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Tillmann, Klaus-Dieter.

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MONTE CARLO CALCULATIONS

Klaus-Dieter Tillmann

March 5, 1970

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March 22, 1971

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To: All recipients of UCRL-19719
From: Technical Information Division
Subject: "Energy Spectra of Backscattered Electrons and Positrons
by Monte Carlo Calculations," by Klaus-Dieter Tillman,
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Figures 4-14 have been inserted in reverse order. All
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ENERGY SPECTRA OF BACKSCATTERED ELECTRONS AND POSITRONS

BY MONTE CARLO CALCULATIONS

Klaus-Dieter Tillmann

Department of Physics and Lawrence Radiation Laboratory
University of California, Berkeley, California

March 5, 1970

ABSTRACT

I present the energy spectra and angular distribution of backscattered electrons and positrons from thick targets for Li, Be, Al, Fe, Cu, and Sn for 1.0 MeV particles; and for Be and Sn for 0.5 MeV and 2.0 MeV particles normally incident on the targets. These spectra have been calculated by a random-sampling Monte Carlo program on a CDC 6600 digital computer. Backscattering coefficients are found to fit a $\sqrt{Z}$ -dependence for $Z > 13$. The shapes of the energy spectra depend strongly on the atomic number Z and are found to be in good agreement with experimental data. The angular distribution fits a $\cos(\pi-\alpha) \cdot e^{-(\pi-\alpha)^2/2}$ -dependence. The energy spectra, angular distributions, and backscattering coefficients depend on the angle β between the initial beam and the target surface. For decreasing angles β , the most probable energy shifts toward the initial energy.

I. INTRODUCTION

This paper presents the energy spectra and angular distributions of backscattered electrons and positrons from infinitely thick targets. For many experiments on nuclear beta decay, knowledge of the amounts and energy spectra of backscattered electrons is important, for example, in making corrections due to backscattered electrons in angular correlation experiments involving beta particles, or in making the backscattering correction of many energy spectra observed with semiconductor detectors.

The diffusion of electrons has been considered since 1893 (Lenard, "Lenard window"). Since then it has become clear that, in a scattering process of charged MeV-particles with potentials (potential scattering), five processes must be considered to treat the problem theoretically:

(1) Single elastic scattering in a Coulomb field. The particle enters a Coulomb field $V(r)$ of an atom. The scattering process occurs close to the nucleus. Then the particle is backscattered by an angle¹ $\chi > \pi/2$. The projectile loses little energy.

(2) Multiple scattering in a Coulomb field. The projectile collides with an electron in the outer shell and is deflected by a small angle, $\chi \ll \pi/2$. Hundreds of thousands of collisions may occur before the particle is backscattered. Small amounts of kinetic energy are transferred to the atomic-bounded electron, which may be separated from its nucleus.

(3) Plural scattering in a Coulomb field. This is the link from single elastic scattering to multiple inelastic scattering. The scattering angle is $20^\circ < \chi < 90^\circ$. The scattering process occurs in the inner shells of the atom. More than one, but not too many, collisions are necessary to backscatter the projectile.

(4) Statistical variation of rate of ionization energy loss.

(5) Energy loss due to Bremsstrahlung.

The theory of plural and multiple scattering is based on the diffusion theory developed by W. Bothe.² He assumed that the backscattered electrons were lower in energy than the primary ones, that the backscattering coefficient p increases with Z , and that the shape of the spectra depends strongly on the target (Z) but little on the primary energy (for $E_0 < 11.0 \text{ MeV}$).

In 1947 and 1948, G. Molière developed his theory of multiple scattering.³ The advantages of his theory over that of Bothe are that it is analytic almost to the very end, and that no special form of the differential cross section is assumed. For large target thicknesses he was able to expand his scattering probability $f(\theta)\theta d\theta$; that is the number of electrons in the spatial interval $(\theta, \theta+d\theta)$, in a power series, which is valid throughout the total angular interval $(0, \infty)$, including the transition from single elastic scattering to multiple scattering. This power expansion

$$f(\theta)\theta d\theta = \delta d\delta (f^0 + B^{-1}f^1 + B^{-2}f^2 + \dots) \quad (1)$$

is the basic relation in this paper. Molière's theory is used in this work to calculate the distribution probabilities for a given

particle of defined energy and directional cosine in a thick target. This expansion is easy to apply, as Molière evaluated the functions f^0 , f^1 , and $f^2(\delta)$ and $B(E, \delta)$ [for definitions of δ , f 's, and B see section II.A.].

Closely related to Molière's theory are L. I. Schiff's "Approximation Method for High-Energy Potential Scattering," published in 1956⁴ and the improvement by Saxon and Schiff one year later.⁵ This method expresses the scattered amplitude in terms of a quadrature by summing the infinite Born series after approximating each term by the method of stationary phase.

H. A. Bethe simplified Molière's theory in 1953⁶ and corrected and expanded Molière's numerical computations. Bethe's values are used in this paper for the scattering functions since they are more complete than Molière's tables.

The region of plural scattering has attracted attention the last few years. This region is included in Molière's theory, but because of some approximations in his numerical computations of the f 's, the error increases with increasing scattering angle χ . This region is difficult to deal with in a closed analytic form.

Experimental data on the energy spectra of backscattered electrons are found from W. Bothe⁷ for 0.68 MeV electrons for C, Al, Cu, Sn, and Pb, and from H. Frank⁸ for 1.75 MeV electrons for Al, Cu, and Pb.

The ionization energy loss of beta's in collisions with electrons was found empirically by R. Whiddington in 1912.⁹ Using the Bhabha cross section, Bethe¹⁰ and Landau¹¹ quantum-mechanically developed the calculation of the average energy loss which a scattering particle

suffers; Rohrlich and Carlson¹² and Birkhoff¹³ developed the slightly differing relativistic formulae for electron and positron collisions. Numerical values in agreement with the Bethe-Bloch formula are taken from A. T. Nelms,¹⁴ who also considers density effects.

The scattering process can be simulated by computing each collision. So the particle is followed through the material from collision to collision; the energy and angle with which the particle leaves one collision becoming the input parameters of the next collision. Using random numbers distributed on an interval corresponding to Molière's scattering distribution, I treat this scattering problem as a "game of probabilities" (Monte Carlo Method).

Monte Carlo calculations of electron scattering have been done by J. E. Leiss, S. Penner, and C. S. Robinson in carbon¹⁵; by D. F. Hebbard and P. R. Wilson for gold and silver for 1.0 MeV electrons¹⁶; by C. MacCallum¹⁷ and by J. F. Perkins¹⁸ for C, Al, and Cu. But up to now no energy spectra of the backscattered electrons have been computed.

II. THEORETICAL ANALYSIS

A. Description of Fundamental Multiple Scattering Processes

This paper follows Molière's theory of multiple scattering.³ Most scattering processes occur in the outer shell of the scattering atom, where the potential $V(r)$ of the nucleus is screened off by the electrons in the inner shells. The problem is to find both an appropriate expression for the cross section which takes screening into account, and a probability distribution for the scattering angle.

An inhomogeneous potential $V(r)$ can be regarded as a source of diffraction for a particle wave. The wave will be distorted while it penetrates through the potential. If the wave function ψ_{sc} of the scattered particle on a plane far away from the center of diffraction can be found, then the differential cross section for scattering into the spatial interval $(\chi, \chi + d\chi)$ is defined by

$$\sigma(\chi) = r^2 |\psi_{sc}(r, \chi)|^2 . \quad (2)$$

If $k(r)$ is the wave number of the particle wave inside the potential and k_0 the wave number outside the potential, then Molière shows that, from the pseudo-diffraction index $n = k(r)/k_0$, the relation

$$k(r) = k_0 (1 - \alpha \chi_0) \quad (3)$$

can be derived, where α determines the influence of the screening on the scattering amplitude [$\alpha = zZe^2/(hv) = zZ/(137\beta)$; $\beta = v/c$; $\chi_0 = 1/(k_0 a)$; $a =$ Thomas-Fermi atom radius]. From the WKB approximation, Molière obtains, for the ratio of the actual cross section to the Rutherford

cross section, the relation

$$\frac{\sigma_{\text{act}}(\chi)}{\sigma_{\text{Ruth}}(\chi)} = q(\chi) = \frac{\chi^4}{(\chi^2 + \chi_a^2)^2} \quad (4)$$

This ratio can be expressed in terms of a single parameter, the screening angle χ_a (German "Abschirmwinkel"). This relation is valid for any appropriate potential, e.g., for the Thomas-Fermi potential,

$$\chi_a = \chi_0 \sqrt{1.13 + 3.76\alpha^2} \quad (5)$$

The ratio $q(\chi)$, rather than the cross section itself, will be used in the derivation of the statistical distribution of scattering angles.

Introducing θ for the total scattering angles after n collisions — $\theta_n = \theta_{n-1} + \chi$ — the probability for n -time scattering can be obtained by a recursion formula:

$$f_n(\theta) = \frac{1}{n} \iint_{\text{surface}} f_{n-1}(\theta - \chi) P(\chi) \frac{d\sigma}{2\pi} \chi, \quad (6)$$

where $P(\chi)$ is the probability that a particle suffers a single deflection into the surface element σ_χ . With

$$f(\theta) = \sum_{n=1}^{\infty} f_n(\theta), \quad (7)$$

Molière obtains an integral expression which he could expand in a power series. The result is [the probability that a particle is scattered into a spatial angular interval $(\theta, \theta + d\theta)$]:

$$f(\theta)\theta d\theta = \delta d\delta (f^0 + B^{-1}f^1 + B^{-2}f^2 + \dots) \quad (1)$$

where $\delta = \theta/(\chi_c \sqrt{B})$; $\chi_c^2 = 4\pi N t z Z(Z+1)e^4/(pv)^2$; t = length of trajectory of a particle; N = number of atoms per unit volume; and B = solutions of the transcendental equation $b = B - \log B$.

The f 's are functions of δ . Molière evaluated the first three terms — f^0 , f^1 , and f^2 — numerically. These functions are given in Fig. 1. Molière showed that, to an accuracy of 1%, the angular distribution is given by these three terms.

B. Theoretical Results of Scattering Probabilities

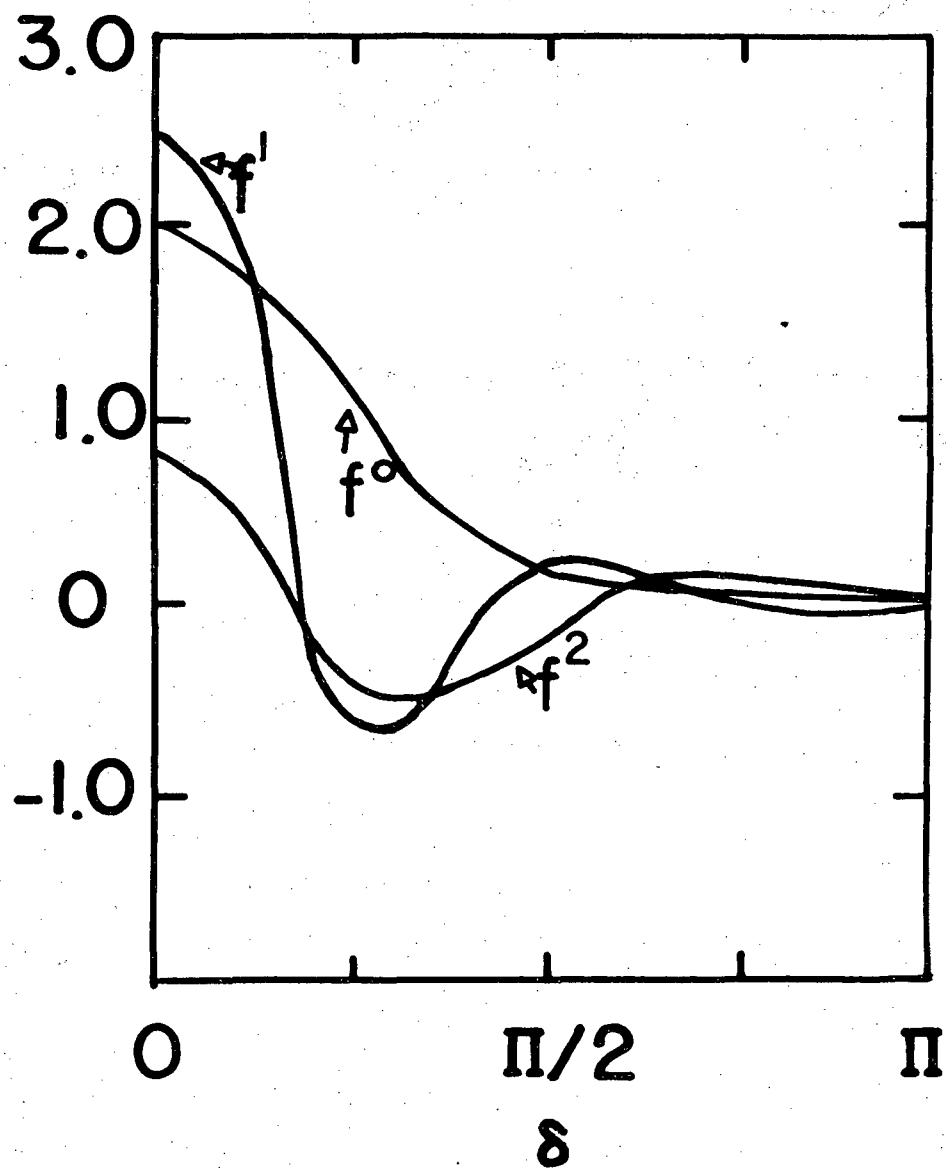
The scattering probabilities in the interval $(0, \pi)$ were calculated for Li, Be, Al, Fe, Cu, and Sn for 20 ranges of energy and angles increasing by 0.01 radians. The probability function $P(x) = f(\theta)\theta$ was normalized so that

$$\int_0^{\pi} P dP = 1 \quad . \quad (8)$$

The distributions for different energies and targets do not vary much except for small t . But here the theory is limited by B (see Molière). A typical distribution is given in Fig. 2 for $B = 10$.

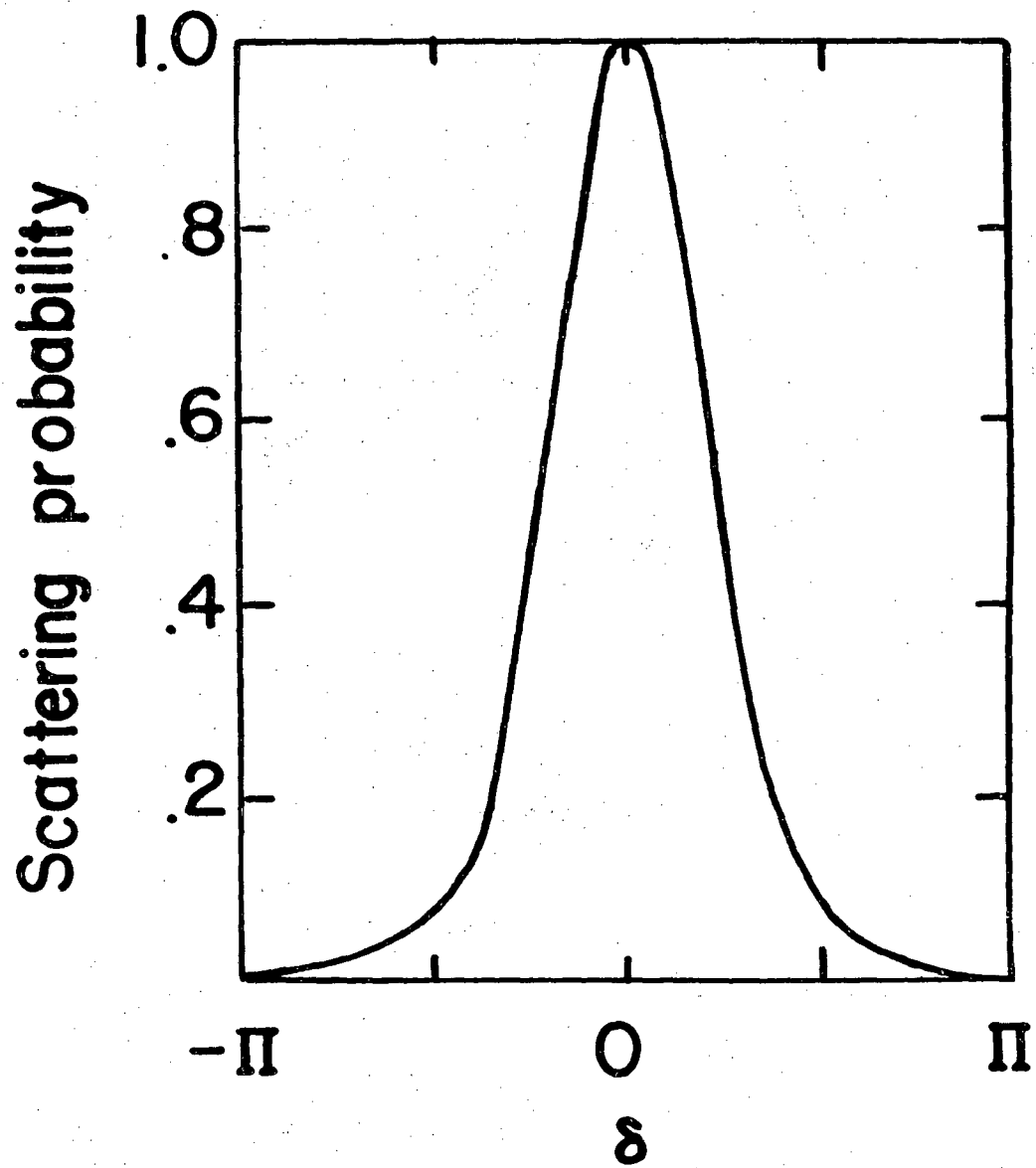
The main source for systematic errors is given by approximations in the numerical computations in the region of plural scattering. Plural scattering has not been considered sufficiently. No spin interactions have been considered, no surface properties of the targets have been taken into account.

The theory is valid for energies $E_{\text{kin}} > Z^{4/3} \cdot 10^2$ eV (see Molière), so likewise for energies from a few keV to a few MeV.



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Fig. 1. The functions f^0 , f^1 , and f^2 in Molière power expansion (Eq. 1).



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Fig. 2. A typical probability distribution $\int_{\delta}^{\delta+d\delta} f(\theta)\theta d\theta$.
This distribution was calculated for $B = 10$.

III. MONTE CARLO PROCEDURE

A. Fundamentals

As the parameters of the trajectory of a single electron or positron in an infinitely thick target cannot be treated analytically because of the complexity involved, the problem is simulated by a straightforward random-sampling Monte Carlo Method. A Monte Carlo program is basically a game whose rules are given by the scattering probabilities.

1. Angular Distribution

As an electron penetrates into a metal a few millimeters only, it is not important to know the exact exit position of the backscattered projectile, but rather its penetration depth and its directional cosine.

If $P(x)$ is the weight of a function $f(x)$ on an interval $(x, x+dx)$, then the integral

$$\int_x^{x+dx} P(x)f(x)dx = r \quad (9)$$

[where r is a random number equally distributed on the interval $(0,1)$] has to be solved numerically for the upper limit. $f(\theta, t)\theta d\theta$ is the probability that a particle is scattered into a spatial angular interval $(\theta, \theta + d\theta)$, where $f(\theta) = f(\theta, t, E)$ and $\theta = f(\delta)$. $f(\theta)$ is not known analytically but can be evaluated for special values of t , E , and δ . So the integrals are solved, iteratively and approximately, for 20 intervals of E and t and 314 intervals of δ .

As it is almost impossible to generate a random number r first and then find the upper limit of the integral because of time

consumption, the integrals are solved for 314 upper limits and normalized so that

$$\int_0^{\pi} P(x)f(x)dx = 1 \quad (10)$$

Then the random number is generated, and by table-search the closest integral value is associated with this random number and the corresponding upper limit taken. By this method the random numbers r which are equally distributed on $(0,1)$ are rearranged on $(0,1)$, corresponding to Molière's scattering probability. The azimuth

$$\phi = 2\pi r \quad (11)$$

does not need to be computed, but it is projected on the x-y plane; x is the coordinate perpendicular to the target surface. x and θ are the parameters which determine the penetration depth t and the direction of the moving particle.

2. Energy Loss

From the penetration depth t , the energy loss $-dE/dx$ and the remaining energy are calculated. The energy loss is found by adding the statistical variations of ionization energy loss shown by Nelms' table¹⁴ (including range straggling) to the radiation losses due to Bremsstrahlung. To save computer time these losses have been precalculated for twenty steps of energy.

3. Backscattering and Absorption

If the remaining energy becomes smaller than zero the run is stopped

and a new particle with the initial conditions is introduced (absorption). Penetration is not considered, as infinitely thick targets are assumed. If θ becomes greater than $\pi/2$ the particle moves backward toward the surface of the target. If then x becomes smaller than zero, both the integers associated with the directional cosine and the energy of the particle are kept in memory, and the corresponding matrix element in the energy-angle matrix [(20,17) matrix] is increased by one. Then a new electron or positron is introduced. After all particles have been calculated, the matrix is printed.

B. Execution of the Program

This program was performed on a CDC 6600 digital computer of the Lawrence Radiation Laboratory at the University of California at Berkeley. Each target required 4 to 5 minutes for 2000 particles; the program was 120 000 cores long.

Random numbers were generated by a subroutine RGEN(X), whose sequence was infinite compared to how often RGEN was called (~1 000 000 times per target). For different runs RGEN was called several times before the program started, to avoid always calling the same sequence.

The function RGEN(X) produces a random number from the set of numbers uniformly distributed on (0,1). RGEN(X) is called as a FORTRAN function where X is a dummy variable. This generator uses the power-residue method sequentially.¹⁹ Pseudo-random numbers can also be generated by a congruence method: $r_{n+1} = 23 \cdot r_n \pmod{T}$; $r_0 = 47594118$; $T = 10^8 + 1$ (example of a random number generator).

The main problem in developing this Monte Carlo program was the

time consumed by the computations. To compute every collision with all parameters would take hours even on the CDC 6600. Therefore it was necessary to compute many successive collisions at one time. The trajectories of the particles were broken into small segments containing a few hundred scattering atoms. The particles were treated as if they suffered just one collision in such a segment, losing the energy they would normally lose in all collisions in this segment. As mostly small-angle scattering occurs, the cumulative result should not depend too much on the lengths of the bins. The number of collisions in various bins was found to have little influence on the shapes of the energy spectra.

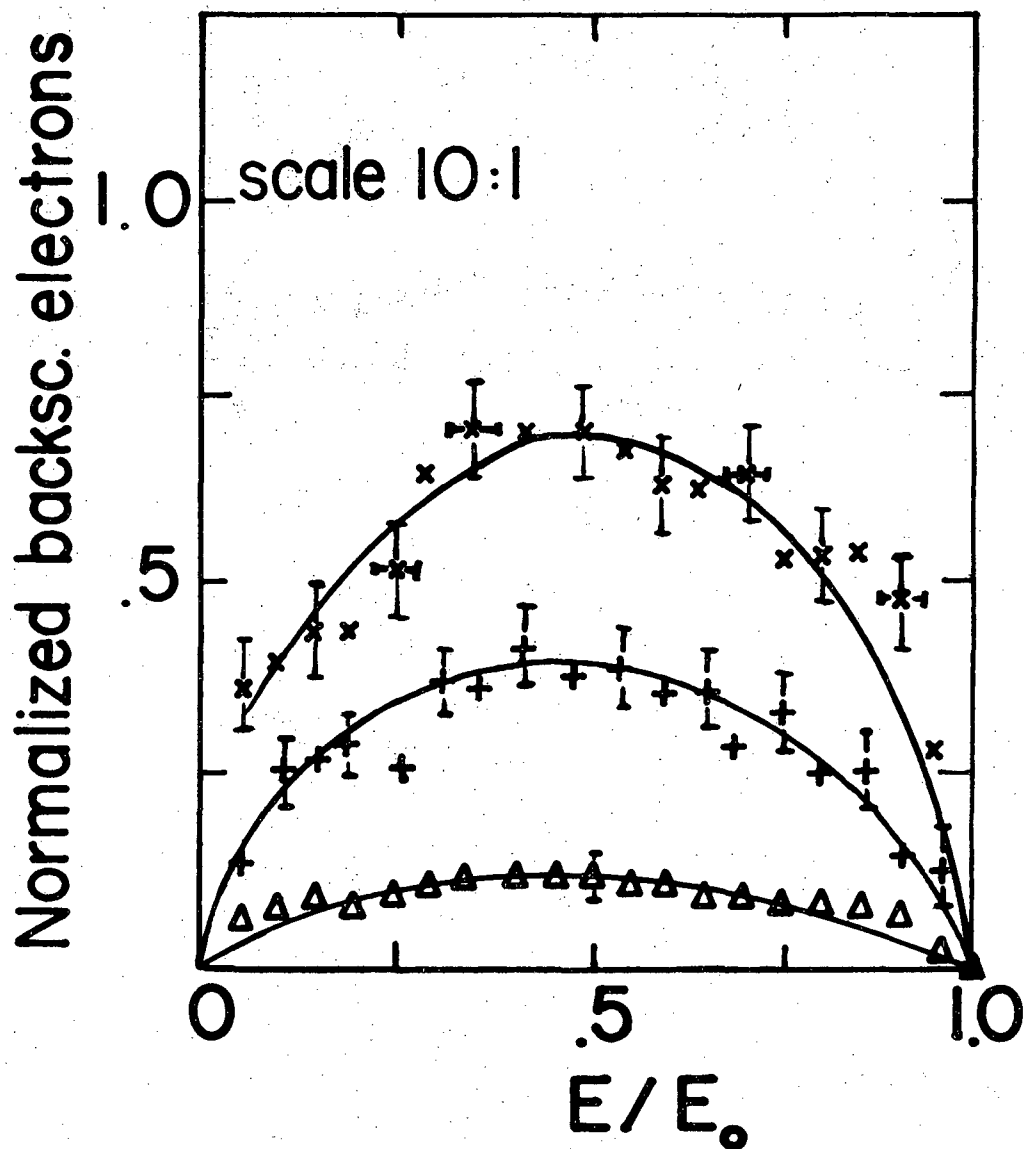
IV. RECORDING OF RESULTS

The program was run for both electrons and positrons on the following six different targets: lithium, at density $\rho = .53 \text{ g/cm}^3$; beryllium at 1.80; aluminum at 2.72; iron at 7.87; copper at 8.93; and tin at 7.30. Results are presented for the following initial energies: $E_{01} = 0.5 \text{ MeV}$ for Be and Sn; $E_{02} = 1.0 \text{ MeV}$ for Li, Be, Al, Fe, Cu, and Sn; and $E_{03} = 2.0 \text{ MeV}$ for Be and Sn.

The incoming beam was always assumed to be at normal incidence to the target (i.e., perpendicular), except in the last two figures (Fig. 11 and 12). For the energy spectra of light elements (lithium and beryllium) the program was run for 10 000 particles; for all other targets, 2000 were run.

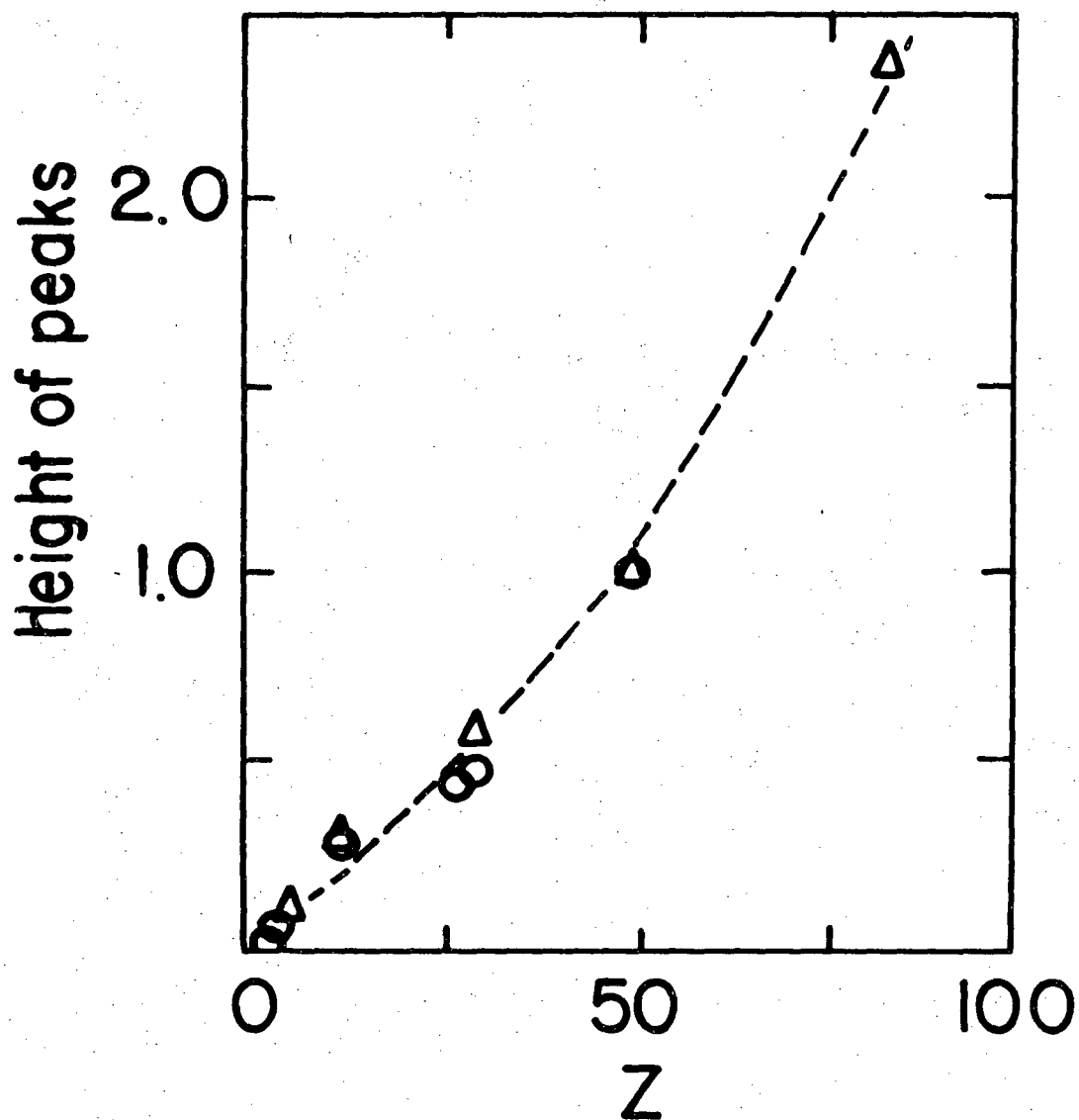
Figures 3, 5, and 7 show the energy spectra for electrons; Figures 4, 6, and 8 those for positrons; Figure 9 shows the Z-dependence of the backscattering coefficients for electrons; Figure 10 shows the angular distribution of the backscattered electrons for all targets (perpendicular beam).

Figure 11 shows the energy spectra of copper for initial energy $E_{04} = 1.75 \text{ MeV}$ for electrons and angles $\beta = 30^\circ$, 60° , and 90° between incoming beam and target surface (see Fig. 11 for β). Figure 12 shows the change of the angular distribution for Cu and angles $\beta = 30^\circ$, 60° , and 90° for the same energy (1.75 MeV). All energy spectra are normalized with respect to the height of the 1.0 MeV electron peak of the tin spectrum in Fig. 7.



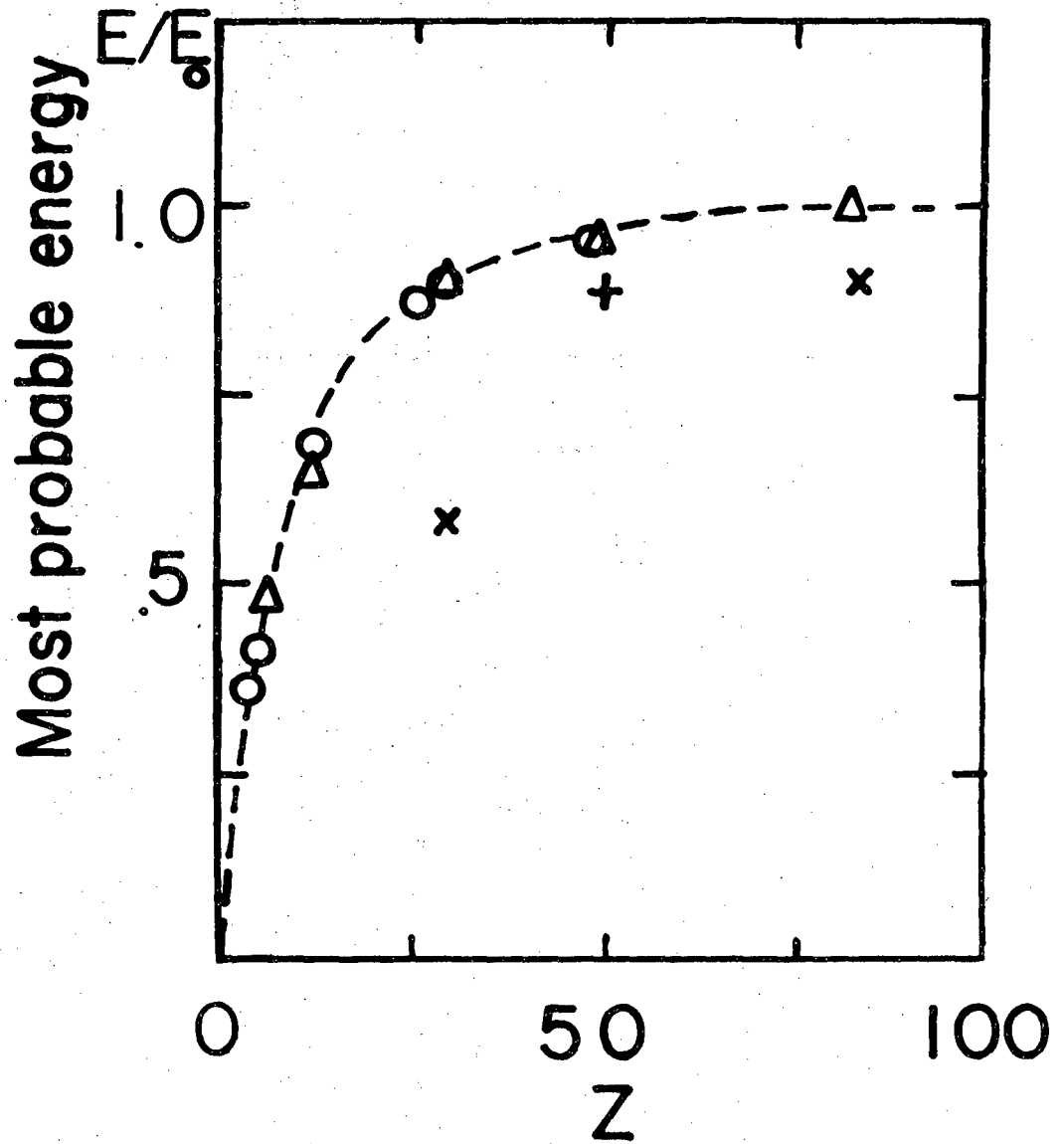
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Fig. 3. Energy spectra of beryllium for initial energies 0.5 MeV, 1.0 MeV, and 2.0 MeV for electrons; normalization corresponding to the height of the tin peak (=1). x = 0.5 MeV; + = 1.0 MeV; Δ = 2.0 MeV.



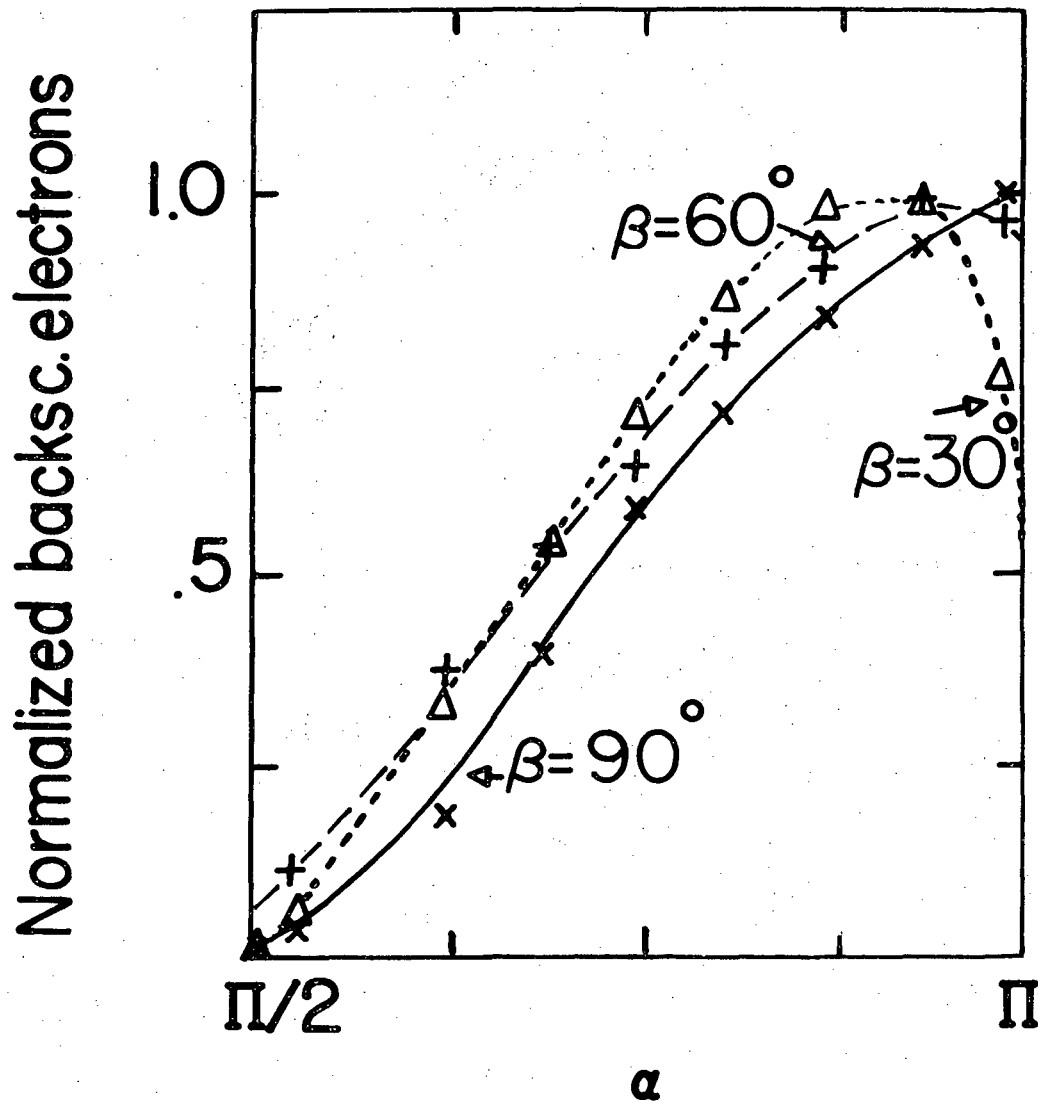
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Fig. 4. Energy spectra of beryllium for initial energies 0.5 MeV, 1.0 MeV, and 2.0 MeV for positrons.
x = 0.5 MeV; + = 1.0 MeV; Δ = 2.0 MeV.



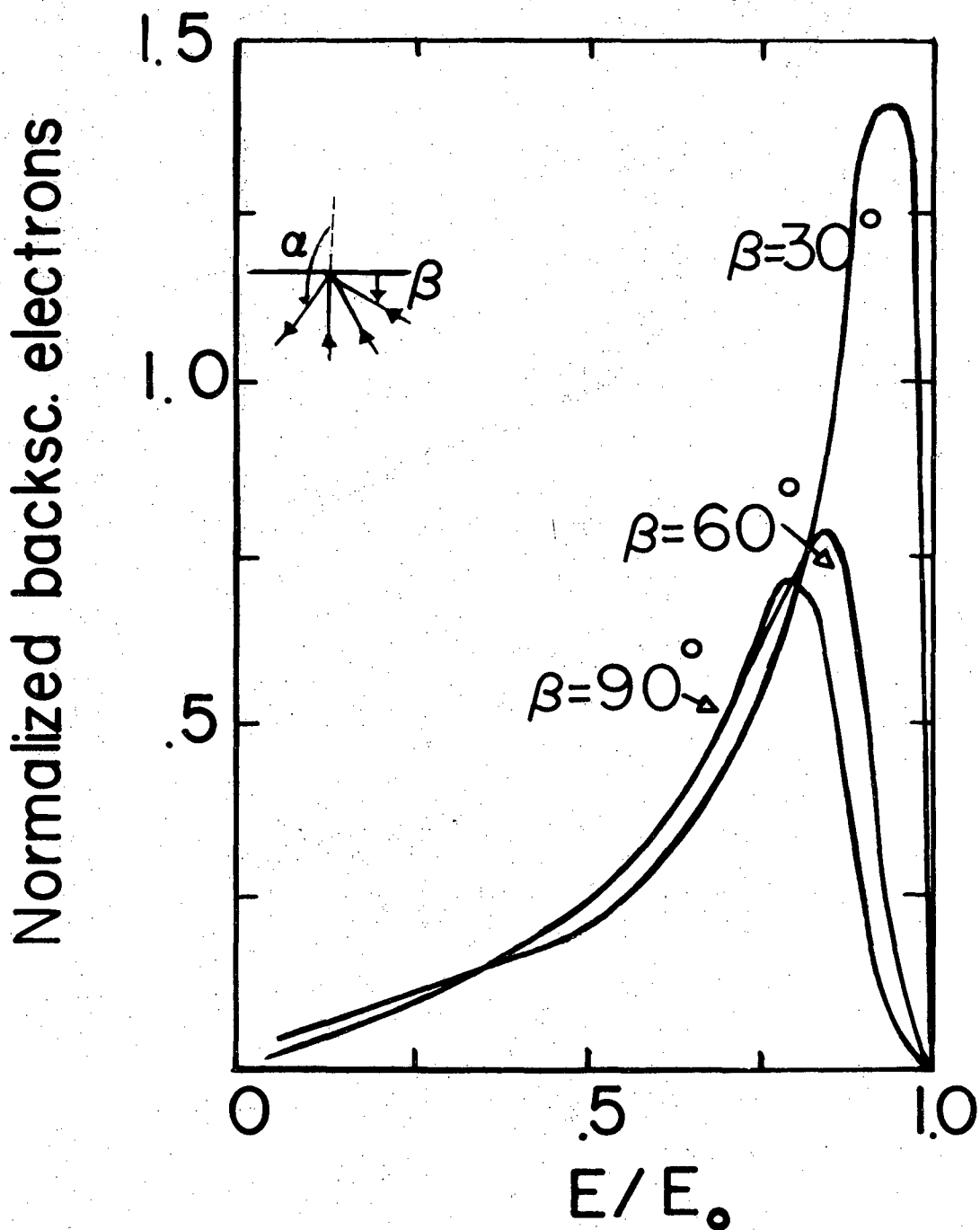
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Fig. 5. Energy spectra of tin for initial energies 0.5 MeV, 1.0 MeV, and 2.0 MeV, x = 0.5 MeV; + = 1.0 MeV; Δ = 2.0 MeV; — = Monte Carlo for 0.5 MeV and 1.0 MeV; · · · = Monte Carlo for 2.0 MeV; --- = Bothe⁷ for 0.68 MeV.



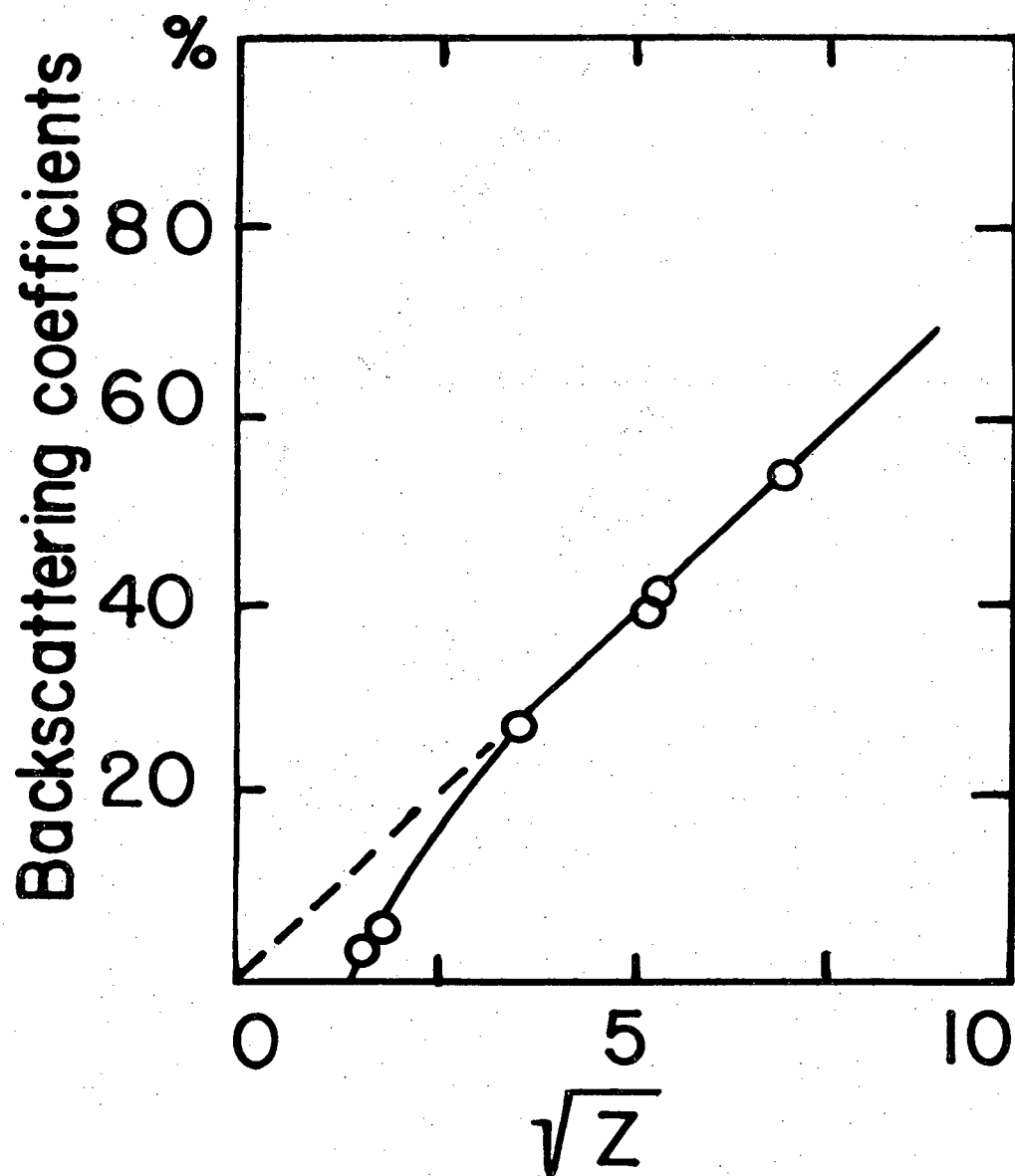
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Fig. 6. Energy spectra of tin for initial energies 0.5 MeV, 1.0 MeV, and 2.0 MeV for positrons.
 x = 0.5 MeV; $+$ = 1.0 MeV; Δ = 2.0 MeV;
 — = Monte Carlo for 0.5 MeV and 1.0 MeV;
 . . . = Monte Carlo for 2.0 MeV.



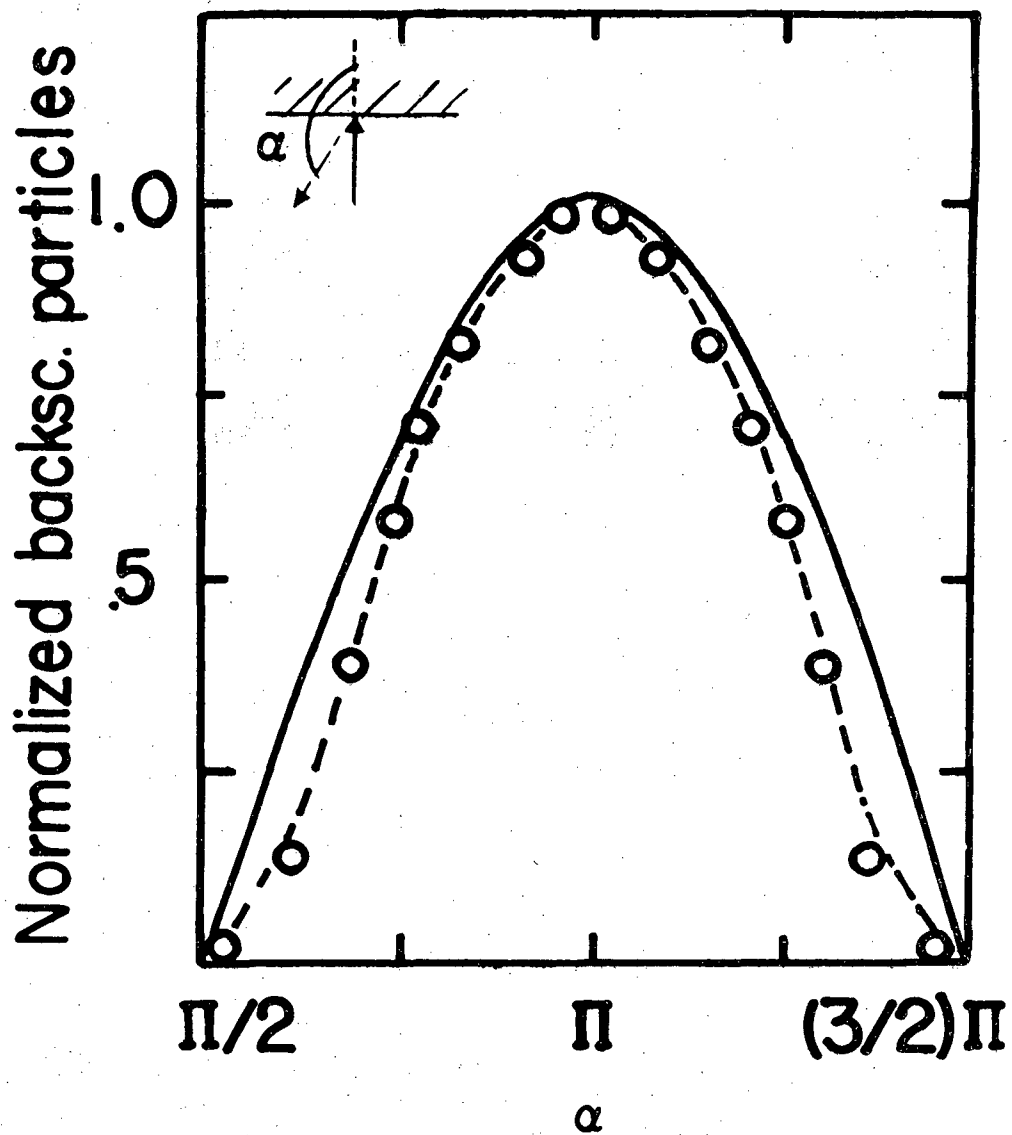
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Fig. 7. Energy spectra of 1.0 MeV electrons for lithium, beryllium, aluminum, iron, copper, and tin. All normalizations in Figs. 3-8 correspond to this tin peak, which has been set equal to 1.



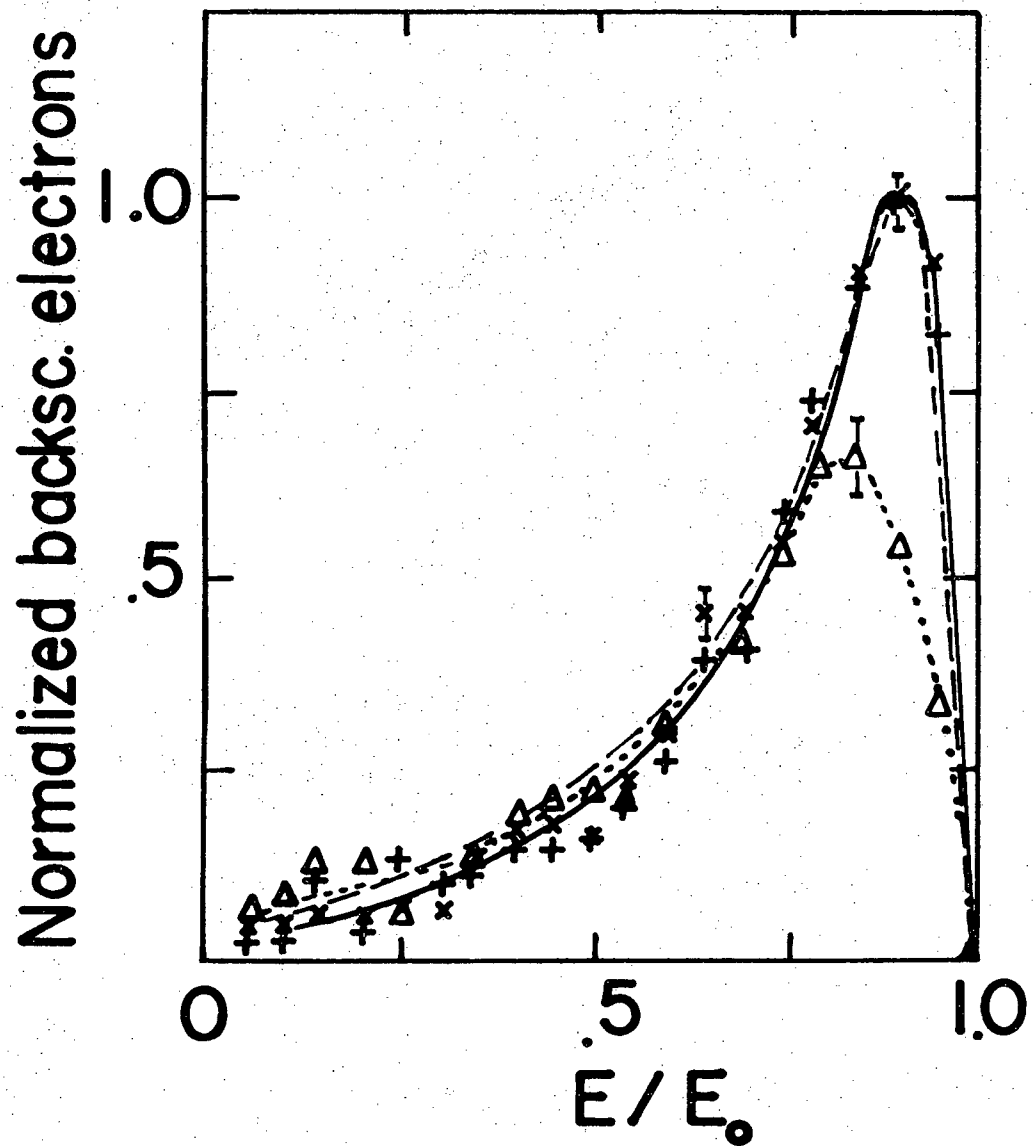
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Fig. 8. Energy spectra of 1.0 MeV positrons for lithium, beryllium, aluminum, iron, copper, and tin.



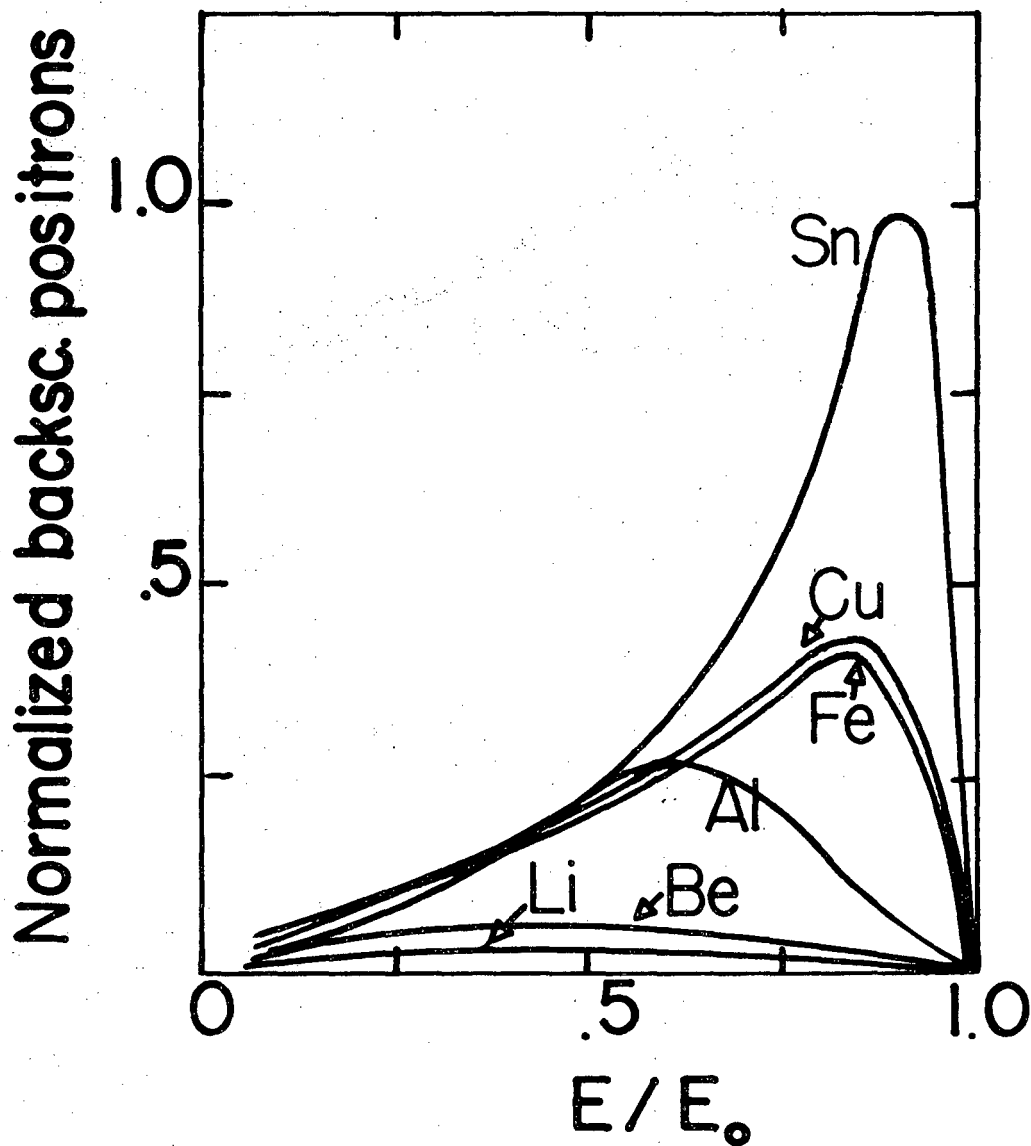
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Fig. 9. Z-dependence of the backscattering coefficients of electrons at 1.0 MeV.



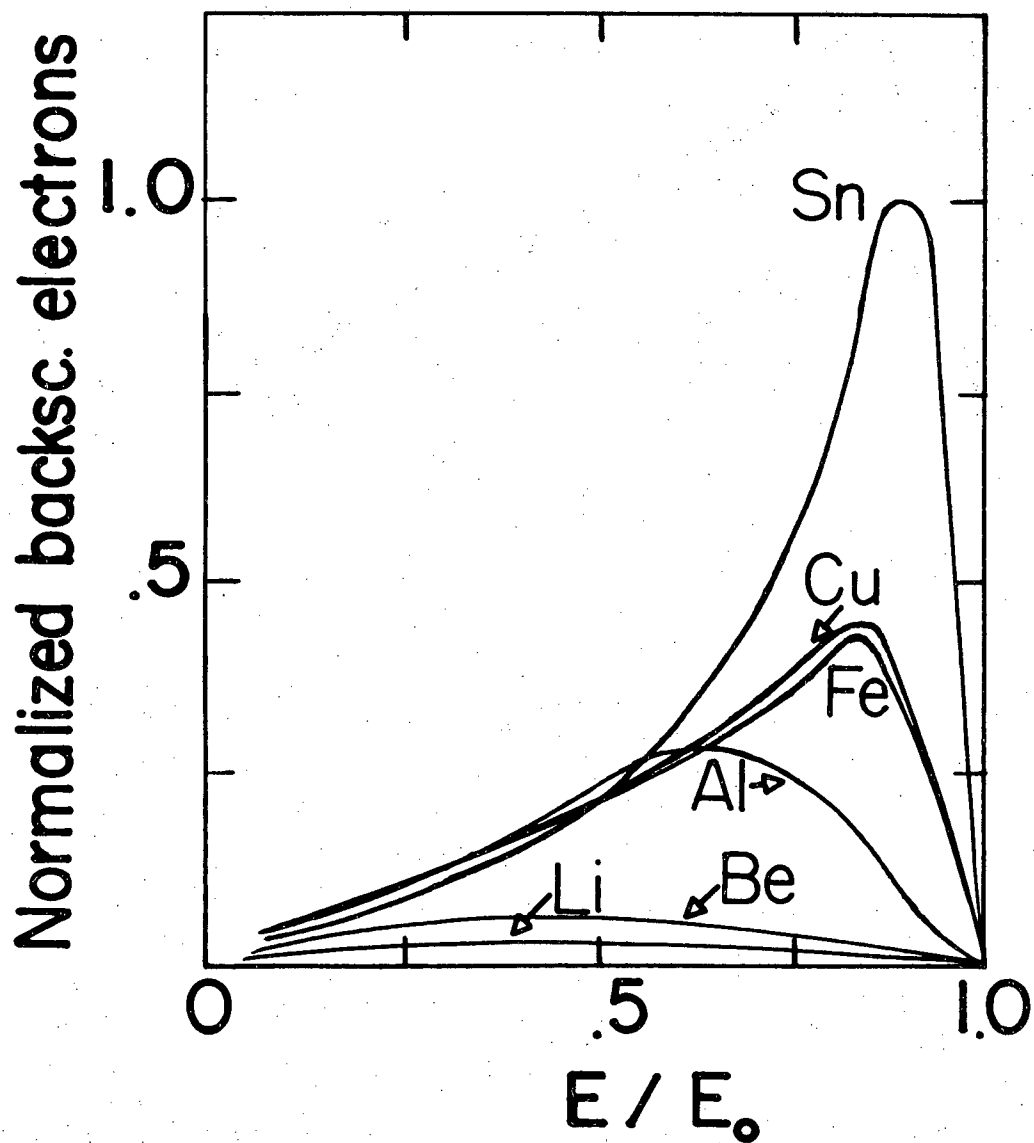
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Fig. 10. Angular distribution of backscattered electrons and positrons for all calculated values (beam perpendicular to target). O = Monte Carlo; — = $\cos(\pi-\alpha)$; --- = $\cos(\pi-\alpha)e^{-(\pi-\alpha)^2/2}$.



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Fig. 11. Energy spectra of copper for electrons with initial energy $E_0 = 1.75$ MeV; initial beam at $\beta = 30^\circ$, 60° , and 90° to the target surface.



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Fig. 12. Angular distributions of backscattered electrons with initial energy $E_0 = 1.75$ MeV. The initial beam was at angles $\beta = 30^\circ$, 60° , and 90° to the target surface (Cu).

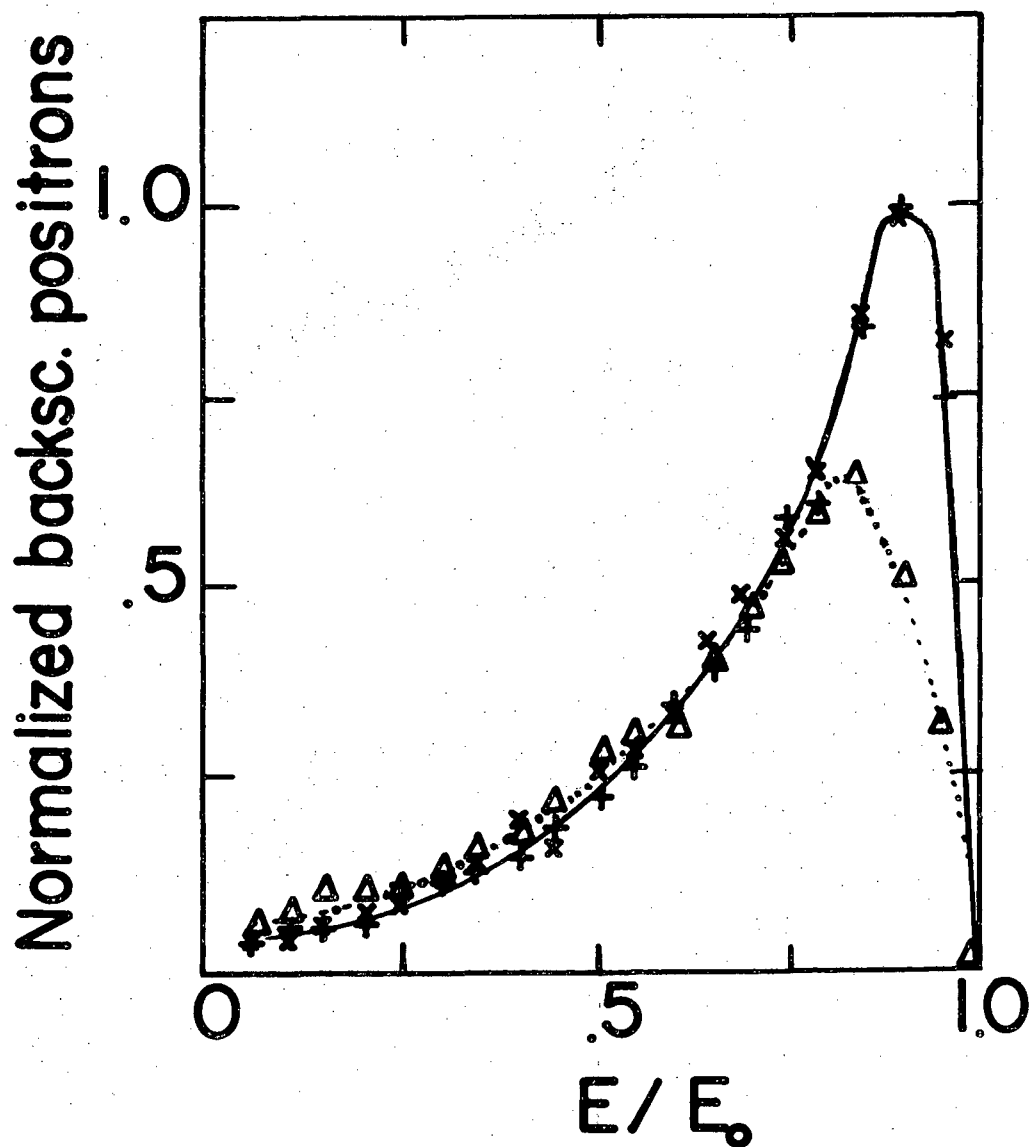
V. ANALYSIS OF MONTE CARLO RESULTS

A. Shapes of the Energy Spectra

The energy spectra are asymmetric with one most probable energy (peak). This maximum shifts toward higher energies with increasing Z . This means that in heavier elements the projectiles are reflected within layers close to the surface, whereas for lighter elements they penetrate deeper into the targets. The peaks for positrons are in general slightly lower in energy than those for electrons, due to the lower backscattering coefficients. Figure 13 shows the most probable energy E/E_0 (peak) as a function of Z . For small Z , E/E_0 increases sharply from 0 to 0.9 at $Z = 30$, and behaves asymptotically to $E/E_0 = 1$ for high Z .

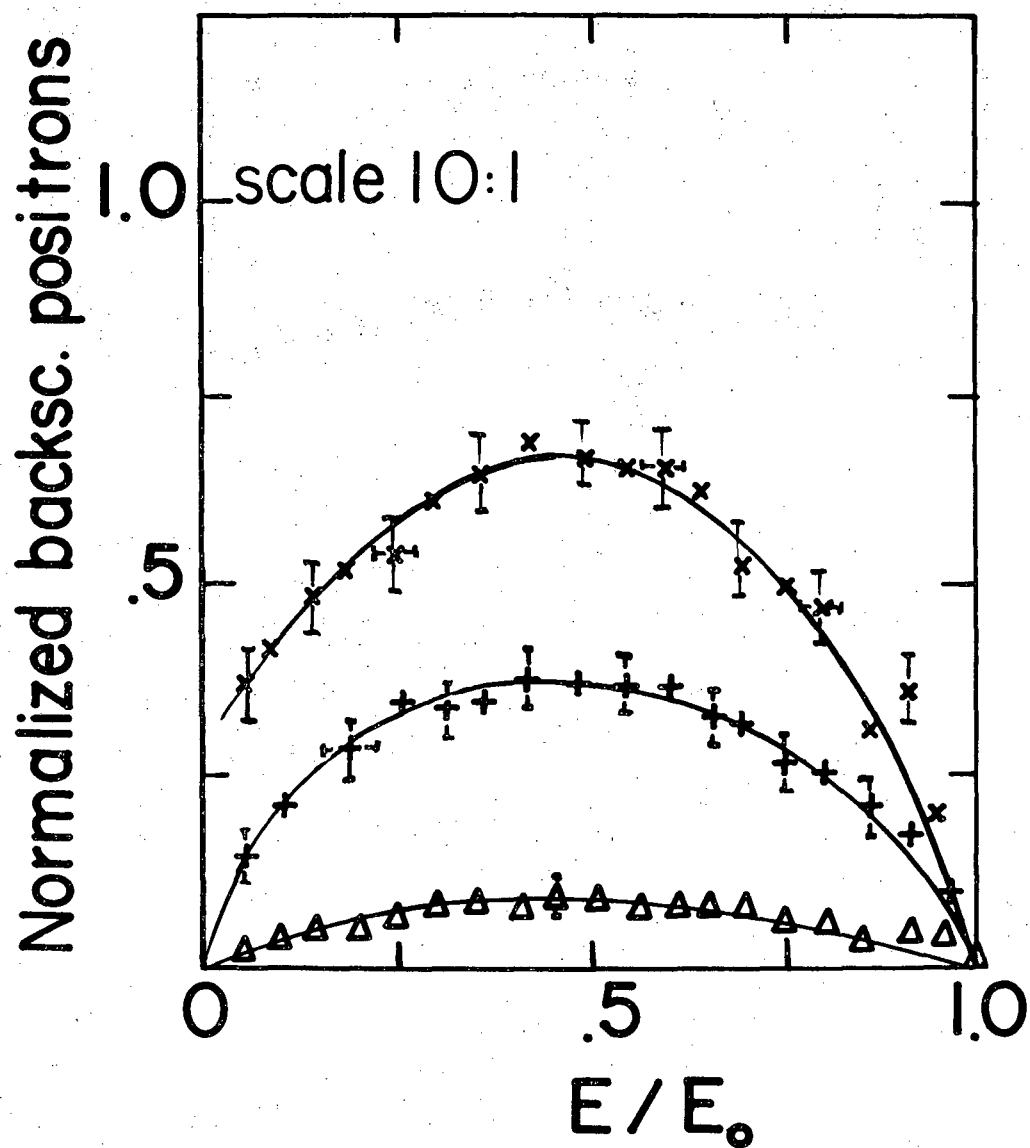
With increasing Z the heights of the maxima increase, corresponding to the increased backscattering coefficients. The peaks themselves become narrower for higher Z , where most backscattered particles have energies close to the initial energy. For high Z , fewer particles are backscattered with low energies than for targets of low Z . The heights of the peaks as a function of Z are shown in Fig. 14.

Previously it has been reported that the backscattering coefficients and the shapes of the energy spectra change only slightly with the initial energy, if $E_0 \approx 1.0$ MeV. The spectra have been computed for two other energies: $E_0 = 0.5$ MeV for beryllium, and $E_0 = 2.0$ MeV for tin. Within the accuracy of these results no difference can be noticed in the shapes of the energy spectra at 0.5 MeV and 1.0 MeV. From 1.0 MeV to 2.0 MeV the backscattering coefficient decreases, as does the most



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Fig. 13. Most probable energy E/E_0 of electrons and positrons as a function of Z . O = Monte Carlo at $E_0 = 1.0$ MeV; Δ = Bothe at $E_0 = 0.68$ MeV;⁷ x = Frank at 1.75 MeV;⁸ + = Monte Carlo at 2.0 MeV.



XBL 7010-6846

Fig. 14. Height of peaks as a function of Z of 1.0 MeV electrons and positrons. $\circ$ = Monte Carlo; Δ = Bothe.

probable energy and the height of the peak. This is in good agreement with the results of H. Frank.⁸

B. Angular Distribution

In all cases the angular distribution of the backscattered particles comes close to a cosine distribution with maximum at $\alpha = 180^\circ$ for initial beams perpendicular to the target. For small angles, however, the deviation from the cosine function cannot be neglected. One can see that a cosine and a Gaussian fit all calculated values. The function

$$N = N_0 \cos x \cdot e^{-x^2/2}, \quad (12)$$

where $x = \pi - \alpha$, fits all values exactly. In the program, of course, it was already assumed that the distribution is the same for all azimuths ϕ and symmetric around $\alpha = 180^\circ$ (see Fig. 10).

C. Backscattering Coefficients

The backscattering coefficient is defined as

$$p = \frac{1}{N_0} \int_{\pi/2}^{3\pi/2} N(\alpha) d\alpha = \frac{1}{N_0} \int_0^{E_0} N(E) dE, \quad (13)$$

where N_0 = number of initial particles, $N(\alpha)$ and $N(E)$ = number of backscattered particles at angles α or energy E .

It has already been observed by A. Bisi and L. Braicovich²⁰ that p is a function of Z . In all papers $p^+ < p^-$, and $p^-/p^+ \approx 1$ for low Z and ≈ 1.3 for high Z . The backscattering coefficients of this work fit the lines

$$p^- = (0.076 \pm 0.002) \sqrt{Z} \quad \text{for } Z > 13$$

and

$$p^+ = (0.072 \pm 0.002) \sqrt{Z} \quad \text{for } Z > 13.$$

(14)

These functions extrapolated to $Z = 0$ give $p = 0$; but for small Z , p decreases faster, so that $p = 0$ for $Z \approx 3$. This was also calculated by A. Cambieri and R. Pappalardo.²¹ Their data, extrapolated, would give $p = 0$ for $Z = 4$.

D. Geometry-Dependence of Spectra

The energy spectra change strongly with the angle between initial particles and target surface. If β is increased from 0 to $\pi/2$ the most probable energy shifts from $E/E_0 = 1$ to $E/E_0 = 0.8$ in the case of copper and 1.7 MeV electrons (see Fig. 11). The backscattering coefficients decrease from $p^- = 0.64$ at $\beta = 30^\circ$ to $p^- = 0.52$ at $\beta = 60^\circ$ and $p^- = 0.39$ at $\beta = 90^\circ$ (for Cu and $E_0 = 1.75$ MeV). The angular distribution of the backscattered electrons has a peak at $\alpha = 166^\circ$ for $\beta = 30^\circ$, at $\alpha = 172^\circ$ for $\beta = 60^\circ$, and at $\alpha = 180^\circ$ for $\beta = 90^\circ$ (see Fig. 12).

E. Comparison with Experimental Data

Experimental data on the energy spectra of suitable monoenergetic electrons are found for beta decays in Bothe⁷ for C, Al, Cu, Sn, and Pb, and for electron accelerators in Frank⁸ for Al, Cu, and Pb. As Bothe's data of the Sn peak have been calculated by a resolution function,²² the given error further increases when normalizing Bothe's graphs and those of this work by taking the heights of the peaks of tin to be equal. But by doing so, the shapes of Bothe's spectra agree well with these Monte Carlo results. His spectrum of tin has been plotted in Fig. 5

(0.68 MeV electrons). Frank measured energy spectra $dN/dE|_{\alpha}$ in angular intervals at $\alpha = 120^\circ$, 150° , and 180° for 1.75 MeV electrons. As the total number of electrons computed in this program was not very high (e.g., 2000 for Cu), the number of electrons backscattered into a spatial interval $\Delta\alpha = 10^\circ$ with energy $\Delta E = E_0/20$ was usually between 10 and 20. To compare the results of this Monte Carlo work with Frank's data, a larger sample of electrons must be computed. However, although these "differential" energy spectra do not agree with Frank's data in details, they agree in all trends.

The backscattering coefficients agree with Bothe's⁷ and Bisi's and Braicovich's²⁰ results for electrons. The coefficients for positrons seem to be too high by a factor of 1.23. The coefficients for electrons are given in Table I below:

Table I. Backscattering coefficients of electrons.

Z	Bothe	Bisi- Braicovich	Monte Carlo		
	$E_0 = 680 \text{ keV}$	$< 540 \text{ keV}$	0.5 MeV	1.0 MeV (± 0.02)	2.0 MeV
3	.03			.02	
4	.06		.09	.05	.02
13	.25 .30 ^a	.25		.27	
26	.405	.39		.38	
29	.43	.41		.42	
50	.58	.57	.56	.53	.46
82	.68	.71		.69 ^b	

^a A. F. Kovarik, Phil. Mag. 20, 849 (1910).

^b Extrapolated from Figure 9.

F. Statistical Spread of Computed Values

Basically this program is a simulation of a beta decay, so that for the statistical spread of the computed values the formula

$$\delta N = \sqrt{\frac{T}{N}} \quad (15)$$

can be used as the percentage error.

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APPENDIX A

List of Symbols

χ	scattering angle of a single collision
θ	total scattering angle
δ	total scattering angle $\sim \theta$
δ_χ	surface element
E_0	initial energy of particles before scattering
E	energy of backscattered particles
$k(r)$	wave number
a	Thomas-Fermi atom radius
α	screening parameter
$\delta(\chi)$	differential cross section
$\delta_{\text{Ruth}}(\chi)$	Rutherford cross section
$\delta_{\text{act}}(\chi)$	actual cross section calculated by WKB approximation
$q(\chi)$	ratio of actual cross section to Rutherford cross section
$P(\chi)$	probability function
$f(\theta)$	total scattering probability function
t	length of trajectory of a particle in a layer; penetration depth
N	number of atoms in target per unit volume
$N(\)$	number of backscattered particles (function of α or E)
N_0	number of initial particles
δN	percentage error of N
Z	atomic number
z	multiple of electron charge
r	random number uniformly distributed on (0,1)

$P(x)$	weight function
p^+	backscattering coefficient for positrons
p^-	backscattering coefficient for electrons
$RGEN(X)$	random number generator
X	dummy variable
ρ	density
x	coordinate perpendicular to target surface
α	angle between backscattered particle and x-axis
β	angle between incoming particle and surface (y-axis)
ϕ	azimuth
ψ	wave function

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