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Relativistic treatment of light quarks in D and B mesons and W-exchange weak decays

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A relativistic bound-state equation is applied to formulate the nonspectator (or W-exchange) decay rates of D, D_s, and B mesons, and their leptonic decay constants. The nonspectator decay rates are also calculated here in the Dirac-equation limit. In this limit we show that up to a logarithm the nonspectator rate grows with heavy-quark mass like m^6, faster than the spectator rate of m^5. Including an analysis of experimental $D^\pm$, D^0, and D_s decay rates shows that the enhancement of D^0 decays from the nonspectator diagram is about a quarter of the contribution from the spectator diagram. The relativistically calculated weak decay constant f_B is in the 260–300-MeV range. Predictions are also made for nonspectator B, B_s, and B_c decay rates including the ratio $\tau_{B^\pm}/\tau_{B^0} \simeq 2$.

I. INTRODUCTION

In order to evaluate the relativistic corrections to bound-state spectra in QCD, and to understand and establish the relativistic nature of the parts of the QCD potential, a relativistic expansion of the Dirac field equation was formulated by Bander, Silverman, Klima, and Maor[1,2] using a valence-quark–antiquark pair in the lowest order. This type of equation applying a multiquantum intermediate-state expansion has a long history from Greenberg,[3] Gross,[4] and Johnson.[5] It includes successful QED calculations[6] of the $O(\alpha^6)$ contributions to the hyperfine splitting in positronium and muonium (μ^+e^-). In the relativistic valence-quark bound-state equation we used a relativistic vector-gluon exchange corrected by an asymptotically free coupling strength, along with a linear confining potential which was established to have a scalar nature. The resulting equation, after partial-wave projection, is a single-variable integral equation for the radial wave functions in momentum space with the mass of the state as the eigenvalue. It reduces to the Breit-Fermi interaction in the Schrödinger-equation limit, and to the Dirac equation in the limit where one quark mass is much heavier than the other. In the first papers,[1,2] the studies were concerned mainly with the spectra, emphasizing the relativistic spin-spin and spin-orbit effects. The spectra and wave functions were calculated for all mesons except the chiral-symmetry-dominated π and K mesons.

The relativistic nature is most important in short-distance or high-momentum phenomena and in cases involving light-mass quarks where the Schrödinger equation is a poor approximation since the momenta are typically larger than the light-quark (u, d, or s) constituent masses.

In the annihilation decays of mesons we are examining short-distance processes, and in the case of mesons with one or both quarks being light, a four-component relativistic treatment. In this paper we formulate and calculate the nonspectator decay rate of D^0, $D_s^\pm$, B^0, and $B^\pm$ mesons, which involve a short-range exchange of a virtual W boson between the constituent c and $\bar{u}$ quarks (in the D^0) converting them into light s and $\bar{d}$ quarks and emitting a gluon to avoid helicity suppression. We also calculate the weak decay constants for D, D_s, B, and B_s mesons.

A number of experiments have shown that D^0 mesons decay faster than $D^\pm$ mesons by an excess decay rate about the same as that of $D^\pm$ itself. The weak decay of a charmed quark by itself, or spectator contribution,[7] is present for both mesons, Fig. 1. The D^0 also has the internal-W-exchange graph, which does not exist in $D^\pm$ decay, with an emitted gluon to avoid helicity suppression, Figs. 2(a) and 2(b). This is called the nonspectator (or W-exchange) contribution.[8] The original calculation of this contribution by Bander, Silverman, and Soni[8] was approximated in an extreme nonrelativistic mode, with the virtual-quark propagator taken with momentum considered small with respect to m_u, and f_D estimated only from a nonrelativistic wave function. In this paper we remedy both of these approximations with first, relativistic wave functions, where the equivalent to the nonrelativistic $\psi(0)$ or f_D is embedded in the calculation and second, with a relativistic propagator for the u quark which includes both positive- and negative-energy poles. We find that the nonspectator decay rate is then rather insensitive to the up-quark mass since the propagator is dominated by the larger up-quark momentum.

The D_s meson also has nonspectator diagrams of the constituent c and $\bar{s}$ quarks annihilating into a virtual W with the emission of a gluon to avoid the helicity suppression, which are exactly the same as the Fierz transforms of the nonspectator D decay graphs.

The rates of the nonspectator D, D_s, B, and B_s decays depend on the value of the strong coupling constant α_s due to the emission of a gluon from the quark. More work will be required to find the appropriate value of α_s including the calculation of QCD radiative corrections to the decay. We call α_{eff} the product of $\alpha_s(m_D)$ multiplied by the higher-order QCD corrections and find that agreement with the data for D decays occurs for $\alpha_{\text{eff}} \simeq 0.39$. In this analysis, the nonspectator process gives about a 25%

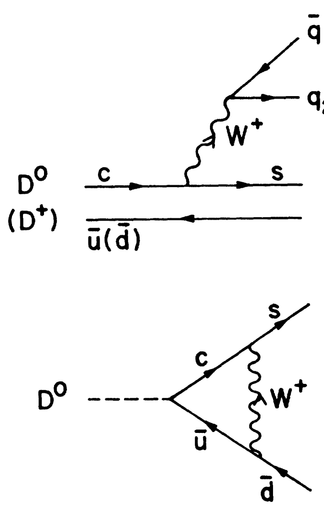

FIG. 1. Spectator diagram for D-meson decays.

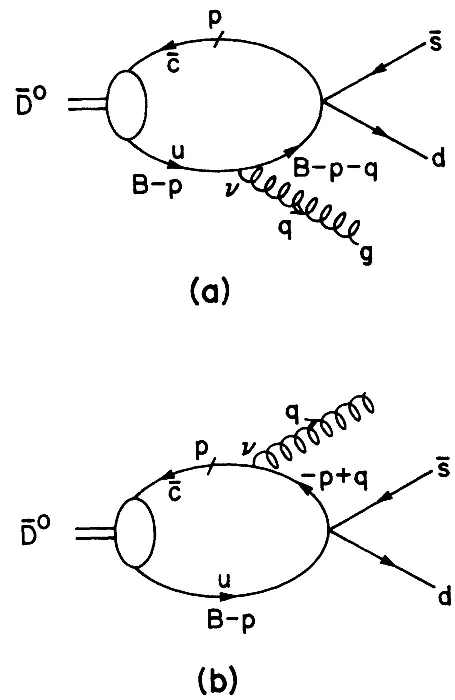

FIG. 2. The W-exchange or nonspectator diagram for $\bar{D}^0$ decay with gluon emission from (a) the u quark and (b) the $\bar{c}$ quark.

enhancement to the D^0 decay, and for $D^{\pm}$ decays a 60% reduction from the spectator rate due to Pauli interference is required. In the case of the nonspectator B^0 decay we predict a ratio $\Gamma(B^0)/\Gamma(B^-)\simeq 1.9$, ignoring the interference effect which has been found to be very small here.

In Sec. II we summarize the relativistic bound-state equation that was previously formulated and used in calculations of the mesonic spectra. In Sec. III we formulate the decay amplitudes for the nonspectator decays of D^0, $D_s^{\pm}$, B^0, and B_s^0 mesons and present our calculations in terms of the wave functions derived from the fits to the spectra. In Sec. IV the weak-leptonic-decay constants f_D, f_{D_s}, f_B, and f_{B_s} are formulated in terms of the relativistic wave functions. We then calculate the leptonic weak decay rates of D, D_s, B, and B_s mesons. In Sec. V we apply the results of the rates of nonspectator decays and the leptonic decays to explain the lifetime differences among D^0, $D^{\pm}$, and $D_s^{\pm}$ mesons, and to predict the lifetime difference between B^0 and $B^{\pm}$ mesons. We also calculate the rate and photon spectrum for $D^0\rightarrow\gamma$ +anything.

II. FORMULATION OF THE RELATIVISTIC BOUND-STATE EQUATION

A. Formulation of the integral equation

The relativistic bound-state equation[1,2] we use is based on the equation for the quark field $\psi(x)$ with mass m_1, coupled to a vector gauge potential $A_\mu(x)$ and a scalar potential $S(x)$:

$$(i\nabla\!\!\!/ - m_1)\psi(x)=[g\not{A}(x)+S(x)]\psi(x)\ . \tag{2.1}$$

The bound-state equation is obtained from the matrix element of Eq. (2.1) between the bound state of four-momentum B and an antiquark state of mass m_2, momentum p, and spin λ:

$$(i\nabla\!\!\!/ - m_1)\langle \mathbf{p},\lambda\,|\,\psi(x)\,|\,B\rangle = \sum_n \langle \mathbf{p},\lambda\,|\,g\not{A}(x)+S(x)\,|\,n\rangle\langle n\,|\,\psi(x)\,|\,B\rangle\ , \tag{2.2}$$

where a complete set of states has been inserted. Up to this point the equation is exact. The approximation we are going to apply consists of keeping only the antiquark state $|\,\mathbf{p},\lambda\rangle$ in the sum. The justification for this approximation is that, at least at larger distances, the valence-quark model appears to work quite well; in this model the mesons are made up of only a quark and an antiquark which is put on mass shell. This then leads to a linear integral equation for the matrix elements $\langle \mathbf{p},\lambda\,|\,\psi_\alpha(x)\,|\,B\rangle$. By using the gluon propagator[9] and source,

$$A^{\mu}(x)=g\int d^4x'D_F^{\mu\nu}(x-x')\bar{\psi}(x')\gamma_\nu\psi(x') \tag{2.3}$$

with a similar equation for $S(x)$, and converting to momentum space, we obtain

$$(\not{B}-\not{p}-m_1)\Psi(p,\lambda)=\sum_{\lambda'}\int\frac{d^3p'm_2}{(2\pi)^3\omega'}[V_V((p-p')^2)\gamma_\mu\Psi(p',\lambda')\bar{v}(p',\lambda')\gamma^\mu v(p,\lambda)+V_S((p-p')^2)\Psi(p',\lambda')\bar{v}(p',\lambda')v(p,\lambda)]\ , \tag{2.4}$$

where

$$\Psi(p,\lambda)=(2\pi)^3\left[\frac{2\omega_B\omega(p)}{m_2}\right]^{1/2}\langle \mathbf{p},\lambda\,|\,\psi(0)\,|\,B\rangle\ ,$$
$$\omega_B=(\mathbf{B}^2+M^2)^{1/2}\ \text{ and }\ \omega=(\mathbf{p}^2+m_2^2)^{1/2}\ . \tag{2.5}$$

In the meson's rest frame Eq. (2.4) is a linear eigenvalue problem for the bound-state mass $B^0=M$. The interactions are denoted by V_V and V_S. Since $\Psi(p',\lambda')$ always appears summed with $\bar{v}(p',\lambda')$, it is appropriate to introduce a 4×4-component wave function Φ where the rows are the components of the quark and the columns that of the antiquark:

$$\Phi_{\alpha\beta}(p)=\sum_{\lambda}\Psi_\alpha(p,\lambda)\bar{v}_\beta(p,\lambda)\ . \tag{2.6}$$

The wave equation for Φ becomes, by Eqs. (2.4) and (2.6),

$$(\not{B}-\not{p}-m_1)\Phi(p)=\int\frac{d^3p'}{(2\pi)^3 2\omega'}[V_V((p-p')^2)\gamma_\mu\Phi(p')\gamma^\mu+V_S((p-p')^2)\Phi(p')](\not{p}-m_2)\ . \tag{2.7}$$

We introduce the analogs of Dirac components in 2×2 submatrices:

$$\Phi=\begin{bmatrix}\tilde{G}_u & \tilde{G}_d\\ \tilde{F}_u & \tilde{F}_d\end{bmatrix}\ , \tag{2.8}$$

where u and d stand for upper and lower $\bar{q}$ components and G and F stand for upper and lower quark components. We note the restrictions arising from Eq. (2.6) that

$$\Phi(p)(\not{p}+m_2)=0\ , \tag{2.9}$$

which means that $\tilde{G}_u$, $\tilde{F}_u$ can be taken as dependent on $\tilde{G}_d$, $\tilde{F}_d$ by

$$\tilde{G}_u=-\tilde{G}_d\frac{\sigma\cdot\mathbf{p}}{\omega+m_2},\quad \tilde{F}_u=-\tilde{F}_d\frac{\sigma\cdot\mathbf{p}}{\omega+m_2}\ . \tag{2.10}$$

In order to do the spin and angular momentum decomposition we convert the $\bar{2}$ representation for the antiquarks to a 2 representation by defining $G_d=\tilde{G}_d\sigma_y$ and $F_d=\tilde{F}_d\sigma_y$. We then expand $G_d(p)$ and $F_d(p)$ in terms of $Y_L^{M_L}(\hat{\mathbf{p}})$ and 2×2 projection operators for $S=0$ and $S=1$ states, coupling them to the total J, M_J. For the unnatural-parity 0^- states we will be treating here, since $J=0$ the expansion simplifies into two wave-function components:

$$F_d(\mathbf{p})=f_+(p)|\,L=J+1=1,S=1\rangle\langle L=J+1=1,S=1\,|\ ,$$
$$G_d(\mathbf{p})=g_0(p)\,|\,L=J=0,S=0\rangle\langle L=J=0,S=0\,|\ . \tag{2.11}$$

Expanding the interactions in Legendre polynomials of $\cos\theta=\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'$ and performing the angular integrations, we obtain the desired integral equations of the wave functions containing the bound-state mass as an eigenvalue

$$\begin{bmatrix}M-\omega+m_1 & -p\\ -p & M-\omega-m_1\end{bmatrix}\begin{bmatrix}f_+\\ g_0\end{bmatrix}(p)=\int dp'K_0(p,p')\begin{bmatrix}f_+\\ g_0\end{bmatrix}(p')\ , \tag{2.12}$$

where $K_0(p,p')$ is the 2×2 kernel matrix[1] containing the angular momentum projections of the interactions for $J=0$.

For the interactions we start with the QCD gluon interaction which is considered to have a vector propagator modified by an asymptotically free coupling strength,

$$V_R(q^2)=\frac{-4\pi\frac{4}{3}(4\pi/9)}{-q^2\ln(1-q^2/\Lambda_R^2)}\ , \tag{2.13}$$

where q^2 is the four-momentum transfer squared. While the large-q^2 (or small-distance) modified Coulomb part is due to a vector exchange, the long-distance infinitely rising part cannot be pure vector since it would lead to a Klein paradox. To avoid this we subtract off the linear part from V_R. The resulting vector interaction is

$$V_V(q^2)=-4\pi\left[\frac{16\pi}{27}\right]\times\left[\frac{1}{-q^2\ln(1-q^2/\Lambda_R^2)}-\frac{\Lambda_R^2}{(q^2)^2}\right]\ . \tag{2.14}$$

For the scalar interaction we have the linear confining part with slope κ. This interaction is cut off at a distance b or equivalently at a height κb for numerical calculations: i.e.,

$$V_L(r)=\kappa r\theta(b-r)+\kappa b\theta(r-b)\ . \tag{2.15}$$

We chose ramps at values of $\kappa b\geq 3$ GeV and the results were insensitive to the value of the height at or above this value. Taking the Fourier transform of Eq. (2.15) and generalizing to $\mathbf{q}^2\to -q^2$ we have

$$V_S(\kappa,q=(-q^2)^{1/2})=(2\pi)^3\kappa b(\omega/m)\delta^3(\mathbf{q})+(4\pi\kappa/q^4)[bq\sin(bq)+2\cos(bq)-2]\ . \tag{2.16}$$

The integral equation diverges slowly at large momentum no worse than logarithmically. Before that becomes important the neglected multiquanta states become important in competing for probability or amplitude with the valence-quark channel. We shall approximate this effect by introducing a large momentum, or in effect, a small-distance cutoff. We introduce into the kernel of Eq. (2.12) and into all other interactions of the valence-antiquark states a cutoff function $S_\varepsilon(\Lambda_0,p)$, with $\varepsilon>0$, which approaches 0 for $p\gg\Lambda_\varepsilon$ and approaches 1 for $p\ll\Lambda_\varepsilon$:

$$S_\varepsilon(\Lambda_0,p)=\left[\frac{\Lambda_\varepsilon^2}{\Lambda_\varepsilon^2+p^2}\right]^{1+\varepsilon} \tag{2.17}$$

with $\Lambda_\varepsilon=\Lambda_0(1+\varepsilon)^{1/2}$. Expanding this for small $p^2/\Lambda_\varepsilon^2$,

$$S_\varepsilon(\Lambda_0,p)=\frac{1}{(1+p^2/\Lambda_\varepsilon^2)^{1+\varepsilon}}\simeq\frac{1}{1+(1+\varepsilon)p^2/\Lambda_0^2(1+\varepsilon)}=\frac{1}{1+p^2/\Lambda_0^2} \tag{2.18}$$

shows that the cutoff, in the predominant region where $p^2<\Lambda_0^2$, is independent of ε.

The parameters used in the fits to the spectra of the mesonic systems are, hence, the quark masses m_1, m_2, the QCD potential scale Λ_R, the linear slope κ_s, and a cutoff $S_\varepsilon(\Lambda_0,p)$ on the slowly diverging integral equations. We have found that $\varepsilon=1.0$ is sufficient to give convergent results for all physical quantities, and that the results vary little for $1<\varepsilon<2$. The value $\varepsilon=1.0$ is used for calculations throughout the paper. The values from the spectral fits were $\Lambda_R=0.4$ GeV, $\kappa=0.15$ GeV^2, $\Lambda_0(D)=3.0$ GeV, and $\Lambda_0(B)=3.7$ GeV.

B. Pseudoscalar-meson case

For the pseudoscalar-meson case, the wave function $\Phi(p)$ can be easily written in Dirac form. From the subsidiary condition $\Phi(p)(\not p+m)=0$, Eq. (2.9), and since the wave function $\Phi(p)$ obeys the integral equation (2.7) and has a definite parity, $\Phi(p)$ for 0^- mesons can be expressed as (we denote $m_2=m$ from here on)

$$\Phi(p)=\gamma_5[a(p)+b(p)\not B/m](\not p-m)\ . \tag{2.19}$$

Reducing this to the form of Eq. (2.8) yields

$$\begin{aligned}\tilde G_d&=(\omega+m)\left[-a(p)+\frac{M}{m}b(p)\right],\\ \tilde F_d&=-p\left[a(p)+\frac{M}{m}b(p)\right]\sigma\cdot\hat{\mathbf{p}}\ .\end{aligned} \tag{2.20}$$

Using Eq. (2.11) and expanding $G_d(\mathbf{p})$, $F_d(\mathbf{p})$ by using Clebsch-Gordan coefficients gives

$$\begin{aligned}G_d(\mathbf{p})&=g_0(p)Y_0^0(\hat{\mathbf{p}})\,|S=0,m_s=0\rangle\langle S=0,m_s=0|\ ,\\ F_d(\mathbf{p})&=f_+(p)\left[\frac{1}{\sqrt3}Y_1^1(\hat{\mathbf{p}})\,|S=1,m_s=-1\rangle\langle S=1,m_s=-1|-\frac{1}{\sqrt3}Y_1^0(\hat{\mathbf{p}})\,|S=1,m_s=0\rangle\langle S=1,m_s=0|\right.\\ &\qquad\left.+\frac{1}{\sqrt3}Y_1^{-1}(\hat{\mathbf{p}})\,|S=1,m_s=-1\rangle\langle S=1,m_s=-1|\right]\ .\end{aligned} \tag{2.21}$$

Working out the projection operators gives

$$\begin{aligned}|S=0,m_s=0\rangle\langle S=0,m_s=0|&=\frac{1}{\sqrt2}\begin{bmatrix}0&1\\-1&0\end{bmatrix}=\frac{1}{\sqrt2}i\sigma_y\ ,\\ |S=1,m_s=1\rangle\langle S=1,m_s=1|&=\begin{bmatrix}1&0\\0&0\end{bmatrix}=\tfrac12(I+\sigma_z)\ ,\\ |S=1,m_s=0\rangle\langle S=1,m_s=0|&=\frac{1}{\sqrt2}\begin{bmatrix}0&1\\1&0\end{bmatrix}=\frac{1}{\sqrt2}\sigma_x\ ,\\ |S=1,m_s=-1\rangle\langle S=1,m_s=-1|&=\begin{bmatrix}0&0\\0&1\end{bmatrix}=\tfrac12(I-\sigma_z)\ .\end{aligned} \tag{2.22}$$

Substituting Eq. (2.22) into Eq. (2.21) we find

$$\begin{aligned}G_d&=\frac{g_0}{\sqrt{8\pi}}\sigma_y\quad\text{or}\quad\tilde G_d=\frac{g_0}{\sqrt{8\pi}}\ ,\\ F_d&=-\frac{f_+}{\sqrt{8\pi}}\sigma\cdot\hat{\mathbf{p}}\sigma_y\quad\text{or}\quad\tilde F_d=-\frac{f_+}{\sqrt{8\pi}}\sigma\cdot\hat{\mathbf{p}}\ .\end{aligned} \tag{2.23}$$

Comparing with Eq. (2.19) then gives $a(p)$ and $b(p)$ in terms of the solutions to the bound-state integral equation $f_+(p)$ and $g_0(p)$:

$$\begin{aligned}a(p)&=\frac{1}{4\sqrt{2\pi}}\frac{1}{\omega+m}\left[-g_0(p)+\frac{\omega+m}{p}f_+(p)\right],\\ b(p)&=\frac{1}{4\sqrt{2\pi}}\frac{1}{\omega+m}\frac{m}{M}\left[g_0(p)+\frac{\omega+m}{p}f_+(p)\right].\end{aligned} \tag{2.24}$$

The normalization condition[1,2]

$$\frac{m^2}{(2\pi)^3M}\int\frac{p^2dp}{\omega(p)}\frac{1}{\omega(p)+m}(|g_0|^2+|f_+|^2)=1 \tag{2.25}$$

becomes

$$\frac{(2m)^2}{M}\int\frac{d^3p}{(2\pi)^3}\left[a(p)^2+\left[\frac{M}{n}b(p)\right]^2-2\frac{M}{\omega}a(p)b(p)\right]=1\ . \tag{2.26}$$

III. FORMULATION OF THE AMPLITUDES FOR THE NONSPECTATOR D, D_s, AND B DECAYS

A. Ratio of D-meson lifetimes and the weak Hamiltonian

The first measurements of the ratio of D lifetimes relied on the fact that the weak Hamiltonian for a $c\to sl^+\nu_l$ transition is the same for D^0 and D^+ in the spectator model, so that $\Gamma(D^0\to l^+X)=\Gamma(D^+\to l^+X)$. With this equality, the ratio of inclusive semileptonic branching ratios for the D^0 and D^+ may be related to the ratio of their lifetimes:

$$\frac{B(D^+\to l^+X)}{B(D^0\to l^+X)}=\frac{\Gamma(D^+\to l^+X)}{\Gamma(D^0\to l^+X)}\frac{\Gamma_{\rm tot}(D^0)}{\Gamma_{\rm tot}(D^+)}=\frac{\Gamma_{\rm tot}(D^0)}{\Gamma_{\rm tot}(D^+)}=\frac{\tau(D^+)}{\tau(D^0)}\ . \tag{3.1}$$

Recently, the Mark III Collaboration[10] at the SLAC e^+e^- storage ring SPEAR performed a high-statistics measurement of the inclusive semileptonic branching ratios by an analysis of the electron content in the recoil spectrum from fully reconstructed D decays of known charm and charge. The results are

$$\begin{aligned}&B(D^+\to l^+X)=(17.0\pm1.9\pm0.7)\%\ ,\\&B(D^0\to l^+X)=(7.5\pm1.1\pm0.4)\%\ ,\\&\frac{\tau(D^+)}{\tau(D^0)}=2.3^{+0.5+0.1}_{-0.4-0.1}\ .\end{aligned}\tag{3.2}$$

The lifetimes of the D^0 and D^+ have also been obtained directly by several collaborations by measuring the decay distance in bubble chambers, silicon strip vertex detectors, and proportional tube vertex chambers. The world-average values of the D^0 and D^+ lifetimes from these experiments are

$$\begin{aligned}&\tau(D^+)=(10.29^{+0.54}_{-0.43})\times10^{-13}\ \text{sec}\ ,\\&\tau(D^0)=(4.43^{+0.19}_{-0.17})\times10^{-13}\ \text{sec}\ ,\\&\frac{\tau(D^+)}{\tau(D^0)}=2.39^{+0.16}_{-0.14}\ .\end{aligned}\tag{3.3}$$

This is in very good agreement with the ratio given above based on the inclusive semileptonic branching ratios.

The charm-changing weak Hamiltonian[11] is

$$H_{w,\Delta C=-1}=\frac{G_F}{2\sqrt2}\{(f_++f_-)[\bar s\,^m\gamma_\mu(1-\gamma_5)c_m\bar u\,^n\gamma^\mu(1-\gamma_5)d_n]+(f_+-f_-)[\bar s\,^m\gamma_\mu(1-\gamma_5)d_m\bar u\,^n\gamma^\mu(1-\gamma_5)c_n]\}\ . \tag{3.4}$$

From renormalization-group techniques,[11,12] we have

$$f_\pm\approx[\alpha_s(\mu^2)/\alpha_s(M_W^2)]^{\gamma_\pm}\ , \tag{3.5}$$

where

$$\gamma_+=-\frac{6}{33-2N_f}\ , \tag{3.6}$$

$$\gamma_-=\frac{12}{33-2N_f}\ . \tag{3.7}$$

For the decay of the charmed quark, there are three lighter quarks which may participate ($N_f=3$) so $\gamma_+=-\frac{2}{9}$ and $\gamma_-=\frac{4}{9}$.

B. Spectator decay rate

Before proceeding with the nonspectator calculation, we review the spectator decay calculation since we will be using its result. If only Cabibbo-favored spectator amplitudes are included, the nonleptonic decay proceeds via $c\to s\bar u d$, and the semileptonic decay via $c\to sl^+\nu_l$. For the nonleptonic decay, it is useful to rewrite the Hamiltonian again as a sum of color-singlet and -octet parts in the $c\bar s$ channel. To do this, we use the Fierz identity

$$[\gamma_\mu(1-\gamma_5)]_{\alpha\beta}[\gamma^\mu(1-\gamma_5)]_{\gamma\delta}=-[\gamma_\mu(1-\gamma_5)]_{\alpha\delta}[\gamma^\mu(1-\gamma_5)]_{\gamma\beta}\ , \tag{3.8}$$

$$(\lambda^a)_{mk}(\lambda^a)_{nl}=-\tfrac{2}{3}\delta_{mk}\delta_{nl}+2\delta_{ml}\delta_{nk}$$

on the (f_+-f_-) term in Eq. (3.4) to get

$$H_{w,\Delta C=-1}=\frac{G_F}{\sqrt2}[\tfrac13(2f_++f_-)\bar s\,^m\gamma_\mu(1-\gamma_5)c_m\bar u\,^n\gamma^\mu(1-\gamma_5)d_n+\tfrac14(f_+-f_-)\bar s\,^m\gamma_\mu(1-\gamma_5)\lambda^a_{mk}c_k\bar u\,^n\gamma^\mu(1-\gamma_5)\lambda^a_{nl}d_l]\ . \tag{3.9}$$

The first term of H_w is the color-singlet part and the second term is the color-octet part. Hence, they do not interfere. To simplify the notation, write $H^{(1)}$ for the first term of H_w and $H^{(8)}$ for the second term. In order to calculate the decay rate, it is necessary to separately square $H^{(1)}$ and $H^{(8)}$, sum the results over the final-state spins and colors, and average over the initial-state spin and color. After performing the color calculations,

$$|H^{(1)}|^2 \propto \frac{1}{3} \sum_{mknl} |\delta_{mk}\delta^{nl}|^2 = \frac{1}{3} \sum_{mknl} (\delta_{mk}\delta^{km}\delta_{ln}\delta^{nl}) = \frac{1}{3}\times 3 \times 3 = 3 \,,$$

$$|H^{(8)}|^2 \propto \frac{1}{3} \sum_{k} \sum_{mln} \Big| \sum_{a} \lambda^a_{mk}\lambda^a_{nl} \Big|^2 = \frac{1}{3} \sum_{ab} \left[\sum_{km} \lambda^a_{mk}\lambda^b_{km} \right] \left[\sum_{ln} \lambda^a_{nl}\lambda^b_{ln} \right] = \frac{1}{3}\times 4 \sum_a \sum_b \delta_{ab}\delta^{ab} = \frac{32}{3} \,. \tag{3.10}$$

Following the notation of (3.9) we have

$$|H_{w,\Delta C=-1}|^2 \propto 3[\tfrac{1}{3}(2f_+ + f_-)]^2 + \tfrac{32}{3}[\tfrac{1}{4}(f_+ - f_-)]^2 = 2f_+^2 + f_-^2 \,, \tag{3.11}$$

$$|H_{w,\Delta C=-1}|^2 = \left[\frac{G_F^3}{\sqrt{2}} \right]^2 (2f_+^2 + f_-^2)[\bar{d}\gamma^\sigma(1-\gamma_5)u\, \bar{c}\gamma^\sigma(1-\gamma_5)s \times \bar{s}\gamma_\mu(1-\gamma_5)c\, \bar{u}\gamma^\mu(1-\gamma_5)d] \,.$$

The spin calculations must now be performed and the result integrated over the available phase space. This task is greatly simplified by noting that, if the u and d quarks are considered massless, the Hamiltonian above is equivalent to that of the decay[9] $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. The spectator decay rate is then

$$\Gamma(c \to su\bar{d}) = (2f_+^2 + f_-^2) \frac{G_F^2 m_c^5}{192\pi^3} g(m_s/m_c) \,. \tag{3.12}$$

The function $g(m_s/m_c)$ is a phase-space factor which corrects[13,14] the decay rate for the nonzero strange-quark mass.

The semileptonic decay rate is again like that for μ decay giving

$$\Gamma(c \to sl^+\nu_l) = \frac{G_F^2 m_c^5}{192\pi^3} g(m_s/m_c) \,. \tag{3.13}$$

The total D^+ decay rate is given by the sum of the nonleptonic and semileptonic decay rates

$$\Gamma_{\rm tot}(D^+) = \Gamma(c \to su\bar{d}) + \Gamma(c \to se^+\nu_e) + \Gamma(c \to s\mu^+\nu_\mu) = (2f_+^2 + f_-^2 + 1 + 1)\frac{G_F^2 m_c^5}{192\pi^3} g(m_s/m_c) \,. \tag{3.14}$$

The semileptonic branching ratio for D^+ is

$$B(c \to sl^+\nu_l) = \frac{\Gamma(c \to sl^+\nu_l)}{\Gamma_{\rm tot}(D^+)} = \frac{1}{2f_+^2 + f_-^2 + 2} \,. \tag{3.15}$$

If the values from the leading-logarithm calculation, $f_+ = 0.74$ and $f_- = 1.8$, are used, the branching ratio is $B(c \to sl^+\nu_1) = 16\%$. The semileptonic branching ratio obtained here is in agreement with that of the D^+ [recall by Eq. (3.2), $B(D^+ \to l^+X) = (17.0 \pm 1.9)\%$], but very far from that of the D^0 [i.e., $B(D^0 \to l^+X) = (7.5 \pm 1.1)\%$].

There are two conventional mechanisms for correcting this problem. One is the destructive (Pauli) interference,[15] due to Fermi statistics, between the spectator antiquark $\bar{d}$ in $D^+(c\bar{d})$ decay and the identical $\bar{d}$ arising in the final state from the nonleptonic c decay ($c \to su\bar{d}$). This interference increases the D^+ lifetime, while the D^0 is totally unaffected. The second mechanism is an important enhancement in nonleptonic decays of the D^0 since there is an amplitude for D^0 decay which is forbidden to the D^+. This is the so-called nonspectator diagram (or W-exchange diagram) shown in Fig. 2, which in lowest order should be small due to helicity suppression at the light-quark vertex. A distinct machanism has been advanced for the lifting of helicity suppression[8] by Bander, Silverman, and Soni. It is the explicit radiation of soft gluons shown in Figs. 2(a) and 2(b), which produces, nonrelativistically, a contribution to the D^0 width given by[8]

$$\frac{\Gamma_{\rm NS}}{\Gamma_{\rm sp}} = \left[\frac{m_D}{m_c} \right]^5 \frac{2\pi\alpha_s}{27} \frac{(f_+ + f_-)^2}{2f_+^2 + f_-^2 + 2} \left[\frac{f_D}{m_u} \right]^2 \simeq (0.35) \left[\frac{\alpha_s}{0.53} \right] \left[\frac{f_D}{m_u} \right]^2 \simeq 0.13 - 3.6 \,. \tag{3.16}$$

The factor of m_u comes from the propagator $1/(\not{p} - m_u)$ and the extreme assumption of $p_u \ll m_u$. f_D comes from the wave function at the origin. The values used in Eq. (3.16) were[8] $m_u = 250$ MeV and $f_D = 150-800$ MeV. Since this calculation was[8] an approximation, and it was performed in the extreme nonrelativistic limit, we need to have a precise relativistic calculation, which can give us more accurate results.

C. Calculations of relativistic amplitudes for the nonspectator D, D_s, B, and B_s decays

1. D^0 mesons

In this section we are going to calculate the decay rate of the nonspectator diagrams of Figs. 2(a) and 2(b). From the reduction formula for gluons,[9] a gluon with momentum k', spin λ', can be removed from the final state giving for the nonspectator D^0 meson decay amplitude

$$\langle \bar{s}^l d^k g^a | \bar{D}^0 \rangle = \frac{-i}{[(2\pi)^3 2q_0]^{1/2}} \epsilon_\nu^*(q,\lambda') \times \int d^4x\, e^{iq\cdot x} \langle \bar{s}^l d^k | J_\nu^a(x) | \bar{D}^0 \rangle , \tag{3.17}$$

where l, k, and a are color indices of the $\bar{s}$, d, and gluon g, and J_ν^a is the color-current operator. At first we treat the current of the u quark, Fig. 2(a),

$$J_\nu^a(x) = \tfrac{1}{2} g_s \bar{\psi}_u^i(x) \lambda_{in}^a \gamma_\nu \psi_u^n(x) , \tag{3.18}$$

where ψ_u represents the u-quark field. By inserting the intermediate c-antiquark states, the current element can be written as

$$\langle \bar{s}^l d^k | J_\nu^a(x) | \bar{D}^0 \rangle = g_s \sum_j \sum_{s_c} \int d^3p \langle \bar{s}^l d^k | \bar{\psi}_u^i(x) | \bar{c}^j p s_c \rangle \tfrac{1}{2} \lambda_{in}^a \gamma_\nu e^{ip\cdot x} \langle \bar{c}^j p s_c | \psi_u^n(0) | \bar{D}^0 \rangle e^{-iB\cdot x} , \tag{3.19}$$

where s_c denotes the spin, p is the momentum of the inserted c-antiquark state, and B is the four-momentum of the bound-state D meson. Hence, the decay amplitude becomes

$$\langle \bar{s}^l d^k g^a | \bar{D}^0 \rangle = \frac{-ig_s}{[(2\pi)^3 2q_0]^{1/2}} \tfrac{1}{2} \epsilon_\nu^*(q,\lambda') \sum_{s_c} \int d^3p\, d^4x \langle \bar{s}^l d^k | \bar{\psi}_u^i(x) | \bar{c}^j p s_c \rangle \lambda_{in}^a \gamma_\nu \langle \bar{c}^j p s_c | \psi_u^n(0) | \bar{D}^0 \rangle e^{i(q+p-B)\cdot x} . \tag{3.20}$$

The bound-state wave function appears in $\langle \bar{c}^j p s_c | \psi_u^n(0) | \bar{D}^0 \rangle$. We know the relativistic bound-state wave function already from the bound-state equations (2.5), (2.6), (2.12), and (2.24). By Eq. (2.6) and the definition of $\Psi(p,\lambda)$ in Eq. (2.5), we can express the wave function as

$$\langle \bar{c}^j p s_c | \psi_\alpha^n(0) | \bar{D}^0 \rangle = \frac{-1}{(2\pi)^3} \frac{1}{\sqrt{3}} \delta_{jn} \left[\frac{m_c}{2\omega_D \omega(p)} \right]^{1/2} \Phi_{\alpha\beta}(p) v_\beta(p,s_c) . \tag{3.21}$$

We repeat the reduction formula on the matrix element $\langle \bar{s}^l d^k | \bar{\psi}_u^i(x) | \bar{c}^j p s_c \rangle$ until we remove all particles from in and out states and reach the vacuum expectation value of a product of field operators:

$$\langle \bar{s}^l d^k | \bar{\psi}_u^i(x) | \bar{c}^j p s_c \rangle = -i^3 \int d^4y\, d^4z\, d^4w [\bar{U}_d(z)(i\overrightarrow{\nabla} - m_d)][\bar{V}_c(y)(i\overrightarrow{\nabla} - m_c)] \times \langle 0 | T[\bar{\psi}_u^i(x) \psi_c^j(x) \psi_d^k(z) \bar{\psi}_s^l(w)] | 0 \rangle (i\overleftarrow{\nabla} - m_s) V_s(w) , \tag{3.22}$$

where

$$\bar{V}_c(y) = \frac{1}{(2\pi)^{3/2}} (m_c/E_c)^{1/2} \bar{v}(p,s_c) e^{-ip\cdot y} ,$$
$$\bar{U}_d(z) = \frac{1}{(2\pi)^{3/2}} (m_d/E_d)^{1/2} \bar{u}(p_d,s_d) e^{ip_d\cdot z} , \tag{3.23}$$
$$V_s(w) = \frac{1}{(2\pi)^{3/2}} (m_s/E_s)^{1/2} v(p_s,s_s) e^{ip_s\cdot w} ,$$

and $E_c = \omega(p)$, E_d, and E_s are the energies of the c, d, and s quarks. Now we will evaluate

$$\tau(x,y,z,w) \equiv \langle 0 | T[\bar{\psi}_u^i(x) \psi_c^j(x) \psi_d^k(z) \bar{\psi}_s^l(w)] | 0 \rangle$$

to first-order perturbation theory in the weak Hamiltonian from Eq. (3.4).

There is no term in $(f_+ - f_-)$ here, because it is associated with

$$\delta_{ij}\delta_{jn}\lambda_{in}^a = \delta_{in}\lambda_{in}^a = \sum_i \lambda_{ii}^a = 0 .$$

This is because the gluon is a color octet and the final $\bar{s}d$ must be in a color octet to make a colorless initial state. Referring to Eq. (3.4), $\bar{s}$ and d quarks are partly in the color octet in the $(f_+ + f_-)$ term, but are pure color singlet in the $(f_+ - f_-)$ term.

After using the standard method we obtain

$$\langle \bar{s}^l d^k | \bar{\psi}_u^i(x) | \bar{c}^j p s_c \rangle = \frac{G}{2\sqrt{2}} \frac{g_s}{(2\pi)^{9/2}} (m_c m_d m_s / E_c E_d E_s)^{1/2} e^{i(p_d+p_s-p)\cdot x} (f_+ + f_-)$$
$$\times \left[\delta_{ij}\delta_{lk} l_\mu \bar{v}(p,s_c)\gamma_\mu(1-\gamma_5)\frac{1}{\not{B}-\not{p}-\not{q}-m_u+i\epsilon} \right], \tag{3.24}$$

where

$$l_\mu = \bar{u}(p_d,s_d)\gamma_\mu(1-\gamma_5)v(p_s,s_s) .$$

Finally, substituting Eq. (3.24) into (3.20), the decay amplitude becomes

$$\langle \bar{s}^l d^k g^a | \bar{D}^0 \rangle_u = \bar{A}^a_{kl}\epsilon_\nu(q,\lambda')l_\mu \sum_{s_c} \int d^3p \left[\frac{m_c}{E_c}\right]^{1/2} \bar{v}(p,s_c)\gamma_\mu(1-\gamma_5)\frac{1}{\not{B}-\not{p}-\not{q}-m_u+i\epsilon}\gamma_\nu\langle \bar{c}^j p s_c | \psi_u^n(0) | \bar{D}^0\rangle , \tag{3.25}$$

where

$$\bar{A}^a_{kl} = \frac{G}{2\sqrt{2}} \frac{g_s}{(2\pi)^{9/2}} (m_d m_s / E_d E_s)^{1/2} \frac{1}{2(2q_0)^{1/2}(2\pi)^6} (f_+ + f_-)\lambda^a_{kl}(2\pi)^4\delta^{(4)}(q+p_d+p_s-B) . \tag{3.26}$$

Using Eqs. (3.21) and (3.25), we finally obtain the decay amplitude in terms of the wave function of the D^0:

$$\langle \bar{s}^l d^k g^a | \bar{D}^0 \rangle_u = A^a_{kl}\epsilon_\nu(q,\lambda')l_\mu \int d^3p \left[\frac{m_c}{E_c}\right] \mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)\frac{1}{\not{B}-\not{p}-\not{q}-m_u+i\epsilon}\gamma_\nu\Phi(p)\right], \tag{3.27}$$

where

$$A^a_{kl} = \frac{1}{(2\pi)^3\sqrt{2M_D}}\frac{1}{\sqrt{3}}\bar{A}^a_{kl} .$$

This amplitude can also be deduced from Feynman-graph rules that include the relativistic bound-state wave function by the additional rules:

closed fermion loop	Trace
on-shell quark in loop	$\int \frac{d^3p}{(2\pi)^3}\left[\frac{m}{E}\right]$
bound-state wave function	$\Phi(p)$

Referring to Fig. 2(b), we now analyze the charm-quark color-current term

$$J^a_\nu(x) = \tfrac{1}{2}\bar{\psi}^i_c(x)\lambda^a_{in}\gamma_\nu\psi^n_c(x) . \tag{3.28}$$

By the same perturbation method, we obtain

$$\langle \bar{s}^l d^k g^a | \bar{D}^0 \rangle_c = A^a_{kl}\epsilon_\nu(q,\lambda')l_\mu \int d^3p \left[\frac{m_c}{E_c}\right] \mathrm{Tr}\left[\gamma_\mu(1-\gamma_5)\Phi(p)\gamma_\nu\frac{1}{-\not{p}+\not{q}-m_c+i\varepsilon}\right]. \tag{3.29}$$

Again, this can be derived from the Feynman-graph rules. Using the subsidiary condition $\Phi(p)(\not{p}+m_c)=0$ and the fact that the c antiquark is on mass shell, the amplitudes of Eqs. (3.27) and (3.29) become

$$\langle \bar{s}^l d^k g^a | \bar{D}^0 \rangle_u = A^a_{kl}\epsilon_\nu(q,\lambda')l_\mu \int d^3p \left[\frac{m_c}{E_c}\right] \frac{1}{(B-p)^2-2q\cdot(B-p)-m_u^2+i\varepsilon}$$
$$\times \mathrm{Tr}\left[\gamma_\mu[-\not{q}\gamma_\nu+2(B-p)_\nu-\gamma_\nu(\not{B}-\not{p}-m_u)]\Phi(p)\right.$$
$$\left. -\gamma_\mu\gamma_5\left[-\not{q}\gamma_\nu+2(B-p)_\nu-\frac{(B-p)^2-m_u^2}{q\cdot B}B_\nu\right]\Phi(p)\right], \tag{3.30}$$

$$\langle \bar{s}^l d^k g^a \,|\, \bar{D}^0 \rangle_c = A^a_{kl} \epsilon_\nu(q,\lambda') l_\mu \int d^3p \left[\frac{m_c}{E_c} \right] \frac{1}{-2p\cdot q + i\varepsilon} \mathrm{Tr}[(\gamma_\mu - \gamma_\mu\gamma_5)\Phi(p)(-2p_\nu + \gamma_\nu \not{q})] \ . \tag{3.31}$$

We have made one modification, which is the replacement of $\gamma_\nu(\not{B} - \not{p} - m_u)$ with $\{[(B-p)^2 - m_u^2]/q\cdot B\} B_\nu$ in the axial-vector term in Eq. (3.30) in order to reach a gauge-invariant amplitude. We did this by the fact that the integral equations of Eqs. (2.4) and (2.7) give, to the lowest order in α_s,

$$(\not{B} - \not{p} - m_u)\Phi \simeq 0 \tag{3.32}$$

or multiplying by $(\not{B} - \not{p} + m_u)$ gives

$$[(B-p)^2 - m_u^2]\Phi(p) \simeq 0 \ . \tag{3.33}$$

If the higher-order intermediate states had been included, the gauge invariance would be obtained without using the approximation of Eq. (3.33).

Now, let us substitute $\Phi(p) = \gamma_5[a(p) + b(p)\not{B}/m_c](\not{p} - m_c)$, from Eq. (2.20), into Eqs. (3.30) and (3.31). The trace parts can be evaluated as

$$\begin{aligned} &\mathrm{Tr}\left[\gamma_\mu[-\not{q}\gamma_\nu + 2(B-p)_\nu - \gamma_\nu(\not{B} - \not{p} - m_u)]\Phi(p) - \gamma_\mu\gamma_5 \left[-\not{q}\gamma_\nu + 2(B-p)_\nu - \frac{(B-p)^2 - m_u^2}{q\cdot B} B_\nu \right] \Phi(p) \right] \\ &\quad = -4i\epsilon_{\rho\mu\sigma\nu} q^\sigma (ap^\rho - bB^\rho) + 4\{[q_\nu - 2(B-p)_\nu + (M_D^2 + m_c^2 - m_u^2 - 2B\cdot p)B_\nu/(qM_D)](ap_\mu - bB_\mu) \\ &\qquad + q_\mu(ap_\nu - bB_\nu) - g_{\mu\nu} q^\rho (ap_\rho - bB_\rho)\} \ , \end{aligned} \tag{3.34}$$

$$\begin{aligned} &\mathrm{Tr}[(\gamma_\mu - \gamma_\mu\gamma_5)\Phi(p)(-2p_\nu + \gamma_\nu\not{q})] \\ &\quad = 4i\epsilon_{\rho\mu\sigma\nu} q^\sigma (ap^\rho - bB^\rho) + 4[(2p_\nu - q_\nu)(ap_\mu - bB_\mu) - q_\mu(ap_\nu - bB_\nu) + g_{\mu\nu} q^\rho (ap_\rho - bB_\rho)] \ . \end{aligned} \tag{3.35}$$

One of the integrals over momentum p can be written, by Lorentz covariance, as a four-vector form in terms of q_μ and B_μ with coefficients F_q and F_B:

$$\int d^3p \left[\frac{m_c}{E_c} \right] \frac{a(p)p_\mu - b(p)B_\mu}{(B-p)^2 - 2q\cdot(B-p) - m_u^2 + i\varepsilon} = F_q q_\mu + F_B B_\mu \ . \tag{3.36}$$

The values of F_q and F_B can be obtained by multiplying q_μ and B_μ times Eq. (3.36). One gets

$$F_B = \frac{1}{q\cdot B} \int d^3p \left[\frac{m_c}{E_c} \right] \frac{a(p)(p\cdot q) - b(p)(q\cdot B)}{(B-p)^2 - 2q\cdot(B-p) - m_u^2 + i\varepsilon} \ . \tag{3.37}$$

An imaginary part appears due to the existence of the pole in the denominator in Eq. (3.36). Writing

$$(B-p)^2 - 2q\cdot(B-p) - m_u^2 = A + B\cos\theta \ , \tag{3.38}$$

where

$$A = 2(-M_D + q)E_c + M_D^2 + m_c^2 - 2M_D q - m_u^2$$

and

$$B = -2pq \ , \tag{3.39}$$

we have

$$\int \frac{d\cos\theta}{A + B\cos\theta + i\varepsilon} = \frac{1}{B} \ln \left| \frac{A+B}{A-B} \right| - i\pi\theta(A-B)\theta(-A-B) \ , \tag{3.40}$$

where θ is the unit step function. Then

$$\begin{aligned} F_B &= \int d^3p \left[\frac{m_c}{E_c} \right] \frac{a(p)(E_c - p\cos\theta)/M_D - b(p)}{A + B\cos\theta + i\varepsilon} \\ &= \frac{2\pi}{M_D} \left[\int dp \frac{m_c}{E_c} \frac{p^2}{B} \left\{ \left[a(p) \left[E_c + \frac{A}{B}p \right] + M_D b(p) \right] \ln \left| \frac{A+B}{A-B} \right| - 2pa(p) \right\} \right. \\ &\qquad \left. - i\pi \int dp \frac{m_c}{E_c} \frac{p^2}{B} \left[a(p) \left[E_c + \frac{A}{B}p \right] + M_D b(p) \right] \theta(A-B)\theta(-A-B) \right] \ , \end{aligned} \tag{3.41}$$

$$F_q=\frac{2\pi}{q}\left[-\int dp\frac{m_c}{E_c}\frac{p^2}{B}a(p)\left[\frac{A}{B}p\ln\left|\frac{A+B}{A-B}\right|-2p\right]+i\pi\int dp\frac{m_c}{E_c}\frac{p^2}{B}a(p)\frac{A}{B}p\theta(A-B)\theta(-A-B)\right]. \tag{3.42}$$

The regions, in terms of q and E_c, where the imaginary parts appear are shown for the D^0 decay in Fig. 3. The pole at low q has positive energy, $(B-p-q)_0>m_u$, and the pole at large q has negative energy, corresponding to the $\bar{c}$ quark decaying to a final state $\bar{u}$ on its mass shell.

Another integral has a tensor form with indices μ and ν which can be expressed as the sum of all possible tensor forms as

$$\int d^3p\left[\frac{m_c}{E_c}\right]\frac{p_\nu[a(p)p_\mu-b(p)B_\mu]}{(B-p)^2-2q\cdot(B-p)-m_u^2+i\varepsilon}=kM_D^2g_{\mu\nu}+lq_\mu q_\nu+mq_\mu B_\nu+nB_\mu q_\nu+rB_\mu B_\nu\ . \tag{3.43}$$

We can find the values of k, l, m, n, and r by the same method. Again, from the pole, all of the F_q, F_B, k, l, m, n, and r will have imaginary parts.

Finally from Eqs. (3.30), (3.34), and (3.36)–(3.43), we get, for the decay amplitude due to the gluon coupled to the u quark, Fig. 2(a),

$$\langle\bar{s}^l d^k g^a\,|\,\bar{D}^0\rangle_u=4A_{kl}^a\epsilon_\nu(q,\lambda')l_\mu[iF_B\varepsilon^{\mu\nu}{}_{\rho\sigma}q^\sigma B^\rho+(S_1q^\mu B^\nu+S_2qM_Dg^{\mu\nu}+S_3B^\mu B^\nu)]\ , \tag{3.44}$$

where

$$\begin{aligned}S_1&=[-2+(M_D^2+m_c^2-m_u^2)/(qM_D)]F_q+F_B-2l+(1-2M_D/q)m\ ,\\ S_2&=(2M_D/q)k-F_B\ ,\\ S_3&=[-2+(M_D^2+m_c^2-m_u^2)/(qM_D)]F_B-(2M_D/q)k-2n+(2-2M_D/q)r\ .\end{aligned} \tag{3.45}$$

For the second amplitude for Fig. 2(b), given in Eqs. (3.31) and (3.35), we use the same method to expand the integral by Lorentz covariance:

$$\int d^3p\left[\frac{m_c}{E_c}\right]\frac{a(p)p_\mu-b(p)B_\mu}{-2p\cdot q+i\varepsilon}=G_qq_\mu+G_BB_\mu\ , \tag{3.46}$$

$$\int d^3p\left[\frac{m_c}{E_c}\right]\frac{P_\nu[a(p)p_\mu-b(p)B_\mu]}{p\cdot q+i\varepsilon}=cM_D^2g_{\mu\nu}+dq_\mu q_\nu+eq_\mu B_\nu+fB_\mu q_\nu+hB_\mu B_\nu\ . \tag{3.47}$$

Again, the values G_q and G_B can be obtained by multiplying q_μ and B_μ in Eq. (3.46). c, d, e, f, and h can be obtained by the same method. Since there is no pole in the denominator $p\cdot q=p_0q_0-\mathbf{p}\cdot\mathbf{q}=E_cq_0-pq\cos\theta=q(\omega-p\cos\theta)$, where $E_c=\omega$ and $q_0=|\mathbf{q}|=q$, all of G_q, G_B, c, d, e, f, and h are real.

Using Eqs. (3.31), (3.35), (3.46), and (3.47), we finally have, for the decay amplitude due to the gluon coupled to the c quark, Fig. 2(b),

$$\langle\bar{s}^l d^k g^a\,|\,\bar{D}^0\rangle_c=4A_{kl}^a\epsilon_\nu(q,\lambda')l_\mu[iG_B\epsilon^{\mu\nu}{}_{\rho\sigma}q^\sigma B^\rho+(T_1q^\mu B^\nu+T_2qM_Dg^{\mu\nu}+T_3B^\mu B^\nu)]\ , \tag{3.48}$$

where

$$T_1=-G_B-e,\quad T_2=G_B-(M_D/q)c,\quad T_3=-h\ . \tag{3.49}$$

Using Eqs. (3.44) and (3.48), the total amplitude is

$$\begin{aligned}\langle\bar{s}^l d^k g^a\,|\,\bar{D}^0\rangle_{u+c}&=\langle\bar{s}^l d^k g^a\,|\,\bar{D}^0\rangle_u+\langle\bar{s}^l d^k g^a\,|\,\bar{D}^0\rangle_c\\ &=4A_{kl}^a\epsilon_\nu(q,\lambda')l_\mu\{i(F_B+G_B)\epsilon^{\mu\nu}{}_{\rho\sigma}q^\sigma B^\rho+[(S_1+T_1)q^\mu B^\nu+(S_2+T_2)qM_Dg^{\mu\nu}+(S_3+T_3)B^\mu B^\nu]\}\ .\end{aligned} \tag{3.50}$$

Gauge invariance, or the vanishing of Eq. (3.50) upon replacing $\varepsilon_\nu\to q_\nu$, implies

$$S_2+T_2=-(S_1+T_1)\ \text{ and }\ S_3+T_3=0\ . \tag{3.51}$$

Hence,

$$\langle\bar{s}^l d^k g^a\,|\,\bar{D}^0\rangle_{u+c}=4A_{kl}^a\epsilon_\nu(q,\lambda')l_\mu\frac{1}{q_0}\{iF_V\epsilon^{\mu\nu}{}_{\rho\sigma}q^\sigma B^\rho+F_A[q^\mu B^\nu+(q\cdot B)g^{\mu\nu}]\}\ , \tag{3.52}$$

where

$$F_V=q_0(F_B+G_B)\ \text{ and }\ F_A=q_0(S_1+T_1)\ . \tag{3.53}$$

The decay rate of the $\bar{D}^0$ through the nonspectator mode is obtained by integrating over q, p_s, and p_d, and summing the result over the gluon state helicities λ' and colors a:

$$\Gamma_{\rm NS}=\mathcal{A}(-g^{\nu\nu'})m_d m_s \int \frac{d^3q\, d^3p_s d^3p_d}{2q_0 2E_s 2E_d}\frac{1}{q_0^2}\delta^{(4)}(p_d+p_s+q-B)\,\mathrm{Tr}(l^\mu l^{\dagger\mu'})$$
$$\times\{iF_V\epsilon_{\mu\nu\rho\sigma}q^\sigma B^\rho+F_A[q_\mu B_\nu+(q\cdot B)g_{\mu\nu}]\}\{-iF_V\epsilon_{\mu'\nu'\rho'\sigma'}q^{\sigma'}B^{\rho'}+F_A[q_{\mu'}B_{\nu'}+(q\cdot B)g_{\mu'\nu'}]\}\ , \tag{3.54}$$

where

$$\mathcal{A}=16\times4\times4\times\frac{G^2}{2}\frac{g_s^2}{3}\frac{1}{(2\pi)^{11}}\frac{1}{2M_D}\left[\frac{f_+-f_-}{2}\right]^2 .$$

The trace term over $l_\mu l_{\mu'}^\dagger$ gives

$$\mathrm{Tr}(l_\mu l_{\mu'}^\dagger)=2[p_{d_{\mu'}}p_{s_\mu}+p_{s_{\mu'}}p_{d_\mu}-g_{\mu\mu'}(p_d\cdot p_s)+i\epsilon_{\mu'\rho\mu\sigma}p_d^\rho p_s^\sigma]/m_d m_s\ . \tag{3.55}$$

To calculate the three-body decay rate, we first integrate over p_d and p_s, and set $Q=B-q$. This gives, by Lorentz covariance,

$$\int\frac{d^3p_s d^2p_d}{2E_s 2E_d}\delta^{(4)}(Q-p_d-p_s)[p_{d_{\mu'}}p_{s_\mu}+p_{s_{\mu'}}p_{d_\mu}-g_{\mu\mu'}(p_d\cdot p_s)+i\epsilon_{\mu'\rho\mu\sigma}p_d^\rho p_s^\sigma]=K_1 g_{\mu\mu'}+K_2 Q_\mu Q_{\mu'}\ . \tag{3.56}$$

K_1 and K_2 can be found by the same method as we used in Eq. (3.36) or (3.43). Multiplying $g_{\mu\mu'}$ and $Q_\mu Q_{\mu'}$ on both sides of Eq. (3.56), we get

$$K_1=-\frac{\pi}{6}Q^2 \ \text{ and } K_2=\frac{\pi}{6}\ . \tag{3.57}$$

Then introducing $\int d^4Q\,\delta^{(4)}(B-q-Q)$ into the integral, we obtain

$$\Gamma_{\rm NS}=\mathcal{A}\int\frac{d^3q}{2q_0^3}\int d^4Q\,\delta^{(4)}(Q+q-B)\frac{\pi}{6}Q^2\left[-g^{\mu\mu'}+\frac{Q^\mu Q^{\mu'}}{Q^2}\right]\{iF_V\epsilon_{\mu\nu\rho\sigma}q^\sigma B^\rho+F_A[q_\mu B_\nu+(q\cdot B)g_{\mu\nu}]\}$$
$$\times\{-iF_V\epsilon_{\mu'\ \rho'\sigma'}^{\ \ \nu}q^{\sigma'}B^{\rho'}+F_A[q_{\mu'}B^\nu+(q\cdot B)g_{\mu'}^\nu]\}\ . \tag{3.58}$$

Now we come across the integral of a two-body decay rate: namely, $B\to q+Q$. In order to handle the integral over the gluon energy part $q_0=|\mathbf{q}|=q$ properly, and simplify the integral, we introduce $\int dS\,\delta^+(S-Q^2)=1$ into the integral. This gives

$$\Gamma_{\rm NS}=\mathcal{A}(\pi^2M_D^3/3)\int_0^{M_D^2}dS\int_0^\infty dq\,\delta\left[q-\frac{M_D^2-S}{2M_D}\right]\frac{1}{2M_D^2}q\frac{M_D^2-2q^2}{(S+q^2)^{1/2}}(M_D^2+S)(F_A^2+F_V^2)\ . \tag{3.59}$$

Finally, the nonspectator decay rate for D^0 is

$$\Gamma_{\rm NS}(\bar{D}^0)=\frac{128G^2\alpha_s}{9(2\pi)^8}\left[\frac{f_++f_-}{2}\right]^2 M_D^2\int_0^{M_D/2}q\left[\frac{M_D}{2}-q\right](F_A^2+F_V^2)dq\ , \tag{3.60}$$

where

$$F_A(q)=\frac{\pi}{M_D}\int_0^\infty dp\frac{m_c}{\omega}p\left\{\left[\frac{p^2A}{qB}a(p)+\frac{M_D(p-\omega)}{q}b(p)\right]\left[\frac{A}{B}\ln\left|\frac{A+B}{A-B}\right|-2\right]\right.$$
$$+\left[-\frac{a(p)p^2}{q}-a(p)\left[\omega+\frac{A}{B}p\right]+M_Db(p)\right]\ln\left|\frac{A+B}{A-B}\right|$$
$$\left.+\left[\frac{m_c^2}{q}a(p)-M_Db(p)\right]\ln\frac{\omega+p}{\omega-p}-2\frac{p\omega}{q}a(p)+4pa(p)\right\}$$

$$-i\pi\frac{\pi}{M_D}\int_0^\infty dp\frac{m_c}{\omega}p\left\{\left[\frac{p^2A}{qB}a(p)+\frac{M_D(p-\omega)}{q}b(p)\right]\frac{A}{B}\right.$$

$$\left.+\left[-\frac{a(p)p^2}{q}-a(p)\left[\omega+\frac{A}{B}p\right]+M_Db(p)\right]\right\}\theta(A-B)\theta(-A-B)\ , \tag{3.61}$$

$$F_V(q)=\frac{\pi}{M_D}\int_0^\infty dp\frac{m_c}{\omega}p\left\{\left[-a(p)\left[\omega+\frac{A}{B}p\right]+M_Db(p)\right]\ln\left|\frac{A+B}{A-B}\right|+M_Db(p)\ln\frac{\omega+p}{\omega-p}\right\}$$

$$-i\pi\frac{\pi}{M_D}\int_0^\infty dp\frac{m_c}{\omega}p\left[-a(p)\left[\omega+\frac{A}{B}p\right]+M_Db(p)\right]\theta(A-B)\theta(-A-B)\ . \tag{3.62}$$

Recall that all of the imaginary parts, in both F_A and F_V, come from the effect of the pole in the propagator in Fig. 2(a), referring to Eqs. (3.36)–(3.43).

The terms

$$\frac{m_c^2}{q}a(p)\ln\frac{\omega+p}{\omega-p}-2\frac{p\omega}{q}a(p) \tag{3.63}$$

in Eq. (3.61) give a logarithmic divergence in $\Gamma_{\rm NS}$ in the $\int dq$ as $q\to 0$. These terms come from the coefficient c in the expansion of the integral in Eq. (3.47). In order to avoid the divergence we need to apply the infrared cutoff on q. This is also necessary because the soft gluons with smaller energy will be trapped into the quark-gluon structure of the final-state hadrons, and cannot be included in the inclusive calculations. The value of the cutoff is taken to be 200 MeV. Our calculation shows that the value of $\Gamma_{\rm NS}$ changes by no more than 10% if we change the cutoff on q from 200 to 100 MeV, and is, therefore, insensitive to its exact value.

In the nonrelativistic limit, $p \ll m_c$, $\omega\simeq m_c$, the $q\to 0$ divergent term becomes

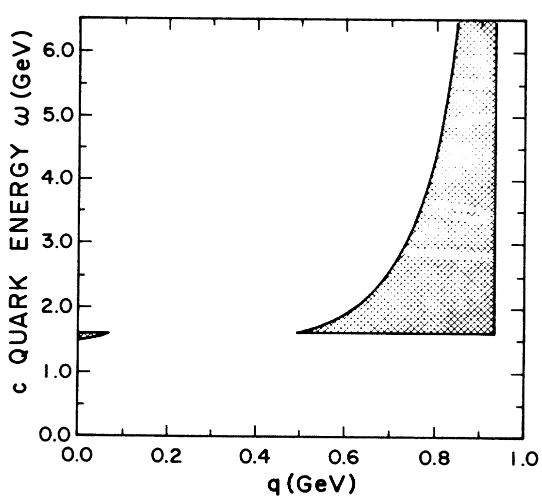

FIG. 3. The regions of q and ω where the imaginary parts of the u-quark propagator contribute to the D-meson nonspectator decay.

$$\left[\frac{m_c^2}{q}\ln\frac{\omega+p}{\omega-p}-2\frac{p\omega}{q}\right]a(p)\simeq\left[\frac{m_c^2 2p}{q\omega}-2\frac{p\omega}{q}\right]a(p)$$

$$=\frac{2a(p)}{q\omega}p(m_c^2-\omega^2)$$

$$=\frac{-2a(p)p^3}{qm}\to 0 \tag{3.64}$$

compared to the other terms in Eq. (3.61). Hence, there is no divergence problem in the nonrelativistic case.

The $\Gamma_{\rm NS}$ in Eq. (3.60) contains the strong coupling constant α_s. It should also be multiplied by the higher-order QCD corrections to the decay. This combination we will call $\alpha_{\rm eff}$ which we now replace α_s by. After substituting the wave functions $a(p)$ and $b(p)$, obtained from the spectral fits to the D and D^* mesons,[1] into Eq. (3.60), we finally find the nonspectator decay rate as

$$\Gamma_{\rm NS}(D^0)=\alpha_{\rm eff}(f_++f_-)^2\ 0.87\times 10^{-13}\ {\rm GeV}\ . \tag{3.65}$$

To check the extreme nonrelativistic results ($p \ll m_u$) from our relativistic formula, we may make a reduction by applying $p\ll m_c$ and $\omega\simeq m_c$. Using the relation[8] $\psi^2(0)=(M_D/12)f_D^2$, we have

$$F_A\simeq\frac{2\pi^3}{\sqrt{M_D}}\left[\frac{M_D}{m_cm_u}\right]\psi(0)\ ,$$

$$F_V\simeq\frac{2\pi^3}{\sqrt{M_D}}\left[\frac{m_c-m_u}{m_cm_u}\right]\psi(0)\ , \tag{3.66}$$

and

$$\Gamma_{\rm NS}^{\rm NR}\simeq\left[\frac{f_++f_-}{2}\right]^2\frac{G^2\alpha_sf_D^2}{648\pi^2m_u^2}M_D^5\ .$$

These are all consistent with the previous nonrelativistic results.[8] The nonrelativistic results are poor approximations since the momentum in the propagator is not small compared to m_u.

In the nonrelativistic case $A<0$, $B<0$, and $A-B\simeq[0.018-0.057q-(M-q)p^2/(2m_c)+2pq]<0$ for $p\leq 0.28$, so that $\theta(A-B)$ gives zero. Thus, all of the imaginary parts in F_A and F_V disappear in the nonrelativistic limit.

When the value of m_u is varied extremely from 0 to 1.57 GeV, the relativistic Γ_{NS} is only changed by a factor of 2.6, and shows that Γ_{NS} is much less sensitive to m_u than in the extreme nonrelativistic limit, Eq. (3.68), where Γ_{NS} behaves as $1/m_u^2$. Around the fitted value of $m_u=0.25$ GeV there is very little variation of Γ_{NS} with changes in m_u.

For comparison with the calculation with a heavy antiquark and a light quark we deduce the Dirac bound-state equation from the relativistic bound-state equation by setting

$$M_D=m_c+E' \text{ and letting } M_D \text{ and } m_c\to\infty \tag{3.67}$$

such that E' is fixed. From the normalization condition (2.26) we see that in this limit $g_0/\sqrt{m_c}$ and $f_+/\sqrt{m_c}$ are normalized independent of m_c. Thus, we find that $F_A=O(m_c^{3/2}/q)$ and $F_V=O(m_c^{-1/2})\ll F_A$ and, hence, F_V can be neglected. The forms of F_A and F_V for the Dirac case are then $F_V=0$ and

$$F_A(q)=-\pi\frac{m_c}{q}\left[\int_0^\infty dp\,p\left[\frac{A}{B}\ln\left|\frac{A+B}{A-B}\right|-2\right]b(p)-i\pi\int_0^\infty dp\,p\frac{A}{B}b(p)\theta(A-B)\theta(-A-B)\right]. \tag{3.68}$$

In this limit $A=E'^2-p^2-m_u^2-2E'q$ and $B=-2pq$. From the rate equation (3.60) and the fact that the meson mass $m_D\simeq m_c$ we see that q is of $O(m_c)$ also. In the Dirac limit of $m_D\simeq m_c\to\infty$, the potential is independent of m_c, and by the normalization conditions Eqs. (2.25) or (2.26), the scaled wave functions $g_0/\sqrt{m_c}$, $f_+/\sqrt{m_c}$, $\sqrt{m_c}a$, and $\sqrt{m_c}b$ are normalized independent of m_c. In the Dirac limit, A and B are proportional to $q=O(m_c)$ and their ratio, $A/B\to p/E'$ is independent of m_c. F_A then becomes

$$F_A(q)=-\frac{M_D^{3/2}}{q}\frac{\pi}{4\sqrt{2\pi}}\left[\int_0^\infty dp\left[\frac{E'}{p}\ln\left|\frac{-E'-p}{-E'+p}\right|-2\right]\frac{f_+}{\sqrt{M}}-i\pi\int_{|E'|}^\infty dp\frac{E'}{p}\frac{f_+}{\sqrt{M}}\right]\equiv\frac{M_D^{3/2}}{q}C_A\ , \tag{3.69}$$

where C_A is independent of q. Although this asymptotic form behaves as $1/q$, the large m_c form, Eq. (3.68), does not have a $q\to 0$ singularity, so we use (3.69) and cut off the q integration in (3.60) at $q\simeq\Lambda_{QCD}$. This gives, in the $M_D\to\infty$ limit,

$$\Gamma_{NS}^{Dirac}(D^0)=\frac{128G^2\alpha_s}{9(2\pi)^8}\left[\frac{f_++f_-}{2}\right]^2C_A^2\times\frac{M_D^6}{2}\left[\ln\left[\frac{M_D}{2\Lambda_{QCD}}\right]-1\right]. \tag{3.70}$$

We see the important result that, up to logarithms,

$$\Gamma_{NS}(m_Q)\to O(m_Q^6)\ .$$

Thus, the ratio of nonspectator to spectator becomes

$$\frac{\Gamma_{NS}(m_Q)}{\Gamma_{sp}(m_Q)}\to O(m_Q)$$

and grows as the heavy-quark mass increases. We, thus, expect and find that this ratio is larger for the B meson than the D meson, and expect further enhancements for the T or top-containing mesons.

The nonspectator decay rate Γ_{NS} in the Dirac limit from Eq. (3.68) is

$$\Gamma_{NS}^{Dirac}(D^0)=\alpha_{eff}(f_++f_-)^2 0.42\times10^{-13}\text{ GeV}\ . \tag{3.71}$$

This is about one-half of the relativistic result, Eq. (3.65).

In the relativistic case we find that the contributions to Γ_{NS} come more from F_V than from F_A, and that the real parts in F_A and F_V contribute more than the imaginary parts.

The lower-component wave function $f_+(p)$ is close to $g_0(p)$ for $p\geq 1$ GeV. $f_+(p)$ and $g_0(p)$ give almost the same contribution to ReF_A and ImF_A. This shows the importance of the relativistic treatment. The contributions to ReF_V and ImF_V are still dominated by $g_0(p)$.

In the Dirac case $g_0(p)$ always comes along with $1/(\omega+m)$ in the expression of the wave functions $a(p)$ and $b(p)$, Eq. (2.24). Hence, in the Dirac limit as $m_c\to\infty$ no contribution to F_A will come from $g_0(p)$.

2. $D_s^\pm$ mesons

The nonspectator process for $D_s^\pm$ meson decay by $c\bar{s}$ annihilation into a QCD-dressed W and an accompanying gluon uses the same diagrams as in Figs. 2(a) and 2(b) with the bound-state u quark replaced by an s quark, and the final-state $\bar{s}$ by a $\bar{u}$ quark. The associated gluon-corrected weak Hamiltonian is the same one as in the case of D mesons. From the requirement of a color singlet in the final state for $u,\bar{d}$ quarks and gluon, the $u\bar{d}$ pair should belong to the octet state, which is included in the (f_+-f_-) term in H_W, Eq. (3.4). Hence, only the term in (f_+-f_-) will survive in the decay amplitude. These annihilation amplitudes are the same as the Fierz transformations of the nonspectator amplitudes. We can write down the decay rate at once from Eq. (3.60):

$$\Gamma_{NS}(D_s^{\pm})=\frac{128G^2\alpha_s^2}{9(2\pi)^8}\left[\frac{f_+-f_-}{2}\right]^2 M_{D_s}^2 \times\int_0^{M_{D_s}/2} q\left[\frac{M_{D_s}}{2}-q\right](F_A^2+F_V^2)dq\ , \quad (3.72)$$

where F_A and F_V are given by Eqs. (3.61) and (3.62) and M_D is replaced by M_{D_s}.

Since D and D_s mesons have the same mass scales, they will have the same values of f_+ and f_- from the theoretical expressions of Eq. (3.6). By Eq. (3.72), we obtain the nonspectator decay rates for $D_s^{\pm}$ (see also Table I):

$$\Gamma_{NS}(D_s^{\pm})=\alpha_{\text{eff}}(f_+-f_-)^2 0.50\times10^{-13}\ \text{GeV}\ ,$$

$$\Gamma_{NS}(D_s^{\pm})=\alpha_{\text{eff}}(f_+-f_-)^2 0.11\times10^{-13}\ \text{GeV} \quad \text{(Dirac case)}\ . \quad (3.73)$$

The contribution from F_A to Γ_{NS} is much less than that from F_V, due to the cancellations between f_+ and g_0 in F_A. The fact that most of the contributions to F_V are from $g_0(p)$ shows that the D_s meson is close to a nonrelativistic case.

3. B^0, B_s^0, and $B_c^{\pm}$ mesons

The decay-rate calculations for B^0 and B_s^0 are the same as for the D^0 case and that for $B_c^{\pm}$ is the same as for the $D_s^{\pm}$ case. One difference is the values of f_+ and f_-. For the B^0 and B_s^0 mesons, they are given for four flavors by[7]

$$f_{\pm}=\left[1+\frac{25}{6\pi}\alpha_s(m_b)\ln\left[\frac{M_W}{m_b}\right]\right]^{\gamma_\pm}, \quad \text{where } \gamma_+=-\tfrac{6}{25} \text{ and } \gamma_-=\tfrac{12}{25}\ , \quad (3.74)$$

$$f_+=0.89,\quad f_-=1.26$$

for $m_B=4.9$ GeV, $\alpha_s(m_b)=0.17$, and $M_W\simeq80$ GeV.

Another difference is that we need to add the Kobayashi-Maskawa (KM) mixing element $|U_{bc}|^2$. Our numerical values for B^0 are (where we have used the value[16] for $U_{bc}=0.045\pm0.008$)

$$\Gamma_{NS}(B^0)=\left[\frac{U_{bc}}{0.045}\right]^2\alpha_{\text{eff}}(f_++f_-)^2 4.3\times10^{-13}\ \text{GeV}\ ,$$

$$\Gamma_{NS}(B^0)=\left[\frac{U_{bc}}{0.045}\right]^2\alpha_{\text{eff}}(f_++f_-)^2 \times4.4\times10^{-13}\ \text{GeV} \quad \text{(Dirac case)}\ . \quad (3.75)$$

For B_s^0, the nonspectator process is $b+\bar{s}\to c\bar{c}g$ via W exchange with KM amplitude U_{bc} and rate

$$\Gamma_{NS}(B_s^0)=\left[\frac{U_{bc}}{0.045}\right]^2\alpha_{\text{eff}}(f_++f_-)^2 \times2.0\times10^{-13}\ \text{GeV}\ ,$$

$$\Gamma_{NS}(B_s^0)=\left[\frac{U_{bc}}{0.045}\right]^2\alpha_{\text{eff}}(f_++f_-)^2 \times1.8\times10^{-13}\ \text{GeV} \quad \text{(Dirac case)}\ . \quad (3.76)$$

For $B_c^{\pm}$, we have $b+\bar{c}\to u\bar{d}g$ and $c\bar{s}g$ which contribute equal amounts when neglecting final-state masses to give

$$\Gamma_{NS}(B_c^{\pm})=\left[\frac{U_{bc}}{0.045}\right]^2\alpha_{\text{eff}}(f_+-f_-)^2 \times0.071\times10^{-13}\ \text{GeV}\ ,$$

$$\Gamma_{NS}(B_c^{\pm})=\left[\frac{U_{bc}}{0.045}\right]^2\alpha_{\text{eff}}(f_+-f_-)^2 \times0.062\times10^{-13}\ \text{GeV} \quad \text{(Dirac case)}\ . \quad (3.77)$$

In the relativistic case, the contribution to Γ_{NS} from F_A is much more than that from F_V because B mesons are close to the Dirac limit where we have shown that $F_A=O(m_b^{3/2}/q)$ and $F_V=O(m_b^{-1/2})\ll F_A$.

Since most of F_A comes from the $f_+(p)$ part in both

TABLE I. Nonspectator decay rates and leptonic decay constants for D and D_s mesons. $m_u=m_d=0.25$, $m_c=1.57$, $m_s=0.41$ GeV. $f_+=0.65$, $f_-=2.37$. $\alpha_{\text{eff}}=0.39$ (normalization such that $f_\pi=132$ MeV).

	Relativistic	Dirac case
	D meson	
Spectator (D^0) (10^{-13} GeV)	12.5	
Nonspectator (D^0) (10^{-13} GeV)	3.1	1.5
f_D (MeV)	580	360
$\Psi_D(0)$ (MeV$^{3/2}$)	230	150
	D_s meson	
Nonspectator (D_s) (10^{-13} GeV)	0.58	0.13
f_{D_s} (MeV)	590	380
$\Psi_{D_s}(0)$ (MeV$^{3/2}$)	240	160

B^0 and B_s^0, the B^0 and B_s^0 mesons are very relativistic cases.

IV. WEAK LEPTONIC DECAY CONSTANTS f_D, f_{D_s}, f_B, AND f_{B_c} AND LEPTONIC DECAY RATES

A. Formulation of the weak leptonic decay constant f_D, f_{D_s}, f_B, and f_{B_c}

The leptonic decay constant is defined through the current matrix element for $D^- \to l^- \nu_l$ decay,

$$\langle 0 | J_\mu | D^- \rangle \equiv \frac{-i}{(2\pi)^{3/2}(2M_D)^{1/2}} f_D B_\mu \sin\theta_C \ , \quad (4.1)$$

where B_μ is the four-momentum of the initial bound-state meson, θ_C is the Cabibbo angle, and f_D is the leptonic decay constant. This matrix element can be expressed in quark fields as

$$\langle 0 | J_\mu | D^- \rangle = \langle 0 | \bar{\psi}_c(x) \gamma_\mu (1-\gamma_5) \psi_d(x) | D^- \rangle \sin\theta_C \ . \quad (4.2)$$

After we insert the intermediate states of an on-mass-shell c antiquark, the amplitude of the current is associated with the bound-state wave function. We have with the $3/\sqrt{3}$ color factor

$$\langle 0 | J_\mu | D^- \rangle = \sum_{s_c} \int d^3p \langle 0 | \bar{\psi}_c(x) | p, s_c \rangle \gamma_\mu (1-\gamma_5) \times \langle p, s_c | \psi_d(x) | D^- \rangle \sqrt{3} \sin\theta_C \ . \quad (4.3)$$

Using

$$\langle 0 | \bar{\psi}_c(x) | \mathbf{p}, s_c \rangle = e^{ip\cdot x} \frac{1}{(2\pi)^{3/2}} \left[\frac{m_c}{\omega} \right]^{1/2} \bar{v}(p, s_c) \ , \quad (4.4)$$

using the expression of the bound-state wave function $\langle \mathbf{p}, s_c | \psi_d(x) | D^- \rangle$ from Eq. (2.5) and (2.6), and summing over the spin states of the c antiquark, we have

$$\langle 0 | J_\mu | D^- \rangle = \sqrt{3} \sin\theta_C \frac{e^{iB\cdot x}}{\sqrt{2M_D}(2\pi)^{9/2}} \times \int d^3p \frac{m_c}{\omega} \mathrm{Tr}[\gamma_\mu (1-\gamma_5) \Phi(p)] \ . \quad (4.5)$$

Replacing

$$\Phi(p) = \gamma_5 \left[a(p) + b(p) \frac{\not{B}}{m_c} \right] (\not{p} - m_c)$$

in Eq. (4.5), and using $\int d\Omega_{\hat{p}} \mathbf{p} = 0$, we get

$$\langle 0 | J_\mu | D^- \rangle = \frac{16\sqrt{3}\pi B_\mu}{(2\pi)^{9/2}(2M_D)^{1/2} M_D} \sin\theta_C \times \int dp \frac{m_c}{\omega} p^2 [-a(p)\omega + b(p) M_D] \ . \quad (4.6)$$

Equating Eqs. (4.1) and (4.6) and from the relations between $a(p)$, $b(p)$ and $g_0(p)$, $f_+(p)$ in Eq. (2.24), we find

$$f_D = \frac{2\sqrt{3}}{\pi^2 M_D} \int dp \frac{m_c}{\omega} p^2 [-a(p)\omega + b(p) M_D]$$

or

$$f_D = \frac{\sqrt{3}}{\pi^2 (8\pi)^{1/2} M_D} \int dp \frac{m_c}{\omega} p^2 \left[g_0(p) - \frac{p}{m_c + \omega} f_+(p) \right] \ . \quad (4.7)$$

(In the Dirac-equation limit, we have

$$f_D = \frac{2\sqrt{3}}{\pi^2} \int dp\, p^2 [-a(p) + b(p)] \ ,$$

$$\sqrt{M_D} f_D = \frac{\sqrt{3}}{\pi^2 \sqrt{8\pi}} \int dp\, p^2 \frac{g_0(p)}{\sqrt{M_D}} \ . \quad (4.8)$$

From the normalization condition (2.25) in the Dirac limit $g_0/\sqrt{M_D}$ is independent of $m_c \simeq m_D$. From Eq. (4.8) we see as $m_c \to \infty$ that $f_D \propto 1/\sqrt{M_D}$ in the Dirac limit.) The calculated values are (see Table I)

$$f_D = 580 \text{ MeV}, \quad f_D = 360 \text{ MeV} \quad \text{Dirac case} \ . \quad (4.9)$$

The Dirac-case value is slightly above the experimental upper bound,[17]

$$f_D \le 290 \text{ MeV} \ , \quad (4.10)$$

and in the range found in a lattice calculation including fermion loops.[18] The fully relativistic value of $f_D = 580$ MeV is perhaps a factor of 2 too large, consistent with the calculation of the analogous vector process of the leptonic decay of the ψ whose amplitude is also a factor of 2 too large.[1] The leptonic decay rate of the Υ came out about right,[1] giving us more confidence in f_B, which is presented later.

The values of f_{D_s}, f_B, and f_{B_c} are given in Tables I and II for the relativistic and Dirac cases. Our conventions in Eq. (4.1) are those in which $f_\pi = 132$ MeV. With these conventions the leptonic decay rate for $D^- \to \mu^- + \bar{\nu}_\mu$ is given by

$$\Gamma_{\text{lep}}(D^- \to \mu^- \bar{\nu}_\mu) = \frac{G^2}{8\pi} f_D^2 m_D m_\mu^2 \left[1 - \frac{m_\mu^2}{m_D^2} \right]^2 \sin^2\theta_C \quad (4.11)$$

TABLE II. Nonspectator decay rates and leptonic decay constants for B, B_s^0, and $B_c^{\pm}$ mesons, $m_u=m_d=0.25$, $m_c=1.57$, $m_s=0.41$, $m_b=4.9$ GeV. $f_+=0.89$, $f_-=1.26$, $\alpha_{\text{eff}}=0.17$, $U_{bc}=0.045$.

	Relativistic	Dirac case
	B meson	
Spectator $(B^0,B^{\pm},B_s^0,B_c^+)$ $(10^{-13}$ GeV)	3.9	
Nonspectator (B^0) $(10^{-13}$ GeV)	3.4	5.8
f_B (MeV)	300	260
$\Psi_B(0)$ $(\text{MeV}^{3/2})$	200	170
	B_s^0 meson	
Nonspectator (B_s^0) $(10^{-13}$ GeV)	1.6	1.4
$\Psi_{B_s}(0)$ $(\text{MeV}^{3/2})$	220	190
	B_c^+ meson	
Nonspectator (B_c^+) $(10^{-13}$ GeV)	0.0017	0.0015
f_{B_c} (MeV)	480	420
$\Psi_{B_c}(0)$ $(\text{MeV}^{3/2})$	350	300

with similar formulas for $D^-\to\tau^-\bar{\nu}_\tau$, and for $D_s^{\pm}$, $B^{\pm}$, and $B_c^{\pm}$ leptonic decays.

V. SPECTATOR, NONSPECTATOR, AND LEPTONIC DECAY RATES OF D, D_s, AND B MESONS

In this section, we are going to apply what we have learned in the previous sections to discuss the difference in lifetimes of the D^0 and $D^{\pm}$ mesons, the total decay rates of $D_s^{\pm}$, the prediction of the difference in lifetimes between B^0 and $B^{\pm}$, and the decay rate and spectrum of $D^0\to\gamma+$anything.

A. Lifetime difference of D^0 and $D^{\pm}$ and the total decay rates of $D_s^{\pm}$

The spectator decay rates of D and D_s are the same as the sum of the decay rates of $\bar{c}\to\bar{s}e^-\bar{\nu}_e$, $\bar{s}\mu^-\bar{\nu}_\nu$, and $\bar{s}\,\bar{u}d$. The decay rate for $\bar{c}\to\bar{s}l^-\bar{\nu}_l$ is given by[13,14]

$$\Gamma(\bar{c}\to\bar{s}l^-\bar{\nu}_l)\equiv\frac{G^2m_c^5}{192\pi^3}\cos^2\theta_C g(m_s/m_c)\ , \qquad (5.1)$$

where $g(x)=1-8x^2-24x^4\ln x+8x^6-x^8$. Assuming that e^-, μ^-, $\bar{u}$, and d quarks are massless compared to c and s quarks, and using $m_s=0.41$ GeV and $m_c=1.58$ GeV, we get

$$g(m_s/m_c)=0.61\ . \qquad (5.2)$$

The spectator decay rate of D or D_s is

$$\begin{aligned}\Gamma_{\text{sp}}(D^0)=\Gamma_{\text{sp}}(D^{\pm})=\Gamma_{\text{sp}}(D_s^{\pm})&=\Gamma(\bar{c}\to\bar{s}e^-\bar{\nu}_e)+\Gamma(\bar{c}\to\bar{s}\mu^-\bar{\nu}_e)+\Gamma(\bar{c}\to\bar{s}\,\bar{u}d)\\&=[2R_{\text{SL}}+(2f_+^2+f_-^2)R_H]g(m_s/m_c)\frac{G^2m_c^5}{192\pi^3}\ .\end{aligned} \qquad (5.3)$$

In Eq. (5.3) we have included the factors R_{SL} and R_H to represent the effect of higher-order QCD corrections to the spectator rate.

As we have mentioned at the end of Sec. III B, there are two mechanisms to explain the lifetime-difference problem in D^0 and $D^{\pm}$ mesons. A calculation of the interference effect[15] in a nonrelativistic approach yields a large effect in reducing the $D^{\pm}$ decay rate. However, in a relativistic quark model[19] or in a nonrelativistic approach in which the Fermi motion of the c quark is taken into account,[20] the effect was estimated to be considerably smaller. Recently, the interference effect was calculated in the relativistic bag model[21] and found to reduce the hadronic rate of D^+ by 41%. The other mechanism is the nonspectator decay of the D^0.

We are hampered in making a unique prediction of the nonspectator rate by not having a well-determined value of α_s to use at this mass scale. Also, the calculations of the interference effect in the $D^{\pm}$ decay range over a wide set of values. There is now, however, a large enough set of data on D^0, $D^{\pm}$, and D_s decays to determine a set of values for the various theoretical contributions. We carry out this analysis below. We use the recently reported results of E691 at Fermilab[22] for the three decays since they have small errors and related systematics. This is important since we will be taking differences of rates to find the nonspectator contribution.

In using the experimental data we just use the semileptonic branching ratio of the D^+ and its total decay rate to find the semileptonic decay rate of the charmed quark:

$$B_{\rm SL}^{\rm expt}(D^+)=0.17\pm0.019\ , \tag{5.4}$$

$$\Gamma^{\rm expt}(D^+)=(6.04\pm0.22)\times10^{-13}\ {\rm GeV} \tag{5.5}$$

to get

$$\Gamma_{\rm SL}(D^+)=(1.03\pm0.12)\times10^{-13}\ {\rm GeV}\ . \tag{5.6}$$

We compare this to the theoretical value of the semileptonic decay rate [Eq. (3.13)]:

$$\Gamma(\bar c\to\bar s l^-\bar\nu_l)=(0.61)\frac{G^2m_c^5}{192\pi}\cos^2\theta R_{\rm SL}$$

$$=(1.30\times10^{-13})R_{\rm SL}\ {\rm GeV}\ ,$$

where[14]

$$R_{\rm SL}=1-\frac{2\alpha_s(m_c^2)}{3\pi}f(m_s/m_c)=1-0.56\alpha_s(m_c^2)\ . \tag{5.7}$$

Agreement of Eq. (5.7) with Eq. (5.6) indicates a satisfactory range of $\alpha_s(m_c^2)$ to be $\alpha_s(m_c^2)=0.37\pm0.16$. For consistency we use the experimental value in Eq. (5.6).

Next, we find the spectator decay rate from the $D_s^\pm$ decay rate which has a very small calculated nonspectator rate. The $D_s^\pm$ decay rate comes from its spectator, nonspectator, and leptonic decay rates

$$\Gamma(D_s^\pm)=\Gamma_{\rm sp}(D_s^\pm)+\Gamma_{\rm NS}(D_s^\pm)+\Gamma(D_s^\pm\to\mu\nu_\mu)$$

$$+\Gamma(D_s^\pm\to\tau\nu_\tau)\ . \tag{5.8}$$

The leptonic decay rates depend on f_{D_s}. We know from the experimental bound on $f_D<340$ MeV that our relativistic calculations of f_D are at least too large by a factor of 2 and this should apply to f_{D_s} as well. Using $f_{D_s}<300$ MeV also gives a small contribution of the leptonic decays of D_s, mostly from $D_s^\pm\to\tau\nu_\tau$ ($D_s^\pm\to\mu\nu_\mu$ is only a tenth of this) and a theoretical range

$$\Gamma(D_s^\pm\to\tau\nu_\tau)=(0.5\pm0.5)\times10^{-13}\ {\rm GeV}\ . \tag{5.9}$$

With our final parameters below we find

$$\Gamma_{\rm NS}(D_s^\pm)\simeq1\times10^{-13}\ {\rm GeV}\ . \tag{5.10}$$

Using the experimental result[22]

$$\Gamma^{\rm expt}(D_s^\pm)=(14.0\pm1.3)\times10^{-13}\ {\rm GeV} \tag{5.11}$$

we find then from Eqs. (5.8)–(5.10) that

$$\Gamma_{\rm sp}(D_s^\pm)=\Gamma_{\rm sp}(D^\pm)=\Gamma_{\rm sp}(D^0)$$

$$=(12.5\pm1.4)\times10^{-13}\ {\rm GeV} \tag{5.12}$$

since the spectator decay rate of the charm quark is by definition the same in each charmed meson.

We now subtract the spectator decay rate from the D^0 decay rate[21] to get the nonspectator rate

$$\Gamma^{\rm expt}(D^0)=(15.6\pm0.5)\times10^{-13}\ {\rm GeV}$$

$$=\Gamma_{\rm sp}(D^0)+\Gamma_{\rm NS}(D^0)\ . \tag{5.13}$$

With Eq. (5.12) this gives

$$\Gamma_{\rm NS}(D^0)=(3.1\pm1.5)\times10^{-13}\ {\rm GeV}\ . \tag{5.14}$$

The nonspectator contribution is, thus, about 25% of the spectator decay rate. This is in agreement with recent treatments of D decays,[23,24] where two-body spectator decay modes accounted for 80% of the D^0 width.

We finally turn back to the D^+ decay rate to find the needed suppression from interference. With the 17% branching into each semileptonic channel, the hadronic branching is (66±2.7)% giving

$$(0.66)\Gamma^{\rm expt}(D^+)=R_I(\Gamma_{\rm sp}(D^+)-2\Gamma_{\rm SL}(D^+))\ , \tag{5.15}$$

where R_I is the interference effect suppression. From Eqs. (5.5), (5.6), and (5.12) we find

$$R_I=0.38\pm0.06\ . \tag{5.16}$$

This is a rather large suppression that is required by the set of data we have used.

In order to calculate the nonspectator decay rates, or to find the experimentally indicated value of $\alpha_{\rm eff}$, we need to know f_+ and f_- separately. For charm quarks the leading-logarithm estimation may not be considered a good approximation. We, nevertheless, carry on using it in the simple form abstracted from Eqs. (3.5)–(3.7), that

$$f_+^2f_-=1\ . \tag{5.17}$$

The spectator decay rate is given by

$$\Gamma_{\rm sp}=\left[2+(2f_+^2+f_-^2)\frac{R_H}{R_{\rm SL}}\right]\Gamma_{\rm SL} \tag{5.18}$$

and using Eq. (5.6) for $\Gamma_{\rm SL}$ and Eq. (5.12) we find

$$(2f_+^2+f_-^2)R_H(f_+^2,f_-^2)=(10.1\pm1.7)R_{\rm SL}\ . \tag{5.19}$$

R_H is given[25] by

$$R_H=1+\frac{2}{3\pi}\alpha_s(Q^2)h\ ,$$

where

$$h=\frac{31}{4}-\pi^2+\frac{19}{4}\frac{f_-^2-f_+^2}{2f_+^2+f_-^2}$$

$$+3\frac{\alpha_s(Q^2)-\alpha_s(M_W^2)}{\alpha_s(Q^2)}\frac{f_+^2\rho_++f_-^2\rho_-}{2f_+^2+f_-^2} \tag{5.20}$$

and for four flavors $\rho_+=-0.47$ and $\rho_-=1.36$. The strong coupling constant which includes two-loop effects is given by

$$\alpha_s=\frac{4\pi}{\beta_0\ln(x)}\left[1-\frac{\beta_1\ln[\ln(x)]}{\beta_0^2\ln(x)}\right]\ , \tag{5.21}$$

where $\beta_0=11-\frac{2}{3}n_f$, $\beta_1=102-\frac{38}{3}n_f$, and $x=Q^2/\Lambda_{\overline{\rm MS}}^2$ ($\overline{\rm MS}$ denotes the modified minimal-subtraction scheme). Solving Eq. (5.19) using (5.17), (5.7), and (5.20) with $Q=1.5$ GeV and $\Lambda_{\overline{\rm MS}}=150$ MeV gives

$$f_-=2.37\pm0.24,\quad f_+=0.65\pm0.04\ . \tag{5.22}$$

For comparison the solution to (5.19) and (5.17) without the QCD correction gives $f_-=3.07\pm0.29$ and $f_+=0.57\pm0.03$. If one uses constituent masses for u and d quarks of 250 MeV, the $g(m_s/m_c)=0.61$ factor in the hadronic decays is reduced[13] to $F(m_c,m_s,m_u)=0.35$ requiring a larger $f_-=3.18$ with $f_+=0.56$. From the result of the relativistic calculation of the nonspectator decay rate, Eq. (3.65), we have

$$\Gamma_{\rm NS}(D^0)=\alpha_{\rm eff}(f_++f_-)^2 0.87\times10^{-13}\ \text{GeV}\ . \tag{5.23}$$

Using Eqs. (5.22) and (5.14) we find

$$\alpha_{\rm eff}=0.39\pm0.19\ , \tag{5.24}$$

which is a reasonable value for α_s times higher-order QCD corrections at this low-mass scale.

To check for consistency that the nonspectator rate for D_s is small, we recall Eq. (3.73):

$$\Gamma_{\rm NS}(D_s)=\alpha_{\rm eff}(f_+-f_-)^2 0.50\times10^{-13}\ \text{GeV}\ .$$

Using Eqs. (5.22) and (5.24) gives

$$\Gamma_{\rm NS}(D_s)=0.58\times10^{-13}\ \text{GeV}$$

consistent with that used in the D_s analysis.

B. The prediction of the lifetime difference between B^0 and $B^\pm$ mesons

For the spectator decays of the B meson, its rate equals the sum of that for $b\to ce^-\bar\nu_e$, $c\mu^-\bar\nu_\mu$, $c\tau^-\bar\nu_\tau$, $c\bar u d$, and $c\bar c s$. The interference effects have been found to be small in the B system.[20] The spectator decay rate is then

$$\begin{aligned}\Gamma_{\rm sp}(B^0)&=\Gamma_{\rm sp}(B^\pm)\\ &=\Gamma(b\to ce^-\bar\nu_e)+\Gamma(b\to c\mu^-\bar\nu_\mu)+\Gamma(b\to c\bar u d)+\Gamma(b\to c\tau^-\bar\nu_\tau)+\Gamma(b\to c\bar c s)\\ &=[(2+2f_+^2+f_-^2)F(m_b,m_c,0)+F(m_b,m_c,m_\tau)+(2f_+^2+f_-^2)F(m_b,m_c,m_c)]\,|U_{bc}|^2\frac{G^2m_b^5}{192\pi^3}\ ,\end{aligned} \tag{5.25}$$

where $F(m_b,m_c,0)$, $F(m_b,m_c,m_\tau)$, and $F(m_b,m_c,m_c)$ are mass correction factors[13,14] calculated to be 0.47, 0.10, and 0.14, respectively. We obtain

$$\Gamma_{\rm sp}(B^0)=\Gamma_{\rm sp}(B^\pm)=\left[\frac{U_{bc}}{0.045}\right]^2 3.9\times10^{-13}\ \text{GeV}\ . \tag{5.26}$$

Combining this spectator decay rate with the nonspectator decay rate from Eq. (3.75) and including $f_\pm$ from Eq. (3.74), we obtain

$$\begin{aligned}\Gamma_{\rm avg}(B)&=\frac{\Gamma_{\rm tot}(B^0)+\Gamma_{\rm tot}(B^\pm)}{2}\\ &=\frac{2\Gamma_{\rm sp}+\Gamma_{\rm NS}}{2}\\ &=\left[\frac{U_{bc}}{0.045}\right]^2(3.9+\alpha_{\rm eff}9.9)\times10^{-13}\ \text{GeV}\ .\end{aligned} \tag{5.27}$$

Equating this to the experimental result [26]

$$\Gamma_{\rm avg}^{\rm expt}(B)=(5.9^{+1.0}_{-0.7})\times10^{-13}\ \text{GeV} \tag{5.28}$$

gives a poorly determined value

$$\alpha_{\rm eff}=0.20\pm0.21\ . \tag{5.29}$$

If we take a typical value at m_b of $\alpha_{\rm eff}\simeq\alpha_s(m_b)\simeq0.17$ for $\Lambda_{\overline{\rm MS}}=150$ MeV, we would expect an increase in $\Gamma_{\rm avg}(B)$ of 43% over the spectator rate from $\Gamma_{\rm NS}$.

The ratio of lifetimes between $B^\pm$ and B^0 from Eqs. (5.26) and (3.75) using $\alpha_{\rm eff}(m_b)\simeq0.17$ is

$$\begin{aligned}\text{ratio}&=\frac{\tau_{B^\pm}}{\tau_{B^0}}=\frac{\Gamma_{B^0}}{\Gamma_{B^\pm}}=\frac{\Gamma_{\rm sp}(B)+\Gamma_{\rm NS}(B)}{\Gamma_{\rm sp}(B)}\\ &=\frac{(3.9+3.4)\times10^{-13}}{3.4\times10^{-13}}\simeq1.9\ .\end{aligned} \tag{5.30}$$

We predict the lifetime ratio between the $B^\pm$ and B^0 mesons is about the same as that for the D mesons, but from different effects. This ratio is roughly independent of the particular $f_\pm$ used.

C. Decay rate of $D^0\to\gamma+$anything

The form of the amplitude of $D^0\to\gamma+$anything is the same as that of the nonspectator diagram except for two different features. One is the color matrix $\frac{1}{2}\lambda^a_{ij}$ is replaced by $3\delta_{ij}$. This will add a factor of $\frac{9}{4}$. The second difference is the requirement of a color singlet in the final state for the decay of $D^0\to\gamma+$anything. $\bar s$ and d quarks are in a pure color singlet in the (f_+-f_-) term and the (f_++f_-) term when crossed to the $\bar s d$ channel using the color identity in Eq. (3.8) has a color singlet in $\bar s d$ with coefficient $\frac{1}{3}$. These two $\bar s d$ singlet contributions are then added. With the electromagnetic coupling constant re-

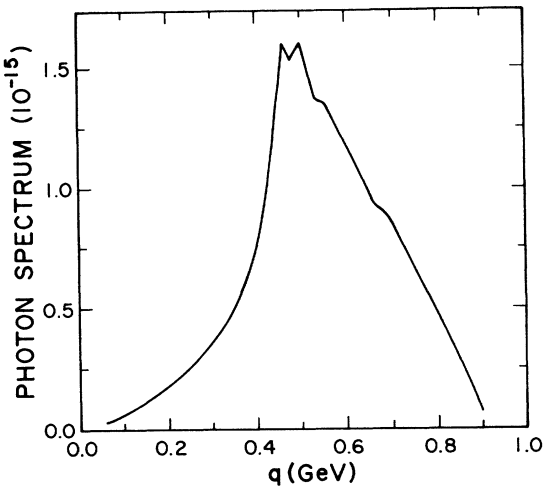

FIG. 4. Photon spectrum $d\Gamma/dq$ for $D^0 \rightarrow \gamma + X$.

placing the strong coupling constant α_{eff}, we have the decay amplitude for $D^0 \rightarrow \gamma$ +anything from Eq. (3.60). The energy spectrum of the photon for $D^0 \rightarrow \gamma$ +anything is, thus, given by

$$\frac{d\Gamma_{\text{NS}}}{dq}(D^0 \rightarrow \gamma + X) = \frac{8G^2\alpha}{(2\pi)^8}[(f_+ - f_-) + \tfrac{1}{3}(f_+ + f_-)]^2 M_D^2 q \times \left[\frac{M_D}{2} - q\right][F_A^2(q) + F_V^2(q)] \quad (5.31)$$

and is depicted in Fig. 4. The total rate is

$$\Gamma(D^0 \rightarrow \gamma + X) = 0.0061 \times 10^{-13} \text{ GeV} . \quad (5.32)$$

This has a branching ratio of the order of 0.04% and is very sensitive to the values of f_+ and f_-.

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[1]M. Bander, D. Silverman, B. Klima, and U. Maor, Phys. Rev. D **29**, 2038 (1984).

[2]M. Bander, D. Silverman, B. Klima, and U. Maor, Phys. Lett. **13B**, 258 (1984).

[3]O. Greenberg, Phys. Rev. **139B**, 1038 (1965); O. Greenberg and R. Genolio, *ibid.* **150**, 1070 (1966).

[4]D. Gross, Phys. Rev. **186**, 1448 (1969).

[5]K. Johnson, Phys. Rev. D **4**, 1101 (1972).

[6]G. P. Lepage, Phys. Rev. A **16**, 863 (1977); W. Caswell and G. P. Lepage, *ibid.* **20**, 36 (1979).

[7]J. Ellis, M. Gaillard, and D. Nanopoulos, Nucl. Phys. **B100**, 313 (1975).

[8]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **44**, 7 (1980); H. Fritzsch and P. Minkowski, Phys. Lett. **90B**, 455 (1980); W. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. C **4**, 257 (1980).

[9]J. Bjorken and S. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[10]R. Baltrusaitis *et al.*, Phys. Rev. Lett. **54**, 1976 (1985).

[11]M. Gaillard and B. Lee, Phys. Rev. Lett. **33**, 108 (1974).

[12]G. Altarelli and L. Maiani, Phys. Lett. **52B**, 351 (1974).

[13]R. Marshak *et al.*, *Theory of Weak Interactions of Particle Physics* (Wiley, New York, 1969).

[14]M. Suzuki, Nucl. Phys. **B145**, 420 (1978); N. Cabibbo and L. Maiani, Phys. Lett. **79B**, 109 (1978).

[15]B. Guberina *et al.*, Phys. Lett. **89B**, 111 (1979).

[16]Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. **170B**, 75 (1986).

[17]J. Adler *et al.*, Phys. Rev. Lett. **60**, 1375 (1988).

[18]H. Hamber, in Proceedings of the Third International Conference on Supercomputing, Boston, 1988 (unpublished).

[19]H. Sawayamagi *et al.*, Phys. Rev. D **27**, 2107 (1983).

[20]G. Altarelli and L. Maiani, Phys. Lett. **118B**, 414 (1982).

[21]N. Bilic, B. Guberina, and J. Trampetic, Nucl. Phys. **B248**, 261 (1984).

[22]J. R. Raab *et al.*, Phys. Rev. D **37**, 2391 (1988), see also J. C. Anjos *et al.*, Phys. Rev. Lett. **58**, 311 (1987); **58**, 1818 (1987).

[23]M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).

[24]A. J. Buras, J.-M. Gerard, and R. Ruckl, Nucl. Phys. **B268**, 16 (1986).

[25]G. Altarelli *et al.*, Phys. Lett. **99B**, 141 (1981); Nucl. Phys. **B187**, 461 (1981).

[26]D. Ritson, in *Proceedings of the XXIII International Conference on High Energy Physics*, Berkeley, California, 1986, edited by S. Loken (World Scientific, Singapore, 1987).