

Bias compensation for UWB ranging for pedestrian geolocation applications

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Abstract—We present an effective bias compensation method to process none-line-of-sight (NLoS) and long-distance line-of-sight (LD-LoS) Ultra-Wideband (UWB) range measurement signals used to aid a pedestrian inertial navigation system (INS). The common UWB bias compensation techniques use machine learning methods to identify and remove the bias in the measurements. These techniques are computationally expensive and require extensive prior data. Here, we propose to use an algorithmic compensation technique that accounts for the bias by estimating it using the Schmidt Kalman filter. Next, we exploit the positivity of the error in the UWB range measurements to propose a novel constrained sigma point based correction filtering that can be used atop the Schmidt Kalman filter for further improvement in the positioning accuracy of the UWB aided pedestrian inertial navigation. Experiments demonstrate the effectiveness of our methods.

Index Terms—Pedestrian geolocation, UWB ranging, bias compensation, NLoS measurements

I. Introduction

This paper devises an effective bias compensation method to process NLoS and LD-LoS UWB range measurement signals that are used in an UWB ranging aided pedestrian inertial navigation system in GPS-denied environments. In this localization method, range measurements via an UWB sensor from beacons with known locations are used to bound drift and correct the estimates in the pedestrian's INS system. In recent years, the UWB time-of-flight based ranging, which under appropriate conditions can reach decimeter level accuracy [1], has been considered as an effective ranging technology in complex environments. This is due to the UWB's capability to take NLoS ranging measurements and its low susceptibility to interfere with coexisting radio signals or UWB signals from other paths. However, the UWB ranging error significantly increases under LD-LoS and NLoS scenarios due to the existence of significant positive bias in the measurements [2]. The degradation of ranging accuracy under these scenarios degrades the performance of the pedestrian localization. To derive accurate location information, in this paper we investigate practical solutions to compensate for the bias in the UWB ranging.

Under LD-LoS ranging, the positive bias due to multipath and path loss of the signal propagation becomes significant and cannot be ignored. The interested reader can find a study on the relation between the distance and the UWB ranging bias, developed using a large number of experiments within a variety of environments, in [3]. The positive bias in the NLoS UWB ranging is due to the extra time that signal takes to travel between the sensor nodes due to the time lost to penetrate through obstructions or traveling a longer non-direct path. To mitigate the effect of NLoS ranging bias on the localization accuracy, one approach in the literature is to identify the NLoS measurements and avoid using them [4]–[6]. However, dropping NLoS measurements limits the effectiveness of the UWB measurement feedbacks in complex indoor environments. Another approach to deal with bias in the NLoS UWB ranging measurements is to use machine learning methods to identify and remove the bias [7]–[11]. For example, [11] uses the classification of obstruction material based on analyzing the statistics of Channel Impulse Response (CIR) to estimate and remove the bias. We used the method of [11] to experimentally test the effectiveness of the classification methods for

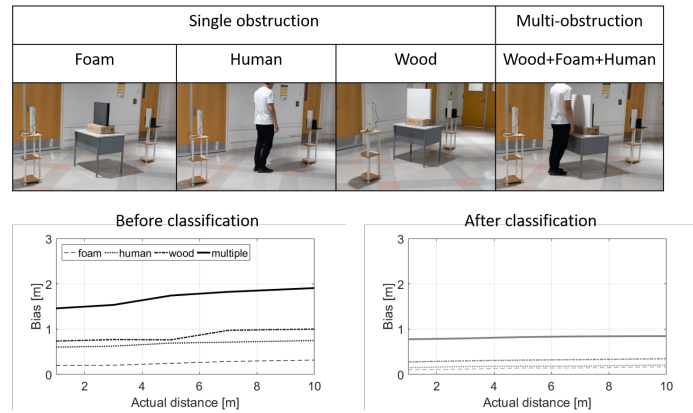


Fig. 1 – Bias in NLoS UWB ranging for different obstructions before and after applying the obstruction classification and bias removal of [11].

bias removal in a series of experiments with different obstruction materials. As shown in Fig. 1, the method works well when there is only a single type of obstruction, however, the residual error is unacceptable if multiple obstructions exist. The classification techniques also come with high computational complexity to analyze the channel statistics and the necessity for collecting a large amount of training data, which makes them impractical for real-time online applications.

In this paper, we take advantage of the state estimation nature of the UWB aided pedestrian inertial navigation to develop bias compensation techniques with low complexity. In our preliminary work [12], we used the covariance inflation method [13] followed by a constrained Kalman filtering [14] to compensate for bias in UWB range measurements in a cooperative localization for human geolocation. Here, we investigate the use of the Schmidt Kalman filtering [15] for bias compensation. Our next contribution is to exploit the positivity of the error in the UWB range measurements to propose a novel constrained sigma point based filtering that can be used atop the Schmidt Kalman filter for further improvement in the positioning accuracy of the UWB aided pedestrian INS. Experimental tests demonstrate the effectiveness of our proposed solution.

II. UWB Aided Pedestrian Geolocation

Let a strapdown INS mechanization [16] mounted on a pedestrian derive an estimate of the ego state $\hat{\mathbf{x}}^-(t) = f(\hat{\mathbf{x}}^+(t-1), \mathbf{u}(t)) \in \mathbb{R}^n$ and the corresponding positive definite error covariance matrix $\mathbf{P}^-(t)$ at each time step $t \in \mathbb{Z}^+$ in an earth-fixed coordinate frame with axes pointing north, east and down. The state $\mathbf{x}(t) \triangleq [\mathbf{p}(t), \mathbf{v}, \boldsymbol{\psi}(t)]^\top$ includes, respectively, position, velocity and attitude (pitch, roll and heading). Because of inherent noises in the INS self-motion measurements $\mathbf{u}(t)$, relying only on INS system results in poor estimation accuracy due to error accumulation. To bound the error and improve the state estimation accuracy, processing of range measurements taken by body-mounted UWB ranging sensor with respect to pre-installed UWB sensors as beacons with known location in the environment is used to correct/update the state estimate to $(\hat{\mathbf{x}}^+, \mathbf{P}^+)$. If the model of the range measurement with respect to the UWB beacons was $z(t) = h(\mathbf{x}(t)) + v(t)$ with $v(t)$ as zero-mean white Gaussian noise, the corrected estimate $(\hat{\mathbf{x}}^+, \mathbf{P}^+)$ could have been obtained using the update stage of a standard Extended Kalman Filter (EKF) at time t . But, the measurement model for the UWB ranging between the pedestrian and a beacon at known position $\mathbf{p}_B(t)$ is

$$z(t) = \underbrace{\|\mathbf{p}(t) - \mathbf{p}_B(t)\|}_{h(\mathbf{x}(t))} + b(t) + v(t), \quad (1)$$

where $b(t)$ is the additive bias and $v(t)$ is the additive zero-mean white Gaussian noise with variance R . The bias is modeled as

$$b(t) = \begin{cases} 0, & \text{short-distance LoS} \\ \phi(t), & \text{long-distance LoS or NLoS} \end{cases}$$

where $\phi(t) > 0$ is the positive bias, modeled as Gaussian noise with mean $\bar{\phi}$ and variance Φ . The NLoS signal propagation can be distinguished from the LoS signal propagation based on a real-time signal power-based approach without any prior information about the environment or the biases [17].

III. Bias Compensation Models

The state estimation update rule when a range measurement $z(t)$ is detected is given by $\hat{\mathbf{x}}^+(t) = \hat{\mathbf{x}}^-(t) + \mathbf{K}[z(t) - \hat{z}(t)]$, where $\hat{z} = h(\hat{\mathbf{x}}^-)$. To simplify the notation, hereafter we only include the time index t when clarification is needed. Given the biased ranging measurement model (1) and using a linear estimate, we obtain

$$\mathbf{e}^+ = \mathbf{x} - \hat{\mathbf{x}}^+ \cong (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{e}^- - \mathbf{K}(b + v) \quad (2)$$

where \mathbf{I} is identity matrix, $\mathbf{e}^- = \mathbf{x} - \hat{\mathbf{x}}^-$, $\mathbf{H} = \frac{dh(\mathbf{x})}{d\mathbf{x}}|_{\mathbf{x}=\hat{\mathbf{x}}^-}$ and

$$\mathbf{P}^+ = \mathbb{E}[\mathbf{e}^+ \mathbf{e}^{+\top}] = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^-(\mathbf{I} - \mathbf{K}\mathbf{H})^\top - (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{C}^-\mathbf{K}^\top - \mathbf{K}\mathbf{C}^{-\top}(\mathbf{I} - \mathbf{K}\mathbf{H})^\top + \mathbf{K}\mathbb{E}[b^2]\mathbf{K}^\top + \mathbf{K}\mathbf{R}\mathbf{K}^\top, \quad (3)$$

where $\mathbf{C}^- = \mathbb{E}[b\hat{\mathbf{x}}^-]$ is the correlation between the state and measurement bias and $\mathbb{E}[b^2] = B = \phi^2 + \Phi$ is the second moment of the bias. The update gain that minimizes the total uncertainty trace(\mathbf{P}^+) is

$$\mathbf{K} = (\mathbf{P}^-\mathbf{H}^\top + \mathbf{C}^-\mathbf{H}^\top)(\mathbf{H}\mathbf{P}^-\mathbf{H}^\top + \mathbf{H}\mathbf{C}^- + \mathbf{C}^{-\top}\mathbf{H}^\top + B + R)^{-1}. \quad (4)$$

Ignoring the bias, i.e., setting $\mathbf{C}^- = \mathbf{0}$ and $\mathbb{E}[b^2] = 0$, mismatches the reality under LD-LoS and also NLoS UWB ranging, leading to inconsistent estimates. The covariance inflation method [13], which

turns the biased measurement to an unbiased measurement $z(t) = h(\hat{\mathbf{x}}(t)) + \bar{v}(t)$ with modified zero mean Gaussian noise \bar{v} with $\bar{R} = \mathbb{E}[\bar{v}^i \bar{v}^{i\top}] = B + R$, is a simple procedure to remove the bias from measurements. That is, the noise covariance is inflated with the second moment of bias such that the bias is covered by the random noise. Despite its simplicity, covariance inflation still has consistency problem as it is equivalent to setting $\mathbf{C}^- = \mathbf{0}$ in (3) (the estimated updated covariance is smaller than the actual covariance, thus the estimates are over-optimistic, resulting in inconsistency).

We use the *Schmidt Kalman filtering* (SKF) method to take into account the cross-covariance term \mathbf{C}^- in update model (3). In the SKF, the state bias correlation is propagated and updated according to

$$\mathbf{C}^-(t) = \mathbf{F}(t)\mathbf{C}^-(t-1), \quad \mathbf{C}^+(t) = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{C}^-(t) + \mathbf{K}B,$$

where $\mathbf{F}(t) = \frac{d\mathbf{f}(\mathbf{x}, \mathbf{u})}{d\mathbf{x}}|_{\mathbf{x}=\hat{\mathbf{x}}^+(t-1), \mathbf{u}=\mathbf{u}(t)}$ and \mathbf{K} is (4). The SKF method considers the bias as a random variable with fixed statistics. To deliver a high performance it needs prior knowledge about the bias statistics (mean and variance), which may not be available with high accuracy. Also, to account for bias in various environments, we need to use the statistics of the worst case bias, which results in conservative estimates. To improve the accuracy of the UWB aided pedestrian INS, next we propose a constrained filtering correction method that is independent of the prior knowledge.

Constrained sigma point based correction method: Under NLoS conditions and also LD-LoS, the UWB measurements are positively biased. The total error in the ranging measurement is always positive because the positive bias is more dominant than the white Gaussian noise [18]. Therefore, the measured distance is always greater than the actual distance, i.e., (recall (1))

$$\|\mathbf{p}(t) - \mathbf{p}_B(t)\| \leq z(t). \quad (5)$$

This relation can be used as an additional information to correct the updated estimate of the pedestrian via the SKF method discussed above. Let $(\hat{\mathbf{x}}_s^+, \mathbf{P}_s^+)$ be the estimate obtained by the SKF update. Implementing the standard constrained filtering [14], the final corrected state $\hat{\mathbf{x}}(t)^+$ is obtained from

$$\hat{\mathbf{x}}^+(t) = \underset{\mathbf{x}}{\operatorname{argmin}}(\mathbf{x} - \hat{\mathbf{x}}_s^+(t))^\top \mathbf{W}(\mathbf{x} - \hat{\mathbf{x}}_s^+(t)), \quad (6a)$$

$$\text{subject to } \|\mathbf{p}(t) - \mathbf{p}_B(t)\| \leq z(t), \quad (6b)$$

where \mathbf{W} is a positive-definite weighting matrix. The standard constrained filtering (6) only corrects the estimates but not the associated error covariance. To improve the uncertainty as well, we propose a constrained sigma point based correction as we describe below. Given the updated SKF state estimate $(\hat{\mathbf{x}}_s(t)^+ \in \mathbb{R}^n, \mathbf{P}_s^+(t) \in \mathbb{R}^{n \times n})$, we calculate $2n + 1$ sigma points as

$$\hat{\chi}_i(t) = \begin{cases} \hat{\mathbf{x}}_s^+(t) & i = 0, \\ \hat{\mathbf{x}}_s^+(t) + [\sqrt{(n+\kappa)\mathbf{P}_s(t)^+}]_i & i = 1, \dots, n, \\ \hat{\mathbf{x}}_s^+(t) - [\sqrt{(n+\kappa)\mathbf{P}_s(t)^+}]_{i-n} & i = n+1, \dots, 2n, \end{cases} \quad (7)$$

where $[\sqrt{(n+\kappa)\mathbf{P}_s(t)^+}]_i$ is the i^{th} column of the matrix square root of $(n+\kappa)\mathbf{P}_s(t)^+$ and $\kappa \in \mathbb{R}$ is the parameter tuning the size of sigma point distribution [19]. The weight associated with each sigma point is $\omega_0 = \kappa/(n+\kappa)$ and $\omega_i = 1/(2(n+\kappa)), i = 1, \dots, 2n$. The sigma points can be used to estimate the covariance matrix accurately up to 3rd order [19]. We pass the sigma points (7) one by one through (6), in

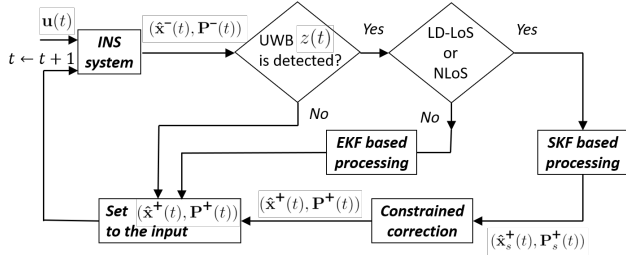


Fig. 2 – UWB ranging aided INS with bias compensation.

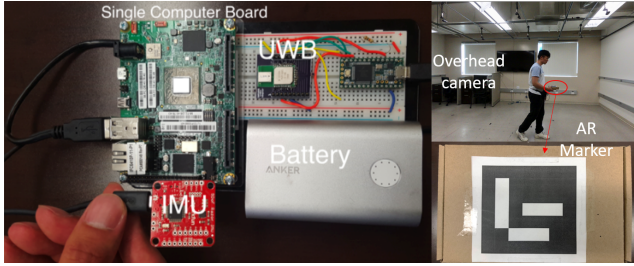


Fig. 3 – The testbed used in the experiments (left); the overhead camera tracking system (right). The device consists of a DWM1000 UWB transceiver developed by DecaWave, a Teensy 3.2 micro-controller as data acquisition interface (the ranging software is mounted on the Teensy), a commercial IMU (SparkFun 9DoF Razor IMU) and an INFORCE 6410PLUS single board computer powered by a portable battery to preform all the online computations.

place of $\hat{\mathbf{x}}_s^+$, and collect the result as $\hat{\chi}_i^+(t)$. Then, the final constrained corrected estimate and its associated covariance are

$$\hat{\mathbf{x}}^+(t) = \sum_{i=0}^{2n} \omega_i \hat{\chi}_i^+(t), \quad (8a)$$

$$\mathbf{P}^+(t) = \sum_{i=0}^{2n} \omega_i (\hat{\chi}_i^+(t) - \hat{\mathbf{x}}^+(t)) (\hat{\chi}_i^+(t) - \hat{\mathbf{x}}^+(t))^T. \quad (8b)$$

Since every $\hat{\chi}_i^+$ respects (6b), which is convex, and $\sum_{i=0}^{2n} \omega_i = 1$, the constrained corrected state (8a) also satisfies the constraint (6b). The UWB aided pedestrian INS with our proposed constrained sigma point based corrected SKF (CS-SKF) bias compensation is shown in Fig. 2.

IV. Experimental Evaluations

We evaluate the effectiveness of our proposed bias compensation method in experiments. Fig. 3 shows the portable localization device used in our experiments. By carrying the device, the pedestrian obtains his/her ego state estimate using the inertial measurement unit (IMU) measurements and ranging measurements with respect to UWB beacons. We employ the signal power based NLoS identification method [17] to identify NLoS measurements from the LoS ones.

First experiment: To evaluate the consistency of our proposed CS-SKF bias compensation method and compare it to other possible solutions, we carried out a Monte Carlo experiment in the Kia Cooperative System Lab of UCI. An UWB beacon is pre-installed in a known location in a next door lab such that the range measurement is NLoS because of the UWB signal propagation through a wall and lab equipment. To track the motion of the pedestrian, we use a well-calibrated overhead camera system along with an Augmented Reality

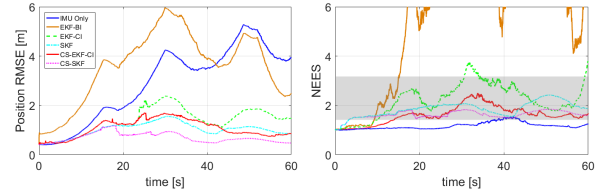


Fig. 4 – The RMSE and NEES plots for the first experimental evaluation.

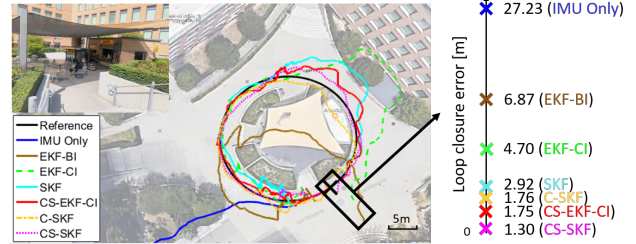


Fig. 5 – The site location, the trajectories due to various UWB aided pedestrian INS algorithms (left) and the corresponding loop closure position errors in the second experimental evaluation test.

(AR) marker that is placed on our portable localization device, see Fig. 3. This system tracks the pedestrian’s trajectory with a position error less than 1cm. This reference trajectory is used as the ground truth to calculate the position root mean square error (RMSE) and the normalized estimation error squared (NEES) [13] from 20 Monte Carlo runs (the pedestrian walks in 20 random trajectories for 60 seconds while being tracked by the overhead camera system). The results are shown in Fig. 4. As the RMSE plot shows, our proposed CS-SKF bias compensation produces the best estimate. The shaded zone in the NEES plot highlights the consistency zone for our experiment, which is the NEES values between [1.22, 2.97]. An EKF method that ignores the bias (EKF-BI plot) results in a NEES measure that goes far beyond the consistency bound, i.e., an over-optimistic estimate. The covariance inflation based EKF method (EKF-CI plot) improves the consistency but it is still not strictly consistent all the time. By applying the constrained sigma point correction atop the covariance inflation based EKF (CS-EKF-CI plot), the consistency is improved. The SKF and the CS-SKF bias compensations both show good consistency property, but CS-SKF has a better RMSE.

Second experiment: In this experiment, shown in Fig. 5, a pedestrian performs a loop closure by walking on a circular reference trajectory around a coffee shop on the UCI campus, which is identifiable on the Google map. The range measurements are taken with respect to a beacon that is placed at the starting point of the trajectory. The range measurements are mostly in NLoS because of a variety of obstructions such as walls, cafe equipment, chairs and tables, bushes, trees, and people. The localization results using various bias compensation methods are shown in Fig. 5. A large deviation from the reference trajectory at around 1 o’clock to 3 o’clock position is observed when the UWB signal is obstructed by the coffee shop structure due to strong multi-layered obstruction. If the bias is naively ignored, the degradation of localization is clearly observed (EKF-BI plot). When the bias is compensated, the localization accuracy is significantly improved. As we can see from the trajectory plots, our proposed CS-SKF bias compensation method outperforms the other approaches. The advantage of our proposed constrained sigma point

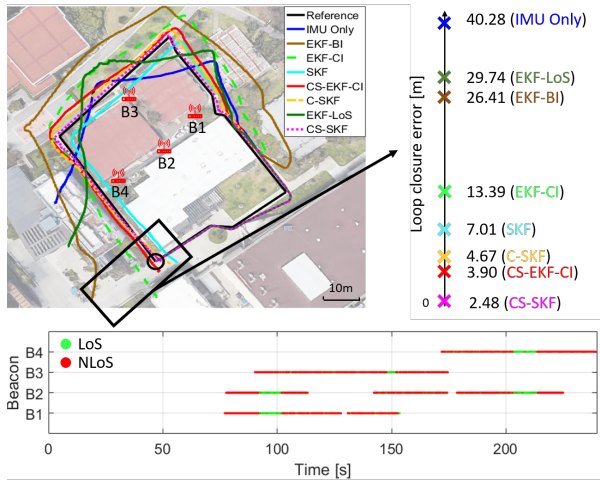


Fig. 6 – The site location with 4 beacons: the trajectories due to various UWB aided algorithms (upper-left), the corresponding loop closure position errors (upper-right) and the detected ranging measurements with respect to each beacon (bottom) in the third experimental evaluation test. based correction is visible when we compare the CS-SKF plot with the SKF and C-SKF plots. Here, C-SKF plot shows the localization result for an SKF bias compensation followed by a standard constrained Kalman filtering.

Third experiment: In this experiment, shown in Fig. 6, the pedestrian performs a loop closure along a longer reference trajectory around Disability Services Center on the UCI campus. Four beacons (B1, B2, B3 and B4) are placed between the buildings such that most of the range measurements are NLoS, as shown in the bottom plot of Fig. 6. In this experiment, we use a higher grade IMU (VectorNav VN-100). As we can see our proposed CS-SKF bias compensation method delivers the best estimation and loop closure result. Here, we can also see that CS-SKF bias compensation outperforms C-SKF bias compensation. We note that if the NLoS measurements are dropped, the localization accuracy and the loop closure error grow significantly (EKF-LoS plot). Lastly, we note that, as expected, the localization accuracy increases when more beacons are used, see Table. 1.

V. Conclusion

We considered the design of an effective bias compensation method to process NLoS and LD-LoS UWB range measurement signals that are used in an UWB ranging aided pedestrian inertial navigation, in GPS-denied environments. We discussed use the Schmidt Kalman filter as an algorithmic bias compensation method and proposed to augment it with a novel constrained sigma point based filtering method to increase the impact. Our experimental results showed that a method consisted of the Schmidt Kalman filter followed by our proposed constrained filtering (CS-SKF method) produces the most effective and consistent solution. The advantage of the proposed method over the existing alternative bias compensation approaches is

Table 1 – Loop closure error in the 3rd experiment with measurements with respect to different number of beacons when CS-SKF method is used.

	B1	B1,B2	B1,B2,B3	B1,B2,B3,B4
Loop closure error [m]	22.35	11.19	5.78	2.48

its low complexity and its low computational cost, making it suitable for online implementation for pedestrian geolocation applications.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of Commerce, National Institute of Standards and Technology (NIST) under Award 70NANB17H192. All work was conducted in the Kia Cooperative Systems Lab at the University of California, Irvine.

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