

UC San Diego

UC San Diego Electronic Theses and Dissertations

Title

An acoustic energy framework for predicting combustion- driven acoustic instabilities in premixed gas-turbines

Permalink

<https://escholarship.org/uc/item/6wn0k48j>

Author

Ibrahim, Zuhair M. A.

Publication Date

2007

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA, SAN DIEGO

**An Acoustic Energy Framework for Predicting Combustion-Driven
Acoustic Instabilities in Premixed Gas-Turbines**

A Dissertation submitted in partial satisfaction of the
Requirements for the degree Doctor of Philosophy

in

Engineering Sciences (Aerospace Engineering)

by

Zuhair M. A. Ibrahim

Committee in charge:

Professor Forman A. Williams, Chair
Professor Steven G. Buckley, Co-Chair
Professor Henry D.I. Abarbanel
Professor Alex Groisman
Professor Stefan G. Llewellyn-Smith

2007

Copyright ©

Zuhair M.A. Ibrahim, 2007

All rights reserved.

The dissertation of Zuhair M.A. Ibrahim is approved, and it is acceptable in
quality and form for publication on microfilm:

Co-Chair

Chair

University of California, San Diego

2007

TABLE OF CONTENTS

SIGNATURE PAGE.....	iii
TABLE OF CONTENTS	iv
LIST OF FIGURES.....	vii
ACKNOWLEDGEMENT	ix
VITA	x
ABSTRACT OF THE DISSERTATION	xii
1. CHAPTER 1	1
Motivation.....	1
Combustion Instabilities	1
Characteristics of Oscillatory Instabilities	2
Amplification of Oscillatory Combustion Instability	5
Damping of Oscillatory Combustion Instability	8
Review of Prior Methods to Study Oscillatory Instability	12
The $(n - \tau)$ Approach	15
An Approach Employing the Rankine-Hugoniot Relations.....	17
Use of Green's Functions.....	22
Approaches making use of a Galerkin Expansion	22
Unsteady Computational Fluid Dynamics Simulation (CFD)	24
Summary of Reviewed Studies	27
Motivation for a New Approach	28
2. CHAPTER 2	29

	INTRODUCTION.....	29
	SUMMARY OF THE BASIC FORMULATION	31
3.	CHAPTER 3	34
	MODELS FOR PERTURBATIONS OF THE RATE OF HEAT RELEASE	
		34
	Pressure-Sensitive Reactors with Spatially Distributed Reactions Having	
	Fixed Time Lags	35
	Pressure-Sensitive Reactors with Spatially Distributed Reactions Having	
	Space-Dependent Time Lags	39
	Pressure-Sensitive Reactors with Spatially Localized Reactions	41
	Velocity-Sensitive Reactors with Spatially Localized Reactions	44
	Velocity-Sensitive Reactors with Spatially Distributed Reactions	50
	Vortex Shedding	51
	Pressure-Sensitive Cylindrical Reactors with Conical Anchored Flames	
		54
	Velocity-Sensitive Cylindrical Reactors with Conical Anchored Flames	
		56
	More Complex Configurations	57
4.	CHAPTER 4	62
	MODELS FOR ATTENUATION OF ACOUSTIC ENERGY	62
	Wall Damping.....	62
	Damping by Perforated Liners	63
5.	CHAPTER 5	67

	COMPARISON WITH EXPERIMENTAL RESULTS	67
	A Single-Injector, Atmospheric-Pressure, Quartz Combustor.....	67
	Single-Injector High-Pressure Combustors.....	78
	A Twelve-Injector, Atmospheric-Pressure, Annular Combustor.....	83
	A Twelve-Injector, High-Pressure, Annular-Combustor Engine.....	91
6.	CHAPTER 6	97
	CONCLUSIONS.....	97
7.	APPENDIX (A)	101
8.	APPENDIX (B).....	102
	REFERENCES	105

LIST OF FIGURES

Figure 1.1: Summary of methods used to study combustion oscillatory instabilities..	14
Figure 3.1: Schematic illustration of a homogenous reactor.....	37
Figure 3.2: Schematic illustration of an anchored flame at a fixed location.....	43
Figure 3.3: Vortex shedding from the fuel spokes in a swirl premixed injector.....	52
Figure 3.4: Schematic illustration of an anchored conical flame.....	56
Figure 4.1: Schematic illustration of a perforated liner sheet [47]	65
Figure 5.1: Quartz rig picture shows the 8” diameter quartz combustor (with a height of 10”), the plenum and the exit plane of the premixer. The actual quartz liner used was 20” in height.	68
Figure 5.2: Stable flame conditions ($W_f=20.39$ lbs/hour, $W_a=0.15$ lbs/s and T - adiabatic= 2766°F).	70
Figure 5.3: Flame oscillation at 250 Hz ($W_f=22.98$ lbs/hour, $W_a=0.15$ lbs/s and T - adiabatic= 3005°F).	71
Figure 5.4: FFT of pressure oscillations at 250 Hz ($W_f=22.98$ lbs/hour, $W_a=0.15$ lbs/s and T -adiabatic= 3005°F).....	72
Figure 5.5: Growth-rate results obtained from Eqs. (11), (34) and (42) with a fixed $n=0.05$	74
Figure 5.6: Growth-rate results obtained from Eqs. (39) and (45).....	75
Figure 5.7: Growth-rate results (zoomed near 250 Hz) from effects of equivalence-ratio fluctuations, as obtained from Eq. (45).....	76
Figure 5.8: Single Injector High Pressure Rig [52].....	79

Figure 5.9: Growth-rate results obtained from Eq. (75) (at a fixed pressure of 7.5 atm and equivalence ratio of 0.65) for the high-pressure experiments, as a function of injection velocity and frequency	82
Figure 5.10: A schematic diagram of the type of production injector used in the experiments	85
Figure 5.11: Pressure Measurements on the Annular Atmospheric Test Cell (courtesy of Solar Turbines Inc.).....	86
Figure 5.12: Dynamic pressure measurements during oscillation for an annular combustor at atmospheric-pressure conditions	87
Figure 5.13: Measured relative pressure amplitudes as functions of circumferential angle	87
Figure 5.14: Growth-rate predictions for the atmospheric-pressure, annular combustor	90
Figure 5.15: Dynamic pressure spectrum obtained during oscillation on a Taurus-70 engine test (full load, 4%pilot flow, and primary zone at 2853°F).....	93
Figure 5.16: Predicted growth-rate results for the annular high-pressure engine test..	95
Figure 6.1: Predicted stability results for the three performed experiments (1-500 Hz)	98
Figure 8.1: Matlab code layout	102
Figure 8.2: Preprocessing steps I	103
Figure 8.3: Preprocessing steps II	103
Figure 8.4: Preprocessing steps III.....	103
Figure 8.5: Stability characteristics.....	104

ACKNOWLEDGEMENT

The work on this dissertation has been an inspiring, often exciting, sometimes challenging, but always interesting experience. It has been made possible by many people, who have supported me and to them I'm very grateful.

I am deeply indebted to my academic advisor, Professor Forman Williams, who has given me the chance to participate in a very interesting research project and attend various international conferences. He has supported me with his encouragement and countless fruitful discussions.

I am also very grateful to my co-advisor, Professor Steven Buckley for his insight and support. Without his help, this work would not be possible.

I would also like to thank the members of my committee. Their advice, patience and recommendations are appreciated.

The research presented here was, to a large extent, funded by a fellowship from the combustion technology group at Solar Turbines. Their collaboration has been instrumental to the success of this work.

Finally, I would like to thank my family for their support. I dedicate this effort to my parents, Dr. Mahmoud and Amna Ibrahim, for their continuous encouragement.

VITA

- 2000 Masters of Engineering, University of Alabama, Huntsville
- 2000-2003 Senior Research Engineer, United Technologies Research Center
- 2002 Masters of Science in Management, Rensselaer Polytechnic Institute
- 2002-2003 Country Consultant, United Nations Development Program
- 2003-2007 Research Assistant, Center for Energy Research,
Department of Mechanical and Aerospace Engineering,
University of California, San Diego
- 2007 Doctor of Philosophy, University of California, San Diego

PUBLICATIONS

Z.M. Ibrahim, F.A. Williams, S.G. Buckley and C.Z. Twardochleb, *A Method for Estimating the Onset of Acoustic Instabilities in Premixed Gas-Turbine Combustors*, ASME Turbo Expo, May, 2007, Montreal, Canada.

Z.M. Ibrahim, F.A. Williams, and S.G. Buckley, *A Study of Combustion-Driven Acoustic Instabilities in Premixed Gas-Turbines Using an Acoustic Energy Framework*, 5th Combustion Institute Meeting, March, 2007, San Diego, CA.

M. Gharavi, Z.M. Ibrahim, M. Borchert, F.A. Williams, S.G. Buckley, and L.O. Arellano, *Tunable Diode Laser Measurements of Equivalence-Ratio Fluctuations for Premixed Gas-Turbine Applications*, 5th Combustion Institute Meeting, March, 2007 San Diego, CA.

Z.M. Ibrahim, F.A. Williams, S.G. Buckley, and J.C.Y. Lee, *An Acoustic Energy Approach to Modeling Combustion Oscillations*, ASME Turbo-Expo, May, 2006, Barcelona, Spain.

Z.M. Ibrahim and D.B. Landrum, *CFD Modeling of Regenerative Cooling in a 15 K Fastrac Rocket Engine*, AIAA Southeast Region Student Conference, May, 2000, Savannah, GA.

FIELDS OF STUDY

Major Field: Engineering

Studies in Gas Dynamics

Professor F.A. Williams

Studies in Combustion

Professor F.A. Williams

Studies in Fluid Mechanics

Professors P.F. Linden, S.G. Llewellyn-Smith and S. Sarkar

Studies in Numerical Methods for Applied Mathematics

Professors T. Bewley

ABSTRACT OF THE DISSERTATION

An Acoustic Energy Framework for Predicting Combustion-Driven Acoustic
Instabilities in Premixed Gas-Turbines

by

Zuhair M.A. Ibrahim

Doctor of Philosophy in Engineering Sciences (Aerospace Engineering)

University of California, San Diego, 2007

Professor Forman A. Williams, Chair

Professor Steven G. Buckley, Co-Chair

For given acoustic frequencies of premixed gas-turbine combustors, a classical method not currently in use is explored for assessing whether acoustically-driven oscillatory combustion will occur. The method involves cataloging and estimating magnitudes of linear amplification and attenuation mechanisms. Linear approximations to nonlinear mechanisms are included in an effort to obtain a reasonably complete description with tractable analysis. A stability index is defined such that oscillation is predicted to occur when the value of the index exceeds unity. The method is tested on the basis of experimental data available in the literature, as well as with new experiments. Moderate success is achieved in rationalizing these experimental results. The objective

of the method is to enable quick and inexpensive decisions to be made for a wide variety of potential design configurations and operating conditions, without the complexity of computational fluid dynamics. The approach therefore may complement other approaches already in use.

1. CHAPTER 1

Motivation

Combustion-generated pollutants, in particular NO_x and carbonaceous soot, may be significantly reduced by use of lean-burning combustion systems in gas-turbine engines. However, as the fuel-air mixture becomes lean, small perturbations in the flow can change the unsteady heat-release pattern, and if the heat release is in-phase with the acoustic pressure fluctuations in the combustor, the fluctuations can grow into high-amplitude oscillations, typically at frequencies in the 100-500 Hz range. These oscillations can cause flame extinction, reduce engine life and/or cause catastrophic structural damage under extreme circumstances. Only to the extent that such oscillations can be controlled and lean mixtures can be burned stably, can emissions be further reduced to meet future standards [1]

Combustion Instabilities

Combustion instabilities arise at different stages of the combustion process and could be grouped into various types [2] Intrinsic instabilities are inherent to the combustion and fluid physics. Chamber instabilities result from the interaction of the combustion process with a combustion chamber. System instabilities involve the interaction of the combustion processes in a chamber with upstream feed lines and/or downstream exhaust. In each of these three categories, different physical processes may contribute to the instability. Another categorization of

instabilities is in terms of the physical processes involved. For example, there are buoyant instabilities, hydrodynamic instabilities, and acoustic instabilities, among others. The focus of this research is on oscillatory types of combustion instabilities that are acoustic chamber instabilities, sometimes with feed-line coupling and with influence of hydrodynamic and intrinsic instability.

Characteristics of Oscillatory Instabilities

The oscillatory instability in gas-turbine combustors arises from the coupling of unsteady heat release with acoustic waves in a chamber, resulting in repeated pressure fluctuations at various characteristic frequencies. The instability frequencies are associated with the geometry of the device and may be influenced by interactions between the device and the flow field. The interactions causing these self-excited oscillations are complex because of the coupling of the flow field with the unsteady (and highly nonlinear) heat release. Some experiments have been interpreted [3 ,4] to indicate that a primary cause for generation of instabilities is an acoustic wave generated by unsteady heat release that trips a Kelvin-Helmholtz instability in the flow [5], where high density gradients, shear, and substantial vorticity exist. The instability modifies both the overall flame structure and the flow (turbulence), and hence an effective closed-loop feedback system is generated.

In 1878 Rayleigh proposed a criterion that has evolved into a clear rule for the potential amplification of an acoustic wave in a combustion system,

essentially that positive correlation of the heat-release and pressure variations over the period of one acoustic cycle results in amplification of oscillations [6],

$$\int_0^T p'(t)q'(t)dt > 0, \quad (1)$$

where p' and q' are the pressure and the heat-release perturbations, respectively, as a function of time t , and T denotes the period of the oscillation. Unfortunately, it is difficult to apply this criteria in a practical setting, as may be observed in studies [7-9] of one-dimensional systems designed to model reheat buzz. For linear oscillatory instability studies, the perturbations of concern can be represented as harmonic oscillations, the real (Re) part of complex functions,

$$p' = \text{Re}\{Pe^{(i\omega t - \alpha)}\} \quad \text{and} \quad q' = \text{Re}\{Qe^{(i\omega t - \beta)}\}, \quad (2)$$

where P and Q are amplitudes of the pressure and heat-release perturbations, respectively, and ω is the frequency. In terms of α and β , the phase angles of the pressure and heat-release perturbations, respectively, it can be shown that equation (1) is satisfied if the phase difference between the two perturbations, $\delta = \alpha - \beta$, lies between 0 and π .

To gain further understanding of the oscillatory instability from first principles, an equation for the acoustic energy may be derived from fundamental fluid mechanics. Starting from the continuity, momentum and energy relations, and considering only linear perturbations, with the brackets denoting an average over the cycle, this equation takes the form

$$\frac{\partial}{\partial t} \int_V \left(\frac{1}{2} \rho \langle u^2 \rangle + \frac{1}{2} \frac{\langle p'^2 \rangle}{\bar{\rho} \bar{c}^2} \right) dV + \int_S (\langle p' u' \rangle - \langle u' \tau' \rangle) dS = \int_V \frac{\gamma-1}{\bar{\rho} \bar{c}^2} \langle p' q' \rangle dV - \int_V \langle \frac{\partial u'}{\partial x} \tau' \rangle dV, \quad (3)$$

where u' , ρ' , τ' , p' and q' are the perturbations for the velocity, density, viscous stress, pressure and heat-release rate respectively. Here dS and dV are the differential surface-area and volume elements respectively, $\bar{\rho}$ is the average density, and $\bar{c}$ is the average speed of sound, given by:

$$\bar{c} = \sqrt{\gamma R \theta}, \quad (4)$$

where γ , R and θ are the ratio of specific heats, the gas constant and the temperature, respectively. For simplicity of notation, tensor contractions are not shown in equation (3), terms being written in the form they would take in a one-dimensional system.

It is worth noting that:

- The first term represents the rate of change in the acoustic energy in the combustor volume.
- The second term is a convection term for energy moving in and out of the control volume through the surface S .
- The third term arises from the coupling between the pressure and heat perturbations (essentially the Rayleigh criteria).
- The fourth term represents the viscous dissipation of the acoustic energy.

The thermodynamic interpretation of the Rayleigh criteria may also be illustrated in terms of the mechanical work done over the period of one cycle by the acoustic energy [10],

$$\oint p' d\nu' = -\frac{\nu}{\gamma p} \oint p' dp' + \oint p' d\nu'^{(q)} = 0 + \int_0^T p' \frac{d\nu'^{(q)}}{dt} dt \sim \int_0^T p' \dot{q}' dt, \quad (5)$$

where p' and ν' are the perturbations in pressure and specific volume, respectively. This work term is split into an isentropic portion, the integral of which vanishes, and a portion, $(\nu'^{(q)})$, resulting from the volume change caused by the heat addition. Depending on the sign of the integral, mechanical work could be added to or extracted from the cycle.

Amplification of Oscillatory Combustion Instability

Several mechanisms have been identified that contribute to acoustic instability amplification, some of which are:

Air/fuel-ratio fluctuations: Recently investigated by Lieuwen and Cho [11], the heat-release perturbations at the flame can cause acoustic waves to propagate upstream into the feed lines and cause perturbations in the incoming air/fuel mixture. These perturbations may be carried by the mean flow and trigger a fluctuation at the base of the flame, closing the instability loop. Several studies have addressed this possible mechanism. For example, Sacarini et al.[12] studied the fuel-air fluctuations in a simple duct and concluded that the strong potential of these fluctuations to drive instabilities justified substantial effort in mitigating

air/fuel fluctuations. In their work [13], they defined a parameter σ as an indicator of the efficiency of the mixing duct,

$$\sigma = \frac{\frac{\phi'}{\phi}}{\frac{u'}{u}}, \quad (6)$$

where $\frac{\phi'}{\phi}$ is the ratio of air/fuel-ratio perturbations to the mean air/fuel-ratio,

and $\frac{u'}{u}$ is the ratio of velocity perturbations to the mean flow velocity.

Their later work focused on trying to get this parameter as close to zero as possible [13], which was accomplished by improving the mixing quality of the reactants by using multiple fuel-injection locations.

Convective-Acoustic Waves: This is a class of perturbations that are carried by the mean flow, such as vortices shed from the flame holder and/or entropy waves propagating downstream, and generating upstream-propagating acoustic waves. The vortex shedding and entropy phenomena are as follows:

Vortex Shedding: Vortices shed from the flame holder were suggested as a cause of combustion instabilities as early as 1956 by Rogers and Marble [14]. More recently, experimental investigation by Poinso et al. [3] looked at this as a possible source of combustion instability. The instability is triggered when the vortices shed at the flame holder entrain unburned mixture, which propagate downstream and causes a sudden heat release at some point downstream. This triggers an acoustic wave propagating upstream, closing the feedback loop. Culick and Magiawalla [15] investigated vortices shed from the flame holder consisting of

pure products, which would impinge on obstacles downstream (e.g. the nozzle) and cause pressure oscillations to intensify. This investigation was of purely acoustic phenomenon with no heat-release contribution. Mateev and Culick [16] recently investigated the formation of vortices behind flame holders and their interaction with flow-field perturbations in premixed combustors, using a newly developed quasi-steady model. In this model, they addressed a dump combustor in which they assumed constant fluid properties, vortex burning as the only source for instability (without vortex-surface interaction), and vortex propagation at the mean flow velocity. They were able to partially validate the model against experimental results for vortex shedding in a non-reacting oscillating cold flow. The authors cited a concern that reacting flows might behave differently and that resulting vortex shedding and interaction in reacting flows might follow a different pattern [16].

Entropy waves: The phenomenon of localized hot spots in a mean gas flow has been known for a number of years. When these hot spots reach the inlet of a choked nozzle, their arrival triggers an upstream acoustic wave propagation that can cause an acoustic instability. The effect of entropy waves on flow-field instabilities was suggested in early work by Chu [17], who considered their influence on combustion instabilities to be minimal except at low frequencies. Polifke et al. [18] recently studied the constructive or destructive coupling of entropy waves with the pressure perturbations. Since the hot spots are transported by the mean flow (usually at low velocity), effects of entropy waves have been assumed to exist at low frequencies [19].

There are other possible sources of combustion oscillatory instability ranging from purely chemical-kinetic to solely fluid-mechanical. Their contributions vary with oscillation modes. It is also possible that some of the oscillation modes are triggered by a combination of perturbations (velocity, temperature, laminar flame speed, etc...).

Damping of Oscillatory Combustion Instability

Little work has been done to include damping effects in the body of literature on combustion instabilities. Most of the previous research has looked at amplification mechanisms and signs of growth or decay of the modes. The few studies looking into damping were independent from research into amplification or were complete engine studies [20].

Williams [2] presented some of the possible damping sources in an oscillating combustor environment.

Wall damping: Wall damping of the velocity parallel to the wall occurs as a result of the oscillating boundary layer with a thickness of order

$$\delta = \sqrt{\frac{\mu_w}{\rho_w \omega}}, \quad (7)$$

where μ_w is the viscosity near wall. Here ω and ρ_w are the oscillatory frequency and the density near the wall, respectively. The wall damping depends on gas molecular properties near the wall and on the surface area of the combustor liner and wall. This energy damping can be found, starting from a simplified time-dependent momentum equation [2],

$$\rho_w \frac{\partial v}{\partial t} = \frac{\partial \left(\mu_w \frac{\partial v}{\partial y} \right)}{\partial y}, \quad (8)$$

where ρ_w, μ_w and v are the density, viscosity and velocity values near the wall.

This equation has a solution,

$$v = \left(\frac{\theta_w}{\theta_c} \right) \text{Re} \left\{ V e^{-(1+i)\sqrt{\frac{\omega \rho_w}{2\mu_w}} y} \right\}, \quad (9)$$

where θ_w, θ_c, V and ω are the temperature at the wall, the temperature at the core, the oscillatory velocity magnitude outside the oscillating boundary layer, and the frequency of the oscillation respectively.

From this velocity solution, an energy dissipation expression, (per unit area of combustor wall), was formulated by Williams [2] as,

$$e_{wd} = \int_0^\infty \frac{1}{2} \rho_w \omega \left[\frac{\theta_w}{\theta_c} V e^{-(1+i)\sqrt{\frac{\omega \rho_w}{2\mu_w}} y} \right]^2 dy = \frac{1}{2} \left(\frac{\theta_w}{\theta_c} \right)^2 |V|^2 \sqrt{\frac{\omega \rho_w \mu_w}{2}}, \quad (10)$$

This solution then needs to be integrated over the wall boundary surfaces to obtain the total rate of dissipation by the wall damping.

Particle damping: This is the sound attenuation by the Stokes drag on small particles (mainly soot in the combustion applications of interest here).

Williams [2] identified the range of contributing particle sizes by

$$r_s \ll \sqrt{\frac{\mu}{\rho \omega}} \ll \lambda, \quad (11)$$

where r_s, μ, ρ, ω , and λ are the mean particle radius, viscosity in the chamber, density, frequency, and the acoustic wave length. The contribution of the particle damping depends on the velocity difference between particles and gas, particle sizes, particle density, number of particles per unit volume and particles distribution.

The total rate of acoustic energy dissipated by solid, spherical particles in the flow can be calculated as

$$e_{pd} = \frac{1}{2} \int_V \frac{1}{\tau} \left\langle n_s \left(\frac{4}{3} \pi r_s^3 \rho_s \right) (v_g - v_s) \cdot v_g \right\rangle dV, \quad (12)$$

where n_s, r_s, ρ_s, v_g , and v_s are the number of particles, particle radius, particle density, gas velocity and particle velocity, and the brackets represent an average over time. The response time, τ , is calculated from

$$\tau = \frac{2r_s^2 \rho_s}{(9\mu)}, \quad (13)$$

where μ is the gas viscosity.

Equating the force decelerating a particle with the Stokes drag and performing a few algebraic operations reduces the coefficient of energy dissipation to

$$\alpha_{pd} = \left(\frac{\omega}{2} \right) \left(\frac{\rho_s}{\rho_g} \right) \int_0^\infty \left\{ \frac{4}{3} \pi r_s^3 (\omega \tau) / [1 + (\omega \tau)^2] \right\} G(r_s) dr_s, \quad (14)$$

where $G(r_s) dr_s$ represents the number of particles per unit volume within radius range dr_s about r_s . The total energy dissipated can be calculated from

$$e_{pd} = e_o \exp(-\alpha_{pd}t) \quad (15)$$

where e_o is the initial acoustic energy.

Relaxation damping: This is a form of homogeneous dissipation that is caused by chemical and vibration relaxation. The contribution of relaxation damping depends on the frequency of the oscillations, the chemical relaxation time, which is a measure of the reaction rate, and the speed-of-sound ratio, $\frac{a_f}{a_e}$,

where a_f and a_e are the frozen and equilibrium sound speeds respectively. Relaxation damping is usually small compared with wall damping, but might become important for large chambers.

Homogeneous damping: This is damping of the acoustic energy caused by viscous dissipation. It primarily depends on velocity gradients, acoustic frequency, and viscous properties. It is believed to have small contribution to acoustic energy attenuation at low frequencies, but this contribution increases as frequencies increase, and it is a possible cause for the absence of very high-frequencies.

Nozzle damping: When choked flow conditions are met, the propagation of acoustic waves is restricted at the nozzle and no propagation of longitudinal waves is allowed upstream from a diverging supersonic-flow section. This boundary can absorb energy depending on the flow conditions.

There are other damping sources to be considered in an actual combustor, such as perforated walls and cooling flows near the wall, and heat-addition mechanisms acting out of phase with the pressure perturbations.

Review of Prior Methods to Study Oscillatory Instability

There is a large body of literature in the area of combustion instabilities. Before instabilities were recognized as problems in the gas-turbine industry, combustion instability was observed as a serious issue during the development of solid rockets, and later also became pronounced for liquid rockets, where much of the earlier research was conducted [21]. The complicated nature of the coupling between the heat release and acoustics continues to make control of instabilities a challenging task for researchers looking into investigating and designing propulsion and power-generation devices.

Figure 1.1 summarizes some of the methods used to analyze and/or predict the onset of combustion instabilities. All approaches to study combustion oscillations start from the Navier-Stokes conservation equations, and may be classified as follows:

- An approach in which the linearized Rankine-Hugoniot equations are expanded around a small Mach number to relate the perturbations across a flame front. The use of network models and the flame jump relations produces a dispersion equation to be solved for the eigenfrequencies of interest.
- Another approach by which simplifying the linearized (N-S) equations produces an inhomogeneous wave equation for pressure perturbations with a source term accounting for combustion heat release. The challenge in this approach becomes finding approximate solutions that satisfy the boundary conditions. The method of weighted residuals and its variants are used to solve the resulting equation.

- A recent approach to numerically integrate the Navier-Stokes equations directly as done by many computational fluid dynamics (CFD) simulations.

The first two approximate methods require a closure term for the heat release term. Closure formulas range from general closure terms relating velocity or pressure perturbations to heat release, to detailed flame models. These will be discussed later. The analysis then either takes the path of finding a dispersion relation for the frequency, via the Rankine-Hugoniot equations, or finding approximate solutions to the inhomogeneous partial differential equations (PDEs) using the Galerkin method. CFD simulations require large computational efforts if they are direct simulations, or need approximations for small turbulent scales as in large-eddy simulations. Examples from these approaches are discussed in more detail below.

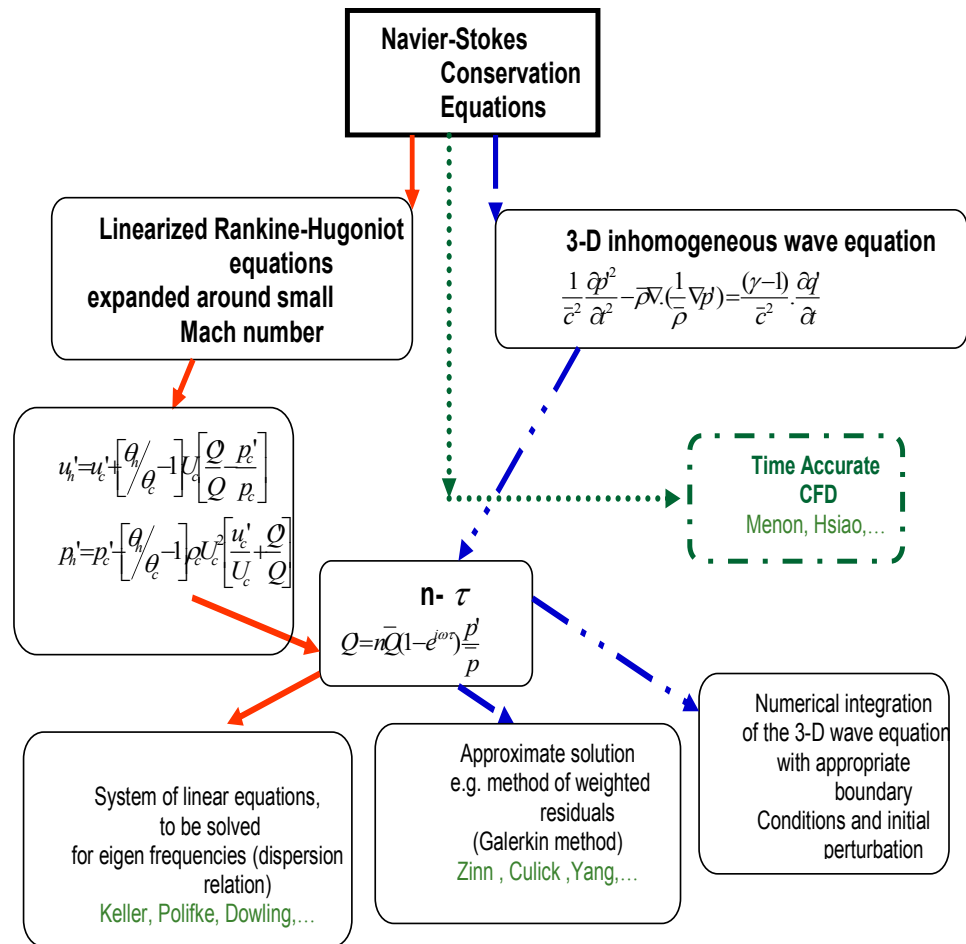


Figure 1.1: Summary of methods used to study combustion oscillatory instabilities

The (n- τ) Approach

Also referred to as the time-lag model, the n- τ approach evolved during research into liquid rocket instabilities by Crocco, Cheng and others [21]. The model provides a way to couple heat perturbations with flow-field perturbations. This is achieved by a pressure-interaction index, n , describing how the pressure oscillations affect combustion oscillations, and a time lag τ between the two fluctuations. The time lag is defined as the interval between the time when the pressure disturbance occurs at the flame to the time when heat is released in that location [6]. Crocco [21] assumed that the heat release is only affected by the pressure and is proportional to p^n , where n is the interaction index. These models have been used for linear and nonlinear perturbations alike [18,22] and are generally formulated to look like some variation of the expression,

$$Q' = n\bar{Q}(1 - e^{i\omega\tau}) \frac{p'}{\bar{p}}, \quad (16)$$

where Q' is the perturbation of the heat release, p' is the pressure perturbation, and $\bar{Q}$ and $\bar{p}$ are the mean heat release rate and pressure respectively. The n- τ model in its original form is open to using empirical correlations between the heat and the pressure perturbation and in general should be viewed as a framework that may include both physical concepts and empirical observations.

Putnam provided a simple way of estimating the time lag for combustion instabilities such as those of interest here [6],

$$\tau = \frac{\delta}{V_p}, \quad (17)$$

where δ is the distance between the fuel injector to the flame front and V_p is the mean fuel velocity. This τ in essence is a mean convection time between the fuel port and the flame front. He later [6] modified the formula to account for an additional time lag based on observations from experimental results. The modified time-lag expression is

$$\tau = \frac{\delta}{V_p} + \left(\frac{a}{3}\right)\left(\frac{V_p}{S_L}\right)\left(\frac{1}{V_p}\right) = \frac{\delta}{V_p} + \frac{a}{3S_L}, \quad (18)$$

where a is the duct diameter and S_L is the laminar flame velocity. The additional time is intended to account of the transverse flame propagation across the diameter. There are various ways of interpreting the time lag. For example:

- The time lag appropriate for the liquid rocket applications often is thought to be associated with droplet evaporation and the consequent heat release, dependent on reaction rate. These processes are directly affected by the pressure, and hence Crocco determined that τ varies with p^n only.
- For the premixed combustion application considered here, the frequencies of concern are lower, and the time lag is more closely associated with the interval from the injection of reactants to their arrival at the flame front (convection time). This is closer to the expressions proposed by Putnam [6] in equations (17) and (18), which many of the studies on premixed combustor oscillations have adopted.

An Approach Employing the Rankine-Hugoniot Relations

Keller [23] and later Polifke [18] derived a set of algebraic correlations by expanding the Rankine-Hugoniot equations around the mean Mach number. The Rankine-Hugoniot relations are

$$\begin{aligned} \rho_c u_c &= \rho_h u_h \\ p_c + \rho_c u_c^2 &= p_h + \rho_h u_h^2 \\ h_c + \frac{1}{2} u_c^2 + q &= h_h + \frac{1}{2} u_h^2 \end{aligned} \quad , \quad (19)$$

where ρ, u, p, h , and q are the density, velocity, pressure, enthalpy and heat addition respectively. The subscripts c and h represent the region prior to the flame (reactants/cold side) and the region after the flame (products/hot side).

Keeping only the linear terms, they derived a relation between flow perturbations on both side of the flame, taken here to be a discontinuity with a temperature jump,

$$\begin{aligned} u_h' &= u_c' + \left[\frac{\theta_h}{\theta_c} - 1 \right] U_c \left[\frac{Q'}{Q} - \frac{p_c'}{p_c} \right] \\ p_h' &= p_c' - \left[\frac{\theta_h}{\theta_c} - 1 \right] \rho_c U_c^2 \left[\frac{u_c'}{U_c} + \frac{Q'}{Q} \right] \end{aligned} \quad , \quad (20)$$

where u', p', θ, ρ , and U are the velocity perturbations, pressure perturbations, mean temperature, mean density and mean velocity, respectively.

Closure for the $\frac{Q'}{Q}$ term: To provide closure for the resulting algebraic

equations, the term $\frac{Q'}{Q}$, representing the ratio of heat-release perturbations to mean

heat release, was written in terms of pressure and/or velocity perturbations with a time lag τ . An example is the closure term for vortex shedding [18],

$$\frac{Q'}{Q} = \varepsilon e^{i\omega\tau}, \quad (21)$$

where ε is interpreted as the percentage of reacting mixture entrained by the vortex. The model assumes τ to be a constant, which is a common assumption in the literature, along with the approximation that the interaction index n is constant. These approximations may not always be accurate, especially when there is significant flame movement, which would lead to a time varying τ , or different operating conditions, which would lead to a changing n .

An alternative to seeking an overall closure model for the heat-perturbation term $\frac{Q'}{Q}$ is to consider a flame model that incorporates some of the nonlinear effects involving the flame location, shape, area and response to flow-field perturbations. The flame model would affect the rate of heat release, since the total heat release can be related to the flame area by the expression

$$Q = C_p (\theta_h - \theta_c) \rho_c S_L A_{flame}, \quad (22)$$

where C_p , ρ_c , S_L , and A_{flame} are the specific heat at constant pressure, density, laminar flame speed, and flame area respectively. In this thin-flame model, the resulting acoustic perturbations can be represented in terms of flame-area perturbations [24].

Efforts have been made to break the heat-perturbation term $\dot{Q}$ into several contributing components in other ways such as [11,25],

$$\dot{Q}_{total} = \dot{Q}_{\Delta H_r} + \dot{Q}_{S_L} + \dot{Q}_{A_{flame}} \quad (23)$$

where $\dot{Q}_{\Delta H_r}$ is the perturbation resulting from heat-release fluctuations, $\dot{Q}_{S_L}$ is the perturbation caused by variation of the laminar flame speed and $\dot{Q}_{A_{flame}}$ represents fluctuations caused by instantaneous flame-area perturbations. These studies have assumed the flame to be a discontinuity separating reactants from products. In the work of Lieuwen [11], this flame front was tracked using a flame-tracking equation. Transfer functions were assigned to each term and contributions from these transfer functions were then incorporated into the combustion instability model. It was indicated that although the transfer functions were adequate approximations for the regime of small Strouhal number $St \ll 1$, at larger Strouhal numbers there were reported differences between experiment and theory [11]. In these studies, the Strouhal number is defined as

$$S_t = \frac{\omega R}{\overline{S_{L0}}}, \quad (24)$$

where ω is the frequency, R is the radius of the combustor inlet, $\overline{S_{L0}}$ is the mean laminar flame speed at a reference point (e.g. center point).

Stability Studies: In studying the stability of combustion systems, the pressure and velocity perturbations were taken to behave in a harmonic manner, similar to equation (2). Polifke et al. [26] constructed a network model for a

simplified combustion system, in which each component of the system was represented by an acoustic impedance. Assuming negligible contribution from the mean flow (low Mach number approximation), and starting from a fuel plenum with boundary conditions set to zero (no flow perturbations), the one-dimensional coupled system of equations developed in this process accounted for pressure losses and velocity changes caused by duct friction and area changes up to the flame location. Jump conditions were introduced at the flame location as the flame was assumed to be a discontinuity of negligible thickness resulting in an instantaneous temperature jump. The combustor zone after the flame was approximated by a constant-area duct and a choked flow conditions were taken at the combustor exit, Mach number equal to unity, representing maximum mass flow and permitting no flow perturbations to propagate upstream from the turbine.

A small harmonic perturbation was then numerically introduced at the plenum exit. The coupled system of equations was manipulated to derive a dispersion equation in the frequency ω . The roots of the equation are the complex eigenfrequencies [18,27].

A stability criterion was determined by looking at the cycle increment, defined [18] as the percentage by which an infinitesimal amplitude may grow in one cycle,

$$CI(\omega) = e^{\left[-2\pi \frac{\text{Im}(\omega)}{\text{Re}(\omega)}\right]} - 1. \quad (25)$$

Here $CI(\omega)$, the cycle increment for a particular frequency, is an exponential function of the ratio of the imaginary to real part of the eigenfrequency and is a

measure of the growth rate of the amplitude, and negative values of $CI(\omega)$ indicate decay.

As the system of equations grows larger, (adding more mechanisms or including more ducting upstream or down stream from the flame), finding all the eigenfrequencies of the dispersion relation becomes a harder numerical task. Also, the dispersion relations obtained by the above studies do not provide concrete information about frequencies other than the eigenfrequencies, and whether they will grow or damp. As operating conditions of the combustor change and the energy distribution shifts between modes, some of these undetermined eigenfrequencies may be susceptible to growth as well.

An alternative to seeking eigenfrequencies and dispersion relations is the approach of inserting a closure term for the heat-release perturbations term in equations (20). For example, putting $\frac{Q'}{Q} = \varepsilon e^{i\omega\tau}$ with, $\varepsilon = 0.05$ (5% of the reacting mixture is entrained by the vortex), enables pressure and velocity perturbations' amplitudes to be calculated. Inspection of the resulting pressure-perturbation amplitude in time (growth or decay) provides a way of inferring growth or decay information from the time history of the perturbation [18].

In this method the choice of the value of ε is empirical, and it is viewed as the only nonlinear addition to the linearized equations. It then becomes the driving source for possible amplification. This makes predicting instability using this method wholly dependent on the choice of the driving term, presumed to arise from nonlinear phenomena in a formulation which nonetheless is linear.

Use of Green's Functions

An intermediate level of analysis, between the linearized Rankine-Hugoniot equations and a full time-accurate CFD simulation, is the use of approximate methods to solve the inhomogeneous PDE. These approximate solutions attempt to capture more of the physics than simple closure approaches can capture. Starting from the linearized Navier-Stokes conservation equations and ignoring the mean flow (Mach number very small so it can be approximated as being equal to zero), Hegde et al. [28] formulated the problem in the form of an inhomogeneous wave equation for the pressure perturbation,

$$\frac{1}{\bar{c}^2} \frac{\partial p'^2}{\partial t^2} - \bar{\rho} \nabla \cdot \left(\frac{1}{\bar{\rho}} \nabla p' \right) = \frac{(\gamma - 1)}{\bar{c}^2} \frac{\partial q'}{\partial t}. \quad (26)$$

To solve the problem, a Green's function was used to represent the heat perturbation term in equation (26). Overall, their use of this method provided some insight but fell short of yielding a conclusive answer because of their omission of a formal coupling between the heat-release and flow-field perturbations [9].

When there is a formal coupling between the pressure perturbations and the heat perturbations, the use of a Green's function essentially simplifies the inhomogeneous PDE into an integral equation to be solved iteratively.

Approaches making use of a Galerkin Expansion

The approach is a variation of the method of weighted residuals developed for solving PDEs. Powell and Zinn [29] applied a linearized Galerkin technique in their investigation of axial and transverse instabilities in liquid rocket motors. A

similar approach was independently developed and used to analyze solid rockets instabilities by Culick [30].

Again the approach is from the inhomogeneous wave equation for the pressure perturbations p' ,

$$\nabla^2 p' - \frac{\partial^2 p'}{\partial t^2} = h, \quad (27)$$

where the source term h is the contribution of the heat perturbations to the pressure perturbations. Using the notation of Culick [31] this term can be written as

$$h = -\bar{\rho} \bar{N}^2 \cdot (\bar{u} \cdot \nabla u' + u' \cdot \nabla \bar{u}) + \frac{1}{c} \bar{u} \cdot \nabla \frac{\partial p'}{\partial t} + \frac{\bar{\gamma}}{c} \frac{\partial p'}{\partial t} \nabla \cdot \bar{u} - \nabla \cdot \left[\bar{\rho} (u' \cdot \nabla) u' + \bar{\rho} \frac{\partial u'}{\partial t} \right] + \frac{1}{c} \frac{\partial}{\partial t} (u' \cdot \nabla p) + \frac{\bar{\gamma}}{c} \frac{\partial p'}{\partial t} \nabla \cdot u' + \nabla \cdot F + \frac{1}{c} \frac{\partial p'}{\partial t} \quad (28)$$

The boundary condition is

$$n \cdot \nabla p' = -f, \quad (29)$$

in which $n \cdot \nabla p'$ is zero for rigid-wall boundary conditions, and f is the effect of the heat addition.

The solution for equation (27) was approximated by a series expansion in the natural modes of the combustion chamber, which are the solutions of the wave equation with no heat perturbation and rigid-wall boundaries. The solution was given the form:

$$p'(r,t) = \sum_m \eta_m(t) \psi_m(r), \quad (30)$$

where $\eta_m(t)$ are the amplitudes of the pressure perturbations and ψ_m are the acoustic eigenfunctions; these are the geometric natural modes with no flow or heat addition. Given the mode shape, the use of the Galerkin method tracks in time whether the amplitude of each mode m $\eta_m(t)$ will grow or decay.

The Galerkin methodology has been used to look at various longitudinal and transverse modes and has been used for both linear and nonlinear analyses. It has been observed that different modes may or may not grow depending on the number of expansion terms included in the Galerkin approximation [32]. Inclusion of more modal terms from the expansion results in more effort and eventually to a larger nonlinear system of ordinary differential equations. Finally, detailed knowledge of the mode shapes and therefore of the combustor geometry is needed to be able to proceed with this type of analysis.

Unsteady Computational Fluid Dynamics Simulation (CFD)

Direct modeling of combustion processes in general requires extensive computational resources. In addition to solving for the flow field and chemistry, an inherently stiff system of equations, direct solution requires solving for the transport properties of N species produced or consumed during the reaction processes. Unsteady CFD, though far superior to previous approximate methods in capturing details of the flow field, requires extensive time and computational

resources to model the complex flow field of multi species, dimension, and time represented in evolving Navier-Stokes equations.

Instead of computing fine-scale fluid details directly with direct numerical simulation (DNS), the practice has been to limit calculations to large fluid structures, thus reducing the computational requirements. A separate, simple model represents the fine structures and quantifies only their contribution to the flow field. This treatment, referred to as large-eddy simulation (LES), utilizes a subgrid model to indirectly account for the fine-scale turbulent structures, which are assumed to be homogeneous and possess a universal character.

Hsiao et al. [33] studied the instabilities in a 3-D model of an annular combustor using LES. He later used CFD modeling results to incorporate into a low order acoustic model for predicting oscillatory instabilities. Stone and Menon [34] studied the effect of swirl on the stability of premixed combustors, again employing LES, they effectively captured the contribution of large structures to mass, momentum, and energy “vortex structures,” while empirical and or analytic formulas were used for the smaller turbulent structures. Comparisons for various fuel/air ratios showed their effect on imperfect mixing and the result on the combustor instability.

Steele et al. [1] used an LES code to run time-accurate simulations of a model combustor with the approximate dimensions of one of the Solar Turbines Mars combustor family. They predicted a range of instabilities that later were compared relatively well with experimental results. They also used an $(n-\tau)$ model to look at the stability characteristics of that particular combustor. This effort,

using results from CFD along with a low-order model, resulted in what some in the industry term the $(\tau - F)$ criteria, which is essentially a convective time lag multiplied by the frequency of interest. The work enabled them to identify some of the stable/unstable operating regimes for gas turbine engines and served as a quick design guideline to avoiding zones of instability, but never explained why instabilities would or would not occur.

To date much of the unsteady CFD literature has focused on simple geometries. In many cases results from steady CFD have been used to compliment low order models.

Summary of Reviewed Studies

Much of the analysis in the literature has assumed the wave behaves as a one-dimensional acoustic wave. Very little analysis has been done on multi-dimensional acoustic wave propagations and the effects of reflecting and or refracting waves. Lieuwen [35] investigated the acoustic field in the near field of a flame using 2-D flame zone geometry and a boundary element method code. He found that while the pressure field remained almost one-dimensional with very small departures, the velocity field behaved weakly 2-dimensionally at the center of the flame (variation from the 1-D wave approximation increasing slightly with the frequency) returning monotonically to the one-dimensional behavior closer to the wall.

The previous numerical and experimental research has focused on simple geometries such as straight ducts or constant area combustors, and in some cases has used network models, a method by which a complex geometry is broken into a series of different size ducts to compensate for different diameters and area changes [18,27]. This has allowed the simplification of the task of calculating the natural modes and convection times appropriate to the geometry.

Neglecting the mean flow is a common simplification of the problem adopted by all of the approximate methods. Mean flow contribution comes in two types, a velocity contribution to the acoustic wave and additional waves such as vortex shedding and entropy waves [9]. Dowling [36] studied the impact of the mean flow on combustion oscillations and concluded that mean flow effects are

negligible for Mach numbers less than 0.2 (many practical combustors), while for higher Mach number flows, the presence of the mean flow adds the possibility of new mode oscillations.

Motivation for a New Approach

Previous research efforts either have attempted to identify the growth or decay of a particular mode, generally the natural modes of the geometry as in the Galerkin approach, or have attempted to identify the modes that will grow or decay by solving a dispersion relation, as in the time-lag approaches. The susceptibility of particular modes to growth or decay depends on the selection of amplification and/or damping mechanisms included in the governing equations.

The authors couldn't find much prior work approaching the instability issue with a direct acoustic energy conservation approach. This approach allows the flexibility to calculate contributions from amplification and/or attenuation across all frequencies, rather than predicting eigenmodes. It is our belief that a direct acoustic energy approach could be of tremendous benefit in determining the susceptibility of all frequencies to growth or decay as operating conditions in a gas turbine change. This approach may also give designers a better handle on improving passive damping in the initial design phase, as they would have the ability to scan various frequencies and investigate their susceptibility to oscillatory instability at different operating conditions. We expect that this effort will provide a new perspective on and enhanced prediction of combustion oscillatory instabilities.

2. CHAPTER 2

INTRODUCTION

Many current combustor designs in the gas-turbine industry employ lean premixed combustion for the purpose of mitigating pollutant emissions. Such combustors, however, have been prone to combustion oscillations that detract from performance, reduce combustor lifetimes and increase operating expenses. Economical tools to help minimize the chances of encountering oscillatory combustion in new and existing combustor designs for low-emission gas turbines. Economical tools to aid in achieving this objective are worthy of investigation.

Significant advances in computational approaches for predicting oscillatory combustion have been achieved in recent years [37]. Although fully resolved direct-numerical simulation of turbulent combustion in practical-scale combustors remains a task for the future, there are a variety of well-developed modeling approaches that now can be brought to bear on the problem, employing different types of computational fluid dynamics (CFD). One class of approaches works with a flame transfer function which, based on a flame model, ultimately provides both the oscillation frequency and the amplification rate of an acoustic disturbance [18,23,36]. Nonlinear CFD methods, such as approaches employing Galerkin finite-difference techniques [30,38], yield predictions of both frequencies and limiting oscillation amplitudes, dependent, of course, on the turbulence and combustion models employed. Since all such approaches, are computationally intensive, there is motivation for investigating less expensive

approaches that are capable of addressing a wide variety of combustor configurations and operating conditions in a reasonable amount of time.

Advantage may be taken of the fact that observed oscillation frequencies often do not differ greatly from natural acoustic frequencies of the combustion chamber. Determination of the frequency may then be treated separately from determination of whether amplification occurs. In the present work it is assumed that potential oscillation frequencies will be identified independently, from acoustic analysis parallel CFD investigations, experiments or estimates based on experience, and the likelihood of oscillation occurring is therefore assessed as a function of frequency. The likelihood is estimated from analysis of amplification and attenuation rates by different physical and chemical mechanisms presumed to occur within the combustion system. Since acoustic amplification and attenuation is fundamentally linear concepts, linear approximations to nonlinear processes are needed in this approach.

We introduce simplified models of potentially relevant processes are introduced here. These models include, for example, different flame configurations ranging from distributed reactions to localized heat release, and different interaction mechanisms, such as pressure-sensitive combustion and mixture-ratio variations. Time lags play a central, crucial role in these models, which make use of simplified descriptions for approximating mode shapes of acoustic fields. The user must estimate the combustor and operational parameters as inputs to the model to obtain the amplification coefficients and time lags. The output is the stability index, which determines whether amplification or damping

is likely to occur as a function of frequency. The approach is shown to be capable of explaining a number of experimental observations, thereby providing some confidence in the utility of the method in future applications.

SUMMARY OF THE BASIC FORMULATION

Elements of the problem formulation have been presented previously [39]. Beginning with a formal definition of the acoustic energy [2,40], an amplification term, such that the energy increases with time t in proportion to $e^{\alpha t}$, can be identified as

$$\alpha = \left[\frac{- \int_A \langle p' u'_i \rangle dA_i - \int_A \langle e \bar{u}_i \rangle dA_i + \int_V \langle \Phi \rangle dV}{2 \langle \hat{e} \rangle V} \right]. \quad (31)$$

where p and u_i denote pressure and velocity, Cartesian tensor notation with the summation convention being adopted, dA_i is the outward-pointing surface area element of the chamber (which has a total surface area A), V is its volume, e represents the local instantaneous acoustic energy density, and Φ is the difference between the local instantaneous acoustic amplification and attenuation rate per unit volume. Notation adopted throughout will be that primes denote acoustic perturbations, overbars denote local average values, angular brackets denote a time average over an acoustic oscillation cycle, a caret denotes a volume average over the entire chamber, and a tilde denotes acoustic oscillation amplitude. Turbulent fluctuations are not identified explicitly in this formulation, it being convenient to include them implicitly as part of the unperturbed mean flow.

In Eq. (31),

$$e = \frac{p'^2}{2\gamma p} + \frac{\rho}{2} \frac{u'_i u'_i}{\rho} \quad (32)$$

and

$$\Phi = \frac{\gamma - 1}{\rho c} p' q' - \frac{\partial u'_i}{\partial x_j} \tau'_{ij} . \quad (33)$$

The three terms in the numerator of Eq. (31) describe respectively, the boundary work, the acoustic energy advected across the control surface by the mean flow and the net acoustic amplification rate, while the denominator is twice the acoustic energy in the chamber, the average acoustic energy per unit volume given by

$$\langle \hat{e} \rangle = \frac{1}{T} \int_0^T \frac{1}{V} \int_V e dV dt . \quad (34)$$

A simple, generic way of expressing acoustic pressure and velocity fluctuations in cylindrical coordinates in the absence of a radial dependence is

$$\begin{aligned} p'(x, \theta, t) &= \tilde{p} \cos(\omega t) \cos(kx) \cos(m\theta) \\ u'(x, \theta, t) &= \left(\frac{\tilde{p}}{\rho c} \right) \cos(\omega t + \varphi) \cos(kx + \ell) \cos(m\theta + \psi) \end{aligned} \quad (35)$$

where k and m are the wave numbers in the axial and circumferential directions respectively, while φ , ℓ , and ψ denote phase differences between the pressure and velocity perturbations. While radially dependent acoustic perturbations generally involve Bessel functions [2], they typically are of higher frequency and of lesser importance, and therefore when they are included they will be

approximated by additional sinusoidal factors in Eq. (35) dependent on the radius r . The phase angles appearing in Eq. (35) depend on the boundary conditions.

Several mechanisms make contributions to α in Eq. (31). Some contributions are positive, denoted by α_{amp} , and others are negative, and their magnitudes are denoted by α_{damp} . A stability index may then be defined as the ratio of the sum of amplification rates to the sum of attenuation rates,

$$S(\omega) = \frac{\sum \alpha_{amp}}{\sum \alpha_{damp}} , \quad (36)$$

This index may be used to examine all frequencies in the region of interest (typically around the acoustic natural modes of the geometry being studied). With this definition, $S(\omega) > 1$ indicates the possibility of oscillations while $S \leq 1$ indicates a neutral to damped system.

3. CHAPTER 3

MODELS FOR PERTURBATIONS OF THE RATE OF HEAT RELEASE

All terms in the numerator of Eq. (31) in general contribute to α and thereby to $S(\omega)$ of Eq. (36). The last term, however, is of major importance to gas-turbine oscillatory combustion. The first term, the boundary work, is of dominant importance for acoustic instabilities in solid-propellant rockets, but in gas turbines it is at most an attenuation term at the walls, if the walls are lined with Helmholtz resonators, for example. The second term, the difference between the advection of acoustic energy into the chamber at the inlet and the advection out at the exit, typically is not negligible locally, but the inward and outward advection tend to cancel, including turbulent contributions thereto, so that often the net contribution is not dominant. The third term, the difference between the net Rayleigh-criterion amplification and the homogenous acoustic damping according to Eq. (33), generally has components in each of these categories that cannot be neglected. Wall damping, for example, often is not negligible and has an influence described by a boundary-layer treatment of the last term in Eq. (33) [2], and damping by liners, which could be viewed as coming from either the first term in Eq.(31) or the last term in Eq.(33), generally always needs to be taken into account in practical combustors. The main amplification comes from the first term in Eq. (33), and therefore ultimately it requires relating the perturbations in the rate of heat release to the pressure perturbations. If only this term is retained, then Eq. (31) becomes

$$\alpha = \frac{(\gamma - 1)}{2 \bar{\rho} \bar{c}^2 \langle \hat{e} \rangle_V} \int_V \langle p' q' \rangle dV . \quad (37)$$

Different mechanisms for producing this kind of coupling are addressed here.

Pressure-Sensitive Reactors with Spatially Distributed Reactions Having Fixed Time Lags

One of the earliest models considered was the $n - \tau$ model, first introduced by Crocco and Cheng [21] in the 1950's during a period of research aimed at understanding oscillations in liquid-propellant rocket engines. The model starts from the observation that the rate of heat release, q , depends on local pressure, concentrations, and temperatures of reactants. Acoustic pressure fluctuations give rise to fluctuations in the rate of heat release, but it takes some time τ for the heat-release rate to respond to pressure because of finite-rate chemistry or transport. To describe this, the heat-release rate can then be taken as proportional to the pressure raised to a power (i.e. $q \propto p^n$), with the pressure evaluated at a time τ earlier, the value of which is locally fixed by the processes occurring at the point in question. Perturbing the resulting expression and linearizing the outcome gives what has been referred to as the $n - \tau$ model,

$$q' = n \bar{q} \frac{p'(t - \tau)}{\bar{p}} . \quad (38)$$

With this model the heat-release-rate perturbation at any particular position in the chamber may be considered to be proportional to pressure perturbations at the same point at an earlier time.

Pressure variations in acoustic fields are uniquely related to variations of the other state variables, temperature and density, but velocity variations have different phase relationships to pressure variations at any given position, depending on the acoustic mode and the boundary conditions. Therefore, in general, if pressures were replaced by velocities in Eq. (38), the outcome for q' would be different, and the value of α calculated from Eq. (37) would be changed. Models can be constructed in which q' responds to velocity fluctuations u' with a locally determined time lag τ , as will be illustrated later, but (with one important exception) such models tend to be unrealistic for homogenous reactors and are not addressed presently. To define the simplest model for a homogenous reactor, τ is taken to be constant throughout the reactor illustrated in Figure (3.1), and Eq. (38) may then be used in Eq. (37) to show that the corresponding contribution to α is

$$\alpha = \frac{\left[n(\gamma - 1) \frac{\bar{q}}{p} \right] \frac{1}{T} \int_0^T \int_V \frac{p'(t)p'(t-\tau)}{\rho c^2} dV dt}{2\langle \hat{e} \rangle V} . \quad (39)$$

In applying this result, values are needed for both n and τ . The simplest approach is to select constant values for each [39], such as $n=0.05$ and

$$\tau = \frac{L_{inj}}{U_{inj}} , \quad (40)$$

where L_{inj} is the distance from the fuel nozzle to the combustor entrance, and U_{inj} is the average flow velocity in the injector stream, illustrated schematically in Fig. 3.1. Depending on the combustor configuration, instead of employing a design-dependent τ , it

may be better to select a design-independent value based on expected heat-release time lags for the local combustion process.

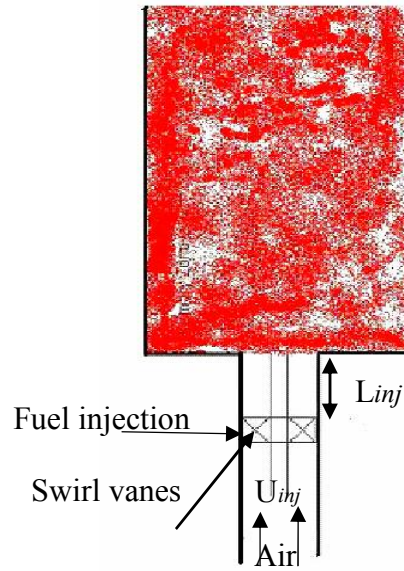


Figure 3.1: Schematic illustration of a homogenous reactor

Irrespective of how τ is determined, as long as it is a known constant, Eq. (39) can be used to calculate this contribution to α once the acoustic field is specified. With the description given in Eq. (35), use may be made of Eqs. (32) and (34) in Eq. (39) to show that

$$\alpha = \left[n(\gamma - 1) \frac{\bar{q}}{p} \right] \frac{\cos(\omega\tau)}{2} . \quad (41)$$

This is a simple but general result for this constant- τ limiting case of the homogenous model which, in turn, represents a limiting case in which the

combustion fills the entire chamber. When the values of the five parameters $n, \gamma, \tau, \bar{q}$ and $\bar{p}$ are known, Eq. (41) may be employed to calculate the associated contribution to α as a function of ω . This contribution may then be positive or negative, depending on the frequency as determined by the sign of $\cos(\omega\tau)$.

There may be interest in applying this model to a reactor in which combustion does not fill the entire chamber. For this extension a factor $f < 1$ must be placed in Eq. (41), where f denotes the ratio of pressure contribution to the acoustic energy over the portion of the chamber in which combustion occurs to the total pressure contribution to the acoustic energy in the chamber. The value of f may be calculated from an acoustic-mode analysis of the chamber, once the portion of the chamber in which combustion occurs is specified. As an especially simple example, for a chamber of length L in which the pressure oscillation is proportional to $\cos\left(\frac{\pi x}{2L}\right)$ and combustion occurs only in the region between $x = 0$ and $x = x_f$, it is found

that $f = \frac{2}{\pi} \left[\left(\frac{x_f}{L} \right) \sqrt{1 - \left(\frac{x_f}{L} \right)^2} + \sin^{-1} \left(\frac{x_f}{L} \right) \right]$. In most configurations a simple

formula like this cannot be obtained for f , and instead a numerical calculation of f is required.

Pressure-Sensitive Reactors with Spatially Distributed Reactions Having Space-Dependent Time Lags

If τ is not constant but instead varies in space, then the value of α obtained from Eq. (39) differs from the simple result in Eq. (41) and depends on both the shape of the acoustic mode and the spatial dependence of τ . In one model of this kind, τ may be assumed to vary linearly with the axial coordinate x through the chamber, the coefficient of variation being $\frac{1}{u_\tau} = \frac{d\tau}{dx}$, which may be positive or negative. By putting

$$\tau = \tau_o + u_\tau^{-1} x \quad (42)$$

in Eq. (38), with τ_o and u_τ both constant, it is found from Eqs. (34), (35) with $m=0$, and (39) that, if

$$k = \frac{\pi}{2L} \quad , \quad (43)$$

then

$$\alpha = \frac{n(\gamma-1)}{2\left(\omega \frac{L}{u_\tau}\right)} \frac{\bar{q}}{\bar{p}} \frac{1}{\left[1 - \left(\frac{\omega L}{\pi u_\tau}\right)^2\right]} \left\{ \left[1 - \left(\frac{\omega L}{\pi u_\tau}\right) - \left(\frac{\omega L}{\pi u_\tau}\right)^2\right] [\cos(\omega \tau_o)] \left[\sin\left(\frac{\omega L}{u_\tau}\right)\right] + \left[\sin(\omega \tau_o)\right] \left[2\left(\frac{\omega L}{\pi u_\tau}\right)^2 - 1 + \cos\left(\frac{\omega L}{u_\tau}\right)\right] \right\} . \quad (44)$$

Equation (43) corresponds to a chamber with an acoustically closed injector end at $x=0$ and an acoustically open end at $x=L$, for which Eq. (35) reduces to

$$\left. \begin{aligned} p'(x, t) &= \tilde{p} \cos(\omega t) \cos\left(\frac{\pi x}{2L}\right), \\ u'(x, t) &= \frac{\tilde{p}}{\rho c} \sin(\omega t) \sin\left(\frac{\pi x}{2L}\right), \end{aligned} \right\} \quad (45)$$

the values $m = 0$, $\psi = 0$, $\varphi = -\frac{\pi}{2}$ and $\ell = -\frac{\pi}{2}$ for example in Eq. (35) producing this result. The formulas are different with different boundary conditions.

The six parameters whose values must be given to use Eq. (44) for finding α as a function of ω are n , γ , τ_0 , $\bar{q}$, $\bar{p}$ and $\frac{u_\tau}{L}$. Although more complicated than Eq. (41), Eq. (44) yields a result which approaches that of Eq. (41) as $\frac{u_\tau}{\omega L}$ approaches infinity. For small values of $\frac{u_\tau}{\omega L}$, however, (high frequencies) the magnitude of the result in Eq. (44) becomes smaller by the factor $\frac{u_\tau}{\omega L}$ as a result of cancellation of contribution from different parts of the chamber. If combustion were to occur only in part of the chamber, then the integration in Eq. (39) would extend only over that part, but a formula as simple as Eq. (44) would not be obtained; in this case, depending on which part of the chamber experiences combustion, the resulting value of α may be decreased or increased.

Mode-dependent results also occur for models that introduce Lagrangian time lags which follow fluid elements. If the fluid is convected downstream, in the x direction, with the constant mean velocity U_c , and if the perturbations to the heat release at position x at time t result only from the pressure perturbation at position $x - U_c \tau_L$, at time $t - \tau_L$, then a space lag must be taken into account in

addition to the Lagrangian time lag τ_L (unless U_c is sufficiently small), causing p' in Eq. (38) to be evaluated at a different spatial position than q' . This result arises from the fact that, in this model, each fluid element moves a distance $U_c \tau_L$ before releasing heat. These complications in general would require the integrals in Eq. (39) to be evaluated numerically, with $p'(t - \tau)$ evaluated at a spatial position different than that of $p'(t)$. This would preclude the derivation of simple analytic formulas such as those given in Eqs. (41) and (44). A wide variety of possible models for reactors with distributed reactions thus exist, the results in Eqs. (41) and (44) representing two special simple limiting cases.

Pressure-Sensitive Reactors with Spatially Localized Reactions

The opposite limit from that of spatially distributed reactions is the limit of spatially localized reactions. It is worthwhile to consider this opposite limiting case, which is simpler to analyze than the general case. In applications, then, judgments can be made as to whether the combustion is approximated better by the distributed-reaction or localized-reaction model, and the result for the model judged to be closest to reality can be used. Alternatively, some components of both limits may be judged to be present, and the combustion may then be partitioned between the two.

The $n - \tau$ description underlying Eq. (38) may still be considered for the localized limit. Results for α in this limit, however, have a greater dependence on the chamber configuration and on the acoustic mode than does the result in Eq. (41). It is simplest to consider a one-dimensional configuration in which the

chamber, of length L , extends from $x=0$ to $x=L$, and a planar flame is anchored at the position $x = x_f$ as shown in Figure 3.2. In this case, Eq. (38) is modified to

$$q' = n \frac{\bar{q}}{\bar{p}} \frac{p'(t-\tau)}{\bar{p}} \delta(x-x_f) L. \quad (46)$$

For the acoustic field of Eq. (45), use of Eqs. (34), (35) and (46) in Eq. (37) results in

$$\alpha = n(\gamma - 1) \frac{\bar{q}}{\bar{p}} \left[\cos \left(\frac{\pi x_f}{2L} \right) \right]^2 \cos(\omega\tau) . \quad (47)$$

Application of Eq. (47) requires knowledge not only of the values of the five parameters n , γ , τ , $\bar{q}$ and $\bar{p}$, which were needed in Eq. (41), but also the value of $\frac{x_f}{L}$. The sign of α , however, again depends only of the value of $\omega\tau$.

The possible choices for τ in this formula parallel those in Eq. (41), although if the design-dependent τ is selected, then Eq. (40) would better be replaced by

$$\tau = \frac{L_{inj}}{U_{inj}} + \frac{x_f}{U_c} , \quad (48)$$

where U_c is the average flow velocity in the chamber. The method of determining the value of n depends on the model selected for the localized heat release. If, for the configuration corresponding to Eq. (45), a model of premixed turbulent flame propagation is selected, with the planar flame stabilized at $x = x_f$, then the average total rate of heat release in the chamber of volume V , with cross-sectional area A , is

$$\bar{q} V = (\overline{Q_r \rho S_T}) A \quad , \quad (49)$$

in which Q_r denotes the heat released per unit mass of mixture consumed and S_T is the turbulent burning velocity. With $q \propto \delta(x - x_f)L$, the corresponding perturbation is [41]

$$\frac{q'}{q} = \frac{Q_r'}{Q_r} + \frac{\rho'}{\rho} + \frac{S_T'}{S_T} \quad , \quad (50)$$

indicating that perturbations in Q_r , ρ and S_T all contribute to n in Eq. (46). The pressure dependences of these three parameters therefore affect n in this model. Expressions for n would be different in other models, and the simplest approach would be to merely select a value, as indicated above Eq. (40).

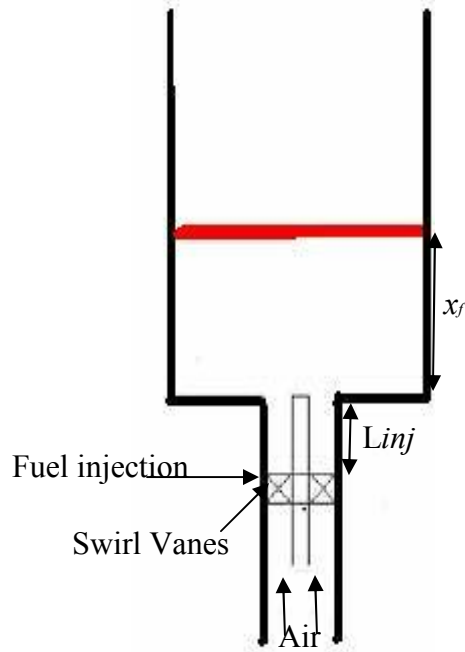


Figure 3.2: Schematic illustration of an anchored flame at a fixed location

Velocity-Sensitive Reactors with Spatially Localized Reactions

For localized reactions, there are more physically reasonable models with velocity-sensitive responses than there are for distributed reactors. Results necessarily will be configuration-dependent and mode-dependent, and for the configuration that led to Eq. (46), that equation becomes

$$q' = n \bar{q} \frac{\gamma u'(t - \tau)}{c} \delta(x - x_f) L, \quad (51)$$

where in Eq. (38) $\frac{p'}{p}$ is replaced by $\frac{\gamma u'}{c}$, u being the velocity in the x direction.

This last selection is better than $\frac{u'}{u}$ because in acoustics it is $\frac{\gamma u'}{c}$ rather than

$\frac{u'}{u}$ that is of the same order of magnitude as $\frac{p'}{p}$.

Equation (37) still applies for this contribution, and for the mode considered, which has $\langle \hat{e} \rangle = \frac{|\tilde{p}^2|}{(2\gamma \bar{p})}$, Eqs. (5) and (21) result in

$$\alpha = \frac{n(\gamma - 1)}{4} \frac{\bar{q}}{p} \sin\left(\frac{\pi x_f}{L}\right) \sin(\omega\tau), \quad (52)$$

which differs from Eq. (47) in that $\left\{ \left[\cos\left(\frac{\pi x_f}{2L}\right) \right]^2 \cos(\omega\tau) \right\}$ is replaced

by $\left\{ \frac{1}{4} \sin\left(\frac{\pi x_f}{L}\right) \sin(\omega\tau) \right\}$ as a consequence of the pressure sensitivity being

replaced by the velocity sensitivity. Although the results are generally similar, the amplitudes are somewhat less for the same value of n (the maximum possible

value being $\frac{1}{4}$ of the previous maximum), and the phase difference is altered, amplification now peaking at $\omega\tau = \pi/2$ instead of at $\omega\tau = 0$.

Flamelet-Area Fluctuations

In the turbulent-flame model of Eqs. (49) and (50), explicit velocity sensitivity can arise for the flamelet regime. In this regime, as is well known [2],

$$S_T A = S_L A_f, \quad (53)$$

where S_L is the laminar burning velocity and A_f is the wrinkled flame area, and the wrinkled flame area may be considered to be proportional to the velocity fluctuations, whence

$$\frac{S_T'}{S_T} = \frac{S_L'}{S_L} + \frac{A_f'}{A_f} = \frac{S_L'}{S_L} + \frac{u'}{u}. \quad (54)$$

The only velocity-dependent term that arises when Eq. (54) is substituted into Eq. (50) is

$$\frac{q'}{q} = \frac{u'}{u}, \quad (55)$$

which, in Eq. (51), corresponds to

$$n = \frac{\bar{c}}{\gamma \bar{u}}, \quad (56)$$

which is a large number even if $\bar{u}$ is taken to be the mean convection velocity. In many models, the $\bar{u}$ in Eq. (55) should instead be the root-mean-square turbulent

fluctuation velocity, which causes n in Eq. (56) to be even larger. Equation (55), however, assumes that the acoustic velocity fluctuations are fully effective in increasing the wrinkled flamelet area. In many models only the turbulent velocity fluctuations increase the flamelet area, and the acoustic fluctuations may have a small or negligible effect on the turbulent fluctuations. A correction factor that is generally small therefore belongs on the right-hand side of Eq. (55), resulting in a corresponding small factor in Eq. (56). This model nevertheless can lead to responses having magnitudes larger than those associated with pressure-sensitive responses. Clearly there are many uncertainties concerning the magnitude of n for this model.

The appropriate time lag τ for this model can be of the order of a large-eddy turnover time,

$$\tau = d / \bar{u} \quad , \quad (57)$$

where d is a large-eddy diameter and $\bar{u}$ a representative turbulent fluctuation velocity, resulting in a numerical value that may often be comparable with values obtained from Eq. (40). As with n , however, there are significant uncertainties in expressions and values for τ .

Since most gas-turbine conditions tend to lie in the thin-reaction-zone regime rather than in the flamelet regime, this potentially large effect may not be representative for most applications. In the thin-reaction-zone regime, Eq. (53) does not apply, but instead the turbulent burning velocity is proportional to the square root of a turbulent diffusivity which if proportional to u' would produce a

factor of $\frac{1}{2}$ in Eq. (56) and would again require a small multiplicative factor if the acoustics are ineffective in generating turbulent diffusivity fluctuations.

Equivalence-Ratio Fluctuations

A different mechanism that has been identified as often being an important candidate for a velocity-sensitive response concerns equivalence-ratio fluctuations. When the fuel and air flow rates of the injector respond differently to acoustic oscillations, the instability generates local fluctuations of the equivalence ratio, which are convected downstream to the turbulent flame, where they produce fluctuations in the heat-release rate.

Specifics depend on details of the injection process, but in many cases the fuel feed pressure is high enough that the injector can be considered to be choked, giving a constant flow rate, while the air flow rate past the injector responds to the acoustic oscillations. If the air flows in the x direction at velocity u at the injector, then the corresponding fluctuations in the equivalence ratio, ϕ , are given by [37]

$$\frac{\phi'}{\phi} = -\left(\frac{u'}{u} + \frac{\rho'}{\rho}\right), \quad (58)$$

in which the local density fluctuations at the injector are usually of a lesser importance and can be neglected, giving a velocity-sensitive response with the time lag of Eq. (48). Since

$$\frac{q'}{q} = \left(\frac{dq}{d\phi} \frac{\bar{\phi}}{q}\right) \frac{\phi'}{\phi}, \quad (59)$$

with this mechanism, in Eq. (21)

$$n = \left(\frac{dq}{d\phi} \right) \left(\frac{\bar{\phi}}{\bar{q}} \right) \frac{1}{\gamma M_{inj}} \quad , \quad (60)$$

where the air Mach number at the injector is

$$M_{inj} = \frac{U_{inj}}{c_{inj}} \quad , \quad (61)$$

and $u'(t - \tau)$ is to be evaluated at the injector, leading to p' and q' being evaluated at different spatial locations in Eq. (37).

That this can be a large effect is evident from the presence of the Mach number in the denominator in Eq. (60). It does, however, become necessary to address the acoustic velocity fluctuation u' in the injection region, and this can be small, offsetting the influence of the Mach number. For example, for the simple model configuration considered in deriving Eq. (44), if the injection is at $x=0$, then $u'=0$ there, and equivalence-ratio fluctuations (which also can be called mixture-ratio or mixture-fraction fluctuations) occur only through the density fluctuations (related to the pressure fluctuations), which have been neglected as being small. If the acoustic field is given by Eq. (45) and the injection of fuel is at $x = x_{inj}$, then use of Eqs. (51) and (60) in Eq. (37) gives

$$\alpha = - \left[\frac{(\gamma - 1)}{2 \gamma M_{inj}} \left(\frac{dq}{d\phi} \frac{\bar{\phi}}{\bar{q}} \right) \frac{\bar{q}}{\bar{p}} \right] \left[\cos \left(\frac{\pi x_f}{2 L} \right) \sin \left(\frac{\pi x_{inj}}{2 L} \right) \right] \sin (\omega \tau) \quad . \quad (62)$$

Comparison of Eq. (62) with Eqs. (47) and (52) shows that if the fractional change in the rate of heat release with the fractional change in the equivalence ratio is of order unity, then this effect can be relatively large. It produces amplification for

$0 < \omega\tau < \pi$ (attenuation for $\pi < \omega\tau < 2\pi$) when $\frac{\pi x_{inj}}{2L}$ is small and negative and attenuation for $0 < \omega\tau < \pi$ when it is small and positive. Expressions will be different for other modes or other configurations.

To use this result numerically, it is necessary to evaluate $\frac{dq}{d\phi}$. For the

flamelet regime, from Eqs. (49), (50) and (54) it may be seen that

$$\frac{dq}{d\phi} \frac{\bar{\phi}}{q} = \frac{dQ_r}{d\phi} \frac{\bar{\phi}}{Q_r} + \frac{dS_L}{d\phi} \frac{\bar{\phi}}{S_L} + \frac{d\rho}{d\phi} \frac{\bar{\phi}}{\rho} + \frac{dA_f}{d\phi} \frac{\bar{\phi}}{A_f} \quad , \quad (63)$$

while for the thin-reaction-zone regime the last term in this approximation is replaced by a corresponding fractional change in diffusion coefficient. In both regimes, the first two terms are dominant and may be evaluated from thermodynamic data and from data on laminar burning velocities.

Turbulent diffusion can be an important phenomenon in modifying influence of equivalence-ratio fluctuations. The mixture non-uniformities introduced at the injector tend to be washed out by diffusion processes as a fluid element flows downstream. This phenomenon has been addressed previously[42,43] by models more complex than the simple estimate to be adopted here. If Eq. (57) is employed as an estimate of a decay time τ_d for equivalence-ratio fluctuations, then a result such as Eq. (62) requires an additional factor of $e^{-\frac{t_L}{\tau_d}}$, taken here as a reasonable approximation accounting for this effect, where t_L is the space-dependent Lagrangian transit time for the fluid element, given for example by Eq. (48). This factor accounts for the reduction in

amplitude of the equivalence-ratio fluctuations by diffusion and causes the effect to diminish as the distance downstream increases.

Velocity-Sensitive Reactors with Spatially Distributed Reactions

The equivalence-ratio fluctuations just described also can be important when there are distributed reactions. This would involve effectively a spatially dependent time lag, as in Eq. (42), and it would give, for the configuration to which Eq.(62) applies, the formula

$$\alpha = -\frac{(\gamma-1)}{\pi\gamma M_{inj}} \left(\frac{dq}{d\phi} \right) \frac{\bar{q}}{\bar{p}} \frac{\sin\left(\frac{\pi x_{inj}}{2L}\right)}{\left[1 - \left(\frac{2\omega L}{\pi u_i}\right)^2\right]} \quad (64)$$

$$\left\{ \cos\left(\frac{\omega L}{u_\tau}\right) \sin(\omega\tau_0) - \left(\frac{2\omega L}{\pi u_\tau}\right) \left[1 - \sin\left(\frac{\omega L}{u_\tau}\right)\right] \cos(\omega\tau_0) \right\} ,$$

in which τ_0 and u_τ may best be obtained from Eq. (48). For small values of $\frac{\omega L}{u_\tau}$,

this result is quite similar to that of Eq. (62), the difference being that

$\cos\left(\frac{\pi x_f}{2L}\right)$ is now replaced by $\frac{2}{\pi}$, but for large values of this parameter the

magnitude becomes smaller by a factor $\frac{u_\tau}{\omega L}$, as in Eq. (44). If combustion were to

occur only in part of the chamber, then again the integral in Eq. (37) would extend

only over that part, precluding a result as simple as that in Eq. (64) from being

derived and, in general, requiring numerical integration for evaluating α . The

limiting case in which the distributed combustion is mainly localized towards the

upstream end of the chamber would correspond to Eq. (62) with $\cos\left(\frac{\pi x_f}{2L}\right) = 1$.

Equation (64) does not take into account the reduction in amplitude of equivalence-ratio fluctuations by diffusion. When this effect is included, the formula becomes more complicated than Eq. (64), and α is best calculated by numerical integration. The effect can be very substantial, lessening the influence of the downstream portion of the chamber greatly and thereby removing most of the destructive interference effects that lead to rapid variations of α with ω .

Vortex Shedding

Vortex shedding is basically a nonlinear phenomenon, and a linear approximation is needed to apply the present approach. The viewpoint to be adopted is that vortices are shed from an object with a characteristic dimension d , a rod diameter (as in the case of fuel spokes in an injector, e.g. shown in Fig.3.3) or a boundary-layer thickness at an edge, and with a characteristic length L_v , the length of the rod or of the edge. The gas entrained in the vortex burns at a time τ after the vortex is shed, where τ may, for example, be considered to be an induction time for ignition, and the heat is released over a period of time comparable with the vortex turnover time. The shedding frequency ω in the absence of the acoustic field is determined by the Strouhal number,

$$St = \frac{\omega d}{U_c}, \quad (65)$$

a representative value of which is about 0.2 [44].

If this were all that were involved, then there would be no contribution to acoustic amplification because there would be no coupling between the phase of the shedding and of the acoustics. Coupling is provided by the fact that the

shedding frequency increases with the velocity, as implied by Eq. (65). The velocity increase associated with the acoustics promotes shedding, so that shedding can be triggered in the high-velocity portion of the acoustic oscillations, thereby synchronizing the shedding with the acoustic field. This coupling clearly is nonlinear since it depends on the magnitude of the acoustic velocity, and the shedding phase thus will be a function of the acoustic amplitude. Moreover, by modifying the shedding frequency through Eq. (65), the nonlinearity can lead to synchronization of shedding over a range of frequencies.

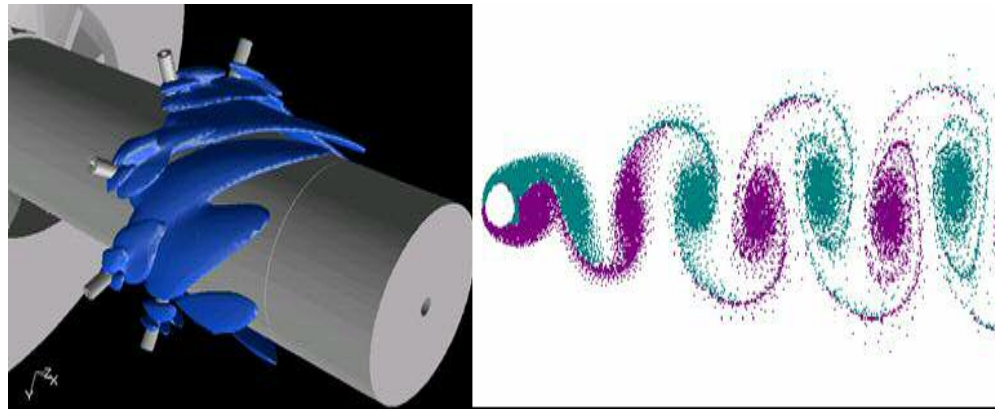


Figure 3.3: Vortex shedding from the fuel spokes in a swirl premixed injector.

Since the acoustics, however, under combustor operating conditions of greatest practical interest, may be expected not to make a large change in the shedding, as an approximation it may be assumed that interaction will occur only over frequencies differing by a small factor r , perhaps about 10%, from the natural shedding frequency of Eq. (65) for the mean velocity U_c . The frequency dependence of the magnitude of the fluctuation in the heat-release rate per unit volume is therefore taken to be

$$|q'| = |q'|_{\max} \left[1 - \left(\frac{\omega - \omega_0}{r \omega_0} \right)^2 \right] \quad (66)$$

for $(1-r)\omega_0 < \omega < (1+r)\omega_0$ and 0 otherwise, where ω_0 is the value obtained from Eq. (65) with U_c being the average velocity at the shedding point. Equation (66) is preferred over a Gaussian, for example, because there is no interaction if the frequencies are too different.

To use Eq. (66), it is necessary to evaluate $q'_{\max}$, the value that occurs when the acoustic and shedding frequencies coincide. The value of q' may be approximated as being zero outside the vortex and constant inside. If the vortex is approximated as having diameter d and length L_v , and if the circulation is estimated as $U_c \left(\frac{U_c}{\omega} \right) = U_c^2 d / St$, then its turnover time, d^2 divided by its circulation, is $d St / U_c$, so that

$$|q'|_{\max} = \frac{Q_r \rho U_c}{d St} . \quad (67)$$

If x_v denotes the value of x at which the vortex is shed, then Eqs. (66) and (67) give

$$q'(x_v + U_c \tau, t) = \frac{Q_r \rho U_c}{d St} \left[1 - \left(\frac{\omega - \omega_0}{r \omega_0} \right)^2 \right] \frac{u'(x_v, t - \tau)}{0.1 U_c}, \quad (68)$$

where the magnitude of the velocity fluctuations has been approximated as 10% of the mean velocity. The final factor in Eq. (68) couples the heat-release fluctuations with the acoustic-triggered shedding. Equation (68) applies over a

volume of diameter d and length L_v centered at position $x_v + U_c \tau$, and q' is zero outside this volume, in this approximation.

For the configuration and acoustic conditions of Eq. (45), with the rod or edge normal to the flow direction, use of Eq. (68) in Eq. (37) results in

$$\alpha = \frac{\pi (\gamma - 1) d L_v Q_r}{0.8 St V c} \left[1 - \left(\frac{\omega - \omega_0}{r \omega_0} \right)^2 \right] \sin(\omega \tau) \sin\left(\frac{\pi x_v}{2 L}\right) \cos\left(\frac{\pi (x_v + U_c \tau)}{2 L}\right), \quad (69)$$

provided that the vortex diameter is small compared with the wavelength of the acoustic field. If there are more than one rod or edge in the system, then Eq. (69) is to be applied separately for each, the sum contributing to α . Since the derivation neglects vortex growth through entrainment during its transit, if $U_c \tau$ is large enough then an entrainment factor, perhaps $1 + 0.1 U_c \tau / d$, may appropriately be included in Eq. (69). The result predicts amplification if $x_v > 0$ for $0 < \omega \tau < \pi$ when ω is close enough to ω_0 (α is zero for $|\omega - \omega_0| > r \omega_0$), and the amplification increases with increasing ratio of vortex volume to chamber volume.

Pressure-Sensitive Cylindrical Reactors with Conical Anchored Flames

This case is between two limiting cases discussed earlier (the distributed-reaction limit and the spatially-localized-reaction limit) and is more appropriate for some applications. The flame in this example is anchored between two axial and radial locations, (x_{f1}, r_{f1}) and (x_{f2}, r_{f2}) as shown in Figure 3.4.

The modeling challenges for this case are similar to those of the spatially localized flame, in that it may be difficult to determine the exact location of the reaction zone. The heat release is assumed to occur only at the flame sheet, and the heat-release rate is written as

$$q' = n \bar{q} \left[\frac{p'(t - \tau)}{\bar{p}} \right] \delta(\eta) \frac{R^2 L \sin \beta}{(r_{f2}^2 - r_{f1}^2)} \quad (70)$$

where η is a coordinate perpendicular to the flame sheet defined by

$$\eta = (r - r_f) \cos \beta - (x - x_f) \sin \beta \quad (71)$$

obtained by transforming the axial (x) and radial (r) coordinates into a system perpendicular to and parallel to the flame sheet. The heat-release perturbation is taken here to be proportional to the pressure perturbations occurring at the flame location, evaluated at a time τ earlier, as in Eq. (46). It clearly also would be possible to address, in this geometry, velocity-sensitive heat release, as in Eq. (51), and the result from equivalence-ratio fluctuations is to be discussed below.

The use of Eqs. (70) and (71) in Eq. (37) gives

$$\alpha = n(\gamma - 1) \frac{\bar{q}}{\bar{p}} \left(1 + \frac{[\cos^2(\xi_2) - \cos^2(\xi_1)] + [(\xi_2 - \xi_0) \sin(2\xi_2) - (\xi_1 - \xi_0) \sin(2\xi_1)]}{[(\xi_2 - \xi_0)^2 - (\xi_1 - \xi_0)^2]} \right) \cos(\omega\tau), \quad (72)$$

where $\xi = \frac{\pi x_f}{2L}$, and x_{f0} is the point where the extrapolated flame meets the x axis

(forming an angle β). Following Putnam [6] in Eq. (72) one may consider putting

$$\tau = \frac{L_{inj}}{U_{inj}} + \frac{x_{f1}}{U_c} + \frac{x_{f2} - x_{f1}}{3U_c} \quad (73)$$

as an approximation in the spirit of Eq. (48), although the resulting value for τ often is too large to produce reasonable results, so that a smaller, design-independent value is preferred. The growth-rate results obtained by this model tend to fall between the two limits discussed before and can be positive or negative depending on the frequency.

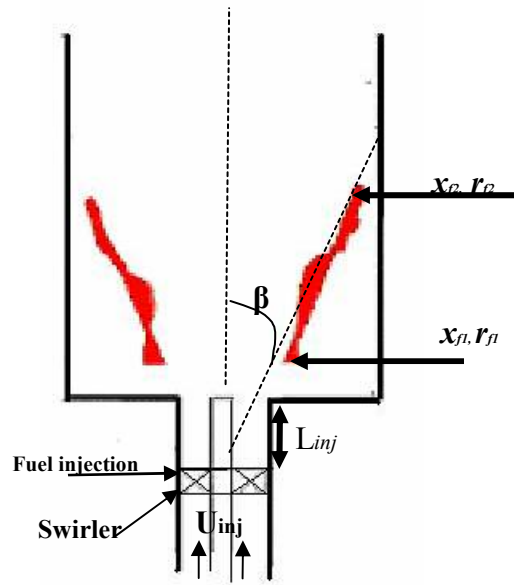


Figure 3.4: Schematic illustration of an anchored conical flame

Velocity-Sensitive Cylindrical Reactors with Conical Anchored Flames

If the heat-release perturbations in the above example are assumed to be sensitive to velocity perturbations only, then in the spirit of Eq. (51) these perturbations can be approximated by

$$q' = n \bar{q} \left[\frac{\gamma u'(t - \tau)}{\bar{c}} \right] \delta(\eta) \left(\frac{R^2 L \sin \beta}{(r_{f2}^2 - r_{f1}^2)} \right). \quad (74)$$

For equivalence-ratio fluctuations the u' in Eq. (74) is evaluated at the injector location, and the growth rate for a variable time lag, which is more physically plausible because combustion occurs along the flame sheet, can be expressed analytically if turbulent diffusion is neglected, although the formula is complicated, even more so than Eq. (72), so that it is simpler to calculate α numerically from the formula

$$\alpha = \left[\frac{(\gamma - 1)}{2\gamma M_{inj}} \left(\frac{dq}{d\phi} \frac{\bar{\phi}}{q} \right) \frac{\bar{q}}{p} e^{-\tau_0/\tau_d} \sin(\beta) \sin(\xi_{inj}) \right] \left(\frac{2L}{\pi R} \tan(\beta) \right)^2 \left\{ \begin{aligned} & \sin(\omega \tau_0) \int_{\xi_1}^{\xi_2} (\xi - \xi_0) e^{-c\xi} \cos(\xi) \cos(b\xi) d\xi + \\ & \cos(\omega \tau_0) \int_{\xi_1}^{\xi_2} (\xi - \xi_0) e^{-c\xi} \cos(\xi) \sin(b\xi) d\xi + \end{aligned} \right\}, \quad (75)$$

in which $b = \frac{2\omega L}{\pi u_\tau}$, $c = \frac{2L}{\pi u_\tau \tau_d}$, $\tau_0 = \frac{L_{inj}}{U_{inj}}$, $u_\tau = U_c$ and $\tau_d = \frac{d}{u}$. In this

expression, the reduction in amplification through turbulent diffusion has been included, d being the size of the large eddies and $\bar{u}$ the turbulent fluctuation velocity.

More Complex Configurations

Equation (75) involved performing a numerical integration along the length of the flame. The amplification rate, α , in general can be evaluated by numerical integration when the necessary information is available, and in most complex configurations numerical integration is required. In flame-sheet

approximations the integration is carried over the area of the sheet, and in distributed-reaction approximations the integration is carried over the volume of the chamber. The later situation is the most general in that a flame sheet can always be approximated as a flame of nonzero thickness, and q' can be taken to be large inside this thickness and zero outside. The information that is required for determining α should be clear from Eq. (37).

First of all, it is necessary to know the average acoustic energy per unit volume defined in Eq. (34). Being proportional to the square of the pressure amplitude, $\tilde{p}^2$, this formally involves knowing the amplitude of the acoustic fluctuations, but in fact that is irrelevant since $\tilde{p}^2$ also occurs in the numerator of Eq. (37) and so cancels out from α . In other words, the amplitude can be assigned an arbitrary value at a selected position. What is needed for calculating $\langle \hat{e} \rangle$ by numerical integration therefore is only the shape of the acoustic mode under consideration. This should be available from chamber-acoustic programs, so that $\langle \hat{e} \rangle$ can be calculated for the given amplitude.

The only other factor in α in Eq. (37) is the ratio of the volume integral (over the entire chamber) of the amplification rate per unit volume to the chamber volume. This ratio also can be evaluated by numerical integration (for the selected amplitude), given the acoustic mode and the information concerning the amplification mechanism that determines q' . This information is, of course, quite different for different mechanisms. Equation (38) provides the simplest example. Time lags often are involved, and Eq. (42) often will be a reasonable

approximation, in which x becomes the distances along a particle path and u_τ the particle velocity along the path. If this velocity varies appreciably in the model, then $u_\tau^{-1}x$ is better replaced by $\int u_\tau^{-1}dx$. To complete the specification of q' , either the amplitude factor n is simply given, or a formula such as Eq. (50), Eq. (59) or Eq. (68) is used, with quantities appearing there given, for example, by Eq. (54) or Eqs. (58) and (63).

Depending on the mechanism, the shape of the acoustic mode will appear in different ways in the evaluation of q' for performing the integrations. The integration over the volume is, however, straightforward once the acoustic mode, the mechanism and the values of the associated parameters are known. The computational times will vary depending on the geometry and mechanisms of choice but are much shorter than the times required for approaches implementing computational fluid dynamics, although the later, of course, give nonlinear behaviors, which the present approach is incapable of doing.

Because the present approach is only an approximate one, it is consistent with the spirit of the method to address various additional simplified approximations of the acoustic field, the chamber and the flame configurations, the physical mechanisms of the heat-release perturbations and the time lags. One prevalent design configuration is an annular chamber with a number N of axially directed injectors distributed uniformly around the annulus. Acoustic modes observed in such units often are circumferential, so that Eq. (35) can be applied with $k = \ell = 0$. If interactions of flames from different injectors, tendencies of

flames to spread conically and distributed-reaction effects are neglected, then the flame from each injector can be approximated by a circular cylindrical sheet of radius r_f , centered on the injector axis. Moreover, r_f may be considered small enough that the phase of the acoustic field for the entire flame of injector k may be approximated as the phase of the azimuthal angle θ_k of the center of that injector. With these simplifications, the analog of Eq. (46) for pressure-sensitive reactors becomes

$$q' = n \frac{\bar{q}}{p} \sum_{k=1}^N p'(\theta_k, t - \tau) \delta(r_k - r_f) \frac{L(R_o - R_i)}{N r_f}, \quad (76)$$

where L is the length of the annular combustor, R_o and R_i are its outer and inner radii, respectively, and r_k is a radial coordinate from the axis of the injector k , appearing in the argument of the delta function.

If Eq. (42) is employed for τ , then use of Eq. (76) in Eq. (37) gives, for the tangential mode considered,

$$\alpha = \frac{(\gamma - 1)n \bar{q} u_\tau}{\omega N \bar{p}(R_o + R_i)} \left[\sum_{k=1}^N \cos^2(m \theta_k) \right] \left\{ \cos(\omega \tau_0) \sin\left(\frac{\omega L}{u_\tau}\right) - \sin(\omega \tau_0) \left[1 - \cos\left(\frac{\omega L}{u_\tau}\right) \right] \right\} \quad (77)$$

where m is an integer ($m=1$ for the fundamental mode). In the limit of $u_\tau = 0$, so that τ is constant, the braces in Eq.(77) approach $\frac{\omega L}{u_\tau} \cos(\omega \tau_0)$. With these

same geometrical approximations applied to equivalence-ratio fluctuations, Eqs. (59) and (63) continue to hold, and

$$\alpha = \frac{B(\gamma-1)n\bar{q}u_\tau}{\omega N \bar{p}(R_o + R_i)} \left\{ \sum_{k=1}^N \cos(\psi) \cos^2(m\theta_k) - \sin(\psi) \cos(m\theta_k) \sin(m\theta_k) \right\} \left[\cos(\varphi) \left[\cos(\omega\tau_0) \sin\left(\frac{\omega L}{u_\tau}\right) - \sin(\omega\tau_0) \left\{ 1 - \cos\left(\frac{\omega L}{u_\tau}\right) \right\} \right] + \sin(\varphi) \left[\sin(\omega\tau_0) \sin\left(\frac{\omega L}{u_\tau}\right) + \cos(\omega\tau_0) \left\{ 1 - \cos\left(\frac{\omega L}{u_\tau}\right) \right\} \right] \right] \right\}, \quad (78)$$

in which the phase angles ψ and φ are those that appear in Eq.(35), and B , less than unity but often of order unity, denotes the ratio of the axial velocity fluctuations in the injector to the tangential velocity fluctuations in the chamber.

For a traveling wave, $\psi = \varphi = 0$, while a standing wave can be described by

putting $\varphi = 0$ and $\psi = -\frac{\pi}{2}$, for example, in both cases eliminating the

$\sin(\varphi)$ term in Eq.(78), and essentially reverting to Eq.(77) in the first case and replacing $\cos^2(m\theta_k)$ in Eq.(47) by $\cos(m\theta_k)\sin(m\theta_k)$ in the second case.

These results neglect turbulent dissipation, and although analytic results could be obtained including that effect, numerical evaluation of the integral over x is just as easy.

4. CHAPTER 4

MODELS FOR ATTENUATION OF ACOUSTIC ENERGY

Acoustic-energy dissipation occurs through a variety of mechanisms in practical combustion systems, the most common of which are wall damping and damping by perforated liners that target certain acoustic modes. Other mechanisms of acoustic attenuation are generally less significant and have been addressed reasonably thoroughly in the literature [2,37]. These other mechanisms therefore are not addressed here.

Wall Damping

This damping is due to the oscillatory boundary layer near the wall, and it depends on the frequency, the surface area and the flow properties near the wall. Detailed analysis of wall damping mechanisms is present in several references [2,40,45]. An expression is derived by Williams [2] starting from a simplified time-dependent momentum equation (with t time and y distance normal to the wall), namely

$$\rho_w \frac{\partial v}{\partial t} = \frac{\partial \left(\mu_w \frac{\partial v}{\partial y} \right)}{\partial y}, \quad (79)$$

where ρ_w , μ_w and v are the density, viscosity and velocity near the wall. This equation has a solution

$$v = \left(\frac{\theta_w}{\theta_c} \right) \text{Re} \left\{ V e^{-(1+i)\sqrt{\frac{\omega\rho_w}{2\mu_w}}y} \right\}, \quad (80)$$

where θ_w, θ_c, V and ω are the temperature at the wall, the temperature in the core, the complex oscillatory velocity magnitude outside the oscillating boundary layer, and the frequency of the oscillation respectively.

From this velocity solution, an energy-dissipation expression, (per unit area of combustor wall), can be derived as

$$e_{wd} = \int_0^\infty \frac{1}{2} \rho_w \omega \left[\frac{\theta_w}{\theta_c} V e^{-(1+i)\sqrt{\frac{\omega\rho_w}{2\mu_w}}y} \right]^2 dy = \frac{1}{2} \left(\frac{\theta_w}{\theta_c} \right)^2 |V|^2 \sqrt{\frac{\omega\rho_w\mu_w}{2}}, \quad (81)$$

This solution then needs to be integrated over the wall boundary surfaces to obtain the total rate of dissipation by the wall damping

$$\alpha_{wd} = \frac{\int_A e_{wd} dA}{2eV}. \quad (82)$$

Damping by Perforated Liners

Perforated liners are common in gas-turbine applications; their primary use is the provision of cooling to prevent severe temperatures from damaging the hardware. This cooling is provided by air impinging on the liner external walls and/or by film cooling to the interior walls. The latter can contribute significantly to acoustic-energy attenuation, as has been explained by Howe [45,46], Hughes and Dowling [47] and Eldredge and Dowling[48]. The physical mechanism for the removal of acoustic energy is vortex shedding at the edge of

the apertures through which there is a mean flow rate. The vortices are removed away from the liner by the mean flow and eventually decay to turbulence.

If the cavity depth (distance from the perforated liner to the backed wall) is larger than the aperture diameter, and if both of these dimensions are much smaller than the acoustic wave length, then, the acoustic properties of an aperture can be expressed as[47]

$$\frac{\partial p'}{\partial y} = \eta [p']_{y=-0}^{y=+0} \quad \text{at } y = 0 , \quad (83)$$

where y denotes distance normal to the aperture pointing into the chamber, and η is an effective compliance found from an expression developed by Howe [46] based on the Rayleigh conductivity for an aperture and can be expressed as

$$\eta = \frac{2 a \chi}{d^2} , \quad (84)$$

in which a is the aperture radius and d the distance between the centers of the apertures shown in Figure 4.1. Here

$$\chi = \lambda - i\delta , \quad (85)$$

where

$$\lambda = \frac{I_1^2(St) \left[1 + \frac{1}{St} \right] + \frac{4}{\pi^2} e^{2ka} \cosh(St) K_1^2(St) \left[\cosh(St) - \frac{\sinh(St)}{St} \right]}{I_1^2(St) + \frac{4}{\pi^2} e^{2St} \cosh^2(St) K_1^2(St)} \quad (86)$$

,

and

$$\delta = \frac{\left[\frac{2}{\pi St} \right] I_1(St) e^{2St} K_1(St)}{I_1^2(St) + \frac{4}{\pi^2} e^{2St} \cosh^2(St) K_1^2(St)}, \quad (87)$$

in which $I_1(St)$ and $K_1(St)$ denote modified Bessel functions of the first and second kind[49]. The Strouhal number is defined here as

$$St = \frac{\omega a}{U}, \quad (88)$$

with U being the mean gas velocity through the aperture.

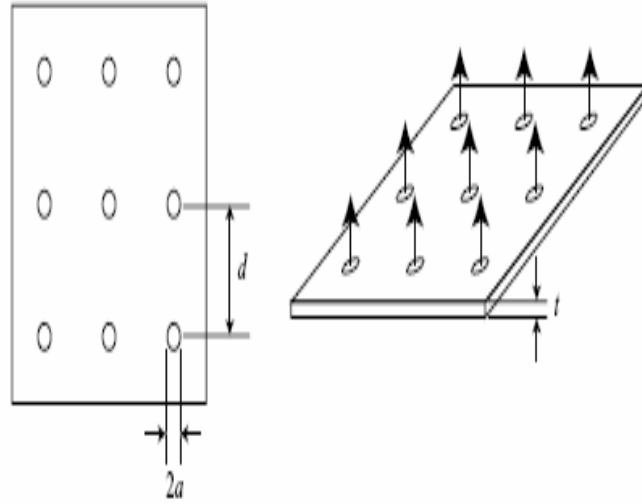


Figure 4.1: Schematic illustration of a perforated liner sheet [47]

An expression for the attenuation rate can be developed by looking at the acoustic admittance (the inverse of acoustic impedance). Following Williams [2], if the acoustic admittance is defined as $y = \frac{\tilde{u}}{\tilde{p}}$, in which $\tilde{u}$ refers to the complex amplitude of the average outward normal velocity, then an average rate of acoustic

energy extraction can be expressed as $\overline{p} \langle p' u' \rangle . n = \left[\frac{p'^2}{2} \right] \text{Re} \{y\}$. A

nondimensional measure of the admittance can be derived [2] as $Y = y \frac{\overline{p}}{c}$, in

which $\frac{\overline{p}}{c}$ gives the characteristic impedance of the internal gaseous medium.

Upon employing the above formulations in Eq. (31), it follows that the contribution of the perforated liner to the damping is

$$\alpha_{p\ell} = \frac{\gamma c}{2} \frac{\int_A |p(x_i)|^2 \text{Re} \{Y\} dA}{\int_V |p(x_i)|^2 dV} \quad . \quad (89)$$

Furthermore if we define a Helmholtz number as

$$\hbar = \frac{\omega \cdot l}{c} \quad , \quad (90)$$

where l is the distance from the aperture to the backing wall, and introduce

$$\beta = \frac{\omega \cdot d^2}{2 a c} \tan(\hbar) \quad , \quad (91)$$

then use of the results of Hughes and Dowling[47], given in the first part of their paper, results finally in

$$\alpha = \frac{\int_A |p(x_i)|^2 \left[\frac{\beta \delta \tan(\hbar)}{\beta^2 + \lambda^2 + \delta^2 - 2\beta\lambda} \right] dA}{\int_V |p(x_i)|^2 dV} \quad . \quad (92)$$

5. CHAPTER 5

COMPARISON WITH EXPERIMENTAL RESULTS

Experiments in four different, increasingly more complex types of combustor configurations are addressed here as means for testing the usefulness of the method that has been proposed. While most of the experiments were performed as part of the present work, some results are taken from the existing literature. The experiments include both atmospheric-pressure and high-pressure rigs and address both single-injector chambers and chambers with full annular sets of injectors. Wide ranges of conditions thus are represented. The approach will be to investigate how well the experimental results can be interpreted on the basis of the preceding models.

A Single-Injector, Atmospheric-Pressure, Quartz Combustor

Experiments were performed with a transparent combustor for the purpose of testing the utility of the method being developed here. The intent was to observe under what conditions oscillatory combustion occurred and to attempt to determine which of the preceding models, if any, could lead to rational interpretations of the experimental results. The experiments were performed with an experimental single-injector quartz combustor system as shown in Figure 5.1. The configuration consisted of a lean premixing fuel injector feeding a dump combustor. The fuel injector is typical of Solar's design with dual flow circuits (main and pilot). In this study, the pilot circuit was shut off. Mixing is provided

by physical placement of the fuel injection points and by the confined swirling flow. A 20" long and 8" diameter quartz liner was used as the dump combustor for ease of flame visualization. The flame was stabilized in the recirculation zone formed downstream by the sudden expansion and by the swirl breakdown.

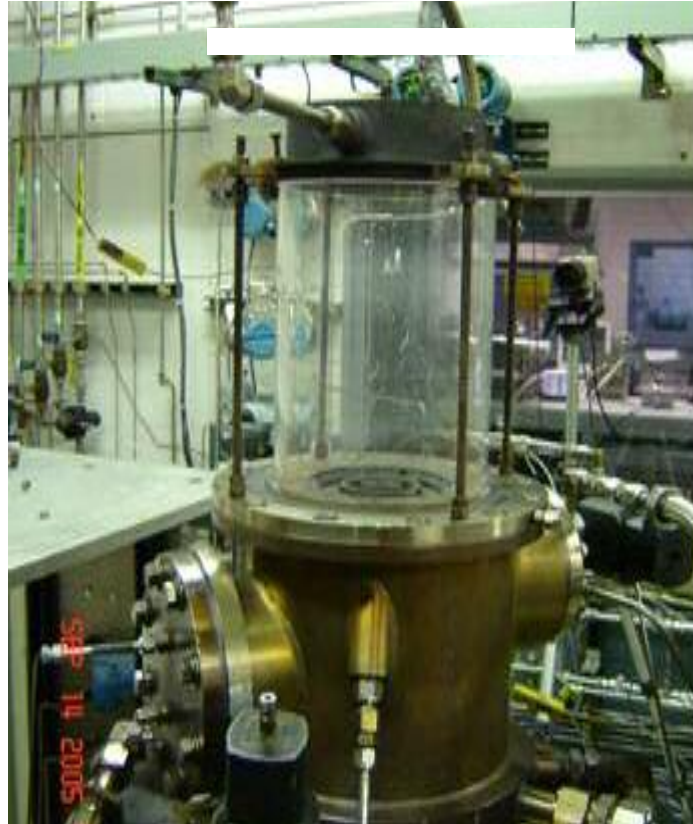


Figure 5.1: Quartz rig picture shows the 8" diameter quartz combustor (with a height of 10"), the plenum and the exit plane of the premixer. The actual quartz liner used was 20" in height.

Flow Conditions

Air was preheated to 283°F. Airflow was varied between 0.12-0.17 lbs/second and the fuel flow was varied between 20-24 lbs/hour. Fuel and air premixing was

provided by the lean premixing fuel injector. The degree of fuel and air premixing was fairly uniform in both the radial and circumferential directions. The uniformity was not measured quantitatively, and the qualitative assessment of uniformity was based on flame observations and previous experience with the premixer. The premixed reactants were then delivered to the combustor, ignited and the combustor was maintained at atmospheric pressure.

Instrumentation

Pressure oscillations were detected using piezoelectric type transducers and microphones mounted at

- The plenum, the probe was mounted on a tee off a 6"long, $\frac{1}{4}$ " diameter, tube with a semi-infinite copper coil to minimize reflection of the acoustic wave [50].
- The combustion chamber, the piezoelectric transducer was mounted on a tee off of a $\frac{1}{4}$ " tube.
- On a stand at 15"height and 10" away in the radial direction from the combustor walls. This helped verify the readings from the two piezoelectric transducers.

The dynamic pressure signal was captured and processed with a spectrum analyzer for frequency and amplitude analyses.

Flame oscillations were also recorded by using videography employing a high-speed camera as well as a standard-speed CCD camera. Images are shown here to illustrate the changes in flame shapes during stable and unstable operations.

Experiment Results

Oscillating frequencies were detected around 300, 250 and 227 Hz at slightly different flow conditions. Elsewhere the combustor was fairly quiet, except for low background noise (due to room acoustics and electrical noise). Figure 5.2 shows an image of a stable flame at the described flow conditions. The flame was observed to extend $\sim 30\%$ of the chamber length, anchored at an angle $\sim 25^\circ$ to the centerline.



Figure 5.2: Stable flame conditions ($W_f=20.39$ lbs/hour, $W_a=0.15$ lbs/s and $T_{adiabatic}=2766^\circ\text{F}$).

Figure 5.3 shows an image during a 250 Hz oscillation. The flame becomes almost flat in shape. This is a drastic change in the flame shape, volume and area from a well-anchored flame. The flame goes through quick cycles of what seems to be extinction and re-ignition at the oscillation frequency (as observed from images taken using a high-speed camera and were not included in this paper).



Figure 5.3: Flame oscillation at 250 Hz ($W_f=22.98$ lbs/hour, $W_a=0.15$ lbs/s and $T_{\text{adiabatic}}=3005^\circ\text{F}$).

Figure 5.4 shows the fast Fourier transform for the dynamic pressure signal during the oscillation frequency. The dominant frequency is 250 Hz with amplitude of ~ 0.07 psi. Smaller peaks indicating electrical noise at 60 and 180 Hz, not related to the combustion oscillation, were also detected, there are even smaller peaks, having amplitudes about 10% of the maximum peak, the nature of which was not identified.

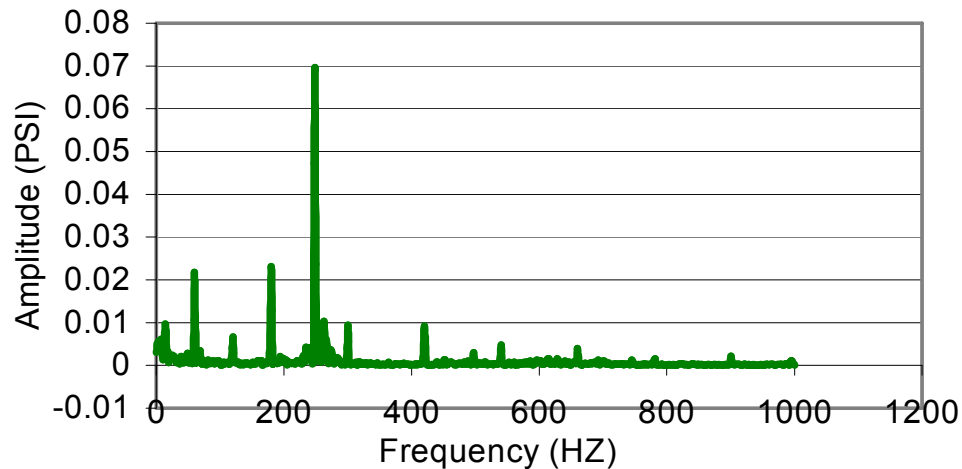


Figure 5.4: FFT of pressure oscillations at 250 Hz ($W_f=22.98$ lbs/hour, $W_a=0.15$ lbs/s and $T_{\text{adiabatic}}=3005^\circ\text{F}$).

The vertically oriented combustion chamber, 8" in inside diameter and 20" long, was open to the exhaust system at the top and fed from the bottom by a lean mixture of air and natural gas, injected through a model single-element swirl premixer about 2" in diameter, having 20 fuel spokes. The air, preheated to 283°F , entered the premixer at 0.12 to 0.17 lbs/s from a cylindrical plenum, coaxial with the combustor and acoustically isolated from it by annular plates, so that the acoustic field in the combustor could reasonably be approximated by Eq. (45). The fuel flow rate was between 0.005 and 0.007 lbs/s, and the pressure in the combustor was maintained at normal atmospheric. Oscillations were detected by measuring pressure variations, monitored through piezoelectric transducers and microphones, and by visual, high-speed-camera and CCD-camera observations.

The fundamental acoustic frequency in the chamber during combustion was 250 Hz. At the lower fuel flow rates oscillatory combustion did not occur, and the flame appeared to be approximately conical, as illustrated in Figure 3.4.

Oscillations at 250 Hz were recorded at the higher fuel flow rates, and under oscillatory conditions on the average the flame appeared to fill approximately the entire lower portion of the combustor.

Results for α based on different models which can be used in attempts to interpret these observations appear in Eqs. (41), (44), (47), (52), (62), (64), (69), (72) and (75), and possible variations of these results have been discussed previously in connection with the equations. One of the first questions is to ask whether a model can predict amplification at the 250 Hz frequency. Some indeed can, as may be seen from results for Eqs. (41), (64) and (72) plotted in Fig. (4). The results for Eqs. (31) and (72) include modifications for the effect of wall damping from Eq.(82), as described previously[39]. Even without turbulent dispersion taken into account to diminish effects of equivalence-ratio fluctuations, the destructive interference from the variable time lag of Eq. (48) eliminates amplification from Eq. (64) for this distributed-reaction model. The pressure-sensitive result for the simple distributed-reaction model of Eq. (41) is seen to produce amplification. It is more appropriate, however, from the flame-shape observations, to address the conical-flame model of Eq.(72), and while the second amplification region for this curve does not exactly coincide with the observed frequency, the inaccuracies are such that the model could well predict amplification where observed. It seems better to base this linear stability analysis on the conical flame rather than the distributed reaction because that is the configuration which is seen prior to the onset of oscillations, so that is the configuration which would have to be linearly unstable.

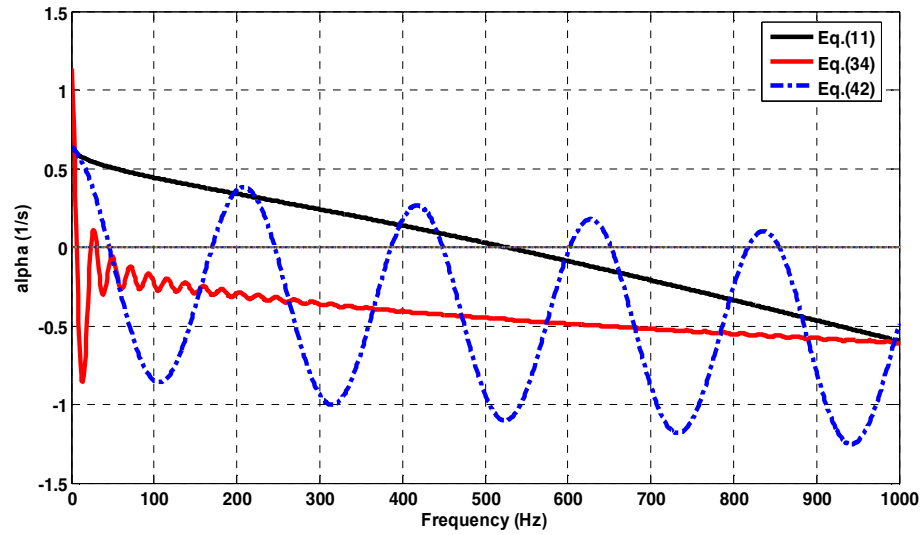


Figure 5.5: Growth-rate results obtained from Eqs. (11), (34) and (42)

with a fixed $n=0.05$

It is widely believed that equivalence-ratio fluctuations tend to be the primary cause of oscillations. For the conical flame, this effect is described most accurately by Eq. (75), the prediction of which is plotted in Figure 5.6 and Figure 5.7, along with the vortex-shedding prediction of Eq. (69). The values of parameters employed for these calculations were 0.5 cm for d , 0.95 cm for L_V , 5 cm for $L_{inj} = -x_v$, 0.63 for ϕ , 14.5 m/s for U_{inj} , 1.8 m/s for U_c , 10% of U_c for $\bar{u}$ and 1.4×10^7 J/kg for Q_r in Eq.(79), and 2.54 cm for x_{f1} , 17.8 cm for x_{f2} , 4.78 cm for r_{f1} , 10.16 cm for r_{f2} and 3.4 cm for d , along with other parameters having the same values as above, in Eq.(75). In evaluating ξ_{inj} , the positive value $x_{inj} = L_{inj}$ was employed, it being clear that the phase of the oscillations at the injector is the same as that near the upstream end of the chamber and

L_{inj} representing the order of magnitude of the displacement away from the velocity node. It is seen from the figures that the equivalence-ratio fluctuations for the conical fame do indeed provide a small amplification effect at the observed frequency, and although the vortex-shedding effect is larger, its potential frequency range is too high, unless U_c is taken to be unreasonably small or d unreasonably large. The small wiggles of the curve in Figure 5.6 arise from interference of the amplification effects of different parts of the conical flame.

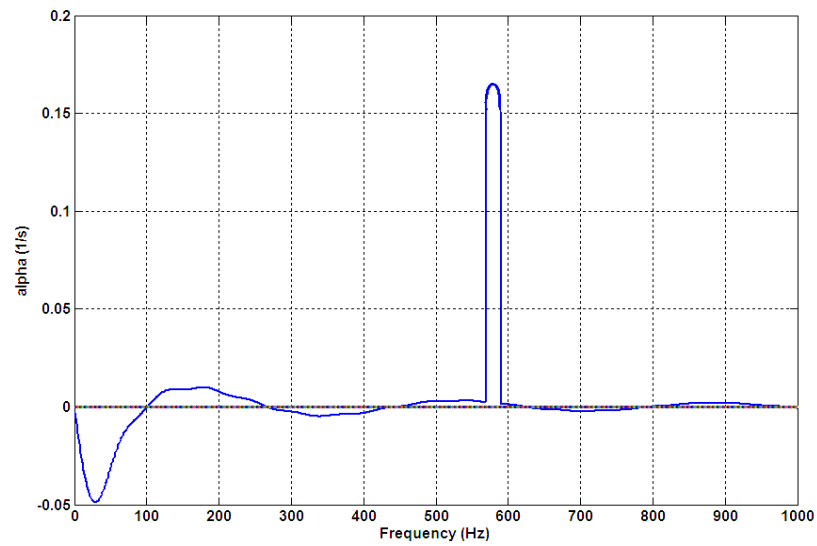


Figure 5.6: Growth-rate results obtained from Eqs. (39) and (45)

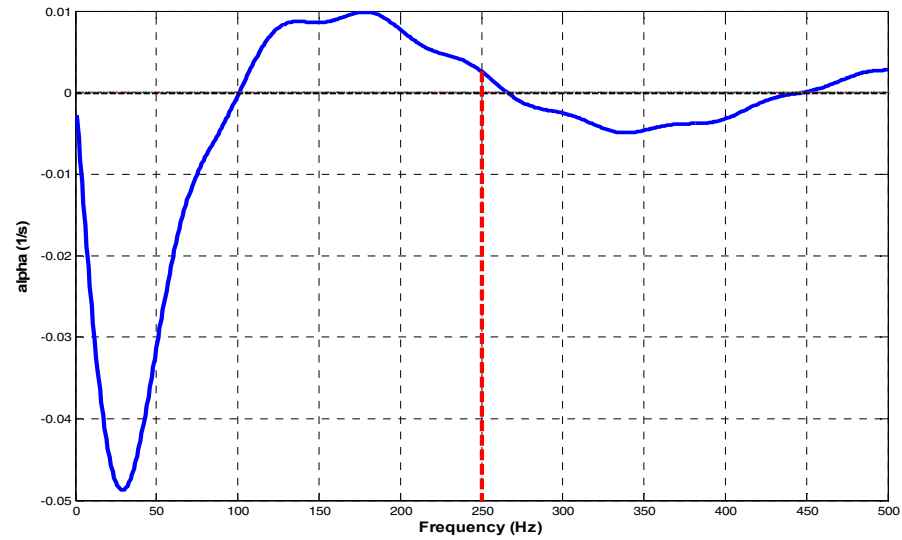


Figure 5.7: Growth-rate results (zoomed near 250 Hz) from effects of equivalence-ratio fluctuations, as obtained from Eq. (45)

These exercises in testing the predictions of different mechanisms against the present experimental results serve to provide experience in ascertaining how useful the various models may be. Predictions of the vortex-shedding model depend strongly on the shedding frequency, which is seen from Eq. (75) to increase with increasing velocity and with decreasing size of the elements from which the shedding occurs. The shedding frequencies estimated here (as well as for the experiments discussed later) are too high for this phenomenon to be significant; they would correspond to a higher mode of oscillations, for which damping effects would be stronger, or they would operate only over a frequency range too narrow to correspond to an acoustic mode. Even though there are uncertainties in estimating sizes, very large elements would be needed to reduce the frequencies into the range observed, yet after oscillation develops vortex

shedding may be involved in the limit cycle. A general aspect of the predictions of all of the other mechanisms is that the magnitude of the effect tends to increase with increasing fuel flow rate. This occurs because over the range of equivalence ratios tested, the only parameter that varied significantly with increasing fuel flow rate was the heat release, expressed per unit mass of mixture through Q_r in Eq. (79) and per unit volume of mixture through $\bar{q}$ in the other equations. The other mechanisms thus are all consistent with the experimental observation that combustion oscillations occurred only at the higher fuel flow rates, provided that the mechanism does predict amplification rather than attenuation at the frequency of interest.

The tentative explanation of the experimental results is that fluctuations of the equivalence ratio induced the oscillations through the amplification seen in Figure 5.6. In this same experimental apparatus, tunable diode lasers were employed to detect oscillations in methane concentration at the injector outlet, and equivalence-ratio fluctuations were established to exist [51]. There is, however, some uncertainty as to whether they were sufficiently large to drive the instability. The values of the parameters employed in using Eq.(75), which correspond to $n=2.94$ in Eq.(74), are seen from Figure 5.7 to provide growth times as long as a few minutes, even though damping phenomena other than turbulent dispersion were not taken into account. The experiences during the experiment do not rule out growth times that long, and moreover, uncertainties in values of parameters are large enough that predicted growth times could be appreciably shorter.

This underscores the rough nature of the general approach proposed here and the possibility that a priori predictions by this method will be inconclusive, especially in marginal cases, which therefore might best be avoided in design selections. An observation that can be made on the basis of the present theoretical work is that in the present tests, if the experimental conditions had been closer to lean blowout, then other parameters, such as the burning-velocity variation in Eq.(63), could have been larger, leading to oscillations being associated with decreasing (rather than increasing) fuel flow rate, if complete blowout could be avoided.

Single-Injector High-Pressure Combustors

Since practical gas turbines operate at elevated pressures, efforts were made to study measurements made in experiments like those of the quartz combustor but instead run at high pressures. An extensive program of such measurements has been completed [52-54] on a setup sketched in Figure 5.8. The inside diameter of the cylindrical combustor, 7.8", was quite close to that of the quartz combustor, and the preheated air was supplied at 600°F. Chamber pressures in the tests ranged from 5 to 10 atm and equivalence ratios of the natural gas from 0.59 to 0.77. An adjustable exhaust plug allowed the fundamental longitudinal frequency to be changed, but most results were obtained with this frequency between about 200 and 250Hz, comparable to that of the quartz combustor. The fuel and air entered the combustor coaxially through an annular premixer with swirl vanes and fuel spokes, having inner and outer diameters of 1.5" and 2.5",

respectively. Because of the exhaust plug and the characteristics of the premixer, a better approximation than Eq. (45) to the fundamental acoustic field is

$$\left. \begin{aligned} p'(x, t) &= \tilde{p} \cos(\omega t) \sin\left(\frac{\pi x}{2L}\right), \\ u'(x, t) &= \left(-\frac{\tilde{p}}{\rho c}\right) \sin(\omega t) \cos\left(\frac{\pi x}{2L}\right), \end{aligned} \right\} \quad (93)$$

corresponding to a pressure node at the chamber entrance and a velocity node at the chamber exit. This leads to modifications in the formulas for α , for example replacing $\cos(\omega\tau)$ by $\sin(\omega\tau)$ in Eq. (41).

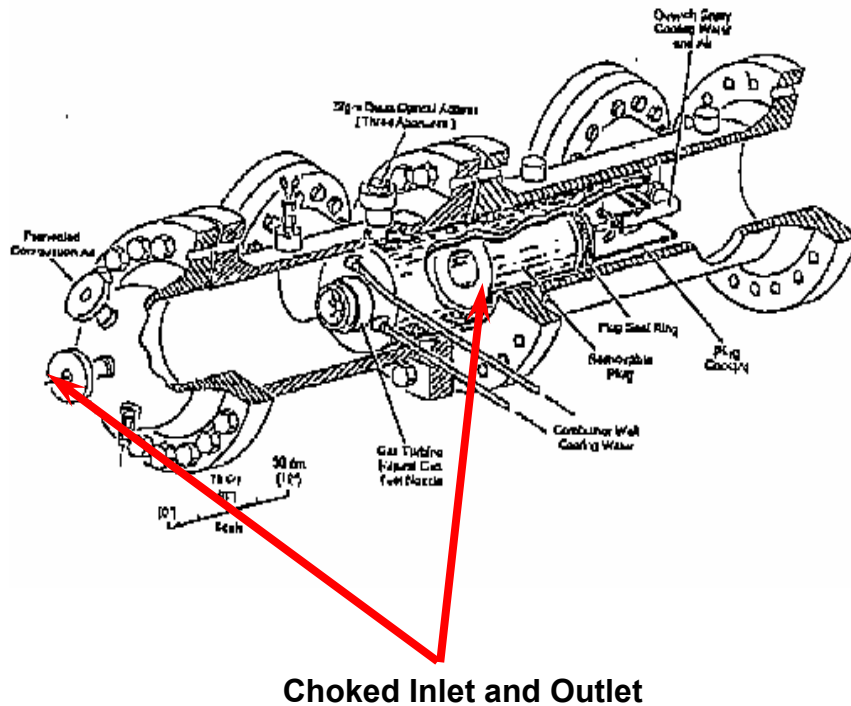


Figure 5.8: Single Injector High Pressure Rig [52].

The primary results of measurements that were reported in the references were amplitudes and frequencies of pressure oscillations as functions of operating

conditions. Extensive variations of parameters were made, especially in the premixer configuration and fuel-injection pattern. There are, however a number of irregularities in the results, such as the observation of different amplitudes on different days, oscillations at apparently sub-acoustic frequencies under certain conditions, and differences in effects of equivalence ratios under different conditions, over the range of equivalence ratios tested. Many of these irregularities are likely attributable to nonlinearities that are not addressed in the present analysis. Since no systematic effect of equivalence ratio or pressure could be extracted from the results over the range tested, it is assumed here that the results are representative of an average condition, 7.5 atm and an equivalence ratio of 0.65. The main variable that clearly did affect the oscillation amplitude was the mean velocity in the premixer. In general, amplitudes were large at injection velocities of 40m/s and below and (except at sub-acoustic frequencies) small at injection velocities of 50 m/s and above. The task of the present analysis therefore is to develop an interpretation for this effect of injection velocity.

Just as for the quartz combustor, it is assumed that high-amplitude oscillations will develop only if the combustor with the flame under nonoscillating conditions is linearly unstable. In this experiment, however independent information on the shape of this flame is not available. It nevertheless seems reasonable to assume a conical flame shape, since swirl-stabilized flames often are conical. This focuses attention on Eqs.(72) and (75), although it also was of interest to investigate predictions for distributed reactions, such as Eq.(64). The results for constant time lags, as in Eq. (41), etc., seem likely to be less relevant

for this experiment. The predictions from Eqs. (64) and (72) did not provide a very good correspondence to the experimental results. The results for Eq. (75), however, are much more promising, as may be inferred from Fig. (13) to be explained below. This figure was obtained employing the values 4.8 cm for $L_{inj} = -x_v$, 0.65 for ϕ , 20-70 m/s for U_{inj} , 0.22-0.77 m/s for U_c , 10% of U_c for $\bar{u}$, 1.4×10^7 J/kg for Q_r , 2.5 cm for x_{f1} , 17.0 cm for x_{f2} , 4.5 cm for r_{f1} , 10.3 cm for r_{f2} and 3.5 cm for d , again with ξ_{inj} positive and $x_{inj} = L_{inj}$. The interaction index n was obtained from Eq. (60) by use of Eq. (63), in which the contribution to n from the laminar burning velocity was 3.0, that from the heat release was 0.9, and that from the other terms was negligible. The agreement to now be described is consistent with the prevailing viewpoint that equivalence-ratio fluctuations were responsible for the oscillatory combustion observed in these experiments.

Figure 5.9 shows the amplification rate evaluated from Eq. (75) in a plane of frequency and injection velocity. To aid in visualizing regions where amplification may occur, a plane is passed through the three-dimensional figure at zero amplification rate, and only regions above that plane are highlighted. In general, there are mountain chains of amplification that peak at frequencies which increase with increasing injection velocities. This is due to the decrease in the time lag τ_0 with increasing injection velocity. At low injector velocities, the fundamental longitudinal frequencies of the combustor fall within the first mountain chain. This first mountain chain passes to higher frequencies at an injection velocity of about 50 m/s, which is where the large pressure amplitudes

are observed experimentally to disappear. This calculation therefore provides a potential interpretation of the experimental observation. Explanations based on other previous equations do not work as well. For example, the vortex-shedding result of Eq. (69) gives amplification only for frequencies that range from about 400 Hz at an injection velocity of 20 m/s to about 1400 Hz at an injection velocity of 70 m/s, all much higher than the observed frequencies. Stronger damping mechanisms at these higher frequencies are likely to prevent oscillations from occurring there. Of all of the present results, therefore, Eq. (75) is most attractive for these experiments, as well as for the experiments with the quartz combustor.

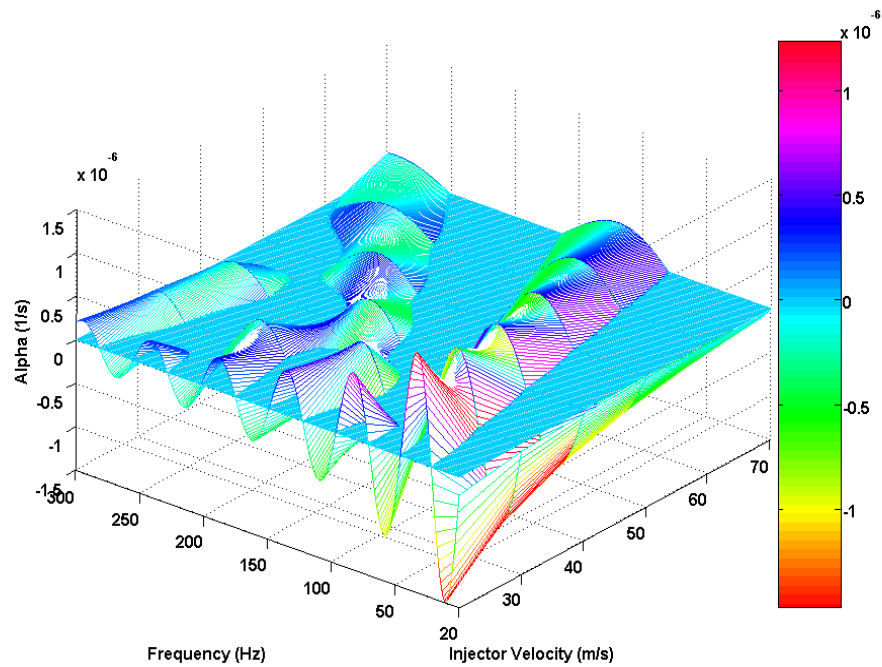


Figure 5.9: Growth-rate results obtained from Eq. (75) (at a fixed pressure of 7.5 atm and equivalence ratio of 0.65) for the high-pressure experiments, as a function of injection velocity and frequency

A Twelve-Injector, Atmospheric-Pressure, Annular Combustor

To investigate more complex configurations, measurements were made on an annular combustor having twelve production premixed injectors, equally spaced around the annulus and directed axially downstream. The combustor, which had an effusion-cooled liner, operated at normal atmospheric pressure in a test rig that had a 1 ft space, open to the laboratory, between the combustor exit and the exhaust system that deflected the exhaust gases upward. Although the setup, which has been used to test various injector designs, was equipped with four rakes to collect temperature profiles at four different angles across the open area and a video camera facing forward to view the flame, the main data were obtained from twelve pressure transducers, one mounted on each injector, and from visual observation of the flame, which could be seen by peering upstream at an angle through the open area. The piezoelectric transducers, dynamic data from which were acquired by an Alta-Solutions Spectra-Shield box, were mounted in tees off the spuds of the pilot fuel line, seen schematically in Figure 5.10 and Figure 5.11. The transducers were connected to 50 ft copper coils in efforts to minimize acoustic interference.

The annular combustor was 0.38 m long, 0.26 m in inner radius and 0.34 m in the outer radius. Each injector, of outer diameter 0.1 m and shown schematically in Figure 5.10, had 10 swirl vanes equipped with 8 fuel-injection holes each, an inner radius of 0.04 m and an axial distance of 0.05 m between the fuel-injection point and the injector's interface with the combustion chamber.

The flow rates were selected to keep the equivalence ratio close to 0.63 where oscillations could be observed. The air flow rate, through a preheater providing air at 283°F, was fixed at 3.07 lbs/sec and the fuel flow was varied between 140 and 199 lbs/hour, with oscillations observed to occur near 194 lbs/hour, the values for which data are addressed here. Pilot flow rates from about 2% to 10% were explored, and oscillations were observed at the lower pilot flow rates, 2% to 4%, as expected. Measurements were made at 4% pilot flow rates, for which representative pressure amplitudes are shown in Figure 5.12. The dominant frequency observed, 250 Hz, is consistent with a first fundamental circumferential mode, calculated from $\frac{\bar{c}}{(2\pi R)}$ (where R is the arithmetic mean of the radius of the annulus, and $\bar{c}$ = 826 m/s) to be 247 Hz.

Relative amplitudes and phases of the twelve different pressure signals were obtained from Spectra Shield software in typically ten snapshots recorded at 3 Hz. The phase results, within the accuracy of the measurements, were consistent with an assumption that the combustor experienced a standing circumferential wave; there were very little variations of relative phases between different snapshots for most transducers, and indicated phase differences were small, most within 20° and all within 70°, these differences corresponding to time differences on the order of 1 ms and likely attributable to small differences in acoustic path lengths and time synchronization for different transducers. The relative amplitude data, shown in Figure 5.13, are also consistent with the standing fundamental circumferential wave (shown by a curve in the figure), within the accuracy of the data. In this figure, the angle $\theta = 0$ has been adjusted to coincide with the location

of the torch employed for ignition, the presence of which may anchor the velocity node.

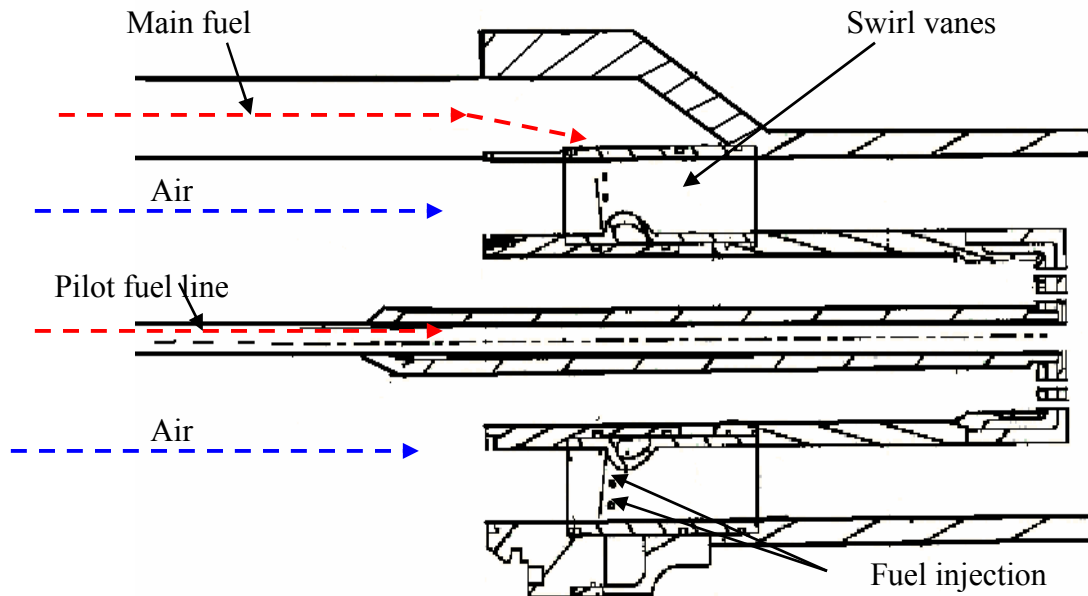


Figure 5.10: A schematic diagram of the type of production injector used in the experiments

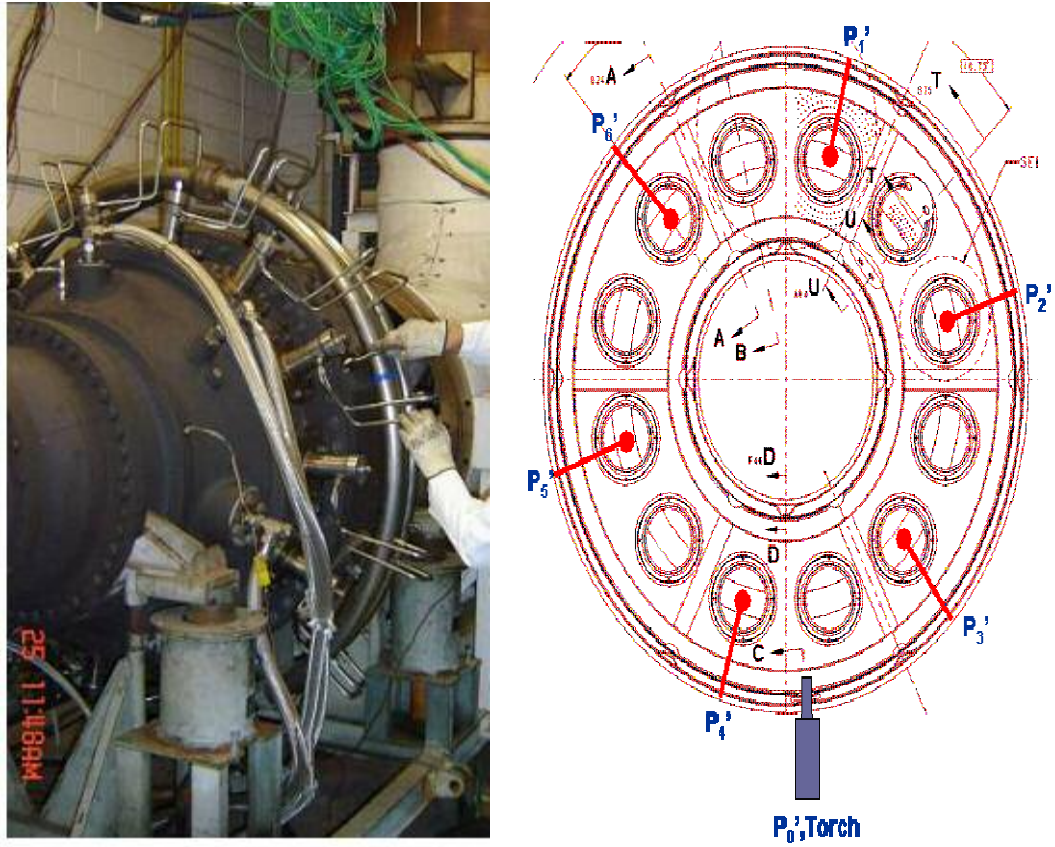


Figure 5.11: Pressure Measurements on the Annular Atmospheric Test Cell
(courtesy of Solar Turbines Inc.)

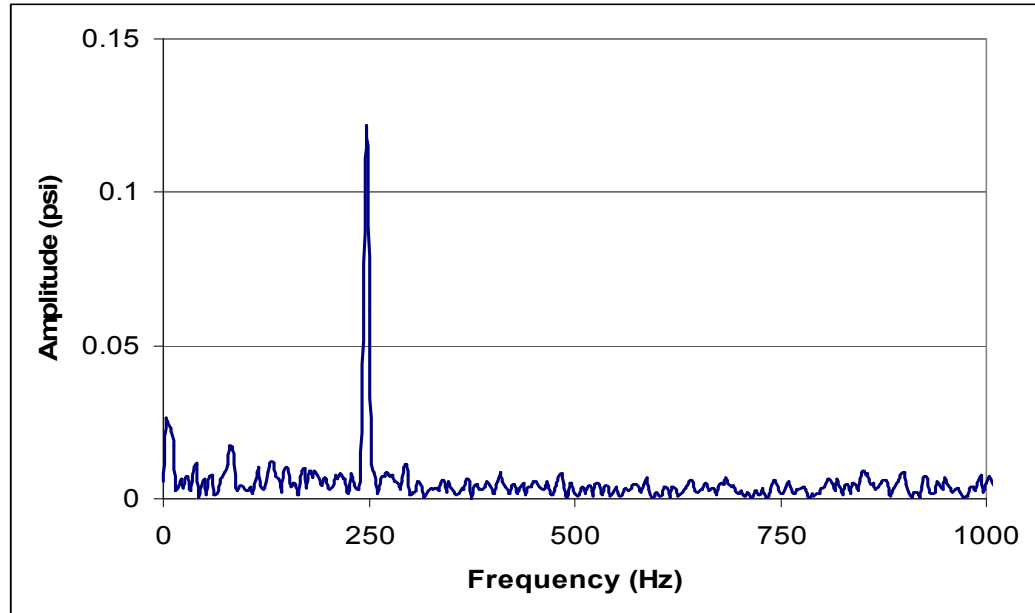


Figure 5.12: Dynamic pressure measurements during oscillation for an annular combustor at atmospheric-pressure conditions

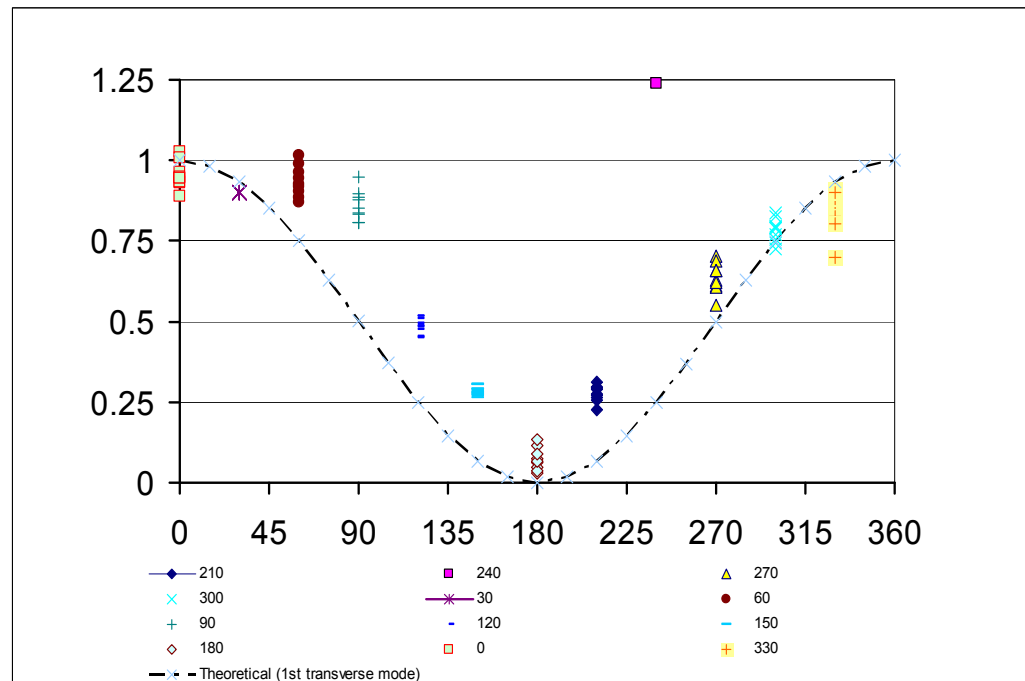


Figure 5.13: Measured relative pressure amplitudes as functions of circumferential angle

For the assumed mode, from Eq.(35), the acoustic field is given by

$$\left. \begin{aligned} p'(\theta, t) &= \tilde{p} \cos(\omega t) \cos(\theta) \\ u'(\theta, t) &= \frac{\tilde{p}}{\rho c} \sin(\omega t) \sin(\theta) \end{aligned} \right\} \quad (94)$$

where u' is now the component of velocity in the θ direction. To estimate amplification, the flame-shape approximations that led to Eq.(76) are adopted. Visual observation suggested that the flame shape may best be approximated by solid cones, but within the accuracy of the present crude approach, differences from the assumed hollow cylindrical shape are not likely to affect predictions substantially. Consideration of possible amplification mechanisms led to the belief that, just as in the previous two types of experiments, mixture-ratio variations seemed likely to contribute the main source. With this in mind, a velocity-sensitive model was addressed for which

$$q' = n \frac{\gamma \bar{q}}{c} \sum_{\kappa=1}^N u'(\theta_{\kappa}, t - \tau) \delta(r_{\kappa} - r_f) \frac{L(R_o - R_i)}{N r_f}. \quad (95)$$

Although u' here is the acoustic velocity in the circumferential direction, it is assumed that this same velocity is transmitted locally to the axial flow in the injector, thereby generating the equivalence-ratio oscillations. Axial velocity fluctuations at twice this frequency also may be expected, but for the standing wave there also should be a contribution at this fundamental frequency. An efficiency of transmission at the fundamental, however, be introduced, for example by including an additional factor less than unity in n .

Rather than working with an explicit formula such as Eq.(78), a numerical integration of Eq.(37) was performed utilizing the expressions obtained in

Eqs.(94) and (95) with the time-lag expression in Eq.(40), and employing the values 5 cm for $L_{inj} = -x_v$, 0.63 for ϕ , 22.8 m/s for U_{inj} , 8.5 m/s for U_c , 10% of U_c for $\bar{u}$, 2.407×10^7 J/kg for Q_r , 1.8 cm for x_{f1} , 13.2 cm for x_{f2} , 3.8 cm for r_{f1} and 33.5 cm for r_{f2} . The interaction index n was obtained from Eq. (60) by use of Eq. (63), in which the contribution to n from the laminar burning velocity was 2.9, that from the heat release was 0.95, and that from the other terms was negligible. In the numerical integration, besides these equivalence-ratio fluctuations, effects of vortex shedding, wall damping and damping due to the perforated wall lining were taken into account, using data given in the appendix. The results for the growth rate, shown in Figure 5.14, indicate a possible instability peaking around the combustion chamber's fundamental frequency of 250 Hz. The vortex shedding frequency was found to be around 915 Hz and had only a small impact on the overall growth rate, while the liner and wall damping had a larger effect and were mainly responsible for the negative slope seen in the figure at the higher frequencies.

Figure 5.14 therefore is consistent with most of the amplification at the fundamental frequency being due to the equivalence-ratio fluctuations.

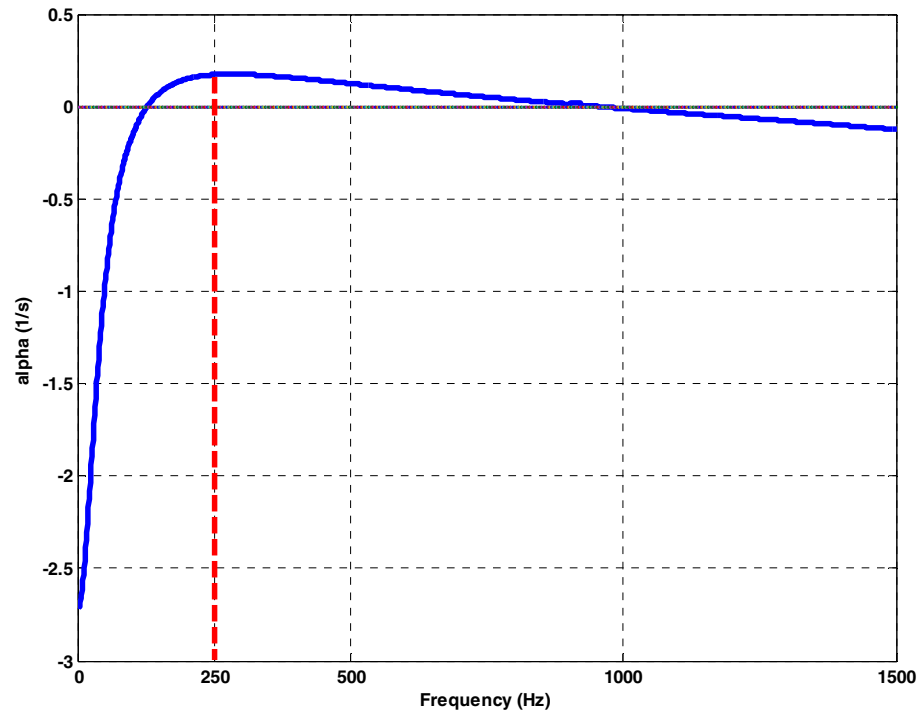


Figure 5.14: Growth-rate predictions for the atmospheric-pressure, annular combustor

Other modes in this combustor are at higher frequencies and generally would be predicted to be stable according to Figure 5.14. For example, the frequency of the first standing axial-mode frequency is estimated from $\bar{c}/(2L)$ to be 1094 Hz, which is beyond the range of positive amplification. Since the liner was designed only for cooling, not for acoustic attenuation, a modified liner design may have eliminated the oscillation at the fundamental azimuthal frequency.

A Twelve-Injector, High-Pressure, Annular-Combustor Engine

To investigate the effect of high pressure in a multiple-injector, annular configuration, an actual production engine was tested. The platform chosen was the Solar Turbines Taurus-70 (a 7.5 MW package). The engine test is an extension of the previous atmospheric annular test in that it uses the same set of injectors and a similar liner, but it operates at a higher pressure. The engine has a fourteen-stage axial compressor (instead of the air preheater) and a few components downstream (turbine and exhaust), all of which might have had a contribution to the overall recorded oscillation and thereby introduce added uncertainty in estimates. The experiment, however, does serve to illustrate application of the approach in a practical engine environment.

The instrumentation, similar to that adopted for the annular atmospheric rig, used dynamic pressure sensors mounted as before on the pilot fuel line to monitor oscillations. In this test there were thirteen dynamic pressure sensors, an extra one having been mounted on the torch to give phase information with respect to the torch. Dynamic pressure data were collected using the same instrumentation described previously.

The flow rates tested were adjusted to keep the equivalence ratio between 0.60 and 0.72, where oscillations could be observed. The air flow rate was fixed at 54 lbs/sec, with a compressor outlet pressure between 160 and 240 psi, and the fuel flow was varied between 1800 and 3500 lbs/hour, with the dominant oscillations observed to arise near 3400 lbs/hour. This is consistent with the predictions, indicated earlier, that amplification rates are largest at the highest

heat-release rates. Pilot flow rates from about 2% to 10% were explored, and oscillations were observed at the lower pilot flow rates, 2% to 4%, just as with the atmospheric rig. Measurements were made at 4% pilot flow rates, for which representative pressure amplitudes are shown in Fig. (19), where the dominant frequency, seen to be about 387 Hz, is consistent with a first mixed (axial/circumferential) mode.

This oscillation, at a frequency on the order of 400 Hz, occurring at full load and low pilot percentage, was the highest-intensity oscillations observed under any conditions. To map out the instability characteristics further, measurements were also made at idle, 50% load and 75% load, with variable pilot percentages, and under different conditions there were indications of low-frequency oscillations (rumble at ~ 37 Hz), of oscillations at a mid-range frequency (200 Hz) and of a low-amplitude high-frequency instability (screech at ~ 1050 Hz), but all of these were much less intense than the 400 Hz oscillation, which occurred at both 75% and full load. Unlike in the atmospheric rig, there was no indication of a purely circumferential mode, at a frequency on the order of 250 Hz, possibly because of damping influence of the guide vanes at the entrance to the turbine in the engine, which would tend to impede circumferential motion. In the present work, therefore, attention was focused on the first mixed mode, the presence of which was consistent with observed irregular and variable phase relations between signals from the different pressure sensors and with corresponding relative amplitude results. With so many rotating parts, namely compressor, turbine, generator (fixed at 60 Hz), etc., it is difficult to isolate the

combustion-related oscillations. A mixed mode, standing axially but rotating circumferentially, therefore finally was assumed for purposes of comparison with predictions.

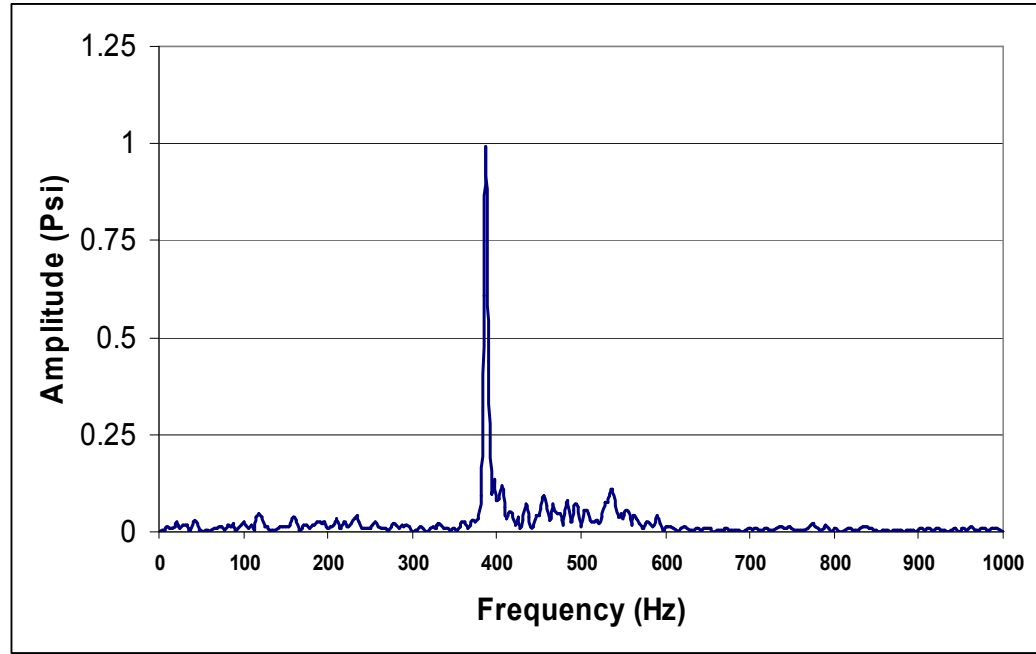


Figure 5.15: Dynamic pressure spectrum obtained during oscillation on a Taurus-70 engine test (full load, 4%pilot flow, and primary zone at 2853°F)

In modeling the engine, the boundary conditions for the combustor-premixer assembly were assumed to provide a pressure node at the injector and a velocity node at the combustor exit (taken at the inlet guide vane to the turbine), where the flow is assumed to approach choked conditions. Equation (5) for the first mixed mode then simplifies to

$$\left. \begin{aligned} p'(x, \theta, t) &= \tilde{p} \cos(\omega t) \sin\left(\frac{\pi x}{2L}\right) \cos(\theta) , \\ u'(x, \theta, t) &= \left(-\frac{\tilde{p}}{\rho c}\right) \sin(\omega t) \cos\left(\frac{\pi x}{2L}\right) \cos(\theta) , \end{aligned} \right\} \quad (96)$$

with u' again representing the circumferential component, which is proportional to the axial component. Numerical integration of Eq. (37), utilizing expressions obtained in Eqs.(95) and (96) with the time lag expression in Eq.(40), and employing 0.63 for ϕ , 89m/s for U_{inj} , 2.1m/s for U_c , 10% of U_c for $\bar{u}$, 2.407×10^7 J/kg for Q_r , and the same dimensions as those used for the annular atmospheric rig, gives the results shown in Figure 5.16. The interaction index n was obtained from Eq. (60) by use of Eq. (63), in which the contribution to n from the laminar burning velocity was 2.9, that from the heat release was 0.95, and other terms were negligible, just as for the atmospheric rig. In the numerical integration, besides these equivalence-ratio fluctuations, effects of vortex shedding, wall damping and damping due to the perforated wall lining were taken into account, using the data given in the appendix. The results for the growth rate, shown in Figure 5.16, indicate a possible instability around the frequency of the first mixed mode of the combustion chamber, about 400 Hz. The vortex-shedding frequency was found to be much higher (over 1500 Hz) and had a small impact on the overall growth rate. Figure 5.16 therefore is consistent with most of the amplification being due to the equivalence-ratio fluctuations, just as in the other experiments.

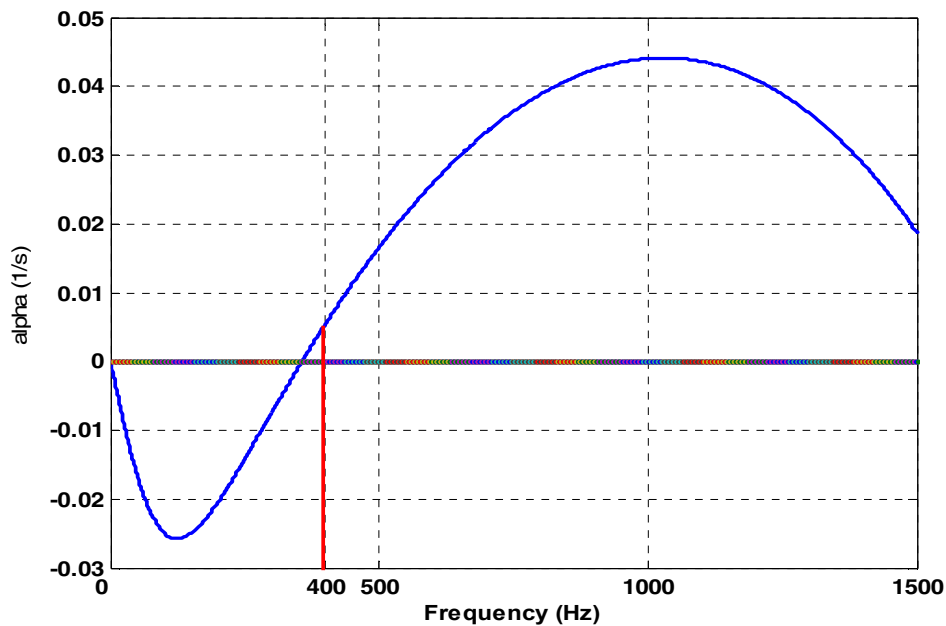


Figure 5.16: Predicted growth-rate results for the annular high-pressure engine test

Except for the difference in the acoustic mode, the preceding model for the engine is quite similar to that for the atmospheric rig. There are, however, significantly greater uncertainties in the modeling results for the engine. The amplification rate at 400 Hz, seen in Figure 5.16, is small, comparable with that found in for the atmospheric rig. Moreover, much higher amplification rates are seen to be predicted at higher frequencies, and it is unlikely that the increased damping for these higher acoustic modes can be large enough to offset the large predicted increase in amplification. Even the vortex-shedding mechanism, with a frequency above 1500 Hz, might be thought to result in screech amplitude in excess of the observed oscillation at 387 Hz. One aspect of Figure 5.16 that may be considered favorable is the predicted attenuation at 250 Hz, which makes it

unnecessary to appeal to extraneous factors such as guide vanes for eliminating the fundamental circumferential mode. This difference from results shown in Fig. (18) for the atmospheric rig is a consequence of the different mode, including now an axial component and also a traveling rather than standing circumferential component. Another criticism of these predictions is that, with a traveling circumferential component (perhaps in the direction of rotor rotation), it is less likely that velocity fluctuations at this frequency can be transferred efficiently to the axial-component fluctuations in the injectors, needed to generate equivalence-ratio fluctuations; double-frequency oscillations in the axial velocities in the injectors may be expected to be more pronounced, calling into question Eq. (95). Equivalence-ratio fluctuations may well be the driver of the instability, but the mode shape assumed in Eq. (96) may be incorrect, the phase information from the pressure transducers being inconclusive. There may, for example, be a standing circumferential component that helps drive the equivalence-ratio oscillations. Even without this, however, the axial component can generate axial velocity fluctuations in the injectors, leading again to Eq. (95), given the same types of assumptions that were employed in its derivation. The main advantage of Eq. (96) is that its associated natural oscillation frequency is close to the observed frequency, but there are other possible mode shapes with this frequency that would modify Fig. (20). These observations underscore the importance of investigating all possible mode shapes, in general, in applying the approach developed here during design investigations.

6. CHAPTER 6

CONCLUSIONS

The paper presents a simplified approach to analyzing combustion-driven acoustic oscillations in premixed gas-turbine combustors. The method works by examining the acoustic energy content in a control volume of interest (usually an injector/combustor assembly). It identifies physical mechanisms that contribute to the growth and decay of acoustic energy in the volume, calculates growth/attenuation rates for the described mechanisms and thus predicts whether the combustion system will experience oscillations. The approach, as described, is linear and utilizes linear approximations when nonlinear events are encountered.

Simplified models for equivalence-ratio fluctuations (including turbulent dissipation) and vortex shedding were developed, along with other models. Results from the first of these models (namely, that for equivalence-ratio fluctuations) were shown to compare reasonably well with experimental results obtained from a single-injector quartz combustor operating at atmospheric conditions as well as from a single-injector high-pressure combustors. It was found to be possible to use this same mechanism to rationalize results obtained from experiments on a twelve-injector, annular, atmospheric-pressure rig. Measurements made on an engine with this same combustor configuration were, however, less conclusive, although they could be consistent with the same mechanism. A stability-index chart, based on Eq. (36), for all three experiments performed in connection with the present work is shown in Figure 6.1. The figure illustrates the strong tendency for oscillations to occur in the twelve-injector, atmospheric

rig at frequencies between 120 and 500 Hz and indicates a comparatively marginal instability range for the quartz combustor under the experimental conditions. Higher fuel-flow rates would have increased the strength of and range of predicted instability for the quartz combustor, while modified liner designs may have reduced the potential instability for the atmospheric rig (as well as for the engine).

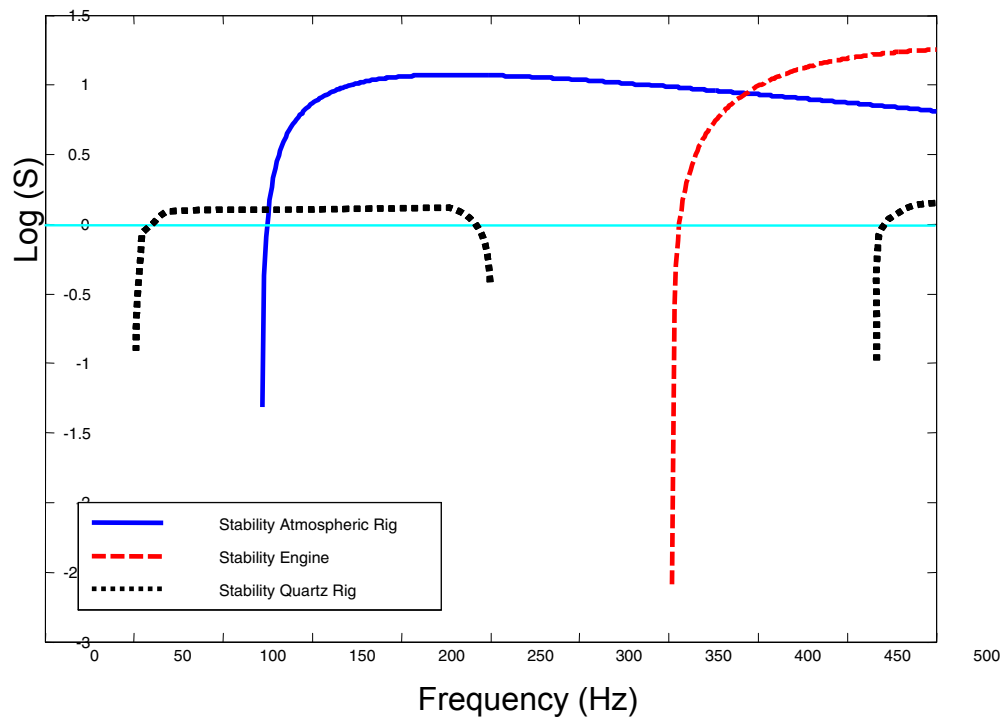


Figure 6.1: Predicted stability results for the three performed experiments (1-500 Hz)

All of the tests of the approach that were reported here involved efforts to explain, after the fact, what was observed experimentally. This would not be possible in using the approach as a design tool. In design, it would be necessary to identify all of the possible acoustic frequencies and acoustic mode shapes in the

combustion chamber, and for each of these to address each of the amplification and damping mechanisms, to determine from Eq.(36) whether oscillations is predicted to occur. This entails appreciable effort and also involves significant uncertainties. The nature of the uncertainties has been illustrated in the four different applications to experiments reported here. Not only do dimensions and operating conditions need to be estimated, but also expected flame shapes need to be hypothesized. As a result, there are likely to be significant ranges of uncertainties in the predictions, and value judgments will need to be made. Similar kinds of uncertainties arise in early applications [30,38] of the Galerkin approach, where the accuracy of the mode description, as well as the chosen model for the heat-release perturbations, affect the predictions to a great extent. While the Galerkin method works by identifying a set of acoustic modes then following their growth or decay by marching in time, the current approach is simpler, in that the mode shapes are approximated in a less complicated fashion, and predictions are made only for a growth rate, which does not involve integrating numerically in time. This makes the calculations simpler and less expensive.

The main advantage of the approach is that, beyond helping to increase the understanding of specific aspects of the types of oscillations that may occur, each individual calculation can be performed much more quickly and less expensively than calculations requiring CFD or other numerically demanding approaches. Since there are also uncertainties in other methods of prediction, the method

proposed could find a useful place within tools used in premixed gas-turbine combustor design.

7. APPENDIX (A)

The values used to calculate the damping rate by the perforated-wall liner are as follow:

Chamber properties and liner dimensions:

- The annular combustor was 0.38 m long
- Inner radius of the combustor was 0.26 m
- Outer radius of the combustor was 0.34m
- Mean Temperature of 1273°K calculated from the temperature of the fresh reactants and the adiabatic flame temperature.
- Perforated wall holes were of two types:
 - Radius 0.0013m spaced at 0.0219m apart
 - Radius 0.0020m spaced at 0.0585m apart
- Both sets of holes were 0.01 m from the backing wall.

8. APPENDIX (B)

Numerical code layout (in Matlab)

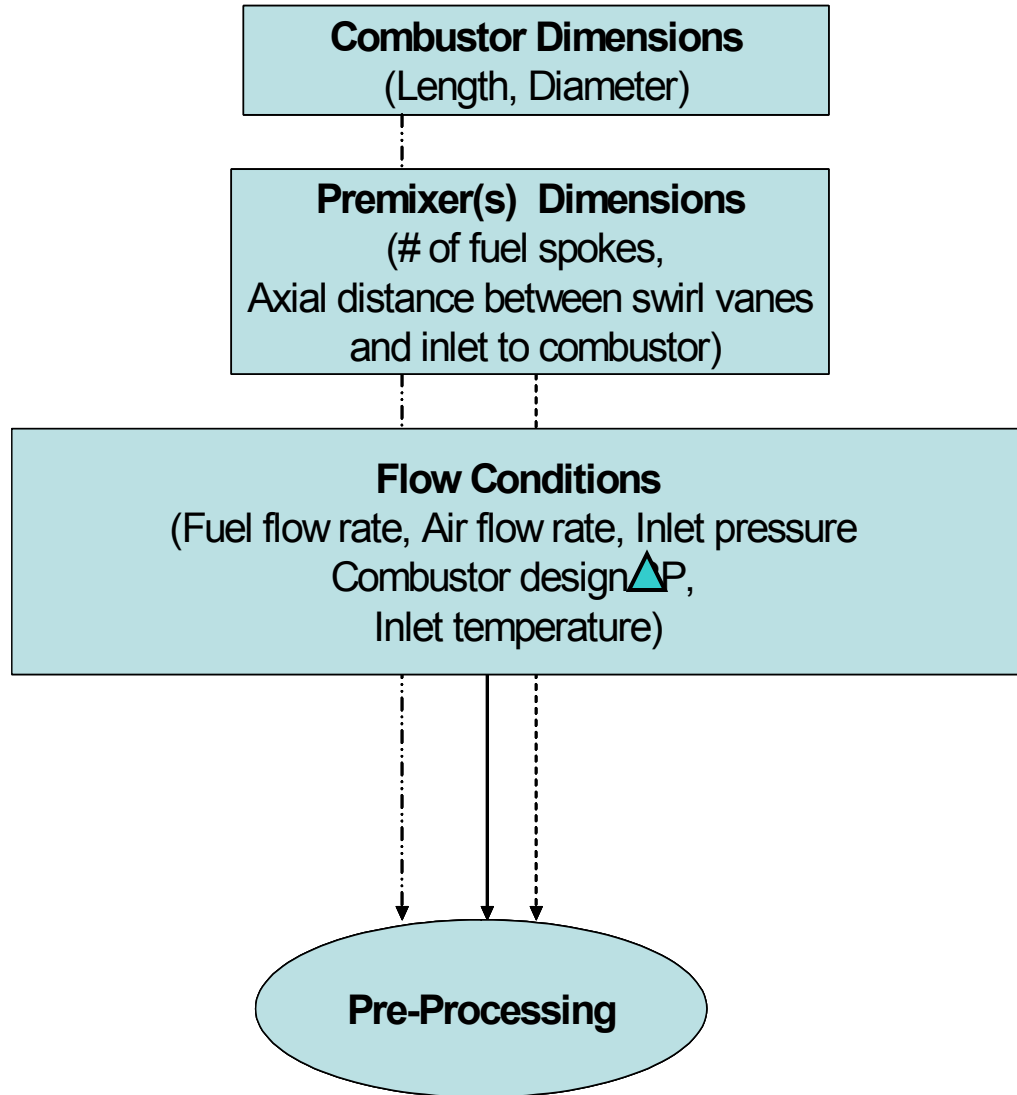


Figure 8.1: Matlab code layout

Pre-Processing

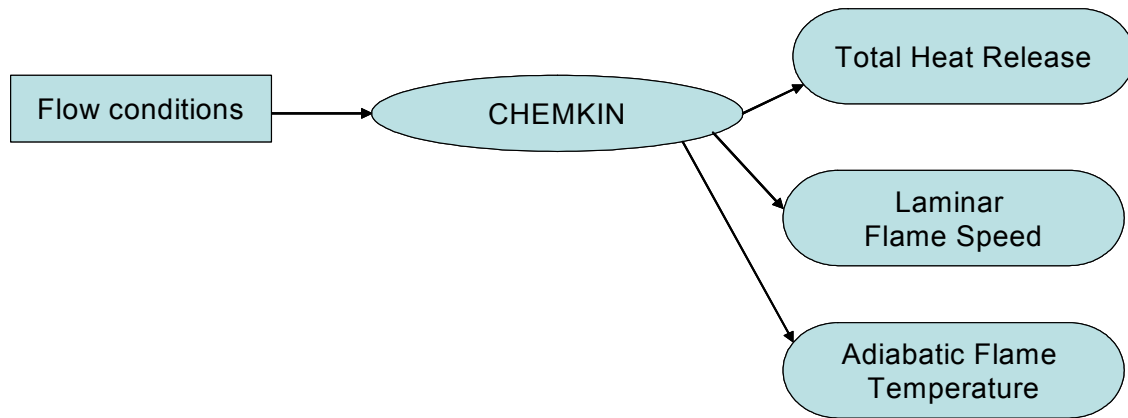


Figure 8.2: Preprocessing steps I

This will evaluate flow conditions near the flame zone and calculates the sensitivity of laminar flame speed and heat of reaction to equivalence-ratio perturbations to evaluate the interaction index n .

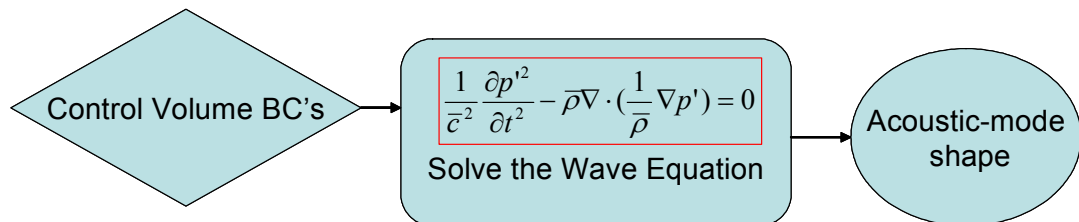


Figure 8.3: Preprocessing steps II

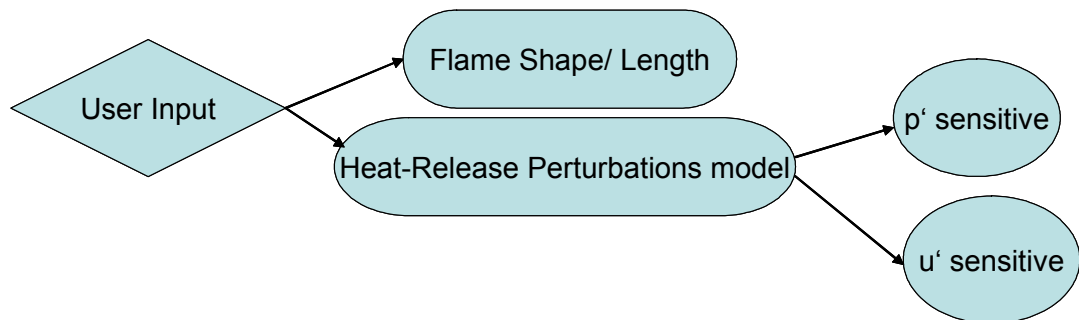


Figure 8.4: Preprocessing steps III

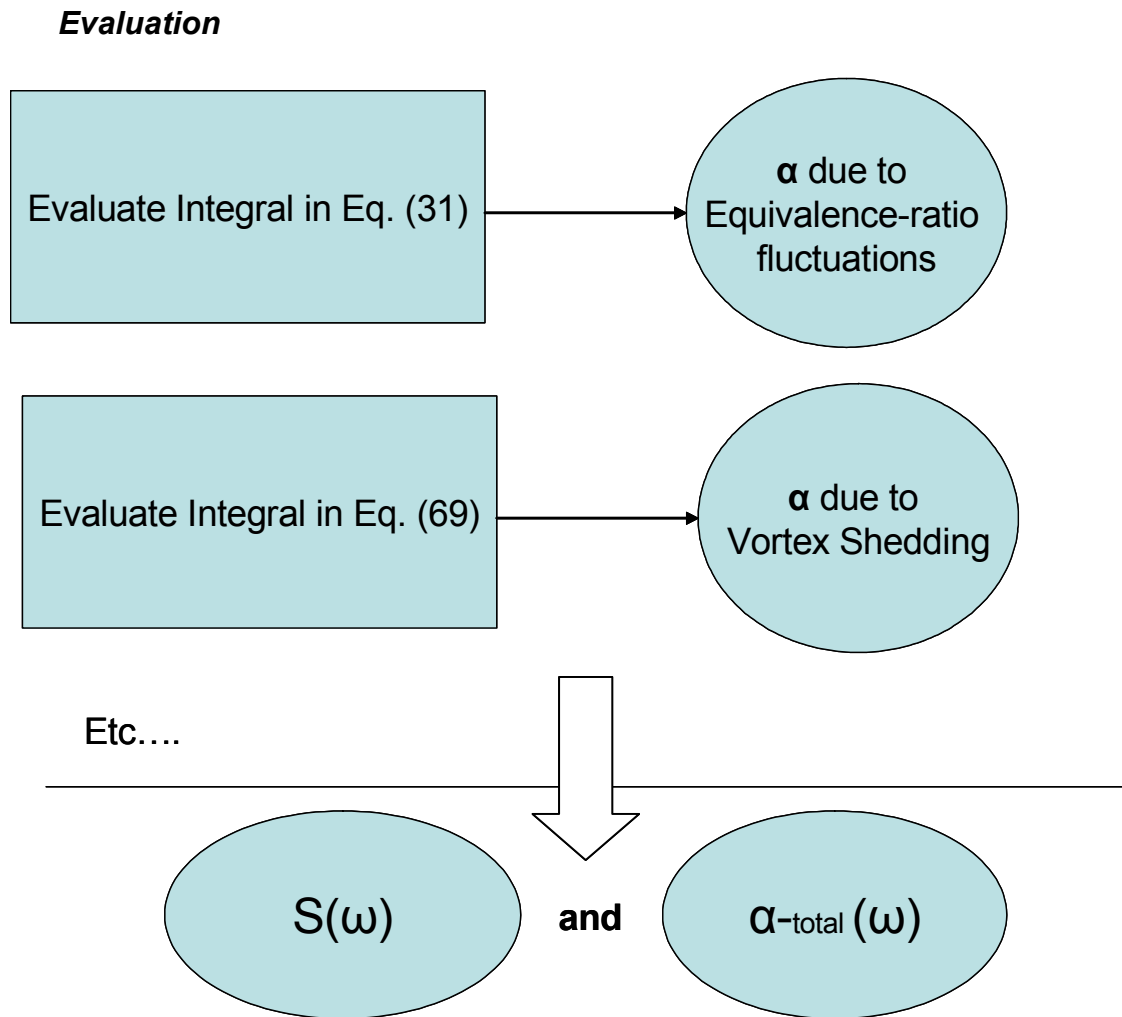


Figure 8.5: Stability characteristics

REFERENCES

- [1] Steele, R. C., Cowell, L.H., Cannon, S.M. and Smith, C.E., "*Passive Control of Combustion Instability in Lean Premixed Combustors*," Journal of Engineering for Gas Turbines and Power, 2000. Vol. 122: pp. 412-419.
- [2] Williams, F. A., "*Combustion Theory*," Second ed. 1985, Redwood City, CA: Addison-Wesley.
- [3] Poinso, T. J., Trounev, A.C., Veynate, D.P., Candel, S.M. and Esposito, E.J., "*Vortex Driven Acoustically Coupled Combustion Instabilities*," Journal of Fluid Mechanics, 1987. Vol. 177: pp. 265-292.
- [4] Yu, K., Trounev, A. and Daily, J., "*Low -frequency Pressure Oscillation in A Model Ramjet Combustor*," Journal of Fluid Mechanics, 1991. Vol. 232: pp. 47-72.
- [5] Lawn, C. J., Evesque, S. and Polifke, W., "*A Model For The Thermoacoustic Response of A premixed Swirl Burner, Part I, Acoustic Aspects*," Journal of Combustion Science and Technology, 2004. Vol. 176: pp. 1331-1358.
- [6] Putnam, A. A., "*Combustion –Driven Oscillations in Industry*." 1971, New York, NY: American EL Sevier Publishers.
- [7] Langhorne, P. J., "*Reheat Buzz: An Acoustically Coupled Combustion Instability, Part I, Experiment*," Journal of Fluid Mechanics, 1988. Vol. 193: pp. 417-443.
- [8] Bloxsidge, G. J., Dowling, A.P. and Langhorne, P.J., "*Reheat Buzz: An Acoustically Coupled Combustion Instability Part II, Theory*," Journal of Fluid Mechanics, 1988. Vol. 193: pp. 445-473.
- [9] Dowling, A. P., "*The Calculation of Thermoacoustic Oscillations*," Journal of Sound and Vibrations, 1995. Vol. 180: pp. 557-581.
- [10] Polifke, W., "*Combustion Instabilities*," Von Karman Institute Lecture Series, Brussels, Belgium, 2004, pp. 1-44.
- [11] Lieuwen, T. C., and Cho, J.H., "*Modeling the Response of Premixed Flames Due to Mixture Ratio Perturbations*," ASME Turbo Expo 2003, Atlanta, Georgia , U.S.A., 2003, GT-2003-38089.
- [12] Sacarini, T., and Freeman, C., "*The Propagation of A Fuel-Air Ratio Disturbance in A Simple Premixer and Its Influence on Pressure Wave Amplification*," ASME Turbo Expo 2000, Munich, Germany, 2000, GT-2000-0106.

- [13] Sacarini, T., Freeman, C. and Day, I., "*Passive Control of Combustion Instability in Low Emission Aeroderivative Gas Turbine*," *ASME Turbo Expo 2004*, Vienna, Austria, 2004, GT-2004-53767.
- [14] Rogers, D. E., and Marble, F.E., "*A Mechanism for High Frequency Oscillations in Ramjet Combustors and Afterburners*," *Jet Propulsion*, 1956. Vol. 1: pp. 456-462.
- [15] Culick, F. E. C., and Magiawalla, K., "*Excitation of Acoustic Modes in A Chamber by Vortex Shedding*," *Journal of Sound and Vibrations*, 1979. Vol. 64: pp. 455-457.
- [16] Mateev, K., and Culick, F.E.C., "*A Model for Combustion Instability Involving Vortex Shedding*," *Combustion Science and Technology*, 2003. Vol. 175: pp. 1059-1083.
- [17] Chu, B. T., "*On the Energy Transfer to Small Disturbances in Fluid Flow. Part I*," *Acta Mechanica*, 1965. Vol. 1: pp. 215-234.
- [18] Polifke, W., Paschereit, C.O. and Dobbeling, K., "*Constructive and Destructive Interference of Acoustic and Entropy Waves in Premixed Combustor With A Choked Exit*," *International Journal of Acoustics and Vibration*, 2001. Vol. 6: pp. 135-146.
- [19] Candel, S. M., and Poinso, T.J., "*Interactions Between Acoustics and Combustion*," *Proceedings of the Institute of Acoustics*, 1988, pp. 103-153.
- [20] Gysling, D., Procia, W. and McCormic, D., "*Combustion System Damping Augmentation With Helmholtz Resonators*," *Journal of Engineering for Gas Turbines and Power*, 2000. Vol. 122: pp. 269-274.
- [21] Crocco, L., and Cheng, S.I., "*Theory of Combustion Driven Instabilities in Liquid Rocket Motors*." 1956, London, United Kingdom: Butterworths Scientific Publications.
- [22] Lieuwen, T. C., "*A Mechanism of Combustion Instability in Lean Premixed Gas Turbine Combustors*," *Journal of Engineering for Gas Turbines and Power*, 2001. Vol. 123: pp. 182-189.
- [23] Keller, J. J., "*Thermoacoustic Oscillations in Combustion Chamber of Gas Turbines*," *AIAA Journal*, 1995. Vol. 33: pp. 2280-2287.
- [24] Clavin, P., "*Premixed Combustion and Gas Dynamics*," *Annual Review of Fluid Mechanics*, 1994: pp. 321-352.

- [25] Hubbard, S., and Dowling, A.P., "*Acoustic Instabilities in Premixed Burners*," 4th AIAA/CEAS Aeroacoustics Conference, Toulouse, France, 1998, AIAA paper 1998-2272.
- [26] Polifke, W., Kopitz, J. and Serbanovic, A., "*Impact of the Fuel Time Lag Distribution in Elliptical Premix Nozzles on Combustion Instability*," 7th AIAA/CEAS Aeroacoustics Conference, Maastricht, Netherlands, 2001, AIAA paper 2001-2104.
- [27] Stow, S. R., and Dowling, A.P., "*Thermoacoustic Oscillations in An Annular Combustor*," ASME Turbo Expo 2001, New Orleans, Louisiana, 2001, GT-2001-0037.
- [28] Hegde, U., Reuter, D. and Zinn, B.T., "*Sound Generation by Ducted Flames*," AIAA Journal, 1988. Vol. 26: pp. 532-537.
- [29] Powell, E. A., and Zinn, B.T., "*Theoretical Investigation of Non-Linear Three Dimensional Instabilities in Liquid Rockets with Real Nozzles*," Acta Astronautica, 1974. Vol. 1: pp. 1051-1073.
- [30] Culick, F. E. C., "*Stability of Longitudinal Oscillations With Pressure and Velocity Coupling in A Solid Propellant Rocket*," Combustion Science and Technology, 1970. Vol. 2: pp. 179-201.
- [31] Culick, F. E. C., "*Combustion Instabilities in Liquid-Fuelled Propulsion Systems-An Overview*," AGARD-CP 450, London, UK, 1988.
- [32] Ananthkrishnan, N., Shardul, Deo and Culick, F.E.C., "*Reduced-Order Modeling and Dynamics of Acoustic Waves in A Combustion Chamber*," Combustion Science and Technology, 2005. Vol. 177: pp. 221-247.
- [33] Hsiao, G. C., Pandalai, R. P., Hura, H. S. and Moniga, H. C., "*Combustion Dynamic Modeling for Gas Turbine Engines*," 34th AIAA/ASME/SAE/ASEE Joint. Propulsion Conference and Exhibit, Cleveland, OH, 1998, AIAA paper 1998-3381.
- [34] Stone, C., and Menon, S., "*Open-Loop Control of Combustion Instabilities in A Model Gas Turbine Combustor*," Journal of Turbulence, 2003. Vol. 4: pp. 1-14.
- [35] Lieuwen, T. C., "*Investigation of Combustion Instability in Premixed Gas Turbines*," Ph.D., Georgia Institute of Technology, 1999, Atlanta, Georgia.
- [36] Dowling, A. P., and Stow, S.R., "*Acoustic Analysis of Gas Turbine Combustors*," Journal of Propulsion and Power, 2003. Vol. 19: pp. 751-764.

- [37] Lieuwen , T., and Yang, V, "*Combustion Instabilities in Gas Turbine Engines: Operational Experience, Fundamental Mechanisms, and Modeling.*" AIAA. Vol. 210. 2006, Reston, VA: AIAA.
- [38] Zinn, B. T., and Lores, M., "*Application of the Galerkin Method in the Solution of Non-linear Axial Combustion Instability Problems in Liquid Rockets,*" Combustion Science and Technology, 1972. Vol. 4: pp. 269-278.
- [39] Ibrahim, Z. M., Williams, F.A., Buckley, S.G. and Lee, J.C.Y., "*An Acoustic Energy Approach to Modeling Combustion Oscillations,*" ASME Turbo-Expo 2006, Barcelona, Spain,2006, GT-2006-90096.
- [40] Landau, L. D., and Lifshitz, E.M., "*Fluid mechanics,*" Second ed. Vol. 6. 1987, Oxford, United Kingdom: Pergamon Press.
- [41] You, D., Huang, Y. and Yang,V., "*A Generalized Model of Acoustic Response of Turbulent Premixed Flame and Its Application to Gas-Turbine Combustion Instability Analysis,*" Combustion Science and Technology, 2005. Vol. 177: pp. 1109-1150.
- [42] Scarinci, T., and Freeman, C., "*The Propagation of A Fuel-Air Ratio Disturbance in A Simple Premixer and Its Influence on Pressure Wave Amplification,*" ASME Turbo Expo 2000, Munich, Germany,2000, GT-2000-0106.
- [43] Sattelmayer, T., "*Influence of The Combustion Aerodynamics on Combustion Instabilities From Equivalence Ratio Fluctuations,*" ASME TurboExpo 2000, Munich, Germany,2000, GT-2000-0082.
- [44] Wang, A. B., Travnicsek, Z. and Chia, K.C., "*On the Relationship of Effective Reynolds Number for the Laminar Vortex Shedding of a Heated Circular Cylinder,*" Physics of Fluids, 2000. Vol. 12: pp. 1401-1416.
- [45] Howe, M. S., "*Acoustics of Fluid-Structure Interactions.*" 1998, Cambridge, United Kingdom: Cambridge University Press.
- [46] Howe, M. S., "*On the Theory of Unsteady High Reynolds Number Flow Through a Circular Aperture,*" Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, 1979. Vol. 366: pp. 205-223.
- [47] Hughes, I. J., and Dowling, A.P., "*The Absorption of Sound by Perforated Linings,*" Journal of Fluid Mechanics, 1990. Vol. 218: pp. 299-335.
- [48] Eldredge, J. D., and Dowling, A.P., "*The Absorption of Axial Acoustic Waves by a Perforated Liner With Bias Flow,*" Journal of Fluid Mechanics, 2003. Vol. 485: pp. 307-335.

- [49] Abramowitz, M., and Stegun, I., "*Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*," 9th ed. 1964, New York, NY: Dover.
- [50] Salikuddin, M., Burrin, R. H., Brown, W. H., "*Design and characterization of a high temperature and high frequency infinite-line pressure probe*," 12th AIAA, Aeroacoustics Conference, San Antonio, TX, 1989.
- [51] Gharavi, M., Ibrahim, Z.M., Bochr, M., Williams, F.A., Buckley, S.G. and, Arellano, L., "*Tunable Diode Laser Measurements of Equivalence-Ratio Fluctuations for Premixed Gas-Turbine Applications*," 5th US Combustion Meeting, San Diego, CA, 2007, E08.
- [52] Richards, G. A., and Janus, M.C., "*Characterization of Oscillations During Premix Gas Turbine Combustion*," Journal of Engineering for Gas Turbines & Power, 1998. Vol. 120: pp. 294-302.
- [53] Straub, D. L., and Richards, G.A., "*Effect of Fuel Nozzle Configuration on Premix Combustion Dynamics*," International Gas Turbines and Aeroengine Congress and Exhibition, Stockholm, Sweden, 1998, GT-1998-492.
- [54] Straub, D. L., and Richards, G.A., "*Effect of Axial Swirl Vane Location on Combustion Dynamics*," ASME Turbo-Expo 1999, Indianapolis, Indiana, 1999, GT-1999-109.