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and Structural Mechanics

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ABSTRACT

A theoretical investigation of the mechanical, thermal and thermal-mechanical properties of two types of reinforced solid propellants is reported. The two materials that are considered are wire screen and continuous wire wound reinforced propellants. Predictions based upon the unit cell concept are made for the properties of the reinforced propellants. Computer programs are listed and described for the evaluation of the expressions given by the theoretical predictions. The accuracy of the predictions is discussed in relationship to other theoretical and to experimental results.

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INTRODUCTION

The usage of solid propellants for military and space applications has necessitated the consideration of their structural response characteristics. For certain applications their structural capabilities have been found to be inadequate; it has been suggested [1,6]* that the overall mechanical behavior of solid propellant could be substantially improved by reinforcing it with wire. (The inapplicability of classical methods of analysis to such reinforced solid propellant grains has been discussed by Freudenthal [7].) In order that an adequate judgement may be made as to the effectiveness and consequences of reinforcing propellants, procedures must be available for evaluating the properties of the resulting composite materials. It, however, has been found to be very difficult to experimentally determine the complete array of mechanical properties of wire reinforced materials. An alternative approach is to theoretically predict the composite properties from a knowledge of the properties of the individual constituent materials; the theoretical approach is utilized in this report. The composite properties of a reinforced material are those properties associated with the phenomenological response of the material. The phenomenological response is the behavior observed when the material is viewed on a scale sufficiently large so that the contributions of the individual constituents are not discernible (only the average response is evident).

Even for those composites which are intended to be uniformly reinforced the actual composite properties will vary somewhat from point to point due to inevitable incorrect positioning of the reinforcing elements. In the following development it is assumed that the reinforcing elements have been

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*Numbers in brackets refer to references listed on page 23 of report.

located with precision, thus, the predicted properties will at best only represent averages of the actual properties. Throughout the remainder of this report references to composite properties will refer to those properties that would exist in the ideally reinforced material. For an ideally reinforced material the composite properties may be predicted by considering the average behavior of a "unit cell" of the material; the unit cell may be considered as the fundamental building block of the material.

There are two basic approaches to the prediction of composite properties. One of the approaches [2] (herein referred to as the energy approach) consists of utilizing the theorems of minimum potential energy and minimum complementary energy to predict bounds on the values of the composite properties. In the development of these predictions it is necessary to obtain approximate solutions to certain boundary value problems. These approximate solutions must in one instant satisfy the admissibility conditions of the Theorem of Minimum Potential Energy and in the other instant the conditions of the Theorem of Minimum Complementary Energy. In the other approach [3] (herein referred to as the "direct approach") the composite properties are directly computed from the solutions to a series of boundary value problems. If the boundary value problems for both procedures are solved exactly the bounds predicted by the energy procedure coincide and are identical to the properties predicted by the direct approach. For complicated reinforcement configurations, however, one may only be able to obtain approximate solutions to the unit cell boundary value problems and hence only find bounds from the energy procedure and approximate properties from the direct approach.

In the authors' judgement the energy procedure has two serious shortcomings which are not present in the direct approach. The energy procedure does not lead to a direct prediction of bounds for all those properties which are needed for structural analysis purposes but rather it directly yields bounds for certain algebraic combinations of the desired quantities. Suitable algebraic manipulation of these expressions lead to bounds for the desired quantities, unfortunately, the resulting bounds are often so far apart (divergence is caused by the algebraic manipulation of the inequalities) that the results are essentially meaningless. The other undesirable feature of the energy approach is that the class of approximate solutions that is admissible is rather restrictive, thus, making it difficult to obtain accurate approximations. It should also be noted that the bounds predicted by the energy procedure are for the ideal composite properties which are in turn approximations to the actual properties, hence, the predicted quantities do not necessarily bound the actual properties.

Because of the above mentioned difficulties with the energy approach it is the authors' opinion that a greater degree of accuracy can be achieved with the direct approach than with the energy approach. It is also the authors' opinion that the desirability of accuracy overshadows the disadvantage of, in general, not being able to say whether the magnitude of the predicted properties are above or below the idealized properties. The direct approach has been employed in the following investigation of two reinforcing configurations.

UNIT CELL CONCEPT

If the reinforcing pattern is repeated a sufficiently large number of times the material can be considered to be homogeneous at the phenomenological level (or as an inhomogeneous material in which the properties vary gradually due to a gradual change in the reinforcing pattern). The consideration of the reinforced material as homogeneous at the phenomenological level is analogous to the consideration of a microscopically crystalline material as macroscopically homogeneous, etc. The reinforced material when viewed at the composite level will, in general, exhibit anisotropic behavior.

For a material that has a regular reinforcing pattern one can, in general, find a small unit of the material which when repeated in all directions results in the actual material configuration; in this report this fundamental building block is called the "unit cell". The average response of the unit cell to a homogeneous phenomenological state of stress or strain is the same as the composite response of the material. Hence, the desired composite properties may be calculated from a consideration of the behavior of the unit cell.

CONTINUOUS WIRE REINFORCED PROPELLANT

The composite material considered in this section is a propellant reinforced by the winding of continuous wires into it. The reinforcing pattern is idealized as successive layers of wire that are alternately inclined from the x_2 axis by plus and minus the wrap angle. Two such layers are shown in Figure 1. If the individual wire layers do not lie in planes then the x_1 , x_2 and x_3 axes are curvilinear (and the material will exhibit curvilinear anisotropy). The composite material is idealized as a succession of layers of the form shown in Figure 5; this idealization, of course, introduces a slight approximation at the surfaces of the grain. If the composite material is subjected to a homogeneous strain state the top, bottom and middle surfaces of the layer shown in Figure 5 are planes of deformation symmetry. Noting these planes of symmetry and the regularity of the reinforcing pattern in the x_1 and x_2 directions it is seen that the unit cell has the form shown in Figure 2. When the composite material is subjected to a homogeneous stress or strain state those cells which are sufficiently far removed from the surface of the material to avoid surface effects must satisfy the following conditions, 1) all unit cells will exhibit identical deformation and stress states, 2) the averages of the unit cell stresses and strains are equal to the phenomenological stresses and strains of the composite and 3) there must be continuity of the displacement and traction vectors across all cell interfaces. Consideration of these conditions lead to the descriptions of the unit cell boundary value problems and to the relating of the composite properties to their solutions.

The anisotropic phenomenological behavior of the composite material (i.e., the ideally reinforced propellant) may be described by the following equations:

Stress-Strain-Temperature Change Relationships:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & & & \\ A_{12} & A_{22} & A_{23} & & & \\ A_{13} & A_{23} & A_{33} & & & \\ & & & A_{44} & & \\ & & & & A_{55} & \\ & & & & & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} - \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T$$

Heat Conduction Equation:

$$\rho^* \, c^* \, \dot{T} = k_1 \, T_{,11} + k_2 \, T_{,22} + k_3 \, T_{,33} \tag{2}$$

Heat Flux Equation:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = - \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} T_{,1} \\ T_{,2} \\ T_{,3} \end{bmatrix} \tag{3}$$

The mechanical properties of the reinforced propellant will, of course, be viscoelastic due to the time dependent nature of the propellant properties. One possible way of considering the viscoelastic nature of the composite, for analysis purposes, is to use the correspondence principle to replace the viscoelasticity problem with a corresponding elasticity problem. In the corresponding elasticity problem the transforms of the viscoelastic properties appear as pseudoelastic properties (A_{ij} would then be interpreted as the transforms of the relaxation functions). To obtain the corresponding elastic properties, from the analysis described herein, use would be made of the transformed propellant shear and bulk relaxation moduli. The transforms of the composite properties are used in the solution of the corresponding elasticity problem; the viscoelasticity solution is found by employing one of the several available approximate inversion techniques. The A_{ij} quantities can also be interpreted as the integral operators which enter into the integral representation of the composite anisotropic viscoelastic stress-strain law; the integral representation is utilized in the step-by-step solution of viscoelasticity problems.

The composite properties (A_{ij} , Ψ_i , etc.) may be theoretically predicted by considering the average response of the unit cell to various homogeneous composite strain states (the strain in the unit cell, of course, will not be homogeneous). The determination of the average behavior of the unit cell requires the solution of a series of complicated three-dimensional boundary value problems. The most accurate means for obtaining solutions to these boundary value problems is the finite element method [4]; the use of the finite element procedure for this analysis, however, was not practical because of the excessive amount of computer time required. The alternative that was employed was to develop some rather simple approximate solutions to the boundary value problems. The error introduced from these approximate solutions is felt to be of the same order as that introduced by the several approximations that have been made to this point in the analysis. In the development of the approximate solutions to the unit cell boundary value problems the following conditions were noted: 1) The percentage of reinforcement is small (of the order of 15% by volume). 2) The shear stiffness of the propellant is much less than that of the wire. 3) The shear stiffness of the propellant is much less than its bulk stiffness. The composite properties were predicted by successively considering a number of simple composite strain and temperature states and approximately calculating the average response of the unit cell (the descriptions of the unit cell boundary value problems may be developed in a manner similar to that employed in [3]). A detailed description of the first of these considerations is given below; the solutions for all the unit cell boundary value problems are tabulated in Appendix A and computer programs written for their evaluation are given in Appendix B.

Consider the following composite strain state:

$$\varepsilon_2 = \varepsilon \quad , \quad \varepsilon_1 = \varepsilon_3 = \gamma_{12} = \gamma_{13} = \gamma_{23} = \Delta T = 0 \tag{4}$$

Equation (1) yields

$$A_{12} = \frac{\sigma_1}{\varepsilon} \tag{5}$$

$$A_{22} = \frac{\sigma_2}{\varepsilon} \tag{6}$$

$$A_{23} = \frac{\sigma_3}{\varepsilon} \tag{7}$$

The composite stress σ_1 is equal to the average of the unit cell normal stress (τ_{11}) acting on the unit cell face perpendicular to the x_1 axis (i.e., $\sigma_1 = \frac{1}{S_3 S_2} \int_0^{S_3} \int_0^{S_2} \tau_{11}\, dx_2\, dx_3$) , etc. Within the unit cell (Figure 2) the stress and strain states must satisfy a) the equilibrium equations, b) the strain displacement relationships and c) the constitutive equations for the constituent materials. The boundary conditions for this particular unit cell boundary value problem (i.e., the boundary value problem corresponding to the composite strain state given above) are ($i = 1,2,3$; τ_{ij} , u_i and ε_{ij} represent the local stress, displacement and strain components within the unit cell):

$$u_i(x_1,0,x_3) = u_i(x_1,S_2,x_3) - \delta_{i2}\varepsilon S_2 \tag{8}$$

$$u_i(x_1,0,x_3) = -(1 - 2\delta_{2i})u_i(S_1 - x_1,0,S_3 - x_3) \tag{9}$$

$$\int_0^{S_3} \int_0^{S_1} u_i(x_1,0,x_3)\, dx_1\, dx_3 = 0 \tag{10}$$

$$\tau_{2i}(x_1,0,x_3) = \tau_{2i}(x_1,S_2,x_3) \tag{11}$$

$$\tau_{2i}(x_1,0,x_3) = -(1 - 2\delta_{2i})\tau_{2i}(S_1 - x_1,0,S_3 - x_3) \tag{12}$$

$$\int_0^{S_3} \int_0^{S_1} \tau_{2i}(x_1,0,x_3)\, dx_1\, dx_3 = \delta_{2i}\sigma_2 S_1 S_3 \tag{13}$$

$$u_i(0,x_2,x_3) = u_i(S_1,x_2,x_3) \tag{14}$$

$$u_i(0,x_2,x_3) = -(1 - 2\delta_{1i})u_i(0,S_2 - x_2,S_3 - x_3) + \delta_{2i}\varepsilon S_2 \tag{15}$$

$$\int_0^{S_3} \int_0^{S_2} u_i(0,x_2,x_3)\, dx_2\, dx_3 = \delta_{i2}\,\frac{1}{2}\,\varepsilon(S_2)^2\, S_3 \tag{16}$$

$$\tau_{1i}(0,x_2,x_3) = \tau_{1i}(S_1,x_2,x_3) \tag{17}$$

$$\tau_{1i}(0,x_2,x_3) = -(1 - 2\delta_{1i})\tau_{1i}(0,S_2 - x_2,S_3 - x_3) \tag{18}$$

$$\int_0^{S_3} \int_0^{S_2} \tau_{1i}(0,x_2,x_3)\, dx_2\, dx_3 = \delta_{1i}\sigma_1 S_2 S_3 \tag{19}$$

$$u_3 = \tau_{13} = \tau_{23} = 0 \quad \text{for} \quad x_3 = 0,S_3 \tag{20}$$

$$\Delta T = 0 \quad \text{throughout the cell} \tag{21}$$

The stress-strain law for the matrix (propellant) is written in the form (τ_{ij}^{m} denotes stress in the matrix, etc.):

$$\tau_{ij}^{m} = \frac{3K - 2\mu}{3} \theta^{m}\delta_{ij} + 2\mu\varepsilon_{ij}^{m} - 3\alpha_{m}K\Delta T\delta_{ij} \tag{22}$$

where

$$\theta^{m} = \varepsilon_{kk}^{m} = \varepsilon_{11}^{m} + \varepsilon_{22}^{m} + \varepsilon_{33}^{m} \tag{23}$$

If the matrix is elastic K and μ respectively denote the bulk and shear moduli of the matrix, if it is viscoelastic their definitions depend upon the manner in which the time dependence of the analysis is being treated (for example if the correspondence principle is being employed they will represent the transforms of the viscoelastic bulk and shear properties). The stress-strain law for the wires is written in the form:

$$\varepsilon_{ij}^{w} = \frac{1}{E}\left[(1 + \nu)\tau_{ij}^{w} - \nu\tau_{kk}^{w}\delta_{ij}\right] + \alpha_{w}\Delta T\delta_{ij} \tag{24}$$

The symbols E and ν denote the elastic properties of the wire, see Appendix A.

The approximate solution to this unit cell boundary value problem is developed in the following steps:

1) Initially consider the following assumptions for the strain state within the matrix:

$$\varepsilon_{22}^{m*} = \varepsilon\frac{V}{V_m} \quad , \quad \varepsilon_{11}^{m*} = 0 \quad , \quad \varepsilon_{33}^{m*} = 0 \quad , \quad \varepsilon_{ij}^{m*} = 0 \quad i \neq j \tag{25}$$

2) The normal stress components τ_{11}^{m*} , τ_{22}^{m*} , τ_{33}^{m*} are calculated from Equation (22).

3) The companion assumptions for the wire are (the s and n directions are shown in Figure 2):

$$\varepsilon_{ss}^{w*} = \varepsilon \cos^2 \alpha \ , \ \tau_{33}^{w*} = \tau_{33}^{m*} \ , \ \tau_{nn}^{w*} = \tau_{nn}^{m*} \ , \ \tau_{ns}^{w*} = 0 \ ,$$

$$\tau_{n3}^{w*} = 0 \ , \ \tau_{s3}^{w*} = 0 \tag{26}$$

4) From Equation (24) the following quantities are computed:

$$\tau_{ss}^{w*} \ , \ \varepsilon_{33}^{w*} \ , \ \varepsilon_{nn}^{w*}$$

Assumptions 1) and 3) result in an approximate solution which does not maintain volume consistency between the composite strain state and the approximate unit cell strain state; the amount of the inconsistency is given by the expression

$$\delta = \varepsilon V - \theta_m^* V_m - \theta_w^* V_w \tag{27}$$

where

$$\theta_w^* = \varepsilon_{33}^{w*} + \varepsilon_{ss}^{w*} + \varepsilon_{nn}^{w*}$$

The symbols V , V_m and V_w denote the total volume of the unit cell, the portion of the unit cell volume occupied by propellant, etc. This discrepancy could be ignored by assuming that the matrix and/or the wire experiences an additional volume change of such a magnitude as to eliminate the inconsistency, however, this volume change would be accompanied by rather large stresses as both the matrix and the wire are very resistant to dilatation; the resulting stresses would not be negligible. To avoid this difficulty the final stress and strain states are obtained by adding to the first approximate solution a second such that the volume inconsistency is eliminated. The development of the second approximation has as its basis the condition that the shear stiffness of the matrix is much less than its bulk stiffness. The resulting total solution does not satisfy all the displacement boundary and interface conditions, however, they could be satisfied by adding a third solution which consists of rearranging the matrix by a distortional strain state; as the propellant is relatively soft in shear it is assumed that the resulting distortional stress field is negligible.

5) Within the matrix the following additional stresses are assumed:

$$\tau_{11}^{m**} = \beta_1 \ , \ \tau_{22}^{m**} = \beta_1 \ , \ \tau_{33}^{m**} = \beta_1 \ , \ \tau_{ij}^{m**} = 0 \qquad i \neq j$$

6) The additional normal strains ε_{11}^{m**} , ε_{22}^{m**} , ε_{33}^{m**} are calculated from Equation (22). Note:

$$\theta^{m**} = \varepsilon_{11}^{m**} + \varepsilon_{22}^{m**} + \varepsilon_{33}^{m**}$$

7) The following additional assumptions are made for the wire:

$$\varepsilon_{ss}^{w**} = 0 \ , \ \tau_{33}^{w**} = \beta_1 \ , \ \tau_{nn}^{w**} = \beta_1 \ , \ \tau_{ns}^{w**} = \tau_{3s}^{w**} = \tau_{3n}^{w**} = 0$$

8) The following quantities are calculated from Equations (24).

$$\tau_{ss}^{w**} , \varepsilon_{33}^{w**} , \varepsilon_{nn}^{w**}$$

9) The quanity β_1 is calculated such that the volume inconsistency vanishes, i.e., from the equation:

$$\delta = \theta^{m**} V_m + \theta^{w**} V_w$$

10) The final approximate solution is obtained by adding the above two solutions, i.e.,

$$\tau_{22}^{m} = \tau_{22}^{m*} + \tau_{22}^{m**} \qquad \text{etc.}$$

and $$\sigma_3 = \frac{1}{S_1 S_2} \int_0^{S_2} \int_0^{S_1} \tau_{33}^{m} \, dx_1 \, dx_2 \qquad \text{etc.}$$

The final results of this solution are given in Appendix A.

SCREEN REINFORCED PROPELLANT

The composite material considered in this section consists of a solid propellant reinforced by wire screen. In the analysis it is assumed that one set of the screen wires are straight and that the other set are warped, Figure 3. The ideally reinforced material is defined such that if a section is passed through the material perpendicular to the straight screen wires and containing the centerline of one of the warped wires the configuration shown in Figure 6 would be revealed. The unit cell for the ideally reinforced material is shown in Figure 4.

When viewed at the composite level the screen reinforced material is governed by Equations (1,2,3). The composite properties are predicted by considering the average response of the unit cell to various homogeneous composite strain states (see the previous section). Because of the complexity of the unit cell it was necessary to be content with approximate solutions to the unit cell boundary value problems. In the development of these approximate solutions it was assumed 1) that the volume percentage of reinforcement is small ($\leq$ 15%), 2) that the shear stiffness of the propellant is much smaller than its bulk stiffness and much smaller than the shear stiffness of the wire, 3) that the value of R_1 is smaller than R_2 and 4) that the unit cell centerline length "ℓ_s" of the small wire could be approximated by the following expression:

$$\ell_s = 2\left\{\sqrt{\left(\frac{S_1}{2}\right)^2 - (R_1 + R_2)^2} + (R_1 + R_2)\sin^{-1}\left[\frac{2(R_2 + R_1)}{S_1}\right]\right\} \quad (28)$$

The approximate solutions to the unit cell boundary value problems were derived in much the same manner as was the solution considered in the previous section; the solutions and accompanying computer program are given in Appendices A and B.

EXAMPLES AND COMPARISONS

The following example is presented as an illustration of the character of the results obtained from the analysis. The composite properties were predicted for a typical propellant which was in the first case reinforced with wire screen and in the second with continuous wires; each of the two hypothetical composite materials had 10% reinforcement by volume, the wrap angle for the second material was 20°. For the sake of brevity the screen reinforced propellant is referred to as the "S" material and the continuous wire reinforced propellant as the "CW" material. The following typical properties for propellant and wire were utilized in the analysis:

MATRIX-

BULK MODULUS=450000.

SHEAR MODULUS=200.

THERMAL CONDUCTIVITY=5.00

THERMAL COEFFICIENT OF EXPANSION=0.0000500

DENSITY=0.0634

SPECIFIC HEAT=0.281

WIRE-

YOUNG MODULUS=10300000.

POISSON RATIO=0.330

THERMAL CONDUCTIVITY=810.

THERMAL COEFFICIENT OF EXPANSION=0.0000130

DENSITY=0.0950

SPECIFIC HEAT=0.210

For the S material the following additional quantities were utilized:

CELL DIMENSIONS

S_1=0.0185

S_2=0.0379

S_3=0.0120

WIRE RADII

R_1=0.0013

R_2=0.0025

For the CW material the following properties were utilized:

CELL DIMENSIONS

S_1=0.0348

S_2=0.0957

S_3=0.0060

WIRE RADIUS=0.0025

WRAP ANGLE=20.0°

The predicted properties (the nomenclature is explained in Appendix B) are listed in Figure 7. The stress-strain law may be inverted to yield the strain-stress law in the form:

$$[\varepsilon] = [C][\sigma] + [\alpha][\Delta T]$$

where $[C] = [A]^{-1}$ etc.

The inverse properties for this example are listed in Figure 8.

From a consideration of the results of this example several interesting observations may be made. The S material has higher moduli (E_2 and E_1) in both the major and minor reinforcing directions than the CW material; this observation is explained by the fact that when the CW material is subjected to a uniaxial stress in either the x_2 or the x_1 directions it can deform relatively easily by developing a distortional strain state in the propellant which permits the wires to rotate instead of elongating. The wire rotation (in the CW material) produced by a positive strain in the x_2 direction results in a large negative strain in the x_1 direction (in fact $-\varepsilon_1 > \varepsilon_2$) and, hence, a value of Poisson's ratio (ν_{12}) much greater than one; in order that the overall volume change of the propellant will be small the material must experience an accompaning positive strain in the x_3 direction and as a result a negative Poisson's ratio (ν_{32}) (the same effect is present to a lesser degree in ν_{23} and ν_{13}). The rotation of the wires can, for certain configurations, result in negative coefficients of thermal expansion for the x_2 and x_3 directions. A relatively large value for the modulus (E_3) in the direction perpendicular to the layers of reinforcement is predicted for both materials. This prediction may be explained by the fact that when the composite is subjected to a stress in the x_3 direction in order for elongation to take place the propellant needs to either decrease its x_1 and x_2 dimensions (Poisson's effect) or it must experience a large dilatation. A decrease of the x_1 and x_2 dimensions, of course, also requires the shortening of the reinforcing wires. Both shortening of the wires and propellant dilatation are strongly resisted, hence, a high modulus results.

The second example that is considered is the filament-resin system analyzed in reference [3]. Essentially exact solutions were obtained for the unit cell boundary value problems in reference [3], hence, the predicted properties are "exact" (i.e., they are the properties of the ideally reinforced material). The composite material configuration (square packed parallel fibers) considered in [3] is somewhat simpler than the configurations considered herein, thus, permitting "exact" solution of the unit cell boundary value problems. The screen reinforced propellant configuration becomes identical to that considered in [3] when $R_1 = 0$ and $S_1 = S_3$, hence, the equations developed herein may be used to approximately predict the filament-resin properties given in [3]. It should be noted that the system considered in [3] does not satisfy some of the conditions upon which the approximate solutions derived herein are based, hence, the prediction of the properties of [3] represents a very severe test of the accuracy of this analysis. The conditions which are not satisfied by the filament-resin system are a) that the percentage of reinforcement be small and b) that the ratio of the shear stiffness of the matrix to that of the filaments should be extremely small. A comparison of the predicted properties to those given in [3] is presented in Figure 9. (μ_w = wire shear modulus, $\psi_w = \alpha_w \frac{1 + \nu}{1 - 2\nu}$). The agreement is quite satisfactory.

The third example consists of a consideration of some of the experimental results given in reference [5]. In reference [5] an account is given of an experiment developed to evaluate some of the orthotropic properties of reinforced propellants. The test consisted of the simultaneous inflation and elongation of a thin-walled cylinder of continuous wire wrapped reinforced propellant. Measurements were made of the circumferential and axial growth, internal pressure and total axial load.

From the measured results the values of four of the orthotropic properties were calculated. The results, as the authors noted, were not entirely consistent as they did not satisfy the symmetry requirement of the stress-strain law. Because of this inconsistency it is not possible to have complete agreement between the predicted properties (which, of course, satisy the reciprocity requirement) and the experimental results. The experimental results reported in [5] are given in Figure 10; the values in parentheses were measured for a compressive stress field, the other values were measured for a tensile stress field. The symmetry requirement

$$-\frac{\nu_{\theta z}}{E_z} = C_{\theta z} = C_{z\theta} = -\frac{\nu_{z\theta}}{E_\theta}$$

yields

$$C_{\theta z} = -1.08 \times 10^{-5} \neq -3.62 \times 10^{-5} = C_{z\theta}$$

The predicted properties vary somewhat depending upon the values selected for the layer thickness S_3 (Figure 2) and for the bulk modulus of the propellant. No substantial effort was made to select these parameters so as to exactly duplicate any of the experimental results, however, several different combinations of values of the properties were considered; the results that gave the best overall agreement are presented in Figure 10. The properties that were used for the predictions are listed below:

CELL DIMENSIONS

S_1=0.0630

S_2=0.0880

S_3=0.0055

WIRE RADIUS=0.0025

WRAP ANGLE=35.3°

MATRIX-

BULK MODULUS=700000.

SHEAR MODULUS=1170.

DENSITY=0.062

WIRE-

YOUNG MODULUS=10500000.

POISSON RATIO=0.330

DENSITY=0.0980

The agreement between the moduli is quite good. The agreement between the values of the Poisson's Ratios, although showing the same trends leave something to be desired; the lack of agreement, of course, can not be avoided because of the fact that the experimental results do not satisfy the reciprocity requirement. It is interesting to note that the average of the terms $C_{z\theta}$ and $C_{\theta z}$ as determined by the experiment is -2.35×10^{-5} which is rather close to the value of -2.41×10^{-5} predicted by the analysis.

Conclusions

Procedures and accompanying computer programs were developed for the prediction of the composite properties of two types of reinforced solid propellant. An example was given to illustrate the characteristics of reinforced propellant properties. From the results of this example it was observed that the screen reinforcement is more effective in the major and minor reinforcement directions than is the continuous wire wrapped reinforcement. It was also observed that a very considerable degree of reinforcing is achieved in the direction normal to the reinforcement layer. Comparisons were made of the predicted properties to theoretical and experimental results; the comparisons indicate that the procedure presented herein gives reliable estimates of the composite properties, however, before complete reliability may be established a considerable number of additional comparisons will need to be made.

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APPENDIX A

EQUATIONS FOR PREDICTION OF COMPOSITE PROPERTIES

1. Notation:

α	Wrap angle
α_w	Wire thermal coefficient of linear expansion
α_m	Matrix thermal coefficient of linear expansion
c	Wire specific heat
$\bar{c}$	Matrix specific heat
E	Young's modulus of the wire
γ_v	Percent reinforcement by volume
K	Bulk modulus of matrix
k	Wire thermal conductivity
$\bar{k}$	Matrix thermal conductivity
μ	Matrix shear modulus (for viscoelastic materials, special consideration must be given to this value; see the main body of the report)
ν	Poisson's ratio of the wire
ρ	Density of the wire
$\bar{\rho}$	Density of the propellant
S_1, S_2, S_3	Cell dimensions (see main body of the report)
$V =$	$S_1 S_2 S_3$
$V_m =$	$(1 - \gamma_v) V$
$V_w =$	$\gamma_v V$

2. Wire Reinforced Solid Propellant:

$A_W = \pi R^2$

R = Wire radius

$d = \sqrt{A_W}$

$$A_{12} = \frac{A_W}{S_2 S_3}\left\{ E \cos^2\alpha + 2\nu\left\{ \frac{V}{V_m}\left[K + \mu\left(\frac{1}{3} - \cos^2\alpha\right)\right] + \beta_1 \right\} \right\} \sin\alpha$$

$$+ \frac{V}{V_m}\left(K - \frac{2}{3}\mu\right) + \beta_1 \qquad (A-1)$$

$$A_{22} = \frac{A_W}{S_1 S_3}\left\{ E \cos^2\alpha + 2\nu\left\{ \frac{V}{V_m}\left[K + \mu\left(\frac{1}{3} - \cos^2\alpha\right)\right] + \beta_1 \right\} \right\} \cos\alpha$$

$$+ \frac{V}{V_m}\left(K + \frac{4}{3}\mu\right) + \beta_1 \qquad (A-2)$$

where

$$\beta_1 = \frac{V(1-3K) - (1-2\nu) V_m \left[\cos^2\alpha + \frac{2(1+\nu)}{E} \frac{V}{V_m}\left(K + \frac{1}{3}\mu - \mu \cos^2\alpha\right)\right]}{\frac{V_m}{K} + \frac{2 V_W (1+\nu)(1-2\nu)}{E}} \qquad (A-3)$$

The value for A_{11} is obtained from Equation (A-1) when S_1 is replaced by S_2, α by $(\frac{\pi}{2} - \alpha)$ and S_2 by S_1.

$$A_{13} = \frac{1}{S_1 S_2}\left\{ \left(K - \frac{2}{3}\mu\right)\left[S_1 S_2 - \frac{d}{\cos\alpha}\left(2S_2 - \frac{d}{\sin\alpha}\right)\right] \right.$$

$$+ \frac{1}{C_2}\frac{d}{\cos\alpha}\left(S_2 - \frac{d}{\sin\alpha}\right)\left[(2S_3 - d)\left(K - \frac{2}{3}\mu\right) + \frac{d\cdot\nu}{1-\nu}\left(K + \frac{4}{3}\mu\right)\right]$$

$$\left. + \frac{1}{C_3}\frac{A_W}{\sin\alpha \cos\alpha}\left[(S_3 - d)\left(K - \frac{2}{3}\mu\right) + \frac{d\cdot\nu}{1-\nu}\left(K + \frac{4}{3}\mu\right)\right] \right\} \qquad (A-4)$$

$$A_{23} = A_{13} \tag{A-5}$$

$$A_{33} = \frac{K+\frac{4}{3}\mu}{S_1 S_2}\left\{ S_1 S_2 - \frac{d}{\cos\alpha}\left(2S_2 - \frac{d}{\sin\alpha}\right) + \frac{S_3 d}{\cos\alpha}\left[\frac{2}{C_2}\left(S_2 - \frac{d}{\sin\alpha}\right) + \frac{1}{C_3}\frac{d}{\sin\alpha}\right]\right\} \tag{A-6}$$

where

$$C_2 = S_3 + \frac{d}{2}\left[\frac{(K+\frac{4}{3}\mu)(1+\nu)(1-2\nu)}{(1-\nu)E} - 1\right]$$

$$C_3 = S_3 + d\left[\frac{(K+\frac{4}{3}\mu)(1+\nu)(1-2\nu)}{(1-\nu)E} - 1\right]$$

$$A_{44} = \frac{1}{S_1 S_3}\left\{ A_W E \cos\alpha \sin^2\alpha + \mu\left[\frac{S_1 S_2 d}{(S_2 - \frac{d}{\sin\alpha})} + S_1(S_3 - d)\right]\right\} \tag{A-7}$$

$$A_{55} = \mu \frac{V}{V_m} \tag{A-8}$$

$$A_{66} = \frac{\mu}{S_1 S_2}\left\{ S_1 S_2 - \frac{d}{\cos\alpha}\left(2S_2 - \frac{d}{\sin\alpha}\right) + \frac{d\, S_3}{\cos\alpha}\left\{ \frac{2(S_2 - \frac{d}{\sin\alpha})}{S_3 + \left[\frac{3\mu(1+\nu)}{E} - 1\right]\frac{d}{2}} + \frac{\frac{d}{\sin\alpha}}{S_3 + \left[\frac{2\mu(1+\nu)}{E} - 1\right] d}\right\}\right\} \tag{A-9}$$

$$\psi_1 = \frac{A_W}{S_2 S_3}\left[(3K\alpha_m - \beta_2)\, 2\nu + E\alpha_w\right] \sin\alpha + 3K\alpha_m - \beta_2 \tag{A-10}$$

$$\psi_2 = \frac{A_W}{S_1 S_3}\left[(3K\alpha_m - \beta_2)\,2\nu + E\alpha_W\right]\cos\alpha + 3K\alpha_m - \beta_2 \qquad (A\text{-}11)$$

$$\psi_3 = 3K\alpha_m - \beta_2 \qquad (A\text{-}12)$$

where

$$\beta_2 = \frac{2(1+\nu)\left[\frac{3K\alpha_m}{E}(1-2\nu) - \alpha_W\right]V_W}{\frac{V_m}{K} + \frac{2V_W(1+\nu)(1-2\nu)}{E}} \qquad (A\text{-}13)$$

$$k_1 = \left(1-\frac{d}{S_3}\right)\bar{k} + \frac{(S_2-d)\,d}{S_2 S_3}\cdot\frac{S_1\bar{k}\,k}{S_1 k + \frac{2R}{\cos\alpha}(\bar{k}-k)} + \frac{A_W \sin\alpha}{S_2 S_3}k \qquad (A\text{-}14)$$

$$k_2 = \left(1-\frac{d}{S_3}\right)\bar{k} + \frac{(S_1-d)\,d}{S_1 S_3}\,\frac{S_2\bar{k}\,k}{S_2 k + \frac{2R}{\sin\alpha}(\bar{k}-k)} + \frac{A_W \cos\alpha}{S_1 S_3}k \qquad (A\text{-}15)$$

$$k_3 = \frac{\bar{k}}{S_1 S_2}\left\{\frac{2A_W S_3}{\left[S_3-\left(1-\frac{\bar{k}}{k}\right)d\right]\sin 2\alpha} + S_1 S_2 - \frac{2dS_2}{\cos\alpha} + \frac{2A_W}{\sin 2\alpha} + \frac{4S_3\left(\frac{dS_2}{\cos\alpha} - \frac{2A_W}{\sin 2\alpha}\right)}{2S_3 - \left(1-\frac{\bar{k}}{k}\right)d}\right\} \qquad (A\text{-}16)$$

$$C^* = \frac{\bar{c}\bar{\rho}V_m + c\rho V_W}{\bar{\rho}V_m + \rho V_W} \qquad (A\text{-}17)$$

$$\rho^* = \frac{V_m\bar{\rho} + V_W\rho}{V} \qquad (A\text{-}18)$$

3. Screen Reinforced Solid Propellant:

R_1 and R_2 denote the radii of the screen wires (see Fig. 4)

$$A_1 = \pi R_1^2 \tag{A-19}$$

$$A_2 = \pi R_2^2 \tag{A-20}$$

$$V_2 = A_2 S_2 \tag{A-21}$$

$$V_1 = V - V_2 - V_m \tag{A-22}$$

ℓ is defined by equation (28)

$$\beta = \sin^{-1}\left(\frac{R_1 + R_2}{S_1/2}\right)$$

$$\phi = 2\left[1 - \frac{2\mu(1+\nu)}{E}\right] \tag{A-23}$$

$$A_{12} = \frac{A_1}{S_2 S_3}\left[K(2\nu-1) + \frac{2}{3}\mu(\nu+1) + \left(\frac{6K-\mu}{3K+\mu}\nu - 1\right)\beta_3\right]$$
$$+ K - \frac{2}{3}\mu + \beta_3 \tag{A-24}$$

$$A_{22} = \frac{A_2}{S_1 S_3}\left[K(2\nu-1) - \frac{4}{3}\mu(\nu+1) + \left(2\nu - \frac{3K-2\mu}{3K+\mu}\right)\beta_3 + E\right]$$
$$+ K + \frac{4}{3}\mu + \left(\frac{3K-2\mu}{3K+\mu}\right)\beta_3 \tag{A-25}$$

$$A_{32} = K - \frac{2}{3}\mu + \beta_3 \tag{A-26}$$

where

$$\beta_3 = \frac{3K+\mu}{3} \cdot \frac{3(V-V_m)E - 2(1+\nu)(1-2\nu)\left\{(3K+\mu)V_1 + \left[3K - 2\mu + \frac{3E}{2(1+\nu)}\right]V_2\right\}}{3EV_m + (1+\nu)(1-2\nu)\left[(6K-\mu)V_1 + (6K+2\mu)V_2\right]} \tag{A-27}$$

$$A_{33} = \frac{2 S_2 S_3}{V_m + V_2}\left(K + \frac{4}{3}\mu\right)\left\{\frac{S_1}{2} - R_2 + \frac{S_3}{C_1}\left[\frac{\frac{\pi}{2} + \sin^{-1}\left(\frac{C_1 R_2}{S_3}\right)}{\sqrt{1 - \left(\frac{C_1 R_2}{S_3}\right)^2}} - \frac{\pi}{2}\right]\right\} \tag{A-28}$$

$$A_{13}^{*} = \frac{S_2}{V_m+V_2}\left\{(K-\tfrac{2}{3}\mu)\left[(S_1-\sqrt{A_2})\,S_3 + \frac{\sqrt{A_2}\,(S_3-\sqrt{A_2})}{1-\frac{C_1\sqrt{A_2}}{2S_3}}\right] + \frac{\nu}{1-\nu}(K+\tfrac{4}{3}\mu)\left(\frac{A_2}{1-\frac{C_1\sqrt{A_2}}{2S_3}}\right)\right\} \quad \text{(A-29)}$$

$$A_{11} = \left\{1-\frac{2R_2}{S_3}+\frac{2S_1}{C_1S_3}\left[\frac{\frac{\pi}{2}+\sin^{-1}\left(\frac{C_1R_2}{S_1}\right)}{\sqrt{1-\left(\frac{C_1R_2}{S_1}\right)^2}}-\frac{\pi}{2}\right]\right\}(K+\tfrac{4}{3}\mu)\left(1-\frac{A_1}{S_2S_3}\right) + \frac{A_1}{S_2S_3}\left[\frac{ES_1\cos\beta}{\ell}+2\nu(K-\tfrac{2}{3}\mu)\left(\frac{1}{1-\frac{C_1R_2}{S_1}}\right)\right] \quad \text{(A-30)}$$

$$A_{31}^{*} = \frac{1}{S_1S_3}\left\{(K-\tfrac{2}{3}\mu)\left[(S_3-\sqrt{A_2})\,S_1 + \frac{A_2\,(S_1-\sqrt{A_2})}{1-\frac{C_1\sqrt{A_2}}{2S_1}}\right] + \left(\frac{\nu}{1-\nu}\right)(K+\tfrac{4}{3}\mu)\left(\frac{A_2}{1-\frac{C_1\sqrt{A_2}}{2S_1}}\right)\right\} \quad \text{(A-31)}$$

where

$$C_1 = 2\left[1-\frac{(1+\nu)(1-2\nu)(K+\frac{4}{3}\mu)}{E(1-\nu)}\right]$$

$$A_{13} = \tfrac{1}{2}\left(A_{13}^{*}+A_{31}^{*}\right) \quad \text{(A-32)}$$

$$A_{44} = \frac{2\mu S_1S_2}{V_m+V_2}\left\{\frac{S_3}{2}-R_2+\frac{S_1}{\phi}\left[\frac{\frac{\pi}{2}+\sin^{-1}\left(\frac{\phi R_2}{S_1}\right)}{\sqrt{1-\left(\frac{\phi R_2}{S_1}\right)^2}}-\frac{\pi}{2}\right]\right\} \quad \text{(A-33)}$$

$$A_{55} = \mu\,\frac{V}{V_m} \quad \text{(A-34)}$$

$$A_{66} = \frac{2\mu S_2S_3}{V_m+V_2}\left\{\frac{S_1}{2}-R_2+\frac{S_3}{\phi}\left[\frac{\frac{\pi}{2}+\sin^{-1}\left(\frac{\phi R_2}{S_3}\right)}{\sqrt{1-\left(\frac{\phi R_2}{S_3}\right)^2}}-\frac{\pi}{2}\right]\right\} \quad \text{(A-35)}$$

$$\psi_1 = \frac{A_1}{S_2 S_3}\left\{\left[\frac{3K(1-2\nu)+\mu(1+\nu)}{3K+\mu}\right]\beta_4 - (3\alpha_m K)(1-2\nu) + \alpha_W E\right\} + 3\alpha_m K - \beta_4 \qquad \text{(A-36)}$$

$$\psi_2 = \frac{A_2}{S_1 S_3}\left\{3\alpha_m K(2\nu-1) + \left[\frac{3K(1-2\nu)-3\mu(1+\nu)}{3K+\mu}\right]\beta_4 + \alpha_W E\right\} + 3\alpha_m K - \frac{3K-2\mu}{3K+\mu}\beta_4 \qquad \text{(A-37)}$$

$$\psi_3 = 3\alpha_m K - \beta_4 \qquad \text{(A-38)}$$

where

$$\beta_4 = \frac{2(3K+\mu)\left[3\alpha_m K(1+\nu)(1-2\nu) - \alpha_W E \nu\right](V_1+V_2)}{3E V_m + (1+\nu)(1-2\nu)\left[(6K-\mu)V_1 + (6K+2\mu)V_2\right]} \qquad \text{(A-39)}$$

$$k_1 = \bar{k}\left(1-\frac{\sqrt{A_2}}{S_3}\right) + \frac{A_1}{S_2 S_3}\left[\left(\frac{S_1}{\ell}\right)k - \bar{k}\right] + \frac{\sqrt{A_2}}{S_3}\left[\frac{S_1 \bar{k} k}{\sqrt{A_2}(\bar{k}-k)+S_1 k}\right] \qquad \text{(A-40)}$$

$$k_2 = \bar{k}\left(1-\frac{\sqrt{A_1}\,\ell}{S_1 S_3}\right) + \frac{A_2}{S_1 S_3}(k-\bar{k}) + \frac{\sqrt{A_1}\,\ell}{S_1 S_3}\left[\frac{S_2 \bar{k} k}{\sqrt{A_1}(\bar{k}-k)+S_2 k}\right] \qquad \text{(A-41)}$$

$$k_3 = \bar{k}\left(1-\frac{\sqrt{A_2}}{S_1}-\frac{\sqrt{A_1}}{S_2}-\frac{A_1}{S_1 S_2}\right) + \frac{1}{S_1 S_2}\left[\frac{A_1 S_3 \bar{k} k}{\ell \bar{k} + (S_3-2R_2-4R_1)k}\right]$$

$$+ \frac{1}{S_1}\left[\frac{\sqrt{A_2}\, S_3 \bar{k} k}{\sqrt{A_2}(\bar{k}-k)+S_3 k}\right] + \frac{1}{S_2}\left[\frac{\sqrt{A_1}\, S_3 \bar{k} k}{\sqrt{A_1}(\bar{k}-k)+S_3 k}\right] \qquad \text{(A-42)}$$

The properties c^* and ρ^* may be calculated from equations (A-17) and (A-18).

APPENDIX B

USER'S MANUALS FOR COMPOSITE PROPERTY PROGRAM

USER'S MANUAL FOR WIRE REINFORCED PROPELLANT PROPERTY PROGRAM (WRG)

The program (FORTRAN IV) described herein is an evaluation of the equations that have been developed for the prediction of the composite properties of wire reinforced solid propellants. The development of the equations is discussed in the main body of this report. The predictions are based upon the unit cell concept. The unit cell is the smallest fundamental building block of the reinforced material. A two-dimensional view of a layer of the reinforced propellant is shown in Figure 1; the unit cell is indicated by heavy lines. A sketch of the three-dimensional geometry of the unit cell is shown in Figure 2; the dimensions of the unit cell are denoted by S_1 , S_2 , S_3 . The governing equations for the phenomenological behavior of the composite material are expressed in the form:

Stress-Strain Law

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & & & \\ A_{12} & A_{22} & A_{23} & & & \\ A_{13} & A_{23} & A_{33} & & & \\ & & & A_{44} & & \\ & & & & A_{55} & \\ & & & & & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} - \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T$$

Heat Conduction Equation

$$\rho^* \, c^* \, \dot{T} = k_1 \, T_{,11} + k_2 \, T_{,22} + k_3 \, T_{,33}$$

Heat Flux Equation

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = - \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} T_{,1} \\ T_{,2} \\ T_{,3} \end{bmatrix}$$

The composite quantities, i.e., stress, strain, temperature, density, specific heat and heat flux are denoted respectively by σ, ε and γ, Ψ, T, ρ^*, c^*, and q .

For structural analysis purposes it is usually sufficient to have a knowledge of the stress-strain law (i.e., $[\sigma] = [A][\varepsilon] - [\Psi] \, T$), however, the strain-stress law is often of more value as an aid to the understanding of the composite behavior of the material; the strain-stress law may be written in the following form:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} + \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T$$

The [C] matrix may be determined by inverting the [A] matrix.

The composite properties depend upon the geometric arrangement of the reinforcing wires, and upon the properties of the wire and of the propellant. In this analysis the reinforcement is considered to be in the form of layers of wires alternately making angles of plus and minus α with respect to the x_2 axis, see Figure 1. The wire spacing is specified by the cell dimensions S_1, S_2 and S_3, Figure 2. The properties of the wire and of the propellant which enter into the various composite property predictions are listed in the description of the program input which is given below.

The following cards are needed as input to the program:

1. Title Card (12A6) - Columns 1 to 72
2. Geometry Data Card (5F10.0)

Columns 1 - 10	S_1 (in.)	Cell Dimensions
11 - 20	S_2 (in.)	Cell Dimensions
21 - 30	S_3 (in.)	Cell Dimensions
31 - 40	R (in.), Wire Radius	
41 - 50	α (Degrees), Wrap Angle	

3. Propellant Data Card (6F10.0)

Columns 1 - 10 Bulk Modulus (psi)

11 - 20 Shear Modulus (psi), see comments below

21 - 30 Thermal Conductivity (Btu-in/hr.-ft^2-^{o}F)

31 - 40 Thermal Coefficient of Linear Expansion (in/in-^{o}F)

41 - 50 Density (lb/in^3)

51 - 60 Specific Heat (Btu/lb-^{o}F)

4. Wire Data Card (6F10.0)

Columns 1 - 10 Young's Modulus (psi)

11 - 20 Poisson's Ratio

21 - 30 Thermal Conductivity (Btu-in/hr.-ft^2-^{o}F)

31 - 40 Thermal Coefficient of Linear Expansion (in/in-^{o}F)

41 - 50 Density (lb/in^3)

51 - 60 Specific Heat (Btu/lb-^{o}F)

The mechanical properties of the reinforced propellant will, of course, be viscoelastic due to the time dependent nature of the propellant properties. One possible way of considering the viscoelastic nature of the composite, for analysis purposes, is to use the correspondence principle to replace the viscoelasticity problem with a corresponding elasticity problem. In the corresponding elasticity problem the transforms of the viscoelastic properties appear as pseudo-elastic properties. These corresponding elastic properties, for the composite material, can be obtained from the program described herein by utilizing as input the appropriate values of the transformed propellant shear and bulk relaxation moduli (these values are called "shear" and "bulk" moduli in the program). The resulting composite properties are then used in the solution of the corresponding elasticity problem; the viscoelasticity solution is found by employing one of the several available approximate inversion techniques.

The program can also be used to predict numerical values of the kernels which enter into the integral representation of the composite anisotropic viscoelastic stress-strain law; the integral representation is utilized in the step-by-step solution of viscoelasticity problems.

The output of the program consists of the composite properties A_{ij} , k_i , Ψ_i , ρ^* , c^* , α_i and C_{ij} (note: C_{44} is the reciprocal of A_{44} etc.). Lastly the values of a number of functions of the A_{ij} matrix are printed. These functions, in addition to the diagonal terms of the A_{ij} matrix, must be positive in order that the resulting stress-strain law should be derivable from a positive definite strain energy function.

The listing for the program is given below:

```
C     ******************************************************
C     *  PROGRAM FOR COMPUTING THE COMPOSITE PROPERTIES    *
C     *  OF WIRE REINFORCED PROPELLANTS                    *
C     ******************************************************


      DIMENSION TITLE(12)
      REAL MUM, NUR, KM, K1, K2, K3
  100 READ (5,800) TITLE
  800 FORMAT (12A6)
      WRITE (6,801) TITLE
  801 FORMAT (1H1,8X,12A6//)
C                  DATA CARD NO.1
      READ (5,110) S1, S2, S3, R, ALPHA,
C                  DATA CARD NO.2
     1             KM, MUM, TCM, ALPHAM, RHOM, CM,
C                  DATA CARD NO.3
     2             ER, NUR, TCR, ALPHAR, RHOR, CR
  110 FORMAT (5F10.0/6F10.0/6F10.0)
      WRITE (6,120) S1, S2, S3, R, ALPHA
  120 FORMAT (1X, 15HCELL DIMENSIONS//11X,3HS1=,F6.4/11X,3HS2=,F6.4/
     1    11X,3HS3=,F6.4/1H0,12HWIRE RADIUS=,F6.4/1H0,12HWRAP  ANGLE=,
     2    F6.2,10H (DEGREES))
      WRITE (6,130) KM, MUM, TCM, ALPHAM, RHOM, CM
  130 FORMAT (1H0, 7HMATRIX-//11X,13HBULK MODULUS=,F10.0/11X,14HSHEAR MO
     1DULUS=,F5.0/11X,21HTHERMAL CONDUCTIVITY=,F8.2/11X,33HTHERMAL COEFF
     2ICIENT OF EXPANSION=,F10.8/11X, 8HDENSITY=,F6.4/11X,14HSPECIFIC HE
     3AT=,F6.4)
      WRITE (6,140) ER, NUR, TCR, ALPHAR, RHOR, CR
  140 FORMAT (1H0,5HWIRE-//11X,14HYOUNG MODULUS=,F10.0/11X,14HPOISSON RA
     1TIO=,F5.3/11X,21HTHERMAL CONDUCTIVITY=,F8.2/11X,33HTHERMAL COEFFIC
     2IENT OF EXPANSION=,F10.8/11X,8HDENSITY=,F6.4/11X,14HSPECIFIC HEAT=
     3,F6.4//)
      SQAR = 1.7724539*R
      AR = SQAR*SQAR
      S1XS2 = S1*S2
```

```
      S1XS3 = S1*S3
      S2XS3 = S2*S3
      RAD = ALPHA/57.2957795
      SINA = SIN(RAD)
      COSA = COS(RAD)
      SIN2A = SINA*SINA
      COS2A = COSA*COSA
      SQ13 = SQAR/COSA
      SQ23 = SQAR/SINA
      VOL = S1XS2*S3
      VOLR = AR*S2/COSA
      VOLM = VOL-VOLR
      T1 = 1.+NUR
      T2 = 1.-2.*NUR
      T3 = T1*T2
      T4 = 1.-NUR
      T5 = KM - 0.66666667*MUM
      T6 = KM + 1.33333333*MUM
      T7 = KM + 0.33333333*MUM
      T8 = VOL/VOLM
      T9 = 3.*KM*ALPHAM
      T10 = ER*ALPHAR
      T11 = 1.-TCM/TCR
      T12 = VOLM/KM + 2.*T3*VOLR/ER
C   (A)
      SA = (-VOLR*T2*(COS2A+2.*T1*T8*(T7-MUM*COS2A)/ER))/T12
      T13 = ER*COS2A + 2.*NUR*(T8*(T7 - MUM*COS2A) + SA)
      A12 = AR*SINA*T13/S2XS3 + T8*T5 + SA
      A22 = AR*COSA*T13/S1XS3 + T8*T6 + SA
C   (B)
      SB = (-VOLR*T2*(SIN2A+2.*T1*T8*(T7-MUM*SIN2A)/ER))/T12
      T14 = ER*SIN2A + 2.*NUR*(T8*(T7 - MUM*SIN2A) + SB)
      A11 = AR*SINA*T14/S2XS3 + T8*T6 + SB
C   (C)
      T15 = T6*T3/ER/T4 - 1.0
      C2 = S3 + 0.5*SQAR*T15
```

```
      C3=C2*2.0-S3
      T16 = S1XS2 - SQ13*(2.*S2 - SQ23)
      A33 = T6*(T16 + S3*SQ13*(2.*(S2 - SQ23)/C2 + SQ23/C3))/S1XS2
      A13 = (T5*T16 + SQ13*((S2 - SQ23)*(T5*(2.*S3 - SQAR) + SQAR*NUR
     1    *T6/T4))/C2 + SQ13*SQ23*(T5*(S3-SQAR) + SQAR*NUR*T6/T4)/C3)
     2        /S1XS2
      A23 = A13
C   (D)
      A44 = (AR*ER*COSA*SIN2A + MUM*(S1XS2*SQAR/(S2 - SQ23)
     1        + S1*(S3 - SQAR)))/S1XS3
C   (E)
      A55 = MUM*T8
C   (F)
      T17 = SQAR*(2.*MUM*T1/ER - 1.0)
      A66 = MUM*(S1XS2 - SQ13*(2.*S2 - SQ23) + S3*SQ13*
     1        (2.*(S2- SQ23)/(S3 + 0.5*T17) + SQ23/(S3 + T17)))/S1XS2
C   (G)
      SG = 2.*T1*(T9*T2/ER - ALPHAR)*VOLR/T12
      T18 = AR*(2.*NUR*(T9 - SG) + T10)
      PSI1 = SINA*T18/S2XS3 + T9 - SG
      PSI2 = COSA*T18/S1XS3 + T9 - SG
      PSI3 = T9 - SG
C   (H)
      K1 = TCM*(1.- SQAR/S3) + SQAR*(S2 - SQAR)*S1*TCM/S2XS3/
     1        (S1 - 2.*R*T11/COSA) + AR*SINA*TCR/S2XS3
C   (I)
      K2 = TCM*(1.-SQAR/S3) + SQAR*(S1 - SQAR)*S2*TCM/S1XS3/
     1        (S2 - 2.*R*T11/SINA) + AR*COSA*TCR/S1XS3
C   (J)
      T19 = T11*SQAR
      K3 = TCM*(SQ13*SQ23*S3/(S3 - T19) + S1XS2 - 2.*S2*SQ13
     1        + SQ13*SQ23 + 4.*S3*SQ13*(S2 - SQ23)/(2.*S3 - T19))/S1XS2
C   (K)
      RHOVOL = RHOM*VOLM + RHOR*VOLR
      RHO = RHOVOL/VOL
```

```
C   (L)
      CC= (CM*RHOM*VOLM + CR*RHOR*VOLR)/RHOVOL
C
      A21=A12
      A31=A13
      A32=A23
  103 COF11 = A22*A33 - A32*A23
      COF12 = A32*A13 - A12*A33
      COF13 = A12*A23 - A22*A13
      COF21 = A31*A23 - A21*A33
      COF22 = A11*A33 - A31*A13
      COF23 = A21*A13 - A11*A23
      COF31 = A21*A32 - A31*A22
      COF32 = A31*A12 - A11*A32
      COF33 = A11*A22 - A21*A12
      DETA = A11*COF11 + A21*COF12 + A31*COF13
      C11 = COF11/DETA
      C12 = COF12/DETA
      C13 = COF13/DETA
      C21 = COF21/DETA
      C22 = COF22/DETA
      C23 = COF23/DETA
      C31 = COF31/DETA
      C32 = COF32/DETA
      C33 = COF33/DETA
      ALPHA1 = PSI1*C11 + PSI2*C21 + PSI3*C31
      ALPHA2 = PSI1*C12 + PSI2*C22 + PSI3*C32
      ALPHA3 = PSI1*C13 + PSI2*C23 + PSI3*C33
      COF44=COF33*COF22-COF23*COF32
      COF55=COF22*COF11-COF12*COF12
      COF66=COF33*COF11-COF13*COF13
      IF (A11.LT.0.0 .OR. A22.LT.0.0 .OR. A33.LT.0.0 .OR. A44.LT.0.0
     1 .OR. A55.LT.0.0 .OR. A66.LT.0.0) WRITE (6,350)
      IF  (COF11.LT.0.0 .OR. COF22.LT.0.0 .OR. COF33.LT.0.0 .OR. COF44
```

```
   1.LT.0.0 .OR. COF55.LT.0.0 .OR. COF66.LT.0.0) WRITE (6,350)
350 FORMAT (1H0,41HERROR-STRAIN ENERGY NOT POSITIVE DEFINITE)
    WRITE (6,104) A11, A12, A13
104 FORMAT (1H0,20HPREDICTED PROPERTIES//11X,6HA11 = ,1PE12.6,5X,
   16HA12 = ,1PE12.6,5X,6HA13 = ,1PE12.6)
    WRITE (6,105) A21, A22, A23
105 FORMAT (1H0,10X,6HA21 = ,1PE12.6,5X,6HA22 = ,1PE12.6,5X,6HA23 = ,
   1         1PE12.6)
    WRITE (6,106) A31, A32, A33
106 FORMAT (1H0,10X,6HA31 = ,1PE12.6,5X,6HA32 = ,1PE12.6,5X,6HA33 = ,
   1         1PE12.6)
    WRITE (6,107) A44, A55, A66
107 FORMAT (1H0,10X,6HA44 = ,1PE12.6,5X,6HA55 = ,1PE12.6,5X,6HA66 = ,
   1         1PE12.6//)
    WRITE (6, 108) K1, K2, K3, RHO, CC
108 FORMAT (1H0, 10X, 5HK1 = ,1PE12.6,5X,5HK2 = ,1PE12.6,5X,5HK3 = ,
   1         1PE12.6,5X,6HRHO = ,1PE12.6,5X,4HC = ,1PE12.6)
    WRITE (6,109) PSI1, PSI2, PSI3
109 FORMAT (1H0,10X,7HPSI1 = ,1PE12.6,5X,7HPSI2 = ,1PE12.6,5X,
   1         7HPSI3 = ,1PE12.6)
    WRITE (6,200) C11, C12, C13
200 FORMAT (1H0,18HINVERSE PROPERTIES//11X,6HC11 = ,1PE12.6,5X,6HC12 =
   1 ,1PE12.6,5X,6HC13 = ,1PE12.6)
    WRITE (6,201) C21, C22, C23
201 FORMAT (1H0,10X,6HC21 = ,1PE12.6,5X,6HC22 = ,1PE12.6,5X,6HC23 = ,
   1         1PE12.6)
    WRITE (6,202) C31, C32, C33
202 FORMAT (1H0,10X,6HC31 = ,1PE12.6,5X,6HC32 = ,1PE12.6,5X,6HC33 = ,
   1         1PE12.6//)
    WRITE (6,203) ALPHA1, ALPHA2, ALPHA3
203 FORMAT (1H0,10X,9HALPHA1 = ,1PE12.6,5X,9HALPHA2 = ,1PE12.6,5X,
   1         9HALPHA3 = ,1PE12.6//////)
    WRITE (6,204) COF33,COF22,COF11,COF44,COF55,COF66,DETA
204 FORMAT (1H0,6HCOF33=,1PE12.6,3X,6HCOF22=,1PE12.6,3X,6HCOF11=,
   1  1PE12.6,3X,6HCOF44=,1PE12.6,3X,6HCOF55=,1PE12.6,3X,6HCOF66=,
   2  1PE12.6/1H0,5HDETA=,1PE12.6)
    GO TO 100
    END
```

USER'S MANUAL FOR SCREEN REINFORCED PROPELLANT PROPERTY PROGRAM (PROP)

The program (FORTRAN IV) described herein is an evaluation of the equations that have been developed for the prediction of the composite properties of screen reinforced solid propellants. The development of the equations is discussed in the main body of the report. The predictions are based upon the unit cell concept. The unit cell is the smallest fundamental building block of the reinforced material. A view of a layer of the reinforced propellant is shown in Figure 3. A sketch of the three-dimensional geometry of the unit cell is shown in Figure 4; the dimensions of the unit cell are denoted by S_1 , S_2 , S_3 . The governing equations for the phenomenological behavior of the composite material are expressed in the form:

Stress-Strain Law

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & & & \\ A_{12} & A_{22} & A_{23} & & & \\ A_{13} & A_{23} & A_{33} & & & \\ & & & A_{44} & & \\ & & & & A_{55} & \\ & & & & & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} - \begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T$$

Heat Conduction Equation

$$\rho^* c^* \dot{T} = k_1 T_{,11} + k_2 T_{,22} + k_3 T_{,33}$$

Heat Flux Equation

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = - \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} T_{,1} \\ T_{,2} \\ T_{,3} \end{bmatrix}$$

The composite quantities, i.e., stress, strain, temperature, density, specific heat and heat flux are denoted respectively by σ , ε and γ , Ψ , T , ρ^* , c^* , and q .

For structural analysis purposes it is usually sufficient to have a knowledge of the stress-strain law (i.e., $[\sigma] = [A][\varepsilon] - [\Psi]\Delta T$), however, the strain-stress law is often of more value as an aid to the understanding of the composite behavior of the material; the strain-stress law may be written in the following form:

$$
\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & & \\ & & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{bmatrix} + \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T
$$

The [C] matrix may be determined by inverting the [A] matrix.

The composite properties depend upon the geometric arrangement of the reinforcement, and upon the properties of the wire and of the propellant. In this analysis the reinforcement is considered to be in the form of layers of wire screen. It is assumed that the screen consists of straight wires parallel to the x_2 axis and warped wires in the x_1 direction, Figure 1. Additionally it is assumed that the wires in the x_1 direction makes up a relatively small percentage of the total volume of reinforcement. The wire spacing is specified by the cell dimensions S_1 , S_2 and S_3 , Figure 4. The properties of the wire and of the propellant which enter into the various composite property predictions are listed in the description of the program input which is given below.

The following cards are needed as input to the program:

1. <u>Title Card</u> (12A6) - Columns 1 to 72
2. <u>Geometry Data Card</u> (5F10.0)

Columns	1 - 10	S_1 (in.)	Cell Dimensions
	11 - 20	S_2 (in.)	
	21 - 30	S_3 (in.)	

31 - 40 R_1 (in.), Radius of warped wires

41 - 50 R_2 (in.), Radius of straight wires

3. Propellant Data Card (6F10.0)

Columns 1 - 10 Bulk Modulus (psi)

11 - 20 Shear Modulus (psi), see comments below

21 - 30 Thermal Conductivity (Btu-in/hr.-ft^2-°F)

31 - 40 Thermal Coefficient of Linear Expansion (in/in-°F)

41 - 50 Density (lb/in^3)

51 - 60 Specific Heat (Btu/lb-°F)

4. Wire Data Card (6F10.0)

Columns 1 - 10 Young's Modulus (psi)

11 - 20 Poisson's Ratio

21 - 30 Thermal Conductivity (Btu-in/hr.-ft^2-°F)

31 - 40 Thermal Coefficient of Linear Expansion (in/in-°F)

41 - 50 Density (lb/in^3)

51 - 60 Specific Heat (Btu/lb-°F)

The mechanical properties of the reinforced propellant will, of course, be viscoelastic due to the time dependent nature of the propellant properties. One possible way of considering the viscoelastic nature of the composite, for analysis purposes, is to use the correspondence principle to replace the viscoelasticity problem with a corresponding elasticity problem. In the corresponding elasticity problem the transforms of the viscoelastic properties appear as pseudo-elastic properties. These corresponding elastic properties, for the composite material, can be obtained from the program described herein by utilizing as input the appropriate values of the transformed propellant shear and bulk relaxation moduli (these values are called "shear" and "bulk" moduli in the program). The resulting composite properties are then used in the solution of the corresponding elasticity problem; the viscoelasticity solution is found by employing one of the several available approximate inversion techniques. The program can also be used to predict numerical values of the kernels which enter into the integral representation of the composite anisotropic viscoelastic stress-strain law; the integral representation is utilized in the step-by-step solution of viscoelasticity problems.

The output of the program consists of the composite properties A_{ij} , k_i , Ψ_i , ρ^* , c^* , α_i and C_{ij} (note: C_{44} is the reciprocal of A_{44} etc.). Lastly the values of a number of functions of the A_{ij} matrix are printed. These functions, in addition to the diagonal terms of the A_{ij} matrix, must be positive in order that the resulting stress-strain law should be derviable from a positive definite strain energy function.

The program listing is given below:

```
C      ***************************************************
C      *  PROGRAM FOR COMPUTING THE COMPOSITE PROPERTIES  *
C      *  OF SCREEN REINFORCED PROPELLANTS                *
C      ***************************************************


       DIMENSION TITLE(12)
       REAL MUM, NUR, KM, K1, K2, K3, LS
       TERM(C,S) = (1.5707963 + ARSIN(C*R2/S))/(SQRT(1. - (C*R2/S)**2))
     1           - 1.5707963
  100  READ (5,800) TITLE
  800  FORMAT (12A6)
       WRITE (6,801) TITLE
  801  FORMAT (1H1,8X,12A6//)
C                   DATA CARD NO.1
       READ (5,101) S1, S2, S3, R1, R2,
C                   DATA CARD NO.2
     1              KM, MUM, TCM, ALPHAM, RHOM, CM,
C                   DATA CARD NO.3
     2              ER, NUR, TCR, ALPHAR, RHOR, CR
  101  FORMAT (5F10.0 / 6F10.0 / 6F10.0)
       WRITE (6,999) S1, S2, S3, R1, R2
  999  FORMAT (1X, 15HCELL DIMENSIONS// 11X,3HS1=,F6.4/11X,3HS2=,F6.4/
     1    11X,3HS3=,F6.4/1H0,10HWIRE RADII//11X,3HR1=,F6.4/11X,3HR2=,
     2    F6.4)
       WRITE (6,998) KM, MUM, TCM, ALPHAM, RHOM, CM
  998  FORMAT (1H0, 7HMATRIX-//11X,13HBULK MODULUS=,F10.0/11X,14HSHEAR MO
     1DULUS=,F5.0/11X,21HTHERMAL CONDUCTIVITY=,F8.2/11X,33HTHERMAL COEFF
     2ICIENT OF EXPANSION=,F10.8/11X, 8HDENSITY=,F6.4/11X,14HSPECIFIC HE
     3AT=,F6.4)
       WRITE (6,997) ER, NUR, TCR, ALPHAR, RHOR, CR
  997  FORMAT (1H0,5HWIRE-//11X,14HYOUNG MODULUS=,F10.0/11X,14HPOISSON RA
     1TIO=,F5.3/11X,21HTHERMAL CONDUCTIVITY=,F8.2/11X,33HTHERMAL COEFFIC
     2IENT OF EXPANSION=,F10.8/11X,8HDENSITY=,F6.4/11X,14HSPECIFIC HEAT=
     3,F6.4//)
       AREA1 = 3.1415927 * R1 *R1
```

```
      AREA2 = 3.1415927 * R2 * R2
      AREA12 = S1 * S2
      AREA13 = S1 * S3
      AREA23 = S2 * S3
      VOL = AREA12 * S3
      BETA = ARSIN(2. * (R1 + R2) / S1)
      LS = SQRT(S1 * S1 - 4. * R2 * R2) + 2. * (R1 + R2) * BETA
      VOLR1 = LS * AREA1
      VOLR2 = S2 * AREA2
      VOLM = VOL - VOLR1 - VOLR2
      T1 = 1. + NUR
      T2 = 1. - 2.*NUR
      T3 = T1*T2
      T4 = KM*T2
      T5 = MUM*T1
      T6 = KM - 0.66666667*MUM
      T7 = KM + 0.33333333*MUM
      T8 = KM + 1.33333333*MUM
      T9 = 2.*KM - 0.33333333*MUM
      T10 = VOL - VOLM
      T11 = VOLM + VOLR2
      T12 = 1.7724539*R1
      T13 = 1.7724539*R2
      T14 = T5 / 3.
      T15 = 3.*ALPHAM*KM
      T16 = ALPHAR*ER
      T17 = TCM*TCR
      T18 = TCM - TCR
      T19 = VOL/VOLM
      F1 = AREA1 / AREA12
      F2 = AREA1 / AREA23
      F3 = AREA2 / AREA13
C
C   (A)
      SA = T7*(T10*ER - 2.*T3*(T7*VOLR1 + VOLR2*(T6 + 0.5*ER/T1)))
     1    /(ER*VOLM + T3*(T9*VOLR1 + 2.*T7*VOLR2))
```

```
C
      A12 = F2*(2.*T14 - T4 + SA*(NUR*T9/T7 - 1.)) + T6 + SA
C
      A22 = F3*(ER - 4.*T14 - T4 + SA*(2.*NUR - T6/T7)) + T8 + SA*T6/T7
C
      A32 = T6 + SA
C  (B)
      C = 2. - 2.*T3*T8/(ER*(1. - NUR))
      F4 = 0.5*C*T13
C
      A13 = S2*(T6*(S3*(S1 - T13) + T13*(S3 - T13)/(1. - F4/S3))
     1    + NUR*T8*AREA2/(1. - NUR)/(1. - F4/S3))/T11
C
      A23 = A13
C
      A33 = 2.*AREA23*T8/T11*(0.5*S1 - R2 + S3*TERM(C,S3)/C)
C  (C)
      A11 = T8*(1. - F2)*(1. - 2./S3*(R2 - S1/C*TERM(C,S1)))
     1     + F2*(ER*S1*COS(BETA)/LS + 2.*NUR*T6/(1. - C*R2/S1))
C
      A21 = (T6*(S1*(S3 - T13) + T13*(S1 - T13)/(1. - F4/S1))
     1     + NUR/(1. - NUR)*T8*AREA2/(1. - F4/S1))/AREA13
C
      A31 = A21
C  (D)
      R = 2. - 4.*MUM*T1/ER
C
      A44 = 2.*MUM*AREA12/T11*(0.5*S3 - R2 + S1*TERM(R,S1)/R)
C  (E)
      A55 = MUM*T19
C  (F)
      A66 = 2.*MUM*AREA23/T11*(0.5*S1 - R2 + S3*TERM(R,S3)/R)
C  (G)
      SG = 2.*T7*(T15*T3 - T16*NUR)*T10/(ER*VOLM + T3*(T9*VOLR1
     1    + 2.*T7*VOLR2))
C
```

```
      PSI1 = F2*(SG*(T4 + T14)/T7 - T15*T2 + T16) + T15 - SG
C
      PSI2 = F3*(SG*(T4 - 2.*T14)/T7 - T15*T2 + T16) + T15 - SG*T6/T7
C
      PSI3 = T15 - SG
C  (H)
      K1 = TCM*(1. - T13/S3) + F2*(S1*TCR/LS - TCM)
     1     + T13*S1*T17/S3/(T13*T18 + S1*TCR)
C  (I)
      K2 = TCM*(1. - T12*LS/AREA13) - F3*T18
     1     + T12*LS*S2*T17/AREA13/(T12*T18 + S2*TCR)
C  (J)
      K3 = TCM*(1. - T13/S1 - T12/S2 - F1) + F1*S3*T17/(LS*TCM
     1     + TCR*(S3 - 2.*R2 - 4.*R1)) + T13*S3*T17/S1/(T13*T18 + S3*TCR)
     2     + T12*S3*T17/S2/(T12*T18 + S3*TCR)
C
      RHOVOL = RHOM*VOLM + RHOR*T10
      RHO = RHOVOL/VOL
      CC = (CM*RHOM*VOLM + CR*RHOR*T10)/RHOVOL
      N = 1
      A12=(A12+A21)/2.0
      A21=A12
      A13=(A13+A31)/2.0
      A31=A13
      A23=(A23+A32)/2.0
      A32=A23
      COF11=A22*A33-A32*A23
      COF12 = A32*A13 - A12*A33
      COF13 = A12*A23 - A22*A13
      COF21 = A31*A23 - A21*A33
      COF22 = A11*A33 - A31*A13
      COF23 = A21*A13 - A11*A23
      COF31 = A21*A32 - A31*A22
      COF32 = A31*A12 - A11*A32
      COF33 = A11*A22 - A21*A12
      DETA = A11*COF11 + A21*COF12 + A31*COF13
```

```
      C11 = COF11/DETA
      C12 = COF12/DETA
      C13 = COF13/DETA
      C21 = COF21/DETA
      C22 = COF22/DETA
      C23 = COF23/DETA
      C31 = COF31/DETA
      C32 = COF32/DETA
      C33 = COF33/DETA
      ALPHA1 = PSI1*C11 + PSI2*C21 + PSI3*C31
      ALPHA2 = PSI1*C12 + PSI2*C22 + PSI3*C32
      ALPHA3 = PSI1*C13 + PSI2*C23 + PSI3*C33
      COF44=COF33*COF22-COF23*COF32
      COF55=COF22*COF11-COF12*COF12
      COF66=COF33*COF11-COF13*COF13
      IF (A11.LT.0.0 .OR. A22.LT.0.0 .OR. A33.LT.0.0 .OR. A44.LT.0.0
     1  .OR. A55.LT.0.0 .OR. A66.LT.0.0) WRITE (6,350)
      IF  (COF11.LT.0.0 .OR. COF22.LT.0.0 .OR. COF33.LT.0.0 .OR. COF44
     1.LT.0.0 .OR. COF55.LT.0.0 .OR. COF66.LT.0.0) WRITE (6,350)
  350 FORMAT (1H0,41HERROR-STRAIN ENERGY NOT POSITIVE DEFINITE)
      WRITE (6,104) A11, A12, A13
  104 FORMAT (1H0,20HPREDICTED PROPERTIES//11X,6HA11 = ,1PE12.6,5X,
     16HA12 = ,1PE12.6,5X,6HA13 = ,1PE12.6)
      WRITE (6,105) A21, A22, A23
  105 FORMAT (1H0,10X,6HA21 = ,1PE12.6,5X,6HA22 = ,1PE12.6,5X,6HA23 = ,
     1        1PE12.6)
      WRITE (6,106) A31, A32, A33
  106 FORMAT (1H0,10X,6HA31 = ,1PE12.6,5X,6HA32 = ,1PE12.6,5X,6HA33 = ,
     1        1PE12.6)
      WRITE (6,107) A44, A55, A66
  107 FORMAT (1H0,10X,6HA44 = ,1PE12.6,5X,6HA55 = ,1PE12.6,5X,6HA66 = ,
     1        1PE12.6//)
      WRITE (6, 108) K1, K2, K3, RHO, CC
  108 FORMAT (1H0, 10X, 5HK1 = ,1PE12.6,5X,5HK2 = ,1PE12.6,5X,5HK3 = ,
     1        1PE12.6,5X,6HRHO = ,1PE12.6,5X,4HC = ,1PE12.6)
      WRITE (6,109) PSI1, PSI2, PSI3
```

```
109 FORMAT (1H0,10X,7HPSI1 = ,1PE12.6,5X,7HPSI2 = ,1PE12.6,5X,
   1        7HPSI3 = ,1PE12.6)
    WRITE (6,200) C11, C12, C13
200 FORMAT (1H0,18HINVERSE PROPERTIES//11X,6HC11 = ,1PE12.6,5X,6HC12 =
   1 ,1PE12.6,5X,6HC13 = ,1PE12.6)
    WRITE (6,201) C21, C22, C23
201 FORMAT (1H0,10X,6HC21 = ,1PE12.6,5X,6HC22 = ,1PE12.6,5X,6HC23 = ,
   1        1PE12.6)
    WRITE (6,202) C31, C32, C33
202 FORMAT (1H0,10X,6HC31 = ,1PE12.6,5X,6HC32 = ,1PE12.6,5X,6HC33 = ,
   1        1PE12.6//)
    WRITE (6,203) ALPHA1, ALPHA2, ALPHA3
203 FORMAT (1H0,10X,9HALPHA1 = ,1PE12.6,5X,9HALPHA2 = ,1PE12.6,5X,
   1        9HALPHA3 = ,1PE12.6//////)
    WRITE (6,204) COF33,COF22,COF11,COF44,COF55,COF66,DETA
204 FORMAT (1H0,6HCOF33=,1PE12.6,3X,6HCOF22=,1PE12.6,3X,6HCOF11=,
   1  1PE12.6,3X,6HCOF44=,1PE12.6,3X,6HCOF55=,1PE12.6,3X,6HCOF66=,
   2  1PE12.6/1H0,5HDETA=,1PE12.6)
    GO TO 100
    END
```

USER'S MANUAL FOR WIRE BUCKLING ANALYSIS PROGRAM (BUCK)

The program (FORTRAN IV) described herein is an evaluation of the equation for the prediction of the buckling load of a wire embedded in a matrix; the prediction is based upon the analysis described in the report "Behavior of Compressively Loaded Reinforcing Wires." In the analysis it is assumed that the ratio of the wire's length to its diameter is large, i.e., the wire is considered as being infinite in length. It is also assumed that the separation between adjacent wires is sufficient so that there is no interaction between their responses; additionally it is assumed that the wire is located sufficiently far from the boundaries of the matrix so that in calculating the supporting action of the matrix upon the wire the matrix may be considered as being infinite in extent. This program employs the foundation model which is referred to as "exact" in the above mentioned report. The effect of transverse wire shear deformation is included in the evaluation (this effect may be neglected by entering a value of zero for the shear deformation coefficient).

An iterative solution procedure is employed, hence, it is necessary to specify the degree of accuracy desired in the solution. The Authors have found that for a typical problem it requires about thirty iterations in order to limit the error in the value of the predicted buckling load to less than 0.1%.

The following cards are needed as input to the program:

1. Title Card (12A6) - Columns 1 to 72
2. First Data Card (I10, 3F10.0)

Columns	1 - 10	Maximum number of iterations
	11 - 20	Shear modulus of the matrix
	21 - 30	Bulk modulus of the matrix
	31 - 40	Young's modulus of the wire

3. <u>Second Data Card</u> (4F10.0)

1 - 10	Poisson's ration of the wire
11 - 20	Radius of the wire
21 - 30	Allowable error in the solution
31 - 40	Shear deformation coefficient for the wire

For a wire with a circular cross-section the value of the shear deformation coefficient is $\frac{7 + 6\nu}{6(1 + \nu)}$; Poisson's ratio of the wire is denoted as ν . If the matrix is viscoelastic the equilibrium values of the bulk and shear moduli should be used.

For each step in the iterative solution the following two quantities are printed, 1) the buckling load divided by the product of the wire's Young's modulus and cross-sectional moment of inertia and 2) the factor β ($\beta = \frac{2\pi}{\ell}$; ℓ is the wave length of the buckled wire).

The program listing is given below:

```
C     **************************************************
C     *  PROGRAM TO DETERMINE THE BUCKLING STRENGTH    *
C     *  OF A WIRE EMBEDDED IN AN INFINITE MATRIX      *
C     **************************************************

      DIMENSION TITLE (12)
      REAL MU, K, NUF, NUM, KO, K1, K2, KS
  100 READ (5,800) TITLE
  800 FORMAT (12A6)
      WRITE (6,801) TITLE
  801 FORMAT (1H1,12A6//)
      READ (5,110) ITMAX, MU, K, EF, NUF, R, E, ALPHAS
  110 FORMAT (I10,3F10.0/4F10.0)
      WRITE (6,115) ITMAX, MU, K, EF, NUF, R, E, ALPHAS
  115 FORMAT (1H0,29HMAXIMUM NUMBER OF ITERATIONS=,I5/
     1    1X,21HMATRIX SHEAR MODULUS=,F10.3/
     2    1X,20HMATRIX BULK MODULUS=,F10.0/
     3    1X,22HYOUNG MODULUS OF WIRE=,F10.0/
     4    1X,22HPOISSON RATIO OF WIRE=,F5.3/
     5    1X,15HRADIUS OF WIRE=,F7.5/
     6    1X,15HACCEPTED ERROR=,F8.6/
     7    1X,30HSHEAR DEFORMATION COEFFICIENT=,F7.4///)
      WRITE (6,129)
  129 FORMAT (10X,22HEXACT FOUNDATION MODEL/
     1     1H0,10X,4HSTEP,15X,4HBETA,15X,4HP/EI/)
      BETAPO=1.0
      DELTAO=10.0
      NUM = 0.5*(3.*K - 2.*MU)/(3.*K + MU)
      T1 = 16.*MU/(EF*R**2)
  121 BETAP = BETAPO
      DELTA = DELTAO
      OLDP = 1.0E+30
      ITER = 0
  130 X = 6.2831853/BETAP
      ITER = ITER + 1
```

```
      IF(ITER.GT.ITMAX) GO TO 240
      CALL BESSKO (X,K0)
      CALL BESSK1 (X,K1)
      K2 = 2.*K1/X + K0
      KS = 8.*ALPHAS*MU*(1.+NUF)/X/EF
      A11 = 2.*K2 + 0.25*KS*NUF*X*X*K1
      A12 = X*(K2*(1.-0.5*NUF*X*X) + NUF*K1*X*(1.-0.25*KS*X))/R
      A21 = K1*(2.-0.5*KS*X)/R
      A22 = X*(X*K2/R - A21)/R
      B1 =-R*(K1*(1.+0.5*NUF*X*X) + K2*((10.-8.*NUM)/X + 0.5*KS*NUF*
     1        X*X))
      B2 = X*K1 - K2*(2.-KS*X)
      DENOM = A11*A22 - A12*A21
      C1 = (B1*A22 - B2*A12)/DENOM
      C2 = 1.0
      C3 = (B2*A11 - B1*A21)/DENOM
      CHIR = 0.5*X*X*ALPHAS*(1.+NUF)*(0.5*C1*K1 + C2*R*K2-C3*0.5*K1*X/R)
      P = (X/R)**2 + R*T1*(CHIR + C1*K2*X + C2*R*((4.-2.*NUM)*K2 +
     1     X*K1))/(C2*K1*R*R - C3*(K2*X - 2.*K1))/X
      BETA = X/R
      WRITE (6,210) ITER, BETA, P
  210 FORMAT (1H ,10X,I3,12X,1PE12.6,6X,1PE12.6)
      IF(ABS(1.-OLDP/P).LT. E) GO TO 220
      IF(P .GT. OLDP) GO TO 215
      OLDP = P
      BETAP = BETAP + DELTA
      GO TO 130
  215 BETAP = BETAP - 2.*DELTA
      IF(BETAP.LE. 0.0) BETAP = BETAPO
      OLDP = 1.0E+30
      DELTA = 0.5*DELTA
      GO TO 130
  220 WRITE (6,230) P
  230 FORMAT (1H0,10X,14HMINIMUM P/EI =,1PE12.6//)
      GO TO 100
  240 WRITE (6,250) ITMAX
```

```
250 FORMAT (1H0,10X,23HDOES NOT CONVERGE AFTER,I4,12H ITERATIONS.//)
    GO TO 100
    END


    SUBROUTINE BESSK0 (X,K0)
    REAL K0, I0
    IF(X.GT.2.0) GO TO 100
    T = X/3.75
    TT = T*T
    I0 = 1.0 + TT*(3.5156229 + TT*(3.0899424 + TT*(1.2067492
   1         + TT*(0.2659732 + TT*(0.0360768 + TT*(0.0045813))))))
    XX = 0.25*X*X
    K0 = -I0*ALOG(0.5*X) - 0.57721566 + XX*(0.42278420
   1 + XX*(0.23069756 + XX*(0.03488590 + XX*(0.00262698
   2 + XX*(0.00010750 + XX*(0.00000740))))))
    GO TO 200
100 XX = 2.0/X
    K0 = (1.25331414 - XX*(0.07832358 - XX*(0.02189568
   1 - XX*(0.01062446 - XX*(0.00587872 - XX*(0.00251540
   2 - XX*(0.00053208)))))))/(SQRT(X)*EXP(X))
200 RETURN
    END


    SUBROUTINE BESSK1 (X,K1)
    REAL K1, I1
    IF(X.GT.2.0) GO TO 100
    T = X/3.75
    TT = T*T
    I1 = X*(0.5 + TT*(0.87890594 + TT*(0.51498869 + TT*(0.15084934
   1 + TT*(0.02658733 + TT*(0.00301532 + TT*(0.00032411)))))))
    XX = 0.25*X*X
    K1 = I1*ALOG(0.5*X) + (1.0 + XX*(0.15443144 - XX*(0.67278579
   1 + XX*(0.18156897 + XX*(0.01919402 + XX*(0.00110404
   2 + XX*(0.00004686)))))))/X
```

```
    GO TO 200
100 XX = 2.0/X
    K1 = (1.25331414 + XX*(0.23498619 - XX*(0.03655620
   1 - XX*(0.01504268 - XX*(0.00780353 - XX*(0.00325614
   1 - XX*(0.00068245)))))))/(SQRT(X)*EXP(X))
200 RETURN
    END
```

USER'S MANUAL FOR BEAM COLUMN ANALYSIS PROGRAM (CHI)

The program (FORTRAN IV) described herein is an evaluation of the equations for the prediction of the beam-column behavior of a wire embedded in a matrix; the prediction is based upon the analysis described in the report "Behavior of Compressively Loaded Reinforcing Wires." For the medium deflection portion of the analysis the deviations of the initial and deformed wire configurations, from straight lines, are considered to be relatively large. It is assumed that the wire's centerline initially lies in the x-y plane and that its equation is given by the expression:

$$Y_o(x) = \chi_o \text{ Cos } \beta x \tag{A}$$

The equation of the centerline of the deformed wire is expressed in the form:

$$Y(x) = (\chi + \chi_o) \text{ Cos } \beta x \tag{B}$$

Hence the deformation is characterized by the parameter χ. The beam-column equation is evaluated in an inverse fashion, i.e., the parameter χ is specified and the column force F_o necessary to produce this deformation is calculated.

In the analysis it is assumed that the ratio of the wire's length to its diameter is large, i.e., the wire is considered as being infinite in length. It is also assumed that the separation between adjacent wires is sufficient so that there is no interaction between their responses; additionally it is assumed that the wire is located sufficiently far from the boundaries of the matrix so that in calculating the supporting action of the matrix it may be considered as being infinite in extent. This program employs the foundation model which is referred to as "exact" in the above mentioned report. The effect of transverse wire shear deformation is not included in the evaluation.

The following cards are needed as input to the program:

1. Title Card (12A6) - Columns 1 to 72
2. Data Card (7F10.0)

   Columns 1 - 10 β ($\beta = \frac{2\pi}{\ell}$, ℓ is the wave length of the wire's centerline, Equation A)

   11 - 20 Shear modulus of the matrix

   21 - 30 Bulk modulus of the matrix

   31 - 40 Young's modulus of the wire

   41 - 50 Radius of the wire

   51 - 60 Poisson's ratio of the wire

   61 - 70 χ_o (Equation A)
3. A Card (F10.0) for each value of χ (Equation B) to be considered

   Columns 1 - 10 χ (Equation B)
4. A blank card

If the matrix is viscoelastic the values of the bulk and shear moduli will, of course, depend upon loading rate.

For each value of χ the following information is printed; a) the value of χ (denoted as CHI), b) the value of the axial load (F_o) carried by the wire, c) the x component of strain (ε) of the composite material (the x axis coincides with the average position of the wire's centerline), d) the effective stiffness ratio of the wire, i.e., $\frac{F_o}{A\varepsilon E}$ (A and E are the cross-sectional area and the Young's modulus of the wire). The above quantities are predicted first from the medium deflection equations and secondly from the small deflection equations.

The listing of the program is given below:

```
C      *****************************************************
C      *  PROGRAM TO DETERMINE THE BEAM-COLUMN BEHAVIOR   *
C      *  OF A WIRE EMBEDDED IN AN INFINITE MATRIX        *
C      *****************************************************

      DIMENSION TITLE (12)
      REAL MUM, KM, NUF, NUM, KO, K1, K2,IF
  100 READ (5,800) TITLE
  800 FORMAT (12A6)
      WRITE (6,801) TITLE
  801 FORMAT (1H1,5X,12A6)
      READ (5,110) BETA, MUM, KM, EF, R, NUF, A
  110 FORMAT (7F10.0)
      WRITE (6,111) MUM, KM, EF, R, NUF, BETA, A
  111 FORMAT (1H0,5X,21HMATRIX SHEAR MODULUS=,1PE12.6/
     1   6X,20HMATRIX BULK MODULUS=,1PE12.6/
     2   6X,19HYOUNG MODULUS-WIRE=,1PE12.6/
     3   6X,12HRADIUS-WIRE=,1PE12.6/
     4   6X,20HPOISSONS RATIO-WIRE=,1PE12.6/
     5   6X, 5HBETA=,1PE12.6/
     6   6X,34HMAGNITUDE OF INITIAL IMPERFECTION=,1PE12.6)
      AF = 3.1415927*R*R
      EFXAF = EF*AF
      IF = AF*R*R/4.0
      NUM = 0.5*(3.*KM - 2.*MUM)/(3.*KM + MUM)
      X = BETA*R
      CALL BESSKO (X,KO)
      CALL BESSK1 (X,K1)
      K2 = 2.*K1/X + KO
      T1 = 16.0*3.1415927*MUM*(1.-NUM)
      T2 = 6.0 - 8.*NUM
      T3 = 4.0 + T2
      T4 = 3.0 - 2.*NUM
      T5 = 6.283185*R*MUM
      BET2 = BETA**2
```

```
      BET4 = BET2**2
      T6 = 0.5*NUF*R*R*BET2
  120 READ (5,130) CHI
C     NOTE- LAST CHI CARD MUST BE BLANK.
  130 FORMAT (F10.0)
      IF(CHI.EQ. 0.0) GO TO 100
      CHIPA1 = CHI + A
      CHIPA2 = CHIPA1**2
      CHIPA3 = CHIPA1**3
      CHIPA4 = CHIPA1**4
      CHIPA5 = CHIPA1**5
      CB2 = CHIPA2*BET2
      DO 240 I=1,2
      IF(I.EQ.2) GO TO 150
      XN = 0.0
      WRITE (6,140)
  140 FORMAT (1H0,///11X,25HSMALL DEFLECTION ANALYSIS)
      GO TO 170
  150 XN = 1.0
      WRITE (6,160)
  160 FORMAT (1H0,10X,26HMEDIUM DEFLECTION ANALYSIS)
  170 A11=2.*K2
      A12 = BETA*K2*(1.-T6) + NUF*R*BET2*K1
      A21 = 2.*K1/R
      A22 = BET2*K2 - 2.*BETA*K1/R
      B1 = -(R*K1*(1.+T6) + T3*K2/BETA)
      B2 = -(2.*K2 - X*K1)
      DENOM = A11*A22 - A12*A21
      C1 = (B1*A22 - B2*A12)/DENOM
      C3 = (B2*A11 - B1*A21)/DENOM
      A1 = R*K1
      A3 = BET2*K2 - 2.*BETA*K1/R
      A4 = 0.5*K1/R - BETA*K2
      A5 = X*K1 + T4*K2
      AS = 0.5*(A3*C3/BETA - A1)
      B = T5*(0.5*BETA*A1*(C1 - BETA*C3)/R + X*K2)/AS
```

```
      CS = T5*R*(C1*A4 - A5 - 0.5*BETA*A1*C3/R**2)/AS
      C = CS*(1.+XN*(3.*CB2/8.0-5.*CB2**2/64.0))
      BS = B*CHI
      F0 = (BS/BET2 + 3.*XN*BS*CHIPA2/8.0 - C*CHI/BETA - EF*IF*BET2
     1     *(CHI-3.*XN*BET2*(CHIPA3-A**3)/8.0))/CHIPA1
      SK2 = 0.5*(BS*CHIPA1 - F0*CB2)/EFXAF
      SK3 = 0.25*(1.5*F0*CB2**2 - BS*CHIPA3*BET2)/EFXAF
      SK4 = 3.*BS*CHIPA5*BET4/16.0/EFXAF
      SK5 = F0/EFXAF
      EPS= SK5+SK5**2/2.0+0.25*(-BET2*CHI*(CHI+2.0*A)+XN*(2.0*SK2*
     1    (1.0+SK5)+SK5*BET2*(A*A+CHIPA2)))+XN*3.0/16.0*(SK2**2
     2    +SK2*BET2*(A*A+CHIPA2)+0.25*BET4*CHI*CHI*(CHI+2.0*A)**2)
      XH=F0/AF/EPS/EF
      WRITE (6,220) CHI,F0,EPS,XH
  220 FORMAT (1H0,15X,4HCHI=,1PE12.6,5X,3HF0=,1PE12.6,5X,8HEPSILON=,
     1    1PE12.6,5X,26HEFFECTIVE STIFFNESS RATIO=,1PE12.6)
  240 CONTINUE
      GO TO 120
      END


      SUBROUTINE BESSK0 (X,K0)
      REAL K0, I0
      IF(X.GT.2.0) GO TO 100
      T = X/3.75
      TT = T*T
      I0 = 1.0 + TT*(3.5156229 + TT*(3.0899424 + TT*(1.2067492
     1         + TT*(0.2659732 + TT*(0.0360768 + TT*(0.0045813))))))
      XX = 0.25*X*X
      K0 = -I0*ALOG(0.5*X) - 0.57721566 + XX*(0.42278420
     1 + XX*(0.23069756 + XX*(0.03488590 + XX*(0.00262698
     2 + XX*(0.00010750 + XX*(0.00000740))))))
      GO TO 200
  100 XX = 2.0/X
      K0 = (1.25331414 - XX*(0.07832358 - XX*(0.02189568
     1 - XX*(0.01062446 - XX*(0.00587872 - XX*(0.00251540
```

```
   2 - XX*(0.00053208)))))))/(SQRT(X)*EXP(X))
200 RETURN
    END


    SUBROUTINE BESSK1 (X,K1)
    REAL K1, I1
    IF(X.GT.2.0) GO TO 100
    T = X/3.75
    TT = T*T
    I1 = X*(0.5 + TT*(0.87890594 + TT*(0.51498869 + TT*(0.15084934
   1 + TT*(0.02658733 + TT*(0.00301532 + TT*(0.00032411)))))))
    XX = 0.25*X*X
    K1 = I1*ALOG(0.5*X) + (1.0 + XX*(0.15443144 - XX*(0.67278579
   1 + XX*(0.18156897 + XX*(0.01919402 + XX*(0.00110404
   2 + XX*(0.00004686)))))))/X
    GO TO 200
100 XX = 2.0/X
    K1 = (1.25331414 + XX*(0.23498619 - XX*(0.03655620
   1 - XX*(0.01504268 - XX*(0.00780353 - XX*(0.00325614
   1 - XX*(0.00068245)))))))/(SQRT(X)*EXP(X))
200 RETURN
    END
```

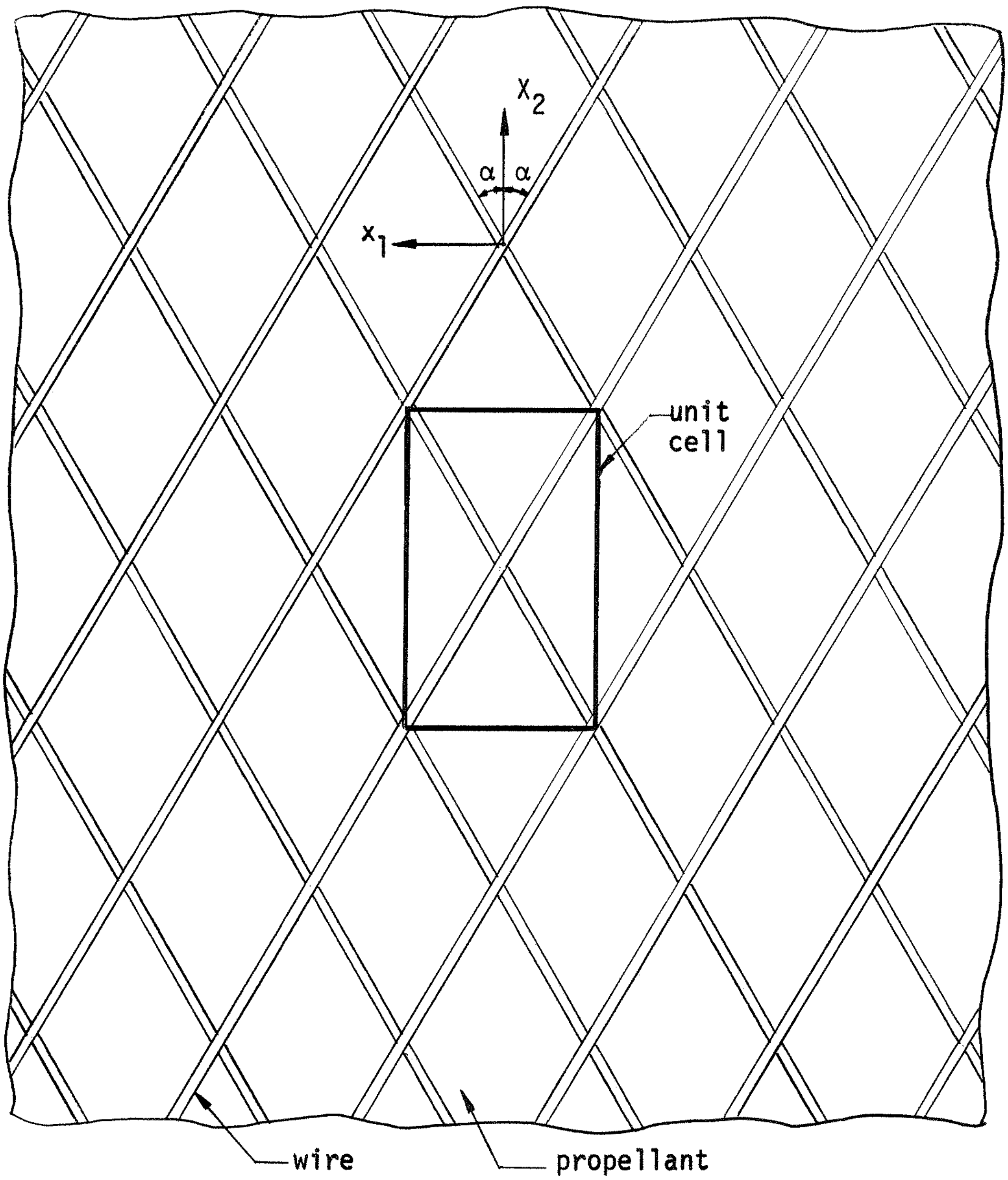

Figure 1 -- Two-Dimensional View of a Layer of Wire Reinforced Propellant

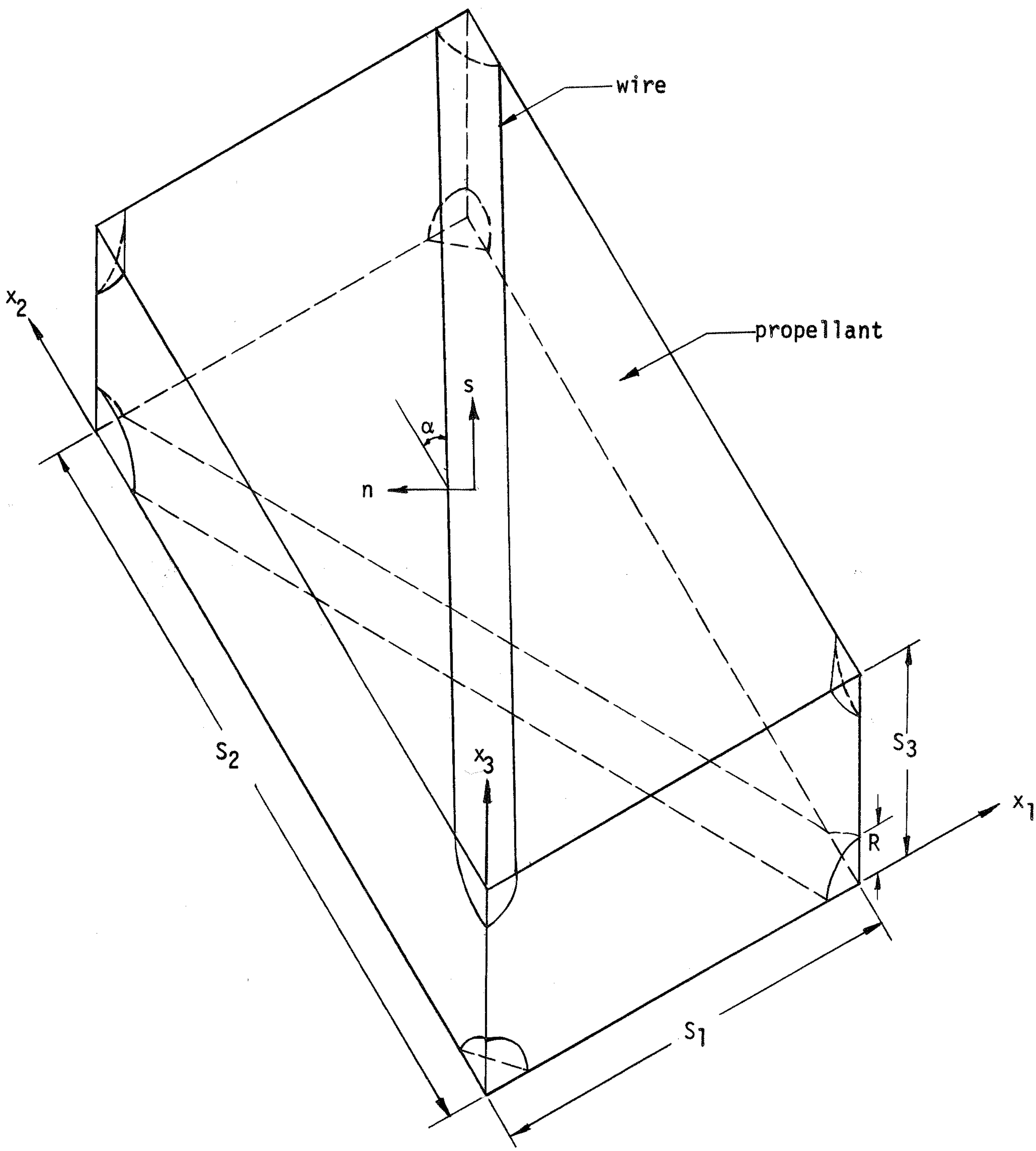

Figure 2 -- Unit Cell

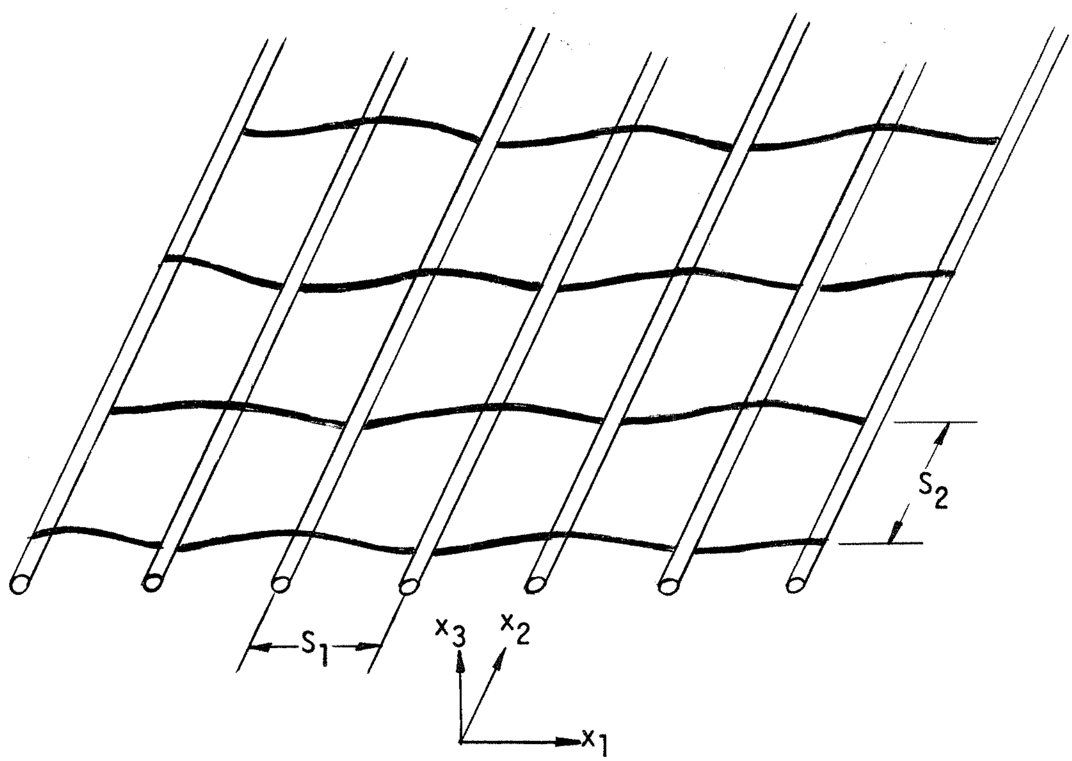

Figure 3 -- Layer of Screen Reinforced Propellant

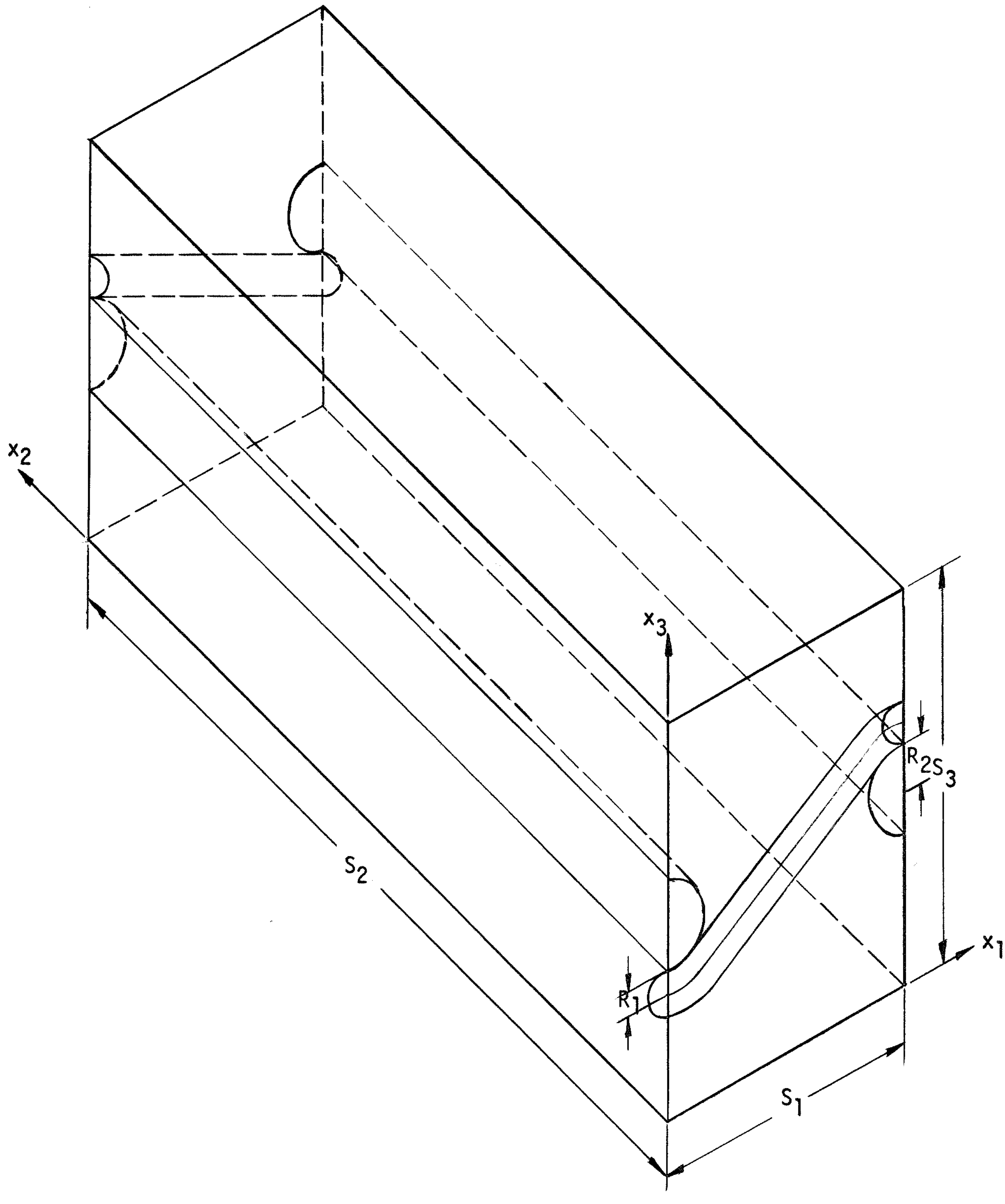

Figure 4 -- Unit Cell

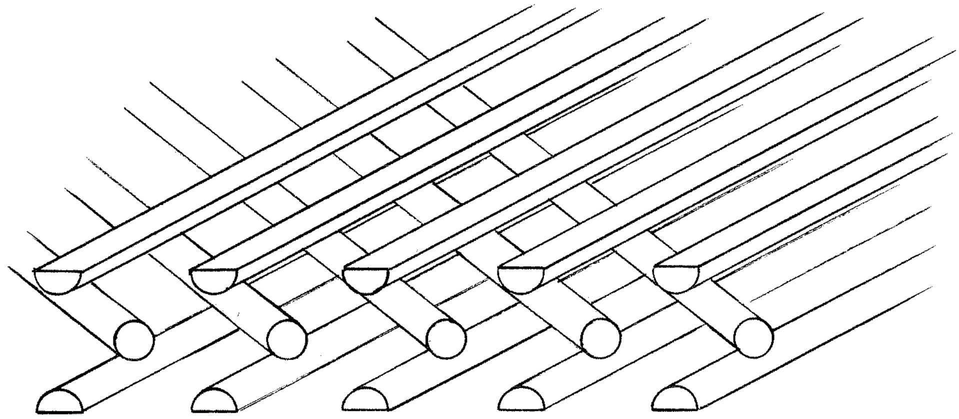

Figure 5. Idealized Layer of Wire Reinforced Propellant

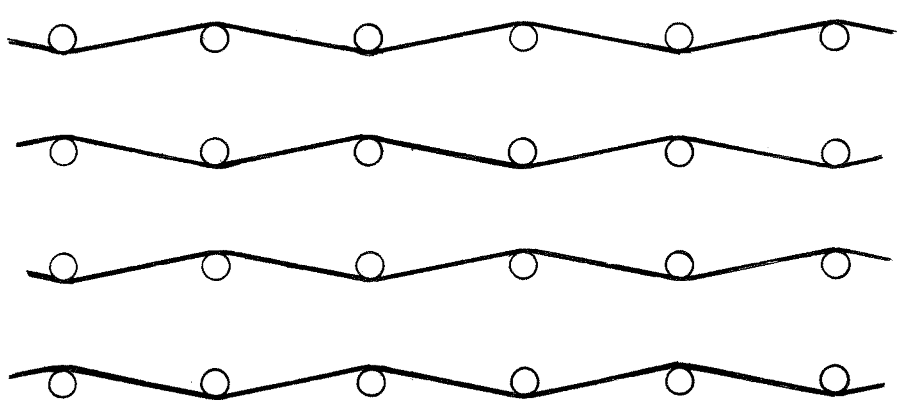

Figure 6. Section Through Idealized Screen Reinforced Propellant

Prop. \ Material	S Material	CW Material
A_{11}	5.89×10^5	5.14×10^5
A_{12}	4.78×10^5	5.93×10^5
A_{13}	4.79×10^5	4.88×10^5
A_{22}	1.38×10^6	1.32×10^6
A_{23}	4.84×10^5	4.85×10^5
A_{33}	5.16×10^5	5.30×10^5
A_{44}	2.26×10^2	1.07×10^5
A_{55}	2.22×10^2	2.22×10^2
A_{66}	2.30×10^2	2.38×10^2
ψ_1	68.8	71.0
ψ_2	77.4	84.8
ψ_3	67.6	68.9
k_1	13.2	14.9
k_2	76.3	76.7
k_3	5.92	5.93
c^*	0.271	0.271
ρ^*	0.0666	0.0666

Figure 7. Predicted Properties for the Typical Reinforced Propellants

Prop. \ Material	S Material	CW Material
A_{11}	5.89×10^5	5.14×10^5
A_{12}	4.78×10^5	5.93×10^5
A_{13}	4.79×10^5	4.88×10^5
A_{22}	1.38×10^6	1.32×10^6
A_{23}	4.84×10^5	4.85×10^5
A_{33}	5.16×10^5	5.30×10^5
A_{44}	2.26×10^2	1.07×10^5
A_{55}	2.22×10^2	2.22×10^2
A_{66}	2.30×10^2	2.38×10^2
ψ_1	68.8	71.0
ψ_2	77.4	84.8
ψ_3	67.6	68.9
k_1	13.2	14.9
k_2	76.3	76.7
k_3	5.92	5.93
c^*	0.271	0.271
ρ^*	0.0666	0.0666

Figure 7. Predicted Properties for the Typical Reinforced Propellants

Prop. \ Material	S Material	CW Material
$E_1 = 1/C_{11}$	1.43×10^5	4.11×10^4
$\nu_{12} = -C_{12}/C_{22}$	0.20	2.22
$\nu_{13} = -C_{13}/C_{33}$	0.74	1.09
$\nu_{21} = -C_{12}/C_{11}$	0.03	0.17
$E_2 = 1/C_{22}$	9.17×10^5	5.48×10^5
$\nu_{23} = -C_{23}/C_{33}$	0.10	-0.12
$\nu_{31} = -C_{13}/C_{11}$	0.90	0.77
$\nu_{32} = -C_{23}/C_{22}$	0.75	-1.12
$E_3 = 1/C_{33}$	1.17×10^5	5.81×10^4
α_1	3.91×10^{-5}	9.83×10^{-5}
α_2	1.40×10^{-5}	8.83×10^{-6}
α_3	8.17×10^{-5}	3.19×10^{-5}

Figure 8. Inverse Properties for the Typical Reinforced Propellants

	Properties From Reference [3]	Predicted Properties
$A_{11}/2\mu_w$	0.43	0.44
$A_{12}/2\mu_w$	0.16	0.16
$A_{13}/2\mu_w$	0.15	0.16
$A_{22}/2\mu_w$	0.91	0.91
$A_{23}/2\mu_w$	0.16	0.16
$A_{33}/2\mu_w$	0.43	0.44
$A_{44}/2\mu_w$	0.095	0.088
$A_{55}/2\mu_w$	0.065	0.063
$A_{66}/2\mu_w$	0.095	0.088
$\psi_1/2\mu_w\psi_w$	1.4	1.3
$\psi_2/2\mu_w\psi_w$	1.3	1.2
$\psi_3/2\mu_w\psi_w$	1.4	1.3
k_1/k	0.85	0.85
k_2/k	0.87	0.87
k_3/k	0.85	0.85

Figure 9. Comparison of predicted properties to the results given in reference [3]

	Experimental Results, [5]	Predicted Properties
E_z	26,200 (18,500)	23,000
$\nu_{\theta z}$	.283 (.212)	.556
$\nu_{z\theta}$	2.17	1.51
E_θ	60,000	62,500

Figure 10. Comparison of predicted properties to the experimental results given in reference [5].

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

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13. ABSTRACT

A theoretical investigation of the mechanical, thermal and thermal-mechanical properties of two types of reinforced solid propellants is reported. The two materials that are considered are wire screen and continuous wire wound reinforced propellants. Predictions based upon the unit cell concept are made for the properties of the reinforced propellants. Computer programs are listed and described for the evaluation of the expressions given by the theoretical predictions. The accuracy of the predictions is discussed in relationship to other theoretical and to experimental results.

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