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Author
Kerns, Quentin A.

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University of California

Ernest O. Lawrence Radiation Laboratory

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Lawrence Radiation Laboratory
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Abstract

The phenomenon of resonance allows a modest power flow to build up a large-amplitude wave. Although resonators are commonly designed for waves varying sinusoidally in time, any waveform describable as a Fourier series

$$f(x) = \sum_{n=1}^{\infty} (A_n \cos nx + B_n \sin nx)$$

can be supported by designing the metallic enclosure for the wave so that resonance occurs at all integer multiples of the fundamental frequency.

The principle is not limited to standing-wave resonators; it can be applied to traveling-wave cavities as well, by incorporating a suitable circulator between output and input ports of the cavity.

Several experimental "superposition" resonators constructed in the course of the Berkeley Accelerator Study are described, together with their rf driving circuits.

Potential applications include, on one hand, continuously operating devices such as sawtooth wave-form bunchers or cavities providing specialized waveforms for particle acceleration, separation, or extraction. Pulsed cavities, on the other hand, by the accumulation of large energy over a period of time for later rapid discharge, can provide fast-rising, shaped pulses of high peak power for track or discharge chambers.

*Work done under auspices of the U. S. Atomic Energy Commission.

Introduction

Electromagnetic fields varying periodically in time have widespread technological application. Low-power electronic systems like telemetry, oscilloscopes, and digital computers exhibit a great variety of waveforms including narrow spikes, flat-topped pulses, sawtooth forms, and many others. In contrast, high-power rf systems in accelerators, radar, and communications tend to rely on waveforms that may be modulated in various ways, but are almost sinusoidal in time.

The reason low-power electronic systems freely utilize waveforms of practically arbitrary shape, whereas high-power systems tend toward sine waves, is largely that cost scales up rapidly with power. Low-amplitude waveforms can be generated by inefficient, wideband devices without the wasted power's being conspicuous; large-amplitude waves similarly generated would be prohibitively expensive in terms of the scale of the apparatus and the lost power, which rises in proportion to the square of the amplitude. Thus, an oscilloscope can easily afford to have a recurrent sawtooth sweep-voltage waveform applied to the plates of its cathode ray tube for beam deflection; the lost power is typically only a few watts. Conversely, if a 100-kV 50-Mc sawtooth wave were similarly generated in RC circuits and applied to deflection plates of 50 pF capacitance, the average power loss in the resistance R would be more than 10 MW.

The results presented here demonstrate that a large-amplitude wave can be obtained via resonance in a low-loss system. Although resonators and waveguide structures are commonly used for supporting sine waves, what is required in this case is a resonator capable of supporting periodic waves of arbitrary shape. It is possible to construct and excite such resonators to obtain any commonly encountered waveform as a time-varying electromagnetic field within the resonator. The method may be described as waveform synthesis by Fourier series superposition in a resonator possessing a harmonically related spectrum of resonances. Although it is impractical to attempt to satisfy completely the infinite trigonometric series, interesting and useful results have been obtained with practical circuits.

Fourier Series Representation

We assume the pulse being considered (i.e., the time function or waveform) has a finite number of points of ordinary discontinuity and a finite number of maxima and minima: the Dirichlet conditions are satisfied. This is certainly true for all commonly encountered waveforms. We further assume the resonator is to handle a continuous train of such pulses, periodic with period T, and defined by its values in the interval 0 to T. If f(x) represents the time variation of the E or H field, the Fourier expression is given by

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{n=\infty} \left(A_n \cos 2n \frac{\pi}{T} x + B_n \sin 2n \frac{\pi}{T} x\right)$$

and the coefficients

$$A_n = \frac{2}{T} \int_0^T f(x) \cos \frac{2n\pi x}{T}\, dx$$

and

$$B_n = \frac{2}{T} \int_0^T f(x) \sin \frac{2n\pi x}{T}\, dx.$$

We can provide the constant term $A_0/2$ in the E or H field by employing dc power supplies, although for many applications, the waveform can be chosen to have zero average value without impairing its usefulness. Thus, for example, a sawtooth waveform in a buncher cavity may as well have a zero average, since no net beam acceleration is required, but only velocity modulation.

Turning our attention now to the terms under the summation, it is clear that the desired circuits must transmit a line spectrum having many lines, extending up to very high frequencies in some cases. The total spectrum space is virtually empty nevertheless, and the commonly used wide-band circuits provide much more bandwidth than is really necessary. If a limited rather than a continuous train of pulses is to be transmitted, more space in the frequency spectrum is required, because the line spectrum of the continuous train goes over into a series of bands. Even here, however, it is rarely that a large fraction of the spectrum is used.

The individual trigonometric terms could be provided by an array of individual sine-wave oscillators, properly adjusted in amplitude and phase, but such an arrangement is unnecessarily cumbersome. We can generate the waveforms directly; for example, grounded grid tubes may be attached to the system in such a manner that their anode circuits appear electrically as a part of the resonator. It is then necessary to modulate the electron stream appropriately; a method of doing this will be discussed.

Resonator Configurations

Figure 1 shows a ring resonator, consisting of a 56-ft length of 125-ohm coaxial line having its ends joined to form a continuous transmission path. (For convenience, the line is arranged in two turns.) The probes couple into the resonator and sample the radial E field between the inner and outer conductors of the coaxial line. Waves are propagated in both senses around the ring.

For such a line, the velocity of propagation v_ϕ is nearly independent of frequency. Cable mode (TEM) resonances occur in the ring at each frequency f_n at which the total line length ℓ is an integral number of wavelengths. The mode spectrum is thus:

$$\lambda_n = \frac{\ell}{n}, \text{ or } f_n = \frac{n}{\ell} v_\phi, \quad \text{for } n = 1, 2, 3, \cdots .$$

Measurements of the mode spectrum made over the range of 15 to 2100 Mc showed that v_ϕ is essentially constant at about 0.885 c. Although measurements were not made higher than 2100 Mc, other evidence indicates that the TEM mode spectrum would be measurable to at least 10 kMc. Modes occur every 15.56 Mc. The pictured resonator has been excited with a variety of waveform, including 2-nsec pulses recurring every 64.3 nsec, a sawtooth of 15.56 Mc fundamental frequency, and flat-topped waveforms. Figure 2 shows a flat-topped wave circulating at 15.56 Mc in the resonator. The exciting pulse, shown below the pulse train in the figure, is set to recur at a subharmonic frequency to illustrate the gradual decay of the stored pulses when the signal is not replenished each cycle. The decay time constant of the fundamental, $\tau = 20/\omega$, is about 4 μsec. The various frequencies do not all decay at precisely the same rate, but the agreement is near enough so the waveform remains recognizable as its amplitude diminishes.

Strip Transmission-Line Resonator

Figure 3 shows a resonator formed of a flat conductor facing a ground plane, shorted to the ground plane at one end and open at the other. The structure has the physical appearance of a $\lambda/4$ resonator, but the width and spacing have been adjusted to give a harmonic mode spectrum. Whereas a uniform $\lambda/4$ resonator has modes at 1, 3, 5, $\cdots$ times the fundamental, the strip line of Fig. 3 varies in characteristic impedance Z_0 inversely as the square of the distance from the open end, achieving the desired property of resonating to all integral multiples of the lowest-mode frequency. Some empirical adjustment to counteract edge effects (which are difficult to compute) is desirable in strip-line resonators. Sawteeth and various other waveforms have been excited on strip-line resonators like Fig. 3.

One feature of a strip-line resonator is that it provides an E-field gap between nearly parallel planes, a feature useful for particle deflection or modulation. Figure 4 shows the contour of a strip-line resonator for which the spacing to the ground plane is a linear function of distance.

Each of the strip-line resonators above could be constructed, as an alternative, in the form of a concentric line. As before, the inverse-square variation of Z_0 with distance must be maintained.

Cylindrical Symmetry

Figure 5 illustrates a harmonic-mode resonator derived from a modification of the $TM_{o,n}$ resonances of the familiar right circular cylinder.

This structure, Fig. 5a, may be regarded as supporting radial TEM modes on a two-conductor line extending outward from the axis to a short circuit around the periphery. Figure 5b illustrates the method of excitation by bunched electron beam. The electron gun excitation arrangement is useful, inter alia, for energy-storage applications where time is available to build up a large-amplitude wave, which is subsequently discharged rapidly. Figure 5c illustrates an application of a hollow electron beam for sustaining the desired waveform in the resonator, which may be used, e.g., for bunching or particle separation.

Traveling-Wave Structures

Harmonic-mode resonators can be made from traveling-wave structures by coupling the output and input ports with a circulator; the ring resonator is a simple example of this technique. If each of the necessary frequency components spans an integral number of wavelengths around the traveling-wave path, waves of arbitrary shape may be excited. In the ring resonator of Fig. 1, a directional coupler could be employed to limit the excitation to a wave traveling in one direction around the ring. Omniharmonic couplers are not commercially available, but there seems to be no reason why they could not be constructed.

Resonator Excitation by Sampling; the Use of Feedback

Since the resonator recirculates the pulse energy, it is possible to excite at a subharmonic mode, as Fig. 2 shows. An extension of the idea of subharmonic excitation is that of sampling a different (short) portion of the stored pulse on each cycle, adding energy as necessary on a sample-by-sample basis. This method of excitation, although quite different from the pulse technique of Fig. 2, was successfully used to excite various waveforms in the resonator of Fig. 1.

The arrangement was as follows: A 1-nsec sampling pulse was introduced at a frequency different by Δf from the 15.56-Mc fundamental frequency. The amplitude of the 1-nsec pulse was modulated at the difference frequency by a wave of the desired form, but lower in frequency by the ratio $15.56/\Delta f$. The Δf was chosen in the range of 0.1 to 0.5 Mc because suitable pulse generators were handy; the period of Δf should be shorter than the resonator decay constant. One advantage of excitation by the sampling procedure is that it allows negative feedback to be used to maintain the desired rf waveform in agreement with a waveform standard of comparison that occurs at a lower frequency. The resonator of Fig. 5, e.g., could well be excited by this technique where accurate waveforms are required; the electron bunch would be kept short, approximating a δ function in time, while the charge in each bunch would be adjusted in accordance with the error signal determined at an earlier sampling instant. The use of feedback makes it possible to maintain a desired voltage waveform despite frequency-dependent load and loss variations.

Energy Storage

The use of harmonic-mode resonators for pulse storage is attractive because of the great flexibility in pulse shape permitted; the pulse shape may easily be programmed electronically, in contrast to the fixed output pulse shape of line-type pulse generators.

Eventually, it is hoped, superconducting resonators will become low enough in cost to consider; they would provide virtually loss-free energy storage.

For low-duty-cycle programmed pulse applications, the resonator energy loss might not be an important consideration. The cavity could be built up to its full amplitude just before switching it into the load. In such applications resonator size can be scaled down and breakdown strength increased by filling the cavity with a uniform isotropic dielectric material.

Summary and Remarks

Waveform generators involving parallel RC circuits cannot be extended to high amplitude at high frequency without encountering serious power limitations, as noted in the Introduction. In the special case of periodic waveforms, however, the circuits need not provide a large instantaneous bandwidth; it is sufficient to provide transmission for a line spectrum of equally spaced frequencies which are integer multiples of the fundamental. Although some waveforms involve only odd harmonics, or only even harmonics, the circuit to transmit arbitrary waveforms must support both even and odd harmonics, e.g., for a sawtooth, $f(t) = 2A(\frac{1}{2} + \frac{1}{\pi} \sin \phi - \frac{1}{2\pi} \sin 2\phi + \frac{1}{3\pi} \sin 3\phi \cdots)$.

Resonators possessing a series of resonances which occur at all integer multiples of a fundamental frequency should, according to the foregoing, be capable of supporting almost any desired waveform; experimentally, a variety of waveforms, including a sawtooth, have been excited in suitable resonators. The advantage of using a resonator is that the power loss associated with producing the waveform is reduced by a factor of 100 to 1000, depending on the resonator Q and shunt resistance for each resonance. The shunt resistance is not a constant with frequency, of course. In general, the shunt resistance tends to fall off with the inverse square root of the harmonic order; on the other hand, for many waveforms (refer to the sawtooth) the required amplitudes of the higher-order terms in the series fall off at least as fast. Thus it is often the case that the presence of the harmonics adds little to the total power lost in the resonator.

The presence of absorption provides a finite bandwidth in the neighborhood of each resonance, an effect which, although it wastes power, permits tuning errors so long as they are much less than 1 part in Q. The half-power bandwidth around each resonance equals the resonant frequency divided by the associated Q.

The resonator geometries which have been discussed here are no doubt only a sample of what could be developed. For one thing, analysis of a given set of boundary conditions to find the resonance spectrum proves to be faster than the synthesis of a resonator shape, given the desired spectrum. A rapid synthesis technique would allow a wider investigation to be made than has been made up to the present.

A principle of similitude may be applied to cavity resonators, viz: if all the linear dimensions of a resonator are changed by a constant factor, the resonant wavelength of all the normal modes will be scaled by the same factor.

Concerning driving circuits, some progress has been made (for other purposes) in the use of gridded power tubes to produce short pulses of anode current. In a cavity like that of Fig. 5, a pair of 8533 triodes could be placed in push-pull across the cavity gap as an alternative to the electron gun shown. The 8533 is capable of producing waveforms having rise times less than 1 nsec.

It is worth mentioning that a sawtooth voltage wave form appears automatically in a harmonic superposition resonator if the driving current waveform approximates a δ function and its recurrence rate is deliberately made slightly different from the resonator fundamental frequency. Since the absorption loss in the cavity can be quite small (high Q), sawtooth waveforms can be excited with practical values of peak current.

The harmonic resonator has an interesting application for small signals apart from driving circuits; as a filter element tuned to a given fundamental frequency it can be used to transmit periodic waves while discriminating against noise. Thus filtering and averaging can be performed on periodic signals without losing detail in the waveform.

In conclusion, we note the converse to the ideas of this paper: since nearly all rf current generators are nonlinear and thus emit a series of harmonics rather than a pure tone, resonant systems for supporting sine waves should be constructed to definitely avoid harmonic relationships between unused resonances and the signal frequency.

Figure Captions

Fig. 1. Ring resonator for Fourier synthesis of periodic waveforms.

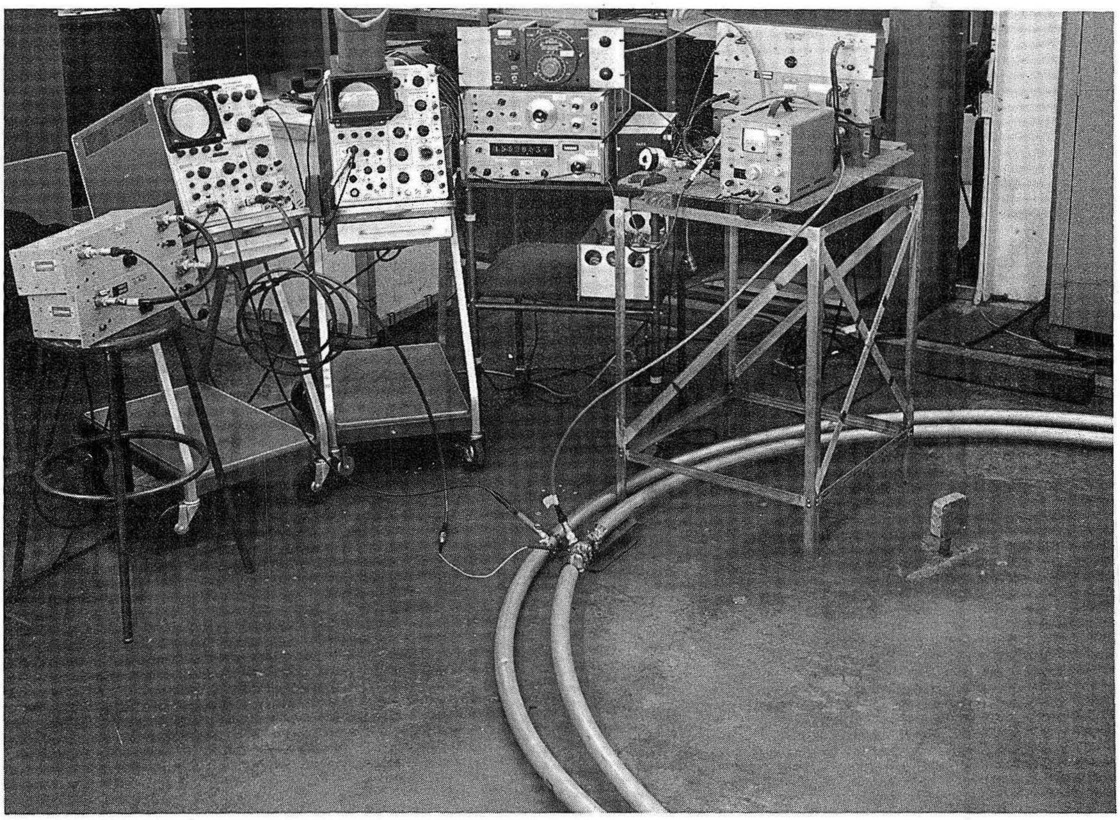
Fig. 2. Pulse storage pattern in ring resonator.

Pulse width 20 nsec, pulse repetition rate, ≈ 15.5 Mc/sec.

Fig. 3. Harmonic resonator in strip-line form.

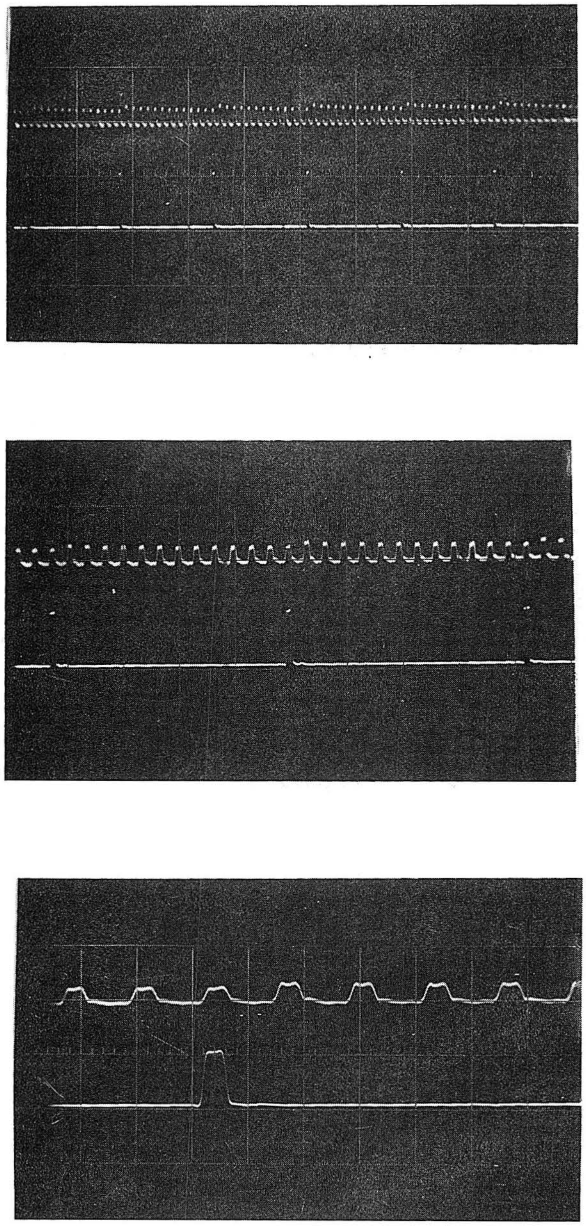
Fig. 4. Harmonic resonator of planar construction.

Fig. 5. Axially symmetric resonator.



ZN-5171

Fig. 1



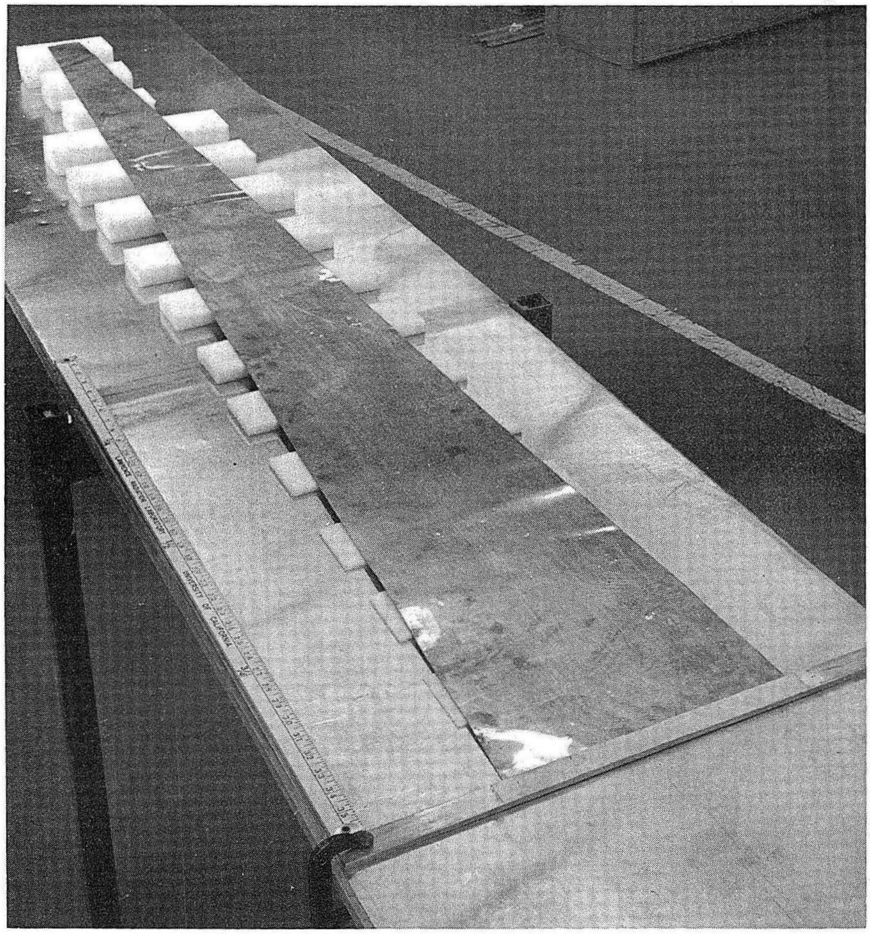
Horizontal: 0.5 μsec/cm

Horizontal: 0.2 μsec/cm

Horizontal: 0.05 μsec/cm

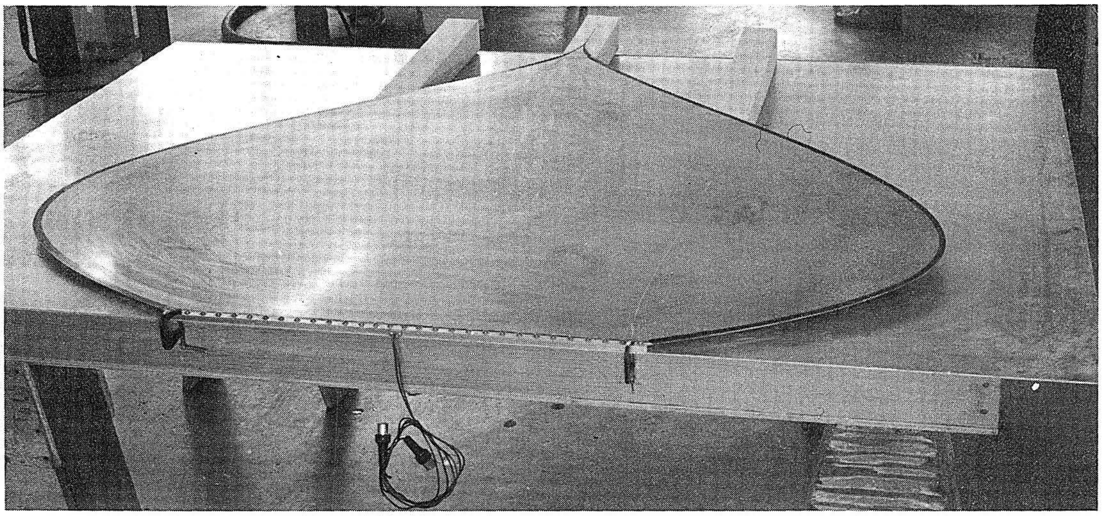
ZN-5174

Fig. 2



ZN-5172

Fig. 3



ZN-5170

Fig. 4

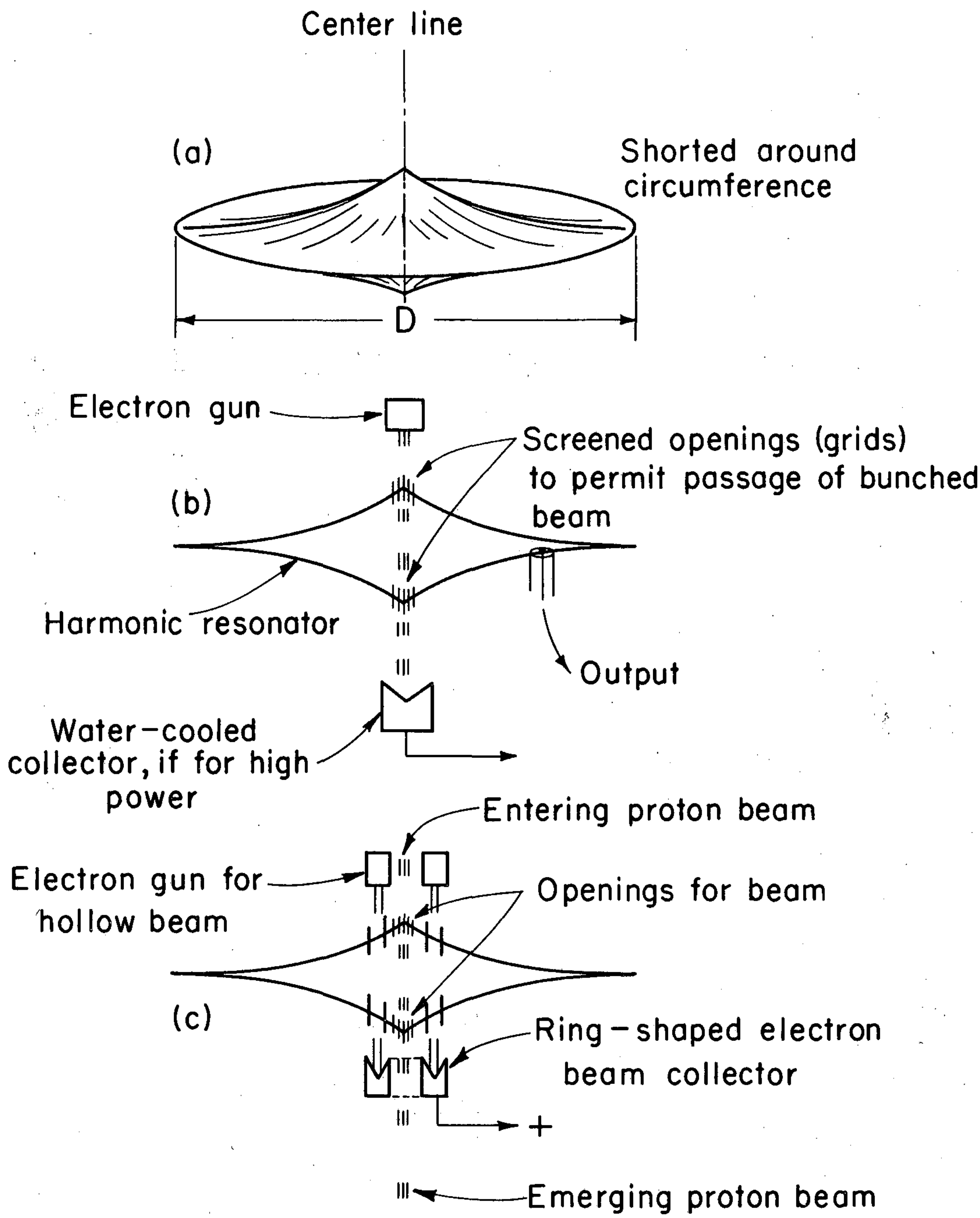


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Fig. 5