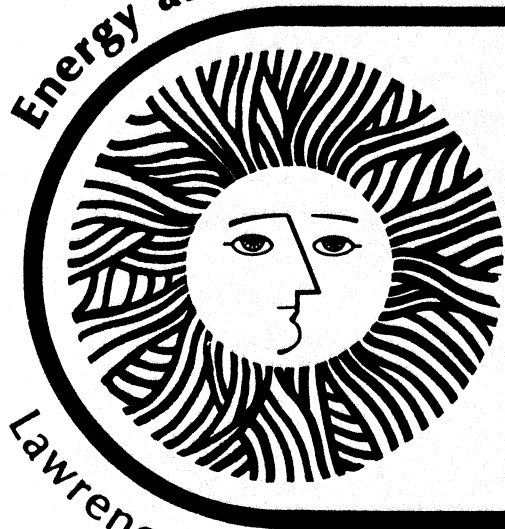


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**Energy and Environment Division**



Convolution Principle As Applied  
To The Heat Transfer Problems Of  
Buildings And Fundamentals Of  
Its Efficient Use

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CONVOLUTION PRINCIPLE AS APPLIED  
TO THE HEAT TRANSFER PROBLEMS OF BUILDINGS  
AND FUNDAMENTALS OF ITS EFFICIENT USE\*

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The major "Energy Utilization Analysis of Buildings" computer programs utilize the Convolution Principle for the following purposes:

- (1) to compute heat gain (loss is considered as negative heat gain) through a wall/roof;
- (2) to compute exterior surface temperature of a wall/roof;
- (3) to compute the time delay between heat gain to a space and resulting loads on the heating, ventilating and air conditioning system.

To understand the use of Convolution Principle, as an example, heat gain into the building through a wall/roof is explained as follows.

The value of heat gain  $Q$  into the building through a wall/roof depends on the present value, and the past history of the temperature difference  $\Delta T$  between the inside air and the outside surface of the wall/roof. In other words, the graph of the schedule of  $Q$  vs. time  $t$  depends on the graph of the schedule of  $\Delta T$  vs.  $t$  as shown in Fig. 1.

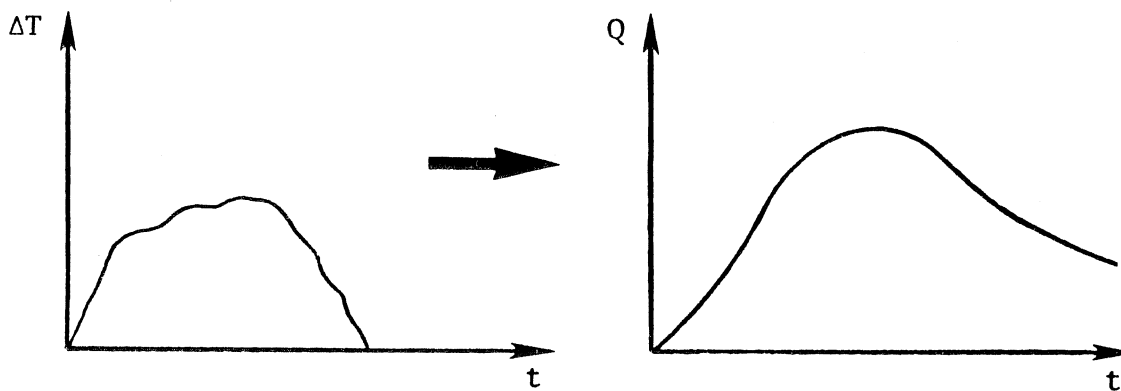


Fig. 1. Dependence of heat gain schedule on temperature difference schedule.

If it were necessary to compute  $Q$  for each hour, on the basis of the hourly history of  $\Delta T$ , the differential equation of heat conduction would have to be repeatedly solved. This time-consuming operation, however, can be simplified so that  $Q$  need be determined as a function of  $t$  for only one temperature difference schedule. When a unit height isosceles triangle temperature pulse is used as one temperature schedule, the values of  $Q$  at successive equal-time intervals elicited by this unit height isosceles triangles, are called the response factors  $r_0, r_1, \dots$  of the wall/roof construction as shown in Fig. 2.

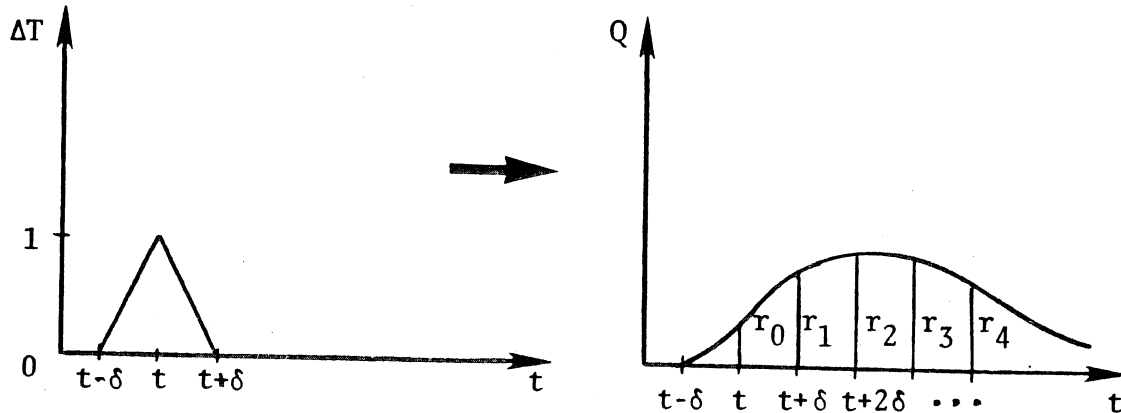


Fig. 2 Heat gain schedule for a unit height isosceles triangle temperature pulse showing response factors.

Any arbitrary schedule of  $\Delta T$  (Fig. 3a) may be approximated by a schedule of  $\Delta T'$ , whose values agree with those of  $\Delta T$  at integral multiples of the time interval  $\delta$ . This schedule of approximate temperature differences  $\Delta T'$ , may be resolved into a series of isosceles triangle pulses ( $\Delta T_1, \Delta T_2, \Delta T_3, \Delta T_4$ , and  $\Delta T_5$  in Fig 3c) which, when added together, give exactly  $\Delta T'$ . Each of these component pulses has a base width, or duration of  $2\delta$ , a peak occurring at each integral multiple of  $\delta$ , and a height equal to the value of  $\Delta T'$  at the time of the pulse's peak. Each pulse alone would elicit its own heat gain schedule as shown in Figs. 3d, 3e, 3f, 3g and 3h. The heat gain schedules elicited by the individual pulses are all the same except for two differences. Their heights are proportional to the heights of the pulses which elicit them, and each is moved to the right on the time axis as far as the pulse which elicited it.

The values of the individual responses,  $Q_1 \dots Q_5$ , may be added at each value of time, to give the curve of sums (Fig. 3i). The superposition theorem asserts that the curve of sums is exactly the heat gain schedule which would be elicited by the approximate temperature difference schedule  $\Delta T'$ . Due to the smoothing effect of the heat transfer process,  $\Delta T$  and  $\Delta T'$  give nearly the same heat gain schedule. Therefore the curve of sums is very nearly the heat gain schedule elicited by the original temperature difference schedule  $\Delta T$ . This method of resolution and recombination is called the Convolution Principle, and can be expressed mathematically by the equation

$$Q_t = \sum_{i=0}^n r_i \Delta T_{t-i}, \quad (1)$$

whereas  $Q_t$  equals the heat gain at the time  $t$ ;  $\Delta T_{t-i}$  equals the temperature difference in hours previous to  $t$ ;  $r_i$  equals the  $t-1$ th response factor for the wall/roof; and  $n$  equals the number of hours of the temperature difference history which significantly effects  $Q_t$ . It should be noted that, the response

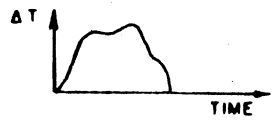


FIGURE 3a

ACTUAL  $\Delta T$  SCHEDULE ACROSS WALL

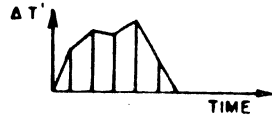


FIGURE 3b

APPROXIMATE  $\Delta T$  SCHEDULE



FIGURE 3c

THE COMPONENT PULSES



FIGURE 3d

FIRST COMPONENT PULSE AND HEAT GAIN IT ELICITS

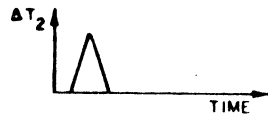


FIGURE 3e

SECOND COMPONENT PULSE AND HEAT GAIN IT ELICITS

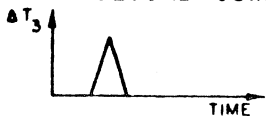


FIGURE 3f

THIRD COMPONENT PULSE AND HEAT GAIN IT ELICITS

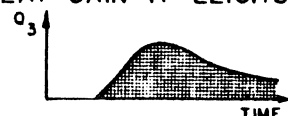


FIGURE 3g

FOURTH COMPONENT PULSE AND HEAT GAIN IT ELICITS

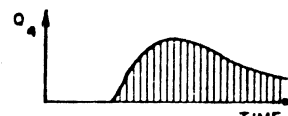


FIGURE 3h

FIFTH COMPONENT PULSE AND HEAT GAIN IT ELICITS

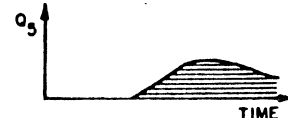


FIGURE 3i  
CURVE OF SUMS

Fig. 3 The convolution principle.

factors are the only information about the wall/roof which appears in Eq. (1). Thus, the response factors characterize completely the thermal properties of the structure of the wall/roof and, alone describe how the structure absorbs and releases heat over a prolonged period of time.

Utilization of Eq. (1), as is, requires extensive computation time at each simulated hour. However, use of a subtle modification of it which allows  $n$  to equal infinity, if it is necessary, saves tremendous amounts of computation time. The explanation of the fundamentals of this efficient technique is given as follows.

At time  $t-1$ , Eq. (1) can be written explicitly as

$$Q_{t-1} = r_0 \Delta T_{t-1} + r_1 \Delta T_{t-2} + \dots + r_m \Delta T_{t-m-1} + r_{m+1} \Delta T_{t-m-2} + r_{m+2} \Delta T_{t-m-3} + \dots \quad (2)$$

If response factors reach a Common Ratio at the  $m^{\text{th}}$  hour after a unit height isosceles triangle pulse is applied, then the Common Ratio  $R$  can be expressed as

$$R = \frac{r_{m+2}}{r_{m+1}} = \frac{r_{m+3}}{r_{m+2}} = \dots \quad (3)$$

Using Eq. (3), Eq. (2) can be written as

$$Q_{t-1} = r_0 \Delta T_{t-1} + r_1 \Delta T_{t-2} + \dots + r_m \Delta T_{t-m-1} + r_{m+1} (\Delta T_{t-m-2} + R \Delta T_{t-m-3} + \dots) \quad (4)$$

Similarly, at time  $t$ , Eq. (1) can be written as

$$Q_t = r_0 \Delta T_t + r_1 \Delta T_{t-1} + \dots + r_m \Delta T_{t-m} + r_{m+1} (\Delta T_{t-m-1} + R \Delta T_{t-m-2} + \dots) \quad (5)$$

Multiplying both sides of Eq. (4) by  $R$ , subtracting the result from Eq. (5), rearranging the terms and solving for  $Q_t$  gives

$$\begin{aligned} Q_t - RQ_{t-1} &= r_0 \Delta T_t + r_1 \Delta T_{t-1} + \dots + r_m \Delta T_{t-m} \\ &\quad + r_{m+1} (\Delta T_{t-m-1} + R \Delta T_{t-m-2} + \dots) \\ &\quad - R r_0 \Delta T_{t-1} - R r_1 \Delta T_{t-2} - \dots - R r_m \Delta T_{t-m-1} \\ &\quad - r_{m+1} (R \Delta T_{t-m-2} + R^2 \Delta T_{t-m-3} + \dots) \\ &= r_0 \Delta T_t + (r_1 - R r_0) \Delta T_{t-1} + \dots + (r_{m+1} - R r_m) \Delta T_{t-m-1} \end{aligned}$$

Let  $r'_0 = r_0$

$$r'_1 = r_1 - R r_0$$

⋮

$$r'_{m+1} = r_{m+1} - R r_m .$$

Then

$$Q_t - R Q_{t-1} = r'_0 \Delta T_t + r'_1 \Delta T_{t-1} + \dots + r'_{m+1} \Delta T_{t-m-1},$$

$$Q_t = R Q_{t-1} + \sum_{i=0}^m r'_m \Delta T_{t-i} . \quad (6)$$

Careful examination of Eq. (6) shows that once  $Q_{t-1}$  is calculated, it can be stored and later used in the calculation of  $Q_t$  along with a few multiplications and additions saving tremendous amounts of computation time compared to repetitive use of Eq. (1) for each simulated hour.

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