Title
Aquifer Management with Logistic Recharge

Journal
Water International, 26(3)

ISSN
0250-8060

Authors
Lóaiciga, Hugo A
Leipnik, Roy B

Publication Date
2001-09-01

DOI
10.1080/02508060108686928

Peer reviewed

Water International

ISSN: 0250-8060 (Print) 1941-1707 (Online) Journal homepage: www.tandfonline.com/journals/rwin20

Aquifer Management with Logistic Recharge

Hugo A. Lóaiciga & Roy B. Leipnik

International Water Resources Association
Water International, Volume 26, Number 3, Pages 358–369, September 2001

Aquifer Management with Logistic Recharge

Hugo A. Lóaiciga *and* **Roy B. Leipnik**, *University of California, Santa Barbara, California, USA*

Abstract: *Theoretical principles of sustainable aquifer management are laid out in this work. The premise of our treatment is that groundwater is a renewable, although exhaustible, natural resource. The theory of this work is aimed at aquifers with a relatively homogeneous recharge that can be approximated by a logistic growth function. Sustainable aquifer exploitation occurs when the rate of groundwater extraction is equal to or less than the natural rate of groundwater replenishment for any level of aquifer storage. There can be many levels of sustainable aquifer exploitation depending on the level of aquifer storage, but there may be only one that maximizes economic returns under a variety of economic and aquifer conditions. Different strategies for sustainable exploitation are derived depending on whether or not the analysis considers tradeoffs among: (i) current and future exploitation; (ii) constant and dynamic aquifer storage conditions; and (iii) regulated and unregulated aquifer exploitation. Key factors affecting sustainable exploitation strategies include: (1) the market price of groundwater; (2) the cost of groundwater extraction; (3) the aquifer storage and natural replenishment characteristics; (4) institutional and environmental regulations on groundwater extraction; and (5) the real discount rate. An example of sustainable groundwater exploitation in Santa Barbara, California, illustrates the methods of this article.*

Keywords: *Aquifer storage, recharge, logistic function, groundwater management, discount rate, net revenue.*

Introduction

Sustainable aquifer exploitation occurs when, at any level of desirable aquifer storage, the rate of aquifer exploitation does not exceed the natural rate of groundwater replenishment. This definition of sustainability does not include any criteria of economic performance, but it does not preclude any either. It will be shown later that sustainable criteria can be made compatible with economic criteria in determining desired rates of optimal aquifer exploitation. This study focuses on homogeneous aquifers, with a strong hydraulic connection to the surface hydrologic cycle (i.e., with an effective groundwater recharge mechanism), and well-delimited recharge and discharge areas. These aquifers are important water sources for small communities and agricultural enclaves throughout the United States and many other regions of the world and their overall contribution to harnessed water resources serving urban and agricultural areas is significant (Solley et al., 1993; Maddock and Hines, 1995). Coastal aquifers which serve small communities (less than 100,000 people) along the California coast are examples of the prototypical aquifer considered in this work (California Department of Water Resources, 1993; Loáiciga and Leipnik, 2000).

Starting with the premise that groundwater is a renewable resource, sustainable aquifer exploitation strategies are developed and analyzed considering: (1) economic factors such as the market price of groundwater and the real discount rate; (2) institutional regulation of groundwater extraction, perhaps motivated by environmental or legal concerns; (3) groundwater extraction costs; (4) time horizons of aquifer exploitation; and (5) the natural groundwater storage and replenishment of aquifers. A case study illustrates the principles of sustainable aquifer exploitation presented in this work. It should be noted that there is a vast literature on the subject matter of groundwater management (e.g., good summaries in Willis and Yeh, 1987; Fetter, 2001). However, analytical/graphical solutions for sustainable aquifer management, as advanced in this article, have received much less attention in the groundwater management literature.

Aquifer Storage Dynamics with Logistic Recharge

Groundwater Storage and Recharge

Consider an aquifer of storage $X(t)$ (units of volume) at time t, driven by an exploitation rate $E[X(t)]$ (units of volume per unit time) and by a natural rate of replenishment $G[X(t)]$ (units of volume per unit time). The time evolution of storage is governed by the following ordinary differential equation:

$$\frac{dX(t)}{dt} = G[X(t)] - E \qquad (1)$$

If the rate of groundwater exploitation is equal to the natural rate of replenishment, i.e., $E = G[(X(t)]$, then the aquifer storage remains constant. The rate of groundwater exploitation is the decision or management variable: one seeks to determine E so as to meet stated objective goals. The rate of natural aquifer replenishment depends on the climatic regime and aquifer characteristics (i.e., hydrostratigraphy, hydraulic conductivity, groundwater storage, and hydraulic head distribution).

The four-year evolution of groundwater storage in a confined coastal aquifer (located in Santa Barbara, California, USA, see Figure 1 for a general location map) from an almost depleted condition in 1991 (i.e., 1,000 acre-foot of groundwater storage remaining, where 1 acre-foot = 1 AF = 1,233 m^3) to near full-storage recovery (i.e., 80 percent of full groundwater storage or 4,000 AF) in 1995 was found to be well described by a logistic function (e.g., France and Thornley, 1994). During 1991 to 1995, no groundwater was extracted from the aquifer. The fitted logistic function was (where storage is expressed in thousands of acre-feet, i.e., $X = 1$ means that groundwater storage is 1,000 AF; and time is expressed in years)

$$X(t) = \frac{\alpha}{1+\beta\, e^{-\lambda t}} \qquad t \ge 0;\ a > 0\ ;\ \beta \ge 1;\ \lambda > 0 \qquad (2)$$

in which the parameters are $\alpha = 5$; $\beta = 4$ (from Equation 2; and $\lambda = 0.69315$. From Equation 2 it is straightforward to establish that the parameter α equals the maximum aquifer storage ($\alpha = X_{max}$), and that $\beta = (X_{max} - X_0)/X_0$, i.e., it is the normalized difference between maximum storage and initial storage ($X_0 = X[t=0]$). In general, if data on aquifer storage $X(t)$ are available as a function of time t during periods in which the aquifer is not being mined, then the parameters in Equation 2 are estimable by statistical methods (e.g., Anderson, 1971; Balakrishnan, 1992).

In the absence of groundwater extraction, the slope ($dX(t)/dt$) of the function in Equation 2, represents the rate of groundwater storage recharge. The shape of the function in Equation 2, which is a special case of a logistic function (Balakrishnan, 1992), encapsulates rather well the key mechanisms of ground recharge in the Santa Barbara confined aquifer. Although Equation 2 must not be interpreted as a general model describing time-dependent aquifer recharge, it appears to be adequate and useful under specific hydrologic conditions. The logistic model is just one possible function suitable for modeling the groundwater recharge mechanism. Its parameters can be calibrated to represent a wide range of observed time-storage groundwater data. The logistic model of Equation 2 is adopted herein as a practical model of groundwater recharge, because, in addition to its easy-to-calibrate nature and acceptable fit to our data, it greatly simplifies the analytical treatment of sustainable aquifer exploitation, which can then be posed in rather general terms, as shown below.

In the absence of groundwater extraction, groundwater storage is driven by its natural rate of recharge, $G[X(t)]$. Assuming that Equation 2 describes the time evolution of storage under no-pumping conditions, then, the time-rate of change of storage $dX(t)/dt = G[X(t)]$ satisfies the following equation

$$G(X) = \lambda X - \frac{\lambda}{\alpha} X^2 \qquad X \le a \qquad (3)$$

in which it is understood that the aquifer storage X is a function of time t.

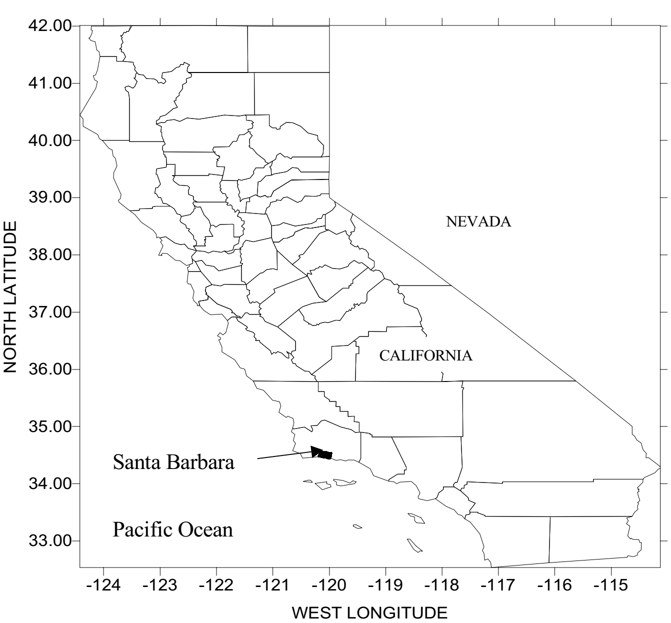


Figure 1. Location map of the Santa Barbara aquifer.

The Dynamic Ground Storage Equation

Substitution of Equation 3 in the right-hand side of Equation 1, followed by factorization of the resulting expression yields the following differential equation for aquifer storage evolution:

$$\frac{dX(t)}{dt} = -\frac{\lambda}{\alpha}(X - A)(X - B) \qquad A < X \le a \qquad (4)$$

with initial condition $X(t_1) = X_1$, and where

$$A = \frac{\alpha}{2}\left(1 - \sqrt{1 - \left(\frac{4}{\alpha\lambda}\right)E}\right) \qquad E < (a\lambda)/4 \qquad (5)$$

and

$$B = \frac{\alpha}{2}\left(1 + \sqrt{1 - \left(\frac{4}{\alpha\lambda}\right)E}\right) \qquad E < (a\lambda)/4 \qquad (6)$$

Equation 4 implies that if the aquifer storage *X(t)* is outside the interval *[A, B]*, then its rate of change is negative; that is, storage decreases as long as that condition persists. On the other hand, if *X(t)* is inside the interval *[A, B]*, then its rate of change is positive, and aquifer storage will increase as long as this condition persists. The condition $E < (a\lambda)/4$ appearing in Equations 5 and 6 is a mathematical necessity in order to keep the variables *A* and *B* real. However, it can be established from Equation 3 that the largest rate of natural replenishment is precisely equal to $(a\lambda)/4$ (this occurs at storage level $X = a/2$). Therefore, the condition $E < (a\lambda)/4$ prevents the exploitation rate from exceeding the largest natural replenishment rate.

The next step in the analysis of aquifer storage is to separate variables in Equation 4, followed by integration from time t_1 to time *t*, to obtain an expression for aquifer storage *X(t)*

$$\left|\frac{X(t)-B}{X(t)-A}\right| = e^{-\frac{\lambda}{\alpha}(B-A)(t-t_1)}$$

$$t \geq t_1 ; \quad A < X(t) \leq a \tag{7}$$

in which |( )| denotes absolute value. All variables in Equation 7 have been previously defined. The right-hand side of Equation 7 tends to zero as $t \to \infty$. Therefore, if aquifer storage starts at a value larger than *B*, then it would tend to *B* as $t \to \infty$. If aquifer storage starts at *B*, then it would remain at that value for all *t*. Furthermore, if storage starts at a value between *A* and *B*, then it would also tend to *B* for large *t*. The nature of aquifer storage as described by Equation 7 for large *t* pre-empts aquifer storage from taking values equal to or less than *A,* thus the condition $A < X(t)$ in Equation 7.

Solving for aquifer storage in Equation 7 yields the following explicit expression for *X(t)*

$$X(t) = \frac{B - \in \cdot A \cdot \left|\frac{X_1 - B}{X_1 - A}\right| e^{-\frac{\lambda}{\alpha}(B-A)(t-t_1)}}{1 - \in \cdot \left|\frac{X_1 - B}{X_1 - A}\right| e^{-\frac{\lambda}{\alpha}(B-A)(t-t_1)}}$$

$$t \leq t_1 ; \quad A < X(t) \leq \alpha \tag{8}$$

where $\in = 1$ when *X(t)* is outside the interval [*A, B*] or else $\in = -1$. Equation 8 shows that aquifer storage *X(t)* depends in a rather complex fashion on aquifer parameters a and λ, on the initial aquifer storage X_1, on the elapsed time $t-t_1$, and on the exploitation rate *E* (which enters in the variables *A* and *B,* see Equations 5 and 6).

A special solution for Equation 4 arises when the exploitation rate *E* takes the value $(a\lambda)/4$, which makes the variables *A* and *B* (see Equations 5 and 6, respectively) equal to each other. In this case, the evolution of aquifer storage can be shown to be given by the following equation

$$X(t)|_{E=(\alpha\lambda)/4} = \frac{\alpha}{2} + \frac{1}{\frac{1}{X_1 - \frac{\alpha}{2}} + \frac{\lambda}{\alpha}(t - t_1)}$$

$$t \geq t_1 ; \quad X(t) > A \tag{9}$$

implying that $X(t) \to \alpha/2$ for $t \to \infty$.

Once the storage evolution is known as a function of time and of the exploitation rate, it is possible to formulate aquifer exploitation strategies that meet pre-specified criteria as shown in a later section.

Sustainable Exploitation: Constant-Storage Case

Let *E(X)* represent the rate of aquifer exploitation at any level of aquifer storage *X* (in units of groundwater storage per unit time). With this and previous definitions, a fundamental conclusion may be now stated about the sustainable rate of aquifer exploitation: for any level of aquifer storage *X* there is one, and only one, rate of sustainable aquifer exploitation which is given by $E(X) = G(X)$. (Note that *G(X)* is given by Equation 3.) Consequently, a sustainable rate of aquifer exploitation must be equal to the natural rate of groundwater replenishment, for any level of aquifer storage. If the rate of aquifer exploitation exceeds the natural rate of replenishment, then the aquifer storage will decline. Conversely, if the rate of aquifer exploitation is less than the rate of natural replenishment, then aquifer storage will be replenished. From the results of the previous section it is known that the rate of sustainable aquifer exploitation may not exceed the rate $G_M = (a\lambda)/4$, and it can be as low as zero.

Sustainable aquifer exploitation as defined in this section implies that aquifer storage either remains at (an acceptable) constant level for a given rate of aquifer exploitation, and this must not be confused with an optimal rate of aquifer exploitation, E^*, which may involve criteria of economic efficiency or environmental constraints not yet discussed. Let us consider the situation that arises when an aquifer is not exploited at sustainable rates. Consider Figure 2, and assume that an aquifer is at storage level X_A. Assume further, that the rate of aquifer exploitation is set at the level E_A $(= G_B)$, which exceeds the sustainable rate G_A. Aquifer storage recedes until it reaches the value X_B in Figure 2. At that point, the rate of aquifer exploitation equals the natural rate of replenishment G_B: aquifer storage will remain at storage level X_B if the exploitation rate

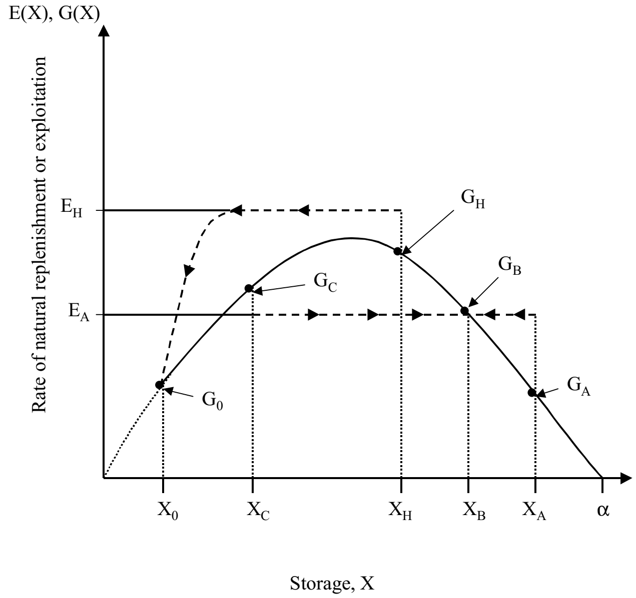


Figure 2. Graphical representation of the relationship among the rate of aquifer replenishment (G(X)), and a sustainable exploitation rate (E(X)), and the aquifer storage (X).

is maintained at the level E_A, in which case this rate of exploitation becomes sustainable. A somewhat asymmetric situation takes place when, starting at storage X_C, an exploitation rate E_A is imposed below the natural replenishment rate G_C . Aquifer storage will increase until it reaches the level X_B, at which point the exploitation rate exactly matches the natural replenishment rate G_B. If the rate of aquifer exploitation remains at level E_A it is sustainable and the aquifer storage stays at X_B. A third case arises when, starting at aquifer storage X_H in Figure 2, a non-sustainable rate E_H is imposed which is larger than the natural rate of replenishment G_H. Aquifer storage drops until it reaches minimum storage X_0, when the rate of aquifer exploitation must drop to the level of the natural rate of groundwater replenishment G_0. Notice that in this third case, the non-sustainable exploitation rate cannot remain at E_H indefinitely, but rather, it declines as the minimum storage level is approached.

Analysis Without Future Tradeoffs Considered

Unregulated Sustainable Exploitation

Let us consider first the case where the future impact of current groundwater exploitation is not taken into account. Assume that the market price for groundwater is *P* ($/unit of groundwater) and that the cost of groundwater extraction as a function of aquifer storage is *C(X)* (in $/unit of time). It is reasonable to make the cost of aquifer exploitation dependent on its storage, since it is well-known, for example, that groundwater extraction costs rise as aquifer storage drops (Willis and Yeh, 1987). The total revenue accruing from exploiting *G(X)* units of groundwater is $TR = P \cdot G(X)$ (in $/unit of time). Therefore, the total revenue curve is simply the natural rate curve *G(X)* scaled by the price *P*, as shown in Figure 3. Notice that by defining revenue as being equal to $TR = P \times G(X)$, it is implied that the exploitation rate equals the natural replenishment rate, i.e., $E(X) = G(X)$, thus implying sustainable exploitation. The total cost curve *TC* is also shown in Figure 3, $TC = C(X)$ (in $/time). The net revenue from extracting and selling groundwater is defined as $F(X) = TR - TC = P \cdot G(X) - C(X)$. The storage value which maximizes net revenue is found by setting the first derivative of the profit function with respect to *X* equal to zero and then solving for the value of *X* that meets that condition. This is the same as solving the equation

$$\frac{d[P \cdot G(X) - C(X)]}{dX} = 0 \qquad (10)$$

The solution of Equation 10 is equivalent to finding an aquifer storage at which, simultaneously, the slopes of the total cost and the total revenue curves are the same, that is, when the marginal cost and the marginal revenue are equal. The *TC* curve in Figure 3 was drawn so that the maximizing storage is *X**. It can be graphically verified from Figure 3 that the slope of the *TR* curve at *X** equals the slope of the *TC* curve at that same storage value. *X** happens to be in this case, by mere coincidence, larger than $\alpha/2$, which is the aquifer storage for which the natural rate of replenishment is greatest. Instead, in this instance, the aquifer storage which maximizes the net revenue from aquifer exploitation requires a sustainable exploitation rate equal to $E(X^*) = G(X^*)$, as shown in Figure 3. By choosing such an exploitation rate the following is achieved: (1) the aquifer storage will remain at the level X*; (2) the exploitation rate is sustainable; and (3) the maximum possible revenue is attained and is given by $F(X^*) = P \cdot G(X^*) - C(X^*)$. The solution meeting these three previous conditions is herein called the optimal, unregulated, and sustainable aquifer exploitation. By unregulated it is meant that no conditions, other than the natural replenishment dynamics of the aquifer and the cost and price schedules, influence the choice of the exploitation rate.

Interesting situations arise when groundwater is exploited above sustainable rates in the unregulated case. Take for example the case where, starting at aquifer storage $\alpha/2$, groundwater is exploited at a rate *E* larger than *G(a/2)*, as illustrated in Figure 3. At this point, a profit is made since the total revenue exceeds the total cost. However, aquifer storage begins to decline due to the non-sustainable exploitation rate imposed on it. As aquifer storage declines, extraction costs rise. In addition, the non-sustainable exploitation rate begins to decline hampered by the increasing adverse extraction conditions encountered as aquifer storage drops. Eventually, the non-sustainable exploitation path intersects the aquifer storage X_E, at which

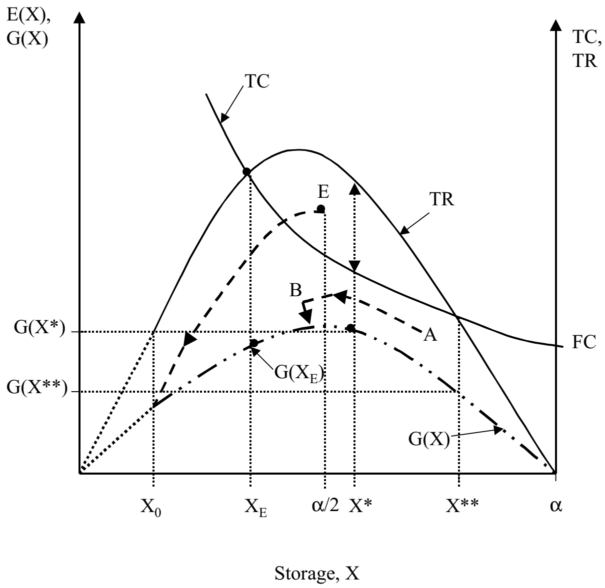


Figure 3. Graphic representation among the rate of aquifer replenishment (G), sustainable exploitation rates (E), total cost of groundwater extraction (TC), total revenue from groundwater sales (TR), and aquifer storage.

total cost of exploitation equals total revenue. To the left of storage X_E in Figure 3, and along the non-sustainable groundwater path which started at point *E*, groundwater exploitation proceeds but incurring a net loss.

Regulated Sustainable Exploitation

Let us now see what is the effect of regulation in the choice of an aquifer exploitation rate. Suppose that two types of regulations are imposed: (1) one of environmental origin, whereby the aquifer storage is not allowed to fall below, say, the level $\alpha/2$ in order to prevent land subsidence, groundwater quality deterioration, and, protect vegetation; and (2) another of institutional origin, whereby the agency managing the aquifer is required to exert average-cost pricing of groundwater, that is, groundwater must be sold so as to exactly recoup all extraction costs. Let us pursue this problem using Figure 3. It can be seen in that figure that the sustainable exploitation rate meeting these two regulatory conditions is that corresponding to aquifer storage X^{**}, that is, $E(X^{**}) = G(X^{**})$, for a zero net revenue since $TR(X^{**}) = TC(X^{**})$, as required by average-cost pricing. Note that average-cost pricing may be attained also at storage level X_E and exploitation rate $E(X_E) = G(X_E)$, but this level of exploitation violates the minimum storage restriction.

Another common type of regulation prescribes that the exploitation rate may not exceed the sustainable rate by more than a certain percentage. Once a pre-set aquifer storage is reached, the exploitation rate must drop to sustainable levels. This would entail, for example, following the exploitation path from A to B in Figure 3 and then drop to the sustainable rate along the *G(X)* curve.

Analysis with Future Tradeoffs Considered

Assume a nominal discount rate *r* (in units of 1/year) that reduces future assets to present worth. If a unit of aquifer storage is extracted during the current time period, that will trigger a future loss of potential revenue that could have accrued if the unit of groundwater would have been preserved for future use rather than being consumed at present. But current consumption of that unit of groundwater also generates a revenue now, which is given by $P - C^*(X)$, where *P* is the market unit price of groundwater (in \$/unit of groundwater) and $C^*(X)$ (in \$/unit of groundwater) is the unit cost of groundwater extraction at storage level *X*. Therefore, a revenue maximizing strategy that considers the tradeoff between foregone future revenue and current revenue must be such that the present worth of the change in future revenue caused by consumption today exactly matches the current revenue stemming from an additional unit of groundwater consumed today. Mathematically (and noticing that $C(X) = C^*(X) \cdot G(X)$)

$$\frac{1}{r}\frac{d[(P - C^*(X))\,G(X)]}{dX} = P - C^*(X) \qquad (11)$$

The left-hand side of Equation 11 represents the present worth of foregone net revenue due to a unit of consumption today (assuming an indefinitely long future impact). The right-hand side of Equation 11 is the net revenue per unit consumption enjoyed presently. Note that in Equation 11 the exploitation rate is sustainable, and equal to *G(X)*. The value of storage X* which satisfies Equation 11 yields the net-revenue maximizing sustainable exploitation rate $E(X^*) = G(X^*)$. Regulatory restrictions on exploitation may be imposed on the fundamental rule expressed by Equation 11, just as it was done for the case where future discounting was not included (see previous section).

In the event that inflation, *f*, is included in the determination of optimal exploitation rates, then one must introduce the real discount rate, r^*, which is given by . When the inflation rate is small the real discount rate is approximated by the nominal discount rate minus the inflation rate, r^* » r-f. In either case, r^* replaces *r* in Equation 11. Carrying out the differentiation of Equation 11, the following rule is obtained for profit maximization with sustainable (i.e., $E(X) = G(X)$) aquifer exploitation considering present-worth discounting

$$\frac{dG(X)}{dX} - \frac{G(X)}{(P - C^*(X))}\frac{dC^*(X)}{dX} = r^* \qquad (12)$$

The storage value, X^*, that satisfies Equation 12 provides the profit maximizing, sustainable, exploitation rate $E(X^*) = G(X^*)$. The fundamental rule for optimal and sustainable groundwater exploitation as written in Equation 12 assumes that the following are known: (1) the function $G(X)$; (2) the marginal cost function $C^*(X)$ and the market price of groundwater P; and (3) the (annual) discount rate r^*. Regulatory constraints may be imposed on Equation 12 as was already illustrated by graphical analysis.

Equation 12 represents the most general formulation of the constant-storage, sustainable, aquifer exploitation problem. In this study we shall consider linear cost-functions for aquifer pumping (Loáiciga and Leipnik, 2000). Thus,

$$C^*(X) = d - b\,X \qquad (13)$$

in which b and d are parameters to be identified from pumping cost data, as shown in the Case Study section below.

Upon substitution of Equation 3, for $G(X)$, and Equation 13 into Equation 12, one obtains a quadratic Equation in terms of aquifer storage. The quadratic equation is

$$X^2 + M \cdot X + N = 0 \qquad (14)$$

in which the coefficients M and N are expressible in terms of model parameters as follows

$$M = \left(\frac{-\alpha}{3b\lambda}\right)\left(\frac{2\lambda}{\alpha}(d-P) + b(2\lambda - r^*)\right) \qquad (15)$$

where the parameters b, d, P, r^*, α, and l have all been previously defined (see Case Study below for numerical values)

$$N = \left(\frac{-\alpha}{3b\lambda}\right)\left((d-P)(r^* - \lambda)\right) \qquad (16)$$

The solutions to Equation 14 under our modeling conditions are given by

$$X^* = \frac{-M \pm \sqrt{M^2 - 4N}}{2} \qquad (17)$$

Once the optimal sustainable storage X^* from Equation 17 is found, the optimal sustainable rate is $G(X^*)$, which is given by Equation 3 and expresses the natural rate of groundwater recharge at a storage level X^*. Equation 17 represents an unconstrained solution to the aquifer management problem formulated in this work. Constraints on aquifer and/or pumping rate levels can be introduced in several ways to obtain constrained solutions to the sustainable aquifer exploitation problem posed in this work. It will be shown in the Case Study that it is advantageous and expeditious to combine (unconstrained) solutions derived from Equation 17 with graphical analysis in the quest for constrained solutions to the aquifer management problem.

Sustainability Revisited: Variable Storage Case and Random Effects

General Formulation

Let us examine now the more complex case in which the aquifer storage is allowed to vary with time within certain bounds stemming from environmental and/or institutional constraints. We must now broaden the definition of sustainable exploitation rates to include those which maintain aquifer storage in the short and long runs within admissible bounds. When the exploitation rate is sustainable and, in addition, meets optimality criteria, then it becomes an optimal exploitation rate for given aquifer conditions, groundwater extraction costs, groundwater market price, and real discount rates.

Consider the present value of the net revenue, R, that accrues from sales of groundwater exploited at a rate E during a period of time t_1 to t. The market price of groundwater is P, the unit cost of groundwater extraction is $C^*(X)$, and the (instantaneous) real discount rate is s, and

$$R = \int_{t_1}^{t} \left[\left(P - C^*(X)\right) \cdot E \right] e^{-s(t' - t_1)}\, dt' \qquad (18)$$

where the storage X is given by Equation 8. In a deterministic context one would seek to find the exploitation rate that maximizes net revenue in Equation 18. Deterministic solutions require perfect knowledge of all variables appearing in Equation 18. This is a rather strong assumption. Fluctuations in discount rates over a long period of time, thirty years for example, may introduce appreciable statistical uncertainty in the level of net revenue to be realized under a chosen aquifer exploitation scheme. If the probability distribution function for the real discount rate s, $f_S(s)$, is known, then the solution for the optimal exploitation rate calls for the maximization of the present value of the expected net revenue with respect to the exploitation rate.

The Rayleigh probability distribution function has been used to model the long-term variations of interest rate in a variety of economic studies (e.g., Arrow and Intriligator, 1986). The Rayleigh distribution is given by

$$f_S(s) = \frac{\varphi^{\gamma+1}}{\Gamma(\gamma+1)} s^{\gamma} e^{-\varphi s} \qquad s \geq 0 \qquad (19)$$

in which g and j are distribution parameters, and G is the gamma function. The maximum present value of the expected net revenue from groundwater sales, R^*, is then given by

$$R^* = \max_{w.r.t.E} \left[\int_0^{\infty} \left\{ \int_{t_1}^{t} \left[\left(P - C^*(X(t))\right) E \left(e^{-s'(t'-t_1)} \right) dt' \right\} f_S(s') ds' \right] \qquad (20)$$

where the pumping cost function is explicitly shown to depend on aquifer storage *X(t)*, and aquifer storage is given by Equation 8.

The right-hand side of Equation 20 represents the maximum present value of the expected net revenue associated with groundwater exploitation, where the expectation is with respect to the real discount rate *s*. The maximization of R^* in Equation 20 may be subject to constraints on storage and exploitation rate.

The Net Revenue in the Case of a Finite Management Time Horizon

In the case of a finite-time horizon ($t \leq \infty$), the integration of Equation 20 leads to the following expression for the present value of expected net revenue

$$R^{*}=\varphi^{\gamma+1}\cdot E\cdot\left[\left(\frac{P-d}{\gamma}\right)\left(\frac{1}{\varphi^{\gamma}}-\frac{1}{(\varphi+t-t_1)^{\gamma}}\right)+b\cdot J\right] \quad (21)$$

where J is given by the following equation

$$J=\frac{\alpha(1+\rho)(\lambda\rho)^{\gamma}}{2\gamma}\left[D^{-\gamma}-\left(D-\ln\bar{\theta}\right)^{-\gamma}\right]+\alpha\lambda^{\gamma}\rho^{\gamma+1}M_0\cdot Z \quad (22)$$

and Z denotes the following integral

$$Z=\int_{\bar{\theta}}^{1}\frac{\left(D-\ln\theta\right)^{-(\gamma+1)}}{1-M_0\cdot\theta}d\theta \quad (23)$$

In addition, the following definitions apply in Equation 22:

$$\rho=\sqrt{1-\frac{4\cdot E}{\alpha\cdot\lambda}} \quad (24)$$

$$\bar{\theta}=e^{-\rho\lambda(t-t_1)} \quad (25)$$

$$D = \rho\ \lambda\ \varphi \quad (26)$$

and, lastly,

$$M_0=\frac{\in\cdot|X_1-\frac{\alpha}{2}(1+\rho)|}{|X_1-\frac{\alpha}{2}(1-\rho)|} \quad (27)$$

In Equation 27, $\in = 1$ when *X(t)* is outside the interval [*A, B*] or else Î = -1. Since *r, , D,* and M_0 depend on the pumping rate *E,* it is clear from Equation 21 that the net revenue R^* is a nonlinear function of the pumping rate. On the other hand, Equation 21 shows that the net revenue is linear on the market price of water *P*, and on the cost parameters *b* and *d.* Constraints (on storage, pumping rate) can be attached to Equation 21 to define a constrained aquifer management problem.

The Net Revenue in the Case of an Infinite Management Time Horizon

A case of particular interest herein is the behavior of net revenue when the management horizon *t* ® μ in Equation 21. In practical terms this implies a sufficiently long time horizon during which an exploitation rate is exerted, eventually leading to steady-state aquifer storage. In this case, the present value of the (expected) net revenue, which is now denoted by R_μ *, takes the following form

$$R_\infty^{*}=\varphi^{\gamma+1}\cdot E\cdot\left\{\frac{P-d}{\varphi^{\gamma}\gamma}+b\cdot\left[\frac{\alpha}{2\gamma}(1+\rho)(\lambda\rho)^{\gamma}D^{-\gamma}+\alpha\lambda^{\gamma}\rho^{\gamma+1}M_0\cdot Z_\infty\right]\right\} \quad (28)$$

in which

$$Z_\infty=\int_{0}^{1}\frac{\left(D-\ln\theta\right)^{-(\gamma+1)}}{1-M_0\cdot\theta}d\theta \quad (29)$$

where all terms have been previously defined. The integral in the right-hand side of Equation 29 can be approximated by numerical integration. Alternatively, the integral is expressible in term of tabulated incomplete gamma functions G(y, z) (e.g., Gradshteyn and Ryzhik, 1980) by using Stieltjes generalized transforms (Erdelyi, 1954). The maximization of the net revenue in Equation 28 with respect to the pumping rate, subject to constraints on aquifer storage and pumping rate, can be pursued by mathematical methods and assisted by graphical analysis. These techniques are illustrated in the Case Study below.

The Special Case When the Exploitation Rate E = *aλ/4*

It was shown in Equation 9 that when the exploitation rate takes the maximum value *aλ/4*, then the aquifer storage evolves in a manner different to that dictated by Equation 8. Using Equation 9 to describe the aquifer storage in Equation 20, and carrying out the integration in Equation 20 when the time horizon $t \to \infty$, one obtains the present value of the (expected) net revenue that would accrue when the pumping rate is *aλ/4*

$$R^{*}=\varphi^{\gamma+1}\cdot\frac{\alpha\lambda}{4}\cdot\left\{\frac{P-d}{\gamma\varphi^{\gamma}}+b\cdot\left[\frac{\alpha}{2\gamma\varphi^{\gamma}}+\frac{k_2{}^{\gamma}\Gamma(\gamma+1)(\varphi k_2)^{-\gamma}}{\Gamma(\gamma+2)k_1}\cdot{}_2F_1\right]\right\} \quad (30)$$

where ${}_2F_1$ denotes the hypergeometric function (Gradshteyn and Ryzhik, 1980), which is evaluated as ${}_2F_1[1;1;\gamma+2;\ 1-(\varphi\ k_2)/k_1]$, with, $k_1 = 1/(X_1 - \alpha/2)$, $k_2 = \lambda/a$, and $\Gamma(\cdot)$ denotes the gamma function (e.g., Gradshteyn and Ryzhik, 1980).

Case Study

General Information

The results on optimal sustainable exploitation will be examined in light of empirical observations in the groundwater basin of the City of Santa Barbara, California, centered approximately at 30° 26′ north latitude and 119° 38′ west longitude. The groundwater basin of Santa Barbara lies within a narrow lowland along the southern slope of the Santa Ynez mountains, a rugged linear range that rises steeply from sea level to crestal altitudes of nearly 1,200 m. The lowland strip consists in most places of elevated terraces that generally lie within 0.5 km to 5 km from the Pacific Ocean coastline, and are separated from it by an alluvial plain. The Santa Barbara area is characterized by a Mediterranean climate of warm, dry, summers and mild, rainy, winters with little frost hazard. Annual mean precipitation in Santa Barbara is about 46 cm. There is a significant increase in precipitation caused by the orographic gradient as altitude raises from sea level to the top of the Santa Ynez mountains, where annual mean precipitation is approximately 76 cm.

Nearly all of the groundwater recharge and surface runoff are derived directly from rainfall. The principal aquifer in Santa Barbara is formed by unconsolidated deposits of Quaternary age (Martin, 1984; Martin and Berenbrock, 1986). These deposits are of marine origin and include fine to coarse sand, silt, clay, with interbedded occasional gravel layers. Sources of groundwater replenishment to the aquifer are seepage from streams, direct infiltration from rainfall, subsurface flow from adjacent mountains, subsurface flow from neighboring groundwater basins, and possible upwelling (and highly mineralized) groundwater from underlying Tertiary bedrock (Freckleton, 1989; McFadden et al., 1991).

From 1987 through 1991, the State of California in general, and the Santa Barbara area in particular, experienced the second worst drought of the century (Loáiciga et al., 1993; Lawrence et al., 1994; Loáiciga and Renehan, 1997). This forced intense mining of the groundwater basin as surface water sources dwindled. The groundwater basin was nearly exhausted by 1991 as groundwater levels dropped significantly, groundwater quality deteriorated, and seawater began encroaching into the coastal aquifers. The water balance in Santa Barbara changed rapidly after 1991, as unusually wet winters followed the dry years (e.g., the 1994 to 1995 rainy season brought in 2.5 times the annual mean annual precipitation in the study area). During the 1991 to 1995 (four-year) interval the groundwater basin was "rested" and during that period its storage rose from X(t=1) = 1 unit to X(t = 4) = 4 units [1 unit of groundwater storage = 1,000 acre feet = 1,000 AF = 1.233 x 10^6 m^3]. The parameters of the time-storage function (see Equation 2) for the Santa Barbara aquifer were calculated earlier as a = 5; b = 4; and l = 0.69315. The market price of groundwater has been determined to be P = $ 1,000,000/unit of groundwater, while the unit cost (in $) per unit of groundwater is $C^*(X) = d - b\,X = 10^6 - 10^5 \times X$ (Loáiciga and Renehan, 1997).

Optimal Sustainable Exploitation Rates: Constant-Storage Case

Aquifer Storage

The solution to Equation 17 yields the optimal constant aquifer storage, which, in turn, defines the optimal sustainable pumping rate, as explained previously. Our results are presented graphically for a number of conditions which illustrate the sensitivity of results to important model parameters.

Figure 4 displays the optimal aquifer storage as a function of the unit price, P, of groundwater and the cost-slope parameter, b, when the real (annual) interest rate is 0 percent. It is seen in Figure 4 that, for a fixed value of the cost-slope parameter, the optimal groundwater storage declines as the unit price increases. In other words, for a fixed cost of groundwater extraction, there is an incentive to extract larger amounts of water, thus leading to lower optimal groundwater storage. It is also seen in Figure 4 that, for a fixed price of groundwater, the optimal groundwater storage tends to decrease as the cost-slope parameter decreases, when the price of groundwater is over $1,200,000/unit. This means that as the cost of extracting groundwater increases (i.e., b decreases), more groundwater is extracted in order to offset pumping costs, thereby leading to relatively lower groundwater storage. However, when groundwater prices fall below $1,200,000/unit, Figure 4 shows that, for fixed P, the aquifer storage increases as the cost-slope parameter decreases. In other words, when the price of groundwater is low, increases in the cost of groundwater extraction call for higher aquifer storage. Notice that any restrictions on aquifer storage level can be immediately outlined graphically in Figure 4, thereby barring inadmissible aquifer levels. For example, no stor-

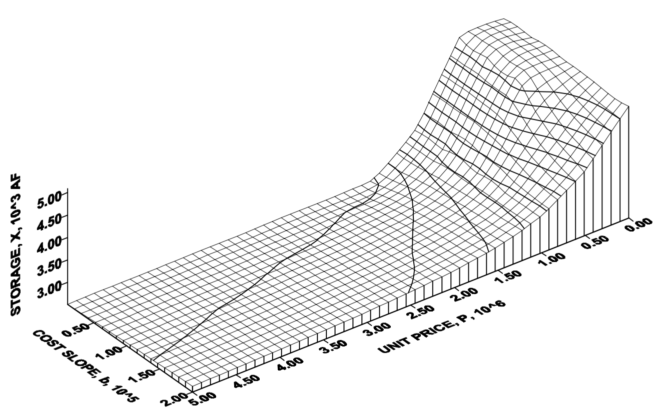


Figure 4. Aquifer storage, X, as a function of pumping cost slope and unit price of groundwater, for a real interest (annual) rate of 0 percent. Minimum contour line is at level X = 2.6 x 10^3 AF and higher ones are drawn with a contour interval of 0.2 x 10^3 AF (1AF = 1,233 m^3).

age below 2.6 units is allowed, then the region to the left of the lowest contour line would be eliminated from the feasible set of solutions displayed in Figure 4. Restrictions on pumping rates can be translated to restrictions on aquifer storage at once, as there is a one-to-one relationship between the sustainable pumping rate and optimal aquifer storage, embodied by Equation 3.

Figure 5 displays the relationship among optimal aquifer storage, cost-slope parameter, and the unit price of groundwater, just as done in Figure 4, except that Figure 5 results correspond to a real interest (annual) rate of 50 percent. The intention here is to contrast groundwater management strategies when the rate of change in the value of money is zero (interest rate is zero), and those that are derived under high rates of change in the value of money over time. The latter are of interest in inflation-ridden economies, which typically exhibit volatile interest rates. The general pattern of the optimal aquifer storage as a function of *b* and *P* in Figure 5 resembles that observed in Figure 4, except that the storage values in Figure 5 are lower than those shown in Figure 4 for any combination of the cost of groundwater extraction (represented by the parameter *b)* and the price of groundwater. This is an important reflection of the fact that, as the real interest rate rises, there is a stronger tendency to mine more of the groundwater resource in the present, thus lowering aquifer storage to lower levels than would otherwise be called for.

Pumping Rates

Let us examine now the behavior of pumping rates in terms of the cost of groundwater extraction and groundwater price. In Figure 6 we show the dependence of the sustainable, and optimal, pumping rate as a function of the cost-slope parameter, *b*, and the unit price of groundwater *P,* when the real interest rate is zero percent. It can be seen in figure 6 that for fixed *P,* the pumping rate tends to decline as the cost of groundwater extraction rises (i.e., *b* decreases). Notice, though, that as the price of groundwater rises above $1,200,000/unit, the pumping rate becomes insensitive to the cost of groundwater extraction. It is also evident in Figure 6 that, for a fixed value of the cost-slope parameter, the optimal (and sustainable) pumping rate increases as the unit price of groundwater increases. The latter pattern of association is intuitive, since it is expected that for a fixed cost of groundwater mining, the pumping rate should increase as the market price of groundwater rises. Figure 7, shows, however, that simple intuition can be misleading when the real (annual) interest is high, say, as high as 50 percent. Figure 7 shows, succinctly, that, for a fixed cost of groundwater extraction (i.e., *b* is constant), the optimal pumping rate increases sharply as the price of groundwater increases, provided that the groundwater price falls below $1,200,000/unit. These high pumping rates corroborate our previous conclusion of high aquifer depletion and present groundwater mining when the real interest rate becomes rather large.

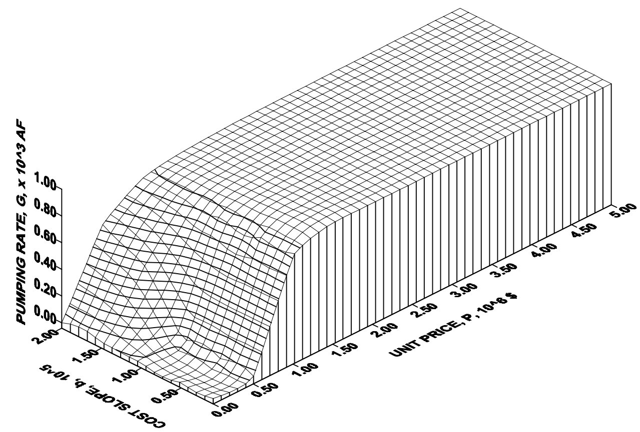


Figure 6. Pumping rate, G, as function of pumping cost slope and unit price of groundwater, for a real interest (annual) rate of 0 percent. Minimum contour line is at level G = 0, and higher ones are drawn with a contour interval of 0.05 x 10^3 AF (1AF = 1,233 m^3).

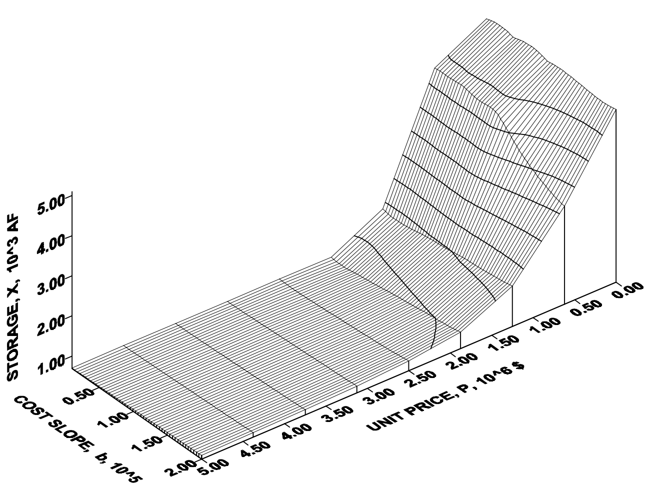


Figure 5. Aquifer storage, X, as a function of pumping cost slope and unit price of groundwater, for a real interest (annual) rate of 50 percent. Minimum contour line is at level X = 1 x 10^3 AF and higher ones are drawn with a contour interval of 0.5 x 10^3 AF (1AF = 1,233 m^3).

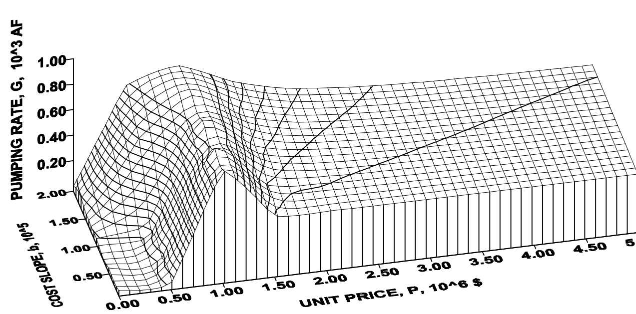


Figure 7. Pumping rate, G, as function of pumping cost slope and unit price of groundwater, for a real interest (annual) rate of 50 percent. Minimum contour line is at level G = 0, and higher ones are drawn with a contour interval of 0.05 x 10^3 AF (1AF = 1,233 m^3).

Interestingly, Figure 7 shows, on the other hand, that, for a fixed cost of groundwater extraction and when the groundwater price exceeds $1,200,000/unit, the optimal pumping rate actually declines as the price of groundwater rises. Consequently, the high cost of groundwater extraction at low aquifer levels forces a drop in pumping rates. It has been established in Figure 5 that those declining pumping rates are associated with very low levels of aquifer storage, and are very likely to be precluded by environmental restrictions on aquifer levels.

Net Revenue

Figure 8 depicts the dependence of net revenue on the market price of groundwater and the cost of groundwater extraction for a real interest rate of zero percent. It is seen in Figure 8 that, for a fixed cost of groundwater extraction, the net revenue increases monotonically as the market price of groundwater rises.. Figure 8 indicates, in addition, that, for a fixed price of groundwater, the net revenue increases as the cost of groundwater extraction drops (i.e., b increases). The largest values of net revenue that theoretically do accrue are on the order of $4,000,000 (on an annual basis). Note that not all of the [*b, P*] domain shown in Figure 8 is feasible, as some combinations of the cost-slope parameter and market price of groundwater lead to inadmissible aquifer storage and/or pumping rates. This has been demonstrated in our previous discussion of Figures 4 through 7. Ignoring constraints on aquifer storage and pumping rates, our calculations show that the highest net revenue that accrues for a real interest rate of zero percent corresponds to $b = 2 \times 10^5$ and $P = \$\ 5{,}000{,}000$/unit. The corresponding aquifer storage (see Figure 4) is 2.638 units of groundwater (2,638 AF), for a pumping rate of 0.864 units (864 AF/year, see Figure 6).

Figure 9 shows the net revenue as a function of the cost of groundwater pumping and the market price of groundwater for a real (annual) interest rate of 50 percent. The general pattern of association among the net revenue, cost of groundwater pumping, and market price of groundwater observed in Figure 9 is similar to that of Figure 8 corresponding to a real interest rate of zero percent. It is evident from Figures 8 and 9, though, that the levels of revenue generated at a 50 percent (annual) interest rate are much lower than those obtained when the real interest rate is zero percent. Our calculations indicate that the largest net revenue generated when r* is 50 percent (i.e., $ 1.9 million) corresponds to $b = 2 \times 10^5$ and $P = \$\ 5{,}000{,}000$, with an associated aquifer storage of 0.775 units (775 AF, see Figure 5) and pumping rate of 0.454 units (454 AF/year, see Figure 7). Even though the pumping rates calculated for r* equal to 50 percent exceed in some instances those obtained when r* is zero percent, the aquifer storages associated with the former tend to be lower than those associated with the latter. Ultimately, the complex interplay between cost of groundwater extraction and revenue from groundwater marketing favors aquifer exploitation under low interest rates: it produces higher storages with healthy pumping rates and larger economic benefits.

Optimal Sustainable Exploitation Rates: The Dynamic-storage Case

Figure 10 shows the behavior of the present value of expected net revenue in terms of optimal pumping rates, for selected values of initial storage, X_1. The results of Figure 10 were developed by solving Equation 28 with: (i) the market price groundwater set at $P = \$1{,}000{,}000$/unit; (ii) the cost parameters $b = 10^5$ and $d = 10^6$; (iii) the dy-

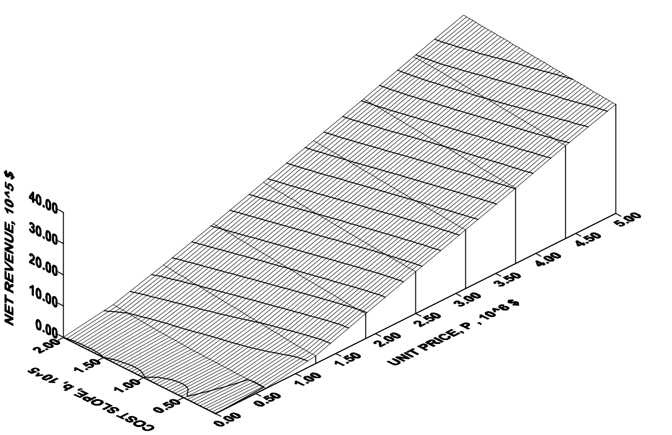


Figure 8. Present value of net revenue from groundwater mining as a function of the cost of groundwater extraction and the market price of groundwater for a real interest (annual) rate of 0 percent. Minimum contour of net revenue is at zero level, and higher ones are drawn with a contour interval of 2×10^5 ($). (1AF = 1,233 m^3).

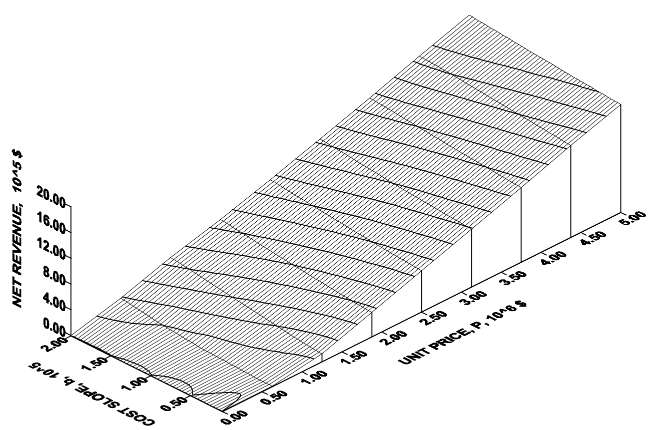


Figure 9. Present value of net revenue from groundwater mining as a function of the cost of groundwater extraction and the market price of groundwater for a real interest (annual) rate of 50 percent. Minimum contour of net revenue is at zero level, and higher ones are drawn with a contour interval of 1×10^5 ($). (1AF = 1,233 m^3).

namic aquifer parameters $a = 5$ and $l = 0.69315$; and (iv) the Rayleigh distribution parameters (which define the distribution of the real interest rate) $g = 0.5625$ and $j = 15.625$ (these imply an average discount rate of 10 percent and standard deviation of 8 percent). The initial storage was varied between its maximum value of five units (=5,000 AF) and a minimum of just above zero, while the pumping rate ranged from its theoretical maximum of 866 AFY (= *al/4)* to zero.

Figure 10 shows that: (1) for a given initial storage, the present value of expected revenue increases with increasing pumping rate up to a level of about 800 AFY, provided that the initial storage exceeds two units (2,000 AF); thereafter, the net revenue falls as the pumping rate approaches its theoretical or feasible maximum; (2) for a given pumping rate, the present value of expected net revenue increases with increasing initial storage; and (3) the maximum pumping rate which is physically realizable for a fixed initial storage decreases with the level of that initial storage. Thus, for example, with an initial storage of five units, the maximum pumping rate equals the theoretical maximum of 866 AFY, while for an initial storage of one unit the maximum feasible pumping rate is on the order of 550 AFY. For a given initial storage and pumping rate, the present values of expected net revenue shown in each of the graphs of Figure 10 are generated over a time period that starts when pumping begins (t_1) and lasts indefinitely. Each feasible combination of initial storage and pumping rate defines a trajectory of aquifer storage which converges asymptotically to a steady-state value. The steady-value depends on the pumping rate, but not the initial storage, and is equal to the variable *B*, which is defined by Equation 6. The actual trajectories of dynamic storage depend on the initial storage and pumping rate (as well as other model parameters, such as *b, d, P,* etc., which are fixed), and can be simulated by means of Equation 8 or Equation 9, as explained before. Each of the net revenue maximizing combinations of initial storage and pumping rate shown in Figure 10 must, therefore, be examined to ensure that restrictions on aquifer storage are not violated. All the mathematical tools needed for this purpose have been developed in this article.

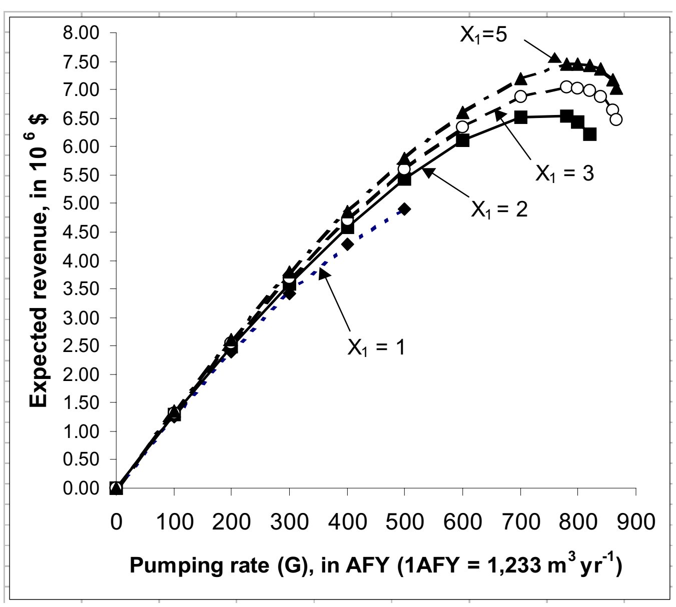


Figure 10. Present value of expected net revenue as a function of the optimal pumping rate, for selected initial aquifer storage, calculated with the dynamic storate model. A Rayleigh distribution was used to describe the real interest rate. The intial storage, X_1, is given in thousands of AF (1 AF = 1 acre feet - 1,233 m^3).

Conclusion

This article has developed an analytical/graphical method for examining the relationship among: (1) economic benefits; (2) groundwater recharge dynamics; (3) groundwater pumping rates; (4) sustainability criteria; (5) cost of groundwater pumping; (6) market price of groundwater; (7) real interest rates, and (8) initial and steady-state values of aquifer storage.

The goal of this work was to examine the cited, complex, relationship in a parsimonious manner, using as few parameters as possible while attempting to capture the essential aspects of the groundwater management problem. The analysis was carried out for constant aquifer storage and dynamic aquifer storage, and general results were obtained for each case, both summarized by objective functions to be optimized in terms of the groundwater pumping rate and a set of key model parameters, while meeting possible constraints.

The theory developed in this work was then illustrated via a case study featuring a specific aquifer, which underlies the City of Santa Barbara and is an important drought back-up water source. Our results elucidated the very highly nonlinear interaction between economic factors and groundwater dynamics, and produced an insight on the way in which the cost of groundwater extraction, market price of groundwater, groundwater recharge, real interest rates, and pumping rates interact to yield economic benefits in the constant-storage case. The specific findings in this respect are too many to repeat here. Nevertheless, a key finding points to the deleterious effect that high real interest rates have on aquifer storage and net revenue accruing from groundwater extraction.

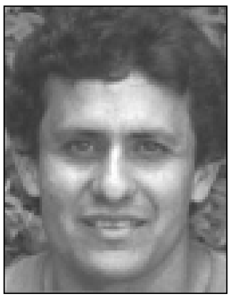
An important set of curves relating the present value of net revenue, pumping rate, and initial storage were developed for the groundwater management problem in the case of dynamic aquifer storage. Perhaps the most important findings derived in this case were: (1) that net revenues do not increase monotonically with increasing pumping rates, but, rather, that they decline after the pumping rates exceed specific thresholds, which are, in turn, a function of initial aquifer storage; and (2) the paramount role that initial aquifer storage has on optimal groundwater management strategies. Initial storage strongly influences the levels of expected net revenue, as well as the feasibility of groundwater pumping rates.

The theory, methods of analysis, and findings of this work hold promise of becoming useful tools for the preliminary screening of groundwater management strategies which consider a variety of economic and hydrogeologic factors.

Acknowledgement

This work was funded in part by the US National Science Foundation grant ATM-9711491.

About the Authors



Hugo A. Lóaiciga is a Professor of Geography in the Department of Geography at the University of California at Santa Barbara, California. He holds a Ph.D. in Engineering from the University of California, Davis. Dr. Lóaiciga teaches and does research in various aspects of surface-groundwater hydrologic interactions and in water and natural resources sustainable utilization. Email:hugo@geog.ucsb.edu.

Roy B. Leipnik is a Professor of Mathemathics in the Department of Mathematics at the University of California, Santa Barbara, California, USA. He holds a Ph.D. from the University of California, Berkeley. Dr. Leipnik teaches and does research in applied mathematics, ranging from mathematical economics to fluid mechanics.

Discussions open until March 1, 2002.

References

Anderson, T.W. 1971. *The Statistical Analysis of Time Series.* New York, New York, USA: John Wiley and Sons.

Arrow, K.J. and M.D. Intriligator, eds. 1986. *Handbook of Mathematical Economics, Volume III*. Amsterdam, Holland: North Holland Publishing Co.

Balakrishnan, N., ed. 1992. *Handbook of the Logistic Distribution*. New York, New York, USA: Marcel Dekker, Inc.

California Department of Water Resources. 1993. *1990-1995 California Water Plan.* Sacramento, California, USA.

Erdelyi, A., ed. 1954. *Tables of Integral Transforms, Volume II*. New York, New York, USA: McGraw-Hill.

Fetter, W. 2001. *Applied Hydrogeology* 4th ed. Englewood Cliffs, New Jersey, USA: Prentice Hall.

France, J. and J.H.M. Thornley. 1984. *Mathematical Models in Agriculture.* London, United Kingdom: Butterworths.

Freckleton, J.R. 1989. "Geohydrology of the Foothill Ground-Water Basin near Santa Barbara, California." *Water Resources Investigations Report 89-4017.* Sacramento, California, USA: United States Geological Survey.

Gradshteyn, I.S. and I.M. Ryzhik. 1980. *Tables of Integrals, Series, and Products*. San Diego, California, USA: Academic Press.

Kennedy/Jenks Consultants, Inc. 1995. "Water Rate Study, City of Santa Barbara, California." Final Report, Public Works Department, City of Santa Barbara, California, USA.

Lawrence, C.H., J.M. Stubchauer, and J.M. Alroth. 1994. "Changing Conditions and Water Elections." *Journal of Water Resources Planning and Management* 120, No. 4: 458–475.

Loáiciga, H.A., L. Haston, and J. Michaelsen. 1993. "Dendrohydrology and Long-term Hydrologic Phenomena." *Reviews of Geophysics* 31, No. 2: 151–171.

Loáiciga, H.A. and S. Renehan. 1997. "Municipal Water Use and Water Rates Driven by Severe Drought: A Case Study." *Journal of the American Water Resources Association* 33, No. 6: 1313–1326.

Loáiciga, H.A. and R.B. Leipnik. 2000. "Closed-form Solution to Coastal Aquifer Management." *Journal of Water Resources Planning and Management* 126, No. 1: 30–35.

Maddock, T.S. and W.G. Hines. 1995. "Meeting Future Public Water Supply Needs: A Southwest Perspective." *Water Resources Bulletin* 31, No. 2: 317–329.

Martin, P. 1984. "Groundwater Monitoring at Santa Barbara, California: Phase 2. Effects of Pumping on Water Levels and on Water Quality in the Santa Barbara Groundwater Basin." *United States Geological Survey Water Supply Paper 2197.* Alexandria, Virginia, USA: Government Printing Office.

Martin, P. and C. Berenbrock. 1986. "Groundwater Monitoring at Santa Barbara, California: Phase 3. Development of a Three-dimensional Digital Groundwater Flow Model for Storage Unit I of the Santa Barbara Groundwater Basin." *Water Resources Investigations Report 86-4103.* Sacramento, California, USA: United States Geological Survey.

McFadden, M., K.G. Polinoski, and P. Martin. 1991. "Measurement of Streamflow Gains and Losses on Mission Creek at Santa Barbara, California, July and September of 1987." *Water Resources Investigations Report 91-4002.* Sacramento, California, USA: United States Geological Survey.

Solley, W.B., R.R. Pierce, and H.A. Perlman. 1993. "Estimated Use of Water in the United States in 1990." *U.S. Geological Survey Circular 1081.*

Willis, R. and W.W. Yeh. 1987. *Groundwater Systems Planning and Management.* Englewood Cliffs, New Jersey, USA: Prentice Hall.