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Journal
Physics of Plasmas, 22(8)

ISSN
1070-664X

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Publication Date
2015-08-01

DOI
10.1063/1.4928436

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PHYSICS OF PLASMAS 22, 082507 (2015)

Ions lost on their first orbit can impact Alfvén eigenmode stability

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(Received 25 June 2015; accepted 24 July 2015; published online 13 August 2015)

Some neutral-beam ions are deflected onto loss orbits by Alfvén eigenmodes on their first bounce orbit. The resonance condition for these ions differs from the usual resonance condition for a confined fast ion. Estimates indicate that particles on single-pass loss orbits transfer enough energy to the wave to alter mode stability. © *2015 AIP Publishing LLC*.
[http://dx.doi.org/10.1063/1.4928436]

I. INTRODUCTION

Alfvén eigenmode stability is a competition between energy extracted from the fast-ion population and energy lost to the background plasma.[1] The energy that a particle exchanges in a single orbit is $\oint e\vec{E}\cdot\vec{v}\,dt$, where e and $\vec{v}$ are the charge and velocity of the particle, $\vec{E}$ is the wave's electric field, and the integral is over the orbit. Customary analysis assumes that fast ions interact with the mode for many wave cycles. In this case, a particle may exchange energy on a single orbit but, after many cycles, only orbits that preserve the wave-particle phase over multiple orbits exchange net energy. These particles are the resonant particles. If the wave frequency ω is much smaller than the cyclotron frequency, the resonance condition is

$$\omega = n\omega_\phi - (m+l)\omega_\theta, \tag{1}$$

where ω_ϕ and ω_θ are the toroidal and poloidal orbital frequencies, n and m are the toroidal and poloidal mode numbers, and l is an integer. Depending on the initial phase, some resonant particles gain energy from the wave and others lose energy. In linear theory, the net exchange of energy between the waves and the particles depends upon the slope of the distribution function at the resonant frequency, as in Landau damping.

In a DIII-D experiment, Chen *et al.* discovered that toroidal Alfvén eigenmodes (TAE)[2] and reversed shear Alfvén eigenmodes (RSAE)[3,4] can deflect newly injected beam ions ~10 cm on their first bounce orbit through the mode.[5,6] In the experiment, the loss detectors are spatially localized, so coherent losses are only measured when the unperturbed orbit passes close to a fast-ion loss detector.[5–7] Presumably, however, the same process occurs for all ions on similar orbits, irrespective of toroidal angle. These lost particles do not satisfy the resonant condition, Eq. (1). Nevertheless, the particles do maintain approximately constant phase on a single pass through the mode, so the net energy exchange $\oint e\vec{E}\cdot\vec{v}\,dt$ is nonzero. Recently, Zhang *et al.*[8] analyzed this situation for a model RSAE and equilibrium. They found that particles that satisfy a single-pass resonant condition

$$\omega = n\langle\dot{\phi}\rangle - m\langle\dot{\theta}\rangle \tag{2}$$

deflect ~5 cm as they orbit through the mode for conditions similar to the experiment. Here, $\langle\dot{\phi}\rangle$ and $\langle\dot{\theta}\rangle$ are the average toroidal and poloidal angular velocities during the portion of the orbit that traverses the mode. Because the orbit is large and the Alfvén eigenmode is of limited spatial extent, $\langle\dot{\phi}\rangle$ and $\langle\dot{\theta}\rangle$ differ considerably from the customary (orbit-averaged) precession and bounce frequencies ω_ϕ and ω_θ.

In both theory and experiment, the magnitude of the deflection depends linearly on the mode amplitude. The direction of the radial "kick" depends on the phase of the wave during the orbit's transit through the mode. Hamiltonian theory implies that, in an interaction with a single mode, the change in particle energy ΔW is related to the change in toroidal canonical angular momentum ΔP_ϕ by the relation

$$n\Delta W = \omega\Delta P_\phi. \tag{3}$$

Ions that are deflected inward to smaller major radius gain energy from the wave; ions that are deflected outward, lose energy.

In a normal interaction with a confined non-resonant particle, a particle that gains energy from the wave eventually gives it back; similarly, a particle that initially loses energy eventually regains it. Irrespective of the initial wave-particle phase, the net energy transfer for non-resonant particles averages to zero. Only resonant particles produce secular terms that survive after averaging over many wave cycles.

A loss boundary destroys this reversibility. If the initial wave-particle phase pushes the ion outward so that it collides with the wall, it loses energy. Because the ion is lost, the return cycle is never completed. Ions that initially move inward gain energy from the wave. Since they move inward, they remain confined until the phase changes sign. They then move outward, collide with the wall, and have lost energy. Irrespective of the initial wave-particle phase, all particles that are near the loss boundary deliver energy to the wave.

The goal of this paper is to assess the impact of this effect on mode stability. The rough estimates presented here show that the power transfer is potentially significant.

II. COMPARISON OF RESONANT AND SINGLE-PASS ENERGY TRANSFER

For fast-ion driven instabilities, the contribution to the growth rate γ_{drive} is proportional to

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$$\gamma_{drive} \propto N_{res} \Delta W f_{net}, \tag{4}$$

where N_{res} is the number of particles that interact with the wave, ΔW is the amount of energy exchanged in an interaction, and f_{net} depends upon the fraction of the particles that lose or gain energy. This section compares resonant interactions with single-pass loss interactions for these three factors qualitatively and quantitatively.

For the quantitative comparison, we use an actual case (discharge #146096 at 365 ms) that is shown in many figures in the experimental papers.[5,6] At 365 ms, this discharge has a circular shape (elongation $\kappa \simeq 1.2$), a line-averaged density of $1.4 \times 10^{19}\,\mathrm{m}^{-3}$, a plasma current of 0.6 MA, and a toroidal field of 2.1 T. The minimum and edge safety factor are $q_{min} = 3.5$ and $q_{95} = 6.0$, respectively. The neutral beams inject 4.6 MW of deuterium neutrals at 74 and 80 keV in the co-current direction, with approximately equal numbers injected by near-perpendicular and near-tangential sources (tangency radii of 74 and 115 cm, respectively). The Larmor radius (banana width) of a typical trapped particle is approximately 5 (33) cm. The beams drive many Alfvén modes unstable, including an $n = 2$, $m = 7$, 117 kHz RSAE with a peak fluctuation amplitude of $\delta T_e / T_e \simeq 1.4\%$ and a radial eigenfunction that extends between $R = 195$–215 cm. (T_e is the electron temperature and R is the major radius.) The calculations in Ref. 8 are based on a model RSAE that resembles this mode.

The first factor that determines the growth rate is the number of resonant fast ions N_{res}. To interact with the wave effectively, an ion must be on an orbit that satisfies either the standard resonance condition (Eq. (1)) or the single-pass resonance condition (Eq. (2)). Figure 1 shows the orbit topology in this equilibrium for ions at the injection energy. The orbits are classified according to their angular momentum P_ϕ and magnetic moment μ. For these large-orbit fast ions, in addition to the usual trapped ions and co-passing and counter-passing circulating ions, there are lost orbits and non-axis encircling orbits.[9] Orbits that satisfy one of the resonance conditions and transit through the measured RSAE eigenfunction are marked on the orbit topology map. For the single-pass resonance, the guiding-center orbit must also pass within 10 cm of the wall. Standard resonances (Eq. (1)) with this mode exist for trapped, co-passing, and counter-passing orbits. Single-pass resonances (Eq. (2)) exist with trapped orbits and occupy a different region of phase space than the standard resonance. The single-pass resonance occurs on the inner (counter-going) leg of the banana orbit, where $\langle \dot{\phi} \rangle$ has opposite sign from the precession frequency ω_ϕ.

In addition to the resonance condition, another factor that influences the number of ions that interact with the wave is the width of the resonance $\Delta\omega$. The standard resonance is much narrower than the single-pass resonance. The calculations in Ref. 8 indicate that $\Delta\omega$ is an order of magnitude wider for the single-pass resonance. For the single-pass resonance, effective wave-particle interaction occurs as long as the phase variation across the interaction region is $\lesssim \pi$, so $\Delta\omega$ is large.[8]

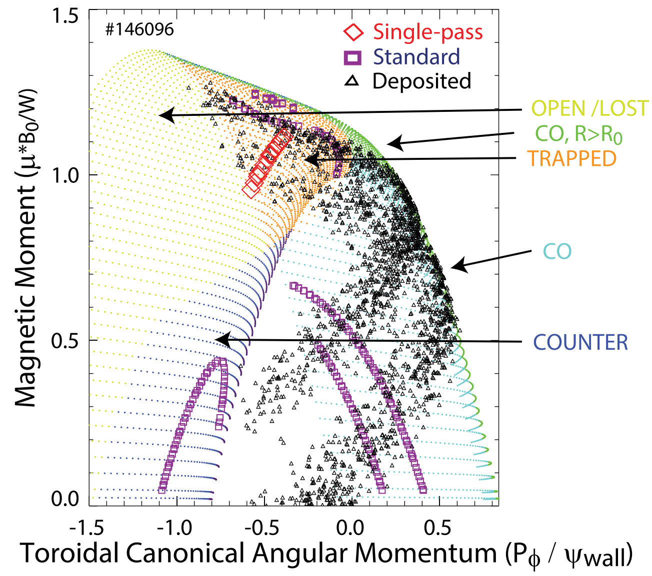

FIG. 1. Orbit topology map for DIII-D discharge #146096 at 365 ms for $W = 75$ keV. The abscissa is the toroidal canonical angular momentum normalized to the poloidal flux at the last-closed flux surface. The ordinate is the magnetic moment normalized by the particle energy and magnetic field at the magnetic axis. Orbits that satisfy the standard resonance condition (Eq. (1)) for $l = -3$ to 3 and that pass through the RSAE eigenfunction are indicated by squares; orbits that satisfy the single-pass resonance condition (Eq. (2)), which pass through the RSAE eigenfunction, and pass within 10 cm of the wall are indicated by diamonds. The small triangles represent orbits that are populated by the injected neutral beams. The various orbit types listed in the legend are lost orbits (including orbits on open field lines), co-passing orbits that do not encircle the magnetic axis, trapped orbits, axis-encircling co-passing orbits, and counter-passing orbits.

In addition to the resonance conditions and resonance widths, the number of ions that interact with the mode depends upon the distribution function. The initial orbits are determined by the beam deposition; subsequently, Coulomb scattering populates other orbits. In Fig. 1, the orbital parameters that are initially populated by the neutral beams are overlaid on the resonance map. The calculation[10] uses measured density and temperature profiles and includes orbits that ionize in the scrape-off layer. The figure shows that the beams populate the co-passing and trapped portions of phase space. As a rough estimate of the number of fast ions that interact with the mode, we count the fraction of deposited orbits that satisfy a resonance condition (within the resonance width). Approximately 2% of the recently ionized fast ions interact with the RSAE through the single-pass resonance condition. Approximately 6% of recently ionized fast ions satisfy a standard resonance condition. The figure shows orbits at the time of fast-ion birth; however, additional ions come into resonance with the mode as they thermalize. Since thermalization causes the banana width to shrink, moving ions away from loss boundaries, additional single-pass resonances from thermalizing ions are rare. In contrast, many confined ions do eventually satisfy a standard resonance condition. To estimate this fraction, we track the deposited ions through phase space as they thermalize, modifying (W, μ, P_ϕ) and hence the orbital frequencies ω_ϕ and ω_θ; at each energy step, we check to see if the tracked ion encounters a resonance with the observed RSAE.

Ultimately, ~20% of the injected ions eventually encounter a resonance with the RSAE. (For this estimate, resonances between $l = -3$ and 3 in Eq. (1) are considered.)

The second factor is the amount of energy exchanged in the interaction ΔW. This depends on the time-averaged value of $\oint e\vec{E}\cdot\vec{v}\,dt$, where the average is over many cycles for the standard resonance and over one cycle for the single-pass resonance. Calculations with a model RSAE (Figs. 2 and 3 of Ref. 8) find that the most effective standard resonances exchange about twice as much energy as the single-pass resonance. In the calculation, single-pass resonant particles exchange ~3.5% of their energy. An independent estimate of the energy exchange can be found from the measured displacement. Experimentally, the single-pass resonant particles are deflected ~10 cm. This deflection implies a change in average flux surface $\Delta\psi/\psi$, where ψ is the poloidal flux. With the assumption that $\Delta P_\phi/P_\phi \simeq \Delta\psi/\psi$, Eq. (3) implies $\Delta W/W \simeq 13\%$.

The third factor is the fraction of resonant particles that gain or lose energy from the wave f_{net}. As argued in Sec. I, virtually all single-pass resonant particles ultimately deliver energy to the wave, so $f_{net} \simeq 1$ for the single-pass resonance. In contrast, for the standard resonance, the fraction of particles that deliver energy to the wave depends upon the slope of the distribution function. Since Alfvén waves are driven by the spatial gradient of the distribution, the relevant slope is $\partial F/\partial P_\phi$. In the nonlinear theory of Landau damping,[11] particles that are trapped by the wave exchange net energy. The fast-ion spatial gradient is steep for some of the resonances shown in Fig. 1 and is shallow for others. For the resonance with co-passing particles at $P_\phi/\psi_{wall} = 0.25$, $\mu B_0/W = 0.3$, a NUBEAM calculation[12] indicates that the slope of the distribution function near the injection energy is $(\partial F/\partial P_{\phi,norm})/F \simeq 10$. Assuming that the wave flattens the distribution over a range of $\Delta P_{\phi,norm} = \pm 0.25$, which corresponds to a spatial step > 10 cm, this implies that twice as many ions lose energy as gain energy, so the effective energy transfer of a typical resonant particle is approximately $(f_{lose} - f_{gain})/(f_{lose} + f_{gain}) \simeq \frac{1}{3}$.

Comparing these factors for the specific case in Fig. 1, the fast-ion drive from the standard resonance is larger than the fast-ion drive from the single-pass resonance by a factor of

$$\frac{\gamma_{standard}}{\gamma_{single}} = \frac{N_{standard}}{N_{single}}\frac{\Delta W_{standard}}{\Delta W_{single}}\frac{f_{standard}}{f_{single}} \sim (10)(2)\left(\frac{1}{3}\right) \simeq 7. \quad (5)$$

For this case, the estimated contribution to the fast-ion drive by the single-pass resonances is sub-dominant but not negligible. It is evident that, for different conditions (particularly different beam deposition), single-pass resonances could provide the dominant drive term.

The estimates given above are for a finite amplitude wave at the nominal experimental amplitude of $\delta T_e/T_e \simeq 1\%$. Since the energy transferred to the wave is proportional to $(\delta B)^2$ for both the standard resonance and the single-pass resonance, the ratio of drive terms is independent of mode amplitude. (Here, δB is the mode amplitude.) For the standard resonance, the width of the trapping region scales as $\sqrt{\delta B}$; assuming that all trapped particles acquire the wave speed yields an energy transfer that scales as $(\delta B)^2 \partial F/\partial v$.[11] For the single-pass resonance, the displacement scales linearly with mode amplitude, which implies that the number of ions that reach the loss boundary N_{res} scales linearly with δB. Through Eq. (3), the energy lost by an ion ΔW also scales linearly with δB. Since the energy transfer is proportional to $N_{res}\delta W$, the transferred energy scales with $(\delta B)^2$.

III. POWER ESTIMATES

This section presents a back-of-the-envelope calculation that demonstrates the plausibility of appreciable power transfer from single-pass resonances.

The measured losses occur during neutral-beam injection. The beams that populate these lost orbits inject a power P_{inj}. The fraction of these injected particles that are lost through single-pass resonance and give energy to the wave is f_{depo}. When a particle is lost, a fraction of its energy $f_E = \Delta W/W_{inj}$ is delivered to the wave. Here, W_{inj} is the injection energy. Putting these factors together, the power delivered to the wave is

$$P_{drive} = f_{depo} f_E P_{inj} = (0.02)(0.13)(4.6e6) = 12\,\text{kW}, \quad (6)$$

for the example of Fig. 1.

To estimate the power required to sustain the mode, assume a saturated state, where $\gamma_{drive} \simeq \gamma_{damp}$. For an Alfvén wave, the energy density u is approximately twice the magnetic energy density (the other half is kinetic energy density), so $u = 2(\delta B)^2/2\mu_0$. The volume of the mode is roughly $V = (2\pi R)(2\pi r)\Delta R$ so the power required to sustain the mode is approximately

$$P_{damp} = \frac{2}{2\mu_0}\left(\frac{\delta B}{B}\right)^2 B^2 (2\pi R)(2\pi r)\Delta R \frac{\gamma_{damp}}{\omega}\omega. \quad (7)$$

For $\delta B/B = 10^{-3}$, $r = 0.2$ m, $\Delta R = 0.1$ m, and $\gamma_{damp}/\omega = 10^{-2}$, $P_{damp} \simeq 30$ kW.

These estimates indicate that it is plausible that single-pass losses help sustain the mode. In comparison, a NOVA-K calculation of the fast-ion drive due to standard resonances[13] finds a stronger contribution to mode stability, $\gamma_{drive} \simeq 6\gamma_{damp}$, for an $n = 2$ DIII-D RSAE. Thus, for the case considered here, we again conclude that single-pass losses make a significant but sub-dominant contribution to mode stability.

IV. DISCUSSION

We have assumed that virtually all single-pass resonant particles ultimately lose energy to the mode. This assumption is reasonable for the example considered here but could be invalid elsewhere. If a particle that gains energy is displaced so that it no longer interacts with the wave (for example, by crossing a topological boundary), it is unable to return the gained energy to the wave when the phase changes. In our case, the loss boundary is nearer than other topological boundaries for most resonant ions but this need not be the case in general.

Nonlinear effects are another neglected complication. Experimentally, when multiple Alfvén eigenmodes produce coherent losses on the fast-ion loss detector (FILD) signals, peaks at sum and difference frequencies often appear in the FILD spectra.[14] Presumably, these nonlinear interactions alter the energy delivered to the fundamental modes; however, since the nonlinear spectral peaks are typically an order of magnitude smaller than the fundamental peaks, the impact on the estimates given here is small.

V. CONCLUSION

In conclusion, a comparison of the factors that influence energy transfer to the wave (Eq. (4)) shows that, under certain conditions, single-pass loss orbits can play an important role in mode destabilization. Rough estimates for an actual DIII-D case suggest a 10%–30% contribution to the fast-ion drive. These estimates motivate more accurate calculations of the actual energy transfer using realistic mode structures, beam deposition, distribution functions, and loss boundaries.

Although loss boundaries have been considered in the previous work, the importance of single-pass resonances that expel particles is new. For example, in their analysis of the energetic particle driven geodesic acoustic mode (EGAM), Berk and Zhou[15] find that the loss boundary for counter-passing particles plays an important role in mode excitation. In a stability calculation using the standard resonance condition, a loss boundary enhances the drive because the slope of the distribution function is infinite at the boundary between confined and lost particles; this effectively increases our f_{net} factor to unity. What distinguishes our work from the previous studies is the recognition that, for a single-pass loss orbit, the resonance condition occupies a different region of phase space. The contribution of single-pass loss orbits to EGAM stability should be considered in future work.

The effect discussed here is unimportant for the main fusion-product population in burning plasmas. In a successful ignited reactor, few fusion products will populate regions near loss boundaries. On the other hand, in present-day magnetic-fusion experiments, neutral beams often deposit appreciable power on orbits that are near loss boundaries. In these plasmas, single-pass resonances can impact mode stability.

ACKNOWLEDGMENTS

The assistance of the DIII-D team and helpful discussions with Ruibin Zhang are gratefully acknowledged. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences, using the DIII-D National Fusion Facility, a DOE Office of Science user facility, under Award Nos. SC-G903402, DE-FC02-04ER54698, and DE-AC0209CH11466. DIII-D data shown in this paper can be obtained in digital format by following the links at https://fusion.gat.com/global/D3D_DMP.

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