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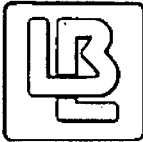**Permalink**
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Abstract

The possibility of nuclear matter undergoing a combined phase transition into abnormal matter and a pion-condensate is investigated. Various Lagrangians for the meson (σ and π_i) fields, based on the σ-models, are used in mean field approximation; and the entire system (mesons + nucleons) is treated fully relativistically. Equilibrium conditions of nuclear matter are obtained with N-N-repulsion, parametrized by the excluded volume approximation. It turns out that the formation of abnormal matter depends crucially on the choice of the σ-model-Lagrangian and considerably less on the additional pion-condensate.

I. Introduction

Exotic states of nuclear and neutron matter, such as pion-condensates or abnormal matter (characterized by a small or zero effective nucleon mass m*) have been the object of considerable interest[1-8]. Calculations have been performed, based on σ-models and on methods that consider the pion-propagator in dense nucleon matter.

These calculations indicate that a pion-condensate could exist in nuclear and neutron matter for densities $\rho \gtrsim 2$-3 ρ_0 ($\rho_0 =$ 0.17 fm^{-3} is the equilibrium density of nuclear matter). However, no convincing experimental proof has been found up to now. Moreover, it is doubtful whether a pion-condensate will significantly influence the equation of state of nuclear matter, once realistic short-range nucleon-nucleon correlations and pion-nucleon vertex cutoffs are taken into account[9,15].

The situation for abnormal matter is more controversial. Whereas Lee[3] and Källman[4] find a critical transition density $\rho_c \sim \rho_0$, Nyman and Rho[5] obtain $\rho_c \sim 10$ ρ_0. In these calculations, the pionic degree of freedom was suppressed. Chanowitz and Siemens[6] include pion-condensation, however in a non-relativistic approximation, and find a combined transition of neutron matter into a pion-condensate and abnormal state at $\rho \sim 1$-2 ρ_0. For a set of σ-models, Moszkowski and Källman[8] do not obtain an abnormal state in neutron matter, even if pion-condensation is taken into account.

Here we would like to present a qualitative study about the influence of pion-condensation on the possibility, that liquid

nuclear matter undergoes a phase transition into abnormal matter. The frame of this survey calculation is the σ-model; various types of Lagrangians within this model are used. Both the meson (σ, π_i) as well as the nucleon system are treated without non-relativistic approximation. The degrees of freedom of $\pi^{\pm}$ resp. π^0-σ-condensation are treated separately with no mixture of them being allowed

Short-range repulsive N-N-correlations in the pion-like channel, Δ-isobars and vertex cutoffs are included in the πN-interaction Lagrangian. That part of the nucleon-nucleon-repulsion, which is not induced by the pion-condensate, is parametrized by an excluded volume, which is the only parameter in addition to the σ-mass m_σ; both are fitted to the equilibrium properties of nuclear matter. The effects of solidification and/or anisotropy of nuclear matter are not considered.

II. σ-model with nucleons and pions

A. Various σ-model Lagrangians

Following the procedure of ref. 10, we start from a model Lagrangian including σ, π_i and nucleons:

$$L = \overline{\psi}\{i\gamma_\mu \partial^\mu - \overline{g}(\sigma + i\pi_j \cdot \tau_j \gamma_5)\}\psi + L_M \qquad (1a)$$

with
$$L_M = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \pi_j \partial^\mu \pi_j + U_i(\phi) - U_i(\phi = 0) + L_{SB} \qquad (1b)$$

The coupling constant $\overline{g}$ is given by $\overline{g} \cdot f_\pi = m = 6.7\, m_\pi$. The pion decay constant is $f_\pi = 94.5$ MeV. For L_{SB} we take the "standard" symmetry breaking term $f_\pi m_\pi^2 \sigma$. Another kind of symmetry breaking

is $(-\frac{1}{2}) m_\pi^2 \pi_j \pi_j$. There is no experimental reason to favor either of these. We make four different choices for $U_i(\phi)$, where $\phi = (<\sigma \cdot \sigma> + <\pi_j \cdot \pi_j>)^{\frac{1}{2}} - \sigma_0$. The vacuum expectation value of $<\sigma>$ is $\sigma_0 = f_\pi$,

$$U_1 = -(\phi + \sigma_0)^4 \frac{m_\sigma^2 - m_\pi^2}{8 f_\pi^2} + (\phi + \sigma_0)^2 \frac{m_\sigma^2 - 3m_\pi^2}{4} \qquad (2a)$$

U_1 has been used in the works of Lee and Margulies[18], Nyman and Rho[5], and Chin[12], since it is renormalisable, which is necessary for dealing with quantum (loop) corrections. However, U_1 gives too large a 3-body-force from the ϕ^3-terms.

The mass m_σ of the σ-meson is not very well known. It is doubtful, whether the "σ-meson" of 550 MeV, which can describe N-N scattering in the $T = 0$ $J^P = 0^+$-channel, is a genuine particle rather than a simulation of the 2π-exchange. The connection of the latter σ-meson to that of the σ-model is unclear. As long as the chiral radius $R = (<\sigma \cdot \sigma> + <\pi_j \cdot \pi_j>)^{\frac{1}{2}}$ equals f_π, m_σ can be taken as infinite. For $R \neq f_\pi$, however, the Lagrangian depends on m_σ. Since this is the case in the present calculation, the value of m_σ is essential for the energy of the mesonic subsystem, and thus for the equation of state of the entire system of mesons and nucleons. We determine m_σ by the equilibrium conditions (11) of nuclear matter. In U_2 the ϕ^3 and ϕ^4-terms are dropped:

$$U_2 = -(4\sigma_0^3 \phi + 6\sigma_0^2 \phi^2) \frac{m_\sigma^2 - m_\pi^2}{8 f_\pi^2} + (\phi + \sigma_0)^2 \frac{m_\sigma^2 - 3m_\pi^2}{4} \qquad (2b)$$

In addition we will consider the form U_3, where the ϕ^3 and ϕ^4 terms are weighted by factors of γ and γ^2 respectively. We take $\gamma = -.55$, a value which has been used by the authors of ref. 8 in order to fit the equilibrium properties of nuclear matter:

$$U_3 = U_2 - (4\gamma\sigma_0 \phi^3 + \gamma^2\phi^4) \cdot \frac{m_\sigma^2 - m_\pi^2}{8f_\pi^2} \qquad (2c)$$

U_2 and U_3 are not renormalisable, since they contain the expression $\sqrt{\sigma^2 + \underline{\pi}^2}$, and can be used only in mean-field approximation. Also Banerjee, Glendenning and Gyulassy[20] have forsaken renormalisability in order to obtain a model, which is in mean-field-approximation consistent with nuclear matter properties. No one has yet succeeded in finding a renormalisable model, which in mean-field satisfies nuclear matter properties and π-π-scattering simultaneously.

U_{1-3} represent the potential energy in mean field-approximation. However, internal pion lines in the nucleon bubbles are treated to a certain degree, since they are partially included in the repulsive N-N-correlations, which are treated within the framework of the Fermi-liquid theory (section II.C). In order to study the influence of lowest order quantum corrections of the σ-field coupling to nucleons, we will adopt the approach of Chin[12] and Lee and Margulies[18]. The nucleon loop contributions in the presence of an external σ-field yield a correction $\Delta E_{q.c.}$

$$\Delta E_{q.c.} = \frac{1}{2\pi^2} \left[\frac{1}{2} m^{*4} \log\left(\frac{m^*}{m}\right) + \frac{1}{2} m^3(m-m^*) - \frac{7}{4} m^2 (m-m^*)^2 + \frac{13}{6} m(m-m^*)^3 - \frac{25}{24} (m-m^*)^4 \right]$$

$$(2d)$$

The sum of U_1 and $\Delta E_{q.c.}$ (called U_4) is the fourth choice for our model Lagrangian.

Further discussion of quantum fluctuations can be found, f.i. in refs. 5b, 18. We are conscious of the fact, that those quantum effects might have an important influence on the equation of state However, the present calculations are not meant to be precise predictions of the equation of state, but to give an idea of how much

the (possible) transition into abnormal matter could be affected by pion condensation.

B. Energy eigenvalues in liquid nuclear matter

The most general ansatz for the mean field mesonic-wavefunctions in <u>liquid</u> (translationally invariant) nuclear matter was given by Dautry and Nyman[13]:

$$\pi^{\pm} = e^{\pm i \vec{k} \vec{r}} \, R \, \sin(\vartheta)$$
$$\sigma \pm i \, \pi^0 = R \, \cos(\theta) e^{\pm i \vec{q} \vec{r}} \tag{3}$$

We would like to treat $\pi^{\pm}$ and π^0-σ-condensation separately. The most general condensation is, of course, a mixture of both, but this gives rise to many complications such as various geometrical arrangements of both condensates and/or solidification.

It should be noted that, although the $\pi^{\pm}$- and π^0-condensates are isospin-symmetric (i.e. have the same threshold for an infinitesimal condensate), the $\pi^{\pm}$ and π^0-σ-condensates are not symmetric, since the latter one has mixed isospin.

In the case of a $\pi^{\pm}$-condensate, there is $\vec{q} = 0$ and consequently $\pi^0 = 0$ as well as $\sigma = R \cos(\theta)$ (the phase factors are chosen in a way to obtain real fields σ and π^0). The $\pi^{\pm}$-fields have the same strength $R \sin(\theta)$ and linear momenta $\pm \vec{k}$. For π^0-σ-condensation there is $\pi^{\pm} = 0$ and consequently $\theta = 0$. Then $\sigma = R \cos(\vec{q}\vec{r})$ and $\pi^0 = R \sin(\vec{q}\vec{r})$. In the second case there is less freedom in the system, since--in the absence of a $\pi^{\pm}$-condensate--the π^0-field has either full strength for $q \neq 0$ or doesn't exist at all for $q = o$.

After subtraction of the vacuum expectation value of the symmetry breaking term the mesonic Hamiltonian H_M is given by

$$H_M = \begin{cases} \frac{1}{2} R^2 k^2 \sin^2(\theta) - m_{\pi}^2 f_{\pi} R \cos(\theta) + m_{\pi}^2 f_{\pi}^2 - U_i(\phi) + U_i(\phi = 0) & \text{for } \pi^{\pm} \quad (4a) \\[2em] \frac{1}{2} R^2 q^2 \left\{ \begin{array}{ll} -m_{\pi}^2 f_{\pi} R + m_{\pi}^2 f_{\pi}^2 & \text{for} \quad q = 0 \\ +m_{\pi}^2 f_{\pi}^2 & \text{for} \quad q \neq 0 \end{array} \right\} - U_i(\phi) + U_i(\phi = 0) & \text{for } \pi^0 \ (4b) \end{cases}$$

where $k = |\vec{k}|$ and $q = |\vec{q}|$.

Note that the energy of the mesonic system is discontinuous at the threshold of π^0-condensation, different from $\pi^\pm$-condensation. This is a consequence of $<\sigma>$ being zero for all non-zero values of $\vec{q}$. A combined isospin and chiral transformation of the nucleonic part of L, which is given in detail f.i. in ref. 10, results in a Hamilton density (including the usual Lagrange-multiplier λ in order to gain baryon conservation):

$$H_{Nucl.} = -i\overline{\psi}\vec{\gamma}\vec{\nabla}\psi + m^*\overline{\psi}\psi - \lambda\overline{\psi}\gamma_0\psi$$

$$- \begin{cases} \frac{1}{2}\cos(\theta)\vec{k}\overline{\psi}\vec{\gamma}\tau_3\psi + \frac{1}{2}\sin(\theta)g_A\vec{k}\overline{\psi}\vec{\gamma}\gamma_5\tau_2\psi & \text{for } \pi^\pm \\ \frac{1}{2}g_A\vec{q}\overline{\psi}\vec{\gamma}\gamma_5\tau_3\psi & \text{for } \pi^0 \end{cases} \qquad (5)$$

The π-N-interaction in eq. (5) is a pure p-wave interaction. The s-wave-interaction, which is important in neutron matter, is very weak in nuclear matter. Moreover, its off-shell behavior is not very well understood. We shall not consider it further.

By a comparison of eq. (1a) and (5) it is obvious, that the field strength $(<\sigma\cdot\sigma>+<\pi_j\cdot\pi_j>)^{\frac{1}{2}}$ of the combined meson fields is the sole source (within this model) of the nucleon mass. Thus the effective mass m* is coupled to the chiral radius R by the expression

$$M^* = \overline{g}R \quad \text{or} \quad \frac{m^*}{m} = \frac{R}{\sigma_0} \qquad (6a)$$

The energy gain of pion-condensation depends much on the axial-vector coupling constant g_A. Unfortunately there is no unique way to obtain g_A. One could either take the experimental value 1.24 or rely on the Goldberger-Treiman relation

$$g_A = \frac{g f_\pi}{m} \quad \text{with} \quad \frac{g^2}{4\pi} = 14.5 \tag{6b}$$

which gives 1.36. In principle, the value of g_A depends on the state of the nuclear matter, and the value 1.24 refers to the vacuum only. There is no experimental information about the behavior of g_A for densities larger than ρ_0 and/or small effective nucleon mass. One possible assumption is, that $g_A = 1.36$ in the entire mass/density region. Another possibility is the application of a modified Goldberger-Treiman relation, where f_π is replaced by R:

$$g_A = \frac{gR}{m} \tag{6c}$$

which gives the conventional value 1.36 in the vacuum ($R = f_\pi$). An argument in favor of eq. (6c) is the linear dependence of the π-N-interaction of g_A in eq. (5). Since the field strength, given by R, is proportional to the π-N-interaction, it doesn't seem unreasonable to couple g_A with R. The latter definition of g_A is different from refs. 6, 13. In the following calculations we will take both choices (hereafter called g_A^I and g_A^{II} (from eq. (6c)). The true value of g_A probably lies between these two extreme possibilities.

The diagonalization[13] of eq. (5) in spin-isospin space yields the eigenvalues (for the positive energy solutions) with subtracted rest mass

$$E_\pm = (p^2 + m^{*2} + \alpha^2 \pm (\beta^2 + 4\alpha^2 p_z^2)^{\frac{1}{2}})^{\frac{1}{2}} - m \tag{7a}$$

$$\alpha^2 = [\cos^2(\theta) + g_A^2 \sin^2(\theta)]\frac{k^2}{4} \qquad \beta = m^* k g_A \sin(\theta) \qquad \text{for } \pi^\pm \tag{7b}$$

$$\alpha^2 = g_A^2 \frac{q^2}{4} \qquad\qquad\qquad \beta = m^* q g_A \qquad\qquad \text{for } \pi^0 \tag{7c}$$

the pion momenta $\vec{q}$ and $\vec{k}$ are oriented in the z-direction. The chemical potential λ for a given set of m*, θ and k (or q) is obtained numerically from

$$\rho = 2 \sum_{\pm} \int \frac{d^3p}{(2\pi)^3} \, \theta \cdot (\lambda - E_{\pm}) \tag{8}$$

The summation $\sum_{\pm}$ is extended over two Fermi-seas of quasiparticles, which are mixtures of protons and neutrons. For further discussion of this point see refs. 9 and 15. Up to now, no nucleon-nucleon-repulsion has been included in the model, but without repulsion the equation of state for nuclear matter has no minimum at $\rho = \rho_0$. Several attempts[7,8,11] have been made to describe this repulsion in a mean-field theory including the ω-meson. However, Pandharipande and Smith[7] found the ω-repulsion too weak to prevent nuclear matter from collapse in the conventional σ-model (U_1). For densities larger than ρ_0 much additional repulsion comes from the Pauli-blocking in the second-order tensor-force contribution to the binding energy. In addition to the ω, heavier mesons might play a certain role. Actually, nothing reliable is known up to now about the equation of state for high densities. As a consequence, we parametrize the repulsion by an excluded volume approximation

$$E_{rep} = E(\bar{\rho}) - E(\rho) \tag{9a}$$

$$\text{with } \bar{\rho} = \frac{\bar{k}_F^{\,3}}{1.5\pi^2} \quad \text{and} \quad E(\bar{\rho}) = \frac{\int_0^{\bar{k}_F} (p^2 + m^{*2})^{\frac{1}{2}} \, d^3p}{\int_0^{\bar{k}_F} d^3p} \tag{9b}$$

where the effective Fermi momentum $\bar{k}_F$ is given by

$$\overline{k}_F = \frac{k_F}{(1 - \frac{k_F}{q_0})} \tag{9c}$$

q_0 is the other parameter (in addition to m_σ) of the presented model. Thus eq. (10) yields the expression for the single particle energy.

$$\rho \cdot E_{s.p.} = 2 \sum_\pm \int \frac{d^3p}{(2\pi)^3} E_\pm \theta(\lambda - E_\pm) + E_{rep} \cdot \rho + H_M \tag{10}$$

The values of m_σ and q_0 are numerically determined from the equilibrium properties of nuclear matter:

i) $E_{s.p.}(\rho_0) = -16$ MeV

ii) $\left. \frac{\partial E_{s.p.}(\rho)}{\partial \rho} \right|_{\rho = \rho_0} = 0$ together with $\left. \frac{\partial E_{s.p.}}{\partial m^*} \right|_{\rho = \rho_0} = 0$ (11)

Pion condensation does not occur for $\rho = \rho_0$, so $k = q = \theta = 0$. m_σ and q_0 are, of course, different for each U_i. The value of q_0, used at $\rho = \rho_0$, is used at higher densities as well. It is an open question how the repulsion behaves for $\rho > \rho_0$. Since any hypothesis regarding this point is little more than speculation, we take a single q_0, which fulfills the minimal requirement of fitting the nuclear equilibrium properties.

C. Nucleon-nucleon correlations and pion-nucleon-vertex cutoff

The essential features of the model up to now are:

(1) a combined σ-π-mean field with self-interactions;
(2) an attractive π-N-interaction;
(3) an attractive σ-N-interaction resulting from $m^*(R)$; and
(4) a repulsive N-N-interaction in the non-pion-like channels, parametrized by an excluded volume.

Not included are:

(1)   short-range repulsive N-N-correlations in the pion-like-channel
(2)   Δ-isobar admixture to the nucleon states
(3)   finite range πN-vertex cutoffs.

These three effects have an important influence on pion-condensa-

tion[9,14,15]. The Landau Fermi-liquid parameter g' determines the

strength of repulsive correlations in the pion-like-channel; most

of them probably arising from ω-exchange in addition to the π or

combined ω-ρ-exchange. Realistic values of g' at $\rho = \rho_0$ lie between

0.6 and 0.7 with 0.5 as the lower bound (in units of m_π^{-2}). Not

very much is known about the behavior of g' at densities larger

than ρ_0, except that it probably smoothly decreases with ρ[17]. We

assume g' = 0.5 in all calculations independent of k (or q), θ,

ρ and m*. This will result in a slight overestimation of the

strength of the pion-condensate and thus its effect on the equation

of state. For a detailed discussion of the g'-problem we refer to

refs. 9 and 15. In a similar way (as in refs. 9 and 15), N-N-

correlations in the pion-like-channel are treated by replacing

$$g_A k \, \sin(\theta) \rightarrow g_A k \, \sin(\theta) - 4g'\tilde{\rho} \qquad (12a)$$

$$g_A q \rightarrow g_A q - 4g'\tilde{\rho} \qquad (12b)$$

in α and β of eqs. (7b) and (7c).

The non-diagonal spin-isospin-density, $\tilde{\rho} = \langle\frac{1}{2} \sigma_z \tau_2\rangle$, should be

evaluated between the quasiparticle eigenvectors belonging to $E_\pm$,

which arise from the chiral transformation and are no longer iden-

tical with either pure protons or neutrons. In addition, the static

pion-nucleon interaction for θ>0 (resp. q≠0) admixes Δ-isobar compo-

nents to the nucleon wavefunctions. Both effects have been treated in

detail in a previous work[9]. Since $\tilde{\rho}$ depends on α and β, it can be determined only numerically in a self-consistent calculation. We choose instead the simple parametrization:

$$\tilde{\rho} = \begin{cases} \rho \sin(\theta)c & \text{for } \pi^{\pm} \\ \rho c & \text{for } \pi^{0} \end{cases} \qquad (13)$$

This formula can be understood this way: $\tilde{\rho}$ is certainly proportional to the diagonal density ρ. In the case of $\pi^{\pm}$, the pion-condensate induces a mixing of proton and neutron states, which is (for small chiral angles θ) proportional to $\sin(\theta)$ (compare eq. A16 in ref. 9b). For π^{0} there is full mixing above the threshold. The energy shift $\frac{E_{c}}{A}$ due to pion-condensation has been determined in a previous work[9a], performing a self-consistent calculation of $\tilde{\rho}$. The parameter c is obtained from a fit to $\frac{E_{c}}{A}$ with 0.9 as the best value. This is in agreement with the fact that $\tilde{\rho} = \frac{9}{10}\rho$ in the high density limit ($\theta = \frac{\pi}{2}$ for $\pi^{\pm}$), as shown in ref. 15. For lower densities than $\sim 2.5\,\rho_{0}$ the repulsion is somewhat underestimated by this choice of c. The subtraction $4g'\tilde{\rho}$ (with the parameter c) in eq. (12) includes the effects of both the N-N-correlations in the pion-like-channel as well as the Δ-admixture, since the latter one has been treated explicitly in ref. 9a, in order to obtain $\frac{E_{c}}{A}$. The limit $\tilde{\rho} = \frac{9}{10}\rho$ is obtained only, if Δ-admixture is included. Otherwise it would be $\tilde{\rho} = \frac{5}{10}\rho$.

Next, we include finite-range pion-nucleon vertex cutoffs by multiplying each pion-nucleon vertex with a monopole form factor

$$f(k) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + k^2} \tag{14a}$$

This is done by replacing

$$g_A \to g_A \cdot f \tag{14b}$$

$$\tilde{\rho} \to \tilde{\rho} \cdot f^2$$

A reasonable value for Λ is 1.2 Gev[16,19]. In order to avoid double counting of the repulsive correlations by introducing $4g'\tilde{\rho}$ in α and β, a term $2g'\tilde{\rho}^2\rho^{-1}$ must be subtracted from $E_{s.p.}$ in (10). Hence, the final expression for the single particle energy with a pion-condensate is given by

$$E_{s.p.} = E_{s.p.} \text{ (of eq. 10) } - 2g'\tilde{\rho}^2\rho^{-1} \tag{15}$$

with the replacements (12) and (14) made.

It should be noted that no experimental evidence for g' and Λ is available for $\rho > \rho_0$ and low effective mass. Only some model calculations[17] of g' up to $\rho \sim 2\rho_0$ exist. The assumption of constant g' and Λ over the entire mass-density range is a severe approximation. To curb this uncertainty, we have made calculations with various values of Λ, and by the choice $g' = 0.5$ we intend not to underestimate the strength of the pion-condensate.

III. Results and Discussion

For a given Lagrangian U_{1-4}, m_σ and q_0 are varied, as long as the equation of state $E_{s.p.}(\rho)$ fits the equilibrium properties (11) of nuclear matter. The condition $\frac{\partial E_{s.p.}}{\partial m^*} = 0$ is required for all sets of ρ, q_0 and m_σ. Since no pion-condensate exists at

$\rho = \rho_0$, it is suppressed by setting $k = q = \theta = 0$. For g_A^I a pion-condensate would occur even at densities lower than ρ_0 for certain combinations of Λ and U_i. The values of q_0 and m_σ, thus obtained, are kept for the entire density range. Above the pion threshold there are further minimum conditions (in addition to $\frac{\partial E_{s.p.}}{\partial m^*} = 0$) to be fulfilled:

$$\frac{\partial E_{s.p.}}{\partial k} = \frac{\partial E_{s.p.}}{\partial \theta} = 0 \text{ for } \pi^\pm$$

$$\frac{\partial E_{s.p.}}{\partial q} = 0 \qquad \text{for } \pi^0 \tag{16}$$

Calculations have been performed for both cases. However, only the results for $\pi^\pm$ are discussed and shown, since the results for π^0 are qualitatively the same, and often even quantitatively similar. For densities well above the pion-condensation threshold the chiral angle approaches $\frac{\pi}{2}$. In this case the results for $\pi^\pm$ and π^0 are identical. For lower densities, however, both types of condensates differ, since the π^0 has one degree of freedom (the chiral angle θ) less than the $\pi^\pm$. Thus the π^0 cannot develop smoothly with an increasing θ, but sets in with full strength. The π^0-threshold is always larger, and the energy gain in the vicinity of the threshold is smaller than for $\pi^\pm$. It should be noted that this is different in neutron matter, where the p-wave attraction for $\pi^\pm$ is reduced by a considerable s-wave-repulsion, which is absent in the case of π^0. Hence, the energy shift in neutron matter due to π^0-condensation will generally be larger as for $\pi^\pm$-condensation.

Let us first regard the results for the equation of state $E_{s.p.}(\rho)$. The result, that a pion-condensate occurs above $\rho \sim 1-2 \rho_0$,

supports previous work on this subject[1,2,9,14,15]. From Fig. 1(a-d)
it is obvious that both the vertex cutoff and the axial vector
coupling constant have a strong influence on the energy shift due
to pion-condensation. This is more pronounced for U_1 than for
U_{2-4}. Deep second minima occur for g_A^I, even for $\Lambda = 1.2$ GeV. In
some cases even the first minimum vanishes, if pion-condensation
starts at $\rho < \rho_0$. There is no experimental evidence either for a
condensation threshold at $\rho \sim \rho_0$ or for an equation of state with
such pronounced second minima. Normal nuclei would have a tendency
to collapse into a high-density state. The coupling constant g_A^{II}
probably comes closer to reality, if combined with $g' = 0.5$ and a
realistic $\Lambda = 1.2$ GeV. An infinite Λ is unrealistic, since experi-
mental evidence as well as theoretical calculations favor a finite
cutoff.

The Lagrangian U_1 produces a second minimum, deeper than the
first one, even for g_A^{II} and $\Lambda = 1.2$ GeV. This was not obtained in
a previous work[9]. However, in ref. 9 m* was predetermined as a
function of ρ with $R = f_\pi$ fixed; different from the procedure in
the present work. In addition, the approximation (13) overestimates
the attraction for about 5 - 10 MeV in the second minimum. A slightly
larger g' together with a self-consistent calculation of $\tilde{\rho}$ in (13)
would flatten this minimum. For U_{2-4}, no second minimum is obtained
for g_A^{II} and $\Lambda = 1.2$ GeV, the curves, however, are rather flat. For
U_3, the compressibility K = 340 MeV comes close to the experimental
value, the other Lagrangians give somewhat "harder" equations of
state.

The corresponding results for the effective mass $m^*(\rho)$ are shown in Fig. 2(a-d). For U_{2-4}, in the absence of a pion-condensate, the effective mass decreases smoothly with the density and no abnormal state occurs. It is however, a matter of definition what could be called an abnormal state. Is it established for all $m^* \lesssim 0.5$ or only if m^* is precisely zero? We do not regard it as justified, to call a state with $m^* = 0$ abnormal and one with $m^* = 0.05$, f.i., normal, since tiny variations of the Lagrangian which describes the system can result in such a small difference. A non-zero but small effective mass is already a clear sign that something substantial has changed in the nucleon system, compared with the $m^* = m$ state. We would like to call a state with $m^* = 0.3$ or smaller an abnormal state. There is, of course, no clear separation between both regions of the effective mass.

For g_A^{II}, pion-condensation has a rather small effect on the effective mass at all densities. There is no unique tendency in the shift of m^* for various cutoffs. For $\Lambda = 1.2$ GeV, m^* can be even larger than in the non-condensed phase. For g_A^{I}, the effects are considerably larger. However, even in this case, which probably overestimates the effects of pion-condensation arbitrarily, no abnormal state is reached.

For U_1, we get a different scenario. Between $\sim 1.2\,\rho_0$ and $\sim 1.6\,\rho_0$ there is in all cases a drastic decrease of the effective mass, and a moderate increase for $\rho > 2.5\,\rho_0$. An abnormal state exists with and without pion-condensation, for all considered values of g_A and Λ. The main effect of the pion-condensate is the reduction of the threshold density for the abnormal state from

$\rho_C \sim 1.6 \ \rho_0$ down to $\rho_C \sim 0.95 \ \rho_0$ for the strongest condensate.

Different from U_{2-4}, the behavior of the effective mass in the vicinity of pion-condensation threshold is rather complicated. There are discontinuities and/or cusps, which result from the crossing of two equations of state, which correspond to two different local minima of $E_{s.p.}$. The effects of pion-condensation on the effective mass are generally larger for U_1 than for U_{2-4}. This was also the case for the single-particle energy. No signs of the drastic drop of the effective mass in the non-condensed phase between $\rho \sim 1.2 \ \rho_0$ and $\sim 1.6 \ \rho_0$ occur in the equation of state for U_1, which has a behavior as smooth as that of U_{2-4} and other conventional equations of state.

From the above results it is apparent that pion-condensation has more influence on the equation of state than on the effective mass. This situation is reversed for the σ-model Lagrangians U_{1-4} in the absence of pion-condensation. They give different results for $m^*(\rho)$, but the equations of state are remarkably similar.

IV. Conclusion

Pion-condensation in the frame of the σ-model is considered in a relativistic calculation up to a density of $\sim 2.5 \ \rho_0$. Above $2.5 \ \rho_0$ it is doubtful whether the above procedure gives an adequate description of the πNN-interaction, even if we had reliable information about the axial vector coupling constant g_A, the πNN cutoff Λ and the Landau parameter g' at high density, which we don't have. For all combinations of g_A, Λ, and g', which give reasonable equations of state without a second minima much deeper than the

first one, pion-condensation has no drastic effect on the effective mass m^*. Only a moderate decrease of the threshold density for the transition into an abnormal state (if at all existing) could be obtained. In this case, the effective mass m^* has a discontinuity in the presence of pion-condensation. In no case --not even for a choice of parameters which give an overestimation of the condensate effect--does the occurrence or nonoccurrence of an abnormal state depend on the existence of the pion-condensate. The existence of an abnormal state depends strongly, however, on the σ-model Lagrangian. Could quantum corrections of the Lagrangian and the possible existence of a combined $\pi^{\pm}$-π^0-condensate (with or without solidification of baryon matter) change this scenario? Nucleon loop corrections of the σ-field do not seem to do this. It is likely that inclusion of higher order quantum effects would result in a mesonic Lagrangian, more realistic than the mean-field approximation. This would, however, not refute the result that the transition to abnormal matter depends strongly on the choice of the Lagrangian. Concerning the combined $\pi^{\pm}$-π^0-condensate, it has been pointed out by Dautry and Nyman[13] that it gives a lower energy than either $\pi^{\pm}$ or π^0 alone. It is unlikely, however, that a combined condensate should have a substantially larger effect on the effective mass than any one of the separate $\pi^{\pm}$ or π^0-condensates.

If an abnormal state in liquid nuclear matter should exist up to $\rho \sim 2.5\ \rho_0$, and if the whole meson-nucleon system should be described adequately by a σ-model, then this abnormal state will

mostly depend on the σ-model Lagrangian and much less on the existence and/or strength of an additional pion-condensate.

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Figure Captions

Fig. 1(a)   The single-particle energy $E_{s.p.}$ as a function of the
nucleon density ρ for the Lagrangian U_1. The full
curve corresponds to $\pi = 0$, the dashed ones to $\pi \neq 0$ with
$\Lambda = \infty$ and the dot-dashed one to $\pi \neq 0$ with $\Lambda = 1.2$ GeV.
The compressibility for $\pi = 0$ at $\rho = \rho_0$ is $K = 450$ MeV.
Two curves correspond to g_A^I, as marked, the others to
g_A^{II}.

Fig. 1(b)   The same as Fig. 1(a) for U_2. The compressibility is
$K = 400$ MeV   .

Fig. 1(c)   The same as Fig. 1(a) for U_3. The compressibility is
$K = 340$ MeV.

Fig. 1(d)   The same as Fig. 1(a) for U_4. The compressibility is
$K = 580$ MeV.

Fig. 2(a)   The effective mass m^* as a function of the nucleon
density ρ for the Lagrangian U_1. Otherwise the same
description as in Fig. 1(a) is applied.

Fig. 2(b)   The same as Fig. 2(a) for U_2.

Fig. 2(c)   The same as Fig. 2(a) for U_3.

Fig. 2(d)   The same as Fig. 2(a) for U_4.
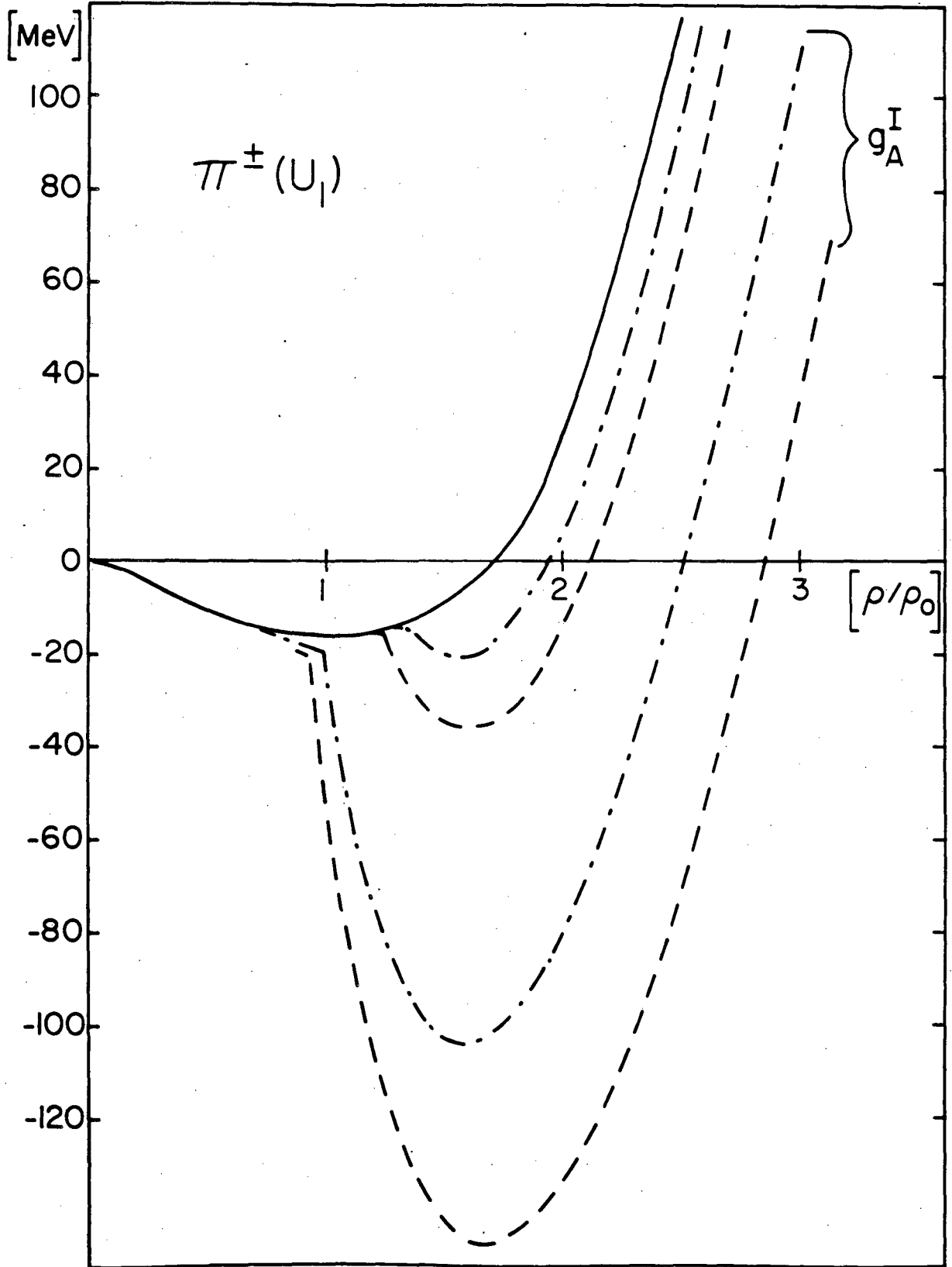
Fig. 1(a)

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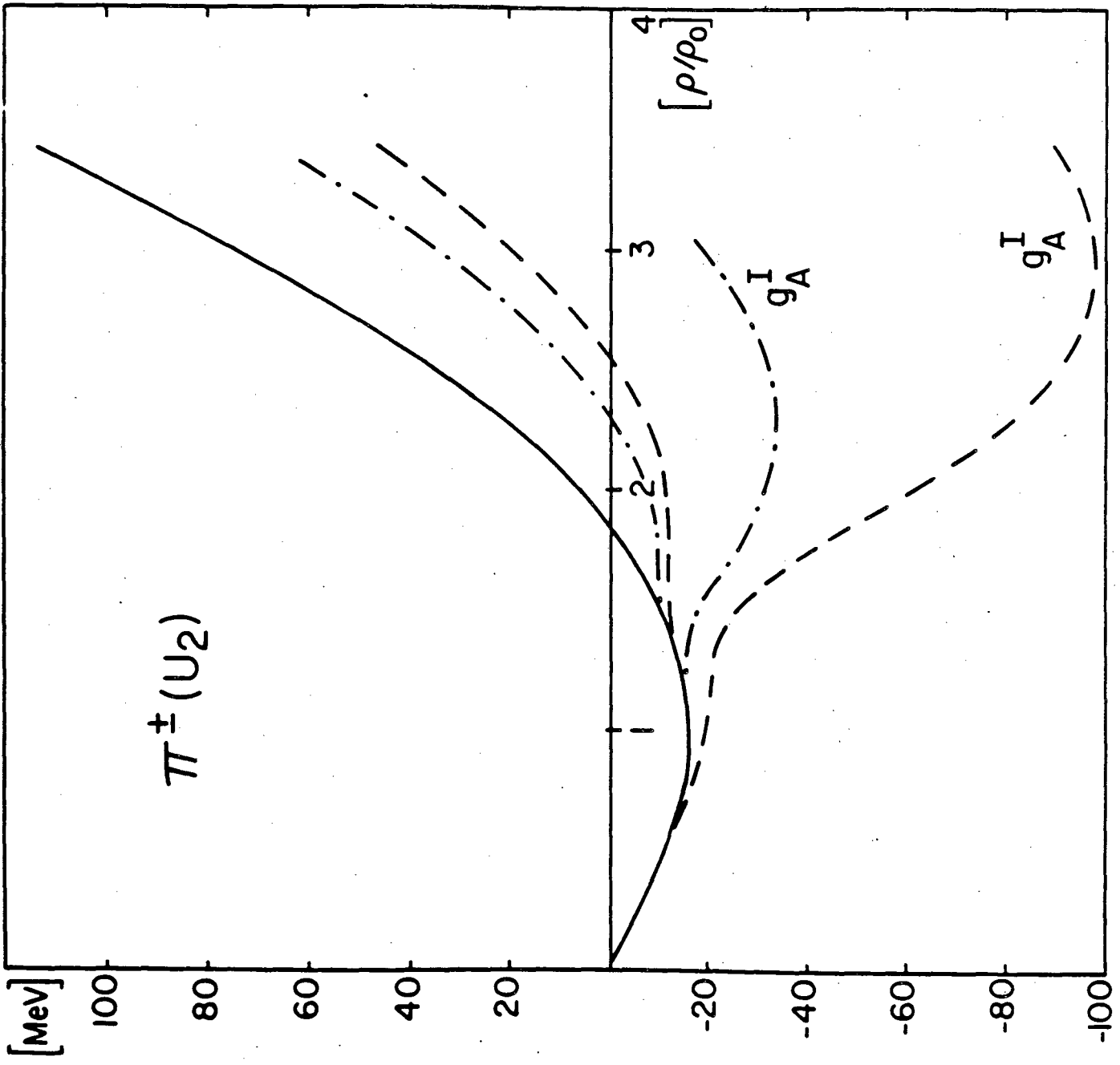
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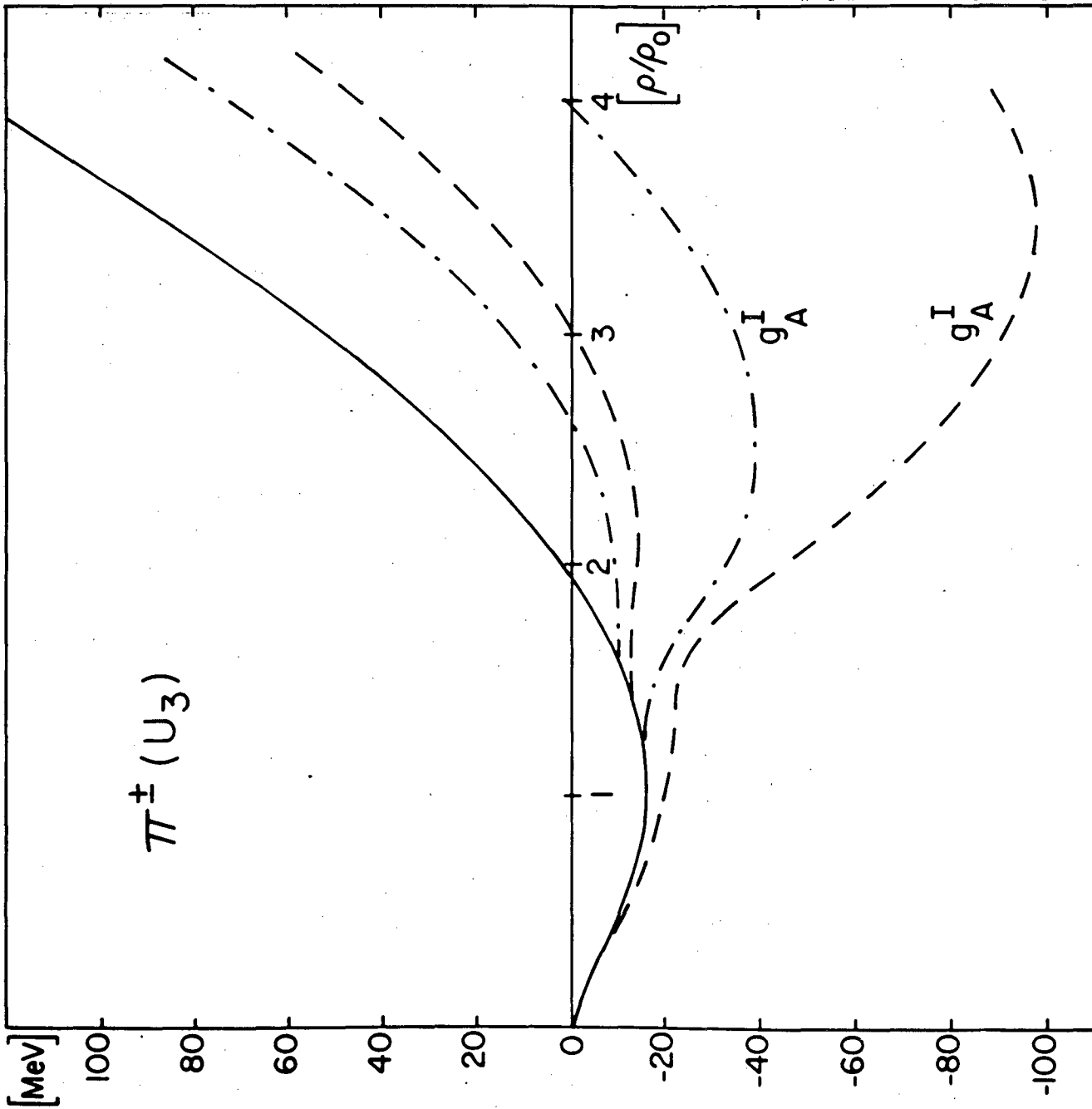
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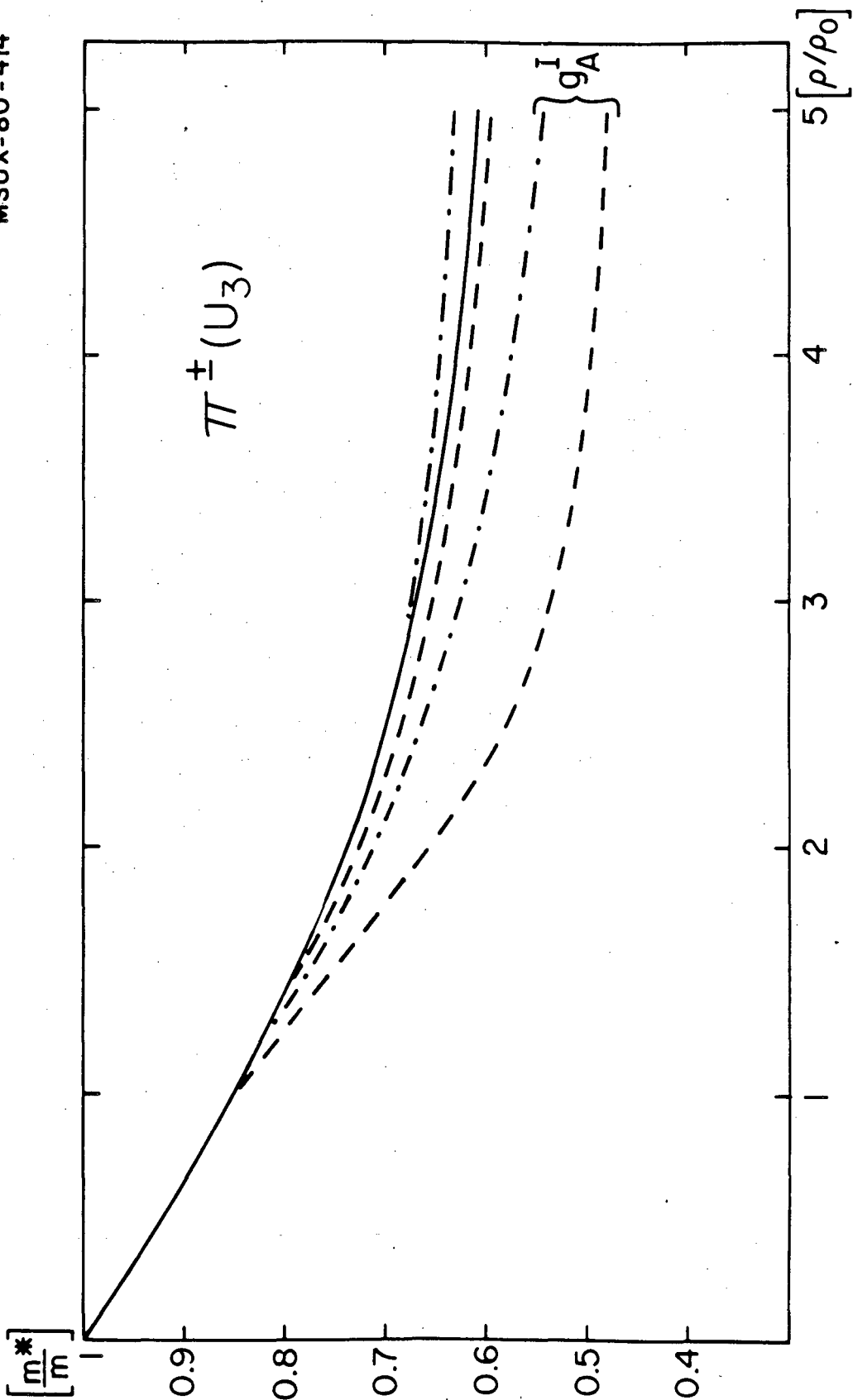Fig. 1(c)
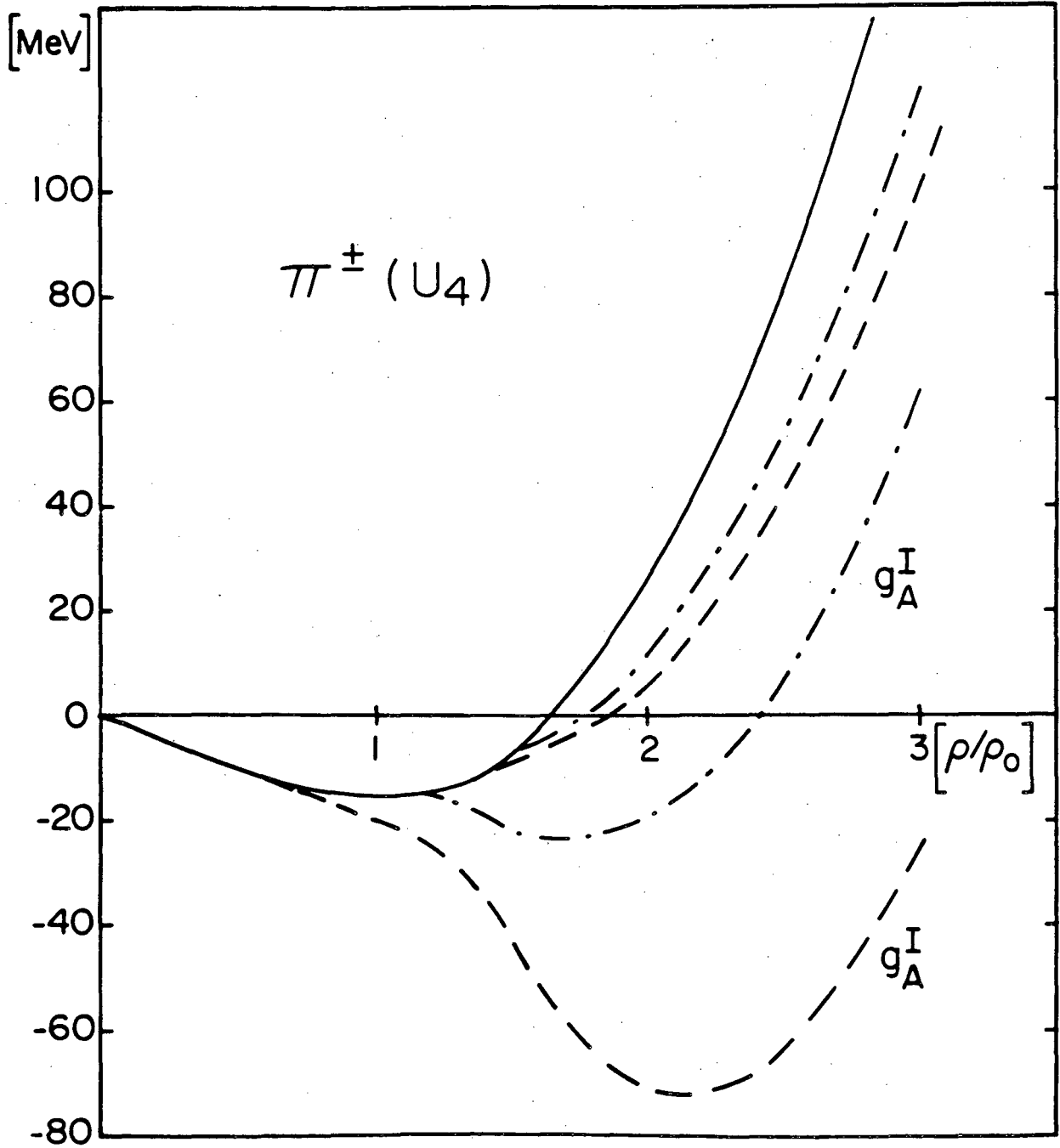
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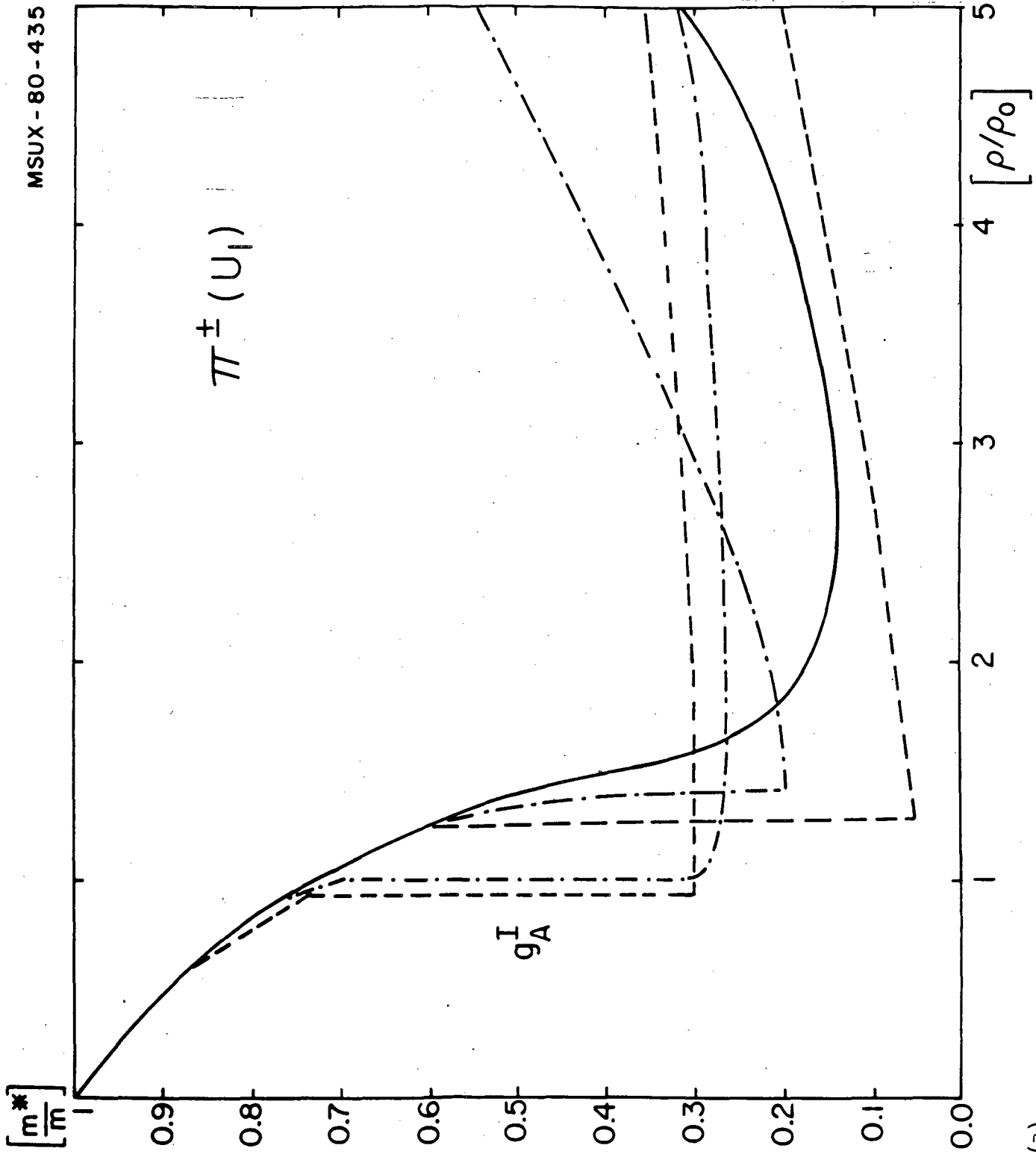
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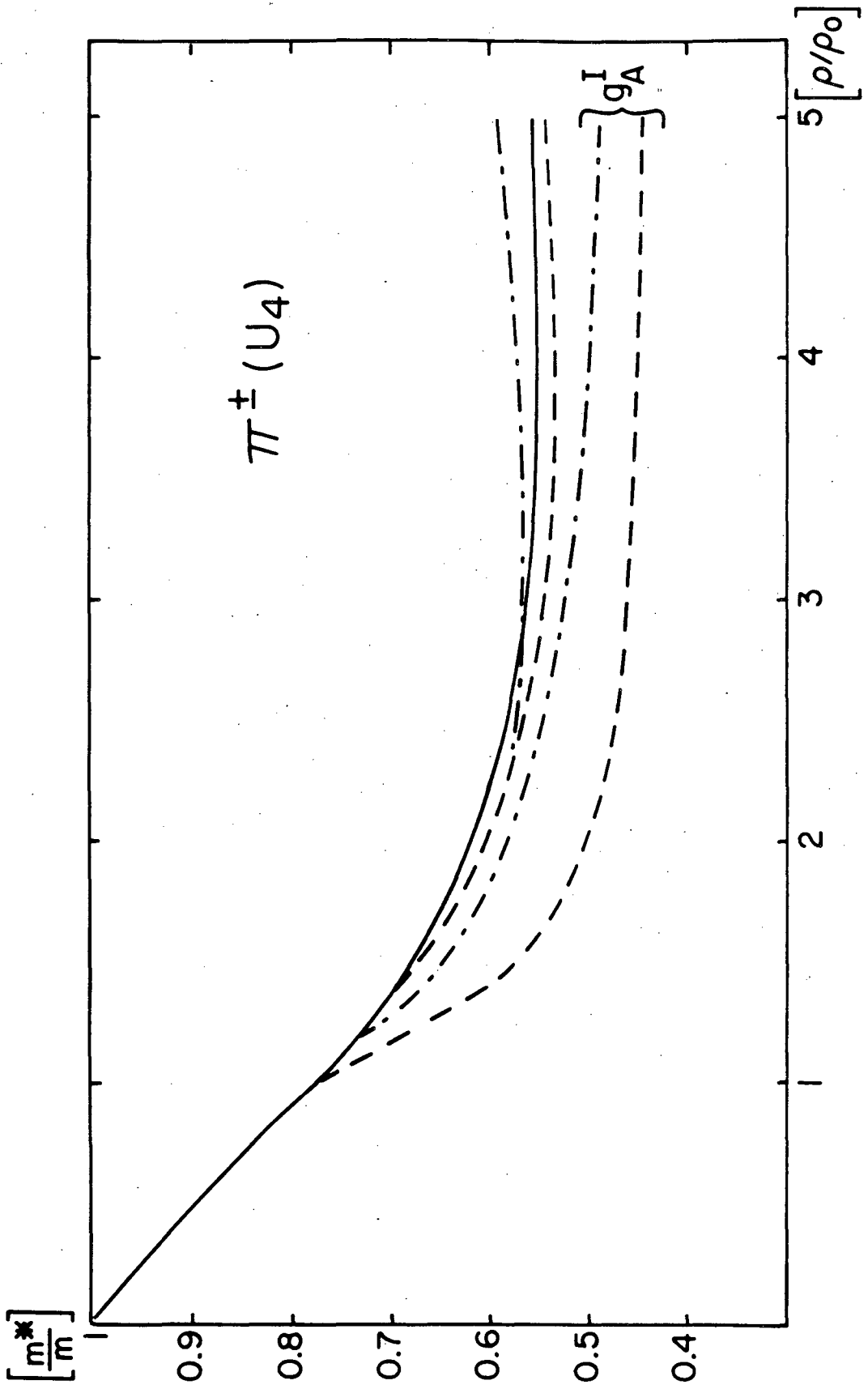
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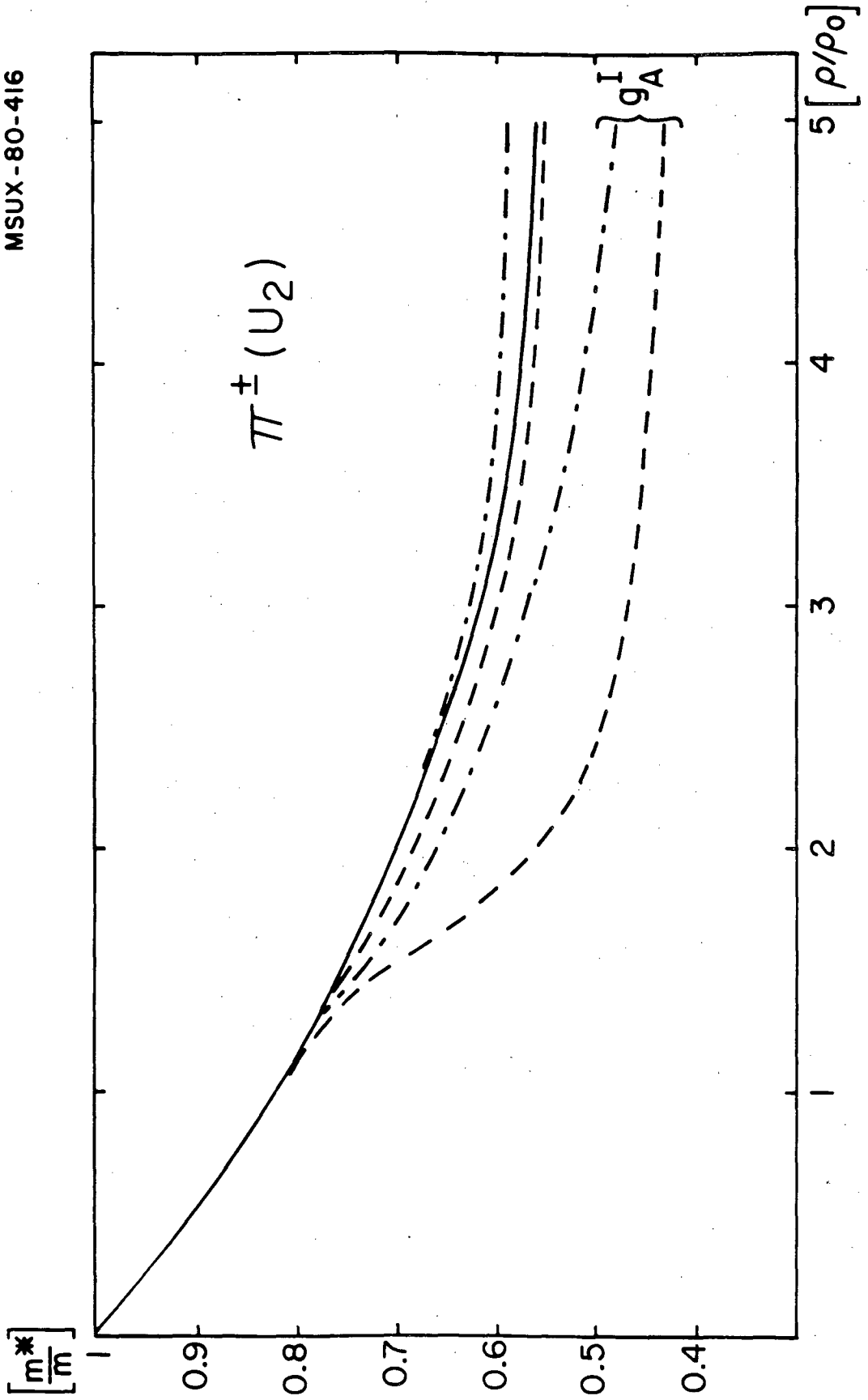
Fig. 2 (b)

MSUX-80-414

Fig. 2(c)

MSUX-80-417

Fig. 2(d)