Title
A theory of time and temporal incidence based on instants and periods

Authors
Vila, Lluis
Schwalb, Eddie

Publication Date
1996-05-06

Peer reviewed

A Theory of Time and Temporal Incidence based on Instants and Periods*

Lluís Vila[†] Eddie Schwalb

Technical Report #96-04

Information and Computer Science Dept.
University of California, Irvine

May 6, 1996

*Contact address: Lluís Vila, 444 CS UCI, Irvine CA 92717; vila@ics.uci.edu; http://www.ics.uci/~vila. **Acknowlegments**: We are thankful to Rina Dechter and anonymous reviewers for remarks that have been helpful for improving this paper.
†Partially supported by a MEC of Spain grant (EX94 77909683).

Keywords: Time, temporal reasoning, ontologies, common-sense reasoning.

Abstract

Time is fundamental in representing and reasoning about changing domains. A proper temporal representation requires characterizing two notions: (1) *time* itself, and (2) *temporal incidence*, i.e. the domain-independent properties for the truth-value of fluents and events throughout time. Formally defining them involves some problematic issues such as (i) the expression of *instantaneous events* and *instantaneous holding of fluents*, (ii) the *dividing instant problem* and (iii) the formalization of the properties for *non-instantaneous holding of fluents*.

This paper discusses how previous attempts fail to address all these issues and presents a simple theory of time and temporal incidence which satisfactorily overcomes all of them.

Our theory of time, called $\mathcal{IP}$, is based on having instants and periods at equal level. Our theory of *temporal incidence* is defined upon $\mathcal{IP}$. Its key insight is the distinction between continuous and discrete fluents.

Contents

1 Introduction

Time has been recognized as an important notion for modelling and reasoning about changing domains. Many frameworks for change and action are built upon a temporal representation [17, 1, 14, 6, 26, 23, 15, 7, 8, 22, 20, 18, 10]. In these frameworks, the domain at hand is formalized by expressing how propositions are true or false throughout time. Commonly there is a distinction between propositions describing the state of the world (*fluents*) and those describing the occurrences that make the world change (*events*). The temporal representation of such frameworks has two basic components: (i) a theory of time, and (ii) a theory of temporal incidence. A *theory of time* describes what are the primitive time elements, the primitive relations between them and their properties (e.g. the ordering relation over instants is transitive). A *theory of temporal incidence* describes the domain-independent properties for the truth-value of fluents and events throughout time (e.g. if a fluent is true during a period it must be true during the instants within that period). Some works that focussed on these issues are [9, 24, 2, 16, 23, 3, 8, 25].

***(I am not sure about next two paragraphs) A proper theory of time and temporal incidence is fundamental for any temporal representation to correctly answer temporal queries such as "Did the event close the relay occur before connecting the battery ?", "At what times were the light on and the door open ?". Consider, for instance, a CLP scheme [12] for representing temporal information: The *theory of time* characterizes the constraint domain which determines the properties (such as completeness) for a constraint solving procedure. The *theory of temporal incidence* characterizes relations among different temporal occurrences at related times and, therefore, has impact in the completeness of the overall proof procedure.

From a practical point of view, they enable identifying implicit, redundant or inconsistent temporal information and, thus, they help with improving efficiency of reasoning algorithms. For example, in the presence of the sentence "the light was on between 8:00 and 9:00" the sentence "the light was on between 8:30 and 9:00" is redundant. As another example, consider two tasks competing on a single resource. The theory of temporal incidence allows inferring that the time periods during which these tasks utilize the common resource do not overlap. As a result, some temporal constraint propagation is enabled.

Formalizing time and temporal incidence is problematic because real world dynamic domains may involve both parameters whose change is modelled as continuous and others whose change is modelled as discrete. We call a model with both types of change a *hybrid model*, and the system a *hybrid system* [10]. For example, consider an electro-mechanical battery charger: recharging a battery can be viewed as a continuous change whereas closing a relay would be better regarded as discrete[1] .

[1]Hybrid systems are interesting since many daily used electro-mechanical devices are suit-

The contribution of this work is identifying the shortcomings of previous approaches and proposing a simple and satisfactory theory of time and temporal incidence. We take what appears to be a rather straight forward approach only in hindsight. We identify the following problematic issues: (i) the expression of *instantaneous events* and *instantaneous holding of fluents*, (ii) the *dividing instant problem* and (iii) the formalization of the properties for *non-instantaneous holding of fluents*. We explain why previous attempts fail to address all of the above and we explain how to overcome them in our approach.

Our theory of time, called $\mathcal{IP}$, is based on having instants and periods[2] at equal level. Our theory of *temporal incidence*, called $\mathcal{CD}$, is defined upon $\mathcal{IP}$. Its key insight is the distinction between continuous and discrete fluents. Although there is a common understanding in the community that they are different, we are not aware of a single satisfactory framework in which the precise details have been worked out properly.

The structure of this paper is as follows. Section 2 introduces some problematic issues to be considered. Section 3 discusses the shortcomings of previous approaches. Section 4 presents our theory of time called $\mathcal{IP}$. Section 5 presents our categorization of propositions and the theory of their temporal incidence. In section 6 we discuss how the above problems are satisfactorily addressed and section 7 presents an example of hybrid model that is described using our approach. Finally section 8 summarizes our contribution.

2 Problematic Issues

In this section we introduce the problematic issues that arise when defining a theory of time and a theory of temporal incidence.

Instantaneous Events The model of a dynamic system often involves some events that cannot be qualified by a duration. Some prototypical examples are "turn off the light", "shoot the gun", "start moving", "sign a contract". Modelling such events can be problematic specially when *sequences* of them occur in presence of continuous change (section 7 discusses it in detail).

Instantaneous Fluent Holding It seems natural to ask queries about whether a fluent is true or not at a certain instant (e.g. was the light red when the car hit John?). Moreover, modelling *continuous change* requires having fluents that may hold at isolated instants. A simple, representative example is the parameter *speed* of a ball tossed upwards in what we call the *Tossed Ball Scenario* (TBS) (see figure 1). The ball moves up during p_1 and down during period p_2. There must be time piece "in between", where the speed of the ball

ably modelled as such.

[2] By period we mean a *time interval.*

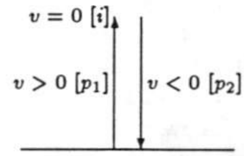

Figure 1: The Tossed Ball Scenario (TBS).

is zero. Such a time piece can only be durationless otherwise it would mean that the ball is stopped for a while. Modelling the TBS requires the ability to talk about the truth value of fluents at durationless times. This, however, may lead to the problem described next.

The Dividing Instant Problem (DIP) Assume that time is made of instants and periods and we need to determine the truth-value of a fluent f (e.g. "the light is on") at an instant i, given that f is true on a period p_1 ending at i and is false at a period p_2 beginning at it (see figure 2) [9, 24, 1, 8].

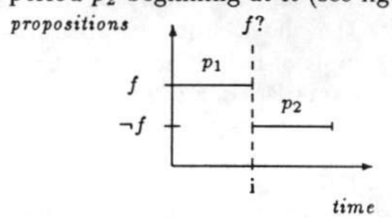

Figure 2: The Dividing Instant Problem.

The problem is a matter of maintaining logical consistency: If intervals are closed then f and $\neg f$ are both true at i which is inconsistent. If they are open we might have a "truth gap" at i. The other two options are open/closed and closed/open intervals which are judged to be artificial [1, 8].

Non-Instantaneous Holding of Fluents Formalizing the properties of temporal incidence for non-instantaneous fluents can be a non-trivial issue. There are two major classes of these properties. The first is the class of properties that relate the holding of a fluent at related times. Important instances of it are:

- *Homogeneity*: If a fluent is true on a piece of time it must hold on any subtime [1, 8].
- *Concatenability*[3]: If a fluent is true on two consecutive pieces of time it must be true on the piece of time obtained by concatenating them. Notice

[3] Most of previous approaches have focussed on properties related to homogeneity whereas very little attention has been paid to concatenability. Nevertheless, it is important since (i)

that there may be different views for the meaning of *consecutive*.

The second is the class of properties relating the holding of contradictory fluents at related times. Two main important instances are:

- *Non-holding*: If a fluent is not true on a piece of time, there must be an subtime where its negation holds [1, 8].
- *Disjointness*: Any two periods such that a fluent is true on one and its negation is true on the other, must be disjoint. There may be different views for the meaning of *disjoint* also.

Non-Atomic Fluents The holding of negation and disjunction of atomic fluents is an issue that needs to be defined [23, 8]. For instance, given a fluent f and a period p, it is necessary to precisely characterize the relation between the holding of $\neg f$ and the non-holding of f on p.

3 Related Work

In this section we survey instant-based approaches, interval-based, approaches where instants are derived from intervals and, finally, an approach with both instants and periods at equal level. For each approach we discuss first the theory of time and then the theory of temporal incidence.

Instant-based Approaches Classical approaches in AI define time as an ordered collection of instants[4]. In section 2 we outlined a number of reasons for having instants as a primitive to properly define temporal incidence. However, several arguments have been put forward against theories of time based on instants:

- *Ontological:* "Our direct experience is with phenomena that take time" [1].
- *Philosophical:* "Instants can have no content: it takes to many of them to make up a durable experience ?" [9, 13], "the point-based, continuous model ... they start with is too rich" [3], "richer than we need. They permit the description of states of phenomenally impossible states of affairs" [9].
- *Logical:* The DIP.

it is a semantical issue, and (ii) has computational benefits since it allows to have a compact representation of fluents holding throughout numerous consecutive periods.

[4]This is the approach classically taken in physics where time is usually isomorphic to the real numbers.

Regarding the theory of temporal incidence, the two most relevant instant-based proposals in AI, McDermott's [17] and Shoham's [23], both include a classification of propositions according to their temporal incidence. McDermott's approach has been criticized for presenting then in an informal and unclear recursive form [21]. A major criticism to both approaches is the lack of attention to the relevant issue of non-instantaneous and non-atomic holding of fluents. In particular, although both agree on the importance of representing continuous change, none of them formally accounts for the essential differences between holding of continuous and discrete fluents.

Interval-based Approaches Allen [1] proposed a theory of time exclusively based on intervals where "instants are modelled as very short intervals". However, neither *instantaneous holding* of fluents nor *instantaneous events* can be properly represented, the reason being that a very short interval does not have the same properties as an instant. In particular, it does not *divide* a period into two meeting ones. For example, in the light on/off scenario (see figure 2), assume that we want to view *switchoff* as an instantaneous event occurring at i. Thus, the period where the light is on (p_1) *meets* the period where it is off (p_2). Clearly i cannot be modelled as a short interval since p_1 would not meet p_2.

Another problem of a time ontology exclusively based on intervals is that defining the theory of incidence becomes much more complex. In Allen's incidence theory, for instance, the axiom for *homogeneity of fluent holding* (H.2) is quite cumbersome, and, in fact, it conflicts with the axiom H.4 for *holding of negated fluents* as shown by [8]. We shall see that instants make defining the incidence behavior over periods very simple and natural.

Deriving Instants from Periods Mathematicians proposed a number of set-theoretic methods for constructing points from intervals that can be used for deriving instants. The following are two examples: (i) the maximal set of intervals that have a non-empty intersection, (ii) the equivalence class of pairs of meeting intervals that meet "at the same place".

The interest of deriving instants from periods is unclear since, as noted by Allen and Hayes [3], "we may end up in the same place" than starting with an instants structure. Indeed we show (section 4) that Allen and Hayes time interval theory [2, 3] together with points derived using the second method above, admits the same models than the period component of our theory of time. As a consequence, any criticism against instants will apply also to the "derived instants".

Now, regarding temporal incidence, the point is whether assertions about occurrence of events and holding of fluents are allowed at the derived instants or not. Allen and Hayes limit their formal development to the theory of time and discuss temporal incidence only informally. As recipient for instantaneous events, they introduce a different type of "instantaneous" entity: the moment [2]. A *moment* is an *indivisible interval*. The problem with moments is the same as with short intervals (subsection 3): since they are intervals, their beginning and end are distinct which does not fit the intuitions about "instantaneous".

Allen and Hayes's reply to the issue of instantaneous holding of fluents is as follows ([3], section 4): "We avoid it by resolutely refusing to allow fluents to hold at points". The reason for that is avoiding the DIP. In section 6 we discuss when the DIP is and is not a problem. Allen and Hayes propose the following alternative: "One could define a notion of a fluent X being true at a point p by saying that X is true at p just when there is some interval I containing p during which X is true". It is easy to see that this does not work for modelling continuous fluents (consider the $v = 0$ fluent in the TBS).

Instants and Periods on the Same Footing The most relevant proposal in this direction is Galton's approach [8]. Its two main features are: (i) The theory of time is defined over instants and periods at equal level (i.e. neither periods are a set-theoretic construction from instants nor viceversa). (ii) Fluents are diversified into instantaneous/durable and *states of position/states of motion*: a state of position can hold at isolated instants; if it holds during a period it holds at its limits (e.g. "a quantity taking a particular value"); a state of motion cannot hold at isolated instants (e.g. "a body being at rest"). The main shortcomings of this approach are the following:

1. The theory formed by the axioms is too weak to properly account for the relations between instants and periods. We discuss it in detail in section 4.

2. It is not clear that Galton's new types of fluents are useful, the reason being that one single type of fluent is not enough to model a continuously changing parameter. Let us illustrate that with the TBS. Consider the fluent $f = (v \neq 0)$. It cannot be modelled as a state of position because f holds on both p_1 and p_2 which must contain the limiting instant i where $\neg f$ holds ($v = 0$). A state of motion cannot be used either because it cannot hold at isolated instants: we are not allowed to say that $\neg f$ is true at i.

4 Time

In this section we present our theory of time, called $\mathcal{IP}$, based on the idea of having instants and periods at equal level[5][6]. Our language for time has two sorts of symbols, the *instants sort* ($\mathcal{I}$) and the *periods sort* ($\mathcal{P}$), which are formed by

[5] Very much with in the same spirit of [8] and [5].

[6] Results on $\mathcal{IP}$ are a summary of those presented in [25].

two infinite disjoint sets of symbols, and three primitive binary relation symbols, $\prec: \mathcal{I} \times \mathcal{I}$ and $\mathtt{begin}, \mathtt{end} : \mathcal{I} \times \mathcal{P}$.

The first-order axiomatization of $\mathcal{IP}$ is as follows:

$\mathbf{IP_1}$	$\neg(i \prec i)$	$\mathbf{IP_{7.1}}$	$\exists i\ \mathtt{begin}(i,p)$
$\mathbf{IP_2}$	$i \prec i' \Rightarrow \neg(i' \prec i)$	$\mathbf{IP_{7.2}}$	$\exists i\ \mathtt{end}(i,p)$
$\mathbf{IP_3}$	$i \prec i' \wedge i' \prec i'' \Rightarrow i \prec i''$	$\mathbf{IP_{8.1}}$	$\mathtt{begin}(i,p) \wedge \mathtt{begin}(i',p) \Rightarrow i = i'$
$\mathbf{IP_4}$	$i \prec i' \vee i \prec i' \vee i = i'$	$\mathbf{IP_{8.2}}$	$\mathtt{end}(i,p) \wedge \mathtt{end}(i',p) \Rightarrow i = i'$
$\mathbf{IP_{5.1}}$	$\exists i'\ (i' \prec i)$	$\mathbf{IP_9}$	$i \prec i' \Rightarrow \exists\ p\ (\mathtt{begin}(i,p) \wedge \mathtt{end}(i',p))$
$\mathbf{IP_{5.2}}$	$\exists i'\ (i \prec i')$	$\mathbf{IP_{10}}$	$\mathtt{begin}(i,p) \wedge \mathtt{end}(i',p) \wedge$
$\mathbf{IP_6}$	$\mathtt{begin}(i,p) \wedge \mathtt{end}(i',p) \Rightarrow i \prec i'$		$\wedge\ \mathtt{begin}(i,p') \wedge \mathtt{end}(i',p') \Rightarrow p = p'$

$IP_1 \div IP_4$ are the conditions for $\prec$ to be an *strict linear order* –namely irreflexive, asymmetric, transitive and linear– relation over the instants[7]. IP_5 imposes unboundness on this ordered set. IP_6 is intended to *order* the extremes of a period. This axiom rules out durationless periods which are not necessary since we have instants as a primitive. The pairs of axioms $IP_{7.\_}$ and $IP_{8.\_}$ formalize the intuition that the beginning and end instants of a period always *exist* and are *unique* respectively. Conversely, axioms IP_9 and IP_{10} close the connection between instants and periods by ensuring the *existence* and *uniqueness* of a period for a given ordered pair of instants.

We now characterize the models by following the simple intuition of an interval being an ordered pair.

Definition 1 ($\mathcal{IP}$-structure) *An $\mathcal{IP}$-structure is a tuple $\langle \mathcal{I}_d, \mathcal{P}_d, <_d, \mathtt{begin}_d, \mathtt{end}_d \rangle$ where $\mathcal{I}_d$ and $\mathcal{P}_d$ are sets of instants and periods respectively, $<_d$ is a binary relation on $\mathcal{I}_d$ and $\mathtt{begin}_d, \mathtt{end}_d$ are binary relations on $\mathcal{I}_d, \mathcal{P}_d$.*

We show that the elements and the pairs of an unbounded linear order $\mathcal{S}$ form a model for $\mathcal{IP}$ and all the models are isomorphic to it.

Theorem 1 (the models) *Given an infinite set $\mathcal{S}$ and an unbounded strict linear order $<$ on it then the $\mathcal{IP}$-structure $\langle \mathcal{S}, \mathtt{pairs}(\mathcal{S}), <, \mathtt{first}, \mathtt{second} \rangle$ forms a model of $\mathcal{IP}$, (`first` returns the first element of the pair and `second` the second). Furthermore, any model $M = \langle \mathcal{I}, \mathcal{P}, \prec, \mathtt{begin}, \mathtt{end} \rangle$ of $\mathcal{IP}$ is* isomorphic *to the structure $\langle \mathcal{I}, \mathtt{pairs}(\mathcal{I}), <, \mathtt{first}, \mathtt{second} \rangle$ where $\mathcal{I}$ and $<$ are the same as in M.*

Corollary 1 *Every model of $\mathcal{IP}$ is characterized by an infinite set $\mathcal{S}$ and an unbounded strict linear order $<$ on it.*

Note that $\mathcal{IP}$ accepts both dense and discrete models of time. The subtheory of dense models, we call it $\mathcal{IP}_{dense}$, is axiomatized by adding the following *denseness* axiom:

$$\mathbf{IP_{11}} \quad i \prec i' \Rightarrow \exists i''\ (i \prec i'' \prec i')$$

[7] Notice that IP_1 is actually redundant since it can be derived from IP_2. We include it for clarity.

Theorem 2 (dense models) *The models of $\mathcal{IP}_{dense}$ are characterized by the set of elements and the set of ordered pairs of distinct elements of an unbounded,* dense, *strict linearly ordered set. Moreover $\mathcal{IP}_{dense}$ is a complete axiomatization for the theory of rationals and rational intervals, namely $Th(\mathcal{Q}, INT(\mathcal{Q}))$.*

Let us see how our theory relates to previous theories of time. Allen and Hayes's theory, let us call it $\mathcal{I}_{\mathcal{AH}}$, is exclusively based on time intervals. To compare to our theory we use the same technique as Ladkin [16] of deriving instants constructed described as follows: first define the notion of pair of meeting intervals, second apply the equivalence relation "having the same meeting point" and, finally, associate an instant to each class. Let us call the resulting theory $\mathcal{I}_{\mathcal{AH}_{\sim_\mathcal{I}}}$. Its class of models is the same as $\mathcal{IP}$, i.e. the theories are equivalent.

Theorem 3 $\mathcal{IP} \equiv \mathcal{I}_{\mathcal{AH}_{\sim_\mathcal{I}}}$

Galton's theory (namely $\mathcal{IP}_\mathcal{G}$) is defined over $\texttt{Within}$ and $\texttt{Limits}$[8] relations. Let I denote a period and i denote an instant:

$\mathbf{I_1}$ $\forall I \exists i\ \texttt{Within}(i, I)$
$\mathbf{I_2}$ $\texttt{Within}(i, I) \wedge \texttt{In}(I, J) \Rightarrow \texttt{Within}(i, J)$
$\mathbf{I_3}$ $\texttt{Within}(i, I) \wedge \texttt{Within}(i, J) \Rightarrow \exists K\ (\texttt{In}(K, I) \wedge \texttt{In}(K, J))$
$\mathbf{I_4}$ $\texttt{Within}(i, I) \wedge \texttt{Limits}(i, J) \Rightarrow \exists K\ (\texttt{In}(K, I) \wedge \texttt{In}(K, J))$

The axioms seem appropriate to avoid DIP-like criticisms and to specify some of the properties for temporal incidence of fluents. However, the theory is too weak. It is easy to identify examples of counter-intuitive models accepted by the theory. For example, let us take a basic model M composed of an infinite set of periods P and Allen's relations satisfying interval axioms, plus an infinite set of instants I which make M satisfy I_1. Now, we take M plus a single instant $i \notin I$ which $\texttt{Limits}$ a certain period $p \in P$ and only that one. The axioms do not force it to limit any of those periods that meet or are met by p. The obvious undesirable consequence of $\mathcal{IP}_\mathcal{G}$ weakness is that some queries will not receive the expected intuitive answers. In the example, given the assertions $\texttt{Within}(i, p), \texttt{Meets}(p, p')$, is not possible to derive an answer for the query $\texttt{Limits}(i, p')$.

The reason of this weakness is the loose connection between instants and periods. In [25] we show how $\mathcal{IP}_\mathcal{G}$ can be properly extended to implement the idea of instants as period the meeting points [25]. We show that the resulting theory is equivalent to $\mathcal{IP}_{dense}$.

5 Temporal Incidence

In this section we present our theory of temporal incidence, called $\mathcal{CD}$, which is based on the following ideas:

[8] $\texttt{Limits}(i, p) \stackrel{def}{=} i = \texttt{begin}(p) \vee i = \texttt{end}(p)$, $\texttt{Within}(i, p) \stackrel{def}{=} \texttt{begin}(p) < i < \texttt{end}(p)$.

1. *We allow fluents to hold at points.* We discuss why this does not, in fact, create any problem. It makes the resulting theory much simpler to define.

2. *We distinguish between continuous and discrete fluents.* We diversify fluents according to whether the change on the parameter they model is *continuous* or *discrete*.

As undelying language we take standard temporal reified first-order one with equality (as in [17, 1, 8]). The decisions made regarding temporal representation are the following:

- **Time theory**: We take $\mathcal{IP}_{dense}$. We define the instant-to-period relations (such as `Within`) and period-to-period (such as `Meets`) upon $\prec$, `begin` and `end`.

- **Reified propositions**: Reified propositions are classified into *continuous fluents*, *discrete fluents*[9] and *events*.

- **Temporal Occurrence Predicates (TOPs)**. We introduce a different TOP for each combination of temporal proposition and time unit (similar to [14, 8]):

$\textsc{Holds}^{\sim}_{on}(f,p)$	$\stackrel{def}{=}$	The continuous fluent f holds throughout the period p
$\textsc{Holds}^{\neg}_{on}(f,p)$	$\stackrel{def}{=}$	The discrete fluent f holds throughout the period p
$\textsc{Holds}^{\sim}_{at}(f,i)$	$\stackrel{def}{=}$	The continuous fluent f holds at the instant i
$\textsc{Holds}^{\neg}_{at}(f,i)$	$\stackrel{def}{=}$	The discrete fluent f holds at the instant i
$\textsc{Occurs}_{on}(e,p)$	$\stackrel{def}{=}$	The event e occurs on the period p
$\textsc{Occurs}_{at}(e,i)$	$\stackrel{def}{=}$	The event e occurs at the instant i

Terminology. Henceforth we use the following notational shorthands. We may use `begin` and `end` in functional form (e.g. $i = \texttt{begin}(p)$). $\textsc{Holds}_{on}$ stands for both $\textsc{Holds}^{\sim}_{on}$ and $\textsc{Holds}^{\neg}_{on}$, and $\textsc{Holds}_{at}$ for $\textsc{Holds}^{\sim}_{at}$ and $\textsc{Holds}^{\neg}_{at}$. Given any two periods p, p' such that $\texttt{Meets}(p,p')$, we define the functions $\texttt{meetpoint}(p,p') \stackrel{def}{=} \texttt{end}(p) = \texttt{begin}(p')$ and $\texttt{concat}(p,p') \stackrel{def}{=} p''$ s.t. $\texttt{begin}(p'') = \texttt{begin}(p) \wedge \texttt{end}(p'') = \texttt{end}(p')$. We use the 13 common qualitative relations between intervals. PR denotes the set of them. We define also

$$\texttt{In} : \mathcal{P} \times \mathcal{P} \stackrel{def}{=} \texttt{Starts} \vee \texttt{During} \vee \texttt{Finishes}$$
$$\texttt{Disjoint}^{\neg}_{on} : \mathcal{P} \times \mathcal{P} \stackrel{def}{=} \texttt{Before} \vee \texttt{After}$$
$$\texttt{Disjoint}^{\sim}_{on} : \mathcal{P} \times \mathcal{P} \stackrel{def}{=} \texttt{Before} \vee \texttt{Meets} \vee \texttt{Met\_by} \vee \texttt{After}$$

[9]We use the equality relation to express a fluent representing a parameter taking a certain value. E.g. the speed of a ball being positive on p is expressed as $\textsc{Holds}(\text{speed} = +, p)$. We omit necessary axioms imposing the exclusivity among the different values of a parameter.

5.1 Axioms of Temporal Incidence

Since instants and periods are both primitive units in $\mathcal{IP}$, we are not forced to accept any assumption on the relation between the holding of a fluent on a period and its holding at the period endpoints. A fluent holds during a period iff it holds at its *inner* instants:

$$\mathbf{CD_1} \quad \text{HOLDS}_{on}(f,p) \iff (\mathtt{Within}(i,p) \Rightarrow \text{HOLDS}_{at}(f,i))$$

From it, nothing can be derived about the holding of f at the p endpoints.

Continuous Fluents A continuous fluent may hold both during a period and at a particular instant without any restriction. This is not the case for discrete ones.

Discrete Fluents The genuine property of discrete fluents is that *they cannot hold at an isolated instant*:

$$\mathbf{CD_2} \quad \text{HOLDS}^{\neg}_{at}(f,i) \Rightarrow \exists p\ (\text{HOLDS}^{\neg}_{on}(f,p) \wedge (\mathtt{Within}(i,p) \vee \mathtt{begin}(i,p) \vee \mathtt{end}(i,p)))$$

Our distinction between continuous and discrete events is different from Galton's distinction between states of position and states of motion. Identifying it as a key property in modelling changing domains is a key insight in this paper.

Non-Instantaneous Events The intuition behind events (both instantaneous and durable) is that of an *accomplishment* that may have relevant consequences over the state of the world. Unlike preceding approaches, *our theory does not include any axiom governing the occurrence of events that take time.* It corresponds to the intuition that two *accomplishments* happening concurrently is plausible or not depending on the abstraction degree of the analysis. For example, the event "programming the program p1" can not occur over two periods that are not disjoint. It is not the case, however, if the event under consideration is merely "programming a program". Therefore, no domain-independent axiom can be stated as part of a general theory of temporal incidence.

Non-Atomic Fluents Our theory directly addresses the issue of the holding of non-atomic fluents with the following axioms:

Negation :	$\mathbf{CD_3}$	$\text{HOLDS}_{at}(\neg f,i) \iff \neg\text{HOLDS}_{at}(f,i)$
Conjunction :	$\mathbf{CD_4}$	$\text{HOLDS}_{at}(f \wedge f',i) \iff \text{HOLDS}_{at}(f,i) \wedge \text{HOLDS}_{at}(f',i)$
Disjunction :	$\mathbf{CD_5}$	$\text{HOLDS}_{at}(f \vee f',i) \iff \text{HOLDS}_{at}(f,i) \vee \text{HOLDS}_{at}(f',i)$

Deriving the properties of non-instantaneous holding of non-atomic fluents from these axioms is straight forward.

6 Revisiting the Problematic Issues

Let us see now how the problems presented in section 2 are addressed using $\mathcal{IP}$ axioms, our set of TOPs and $\mathcal{CD}$ axioms.

Instantaneous Events Since instants are a primitive unit in our theory, we can directly express instantaneous events using the predicate $\textsc{Occurs}_{at}$. In the DIP, for instance, we have $\textsc{Occurs}_{at}(\text{switchoff}, i)$. In section 7 we discuss the more delicate issue of *sequences of instantaneous events'*.

Instantaneous Holding We allow to talk about the instantaneous holding of a fluent at a certain instant by using the predicate $\textsc{Holds}_{at}$. Axiom $\mathbf{CD_1}$ (by which fluents hold over *a priori* "open" periods) ensures we will be able to express the holding of contradictory fluents ending or beginning at that instant without getting in conflict. Furthermore, we can express the holding of a continuous fluent at an solated instants which is required to model continuous change. The TBS scenario, for example, is simply represented as follows:

$$\begin{array}{ll} \textsc{Holds}^{\sim}_{on}(\text{speed} = +, p_1) & \\ \textsc{Holds}^{\sim}_{at}(\text{speed} = 0, i) & \quad \texttt{end}(p_1) = i = \texttt{begin}(p_2) \\ \textsc{Holds}^{\sim}_{on}(\text{speed} = -, p_2) & \end{array}$$

The Dividing Instant Problem The DIP is not a problem for temporal incidence theories where the following two conditions hold:

1. The holding of a fluent over a period does not constrain its holding at the period's endpoints.

2. Instantaneous holding of fluents can be expressed.

These conditions avoid logical contradiction and truth gap at the dividing instant, respectively. In figure 2, the fluent f can be regarded as discrete and the DIP scenario can be formalized as follows:

$$\textsc{Holds}^{\neg}_{on}(\text{light} = \text{on}, p_1) \;\wedge\; \texttt{Meets}(p_1, p_2) \;\wedge\; \textsc{Holds}^{\neg}_{on}(\text{light} = \text{off}, p_2)$$

Given this information only, the query $\textsc{Holds}^{\neg}_{on}(\text{light} = \text{on}, \texttt{end}(p_1))$ simply cannot be answered. Our position is that answering it requires additional domain-dependent information. In some cases we may want to specify that a fluent holds on and at the end of a period. For instance, consider the fluent "being in contact with the floor" for a ball being lifted up. Other fluents will hold at the beginning and on the given period (e.g. "not being in contact with the floor" for a ball that falls on the floor). In the light example, the most appropriate might be having three fluents light=on, light=off and light=changing, where the former two hold over open periods and the latter at an instant.

We believe that this is a domain-dependent issue. Our approach avoids making any commitment about the holding at period's endpoints, whereas provides the means to safely specify what happens *at* the dividing instant.

Non-Instantaneous Fluent Holding A nice feature of our proposal is that the above few axioms are enough to easily derive the fundamental properties of temporal incidence of fluents. For instance, *Allen's Homogeneity* $\textsc{Holds}^{\sim}_{on}(f,p) \iff \mathtt{In}(p',p) \Rightarrow \textsc{Holds}^{\sim}_{on}(f,p')$ is easy to prove from axiom $\mathbf{CD_1}$. The properties for concatenability are as follows:

Theorem 4 (Concatenability of discrete fluents)
If $\mathtt{Meets}(p,p')$ *then*

$$\textsc{Holds}^{\neg}_{on}(f,p) \wedge \textsc{Holds}^{\neg}_{on}(f,p') \iff \textsc{Holds}^{\neg}_{on}(f,\mathtt{concat}(p,p'))$$

Theorem 5 (Concatenability of continuous fluents)
If $\mathtt{Meets}(p,p')$ *then*

$$\textsc{Holds}^{\sim}_{on}(f,p) \wedge \textsc{Holds}^{\sim}_{on}(f,p') \wedge \textsc{Holds}^{\sim}_{at}(f,\mathtt{meetpoint}(p,p')) \iff$$
$$\iff \textsc{Holds}^{\sim}_{on}(f,\mathtt{concat}(p,p'))$$

Concatenability can be regarded as a special case of *joinability*. Given two periods p, p', $\mathtt{join}(p,p')$ is defined as a period p'' such that $\mathtt{begin}(p'') = \mathtt{min}(\mathtt{begin}(p),\mathtt{begin}(p'))$ and $\mathtt{end}(p'') = \mathtt{max}(\mathtt{end}(p),\mathtt{end}(p'))$, where $\mathtt{min}$ and $\mathtt{max}$ are defined according to the ordering relation $\prec$ over instants.

Theorem 6 (Joinability of discrete fluents)
If $\neg\mathtt{Disjoint}^{\neg}_{on}(p,p')$ *then*

$$\textsc{Holds}^{\neg}_{on}(f,p) \wedge \textsc{Holds}^{\neg}_{on}(f,p') \iff \textsc{Holds}^{\neg}_{on}(f,\mathtt{join}(p,p'))$$

Theorem 7 (Joinability of continuous fluents)
If $\neg\mathtt{Disjoint}^{\sim}_{on}(p,p')$, *or*
$\mathtt{Meets}(p,p') \wedge \textsc{Holds}_{at}(f,\mathtt{meetpoint}(p,p'))$, *or*
$\mathtt{Met\_by}(p,p') \wedge \textsc{Holds}_{at}(f,\mathtt{meetpoint}(p',p))$
then

$$\textsc{Holds}^{\sim}_{on}(f,p) \wedge \textsc{Holds}^{\sim}_{on}(f,p') \iff \textsc{Holds}^{\sim}_{on}(f,\mathtt{join}(p,p'))$$

There are also a number properties relating the holding of contradictory fluents at distinct, related times.

Theorem 8 (non-holding of discrete fluents)

$$\neg\textsc{Holds}^{\neg}_{on}(f,p) \iff \exists p'\ \mathtt{In}(p',p) \wedge \textsc{Holds}^{\neg}_{on}(\neg f,p')$$

Theorem 9 (non-holding of continuous fluents)

$$\neg\textsc{Holds}^{\sim}_{on}(f,p) \iff \exists i\ \mathtt{Within}(i,p) \wedge \textsc{Holds}^{\sim}_{at}(f,i)$$

Theorem 10 (disjointness)

$$\text{HOLDS}_{on}(f,p) \wedge \text{HOLDS}_{on}(\neg f,p') \Rightarrow \texttt{Disjoint}^{\sim}_{on}(p,p')$$

At this point one may ask for how long can we go enumerating properties of temporal incidence. To answer this question, let us analyze the issue from a more general perspective. The above properties are, in fact, particular cases of the following general scheme (f is a fluent and $\bar{p}$ denotes the collection of periods $p_1,\ldots,p_n$) :

If $\text{HOLDS}(f,\bar{p})$ and $f \models f'$ then $\text{HOLDS}(f',\bar{p}')$
If $\text{HOLDS}(f,\bar{p})$ and $f \models \neg f'$ then $\text{HOLDS}(\neg f',\bar{p}')$

The scope of this paper goes as far as showing that the most basic properties of this scheme are theorems of our theory. There might be other properties one may be interested to consider such as more sophisticated instances of the above scheme or general temporal properties of some relations basic for a particular task (e.g. the temporal relation between f and f' when $\text{CAUSES}(f,f')$).

7 Example: Modelling Hybrid Systems

In this section we illustrate the application of our theory in qualitative modelling of a physical system. A (qualitative) model of a system is usually the result of an abstraction intended to simplify the analysis. When this abstraction produces discontinuous or discrete behaviors together with continuous ones the result is called a *hybrid model.* There are many instances physical systems such as most electro-mechanical devices (e.g. photocopiers, cars, stereo sets, video cameras) that are suitably modelled as hybrid. There are several approaches to representing discrete changes into a continuous model in the area of qualitative modelling [19, 7, 11, 10].

Some semantical problems have been encountered because of the different nature of discrete and continuous change. We shall see that an adequate theory of time and temporal incidence is fundamental to overcome them. Let us consider a particular example from [10] (we borrow the example, the qualitative model, the intended envisionment and a tentative solution). Figure 3 shows a simple circuit in which electric power is provided to a load by either a solar array or a rechargeable battery.

A part of the continuous behavior of this system is described as follows:

C0: "If the sun is shining and the relay is closed then the solar array acts as a constant current source and the battery accumulates charge."

The discrete events are specified as follows:

D1: "If the relay is closed, when the signal from the controller goes high, then the relay opens."

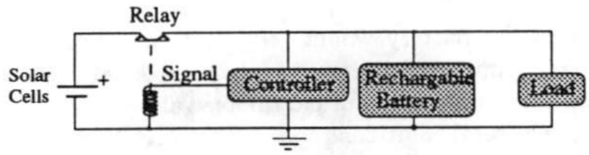

Figure 3: The hybrid circuit example.

D2: "If the relay is open, when the signal from the controller goes low, then the relay closes."

D3: "If the signal is low, when the controller detects that the charge level in the battery has reached the threshold q_2, then the controller turns on the signal to the relay."

Now, let us consider a particular predicted qualitative behavior. This is normally described as a sequence of states that hold alternatively at a point and during an interval. The transition from one state to another is produced either by a continuous change or by a discrete change produced by a discrete event. Quoting Iwasaki *et al*: "...we would like to model discrete events as being instantaneous". Problems arise when sequences of them need to be described, such as "the signal goes high and *immediately after* the relay closes". The following predicted behavior and the explanation about why sequences of discrete events are problematic are from [10]. Given the initial state of our example where the *signal* is *low*, the *relay* is *closed* and the sun is shining, the intended envisionment would be the following sequence of states:

s_1	(t_1, t_2)	$Q_{BA} < q_2$	*signal* = *low*	*relay* = *closed*
s_2	t_2	$Q_{BA} = q_2$	*signal* = *low*	*relay* = *closed*
$s_{2.1}$	$t_{2.1}$	$Q_{BA} =?$	*signal* = *high*	*relay* = *closed*
$s_{2.2}$	$t_{2.2}$	$Q_{BA} =?$	*signal* = *high*	*relay* = *open*
s_3	$(t_{2.2}, -)$	$Q_{BA} < q_2$	*signal* = *high*	*relay* = *open*

The state $s_{2.1}$ is produced by the signal going high, and $s_{2.2}$ by the relay closing. It is not clear how to model the times of s_2, $s_{2.1}$ and $s_{2.2}$ as well as both the time spans and the discrete events between them. Assuming that discrete events take no time leads to logical contradiction because of the way the discrete events are specified. "The antecedent for rules specifying such events often includes the negation of the consequence; this leads to a contradiction when events are treated as implications." In the case of **D1**, for example, the signal is low and goes high, but if the change is instantaneous both values for the signal will be true at the same time.

The alternative is assuming that discrete changes take a very little time interval. It is problematic too since the value of continuous variables changing concurrently becomes unknown after a sequence of actions. In the example, the charge of the battery would keep continuously increasing for a short period. After a number of discrete events these small variations accumulate and complicate the computation of parameter values.

There are several solutions proposed to solve this quandary. They are based on complicating the model of time either by introducing *mythical* time ([19], *direct method*), by extending the real numbers with infinitessimals ([19], *approximation method*)[4], or by using non-standard analysis [10]. We next show that none of them is necessary. We use our theory of instants/periods and continuous fluents/discrete fluents/events as follows:

- Discrete events are modelled as instantaneous events.
- Continuous (discrete) quantities are modelled as continuous (discrete) fluents.

Since HOLDS_{on} is defined as holding at the inner points only, the values of fluent being changed by an instantaneous event will not be defined at the time the event occurs unless there is some specific knowledge about it. The sequence of states representing the intended envisionment becomes simpler:

s_1	(t_1, t_2)	$Q_{BA} < q_2$	$signal = low$	$relay = closed$
s_2	t_2	$Q_{BA} = q_2$	$signal = ?$	$relay = ?$
s_3	$(t_2, \_)$	$Q_{BA} < q_2$	$signal = high$	$relay = open$

Indeed, this solution is much simpler than the previously proposed techniques. Its formalization in terms of event occurrences and holding of continuous and discrete fluents is as follows:

$\widetilde{\mathrm{HOLDS}}_{on}(Q_{BA} < q_2, p_1)$	
$\mathrm{HOLDS}^{\neg}_{on}(signal = low, p_2)$	
$\mathrm{HOLDS}^{\neg}_{on}(relay = closed, p_3)$	
$\widetilde{\mathrm{HOLDS}}_{at}(Q_{BA} = q_2, \mathrm{end}(p_1))$	
$\mathrm{OCCURS}_{at}(turn\_on(signal), \mathrm{end}(p_1))$	$\mathrm{end}(p_1) = \mathrm{end}(p_2)$
$\mathrm{HOLDS}^{\neg}_{on}(signal = high, p_4)$	$\mathtt{Meets}(p_2, p_4)$
$\mathrm{OCCURS}_{at}(open(relay)), \mathrm{end}(p_3))$	$\mathrm{end}(p_2) = \mathrm{end}(p_3)$
$\mathrm{HOLDS}^{\neg}_{on}(relay = open, p_5)$	$\mathtt{Meets}(p_3, p_5)$

8 Conclusion

A theory of time and temporal incidence is the foundation for a proper temporal representation, independently of the method used. In this paper we identified the problematic issues that need to be addressed, namely the expression of *instantaneous events* and *instantaneous holding of fluents*, the *dividing instant problem* and the formalization of the properties *for non-instantaneous holding of fluents*.

The main contribution of this paper twofold. First, we show that no single previous approach satisfactorily addresses all of these issues. Then, we propose a theory of time and temporal incidence (that may look straight forward in hindsight) which does it. The key insights behind our approach are:

1. *The time ontology is composed of both instants and periods.* Several criticisms (such as the DIP) have been put forward against having instants as a primitive. We discussed that instants are needed to describe instantaneous events and instantaneous holding of fluents, specially when we want to model systems with both discrete and continuous change. We have shown that alternatives based on deriving points from intervals are either not appropriate or do not provide any benefit. We explained how is possible to allow instantaneous holding of fluents and avoid the DIP.

2. *Fluents are diversified into continuous and discrete.* The diversification of fluents into continuous and discrete is commonly agreed upon, but there is no previous approach that develops it properly. We propose a theory of temporal incidence based on this distinction and formally present the relevant properties derived from it.

References

[1] J. Allen. Towards a general theory of action and time. *Artificial Intelligence*, 23:123–154, 1984.

[2] J. Allen and P. Hayes. A common-sense theory of time. In *Proc. IJCAI'85*, pages 528–531, 1985.

[3] J. Allen and P. Hayes. Moments and points in an interval-based temporal logic. *Computational Intelligence*, 5:225–238, 1989.

[4] R. Alur, C. Courcoubetis, T. Henzinger, and P. Ho. Hybrid automata: an algorithmic approach to the specification and analysis of hybrid systems. In *Proc. Workshop on Theory of Hybrid Systems*, pages 209–229, 1993.

[5] A. Bochman. Concerted instant-interval temporal semantics i: Temporal ontologies. *Notre Dame Journal of Formal Logic*, 31(3):403–414, 1990.

[6] T. Dean and D. McDermott. Temporal data base management. *Artificial Intelligence*, 32:1987, 1987.

[7] K. Forbus. Introducing actions into qualitative simulation. In *Proc. IJCAI'89*, pages 1273–1278, 1989.

[8] A. Galton. A critical examination of Allen's theory of action and time. *Artificial Intelligence*, 42:159–188, 1990.

[9] C. Hamblin. Instants and intervals. In J. Fraser, editor, *The Study of Time*, pages 325–331. Springer-Verlag, 1972.

[10] Y. Iwasaki, A. Farquhar, V. Saraswat, D. Bobrow, and V. Gupta. Modelling time in hybrid systems: How fast is 'instantaneous'? In *Proc. IJCAI'95*, 1995.

[11] Y. Iwasaki and C. Low. Device modelling environment: An integrated model-formulation and simulation environment for continuous and discrete phenomena. In *Proc. of Conf. on Intelligent Systems Engineering*, 1992.

[12] J. Jaffar and M. Maher. Constraint logic programming: A survey. *Journal of Logic Programming*, (in press), 1996.

[13] H. Kamp. Events, instants and temporal reference. In R. Bäuerle, U. Egli, and A. von Stechow, editors, *Semantics from Different Points of View*, pages 376–417. Springer-Verlag, 1979.

[14] R. Kowalski and M. Sergot. A logic-based calculus of events. *New Generation Computing*, 3, 1986.

[15] B. Kuipers. Qualitative simulation using time-scaled abstraction. *Artificial Intelligence in Engineering*, 3(4):185–191, 1988.

[16] P. Ladkin. Models of axioms for time intervals. In *Proc. AAAI'87*, pages 234–239, 1987.

[17] D. McDermott. A temporal logic for reasoning about processes and plans. *Cognitive Science*, 6:101–155, 1982.

[18] R. Miller and M. Shanahan. Narratives in the situational calculus. *Journal of Logic and Computation - Special Issue on Actions and Processes*, 4(5):513–530, 1994.

[19] T. Nishida and S. Doshita. Reasoning about discontinuous change. In *Proc. AAAI'87*, pages 643–648, 1987.

[20] J. Pinto. Temporal reasoning in the situational calculus. Technical Report KRR-TR-94-1, Dept. of Computer Science, University of Toronto, 1994.

[21] F. Sadri. Three recent approaches to temporal reasoning. In A. Galton, editor, *Temporal Logics and their Applications*, pages 121–168. Academic Press, 1987.

[22] E. Schwalb, K. Kask, and R. Dechter. Temporal reasoning with constraints on fluents and events. In *Proc. AAAI'94*, 1994.

[23] Y. Shoham. Temporal logics in AI: Semantical and ontological considerations. *Artificial Intelligence*, 33:89–104, 1987.

[24] J. van Benthem. *The Logic of Time*. Kluwer Academic, Dordrecht, 1983.

[25] L. Vila. *IP*: An instant-period–based theory of time. In R. Rodriguez, editor, *Proc. ECAI'94 Workshop on Spatial and Temporal Reasoning*, 1994.

[26] B. C. Williams. Doing time: Putting qualitative reasoning on firmer ground. In *Proc. AAAI'86*, pages 105–112, 1986.

9 Rules formalizing the Behavior of the Circuit Example

D1 $\textsc{Holds}^{\neg}_{on}(relay = closed, p) \wedge$
$\wedge\ \textsc{Occurs}_{at}(open(relay), \mathbf{end}(p)) \Rightarrow$
$\Rightarrow \textsc{Holds}^{\neg}_{on}(relay = open), p') \ \wedge\ \mathtt{Meets}(p, p')$

D2 $\textsc{Holds}^{\neg}_{on}(relay = closed, p) \wedge$
$\wedge\ \textsc{Occurs}_{at}(close(relay), \mathbf{end}(p)) \Rightarrow$
$\Rightarrow \textsc{Holds}^{\neg}_{on}(relay = closed), p') \ \wedge\ \mathtt{Meets}(p, p')$

D3 $\textsc{Holds}^{\neg}_{\underset{\sim}{on}}(signal = low, p) \wedge$
$\underset{\sim}{\textsc{Holds}}_{at}(Q_{BA} \geq q_2, \mathbf{end}(p)) \Rightarrow$
$\Rightarrow \textsc{Occurs}_{at}(turn\_on(signal), \mathbf{end}(p))$