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RECOIL EFFECTS IN HEAVY ION REACTIONS^{*}

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Abstract

Expressions for the recoil effects in single nucleon transfer in heavy ion reactions are obtained in first order in the mass ratio (m/M), where m is the mass of the transferred nucleon and M is the mass of either of the nuclear cores. The effect of recoil on the j dependence, the energy dependence and angular momentum of the residual nucleus is predicted.

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1. Introduction

Recent experiments¹⁾ on single nucleon transfer in heavy ion induced reactions seem to indicate that the approximation of neglecting terms of the order m/M^2 , where m is the mass of the transferred nucleon and M is the mass of the projectile or the target, is inadequate. The importance of these recoil terms had been pointed out by Greider³⁾ who had explicitly demonstrated the effect of recoil on the differential cross sections at high energies. An approximate first order treatment of recoil⁴⁾ was proposed by the author, where the recoil corrections were included by making a Taylor series expansion of the distorted waves and retaining only the terms of the order m/M , and the gradient of the distorted waves was evaluated in the W. K. B. approximation. The above theory is in agreement with the kinematic conditions of Brink⁵⁾. The selection rules proposed by Brink for single nuclear transfer at energies above the Coulomb barrier explicitly allow one to estimate the angular momentum carried by the neglected recoil term. The angular momentum conservation rule is given by

$$\lambda_2 - \lambda_1 = \delta(\mu VR) \quad (1.1)$$

where λ_1 and λ_2 are the Z components of the single particle orbital angular momentum in its initial and final bound states, and μVR represents the angular momentum of relative motion of the colliding particles at the distance of closest approach. The local angular momentum is different in the initial and final states because there is a change in the reduced mass due to mass transfer, and a change in the relative velocity and the distance of closest approach. In the approximation of Buttle and Goldfarb²⁾, one assumes that one can neglect

the change in R , so that one would obtain

$$\lambda_2 - \lambda_1 = R\delta(\mu V) \quad (1.2)$$

Thus the recoil momentum is explicitly defined by the quantity

$$\begin{aligned} \lambda_{\text{recoil}} &= (\mu V)\delta(R) \\ &= \frac{m}{M_1} (\mu V R_1) - \frac{m}{M_2} (\mu V R_2) \end{aligned} \quad (1.3)$$

Since the relative angular momentum of the colliding particle is usually large in reactions above the Coulomb barrier, the recoil angular momentum could become of the order of 1 or 2 units at energies of 2 MeV per nucleon above the Coulomb barrier. The recoil term obtained by the author⁴) was identical to the above expression. The Taylor series expansion of the distorted waves yields

$$\chi_{\vec{k}_i}^{(+)}(\vec{r}_i) \cong (1 - i \frac{m}{M_1} \vec{K}_i \cdot \vec{r}_1) \chi_{\vec{k}_i}^{(-)}(\vec{r}) \quad , \quad (1.4a)$$

$$\chi_{\vec{k}_f}^{(-)}(\vec{r}_f) \cong (1 + i \frac{m}{M_2} \vec{K}_f \cdot \vec{r}_2) \chi_{\vec{k}_f}^{(-)}(\vec{r}) \quad , \quad (1.4b)$$

where $\vec{K}_i, \vec{K}_f$ are respectively the local momenta at the distance of closest approach for the initial and final states, $\vec{r}$ is the intercore distance vector, $\vec{r}_1$ and $\vec{r}_2$ are respectively the vectors representing the distances of the transferred nucleon from the outgoing ion and the target, and M_1 and M_2 are the masses of the target and the residual nucleus. If one assumes that the transfer takes place in the peripheral region around the distance of closest approach, the recoil corrections predicted by eqs. (1.3) and (1.4) should be qualitatively similar.

The approximate treatment of recoil. eq. (1.4), has been applied to analyze the experiments on single proton transfer to Pb^{208} with O^{16} and C^{12} ions at two different energies⁶). The analysis indicates the importance of recoil in predicting the j dependence noticed in these reactions as well as the energy dependence. In the first order treatment of recoil, the transfer amplitude is obtained as a sum of two coherent amplitudes. The two amplitudes are determined by angular momentum transfer which are different. The reliability of the spectroscopic factors from the above treatment is thus very sensitive to the relative phase of the no-recoil and the first order recoil amplitudes. In the present paper, we obtain the exact first order recoil amplitude.

2. The Transition Amplitude

We shall use the co-ordinate system used by Buttle and Goldfarb²⁾ and use their terminology throughout this paper. We are interested in the reaction schematically represented by

In a distorted wave Born approximation (DWBA), the transfer amplitude is given by

$$T_{fi}(\vec{k}_f, \vec{k}_i) = \int d^3r \int d^3r_1 \chi_{\vec{k}_f}^{(-)*}(\vec{r}_f) \langle a_2 \alpha_2 c_1 r_1 | V_{c_1 n}(r_1) | a_1 \alpha_1 c_2 r_2 \rangle \times \chi_{\vec{k}_i}^{(+)}(\vec{r}_i) \quad (2.2)$$

where $\chi_{\vec{k}_i}^{(+)}(\vec{r}_i)$ and $\chi_{\vec{k}_f}^{(-)}(\vec{r}_f)$ are respectively the distorted waves in the initial and final channels, and $V_{c_1 n}(r_1)$ is the interaction which binds the neutron in the projectile. We wish to use the vectors $\vec{r}$ and $\vec{r}_1$ as the two linearly independent vectors. We thus expand the initial and final distorted waves in a Taylor series,

$$\chi_{\vec{k}_i}^{(+)}(\vec{r}_i) \cong \left[1 - \frac{m}{M_{a_1}} \vec{r}_1 \cdot \vec{\nabla} \right] \chi_{\vec{k}_i}^{(+)}(\vec{r}) \quad , \quad (2.3a)$$

and

$$\chi_{\vec{k}_f}^{(-)*}(\vec{r}_f) \cong \left[1 + \frac{m}{M_{a_2}} \vec{r}_1 \cdot \vec{\nabla}' \right] \chi_{\vec{k}_f}^{(-)*}(\vec{r}') \quad , \quad (2.3b)$$

where

$$\vec{r}' = \frac{M_{c2}}{M_{a2}} \vec{r}, \quad (2.3c)$$

and m is the mass of the transferred neutron. The transfer amplitude can thus be expressed as

$$T_{fi}(\vec{k}_f, \vec{k}_i) = T_{fi}^{(0)}(\vec{k}_f, \vec{k}_i) + T_{fi}^{(1)}(\vec{k}_f, \vec{k}_i) \quad (2.4)$$

where $T_{fi}^{(0)}$ is the no-recoil amplitude evaluated by various authors³⁾, and $T_{fi}^{(1)}$ is a first order recoil amplitude. One can obtain an expression for $T_{fi}^{(1)}$ in the notation of Tobocman et al.²⁾. However, we shall obtain the expression using the method of Buttle and Goldfarb²⁾ where one assumes that the reaction occurs in the asymptotic region of the final single nucleon bound state. Replacing the final bound state by an exponential decaying spherical Hankel function, one obtains

$$\begin{aligned} T_{fi}^{(1)}(\vec{k}_f, \vec{k}_i) = & \theta_{j_1 l_1}^{(1)} \theta_{j_2 l_2}^{(2)} N_2 \sqrt{2\pi} \sum_{\ell \ell'} \sum_{LM} i^{\ell+\ell'-L} \frac{\hat{\ell}^2 \hat{\ell}_1 \hat{\ell}_2 \hat{j}_1 \hat{j}_2}{\hat{\ell} \hat{L}^3} \\ & \times (-1)^{\ell_1+\ell-1/2-\rho_1} \langle \ell_0 \ell_2 0 | \ell' 0 \rangle \langle \ell_1 0 | 0 | \ell' 0 \rangle \\ & \times U(\ell_1 j_1 \ell_2 j_2; 1/2 L) U(\ell_1 \ell_2 \ell; \ell' L) \langle j_1 - \rho_1 j_2 \rho_2 | LM \rangle \\ & \times \langle j_1 \rho_1 c_1 \gamma_1 | a_1 \alpha_1 \rangle \langle j_2 \rho_2 c_2 \gamma_2 | a_2 \alpha_2 \rangle B_{\ell_1 \ell'} t_{LM}^{(1)}(\vec{k}_f, \vec{k}_i) \end{aligned} \quad (2.5a)$$

where it has been assumed that the single particles are in pure $(n\ell j)$ configurations. The coefficients $U(abcd;ef)$ are related to the W coefficient through the relation

$$U(abcd;ef) = \hat{e}\hat{f} W(abcd;ef) \quad (2.5b)$$

and where we use the notation $\hat{a} = (2a+1)^{1/2}$. The nuclear overlap integral

$B_{\ell_1 \ell'}$ is defined by

$$B_{\ell_1 \ell'} = \int_0^\infty r_1^3 dr_1 j_{\ell'}(i\chi_2 r_1) V(r_1) U_{\ell_1}(r_1) \quad (2.5c)$$

where $j_{\ell'}(i\chi_2 r_1)$ is a spherical Bessel function of order ℓ' , χ_2 is the decay constant of the final bound state, and $U_{\ell_1}(r_1)$ is the radial wave function for the initial bound state. The distorted wave integral $t_{LM}^{(1)}$ is given by

$$\begin{aligned} t_{LM}^{(1)}(\vec{k}_f, \vec{k}_i) = & -\frac{m}{Ma_1} \left\{ b_{LM}^{(1)}(\vec{k}_f, \vec{k}_i) + c_{LM}^{(1)}(\vec{k}_f, \vec{k}_i) \right\} \\ & + \frac{m}{Ma_2} \left\{ b_{LM}^{(2)}(\vec{k}_f, \vec{k}_i) + c_{LM}^{(2)}(\vec{k}_f, \vec{k}_i) \right\} \end{aligned} \quad (2.6a)$$

where

$$b_{LM}^{(1)} = \langle 0010 | L0 \rangle \int d^3r \chi_{\vec{k}_f}^{(-)}(\vec{r}') h_{\ell}^{(1)*}(i\chi_2 r) Y_{LM}^*(\hat{r}) \frac{\partial}{\partial r} \chi_{\vec{k}_i}^{(+)}(\vec{r}) \quad (2.6b)$$

$$c_{LM}^{(2)} = \langle 0010 | L0 \rangle \int d^3r \left[\frac{\partial}{\partial r} \chi_{\vec{k}_f}^{(-)}(\vec{r}') \right] h_{\ell}^{(1)*}(i\chi_2 r) Y_{LM}^*(\hat{r}) \chi_{\vec{k}_i}^{(+)}(\vec{r}) \quad (2.6c)$$

and

$$\begin{aligned} c_{LM}^{(1)} = & \sqrt{2} \sum \langle 0010 | L'0 \rangle U(L'\ell 11; 1L) \langle L'-M'1-\nu | LM \rangle \\ & \times \int d^3r \chi_{\vec{k}_f}^{(-)*}(\vec{r}') \frac{h_{\ell}^{(1)*}(i\chi_2 r)}{r} Y_{L'-M'}^*(\hat{r}) (-1)^{\nu} Y_{L\nu} \chi_{\vec{k}_i}^{(+)}(\vec{r}) \end{aligned} \quad (2.7a)$$

$$\begin{aligned}
(2) = & \sqrt{2} \sum_{\substack{L'M' \\ v'}} \langle \ell 0 1 0 | L' 0 \rangle U(L' \ell 1 1; 1 L) \langle L' -M' 1 -v' | LM \rangle \\
& \times \int d^3 r (-1)^{v'} L_{v'} \chi_{\vec{k}_f}^{(-)*}(\vec{r}') \frac{h_{\ell}^{(1)*}(i\chi_2 r)}{r} Y_{L'-M'}^*(\hat{r}) \chi_{\vec{k}_i}^{(+)}(\vec{r})
\end{aligned} \quad (2.7b)$$

In eqs. (2.6) and (2.7), $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial r'}$ are radial differential operators, and $L_{v'}$, $v'=0, \pm 1$, are the spherical components of the orbital angular momentum operators which operate on the distorted wave functions immediately to their right. The transfer cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} \frac{1}{(2a_1+1)(2c_2+1)} \sum |T_{fi}|^2 \quad (2.8)$$

$$\frac{d\sigma}{d\Omega} \Big|_0 + \frac{d\sigma}{d\Omega} \Big|_{\text{Int}} + \frac{d\sigma}{d\Omega} \Big|_{\text{Recoil}}$$

where $\frac{d\sigma}{d\Omega} \Big|_0$ is the no-recoil cross section, $\frac{d\sigma}{d\Omega} \Big|_{\text{recoil}}$ is the recoil cross-section

and $\frac{d\sigma}{d\Omega} \Big|_{\text{Int}}$ is the interference term. They are given by

$$\frac{d\sigma}{d\Omega} \Big|_0 = \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} \frac{(2a_2+1)}{(2c_2+1)(2j_2+1)} S |A_{\ell_1}|^2 \sum_L \langle j_1 1/2 L 0 \ j_2 1/2 \rangle^2 |t_{LM}^{(0)}|^2 \quad (2.9a)$$

where

$$t_{LM}^{(0)} = \int d^3 r \chi_{\vec{k}_f}^{(-)*}(\vec{r}') h_L^{(1)*}(i\chi_2 r) Y_{LM}^*(\hat{r}) \chi_{\vec{k}_i}^{(+)}(\vec{r}) \quad (2.9b)$$

and

$$S = \left(\theta_{j_1 \ell_1}^{(1)} \theta_{j_2 \ell_2}^{(2)} N_2 \sqrt{4\pi} \right)^2 \quad (2.9c)$$

and

$$A_{\ell_1} = \int r_1^2 dr_1 j_{\ell_1}^* (ix_2 r_1) V(r_1) U_{\ell_1}(r_1)$$

$$\begin{aligned} \frac{d\sigma}{d\Omega} \Big|_{\text{Int}} &= \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} S \frac{(2a_2+1)}{2(2c_2+1)} \sum_{\substack{\ell\ell' \\ LM}} i^{L+\ell'-\ell_1-\ell} \frac{(2\ell_1+1)(2\ell_2+1)}{(2L+1)} \\ &\times \left[\frac{(2\ell_2+1)}{(2\ell'+1)(2L+1)} \right]^{1/2} (-1)^{L-\ell} \langle L0\ell_2 0 | \ell_1 0 \rangle \langle \ell_0 \ell_2 0 | \ell' 0 \rangle \langle \ell_1 0 1 0 | \ell' 0 \rangle \\ &\times U^2(\ell_1 j_1 \ell_2 j_2; 1/2L) U(\ell_1 1 \ell_2 \ell; \ell' L) A_{\ell_1}^* B_{\ell_1 \ell'} t_{LM}^{(0)*} t_{LM}^{(1)} \end{aligned} \quad (2.10)$$

+ complex conjugate

and

$$\begin{aligned} \frac{d\sigma}{d\Omega} \Big|_{\text{Recoil}} &= \frac{\mu_i \mu_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} S \frac{(2a_2+1)}{2(2c_2+1)} \sum_{\substack{\ell\ell' \\ \bar{\ell}\bar{\ell}' \\ LM}} i^{\ell+\bar{\ell}'-\ell-\bar{\ell}} \\ &\times \left[\frac{(2\ell+1)(2\bar{\ell}+1)(2\ell_1+1)(2\ell_2+1)}{(2L+1)^3} \right] \left[\frac{1}{(2\ell'+1)(2\bar{\ell}'+1)} \right]^{1/2} (-1)^{\ell+\bar{\ell}} \\ &\times \langle \ell_0 \ell_2 0 | \ell' 0 \rangle \langle \bar{\ell}_0 \ell_2 0 | \bar{\ell}' 0 \rangle \langle \ell_1 0 1 0 | \ell' 0 \rangle \langle \ell_1 0 1 0 | \bar{\ell}' 0 \rangle \\ &\times U^2(\ell_1 j_1 \ell_2 j_2; 1/2L) U(\ell_1 1 \ell_2 \ell; \ell' L) U(\ell_1 1 \ell_2 \bar{\ell}; \bar{\ell}' L) \\ &\times B_{\ell_1 \ell'}^* B_{\ell_1 \bar{\ell}'} |t_{LM}^{(1)}|^2 \end{aligned} \quad (2.11)$$

Equations (2.9) to (2.11) give the exact expression for the first order recoil corrected cross-sections. As the energy of the projectile becomes larger, one would expect the validity of the Taylor Series expansion of the distorted wave to break down. We feel, however, that the expansion is meaningful because of the convergence of the nuclear overlap integrals. In fact, the overlap integral $B_{\ell_1+\ell', R}$ appearing in the recoil amplitude is related to the integral A_{ℓ_1} appearing in the no-recoil amplitude. If one uses the approximate expression for A_{ℓ_1} given by Buttle and Goldfarb²),

$$A_{\ell_1} = N_1 (-1)^{\ell_1} \frac{\hbar^2}{2m} \frac{\chi_2^{\ell_1}}{\chi_1^{\ell_1+1}} \quad (2.12)$$

one obtains

$$\begin{aligned} B_{\ell_1 \ell'} &= 0 \text{ for } \ell' = \ell_1 + 1 \\ &- i \frac{2\ell_1 + 1}{\chi_2} \text{ for } \ell' = \ell_1 - 1 \end{aligned} \quad (2.13)$$

and the contributions from all the higher order terms in the Taylor Series expansion vanish. The approximation, eq. (2.12), is valid if $\chi_2 R_1 \ll \ell$.

There are a large number of reactions which fall into the above category.

3. The Distorted Wave Integrals

We shall neglect the spin-orbit force in the optical potential. The distorted waves can be expanded into partial waves as follows:

$$\chi_{\vec{k}_i}^{(+)}(\vec{r}) = \frac{4\pi}{k_i r} \sum_{L_i M_i} i^{L_i} \chi_{L_i}(k_i, r) Y_{L_i M_i}^*(\hat{k}_i) Y_{L_i M_i}(\hat{r}) \quad (3.1)$$

$$\chi_{\vec{k}_f}^{(-)*}(\vec{r}') = \frac{4\pi}{k_f r'} \sum_{L_f M_f} i^{-L_f} \chi_{L_f}(k_f, r') Y_{L_f M_f}(\hat{k}_f) Y_{L_f M_f}^*(\hat{r}') \quad (3.2)$$

where the radial waves $\chi_L(k, r)$ have the asymptotic form

$$\chi_L(k, r) \xrightarrow{r \rightarrow \infty} \frac{i}{2} \left[H_L^*(kr) - \eta_L H_L(kr) \right] \quad (3.3)$$

where $H_L = G_L + iF_L$ is the outgoing wave Coulomb function and η_L is the reflection coefficient for the L wave. The integral $t_{LM}^{(0)}$ is given by

$$\begin{aligned} t_{LM}^{(0)}(k_f, k_i) &= \int d^3r \chi_{\vec{k}_f}^{(-)*}(\vec{r}') h_L^{(1)*}(i\chi_2 r) Y_{LM}^*(\hat{r}) \chi_{\vec{k}_i}^{(+)}(\vec{r}) \\ &= \frac{\sqrt{4\pi}}{k_i k_f} \sum_{L_i L_f} i^{L_i - L_f} \hat{L}_f^{2\hat{L}} \left[\frac{(L_f - M)!}{(L_f + M)!} \right]^{1/2} P_{L_f}^M(\theta) \\ &\quad \times I_{L_i L L_f}(k_i, k_f) \langle L_f - M L M | L_i 0 \rangle \\ &\quad \times \langle L_f 0 L 0 | L_i 0 \rangle \end{aligned} \quad (3.4)$$

where

$$I_{L_i L L_f}(k_i, k_f) = \int dr \chi_{L_f}(k_f, r) h_L^{(1)*}(i\chi_2 r) \chi_{L_i}(k_i, r) \quad (3.5)$$

In deriving eq. (3.4), k_i has been chosen along the z-axis and the reaction plane is chosen to be the x-z plane. There are two types of distorted wave integrals that appear in the recoil amplitude. The integral $b_{LM}^{(1)}$, for example, will be given by

$$\begin{aligned}
 b_{LM}^{(1)} &= \langle \ell 0 1 0 | L 0 \rangle \int d^3 r \chi_{\vec{k}_f}^{(-)*}(\vec{r}) h_\ell^{(1)*}(i\chi_2 r) y_{LM}^*(\hat{r}) \frac{\partial}{\partial r} \chi_{\vec{k}_i}^{(+)}(\vec{r}) \\
 &= \frac{\sqrt{4\lambda}}{k_i k_f} \langle \ell 0 1 0 | L 0 \rangle \sum_{L_i L_f} i^{L_i - L_f} \hat{L}_f^2 \hat{L} \left[\frac{(L_f - M)!}{(L_f + M)!} \right]^{1/2} P_{L_f}^M(\theta) \\
 &\quad \times \langle L_f - M M | L_i 0 \rangle \langle L_f 0 L 0 | L_i 0 \rangle \\
 &\quad \times \int dr \chi_L(k_f, r) h_\ell^{(1)*}(i\chi_2 r) \left(\frac{\partial}{\partial r} - \frac{1}{r} \right) \chi_{L_i}(k_i, r)
 \end{aligned} \tag{3.6}$$

$b_{LM}^{(2)}$ will be given by a similar expression. The other integral $c_{LM}^{(1)}$ is given by

$$\begin{aligned}
 c_{LM}^{(1)} &= \frac{-\sqrt{2} \sqrt{4\lambda}}{k_i k_f} \sum_{L_i L_f} \langle \ell 0 1 0 | L' 0 \rangle \langle L_f 0 L' 0 | L_i 0 \rangle i^{L_i - L_f} U(L' \ell 1 1; 1 L) \\
 &\quad \times \hat{L}_f^2 \hat{L}' \left[\frac{(L_f - M)!}{(L_f + M)!} \right]^{1/2} P_{L_f}^{-M}(\theta) [L_i (L_i + 1)]^{1/2} \\
 &\quad \times \langle L_f - M M | L_i 0 \rangle U(L_f L' L_i 1; L_i L) \\
 &\quad \times I_{L_i \ell L_f}(k_i, k_f)
 \end{aligned} \tag{3.7}$$

where $I_{L_i \ell L_f}$ is defined in eq. (3.5). Thus, in addition to the radial-integral of the type, eq. (3.5), one has to evaluate radial integrals of the form appearing in eq. (3.6)

The magnitude of the radial integrals are very sensitive to the value of ℓ characterizing the form factor $h_{\ell}^{(1)*}(i\chi_2\gamma)$, becoming larger for larger ℓ . In the no-recoil approximation, ℓ is determined by the selection rules

$$\left. \begin{aligned} |j_1 - j_2| &\leq \ell \leq j_1 + j_2 \\ |l_1 - l_2| &\leq \ell \leq l_1 + l_2 \\ l_1 + l_2 + \ell &= \text{even} \end{aligned} \right\} \quad (3.8)$$

In the case of the recoil amplitude, we obtain

$$\left. \begin{aligned} (l_2 + l + l_1) &= \text{odd}, \\ (l_1 + l_2 + 1)_{\min} &\leq \ell \leq (l_1 + l_2 + 1) \\ (j_1 + j_2 + 1)_{\min} &\leq \ell \leq (j_1 + j_2 + 1) \end{aligned} \right\} \quad (3.9)$$

Thus the angular momentum ℓ has opposite parities for the no-recoil and the first order recoil amplitude. If the second order term in the Taylor expansion were included, one would once again obtain terms of the same parity as in the no-recoil approximation, but with a wider range of ℓ values. The importance of higher order terms can therefore be estimated by comparing the no-recoil amplitude with the normal parity amplitude in an exact calculation such as those of Devries⁸⁾ or Bock and Yoshida⁹⁾.

4. Discussion

One can attempt to make some predictions on the effect of recoil. From eqs. (3.8) and (3.9), one can show that the maximum angular momentum transfer allowed by the first order recoil term will always be one unit larger than that allowed in the no-recoil approximation. However, one has to consider also the properties of the overlap integral $B_{\ell_1 \ell'}$. From eq. (2.13), it is seen that only the integral corresponding to $\ell' = \ell_1 - 1$ will be non-vanishing if eq. (2.2) is valid. In some test calculations, it was verified that eq. (2.12) is valid only for very small binding energies of the final nucleon bound state, i.e.; for $\chi_2 R_1 \leq 1$. Thus, one can expect this condition to hold for neutron transfer to a final bound state with binding energy of the order of 3 to 4 MeV. In these cases, one obtains a considerable increase in the cross-section due to recoil only for the case with $j_1 = \ell_1 - 1/2$ and $j_2 = \ell_2 - 1/2$. The maximum allowed ℓ transfer, in this case, is $\ell_1 + \ell_2 - 2$ for the no-recoil amplitude and $\ell_1 + \ell_2 - 1$ for the recoil amplitude. The DWBA integral increases with increasing ℓ and hence the recoil amplitude will be large. For all other values of j_1 and j_2 , the ℓ transfer for recoil is one unit less than that allowed for the no-recoil amplitude. The above case applies to the reaction $\text{Pb}^{208}(O^{16}, N^{15})\text{Bi}^{209}$ leading to states of Bi^{209} with $j_2 = \ell_2 - 1/2$. The experiment does exhibit stronger recoil effects for these states than states with $j_2 = \ell_2 + 1/2$.

An obvious effect of recoil is to change the energy dependence of the transfer cross section. In general, if the no-recoil and recoil amplitudes are in phase, the energy dependence of the cross-section will be steeper than predicted by the no-recoil theory. This effect has been verified in the study of proton transfer on Pb^{208} with O^{16} ions where it was found that the discrepancy

between the j lower and j upper states became larger at the higher bombarding energy. The no-recoil and recoil amplitudes were found to interfere constructively for the $j_{<}$ and destructively for the $j_{>}$ states. Finally, one can predict that the recoil effects will be more important for states with large angular momenta. This arises from the properties of the form factor which behaves like a spherical Hankel function. The ratio of the form factors for $\ell + 1$ and ℓ increases with ℓ .

Summary and Acknowledgments

The problem of incorporating the effect of recoil in heavy ion transfer reactions has been attempted. The exact expression for the recoil correction in first order is derived and its consequences analyzed. It should be pointed out that the analysis of recoil effects by a Taylor expansion is, at the moment, academic and one needs to demonstrate the adequacy of such an expansion. However, it is evident from the discussions above that such an analysis sheds light on some of the features of recoil effects which would remain hidden in an exact calculation. It would be interesting to analyze the available experimental data, with recoil effects introduced approximately or exactly.

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