

UC Berkeley

Fisher Center Working Papers

Title

Rents, Prices and Expectations in the Land Market

Permalink

<https://escholarship.org/uc/item/703012gg>

Author

Markandya, Anil

Publication Date

1982-06-01

Peer reviewed



Institute of
Business and
Economic Research

University of
California,
Berkeley

**CENTER FOR REAL ESTATE
AND URBAN ECONOMICS
WORKING PAPER SERIES**

WORKING PAPER 82-51

RENTS, PRICES AND EXPECTATIONS
IN THE LAND MARKET

BY

ANIL MARKANDYA

These papers are preliminary
in nature. their purpose is
to stimulate discussion and
comment. Therefore, they
are not to be cited or quoted
in any publication without
the express permission of
the author.

GRADUATE SCHOOL OF BUSINESS ADMINISTRATION

**CENTER FOR REAL ESTATE AND URBAN ECONOMICS
UNIVERSITY OF CALIFORNIA, BERKELEY**

The Center was established in 1950 to examine in depth a series of major changes and issues involving urban land and real estate markets. The Center is supported by both private contributions from industry sources and by appropriations allocated from the Real Estate Education and Research Fund of the State of California.

INSTITUTE OF BUSINESS AND ECONOMIC RESEARCH

J. W. Garbarino, Director

The Institute of Business and Economic Research is a department of the University of California with offices on the Berkeley campus. It exists for the purpose of stimulating and facilitating research into problems of economics and of business with emphasis on problems of particular importance to California and the Pacific Coast, but not to the exclusion of problems of wider import.

RENTS, PRICES AND EXPECTATIONS

IN THE LAND MARKET

by

Anil Markandya

Working Paper 82-51

June, 1982

This research was done while I was a Visiting Professor at the Center for Real Estate and Urban Economics, University of California, Berkeley. I would like to thank the Center for financial support and participants at the Urban Economics Seminar at Berkeley for helpful comments.

ABSTRACT

RENTS, PRICES AND EXPECTATIONS

IN THE LAND MARKET

by

Anil Markandya

This paper investigates the relationship between the ratio of rents to capital values and expectations of future real interest rate movements and future rates of inflation. It is shown that, in general, the ratio is affected mainly by real interest rate expectations and future real price increases. Using a simple model to generate such expectations, we obtain a relationship between the rent/capital value ratio and parameters that reflect long term expectations and the rate of convergence to such expectations.

Finally, the model is used to examine the movement in land prices and rents in England over the period 1960-1980. It is found that, in general, expectations of real interest rates have been too optimistic and those of real prices too pessimistic in that market.



RENTS, PRICES AND EXPECTATIONS

IN THE LAND MARKET

by

Anil Markandya

1. Introduction

It is widely understood that, just as variations in price earnings ratios are indicative of changing expectations about future earnings in the stock market, so variations in the ratio of rents to land prices contain some information regarding future rents and capital values. During periods when expectations about future inflation rates can reasonably be held to be high, for example, we generally observe rents becoming a lower proportion of the capital value of land. Conversely, when inflationary expectations fall, the ratio appears to rise. However, precisely how expectations are incorporated in this ratio is something which does not appear to have been worked out. If one could build a valid model that identified a relationship between observable variables such as the ratio of rents to capital values and unobservables, such as the expected profile of future rents and interest rates, then it would clearly be of great value. It would help us to see when and how expectations in these variables are systematically biased with respect to the true outcomes and it would help us to predict the movement of prices in this market. The benefits of such an exercise could be a more

efficient functioning of the land market as well as a greater awareness among government agencies of the consequences of their policy actions on the price of land.

This paper is an initial attempt at the construction of such a model. In doing so it exploits the fact that, in the United Kingdom at least, investment in agricultural land is viewed as a long term commitment. Of the stock of such land which is rented out on a commercial basis, figures indicate that over the last decade, around 1 percent appeared on the market in any one year, with the actual amount varying between 0.6 and 1.4 percent. This compares with 11 percent of the housing stock in Great Britain and 60 percent of the value of listed securities on the London Stock Exchange¹ that are traded on average every year. Although this is not conclusive proof that very little trading for quick short term capital gains takes place in this market, it does support such a view and other sources of evidence corroborate this fact.

The implication of a long term view taken by investors in tenanted farmland is that one can concentrate on expectations of future real rents and future real interest rates, and ignore expectations about movements in the capital value of land in the short run. This is, of course, in stark contrast to any modelling of expectations in the stock market, where short term price movements are of great importance. In section two of this paper, we examine how the profile of expected rents and interest rates affects the current

ratio of rent to capital value. In this section it is argued that the important expectational elements in these variables can be captured by concentrating on the expected path of future real interest rates and the expected (constant) difference between the rate of growth of land rents and the price level. This essentially allows us to remove price expectations as a direct variable although, of course, such expectations do influence capital values through their impact on the anticipated path of real interest rates. In section three a simple mathematical model is introduced which imposes the requirement that the rate of change of real interest rates be expected to satisfy a first order linear differential equation. Such an equation can be thought of as the reduced form of some macroeconomic model and, hence, conceivably incorporating rational expectations,² but no attempt is made to establish such a link. The appeal of the model in this context is rather that it allows us to introduce expectations about the first derivative of the real interest rate and hence about the second derivative of the rate of inflation in a convenient and interesting form. Moreover, by imposing the requirement that this equation satisfy the data on the real interest rate at the time that the expectations were being formed, we obtain a simple equation that relates, under a reasonable approximation, the ratio of rents to capital values, to the current rate of change of the real interest rate and to parameters

of the differential equation that have a clear expectational interpretation. A second order differential equation which would incorporate expectations on the third derivative of the rate of inflation can also be analyzed but does not yield a final form that is so analytically tractable or empirically useful. In section four we consider some of the data on land rents and capital values for agricultural tenanted farmland in England over the period 1969-1979. By applying the model developed in the previous section, we can relate the rent/capital price ratio to two variables: the long term (stable) expected real interest rate (a_t^0) and the expected rate of change of the real rent level over time (α_t^0). By looking at the stability conditions of the model, we can place bounds on α_t^0 . These help us to identify when the expectations on variables were particularly unreasonable. In section five we attempt to fit an econometric equation to explain a_t^0 and α_t^0 . Our analysis of the previous sections, as well as other theoretical considerations, suggest that we consider the past pattern of nominal interest rates, the past pattern of growth in rents and consumer prices, and the level of activity in the market as possible explanatory variables. Although the resulting model is not of much use as a predictive equation, the determinants of the expectational variables do yield some insight into the process by which these expectations are formed. Section six concludes the paper and indicates where the major difficulties lie.

2. Modelling expectations in the Land Market

In a free market for used land the price at which transactions take place is so determined that the marginal buyer and seller equate that price to the discounted present value of future rents. If the frequency of rent change and compounding is sufficiently great, then we may represent the discounted present value using a continuous time model as follows. Let³

r_0 = rent level at time 0

$\lambda_{t'}$ = expected rate of growth of prices at time t'

$\alpha_{t'}$ = expected real rate of growth of rents at time t'
(i.e. actual rate of growth of rents less $\lambda_{t'}$)

$\sigma_{t'}$ = expected real interest rate at time t'

P_0 = capital value at time 0

Then, assuming no depreciation,

$$P_0 = r_0 \int_0^{\infty} e^{-\int_0^{t'} (1 + \lambda_{t'} + \alpha_{t'}) dt'} e^{-\int_0^{t'} (\sigma_{t'} + 1 + \lambda_{t'}) dt'} dt \quad (1)$$

$$= r_0 \int_0^{\infty} e^{-\int_0^{t'} \alpha_{t'} dt'} e^{-\int_0^{t'} \sigma_{t'} dt'} dt \quad (2)$$

Equation (2) contains only the real rates of growth of rents and the real interest rates of the future. It is easy to see that the following is a special case of the above. Let

$\alpha_{t'} = \alpha^0$ all t'

$\sigma_{t'} = \sigma^0$ i.e. constant real changes in the rent level and the real interest rate

Then,

$$\frac{r_0}{P_0} = \sigma^0 - \alpha^0 \quad (3)$$

and the rent/capital value ratio is equal to the real interest rate less the rate of growth of the real rent level. If the latter is zero, then the rent/capital value ratio expresses exactly the currently expected long term real rate of interest. When σ_t , is not expected to be constant, however, the relationship between its future profile and r_o/P_o is more complicated. This is explored further in the next section. At this stage it is important to note that inflationary expectations (i.e. those on λ_t) do not influence the ratio of rents to capital values in a situation where continuous discounting is applicable. Where discounting and rent increases take place at discrete intervals, with the interval for the rent increases being longer than that for the discounting of rents, then the expected rate of inflation will influence the ratio of r_o/P_o . In the agricultural land market, for example, rents increase on average about once every three years. If we take this as the actual interval, and assume that rents are just expected to keep pace with a constant expected rate of inflation, then we have the following expression:

$$\frac{r_o}{P_o} = \frac{1 - (1 + \lambda^o)^3 x^{3n}}{1 - x^{3n}} \times x(\sigma^o + \lambda^o) \quad (4)$$

$$x \equiv \frac{n}{n + \sigma^o + \lambda^o} \quad (5)$$

n is the frequency of discounting within a year, and σ^o and λ^o the constant expected real rates of interest and

inflation respectively. To illustrate how λ^0 affects r_0/P_0 consider the case where the real rate of interest is expected to be 2 percent (i.e. $\lambda^0 = 0.02$). From equation (3) we know that with continuous discount $r_0/P_0 = 0.02$. From equation (5) it can be seen that, discounting monthly ($n = 12$) gives $r_0/P_0 = 0.0225$ when $\lambda^0 = 0.05$ and $r_0/P_0 = 0.0275$ when $\lambda^0 = 0.10$. The effect of λ^0 on r_0/P_0 increases as λ^0 increases and even with inflation rates at around 10 percent it is clearly quite substantial.

The effects of inflationary expectations are diminished, however, if the rate of rent increase adjusts to compensate for the discreteness of the increase in rents. Notably, if the rate of rent increase is faster than it would otherwise have been when rents are rising, then to correct for the length of interval in relating r_0/P_0 to σ^0 would be inappropriate. Although most economists would be inclined to believe that market forces will attempt to get around such an institutional constraint as triennial rent increases and that, at least in part, the influence of discreteness on r_0/P_0 will be eliminated, there appears to be no straightforward way of testing this assumption. The higher the inflation rate and the lower the interest rate, the greater the effect has to be on the rate of rent increase to compensate for discreteness. However, without a model to determine the equilibrium rate of growth of rents we cannot know what this difference is in each time period. In this

paper the assumption is made that rent increases adjust sufficiently for the continuous time model to be applicable to this market. As there is no strong reason for this not to happen and there are possible pressures for it to happen it seems reasonable to make this assumption.

Even when the rent increases do adjust for discreteness there does remain one issue that is worth noting. This is that now the observed rate of rent increase will depend upon the rate of inflation. Table I shows, for illustrative purposes, how much α_0 in equation (3) is raised above zero by the discreteness effect for different values of σ_0 and λ_0 . Although this does not affect the theoretical model as such, it does mean that any attempt to "explain" α_0 in terms of past rates of growth of real rents should take account of the fact that the observed values deviate from the theoretical concept because of the expected rate of growth of prices, and that we will need to take account of the latter if we are to obtain a satisfactory econometric equation for α_0 .

In summary, what this discussion suggests is that it is not an unreasonable procedure to use a continuous time model to analyze a land market where rent increases are discrete. However, in the statistical work on these expectational variables it will probably be necessary to introduce price expectations.

TABLE I

Amount by which the Real Rate of Growth of Rents is Raised
Above Zero due to the Discreteness of Rent Changes

λ_0 σ_0	0.05	0.10	0.15	0.20	0.25
0.01	0.002	0.006	0.013	0.022	0.034
0.04	0.004	0.010	0.019	0.031	0.044

3. An explicit expectations model of real interest rates and real rates of growth of rents.

In order to use equation (2) to construct a relationship between r_0/P_0 and the market expectations of $\{\sigma_t\}$ and $\{\alpha_t\}$, it is helpful if the time paths of the latter variables can be represented by some process which is a function of relatively few parameters. In this section we consider a model where expected values of σ_t , can be represented by the following equation:

$$e^{\sigma^0 t} = a^0 + b^0 e^{-\mu^0 t} \quad (6)$$

$$a^0 > \alpha^0 \quad \mu^0 > 0 \quad t \geq 0$$

$e^{\sigma^0 t}$ is the expected value of σ at time t , where the expectations are formed at time 0. The model is assumed to be correct at the time the expectations were formed. Hence,

$$e^{\sigma^0} = \sigma_0 = a^0 + b^0 \quad (7)$$

and

$$\frac{d(e^{\sigma^0 t})}{dt} = \frac{d\sigma_0}{dt} \equiv \sigma_0' = -\mu^0 b^0 \quad (8)$$

The expected rate of change of the real interest rate satisfies a first order linear differential equation. The long term real rate is expected to be a^0 , and μ^0 measures the rate of adjustment from the current real rate to this expected long term rate. As changes in the real rate involve changes in nominal rates and changes in the rate of price inflation,

such an equation is intended to capture market expectations about these rates of change.

As far as the expected rate of growth of real rents is concerned, this is assumed to be constant at any given point in time for the indefinite future, i.e. if $e^{\alpha^0 t}$ is the real rate of growth of rents at time t , expected at time 0 then, $e^{\alpha^0 t} = \alpha^0$, all t . Although the actual real rate of growth does vary quite a lot from year to year, such variations do not seem to be systematically related to expected macro-economic or market situations. In most cases, it seems difficult enough to form a view about the average expected real rate of growth, let alone its time variation, and so no attempt is made to model the latter.

Equation (2) may now be modified to become:

$$\frac{P}{r} = e^{\frac{b}{\mu}} \int_0^{\infty} e^{-(a - \alpha)t} e^{-\frac{b}{\mu} t} e^{-\mu t} dt \quad \mu > 0 \quad (9)$$

The subscript 0 is dropped in equations (9) and (10) for simplification.

Integrating (9) by parts, we obtain:⁴

$$\begin{aligned} \frac{P}{r} = & e^{\frac{b}{\mu}} \left\{ \frac{1}{(a - \alpha)} - \frac{b}{\mu} \left(\frac{1}{\mu + (a - \alpha)} \right) + \right. \\ & \left. \left(\frac{b}{\mu} \right)^2 \frac{1}{2!} \left(\frac{1}{2\mu + (a - \alpha)} \right) - \left(\frac{b}{\mu} \right)^3 \frac{1}{3!} \left(\frac{1}{3\mu + (a - \alpha)} \right) + \right. \\ & \left. \dots \dots \dots \right\} \quad a > \alpha \quad \mu > 0 \quad (10) \end{aligned}$$

From equation (10), we can see that if $b^0 = 0$, then we revert back to $P_0/r_0 = 1/(a^0 - \alpha^0)$, which is the reciprocal of equation (3), and what one would expect if real interest rates were constant. The same holds for $\mu^0 = 0^5$. Equations (7), (8), and (10) give three equations in the unknown parameters a^0 , b^0 , μ^0 , and α^0 . Substituting from equations (7) and (8) into (10) gives us an equation in a^0 and α^0 . In the next section we analyze this equation and observe that, for the particular application we have in mind, the second and subsequent terms in { } on the RHS of equation (10) are numerically insignificant. This permits us to work with a much more simplified form on the solution to equation (9). Although it is not possible to solve for a^0 and α^0 , we can, by imposing the stability conditions, observe the bounds on the values of α^0 that are required to satisfy those conditions. This is quite revealing about market expectations regarding real rent growth. Finally, as the solution to equation (9) is fairly simple, a nonlinear least squares estimation is attempted with a^0 and α^0 being replaced by distributed lags of past values of rates of inflation, rent growth and interest rates. The data used in this section is that of agricultural land rents and prices in England over the period 1960-1979.

4. An Application to the Tenanted Agricultural Land Market

Tenanted agricultural land in England and Wales accounts for around 42 percent of all agricultural land. The percentage has fallen slightly since 1960, when it was 51 percent and considerably since 1910, when it was 88 percent. As stated earlier, around 1 percent of the tenanted area is sold without vacant possession every year. Table II gives the rent and capital value per hectare from 1960-1979 along with the average annual yield on long term government securities and the average annual rate of inflation. For the purposes of equation (10) we need the rental income net of tax, as that determines the willingness to pay for land. Whether or not we should take the real interest rate net of tax is unclear. As there are a large number of ways of escaping or deferring capital taxation through the use of trust and tax shelters, it seems sensible to calculate the parameters both with and without a tax adjusted real interest rate. The tax adjusted rents and nominal interest rates are given in columns (6) and (8) respectively. Columns (7) and (9) give the real interest rates corresponding to columns (4) and (8) respectively.

The solution to equation (11) can be approached in two ways, depending on whether σ_0' is close to zero or not. If it is close to zero, then from equations (7) and (8) either μ^0 or b^0 (or both) are close to zero. By direct integration for the case when either of these parameters is actually zero, we know that the the solution is $1/(a^0 - \alpha^0)$. As there is sufficient continuity

TABLE II

Agricultural Rents, Capital Values and Real Interest Rates in England (1960-1980)

(1) Year	(2) P_t	(3) R_t	(4) r_t (x100)	(5) $\Delta \log(RPI)_t$ (x100)	(6) $r_{t(\text{tax})}$	(7) σ_t (x100)	(8) $r_{t(\text{tax})}$ (x100)	(9) $\alpha_{t(\text{tax})}$ (x100)	(10) σ'_t (x100)
1960	158		5.51	1.01		4.71	3.70	2.69	
1961	168	6.37	6.15	3.44	4.27	2.71	4.13	0.69	-2.00
1962	182	7.13	6.28	4.21	4.78	2.07	4.21	0.00	-0.69
1963	196	7.86	5.49	2.02	5.27	3.47	3.67	1.68	+1.68
1964	256	8.40	5.56	3.28	5.63	2.28	3.73	0.45	-1.23
1965	279	8.90	6.36	4.77	5.96	1.59	4.26	-0.51	-0.96
1966	288	9.52	6.86	3.91	6.38	2.95	4.60	0.69	+1.20
1967	308	10.29	6.97	2.49	6.89	4.41	4.67	2.18	+1.49
1968	328	11.14	7.40	4.69	7.46	2.71	4.96	0.27	-1.91
1969	333	11.97	8.68	5.45	8.02	3.23	5.82	0.37	+0.10
1970	318	13.07	9.48	6.36	9.15	3.12	6.64	0.28	-0.09
1971	362	14.27	9.46	9.44	9.99	0.02	6.62	-2.82	-3.10
1972	737	15.17	8.87	7.08	10.62	1.79	6.21	-0.87	+1.95
1973	988	16.13	10.28	9.18	11.29	1.10	7.12	-2.06	-1.19
1974	808	17.49	13.64	15.99	11.72	-2.35	9.14	-6.85	-4.79
1975	714	20.26	15.67	24.11	13.57	-8.44	10.50	-13.61	-6.76
1976	856	24.66	14.95	16.77	16.52	-1.82	10.02	-6.75	+6.86
1977	1211	29.80	14.60	15.89	19.97	-1.29	9.78	-6.11	+0.64
1978	1539	35.60	12.77	8.28	23.85	4.49	8.51	0.23	+6.34
1979	1960	42.10	13.46	13.35	29.47	0.11	9.42	-3.93	-4.16
1980			14.25	18.07		-3.82	9.98	-8.09	-4.16

Sources: Agricultural Land: Its Ownership, Price and Rent, by P.J. Lund and J.M. Slater, Economic Trends, No. 314, December 1979

Farm Rents in England and Wales and Agricultural Land Prices in England and Wales, Ministry of Agriculture, 1959 onwards

Monthly Digest of Statistics

TABLE II
(continued)

Agricultural Rents, Capital Values and Real Interest Rates in England (1960-1980)

Column Descriptions

- Column (2) Price of land per hectare. From 1969, the actual price recorded is available. From 1960-1968 the price of farms without vacant possession is not given on the same basis. However, during that period the tenanted price for England and Wales was 0.61 percent of the average price and around 10 percent of the acreage sold was tenanted. From this it follows that the tenanted price is 0.634 percent of the average price.
- Column (3) Rent per hectare of agricultural land.
- Column (4) The gross yield on long term government securities. It is the annual average taken for each of the twelve months and then averaged over the year.
- Column (5) The percentage change in the retail price index. Again, this is the average of the annual averages for each of the months.
- Column (6) The tax corrected rental income. The tax rate used is the standard marginal rate applicable in the United Kingdom during that year.
- Column (7) The real interest rate calculated as column (4) - column (5). i.e. not adjusted for taxes in capital.
- Column (8) The tax adjusted gross yield on long term government securities. The same tax correction is made as in column (6).
- Column (9) The real interest rate calculated as column (8) - column (5). i.e. adjusted for taxes in capital.
- Column (10) The yearly change in the real rate, being the first differences of column (7).

in equation (9), we need look for a solution in the neighborhood of that value when σ_o' is very small. Although actually finding the solution may involve several terms on the RHS of equation (10), the final answers will be very close to $1/(a^o - \alpha^o)$. On the other hand, when σ_o' is greater than 0.009 in absolute value, as it is for all but three years of the data in Table II, then the second and subsequent terms in { } on the RHS of equation (10) turn out to be insignificant. From equations (7) and (8) it follows that:

$$\frac{b^o}{\mu^o} = - \frac{(\sigma^o - a^o)^2}{\sigma_o'} \quad (11)$$

$$\mu^o = \frac{\sigma_o'}{(a^o - \sigma^o)} \quad (12)$$

Given σ and σ' , $\frac{b^o}{\mu^o}$ lies in the range of ± 0.2 .

For $\alpha^o = 0$, this makes the second term in { } lie in the range of ± 0.04 with the other terms being even smaller. Such values are, of course, a tiny proportion of the value of the first term (no more than 0.1 percent). Introducing a large negative number for α^o could alter these figures, but within the sort of range for α^o that we are likely to consider, this should not be the case. Hence we can obtain a close approximation to the solution by using the equation:

$$\frac{p_o}{r_o} = \frac{\frac{b^o}{\mu^o}}{(a^o - \alpha^o)} = \frac{e^{-\left(\frac{(\sigma^o - a^o)^2}{\sigma_o'}\right)}}{(a^o - \alpha^o)} \quad (13)$$

Except when $|\sigma_0'|$ is very small in which case:

$$\frac{P_0}{r_0} = \frac{1}{(a^0 - \alpha^0)} \quad (14)$$

Although it is not possible to calculate α^0 and a^0 explicitly without further information, we can use the data in Table II along with equations (13) and (14) to get bounds on the values of α^0 that satisfy the constraint $\mu^0 \geq 0$.

If $\mu > 0$, the model is invalid because either equation (9) is non convergent ($b < 0$) or because real rates of interest are assumed to rise forever ($b > 0$). To find the maximum or minimum value of α^0 that ensures $\mu^0 \geq 0$, we set $a = \sigma_0$.

Then we have:

$$\frac{P_0}{r_0} = \frac{1}{(\sigma_0 - \alpha^0)} \quad (15)$$

or

$$\alpha^0 = \sigma_0 - \frac{r_0}{P_0} \quad (16)$$

It is easy to see from equation (13) that this is a lower bound on α^0 when $\sigma_0' > 0$ and an upper bound when $\sigma_0' < 0$. These bounds are calculated and reported in Table III along with the annual rate of growth of real rents.

Both sets of figures in Table III are strongly indicative of the pessimistic nature of the assumptions regarding real rates of growth of rents.⁶ They are frequently bound above by a negative value. For the nominal interest rate net of tax case, they are never bound below by a positive value.

TABLE III

Year	Bounds Imposed on α^0 by Equation (11)				Real Rate of Growth of Rents
	Nominal Interest Rate Gross of Tax		Nominal Interest Rate Net of Tax		
	$\alpha^0 \geq$	$\alpha^0 \leq$	$\alpha^0 \geq$	$\alpha^0 \leq$	
1961		-0.17		-1.85	+8.91
1962		-0.57		-2.64	+4.88
1963	+0.78		-1.01		+10.18
1964		+0.08		-1.75	+3.06
1965		-0.55		-2.65	-1.33
1966	+0.73		-1.53		+3.89
1967	+2.17		-0.06		+5.78
1968		+0.44		-2.00	+2.83
1969	+0.82		-2.04		+2.93
1970		+0.24		-2.60	+0.65
1971		-2.74		-5.58	-5.27
1972	+0.35		-2.31		-1.25
1973		-0.04		-3.20	-2.79
1974		-3.80		-8.30	-2.72
1975		-10.34		-15.51	-2.39
1976	-3.75		-8.68		+5.22
1977	-2.94		-7.76		+3.57
1978	+2.94		-1.32		+10.29
1979		-1.39		-9.59	+5.57
1880					-1.28

In the gross of tax case, this is so for six of the years, but for all except two, the figure is less than 1 percent. In fact, over the period 1960 - 1980 real rent growth has generally been positive and real rents have increased substantially. This is in spite of the restrictions imposed on rent increases in 1973 and to a lesser extent in 1974 and 1975.⁷

It is hard to account for the market's failure to adjust its expectations of real rents, except perhaps to suggest that investors draw their experiences from a much longer period of time, including the pre-war years, and that they react irrationally when large price rises take place. It is plausible that in 1975, for example, when prices rose over 24 percent, investors thought that rents would only go up by 14 percent, leaving a shortfall of 10 percent. It would be of some interest to see if we could "explain" the variations in α^0 and a^0 in terms of other economic variables. Given, however, that both are unobservable we can only do this indirectly by specifying econometric equations for both a^0 and α^0 and substituting into equations (13) and (14). Then by nonlinear least squares it may be possible to estimate the parameters a^0 and α^0 and, equally interestingly, identify the main variables that influence them. This task is attempted in the next section.

5. Econometric Estimation of a^0 and α^0

The formation of long term expectations on real interest rates (a^0) and real rates of rent growth (α^0) are most likely to depend upon past nominal interest rates and rates of price inflation (in the case of a^0) and upon past rates of increase of rents and of price inflation (in the case of α^0). In addition, one might argue that the number of transactions in the market could have an effect. Recall that it is the marginal trades as expectations that are of relevance. In an active bull market, the marginal trader could have expectations that are more optimistic than average and conversely in a bear market. However, all attempts to include the number of transactions in a^0 and α^0 failed to achieve any well determined coefficients on that variable. Hence, it is excluded from any further estimation reported here.

The form taken by the equations for a^0 and α^0 were:

$$a^0_t = c_1 + c_2 \Delta \log(\text{CPI}_t) + c_3 r_t + u_t \quad (17)$$

$$\alpha^0_t = c_4 + c_5 \Delta \log(\text{CPI}_t) + c_6 \Delta \log(\text{Rent}_t) + v_t \quad (18)$$

where the c_i 's are the coefficients to be estimated, and $\Delta \log(\text{CPI}_t)$ is a measure of price inflation, r_t is a measure of the interest rate and $\Delta \log(\text{Rent}_t)$ is a measure of the rate of increase in rents; u_r and v_r are assumed to be the error terms.

The procedure employed for the estimation was nonlinear least squares, using the TROLL program⁸ for solving such equations. Ideally, one would like to include some long distributed lags in the price, rent and interest rate variables, as is common whenever one is estimating some expectational variables. The difficulty with doing that in this model is twofold: first, that data is only available for nineteen years (1961 - 1979) and so one has limited degrees of freedom; and second, that it is difficult to obtain convergence with many distributed lag coefficients in nonlinear models. This proved to be the case when some experiments were done with the above model, taking a quadratic polynomial lag on $\Delta \log(\text{CPI}_t)$, γ_t and $\Delta \log(\text{Rent}_t)$. Consequently, one is limited to employing some moving average on these variables as a proxy for the influence of their past values on a^0 and α^0 . Several moving averages were experimented with, and in general, those between seven and eleven years performed best. The results reported in Table IV are those for the eleven years moving average and the value of σ computed by using the gross of tax nominal interest rate.

The equation in Table IV is moderately well determined and the Durbin-Watson statistic (which is dubious in this nonlinear context anyway) is inconclusive with regard to serial correlation. The main difficulty with the estimation arose because of c_1 . In general, the other coefficients would settle down fairly quickly in the neighborhood of their

TABLE IV

Equation Estimated

$$\frac{r_o}{P_o} = a^o - \alpha^o \quad \text{if } |\sigma^{o'}| < 0.009$$

$$\log \frac{r_o}{P_o} = \log (a^o - \alpha^o) + \left\{ \frac{(\sigma^o - a_o^o)^2}{\sigma_o} \right\} \quad \text{if } |\sigma^{o'}| \geq 0.009$$

$$a^o_t = c_1 + c_2(\text{MA11DP})_t + c_3(\text{MA11IR})_t + u_t$$

$$\alpha^o_t = c_4 + c_5(\text{MA11DP})_t + c_6(\text{MA11RE})_t + v_t$$

MA11DP = 11 year moving average of the rate of increase of the retail price index.

MA11IR = 11 year moving average of the rate of increase of the yield on long term government securities.

MA11RE = 11 year moving average of the rate of increase of farm rents in England and Wales.

Number of observations = 19

Number of variables = 6

Range = 1961 - 1979

RSQ = 0.524 CRSQ = 0.342 F(15/13) = 2.868

DW(0) = 1.30

Coefficient	Value	T Statistic
c_1	+ 0.088	+ 0.85
c_2	- 0.739	- 0.71
c_3	- 0.450	- 2.21
c_6	- 0.182	- 1.81
$c_8 = (c_1 - c_4)$	+ 0.034	+ 3.96
$c_9 = (c_2 - c_5)$	+ 0.103	+ 0.74

The implied values of c_4 and c_5 are 0.055 and -0.637 respectively.

present value, when the program was started from reasonable initial conditions (otherwise it would not converge). However, c_1 was poorly determined and kept the program from converging. Consequently, a lower convergence criterion was applied to c_1 than to the other parameters.⁹

The main features of note are the magnitudes of c_2 and c_5 and the sizes of c_3 and c_6 . The price coefficients on a^0 and α^0 (c_2 and c_5 respectively) are estimated at -0.739 and -0.637. This indicates that a 1 percent increase in the eleven year moving average of the rate of price inflation will lower the expected real rate of interest by 0.739 of 1 percent and the expected rate of increase in the real rent by 0.637 of 1 percent. Both are less than unity, which suggests that, other things being the same, an increase in inflation would lower the expected real interest rates and real rents by less than their actual amounts.¹⁰ At the same time, however, c_3 and c_6 are negative, indicating that expected real rents and real interest rates fall as the nominal rents and nominal interest rates rise. These latter effects probably arise because of a strong relationship between the nominal rates and the expected rate of price inflation. If, for example, the nominal rate of interest is seen in part as an indicator of higher inflation rates in the future, and if real rates are expected to fall as a result, then a negative value on c_3 may be observed. Similarly, if the nominal rate of rent growth is viewed as an indicator of higher inflation

in the future, with real rents expected to fall, then c_6 could be negative. The negative value of c_6 is, however, a little less plausible than that of c_3 .¹¹ What both do suggest is that the modelling of these expectations probably requires further expectational relationships to be included.

The estimated model is not particularly good as predicting r_o/P_o . As this was not the intention of building the model, no attempt is made to examine its predictive powers. Incidentally, it might be noted that with six period quadratic distributed lags in $\Delta \log(\text{CPI}_t)$, $\Delta \log(\text{Rent}_t)$, r_t and a linear regression of these variables on r_o/P_o , one can get a good predictive equation for r_o/P_o , with almost all the coefficients as well determined.

Finally, we report in Table V the computed values of a^o and α^o for each year from 1961 - 1979. The notable features of this table is how generally pessimistic the market has been about real rent growth and at the same time how much more optimistic its real interest rate expectations have been. Predictions of long term real interest rates of 3 and 4 percent in the 1960s have certainly not been the experience of the 1970s and 1980s. It remains, of course, to be seen whether investors of the late 1970s were nearer the mark for the 1980s and 1990s or not. It should be noted that a market that overestimates a^o and underestimates α^o will generally pay much less than the land is worth as the two errors compound to undervalue land.

TABLE V

Year	Expected Long Term Real Interest Rate (α^o x100) (x100)	Expected Long Term Rate of Growth of Real Rents (α^o x100) (x100)
1961	3.69	1.93
1962	3.91	2.14
1963	4.37	2.43
1964	4.27	2.38
1965	3.94	2.21
1966	3.87	2.16
1967	3.97	2.23
1968	3.81	2.11
1969	3.52	1.92
1970	2.97	1.57
1971	2.28	1.17
1972	1.90	1.04
1973	1.38	0.80
1974	0.04	-0.02
1975	-1.71	-1.49
1976	-2.81	-2.45
1977	-3.85	-3.33
1978	-4.48	-3.84
1979	-5.28	-4.54

6. Conclusions

In this paper, a fairly simple expectations model has been developed, which focusses on the rent to capital value ratio and uses it to identify long term expectations about real rent increases and real interest rates. The model is applied to the agricultural land market in England and yields some interesting results. One is the volatility and pessimism about expected real rent increases. The other is the optimism about real interest rates. It would be interesting to test this model for other similar markets to see if such results were replicated. It was also found that as expected inflation rates could be influenced by current nominal interest rates, the econometric specifications need to include such a relationship if the real interest rate effect is to be separated from the inflation effect in the determination of a^0 .

APPENDIX 1

The integration required is:

$$\int_0^{\infty} e^{-(a - \alpha)t} e^{-\frac{b}{\mu} t} e^{-\mu t} dt \quad (1')$$

Let:

$$u = e^{-(a - \alpha)t} \quad (2')$$

$$v' = e^{-\frac{b}{\mu} t} e^{-\mu t} \quad (3')$$

Then:

$$v = \int v' dt = \int e^{-\frac{b}{\mu} t} e^{-\mu t} \frac{dt}{ds} dt \quad (4')$$

Let:

$$s = \frac{1}{\mu} e^{-\mu t} \quad (5')$$

$$\frac{dt}{ds} = -\frac{1}{\mu s} \quad (6')$$

$$v = -\frac{1}{\mu} \int \frac{e^{-bs}}{s} ds \quad (7')$$

$$= \frac{1}{\mu} \left\{ \log s - bs + \frac{b^2 s^2}{2 \cdot 2!} - \frac{b^3 s^3}{3 \cdot 3!} + \dots \right\} \quad (8')$$

Hence:

$$\int uv' dt = \left[-\frac{e^{-(a - \alpha)t}}{\mu} \left\{ \log s - bs + \frac{b^2 s^2}{2 \cdot 2!} \dots \right\} \right]_{t=0}^{t=\infty} \quad (9')$$

$$= \frac{(a - \alpha)}{\mu} \int_{t=0}^{t=\infty} \left\{ \log s - bs + \frac{b^2 s^2}{2 \cdot 2!} \dots \right\} e^{-at} dt$$

APPENDIX 1
(continued)

Taking the terms in equation (9') we have the first term as

$$\frac{1}{\mu} \log \frac{1}{\mu} - \frac{b}{\mu^2} + \frac{b^2}{2 \cdot 2! \mu^3} - \frac{b^3}{3 \cdot 3! \mu^4} \dots \dots \dots \quad (10')$$

And the second term as:

$$-\frac{1}{\mu} \log \frac{1}{\mu} + \frac{1}{(a - \alpha)} + \frac{(a - \alpha)b}{\mu^2(\mu + (a - \alpha))} - \frac{(a - \alpha)b^2}{\mu^3 2 \cdot 2!(2\mu + (a - \alpha))} \quad (11')$$

Combining equations (10') and (11') we obtain:

$$\begin{aligned} f_{uv}' &= \frac{1}{(a - \mu)} - \frac{b}{\mu(\mu + a - \alpha)} + \frac{b^2}{2! \mu^2(2\mu + a - \alpha)} \\ &\quad - \frac{b^3}{3! \mu^3(3\mu + a - \alpha)} \end{aligned} \quad (12')$$

and

$$\begin{aligned} \frac{P_o}{r_o} &= e^{\frac{b}{\mu}} \left\{ \frac{1}{(a - \alpha)} - \frac{b}{\mu(\mu + a - \alpha)} + \frac{b^2}{2! \mu^2(2\mu + a - \alpha)} \right. \\ &\quad \left. - \frac{b^3}{3! \mu^3(3\mu + a - \alpha)} \right\} \end{aligned} \quad (13')$$

FOOTNOTES

1. The data is available for tenanted farms for the period 1970 - 1980, and refers to the average transacted over the total acreage in the Ministry of Agriculture sample for England and Wales. The housing figures are taken from the 1971 census and the Stock Exchange figures from Financial Statistics 1979.
2. As the expectations are continually and systematically falsified by the performance of the market and as perfect foresight calculations indicate that investors were pessimistic for almost every year of the post-war period, it is hard to accept a rational expectation view to this market.
3. In this paper, we have tried to adhere to the convention that superscripts indicate expected values.
4. Details of the integration are given in Appendix 1.
5. When $\mu^0 = 0$, equation (9) is no longer applicable. We see the derived result by directly using equation (6).
6. The pessimism of the market with regard to real rent growth is also reflected in some calculations that were done on the discounted present value of net of tax rents plus the capital value in 1980 discounted to the year of purchase. This sum was compared to the price paid in that year. For purchases between 1946 and 1960 this figure amounted to between 1.75 and 2.5 times the price paid. From 1961 - 1972 it ranged between 1.05 and 1.55 times the price paid. Since then the period is too short to allow rents a significant role but the figures clearly indicate that to 1972 tenanted farmland has been an excellent investment.
7. From 1961 - 1979 rents went up 5.6 times whereas consumer prices only rose 3.8 times. This represents an average annual rate of increase over that period of 5.9 percent in real rents. The controls of prices and incomes in 1973 - 1975 probably resulted in overcompensation later, particularly in 1978. It might be thought that the high rent growth figures are significantly affected by improvements made by landlords, but the data does not support that. From 1968 - 1971, the Ministry of Agriculture published figures that allowed one to calculate expenditures on improvements per acre on tenanted farms. Amortizing this expenditure over twenty years yields an annual amount that is around 7 percent of the actual increase in rents.

While not negligible, this would make little impression on the conclusions drawn in Table III.

8. See Troll (1980).
9. Convergence was said to take place when the change in the coefficients was less than 0.5 percent in absolute value for c_2 through c_9 , as the program moved from one variation to the next. For c_1 it was 5 percent. An attempt was made to suppress c_1 completely, but this resulted in implausible values of the other coefficients.
10. When we say the "actual" amount, we are referring to an increase in the eleven year moving average. This would of course, require a 1 percent increase to be sustained, for eleven years, or equivalently require an 11 percent increase in one year to be treated as a 1 percent increase.
11. The rent analysis is complicated by the discreteness effect discussed in Table I, which would create a positive relationship between the rate of inflation and of real rent growth (resulting in c_5 being pushed down). In addition, there was the rent control imposed in 1973 and partially in 1974 and 1975. Traill (1980) fits a dummy for these years in a linear rent equation but it was not well determined. We had similar difficulties in some experimental linear equations and so did not pursue this line further.

REFERENCES

Ministry of Agriculture, Fisheries and Food, Farm Rents in England and Wales, various issues, 1959 onwards.

Ministry of Agriculture, Fisheries and Food, Agricultural Land Prices in England and Wales, various issues, 1959 onwards.

Lund, P.J. and Slater, J.M., "Agricultural Land: Its Ownership, Price and Rent," Economic Trends, no. 34, December, 1979.

Traill, W.B., Land Values and Rents, Bulletin 175, Department of Agricultural Economics, University of Manchester, August, 1980.

TROLL/1 Users Guide, National Bureau of Economic Research, Cambridge, Massachusetts, 1980.

CENTER FOR REAL ESTATE AND URBAN ECONOMICS
WORKING PAPER SERIES PUBLICATION LIST

Institute of Business and Economic Research
156 Barrows Hall, University of California
Berkeley CA 94720

The following working papers in this series are available at a charge of \$5.00, which partially covers the cost of reproduction and postage. Papers may be ordered from the address listed above. Checks should be made payable to the Regents of the University of California.

---*---

- 79-1 Kenneth T. Rosen and David E. Bloom. "A Microeconomic Model of Federal Home Loan Mortgage Corporation Activity." April 1979.
- 80-2 Kenneth T. Rosen and Mitchel Resnick. "The Size Distribution of Cities: An Examination of the Pareto Law and Primacy." July 1979.
- 80-3 Jennifer R. Wolch. "Residential Location of the Service-Dependent Poor." August 1979.
- 80-4 Stuart Gabriel, Lawrence Katz, and Jennifer Wolch. "Local Land-Use Regulation and Proposition 13: Some Findings from a Recent Survey." September 1979.
- 80-5 David Dale-Johnson. "Hedonic Prices and Price Indexes in Housing Markets: The Existing Empirical Evidence and Proposed Extensions." January 1980.
- 80-6 Susan Giles Levy. "Consumer Response to High Housing Prices: The Case of Palo Alto, California." January 1980.
- 80-7 Dwight Jaffee and Kenneth T. Rosen. "The Changing Liability Structure of Savings and Loan Associations." February 1980.
- 80-8 Dwight Jaffee and Kenneth T. Rosen. "The Use of Mortgage Passthrough Securities." February 1980.
- 80-9 Stuart A. Gabriel. "Local Government Land-Use Allocation in the Wake of a Property Tax Limitation." May 1980.

- 80-10 Kenneth T. Rosen. "The Affordability of Housing in 1980 and Beyond." June 1980.
- 80-11 Kenneth T. Rosen. "The Impact of Proposition 13 on House Prices in Northern California: A Test of the Interjurisdictional Capitalization Hypothesis." June 1980.
- 80-12 Kenneth T. Rosen. "The Federal National Mortgage Association, Residential Construction, and Mortgage Lending." August 1980.
- 80-13 Lawrence Katz and Kenneth T. Rosen. "The Effects of Land Use Controls on Housing Prices." August 1980.
- 80-14 Kenneth T. Rosen. "The Demand for Housing Units in the 1980s." September 1980.
- 80-15 Konrad Stahl. "A Note on the Microeconomics of Migration." October 1980.
- 80-16 John T. Rowntree and Earl R. Rolph. "Efficient Community Management." August 1980.
- 80-17 John M. Quigley. "Non-linear Budget Constraints and Consumer Demand: An Application to Public Programs for Residential Housing." September 1980.
- 80-18 Stuart A. Gabriel and Jennifer R. Wolch. "Local Land-Use Regulation and Urban Housing Values." November 1980.
- 80-19 F. E. Balderston. "The Structural Option for the Savings and Loan Industry." November 1980.
- 80-20 Kristin Nelson. "San Francisco Office Space Inventory." November 1980.
- 80-21 Konrad Stahl. "Oligopolistic Location under Imperfect Consumer Information." December 1980.
- 80-22 Konrad Stahl. "Externalities and Housing Unit Maintenance." December 1980.
- 81-23 Dwight M. Jaffee and Kenneth T. Rosen. "The Demand for Housing and Mortgage Credit: The Mortgage Credit Gap Problem." March 1981.
- 81-24 David E. Dowall and John Landis. "Land-Use Controls and Housing Costs: An Examination of San Francisco Bay Area Communities." March 1981.

- 81-25 Jean C. Hurley and Constance B. Moore. "A Study of Rate of Return on Mortgage Pass Through Securities." March 1981.
- 81-26 Kenneth T. Rosen. "The Role of Pension Funds in Housing Finance." April 1981.
- 81-27 John M. Quigley. "Residential Construction and Public Policy: A Progress Report." April 1981.
- 81-28 Kenneth T. Rosen. "The Role of the Federal and 'Quasi-Federal' Agencies in the Restructured Housing Finance System." June 1981.
- 81-29 Diane Dehaan Haber and Joy Hashiba Sekimura. "Innovations in Residential Financing: An Analysis of the Shared Appreciation Mortgage and a Comparison of Existing Alternative Mortgage Instruments." June 1981.
- 81-30 Diane Dehaan Haber and Joy Hashiba Sekimura. "Alternative Mortgages Consumer Information Pamphlet." June 1981.
- 81-31 Jean C. Hurley. "A Model for Pricing Pass-Through Securities Backed by Alternative Mortgage Instruments." June 1981.
- 81-32 Kenneth T. Rosen. "The Affordability of Housing in California." September 1981.
- 81-33 Kenneth T. Rosen and Lawrence Katz. "Money Market Mutual Funds: An Experiment in Ad Hoc Deregulation." September 1981.
- 81-34 Kenneth T. Rosen. "New Mortgage Instruments: A Solution to the Borrower's and Lender's Problem." September 1981.
- 81-35 Konrad Stahl. "Toward a Rehabilitation of Industrial, and Retail Location Theory." September 1981.
- 81-36 Frederick E. Balderston. "S&L Mortgage Portfolios: Estimating the Discount from Book Value." October 1981.
- 81-37 Kenneth T. Rosen. "A Comparison of European Housing Finance Systems." October 1981.
- 81-38 Frederick E. Balderston. "Regression Tests of the Relationship between Book Net Worth and Revised Net Worth of S&Ls." October 1981.

- 81-39 Lawrence B. Smith and Peter Tomlinson. "Rent Controls in Ontario: Roofs or Ceilings?" November 1981.
- 81-40 Alan R. Cerf. "Investment in Commercial Real Estate Including Rehabilitation: Impact of the Tax Recovery Act of 1981." November 1981.
- 81-41 Frederick E. Balderston. "The Savings and Loan Mortgage Portfolio Discount and the Effective Maturity on Mortgage Loans." November 1981.
- 82-42 John M. Quigley. "Estimates of a More General Model of Consumer Choice in the Housing Market." January 1982.
- 82-43 Martin Gellen. "A House in Every Garage: The Economics of Secondary Units." March 1982.
- 82-44 John D. Landis. "California Housing Profiles: 1980." March 1982.
- 82-45 Paul F. Wendt. "Perspectives on Real Estate Investment." February 1982.
- 82-46 Kenneth T. Rosen and Lawrence B. Smith. "The 'Used House Market.'" May 1982.
- 82-47 Kenneth T. Rosen. "Deposit Deregulation and Risk Management in an Era of Transition." May 1982.
- 82-48 Steven W. Kohlhagen. "The Benefits of Offshore Borrowings for the S&L Industry." May 1982.
- 82-49 Lawrence B. Smith. "The Crisis in Rental Housing: A Canadian Perspective." June 1982.
- 82-50 Anil Markandya. "Headship Rates and the Household Formation Process in Great Britain." June 1982.
- 82-51 Anil Markandya. "Rents, Prices and Expectations in the Land Market." June 1982.
- 82-52 Kenneth T. Rosen. "Creative Financing and House Prices: A Study of Capitalization Effects." July 1982.
- 82-53 Kenneth T. Rosen and Lawrence B. Smith. "The Price Adjustment Process for Rental Housing and the Natural Vacancy Rate." September 1982.
- 82-54 Frederick E. Balderston. "Analysis of the Viability of S&L Firms." September 1982.