t-1}^{\frac{\theta}{\theta-1}}+(1-\gamma)(p_t^*)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}} \tag{2.34}$$

[51] This is nominal marginal cost again.

[52] See, Christiano et al. (2005) and Erceg et al. (2006) for an alternative specification.

2.2.3 The monetary authority - Central bank

The central bank controls the short-term nominal interest rate i_s. I assume the central bank (CB) follows a Taylor rule (Taylor, 1993) to implement monetary policy and that a zero lower-bound for i_s holds.

$\hat{i}_s = \max\{0, \hat{r}_s^n + \phi_p \pi_s + \phi_y \hat{Y}_s\}$

where $\phi_\pi > 1$ and $\phi_y > 0$. $\hat{r}_s^n = log\beta^{-1} + \hat{\xi}_s - \hat{\xi}_{s+1}$ is the efficient real interest rate or the natural rate of interest (goes to $\hat{r}_s^n < 0$, if there is a large enough negative shock, $\hat{\xi}_{s+1}$). r^n is the steady state interest rate ($i = r^n = log\beta^{-1}$ with zero inflation and no shock).

The objective of the central bank is to achieve zero inflation under normal circumstances. The Central Bank, with a Taylor rule, sets i_s to achieve zero inflation. Given a positive interest rate, it means CB supplies any base money that is demanded at that nominal rate of interest. However, if interest rates are down to zero, then it sets i_s at zero and lets π_s be determined by the equilibrium conditions, which usually puts it down below zero. Central Bank's commitment to a higher future inflation could be an alternative to temporary government expenditure shocks or temporary cuts in taxes in shifting the aggregate demand, therefore.[53] Commitment policy does not require any change in G or tax cut, but it has a credibility problem (Kydland and Prescot (1977), Walsh (2010) chapter 6). I assume the central bank is not able to commit to future policy.

Absent the Keynesian households and given the tax policy, the Taylor principle

[53]See e.g. Krugman (1998), Eggertsson and Woodford (2003) and Eggertsson (2010)

is a necessary and sufficient condition for uniqueness of equilibrium for the linear system of equations for long-run with $\pi = 0$ and $i = r^n = \bar{r}$.[54]

As pointed out by Christiano (2004), if $\hat{i}_s = \hat{r}^n_s$ holds for all periods, from the IS equation, then there is an equilibrium where C stays at its steady-state level and $\hat{\pi}_s = 0$ for all periods. Yet if $\hat{r}^n_s < \beta^{-1} - 1$ - there is a shock to $\hat{r}^n_s$, then $\hat{i}_s = \hat{r}^n_s$ does not hold (it violates ZLB condition).

2.2.4 The fiscal authority - Government

Governments basically have three financing methods: taxation, borrowing (issuing debt) and printing money (seigniorage revenue). At its core, all of these methods are varying forms of taxation. While printing money is taxation of money holdings of the public, borrowing or bond issuance is taxation in the future. I will consider the first two here.

This paper studies the efficacy of the distortionary taxes in eliminating the recession. The government sets different taxes and issues bonds to balance its budget every period. B_s is a government bond. Because the Ricardian equivalence does not hold, the timing of taxation matters. Therefore, we need the government budget constraint as another equilibrium condition.[55] I break Ricardian equivalence by adding

[54]The Taylor principle is a property of the interest rate rule that tells when there is a change in inflation, the nominal interest rate responds more than one for one (more than proportional) to that change in inflation. See, among others, the discussion in Walsh (2010) chapter 8, Gali et al. (2007), Eggertsson and Krugman (2010) and Correia et al. (2011).

[55]If we had only Ricardian households and government expenditures were financed by lump-sum taxes then from the Ricardian equivalence, the timing of taxation would not matter. Fiscal policy would be ineffective. Meaning government expenditure increases or lump-sum tax decreases would be ineffective in raising private comsumption and GDP. This is because, the public would perfectly anticipate future taxes to finance current increases in expenditures.

distortionary taxes and the Keynesian households to the model. Gali et al. (2007) and Erceg and Linde (2010) argue the short-run fiscal multiplier is greater than one if hand-to-mouth consumers are added to the model and private consumption goes up.

Because the cost of stimulus packages is low in a zero interest rate case, Keynesian economists argue it is beneficial for the government to issue bonds to finance its deficit in short-run. I assume the government finances its deficit (from tax cut / expenditure increase) with some bonds in one case and also consider another case where government expenditure is financed by simultaneously increasing another tax.

Government expenditures in the model economy are financed by means of a variety of distortionary taxes such as labor income tax, capital income taxes (both the asset and profit taxes), sales tax, investment tax credit, a lump-sum tax and nominal debt, risk-less one period bonds B_t. Having bonds to finance the government budget constraint means the government does not need to balance its budget each period. Having distortionary taxes to finance the government spending may change the effect of policy instruments, especially with different income and substitution effects on different households (in their consumption behaviors). I will also assume fiscal rules for discretionary fiscal policy, in the case of income-tax changes as in Gali and Perotti (2003). There are many hand-to-mouth agents who consume their current disposable income. If the government decides to have a budget deficit to finance with bonds, it will have significant effect on the aggregate demand due to existence of the Keynesian households who optimize per period. The government budget constraint will be as follows.

$$R_s^{-1}B_{s+1} + P_sT_s + \tau_s^c P_s C_s + \tau_s^w W_s L_s + \tau_s^p P_s Z_s + \tau_s^A B_s = P_s G_s + B_s \tag{2.35}$$

or in real terms,

$$R_s^{-1}B_{s+1}/P_s + T_s + \tau_s^c C_s + \tau_s^w w_s L_s + \tau_s^p Z_s + \tau_s^A B_s/P_s = G_s + B_s/P_s$$

where $R_s^{-1} = \frac{1}{1+i_s}$ is the gross nominal interest rate and $(1 + i_s) = (1 + \pi_{s+1})(1 + r_s)$. Also $T_s = fT_s^k + (1 - f)T_s^r$.

I also include lump-sum taxes which are standard in the New-Keynesian models and change endogenously (as a residual) to keep government budget in balance. We need a fiscal policy rule that shows how lump-sum taxes change (for financing of tax cuts) to keep the government budget in balance and ensure non-explosive debt dynamics.

$$\hat{t}_s = \phi_b \hat{b}_s + \phi_t \hat{\tau}_s^x, \tag{2.36}$$

where $\phi_b > 0$ and $\phi_t > 0$ and $x = c, w, p, A$ or I.

Following Eggertsson (2010) and Gali et al. (2007), I use $\hat{g}_s = (G_s - G)/Y$, $\hat{t}_s = (T_s - T)/Y$, $\hat{c}_s = (C_s - C)/Y$ and $\hat{b}_s = [(B_s/P_s) - (B/P)]/Y$.

I assume a two-state markov process for resetting the labor income taxes exogenously (as is the banking shock to the economy), which is the same idea as having a stochastic process such as the following AR(1) process $\hat{\tau}_s^w = \phi_{tax}\hat{\tau}_{s-1}^w + \epsilon_s$ where ϵ_s would be a normally distributed i.i.d. process. I assume a tax cut rule as,

$$\hat{\tau}_s^w = \phi_t r_s^n, \tag{2.37}$$

with $\phi_t > 0$.

Once we have a tax cut, with probability $(1-\omega)$ the tax cuts converges to steady state $\hat{\tau}^w_s = 0$ and with probability (ω) it stays at the short run level $\hat{\tau}^w_s < 0$.

2.3 The Market Clearing

Factor market clearing (for all s):

The market for capital is in equilibrium since firm-specific capital is accumulated by the firm itself. Demand for capital by the intermediate good firm equals the supply of capital by the intermediate good firm.

At the wage rate set by the households, $W_s(j)$, the labor market will be in equilibrium. Since we assumed the real wage was always higher than the mrs, and labor demand was uniformly distributed among both type of households, labor demand by intermediate good firms equals labor supply by households.

$$\int_0^1 N_s(i)di = N_s = L_s = \left[\int_0^1 L_s(j)^{\theta_l}\right]^{\frac{1}{\theta_l}} \tag{2.38}$$

Market clearing for the dividend payments requires total dividends from all intermediate firms equal total dividend payment to the households.

$$\int_0^1 Z_s(i)di = \int_0^1 Z_s(j)dj \tag{2.39}$$

Bond market clearing[56]:

$$B_s = \left[\int_0^1 B_s(j)\right] = 0 \tag{2.40}$$

[56] In a closed economy, if the government has no bond, then in equilibrium, $B_t = 0$. But in my model, since government has bonds, then $B_t \neq 0$

Market clearing in the intermediate goods sector:

$$c_s^r(i) + c_s^k(i) + g_s(i) + I_s(i) = Y_s^d(i) = Y_s(i) \tag{2.41}$$

Resource constraint:

$$P_s Y_s = P_s C_s + P_s I_s + P_s G_s \tag{2.42}$$

and in real terms. $\mathrm{Y}_s = C_s + I_s + G_s$ where $\mathrm{I}_s = I(\frac{K_{s+1}}{K_s}, \xi_s)K_s$ and $\frac{K_{s+1}}{K_s} = I_s^N$ is the per period net increase in the capital stock, which implies the following equality. $\mathrm{Y}_s = C_s + I(I_s^N, \xi_s)K_s + G_s$

2.4 The Steady State

For simplicity, it is assumed that the steady-state consumption levels are the same across household types, $C^k = C^r = C$. It can be made sure by right choice of T^r and T^k.[57] This paper is not focused on the steady-state differences, we are interested in responses to shocks, thus this assumption (while simplifies analysis a lot) is not affecting results.

I assumed labor market are not perfectly competitive and wages are set by households (or by union), and labor supply is determined by demand of firms (given the wage) and households are supplying any amount of labor demanded by assuming wage is always above the mrs between C and L. Demand for a differentiated labor type j is uniformly distributed among these households, r and k. Therefore, $N_s^r = N_s^k$, $\forall$ s

[57]See e.g. Gali et al. (2007) and Eggertsson and Krugman (2010).

and $N^r = N^k = N$ in the steady state. Labor supplies might diverge because there are distortionary taxes, Walsh (2010).

Indeed, if the labor market was perfectly competitive, then $N_s^r = N_s^k$, $\forall$ s without the need for any other assumption. Since the MRS would have to be equal to the real wage for both type of households.

The steady state has zero inflation ($\pi = 0$), constant taxes (τ^c, τ^w, τ^P, τ^a and τ^I) and the nominal interest rate equals to real rate of interest, and hence to natural rate of interest ($\hat{i} = r^n = ln\beta^{-1}$) since there is no shock and natural rate of interest is positive[58].

I call the steady state ratios $K/Y = \gamma_k$, $C/Y = \gamma_c$, $G/Y = \gamma_g$, and $I/Y = \gamma_i$.[59]

The FOC from the endogenous capital accumulation is again given below.

$$I'(I_s^N(i), \xi_s)(1 + \tau_s^c)(1 + \tau_s^I)(1 - \tau_s^P) = \tag{2.43}$$

$$\mathrm{E}_s Q_{s+1} \Pi_{s+1}(1 - \tau_{s+1}^P) \left[r_s^k(i) + I'(I_{s+1}^N(i), \xi_{s+1}) I_{s+1}^N(i)(1 + \tau_{s+1}^c)(1 + \tau_{s+1}^I) \right.$$

- $\mathrm{I}(\mathrm{I}_{s+1}^N(i), \xi_{s+1})(1 + \tau_{s+1}^c)(1 + \tau_{s+1}^I)$

In the steady state, given that $I(1, \xi) = \zeta$, $I_I(1, \xi) = 1$, and $I_{II}(1, \xi) = \epsilon_x$, I get,

$(1+\bar{\tau}^c)(1 + \bar{\tau}^I)(1 - \bar{\tau}^P) = Q(1 - \bar{\tau}^P)[r^k + (1 + \bar{\tau}^c)(1 + \bar{\tau}^I) - \zeta(1 + \bar{\tau}^c)(1 + \tau^I)]$

Rewriting the equation (and using the steady state value $Q = \beta$), I get

$$r^k = (\beta^{-1} - 1 + \zeta)(1 + \bar{\tau}^c)(1 + \bar{\tau}^I) \tag{2.44}$$

[58]See the footnote below to see why $\pi = 0$ is optimal in the steady-state.
[59]As in Christiano (2004).

The firm's pricing equation in the SS is: $MC = \frac{1}{\mu_{ss}} = \theta$, since $P_t(i) = P_t^* = P_t$ in steady state.

We know that $\frac{R_s^k/P_s}{MPK_s} = MC_s = \frac{R_s^k/P_s}{(\alpha)Y_s/K_s} = \frac{R_s^k K_s}{(\alpha)Y_s P_s}$. In steady state: $MC = \frac{1}{\mu_{ss}} = \frac{r^k K}{\alpha Y}$.

Therefore, $(\frac{K}{Y}) = \gamma_k = (\frac{\alpha}{\mu_{ss} r^k}) = (\frac{\alpha\theta}{r^k})$.

$$\frac{K}{Y} = \gamma_k = \frac{(\alpha)}{\mu_{ss} r^k} = \frac{(\alpha)}{\mu_{ss}[(\beta^{-1} - 1 + \zeta)(1 + \bar{\tau}^c)(1 + \bar{\tau}^I)]} \tag{2.45}$$

And from the same idea,

$$\frac{wN}{Y} = \frac{wL}{Y} = \frac{(1-\alpha)}{\mu_{ss}} \tag{2.46}$$ [60]

The resource constraint:

Y$_s = C_s + I_s + G_s$

Y = C+I+G

since $I_s = I(\frac{K_{s+1}}{K_s}, \xi_s)K_s$ and $I = I(1, \bar{\xi})K = \zeta K$,

Y = C+$\zeta K + G$

therefore, and since we know that $(\frac{K}{Y}) = (\frac{(\alpha\theta)}{r^k}) = (\frac{(\alpha)}{r^k}\mu_{ss})$.

$$\frac{C}{Y} = \gamma_c = 1 - \zeta\gamma_k - \gamma_g = 1 - \gamma_g - \frac{(\zeta\alpha)}{\mu_{ss} r^k} = 1 - \gamma_g - \frac{(\zeta\alpha\theta)}{(\beta^{-1} - 1 + \zeta)(1 + \bar{\tau}^c)(1 + \bar{\tau}^I)} \tag{2.47}$$

Which shows that γ_c does not depend on the fraction of the rule-of-thumb (Keynesian) agents, given γ_g that is exogenous.

[60] We know that $\frac{W/P}{MPL} = \frac{w}{MPL} = MC = \frac{w}{(1-\alpha)Y/L} = \frac{wL}{(1-\alpha)Y}$. In steady state: $MC = \frac{1}{\mu_{ss}} = \frac{wL}{(1-\alpha)Y}$. Therefore, $(\frac{wL_s^k}{Y}) = (\frac{(1-\alpha)Y}{\mu_{ss} Y}) = (\frac{(1-\alpha)}{\mu_{ss}})$.

2.5 Log-Linearization: For Illustration

I use basic FOCs with their steady state versions to work on Dynare, to get the impulse responses. Yet for the sake of illustration and to use it for future works; I will discuss the log-linearization process as well. This section analyzes the log-linear approximation to the structural equations of the model (market clearing and optimality conditions used to analyze equilibrium dynamics). I log-linearize the structural equations of the model around paths of inflation, interest rate and output related to zero-inflation steady-state (which is optimal policy absent any shock), without any shock ($\xi_t = 0$).[61]

Before we start the log linearization, I list some parameters of the model. $\sigma_u = -\frac{u''C}{u'} > 0$ and I call $\sigma = -\frac{u''C}{u'}\frac{Y}{C} = \sigma_u(\gamma_c)^{-1} > 0$, $b = -\frac{u''G}{u'} > 0$, $\eta = \frac{u''L}{u'} > 0$ and $\alpha = -\frac{f''L}{f'}$, for $'$ and $''$ standing for the first and second derivatives.

I use the Hansen method $\hat{x}_s = \frac{X_s - X}{X}$ and $logX_s$/X for log-linearization. A 'hat' over a variable, $\hat{x}_t$, means deviation of a variable (x_t) from its steady state value (x) as a fraction of its steady-state value again. All the aggregate variables that show up in the resource constraint will be linearized as deviation from the steady state value over steady state output level. Which basically means $\hat{C}_t = \frac{C_t - C}{Y}$, $\hat{G}_t = \frac{G_t - G}{Y}$, and $\hat{I}_t = \frac{I_t - I}{Y}$.

[61] A long-run inflation-target of zero is optimal, Woodford (2003) chapter 7. Zero-inflation is needed to get to the efficient production level. A positive inflation means price dispersion (relative price differences among firms), since we have a staggered price setting. This means distortion in the economic activity - allocation of resources - (since demand for each good is depended on its price, as in equation 20. Policy-makers choose (zero) inflation to maximize utility of the representative hh). Zero inflation, $\forall$ firms $P_s(i) = P_s = P_{-1}$, can only be achieved if all firms start initially at the same price (such as P_{-1}) and all firms that have the chance to make a change in their prices choose the same initial price (P_{-1}).

2.5.1 Log-linearization of the households' optimality conditions

Log-linearizing the Ricardian agent's inter-temporal Euler Equation, I get (from the Appendix),

$\hat{C}^r_s = E_s \left\{ \hat{C}^r_{s+1} \right\} - \frac{1}{\sigma} \left[\hat{i}_s - E_s \pi_{s+1} - \hat{r}^n_s \right] - \frac{\chi^c}{\sigma} E_s \left\{ \hat{\tau}^c_s - \hat{\tau}^c_{s+1} \right\} + \frac{\chi^A}{\sigma} (\hat{\tau}^A_s)$

and the log-linearized consumption equation for the Keynesian agents is below

$(\chi^c \hat{\tau}^c_s + \hat{c}^k_s) \frac{(1+\bar{\tau}^c)C^k}{Y} = (-\chi^w \hat{\tau}^w_s + \hat{w}_s + \hat{l}^k_s)(\frac{(1-\bar{\tau}^w)WL^k}{Y}) - \hat{t}^k_s$

where $t^k_s = (\frac{(T^k_s - T^k)}{Y})$ as in Gali et al. (2007). I assume steady state consumption levels are the same across heterogenous households. See, e.g., discussion in Gali et al. (2007). This implies $\hat{l}^k = \hat{l}^r = \hat{l}$, since the MRS between consumption and labor supply is equalized among the heterogenous agents (due to the same wages for both types of households in the labor market). Also $\frac{WL^k_s}{Y} = \frac{(1-\alpha)}{\mu_{ss}}$. See the appendix, for details. then the above log-linearized equation turns to a simpler form.

$(\chi^c \hat{\tau}^c_s + \hat{c}^k_s) \frac{(1+\bar{\tau}^c)C^k}{Y} = (-\chi^w \hat{\tau}^w_s + \hat{w}_s + \hat{l}^k_s)[\frac{(1-\alpha)}{\chi^w \mu_{ss}}] - \hat{t}^k_s$

The wage schedule is given below, which is derived from the (in perfect competition labor market) intra-temporal Euler equations (both) combined with the aggregation equations (for L and C). Gali et al. (2007) shows how we get a log-linear approximation of the form below, from an intra-temporal equation as $W_s = H(C_s, N_s)$ (in an imperfect labor market). $\hat{w}_s - \chi^c \hat{\tau}^c_s - \chi^w \hat{\tau}^w_s = \sigma \hat{c}_s + \eta \hat{l}_s$ The inter-temporal equilibrium condition for the aggregate consumption is below then.

$$\hat{c}_s = E_s \hat{c}_{s+1} + \frac{DD}{BB} E_s \Delta \hat{\tau}^c_{s+1} - \frac{f(1+\eta)(1-\alpha)\chi^c}{BB} E_s \Delta \hat{l}_{s+1} + \frac{AA}{BB} f \frac{\chi^c}{\gamma_c} E_s \Delta \hat{t}^k_{s+1} \quad (2.48)$$

$-\frac{AA.CC}{BB}(\hat{i}_s - E_s\pi_{s+1} - \hat{r}^n_s) + \frac{AA.CC}{BB}\chi^A(\hat{\tau}^A_s)$ Where

$AA = \chi^w \mu_{ss}\gamma_c$

$BB = \frac{\chi^w \mu_{ss}\gamma_c - f\sigma(1-\alpha)\chi^c}{\chi^w \mu_{ss}\gamma_c} = \frac{AA - f\sigma(1-\alpha)\chi^c}{AA}$

$CC = \frac{(1-f)}{\sigma}$

$DD = -f\left(\chi^c(1-\alpha)\chi^c + \chi^c AA\right) + (1-f)\frac{\chi^c AA}{\sigma}$

2.5.2 Log-linearization of the firms' optimality conditions

Appendix D shows that the real MC in log-linearized form is below.

$\hat{MC}^{real}_s = (\frac{\eta}{1-\alpha} + \frac{\alpha}{1-\alpha})(\hat{Y}_s - \hat{K}_s) + \eta\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}^c_s$

Appendix C shows how to derive and log-linearize $\hat{r}^k$,

$\hat{r}^k_s = \mu_y\hat{Y}_s - \mu_y\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}^c_s + \chi^w\hat{\tau}^w_s$

$$\hat{r}^k_s = \mu_y(\hat{Y}_s - \hat{K}_s) + \eta\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}^c_s + \chi^w\hat{\tau}^w_s \tag{2.49}$$

where

$\mu_y = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha}$

$\mu_k = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha} - \eta$

Appendix C shows that the FOC from the endogenous capital accumulation by firms in the log-linearized form is below.

$$\hat{I}^N_s = \beta E_s\hat{I}^N_{s+1} - \sigma_I(i_s - E_s\pi_{s+1} - \hat{r}^n_s - \chi^A\hat{\tau}^A_{s+1}) + \chi E_s\hat{r}^k_{s+1} \tag{2.50}$$

-$\chi^c[\hat{\tau}^c_s - \beta(1-\zeta)E_s\hat{\tau}^c_{s+1}] + \chi^P[\hat{\tau}^P_s - \beta(1-\zeta)E_s\hat{\tau}^P_{s+1}] - \chi^I[\hat{\tau}^I_s - \beta(1-\zeta)E_s\hat{\tau}^I_{s+1}]$

for some,

$\chi = \frac{[1-(1-\zeta)\beta]}{\epsilon_x} = \frac{\beta r^k}{(1+\bar{\tau}^c)(1+\bar{\tau}^I)\epsilon_x}$

$\sigma_I = \frac{1}{\epsilon_x}$

Appendix D shows step-by-step derivation of the following NK phillips curve (to shows how firms set prices),

$$\pi_t = \kappa(\psi + \sigma)\hat{Y}_t - \kappa\sigma\hat{G}_t - \kappa\sigma\gamma_k\hat{I}_t^N + \kappa(\eta - (\psi) - \sigma\gamma_k\zeta)\hat{K}_t + \kappa\chi^c\hat{\tau}_t^c + \beta E_t\pi_{t+1} \quad (2.51)$$

Log-linearization of the production function is shown in the appendix,

$$Y_s = (N_s)^{1-\alpha}(K_s)^{\alpha} \quad (2.52)$$

It equals to,

$$\hat{Y}_s = (1-\alpha)\hat{N}_s + (\alpha)\hat{K}_s \quad (2.53)$$

2.5.3 Log-linearization of the government budget constraint: GBC

Following Eggertsson (2010) and Gali et al. (2007), I assume, $g_s = (G_s - G)/Y$, $t_s = (T_s - T)/Y$, $c_s = (C_s - C)/Y$ and $b_s = [(B_s/P_s) - (B/P)]/Y$. Then the GBC in log-linearized form is below. Appendix shows that the government budget constraint in log-linearized form is as below.

$$\hat{b}_{s+1} = (1+r)[\hat{g}_s - \hat{t}_s - [\hat{c}_s + \frac{\hat{\tau}_s^c}{\bar{\tau}^c}](\bar{\tau}^c\gamma_c) - [\hat{w}_s + \hat{L}_s + \frac{\hat{\tau}_s^w}{\bar{\tau}^w}](\bar{\tau}^w\frac{(1-\alpha)}{\mu_{ss}}) - [\hat{Z}_s + \frac{\hat{\tau}_s^P}{\bar{\tau}^P}](\frac{\bar{\tau}^P Z}{Y}) + (-\chi^A\hat{\tau}^a + \hat{b}_s)(\frac{(1-\bar{\tau}^a)b}{Y})] \quad (2.54)$$

where

$\hat{b}_{s+1} = \frac{(b_{s+1} - b)}{Y}$

$\hat{g}_s = \frac{(G_s - G)}{Y}$

$\hat{t}_s = \frac{(T_s - T)}{Y}$

It also equals to,

$\hat{b}_{s+1} = (1+r)[\hat{g}_s - \hat{t}_s - [\hat{c}_s + \frac{\hat{\tau}^c_s}{\bar{\tau}^c}](\frac{\bar{\tau}^c C}{Y}) - [\hat{w}_s + \hat{L}_s + \frac{\hat{\tau}^w_s}{\bar{\tau}^w}](\frac{\bar{\tau}^w wL}{Y}) - [\hat{Z}_s + \frac{\hat{\tau}^P_s}{\bar{\tau}^P}](\frac{\bar{\tau}^P Z}{Y}) + (-\chi^A \hat{\tau}^a + \hat{b}_s)(\frac{(1-\bar{\tau}^a)b}{Y})]$ where all variables are aggregated over households and firms.

Using the fiscal policy rule assumed earlier and the log-linearized government budget constraint, we get the following equilibrium condition.

$\hat{b}_{s+1} = (1+r)[\hat{g}_s - (\phi_b \hat{b}_s + \phi_t \hat{\tau}^x_s) - [\hat{c}_s + \frac{\hat{\tau}^c_s}{\bar{\tau}^c}](\frac{\bar{\tau}^c C}{Y}) - [\hat{w}_s + \hat{L}_s + \frac{\hat{\tau}^w_s}{\bar{\tau}^w}](\frac{\bar{\tau}^w wL}{Y}) - [\hat{Z}_s + \frac{\hat{\tau}^P_s}{\bar{\tau}^P}](\frac{\bar{\tau}^P Z}{Y}) + (-\chi^A \hat{\tau}^a + \hat{b}_s)(\frac{(1-\bar{\tau}^a)b}{Y})]$

2.5.3.1 Log-linearization of the Resource Constraint

Log-linearizing the resource constraint, we get the following result.

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \hat{G}_t \tag{2.55}$$

since $\hat{X}_t = \frac{X_t - X}{Y}$ for all X = C, G, and I. We also know that[62],

$\hat{I}_t = \gamma_k[\hat{K}_{t+1} - (1-\zeta)\hat{K}_t] = \gamma_k[\hat{K}_{t+1} - \hat{K}_t] + \gamma_k \zeta \hat{K}_t$

using this equation, the above log-linearized resource constraint turns into,

$$\hat{Y}_t = \hat{C}_t + \hat{G}_t + \gamma_k \hat{I}^N_{t+1} + \gamma_k \zeta \hat{K}_t \tag{2.56}$$

2.6 Equilibrium

An equilibrium of model is characterized by stochastic processes for a long list of endogenous variables, including $C_s, L_s, K_s, Y_s,\ P^*_s, P_s, W_s, R^k_s,\ B_s,\ \tilde{W}_s,\ \tilde{\rho}_s,\ \pi_s,\ mc_s,$

[62] From the Appendix Ea

and policy variables $i_s, \tau_s^c, \tau_s^w, \tau_s^A, \tau_s^p, T_s, G_s^N, \tau_s^I$; as well as an initial value for price p_{-1}. The economy have an exogenous sequence ξ_s. All these variables along with earlier created model equations satisfy the household optimality conditions (inter-temporal and intra-temporal Euler-Equations), firm price setting equations (the aggregate price index and optimal price setting equation), firms' endogenous capital accumulation equation, a market clearing equation (the resource constraint) and the government budget constraint. There is no need to keep track of the other budget constraints, because lump-sum taxes adjust to keep the other budget constraints satisfied.

Meanwhile following the shock, $i_s \geq 0$. Also assume that all those firms that haven't yet set their prices (α^{s+1} share of firms) and those that have set it j periods ago ($\alpha^j(1-\alpha)$ share of firms) all have an exogenous price p_{-1}.

An estimation process, that we don't contemplate on here directly, checks for uniqueness of the equilibria, and conditions for a unique equilibrium. Yet, I stick to parameter values that give a unique equilibrium (as in Gali et al. (2007)). Gali et al. (2007) show having rule-of-thumb agents change equilibrium properties a lot, and that high degree of price stickiness and large share of the Keynesian agents together cause indeterminacy (even in case of interest rule satisfying the Taylor principle). Low and average values of 'share of Keynesians' and 'share of firms not able to change their prices' will still give a unique equilibrium (such as share of Keynesians = 1/2 and price stickiness = 0.75, baseline calibration values in Gali et al. (2007)).

2.6.1 Approximate equilibrium

Because I look at multiplier effect of tax cuts, not the optimal fiscal policy, I can get a closed form solution as in Eggertsson (2010).[63] I also assume a short and long-run in the economy which makes it easier to get a closed form solution, even-though we have an infinite horizon problem.

I combine all equilibrium conditions finally and get log-linear equations that characterize the equilibrium dynamics, accompanied by a monetary policy rule. They can be summarized by aggregate demand and aggregate supply equations. The equilibrium equations will include two Euler equations for the AD (the optimal inter-temporal consumption decision of households and optimal investment decision of firms) and one equation for the AS (the firm pricing EE). Equilibrium of the model will be reduced to stochastic processes for the endogenous variables $\hat{Y}_s, \pi_s, \hat{r}_s^n, \hat{\imath}_s$ and fiscal policy rules for $\tau_s^c, \tau_s^w, \tau_s^A, \tau_s^p, \tau_I^p, G_s^N$ that solve the following equations.

When I assume government is using sales tax-cut, for instance, to stimulate the economy, government expenditure and the other taxes do not change and $\hat{G}_s = \hat{G}_{s+1} = 0$ and $\hat{\tau}_s^w = \hat{\tau}_{s+1}^w = 0$.

The Aggregate Demand (AD) is from the optimal inter-temporal consumption decision of households and optimal investment decision of firms. The aggregate demand

[63] As pointed out by Uhlig and Drautzburg (2011), policy-makers usually care about welfare and thus the optimal policies. Although this paper is not directly focusing on optimal fiscal policy, its findings may be used in that direction.

equations are as follows, then.

$$\hat{c}_s = E_s\hat{c}_{s+1} + \frac{DD}{BB}E_s\Delta\hat{\tau}^c_{s+1} - \frac{f(1+\eta)(1-\alpha)\chi^c}{BB}E_s\Delta\hat{l}_{s+1} + \frac{AA}{BB}f\frac{\chi^c}{\gamma_c}E_s\left(\phi_b(\hat{b}_{s+1}-\hat{b}_s) + \phi_t(\hat{\tau}^x_{s+1}-\hat{\tau}^x_s)\right) \tag{2.57}$$

$-\frac{AA.CC}{BB}(\phi_y\hat{Y}_s + \phi_\pi\hat{\pi}_s + \hat{r}^n_s - E_s\pi_{s+1} - \hat{r}^n_s) + \frac{AA.CC}{BB}\chi^A(\hat{\tau}^A_s)$

Where

$AA = \chi^w\mu_{ss}\gamma_c$

$BB = \frac{\chi^w\mu_{ss}\gamma_c - f\sigma(1-\alpha)\chi^c}{\chi^w\mu_{ss}\gamma_c} = \frac{AA - f\sigma(1-\alpha)\chi^c}{AA}$

$CC = \frac{(1-f)}{\sigma}$

$DD = -f\left(\chi^c(1-\alpha)\chi^c + \chi^c AA\right) + (1-f)\frac{\chi^c AA}{\sigma}$

$$\hat{I}^N_s = \beta E_s\hat{I}^N_{s+1} - \sigma_I(\phi_y\hat{Y}_s + \phi_\pi\hat{\pi}_s + \hat{r}^n_s - E_s\pi_{s+1} - \hat{r}^n_s - \chi^A\hat{\tau}^A_{s+1}) \tag{2.58}$$

$+\chi E_s[\mu_y\hat{Y}_s - \mu_y\hat{K}_s + \sigma\hat{C}_s + \chi^c\hat{\tau}^c_s + \chi^w\hat{\tau}^w_s]$

-$\chi^c[\hat{\tau}^c_s - \beta(1-\zeta)E_s\hat{\tau}^c_{s+1}] + \chi^P[\hat{\tau}^P_s - \beta(1-\zeta)E_s\hat{\tau}^P_{s+1}] - \chi^I[\hat{\tau}^I_s - \beta(1-\zeta)E_s\hat{\tau}^I_{s+1}]$

for some,

$\chi = \frac{\beta r^k}{(1+\bar{\tau}^c)(1+\bar{\tau}^I)\epsilon_x}$

$\sigma_I = \frac{1}{\epsilon_x}$

and

$\mu_y = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha}$

$\mu_k = \frac{\eta}{1-\alpha} + \frac{1}{1-\alpha} - \eta$

Aggregate Supply (AS), from the optimal pricing and consumption decision of firms, in log linearized form is below.

$$\pi_t = \kappa(\psi+\sigma)\hat{Y}_t - \kappa\sigma\hat{G}_t - \kappa\sigma\gamma_k\hat{I}^N_t + \kappa(\eta-(\psi)-\sigma\gamma_k\zeta)\hat{K}_t + \kappa\chi^c\hat{\tau}^c_t + \kappa\chi^w\hat{\tau}^w_t + \beta E_t\pi_{t+1} \tag{2.59}$$

where, $\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{(\gamma)}$

$\psi = \frac{\alpha+\eta}{1-\alpha}$ I also need the Monetary policy rule to close the model, which is zero in this specific environment.

$$i_t = 0 \tag{2.60}$$

2.6.2 Discussion of the log-linearized equations above

The aggregate EE for households' consumption decisions is the only log-linear equation involving 'f' coefficient, fraction of the Keynesian households. Including the Keynesian households into the model generates 'direct demand effects' of distortionary tax and employment changes on C and demand. For instance, as a sales-tax is cut temporarily, it increases households purchasing power; and also makes consumption in current period cheaper than that in the future, once taxes go back to their steady-state level. As consumption increases, the aggregate-demand (AD) goes up. The AD increase, expands the output and that increases employment and real wage further. Hence the bigger multiplier effect as in the old-Keynesian theory.

The effect of tax cut on consumption and output will be maximized, if the response of interest rate and change in other taxes is muted. This would be done by appropriate fiscal and monetary policies, Gali et al. (2007).

It should also be noted that, I assumed real wages are always higher than the MRS (between C and L) and hence households are willing to supply any amount of labor demanded. I also assumed both households are supplying every type of labor (both types of households are uniformly distributed among labor types), and firms' demand is uniformly distributed among households. This means their labor supply is responding to tax changes in the same way. Thus, the firm's Euler Equation is not affected through labor supply differences.

2.7 Calibration of the Model

Theoretical background of the calibration of the model economy to the data and selection of the parameter values follows Eggertsson (2010), Christiano (2004), Woodford (2003), Gali et al. (2007) and Walsh (2010). I follow Eggertsson (2010) and use parameter and shock values to match an output contraction of 30 percent and a deflation of 10 percent, both of which are statistics from the first quarter of 1933 in the U.S. economy. This was trough of the Great Depression with a zero nominal interest rate. This benchmark is to strengthen the argument that the fiscal stimulus in 2009 was, as argued by many economists, more like a reaction to avoid another great depression due from the banking shock in 2008. The magnitude of the crash due to the 2008 crisis was comparable to the Great Depression according to Reis (2010). Great depression is the main example for any liquidity trap analysis according to Krugman (1998).

I use parameter values close to the benchmark Eggertsson (2010) model, as much as possible, to make sure I have a good comparison between my and his model. Each time period in the model is a quarter of a year. $\beta = 0.995$ is the discount factor for the Ricardian households and imply a long-run real-rate of interest $\overline{r} = r_{ss}$ equal to 2 percent.[64] κ is consistent with the empirical estimates from Rotemberg and Woodford

[64] $\beta = (1 + r_{ss})^{-\frac{1}{4}}$ and $(1 + 0.02)^{-\frac{1}{4}} = 0.995$

(1997). σ_u, coefficient of relative risk aversion is 1.1599. α and ζ are standard in the literature.

$\eta = 1.5692$ is the inverse Frisch elasticity of labor supply. It is very high in Woodford (2003) - ZLB does not bind in small-shock case according to Christiano (2004), and he keeps it at a standard value (for 1) and shows that with a relatively smaller value, even a big shock is not causing ZLB to bind (never binds). Gali et al. (2007) sets it to $\eta = 0.2$. If $\eta = 1$ then in no-investment case same results as in Eggertsson and Woodford are obtained, yet for investment case, results are very sensitive to the value of η. Christiano (2004) argues if parameters in Woodford (2003) are used, in a model with investment, then probability of output collapse and negative inflation is reduced a lot. An elasticity of 100 (very high compared with literature) is needed for the worst case scenario to happen. Christiano (2004) sets $G/Y = \gamma_g = 0.18$, the average government expenditure - output ratio in the post-WWII US economy. While Gali et al. (2007) take it as (0.2).

$'1 - \omega_w'$ shows the probability that the economy converges to its steady-state equilibrium each period. γ (fraction of firms that keep their prices fixed) imply prices are fixed for $\frac{1}{1-\gamma}$ periods on average. Depreciation rate ζ and β are standard as in RBC models. Markup $\mu = \frac{1}{\theta_p}$ means a markup of price on marginal cost in the steady state.

Eggertsson and Woodford (2003) has no capital accumulation (no investment), therefore $\epsilon_x = \infty$. Woodford (2003), on the other hand, suggests $\epsilon_x = 3$ s.t. for a small shock ($r_t^n = -2$ from SS value 4 percent) ZLB is not binding. Eggertsson (2010) chooses ϵ_x such that output contraction in the fourth quarter the depression is -30 percent (he assumes consumption and investment decline in the same proportion). Gali et al. (2007) choose a value equivalent to 1 in baseline calibration.

Calibration takes place in accordance with the following target values for the benchmark economy:

Targets	Values
$\hat{Y}_s$	-30 percent
π_s	-10 percent

Parameter values of policy rules are as follows. $\phi_p = 1.5$, the Taylor principle, as commonly assumed and $\phi_y = 0.5/4$ as in Taylor (1993). Choice of fiscal policy (rule) parameters, ϕ_b and ϕ_t, affect aggregate consumption and hence demand in the economy. These parameters will also be changed or sensitivity analysis.

Gali et al. (2007) - estimated averages from VAR: $\phi_b = 0.33$ and $\phi_t = 0.1$, their ϕ_g.

Table 2: Parameter values of the model economy

Parameters	Description	Values
σ_u	coefficient of relative risk aversion	$1.1599\gamma_c$
β	subjective discount factor	0.995
η	inverse elasticity of labor supply	1.5692
γ_l	Calvo hazard rate	0.75
γ_p	Calvo hazard rate	0.75
θ_p	degree of MC in goods market	0.1
θ_l	degree of MC in labor market	0.1
ζ	depreciation rate of capital	0.025
α	capital share of output	0.4
ϵ_x	degree of adjustment cost parameter	71.9
ϕ_p	coefficient on $\hat{\pi}_s$	1.5
ϕ_y	coefficient on $\hat{Y}_s$	0.5/4
ϕ_b	coefficient on $\hat{b}_s$ in fiscal rule	0.33
ϕ_t	coefficient on $\hat{t}_s$	0.1
r_s^n	natural rate of interest rate	-0.0104
$\bar{\tau}^c$	SS value of consumption tax	0.05
$\bar{\tau}^w$	SS value of labor-income tax	0.2
$\bar{\tau}^A$	SS value of capital tax	0
$\bar{\tau}^P$	SS value of profit tax	0.3
$\bar{\tau}^I$	SS value of investment-tax credit	0
f	measure of Keynesian households	between 0.5 and 0
ω_w	prob. of $\hat{r}_t^n$ not returning to its SS value	0.9030
γ_g	average share of G in GDP	0.2
γ_i	average share of I in GDP	0.2

2.8 Discussion of the Model Calibration

A key assumption here, as is usually assumed in all DSGE models, is that the economy faces a shock that takes the economy into a liquidity trap with zero nominal interest rate. I assume, given all the fiscal variables stay at their steady state values (no intervention) the economy faces an output collapse comparable to the Great Depression after a shock. The Great Depression is a useful benchmark for my model for two main reasons. The first is the trivial case that it is a good benchmark for any liquidity trap analysis, as pointed out by Krugman (1998); and secondly, as pointed out by Eggertsson (2010) among others, economic models show if the government did not use its fiscal tools to intervene into the economy in 2008; the US would have faced another Great Depression.

The paper analyzes the question that; if we cut taxes by 1 percent (from their steady-state values), how much does the output change. Since all the aggregate variables including G are log-linearized as a fraction of steady state output level, a one percent change in taxes on the output and hence consumption and labor income will be of the same effect as a G effect. Therefore, comparison of multipliers makes sense as in Eggertsson (2010).

In a crisis period, such as that studied here, what we observe is that, initially the (log) natural rate of interest goes down to the negative territory unexpectedly; and then it goes back to its long-run (SS) value ($\bar{r} > 0$) with some fixed probability, $'1-\omega_w'$ each period.[65] If inflation target is zero, as assumed here, even when the central-bank decreases nominal interest rate to zero, the real interest rate is positive because of expectations of deflation. However, in a negative natural rate of interest case, if the central-bank has a large enough inflation target to get zero interest (for instance $r_s^n = -2$ percent and $\pi^* = 2$ percent), then it is enough to close the output gap and keep inflation on target. However, as discussed earlier, I assume the central bank is not able to commit to a future positive inflation rate.

I assume the economy is initially in a deterministic steady state (with no shock)

[65] If $a < 1$, then $log(a) < 0$.

until period (t-1). At period $'t'$ there is, unexpectedly, a shock hits r_t^n and decreases it to maybe even negative. Each period after that, $\hat{r}_t^n$ stays low with a probability $'\omega'$ and increases to its steady-state value with probability $'1-\omega'$.[66]

I also assume 'T' is the period, when r_s^n goes back to its steady state value with no exogenous shock. Then, the short-run is the period where we observe a preference shock s.t. $r_s^n = \frac{1}{\beta}\frac{\xi_s}{\xi_{s+1}} < 1$ for $s < T$; and the long-run is defined by $r_s^n = \frac{1}{\beta} > 1$ for $s \geq T$ (This is because $\frac{\xi_s}{\xi_{s+1}} < \beta$ for $t < T$, and $\frac{\xi_s}{\xi_{s+1}} = 1$ for $t \geq T$ holds exogenously). And $1 + i_s = 1$ for $s < T$, and $i_s = r_s^n = \bar{r} = \frac{1}{\beta}$ for $s > T$.

In the log-linearized form, it will be as $r_s^n = log\beta^{-1} + \hat{\xi}_s - \hat{\xi}_{s+1}$. Where $log\beta^{-1} \cong r^n$, when there is no shock, the steady state real risk-free interest rate. $\hat{\xi}_s - \hat{\xi}_{s+1}$ is a measure of risk from the exogenous shock and ξ is the shock that enters the utility function. In a demand shock case r_s^n is a function of the shock (ξ) that enters the utility function. Where a lower demand (negative demand shock) would increase savings and then the real interest rate would go up. ξ is a vector of disturbances that covers an external shock such as a changing technology (A) or preference.

I assume, as in Correia et al. (2011), that the nominal interest rates are always set to zero whenever (log) natural rate of interest is negative ($r_s^n < 0$). And they start increasing when natural rate of interest becomes positive. This means, when we have a negative natural rate of interest, deflation and thus positive real interest rate at the ZLB. This is a Liquidity trap case. And deflation means an output contraction as shown

[66] Higher $\hat{r}_s^n$ means lower β, and thus higher consumption since agents value future consumption less. A lower $\hat{r}_t^n$, means higher β and therefore less current consumption relative to future. The shock in this economy makes $\hat{r}_t^n$ negative (in log form).

in a NK IS curve.

In order to increase spending and demand in the economy, we need negative real interest rate. If we keep taxes constant, the only way to decrease the real interest rates (to negative) is to generate positive inflation as offered by Krugman (1998) and Eggertsson and Woodford (2003). Yet we know that positive inflation means relative price dispersions in economy; and given staggered price setting decision of firms, it creates distortions in real economy. We therefore, make future inflation announcements not-credible.

However, by changing taxes, as in Correia et al. (2011), it is possible to get back to an efficient outcome level even when the ZLB binds. However, to get the efficient outcome level, it is necessary that all distortionary taxes adjust simultaneously. So we can set nominal interest rate equal to natural rate of interest whenever the latter is positive, and set nominal rate to zero whenever the natural rate of interest is zero. Then we use the inter-temporal and intra-temporal Euler-Equations to set the taxes such that all the distortions are eliminated. Price level will be kept constant of course.

We assume a Markovian process for the natural rate of interest. In the short run if the shock occur, $r_s^n < 0$, then recession occurs and $i_s = 0$ (zero lower bound binding). I assume the government uses distortionary tax cuts to stimulate the economy, e.g. $\hat{\tau}_s^w < 0$ in the short-run. $\hat{\tau}_s^w < 0$ is reversed to the steady-state value $\hat{\tau}_s^w = 0$ with probability $(1 - \omega)$ each period in the short-run, once we have a cut. The two state Markov process for the shock (the assumption that the shock goes back to its long-run value each period in short-run with probability $(1 - \omega)$ each period) means inflation and output goes back back to their long-run values with the same probability $(1-\omega)$. Which basically means,

I only consider the case where condition $C1$ and $C2$ in the Eggertsson (2010) model holds and we have a unique and bounded equilibrium. Including the Keynesian households, we need to make sure the parameter values are again consistent with a uniques equilibrium. That is the case that we have zero interest rates in the short-run, due to the banking shock. In the long-run there is a unique bounded solution (if only Ricardians existed) with $\pi_s = 0$, $\hat{Y}_s = 0$ and $i_s = r_s^n = \bar{r}$.

$i_s = r_s^n = \bar{r}$ for $s \geq T$

$i_s = 0$ for $s < T$

Assuming $s < T$ and there is a banking shock, in the next period, $'s+1'$, we have the following case.

$\mathrm{E}_s\hat{Y}_{s+1} = \omega\hat{Y}_s + (1-\omega)0,$ $\qquad E_s\pi_{s+1} = \omega\pi_s + (1-\omega)0$

and the tax cut rule is,

$(\hat{\tau}_s^c, \hat{\tau}_s^w, \hat{\tau}_s^A, \hat{\tau}_s^P, \hat{\tau}_s^I, \hat{\tau}^{GN}_s) = (1,1,1,1,1,1)$ $\qquad$ s

$(\hat{\tau}_s^c, \hat{\tau}_s^w, \hat{\tau}_s^A, \hat{\tau}_s^P, \hat{\tau}_s^I, \hat{\tau}^{GN}_s) = (0,0,0,0,0,0)$ $\qquad$ s$\geq T$ $\qquad$ $\mathrm{r}_s^n = \bar{r}$

Long-run (or steady state) in this paper is the case that the shock r_s^n goes back to the steady state $r^n = \bar{r}$. Short run, on the other, is the case that the economy faces a shock, $r_s^n < 0$. In the short-run, given that the shock has occured, the shock goes back to its steady state value with probability $(1-\omega)$.

Because I am looking at the special case where the zero bound binds (due to existence of the shock), all i_s are equal to zero.

$E_s\hat{Y}_{s+1} = \omega\hat{Y}_s + (1-\omega)0 = \omega\hat{Y}_s$ and $E_s\pi_{s+1} = \omega\pi_s + (1-\omega)0 = \omega\pi_s$.

$E_s\hat{\tau}^c_{s+1} = \omega\hat{\tau}^c_s + (1-\omega)0 = \omega\hat{\tau}^c_s$ and $r^e_s < 0$.

I weigh the two multipliers from the EE from firm's investment decision and the EE from household's consumption decisions. Steady state (or long-run) share of investment in total GDP γ_i will be used for the multiplier from equation 128 that comes from firms' investment decision problem; and steady state share of consumption in total GDP γ_c will be used to weigh the multiplier from households' consumption EE.

The aggregate multiplier is,

$$\gamma_c M_{hh} + \gamma_i M_{firm} = M_{total} \tag{2.61}$$

where γ_c and γ_k were given earlier as $\gamma_c = 1 - \gamma_g - \frac{\zeta\alpha\theta}{r^k}$ and $\gamma_k = \frac{\alpha\theta}{r^k}$. From the resource constraint in steady state,

$Y = C + I + G \Rightarrow \frac{Y}{Y} = \frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y} \Rightarrow 1 = \gamma_c + \gamma_i + \gamma_g$

Then $\gamma_i = 1 - \gamma_g - \gamma_c$.

2.8.1 Labor tax cuts

We are interested in a fiscal policy tool that increases demand and hence output in the economy such that it brings an end to the recession caused by missing demand. In the short-run, $s < T$, $\hat{\tau}^w_s < 0$ and in the long-run, $s \geq T$, $\hat{\tau}^w_s = 0$. As before, we assume, each period tax cut goes back to steady-state with probability $'1-\omega'$. This Markov process has the same implications as a stochastic process, $\hat{\tau}_s = \mu_s\hat{\tau}_{s-1} + \epsilon_s$ with

ϵ_s iid and normally distributed, assumed in Gali et al. (2007) and Eggertsson (2010).

As discussed earlier, labor tax cuts in this model are initially the labor-income taxes paid by workers rather than payroll taxes paid by firms. Thus workers are directly affected by this cut. This is both because wages are fixed in this model and nominal wages on contracts exclude any taxes.

The labor-income tax cut, $\hat{\tau}^w_s < 0$, impact and long-run multipliers will be as follows.

$\frac{\Delta \hat{Y}_s}{-\Delta \hat{\tau}^w_s} > 0$

and

$\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}^w_s} > 0$

where Δ means change relative to steady-state of no variation.

Fiscal policy is as below,

$\hat{\tau}^w_s = \phi_s r^n_s \qquad s < T$

$\hat{\tau}^w_s = 0 \qquad s \geq T$

2.9 Results

By including nominal rigidities and hand-to-mouth agents (via the direct demand effect) into the model, I primarily focus on and expect to see the positive effect from this countercyclical discretionary fiscal policy that has been controversial in recent studies. As a matter of fact, compared to the benchmark Eggertsson (2010) model, the paper finds significant effects for consumption and / or labor-income taxes.

Table 4: Benchmark Model Outcomes

Targets	Values initially	Value after the ZLB binds
$\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}_s^w}$	$-1,2$ percent	- 2,1percent

I find $\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}_s^w} = 2,1$, which means: if the fiscal authority cuts tax rate $\hat{\tau}_s^w$ by 1 percent, output increases by 2,1 percent. In dollar terms, it means in the steady state, when government cuts taxes by 1 dollar, it increases output by 2,1 dollars.

The idea is that, when there is a tax cut, nominal income from work goes up and that increases willingness to work more, to get more money for each unit of labor supply. Increasing labor supply, decreases real wages (marginal cots down). Lower real wages means lower input cost which increases supply and decreases prices. Therefore, we observe a deflationary pressure. Deflationary expectations, in return, increase the real interest rate which decreases demand and spending in the current period.

If we had positive interest rate, under normal circumstances - absent any shock, the monetary authority would cut taxes aggressively (more than proportional) in order to decrease real interest rates, and thus increase the demand in economy (CB following the Taylor principle). However, if the ZLB binds, the monetary authority is not able to cut the nominal rates to change the real rate of interest. Therefore, the AD curve becomes upward sloping. This means a low inflation will always imply a higher real rate of interest and thus lower demand, and a high inflation will give low real rate because central bank is not able to respond.

Eggertsson (2010), in a model with endogenous investment, finds a multiplier equal to 0.16 in positive interest rate, and another equal to -1.2706 for the case that the ZLB binds.[67] My multiplier is much higher since I am including the direct demand effect via the the Keynesian agent setup. Based on the multipliers he estimates, Eggertsson (2010) suggests a balanced budget (GBC) stimulus package with temporary sales-tax cuts and/or investment tax credits; financed by again temporary labor-tax and/or capital income tax increases. But, don't increase labor taxes!, because some agents consume all of their current after-tax income.

Meanwhile, Eggertsson (2010) finds adding capital does not change results a lot. Which contradicts with findings here and those in Christiano (2004). One reason could be that the same shock is included in both utility function (and thus C EE) and investment adjustment cost (hence in the I EE). Another reason is that Eggertsson (2010) does not estimate model parameters again when he adds capital to the model. Instead, he uses his same estimates from the model without endogenous capital accumulation. Yet, his paper claims, change in multipliers would be even smaller if he did reestimation of the model parameters after he adds capital.

Eggertsson and Woodford (2003) has no capital accumulation (no investment), therefore $\epsilon_x = \infty$. Woodford (2003), on the other hand, suggests $\epsilon_x = 3$ s.t. for a small shock ($r_t^n = -2$ from SS value 4 percent) ZLB is not binding. Eggertsson (2010) chooses ϵ_x such that output contraction in the fourth quarter the depression is -30 percent. Gali et al. (2007) choose (they call it η) 1 in baseline calibration.

[67] $\Delta \hat{\tau}_s^w = -1$ percent $\Rightarrow \Delta \hat{Y}_s = -1.2706$ percent (because everything is in logs).

2.9.1 Robustness check of fiscal multipliers - Sensitivity analysis

In an effort to check for robustness of the fiscal multipliers, I follow the literature and make a few adjustments. For instance, Uhlig and Drautzburg (2011), change capital share to 0.35. He would estimate median estimates for the Calvo parameter for prices and wages at 0.81 and 0.83 respectively, in order to increase the price stickiness. The fiscal multipliers are also sensitive to the duration of the ZLB as discussed in the literature review. $'1 - \omega_w'$ shows the probability that the economy converges to its steady-state equilibrium each period. γ (fraction of firms that keep their prices fixed) imply prices are fixed for $\frac{1}{1-\gamma}$ periods on average. Depreciation rate ζ and β are standard as in RBC models. Markup $\mu = \frac{1}{\theta_p}$ means a markup of price on marginal cost in the steady state.The benchmark duration is initially set at 8 quarters and then change it to 12 quarters and in another case endogenize it. A longer duration decreases the fiscal multiplier to -0.03 or -0.19 respectively.

Results are very sensitive to the value of η. For high values, as it is not possible to observe the ZLB, the fiscal multiplier will be very slow; whereas, for very small eta values, as it is very easy to get into a liquidity trap case, the fiscal multipliers get very high. η is very high in Woodford (2003) - ZLB does not bind in small-shock case according to Christiano (2004), and he keeps it at a standard value (for 1) and shows that with a relatively smaller value, even a big shock is not causing ZLB to bind (in a way, the ZLB case never binds). Gali et al. (2007) sets it to $\eta = 0.2$. If $\eta = 1$ then in no-investment case same results as in Eggertsson and Woodford are obtained, yet

Table 4: Robustness Tests for Various Parameter Values

Targets	Initial value ($f = 0,3$)	at the ZLB	ω_w up to $0,95$	f $= 0,5$	$f = 0,7$
$\frac{\Delta \hat{Y}_{s+k}}{-\Delta \hat{\tau}_s^w}$	$1,2$ percent	2,1	3,35	4,05	4,70

for investment case, results are very sensitive to value of η. Christiano (2004) argues if parameters in Woodford (2003) are used, in a model with investment, then probability of output collapse and negative inflation is reduced a lot. Yet an elasticity of 100 (very high compared with literature) is needed for the worst case scenario to happen.

Sensitivity analysis for the fiscal multipliers (model features that change the multiplier):

f, share of the Keynesian households, is first set to 1/3 and then will be changed for sensitivity analysis. I only use range of f values consistent with a unique equilibrium. Campbell and Mankiw (1989) find a fraction of 1/2 captures the importance of the rule-of-thumb behavior in the industrial economies. Forni et al. (2009) find a fraction of non-Ricardian agents around 30 to 40 percent for the Euro area. While Gali et al. (2007), in an estimated DSGE model for the Euro area, finds (baseline 1/2) fraction of Keynesians over 1/4 is needed for a positive response of C to fiscal shocks in a monopolistically competitive labor market (for perfect competition case in labor market, very high-unrealistic- fractions of K are needed). Uhlig and Drautzburg (2011) take the fraction of the constrained households between $(0, 0.5)$.

Instead of considering pure myopic Keynesian agents, Uhlig and Drautzburg (2011) consider rates of time preferences ranging between 7 and 30 percent higher for the credit-constrained households, compared to unconstrained households. Which means a higher β for the unconstrained households again. They also find that with the rates of time preferences around or higher than 20 percent, the constrained agents get substantial positive welfare gains.

Gali et al. (2007) find the impact multipliers are changing by the degree of price stickiness γ, with a higher stickiness meaning higher multiplier and the multiplier changing in nonlinear way (increasing), and that $\gamma > 0.5$ are consistent (with micro evidence and) with positive multiplier resulting from stronger consumption response. Fiscal multipliers are also sensitive to the capital adjustment cost (ϵ_x), but not affected by elasticity of substitution for labor η. Rise in capital adjustment cost, increases consumption further and decreases the negative impact on investment and thus a higher output is observed.

Sensitivity to policy parameters: a higher ϕ_π means stronger response to increasing inflation, and thus a higher real interest rate, r. Higher real interest rate, decreases consumption (of the Ricardians) and thus the output, therefore it negatively affects the multiplier. Gali et al. (2007) further find that positive co-movement of C and output requires a high response of debt financing, ϕ_b in the fiscal rule, and a low response of tax - ϕ_t. This basically means, the more tax cuts are financed by debt finance in future, the more better off the Keynesians are and therefore the higher C and Y response we get.

2.10 Extensions & Future Work

One problem is that since we have some Keynesian agents in the model, implementation lags matter, as claimed in the Keynesian theory. Timing of tax cuts matters since some of the agents make their decisions per period. In models where we have only Ricardian agents, implementation lags would not matter that much because households optimize inter-temporally and take all future policies into account. So its the announcement of a policy, rather than timing of implementation that matters.[68]

The other issue is, as Krugman (1998) points out, if current income has very high effect on spending, then economy could have multiple equilibria as well. However, there is no evidence for such multiple equilibria. If it is the case, this could lead a sufficient temporary fiscal stimulus to take the economy out of the trap (where conventional MP is effective again) and thus a temporary fiscal shock have permanent effects. But it should not mean that FP was not effective and should not be used, as he suggests both the fiscal policy and inflation expectation.

Extensions of this study could include income and consumption tax cuts. I may follow Hall and Woodward (2008), Feldstein (2002) and Correia et al. (2010) who use the following setup. Decrease current consumption taxes and increase future taxes. Increase in taxes goes on until the recession is over. Meanwhile, labor taxes go down not to have any distortionary affect through MRS between consumption and labor.

$\mathrm{E}_s \hat{\tau}^c_{s+1} - \hat{\tau}^c_s = r^n_s$

[68] See also, the arguement by Christiano et al. (2009) and Eggertsson (2010), expectations of a future policy matter if agents expect a policy in all future states of the world.

s.t. in the IS curve, it will satisfy $\hat{Y}_s = E_s\hat{Y}_{s+1} = 0$ and $E_s\pi_{s+1} = 0$, and

$$\hat{\tau}_s^w = -\hat{\tau}_s^c \tag{2.62}$$

s.t. in the Phillips curve, it will satisfy $\hat{Y}_s = 0$ and $\pi_s = E_s\pi_{s+1} = 0$, and another condition for capital taxes,

$\hat{\tau}_s^A = -\hat{\tau}_s^c$

I use debt payment for current tax cuts and assume tax cuts are financed by current or future lump-sum taxes. An alternative approach, as in Correia et al. (2011), is to use other distortionary taxes to finance current tax cuts in future.

2.10.1 Other distortionary taxes

Multiplier of 'investment tax credit': 0.31 in Eggertsson (2010). One percent decrease in investment tax credit (that decreases net purchaing price), increases output by 0.31 percent. $\hat{\tau}_s^I = -1$ allows firms to subtract one percent of the 'purchaing price' (with sales tax added) of their investment from their profits (that will be taxed again with τ_s^P). Investment tax credit makes current investment cheaper, and thus gives an incentive to firms to invest today. Therefore increases the spending and demand that is needed.

Multiplier effect of cutting taxes on saving $'\tau_s^{A}{}'$: -0.0752 in Eggertsson (2010). One percent decrease in taxes on saving, decreases output by 0.0752 percent. $\hat{\tau}_s^A = -1$ gives incentive to the Ricardian consumers to save more. The aggregate income goes down, however, since everybody tries to save and demand goes down. Lower demand

and hence income means lower saving. Therefore, when everybody is trying to save more, the aggregate saving goes down. Which is the tarditional Keynesian 'paradox of thrift'.

Multiplier effect of cutting taxes on profit $'\tau_s^P'$: -0.4670 in Eggertsson (2010). One percent decrease in taxes on profit, decreases output by 0.467 percent. $\hat{\tau}_s^A = -1$ gives incentive to the firms to pay the profit as divident today and thus decrease investment today. It is therefore, more reasonable to increase taxes on profit today to give incentives to firms to increase their investment, rather than giving out all cash as divident to the shareholders.

Multiplier effect of cutting taxes on consumption $'\tau_s^c'$: 2.73 in Eggertsson (2010). One percent decrease in taxes on sales, increases output by 2.73 percent. $\hat{\tau}_s^c = -1$ gives incentive to the consumers to increase their spending and thus increases the demand in the economy.

Part II

Efficacy of Monetary Policy

Chapter 3

Efficacy of Unconventional Monetary Policy

Executive Summary

This paper aims to provide a better understanding of the efficacy and macroeconomic benefits of an unconventional monetary policy for a closed economy. The paper uses a partial equilibrium, standard New-Keynesian DSGE model, but incorporates the credit frictions proposed by Kiyotaki and Moore (2008) to better capture the real world implications of macro policies. The central bank of the closed economy employs quantitative easing (credit easing policies in particular) to stimulate the economy. The paper deals with efficacy of this most controversial tool at the central banks' discretion, to deal with a financial crisis and analyzes how it affects the macroeconomic (C, I and Y) and financial variables (asset prices such as the q ratio) of a country. In particular,

it analyzes how the interaction of the zero-lower-bound with credit frictions affect the economy. Simulation analysis show the quantitative easing (QE) policies do actually work and are able to stimulate the macro variables significantly.

" *...these policies (QEs) are not beggar-thy-neighbor but rather are positive-sum, enrich-thy-neighbor actions.*" Ben Bernanke, March 25, 2013 March.[1]

3.1 Introduction

The Great Recession and the following European debt crisis has taught a new lesson: Even at the zero-lower-bound (ZLB), on the short-term nominal interest rates (e.g. the federal funds rate), there is still much monetary policy can accomplish. The new tool, quantitative easing (QEs), used by the central banks in advanced economies (namely the Fed, the BOJ, the BOE and finally the ECB) was all new. It has been argued, since then, that the recovery across advanced economies owes its success to the determination and aggressive policy actions of the central banks[2].

Understanding these policies, requires a profound understanding of the problem during the crisis. It has long been argued (see, among others Brendon et al., 2011) that private assets (e.g. mortgage backed securities - MBSs) and the related housing sector were at the core of the 2008 financial crisis (as is evident in the failure and the following effects of Bear Sterns and Fannie Mae). It would, hence, not be possible to increase the economic activity without a significant improvement in the housing sector. Naturally, the QEs should have focused on the MBSs mainly. As a matter of fact, they mostly did. Even during the most recent one in the U.S., in September 2012, when the Fed announced QE III (or LSAP 3), it was aimed at agency MBSs in particular. It

[1]The idea that there is no currency war and that the QE policies are to the benefit of the whole world economies.

[2]See e.g. Eichengreen et al. (2011) and Del Negro et al. (2013) among others.

brought forth a new $40 billion worth asset purchases each month (with no end date).[3] Main goal was said to increase the economic activity and bring down the unemployment rate. It is supposed to continue until the economy (particularly the labor markets) shows significant improvement.[4]

Up until December 2013, the Fed was buying $85bn (then started decreasing it by $10bn a month) worth assets a month and funded these purchases with newly created money.[5] Meanwhile, the BOJ has recently announced it would accelerate its bond purchases to nearly 84 trillion yen this year, equivalent to 17 percent of GDP in Japan. In Europe, on the other hand, the ECB is planning to (at least mostly expected to) start active purchase of assets during the spring of 2015. This paper focuses on effects of these QE policies and discusses whether these QE policies implemented by the advanced economy central banks are a simple 'beggar-thy-neighbour' (zero-sum) actions or a (positive-sum) 'enrich-thy-neighbour' policy actions, as proposed by Bernanke (in a March 2013 briefing). In a simple New Keynesian DSGE model setup, incorporated by Kiyotaki and Moore (2008) type credit frictions, I hope to better capture fluctuation in the macroeconomic variables and show how its real world economy effects work.

It should also be noted that while the Fed has very recently began tapering its QE stimulus packages, in the EU and Japan, central banks are still easing. Hence, unlike the Bank of England and the Federal Reserve, the ECB and the BOJ are more

[3] This $40 billion agency MBSs was in addition to the ongoing long-term asset purchases of around $85 billion per month (which includes $45 billion worth Operation twist that ended in December of 2012).

[4] QE III was told to bring in a new language in Fed statements, in the sense that it recalls importance of employment increase within the central bank's dual mandate: price stability and full-unemployment.

[5] The latest range of QE was funded by newly created money, but in the past, it used different measures to fund its asset purchases. See the introduction section.

into further looser monetary policies in order to keep their economies out of the deflation risks. Additionally, given low inflation and even deflation problem in advanced economies, QEs are still very likely to be on the agenda for a foreseeable future, in particular in the developed economies. In 2014, the ECB decreased the rates to the negative territory, as a means to fend off deflation. Even in the USA, as Mrs Yellen put it during the talk at her nomination in late 2013, still, 'more needs to be done to strengthen the recovery', and 'too many Americans still can't find a job and worry how they'll pay their bills and provide for their family.' This is a clear sign that there is still a lot to be done in terms of monetary expansion and that labor markets are (and should be) at the core of the recent policy actions of the Fed.

3.1.1 Do the QEs really work?

Broadly speaking, the usual concern about the effects of QE policies is their domestic effects as they were implemented by advanced economies to mainly deal with their domestic issues. QEs work via a few channels, to affect the economies they are implemented in. They may, for instance, cause an excessive decline in exchange rates and even asset or real-estate bubbles (through capital flows). They have usually sparked concerns of currency war. Many monetary economists, including economic historian Eichengreen, oppose these claims though. On the other hand, if investors (that sell their securities to the central banks) buy foreign assets, domestic currency also weakens and NX increases. Decreasing currency value in rich-world (dollar depreciating for instance) causes capital inflow to the rich world. Neely (2010), for instance, find that the central bank purchases of LSAPs had significant effects on the US and foreign long-term bond yields and on the exchange-rates.[6]

Expansionary monetary policies in advanced economies were reported to increase price of the other financial and real estates (since increasing money supply increases the demand for these financial assets) such as the stock prices. Some were even expecting a bubble in real estate market (similar concerns are voiced in the US as well, every now and then). Yen was reported to go down against the US Dollar after the QE announcement by the BOJ. Increasing asset value, increases purchasing power of the households and firms. Increasing demand and consumption, in return, increases economic activity, and hence the taxes and income for government. In Japan, since the beginning of 2013 when the QQE was initially applied, a 4% growth was observed in the first 2 quarters and another 2 percent growth in the third quarter. Devaluation, following a huge supply of Yen, depreciates (weakens) Yen against other major foreign currencies and increases competitiveness of the domestic producers and hence increase exports. Increased exchange rate (devaluation in SOE currency) will (in Japan for instance, since the Yen is depreciated) also help increase inflation that is needed through imports.

Likewise, after the latest (September 2012) round of the QE policies in the US, the value of US dollar was decreased, since total supply of dollar is increasing. On the other hand, the stock indices were reported to jump by %1.5 with the $S\&P$ 500 hitting its highest close since 2007. The real test will be the behavior of spending, income and

[6] in Gagnon and Sacks(2010).

employment in a longer time period. Del Negro et al. (2010) find, similar to the very high government expenditure (and other fiscal multipliers as in Eggertsson (2010) and Christiano et al. (2005)), that non-standard OMOs (liquidity policies) have very large effects in a liquidity trap (in a binding ZLB case). Even as high as preventing another Great Depression (which correspondents to an up to %30 percent drop in output). Elias and Jorda (2013), based on Jorda et al. (2012), show, had a central bank responded to the 1907 panic the same way they did at the earlier stages of the 2008 crisis, the GDP would have contracted by 2 percent less. They show both the liquidity shock (the financial shock) and unconventional policy can have large quantitative effects. In absence of the unconventional monetary policies implemented by the central banks, they argue, the economy would have faced a second great depression.

3.1.2 Focus on heterogeneity

An important feature of the model in this paper is the heterogeneity it employs. The models that distinguish between savers and borrowers, as in Bernanke et al. (1999), Eggertsson and Krugman (2010) and Brendon et al. (2011), are usually more useful in policy analysis and discussions, today, than a very basic representative-agent model. Most basically, if borrowers (or debtors) are debt-constrained, as in Eggertsson and Krugman (2010), than the Ricardian equivalence (which itself is an outcome of the models rather than being an empirical reality) does not hold. In the representative household models, since the household holds all the government debt and provides all the money and funds, borrowing constraints (that were at the core of the crisis) do not bind and thus do not matter. However, if we assume heterogenous households as in Bernanke et al. (1999) or more recently as in Eggertsson and Krugman (2010) or Gali et al. (2007), the constraints will matter. The effect of OMOs will depend on the type of assets bought, what kind of liabilities are issued and more importantly the way and the time the central bank or government profits (transfers) are rebated to the public.

Likewise, papers using the simple representative agent assumption ignore the complications of real life such as varying liquidity of different papers and various interest rates that co-exist. Brendon et al. (2010) use a sticky price business-cycle model with some entrepreneurs constrained with collateral constraints. While they find no role for quantitative easing in general, their model show, given some heterogeneity and differing liquidity of papers, they find substantial use for credit easing policies (the central bank buying securitized loans from banks - that were aimed at entrepreneurs). Therefore, one of the key points of the paper is the differing liquidity of private and public papers (different assets have different liquidities), as in Tobin (1969). The paper, with its focus on liquidity (as opposed to nominal rigidities only, usually studied in the NK models), has an alternative interpretation of the Keynesian interventionist policies, as in Kiyotaki and Moore (2012). With its focus on the effect of LSAPs on market interest rates, the paper analyzes whether the LSAPS decreased the market interest rates and created any spreads.

This paper, via the entrepreneurial setup, have a banking sector (to make sure there isn't any liquidity trap and money supply is effective). This type of financial intermediaries (that function as banks) are key to the money multiplier process as well. If we believe monetary base and aggregate money supply will be different, then there should be a banking sector in the model that uses that base money supplied and via a money creation process (credit and lending) it causes a much higher money supply that is needed to avoid any credit constraints and any decrease in the aggregate demand. Gertler and Karadi (2012) argue LSAPs show the central bank as a financial intermediary. This would work if there was any limit to arbitrage for the private financial intermediaries.

3.1.3 Relation to the literature

The paper uses a standard NK DSGE model (in line with Christiano et al. (2005) and Smets and Wouters (2007)) and incorporates the Kiyotaki and Moore (KM) credit frictions to account for real effects from an unconventional MP. Other frictions in the model are a result of sticky wage and prices, and capital adjustment costs (as is common in the RBC models). In response to a shock that decreases the aggregate demand and tightens the credit constraints, the central bank uses (unconventional) monetary expansion to stimulate the demand and loose the credit constraint by increasing the collateral values.

The paper is in line with literature that incorporates financial frictions, nominal rigidities and a shock reminiscent of liquidity problems during the 2008 crisis into an otherwise standard DSGE model to analyze efficacy of various policies. This literature includes, but not all, Christiano et al. (2009), Bernanke et al. (1999), and Curdia and Woodford (2009) and Kiyotaki and Moore (2012). The paper adds nominal rigidities to Kiyotaki and Moore (2008) model. Sticky price and wages are important because they allow output to drop. Absent sticky prices, only investment would be negatively affected by a liquidity shock. Interaction of financial frictions (shocks), nominal frictions (rigidities) and the binding ZLB constraint (that constrains the central bank to respond to the shock or to accommodates to shock) creates this results.

The model has heterogeneity in the household sector. The financial intermediation that is introduced in the model is crucial (and matters) for better allocation of resources.[7] The paper, in this sense, is in line with a series of papers including Williamson (1987), Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) in Kiyotaki and Moore (2008) that include the banking sector in their business cycle model. It analyzes the real effects of the quantitative easing policies implied by the central banks of the rich economies around the world, mainly the Fed and the BOE, BOJ and most recently the ECB.

Gertler and Karadi (2009) show an active rule for buying bank equities, as in the latest credit easing policy announced, substantially decreases the response of the output gap and the inflation. Curdia and Woodford (2010) show if the interest rates follow a Taylor rule, instead of an optimal policy for setting interest rates, there is an important role for central bank intermediation (for purchasing the private sector assets/loans). The Central Bank purchases loans made by banks to the impatient consumers (or entrepreneurs). QE1s and QE3, that I focus here, were in line with suggestions in Bernanke, Reinhart and Sack (2004). Gertler and Karadi (2012) argue

[7] This is in contrast to Wallace (1981), Krugman (1998) and Eggertsson and Woodford (2003) and more earlier to Barro (19777) for his work on the RE theory.

the CB interventions through the LSAPs were a basic re-intermediation process where the central bank is replacing the private intermediaries that face limits to arbitrage.

3.2 The Model

I use a partial equilibrium, standard New-Keynesian (NK) DSGE model with Calvo (1983) type nominal frictions and Kiyotaki and Moore (2008) type financial frictions.[8] The interaction of ZLB with credit frictions is key to this paper and I analyze how it affects the economy. Given these nominal frictions (and the recessionary case), actual labor and output will be demand determined. In other words, firms and workers commit to supply any amount of goods and labor demanded at the prices set by the corresponding agents.

All agents in the model are optimizing (except the central bank that follows a Taylor rule in normal times and is bounded by the ZLB on interest rates otherwise). Up to this point, this is a standard New-Keynesian DSGE model. However, I use two type of infinitely lived agents: workers and entrepreneurs (to represent the financial sector), in line with Bernanke et al. (1999), and the central bank conducts unconventional policies in addition to its conventional Taylor rule.[9] The households are accompanied by a continuum of intermediate good producers and a representative final good producer. Capital producers, convert consumption goods to investment and then create the capital for the economy. Additionally, there is a central bank conducting monetary policy and a government as the fiscal authority. Time is discrete and the only uncertainty comes from a shock to the liquidity of private paper (ϕ). Another shock to the productivity,

[8] Indeed, all DSGE models are built on RBC models with some form of frictions and some shocks for dynamics.

[9] We need entrepreneurs (financial markets), because we want saving and borrowing to occur between private agents.

will also be discussed. The economy is cashless as is standard in the NK literature (See Woodford, 2003).

The model adds banks (i.e. financial intermediaries (entrepreneurs) that intermediate funds between savers and borrowers) to the standard NK model setup. These constrained agents (borrowers or impatient agents) borrow from the unconstrained agents (savers or patient agents or workers).[10] An important assumption is that each member of the household works for the benefit of the whole household (family), such that, because of these preference differences the entrepreneurs focus on production while workers specialize in supplying labor force and buying the consumption goods needed for the family.

The model has no money. Instead, I have bonds (government issued nominal assets) as the liquid assets (as in Woodford, 2003). This makes the model different than the original Kiyotaki and Moore (2008) model. Having bonds, instead of money, and an accompanying (gross) nominal interest rate, R_t, makes it possible to use a conventional monetary policy such as a standard nominal interest rate setting rule. It is also needed to show the ZLB case. B_t represents all liquid assets in the market.

3.2.1 Households' problem

As in Woodford (2003), I assume a representative household setup with infinite number of identical households members living in the economy (normalized to one as in representative household setup). However, I follow Bernanke et al. (1999) and assume,

[10]It could alternatively be set as $\beta^e < \beta$ (β of workers is higher than that of the entrepreneurs). This is why they are saving.

each household is composed of two sets of a continuum of members: workers (like Keynesians) and entrepreneurs (like Ricardians). Each period they face a random draw that determines their type. Probability of being a worker is 'f'. This is also the fraction of workers in the economy, by law-of-large-numbers, at any point in time.

I also assume a continuum of household members, indexed by $j \in [0, 1]$, who are monopolistically competitive in their labor supply as in Erceg et al. (2000). The growth rate of population is zero and the population is normalized to one.[11] Each worker member of household is infinitely-lived and provides a differentiated labor service $l_t(j)$ to the (single, economy-wide) factor market. Following Erceg et al. (2000), I assume an (zero-profit) employment agency (labor aggregator agency) combines all the imperfectly substitutable labor supply by different household in accordance with firms' demand. The employment agency's demand for each particular labor type will be equal to the total demand for that particular labor type by all firms. The aggregate labor index has the Dixit-Stiglitz (1977) form as in equation (1). I assume, the workers are uniformly distributed across labor types. The aggregate-labor-index is below.

$$L_s = \left[\int_0^1 L_s(j)^{\theta_l} dj \right]^{\frac{1}{\theta_l}} , \tag{3.1}$$

where $0< \theta_l <1$ and $L_s(j)$ is labor supply of type j.

The employment agency takes $W_s(j)$ - the nominal wage chosen by labor unions - and W_s as given and maximizes profit subject to L_s equation above and with respect $L_s(j)$. In other words, it minimizes the cost of producing the aggregate-labor-index L_s demanded by firms. The Aggregator's problem is below.

[11]Good, since we're focusing on short-run.

$\max_{L_s(j)} \Pi_s = W_s L_s - \int_0^f W_s(j) L_s(j) dj$

The cost minimization (or profit maximization) problem for the labor aggregator agency results in the following overall demand (across all firms, for household j's labor). This will be equivalent to the employment agency's demand for that particular labor type ($L_s(j)$).

$$L_s(j) = \left[\frac{W_s(j)}{W_s}\right]^{\frac{1}{\theta_l - 1}} L_s \tag{3.2}$$

The wage-aggregator (or the aggregate wage index) for the composite labor index L_s, by using equation (3) in (1) is below.

$$W_s = \left[\int_0^1 W_s(j)^{\frac{\theta_l}{(\theta_l - 1)}} dj\right]^{\frac{(\theta_l - 1)}{\theta_l}} \tag{3.3}$$

The household's (or a typical household member's) decision problem is two stages. The first step is cost minimization and the second step is utility maximization. The final-good producer, we will see later, deals with the first step by minimizing the cost of producing a composite consumption good C_s. I show that part below for illustration only. The households basically face the following cost minimization problem.

$$\min_{c_s(i)} \int_0^1 P_s(i) c_s(i) di \tag{3.4}$$

subject to achieving a Dixit-Stiglitz aggregate consumption level C_s, where

$\mathrm{C}_s = \left[\int_0^1 (c_s(i))^\theta di\right]^{\frac{1}{\theta}}$

is the Dixit-Stiglitz composite consumption index and

$P_s = \left[\int_0^1 (P_s(i))^{\frac{\theta}{(\theta-1)}} di\right]^{\frac{(\theta-1)}{\theta}}$

is the corresponding Dixit-Stiglitz composite price index.

A typical household holds bonds and own a share of firms. They buy composite consumption goods, and supply labor to the intermediate-goods sector. All of the households earn a labor income as $\int_0^f W_s(j)L_s(j)$. The agents (or their union) decide $W_s(j)$ -the nominal wage for type j labor- endogenously. Different than the fiscal paper, here, we have capital producers that convert consumption good to investment. All firms are owned by households; therefore, all the profits go to households as D_s (from the intermediate good firms) and D_s^I (from the capital producing firms). The utility maximization problem of a typical representative agent is below. They maximize the expected present discounted value of their inter-temporal utility with respect to (wrt) $C_s(j)$, $B_{s+1}(j)$, $N_{s+1}(j)$ and $I_s(j)$.

$$\max_{\{C_s(j),I_s(j),B_{s+1}(j),N_{s+1}(j)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s(j), l_s(j))\right] \tag{3.5}$$

As in the fiscal paper, I assume a separable period utility function for agents. This is mainly for simplicity and to account for importance of heterogeneity in agents as in Campbell and Mankiw (1989), and Bernanke et al. (1999). The utility function is a common labor-leisure decision utility function consistent with stylized facts (balanced growth path). All agents are identical in their preferences and have the following identical period utility function (and the period disutility from working).

$\mathrm{U}(\mathrm{C}_s, L_s(j)) = \frac{(C_s)^{1-\sigma_u}}{1-\sigma_u} - \frac{\omega}{1+\eta}\int_0^1 (L_s(j))^{1+\eta} dj$

A household member maximizes the utility function (6) subject to the inter-temporal version of the following period budget constraint (in real terms),

$$C_s(j) + P_s^I I_s(j) + q_s(N_{s+1}(j) - I_s(j)) + \frac{R_s^{-1} B_{s+1}}{P_s} \tag{3.6}$$

$$= (r_s^k + (1-\zeta)q_s)N_s(j) + \frac{B_s}{P_s} + D_s + D_s^I + \frac{W_s(j)L_s(j)}{P_s} - T_s(j)$$

where $L_s(j)$ is amount of labor of type j supplied. B_s, the beginning of period bond holding, is a one period risk-less bond issued by the government. $D_s(i)$ and $D_s^I(i)$ are lump-sum profit distributed between households (households have equal share of firms therefore $D_s(i) = D_s$ and $D_s^I(i) = D_s^I$). $W_s(j)$ is nominal wage for the labor supplied. T_s are lump-sum taxes or (if negative) transfers from the government.

The budget constraint above, shows all the household income (from labor income, profits from their ownerships at firms - intermediate good and capital producing etc, interest incomes over bonds and other assets) flows at the beginning of the period. Household members allocate their funds/resources among non-storable consumption, investment in new capital and saving in bonds or equities. $R_s = (1 + i_s)$ is the gross nominal interest rate. Households take prices, transfers (or taxes) from government and all the aggregates as given and maximize utility subject to the budget constraint and the demand function for their labor.

Households maximize inter-temporal utility function, equation (5), subject to equations (7) (the period budget constraint) and (2) (demand for its labor supply) and the investment function (equation 8) below. Each household member type, j, has monopoly power over his/her nominal wage ($W_s(j)$) such that he/she (or the union representing them) resets his wage at the end of the contract periods (which have random durations). This process is analogous to the price setting process for firms for their output, which we will see in the next section. The Aggregate household budget constraint (combining that of workers and entrepreneurs) is below.

$$C_s + P_s^I I_s + q_s(N_{s+1} - I_s) + \frac{R_s^{-1} B_{s+1}}{P_s} \tag{3.7}$$

$$= (r_s^k + (1-\zeta)q_s)N_s + \tfrac{B_s}{P_s} + D_s + D_s^I + \int_0^f \tfrac{W_s(j)L_s(j)dj}{P_s} - T_s$$

The investment equation from the Entrepreneurs problem is given below:

$$I_s = (1-f)[\frac{(r_s^k + (1-\zeta)q_s\phi_s)N_s(j) + \frac{B_s^r}{\pi_s} + (Y_s - w_s L_s - r_s^k K_s)}{P_s^I - \theta q_s} \tag{3.8}$$

$$+(1-f)\tfrac{(p_s^I I_s - I_s(1+S(\frac{I_s}{I}))) - T_s}{P_s^I - \theta q_s}]$$

where $C_s = \int_0^1 C_s(j)dj,\ N_{s+1} = \int_0^1 N_{s+1}(j)dj,\ B_{s+1} = \int_0^1 B_{s+1}(j)dj$ and we used the identities $D_s = Y_s - w_s L_s - r_s^k K_s$ and $D_s^I = p_s^I I_s - I_s(1 + S(\frac{I_s}{I}))$.

The first-order necessary conditions for the households' optimality, by maximizing the utility function subject to the households inter-temporal budget constraint and the investment equation with respect to their choice variables C_s, B_{s+1}, N_{s+1} and I_s, will be demonstrated in the appendix. Here, I just show the FOC wrt investment (I_t): $\lambda_t(q_t - p_t^I) - \eta_t = 0$ where λ_t is the Lagrange multiplier on the household budget constraint, and η_t is the Lagrange multiplier on the investment equation. We focus on a constrained equilibria, where $q_t > p_t^I$ (the market price of equity is bigger than the installation cost) holds both inside and outside the steady-state.[12] This represents a tight financial constraint on investment.

Given this, the following equalities will hold for all entrepreneurs.

$$N_{t+1}(j) = (1-\theta)I_t(j) + (1-\phi_t)(1-\zeta)N_t(j) \tag{3.9}$$

[12] So that $\eta_t > 0$ always.

$$B_{t+1}(j) = 0 \tag{3.10}$$

$$C_t(j) = 0 \tag{3.11}$$

$$I_s = (1-f)\frac{(r_s^k + (1-\zeta)q_s\phi_s)N_s(j) + \frac{B_s^r}{\pi_s} + D_s + D_s^I - T_s}{P_s^I - \theta q_s} \tag{3.12}$$

We have two Euler equations (EE) for inter-temporal consumption allocation, which links the marginal cost of consumption today to the expected marginal benefit of consumption in the future period.

EE for equity holdings:

$$C_t^{-\sigma_u} q_t = \beta E_t\{C_{t+1}^{-\sigma_u}[(r_{t+1}^k + (1-\zeta)q_{t+1}) + \frac{(1-f)(q_{t+1} - p_{t+1}^I)(r_{t+1}^k + (1-\zeta)q_{t+1}\phi_{t+1})}{p_{t+1}^I - \theta q_{t+1}}]\} \tag{3.13}$$

EE for bond holdings:

$$\frac{C_t^{-\sigma_u} R_t^{-1}}{P_t} = \beta E_t\{C_{t+1}^{-\sigma_u}[\frac{1}{P_{t+1}} + \frac{(1-f)(q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}}\frac{1}{P_{t+1}}]\} \tag{3.14}$$

and the Transversality conditions (TVCs) - no-ponzi condition. $\lim_{s\to\infty} E_s\lambda_s B_{s+1} = 0$ $\lim_{s\to\infty} E_s\lambda_s N_{s+1} = 0$ where $0 < \beta < 1$ is the discount factor, and E_s is the conditional expectation based on information available at time s.

Wage setting is determined separately.[13] Wages of households are set in a staggered fashion, *à* la Calvo (1983). Each period a fraction of unions (representing a type 'j' worker) are able to reset their wages. The probability of each household resetting its wage in each period is the same. I will call this probability as $'1-\gamma_l'$.[14] They choose wage that maximizes the utility:

[13] We could have included it into the household FOCs - the optimality conditions.

[14] Each period, γ_l fraction of agents are not able to reset their wages again.

$\max_{\{\tilde{W}_t(j)\}} E_t \sum_{s=t}^{\infty} (\gamma_l \beta) \left[\frac{(C_s)^{1-\sigma_u}}{1-\sigma_u} - \frac{\omega}{1+\eta} (L_s(j))^{1+\eta} \right]$

Given all the information available, when an agent (union) is allowed to set the optimal wage W_s^*, he/she chooses the wage $\tilde{W}_s(j)$ that maximizes his utility - equation (5) - subject to the period budget condition and demand for its labor, taking into account that he might not even have another chance ever to re-optimize ($\tilde{W_{s+1}}(j) = W_s^*$, $\forall$ future periods beyond s). This probability of not being able to reset the wage, changes his discount rate from $'\beta'$ to $'\gamma_l \beta'$. Unions take L_s and W_s as given. E_s is again a conditional expectation based on information available at time 's'. I will assume wages are not changing at all when a household type is not allowed to reset its wage for a period. First order condition (FOC) of the above utility maximization problem, subject to the the two conditions and, with respect to W_s^*, then, is as follows.

$$E_t \left\{ \sum_{s=t}^{\infty} (\gamma_l \beta)^{s-t} L_s(j) \left[\frac{W_t^*}{P_s} - \frac{\omega}{\theta_l} \left(\frac{L_s^{\eta}(j)}{C_s^{-\sigma_u}} \right) \right] C_s^{-\sigma_u} \right\} = 0 \tag{3.15}$$

or

$\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} (\gamma_l \beta)^{s-t} L_s(j) \left[\frac{w_t^*}{\Pi_{t,s}} + \frac{\omega}{\theta_l} (MRS^{real}) \right] C_s^{-\sigma_u} \right\} = 0$

where $MRS_s = -\frac{U_{L_s}}{U_{C_s}} = -\frac{C_s^{-\sigma_u}}{L_s^{\eta}(j)}$ and the markup $\mu = \frac{1}{\theta_l}$. $\Pi_{t,s} = \Pi_{t+1} * \Pi_{t+2} * ... * \Pi_s$ and $\Pi_t = (1 + \pi_t)$. In other words, wages are set such that the expected discounted marginal benefits are equal to the expected discounted marginal disutility from working.

Equation 15 may also be written in recursive format, as shown below, for the sake of simplicity. For two new variables ww_t^1 and ww_t^2:

$\mathrm{ww}_t^1 = L_t^{\eta+1} + \beta \gamma_l E_t ww_{t+1}^1$

$\mathrm{pp}_t^2 = L_t C_t^{-sigma_u} \frac{W_t^*}{P_t} + \beta\gamma_l E_t ww_{t+1}^2$

Equation 15 is actually equal to:

$$ww_t^1 = \frac{\theta_l}{\omega} ww_t^2 \tag{3.16}$$

If I assume a perfectly flexible case, then $\gamma_l \rightarrow 0$ (meaning there is no agent that is not allowed to re-optimize) and the above equation reduces to,

$$w_t^* = \frac{1}{\theta_l} MRS_s^{real} \tag{3.17}$$

By taking γ_l as the fraction of agents that keep their wages fixed each period, by the law of large numbers, I get the following wage index.

$$w_t = [\gamma_l(\frac{w_{t-1}}{\pi_t})^{\frac{\theta_l}{\theta_l - 1}} + (1-\gamma_l)(w_t^*)^{\frac{\theta_l}{\theta_l - 1}}]^{\frac{\theta_l - 1}{\theta_l}} \tag{3.18}$$

where w_t is real wage[15].

3.2.2 The equity market

N_t^o represents claims, by the home country household, on the other households' equity holdings, and N_t^I shows claims, on home households' assets, by the other households.

The two 'liquidity constraints' (or financing constraints) entrepreneurs face are below.

$$N_{s+1}(j) \geq (1-\theta)I_s(j) + (1-\phi_s)(1-\zeta)N_s(j) \tag{3.19}$$

$$B_{s+1}(j) \geq 0 \tag{3.20}$$

[15] Wage inflation π^w depends on the real marginal cost, which is equal to the gap between real wage and marginal rate of substitution between consumption and leisure

Additionally, tho following non-negativity constraint should also be holding.

$$C_{s+1}(j) \geq 0 \tag{3.21}$$

Additionally, I define a real bond to represent the real value of the assets at the end of the period.

$$B^r_{t+1} = \frac{B_{t+1}}{P_t} \tag{3.22}$$

3.2.3 Entrepreneurs' problem

Probability of being an entrepreneur, each period, is '$1 - f = \xi$'. Hence, the entrepreneurs constitute '1-f' portion of the household members each period (by the LLN). They do not work, but make investment. Using the budget constraint for each household member above (equation 6), and focusing on the constrained equilibrium where the 'market price of equity' is higher than the 'installation cost of a unit of capital' ($q_t > p^I_t$)[16], we should get:

$\mathrm{N}_{t+1}(j) = (1-\theta)I_t(j) + (1-\phi_t)(1-\zeta)N_t(j)$

for the financing constraint on equity holdings, and the following financing constraint on bond holdings,

$\mathrm{B}_{t+1}(j) = 0$

plus an identity showing consumption of Es cannot be below zero (non-negativity).

$\mathrm{C}_t(j) = 0$

along with a defined variable $B^r_{t+1} = \frac{B_{t+1}}{P_t}$. Using all these equalities inside the following

[16]This also means, return on physical investment will be higher than return from holding interest bearing bonds or equities

basic budget constraint,

$$C_s(j) + P_t^I I_t(j) + q_t(N_{t+1}(j) - I_t(j)) + \frac{R_s^{-1} B_{s+1}}{P_s}$$

$$= (r_s^k + (1-\zeta)q_s)N_s(j) + \frac{B_s}{P_s} + D_s + D_s^I + \frac{W_s(j)L_s(j)}{P_s} - T_s(j)$$

aggregating over all the entrepreneurs, we should eventually get the following investment equation:

$$I_s = (1-f)\frac{(r_s^k + (1-\zeta)q_s\phi_s)N_s(j) + \frac{B_s^r}{\pi_s} + D_s + D_s^I - T_s}{P_s^I - \theta q_s} \tag{3.23}$$

where $(1-f) = \xi$ will be called the probability of receiving an investment opportunity or the fraction of entrepreneurs, by the law of large numbers.

The two type of financial frictions included in the paper are the basic borrowing constraint and the more recently added resaleability constraint (both due to Kiyotaki and Moore (2008)). The borrowing constraint tells that at any point in time, the entrepreneurs are able to borrow up to a fraction of the expected NPV of their assets (or investment). While the resaleability constraint means only a fraction of the illiquid assets are sold at any period.

The primary source of the financial crisis (the shock), in the model, is a change in the resaleability constraint (the same as that in Kiyotaki and Moore (2008), Del Negro et al. (2010), and Brendon et al. (2011)) and another productivity shock as is common in RBC and NK DSGE models. The borrowing constraint is constant and not effected by any shocks.

After the shock, resaleability constraint is stricter and funding is more scarce. This fall in funding, decreases production through its affect on all the markets (labor, goods and asset markets). The government papers, bonds (and money), don't have any resaleability constraints. They are liquid therefore.

Unconventional MP directly affects the liquidity of private paper by buying illiquid papers in private sector's portfolio. However, this purchase is just decreasing the total supply of private papers. In other words, central bank is not buying private papers from individual entrepreneurs. It is not subsidizing private sector, but the government is just buying assets to earn money.

Entrepreneurs borrow against expected NPV (net present value) of their asset holdings (e.g. commercial real estate), and do the production of output. However, this borrowing is only up to a fraction of that expected NPV, hence the collateral constraints. The shock is to this fraction that the entrepreneurs are able to borrow.

Investment equation can be alternatively written as:

$$I_s = (1-f)\left\{\frac{(r_s^k + (1-\zeta)q_s\phi_s)N_s(j) + \frac{B_s^r}{\pi_s} + (Y_s - w_sL_s - r_s^kK_s) + (p_s^I I_s - I_s(1+S(\frac{I_s}{I}))) - T_s}{P_s^I - \theta q_s}\right\} \tag{3.24}$$

At this point, a few crucial assumptions about the model are listed below. I first assume that $\phi_t^I = \phi_t^o = \phi_t$. Later on we change this for robustness analysis. Also, $\beta > (1-\delta)$. Another assumption is that, as we discussed above, the paper focuses on the constrained equilibria that $q_t > p_t^I$. It should hold in the SS first, and then should hold in the numerical experiment as well. At the end of the period, all members of the household bring all their $C_t(j)$ together, and it gets distributed equally between members. Finally, at the end of each period, the household also shares all assets accumulated during the period among its members.

3.2.4 Firms' problem

A perfectly competitive representative final-good-firm produces composite consumption goods and a continuum of intermediate goods, indexed by $i \in [0,1]$, are produced by a continuum of monopolistically competitive firms indexed by the good they produce. Final-good firms buy intermediate goods, assemble them in the same proportions as consumers (households, firms, capital producers and the government) demand and sell it to the private sector and the government in a competitive market. They do not use any labor, and therefore pay only for the intermediate good they use. Following Chari, Kehoe and McGrattan (2000), I use the following Dixit-Stiglitz form production function for the final good produced by a perfectly competitive firm.

$$Y_s = \left[\int_0^1 Y_s(i)^{\theta_p} di \right]^{\frac{1}{\theta_p}}, \tag{3.25}$$

where 0< θ_p <1. Final-good firms choose $Y_s(i)$ (demand for good 'i') to maximize profit below:

$$\max_{Y_s(i)} \Pi_s = P_s Y_s - \int_0^1 P_s(i) Y_s(i) di \tag{3.26}$$

subject to equation (20) for Y_s, where $Y_s(i)$ is an intermediate good produced by firm i.[17] Final good producers face a perfectly competitive market for both their output and

[17] Constant elasticity of substitution (CES) between the intermediate goods is $\frac{1}{1-\theta_p}$. As $\theta_p \to 0$, ES goes to 1 (unit elastic). Constant markup over the marginal cost for monopolistically competitive firms is $\frac{1}{\theta_p}$.

the intermediate goods they need for production. They take the prices of final good P_s (comes out of zero profit condition) and intermediate good $P_s(i)$ as given, and choose the amount of intermediate good $Y_s(i)$ for production in order to meet the demand for final good Y_s. Profit maximization of the final good producers results in the following demand function for each intermediate good. This is the sum of the demand for a particular intermediate good, by consumers in the economy, since final-good producers assemble intermediate goods according to the demand in the market.

$$Y_s(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta_p}} Y_s \tag{3.27}$$

The perfect competition in final goods market and the following zero-profit condition for the final good producers gives the price of final good (or the aggregate price index), which is equal to the marginal cost of production.

$$P_s = \left[\int_0^1 P_s(i)^{\frac{\theta_p}{(\theta_p-1)}} di\right]^{\frac{(\theta_p-1)}{\theta_p}} \tag{3.28}$$

The monopolistically competitive (in output they produce) intermediate good firms produce intermediate goods, $Y_s(i)$. Each intermediate-good firm uses capital it bought from entrepreneurs and labor rent from labor-agency (from other members of household, hence), and produces a differentiated good $Y_s(i)$. All producers hire the same kind of labor and face the same wages, W_s (cost of composite labor they take from labor agency). They face the demand function in equation (22) for their output (they all have the same constant demand elasticity). All of the firms are owned by households (each household own an equal share of all firms and capital stock as in Erceg at al. (2000)). Therefore, all the profit goes to households as the dividend payment. The production function for intermediate goods has a usual CRS Cobb-Douglass form. All intermediate-good firms have the same following production function

$$Y_s(i) = F(K_s(i), L_s(i)) = A_s K_s(i)^{\alpha} L_s(i)^{1-\alpha} \tag{3.29}$$

where $L_s(i)$ and $K_s(i)$ are labor and capital inputs for production of the intermediate goods. A_s is the total factor productivity (TFP). They face a perfectly competitive factor market, as did the final-goods firms, in contrast to the imperfect goods market for their output. They take the purchasing price for capital R_s^k and the aggregate wage W_s as given and maximize profit function w.r.t. $P_s(i)$, $K_s(i)$, $L_s(i)$ and subject to the demand function (22). The intermediate-good firms maximize profit (or minimize cost) below.

$$\max_{\{L_s(i), K_s(i)\}} \Pi_s(i) = P_s(i) Y_s(i) - W_s L_s(i) - R_s^k K_s(i)^{18} \tag{3.30}$$

The nominal marginal cost for all firms (common, because of the CRS property of the production function) and per output is below.

$\mathrm{MC}_s{}^{19} = \frac{w_s}{MPL_s} = \frac{r_s^k}{MPK_s}$

From the FOCs of the profit maximization

$\frac{K_s(i)}{L_s(i)} = \frac{\alpha}{1-\alpha} \frac{w_s}{r_s^k}$

using this in one of the FOCs

$\lambda_s(i) = MC_s(i) = MC_s = \frac{1}{A_s} (\frac{r_s^k}{\alpha})^{\alpha} (\frac{w_s}{1-\alpha})^{1-\alpha}$

or since

[18] Agents sell units of labor index L_s at the cost W_s to the intermediate-goods sector. $w_s = \frac{W_s}{P_s}$ is real wage.

[19] MC_s is real marginal cost and w_s (small) is real wage.

$MPK_s = \alpha K_s^{\alpha-1} N_s^{1-\alpha} = \alpha Y_s / K_s$

with K_t and L_t are aggregate capital and labor stocks. This implies

$MC_s(i) = \frac{r_s^k(i)}{MPK_s(i)} = \frac{\alpha}{1-\alpha}\frac{K_s(i)}{\alpha Y_s(i)}\frac{L_s(i)}{K_s(i)} W_s(i) = \frac{L_s(i)}{(1-\alpha)Y_s(i)} MRS_s = \frac{W_s(i)}{R_s^k(i)}$

or alternatively

$MC_s = \frac{L_s(i)}{(1-\alpha)Y_s(i)} MRS_s = \frac{L_s(i)}{(1-\alpha)Y_s(i)}\frac{(l_s)^\eta}{(C_s)^{\sigma u}}$

Intermediate good firms need also consider profit maximizing price setting P_t^* in addition to selection of optimal amount of input process. Intermediate-good producers have a monopoly power over the price of the differentiated good they produce. Prices for intermediate goods are set in a staggered fashion *à* la Calvo (1983) again. Each period a fraction of firms are able to reset their prices. The probability of each firm resetting its price in each period is the same. I will call this probability, $'1-\gamma'$. Intermediate-good firms set their price and then decide the amount of labor they need for production. All of the profit of intermediate-good firms, D_t, is going to the households, since they own all firms.

The profit maximization problem for a typical intermediate-good firm resetting its price $p_t^* = \tilde{p}_t(i)$ is below.[20] When a firm is allowed to set the optimal price, given all the information available, it chooses the price that would maximize its profit even if it never has another chance to re-optimize. E_t is conditional expectation based on the information available at time t.

[20]All intermediate good firms face the same problem. In a symmetric equilibrium, all firms will choose the same price $p_t^* = \tilde{p}_t(i)$.

$$\max_{\{P_t^*\}} E_t \left\{ \sum_{s=t}^{\infty} (\gamma_p \beta)^{s-t} Q_{t,s} \left[(\frac{p_t^*}{P_s}) - (\frac{MC_s^{real}(i)}{1}) \right] Y_s(i) \right\}$$

subject to

$\mathrm{Y}_s(i) = \left[\frac{P_s(i)}{P_s}\right]^{\frac{-1}{1-\theta_p}} Y_s$

where $Q_{t,s} = (\frac{C_s}{C_t})^{-\sigma_u}$ is hh's inter-temporal MRS. $Q_{t,s} = \frac{\beta E_t \lambda_{t+1}}{\lambda_t} = \frac{R_t^{-1}}{P_t} \frac{P_{t+1}}{1} \frac{1}{1+\frac{(1-f)(q_{t+1}-p_{t+1}^I)}{(p_{t+1}^I-\theta q_{t+1})}}$.

Where $\frac{R_t^{-1}}{P_t}\frac{P_{t+1}}{1} = \frac{1}{1+r_t}$.[21]

I will assume prices are not changing at all when a firm is not allowed to reset its price. The first order condition (FOC) of the above profit maximization problem wrt to p_t^*, is

$$E_t \left\{ \sum_{s=t}^{\infty} (\gamma_p \beta)^{s-t} Q_{t,s} \left[(\frac{p_t^*}{P_s}) - \frac{1}{\theta_p} (\frac{MC_s^{real}}{1}) \right] Y_s(i) \right\} = 0 \tag{3.31}$$

which equals

$$\mathrm{E}_t \left\{ \sum_{s=t}^{\infty} (\gamma_p \beta)^{s-t} Q_{t,s} \left[(\frac{\tilde{\rho}_t}{\pi_{t,s}}) - \frac{1}{\theta_p} (\frac{MC_s^{real}}{1}) \right] (\frac{\tilde{\rho}_t}{\pi_{t,s}})^{\frac{1}{\theta_p - 1}} Y_s \right\} = 0$$

Equation 31 may also be written in recursive format, as shown below, for the sake of simplicity. For two new variables pp_t^1 and pp_t^2:

$\mathrm{pp}_t^1 = Q_{t,t} MC_t Y_t + \beta \gamma_p E_t pp_{t+1}^1$

$\mathrm{pp}_t^2 = Q_{t,t} \frac{P_t^*}{P_t} Y_t + \beta \gamma_p E_t pp_{t+1}^2$

Equation 31 is actually equal to:

$$pp_t^1 = \theta_p pp_t^2 \tag{3.32}$$

If I assume a perfectly flexible case, then $\gamma \to 0$ (meaning there is no firm that is not allowed to reoptimize) and the above equation reduces to,

$$\rho_t^* = \frac{1}{\theta} MC_s \tag{3.33}$$[22]

[21] Actually, $Q_{t,s} = Q_{t,t+1}$ here.

[22] This is nominal marginal cost again.

By taking γ as the fraction of firms that keep their prices fixed each period. We will get the following price index.

$$1 = \gamma_p(\frac{1}{\pi_t})^{\frac{\theta_p}{\theta_p - 1}} + (1 - \gamma_p)(\tilde{\rho}_t)^{\frac{\theta_p}{\theta_p - 1}} \tag{3.34}$$

or alternatively,

$\mathrm{P}_t = [\gamma P_{t-1}^{\frac{\theta}{\theta-1}} + (1-\gamma)(p_t^*)^{\frac{\theta}{\theta-1}}]^{\frac{\theta-1}{\theta}}$

Using the same idea for the real relative price, we used above, the aggregate production function turns into (showing aggregate demand equals aggregate supply)

$$Y_s = A_s K_s^{\alpha} L_s^{1-\alpha} = \int_0^1 Y_s(j)dj = \sum_{s=t}^{\infty} \gamma_p (1-\gamma_p)^{s-t} (\frac{\tilde{\rho}_t}{\pi_{t,s}})^{\frac{1}{\theta_p - 1}} Y_s \tag{3.35}$$

3.2.5 Capital producers

In contrast to the fiscal paper that includes endogenous capital variations (where firms consider their capital accumulation process too) into the model, this paper assumes perfectly competitive capital producers that transform consumption goods into investment good. The profit maximization problem of firm is below.

$$\max_{\{I_t\}} D_t^I = I_t \left[p_t^I - (1 + S(\frac{I_t}{I})) \right] \tag{3.36}$$

where p_t^I is given due to perfect competition. The FOC with respect to I_t chosen for time $'t'$ is below.

$$p_t^I = 1 + S(\frac{I_t}{I}) + S'(\frac{I_t}{I})\frac{I_t}{I} \tag{3.37}$$

where $p_t^I \neq p_t$ since there is an adjustment cost $S(\frac{I_t}{I})$. I will also assume that in the steady-state, $S(1) = S'(1) = 0$ and $S''(0) > 0$ [23].

[23]Consistent with balanced growth path and close to a form like $S(\frac{I_t}{I}) = \frac{\kappa}{2}(\frac{I_t}{I} - \Lambda_x)^2$.

3.2.6 The monetary authority - Central bank

The central banks are probably the most important institutions in today's financial world. Their balance sheets have had more attention after the recent shift in their policies. They hold government securities, foreign exchanges and private securities (with the latter becoming more dominant recently) on the asset side of their balance sheet; and their liabilities are the currencies in circulations (that they have issued) and the reserves on the other central banks'. This paper is on the most recent increase in private asset purchases on the balance sheet. In the model, the central bank, buys private assets and, provides liquidity to the whole market. It doesn't relax constraints of a specific firm.

The model in this paper has no money, instead I have bonds (government issued nominal assets) as the liquid assets. Indeed, standard NK models don't have money. This makes the model different than the original Kiyotaki and Moore (2008) model. Having bonds, instead of money, and an accompanying (gross) nominal interest rate, R_t, makes it possible to use conventional monetary policy as a standard nominal interest rate setting rule, as is standard in the NK literature such as Woodford (2003) (and because I look at the ZLB case).

We need the conventional monetary policy (an interest rate rule) due to existence of the two (sticky wage and prices) nominal rigidities (Leverage for conventional monetary policy). The central bank conventionally controls the interest rate i_s. The central bank (CB) follows a Taylor rule (Taylor, 1993) to implement monetary policy and that a zero lower-bound for i_s holds.

$$R_s = \max\{0, R(\Pi_t)^{\phi_p}(Y_t)^{\phi_y}\} \tag{3.38}$$

which in log-linearized form can be written as

$\hat{i}_s = \max\{0, \hat{r}_s^n + \phi_p \pi_s + \phi_y \hat{Y}_s\}$

where $\phi_\pi > 1$ and $\phi_y > 0$. $r_s^n = log\beta^{-1}$ is the real interest rate or the natural rate of interest ($i = r^n = log\beta^{-1}$ with zero inflation and no shock). The Central Bank, under normal circumstances, sets i_s to achieve zero inflation. However, if interest rates are down to zero, then it sets i_s at zero and lets π_s be determined by the equilibrium conditions.[24]

This paper assumes commitment is NOT possible. I therefore, ignore policy suggestions such as an inflation targeting, which is subject for another paper. The discretion policy requires that, at the ZLB when you can't substitute cutting ST rates for managing expected future ST rates, you will need to use unconventional policies.

The central bank, unconventionally, implements the monetary policy via open market purchase of private assets. The central bank, by changing the composition of assets held by households, increases the liquidity in the economy, changes the equilibrium outcomes and affects the real activity.

Unconventional monetary policy will be represented by government purchase

[24]Eggertsson (2010) argues that the Central Bank's commitment to higher future inflation could be an alternative to temporary government expenditure shocks or temporary cuts in taxes in shifting the aggregate demand. Commitment policy does not require any change in G or tax cut, but it has a credibility problem (Kydland and Prescot (1977), Walsh (2010) chapter 6).

of private papers (N^g_{t+1}) as a function of its liquidity (ϕ_t).

$$\frac{N^g_{t+1}}{K} = \psi_k(\frac{\phi_t}{\phi} - 1) \tag{3.39}$$

where $\psi_k < 0$.

3.2.7 The fiscal authority - Government

The paper studies the efficacy of the unconventional policies in eliminating the recession. Government uses OMOs to intervene in the equity market. Relative size of government is large enough to affect prices p_t and q_t. B_s is a government bond. Because Ricardian Equivalence does not hold, the timing of policy interventions matters. Therefore, we need the government budget constraint as another equilibrium condition. Because the cost of stimulus packages is low in a zero interest rate case, many Keynesian economists argue it is beneficial for the government to issue bonds to finance its deficit in short-run. I assume the government finances its deficit with some bonds.

Having bonds to finance the government budget constraint means the government does not need to balance its budget each period. I will also assume a fiscal rule for discretionary fiscal policy, in the case of income-tax changes as in Gali and Perotti (2003). The public sector BS or the real government flow budget constraint is as follows (government spending is zero, not positive as in Kiyotaki and Moore (2008)):

$$q_t(N^g_{t+1} - (1-\zeta)N^g_t) = r^k_t N^g_t + R^{-1}_t B^r_{t+1} - \frac{B^r_t}{\pi_t} + \tau_t \tag{3.40}$$

where q is relative price of equities in terms of consumption goods. B^r_t is for 'real bond balances'.

The government follows the following fiscal rules for inter-temporal solvency:

$$\tau_t - \tau = \psi_t(\frac{B_t^r}{\Pi_t} - B^r - q_t N_t^g) \tag{3.41}$$

for a positive value of ψ_t.

3.3 The Market Clearing

All of the markets should clear out in that total supply should be equal to the total demand in that market. Factor market clearing (for all s) requires that the market for capital is in equilibrium when demand for capital by the intermediate good firms equals the supply of capital by the entrepreneurs.

$$K_s^s = K_s = \int_0^1 K_s^d(i)d_i \tag{3.42}$$

and the equation for aggregate capital stock accumulation is:

$$K_{s+1} = (1-\zeta)K_s + \int_f^1 I_s(j)d_j \tag{3.43}$$

Meanwhile, the capital stock in the economy is divided between households and the government (in the form of equity):

$$K_{s+1} = N_{s+1} + N_{s+1}^g \tag{3.44}$$

where

$$N_{s+1} = \int_0^1 N_{s+1}(j)d_j{}^{25} \tag{3.45}$$

[25] since both E and W hold equity

At the wage rate set by the labor unions, $W_s(j)$, the labor market will be in equilibrium. Labor demand by intermediate good firms equals labor supply by labor unions.

$$L_s^s = L_s = \int_0^1 L_s^d(i) d_i \tag{3.46}$$

Bond market clearing requires

$$B_{s+1} = \int_0^f B_{s+1}(j)^{26} \tag{3.47}$$

Market clearing in the intermediate goods sector:

$$Y_s^d(i) = Y_s^s(i) \tag{3.48}$$

Resource constraint:

$$P_s Y_s = P_s C_s + P_s I_s[1 + S(\frac{I_t}{I})] \tag{3.49}$$

and in real terms: $\mathrm{Y}_s = C_s + I_s[1 + S(\frac{I_t}{I})]$

Supply equals demand:

$$A_t(K_t)^\alpha (L_t)^{1-\alpha} = \int_0^1 Y_t(i)di = \sum_{s=0}^{\infty} \gamma_p (1-\gamma_p)^{s-t} (\frac{\tilde{\rho}_t}{\pi_{t,s}})^{\frac{1}{\theta_p - 1}} Y_s \tag{3.50}$$

Once all these market clearing conditions ar satisfied and the government budget condition is satisfied, the household budget condition will hold trivially by the famous Walras' Law.

3.4 Equilibrium

The recursive competitive equilibrium has 9 endogenous quantities and 9 endogenous prices that satisfy 18 equilibrium conditions, as a function of state variables

[26] since Es don't hold any bonds

(exogenous state variables A_s and ϕ_s; and endogenous state variables predetermined K_s, N_s^g, B_s^r and w_{s-1}). The endogenous equilibrium process of prices q_s, p_s^I, $\tilde{w}_s$, w_s, $\tilde{\rho}_s$, π_s, r_s^k, mc_s^r and R_s is such that; households (entrepreneurs and workers) choose I_s, C_s, Y_s, L_s, K_{s+1}, N_{s+1} to maximize the utility function; government choose N_{s+1}^g, B_{s+1}^r and τ_s to satisfy the government budget constraint and the fiscal rule. Firms chose capital and labor to maximize profit.

Model's equilibrium conditions are approximated, up to first order, around a steady state with binding equations 5 and 6 (liquidity constraints). Binding ZLB on nominal interest rate causes a non-linearity which will be taken care of by using a special solution method due to Eggertsson (2010).

The solution method requires that the $\hat{\phi}_t$ follow a two-state Markov process[27]. In the crisis period, $\hat{\phi}_t < 0$ (resaleability constraint is below its SS value). In the normal state, on the other hand, $\hat{\phi}_t = 0$ (the resaleability constraint is back to its SS value).

The economy starts at SS (where $\hat{\phi}_t = 0$), at time 0. At period 1, there is a shock to the resaleability constraint (ϕ_t). After period one, the crisis period continues with probability $1 - \omega_w$ (and goes back to its SS value with probability ω_w).

We define two other variables:

$\mathrm{R}_s^q = E_s[\frac{r_{s+1}^k + (1-\zeta)q_{s+1}}{q_s}]$

for the return on equity, and

$\mathrm{R}_s^k = E_s[r_{s+1}^k + (1-\zeta)]$

for the standard return on capital.

[27] $\hat{\phi}_t = \frac{\phi_t - \phi}{\phi}$ or $\hat{\phi}_t = ln(\frac{\phi_t}{\phi})$.

3.5 The Steady State

I assumed, for the sake of simplicity, that the government debt and government equity holdings are zero in the steady state. This assumption makes sure that inflation does not show up in the log-linear equation for debt. And, as pointed out by Brendon et al. (2011), that inflation cannot be used as a shock absorber. I use the following steady state nominations $K/Y = \gamma_k$, $C/Y = \gamma_c$, $I/K = \gamma_{ik}$, $K/L = \gamma_{kl}$ and $I/Y = \gamma_i$.[28]

The firm's pricing equation in the SS is: $MC = \frac{1}{\mu} = \theta_p$.

Since $\frac{R_s^k/P_s}{MPK_s} = MC_s = \frac{r_s^k}{(\alpha)Y_s/K_s} = \frac{r_s^k K_s}{(\alpha)Y_s}$, in steady state: $MC = \frac{1}{\mu_{ss}} = \frac{r^k K}{(\alpha)Y}$.

Meanwhile,

$MC = \frac{1}{A}(\frac{r^k}{\alpha})^{\alpha}(\frac{w}{1-\alpha})^{1-\alpha} = \theta_p$

Real wage is obtained from

$w = (1-\alpha)(A\theta_p)^{\frac{1}{1-\alpha}}(\frac{\alpha}{r^k})^{\frac{\alpha}{1-\alpha}}$

$\frac{Y}{K} = 1/\gamma_k = \frac{r^k}{\alpha}\frac{1}{\theta_p}$

$\frac{K}{L} = \gamma_{kl} = \frac{\alpha}{1-\alpha}\frac{w}{r^k}$

$\frac{I}{K} = \gamma_{ik} = \zeta$

The resource constraint:

$Y_s = C_s + I_s(1 + s(\frac{I_s}{I}))$

$\frac{C}{Y} = \gamma_c = 1 - \zeta(\gamma_k)^{-1} = 1 - \frac{(\zeta r^k)}{\alpha\theta_p}$

and $r^k = (r+\zeta)$ for $r = \beta^{-1} - 1$. This shows that γ_c does not depend on the fraction of the rule-of-thumb (keynesian) agents. Y level is determined by

$w = \frac{1}{\theta_p}[\frac{L^{\eta}}{C^{-\sigma_u}}]$

[28] As in Christiano (2004).

The two other variables that were created show that:

$R^q = \frac{r^k + (1-\zeta)q}{q}$

for the return on equity (in simple form - where $q = 1$ - it gives $R^q = R = r^k + (1-\zeta)$, which is the usual r we would observe if there are no arbitrage opportunities in capital markets), and

R$^k = r^k + (1-\zeta)$

for the standard return on capital.

3.6 Log-Linearization: For Illustration

Although, I use Dynare for impulse response calculations and do not need log-linearization directly; for the sake of illustration I will show log-linearization process below. The Hansen method $\hat{x}_s = logX_s/$X, where a 'hat' shows log deviation of a variable from a steady-state. All the aggregate variables that show up in the resource constraint will be linearized as deviation from their steady state value over 'steady state output level'. Which basically means $\hat{C}_t = \frac{C_t - C}{Y}$, $\hat{G}_t = \frac{G_t - G}{Y}$, and $\hat{I}_t = \frac{I_t - I}{Y}$.

Before I do any log linearization, I list some parameters of the model that we will be using. $\sigma_u = -\frac{u''C}{u'} > 0$ and I call $\sigma = -\frac{u''C}{u'}\frac{Y}{C} = \sigma_u(\gamma_c)^{-1} > 0$, $b = -\frac{u''G}{u'} > 0$, $\eta = \frac{u''L}{u'} > 0$ and $\alpha = -\frac{f''L}{f'}$, for $'$ and $''$ standing for the first and second derivatives.

3.7 Calibration of the Model

Model economy is calibrated to the data for a small-open-economy at quarterly frequencies. Most of the standard parameter values of the model economy follow Woodford (2003), Christiano (2004), Eggertsson (2010), Gali et al. (2007), Kiyotaki and Moore (2008) and Del Negro et al. (2010). As in the fiscal policy paper, I use parameter and shock values to match an output contraction of 30 percent and a deflation of 10 percent, both of which are statistics from the first quarter of 1933 in the U.S. economy, trough of the great depression with a zero nominal interest rate. I choose the great depression as the benchmark because my argument is that the unconventional monetary policies during the 2008 -2010 period were, as argued by many, better be regarded as a reaction to avoid another great depression due from the banking shock in 2008. The magnitude of the crash due to the 2008 crisis was comparable to the Great Depression according to Reis (2010). Therefore, calibration takes place in accordance with the following target values for the benchmark economy:

Targets	Values
$\hat{Y}_s$	-30 percent
π_s	-10 percent

The standard (conventional) parameters are as follows. The discount factor (β), which is quarterly and implies a steady-state short-term interest rate of %2 (since $\beta = \frac{1}{1+r}$ and $(1+0.02)^{-\frac{1}{4}} = 0.995$)[29] is 0.995; σ_u, coefficient of relative risk aversion is 1.1599; the depreciation rate (ζ) is 0.025 (annual 10 percent); the capital share (α) is

[29] Consistent with the ZLB literature and increases the likelihood that the ZLB binds in the 'crisis experiment' here.

0.4, elasticity of substitution between goods and labor supplied ($\frac{1}{1-\theta_p}$) is $1.1/0.1 = 11$[30], the relative utility weight on labor or the parameter that pins down the steady state level of hours (ω) is 1 (it is set to get a steady state share of working hours), the Frisch elasticity of labor supply (η^{-1}) is 1.5692, the measure of price and wage rigidity (γ_p or γ_l) is 0.75 implying an average duration of 4 quarters (a year) for wage or price contracts.[31].

For the conventional policy rule the standard estimates are those from the Taylor rule, Taylor (1993). The feedback coefficient on inflation (ψ_p) is 1.5, and that on output-gap/output (ψ_y) is 0.5. Coefficient in the fiscal rule that determines how transfer/taxes respond to changes in net government debt position is chosen to be $\psi_t = 0.1$ which means transfers/taxes respond very slowly to changes in public debt. This small coefficient in a way tells that the government funds its interventions by public debt (bonds), in the SR; but in the LR, with smooth increase in taxes, the intervention will be funded by taxes eventually.

Calibration of the central bank intervention (asset purchases) is in accordance with the more than \$1.5trillion increase in the Fed's balance sheet between late 2008 and early 2009, during the first QE. The steady-state value of government-debt-to-GDP ratio ($\frac{B}{PY} = \frac{B^r}{Y}$) is taken as 40%, as estimated by Del Negro et al. (2013) and consistent with flow of funds measure of real government paper to GDP ratio. The degree of monopolistic competition is calibrated using the steady-state markup of 10%

[30] $\theta_p = 1/1.1$ is a measure of the degree of monopolistic competition in labor and product markets and $\frac{1}{\theta_p} = \frac{1.1}{1} = 1.1$, which implies a 10% markup.

[31] And $\frac{1}{1-\gamma_p} = 4$ quarters.

($\theta_p = \theta_l = 0.1$). This is relatively high compared to the literature (such as Bils and Klenow (2004)). It in a way is high to account for real rigidities that are ignored.

The main challenge in terms of model parameters is selection of the financial friction parameters, namely ϕ and θ (actually their steady state - pre-crisis - values). ϕ is calculated, in Del Negro et al. (2013), by looking at the average liquidity share in the US economy, between 1952 and 2008 (data from the U.S. flow of funds). $L_t = \frac{B_{t+1}}{B_{t+1}+P_t q_t K_{t+1}}$ (ratio of the liquid government liabilities to the total assets in the U.S. economy) is about 13% on average. Shock to this variable is a shock to ϕ, that we are analyzing here.

The model-specific parameters will be as follows. The arrival rate of investment opportunity per-period ($1 - f = \xi$) is 0.05, which is also the fraction of entrepreneurs, as in Kiyotaki and Moore (2011) and Del Negro et al. (2013).[32]

The steady state value of the two financial friction parameters is another crucial model specific parameterization. The borrowing constraint parameter (mortgageable fraction of new investment, θ) is 0.19; the reasaleability constraint parameter (resaleable fraction of equity in the steady state, ϕ) is 0.185.[33] The choice of $\phi = 0.185$ corresponds to a liquidity premium of around 1.5% (and the liquidity premium goes up as ϕ goes down), and a return on liquid assets equal to 2.2%, both of which are real returns (as ϕ goes down, return on liquid assets goes down since liquid assets are becoming more valuable and demand going up).

[32] This is meanwhile the arrival rate of investment opportunity per-period (ξ). Since WWII, the fraction of entrepreneurs (houshold members doing financial intermediation) in the US has increased from 2% to 8%, according to Del Negro et al. (2013). It is as in Doms and Dunne (1998) and Gourio and Kashyap (2007); Cooper et al. (1999) take it as 10% per quarter and 40% per year.

[33] Consistent with the 13% liquidity share in steady-state, as is evident from the data. It basically means an Entrepreneur can resale up to 18.5% of his equity holding within a period/quarter, and $1 - (0.815)^4 = 56\%$ of his equity within a year.

Model shocks are, a shock to the productivity (used in many RBC and NK DSGE models) A_t (the aggregate productivity), another shock to liquidity of private paper (as used in Kiyotaki Moore (2008) and Del Negro et al. (2013)) ϕ_t (the liquidity of equity) and a shock to the borrowing constraints (as in Brendon et al. (2011)) are the three shocks we will be analyzing.[34] I assume these shocks capture events that occured during the 2008 crisis. They all jointly follow a stationary Markov process (a stochastic process) in the neighborhood of their steady-state values (A, ϕ). A shock to A_t is called a productivity shock, while that to ϕ_t is called a liquidity shock.

The primary shock to the economy is a financial shock to these frictions (a reduction in resaleability of the private paper - or liquidity in the secondary market for private papers drying up as pointed out by Gorton and Metrick (2010)) similar to the one Del Negro et al. (2013) uses; while Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) use a real shock that decreases the capital stock.[35]

Key parameters that determine the calibrated values are the magnitude of shock ($\hat{\phi}_L$, where L stands for crisis period), the duration ($1/\omega_w$) of the shock, and the intensity (ψ_k) of the government response to the shock. The persistence parameters ρ_a, ρ_θ and ρ_ϕ (in the functions for exogenous changes in productivity and resaleability constraints) are both/all equal to 0.95. The innovations of the shocks are e_a and u_ϕ, and they follow a normal distribution, $e_a- > N(0, \sigma)$ and $u_\phi- > N(0, \sigma)$. The aggregate

[34] I add the third one following Brendon (2011) that claim the shock to the pledgability ratio (fraction of the expected NPV of assets the entrepreneurs are able to borrow against) captures the fact that the crisis was originated in the financial markets.

[35] If we take $1 - \phi_t$ as the haircut on private assets in the secondary market, as pointed out in Gorton and Metrick (2010), then the size of the shock is in a way a shock/increase to these haircut. Haircuts on repo has taken place of run on deposits of the past decades of bank runs indeed.

productivity (A) increases by 1%.

Magnitude of the shock to the resaleability constraint is captured by increase in liquidity share at the trough of the crisis (right after the collapse of Lehman).[36] During the fourth quarter of 2008, liquidity share increased by 26.6% according to Del Negro et al. (2013). Size of the government intervention is also known. Then the parameters $\hat{\phi}_L = -0.6$ and $\psi_k = -0.063$ (The size of government response or intensity of the government response to the shock) are calibrated to match the 26.6% increase (on impact) in the liquidity share and an unconventional policy intervention of 10% of GDP.[37]

The only key parameter left to be determined is the expected duration of the shock. For that I use $\omega_w = 0.167$, which corresponds to 6 quarters of ZLB binding or crisis continuing[38].

Average share of liquidity and the steady-state returns on government and private assets (in addition to the other benefits) also provide information on the importance of financial (credit) frictions.

[36]We talk of an increase, because during a crisis, value of bonds - liquid assets- increases while that of illiquid equities decreases. Meanwhile, government will also be increasing quantity of bonds.

[37]Consistent with change in the asset side of The CB's BS after the collapse of Lehman. $\hat{\phi}_L = -0.6$ basically means, the resaleability of equity in the secondary market falls by 60%.

[38]ω_w is the probability that the crisis will end. The expected duration of shock (or ZLB to bind): $\frac{1}{1-(1-0.167)} = \frac{1}{0.167} = 6$ quarters. At baseline $\omega_w = 0.167$, and at great escape=0.100.

Table 2: Parameter values of the model economy

Parameters	Description	Values
σ_u	coefficient of RRA	1.1599
β	subjective discount factor	0.995
η	inverse elasticity of labor supply	1.5692
γ_p	Calvo hazard rate for price	0.75
γ_l	Calvo hazard rate for wage	0.75
θ_p	the degree of MC in goods market	1/1.1
θ_l	the degree of MC in labor market	1/1.1
ζ	depreciation rate	0.025
α	elasticity of output with respect to capital	0.4
$S''(1)$	adjustment cost parameter	1
ψ_p	coefficient on inflation in Taylor rule	1.5
ψ_y	coefficient on output in Taylor rule	0.5/4
ψ_t	coefficient on debt in fiscal rule	0.1
ψ_k	size of government response	-0.063
r_s^n	natural rate of interest rate	-0.0104
θ	SS borrowing constraint	0.19
ϕ	SS resaleability constraint	0.185
ω_w	probability that crisis will end	0.167
$\hat{\phi}_L$	size of the liquidity shock	-0.6
$1 - f = \xi$	fraction of entrepreneurs	0.05
B^r/Y	SS government debt-GDP ratio	0.4
r^b	return on liquid assets	0.022
ω	relative utility weight on labor	1

3.8 Discussion of Modeling

This paper builds a more complicated and realistic agent setup and aims to generalize implications of the effects of QE policies. It studies the effect of unconventional policies on the home country applying the controversial QE policies. In a relatively simpler setup, Del Negro et al. (2011), for an advanced economy and employing QE policy, find that both the liquidity shock (shock to the resaleability constraint) and following Fed intervention in early 2009 have had significant real effects, particularly because of the binding ZLB. They find very close results to the data on inflation and output gap for the period. Similar studies find that the credit easing policies at the time have prevented both the inflation and output not to decrease by another %30. The quantitative impact of the shocks and the following interventions depend on the duration of the credit market froze as well. Absent the Fed intervention, (and given enough time) the economies could even have experienced another great depression.

In these model setups, in the short run, if the shock occur, $r_s^n < 0$; then recession occurs and $i_s = 0$ (zero lower bound binding). I assume the government uses unconventional QE policies (that is, it prints money to buy assets from the market) to stimulate the economy. $\hat{\tau}_s^w < 0$ in the short-run. $\hat{\tau}_s^w < 0$ is reversed to the steady-state value $\hat{\tau}_s^w = 0$ with probability $(1-\omega_w)$ each period in the short-run, once we have a cut. The two state Markov process for the shock (the assumption that the shock goes back to its long-run value each period in short-run with probability $(1-\omega)$ each period) means inflation and output goes back back to their long-run values with the same probability $(1-\omega)$. Which basically means,

$$\mathrm{E}_s \hat{Y}_{s+1} = \omega \hat{Y}_s + (1-\omega)0 \qquad , \qquad E_s \pi_{s+1} = \omega \pi_s + (1-\omega)0$$

Long-run (or steady state) in this paper is the case that the shock r_s^n goes back to the steady state $r^n = \bar{r}$. Short run, on the other, is the case that the economy faces a shock, $r_s^n < 0$. In the short-run, given that the shock has occurred, the shock goes back to its steady state value with probability $(1-\omega)$.

I only consider the case where condition $C1$ and $C2$ in the Eggertsson (2010) model holds and we have a unique and bounded equilibrium. That is the case that we have zero interest rates in the short-run, due to the banking shock. In the long-run there is a unique bounded solution with $\pi_s = 0$, $\hat{Y}_s = 0$ and $i_s = \bar{r}$.

$i_s = r_s^n = \bar{r}$ in the long run.

$i_s = 0$ in the short run.

3.9 Results

The impulse responses show, the initial shock decreases all the main macro and financial variables with output and inflation leading the decline and taking the economy into a deep depression. Private consumption (C) and business investments (I) are negatively affected in line with the output (Y) movement. As is expected, though, I declines more than C (consumption). Indeed, this is the usual trend during the financial crisis. If 'I' does not fall as much as expected, this is most probably because of the absence of a strong residential sector and hence the residential investment. Considering the importance of the housing sector during the great recession period, the model outcomes would not make a sense. Overall, this outcome shows we are on the right track.

As discussed earlier, high real impacts of the liquidity shocks are related to existence of both the nominal and real rigidities of the model economy as well as the entrepreneurial setup and the binding ZLB. Following the shock, output, private consumption and the investment go down and then gradually recover over time. The gradual recovery process owes to the persistence of the shock, and also adds to the model dynamics.

The fall in private consumption is meanwhile similar to what the data and the literature shows. If fall in 'C' was different than the decrease in the data, it would be basically be due to absence of stimulative fiscal policies used that the model ignores (namely the American Recovery Act).

Table 3: Initial impulse response of variables to liquidity shock (with or without QE)

variable	No Intervention Case	Under QE
Y	-31,2%	-17,0%
π	-13,7%	-14,3%
i	-5	-1%

The model delivers simultaneous drop in output, in inflation, and interest rates (goes far below the ZLB, if there is no ZLB) during the crisis. Yet, as argued by the Keynesian business cycle theory, the drop in I (investment) is much higher compared to private sector investment. Even under QE, this difference is still higher but the recovery period for I is shorter compared to both the general output (Y) and the private sector consumption (C).

Comparing figures 1 and 2, we observe that responding to the financial crisis (that is the liquidity crunch) with money creation and bond purchases from the market (using the QE policies or the credit easing policies as we focus on in this paper); hence providing the much needed money to the market, both decreases the negative outcomes of the crisis and the period of recovery after the shock is over.

In a way, the outcomes show, non-standard monetary policies of the central banks of the advanced economies have actually been working and helping economies suffer much less compared to what would occur otherwise.

3.10 Robustness Analysis

Following the initial outcomes of the model, I run a couple of robustness tests. These tests are to check the significance of the quantitative results to the selected parameter values. As has already been mentioned in the earlier sections of the paper, a few crucial features of the model are the nominal rigidities, the heterogenous household setup and the adjustment costs usually assumed in the RBC literature, due mainly to the time required to accumulate capital and to better capture the dynamics of the model variables. The literature, usually checks for alternative variables σ and β values as well. I run a couple more tests to see whether the nominal rigidity, adjustment cost parameter and the likes also matter for the model outcomes.

For instance, I change the the degree of wage and price rigidities parameter γ to play with the time it takes to change prices for a group of firms. γ_p and γ_l are originally set at 0.75, as commonly used in the literature. I change them down to 0.60 and up to 0.85. I then change the relative risk aversion level parameter, σ_u. It was set at originally at 1, corresponding to compensating income and substitution effects of delaying a unit of consumption. Changing this parameter to 2 as in Del Negro et al. (2011) seems to matter as well. Yet assuming a stable 1 value still makes sense as the outcome is not significant.

And the adjustment cost parameter, $S''(1)$. Originally at 1; following the literature, I change it to 0 and to 3 to see its impacts. The feedback coefficient (ψ_k) in the policy rule for asset purchases, and the size of the shock ($\hat{\phi}_t$) are adjusted, each time (for each new parameter value), to catch the \$1.5 tr increase in Fed balance-sheet, and a 26.6% increase in liquidity share. For a more severe financial crisis, the expected probability of returning to the steady state might also be increased.

Table 4: Robustness Tests for Various Parameter Values

Targets	Initial Value	After QE	Nom. Rigidity	$\Delta CRRA$	Adj. Cost
$\hat{Y}_s$	-31 percent	-19	-25	-30	-30
π_s	-14 percent	-14	-11	-13	-13
C_s	-28 percent	-12	-25	-28	-28
I_s	-58 percent	-38	-46	-56	-52

As is clear from the table above, the effects of the QE policies makes the most difference with changing nominal rigidity parameter, which is missing in many related studies including in Kiyotaki and Moore (2008). The probability of the recession to end, and the fraction of or probability of being an entrepreneur (that is the heterogeneity of household setup) are also important and needs to be analyzed further.

The table above shows, while, the Fed intervention has been effective, changing CRRA and the adjustment cost parameters do not effect the results much. Yet, fraction of entrepreneurs and the degree of nominal rigidities are still vital for the quantitative model outcomes.

3.11 Concluding Remarks

With the ECB most recently announcing its intention to buy private sector bonds in Europe, the earlier bond and asset purchases by the Fed, the BOE and the BOJ has regained importance. While there are already a number of papers on domestic impacts of these policies, literature misses a sticky price, credit constrained NK model for credit easing policy policy analysis.

The paper's main result is that the unconventional Fed policies in this sticky wage, sticky price setup with a binding ZLB constraint have actually prevented a huge output loss the countries that have benefited. Additionally, the nominal rigidities (sticky price and/or sticky wages) provide rationale for conventional monetary policy (a rule for setting the interest rate).

Nominal rigidities are needed to get real effects. Absent nominal rigidities, only 'I' goes down (C rise cancels I change out so Y is unchanged). With the sluggish prices both C and Y go down. This is because, with binding ZLB, nominal interest can't fall, so real rates cannot go down to increase C.

This paper has provided an economic analysis of an 2008-like financial crisis. It aimed to provide a better understanding of the competitive advantages of QE policies implemented by advanced economies during the Great Recession. The NK DSGE model had a liquidity shock (decreasing resaleability of the private paper) to capture what happened in 2008.

The Key factor in the model is varying liquidity in private and government papers, a distinction reminiscent of the difference between the federal funds rate and the private borrowing rate.

The model incorporates nominal rigidities, a few financial frictions and the ZLB to account for quantitative impacts of the QE policies. Adding nominal rigidities provides an advantage to capture the real effects of the asset purchases as nominal rigidities provide basis for real movements in response to nominal changes.

A theoretical small-open economy with two country model with one country imposing the QE and the other one bearing its effects is an important extension of this paper that I intend to do in the future. Meanwhile, a more comprehensive robustness analysis with different key parameter values and estimation of the model parameters also needs to be done in order to better capture the impulse responses.

Part III

Efficacy of Structural Policy

Chapter 4

Macroeconomic Implications of a Structural Change

Executive Summary

The Turkish social insurance system has been feverishly debated for years, particularly through its burden on the economy. The most recent reform is an attempt to neutralize this deterioration within social security system and its effects on the economy. After the recent reform, the way that retirement benefits are calculated is changed unfavorably for workers and the minimum age for retirement is increased. In particular, for an agent with 25 years of social security tax payments, the replacement rate is down from 65 percent to 50 percent. On the other hand, retirement age is up from 60 to 65. The aim of this paper is to investigate the macroeconomic effects of these changes using an OLG model. My findings indicate that labor supply, output and capital stock increase

when the changes mentioned above are applied to the benchmark economy calibrated to the Turkish economy data in 2005. A critical change with the current reform is that the marginal benefit of working has become uniform over ages. In another simulation exercise, I change the marginal retirement benefit in the benchmark economy to be uniform over ages while keeping the size of social security unchanged. As a result of that scenario, the benefit of retiring in a later period is increased. However, uniform distribution of the marginal benefits decreases both the capital stock and output of the economy. Increasing the retirement age, on the other hand, has positive effects on the economy since agents obtain retirement benefits for fewer number of years and at an older age. Age increase has substantial positive effects on labor supply, capital stock, and the output.

" *...the unsustainable social security system deserves a large part of the blame for Turkeys fiscal challenges over the past decade. In this context, a social security reform is essential.*" IMF Turkey Report, 2006.[1]

4.1 Introduction

The Turkish social insurance system has been an active area of debates for its generosity and deficits in social security budget, especially after 1980s. This is particularly because the public sector deficits are the main challenges of the Turkish economy. Although a significant portion of the deficit stems from deficit in the public budget, deficit in the social security systems is another important source (Sayan and Kiraci, 2001). The social insurance budget deficits are mainly due to early retirement and unofficial employment[2](Alper, Imrohoroglu and Sayan, 2004). Both early retirement and unofficial employment are basically caused by no minimum age requirement to be entitled to the pension payments and lower number of payment days of premium (Akbulak and Akbulak, 2004). According to OECD-Economic Outlook statistics, Turkey ranks quite high in OECD countries in accordance with individual tax burden. Social security taxes accounts for 40 percent on average, for instance. High taxes over income, or for social security, encourages informal economy and discourages economic activity and employment (Ozbek, 2006).

According to the IMF calculations, despite all the favorable demographics,

[1]IMF Turkey program required Turkey to take effective policy actions to deal with the public sector deficits.

[2]Which is still about 46.9 percent according to the latest TUIK statistics

the total amount of the social security system deficit amounted to 475 billion YTL between 1994 and 2004. This is equivalent to 110% of the total GDP of Turkey or 1.5 times the total debt stock, as of the end of 2004. It therefore deserves a large part to blame for the Turkey's fiscal challenges. Naturally, there have been several attempts to reform the social security system in the past. Initially, in 1999, the first reform temporarily decreased the deficit slightly. Later on, in 2006, the three separate social security institutions would be united.[3]

Transfer payments to the social security institutions from the public budget amounts 4.5 percent of GDP per year, as of 2005[4]. This is a heavy burden for the fragile Turkish economy and causes economic instability. Particularly considering almost 85 percent of the population in Turkey has social insurance record (Ministry of Labor and Social Security statistics of 2005), gravity of the problem with the former social security system gets more clear.

According to the ILO (International Labor Organization), IMF and the MLSS statistics (TUSIAD, 2004), Turkey is among the most rapidly aging countries because of its current younger population and relatively high growth rates (Ministry of Labor and Social Security (MLSS) reform book, 2008). Statistics show that along the following 20 years, active labor force population will increase (TUSIAD, 2004). Yet following that period the dependency ratio is expected to rise. Assuming no reform on the current pension system, the total deficit of the social security institutions is expected to rise to

[3] The 3 separate social security institutions prior to 2006 were: SSK, for private and public sector workers; Emekli Sandigi (ES), for civil servants; and Bag-Kur, for self-employed workers and farmers.

[4] Ministry of Labor and Social Security statistics, 2005.
See Figure 1, in the Appendix, for the deficits of the Social Security Institutions between 1994 and 2004.

6% by 20150, and further up to 7% by 2070.[5]

In order to benefit from this demographic opportunity, Imrohoroglu (2004) suggests that Turkey have a reform to deal with the upcoming deterioration in the demographic profile. Due to the higher economic growth anticipated for the following years, it is suggested that savings and funds of the social insurance institutions should be increased along this period (TUSIAD, 2004). Sayan and Kiraci (2001), on the other hand, offer control over deterioration in dependency ratio in to change minimum retirement ages, and to change the contribution and replacement ratios in order to deal with deficits in pension system[6].

Despite Eldred‘s classification of social security as overcharging some while undercharging some others to have ’social adequacy’ while having its budget balances in equilibrium (Eldred, 1981), the Turkish social security system almost overcharges the majority of its participants. Replacement ratio[7] in the Turkish public insurance system is quite high compared to its European and other developed counterparts. Currently the replacement ratio is 2.6 percent on average for the first 25 years (Articles 506, 5434, 1479 and 5510). The world average, however, is 1.5 percent per year. The replacement ratio in aggregate may be over 100 percent in Turkey while its OECD counterparts‘ average is 68.7 percent.(OECD country statistics, oecd.com/economics)

On average a social insurance system should have 4 participants for each retiree, the world average for the dependency ratio. Turkey, on the other hand, has 1.9

[5]See Figure 2 in the Appendix.
[6]The "replacement rate" is: (retirement benefits) / (past mean earnings)
[7]I will use Replacement rate and Replacement ratio interchangeably.

participants for each retiree (MLSS reform book, 2008). Sayan and Kiraci (2001) point to the increasing dependency ratio (ratio of retirees to workers) as the sign of financial difficulties in pension systems.

There are 2 ways to cope with related problems with the social insurance system; increase in tax collection or decrease in retirement benefits (lower replacement rate). Recent reform decreases retirement benefit calculation formula. Formerly, there was 65 percent replacement rate for 25 years of contribution payment whereas the new act requires a 50 percent pension payment for the same period. Marginal retirement benefit was decreasing by years in labor force previously. Benefit calculation in benchmark economy was sum of 3.5 per every year of the first ten years; 2 percent per each year of the following fifteen years and 1.5 percent per each year thereafter; the reform economy requires a uniform contribution to the replacement ratio per each year of work. Marginal retirement benefit becomes uniform over ages. Reform also increases minimum age for retirement benefit collections. Retirement age is increased from 60 to 65.

This paper employs a dynamic model of Overlapping Generations (OLG) model to examine the macroeconomic effects of three major changes by the recent, extensive social insurance reform. I develop a partial equilibrium life-cycle model. This model mostly follows the model used in Huggett and Ventura (1997). Agents start out as workers and they are allowed to make labor supply and saving decisions. After being entitled for retirement benefits (25 years of work), workers face utility costs if their labor supply is positive. Agents labor-leisure decisions after this period depends on this utility cost they face. Labor productivity of agents changes deterministically by age.

I evaluate two alternative economies in this paper. In the first alternative economy, calculation of benefit payments and therefore replacement rate for retirement benefits is decreased. Second alternative economy has calculation of benefit payments changed while social security system taxes and retirement benefits are kept at its benchmark economy level. The macroeconomic effects of the changes are demonstrated by steady-state comparison of the benchmark and reform economies. I apply these changes individually and then compare macroeconomic variables to capture effects of each change. The pay-as-you-go (PAYG) property of social security system is kept through all alternative economies.

The main results of the paper are as follows. Decreasing replacement rate, results in decrease in retirement benefit for the same periods of contribution payments. Hence, agents work for more time and make more savings before retirement. Changing replacement rate decreases size of social security system. Social security tax rate decrease to $\theta = 15.20$ from its benchmark value, $\theta = 17.35$. Output change by 15.38 percent and capital stock increase by 12 percent are substantial responses to replacement rate modification while hours in work and average retirement ages changes slightly.

Secondly, I investigate the case with changing only the distribution of marginal benefits of retirement (contributions to replacement rate). Marginal benefits of retirement are kept uniform without changing social security taxes rate (θ =17.35) and benefit payments. This time benefit of getting retired in a late period is increased. After the first 25 years, agents get extra benefit payment for each year in work. Changing only distribution of replacement rate, surprisingly, decreases economic activity, however. Output falls by 3.75 percent while capital stock decreases by 5.6. Hours of work also decrease, but average retirement age is increased slightly (1.36 percent).

4.1.1 Literature review

Macroeconomic effects of social security reforms is not a common issue in the literature over social insurance in developing countries (Glomm, 2006). Ferreira has studied social security reforms in the Brazilian economy (Ferreira, 2004 and Ferreira, 2005). He reveals contributions of the reform to economic recovery of Brazil as a developing country. Glomm (2005 and 2006) on the other hand, concentrates on the large scale implications of the generous public sector pensions in Brazil. Glomm‘s findings regarding early retirement effects of generous public sector pensions is an essential step in social security reform analysis of any other developing country.

Macroeconomics effects of social security is an expended area of study in developed countries, however. Elder and Holland (2002) study macroeconomic effects of social security on interest rate through investment of social security funds to the bonds or equities market. They examine the effect of the size and portfolio distribution of the social security funds over the interest rates and model the relationship between the two. They find that as the size of US Social Security Trust Funds or the portfolio share of bonds or equities increases, interest rate over that investment is decreased (Elder and Holland, 2002).

Kydland and Prescott‘s revolutionary 1982 paper, time to build and aggregate fluctuations, is a classic reference to get better understanding of reel economy effects of a reel of nominal policy sock including in an OLG model case. We therefore, benefit from their analysis of the reel economy effects of an external shock. Meanwhile, Auerbach and Kotlikoff's 1987 book "Dynamic Fiscal Policy" is a reference book in studies over fiscal policy analysis, an in particular for those in an overlapping generations context.

The discussion in this paper links up well with the literature by Sayan and Kiracı (2001), Huggett and Ventura (1997) and Kaygusuz (2007) in its modeling the social security system in Turkey.

As part of a study for TUSIAD, Imrohoroglu compares the Turkish social insurance system with its OECD countries counterparts and introduces a general equilibrium model for the Turkish insurance system reform (Alper, Imrohoroglu, and Sayan, 2004). According to Imrohoroglu (2004), the current distributive Pay-As-You-Go social security system deters savings as well as decrease in labor supply and employment and, thus, reduces real wages and GDP of a country, as it is in Turkey. Alper, Imrohoroglu and Sayan (2004) present a comprehensive model for the Turkish Social insurance system reform. They point to the potential financial distress and danger in aging of the population in Turkey.

Sayan and Kiraci (2001) study an alternative pension reform with higher retirement age, and changes in contribution and replacement rates to the PAYG system in Turkey after the age requirement arrangement in 2000. They focus on the public pension system deficits and propose options to PAYG system to decrease deficit.

Early retirement is not just a problem in developing countries indeed. Beker, Gruber and Milligan (2003) study the impact of social security on retirement behavior of participants in Canada. Canada's social security system has income security structure that it disables working in older ages. They suggest control over life-time earning that has incentive for retirement in early ages. Gruber also demonstrates the early retirement incentives of the social security systems (Gruber, 1999). Haveman, Holden, Wilson and Wolfe (2003) in their paper " Soacial Security, Age of Retirement, and Economic Well-Being: Intertemporal and Demographic Patterns among Retired-Worker Beneficiaries" focus on effects of early retirement on the economic well-being of retired-workers. They find strong links between accepting early-retirement benefits and poverty in older ages. Although, this is much a problem with the punishment rate for early retirement in the States, the Turkish case with decreasing the replacement rate and initiating more years in labor force is in a way such a punishment for early retirement.

Paper will continue as follows. The next section is the model, which includes household's problem, firms' problem and the definition of equilibrium. Calibration to the Turkish economy data follows in section 3. Then the reform is applied to the model and in section 5 results are revealed. Finally, conclusion section summarizes the paper and fulfils the study, in section 7.

4.2 The Model

This paper describes an economy with agents that differ in their asset holdings, ages, past mean earnings, utility costs, and experience in labor force. I develop a partial equilibrium life-cycle model. This model mostly follows the model used in Huggett and Ventura (1997)[8].

Given particular preferences, production technology and endowments fixed, I will apply a social security reform and then will observe the macroeconomic effects the reform will result in. Social security reform rearranges minimum age for retirement benefits and calculation of replacement rate for retirement benefits. And through that variation, the aggregate effect over the economy is evaluated by steady-state comparison of the two cases.

I have a dynamic model of overlapping generations economy. The economy is populated by a continuum of male agents with total measure one. Agents live through periods 1 to T where each period is five years and total population equals one in each period. Every period a new generation (cohort) is born. Each cohort‘s share in population η_j is calculated by,

$$\eta_j = (1 - I(j)\rho_j)\frac{\eta_{j-1}}{1+n}, and \sum_j \eta_j = 1 \tag{4.1}$$

[8]Some other important overlapping generations models to model the social security are Imrohoroglu, Imrohoroglu, Joines (1995); Rios and Rull (1996), Hubbard and Judd (1987), Cooley and soares (1995, 1996), Conesa and Krueger (1998), Imrohoroglu (1998), Rust and Phelan (1997), Storesletten, Telmer, and Yaron (1997))

where the indicator function,

$$I(j) = \begin{cases} 0 \quad if j \leq 8 \\ \quad\quad 1 \text{ if } 8 < j \leq T \end{cases} \tag{4.2}$$

8, here, is the period corresponding to age 60 where agents begin to face mortality risk and 'j' is age of an agent at a specific date.

There is an age J that agents become entitled to retirement and its benefits, but have to wait until age 'R' to get retirement benefits. Agents retire at age 'R' for sure. Retirees get retirement payment after age 60 ≡ period 8 until age 'T' as long as they survive. Agents will face a mortality risk after age of 60, (ρ). Asset holdings left from people died are distributed to the living agents. This is called the transfer payments from government, TR, and is uniformly distributed to the living agents.

Every period a new generation is born and population grows at rate 'n'. Also, each period an agent is given 1 unit of labor. Agent devotes 'l' proportion of his labor to work and keeps the remaining proportion as leisure (1-l) since he will get utility from both consumption and leisure. Agents will have different productivity levels (z) by their ages. Productivity level will determine labor income agents will get and will change by age.

$$z_j \in Z = (z_1, z_2, ..., z_R)$$

Agents will get income from labor equal to $z_j.l.w$. Where 'w' is real wages. There is a consumption tax τ^c and social security tax θ as well as some income tax τ

over the total income from labor and assets (a). Asset holdings will provide an interest payment at rate 'r'.

The utility function of agents at any period is given below. Utility function I use here is a common labor-leisure decision utility function consistent with stylized facts which was also used by Kaygusuz (2007). All agents are identical in their preferences and have identical utility function.

$$U_j(c, 1-l) = log(c) - \sigma_1 \frac{(l)^{1+\sigma_2}}{1+\sigma_2} - \mu(l,j)\Pi^{j-J} q_J \tag{4.3}$$

For some,

$$\mu(l,j) = \begin{cases} 1 & if \text{l¿0} and \text{j¿ J} \\ 0 & if \text{o/w} \end{cases} \tag{4.4}$$

Each agent has some utility cost when he is born. After age J, agents face this idiosyncratic utility cost (q) that will affect agent's decision regarding working attitudes. Utility cost is from an exponential distribution, where $\bar{q}$ is calibrated and, and q changes by age, that is:

$f(q) = \frac{1}{\bar{q}} e^{-\frac{q}{\bar{q}}}$

and utility cost,

$q_t = \Pi^{j-J} q_J$ given $t > J$.

Which briefly means: Some agents will prefer not to work after facing high utility costs and wait for their retirement benefit payments, while some others may prefer to keep working until age R depending on the utility cost they will face.

4.2.1 Pension earnings

Retirees get a benefit payment $b(\bar{e})$ after age R if they have completed their social security payments and are entitled to retirement. $\bar{e}$ is the average past labor income of an agent.

Former social security system requires the following benefit payments after age R - with 'j' age and 'i' number of years worked (experience):

$$b(\bar{e},h) = \begin{cases} (\psi_1 h)\bar{e} & if \text{h} \leq i_1 \\ (\psi_1 i_1 + \psi_2(h - i_1))\bar{e} & if \text{i}_1 < h \leq i_2 \\ (\psi_1 i_1 + \psi_2(i_2 - i_1) + \psi_3(h - i_2))\bar{e} & if \text{h} \geq i_2 \end{cases} \tag{4.5}$$

Where, $\bar{e}$ is the average past labor income, h is years of experience, $i_1 = 10$, $i_2 = 25$, and ψ_1, ψ_2, ψ_3, are marginal retirement benefits corresponding to 3.5, 2 and 1.5 percents respectively.

The new social security system, however, has the following benefit formula:

$$b(\bar{e},h) = \begin{cases} (\gamma h)\bar{e} & if \text{h} \geq 0 \end{cases} \tag{4.6}$$

where $\bar{e}$ is again the average past labor income, and γ is 2 percent marginal retirement benefit added to replacement rate per years of work.

The new social security system, as is clear from the benefit formula, have redistributed and decreased the replacement rate (the benefit payment coefficients) in order to encourage workers remain in labor force and pay more social security taxes.

4.2.2 Households' problem

Households differ in their ages (j), productivity levels (dependent on age) (z_j), average past earnings $\bar{e}$, idiosyncratic utility costs q_j and asset holdings (a). Each period, agents observe their assets (a), number of periods worked (i) and past mean earnings ($\bar{e}$) and given their utility costs q_j they face between ages J and R, they will decide whether to work more or have more leisure.

Households at age 1 has zero asset holding, zero initial wealth. I have the state variables a,j,$\bar{e}$,q, i and control variables 'a' and 'l'. Bellman equation for household's problem is as follows,

V(a,j,$\bar{e}, q, h) = max_{a' \geq 0, l} U(c, 1-l) + \beta(1 - I(j)\rho_{j+1})V(a', j+1, \bar{e}', q, h')$

subject to,

$(1+\tau^c)c + a' = z_j lw - \theta(z_j lw) - \tau(z_j lw + ra) + (1+r)a + I(j)b(\bar{e}, h) + TR$

recalling that $I(j)$ was as follows,

$$I(j) = \begin{cases} 0 \quad if \text{j} \leq R \\ \quad\quad 1 \text{ if } R < j \leq T \end{cases} \tag{4.7}$$

and

$h = 0$ if $R < j < T$

$h \in [0, 1]$ if $j \leq R$.

Since 'i' is the sum of years worked until age 'j', then, the average past income at time 'j+1' is as follows,

$$\bar{e}' = \begin{cases} \frac{\bar{e}h+z_j wl}{h'} & if \text{j} \leq R \\ \bar{e} & if \text{j} \geq R \end{cases}$$

and,

$$\text{h}' = \begin{cases} h+1 & if \text{l} \geq 0 \\ \text{h} & \text{if } l = 0 \end{cases}$$

$q' = \Pi q_J$

$l = 0$ if $R < j < T$

$l \in [0, 1]$ if $j \leq R$.

4.2.3 Firm's problem

I have a constant return to scale (CRS) type Cobb-Douglas production function and a representative firm in this economy. K is the aggregate capital and L is aggregate labor supply.

Production function:

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha} \tag{4.8}$$

Where A is normalized to 1. $\alpha \in (0, 1)$ is capital share of output and will be constant, and $\delta \in (0, 1)$ will be the capital depreciation rate for the economy. Firm's maximize their profit;

$$max_{K,L}F(K, L) - wL - rK \tag{4.9}$$

given (w,r).

4.3 Calibration

This section studies calibration of the model economy to the data from the Turkish economy and selection of the parameter values of the model economy. Simulation of the economy is examined through selecting values of demographic, production and preference parameters, and then parameterizing social security system.

4.3.1 Demographics

The model economy is calibrated to the Turkish economy data in 2005. Each period is 5 years. And each agent, through periods 1 to T, lives for 13 periods. Agents are born and economically active at age 20. Agents live through ages 20 to 85 and die for sure at age 85 (T=85), $\rho_T = 1$. Each agent is able to work through ages 20 to 60 (R=60 in benchmark economy). Thus, they are economically active at age 20 and can not work after the age 60. At age J=45, each agent with 5 periods of experience, 'i', is entitled to retirement benefits. Demographic variables are set for a period of 5 years. Population growth rate n is set equal to the average growth rate in Turkey between 1985 and 2005 (data from the Turkish Statistical Institute, TUIK) which equals 1.8 percent. Mortality rate after age 60 is set so that the fraction of population over 60 to population over 20 equals 14.9 percent ($\rho = 0.233$).[9]

[9] Data from the Turkish Social Insurance Institute (SII) statistics and the Turkish Statistical Institute (TUIK)

Table 1: Productivity by Age

Age	Productivity
1	0.570
2	0.808
3	1.012
4	1.129
5	1.201
6	1.232
7	1.134
8	0.858
9	0.697

4.3.2 Productivity

Considering agents of ages between 20 and 60 (and 65, for the reform economy), the market productivity levels should also be determined. Productivity levels will change by age. Mean hourly wages are calculated as in Kaygusuz (2007). Productivity level z_j and its distribution is derived from household‘s labor force data.[10] Weekly working hours and wages from 1985 to 2005 for each group of agents are derived from the database. Then, hourly wages are evaluated, where mean hourly wage is 3.2274.[11]

4.3.3 Production technology

Recalling our production function:

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$$

[10]The data of household labor force database from the Turkish Statistical Institute (TUIK).

[11]Hourly wage is simply, wages divided by 4 (weekly payments) and then divided by working hours per week. Mean hourly wage will be average hourly wage for those working over 30 hours a week, that is of full-time workers.

Production parameters for the Turkish economy follow study by Imrohoroglu (as a part of Alper, Imrohoroglu and Sayan, 2004). The technology level A is normalized to 1. α, the capital share of output, is set to be 0.35. And the depreciation rate, δ, is set equal to 0.055.[12]

4.3.4 Preferences

Utility function of agents is as follows:

$$U_j(c, 1-l) = log(c) - \sigma_1 \tfrac{(l)^{1+\sigma_2}}{1+\sigma_2} - \mu(l, j)\Pi^{j-J} q_J$$

Regarding preferences, we have the discount factor parameter β to be set, which is used to evaluate the steady state capital to output ratio to be consistent with the value in data. Capital output ratio is 2.73, which is calculated from the data at the State Planning Organization (DPT). $\frac{l}{\sigma_2}$, Frisch elasticity of labor supply is set to be 0.5, as in its literature estimates by Blundell and MaCurdy (1999) and MaCurdy (1981). Imrohoroglu (2004), on the other hand, use capital -output ratio equal to 2.52 which is indeed quite close to our estimates over data from the Turkish State Planning Organization (DPT). Also, I have σ_1 (the coefficient of relative risk aversion) that will also be calibrated to match hours per week data.

Calibration takes place in accordance with the following target values for the benchmark economy:

[12]See paper by Alper, Imrohoroglu, Sayan (2004) from TUSIAD

Targets	Values
K/Y	2.73 per year
Hours	52.1 per week
average Retirement	55 years

4.3.5 Utility cost

Labor force participation of agents between ages J=45 and R=60 depend on the distribution of the level of utility cost agents face, $\phi(q, j)$. Utility cost might be the utility agents would get from rest at home instead of working or sometimes the benefit participants would get from informal employment, as it is in many developing countries that people keep working without any social insurance record. Which is indeed beneficial both to employer and the employee. Agents have their utility costs when the are born, but face this utility cost at the age of J. Utility costs are idiosyncratic and also change by age, once they occur. Utility cost is from an exponential distribution, where $\bar{q}$ will be calibrated and,

$f(q) = \frac{1}{\bar{q}} e^{-\frac{q}{\bar{q}}}$

Then the distribution of the utility cost is as follows:

$$q_j = \Pi^{j-J} q_J$$

given $j > J$.

where Π is calibrated from the model such that together with mean utility cost $\bar{q}$, they match half of agents that continue working after age 45 (period 5), retire by age 55. J=45, here, is the age participants get entitled to retirement benefits and j is age of the agent. Mean utility cost, $\bar{q}$ will also be calibrated.

4.3.6 The Social security system

The social security system should be in balance at all periods. Income of the social security system is from to the social security taxes θ and payments are in accordance with the replacement rate and past mean earnings, $\bar{e}$. In this model, I use the given replacement rate and decide the social insurance tax that balances social security budget. Benefit functions are given for both the benchmark and the reform economies and I analyze the equilibrium values of social security taxes that adjust to have the budget balanced.

Benchmark economy replacement rate calculation is as follows:

$$b(\bar{e},h) = \begin{cases} (\psi_1 h)\bar{e} & if \text{h} \leq i_1 \\ (\psi_1 i_1 + \psi_2(h - i_1))\bar{e} & if \text{i}_1 < h \leq i_2 \end{cases}$$

$$(\psi_1 i_1 + \psi_2(i_2 - i_1) + \psi_3(h - i_2))\bar{e} \qquad if \text{h} \geq i_2$$

(4.10)

The Reform economy benefit calculation formula, on the other hand, is as follows:

$$\text{b}(\bar{e},h) = \begin{cases} (\gamma h)\bar{e} & if \text{h} \geq 0 \end{cases}$$

Where, past mean earnings ($\bar{e}$) and experience (h) of agents change as follows:

$$\bar{e}' = \begin{cases} \frac{\bar{e}h + z_j wl}{h'} & if \text{j} \leq R \\ \bar{e} & if \text{j} \geq R \end{cases}$$

and,

$$\text{h}' = \begin{cases} h+1 & if \text{l} \geq 0 \\ \text{h} & \text{if } l = 0 \end{cases}$$

Above are calculations of replacement rates for two cases of social insurance system. Regarding the benchmark social insurance system‘s replacement rate, I have ψ_1, ψ_2 and ψ_3 that equals 0.035, 0.02 and 0.015 per years of experience respectively. Which is indeed, on average, 13 percent per each period in first 5 periods. Reform in social insurance system changes the distribution of the marginal retirement benefit. γ is 0.02 for each year of social security payments after the reform. Which is actually, 10 percent per period.

Premium ratio is 40 percent on average in Turkey. However, approximately 17 percent of this payments are done by agents. The exact amount changes by social security institution from 15 to 19 percent. Maximum taxable labor income E_max is 3.802,50 YTL in Turkey, which is six times the wage floor in 2006.[13]

4.3.7 Interest rates

I use the capital-GDP ratio from DPT statistics to decide the interest rate, r. Which is simply derived from first order conditions of the production function with respect to capital and labor.

4.3.8 Income and consumption taxes

There are two additional taxes paid apart from the social security tax: the income tax, τ, and the consumption tax, τ^c. Income tax is paid over labor income plus interest income while consumption tax is proportional to the consumption at each

[13] Wage floor in 2006 is 585.00YTL (from TUIK statistics)

period. Income tax, τ, equals 6.6 percent on average from statistics of Maliye Bakanligi (2005). Income tax is derived by formula below.

Income tax= (Total income tax paid)/(Total income(Labor income+ Interest income))

Consumption tax, τ^c, on the other hand is 13.6 percent, again from statistics of Maliye Bakanligi in 2005.[14]

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[14]Maliye Bakanligi, "Genel Faaliyet Raporu - 2006", www.maliye.gov.tr - June 2007.

Table 2: Parameter values of the model economy

Parameters	Values
α	0.35
δ	0.055
β	0.952
r	0.073
n	0.093
ρ	0.233
τ	0.066
τ^c	0.136
K/Y	0.546
Π	1.15
$\bar{q}$	0.65
σ_1	10
$\frac{1}{\sigma_2}$	0.5

4.4 Reforms

This section studies the reforms, that is changes by the social insurance reform. I will examine 2 alternative economies where each economy reflects a different change by reform in order to get a better understanding of individual effect of each change. There are two types of variations by the reform that I will study. The study takes place as follows: First, each change is applied independently and then, secondly, three variations are applied together to see the aggregate effect over the economy. Benchmark economy variables are then compared to the reform economy values, and results are driven.

Two types of changes to the social security system, are applied in four different scenarios, are as follows. First of all, calculation of benefit payments is changed. Then, the distribution of marginal benefit of retirement is changed. The third alternative economy is the case where only the age increase is applied. And finally, the fourth scenario, where we examine the aggregate effect of both replacement rate and age changes.

4.4.1 Reform - 1

I initially focus on the alternative economy where calculation of benefit payments is changed and agents get less benefits payments for the same years of experiences. Where the social security system includes the age of 60 for calculation of retirement benefit payments and marginal benefit of retirement is 2 percent for each years of work.

Replacement rate formula for benchmark economy is as follows.

$$b(\bar{e}, h) = \begin{cases} (3.5xh)\bar{e} \quad if \text{h} \leq 10 \\ \qquad (3.5\text{x}10\text{+}2\text{x(h-10))}\bar{e} \qquad if 10 \leq h \leq 25 \end{cases}$$

$$(3.5x10 + 2x15 + 1.5x(h - 25))\bar{e} \qquad if \text{h} \geq 25$$

(4.11)

Where, $\bar{e}$ is the average past labor income, h is years of experience.

The new social security system, however, has the following benefit formula:

$$b(\bar{e}, h) = \begin{cases} (2xh)\bar{e} \quad if \text{h} \geq 0 \end{cases} \tag{4.12}$$

where $\bar{e}$ is again the average past labor income, 2 is marginal retirement benefit added to replacement rate per years of work.

Findings

This first reform results in participants‘ benefit from the payments for less time by the reform and with less replacement rate. Also social security tax, θ is decreased. Which means size of the social security system will be minimized in aggregate.

Social security taxes in benchmark economy is on average $\theta = 0.1735$. On the other hand, by the first reform, the reform economy social security taxes decrease to

Variable	Percent Change
Output	12.05
Capital	15.38
Hours	0.1
Retirement	2

$\theta = 0.1520$. Decrease in social security system, decreases tax payments and minimizes the size of social security system.

Applying changes in calculation of replacement rate to the social security system has a substantial effect on output of economy. Output increases by 12 percent approximately.

Capital of economy has an even larger response to the reforms. Capital level increases by 15.38 percent. However, the average retirement age in economy and hours in labor do not show substantial changes surprisingly. Hours in work per agent in a week remain the same almost.

4.4.2 Reform - 2

Secondly, I study the alternative economy where the social security tax payment θ and therefore the size of social security system is not changed. I will only focus on change in the distribution of marginal benefit of retirement. That is agents pay the same taxes for more time and do not benefit from the reform while working. On the other hand, the replacement rate is increased by a coefficient, which was 2 percent for each years of social security payments after the reform.

The new social security system has the following benefit formula this time:

Variables	Percent Change
Output	-3.75
Capital	-5.6
Hours	-0.24
Retirement	1.36

$$b(\bar{e}, h) = \left\{ (2.33xh)\bar{e} \quad if \text{h} \geq 0 \right. \tag{4.13}$$

where $\bar{e}$ is the average past labor income and 2.33 is the adjusted marginal retirement benefit added to replacement rate per years of work.

This change in benefit formula corresponds to 16.36 percent increase in marginal benefit of retirement. In this way, I investigate just the effect of changing distribution of marginal benefit of retirement.

Findings

Given the table for results of second reform,

The social security tax payment, θ, is naturally not changed. θ is constant as its value in benchmark economy, $\theta = 0.1735$. To get this result, we need a coefficient for the replacement rate equal to 1.1636, which means marginal utility from retirement should be increased by 16.36 percent per each year of experience.

Output of the economy is decreased surprisingly, in this case. Whereas, hours in labor force and retirement ages are not varied too much. Capital level, in contrast to the preceding reform, is also decreasing slightly. Changing the distribution of marginal benefit of retirement for each year has proven to be negatively effective on economic

Variables	Percent Change
Output	27.7
Capital	42.2
Hours	0.6
Retirement	3

activity. This is related to increasing benefit of retirement which results in agents saving less and producing less.

4.4.3 Reform - 3

The third alternative economy is the case where only the age increase is applied. Replacement rate formula is the same as its benchmark economy formula.

$$b(\bar{e}, h) = \begin{cases} (3.5xh)\bar{e} & if \text{h} \leq 10 \\ (3.5\text{x}10+2\text{x}(\text{h}-10))\bar{e} & if 10 \leq h \leq 25 \end{cases}$$

$$(3.5x10 + 2x15 + 1.5x(h - 25))\bar{e} \qquad if \text{h} \geq 25$$

(4.14)

Minimum age for retirement benefits collection, on the other hand, is increased from 60 to 65.

Findings

Here is the output of the new reform economy.

It is clear from the above table that minimum age requirement for benefit collection change has more impact than the change in marginal benefits. Output increases by 27.7 percent when age requirement for retirement benefits is increased to 65. Capital increase is even greater than output response. Aggregate capital increases by 42.2. Hours of work per week and average retirement ages, however, are slightly changed. Therefore, it is quite easy to assert that even without changing hours in work it is possible to have substantial changes in aggregate economic activity, by prompting more years in labor force. The social security tax payment, θ, is dropped to 14.14. Since agents are working for more time and will get retirement benefits for fewer years at retirement, size of social security and and therefore social security tax payments are decreased.

4.4.4 Reform - 4

The final alternative economy examines the aggregate effect of both replacement rate and age changes. This final reform economy has the following benefit formula.

$$b(\bar{e}, h) = \begin{cases} (2xh)\bar{e} \quad if \mathrm{h} \geq 0 \end{cases} \tag{4.15}$$

where $\bar{e}$ is again the average past labor income, 2 is marginal retirement benefit added to replacement rate per years of work.

And minimum age for retirement benefits collection is increased from 60 to 65.

Findings

This final reform economy has the following outputs.

The above table shows the case with both reforms (replacement rate and age modifications) in effect. Changing only minimum age for retirement benefits was shown to have considerable effect on economic activity of the economy. Applying both reforms

Variables	Percent Change
Output	32.4
Capital	50.2
Hours	0.3
Retirement	4.5

to the model economy is analogous in many respects to the age requirement increase, but of course more influential in some respects.

Output increases by 32 percent and capital stock of economy is increases by 50 percent, with an almost half impact. Hours in work in aggregate seems almost not to change while average retirement age is increased by 4.5 percent. Aggregate economic activity is shown to change with even greater response to both age and replacement rate modifications of the reform.

4.5 Results

This section analyzes the effects of the reforms over the economy and the outcomes listed in tables in above section. Details of results are demonstrated in tables above. I will take each change to the social insurance system individually and then compare them with each other.

First reform is the one that the formula for benefit payments calculation is changed. Replacement rate for the same years of experience is down, and this decreases social security expenditures. Agents, on the other hand, work for more periods to compensate this decrease in periodical contribution to the replacement ratio. Decreasing marginal benefits for retirement to replacement rate for first 10 years from 3.5 percent to 2 percent, and prompting staying in labor force after even getting entitlement for retirement benefits has positive effect on labor supply, output and capital stock of the economy. Output and capital stocks increase by 12 percent and 15 percent respectively. Hours in work and average retirement ages per agent do not show outstanding changes, however. Social security tax in reform economy with replacement rate changes, θ, is 15.20 percent on average.

Then the marginal benefit of retirement per each years of experience is changed in its distribution. Changing only the distribution of the replacement rate, that is increasing benefit of getting retired in a later period and decreasing contribution of each period in first 8 periods (age of 60), increases tendency to work. Thus, decreases dependency ratio and increases inflows to the social security system. Change in distribution of replacement rate is the only negative effect on the economic activity of economy in question. Both output and capital are down by 3.75 and 5.6 percents respectively. Which briefly means savings are decreased, but labor supply is increased. Decrease in economic activity is because of dominance of savings fall over labor supply increase.

The third change is that the minimum age for collection of retirement benefits is increased to 65. Increase in minimum age for retirement benefits prompts more days in labor force and more social security premium payments. Also agents get retirement benefits for less time. Age increase for retirement benefits collection is shown above to be more effective than the basic replacement rate change in previous reforms.

Applying both age increase and change benefit calculation formula modifications reflects aggregate effect of both changes to the economy. The last alternative economy shows substantial changes both in capital stock and output of the economy. This is because increasing minimum age for benefit payment payments is more effective and dominates change in calculation of benefit payments. The model provides a new rate in equilibrium, by the new social security system. In a way, the social security reform will encourage more time in labor force and therefore more tax payments to the system. And this will help decrease the social security taxes. Social security taxes in reform economy with both reforms applied (age and benefit formula) decreases to around 13 percent.

4.6 Concluding Remarks

This paper employs an OLG model to study the quantitative implications of the changes by the reform in the Turkish Social Insurance System in 2008. The 2008 reform in the social insurance system affects the replacement rate for the persons entitled to the pension benefits such that it is decreased in aggregate; and the distribution of the marginal retirement benefit is also changed and minimum age to begin collecting the old-age pensions is constant at 60 years.

This paper shows that after the reform, pensioners work for more time and make more savings before retirement. Benefit of getting retired in a later period is increased by the reform. Post-the-reforms, people get pensions for less time and get their pensions in an older-age. Although hours in work per agent and average retirement ages are not changed much; prompting more years in labor force is shown to have positive effect on economic activity via increasing labor supply, output level and capital stock of economy.

Regarding all benefit payment collection formula changes in the social insurance system, the model demonstrates that social security tax, θ, is decreased to 15.20 percent from its benchmark value 17.35 percent. Benefit payments are also decreased (Replacement rate for 25 years of contribution payment decreased from 65 percent to 50 percent) since the replacement rate is decreased for an average agent. In a way, the size of the social security system is minimized.

Alternatively, considering just change in the distribution of marginal benefits of retirement, economic activity shows decrease in output and capital stock. Which means the social security tax, θ, is constant at 17.35 just as its value in the benchmark economy value, but marginal benefit of initial years is decreased and that of later years is increased. The output and capital stock response to changing distribution of marginal contributions to replacement rate is negative.

Although distribution of marginal contributions to the replacement rate shows negative effect on the economic activity; aggregate change in replacement rate and increase in age requirement for retirement benefits, compensates this decrease and has even outstanding increases in labor supply, capital stock and the overall output of the economy.

This model, therefore, demonstrates that the recent reform in the Turkish social insurance system have positive effects on aggregate economic activity and saving behavior or agents, and thus the longer term capital stock in economy by prompting saving more. Since this reform is just launched, it will take time for reform to be effective in all respects. Hence, the reform is considered to be beneficial for the Turkish economy in the long run.

Future studies and extensions of this study, may include differentiation among the social security institutions for agents from varying areas of work. That is differentiating between SSK, BAĞ-KUR and Emekli Sandığı, the three branches of the Turkish social security system. And the effects of the reform over all these social security systems should be examined. Reform in the social security system also aims to include those without any social security record, those that previously held the green cards (the-freeriders). If so, then, effects of including those funded from public budget should also be of interest for future studies.

Finally, this paper, have assumptions like everybody has the same minimum age to be entitled to and to get retirement benefits. Whereas, in reality, agents face different age requirements depending on the first years of their social security records. Discrimination between agents of differing restrictions for retirement benefits might be useful for the medium-run. As it is stated in the reform bulletin (MLSS reform bulletin, 2008), the reform indeed will fully take effect, in all sides, after 2048.

Chapter 5

Conclusion

This thesis has focused on understanding the recessions; its causes and policies to deal with it, as well as the effectiveness of policies have been used by policymakers. It skims through a wide range of literature and provides a comprehensive review. Meanwhile, the research has as few assumptions as possible to better capture the real life implications different shocks and efficacy of various policies implemented to deal with these divergences from the long-run trends (i.e. the potential growth trends).

As in the RBC literature, that uses the method of calibration to work out a detailed numerical example of the theory, I also use a model based on RBC theory.The chapter of the thesis provides micro foundations, as in RBC model, for macro models.

The first chapter analyzed efficacy of a fiscal policy tool, a tax cut in particular, in a liquidity trap scenario where monetary expansion is ineffective. It basically answers a question, as to: When the zero-lower-bound, on nominal interest rates is binding and the conventional monetary policy is not working, are the discretionary fiscal policy instruments really ineffective (as argued by many including Eggertsson (2010))? It digs further into discussions on: whether the paradoxes of toil, flexibility, and thrift really hold in a heterogeneous agent model with binding ZLB and some other real and nominal constraints. Money does not have any real effects and even money neutrality doesnt work since money is not changing even price levels in this environment.

The second chapter focused on unconventional monetary policies in a closed economy. This chapter researches on a question as to whether certain assumptions regarding constraints and rigidities amplify or mitigate the macroeconomic or real effects of unconventional monetary policy (QE1 and QE3 cases), credit-easing policies in particular. Under certain assumptions and depending on whether economies are open or not, QEs are could be ineffective in advanced economies, and have negative effects on EMs. Meanwhile, certain ties of QEs could have positive effects on DEs, but have negative effects on EMs. This chapter also studies the comparative advantage the QE policies provide for a country. It indirectly accounts for why other countries oppose the QE policies implemented by major economies?

The third chapter examines the macroeconomic effects of a social security reform. The Turkish social insurance system has been at the center of debates over the deficits and the fragile economic structure. This chapter, analyzes different forms of changes and quantifies the macroeconomic implications of various reforms. Scenario analysis reveal positive effects for labor supply, capital stock and output. Structural reform chapter has a rather longer term focus. Instead of the first two chapters on efficacy of short-term macroeconomic policy analysis, the third chapter looks at mediumterm structural reforms for shifts in economies to deal with certain issues.

Possible extensions for this research includes, but not all, an inflation targeting chapter to analyze a commitment case for a monetary authority. In a zero or close to zero nominal interest case, a central bank can use a high inflation target to stimulate the aggregate demand (through lower real interest rates) and hence increase the economic activity. In that sense, the paper would question the earlier argument of optimal inflation for an economy, and ask whether a zero or low inflation is really the optimal one? An increase in inflation expectations makes current spending more attractive than the future consumption (by decreasing real interest rate). This is how monetary policy (e.g. monetary expansion) works in general. Currently all of advanced economies have a 2% or close 2% inflation target (including the Fed, the ECB, the BOE and the BOJ).

The main problem the world economies (mainly advanced economies) are facing today is extremely low inflation rates. The average inflation rate in the rich countries club, OECD, is around 1-1.5%, down from an average of 2.2% in 2012. This is much lower than the central banks official targets (typically around 2% or just under that). This low inflation case is particularly evident in the euro area where annual consumer-price inflation is below 0.5%, down from 2.5% a year ago. A high inflation also decreases the burden of debt for borrowers. Hence increases demand on the part of borrowers that were highly leveraged (that were looking for deleveraging).

Another relevant extension would be a monetary policy and fiscal policy coordination. An analysis of what the fiscal policy (and hence the fiscal authority) and the monetary policy (hence the monetary authority) should be in charge of in an economy to achieve both price-stability and increased economic activity. In a way, the chapter would focus on the role of monetary policy and fiscal policy to work together for a common goal. The fiscal theory of price level (FTPL) literature has achieved this goal to a certain degree. The new paper would extend that literature in terms of heterogeneity and certain nominal and real rigidities as well as the recent trend on financial constraints.

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Appendix A

Fiscal Appendix

Appendix A, the 'FiscalAppendix' attachment, shows derivation of most of the key equation of the fiscal chapter. Equations listed include, but not all, the FOCs, steady-state equations and some key logarithmic equations. I also list the equilibrium equations.

Appendix B

Monetary Appendix

Appendix B, the 'MonetaryAppendix' attachment, again shows how we derive the key monetary paper equations. In addition, it lists some key impulse response functions; as well as some illustrative key central bank balance sheets, to show how central banks around the advanced economies responded to the Great Recession of 2008-09.

Appendix C

Structural Appendix

Appendix C, the 'StructuralAppendix' attachment, on the other hand has two key figures to show how important the 2008 reform on the social security system was for the balanced growth of the Turkish economy.