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Journal

Astrophysical Journal, 452(1)

ISSN

0004-637X

Authors

Jedamzik, K

Fuller, GM

Publication Date

1995-10-10

DOI

10.1086/176278

Peer reviewed

NUCLEOSYNTHESIS IN THE PRESENCE OF PRIMORDIAL ISOCURVATURE BARYON FLUCTUATIONS

KARSTEN JEDAMZIK¹ AND GEORGE M. FULLER²

Received 1994 October 12; accepted 1995 April 13

ABSTRACT

We study big bang nucleosynthesis in the presence of large mass scale, nonlinear entropy fluctuations. Overdense regions, with masses above the local baryon Jeans mass, are expected to collapse and form condensed objects. Surviving nucleosynthesis products therefore tend to originate from underdense regions. We compute expected survival light-element (²H, ³He, ⁴He, and ⁷Li) abundance yields for a variety of stochastic fluctuation spectra. In general, we find that spectra with significant power in fluctuations on mass scales below that of the local baryon Jeans mass ($\sim 10^5 M_\odot$) produce nucleosynthesis yields which are in conflict with observationally inferred primordial abundances. Only when this small-scale structure is absent or suppressed, and the collapse efficiency of overdense regions is high, can there exist a range of fluctuation spectral characteristics which conceivably could meet all primordial abundance constraints. Even then, the primordial abundance of ⁷Li would have to be larger than ${}^7\text{Li}/\text{H} \approx 3 \times 10^{-10}$. In such models, abundance constraints could be met even when the precollapse baryonic fraction of the closure density is $\Omega_b \approx 0.2 h^{-2}$ (h is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$). Nucleosynthesis in these models is characterized by high ²H/H and low ⁴He mass fraction relative to a homogeneous big bang at a given value of $\Omega_b h^2$. A potentially observable signature of these models is the production of intrinsic primordial abundance variations on baryon mass scales up to 10^{10} – $10^{12} M_\odot$.

Subject headings: cosmology: theory — early universe — large-scale structure of universe — nuclear reactions, nucleosynthesis, abundances

1. INTRODUCTION

In this paper we calculate the nucleosynthesis to be expected in the presence of primordial isocurvature baryon number (hereafter PIB) fluctuations. Such fluctuations have recently been proposed as possible seeds for large-scale structure formation (Peebles 1987a, b; Sugimoto & Suto 1992; Cen, Ostriker, & Peebles 1993). Density fluctuations in PIB models are essentially *entropy* fluctuations. In an earlier series of papers (Jedamzik & Fuller 1994; Jedamzik, Fuller, & Mathews 1994) we examined the evolution and nucleosynthesis effects of small mass scale nonlinear entropy fluctuations. In the present paper we extend our nucleosynthesis study to larger mass scale fluctuations, including those relevant for PIB models.

Proposed PIB models invoke a spectrum of entropy fluctuations characterized by increasing amplitude with decreasing fluctuation mass scale. In these models the relationship between the fluctuation amplitude, $\delta\rho/\rho$, and the mass contained inside the region of the fluctuation, M , can be written as

$$\frac{\delta\rho}{\rho} \sim M^{-1/2-m/6}, \quad (1)$$

where m is a spectral index. Spectra of this form can be specified by giving the index, m , and the mass scale, M_{unity} , on which $\delta\rho/\rho = 1$. Typical spectral indices employed in PIB models include $m = 0$ and $m = -1$, where accompanying values of M_{unity} are

$$10^{11} \gtrsim \frac{M_{\text{unity}}}{M_\odot} \gtrsim 10^6. \quad (2)$$

We have no compelling microscopic theory for how PIB fluctuations could be generated in general, though there are some cogent suggestions. Yokoyama & Suto (1991) and Dolgov & Silk (1993) have proposed a microscopic mechanism for the generation of baryon number (entropy) fluctuations. These schemes exploit quantum fluctuations in a charge parity-violating angle to effect a spatial variation in baryon-to-photon number. The resulting fluctuations can be on the large mass scales of interest in PIB models if correlations in a field, which later on drives baryogenesis, are laid down before, or during, an inflationary epoch.

A common, though not necessarily inevitable, feature of inflationary baryogenesis models is the production of both matter and antimatter domains separated by astrophysically significant length scales. The production of antimatter domains can be avoided if the baryon number fluctuation amplitudes are below unity on all length scales. In this case, however, the resulting light-element abundance yields from nucleosynthesis will be only slightly altered from abundance yields in homogeneous big bang nucleosynthesis (HBBN) at the same average baryon-to-photon ratio η (for a discussion of small-amplitude fluctuations in η and primordial nucleosynthesis yields see Epstein & Petrosian 1975).

Though large-scale structure considerations in PIB models utilize only the large mass scale, linear ($\delta\rho/\rho < 1$) end of the spectrum in equation (1), successful models may well require some small mass scale, nonlinear ($\delta\rho/\rho > 1$) structure in order to produce collisionless dark matter and/or to effect late reionization of the universe (Cen et al. 1993). Of course, the biggest hurdle which PIB models face may well be stringent observational limits on cosmic background radiation anisotropy (Gorski & Silk 1989; Chiba, Sugiyama, & Suto 1994; Hu & Sugiyama 1994).

If any PIB model for structure formation must invoke a

¹ Physics Research Program, Institute for Geophysics and Planetary Physics, University of California, Lawrence Livermore National Laboratory, Livermore, CA 94550.

² Department of Physics, University of California, San Diego, La Jolla, CA 92093-0319.

nonlinear lower mass scale fluctuation tail in a spectrum like that in equation (1), or if such a tail is an inevitable consequence of some microscopic fluctuation generation mechanism, then primordial nucleosynthesis effects may well provide constraints on the model. These constraints could be complementary to cosmic background anisotropy constraints, as nucleosynthesis probes a different end of the fluctuation spectrum. In this paper we aim to see under what circumstances such constraints on PIB-like models could be found. Our study and our derived constraints will generally apply to *any* models which have nonlinear entropy fluctuations—not just PIB-like models for large-scale structure formation. It should be noted that our study does not derive constraints on models where the entropy per baryon is constant, but rather the entropy per dark matter varies.

Studies of HBBN only allow a narrow range for the fraction of the closure density that can be contributed by baryons:

$$0.01 \lesssim \Omega_b h^2 \lesssim 0.015, \quad (3)$$

where h is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see Walker et al. 1991; Smith, Kawano, & Malaney 1993). Models of inhomogeneous big bang nucleosynthesis (IBBN) with very small scale, nonlinear entropy fluctuations, such as are expected to emerge from a first-order QCD transition, cannot significantly change the conclusions of HBBN as to the upper limit on $\Omega_b h^2$ (see Jedamzik et al. 1994).

However, there has been a longstanding suggestion that large mass scale entropy fluctuations might provide a way to circumvent the HBBN upper bound on $\Omega_b h^2$ (Harrison 1968; Zel'dovich 1975; Rees 1984; Sale & Mathews 1986). In these schemes, regions of relatively large baryon-to-photon ratio (η) collapse to form inert remnants and therefore remove their “bad” nucleosynthesis products from the primordial medium. The nucleosynthesis products which *survive* in these models reflect the relatively lower baryon density of regions which do not collapse. In principle, the precollapse $\Omega_b h^2$ could be considerably larger than the value of this quantity (presumably $\Omega_b h^2 \approx 0.01\text{--}0.015$) which characterizes the low-density medium surrounding collapsed regions. Alternatively, it may be that if overdense regions do not collapse as units, but fragment during collapse, the likelihood of early star formation may be high. In this case, a large supernova rate could imply the early enrichment of the cloud with heavy elements. For the observational determination of primordial abundances, it is common practice to determine abundances as a function of metallicity in *low-metallicity* environments and extrapolate to zero metallicity. Such a primordial abundance determination is therefore highly biased toward low-metallicity objects and may, for the above-mentioned reasons, exclude the determination of abundances in overdense, super-Jeans mass size clouds.

Studies of the fate of large-mass entropy fluctuations in the epoch between primordial nucleosynthesis and some time after recombination indicate that overdense regions would collapse with high efficiency (Hogan 1978, 1993; Kashlinsky & Rees 1983; Loeb 1993). These studies show that the evolution of large entropy fluctuations is dominated by photo-electron Thomson drag (Hogan 1993; Loeb 1993). Any fluctuation with a baryon mass larger than about

$$M_J^b \approx 3 \times 10^5 M_\odot \left(\frac{\Omega_b h^2}{0.1} \right)^{-1/2} \quad (4)$$

will shrink slowly, with Thomson drag providing an efficient mechanism for cooling the collapsing protons and electrons, and also damping rotation. The end result of this process will be the production of a condensed object, either a black hole or possibly, if fragmentation occurs, many small brown dwarfs (Kashlinsky & Rees 1983; Hogan 1993). Note that the mass scale in equation (4) is essentially coincident with the local baryon Jeans mass. Of course, the true Jeans mass in a radiation-dominated environment is close to the horizon mass. By “local baryon Jeans mass” we mean the effective Jeans mass one would calculate inside a fluctuation by neglecting all photon stresses.

We note, however, that the studies of Harrison (1968), Zel'dovich (1975), Rees (1984), and Sale & Mathews (1986) provide what are at best simplistic treatments in their calculations of nucleosynthesis yields in the presence of collapsing regions. In fact, all of these studies except Sale & Mathews (1986) treat the distribution of η in a two-phase, bimodal fashion. All of these studies neglect the significant complication that fluctuations may reside inside larger mass scale fluctuations, the so-called cloud-in-cloud problem. None of these prior studies addresses the nucleosynthesis effects of fluctuations below that in equation (4)—such fluctuations are damped by expansion against Thomson drag (see Hogan 1993; Jedamzik & Fuller 1994), but on timescales that allow significant nucleosynthesis effects (Alcock et al. 1990; Jedamzik et al. 1994).

Recently, Gnedin, Ostriker, & Rees (1995) have reexamined the problem of nucleosynthesis with collapsing entropy fluctuations. These authors provide a more sophisticated numerical treatment of nucleosynthesis from regions of varying density and, in addition, attempt to take account of light-element “reprocessing” effected by accretion on black holes (Gnedin & Ostriker 1992). They conclude that significant power on fluctuation scales below the limit in equation (4) could lead to unacceptable nucleosynthesis which, in turn, could wreck any attempt to employ collapsing high mass scale fluctuations to circumvent HBBN bounds on $\Omega_b h^2$. They do, however, provide for several loopholes in this conclusion: phase correlations in fluctuations and invocation of very finely tuned IBBN scenarios for small mass scale fluctuations.

However, Gnedin et al. (1995) do not attempt to take account of the effects of the cloud-in-cloud problem. Furthermore, their suggestion that baryon diffusive effects on small scales may provide a loophole to the $\Omega_b h^2$ limit is suspect, as it requires an extreme, though not impossible, degree of fine-tuning.

In this paper we attempt a detailed numerical treatment of nucleosynthesis with various stochastic entropy fluctuation spectra. We explicitly treat the cloud-in-cloud problem. We find that any significant power in stochastic fluctuations on mass scales smaller than that in equation (4) inevitably leads to overproduction of ^4He and ^7Li relative to observationally inferred limits on these abundances. Furthermore, we argue that persistence of fluctuations down to mass scales on which baryon diffusive effects are significant is unlikely to change our conclusions. Finally, we show that only under a restrictive set of circumstances (i.e., no significant power in fluctuations on mass scales below M_J^b and high collapse efficiency for scales above M_J^b) is it possible to evade HBBN bounds on $\Omega_b h^2$. We note that evasion of the bound on $\Omega_b h^2$ in these models also usually requires that the primordial abundance of ^7Li be close to the Population I value of $^7\text{Li}/\text{H} \approx 10^{-9}$. We emphasize that

such models imply the existence of intrinsic spatial variations in the primordial abundances.

In § 2 we discuss our model for fluctuation evolution and nucleosynthesis and the numerical techniques employed. In § 3 we present results for nucleosynthesis yields for various fluctuation spectra and assumed fluctuation evolution parameters. We compare these to observationally inferred abundances. We give conclusions in § 4.

2. SIMULATIONS OF BARYON-TO-PHOTON FLUCTUATIONS

In this section we discuss our numerical calculations for fluctuation evolution and nucleosynthesis. The basis of our models is a stochastic distribution of fluctuations in baryon-to-photon ratio on various scales. These scales will be selected in such a manner as to roughly approximate the density distribution in an overall spectrum like that in equation (1). Note, however, that much is hidden in a simplistic power-law density distribution like equation (1). Going from equation (1) to a numerical representation of fluctuation amplitude and mass-scale distribution is not a unique procedure. In what follows we present our numerical approach to this problem and discuss the statistics of our numerically generated distributions of baryon-to-photon number.

2.1. Numerical Models

As a first step, we generate a stochastic distribution of baryon-to-photon number by employing a spatially inhomogeneous Gaussian random variable, $A(x)$. This Gaussian random variable can be Fourier decomposed to give

$$A(x) = 2 \sum_k A_k \cos(kx + \phi_k), \quad (5)$$

where x is a spatial coordinate. In this expression, the amplitudes A_k and phases ϕ_k are chosen randomly according to the distribution functions

$$P_{\phi}(\phi_k) = \frac{1}{2\pi}, \quad \phi_k \in [0, 2\pi], \quad (6a)$$

$$P_A(A_k) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_k} \exp\left(-\frac{1}{2} \frac{A_k^2}{\sigma_k^2}\right), \quad A_k \in [-\infty, \infty]. \quad (6b)$$

Note that our adoption of the distribution function in equation (6a) implies that there will be no correlations between the phases of different Fourier modes—of course, this is the definition of a Gaussian random variable. We adopt a power-law dependence on wavenumber for the variance, σ_k ,

$$\sigma_k^2 \sim k^n, \quad (7)$$

where n is a spectral index. Below we will discuss how we replace the spectral index n in equation (7) with either n_1 or n_3 .

The spatially dependent baryon-to-photon ratio, $\eta(x)$, is taken to be a function of the spatially fluctuating random variable $A(x)$. We consider three different models:

$$\eta(x) = \eta_N A^2(x), \quad (8a)$$

$$\eta(x) = \eta_N A^{10}(x), \quad (8b)$$

$$\eta(x) = \eta_N [A^{10}(x) + a], \quad (8c)$$

where η_N and a are constants. Note that the functions in equations (8a)–(8c) were deliberately chosen to be positive definite. This choice guarantees that antimatter domains are avoided.

Our choice of the spatial distributions in equations (8a)–(8c) is not based on specific baryogenesis scenarios. Rather, we have chosen these models simply to generate a wide variety of stochastic baryon number distributions. It remains to be shown if baryon number distributions with these, or similar characteristics, could arise naturally during the evolution of the very early universe.

In this paper we analyze a one-dimensional analog to a three-dimensional distribution of entropy (or, equivalently, η). By using a one-dimensional distribution of η we are able to investigate a much wider range of mass scales in a single simulation. The one-dimensional analogy to the three-dimensional theory is attained by replacing the spectral index n in equation (7) with n_1 . The three-dimensional case can be obtained by replacing the spectral index n in equation (7) by n_3 . The relationship between the spectral indices for the one-dimensional analog and the full three-dimensional case is $n_1 = n_3/3$. For example, $n_3 = -3$ corresponds to a scale-invariant (“equal power on all scales”) spectrum in the three-dimensional theory, whereas $n_1 = -1$ corresponds to the “same” scale-invariant spectrum in one dimension.

In what follows we will always refer to the three-dimensional spectral index when we wish to characterize a particular one-dimensional distribution in η . In a one-dimensional calculation we compute the “mass” of a region by multiplying “density” (or amplitude $\delta\rho/\rho$) by the length scale of the region. In three dimensions, masses are the product of $\delta\rho/\rho$ and a volume. The transformation of spectral indices $n_3 \mapsto n_1 = n_3/3$ insures that the functional dependence of $\delta\rho/\rho$ on mass scale (e.g., eq. [1]) that we derive in our one-dimensional calculation is the same as that in an equivalent ($n_1 \mapsto n_3$) three-dimensional case.

It is well known that diffusive and hydrodynamic processes occurring during the epoch of primordial nucleosynthesis can significantly alter the light-element abundance yields. Diffusive processes will play an important role during nucleosynthesis when significant small-scale fluctuations on mass scales between $M \sim 10^{-21} M_\odot$ and $M \sim 10^{-11} M_\odot$ are present. Nuclear abundance yields in such scenarios will certainly be highly geometry dependent, so that a full three-dimensional treatment would be needed. (Mathews et al. 1990; Meyer et al. 1991). However, we will suggest below that, for a truly stochastic baryon number distribution (e.g., minimal phase correlations between different Fourier modes of the distribution), the presence of such small-scale fluctuations cannot be reconciled with the observationally inferred primordial abundance limits. We note here that a study of stochastic fluctuations which includes the effects of diffusive processes is currently being performed (Kurki-Suonio, Jedamzik, & Mathews 1995). We will not be concerned with diffusive processes in this paper.

In the case in which diffusive and hydrodynamic processes during nucleosynthesis are unimportant, average light-element abundance yields are determined by a weighted average over the standard homogeneous big bang yields of separate regions at different baryon-to-photon ratios. In this study we obtain the light-element abundance yields by employing the HBBN code of Wagoner, Fowler, & Hoyle (1967), as updated by Kawano (1992).

In our calculations the light-element contributions from overdense regions, which are expected to collapse and form compact objects or undergo early star formation, are excluded from the abundance average. We choose all regions which are overdense on average by some critical amount Δ_{cr} to be “destined” for collapse. We define the overdensity parameter,

Δ_λ , to be

$$\Delta_\lambda = \eta_\lambda(x) - \langle \eta \rangle. \quad (9)$$

In the above expression, $\langle \eta \rangle$ denotes the cosmic average baryon-to-photon ratio and $\eta_\lambda(x)$ represents the average baryon-to-photon ratio within a region of size λ around space coordinate x . In our numerical prescription only regions with $\Delta_\lambda = \Delta_{\text{cr}}$ are taken to collapse. If a region is found to have $\Delta_\lambda > \Delta_{\text{cr}}$ we look for a larger region, which contains the original, for which $\Delta_\lambda = \Delta_{\text{cr}}$. This larger region is taken to collapse. In other words, we take the *largest* scale for which $\Delta_\lambda = \Delta_{\text{cr}}$ to collapse and take everything inside to oblivion.

The above prescription for determining which regions collapse is a widely used analytic tool in the study of structure formation (see Press & Schechter 1974). This procedure is also confirmed by comparison of the analytic results to numerical simulations (Efstathiou et al. 1988). Note that the Press-Schechter analysis has been recently modified in order to account properly for the cloud-in-cloud problem (Jedamzik 1994).

The epoch at which an overdense region collapses is determined, for a given cosmological model, predominantly by its initial average overdensity. For example, the formation of supermassive black holes in PIB models requires early collapse and high initial overdensities (Hogan 1993).

In part, this requirement arises because Thomson drag on background photons becomes a less efficient mechanism for entropy transport and rotational damping at lower cosmic temperature. A certain determination of the fate of a collapsing cloud would require a full three-dimensional hydrodynamic simulation. Determining the fate of overdense regions is important since we need to know whether to count their freeze-out big bang nucleosynthesis products in the final abundance yields of the diffuse, “uncollapsed,” surviving background of primordial gas.

The value of Δ_{cr} where the efficiency of collapse becomes large depends on the cosmological model and the ionization history of the universe (Hogan 1993). We define the collapse efficiency parameter, f , to be the fraction of regions with $\Delta_\lambda = \Delta_{\text{cr}}$ which actually do collapse. In this paper we will simply treat Δ_{cr} and f as parameters to be varied. We will determine the sensitivity of our calculated light-element abundance yields, and ratio of diffuse baryons to dark baryons, to these parameters.

To determine the light-element abundances in the presence of fluctuations it is not sufficient to consider only the average densities of regions. Rather, in principle, a detailed knowledge of the baryon number distribution on all scales is required. Toward meeting this requirement, we define $P_\eta(x)$ to be the probability that the region at point x has baryon-to-photon ratio η . We will find, ultimately, that all small mass scale fluctuations ($M < M_J^b$) would have to be suppressed in order for a PIB-like fluctuation spectrum to be able to produce nucleosynthesis consistent with observational constraints. If we utilize this result and simply assume that on small enough scales fluctuation amplitudes go to zero, then we can exploit the overall translational symmetry of the universe to write $P_\eta(x) = P(\eta)$. In other words, the probability of finding a region with baryon-to-photon ratio η is independent of x ; mathematically, $P(\eta)$ is the probability of finding any point to have baryon-to-photon ratio η . By translational symmetry in this

argument, we mean that any one *large* region of the universe (large enough to have its average η equal to the cosmic average) must be fully equivalent and similar, in a statistical sense, to any other such region.

Note that writing $P(\eta)$ makes sense only if there is a small-scale cutoff in structure. In our numerical calculations, any scale smaller than this cutoff scale is taken to be homogeneous. The probability for finding any such spatial zone to have baryon-to-photon ratio η is $P(\eta)$. The probability that a given scale λ has *average* baryon-to-photon ratio η is defined to be $P_\lambda(\eta)$. Clearly, with this definition $P_\lambda(\eta) = P(\eta)$ for all $\lambda \leq \lambda_c$, where λ_c is the cutoff scale. We will discuss the statistical relationship between $P_\lambda(\eta)$ and $P(\eta)$ for larger scales below.

It is quite important for what follows to ascertain that $P(\eta)$, and any mass-based statistical criterion for describing mass distributions, will be invariant under the transformation from one dimension to three dimensions as outlined above. An example of an invariant mass-based distribution function is the following: the distribution of masses which have overdensities equal to Δ_{cr} . Let us denote the probability function for finding a region with baryon-to-photon ratio η which is not included in a collapsing region and therefore will contribute to the abundance average by $P_{\text{diff}}(\eta)$. If we could show that $P_{\text{diff}}(\eta)$ is invariant under the transformation from one dimension to three dimensions we would have established that the average abundances in a three-dimensional theory are identical to those in our one-dimensional analog. Even though it is not straightforward to prove the invariance of $P_{\text{diff}}(\eta)$, the invariance of $P(\eta)$ and the invariance of the distribution of masses which are overdense by more than Δ_{cr} may suggest that $P_{\text{diff}}(\eta)$ is invariant as well.

We wish to stress the necessity of a numerical treatment for the reliable determination of nuclear abundance yields. Furthermore, there are many subtle pitfalls involved in estimating abundance yields in an inhomogeneous environment. For example, it is not adequate to simply average over nucleosynthesis yields resulting from all regions below some threshold, $\eta \leq \eta_{\text{cr}}$. This procedure is inadequate since such regions could possibly be underdense clouds within collapsing overdense regions. As another example, an analytic computation of $P_\lambda(\eta)$ is difficult (impossible) since the transformation in equations (8a)–(8c) introduces phase correlations between the different Fourier modes of the baryon number distribution. We therefore expect the stochastic baryon number distribution of our model to have a non-Gaussian character. We note here that the analytic study by Gnedin et al. cannot address these subtleties, and therefore, there remains some doubt about the reliability of their predicted abundance yields for specific fluctuation spectra.

We have simulated three types of stochastic baryon number distributions by applying the transformation in equations (8a)–(8c) to a Gaussian random variable. For these simulations we generated 10^5 Fourier modes of the Gaussian random variable. The wavevectors of these Fourier modes were determined by applying periodic boundary conditions in a box extending from zero to one. A small part of these distributions are shown in Figure 1. Note that the value of 10^0 on the logarithmic abscissa corresponds to the cosmic average baryon-to-photon ratio, $\langle \eta \rangle$, in all three panels.

In Figure 1a most of the peaks in the distribution of η are seen to be between values of 5 and 10 (e.g., 5 to 10 times average density), whereas the distribution shown in Figure 1b exhibits peaks with values between 10 and 100. The distribu-

FIG. 1a

FIG. 1.—(a) One-dimensional baryon-to-photon number distribution $\eta(x)$ divided by the average baryon-to-photon ratio $\langle\eta\rangle$ as a function of space coordinate x (solid line). This distribution has been generated by applying transformation eq. (8a) to a Gaussian random variable. Dotted boxes indicate those regions which are overdense on average by the critical amount Δ_{cr} ; here we have used $\Delta_{cr} = 1.5$. Note that the full simulation “volume” extends from $x = 0$ to 1. The figure shows only a small part of the full distribution. (b) Same as (a), but for eq. (8b) and $\Delta_{cr} = 2$. (c) Same as (a), but for eq. (8c) and $\Delta_{cr} = 2$.

tion of η in Figure 1c also exhibits large overdense peaks. However, in this distribution there are no regions with very low density. The dotted boxes in these figures indicate the regions which are overdense by the critical amount, Δ_{cr} . These regions will be expected to collapse very early on. It is obvious from the figures that such overdense regions often are centered on very prominent peaks. Note, however, that these overdense regions can include underdense material at small baryon-to-photon ratios as well. We will analyze the nucleosynthesis resulting from these baryon-to-photon number distributions in § 3.

2.2. Statistics of the Baryon-to-Photon Number Distributions

In this section we will investigate the statistics of the baryon-to-photon number distributions generated by the prescription outlined in § 2.1. Consider first a Gaussian random variable A . The probability distribution $\bar{P}(A)$ for finding a small region to have value A is given by the Gaussian distribution

$$\bar{P}(A) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_{tot}} \exp\left(-\frac{1}{2} \frac{A^2}{\sigma_{tot}^2}\right), \quad (10a)$$

where

$$\sigma_{tot}^2 = \frac{1}{2} \left(\frac{L}{2\pi}\right) \int_0^{k_c} dk \sigma_k^2. \quad (10b)$$

Note the relationship between these expressions and those for

$P_A(A_k)$ and σ_k^2 in equations (6b) and (7). Here $(L/2\pi)^{-1}$ is the volume in wavevector space in which there is one Fourier mode. Equation (10) makes use of the fact that the sum, A , of several normally distributed quantities (e.g., the amplitudes of the uncorrelated Fourier modes, A_k) itself follows a Gaussian distribution by the central limit theorem. The square of the variance, σ_{tot}^2 , of this Gaussian distribution in A is then given by the sum of the squares of the individual variances of the Fourier modes, σ_k^2 . Note that in equation (10b) we introduce a cutoff wavevector, k_c , since we assume that there are no small-scale fluctuations. In particular, we take $\sigma_k \approx 0$ for $k > k_c = 2\pi/\lambda_c$.

With the help of equation (10a) we can derive the probability function $P(\eta)$. Assuming a transformation of the form $\eta(x) = \eta_N[A^{2q}(x) + a]$ we obtain

$$P(\eta) = \frac{1}{\sqrt{2\pi}} \frac{1}{q\sigma_{tot}\eta_N} \left(\frac{\eta}{\eta_N} - a\right)^{(1/2q)-1} \times \exp\left[-\frac{1}{2} \left(\frac{\eta}{\eta_N} - a\right)^{1/q} / \sigma_{tot}^2\right], \quad (11)$$

where η falls in the range $\eta \in [a\eta_N, \infty]$. In this expression, note that q is an integer index while η_N and a are constants.

It should be stressed that it is not possible to obtain the probability function $P_\lambda(\eta)$ by simply employing equation (11) with a modified variance $\sigma_{tot} \mapsto \sigma \sim \int_0^{k_c} dk' \sigma_{k'}^2$. This is because with a positive-definite transformation such as that in equa-

FIG. 1b

FIG. 1c

tions (8a)–(8c) the average η of a region is determined by the large scale Fourier modes with $k' < k$ as well as by the smaller scale Fourier modes with $k' > k$. We therefore must resort to numerical techniques in order to analyze the baryon-to-photon number distribution on larger scales.

We define the integrated variance in the distribution of η on mass scales below the cutoff scale to be $\sigma(M < M_c)$. If M_c is the mass scale corresponding to the cutoff wavevector k_c , then we can define

$$\sigma_\eta(M < M_c) = \left[\frac{1}{\langle \eta \rangle} \int_{a_{\eta N}}^{\infty} d\eta (\eta - \langle \eta \rangle)^2 P(\eta) \right]^{1/2}. \quad (12)$$

The quantity σ_η determines the likely magnitude of fluctuation amplitudes. For the transformation $\eta(x) = \eta_N[A^2 q(x) + a]$ we can derive, for example,

$$\begin{aligned} \sigma_\eta(M < M_c) &= \frac{1}{\langle \eta \rangle} \frac{\eta_N}{\sqrt{\pi}} (2\sigma_{\text{tot}}^2)^q \\ &\times \left[\sqrt{\pi} \Gamma\left(2q + \frac{1}{2}\right) - \Gamma^2\left(q + \frac{1}{2}\right) \right]^{1/2}, \quad (13) \end{aligned}$$

where the average baryon-to-photon ratio is given by

$$\langle \eta \rangle = \frac{\eta_N}{\sqrt{\pi}} \left[(2\sigma_{\text{tot}}^2)^q \Gamma\left(q + \frac{1}{2}\right) + a \Gamma\left(\frac{1}{2}\right) \right]. \quad (14)$$

If we apply equation (13) to the transformation in equation (8a) we obtain $\sigma_\eta(M < M_c) = 2^{1/2}$. By contrast, the fluctuation amplitudes for the transformations in equations (8b) and (8c) are larger and yield $\sigma_\eta(M < M_c) \approx 27$. It should be noted that σ_η is independent of σ_{tot} .

We have investigated numerically the statistics of the baryon-to-photon number distributions on large scales. Figures 2a–2c show the distribution functions $P_\lambda(\eta)$ on different mass scales M (or, equivalently, spatial scales λ) for the three transformations in equations (8a)–(8c). In these figures, the value of 1 on the abscissa corresponds to the average baryon-to-photon ratio, whereas the scale of the ordinate is chosen in each case so that the complete distribution of $P_\lambda(\eta)$ is displayed [i.e., the integrals over $P_\lambda(\eta)$ are not normalized to unity]. The lines on these figures are for the distribution function $P_\lambda(\eta)$ on mass scales $M = M_c, 10M_c, 100M_c$, and $1000M_c$. We will determine the numerical value of M_c in the following section. We show the result from equation (11) for comparison (dash-dotted line). It can be seen from these figures that the distribution functions $P_\lambda(\eta)$ are highly non-Gaussian on small scales but approach a Gaussian distribution character on large scales. Note that the variance, σ_η , decreases as the mass scale increases.

A non-Gaussian distribution $P_\lambda(\eta)$ on small scales could only result if there were phase correlations between the different Fourier modes of the baryon-to-photon number distribution. We note that phase correlations are introduced in our procedure as a result of our application of the transformation equation (8) to the uncorrelated Gaussian random variable. Similar results (e.g., non-Gaussian distributions on small scales and Gaussian distributions on large scales) have been obtained by Yamamoto et al. (1992). These authors analyzed the baryon-to-photon number distribution resulting from the inflationary baryogenesis scenario proposed by Yokoyama & Suto (1991). That baryogenesis scenario assumed that the production of baryon number was proportional to a trigonometric

function of a Gaussian random variable, e.g., $\eta(x) = \eta_0 \sin[A(x)]$.

In Figures 3a–3c we show the fluctuation variance $\sigma_\eta(M)$ as a function of mass scale for the three different transformations in equations (8a)–(8c). In each panel, the dotted line gives $\sigma_\eta(M)$ in the case where the variance of the Fourier amplitudes is taken to be constant, $\sigma_k^2 = \text{constant}$, and the dashed line where $\sigma_k^2 \sim k^{-2.4}$ —here the index $n_3 = -2.4$ is the “three-dimensional” index.

For purposes of comparison in Figure 3 we show the rms mass fluctuation expected for a Gaussian random variable,

$$\sigma_\eta(M) = \left(\frac{\delta M}{M} \right)_{\text{rms}} = \sigma_\eta(M_c) \left(\frac{M}{M_c} \right)^{-(1/2) - (m_3/6)}. \quad (15)$$

Here m_3 is the same index as m in equation (1). We append the “3” on this index to emphasize that we are referring to the three-dimensional case. The solid lines in Figure 3 correspond to $\sigma_\eta(M)$ from equation (15) with $m_3 = 0$ and -1.5 .

2.3. Normalization of the Baryon-to-Photon Number Distributions

We can now determine the absolute magnitude of the cutoff mass scale, M_c , for models with different effective spectral indices, m_3 , and variances, $\sigma_\eta(M_c)$. Primordial baryon-to-photon number fluctuations are constrained by the observed structure of the universe and by observationally inferred primordial light-element abundances. Fluctuations are also constrained by the high degree of isotropy of the cosmic microwave background radiation (hereafter CMBR). However, we will assume here that an early reionization can erase any preexisting anisotropies in the CMBR caused by the baryon-to-photon number fluctuations. It is not at all obvious whether a given PIB spectrum of fluctuations will result in early reionization and thus have a chance of escaping constraints from CMBR anisotropy limits.

The variance, or rms fluctuations in mass, $(\delta M/M)_{\text{rms}}$, at the present epoch has been determined by Davis & Peebles (1983). They find $(\delta M/M)_{\text{rms}} \approx 1$ in a sphere of radius $(8 h^{-1} \text{ Mpc})$. This volume would correspond to a baryonic mass scale of $M_8 \approx 6 \times 10^{13} M_\odot (\Omega_b h^2 / 0.1) h^{-3}$. It is well known that sub-horizon, super-Jeans mass size baryon number perturbations grow proportionally to the scale factor of the universe during a matter-dominated epoch (see Kolb & Turner 1990). This result generally obtains after recombination and when fluctuations are in the linear regime ($\delta M/M \ll 1$). For example, the amplitudes of such baryon number perturbations will have grown by a factor of $1 + z_R$ between the epoch of recombination and the present epoch if a standard recombination scenario with recombination redshift $z_R \approx 1100$ is assumed.

If the universe stays ionized at redshifts $z < z_R$, the growth of perturbations will be inhibited by coupling of photons to baryons. In the limit where the photon mean free path, l_γ , is larger than the fluctuation size, l_f , growth of baryon number perturbations is inhibited by Thomson drag between photons and electrons. This drag force effectively suppresses any perturbation growth for redshifts above $z_{\text{drag}} \approx 200$ –300 (see Peebles 1971). For redshifts below z_{drag} , Thomson drag rapidly becomes inefficient and so cannot hinder the growth of perturbations. In the limit when $l_\gamma \ll l_f$ (and when dissipation is negligible), entropy perturbations with $M > M_J^s$ oscillate around their hydrostatic equilibrium position or simply stay in hydrodynamic equilibrium “frozen in” during the cosmic

FIG. 2a

FIG. 2b

FIG. 2.—(a) Probability distribution function $P_\lambda(\eta)$ of finding a region of size λ (or, equivalently, mass M) with average baryon-to-photon ratio η . These distribution functions are plotted as a function of the ratio $\eta/\langle\eta\rangle$, where $\langle\eta\rangle$ is the cosmic average baryon-to-photon ratio. Distribution functions are shown for the mass scales $M = M_c$ (solid line), $M = 10M_c$ (dotted line), $M = 100M_c$ (short-dashed line), and $M = 1000M_c$ (long-dashed line). M_c is the cutoff mass scale below which baryon-to-photon fluctuations are assumed to be suppressed. The analytically determined distribution function $P(\eta)$ (eq. [11]; dash-dotted line) is shown for comparison. Note that the ordinate scales differently for different distribution functions. The baryon-to-photon distribution has been generated by applying transformation eq. (8a) to a Gaussian random variable. (b) Same as (a), but for eq. (8b). (c) Same as (a), but for eq. (8c).

expansion. Note that the photon mean free path has a co-moving size of roughly $\sim 8 h^{-1}$ Mpc (corresponding to M_8) at a redshift of $z \approx 200$.

In order that cosmic structure in PIB fluctuation models not be “overproduced,” it is necessary that the primordial rms mass fluctuations on the mass scale M_8 be smaller than $(1 + z_r)^{-1}$,

$$\sigma_\eta(M_8) \equiv \left(\frac{\delta M}{M} \right)_{M_8} \lesssim \frac{1}{1 + z_r}, \quad (16)$$

where $1100 \lesssim z_r \lesssim 200$. Here the range in redshifts results from two extreme scenarios: standard recombination and ionization fully maintained down to low redshifts.

Note that equality in equation (16) pertains to the case when the PIB fluctuations are the seeds for the presently observed cosmic structure. It is, however, also conceivable that the formation of structure on large scales is mainly due to, for example, adiabatic fluctuations. In this case perhaps only the

small mass scale fluctuations are dominated by the PIB fluctuation component. The inequality in equation (16) would apply in this latter example.

By using equations (15) and (16) we can derive a limit on the mass cutoff,

$$M_c \lesssim 6 \times 10^{13} M_\odot \left(\frac{\Omega_b h^2}{0.1} \right) h^{-3} [(1 + z_r) \sigma_\eta(M_c)]^{-6/(3 + m_3)}. \quad (17)$$

Our nucleosynthesis results show that this mass cutoff must not fall below M_c^l . If it did, we would produce unacceptable light-element abundances. Furthermore, unless $\sigma_\eta(M_c) \gtrsim 1$, there would be essentially no interesting effects on nucleosynthesis (e.g., no early collapse of overdense regions with significant mass fractions). For a given spectral index it is not always clear whether these two requirements are mutually exclusive.

These considerations are illustrated in the parameter-space

FIG. 2c

diagram of Figure 4. Any choice of the parameters m_3 , $\sigma_\eta(M_c)$, and $1 + z_r$, which falls into the upper left-hand hatched region will either be in conflict with the presently observed rms fluctuations on the mass scale M_8 or violate observational constraints on the primordial abundances of ^4He and/or ^7Li .

2.4. Convergence of Numerical Results

We have performed detailed primordial nucleosynthesis calculations with the numerical techniques outlined above. In Figure 5 we present several measures of convergence accuracy as functions of numbers of Fourier modes employed in our numerical study.

For the results shown in Figure 5a we employ $\langle\eta\rangle = 6 \times 10^{-10}$, $\Delta_{cr} = 1.5$, $n_3 = 0$, and the transformation in equation (8a). In Figure 5a the upper panel shows the ratios of $^2\text{H}/\text{H}$, Y_p , and $^7\text{Li}/\text{H}$ to their converged values as functions of the number of zones employed. Also shown in this figure is the ratio of Ω_b/Ω_{diff} to its converged value. Here Ω_{diff} is the fractional contribution of baryons in surviving, uncollapsed regions to the closure density. It is clear that good convergence is obtained for cases which employ more than a few thousand Fourier modes.

The lower panel of Figure 5a shows $\sigma_\eta(M_c)$ and $20 \times \sigma_\eta(1000 \times M_c)$ as functions of the number of Fourier modes. Convergence of σ_η is good whenever more than a few thousand Fourier modes are employed.

For the computations presented in Figure 5b we employ $\langle\eta\rangle = 1.2 \times 10^{-9}$, $\Delta_{cr} = 2.0$, $n_3 = 0$, and the transformation in equation (8b). The quantities plotted in these figures are the same as in Figure 5a, and the notation is the same. Again, we note the rapid convergence of light-element abundance yields and Ω_b/Ω_{diff} with increasing number of Fourier modes (upper panel). However, we must conclude that the calculation for σ_η remains unconverged, even for 10^5 Fourier modes (lower panel).

The lack of convergence in σ_η for the case in Figure 5b stems from the fact that the calculations of σ_η are dominated by (rare) high- σ events in the Gaussian random variable. Note that this is not the case for the calculations of the light-element abundances. Therefore, we can predict the light-element abundances in this case with confidence, though we are unable to predict accurately the effective spectral index, n_3 , which corresponds to our distribution for η in the cases where the parameters of Figure 5b are adopted. Similar conclusions would obtain if we had employed the transformation law in equation (8c) instead of that in equation (8b).

3. RESULTS

In this section we describe the results of our numerical study of primordial nucleosynthesis in the presence of large mass scale, PIB-like entropy fluctuations. Here we will discuss not only the light-element (^2H , ^3He , ^4He , and ^7Li) nucleosynthesis yields from various models for the distribution of η but also the fraction of baryons in these models which survive collapse and constitute the “diffuse” primordial gas. We will characterize the surviving diffuse component of baryons by its fractional contribution to closure, $\Omega_{diff} h^2$. In what follows, $\Omega_b h^2$ refers to the baryonic fraction of the closure density *prior* to freeze-out from nuclear statistical equilibrium and, thus, prior to any significant amount of collapse. We wish, however, to start with a brief review of the observationally inferred limits on the primordial abundances.

3.1. Observational Constraints on the Primordial Abundances

The situation for the ^4He mass fraction, Y_p , is as follows. The best determination of Y_p is obtained from observations of ^4He recombination lines in metal-poor, extragalactic H II regions. The existing data have been analyzed by a number of authors (see Skillman et al. 1994). It is generally believed that the upper limit on Y_p should be somewhere in the range $Y_p \lesssim 0.24\text{--}0.245$, with the “favored” value for Y_p around 0.23.

However, more recently Skillman et al. (1994) have pointed out that the observational determination of Y_p is subject to several systematic uncertainties which have previously not been well appreciated. When all the systematic uncertainties are taken into account, such as uncertainties associated with the determination of emissivities and corrections due to the possible existence of some neutral helium in H II regions, a *firm* upper limit on Y_p may be as large as $Y_p \lesssim 0.252$. A lower limit on Y_p , which is of less constraining power for PIB-like models than the upper limit, should be somewhere around $Y_p \gtrsim 0.21\text{--}0.22$. It is clear that an accurate determination of the primordial ^4He mass fraction from the existing data is actually not that straightforward.

FIG. 3a

FIG. 3b

FIG. 3.—(a) Variance $\sigma_\eta(M)$ as a function of the ratio of mass to cutoff mass scale (M/M_c). The baryon-to-photon distribution has been generated by applying transformation eq. (8a) to a Gaussian random variable: dotted line, spectral index $n_3 = 0$ of the Gaussian random variable has been used (eq. [7]); dashed line, $n_3 = -2.4$ has been used. For comparison we also show $\sigma_\eta(M)$ from eq. (15) with $m_3 = 0$ (lower solid line) and $m_3 = -1.5$ (upper solid line). (b) Same as (a), but for eq. (8b). (c) Same as (a), but for eq. (8c).

The situation is not very different for the primordial abundances of ^2H and ^3He . It is common practice to give the upper limit for the sum of ^2H and ^3He (Walker et al. 1991). This is done since it is not known to what extent ^2H , the most fragile of the light elements, has been destroyed in stars prior to the formation of the solar system. Before the very recent claims of an extragalactic observation of ^2H by Songaila et al. (1994), the deuterium-to-hydrogen number ratio had been estimated only for the solar system. By considering the sum of ^2H and ^3He , some of the uncertainties in the determination of the individual abundances are evaded since the destruction of ^2H is expected to lead to the production of ^3He via $^2\text{H}(p, \gamma)^3\text{He}$. In any case, it is commonly believed that the upper limit on the sum of these light elements is $(^2\text{H} + ^3\text{He})/\text{H} \lesssim 10^{-4}$. The lower limit for deuterium is usually given at $(^2\text{H}/\text{H}) \gtrsim 1.8 \times 10^{-5}$.

Recently, Songaila et al. (1994) observed an isotope-shifted Ly α absorption line in a Ly α cloud system which lies along the line of sight to a quasar. The existence of an absorption line is well explained if the Ly α cloud has a deuterium-to-hydrogen number fraction as large as $(^2\text{H}/\text{H}) \approx 1.9\text{--}2.5 \times 10^{-4}$. This observation may cast doubt on the previously believed upper limit on primordial $(^2\text{H} + ^3\text{He})/\text{H}$. It may, however, also be that the detected absorption line comes from hydrogen as

opposed to deuterium. This could occur if, by coincidence, a small component of the Ly α cloud system has a small collective velocity relative to the main cloud. In such a case, the existence of an isotope-shifted Ly α absorption line might be mimicked. In contrast, Tytler (1995) has reported on the observation of deuterium in a different Ly α cloud. His observation suggests a deuterium-to-hydrogen number fraction of $(^2\text{H}/\text{H}) \approx 3 \times 10^{-5}$ for this cloud, which may be in conflict with the Songaila et al. result, at least in the context of an HBBN scenario. We hope further observations of Ly α cloud systems will provide us with a reliable primordial deuterium abundance.

The situation for the primordial ^7Li abundance is controversial as well. Spite & Spite (1982) detected a ^7Li abundance plateau for hot, low-metallicity Population II halo stars over some temperature and metallicity range. A commonly held view is that the ^7Li abundance of $(^7\text{Li}/\text{H}) \approx 1.4 \times 10^{-10}$ observed in the “plateau” stars is the primordial one (Spite & Spite 1982; Thorburn 1994). When uncertainties in the employed model atmospheres for the plateau stars and “small” amounts of diffusion-induced ^7Li depletion are taken into account, the upper limit on the primordial value of ^7Li may be larger by a factor between 1 and 3.

The above interpretation of the data precludes the possi-

FIG. 3c

bility of significant ${}^7\text{Li}$ depletion in plateau stars. However, it has been shown by Pinsonneault, Deliyannis, & Demarque (1992) and Chaboyer & Demarque (1994) that ${}^7\text{Li}$ in Population II stars is depleted by up to an order of magnitude, or more, over the entire surface temperature range (including the Spite plateau) when microscopic diffusion and rotation of stars are included in the stellar models. Furthermore, only combined models which include rotation and diffusion can simultaneously explain the very different ${}^7\text{Li}$ depletion patterns in old Population II stars and young Population I stars (Pinsonneault et al. 1992; Chaboyer, Demarque, & Pinsonneault 1995a, b). This may be a powerful argument for the validity of the combined models. Combined models predict a ${}^7\text{Li}$ abundance of $({}^7\text{Li}/\text{H}) \sim 1 \times 10^{-9}$, much larger than the value of the Spite plateau.

It has been pointed out that a possible detection of ${}^6\text{Li}$ in two Population II stars (Smith, Lambert, & Nissen 1993; Hobbs & Thorburn 1994) may provide an argument against significant depletion. This is because ${}^6\text{Li}$, presumably produced by cosmic-ray spallation of heavier elements over the entire history of the universe, would have been depleted below detection level along with ${}^7\text{Li}$. On the other hand, the observations of three highly ${}^7\text{Li}$ -depleted plateau stars (Thorburn 1994) indicates that significant ${}^7\text{Li}$ depletion in plateau stars may occur, at least for some stars.

The situation remains controversial. It should be noted that

FIG. 4.—Parameter-space diagram showing the product of recombination redshift, $1+z_r$, and rms entropy fluctuations (measured on the fluctuation cutoff mass scale), $\sigma_\eta(M_c)$, vs. effective spectral index of the entropy distribution, m_3 . Any combination of parameters which falls into the hatched region is excluded by observations.

the primordial ${}^7\text{Li}$ abundance, when determined with confidence, could be a crucial argument against, or indication for, the existence of small-scale and/or large-scale baryon-to-photon inhomogeneity during the nucleosynthesis epoch.

3.2. Abundance Yields for Different Fluctuation Characteristics

In Figure 6 we present nucleosynthesis results for a model of the distribution of η characterized by the transformation in equation (8a), spectral index $n_3 = 0$ (employed in eq. [7]), $M_c > M_J^b$, and various values of Δ_{cr} . The panels in Figure 6a give the ${}^4\text{He}$ mass fraction, Y_p , and the number ratios ${}^2\text{H}/\text{H}$, ${}^7\text{Li}/\text{H}$, and ${}^3\text{He}/\text{H}$ as functions of $\Omega_b h^2$. Results for standard HBBN are given for comparison (solid lines). In Figure 6b we plot $\Omega_{diff} h^2$ as a function of $\Omega_b h^2$ for the models in Figure 6a.

Note that several general trends are evident in Figure 6. First we note that for all values of Δ_{cr} and $\Omega_b h^2$ considered here, Y_p is lower than the HBBN yield while ${}^2\text{H}/\text{H}$ and ${}^3\text{He}/\text{H}$ are higher than the HBBN yield. Our models clearly retain the well-known “ ${}^7\text{Li}$ dip,” but in general ${}^7\text{Li}/\text{H}$ can be slightly smaller than, comparable to, or larger by up to a factor of 5 than the yield from HBBN at a given $\Omega_b h^2$.

These results are easily understood by comparing the panels in Figure 6a to the graph in Figure 6b. Clearly, ${}^2\text{H}/\text{H}$ is high in inhomogeneous models compared to HBBN at the same $\Omega_b h^2$ because the surviving diffuse baryonic component is characterized by $\Omega_{diff} h^2 < \Omega_b h^2$. In fact, since in HBBN ${}^2\text{H}/\text{H}$ yields rise very steeply with decreasing η , we can identify two competing effects in ${}^2\text{H}$ production. Regions with smaller η produce relatively more ${}^2\text{H}$, but they make a relatively smaller contribution to the total surviving mass of baryons. We find that the regions which are most effective in producing ${}^2\text{H}$ have $\eta \approx 3 \times 10^{-11}$. Since in primordial nucleosynthesis ${}^3\text{He}$ is produced by ${}^2\text{H}(p, \gamma){}^3\text{He}$, it is not surprising that ${}^3\text{He}$, like ${}^2\text{H}$, is always high compared to HBBN at the same $\Omega_b h^2$.

FIG. 5a

FIG. 5b

FIG. 5.—(a) Convergence of numerical results as a function of the number of Fourier modes employed in the simulation. *Upper panel:* Ratios of $(^2\text{H}/\text{H})$ (solid line), ^4He mass fraction Y_p (dotted line), $(^7\text{Li}/\text{H})$ (short-dashed line), and $(\Omega_b/\Omega_{\text{diff}})$ (long-dashed line) to their convergent values as noted. *Lower panel:* Variances $\sigma_n(M_*)$ (solid line) and $20 \times \sigma_n(1000 \times M_*)$ (dotted line). In these simulations we have used transformation eq. (8a), spectral index $n_3 = 0$, and $\Delta_{\text{cr}} = 1.5$. (b) Same as (a), but for eq. (8b), $n_3 = 0$, and $\Delta_{\text{cr}} = 2$.

The situation for ^4He is straightforward. Since Y_p is a rising function of η in HBBN, it is obvious that when $\Omega_{\text{diff}} h^2 < \Omega_b h^2$ inhomogeneous models will produce low Y_p relative to HBBN at the same $\Omega_b h^2$.

By contrast, the behavior of the $^7\text{Li}/\text{H}$ yield in our inhomogeneous models is more complicated. In part, this is due to the two principal production channels for ^7Li . These are $^3\text{H}(\alpha, \gamma)^7\text{Li}$, which is dominant in low-density regions where ^2H (thus also ^3H) is high, and $^3\text{He}(\alpha, \gamma)^7\text{Be}(e^-, \nu_e)^7\text{Li}$, which is dominant in higher density regions. Since, to some extent, our models average over regions which have baryon-to-photon ratios on opposite sides of the dip, our $^7\text{Li}/\text{H}$ yields are high compared to HBBN for a fair range of $\Omega_b h^2$. Note, however, two interesting features of our $^7\text{Li}/\text{H}$ results: (1) the dip is offset to higher $\Omega_b h^2$ than the HBBN dip and (2) $^7\text{Li}/\text{H}$ at higher $\Omega_b h^2$ is lower than the yield in HBBN.

A surprising feature of Figure 6a is that the light-element abundance yields in our inhomogeneous model are actually fairly insensitive to the values of the parameter Δ_{cr} employed. We conclude that, even in cases where both the criterion in Δ_{cr}

required for collapse and the collapse efficiencies are not accurately determined, we can predict the nucleosynthesis for a given PIB-like fluctuation spectrum with fair confidence. This result is quite important since the collapse efficiencies may well depend not only on the fluctuation's overdensity but also on internal geometry, intrinsic angular momentum, and the local environment (e.g., neighboring fluctuations). We therefore expect collapse efficiencies to not change discontinuously from zero to one at some value Δ_{cr} , but rather to approach unity in a continuous fashion over some interval of overdensities.

However, these conclusions depend on the assumption that collapsing regions do not explode and produce large amounts of explosive nucleosynthesis (see Fuller, Woosley, & Weaver 1986). Furthermore, the results in Figure 6 depend critically on having a significant fraction of the baryons collapse. In the case where only a small fraction of the baryons collapse (i.e., Δ_{cr} very large) we would find Y_p and $(^7\text{Li}/\text{H})$ to be increased considerably over the results displayed in Figure 6a.

The implications of the observationally inferred primordial abundance constraints for the model results displayed in

FIG. 6a

FIG. 6b

FIG. 6.—(a) Nucleosynthesis yields as functions of $\Omega_b h^2$ for a model distribution of η characterized by transformation eq. (8a), spectral index $n_s = 0$, $M_c > M_p^0$, and various values of Δ_{cr} . Upper left, ^4He mass fraction Y_p ; lower left, $(^7\text{Li}/\text{H})$ number ratio; upper right, $(^2\text{H}/\text{H})$ number ratio; lower right, $(^3\text{He}/\text{H})$ number ratio. The lines show $\Delta_{cr} = 1.25$ (dotted lines), $\Delta_{cr} = 1.5$ (short-dashed lines), $\Delta_{cr} = 1.75$ (long-dashed lines), and $\Delta_{cr} = 2$ (dash-dotted lines). Solid lines give the results of standard HBBN for comparison. (b) $\Omega_{\text{diff}} h^2$ as a function of $\Omega_b h^2$ for models in (a).

Figure 6 are as follows. There seem to be two possible ranges in $\Omega_b h^2$ which could yield agreement with abundance constraints within the above-mentioned observationally inferred primordial abundance uncertainties. For $\Omega_b h^2 \approx 0.015$ we find from Figure 5a that $(^2\text{H}/\text{H}) \approx 2\text{--}4 \times 10^{-4}$, $Y_p \approx 0.225\text{--}0.235$, and $(^7\text{Li}/\text{H}) \approx 3.5\text{--}4.5 \times 10^{-10}$ depending on the employed Δ_{cr} . To be correct, this scenario would have to assume that there was a modest amount of ^7Li depletion in halo stars and a high primordial deuterium abundance.

For the range $\Omega_b h^2 \approx 0.03\text{--}0.05$, we obtain $(^2\text{H}/\text{H}) \approx 4 \times 10^{-5}$ to 1.5×10^{-4} , $Y_p \approx 0.24\text{--}0.25$, and $(^7\text{Li}/\text{H}) \approx 5 \times 10^{-10}$ to 1.5×10^{-9} . This model would then require significant ^7Li depletion, relatively low deuterium abundance, and high ^4He abundance. For comparison, the preferred $\Omega_b h^2$ in HBBN is somewhere in the range $\Omega_b h^2 \approx 0.005\text{--}0.015$, mostly depending on what primordial deuterium abundance is adopted.

Another generic feature of our models with large-scale inhomogeneity in η is evident from Figure 6b. Even though the abundance yields in these models tend to agree with observationally inferred abundance limits for higher $\Omega_b h^2$ than the allowed range of this quantity in standard homogeneous big bang models, the fractional contribution of the diffuse, “surviving” baryons to the closure density, $\Omega_{\text{diff}} h^2$, tends to be lower than the allowed range of $\Omega_b h^2$ in homogeneous models.

For a good agreement between abundance yields and observationally inferred abundance limits in PIB-like models, the low-density regions must have approximately the preferred average standard homogeneous baryon-to-photon ratio [i.e., $\Omega_b h^2 \approx 0.0125$ for $(^2\text{H}/\text{H}) \lesssim 10^{-4}$]. Therefore, $\Omega_{\text{diff}} h^2$ is lower than the $\Omega_b h^2$ from a homogeneous big bang since only the low-density regions in the universe contribute to the diffuse baryons. The high-density regions do not contribute since these presumably collapse and form condensed, dark objects.

This way of lowering $\Omega_{\text{diff}} h^2$ compared to $\Omega_b h^2$ inferred from HBBN is quite analogous to the scenario proposed by Jedamzik, Mathews, & Fuller (1994). In that paper it was shown that there is no lower limit on $\Omega_b h^2$ in inhomogeneous primordial nucleosynthesis scenarios since it may be that only a certain fraction of space has baryons with baryon-to-photon ratio $\eta \approx 3 \times 10^{-10}$, with the remaining fraction of space depleted in baryons. Such a scenario could provide an explanation for the small amount of baryons observed in luminous form (i.e., galaxies and diffuse intergalactic gas). The fractional contribution of luminous baryons to the closure density is $\Omega_b^{\text{lum}} \approx 0.003\text{--}0.007$. This value can be much smaller than the Ω_b inferred from HBBN. It is interesting to note that inhomogeneous PIB-like models conceivably could provide a natural explanation for the small value of Ω_b^{lum} .

In Figure 7 we present nucleosynthesis results for a model of the distribution of η characterized by the transformation in equation (8b), spectral index $n_3 = 0$, $M_c > M_J^b$, and various values of Δ_{cr} . The notation is as in Figure 6. The assumed values for Δ_{cr} in these calculations are $\Delta_{\text{cr}} = 1.5, 2, 2.5$, and 3 .

It is seen from Figure 7 that most of the general trends observed in the PIB-like models of Figure 6 are retained. It is evident that there is a range in $\Omega_b h^2$ ($0.04 \lesssim \Omega_b h^2 \lesssim 0.07$) for which all the abundance constraints may be satisfied. In this range we find $(^2\text{H}/\text{H}) \approx 9 \times 10^{-5}$ to 3×10^{-4} , $Y_p \approx 0.235\text{--}0.248$, and $(^7\text{Li}/\text{H}) \approx 7 \times 10^{-10}$ to 2×10^{-9} , depending on the value of Δ_{cr} employed. Clearly, such models would be ruled out if the ^7Li abundance of the Spite plateau in halo stars represents the actual primordial abundance. Low values for $\Omega_b h^2 \lesssim 0.02$ are ruled out by deuterium overproduction. While the models of Figure 6 are characterized by collapse ratios $\Omega_b/\Omega_{\text{diff}} \approx 2.5\text{--}10$, a much larger fraction of the baryons would collapse in the models of Figure 7, $\Omega_b/\Omega_{\text{diff}} \approx 25\text{--}75$.

In Figure 8 we present the results for a distribution in η characterized by the transformation in equation (8c), spectral index $n_3 = 0$, and $M_c > M_J^b$. As in Figures 6 and 7 we give results as functions of $\Omega_b h^2$ for various Δ_{cr} . The lines are for $\Delta_{\text{cr}} = 1.5, 2, 2.5$, and 3 .

In this model, the low ^4He mass fraction and high $(^2\text{H}/\text{H})$ number ratio compared to a HBBN model at the same $\Omega_b h^2$ are most pronounced. The collapse ratios, $\Omega_b/\Omega_{\text{diff}}$, are very similar to the collapse ratios in the models of Figure 7, in particular $\Omega_b/\Omega_{\text{diff}} \approx 25\text{--}75$. In the range $\Omega_b h^2 \approx 0.06\text{--}0.2$ we obtain abundance yields of $(^2\text{H}/\text{H}) \approx 4 \times 10^{-5}$ to 4×10^{-4} , $Y_p \approx 0.225\text{--}0.25$, and $(^7\text{Li}/\text{H}) \approx 9 \times 10^{-10}$ to 3×10^{-9} depending on the value of Δ_{cr} . These abundance yields may agree with observationally inferred abundance limits when significant ^7Li depletion in Population II halo stars occurs. It is remarkable that the allowed (albeit with high ^7Li) range of $\Omega_b h^2$ in these models is between 10 and 40 times larger than the allowed range of $\Omega_b h^2$ in HBBN models. The upper end of this range in $\Omega_b h^2$ may even allow for baryons to provide closure density when the Hubble parameter is smaller than $h \lesssim 0.45$.

3.3. Sensitivity of Abundance Yields to the Collapse Efficiency

We have also investigated the nucleosynthesis results in PIB-like models as a function of the collapse efficiency parameter f . This parameter is defined in § 2. In Figures 9a and 9b we present the nucleosynthesis yields and ratios $\Omega_b/\Omega_{\text{diff}}$ for a distribution in η characterized by the transformation in equation (8a), spectral index $n_3 = 0$, $M_c > M_J^b$, and $\Delta_{\text{cr}} = 1.5$. In this figure we vary $\Omega_b h^2$ and the collapse efficiency parameter. The notation in these figures is similar to that in Figures 6, 7, and 8.

The different lines correspond to the collapse efficiencies $f = 100\%$, 97.5% , 95% , 90% , and 80% . Note that the lower dotted lines in Figure 9 represent the results for the same model as the short-dashed lines in Figure 6. These lines are shown for comparison. Note that in Figure 9 the lower dotted lines in the Y_p and $(^7\text{Li}/\text{H})$ panels and in the Ω_{diff} panel are to be associated with the upper dotted lines in the $(^2\text{H}/\text{H})$ and $(^3\text{He}/\text{H})$ panels.

It is not surprising to find that the ^4He and ^7Li abundances increase, and the ^2H and ^3He abundances decrease, when the collapse efficiency parameter decreases. This is because with lower collapse efficiency more high-density regions contribute their nucleosynthesis yields to the diffuse baryons. The high-density regions produce relatively larger amounts of ^4He and ^7Li and smaller amounts of ^2H and ^3He than the low-density regions. In this model, however, it is evident that collapse efficiencies do not have to be extremely close to 100% in order to avoid gross overproduction of ^4He and ^7Li .

In Figures 10a and 10b we present the nucleosynthesis yields and survival ratios $\Omega_b/\Omega_{\text{diff}}$ for a distribution in η characterized by the transformation in equation (8b), spectral index $n_3 = 0$, $M_c > M_J^b$, and $\Delta_{\text{cr}} = 2$. We show models with collapse efficiencies $f = 100\%$, 99% , and 97.5% . Note that the dotted lines in Figure 10 represent the results for the same model as the short-dashed lines in Figure 7.

From Figure 10 it is evident that the collapse efficiencies for these models would have to be very close to 100% in order to avoid overproduction of ^4He and, especially, ^7Li . This conclusion stands in contrast to that derived from the models of Figure 9 and is easily understood by an examination of the results presented in Figure 1b. This figure shows a small part of the distribution in η from which the model results of Figure 10 have been computed. The distribution is characterized by very overdense peaks. If only a very few of these peaks did not collapse, their contributions to the ^7Li component in the surviving diffuse baryons from these high-density regions would be very significant, perhaps even dominant.

Our computation of the abundance yields from a particular model as a function of the collapse efficiency parameter f proceeds in the following manner. First, the algorithm finds all the regions which are overdense by the critical amount Δ_{cr} . Then, the algorithm randomly chooses a fraction $1 - f$ of these overdense regions to not collapse and thus to contribute to the abundance yields of the surviving diffuse baryons.

This procedure may not actually lead to a very accurate assessment of the variations in abundance yields associated with the uncertainties of collapse of particular regions. Realistically, an overdense region may not completely collapse because of its peculiar angular momentum and local environment. However, we expect the very overdense peaks within an overdense region to collapse with high efficiency. A better assessment of the variations in abundance yields due to collapse efficiency uncertainties may be obtained by averaging

FIG. 7a

FIG. 7b

FIG. 7.—(a) Nucleosynthesis yields as functions of $\Omega_b h^2$ for a model distribution of η characterized by transformation eq. (8b), spectral index $n_3 = 0$, $M_c > M_J^b$, and $\Delta_{cr} = 1.5$ (dotted lines), $\Delta_{cr} = 2$ (short-dashed lines), $\Delta_{cr} = 2.5$ (long-dashed lines), and $\Delta_{cr} = 3$ (dash-dotted lines). Remaining notation as in Fig. 6a. (b) $\Omega_{diff} h^2$ as a function of $\Omega_b h^2$ for models in (a).

over the abundance yields for a range of Δ_{cr} . Figures 6–8, which show the abundance yields of different models for different Δ_{cr} , may therefore provide a more realistic estimate for the anticipated magnitude of variations in abundance yields due to collapse efficiency uncertainties. In any case, Figure 10a illustrates that a very large fraction ($\geq 99\%$) of the very overdense peaks of a distribution must collapse in order that ${}^4\text{He}$ and ${}^7\text{Li}$ not be overproduced.

In Figures 11a and 11b we show the nucleosynthesis results and ratios Ω_b/Ω_{diff} for a distribution in η characterized by the transformation in equation (8c), spectral index $n_3 = 0$, $M_c > M_J^b$, $\Delta_{cr} = 2$, and varying collapse efficiencies f . The lines represent $f = 100\%$, 99% , and 97.5% . The conclusions drawn from these figures are quite similar to the conclusions for the model investigated in Figure 10. A large fraction ($f \geq 99\%$) of very overdense peaks has to form dark remnants in order that ${}^7\text{Li}$ and ${}^4\text{He}$ not be overproduced.

3.4. Abundance Yields as a Function of Fluctuation Cutoff Mass Scales

In our study we have assumed so far that the fluctuation cutoff mass scale, M_c , is larger than the mass scale M_J^b . The local baryon Jeans mass, M_J^b , divides fluctuation evolution into

FIG. 8a

FIG. 8b

FIG. 8.—Same as Fig. 7, but for eq. (8c)

two regimes: overdense fluctuations on mass scales $M > M_J^b$ ultimately are expected to collapse, whereas overdense fluctuations on mass scales $M < M_J^b$ will expand, and their nucleosynthesis yields will mix with the diffuse baryons. We have examined the nucleosynthesis yields of PIB-like models when fluctuations exist on mass scales below M_J^b .

In Figures 12a and 12b we show the nucleosynthesis yields and fraction $\Omega_b/\Omega_{\text{diff}}$ for a PIB-like model with a distribution in η characterized by the transformation in equation (8a), spectral index $n_3 = 0$, $\Delta_{\text{cr}} = 1.5$, and collapse efficiency $f = 100\%$. In this figure we have varied the cutoff mass scale M_c . The lines represent $M_c > M_J^b$ and $M_c = M_J^b/3$, $M_J^b/6$, and $M_J^b/10$. Note that the dotted lines represent the results of our “reference” model, which have already been shown by the short-dashed lines in Figure 6 and the dotted lines in Figure 9.

Compared to the reference model, the ${}^4\text{He}$ and ${}^7\text{Li}$ yields increase and the ${}^2\text{H}$ and ${}^3\text{He}$ yields decrease whenever M_c falls below M_J^b . These results are as expected, since sub-Jeans mass size overdense fluctuations which are not included in super-Jeans mass size overdense regions will expand and contribute large amounts of ${}^4\text{He}$ and ${}^7\text{Li}$ to the diffuse baryons. This material from sub-Jeans mass size overdense fluctuations will also dilute the $({}^2\text{H}/\text{H})$ and $({}^3\text{He}/\text{H})$ ratios relative to the refer-

FIG. 9a

FIG. 9b

FIG. 9.—(a) Nucleosynthesis yields as functions of $\Omega_b h^2$ for a model distribution of η characterized by transformation eq. (8a), spectral index $n_s = 0$, $M_c > M_J^*$, $\Delta_{cr} = 1.5$, and various collapse efficiency parameters $f = 100\%$ [lower dotted lines, panels for Y_p and $({}^7\text{Li}/\text{H})$], $f = 97.5\%$ (short-dashed lines), $f = 95\%$ (long-dashed lines), $f = 90\%$ (dash-dotted lines), and $f = 80\%$ [upper dotted lines, panels for Y_p and $({}^7\text{Li}/\text{H})$]. Note that the lower dotted lines in the panels for Y_p and $({}^7\text{Li}/\text{H})$ correspond to the upper dotted lines in the panels for $({}^2\text{H}/\text{H})$ and $({}^3\text{He}/\text{H})$, and vice versa. Remaining notation as in Fig. 6a. (b) $\Omega_{diff} h^2$ as a function of $\Omega_b h^2$ for models in (a). Results shown here by the lower dotted line correspond to results shown by the lower dotted lines in the panels for Y_p and $({}^7\text{Li}/\text{H})$ in (a).

FIG. 10a

FIG. 10b

FIG. 10.—(a) Nucleosynthesis yields as functions of $\Omega_b h^2$ for a model distribution of η characterized by transformation eq. (8b), spectral index $n_s = 0$, $M_c > M_J^b$, $\Delta_{cr} = 2$, and various collapse efficiency parameters $f = 100\%$ (dotted lines), $f = 99\%$ (short-dashed lines), and $f = 97.5\%$ (long-dashed lines). Remaining notation as in Fig. 6a. (b) $\Omega_{diff} h^2$ as a function of $\Omega_b h^2$ for models in (a).

ence model. Note that ^4He and ^7Li (for most of the “interesting” range in $\Omega_b h^2$) will even increase over the HBBN yields as the fluctuation cutoff mass scale is decreased.

It is evident from Figure 12b that the fraction of material included in regions which meet the requirement to have overdensity Δ_{cr} on a mass scale larger than M_J^b decreases as the cutoff mass scale M_c decreases. Equivalently, the ratio Ω_b/Ω_{diff} decreases with decreasing M_c as well. In the limit where Ω_b/Ω_{diff} approaches unity, there is no collapse and the abundance yields would be given by a weighted average over the HBBN yields of all the regions of the entire distribution. The model results displayed by the dash-dotted lines are actually not far removed from that limit ($\Omega_b/\Omega_{diff} \lesssim 2$). For comparison, the same model but *without* any collapse ($\Omega_b/\Omega_{diff} = 1$) would yield $(^2\text{H}/\text{H}) = 1.5 \times 10^{-4}$, $Y_p = 0.242$, and $(^7\text{Li}/\text{H}) = 8.4 \times 10^{-10}$ for $\Omega_b h^2 = 0.01$. These abundance yields differ only slightly from the yields for $\Omega_b h^2 = 0.01$ (dash-dotted line).

It is quite surprising that even for a distribution of η as inhomogeneous as that in Figure 1a, and without any collapse, there exists a narrow interval in $\Omega_b h^2$ where all the observational abundance constraints may be satisfied. In particular, for $0.006 \lesssim \Omega_b h^2 \lesssim 0.0012$ agreement between the computed abundance yields and the observationally inferred abundance

FIG. 11a

FIG. 11b

FIG. 11.—Same as Fig. 10, but for eq. (8c)

constraints is possible within the observational uncertainties. Note that this range for $\Omega_b h^2$ is close to the range for $\Omega_b h^2$ inferred from HBBN. Agreement between abundance yields and observationally inferred abundance limits would require significant ${}^7\text{Li}$ depletion and a high deuterium abundance (${}^2\text{H}/\text{H} \gtrsim 10^{-4}$).

We have implicitly assumed so far that there is a cutoff mass scale which is larger than $M_c \gtrsim 10^{-11} M_\odot$ (approximately 16 orders of magnitude smaller than M_J). If there were fluctuations on mass scales below $M \lesssim 10^{-11} M_\odot$, diffusive and hydrodynamic processes during the epoch of primordial nucleosynthesis would alter the abundance yields from what we have calculated.

It is not straightforward to estimate accurately the effects of diffusive processes during the nucleosynthesis era on the abundance yields of *stochastic* baryon number distributions such as those shown in Figure 1. The nucleosynthesis yields of a regular lattice of fluctuation sites in the mass range $10^{-21} \lesssim M \lesssim 10^{-11} M_\odot$ have been investigated in detail. Depending on the fluctuation characteristics, especially the separation of adjacent fluctuations in the regular lattice, Y_p is usually larger (but can be smaller) than the Y_p of a homogeneous model at the same average $\Omega_b h^2$. The number ratio of (${}^7\text{Li}/\text{H}$) tends to be

FIG. 12a

FIG. 12b

FIG. 12.—(a) Nucleosynthesis yields as functions of $\Omega_b h^2$ for a model distribution of η characterized by transformation eq. (8a), spectral index $n_3 = 0$, $\Delta_{cr} = 1.5$, and various cutoff mass scales $M_c > M_J^b$ (dotted lines), $M_c = M_J^b/3$ (short-dashed lines), $M_c = M_J^b/6$ (long-dashed lines), and $M_c = M_J^b/10$ (dash-dotted lines). Remaining notation as in Fig. 6a. (b) $\Omega_{diff} h^2$ as a function of $\Omega_b h^2$ for models in (a).

much larger in such inhomogeneous models than its HBBN value. On the other hand, most inhomogeneous scenarios *with* diffusion tend to yield slightly lower values for Y_p and $({}^7\text{Li}/\text{H})$ than do inhomogeneous models which neglect diffusive processes.

The abundance yields for a truly stochastic distribution may be roughly approximated by averaging over the abundance yields from different regular lattices of fluctuation sites with varying fluctuation separation lengths. Such an average should always increase Y_p and $({}^7\text{Li}/\text{H})$ relative to their respective HBBN yields (Meyer et al. 1991). Therefore, we expect models which have small-scale fluctuations and diffusive processes during the nucleosynthesis era to produce ${}^4\text{He}$ and ${}^7\text{Li}$ yields far above those shown by dotted lines in Figure 12a. We also expect such models to yield ${}^7\text{Li}$ and ${}^4\text{He}$ slightly below the amounts shown by dash-dotted lines in Figure 12a. The situation for the other light elements, ${}^2\text{H}$ and ${}^3\text{He}$, is more complicated. These issues are investigated in more detailed by Kurki-Suonio et al. (1995). These authors will treat diffusive processes during the nucleosynthesis epoch explicitly and combine their results with the results of the present study.

Note that our arguments implicitly assume that there is some turnover in effective spectral index of the baryon number distribution, from $m_3 = 0$ on large mass scales to an $m_3 = -3$

FIG. 13a

FIG. 13b

FIG. 13.—Same as Fig. 12, but for eq. (8b) and $\Delta_{cr} = 2$

scale-invariant character on small mass scales. If there were not such a turnover in effective spectral index, there would either be no significant fraction of baryons collapsing (since fluctuation amplitudes on M_J -size mass scales are very small) or fluctuation amplitudes on small scales $M \lesssim 10^{-11} M_\odot$ would have to be extremely large ($\Delta \gtrsim 10^8$).

In Figures 13a and 13b we show the nucleosynthesis yields and fraction Ω_b/Ω_{diff} for a distribution of η characterized by the transformation in equation (8b), spectral index $n_3 = 0$, $\Delta_{cr} = 2$, and varying M_c . The lines represent results for $M_c > M_J^b$ and $M_c = M_J^b/3$, $M_J^b/6$, and $M_J^b/10$. In this figure the dotted lines represent the results of our reference model which have already been shown by the short-dashed lines in Figure 7 and the dotted lines in Figure 10.

When the cutoff mass scale is decreased, Y_p and $(^7\text{Li}/\text{H})$ abundance significantly increase, and $(^2\text{H}/\text{H})$ and $(^3\text{He}/\text{H})$ decrease relative to the results of the reference model. Since this particular distribution in η includes very overdense peaks ($\Delta \sim 100$), it is not surprising that the ^4He and ^7Li yields increase over the yields of a HBBN scenario at the same $\Omega_b h^2$.

For the particular distribution investigated here, there seems to be no interval in $\Omega_b h^2$ for which all the abundance constraints may be met. This result implies that models with stochastic large- and small-scale, large-amplitude ($\Delta \sim 100$)

FIG. 14a

FIG. 14b

FIG. 14.—Same as Fig. 13, but for eq. (8c)

inhomogeneities in the baryon-to-photon ratio are ruled out by the observationally inferred primordial light-element abundances. We can only expect possible agreement between the abundance yields of such models and observationally inferred abundance limits when there is a fluctuation cutoff mass scale $M_c \gtrsim M_J^b$. Furthermore, our discussion about the effects of diffusive processes during the nucleosynthesis era should indicate that the existence of small-scale fluctuations down to the mass range $M \sim 10^{-21} - 10^{-11} M_\odot$ is not likely to change these conclusions.

In Figure 14 we present the nucleosynthesis yields from a distribution in η characterized by the transformation in equation (8c), spectral index $n_3 = 0$, and $\Delta_{cr} = 2$ for cutoff mass scales $M_c > M_J^b$ and $M_c = M_J^b/3$, $M_J^b/6$, and $M_J^b/10$. It is evident from these figures that for $M_c < M_J^b$ there seems to be no range in $\Omega_b h^2$ where the production of all the light elements is consistent with observationally inferred abundance constraints. Note that this model is characterized by large-amplitude fluctuations and so is similar to the model considered in Figure 13.

3.5. The Dependence of Abundance Yields on Effective Spectral Index

We have investigated the dependence of nucleosynthesis yields in PIB-like models on the effective spectral index of the

FIG. 15a

FIG. 15b

FIG. 15.—(a) Nucleosynthesis yields as functions of $\Omega_b h^2$ for a model distribution of η characterized by transformation eq. (8a), $M_c > M_J^b$, $\Delta_{cr} = 1.5$, and effective spectral indices $m_3 \approx 0$ (dotted lines) and $m_3 \approx -1.5$ (dashed lines). Remaining notation as in Fig. 6a. (b) $\Omega_{diff} h^2$ as a function of $\Omega_b h^2$ for models in (a).

distribution in η . We have employed spectral indices $n_3 = 0$ and -2.4 in equation (7). The $\sigma_\eta(M)$ (or, equivalently, $\delta\rho/\rho$ as a function of mass scale) for the resultant distributions in η has been presented in Figure 3. These figures illustrate that the effective spectral index of the distribution in η is $m_3 \approx 0$ for $n_3 = 0$ and $m_3 \approx -1.5$ for $n_3 = -2.4$. We note, however, that there is some uncertainty associated with the exact value of m_3 .

In Figures 15a and 15b we show the nucleosynthesis yields and Ω_b/Ω_{diff} for a distribution in η characterized by transformation equation (8a), $\Delta_{cr} = 1.5$, $M_c > M_J^b$, and effective spectral indices $m_3 \approx 0$ and -1.5 . It is surprising to find that there is so little dependence of the nucleosynthesis yields on the effective spectral index of the distribution in η . In general, for decreasing m_3 the ^4He mass fraction, Y_p , decreases slightly while the number ratios ($^2\text{H}/\text{H}$) and ($^3\text{He}/\text{H}$) increase slightly. The number ratio ($^7\text{Li}/\text{H}$) decreases for large $\Omega_b h^2$ and increases for small $\Omega_b h^2$ when m_3 decreases.

These trends can be easily understood on inspection of Figure 15b. For a smaller m_3 , a larger fraction of the baryons collapse. In turn, a larger fraction of collapsing baryons implies that the average $\Omega_{diff} h^2$ is smaller, which ultimately leads to smaller ^4He yields and larger ^2H and ^3He yields. The ^7Li yield increases or decreases for decreasing Ω_b/Ω_{diff} depending on

FIG. 16a

FIG. 16b

FIG. 16.—Same as Fig. 15, but for eq. (8b) and $\Delta_{cr} = 2$

whether $\Omega_b h^2$ is smaller than or larger than the $\Omega_b h^2$ at the ${}^7\text{Li}$ dip. Our results show that the nucleosynthesis yields in PIB-like models do not depend much on the effective spectral index of the distribution of η in the interval $-1.5 \lesssim m_3 \lesssim 0$. Note that PIB models for large-scale structure formation would prefer spectral indices somewhere between $m_3 = 0$ and -1 .

These conclusions are confirmed by the results shown in Figures 16a and 16b and Figures 17a and 17b. These figures show nucleosynthesis yields and Ω_b/Ω_{diff} for models with $\Delta_{cr} = 2$, $M_c > M_J^0$, and spectral indices $m_3 \approx 0$ and -1.5 . The calculations described by Figure 16 employ the transformation in equation (8b) for the generation of the distribution of η , whereas the calculations described by Figure 17 employ the transformation in equation (8c).

3.6. Intrinsic Spatial Variations of Primordial Abundances

There is a unique prediction of cosmological models which contain nonlinear, intermediate- to large-scale PIB fluctuations. Since such models are characterized by inhomogeneity in the baryon-to-photon ratio during the nucleosynthesis epoch, we expect the production of *intrinsic* spatial variations in the primordial light-element abundances. In contrast, HBBN models predict a universal set of cosmic light-element abundances.

FIG. 17a

FIG. 17b

FIG. 17.—Same as Fig. 16, but for eq. (8c)

To illustrate this point, we define the probability distribution function $P_\lambda(Y_p)$. This function gives the probability of finding a region of size λ having average ^4He mass fraction Y_p . Note that the definition of $P_\lambda(Y_p)$ is analogous to the definition of $P_\lambda(\eta)$ in § 2. In a similar fashion we define probability functions $P_\lambda(^2\text{H}/\text{H})$, $P_\lambda(^3\text{He}/\text{H})$, and $P_\lambda(^7\text{Li}/\text{H})$ to give the probability of finding a region of size λ with average number ratios of $(^2\text{H}/\text{H})$, $(^3\text{He}/\text{H})$, and $(^7\text{Li}/\text{H})$, respectively.

In Figures 18a–18c we display these numerically determined probability functions for various scales λ and different models of the distribution of η . The panels display $P_\lambda(Y_p)$, $P_\lambda(^7\text{Li}/\text{H})$, $P_\lambda(^2\text{H}/\text{H})$, and $P_\lambda(^3\text{He}/\text{H})$. Within each panel we show four probability distribution functions determined for the scales $\lambda < \lambda_c$ and $\lambda = 100\lambda_c$, $1000\lambda_c$, and $10000\lambda_c$. Note that in our “one-dimensional” theory the length scale λ is proportional to the mass scale M .

In Figure 18a we show probability distribution functions for a model spatial distribution of η characterized by the transformation in equation (8a), spectral index $n_3 = 0$, $\Delta_{cr} = 1.5$, and $M_c > M_J^b$. Furthermore, we assume a cosmic average baryon-to-photon ratio of $\langle \eta \rangle = 6 \times 10^{-10}$. This corresponds to a value of $\Omega_b h^2 = 0.0224$.

Figure 18a clearly illustrates that there is a finite width to the abundance probability distribution functions. In other words,

FIG. 18a

FIG. 18.—(a) Probability distribution functions as defined in the text: upper left, $P_\lambda(Y_p)$; lower left, $P_\lambda(^7\text{Li}/\text{H})$; upper right, $P_\lambda(^2\text{H}/\text{H})$; lower right, $P_\lambda(^3\text{He}/\text{H})$, respectively. Probability distribution functions are determined on different scales: solid lines, $\lambda = 10,000\lambda_c$; dotted lines, $\lambda = 100\lambda_c$; short-dashed lines, $\lambda = 1000\lambda_c$; long-dashed lines, $\lambda = 10,000\lambda_c$. Here the distribution functions have been computed from a model spatial distribution in η characterized by transformation eq. (8a), spectral index $n_3 = 0$, $M_c > M_J^b$, $\Delta_{cr} = 1.5$, and average baryon-to-photon ratio $\langle\eta\rangle = 6 \times 10^{-10}$. (b) Same as (a), but for eq. (8b), $\Delta_{cr} = 2$, and $\langle\eta\rangle = 1.2 \times 10^{-9}$. (c) Same as (b), but for eq. (8c) and $\langle\eta\rangle = 3 \times 10^{-9}$.

there is a finite probability of finding regions of mass scale M having abundances smaller, or larger, than average cosmic abundances. The widths of the distribution functions decrease as the mass scale (or, equivalently, λ) increases. On the smallest mass scales (solid lines), there is a fairly wide range of about equally probable primordial abundances. On the largest mass scales (long-dashed lines), the intrinsic widths of the probability distribution functions are approximately $4 \times 10^{-3}\langle Y_p \rangle$ for the ^4He mass fraction, $10^{-1}\langle (^2\text{H}/\text{H}) \rangle$ for the deuterium-to-hydrogen number ratio, $5 \times 10^{-2}\langle (^3\text{He}/\text{H}) \rangle$ for the ^3He -to-hydrogen number ratio, and $7 \times 10^{-2}\langle (^7\text{Li}/\text{H}) \rangle$ for the ^7Li -to-hydrogen number ratio. Here the brackets denote the cosmic average abundances for this particular model for the distribution of η .

In Figure 18b we show probability distribution functions for a spatial distribution of η characterized by the transformation in equation (8b), spectral index $n_3 = 0$, $\Delta_{cr} = 2$, and $M_c > M_J^b$. In this model we assume a cosmic average baryon-to-photon ratio of $\langle\eta\rangle = 1.2 \times 10^{-9}$. We observe the same trends and

features in this figure as were observed in Figure 18a. However, the intrinsic width in the distribution functions for the largest scales ($\lambda = 10^4\lambda_c$) are much larger than those in Figure 18a. This is because the distribution investigated in Figure 18b includes large-amplitude fluctuations in η , whereas the fluctuation amplitudes of the distribution investigated in Figure 18a are moderate. On the largest scales we find approximate widths of the distribution functions to be $2 \times 10^{-2}\langle Y_p \rangle$ for the ^4He mass fraction, $0.4\langle (^2\text{H}/\text{H}) \rangle$ for the deuterium-to-hydrogen number ratio, $0.25\langle (^3\text{He}/\text{H}) \rangle$ for the ^3He -to-hydrogen number ratio, and $0.3\langle (^7\text{Li}/\text{H}) \rangle$ for the ^7Li -to-hydrogen number ratio.

In Figure 18c we display distribution functions for a spatial distribution of η characterized by the transformation in equation (8c), spectral index $n_3 = 0$, $\Delta_{cr} = 2$, and $M_c > M_J^b$. For this calculation we have assumed a cosmic average baryon-to-photon ratio of $\langle\eta\rangle = 3 \times 10^{-9}$. For the smallest scales, the probability distribution functions display a peak which is not centered at the cosmic average abundances. This is easily understood by inspection of Figure 1c, where it is obvious that

FIG. 18b

in the present model there exists a minimum baryon-to-photon ratio $\eta_{\min}$. A large fraction of the universe in this model has $\eta_{\min}$, so that the probability of finding a small region with abundance yields pertaining to $\eta_{\min}$ is large. The intrinsic widths of the distribution functions on the largest scales are somewhat smaller than those found in Figure 18b.

These results imply that PIB-like models which have an intermediate-scale, nonlinear fluctuation component would lead to intrinsic primordial abundance variations on small, as well as large, mass scales. Note that a mass scale of $M = 10^4 M_c$ assumed for the probability distribution functions shown in Figure 18 can easily correspond to mass scales as large as $M \sim 10^{11} - 10^{12} M_\odot$, depending on the assumed value of the cutoff mass scale, M_c . We do not expect small- to intermediate-scale primordial abundance variations to survive to the present epoch. This is because mixing mechanisms, such as shock waves induced by supernova explosions, should be efficient enough to mix the primordial material on intermediate mass scales.

It is, however, questionable whether mixing could erase preexisting primordial abundance variations on mass scales as large as $M \sim 10^9 - 10^{11} M_\odot$. In fact, it is well known that the ^4He mass fraction observed in metal-poor, extragalactic H II

regions exhibits significant variation between objects of the same metallicity. Typical ^4He mass fractions are in the range 0.22–0.26. It has been suggested that this spread is not due to observational uncertainties, but rather represents a real physical variation. However, even if these suggestions are correct, there are several chemical evolution effects which may produce such a spread (Campbell 1992).

In any case, the intrinsic spread in the ^4He mass fractions observed in metal-poor H II regions may be used to put upper limits on the magnitude of preexisting primordial abundance variations. In order to derive useful upper limits on large-scale inhomogeneity, the details of the mixing of primordial material would have to be considered. In this way, cosmological models which include primordial, nonlinear, isocurvature baryon number fluctuations could be further constrained.

One might argue that the recent observations of two different $(^2\text{H}/\text{H})$ ratios observed in different high-redshift Ly α cloud systems may already hint the existence of spatial abundance variations. It is widely believed that deuterium abundances observed in high-redshift Ly α clouds directly reflect the primordial deuterium abundance since chemical evolution of this isotope could not have been significant. We wish to caution, however, that the high $(^2\text{H}/\text{H})$ ratio inferred by Songaila et al.

FIG. 18c

may be the result of a hydrogen interloper. Since the probability of hydrogen interlopers may not actually be very large ($\sim 10\%$), we hope future observations of $(^2\text{H}/\text{H})$ ratios in different Ly α clouds may shed light on the existence/absence of spatial abundance variations.

4. CONCLUSIONS

We have examined the primordial nucleosynthesis process in the presence of large mass scale, nonlinear entropy fluctuations. A variety of fluctuation spectra were considered, including some which have the characteristics of the spectra of PIB models extrapolated to smaller mass scales. Our computations provide for the collapse of overdense regions with masses above the local baryon Jeans mass, M_J^b (eq. [4]). The baryons, and hence the nucleosynthesis products, which avoid incorporation into condensed objects were found to originate mostly from underdense regions. A complicating feature of nucleosynthesis calculations in a field of stochastically distributed fluctuations arises from the fact that a particular underdense region may reside inside a larger overdense region which, in turn, might be destined for collapse. In our computations we have included a detailed numerical treatment of this cloud-in-cloud problem, and we have found that such a treatment was

important for the accurate estimation of light-element (^2H , ^3He , ^4He , and ^7Li) abundance yields.

In general, we have found that PIB-like spectra with significant small-scale structure (i.e., structure on mass scales $M < M_J^b$) produced light-element abundance yields which were in conflict with observationally inferred limits for almost any pre-nucleosynthesis Ω_b . This may represent an important nucleosynthesis-based constraint on PIB models for large-scale structure formation, if the underlying microscopic mechanisms which generated fluctuations in these models somehow demanded the presence of nonlinear small-scale structure.

However, if such small-scale fluctuations were absent or suppressed, then our computations have shown that there exists a range of fluctuation spectral characteristics which produce light-element abundance yields in agreement with observationally inferred limits. Particular distributions of baryons were found to produce acceptable nucleosynthesis yields even when the pre-nucleosynthesis baryonic fraction of the closure density was as high as $\Omega_b \approx 0.2 h^{-2}$ (i.e., roughly closure density for small Hubble parameter). However, we have found that such a relaxation of the homogeneous big bang limit on Ω_b would usually require that the primordial abundance of ^7Li be closer to the observed Population I value than to the Spite

“plateau” value. In turn, this would demand that there had been a fair amount of destruction of ${}^7\text{Li}$ in the Spite plateau stars, a conceivable though controversial possibility. Future observations may resolve this issue.

In our models which met abundance constraints, we found that ${}^2\text{H}/\text{H}$ was high and the ${}^4\text{He}$ mass fraction low relative to a homogeneous big bang at a given value of $\Omega_b h^2$. Obviously, this would have to be the case if these models were to meet abundance limits for values of Ω_b that were larger than the limit on this quantity from the homogeneous big bang. The important point is that PIB-like models possessing small-scale fluctuation cutoffs could alter the relative abundances of ${}^2\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$, and ${}^7\text{Li}$ over those from a homogeneous big bang while still meeting abundance constraints within observational uncertainties. More accurate observational determinations of any two of these light-element abundances might provide a signature for or, more likely, a constraint on such PIB-like models.

We have pointed out that a potential signature of a PIB-like distribution of entropy at the nucleosynthesis epoch would be the observation of a significant and *intrinsic* variation in the primordial abundances of the light elements, especially deuterium. Our calculations have predicted that such PIB-like models could produce light-element abundance variations on baryon mass scales up to 10^{10} – $10^{12} M_\odot$.

Note added in manuscript.—It has been brought to our attention that Copi, Olive, & Schramm (1995) have independently examined the problem of primordial nucleosynthesis in the presence of large-scale baryon-to-photon fluctuations. As in our study, these authors apparently find that significant fluctuations on sub-baryon Jeans mass scales produce unacceptable primordial abundance yields. Their work, however, differs from ours in its statistical treatment of the distribution of large-scale fluctuations (i.e., the cloud-in-cloud problem). Furthermore, the authors claim that even in the absence of significant sub-baryon Jeans mass scale fluctuations, nucleosynthesis yields in their work are incompatible with observationally inferred limits. We would reach the same conclusions as they do if we adopted their observationally inferred primordial abundance of ${}^7\text{Li}$ (${}^7\text{Li}/\text{H} \lesssim 2 \times 10^{-10}$).

We would like to acknowledge useful conversations with Craig Hogan and Grant Mathews. K. J. wishes to acknowledge the hospitality of the Department of Astronomy and Astrophysics at the University of Chicago. This work was supported in part by NSF grant PHY 91-21623. It was also performed in part under the auspices of the US Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48 and DoE Nuclear Theory grant no. SF-ENG-48.

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Note added in proof.—We want to draw to the reader's attention that for comparison purposes we have used the reaction rates of Kawano (1992). Recent updates of nuclear reaction rates give very slight differences in calculated light-element abundance yields over what is presented in this paper. The biggest discrepancy would be in the ${}^4\text{He}$ abundance yield increasing Y_p by $\Delta Y_p \sim +0.002$.