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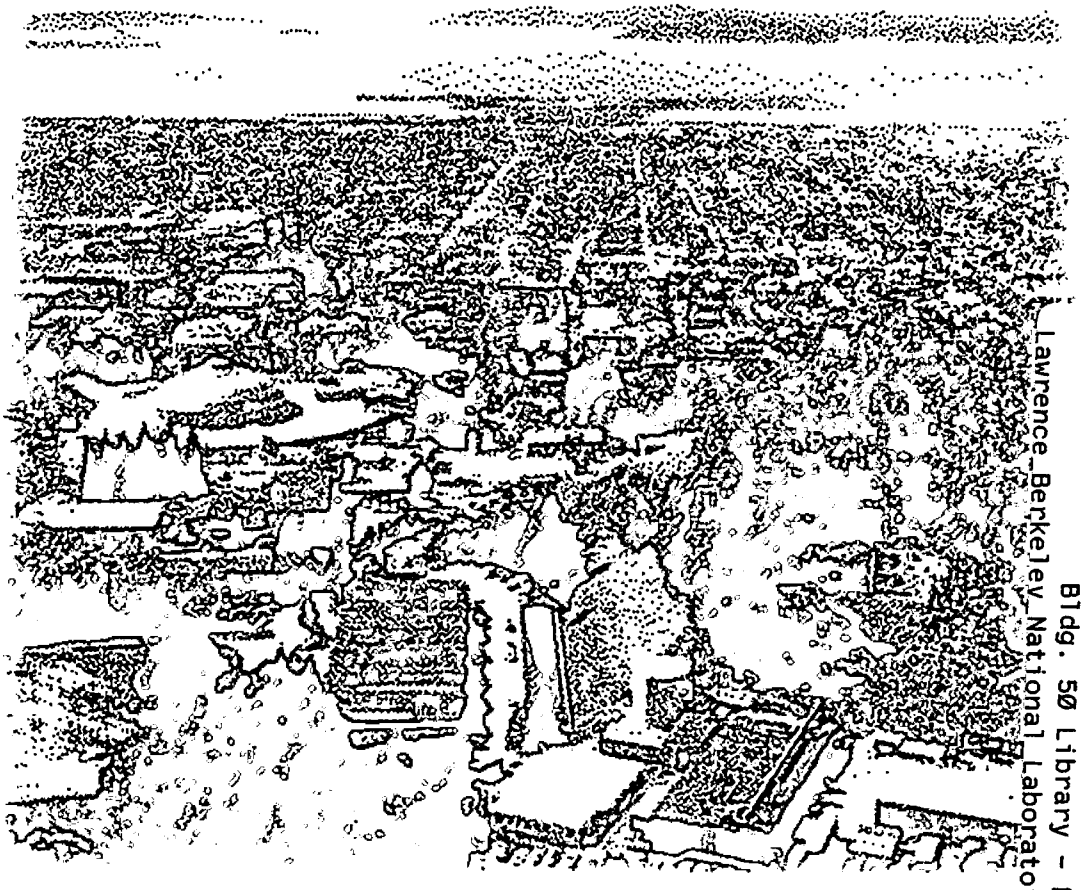
CP Violating $Zt\bar{t}$ and $\gamma t\bar{t}$ Couplings at a Future e^+e^- Collider

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Physics Division

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CP violating $Zt\bar{t}$ and $\gamma t\bar{t}$ Couplings at a Future e^+e^- Collider

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Abstract

The effect of new operators that give rise to CP–violating couplings of the type $Zt\bar{t}$ and $\gamma t\bar{t}$ are examined at future electron positron Linear Colliders (FLC). The impact of these CP-violating interactions over Standard Model predictions was studied for the process $e^+e^- \to t\bar{t}$ with the subsequent decays $t \to bl^+\nu_l$ and $\bar{t} \to \bar{b}l^-\bar{\nu}_l$, called as dilepton mode, and $t \to bl^+\nu_l$ and $\bar{t} \to \bar{b}\bar{q}q'$ or $t \to bq\bar{q}'$ and $\bar{t} \to \bar{b}l^-\bar{\nu}_l$, called as single lepton mode, where the final leptons are $l^\pm = e^\pm$ or $\mu^\pm$, and the final quarks are $q(q') = u(d)$ or $c(s)$. Polarized electron beam and CP observables and asymmetries are used to impose bounds on the anomalous couplings.

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I. INTRODUCTION

Top quark is the heaviest elementary particle observed to date and is hence most sensitive to the mechanism of electroweak symmetry breaking. The top quark couplings to gauge bosons probe the nature of electroweak symmetry breaking and other not well understood aspects of the electroweak interactions [1]. In particular, it would be interesting to investigate whether top couplings conserve CP, a symmetry so far known to be violated only in K-meson system. Possible CP violating couplings of fermions are electric dipole type interactions with the electromagnetic field and the analogous "weak" dipole coupling to the Z field. These can arise, for instance, in certain models of CP violation like the two-Higgs-doublet model [2], in the minimal supersymmetric standard model (MSSM) at one-loop level [3] or in its next-to minimal extension (NMSSM) at tree level [4] even though the order of magnitudes of their estimate is probably well below the experimental sensitivity.

We, however, find that the supersymmetric contribution to the CP-violating top couplings can be sizable. For instance, a gluino exchange together with the stop left-right mixing would produce the electric dipole moment of the order of $e(\alpha_s/\pi)\mathrm{Im}(A^*M_3)m_t/m_{\tilde{t}}^4$ and the form factors defined in Section II can easily be of a few percents if $m_{\tilde{t}} \sim m_t$ which is still allowed. Note that the constraints from the neutron and electron electric dipole moments do not restrict the trilinear coupling A for the stop unless specific assumptions such as the universal trilinear coupling is made.

In this paper, we do not restrict ourselves to any particular model, but parametrize the CP violation in terms of convenient effective form factors proportional to the electric and weak dipole moments of the top-quark.

A high-energy future linear e^+e^- collider (FLC) will provide a very impressive tool to investigate the properties of the top-quark. Since the mass of the top-quark is very high ($m_t = 174.3 \pm 5.1$ GeV) [5], its weak decay takes place before it can hadronize and hence it can be studied in a much cleaner way than other quarks. Moreover, since all theories involving CP violation effects in the electroweak coupling of fermions are expected to be proportional to their mass, the top-quark is a privileged candidate for observing such effects [6].

In this paper we will study possible CP violating effects due to anomalous form factors to the vertex $(Z,\gamma)t\bar{t}$ [7,8] in the top-quark production at an e^+e^- collider, *i.e.*, $e^+e^- \to Z,\gamma \to t\bar{t}$. These form factors are presented in Section II.

There have been several studies to measure possible CP violating effects due to non standard $Zt\bar{t}$ and $\gamma t\bar{t}$ couplings. Various experiments have been suggested to perform these measurements by making use of CP-odd quantities (see Ref. [6,9] and references therein). In this paper we study the impact of CP violating $Zt\bar{t}$ and $\gamma t\bar{t}$ couplings using two sets of CP-odd observables [6,9,10], by studying their expectation values and their corresponding asymmetries defined in Section III.

Moreover, effects of a possible highly polarized electron beam ($\pm 90\%$) at FLC will be considered in our analyses of CP violating $Zt\bar{t}$ and $\gamma t\bar{t}$ couplings. Our results are presented in Section IV. Finally, we draw our conclusions in Section V.

II. THE GENERAL FORM FACTORS

In order to study the effects of CP violating form factors to the vertex $(Z,\gamma)t\bar{t}$, we use the most general form factors for the coupling of t and $\bar{t}$ with either Z or γ defined in Ref. [7],

$$\Gamma^{\mu}_{Vt\bar{t}} = ig\Big[\gamma^{\mu}(F_1^{V(L)}P_- + F_1^{V(R)}P_+) - \frac{i\sigma^{\mu\nu}k_\nu}{m_t}(F_2^{V(L)}P_- + F_2^{V(R)}P_+) + k^{\mu}(F_3^{V(L)}P_- + F_3^{V(R)}P_+)\Big], \tag{1}$$

were $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$, $i\sigma^{\mu\nu} = -\frac{1}{2}[\gamma^{\mu},\gamma^{\nu}]$, m_t is the top mass, k^{μ} is the momentum of the gauge boson V and is taken by convention to be directed into the vertex. V can be the Z gauge boson or photon A, and the F's are the form factors for V. When $V = A$, $F_3^{A(L)}$ and $F_3^{A(R)}$ have to vanish as a result of gauge invariance (or current conservation). For a Z boson which is on shell or coupled to massless fermions, the $F_3^{Z(L)}$ and $F_3^{Z(R)}$ contributions vanish. In our case we will ignore these F_3 contributions. The Standard Model values for the form factors at tree level are:

$$\begin{aligned} F_{1_{SM}}^{Z(L)} &= \frac{1}{\cos\theta_W}\left[\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right] \ , \ F_{1_{SM}}^{Z(R)} = \frac{1}{\cos\theta_W}\left[-\frac{2}{3}\sin^2\theta_W\right] \ , \\ F_{1_{SM}}^{A(L)} &= F_{1_{SM}}^{A(R)} = \frac{2}{3}\sin\theta_W \ , \\ F_{2_{SM}}^{Z(L)} &= F_{2_{SM}}^{Z(R)} = F_{2_{SM}}^{A(L)} = F_{2_{SM}}^{A(R)} = 0 \ , \end{aligned} \tag{2}$$

where θ_W is the weak mixing angle.

Applying the Gordon decomposition, equation (1) becomes

$$\Gamma^{\mu}_{Vt\bar{t}} = \frac{ig}{2}\left[\gamma^{\mu}(A^V - B^V\gamma^5) + \frac{t^{\mu} - \bar{t}^{\mu}}{2}(C^V - D^V\gamma^5)\right], \tag{3}$$

where

$$\begin{aligned} A^V &= F_1^{V(L)} + F_1^{V(R)} - 2(F_2^{V(L)} + F_2^{V(R)}), \\ B^V &= F_1^{V(L)} - F_1^{V(R)}, \\ C^V &= \frac{2}{m_t}\left(F_2^{V(L)} + F_2^{V(R)}\right), \\ D^V &= \frac{2}{m_t}\left(F_2^{V(L)} - F_2^{V(R)}\right). \end{aligned} \tag{4}$$

In equation (3), t^{μ} ($\bar{t}^{\mu}$) is the momentum of the outgoing t ($\bar{t}$). The Standard Model values at tree level of these last set of form factors are

$$\begin{aligned} A_{SM}^Z &= \frac{1}{\cos\theta_W}\left[\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right] \ , \ A_{SM}^A = \frac{4}{3}\sin\theta_W \ , \\ B_{SM}^Z &= \frac{1}{2\cos\theta_W} \ , \ B_{SM}^A = 0 \ , \\ C_{SM}^Z &= C_{SM}^A = D_{SM}^Z = D_{SM}^A = 0. \end{aligned} \tag{5}$$

Beyond the tree level, all of them except D^V ($V = Z$ or A), which controls the CP violation, have contributions due to loop corrections in the SM provided we ignore the small CP-violating effects which reside in the Yukawa couplings that govern the interactions between the Higgs boson and the quarks [8]. These CP-violating amplitudes in the Cabibbo-Kobayashi-Maskawa model are typically suppressed by a factor of order 10^{-12} [11].

Since we are interested in possible non-standard CP-violating effects of the vertex $(Z,\gamma)t\hat{t}$, we will analyze the impact of the form factor D^V in the process $e^+e^- \to (Z,\gamma) \to t\bar{t}$. For convenience, we define a dimensionless CP-violating coupling constants $d^V = (m_t/2)D^V$ and from equation (4) it is obvious that

$$d^V = (F_2^{V(L)} - F_2^{V(R)}). \tag{6}$$

The Standard Model value at tree level for d^V is zero. The impact of non-vanishing values for d^V in the processes (7) will be studied here through the analysis of the processes

$$e^+e^- \to t(\to bl^+\nu_l)\ \ \bar{t}(\to \bar{b}l^-\bar{\nu}_l)\ , \tag{7}$$

where the final leptons $l^\pm$ are $e^\pm$ or $\mu^\pm$, called dilepton mode and,

$$e^+e^- \to t(\to bq\bar{q}')\ \ \bar{t}(\to \bar{b}l^-\bar{\nu}_l)\ , \tag{8}$$
$$e^+e^- \to t(\to bl^+\nu_l)\ \ \bar{t}(\to \bar{b}\bar{q}q')\ , \tag{9}$$

where the final quarks $q(q')$ are the up(down)-quarks $u(d)$ or $c(s)$, called single lepton mode. Eq. (8) will be called sample $\mathcal{T}$, while Eq. (9) will be called sample $\bar{\mathcal{T}}$ of the single lepton decay mode.

In order to compute these contributions, we have incorporated all anomalous couplings in HELAS–type [12] Fortran subroutines. These new subroutines were used to adapt a Madgraph [13] output to include all the anomalous contributions. We have checked that our code is able to reproduce the results for helicity amplitudes Eq. (2.8) of Ref. [8]. We employed Vegas [14] to perform the Monte Carlo phase space integration with the appropriate cuts to obtain the differential and total cross sections of the processes (7), (8), and (9).

III. OBSERVABLES AND ASYMMETRIES

The effects of CP-violating form factors of the vertex $(Z,\gamma)t\bar{t}$ can be traced through the analysis of the behavior of some convenient CP observables. For the dilepton decay channel of the top-quark pair production at FLC we will consider two sets of observables.

The first set of observables was defined in Ref. [6,9] in order to study the impact of CP-invariant form factor of the vertex $Vt\bar{t}$ ($V = Z,\gamma$). It consists in the following two observables:

$$O_1 = (\hat{p}_b \times \hat{p}_{\bar{b}}) \cdot \hat{p}_{e^+}\ , \tag{10}$$
$$O_2 = (\hat{p}_b + \hat{p}_{\bar{b}}) \cdot \hat{p}_{e^+}\ , \tag{11}$$

where $\hat{p}_b$ and $\hat{p}_{\bar{b}}$ are the b, $\bar{b}$ momentum directions in the e^+e^- CM frame and $\hat{p}_{e^+}$ is the momentum direction of the positron. The observable O_1 is CP odd but CPT even, and

probes the imaginary part of the CP-violating form factors $[Im(d^{Z,\gamma})]$, while the observable O_2 is both CP and CPT odd and probes the real part of the CP-violating form factors $[Re(d^{Z,\gamma})]$. A CPT-odd observable can only have a non-zero value in the presence of an absorptive part of the amplitude [15].[1]

The second set was defined in Ref. [10] in order to study effects of Higgs sector CP violation in top-quark pair production. It consists in the following two observables:

$$Q_1 = \hat{p}_t \cdot \hat{q}_+ - \hat{p}_{\bar{t}} \cdot \hat{q}_- \ , \tag{12}$$

$$Q_2 = \frac{1}{2}(\hat{p}_t - \hat{p}_{\bar{t}}) \cdot (\hat{q}_- \times \hat{q}_+) \ , \tag{13}$$

where $\hat{p}_t$ and $\hat{p}_{\bar{t}}$ are the t, $\bar{t}$ momentum directions in the e^+e^- CM frame and $\hat{q}_-$ and $\hat{q}_+$ are the l^+, l^- momentum directions in the t and $\bar{t}$ rest frames, respectively. The observable Q_1 is CP odd but T even, i. e. do not change sign under a naive T transformation, and probes the real part of the CP-violating form factors $[Re(d^{Z,\gamma})]$, while the observable O_2 is both CP and T odd and probes the imaginary part of the CP-violating form factors $[Im(d^{Z,\gamma})]$.

For the single lepton decay channel of the top-quark pair production at FLC we will also consider two sets of observables. The first set of observables consists of the observables of Eqs. (10) and (11), *i.e.*, the same set of observables for the dilepton decay channel.

However, the second set of observables must not be the same of the dilepton decay channel [Eqs. (12) and (13)]. Instead we use the following observables:

For the sample $e^+e^- \to t(\to bl^+\nu_l)\bar{t}(\to \bar{b}q\bar{q}')$ (sample $\mathcal{T}$) they define

$$Q_1^{(t)} = \hat{p}_t \cdot \hat{q}_+ \ , \tag{14}$$

$$Q_2^{(t)} = \hat{p}_t \cdot (\hat{q}_+ \times \hat{q}_{\bar{b}}) \ , \tag{15}$$

where $\hat{q}_{\bar{b}}$ is the momentum direction of the $\bar{b}$ quark jet in the $\bar{t}$ quark rest frame, while for the process $e^+e^- \to t(\to b\bar{q}q')\bar{t}(\to \bar{b}l^-\bar{\nu}_l)$ (sample $\bar{\mathcal{T}}$),

$$Q_1^{(\bar{t})} = \hat{p}_{\bar{t}} \cdot \hat{q}_- \ , \tag{16}$$

$$Q_2^{(\bar{t})} = \hat{p}_{\bar{t}} \cdot (\hat{q}_- \times \hat{q}_b) \ , \tag{17}$$

where $\hat{q}_b$ is the momentum direction of the b quark jet in the t quark rest frame. Taking both samples one can define the quantities

$$\epsilon_1 = < Q_1^{(t)} > - < Q_1^{(\bar{t})} > \ , \tag{18}$$

$$\epsilon_2 = < Q_2^{(t)} > + < Q_2^{(\bar{t})} > \ . \tag{19}$$

The quantity ϵ_1 probes the real part of the CP-violating form factors $[Re(d^{Z,\gamma})]$, while ϵ_2 probes the imaginary part of the CP-violating form factors $[Im(d^{Z,\gamma})]$.

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[1]Here and below, we have the "naive T" in mind where spins and momenta are reversed but the initial and final states are not interchanged. Therefore, CPT-odd observables do not imply the true CPT violation which is of course impossible in quantum field theories.

We also define corresponding asymmetries which should be experimentally more robust than equations (10,11,12,13, 18,19), because only the signs of $O_{1,2}$, $Q_{1,2}$, and $\epsilon_{1,2}$ have to be measured. For the first set of observables, we define the asymmetry for both single and di-lepton decay channels, as follows

$$A_{O_{1,2}} = \frac{N(O_{1,2} > 0) - N(O_{1,2} < 0)}{N(O_{1,2} > 0) + N(O_{1,2} < 0)} \ , \tag{20}$$

where N is the number of $t\bar{t}$ events in the single and di- lepton decay channels.

For the second set of observables we define the asymmetry as follows: for the dilepton decay channel,

$$A_{Q_{1,2}} = \frac{N(Q_{1,2} > 0) - N(Q_{1,2} < 0)}{N(Q_{1,2} > 0) + N(Q_{1,2} < 0)} \ , \tag{21}$$

where N is the number of $t\bar{t}$ events in the dilepton decay channels.

For the single lepton decay channel,

$$A(\epsilon_1) = \frac{N_{\mathcal{T}}(Q_1^{(t)} > 0) - N_{\mathcal{T}}(Q_1^{(t)} < 0)}{N_{\mathcal{T}}} - \frac{N_{\bar{\mathcal{T}}}(Q_1^{(\bar{t})} > 0) - N_{\bar{\mathcal{T}}}(Q_1^{(\bar{t})} < 0)}{N_{\bar{\mathcal{T}}}} \ , \tag{22}$$

$$A(\epsilon_2) = \frac{N_{\mathcal{T}}(Q_2^{(t)} > 0) - N_{\mathcal{T}}(Q_2^{(t)} < 0)}{N_{\mathcal{T}}} + \frac{N_{\bar{\mathcal{T}}}(Q_2^{(\bar{t})} > 0) - N_{\bar{\mathcal{T}}}(Q_2^{(\bar{t})} < 0)}{N_{\bar{\mathcal{T}}}} \ , \tag{23}$$

where $N_{\mathcal{T}}$ and $N_{\bar{\mathcal{T}}}$ are the number of $t\bar{t}$ events in samples $\mathcal{T}$ and $\bar{\mathcal{T}}$, respectively.

The sensibility of non-null values of the CP-violating form factors $d^{Z,\gamma}$ over these two sets of observables and correspondent asymmetries are summarized in TABLE I.

IV. RESULTS

The impact of the CP-violating form factors described in Section II in the top-quark pair production and subsequent decay into 2 jets plus 2 leptons (dilepton mode), and into 4 jets plus 1 lepton (single lepton mode) is analyzed for a FLC with CM energy of 500 GeV. Polarization effects of the electron beam is also considered. We assume two runs at FLC, one with 90% left hand polarized electrons ($\mathcal{P}_{e^-}^-$) and the other run with 90% right hand polarized electrons ($\mathcal{P}_{e^-}^+$), both with integrated luminosity of 50 fb^{-1}. We have considered $m_t = 175$ GeV in our analysis.

A discussion concerning event selection and backgrounds, that can be found in Ref. [16] and references therein, is briefly summarized here. The $t\bar{t}$ cross section at an FLC with $\sqrt{s} = 500$ GeV is roughly 0.5 pb. On the other hand, the cross section for lepton and light quark pairs is about 16 pb, while for W^+W^- production is about 8 pb. The emphasis of most event selection strategies has been to take advantage of the multi-jet topology of the roughly 90% of $t\bar{t}$ events with 4 or 6 jets in the final state. Therefore, cuts on thrust or number of jets drastically reduces the light fermion pair background. In addition, one can use the multi-jet mass constraints M(jet-jet) $\approx M_W$ and M(3-jet) $\approx m_t$ for the cases involving $t \to bqq'$. The background due to W-pair production is the most difficult to eliminate. However, in the limit that the electron is fully right-handed polarized, the W^+W^- cross section is reduced

to about 30 fb. Hence, even though the beam polarization will not reach 100%, this allows for experimental control and measurement of the background. Another important technique that can be used is that of precision vertex detection. The small and stable interaction point of linear e^+e^- colliders, along with the small beam sizes and bunch-structure timing, make then ideal for pushing the techniques of vertex detection.

The Standard Model total cross sections of $t\bar{t}$ production at FLC with $\sqrt{s} = 500$ GeV obtained by our Monte Carlo simulation are:

$$\sigma_{e^+e^-\to t\bar{t}}(\mathcal{P}_{e^-}^-) = 777.3(3) \text{ fb} , \tag{24}$$

$$\sigma_{e^+e^-\to t\bar{t}}(\mathcal{P}_{e^-}^+) = 373.8(1) \text{ fb} . \tag{25}$$

We conservatively assume $[W^-(\to l^-\bar{\nu}_l)W^+(\to l^+\nu_l)]$, $[W^-(\to l^-\bar{\nu}_l)W^+(\to q\bar{q}')]$, $[W^-(\to \bar{q}q')W^+(\to l^+\nu_l)]$ tagging efficiencies of about 80% and a b and $\bar{b}$ tagging efficiency also of 80%. The overall $b-$, $\bar{b}-$, and $W-$ tagging efficiency would then be about $(80\%)^3 = 51.2\%$. In our calculations we consider an overall tagging efficiency of 50% ($f_{eff} = 0.5$). Considering the leptons being only electron and muon, the branching ratio of the dilepton decay mode is $BR = \frac{4}{81}$. For the single lepton decay, when the final quarks are the quarks up, down, charm, and strange, the branching ratio is $BR = \frac{24}{81}$. The number of events in each decay mode of the top-quark pair production at FLC is given by $N = \sigma.\mathcal{L}.f_{eff}.BR$ and is shown in TABLE II.

A. Expectation Values

The observables defined in Section III acquire non-vanishing expectation values in the presence of CP-violating anomalous couplings $Zt\bar{t}$ and $\gamma t\bar{t}$. Expectation values of observables are defined as usual by

$$\langle \mathcal{O} \rangle = \frac{\int d\sigma \mathcal{O}}{\int d\sigma} . \tag{26}$$

To be statistically significant, the expectation values of an observable $\mathcal{O}$ must be larger then its expected natural variances $\langle (\mathcal{O} - \langle \mathcal{O} \rangle)^2 \rangle$.[2] A signal of η standard deviations is obtained for a sample of N events if

$$\langle \mathcal{O} \rangle \geq \eta \sqrt{\frac{\langle \mathcal{O}^2 \rangle}{N}} . \tag{27}$$

In order to obtain bounds on the anomalous form factors $d^{Z,\gamma}$ we have evaluated numerically, for the first set of observables, the fraction

$$\mathcal{F}_{O_{1,2}} = \frac{\langle O_{1,2} \rangle}{\sqrt{\langle O_{1,2}^2 \rangle}} , \tag{28}$$

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[2] Of course $\langle \mathcal{O} \rangle = 0$ in the Standard Model.

for different values of the form factors $d^{Z,\gamma}$ for both dilepton and single lepton modes.

For the second set of observables we have evaluated numerically, for the dilepton mode, the fractions,

$$\mathcal{F}_{Q_{1,2}} = \frac{\langle Q_{1,2} \rangle}{\sqrt{\langle Q_{1,2}^2 \rangle}} \ , \tag{29}$$

while for the single lepton mode we have evaluated numerically the fractions,

$$\mathcal{F}_{Q_{1,2}^{(t)}} = \frac{\langle Q_{1,2}^{(t)} \rangle}{\sqrt{\langle Q_{1,2}^{(t)\,2} \rangle}} \quad , \quad \mathcal{F}_{Q_{1,2}^{(\bar{t})}} = \frac{\langle Q_{1,2}^{(\bar{t})} \rangle}{\sqrt{\langle Q_{1,2}^{(\bar{t})\,2} \rangle}},$$

respectively for the samples $\mathcal{T}$ and $\bar{\mathcal{T}}$ for different values of the form factors $d^{Z,\gamma}$. Then we evaluate the following quantities,

$$\mathcal{F}_{\epsilon_1} = \mathcal{F}_{Q_1^{(t)}} - \mathcal{F}_{Q_1^{(\bar{t})}} \ , \tag{30}$$

$$\mathcal{F}_{\epsilon_2} = \mathcal{F}_{Q_2^{(t)}} + \mathcal{F}_{Q_2^{(\bar{t})}} \ . \tag{31}$$

A 95% CL bound is obtained when $\eta = \pm 1.96$, so calling by $\mathcal{F}$ the quantities of Eqs. (28, 29, 30, 31), we have to observe

$$|\mathcal{F}| \geq \frac{|\eta|}{\sqrt{N_{events}}} = \frac{1.96}{\sqrt{N_{events}}}, \tag{32}$$

where the total number of events of both samples, for each polarization mode of the electron beam, which is presented in TABLE II. Our results are presented in TABLE IV for the dilepton decay mode and in TABLE VI for the single lepton decay mode.

B. Asymmetries

The asymmetry in the observable $\mathcal{O}$ is defined by

$$A_{\mathcal{O}} \equiv \frac{N(\mathcal{O} > 0) - N(\mathcal{O} < 0)}{N(\mathcal{O} > 0) + N(\mathcal{O} < 0)} \, . \tag{33}$$

The asymmetry is predicted to be zero in the Standard Model for all observables defined in Section III. The Gaussian fluctuation in the asymmetry is given by

$$\langle (A_{\mathcal{O}} - \langle A_{\mathcal{O}} \rangle)^2 \rangle = 4 \frac{N(\mathcal{O} > 0) N(\mathcal{O} < 0)}{(N(\mathcal{O} > 0) + N(\mathcal{O} < 0))^3} = \frac{1}{N_{events}}, \tag{34}$$

where vanishing asymmetry $N(\mathcal{O} > 0) = N(\mathcal{O} < 0)$ was assumed in the last equality.

Hence, from Eqs. (20,21,22, 23), and (34), a 95% CL deviation is obtained when one measures the asymmetry

$$A_{\mathcal{O}}^{95\%CL} = \frac{\pm 1.96}{\sqrt{N_{events}}} \ . \tag{35}$$

We present in TABLE III the value of the quantity $A_{\mathcal{O}}$ needed to obtain a 95% CL deviation from the Standard Model prediction considering the total number of events N presented in TABLE II for each decay channel mode of the top-quark pair production at FLC. Once again, we have evaluated numerically the quantity $A_{\mathcal{O}}$ for different values of the form factors $d^{Z,\gamma}$ in order to obtain a 95% CL CP violating signal. Our results are presented in TABLE V for the dilepton decay mode and in TABLE VII for the single lepton decay mode.

C. Improving the Limits

In order to improve the limits obtained for each polarization mode of the electron beam, we combine the results of both modes. We define,

$$\mathcal{F}^{\pm} = \mathcal{F}(\mathcal{P}_{e^-}^-) \pm \mathcal{F}(\mathcal{P}_{e^-}^+) \ ,$$
$$A_{\mathcal{O}}^{\pm} = A_{\mathcal{O}}(\mathcal{P}_{e^-}^-) \pm A_{\mathcal{O}}(\mathcal{P}_{e^-}^+) \ .$$

The number of events for these new quantities is

$$N_{events} = N_{events}(\mathcal{P}_{e^-}^-) + N_{events}(\mathcal{P}_{e^-}^+).$$

TABLES IV, V, VI, and VII show the improved limits for these quantities.

V. CONCLUSIONS

The effect of new operators that give rise to CP–violating couplings of the type $Zt\bar{t}$ and $\gamma t\bar{t}$ were examined at future electron positron Linear Colliders (FLC). The impact of these CP-violating interactions over Standard Model predictions was studied for the process $e^+e^- \to t\bar{t}$ with the subsequent decays into a pair of b jets plus four leptons (dilepton mode), and decays into a pair of b jets plus a pair of light quark jets plus a pair of leptons (single lepton mode).

Polarized electron beam and two set of CP observables and asymmetries were used to impose bounds on the anomalous couplings. The first set of observables was defined in Ref. [6,9], while the second one was defined in Ref. [10].

Our evaluations show that, for the dilepton mode, the second set of observables provides better results than the first one. This is more evident for the real part of the anomalous form factors $d^{Z,\gamma}$, as one can see in Tables IV and V. However, for the single lepton mode, the first set of observables is the one that provides better results. Once again, this is more evident for the real part of the anomalous form factors $d^{Z,\gamma}$, as shown in Tables VI and VII.

According to the statement that the study of the asymmetries is experimentally more robust than evaluation of expectation values because only the signs of the observables have to be measured, the measurement of asymmetries can be an important tool in the search for CP-violating effects in $t\bar{t}$ production at a future linear e^+e^- collider. Our results show that the bounds obtained for the expectation values analyses on Tables IV and VI and the bounds from the asymmetry analyses on V and VII are very similar. So we conclude that the study of the asymmetries should be experimentally easier, with good results. Furthermore the sensitivity approaches the order of magnitudes which can arise in supersymmetric theories.

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REFERENCES

[1] R. D. Peccei and X. Zhang, Nucl. Phys. **B337** (1990) 269; R. D. Peccei and X. Zhang, Nucl. Phys. **B349** (1991) 305; E. Boos, L. Dudko and T. Ohl,Eur. Phys. J. **C11** (1999) 473; F. del Aguila, hep-ph/9911399.

[2] T. D. Lee, Phys. Rev. **D8** (1973) 1226; T. D. Lee, Phys. Rep. **C9** (1974) 143; G. C. Branco and M. N. Rebelo, Phys. Lett. **B160** (1985) 117; J. Liu and L. Wolfenstein, Nucl. Phys. **B289** (1987) 1; S. Weinberg, Phys. Rev. **D42** (1990) 860.

[3] K. S. Babu, C. Kolda, J. March-Russell and F. Wilczek, Phys. Rev. **D59** (1999) 016004; A. Pilaftsis, Phys. Lett. **B435** (1998) 88.

[4] M. Matsuda and M. Tanimoto, Phys. Rev. **D52** (1995) 3100; N. Haba, Prog. Theor. Phys. **97** (1997) 301.

[5] C. Caso *et al.*, Eur. Phys. J. **C3** (1998) 1, and 1999 off-year partial update for the 2000 edition available on the PDG WWW pages (URL: http://pdg.lbl.gov/)

[6] F. Cuypers, hep-ph/9510255.

[7] G. L. Kane, G. A. Ladinsky and C. -P. Yuan, Phys. Rev. **D45** (1992) 124.

[8] G. A. Ladinsky and C. -P. Yuan, Phys. Rev. **D49** (1994) 4415.

[9] F. Cuypers and S. D. Rindani, Phys. Lett. **B343** (1995) 333.

[10] W. Bernreuther, A. Brandenburg and M. Flesch, hep-ph/9812387.

[11] C. Jarlskog, Phys. Rev. **D35** (1987) 1685.

[12] H. Murayama, I. Watanabe and K. Hagiwara, KEK report 91–11 (1992), unpublished.

[13] T. Stelzer and W. F. Long, Comput. Phys. Commun. **81** (1994) 357.

[14] G. P. Lepage, J. Comp. Phys. **27** (1978) 192.

[15] B. Ananthanarayan and S. D. Rindani, Phys. Rev. **D50** (1994) 4447; B. Ananthanarayan and S. D. Rindani, Phys. Rev. Lett. **73** (1994) 1215.

[16] R. Frey, hep-ph/9606201; R. Frey *et al.*, hep-ph/9704243.

TABLES

Form Factor	Dilepton Mode	Single Lepton Mode
$Re[d^{Z,\gamma}]$	O_2 and Q_1	O_2 and ϵ_1
$Im[d^{Z,\gamma}]$	O_1 and Q_2	O_1 and ϵ_2

TABLE I. Sensibility of the observables $O_{1,2}$, $Q_{1,2}$ and $\epsilon_{1,2}$ to the CP-invariant form factor $d^{Z,\gamma}$.

Polarization Mode	Dilepton Mode	Single Lepton Mode
$\mathcal{P}^-_{e^-}$	960	5758
$\mathcal{P}^+_{e^-}$	461	2769
$\mathcal{P}^-_{e^-} + \mathcal{P}^+_{e^-}$	1421	8527

TABLE II. Expected number of events per each channel decay mode of $t\bar{t}$ production at FLC with $\sqrt{s} = 500\text{GeV}$, $\mathcal{L} = 50\text{fb}^{-1}$, and a conservative overall tagging efficiency of 50%.

Polarization Mode	Dilepton Mode	Single Lepton Mode
$\mathcal{P}^-_{e^-}$	± 6.33 %	± 2.58 %
$\mathcal{P}^+_{e^-}$	± 9.13 %	± 3.72 %
$\mathcal{P}^-_{e^-} + \mathcal{P}^+_{e^-}$	± 5.20 %	± 2.12 %

TABLE III. Expected values for the fraction $\mathcal{F}$ or for the asymmetry A_O of a CP-observable for a 95% CL deviation from the Standard Model prediction.

Expected Value $\mathcal{F}$	$\mathcal{P}^-_{e^-}$	$\mathcal{P}^+_{e^-}$	$\mathcal{P}^-_{e^-} + \mathcal{P}^+_{e^-}$	$\mathcal{P}^-_{e^-} - \mathcal{P}^+_{e^-}$
$Im(d^\gamma)$ from O_1	(−0.130 , 0.129)	(−0.181 , 0.178)	(−2.59 , 2.54)	(−0.053 , 0.052)
$Im(d^\gamma)$ from Q_2	(−0.119 , 0.121)	(−0.173 , 0.173)	(−0.049 , 0.050)	(−158 , 160)
$Im(d^Z)$ from O_1	(−0.192 , 0.193)	(−0.260 , 0.257)	(−0.077 , 0.076)	(−2.19 , 2.11)
$Im(d^Z)$ from Q_2	(−0.192 , 0.188)	(−0.366 , 0.366)	(−0.624 , 0.612)	(−0.091 , 0.088)
$Re(d^\gamma)$ from O_2	(−0.300 , 0.299)	(−0.260 , 0.259)	(−0.093 , 0.092)	(−0.372 , 0.370)
$Re(d^\gamma)$ from Q_1	(−0.127 , 0.128)	(−0.176 , 0.174)	(−0.051 , 0.051)	(−1.96 , 1.91)
$Re(d^Z)$ from O_2	(−0.472 , 0.461)	(−0.516 , 0.517)	(−1.25 , 1.29)	(−0.169 , 0.164)
$Re(d^Z)$ from Q_1	(−0.184 , 0.184)	(−0.387 , 0.383)	(−0.487 , 0.495)	(−0.090 , 0.089)

TABLE IV. Expected 95% CL bounds on $d^{Z,\gamma}$ from the expectation value (fraction $\mathcal{F}$) of the observables at FLC for the dilepton decay mode.

Asymmetry	$\mathcal{P}_{e^-}^-$	$\mathcal{P}_{e^-}^+$	$\mathcal{P}_{e^-}^- + \mathcal{P}_{e^-}^+$	$\mathcal{P}_{e^-}^- - \mathcal{P}_{e^-}^+$
$Im(d^\gamma)$ from $A(O_1)$	(−0.155 , 0.157)	(−0.201 , 0.199)	(−1.04 , 1.01)	(−0.061 , 0.060)
$Im(d^\gamma)$ from $A(Q_2)$	(−0.118 , 0.120)	(−0.129 , 0.129)	(−0.041 , 0.042)	(−0.273 , 0.273)
$Im(d^Z)$ from $A(O_1)$	(−0.234 , 0.236)	(−0.322 , 0.318)	(−0.094 , 0.093)	(−3.35 , 3.24)
$Im(d^Z)$ from $A(Q_2)$	(−0.183 , 0.180)	(−0.283 , 0.282)	(−2.03 , 1.99)	(−0.079 , 0.076)
$Re(d^\gamma)$ from $A(O_2)$	(−0.362 , 0.366)	(−0.311 , 0.308)	(−0.111 , 0.111)	(−0.435 , 0.423)
$Re(d^\gamma)$ from $A(Q_1)$	(−0.170 , 0.168)	(−0.164 , 0.162)	(−0.057 , 0.055)	(−0.281 , 0.277)
$Re(d^Z)$ from $A(O_2)$	(−0.534 , 0.530)	(−0.611 , 0.612)	(−1.71 , 1.72)	(−0.194 , 0.193)
$Re(d^Z)$ from $A(Q_1)$	(−0.223 , 0.218)	(−0.357 , 0.353)	(−1.68 , 1.66)	(−0.098 , 0.093)

TABLE V. Expected 95% CL bounds on $d^{Z,\gamma}$ from the asymmetry of the observables at FLC for the dilepton decay mode.

Fraction $\mathcal{F}$	$\mathcal{P}_{e^-}^-$	$\mathcal{P}_{e^-}^+$	$\mathcal{P}_{e^-}^- + \mathcal{P}_{e^-}^+$	$\mathcal{P}_{e^-}^- - \mathcal{P}_{e^-}^+$
$Im(d^\gamma)$ from O_1	(−0.053 , 0.053)	(−0.075 , 0.074)	(−1.49 , 1.42)	(−0.022 , 0.021)
$Im(d^\gamma)$ from ϵ_2	(−0.056 , 0.058)	(−0.067 , 0.071)	(−0.020 , 0.023)	(−0.239 , 0.257)
$Im(d^Z)$ from O_1	(−0.079 , 0.078)	(−0.106 , 0.102)	(−0.032 , 0.029)	(−0.766 , 0.724)
$Im(d^Z)$ from ϵ_2	(−0.095 , 0.088)	(−0.176 , 0.169)	(−0.232 , 0.316)	(−0.046 , 0.039)
$Re(d^\gamma)$ from O_2	(−0.126 , 0.125)	(−0.107 , 0.107)	(−0.038 , 0.038)	(−0.149 , 0.150)
$Re(d^\gamma)$ from ϵ_1	(−1.12 , 1.12)	(−1.11 , 1.11)	(−0.375 , 0.375)	(−2.03 , 2.02)
$Re(d^Z)$ from O_2	(−0.189 , 0.179)	(−0.248 , 0.243)	(−1.80 , 1.85)	(−0.076 , 0.069)
$Re(d^Z)$ from ϵ_1	(−1.52 , 1.52)	(−2.76 , 2.73)	(−6.09 , 6.25)	(−0.702 , 0.687)

TABLE VI. Expected 95% CL bounds on $d^{Z,\gamma}$ from the expectation value (fraction $\mathcal{F}$) of the observables at FLC for the single lepton decay mode.

Asymmetry	$\mathcal{P}_{e^-}^-$	$\mathcal{P}_{e^-}^+$	$\mathcal{P}_{e^-}^- + \mathcal{P}_{e^-}^+$	$\mathcal{P}_{e^-}^- - \mathcal{P}_{e^-}^+$
$Im(d^\gamma)$ from $A(O_1)$	(−0.063 , 0.064)	(−0.083 , 0.082)	(−0.487 , 0.469)	(−0.025 , 0.025)
$Im(d^\gamma)$ from $A(\epsilon_2)$	(−0.055 , 0.056)	(−0.061 , 0.064)	(−0.019 , 0.021)	(−0.158 , 0.172)
$Im(d^Z)$ from $A(O_1)$	(−0.092 , 0.095)	(−0.134 , 0.139)	(−0.039 , 0.038)	(−7.07 , 7.71)
$Im(d^Z)$ from $A(\epsilon_2)$	(−0.101 , 0.094)	(−0.158 , 0.152)	(−0.870 , 0.858)	(−0.045 , 0.039)
$Re(d^\gamma)$ from $A(O_2)$	(−0.153 , 0.156)	(−0.128 , 0.127)	(−0.046 , 0.046)	(−0.172 , 0.166)
$Re(d^\gamma)$ from $A(\epsilon_1)$	(−1.18 , 1.18)	(−1.23 , 1.23)	(−0.406 , 0.406)	(−2.52 , 2.52)
$Re(d^Z)$ from $A(O_2)$	(−0.207 , 0.204)	(−0.238 , 0.238)	(−0.680 , 0.694)	(−0.075 , 0.075)
$Re(d^Z)$ from $A(\epsilon_1)$	(−1.57 , 1.57)	(−9.78 , 9.93)	(−1.69 , 1.65)	(−1.03 , 1.06)

TABLE VII. Expected 95% CL bounds on $d^{Z,\gamma}$ from the asymmetry of the observables at FLC for the single lepton decay mode.

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