Centering Students' Perspectives:
A Multifocal Mixed Methods Investigation of Participatory Equity in a Distance Learning Calculus Class

By

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Abstract

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Advancing participatory equity – cultivating classrooms with fair (not necessarily equal) opportunities to participate – requires a deep understanding of students' varied learning experiences. Who is invited to participate and in what ways? Who has *genuine* opportunities to engage with rich mathematics? Who feels they have space to contribute? What, if anything, is holding students back? Every student has meaningful mathematical contributions to make, but not every student is invited and supported to share them. Gender and race shape students' opportunities to participate, with females and students from minoritized backgrounds often positioned as less capable and less likely to succeed, leading to disparate and stratified opportunities to engage in mathematics. Understanding the complexity of how participatory inequities are constructed and play out through classroom interactions calls for a much broader, multi-faceted inquiry than is currently found in the literature.

This dissertation digs beneath the surface of the commonly used phrase "equitable participation" to craft a more nuanced and comprehensive understanding of participatory equity. Three different sets of experiences in a distance learning high school calculus classroom are juxtaposed: (A) the perspectives of the teacher and student teacher, (B) the students' perspectives, and (C) the researcher's perspective, informed by lesson observations and video-stimulated interviews, and triangulated with various analytic methods including both qualitative and quantitative equity-focused analyses. In this context, student voice is central. Privileging students' perspectives provides a deeper sense of why students participated in the ways that they did, which also clarifies the implications of classroom interactions for students' content and identity development. Key questions include: What did participants notice, and what did they value? How did students' participatory experiences differ from one another, and how did these differences align with gender and/or race?

Analyses support four primary claims: (1) Participants' articulation of participation issues ("boys talking too much" or "girls not speaking up") focused on symptoms of problems as opposed to underlying causes. The underlying causes of unfairly distributed opportunities to participate were

structural in nature and carried out through interactions (e.g., having only students with complete and correct homework solutions present to the class). (2) Intertwined gendered and racialized storylines about mathematical competence shaped participants' experiences with participation in inequitable ways. Storylines affected how barriers to participation functioned, how participatory expectations were assigned, and how teacher-student interactions played out. (3) Participants' views of participation issues were consistent, and yet were different from the researcher's view based on semester-long participation metrics. Specifically, participants talked about male students dominating discussions but said little about race, whereas contribution metrics indicated that white dominance superseded male dominance, with male students of color having the fewest opportunities for mathematically meaningful participation. (4) Participants' views of small-group experiences were consequentially different from each other. In the focal group, three male students experienced the task positively, while the only female student experienced the opposite. The female student's feelings of not having space to contribute were sensed by the student teacher and supported by interaction analysis but were not noticed by other participants.

By bringing multiple perspectives to light and reflecting on the tensions revealed, this dissertation aimed to help unpack the challenges the field faces in grappling with issues of participatory equity in mathematics classrooms.

Table of Contents

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Chapter 1: Introduction

My commitment to pursuing equity in mathematics participation and learning has taken many forms over the years, from classroom teaching to coaching, parenting, and now research. This dissertation aims to support the field in moving one step closer to understanding what "equitable participation" is and how we, as educators and researchers, can work toward attaining this multi-dimensional, extremely complex goal. I begin this introduction by sharing a brief history of my experiences as a learner and teacher of mathematics, which provides grounding and direction for my work as a developing mathematics education researcher. Next, I describe how my understandings of equity have grown and shifted over time and how my commitments to participatory equity informed the design of my dissertation. I then share the goals and purpose of this research project, before concluding with an overview of the dissertation.

My Personal History with Mathematics Education

I have always loved math. As a child I enjoyed logic puzzles, number games, and even rote procedural processes. I believed I was good at math, and I believed I could understand complex mathematical problems. As a child I was also aware that not everyone loved math as much as I did, that many of my peers did not believe they were good at math, and that math enjoyment and perceptions of smartness seemed to be aligned with gender; math was more popular with boys than with girls. This pattern of underrepresentation of females in mathematics continued into college and into my first job in technology consulting. I felt like an exception to the rule.

I had observed the pattern of male dominance in mathematics for many years but did not think much about the reasons behind it until I enrolled in a teacher credential program at Mills College. This program opened my eyes to the political nature of mathematics education, to the ways traditional power structures organize and sort people to maintain the status quo, and to the ways mathematics education tends to marginalize not just female students but also students of color. I began to understand that students' enjoyment of and success with math was not simply based on students' preferences or aptitudes, but rather was a complex function of how society tends to position certain groups of people, specifically females and people of color, as less capable and less likely to succeed in mathematics. Thinking back to my STEM-focused college courses and early jobs, I was indeed in the minority in terms of gender, but I was in the *majority* in terms of race. Through my teaching experiences, I became more aware of my Whiteness and its associated privileges, and I began to understand some of the complexities involved in me being a White woman wanting to teach and serve communities of color.

Supporting the development of positive mathematical identities for students has always been an important goal for me. I believed every one of my middle school students over my 10 years of teaching was capable of engaging in rich mathematics and contributing to the collective learning of their class, but many students (as well as some teachers) did not share in these beliefs. I was troubled by the fact that my 7th grade "advanced" math classes were composed of mostly White and Asian students, while my lower level "support" classes included most of my Black and Hispanic students and students learning English as a second or third language. I felt I had no influence over who was in my class, but I had some control over the in-class experiences of students who ended up with me. It was important for my students to learn the intended mathematical content, but it was just as important (if not more) for my students to feel confident in their mathematical abilities, to believe they could solve complex math problems, and to see

themselves as successful in mathematics. Mathematical identity concerns were not limited to students in the lower-level math courses. There were numerous students in the regular and advanced math classes who suffered from low confidence and insecurities as well, often manifesting as hesitancy to take mathematical risks out of fear of being wrong.

I knew course enrollment was problematic, but I felt there was nothing I could do at that point to change the tracking procedures in place[1]. Therefore, I focused on what I could do as a teacher during class time with the students assigned to my classes. I grappled with the question, what can I do to help my students develop both strong content understandings and positive mathematical identities? One strategy I used was to design and implement daily small-group tasks intended to promote dialog among students and to create opportunities for students to participate together in various ways. I felt there was great potential for groupwork to elicit a wider array of productive types of participation from a wider array of students, but I also learned quickly that implementing effective groupwork was not easy. Issues of status were prevalent and influenced students' interactions with one another, positioning some students as more capable than others. I paid attention to whose ideas were taken up, who students looked to for guidance and help, and how students challenged and questioned each other when they disagreed. And I did what I could to manage interactions in-the-moment, attempting to mitigate status hierarchies and imbalanced power among students. I believe there were positive shifts in participation patterns, but overall, it seemed White and Asian male students continued to dominate. Countless students were not seeing themselves, and/or were not seen by others, as being mathematically capable. I was left feeling that I did not fully understand the issues at play, and therefore, was not doing enough to address them. I wanted to understand more about how and why students participate in the ways that they do and what teachers can do to support more mathematical engagement for more students. I felt I did not have the time nor the resources to investigate these questions while working as a classroom teacher, so I decided to enroll in graduate school.

My Developing Understandings of Equity

Through my personal and professional experiences, my understandings of issues related to equity, inclusion, and belonging and my commitment to building empowering learning communities has deepened. Teaching middle school math for 10 years in two vastly different public-school districts opened my eyes to the day-to-day challenges and triumphs of students living lives very different from my own. I learned that children's lives outside of school play major roles in their readiness to participate and learn. It was not until I really got to know individual students that I began to understand the depth and breadth of the inequities my students faced. It is not simply that "the playing field" is uneven; it is that the playing field is severely unjust. Many of my students faced barriers to participation that were far beyond my experience, barriers related to food insecurity, incarcerated family members, and racial discrimination. I wanted my students to think deeply about mathematics and to participate as their full selves in my class, but was it fair for me to expect that of them? Or would it have been unfair for me to expect any less of them? I constantly considered the question of what I, an upper middle class White woman, could and should do to support my students' engagement with mathematics, specifically the engagement of my students of color.

[1] I became an instructional coach after 10 years of teaching, working out of the school district office. In that role, I tried to change the course enrollment / tracking procedures to be more inclusive (and less racist) but to no avail.

I always considered myself an "equity-focused" educator, but it was not until my time at Berkeley that I deeply considered the meaning of equity. I have come to realize the word "equity" is used to mean many different things to many different people, and it is often used without definition. In graduate school I have thought hard about questions such as, what does it mean to strive for equity in and through mathematics education? What does equity look like, sound like, and feel like? And by whose standards should equity be judged? I do not have any certain or complete answers to these questions, but I believe my dissertation furthers the conversation about what equity means with respect to student participation in mathematics classrooms.

I have found Rochelle Gutiérrez's conceptualization of equity helpful to organize the various ways people think about and study equity (Gutiérrez, 2012). She divides equity perspectives into four dimensions (access, achievement, identity, and power), with the first two dimensions grouped together on the dominant axis and the second two on the critical axis. She emphasizes learning outcomes related to school settings, in addition to learning outcomes related to life more generally. When I reflect on the priorities I set and the decisions I made as a teacher, I characterize my equity perspective at that time as focused primarily on access with some attention to identity and some inkling that power was important. Even though achievement was not part of my focus, it still seems that overall, I conceptualized equity from a dominant perspective. I was working to create opportunities for more students to engage in mathematics, but the mathematics I was promoting was that which reflects the status quo in society. Although I was beginning to recognize that not all students entered the classroom in the same ways, I was still mostly focused on issues of access, including access to resources, rich math tasks, and positive mathematical experiences. In Gutiérrez's words, I was helping more students "play the game called mathematics" (Gutiérrez, 2012, p. 20), as opposed to working to change the game being played.

Through my time at Berkeley my understanding of and focus on equity has shifted to include more critical aspects. With my research, I acknowledge the need to address social and political issues that extend beyond classroom walls and that undoubtedly shape the interactions that occur within classroom contexts. My work is centered around Indigo Esmonde's definition of equity - a fair distribution of opportunities to participate and learn (Esmonde, 2009). While I continue to focus on opportunities for students to participate, my goals now extend beyond access. I think about educational inequities as existing in both learning outcomes and learning processes, with extra emphasis on the moment-to-moment classroom interactions through which inequities are constructed and experienced by students. I also think about how students' opportunities to participate shape, and are shaped by, the ways in which they are positioned with respect to mathematics and each other. In my dissertation, I consider how aspects of student identity played into the co-construction of opportunities to participate in ways that either supported content and identity development or failed to do so. I look at how individual student participation counts were connected to the construction of opportunities to participate, shifting the blame for limited verbal participation away from particular students and toward the interactional context. I explore what counted as *genuine* opportunities to participate and how genuine opportunities differed across students and contexts. I center the voices and experiences of students, specifically those who have been marginalized in mathematics classrooms, to understand how students made sense of their own and their peers' participation. And I privilege students' "insider" perspectives when determining to what extent and in what ways classroom

participation was equitable, acknowledging that it does not make sense to assess equity from researchers' "outsider" perspectives alone.

My current thinking about equity is reflected in the design of my dissertation. This study is guided by the following three principles in support of a comprehensive, multidimensional examination of participatory equity in a high school classroom context.

- Participatory equity is conceptualized in terms of both outcomes and processes, with an analytic focus on connections between individual student participation metrics and the interactional construction of opportunities to participate.
- Students are thought about as unique individuals who participate in classroom communities operating within broader social contexts.
- "Insiders" (students and teachers) and "outsiders" (researchers) offer complementary views of students' learning experiences, both of which are essential.

Elaborations of these three principles are included in Chapter 2: Theoretical and Conceptual Framing Supported by Prior Research.

Goals and Purpose of this Dissertation

Advancing participatory equity – cultivating classrooms with fair (not necessarily equal) opportunities to participate (Ernest et al., 2019; Esmonde, 2009; Secada, 1989; Shah & Lewis, 2019) – requires a deep understanding of students' varied learning experiences. Do students feel they have space to contribute? What, if anything, is holding them back from participating? Who is invited to participate and in what ways? Who has genuine opportunities to engage with mathematics?

Every student has meaningful mathematical contributions to make, but not every student is invited and supported to share them. Students' opportunities to develop rich content understandings and positive mathematical identities are differentially constructed through often subtle classroom interactions that distribute power unfairly among students (Gutiérrez, 2012; Leyva et al., 2021; Martin, 2009). Students who are positioned with competence and/or authority tend to have more opportunities to participate in consequential and influential ways, and therefore have better access to rich mathematical learning experiences and more academic power and status in the classroom (Herbel-Eisenmann et al., 2015; Langer-Osuna, 2011).

Gender and racial identity are primary shapers of students' positionings in mathematics classrooms, with female students and students from minoritized backgrounds often positioned as less capable and less likely to succeed (Gresalfi & Hand, 2019; Martin, 2000). Too often, problematic participation is attributed to deficiencies in individual students as opposed to problematic patterns of classroom interactions (McDermott, 2010). To disrupt participatory inequities, we first need to understand how these inequities take shape, how they are perceived by both teachers and students, and how classroom participants' perceptions align or misalign. If, for example, a teacher does not see patterns of inequity felt by the students, those patterns will go unaddressed. Likewise, if students do not understand the impact of their problematic behaviors, those behaviors are not likely to change.

Understanding this complexity calls for a much broader, multi-faceted inquiry than is currently found in the literature. Patterns of inequitable opportunities or positioning have

typically been documented by external observers, through participation counts or discourse or sociological analysis. But social interactions are highly complex; classroom participants (teachers, others who support instruction, and the students) may perceive or interpret the same events differently from each other and from external observers. As much as possible, understanding the participatory opportunities offered, taken, and not taken calls for the analysis of interactions and obtaining the perspectives of all actors involved, especially the perspectives of the students themselves. Only when these are all understood is it possible to understand the impact of proposed interventions.

This dissertation seeks to bring to light and juxtapose three different sets of experiences in a high school calculus classroom: (1) the perspectives of the teacher and student teacher, as documented by observations and interviews; (2) the students' experiences and perspectives, as seen in whole-class and small-group sessions, and documented in a range of interviews; and (3) the researcher's perspective, informed by observations and interviews, and triangulated with various analytic methods including quantitative equity-focused analyses.

In this context, student voice is central. Highlighting students' perspectives in conversations about participation and equity provides an opportunity to consider the reasons why students participate in the ways that they do. Hearing directly from students leads to a better understand of the implications of participatory interactions for students' content and identity development. Understanding factors affecting student engagement requires talking to and listening closely to what students have to say. They are the experts on what they do and why. Key questions include: What do participants notice, and what do they value? How do students' participatory experiences differ from one another, and how do these differences align with gender and/or race?

It goes without saying that the views of the instructional staff in this context (here, the teacher and student teacher) are essential to understand. The instructors orchestrate students' learning experiences, so what they believe about instruction and about their students, and how they implement those understandings in large measure determines the affordances of the learning environment for students. (How much and in what ways those affordances are taken advantage of is another matter; that is why the student perspective is essential.)

Finally, the more distanced (but not necessarily unbiased) view of the researcher provides a way to mediate the sometimes contrasting perceptions of individual participants, and to document some objective realities regarding participation that may be easier to see from an outside perspective.

Any one study is necessarily incomplete, but my hope is that bringing these multiple perspectives to light and reflecting on the tensions revealed will help to unpack the challenges the field faces in grappling with issues of participatory equity.

Dissertation Overview

This dissertation digs beneath the surface of the commonly used phrase "equitable participation" to craft a more nuanced and comprehensive understanding of participatory equity by examining what classroom participation looked, sounded, and felt like for students in one particular context, in this case, a distance learning high school calculus class. The overarching question addressed in this study is: *To what extent, in what ways, and from whose perspective was participation equitable?* Sub-questions address how students participated, how opportunities to participate were constructed, and how classroom participants made sense of

student participation patterns. The chapters that follow situate this study with respect to relevant theories, concepts, and literature (Chapter 2), provide a methodological overview of study design, data collection, and analysis (Chapter 3), present analytic results related to participants' perceptions of participation, along with qualitative and quantitative analyses of whole-class and small-group participation (Chapters 4 – 6), and finally, offer a discussion that presents conclusions and implications of this work.

Chapter 2 lays out the theoretical and conceptual foundations underlying the analyses, commentary, and implications presented in this dissertation. This chapter includes definitions for participation, learning, and participatory equity, an overview of positioning theory and its connections to participation and learning, and a conceptual framework designed to support examination of participatory equity in a classroom context. This chapter concludes by using prior research to describe and rationalize three principles that guided the design of this dissertation (focusing on participatory outcomes and processes, considering multiple levels of context, and integrating outsider and insider views of participation).

Chapter 3 describes the context of the study and the methodological approach taken with data collection and analysis. This chapter includes a statement about researcher positionality, a description of a pilot study that informed the design of this dissertation, the methods used to collect data across one school semester (spring 2021), and the various methods used for analysis organized by analytic chapter. The two focal classes in this study (12^{th} grade AB Calculus classes) were taught by a White female teacher with 30+ years of experience teaching high school mathematics, supported by a Mexican male student teacher enrolled in a teacher credential program at a local university. Integrating data from video-recorded class observations and interviews with classroom participants, I use a mixed methods approach to examine and triangulate whole-class and small-group participation outcomes (individual student participation metrics) and processes (construction of opportunities to participate).

Drawing from the whole corpus of data (lessons, lesson debrief sessions, interviews), Chapter 4 presents analysis related to participants' perceptions of classroom participation, including their identification of barriers, articulation of issues, and celebration of successes. The question guiding analysis in Chapter 4 was: *How did participants talk about connections between participation and various classroom, institutional, and societal factors that shaped student engagement in the course?* Analytic results indicated that distance learning and issues related to gender and/or race presented barriers to participation for many students. Interestingly, the teacher, student teacher, and students talked repeatedly about participatory issues related to gender but rarely talked about race. Specifically, they talked about how male students spoke more than female students. Some students were identified by their peers and their teacher as having achieved some level of participatory success, but even those "successful" female students and/or students of color talked about the barriers to participation they continued to face on a daily basis.

Chapter 5 presents analyses assessing participatory equity during whole-class discussions, including both qualitative and quantitative analyses of student participation (Creswell & Creswell, 2017). Whole-class participation data include video-recorded lesson observations and student contribution-level data collected in real-time using a tool called EQUIP (Equity QUantified In Participation) (Reinholz & Shah, 2018). The question guiding analysis in Chapter 5 was: *In what ways, to what extent, and from whose perspectives was classroom participation equitable during*

whole-class discussions in one high school calculus course? Analytic results showed that White dominance superseded gender dominance during whole-class discussions, with White males having the most robust opportunities to participate, followed by White females, female students of color, and finally male students of color. Participants' gender-focused observations of participation issues are juxtaposed with semester-long participation analyses that showed significant differences by racial groups but not gender groups.

Chapter 6 presents analyses of participatory equity during a small-group math task. The participation of one group of four students (Guadalupe, Yonas, Hosein, and Elijah) is examined by combining micro-interaction analysis of a 10-minute video of the group task with stimulated-recall observations made by the four students, their teacher, and their student teacher (Erickson, 2006). This chapter includes a close look at how Guadalupe's experience compared to the experiences of her peers. The question guiding analysis in Chapter 6 was: *In what ways, to what extent, and from whose perspectives was classroom participation equitable for one group of four students working on one calculus task?* Analytic results showed that participants' assessments of the small-group task differed in important ways, and overall, participation was not equitable for this group of four students. Opportunities to participate seemed to be distributed fairly among the three male students, but Guadalupe did not get her fair share. Analysis suggested specific interactional moments, differing thresholds for verbal participation between students, and gender imbalance in the group contributed to the inequities.

Chapter 7 synthesizes analyses from the previous three chapters and offers reflections on conceptualizing and assessing participatory equity in mathematics classrooms. Claims and implications of this study are organized according to the three guiding principles that informed the design of the study - a focus on participatory outcomes and processes, consideration of multiple levels of context, and integration of outsider and insider views.

- **Claim 1 (Outcomes & Processes):** Participants' articulation of participation issues ("boys talking too much" or "girls not speaking up") focused on symptoms of problems (outcomes) as opposed to underlying causes (processes). Analysis revealed that the underlying causes of unfairly distributed opportunities to participate were structural in nature and carried out through interactions (e.g., having only students with complete and correct homework solutions present to the class).
- **Claim 2 (Multiple Context Levels):** Intertwined gendered and racialized storylines about mathematical competence shaped participants' experiences with participation in unfair ways. Storylines affected how barriers to participation functioned (e.g., seemingly "successful" students faced ongoing obstacles related to not feeling valued due to being female and a student of color), how participatory expectations were assigned (e.g., students who understood were expected to share, especially female students "with skills"), and how teacher-student interactions played out (e.g., Ms. B felt she had the right to push her high-achieving female students).
- **Claim 3 (Insider vs. Insider Views):** Insiders' views of a shared small-group learning experience were consequentially different from each other. The three male students had numerous opportunities to participate in mathematically meaningful

ways; the one female student (Guadalupe) did not. In addition, participants' perceptions of other participants' experiences during the task varied considerably. The student teacher was the only one who correctly sensed that Guadalupe felt left-out and unappreciated. The teacher and two male students commented on Guadalupe's lack of talk, but none connected Guadalupe's participation to problematic interactions that should be addressed.

- **Claim 4 (Insiders vs. Outsider Views):** Insiders' views of participation issues during whole-class discussions were consistent with each other, and yet were consequentially different from an outsider's view as represented by participation metrics. Participants talked about the issue of male dominance in class conversations but said little about race, whereas semester-long participation metrics indicated White dominance superseded male dominance, with male students of color having the fewest opportunities for mathematically meaningful participation.

Chapter 7 concludes with a discussion of study limitations. My hope is that despite its limitations, this dissertation paves the way for future studies to explore the consequential issues it has uncovered.

Chapter 2: Theoretical and Conceptual Framing Supported by Prior Research

This chapter lays out the theoretical and conceptual foundations underlying the analyses, commentary, and implications presented in this dissertation. I begin by articulating definitions for important concepts, including participation, learning, and participatory equity. I then discuss how positioning theory informs this study and present the conceptual framework I designed to support the examination of participatory equity. The conceptual framework builds on sociocultural and situated theories of learning and positioning theory to illustrate connections between interactional processes, individual participation metrics, and student outcomes. I conclude this chapter by articulating and rationalizing three principles that guided the design of this dissertation. Those principles address the need to focus on participatory outcomes *and* processes, to consider multiple levels of context, and to integrate outsider and insider views of participation.

Participation, Learning, and Participatory Equity

This dissertation is informed by sociocultural and situated theories that claim learning happens as people interact and participate in communities of practice (Lave & Wenger, 1991; Rogoff, 1990; Vygotsky, 1978; Wenger, 1998). While language and discursive practices are considered central to developmental processes (Lerman, 2001), participation in classroom activities includes more than just talk, meaning that non-verbal forms of communication are considered valuable for learning as well (Esmonde, 2009; Hinestroza, 2022). Learning is defined as changes in students' participation in collective classroom practices (Lave & Wenger, 1991). Learning encompasses both the development of content proficiencies and the development of competent positional identities (Esmonde, 2009). For example, it is important for students to learn mathematical content and practices as described in the Common Core State Standards, but it is also important for students to grow to see themselves, and to be seen by others, as competent learners and doers of mathematics (Schoenfeld, 2014). Content development is a function of students' engagement in active learning practices that include sharing thinking, questioning, and working with peers to develop novel problem-solving strategies (National Council of Teachers of Mathematics, 2000; NRC 2001), leading to better learning outcomes (Boaler & Staples, 2008). Positional identities refer to the ways students position themselves and are positioned by others with respect to curricular content, their peers, and society at large (Davies & Harré, 1990). Identity development is dynamic, interactive, and shaped by social contexts (Holland et al., 2001). Positive mathematical identities are associated with students feeling supported by their learning communities and having their personal needs met (Langer-Osuna, 2011).

Participatory equity is defined as a fair distribution of opportunities for students to participate and learn (Ernest et al., 2019; Esmonde, 2009). The focus on *opportunities* to participate highlights the situated and social nature of participation and learning (Gresalfi et al., 2009). Problematic patterns can and do exist in student participation and learning outcomes, but participatory inequities are created and perpetuated through the social construction of disparate and stratified opportunities for students to engage in meaningful learning experiences (Sullivan & Wilson, 2015). The focus on a *fair* distribution distinguishes equity from equality. Cultivating classrooms with fair (not necessarily equal) opportunities to participate (Gutiérrez, 2012; Secada, 1989) requires understanding how students in specific contexts perceive what is fair (McDermott & Roth, 1978). While not always visible to an outside observer, fairness is certainly felt by

participants and every classroom participant has their own perspective (Esmonde et al., 2009; Rocca, 2010). The teacher's view on students' learning needs may not align with students' views, and what one student feels they need may be different from what another student feels they need. For example, a teacher may believe that giving students a challenging groupworthy problem without prior formal instruction is a *fair* way to "even the playing field" and improve access to mathematical content (Cohen & Lotan, 2014). On the other hand, a student may feel it is *unfair* for the teacher to give them a problem which they have not yet been shown how to do. Or, a White male student may feel classroom participation is *fair* because the teacher calls on students who raise their hands, but a female student of color may feel participation is *unfair* because class discussions are dominated by White male voices.

Positioning Theory

Positioning theory is used to examine classroom participation from an interactional perspective (van Langenhove & Harré, 1999). Positioning theory states that through interactional *acts of positioning*, people are assigned specific rights and duties aligned with certain *positions*. Rights refer to "what others must do for [a person]" and duties refer to "what [that person] must do for others" (Herbel-Eisenmann et al., 2015, p.190). These various positions correspond to different characters in the overarching *storylines* that are in play in a given situation, such as a classroom. Teachers and students co-construct participation expectations by drawing on various classroom and society-level storylines about what each person can and should do with respect to learning tasks. Positioning is a mutually constitutive process. For example, for a student to be positioned as an "expert," that student needs to position themself as someone with expertise worth sharing *and* this positioning needs to be endorsed by other participants (Holland et al., 2001). Power is distributed, often unfairly, among students as participatory rights and duties are negotiated within the context of conscious and unconscious biases (e.g., racism, sexism, ableism) (Esmonde & Langer-Osuna, 2013; Reinholz & Shah, 2018).

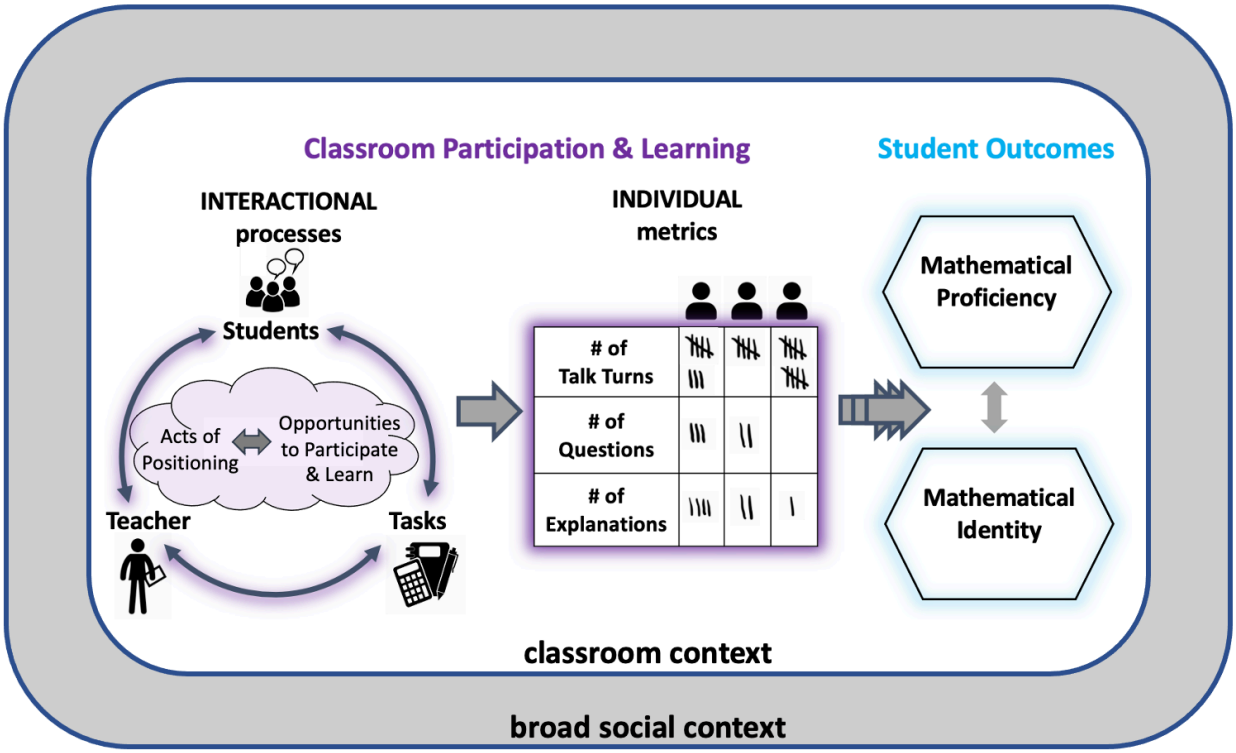
Over time, patterns of positioning emerge and "thicken" (Leander, 2002) as participation and learning narratives are constructed, contested, and amended through interactions among interlocutors. These narratives are influenced both by large-scale societal beliefs and expectations and more localized classroom philosophies and norms of behavior (Louie, 2018). The varying ways different students get called on to participate in mathematical tasks depend on which storylines are evoked and what roles specific students play in the stories (Langer-Osuna, 2011), leading to trajectories of positional identity and engagement over time (Holland et al., 2001; Wortham, 2004). If storylines are racialized and gendered, then the roles students play will be racialized and gendered as well (Gholson & Martin, 2019; Nasir & Vakil, 2017). Equitable learning processes require that every student be positioned as a competent learner and doer of mathematics who has ideas worth sharing and from whom peers can learn (Schoenfeld, 2014). Students positioned with competence and authority have more genuine opportunities to participate in consequential and influential ways, and therefore have better access to rich mathematical learning and identity development (Cohen & Lotan, 1995; Engle et al., 2014; Gresalfi et al., 2009; Langer-Osuna, 2011).

Conceptual Framework for Studying Participatory Equity

To support the examination of participatory equity in this dissertation from both outsider and insider perspectives, I designed a conceptual framework that illustrates connections

between interactional processes, individual participation metrics, and student outcomes (Figure 2a). The framework draws on sociocultural and situated theories of learning and positioning theory.

Figure 2a. Conceptual Framework for Studying Participatory Equity

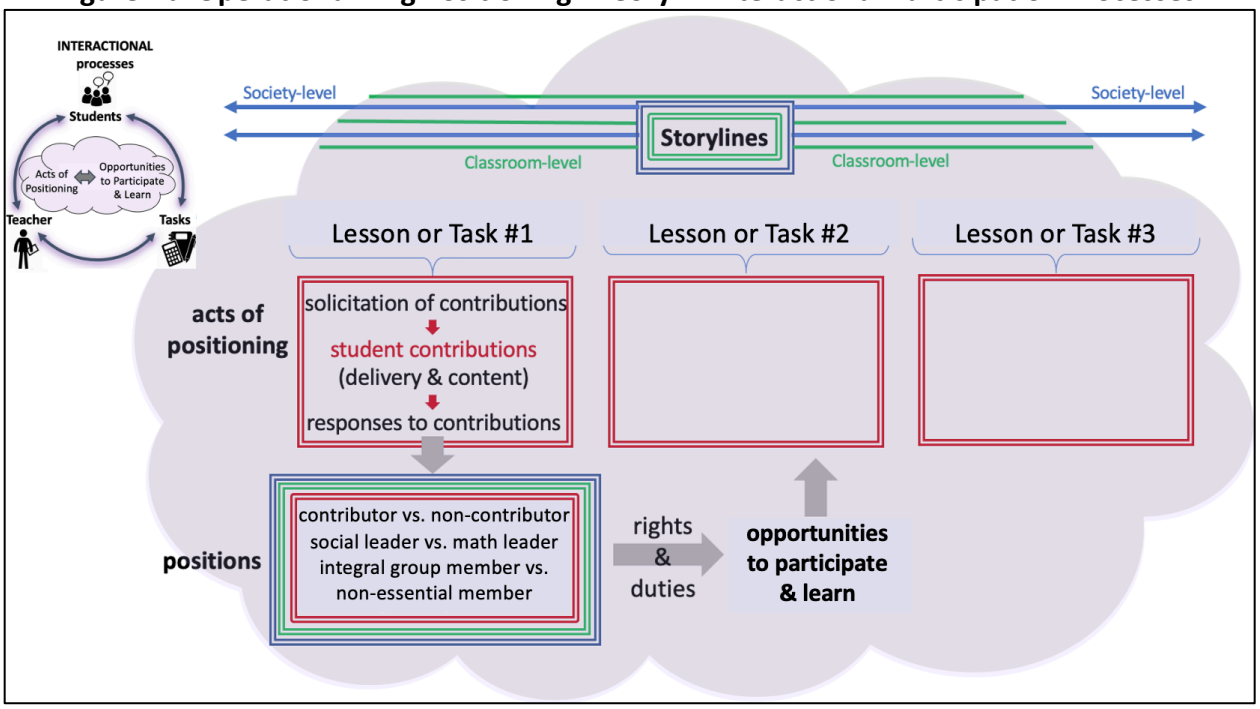


Opportunities for participation are created through, and influenced by, acts of positioning that occur in interactions between students, teachers, tools, and tasks (left-hand side of Figure 2a). Interactional processes result in individual students talking and acting in quantifiable ways, which can be captured in the form of individual participation metrics (center of Figure 2a). Over time, participation shapes student outcomes – how students understand content (mathematical proficiency) and how students come to see themselves and each other as learners and doers of mathematics (mathematical identity). Expanding outward, participatory processes take place within specific classroom contexts that operate within broader social contexts. Contextual factors, both at the classroom-level and society-level, can bolster or constrain classroom participation in various ways by creating supports for participation or barriers to participation. These supports and barriers may be common across all students in a given class or they may only pertain to a subset of students. Generally speaking, supports for and barriers to participation do not affect all students in the same ways due to students' varying backgrounds and experiences.

Figure 2b expands on the purple "cloud" in Figure 2a, providing further details as to how positioning theory is operationalized in the context of this dissertation to study interactional participation processes. This figure shows how acts of positioning shape the construction of opportunities to participate and learn, and how opportunities to participate and learn

consequently shape future acts of positioning, all within the context of activated classroom-level and society-level storylines.

Figure 2b. Operationalizing Positioning Theory in Interactional Participation Processes



Multiple, interwoven storylines shape what happens and how people make sense of what happens during interactional participation processes, denoted by green and blue lines in Figure 2b. Storylines occur both at the society-level (e.g., what genders and/or races of students are perceived to be good at math generally in society) and classroom-level (e.g., which students are perceived to be competent or successful in this specific class). Acts of positioning, positions, and the construction of opportunities to participate are influenced by the various storylines that are activated by and about certain students (Herbel-Eisenmann et al., 2015). For example, if a society-level storyline about boys being better at math than girls is activated (participants are thinking and/or talking about this "story"), the opportunities female and male students have to participate will likely be different because they will be assigned different rights and duties according to their positions (boys as primary mathematical contributors and girls as secondary or non-contributors). This positioning could occur through acts of self-positioning (e.g., girls volunteering less frequently than boys because they do not feel they have anything to contribute) or positioning by others (e.g., students who have a question, turn to a male peer to ask for help). This gendered societal-level storyline may be challenged (or supported) by related classroom-level storylines, such as a teacher making explicit statements about female students having just as much, if not more, to contribute to class discussions as male students. Conflicting storylines are negotiated through classroom interactions, with the associated positions and resulting opportunities to participate shifting over time.

The red boxes in Figure 2b denote acts of positioning. Soliciting a student contribution is an act of positioning (Langer-Osuna, 2011; Radinsky, 2008). The teacher, other students, and the

task/tools position the contributing student in certain ways depending on how/if the contribution was solicited or invited. Was the student called on by the teacher or did the student raise their hand and volunteer? Was the contribution invited by a peer's question or did the contribution interrupt another student's thoughts? Students who offer seemingly unsolicited contributions or contributions in response to open invitations to the class (e.g., Does anyone have any ideas of what to do first?) position themselves as confident contributors. When the teacher asks a specific student to share their explanation from the previous night's homework, the teacher is positioning the contributing student as a competent mathematical contributor. More than one student may be positioned through a single contribution solicitation. For example, if a student interrupts another student while they are explaining their thinking, the student doing the interrupting is positioning themself as a worthy contributor and they are positioning the person they are interrupting as a relatively less worthy contributor.

The act of making a contribution is also an act of positioning (Gholson & Martin, 2014). The contributing student positions themself through the delivery (method, tone, timing) and the content of their contribution. Was this contribution spoken out loud or written down? Was this contribution delivered with a smile and positive tone or shared reluctantly? Did the contribution offer a possible solution supported by mathematical reasoning, or was the contribution reading a problem aloud? Did the student offer an appropriate answer, or did they decline to answer the question? Was the contribution a mathematical contribution or a social contribution? Depending on how the contribution is made and what the contribution is, the contributing student positions themself differently, perhaps as a social leader in the class and/or as a mathematical leader.

Responding to other students' contributions is a third act of positioning (Anderson, 2009; Hand, 2010). Classroom participants position the contributing student in certain ways depending on how they react and respond to a contribution that was made. Did a student's mathematical contribution receive verbal affirmation for their peers or from their teacher? Did a student's social contribution result in smiles from other participants? Did other students build on the ideas presented in a student's contribution? Responses to contributions communicate whether contributions were welcomed and appreciated by other participants or not, positioning some students as integral members of the community and other students as outsiders to the group and non-essential to the task at hand.

The nested red, green, and blue boxes in Figure 2b are intended to signify that the combination of multi-level storylines and acts of positioning define and assign particular positions to particular students. Within activated storylines, positions are linked to certain participatory rights and duties. For example, a student positioned as a worthy contributor has the right to have time and space to share their thinking and has the duty to answer their peers' questions. Opportunities for students to participate are constructed and distributed based on the participatory expectations established through assigned rights and duties. A worthy contributor is likely to have more frequent and more robust opportunities to participate in mathematically competent ways, which in turn, shapes future acts of positioning.

Guiding Principles

In light of the preceding theoretical and conceptual framing of participation and learning, the following three principles guided the design of this dissertation:

- Participatory equity is conceptualized in terms of both outcomes and processes, with an analytic focus on connections between individual student participation metrics and the interactional construction of opportunities to participate.
- Students are thought about as unique individuals who participate in classroom communities operating within broader social contexts.
- "Insiders" (students and teachers) and "outsiders" (researchers) offer complementary views of students' learning experiences, both of which are essential.

Participatory Outcomes and Processes

To support the identification and mitigation of inequities in classroom participation, equity is conceptualized in terms of participatory outcomes *and* participatory processes. Both shorter-term outcomes (individual student participation metrics) and longer-term outcomes (content proficiency and identity) are considered. Differences in students' participation metrics (e.g., # of explanations shared) and content proficiency measures (e.g., students' test scores) suggest the existence of participatory inequities due to presumed connections between students' participation and content understandings (Bianchini, 1997; Ing et al., 2015; Otten & Soria, 2014). However, simply knowing that participatory inequities exist does not explain the nature of these inequities, how they were constructed, or how educators might go about trying to address them. Inequitable participatory outcomes are a symptom of underlying issues with participatory processes, specifically, that opportunities to participate are unfairly distributed among students (Esmonde, 2009).

When focusing only on outcomes, the tendency is to treat participation and learning issues as residing within individual students (e.g., this student does not talk enough during class or this student's Algebra skills are weak) and to implement solutions that blame the victim and try to "fix" the child (e.g., telling the child to speak up more or assigning a student extra skills practice) without understanding how these outcomes came to be (McDermott & Varenne, 1995). The alternative is to recognize that students' participatory outcomes are shaped by the opportunities students have to participate and learn (Gresalfi et al., 2009), which are co-constructed through classroom interactions. With this approach, participation issues are positioned as residing within the classroom interactions and therefore solutions are targeted at shifting interactional patterns to support more equitable opportunities for participation (e.g., changing participation structures, pedagogical questioning strategies, support for student-to-student interactions, or task design).

Another reason to look beyond outcomes when examining participatory equity is that inequities do not always show up in traditional outcome measures. For example, both Hinchman and Young (2001) and Langer-Osuna (2011) identified inequitable patterns of participation despite the focal students in their studies being recognized as high-achieving students. In addition, Sengupta-Irving and Enyedy (2015) studied two different instructional approaches and found differences in students' participation that were consequential for their learning and engagement, despite the same levels of achievement. One final example comes from the pilot study that informed this dissertation[2] (Fink, 2019). In that study, two students (Becca and Kyle) had similar individual participation metrics while working on a small-group task; they both spoke

[2] Details of the pilot study that informed the design of this dissertation are included in Chapter 3: Methodology.

relatively few words compared to their two peers. Yet, interactional analysis suggested the two students were positioned in consequentially different ways. Kyle was positioned as an active, knowledgeable contributor while Becca was positioned as a passive, confused non-contributor, which had implications for mathematical identity development. Since classroom participation should support not only content proficiency but also positional identity (right-hand side of Figure 2a), it is essential to look beyond outcome measures and look closely at participatory processes when identifying and assessing educational inequities.

Often studies of classroom participation approach participation from an individual perspective, meaning that researchers track, measure, and compare what individual students do during learning tasks. Individual participation metrics tend to be low inference counts of discourse dimensions, including # of talk turns per student (O'Connor et al., 2017), # and type of questions asked/answered per student (Reinholz & Shah, 2018), and # of explanations shared per student (Ing et al., 2015). Individual participation studies embrace quantitative methods of analysis and often seek to understand a) connections between individual student participation and individual performance on content assessments (Ing et al., 2015; O'Connor et al., 2017), or b) patterns of individual participation across specific groups of students, such as by race or gender (e.g., Reinholz & Shah, 2018; Tatum et al., 2013).

Studies that approach participation from an interactional perspective focus on how groups of students organize their collective participation, how students react to and invite particular kinds of participation from peers, and how teachers intervene and support student learning. Compared to individual participation and learning metrics, interactional measures are more nuanced and not as easy to document in real-time. Examples include teacher and student responses to students' mathematical contributions (Anderson, 2009), teacher and student responses to student actions (Gresalfi et al., 2009; Hand, 2010), and student responses to their peers' bids for participation (Langer-Osuna, 2011; Radinsky, 2008). Interactional participation studies embrace qualitative methods of analysis that consider various verbal and non-verbal modes of communication, often seeking to understand a) how students are positioned through classroom interactions as certain "kinds of people" (Gee, 1996), such as a "quiet" student (Anderson, 2009) or a "competitive challenger" (Radinsky, 2008), or b) how trajectories of student engagement and identity develop over time (Gholson & Martin, 2014; Langer-Osuna, 2011).

Students as Individuals in Classrooms within Broader Social Contexts

To understand how participatory inequities are constructed and play out in specific classrooms, students are thought of as unique individuals who participate in classroom communities which operate within broader social contexts. Conceptualizing individual students as completely independent units for analysis is neither accurate nor helpful because student participation does not happen in isolation; contextual factors are ever-present and undoubtedly influence how classroom participants engage with content and with each other (Nasir & Hand, 2008). Some contextual factors operate at a classroom level (e.g., how competence is defined, how tasks are designed, how "successful" participation is perceived). Classroom-level storylines affect the positions available to various students (Horn, 2008; McDermott & Varenne, 1995). For example, mathematical competence is co-constructed through negotiated interactions in a given classroom context, as opposed to being a set of individual student attributes that can be defined outside of the classroom (Gresalfi et al., 2009). In one classroom a student who repeats

mathematical procedures exactly as demonstrated by the teacher may be labeled a highly competent student, and a student who explores their own method of solving may be considered less competent; in a different classroom, the positions could be reversed if the class community values mathematical exploration and multiple solutions more than precision and efficiency. The same is true for "productive" or "successful" participation; expectations for and assessments of participation are co-constructed through negotiated interactions. Universal definitions for these labels do not exist.

Other contextual factors operate at a broader societal level (e.g., narratives about race and gender, representation of females and people of color in advanced mathematics). Society-level storylines shape expectations for the ways students can and should participate, privileging some students and marginalizing others (Agarwal, & Sengupta-Irving, 2019; Esmonde & Langer-Osuna, 2013; Gholson & Martin, 2019; Leyva et al., 2021). Historically (and currently), the opportunities White, male, and affluent students have to participate in mathematics are more robust than the opportunities granted to their Black, Latinx, Indigenous, female, and poor peers (Martin, 2019; Zavala, 2014). Racialized and gendered narratives about belonging and competence in mathematics result in students from non-dominant groups having to face additional barriers to participation as they fight against stereotypes and try to prove their worthiness to those around them and sometimes to themselves as well (Gholson & Martin, 2014; Rainey et al., 2018; Sengupta-Irving & Vossoughi, 2019).

Figure 2a illustrates how classroom participation is connected to students' mathematical identities - how students see themselves and are seen by others with respect to learning and doing mathematics (e.g., Horn, 2008; Schoenfeld, 2014). However, students' mathematical identities are deeply intertwined with other ways they identify themselves and are identified by others, including racial identities (English-Clarke et al., 2012), gender identities (Moore, 2021), and/or able-oriented identities (Heyd-Metzuyanim, 2013). The interconnected nature of identities implies classroom participation is connected not only to mathematical identity, but to student identity more generally (Hand & Gresalfi, 2015). These identity connections also suggest that studying participation with respect to intersectional identity groups is a better assessment of students' experiences than studying participation with respect to a single identity group alone (e.g., Collins et al., 2020; Gholson & Martin, 2014; Leyva, 2017; Leyva, 2021). In the words of Crenshaw (1991), "Women of color experience racism in ways not always the same as those experienced by men of color and sexism in ways not always parallel to experiences of white women, antiracism and feminism are limited, even on their own terms" (p. 1252).

Studies of classroom participation are designed according to the types of contextual factors they highlight when trying to make sense of student participation. Some researchers consider only student-level attributes, such as standardized test scores (e.g., Ladd et al., 2000) or responses to teacher questions (e.g., Webb & Farivar, 1994). Student-level studies are most often focused on understanding connections between student participation and achievement outcomes and are *not* framed with respect to equity (e.g., Ing et al., 2015; Mulyran, 1992; Valiente et al., 2008). Other researchers consider elements of the classroom communities in which student participation took place, such as how systems of competence were constructed (e.g., Gresalfi et al., 2009) or how community practices were established and taken up (e.g., Nasir & Hand, 2008). Classroom-level studies have looked at participation outcomes and participation processes, some of which mention equity (e.g., Gresalfi et al., 2009; Sengupta-Irving & Enyedy,

2015) but many of which do not (e.g., Otten & Soria, 2014; Radinsky, 2008). Still other researchers consider the impact of broader societal narratives on students' participation, such as race and/or gender (e.g., Esmonde & Langer-Osuna, 2013; Fassinger, 1995). Society-level studies focus more often on participation processes than outcomes and almost always are framed with respect to equity (e.g., Langer-Osuna, 2011; Shah & Crespo, 2018; Turner et al., 2013).

Insiders' and Outsiders' Complementary Views

To form a comprehensive picture of how and why students participate in various ways and how classroom participation impacts student outcomes, it is essential to include perspectives from both insiders (students and teachers) and outsiders (researchers) – to the degree that one can attribute them. Each classroom participant has their own perspective and experiences their own reality, all of which are important to understand when trying to make sense of students' collective and individual learning experiences. David Clarke justified the need to include participants' varied and complementary views in the design of the Learner's Perspective Study (Clarke, 2001) as follows.

> We need to acknowledge the multiple potential meanings of the situations we are studying by deliberately giving voice to many of these meanings through accounts both from participants and from a variety of "readers" of those situations. The implementation of this approach requires the rejection of consensus and convergence as options for the synthesis of these accounts, and instead accords the accounts "complementary" status, subject to the requirement that they be consistent with the data from which they are derived, but not necessarily consistent with each other, since no object or situation, when viewed from different perspectives, necessarily appears the same (Clarke, 2001, p. 1).

However, there are some (relatively) objective truths when it comes to making sense of classroom participation, such as who spoke during a particular class period or whose ideas were built on by other students (Anderson, 2009; Engle & Conant, 2002; Esmonde & Langer-Osuna, 2013). These patterns in participation are easier to identify from an outsider's perspective and are presumably less biased (Reinholz & Shah, 2018). Through detailed qualitative examination and/or longer-term quantitative tracking, researchers' views address important questions pertaining to how students participated in-the-moment and over time (Gresalfi et al., 2009; Kovalainen & Kumpulainen, 2007; Yackel et al., 1991). Based on theory, researchers can also offer possible explanations for why students participated in the ways that they did and offer possible implications of participation for students' content proficiency and identity development. Researchers can reflect on learning experiences from the perspectives of students by contemplating answers to questions like, "Am I invited to explain and present my ideas?" or "Do I get to participate in meaningful mathematical learning?" (Schoenfeld, 2018). However, researchers cannot know through observation alone when a student *feels* they are invited to contribute to a mathematical discussion or what a student *thinks* is meaningful for them.

Insiders' reflections on their experiences need to be included when examining the reasons behind and the meanings of classroom participation, especially when issues of equity are central to the investigation. Insiders include both teachers and students, although special attention is paid to students' perspectives since they are the ones most often left out of participation assessments (Hinestroza, 2022), and they are the ones whose interests researchers are trying to

serve. This dissertation aims to support the cultivation of classrooms with fair (not necessarily equal) opportunities to participate (Gutiérrez, 2012; Secada, 1989), which requires understanding how individual students in specific classroom contexts perceive what is fair and just in their eyes (McDermott & Roth, 1978). Given the Whiteness of the academic community (Preston, 2014) and K-12 teachers (Picower, 2009), myself included, it is particularly important to seek and center the voices of students who identify as members of historically marginalized groups (Martin & Garza, 2020). It would be a stretch for me, a White, cis-gender, affluent, able-bodied woman to presume to know how a similarly identifying student was feeling or thinking in a mathematics classroom. But it would be beyond reasonable, and in fact severely problematic, for me to presume to know how a Black male, recent immigrant, queer youth, or student learning English was experiencing participatory interactions in their class.

Most studies of classroom participation use video-recorded observations as a primary source of data, but if and how these data are supplemented with other sources tends to be connected to whose perspectives are privileged. For many researcher-perspective studies, lesson observations are the only data considered (e.g., Gresalfi et al., 2009; Kovalainen & Kumpulainen, 2007), although sometimes observations are supplemented with student work (e.g., Esmonde & Langer-Osuna, 2013; Yackel et al., 1991), lesson artifacts like computer log files (e.g., Lipponen et al., 2003), and/or standardized test scores (e.g., Ing et al., 2015). While these additional data provide more information about students' experiences, they are all outsiders' views. Participation studies that include insiders' views typically supplement lesson observations with one-on-one participant interviews and/or individual participant surveys. Occasionally, participants' perspectives on classroom interactions are prompted through stimulated-recall methods (e.g., Clarke, 2004; Geiger et al., 2016; Xie, 2009). Sometimes insider perspective studies focus on both teachers' and students' perspectives (e.g., Black, 2004; Gholson & Martin, 2014; Hand, 2010) and sometimes they focus on only teacher perspectives (e.g., Horn, 2007; Wager, 2014) or only student perspectives (e.g., Esmonde et al., 2009; Hinchman & Young, 2001). A very limited number of studies have examined parent perspectives as well (e.g., Martin, 2018).

Chapter 3: Methodology

Research Goals and Methodology Overview

My research is motivated by the question: *How can teachers support more equitable learning experiences for students in mathematics classrooms?* With this dissertation, I aim to provide contextualized multidimensional understandings of the concept of "equitable participation" by exploring questions such as: What factors seem to shape how and why students participate in the ways that they do? How are individual student participation metrics connected to interactional participation processes? How do classroom participants' "insider" perspectives on student participation compare to observations from "outsider" perspectives? A major goal of this study was to connect micro-level interactions (e.g., who said what and to whom), meso-level stories (e.g., how teachers and students made sense of participation patterns), and macro-level narratives (e.g., who is expected to be smart at math). Specifically, I wanted to understand how gendered and racialized discourses shaped expectations for and assessments of student participation. In addition, I wanted to examine how various forms of student participation were supported or inhibited through the construction of opportunities to participate and how they differed between intersectional groups of students (e.g., Female students of color vs. White female students). Throughout the research process, I had an explicit goal of centering classroom participants' perspectives, especially the thoughts, feelings, and observations of students, and even more specifically, the experiences of students from marginalized communities.

To address these research goals, it was important to draw data from multiple sources and from multiple perspectives over an extended period of time. Data included participant interviews and lesson observations. I conducted multiple one-on-one interviews with the teacher, student teacher, and focal students, some of which prompted participants to reflect on previous lessons using video-stimulated recall methods. I wanted to supplement classroom participants' general observations about their mathematical experiences with their specific observations of in-the-moment classroom interactions. Video-stimulated one-on-one interviews with the teacher and focal students allowed me to privilege participants' perspectives on classroom interactions and provided insight into how students made sense of what happened, why it happened, and the significance of certain interactions (Erickson, 2006). By grounding students' accounts of their own and their peers' learning experiences in video-recordings, clearer connections could be made between what happened and how it affected students' perceptions of their own and their peers' mathematical participation. In addition to interviews, I observed two periods of AB Calculus taught by the same teacher for one semester. I documented my observations real-time during the lessons, and I video-recorded the lessons and the lesson debrief sessions with the teacher and student teacher immediately following the lessons. I wanted to capture what was happening in the lessons and how the teacher and student teacher were reflecting on the lessons each day. Observing lessons over an entire semester allowed me to look at participation patterns over time and to see how the lessons selected for in-depth analysis fit within the bigger picture of what was happening in the classroom.

To address these research goals, it was also important to coordinate multiple forms of qualitative and quantitative analyses to examine both individual and interactional aspects of student participation (Clark, 2019; Creswell & Creswell, 2017). Analyses included the narrative (qualitative) analysis of semi-structured participant interviews and lesson debrief sessions, the

(qualitative) micro-interaction analysis of one whole-class discussion and one small-group task, and the (quantitative) statistical comparisons of student participation by gender and race.

Participant talk data, including interviews and lesson debrief sessions, were analyzed using an open-coding, iterative process allowing me to identify common themes, areas of discrepancy, and nuanced understandings across participants' perspectives (Corbin & Strauss, 2014; Maxwell, 2013). The observations and experiences of the teacher, student teacher, and students guided the analysis, as opposed to my own preconceived ideas, which helped to center participants' perspectives.

Lesson observation data were analyzed by looking at both individual student participation metrics and interactional participation processes. Examining both individual and interactional aspects of student participation provided complementary views of students' learning experiences. Examining individual participation metrics provided a summative picture of each student's participation outcomes, helping to highlight participation differences among students for a given task and for the same students over time (e.g., Hufferd-Ackles et al., 2010; Reinholz & Shah, 2018). Counting the number of times each student shared homework solutions, responded to questions, or volunteered during class discussions facilitated participation comparisons between individual students and among groups of students (e.g., female students vs. male students). These comparisons allowed me to answer questions such as who dominated conversations in each of the classes, whose mathematical ideas were absent from the conversations, and how often did certain students or groups of students volunteer to share ideas compared to others?

Examining interactional participation processes provided insight into how opportunities for participation were constructed and played out through moment-to-moment interactions, helping to determine if participation differences were problematic and providing insight into how we might go about shifting problematic participation patterns (e.g., Gresalfi et al., 2009; Langer-Osuna, 2011). Looking at how students were invited to contribute during learning tasks allowed me to understand more about how classroom participants coordinated their actions and positioned each other. Participatory actions of students are shaped by the actions of other students and their teacher. This is where positioning theory comes in (van Langenhove & Harré, 1999), since students who are positioned as competent learners and doers of mathematics have more opportunities to engage deeply with mathematical ideas, bolstering the development of conceptual knowledge and positive mathematical identities (Gresalfi et al., 2009).

Researcher Positionality

It is important for me, as the designer of this study and author of this dissertation, to acknowledge my position as a White woman who is seeking an advanced degree from an elite institution. My positionality affords me certain privileges that are different from many of the participants in my study. I seek to center the experiences of students, specifically the experiences of students from marginalized communities, but I acknowledge that the act of me being the one to amplify students' voices is inherently shifting focus away from the students and toward me as the researcher. I also acknowledge the unequal power dynamics that play out during classroom observations and one-on-one interviews. Regardless of the relationships I've built with study participants over time, unequal power relations still exist. What participants say and do is likely shaped by these power differentials. In addition, I acknowledge that the design of my study and the conclusions I draw are shaped by my own experiences and unconscious biases. I welcome

critiques and reflections from readers that will help me reflect on my own prejudices in ways that strengthen the rigor of my work and further my goal of centering students' experiences.

Pilot Study

This dissertation is informed by a pilot study conducted several years earlier in a public high school in the same region of the United States (Fink, 2019). The goal of the pilot was to gain a better understanding of how Becca, a student labeled as "quiet" by her teacher, engaged in a small-group mathematics task through interaction with her peers. I examined individual participation metrics (e.g., # of talk turns, # of explanations shared) in relation to interactional participation processes (e.g., responses to student actions).

Results from the pilot study indicated the potential power of taking an integrated individual-interactional approach to classroom participation, and the analytic design of the study provided a methodological foundation from which to build my dissertation. There were two primary findings: 1) conventional individual ways of assessing and measuring Becca's participation obscured her mathematical contributions and indicated - incorrectly! - that she did not engage in mathematical sense-making, and 2) Becca's limited opportunities to participate and her resulting "quiet" participation were co-constructed through peer interactions. Findings suggested that during this particular small-group task, the opportunities Becca had for rigorous mathematical engagement and positive mathematical identity development were substantially limited in comparison to those of her more vocal peers. In addition, findings suggested that the perceived issue of low verbal participation did not reside within Becca, but rather was the byproduct of inequitable participatory processes that played out through peer interactions.

An integrated participation approach allowed me to examine Becca's lack of talk in relation to her mathematical contributions and to understand how disparate opportunities for participation were constructed through interactions. Analysis showed that Becca's quantity of talk was similar to that of her groupmate, Kyle. However, the opportunities Becca had to participate in consequential and influential ways were notably more limited than Kyle's due to how they each were positioned. Becca was positioned as an insecure, less knowledgeable non-contributor to the collective learning despite valid contributions; Kyle was positioned as a confident, knowledgeable contributor in accordance with his contributions. Becca and Kyle were described differently by their teacher. Becca was described as "really quiet," "very shy," and "uncomfortable in a group." Kyle was described as "independent" and "not liking groupwork." Although both descriptions set low expectations for student talk, the underlying assumptions about students' competences and developing mathematical identities were quite different. A focus on individual participation metrics without considering the underlying interactional processes would have resulted in an incomplete and misleading understanding of students' participation and the opportunities they had for rich content engagement and positive identity development.

In the pilot study I examined one group of four students working on one 15-minute task. I interviewed the teacher but no students. I knew little about the class norms and context. Through expanded data collection and analysis, this dissertation study builds on the ideas explored in the pilot study in ways that result in a more comprehensive and contextualized understanding of high school students' participatory experiences in mathematics classes.

Data Collection

Data collection took place during the spring semester of the 2020-2021 school year at Evergreen High School (pseudonym) for the primary purpose of supporting my dissertation research project. The data collection school site employed a distance learning model of instruction for the duration of that school year. UC Berkeley's IRB approved the research procedures on 11/18/2020 (CPHS protocol #: 2016-07-8971), and the project was approved by the participating school district and the selected school site on 12/8/2020. All study participants (i.e., the teacher, student teacher, and students/guardians) signed formal consent forms prior to data collection.

Distance Learning Context

Due to the Covid-19 pandemic, instruction for Evergreen High School (EHS) was remote during the 2020-2021 school year. The district worked to ensure all students had access to appropriate distance learning resources, including computers and internet access. Classes were held daily using Zoom technology. Students logged on for required 60-minute instructional lessons on Mondays, Tuesdays, Thursdays, and Fridays. Teachers held optional 50-minute "Help Sessions" on Wednesdays, also through Zoom. Intended to lighten the load on teachers and students, during the 2020-2021 school year EHS used an alternating course schedule by term that resulted in students taking a maximum of four courses at a time, switching every 4-5 weeks as shown in Table 3a. For example, students enrolled in Period 1 AB Calculus, attended this course during terms 1, 3, 5, and 7 but not during terms 2, 4, 6, and 8.

Table 3a. 2020-21 Term Schedule for Evergreen High School

Evergreen HS: 2020-21 Distance Learning				
Semester	**Quarter**	**Term**	**Periods**	**Dates**
1	1	1	1, 3, 5, 7	8/17-9/11
		2	0, 2, 4, 6	9/14-10/9
	2	3	1, 3, 5, 7	10/13-11/13
		4	0, 2, 4, 6	11/16-12/18
2	3	5	**1**, 3, 5, 7	1/4-2/5
		6	0, **2**, 4, 6	2/8-3/12
	4	7	**1**, 3, 5, 7	3/15-4/23
		8	0, **2**, 4, 6	4/26-6/3

The bolded class periods (i.e., Periods 1 & 2 in Semester 2) correspond to the two AB Calculus class periods included in data collection for this study. In-person meetings between teachers and students were forbidden by the school district until April 2021, at which time the district asked teachers to offer optional *in-person* support sessions to students in addition to the Monday-Friday class sessions they were already teaching. Starting in May, the optional sessions became required sessions due to a change in district protocol; students could choose to attend these additional help sessions either in-person or through Zoom. Data collection included required regular lessons and optional/required help session that were held through Zoom. Due to participant consent restrictions, data collection did not include any of the in-person help sessions held in May.

Participant Recruitment

During the fall semester of the 2020-2021 school year, I supported instruction at EHS as a district-approved classroom volunteer in two "English Language Learner Math 1" classes (Periods 4 & 5). Both classes were taught by the teacher participant in this study (Ms. B), though neither of these classes were included in my dissertation study. Since I had a pre-existing relationship with Ms. B through my volunteering, she was the first (and only) teacher I asked to participate in this study once project approval had been granted at the end of the fall semester. She and I agreed that it made sense to focus my dissertation data collection on her 12th grade AB Calculus classes instead of the Math 1 classes in which I was volunteering so my role as an observer / researcher (as opposed to an instructional helper) would be more clearly defined for us and for the students. Ms. B taught three AB Calculus class periods at the time, but we decided I would only recruit students in her Period 1 & Period 2 classes for study participation (and not Period 3) for feasibility purposes. I continued volunteering in Periods 4 & 5 for the duration of the school year, and I did not engage with the students in Ms. B's Period 3 class.

Student recruitment for Period 2 was completed prior to recruitment for Period 1 due to the timing of the dissertation project approval (early December 2020) and the alternating distance learning term schedule. Student recruitment for Ms. B's Period 2 AB Calculus class was completed during the week of December 14-18, 2020, and for Ms. B's Period 1 AB Calculus class was completed during the week of January 4-8, 2021. All students in the classes (and parents for students <18 years old) were asked to consent to recorded class observations and one-on-one interviews separately, meaning it was possible for participants to opt-in for one form of study participation and opt-out for the other. Course enrollment and consent numbers are shown in Table 3b.

Table 3b. Course Enrollment and Research Consent Numbers for Periods 1 & 2

		Course Enrollment	Consented to...			
			Recorded Observations & Interviews	Recorded Observations Only	Interviews Only	Neither Recorded Observations nor Interviews
Period 1		28	12	8	1	7
Female	**Students of Color**	3	2	1	0	0
	White	5	1	1	0	3
Male	**Students of Color**	8	4	2	0	2
	White	12	5	4	1	2
Period 2		32	15	9	1	7
Female	**Students of Color**	6	2	3	1	0
	White	6	6	0	0	0
Male	**Students of Color**	6	2	2	0	2
	White	14	5	4	0	5

Because some analysis presented in this dissertation examined student participation according to racial-gender groupings, consent numbers are presented in this way as well to give a sense of how well represented certain groups of students were in data collection. Just under half of the students in each class consented to participation in both the observations and interviews (12/28 in Period 1, 15/32 in Period 2). Over 70% of students in each class consented to recorded observations (20/28 in Period 1 and 24/32 in Period 2). When students who did not consent to recorded observations spoke during whole-class discussions, the Zoom recording was paused. By visually tracking who was unmuting and when, I was able to anticipate when non-consenting

students were about to speak. I documented real-time observational notes about whole-class participation for all students, meaning I had a record of when non-consenting students spoke, but the actual words they said, their voices, and their images were not captured. I did not observe any non-consenting students during small-group Zoom breakout rooms. One small-group was selected each day to observe and record; the group was eligible for selection only if all group members had given permission for recorded observations.

Study Location & Participants

The study took place at Evergreen High School (pseudonym), an urban high school in the United States with 3000+ students in grades 9-12. Approximately 58% of the student population is classified as "minority enrollment" and 26% is classified as "economically disadvantaged" (US News & World Report).

Teacher (Ms. B)

Data collection occurred in two AB Calculus classes (Period 1 & Period 2) taught by Ms. B (pseudonym), a veteran teacher with 30+ years of experience teaching secondary mathematics, 15 years at EHS. She had taught the AB Calculus course numerous times before, though never in a distance learning synchronous format. Ms. B taught every lesson during the 2020-2021 school year by logging into Zoom from a laptop computer positioned in front of a whiteboard in her EHS classroom. Ms. B taught three additional classes during the year that were not included in this study, consisting of another section of AB Calculus and two sections of an EL Math 1 course. Ms. B majored in mathematics at Cornell University, received her secondary mathematics teaching credential at Queens college (CUNY), and described herself as a White female. Ms. B also has two White sons who had previously graduated from EHS.

Student Teacher (Mr. K)

Mr. K (pseudonym) joined all five of Ms. B's classes in January 2021 as a student teacher. He was enrolled in a local university as a fourth-year undergraduate student, majoring in mathematics while also working toward a secondary mathematics teaching credential. Mr. K and Ms. B knew each other prior to this student teaching assignment since Mr. K had observed some of Ms. B's math classes the prior school year as part of a university course. Mr. K took on the primary teaching responsibilities in Ms. B's Period 3 AB Calculus class (not included in this study) and served as an instructional helper in Ms. B's other four classes, checking in with students during small-group breakout sessions and offering additional ideas and examples during whole-class discussions. He attended regular lessons and Wednesday "Help sessions" daily from January – June 2021 and participated in lesson debrief sessions with Ms. B immediately following lessons. Mr. K was committed to get his single subject math teaching credential, though not sure if he wanted to pursue a full-time teaching position right away. He described himself as a Mexican male.

Researcher (Ms. Fink)

Known to the students as "Ms. Fink," I attended (via Zoom) all but six class sessions during semester 2. I chose not to attend on the four days that Ms. B gave students individual written assessments (two sessions for Period 1 AB Calculus and two sessions for Period 2 AB Calculus). In addition, I missed one class session due to a personal conflict and one class session due to a power outage at my home. My primary role in Ms. B's Period 1 and Period 2 AB Calculus classes was as an observer and researcher. My secondary role was as a thinking partner for Ms. B and

Mr. K during lesson debriefs. Ms. B would sometimes ask me for feedback on a particular lesson or my thoughts about a particular student. I am also a former math teacher. I spent ten years as a middle school math teacher and worked as an instructional coach supporting teachers with math instruction, so I am comfortable talking with students and teachers about math. However, I have never taught calculus and have not taken a calculus class in over 25 years. At times the content seemed familiar to me and at other times I felt like I was learning it for the first time. Most students ignored me, but some students would routinely greet me when I joined their small-group session. Occasionally, students would address content-related questions to me while I was observing their small-groups. When this happened, I suggested they check in with their peers, Ms. B, or Mr. K. I identify as a White woman, an experienced math teacher, and an early career researcher.

Students

There were 28 students in the Period 1 class and 32 students in the Period 2 class. Table 3c shows the distribution of students enrolled in Ms. B's AB Calculus classes across gender and racial groups[3]. Students of color included students who identified as Black, Chinese, Filipino, Indian, Iranian-American, Korean, Mexican, Mexican / Middle Eastern, Mixed, Nicaraguan/White, and Vietnamese.

Table 3c. Student Enrollment Numbers by Gender and Racial Groups

		# of Students	
Period 1		28	
Female	**Students of Color**	3	11%
	White	5	18%
Male	**Students of Color**	8	29%
	White	12	43%
Period 2		32	
Female	**Students of Color**	6	19%
	White	6	19%
Male	**Students of Color**	6	19%
	White	14	44%

Students were in 12th grade and were either 17 or 18 years old at the time of the study. All names used in this dissertation are pseudonyms; some of the names were chosen by the students themselves and others were chosen by the researcher. All students in Ms. B's Period 1 & 2 AB Calculus classes were invited to participate in the study. Forty-four students consented to recorded lesson observations and 29 students consented to interviews (see Table 3b). Focal students were selected from the subset of fully consenting students based on participation patterns and questions that emerged through lesson observations and conversations with the teacher. In addition, the frequency with which the students were recorded during groupwork was also considered in the selection of focal students. Due to logistical constraints, not all 29 consenting students were interviewed as part of this study. Seventeen students across the two

[3] Students' gender and race classifications were initially based on information provided by the teacher drawn from her experiences with these students. Some of the classifications were refined and/or changed based on additional information the researcher gathered through one-on-one interviews with students.

calculus classes participated in interviews (nine students from Period 1 and eight students from Period 2). Student interviewees consisted of five female students of color (Iranian-American, Mexican, Mexican/Middle Eastern, Mixed, Nicaraguan/White), four White female students, five male students of color (Black, Chinese, Filipino, Mixed, Vietnamese), and three White male students. Each interviewee was asked to describe their gender and their "race, ethnicity, culture" at the end of their first one-on-one interview. Therefore, racial and gender identities used throughout this paper are self-stated for participant interviewees. Racial and gender identities are teacher-ascribed for students who did not participate in interviews.

The phrase "students of color" is used to group non-White identifying students together for analysis in this paper, though the term is used with hesitation. I acknowledge students of color are not a homogenous group, and important distinctions exist within this classification. The phrase "students of color" was selected for use based on the ways students talked about themselves in relation to their peers during interviews. Multiple students used the phrase "people of color" or "PoC" to refer to themselves in comparison to their White classmates, such as when Guadalupe said, "it's different with PoC girls and the White girls in the class…" I have chosen to capitalize "White" in this paper to draw attention to the power this classification holds, though I acknowledge there are valid reasons other scholars have chosen not to capitalize the word. My decision follows the rationale of Eve Ewing, a Black writer and sociologist of education who studies racism and social inequality. She shared the following in an online reflection explaining her decision to start capitalizing "White" in her writing.

> Whiteness… is a specific social category that confers identifiable and measurable social benefits… When we ignore the specificity and significance of Whiteness — the things that it is, the things that it does — we contribute to its seeming neutrality and thereby grant it power to maintain its invisibility (Ewing, 2020).

I have selected the identity labels and classifications used in this dissertation with intention, though I acknowledge they are imperfect and are not set in stone. Again in Ewing's words, "Language and racial categories have some important things in common: They are fluid, they are inherently political, and they are a socially constructed set of shared norms that are constantly in flux as our beliefs and circumstances change" (Ewing, 2020).

Lesson Observations

Almost all lessons for the Period 1 and Period 2 AB Calculus classes during the 2020-2021 spring semester were recorded using Zoom functionality and observed by me, the researcher. Formal lesson observations included handwritten notes (see Appendix A for the template), a typed summary of each lesson recorded in Excel at the end of the lesson, and real-time logging of student contributions using the EQUIP observation tool (Reinholz & Shah, 2018; details included later in this chapter). Lesson Observations included both "regular lessons" and "help lessons." Most lesson observations were followed immediately by videorecorded observations of lesson debrief sessions with the teacher and student teacher. The number of school days, lesson observations, and lesson debrief sessions for each class period are shown in Table 3d.

Table 3d. Number of School Days and Observations for Periods 1 & 2

	AB Calculus	
	Period 1	Period 2
# of School Days (Semester 2)	48	50
# Total Lesson Observations	45^	47^
# EQUIP Observations	40*	40"
# Regular Lesson Obs	30	32
# Help Lesson Observations	10	8
# of Lesson Debrief Observations	30	45

^ 2 days - student assessments; 1 day - researcher absence

* 5 days - groupwork only, no whole-class (WC) discussions

" 6 days - groupwork only, no WC; 1 day - poor internet connection

Due to the distance learning alternating term schedule, Period 1 only met during the odd-numbered Terms and Period 2 only met during the even-numbered Terms. The way the scheduled was designed resulted in a total of 48 school days across Terms 5 and 7 during for the Period 1 AB Calculus students and a total of 50 school days across Terms 6 and 8 for the Period 2 AB Calculus students. This schedule resulted in classes having roughly half as many lessons per semester as they would have had in a typical year. Therefore, the 45 lesson observations of the Period 1 class and 47 lesson observations of the Period 2 class represent almost all the AB Calculus lessons that took place during semester 2. Two lessons in each class were not observed due to student assessments and one lesson in each class was not observed due to the researcher's absence.

Audio/Video-Recorded Lesson Observations

Sixty-minute regular lessons (Mondays, Tuesdays, Thursdays, and Fridays) and 50-minute optional help lessons (Wednesdays) were recorded throughout Semester 2. Except for individual assessments days, every regular lesson consisted of both whole-class and small-group formats. Help lessons were conducted in a whole-class format, though the "whole-class" included fewer students since these lessons were optional for most of the year. Attendance ranged from 3 to 9 students on any given Wednesday from January to April. In May, when the district policy changed making it required for students to attend help sessions either through Zoom on Wednesdays or in-person on Friday afternoons, the attendance at the Wednesday lessons increased to about 20 students.

Distance learning lessons (both regular lessons and help lessons) were recorded using Zoom functionality. Analyses of whole-class interactions were based on video and/or audio-recordings of the lesson, in addition to EQUIP observations and handwritten notes taken by me. Video-recordings focused on the view of the teacher, allowing visibility to what Ms. B was writing on the whiteboards in her classroom. Student voices were still captured through Zoom, though they typically did not show up in the view of the recording. There were eight students in each class period who did not consent to observation recordings, so I kept a post-it note with those eight names taped to my computer every day. I was able to anticipate when non-consenting students were about to speak since students needed to manually unmute before speaking out during whole-class discussion. When a non-consenting student spoke, I paused the Zoom recording. Analyses of small-group interactions were based on video-recordings of the lesson and handwritten notes. The EQUIP observation tool was not used during small-group sessions because it was too difficult to log each student contribution reliably in real-time. Small-group

interactions were not nearly as regulated or smooth as whole-class interactions. Instead, observational field notes were taken during small-group observations pertaining to how each student participated. One small-group per lesson was selected for observation based on which group(s) contained students who had all consented to recorded lesson observations.

I logged into each regular lesson and help lesson about 5 minutes before the official start time of the class. When I joined the class, it was common for a few students to already be in the Zoom session chatting with Ms. B. I did not want to make the students feel uncomfortable during these casual conversations, so I did not begin the lesson recording until after the official start of class. At the beginning of every class, Ms. B greeted her students by saying "Good morning, everyone!" and students would unmute and say, "Good morning" in response. It was after these greetings that I would begin recording.

Ms. B enforced a policy requiring every student to have their video turned on for the duration of the class session unless the student notified her and gave a reason for having the video turned off. Typically, on any given day two to four students would attend class with no video; the other students had their videos on the entire time. Reasons given for no video included internet trouble, device limitations, and illness. Ms. B shared with me that other teachers at EHS often complained to her about having to teach their classes while most students had their videos off. Students in Ms. B's class also shared with me that they often turned their videos off during their other classes, but not during math. Most of the students captured in the video-recorded small-group lessons had their videos turned on.

EQUIP Observations

EQUIP (Reinholz & Shah, 2018) was used during whole-class discussions to capture student contributions in real-time, supplementing lesson recordings. EQUIP, which stands for Equity QUantified In Participation, is a classroom observation tool that tracks user-defined classroom discourse metrics at the student contribution level. Every time an individual student contributed during a whole-class discussion, the contribution was logged in real-time using EQUIP. Data were captured at the student contribution level and then aggregated for class periods and for individual students. Often the tool is used to help uncover teachers' implicit biases related to how teachers solicit student contributions and how teachers respond to students' contributions based on gender and/or race. Tool setup required me to define the discourse dimensions that would be captured during lesson observations for each student contribution. I defined both the category and the possible responses for each discourse dimension. Before beginning formal data collection, I went through a series of trial observations to define and redefine dimensions in ways that captured student participation as accurately and comprehensively as possible.

For this study, student contributions included speech that occurred during the class period about mathematical content or logistics related to the class, comments typed into the Zoom chat window accessible to everyone in the class, and non-verbal actions, such as sharing a computer screen, that supported the lesson. Typically, one student talk turn resulted in one student contribution. However, if a student made more than one type of contribution during a talk-turn (e.g., they answered the teacher's question then asked a question of their own), then a new contribution would be logged for that student for each distinct contribution. Informal conversations between the teacher and students before or after the official class period were not included as contributions. Since EQUIP users essentially code in real-time (i.e., assigning

discourse dimension values to each student contribution as the lesson occurs), data collection and data analysis were happening simultaneously. Definitions for the discourse dimensions and their values used in this study are included later in this chapter in the Data Analysis section (Tables 3g & 3h).

Not all students (and/or their parents) gave consent for recorded lesson observations. In those cases, students' contributions were logged in real-time using EQUIP, but were not recorded on video or audio. The Zoom recording was paused while non-consenting students were contributing. These omissions are noted when the transcript from a whole-class discussion is shared in Chapter 5.

Video-Recorded Lesson Debrief Sessions

Typically, after students were excused from each class period, the teacher, student teacher, and I (the researcher) remained on the Zoom call to touch base and check in. There were occasions when either I or the teacher had to leave right away because of another commitment and on those days, debrief sessions were not observed. Debrief sessions for Period 1 ranged from less than 1 minute up to 11 minutes, averaging approximately 4 minutes. Debrief sessions for Period 2 ranged from 2 minutes to 27 minutes and tended to be more frequent and longer than Period 1 debriefs, averaging approximately 10 minutes. Ms. B taught another class after the Period 1 AB Calculus class and therefore needed to prepare for that next class. However, she had a planning period following the Period 2 AB Calculus class, and therefore had more time to talk and reflect after the class.

Participant Interviews

To supplement the researcher's lesson observations, participants' perspectives on experiences with mathematics, specifically related to classroom participation, were elicited through one-on-one interviews with the teacher, student teacher, and a subset of consenting students. Initial interviews took place at the beginning of the semester, followed by a series of video-stimulated interviews throughout the semester and final interviews after the semester ended. Interviews focused on participants' previous and current experiences and feelings associated with mathematics and their observations, reflections, and assessments of their own and other students' classroom participation. See interview protocols in Appendices B & C. In addition to formal semi-structured interviews, informal conversations with the teacher and student teacher (through Zoom or email) were included in data collection. The intent of the formal interviews and informal conversations was to provide additional insight into participatory interactions related to students' developing perceptions of themselves and their peers as learners and doers of mathematics. Hearing participants' perspectives helped illuminate how barriers to and opportunities for participation were differentially constructed for students in Ms. B's two focal classes.

Study participants took part in different numbers of formal interviews depending on their availability, their interest in study participation, and their relevance to emerging analytical themes. Figure 3a offers a detailed break-down of the interviews conducted with participants.

Figure 3a. Types of Interviews Conducted with Study Participants

	Total # of Students Interviewed	1 interview only	2+ interviews (incl. video reflection)
Period 1	9	5	4
Period 2	8	6	2

	Beginning of Semester Interview	End of Semester Interview	Video Reflection 2.2.21	Video Reflection 3.2.21	# of Additional Video Reflections
Teacher	✓	✓	✓		3
Student Teacher	✓	✓	✓		3
Yonas	✓		✓	n/a	0
Guadalupe	✓	✓	✓	n/a	1
Hosein	✓	✓	✓	n/a	4
Elijah	✓	✓	✓	n/a	1
Alison	✓		n/a	✓	0
Nate	✓		n/a	✓	0

The teacher and student teacher participated in the beginning and end of semester interviews, in addition to video-stimulated reflections of four group lessons. All video reflections involved groups of students in the Period 1 AB Calculus class. Four students in Period 1 participated in more than one interview: Yonas, Guadalupe, Hosein, and Elijah. These are the four students featured in the small-group analysis in Chapter 6, which includes their video-stimulated reflections of the lesson that took place on 2.2.21. All but Yonas also took part in the end of semester interview and at least one additional video-stimulated reflection. Two students in Period 2 participated in more than one interview as well: Alison and Nate. They each participated in exactly two interviews: the beginning of semester interview and a video-stimulated interview reflecting on a group lesson that took place on 3.2.21.

Formal Beginning of Semester Interviews

All beginning of semester interviews took place over Zoom and consisted of the interviewer and one study participant at a time. Nine students in Period 1 and eight students in Period 2 participated in initial interviews. These interviews lasted between 16 and 54 minutes (average = 32 minutes) depending on the length of students' responses to the questions asked. The shortest interview was with Elijah and the longest interview was with Guadalupe, both are focal students featured in Chapter 6. The first formal interview with the teacher lasted 38 minutes and the first formal interview with the student teacher lasted 40 minutes. During these initial interviews all participants were asked about their beliefs pertaining to teaching/learning math, their past experiences with math, goals for the semester, and about recent successes and current challenges with their calculus class. Participants were also asked to describe their gender and their race, ethnicity, culture. At the end of the interview, each participant was asked, "Is there anything else that you can think of related to your participation or your experiences in this class that you feel would be helpful for me to know?" Some participants offered additional thoughts and others did not.

Formal Stimulated-Recall Interviews

In one-on-one interviews the teacher, student teacher, and six students (four in Period 1 and two in Period 2) were asked to reflect on portions of video-recorded small-group sessions using stimulated-recall methods (Erickson, 2006). Participants were asked to watch and reflect on videos of groups in which they took part, prompted by the question, "What do you find noteworthy about this groupwork session?" Follow-up questions included topics such as ease of participation, motivation, goals, and enjoyment. Participants were given the option to stop the video partway through to share comments or to watch the entire video before commenting. The student teacher was the only participant who chose to stop the video and share comments

periodically throughout the video. Conversation length varied depending on the breadth and depth of participants' reflections. For example, conversations with the four students about the 2.2.21 video featured in Chapter 6 ranged from 5-18 minutes, the conversation with the teacher was 16 minutes, and the conversation with the student teacher was 35 minutes. One of the factors that contributed to a longer conversation with the student teacher was his choice to stop the video multiple times and comment as he went. He ended up commenting on more specific interactions, whereas the other participants tended to make more holistic comments about the entire experience. Some of the comments participants made were about the group as a whole, and other comments pertained to individual participants. Reflections provided insight into how each participant made sense of what happened, why it happened, and the significance of certain interactions from the participant's perspective.

Formal End of Semester Interviews

End of semester interviews were conducted with the teacher, student teacher, and three students in the Period 1 AB Calculus class (Guadalupe, Hosein, and Elijah). These one-on-one interviews took place after the spring semester was over and were relatively short, ranging from 8 to 15 minutes. Participants were asked to share their overall reflections of the AB Calculus class. Responses included thoughts on distance learning challenges, aspects of the course that went well, and plans for next year. These three students plus Yonas were invited to participate in the end of the year interviews because they are the four students featured in the small-group task analysis in Chapter 6. Yonas expressed interest in participating in this final interview but was unable to make it work with his schedule.

Informal Teacher and Student Teacher Conversations

Occasionally, I (the researcher) was included in conversations with the teacher and/or student teacher outside of class time. These conversations were similar to lesson debrief sessions and included topics such as student group composition, lesson plans, and observations about specific students. There were eight informal Zoom conversations that took place averaging 11 minutes each; six conversations included the teacher, student teacher, and researcher, and two included just the teacher and researcher. There were also twelve informal email conversations; three conversations included the teacher, student teacher, and researcher, and the other nine included just the teacher and researcher.

Data Analysis

Qualitative and quantitative analytic methods were integrated in this dissertation to offer complementary views of students' participatory experiences. Qualitative methods were intended to provide "insiders'" perspectives (Patton, 1990) on what happened for students in Ms. B's AB Calculus classes. The goal was "to produce in-depth and illustrative information in order to understand the various dimensions of the problem under analysis" (Queirós et al., 2017, p.370). Qualitative methods were drawn from interaction analysis (Bucholtz & Hall, 2005; McDermott & Roth, 1978) and narrative analysis (Schiffrin, 1996; Smith, 2000). Quantitative methods were intended to provide a less biased "outsider's" perspective on what happened for students in Ms. B's AB Calculus classes. Quantitative analyses were descriptive and comparative in nature and included counts of student participation metrics, data displays such as box plots, and mean comparison tests (Punch & Oancea, 2014). Typically, the goal of quantitative research is "to objectively measure reality" (Williams, 2007, p.66). However, I take the approach that "reality"

is multidimensional, not uniformly perceived, and socially constructed. Students in the same classroom can have different realities based on how they interpret and experience classroom interactions. In this study, the goal of the quantitative analyses was to provide a relatively more objective, complementary lens through which to see and understand patterns in student participation. I acknowledge quantitative analyses are not completely objective because I, the researcher, defined the relevant variables and collected the data, and I have my own biases, subconscious or not, which are ever-present.

Although there were some similarities and overlap in analytic methods used in various stages of the study, the analytic results presented in each of the three findings chapters are quite different from one another. Therefore, the details of analyses are explained chapter by chapter.

Chapter 4 – Participants' Perspectives on Participation

Preliminary Exploration of Participant Talk Data

The first step in analyzing participants' perspectives on student participation was to identify which data sources were worth exploring further. I asked myself the question, "Where should I look to find information regarding how participants talked about the factors that shaped their own or other classroom participants' participation?" The answer was, "Everywhere," so I started out considering the whole corpus of data. Table 3e displays descriptions for the four data types included in this participant talk analysis: lessons, lesson debrief sessions, one-on-one participant interviews, and informal conversations.

Table 3e: Data Used for Participant Talk Analysis

Source Type	With Whom?	Quantity	Total Time (hours)
Lessons	Pd.1 class	45 observations (semester 2)	45
	Pd.2 class	47 observations (semester 2)	47
Lesson Debriefs (following lessons)	teacher & student teacher	30 Pd.1 debrief sessions	2.0
		45 Pd.2 debrief sessions	7.7
Participant Interviews	teacher	1 teacher, 4 interviews	3.6
	student teacher	1 student teacher, 3 interviews	1.9
	Pd.1 students	9 students, 16 interviews	9.4
	Pd.2 students	8 students, 11 interviews	6.0
Informal Conversations	teacher & student teacher	8 recorded Zoom conversations	1.8
		12 email chains	n/a

There were 92 hours of lessons, almost ten hours of lesson debrief sessions, about 21 hours of participant interviews, and an additional 2 hours of informal conversations plus email exchanges. From these, I needed to identify the times, within this huge amount of data, when participants talked about factors affecting student participation.

I began by reading through all the hand-written and typed daily lesson observation notes for both the Period 1 and Period 2 calculus classes, looking for instances when the teacher talked to the students about their participation during the class. Talk about student participation included times when the teacher described general expectations for student participation and in-the-moment prompts for certain kinds of participation from certain students (e.g., "Sorry to be blunt, but can we have a girl guess?").

Next, I created content logs while watching all video-recorded lesson debrief sessions, participant interviews, and video-recorded informal conversations and while reading the messages from the informal email exchanges. The length of the sessions, the participants present, the topics discussed, and the names of students mentioned were all documented, along with additional detailed notes about the content of what was said.

Themes, Episodes, and Transcriptions

General themes were identified by reading through the content logs and looking for patterns in participants' talk. The most common topics participants talked about related to student participation were distance learning, gender, and race. Once these three general topics of discussion were identified, all references to distance learning, gender, or race in classroom participant dialog during the spring semester 2021 were identified from re-reading content logs, re-watching selected video clips of lesson debriefs, participant interviews and informal recorded conversations, searching typed lesson observation notes, and re-reading email exchanges. All episodes in the video and email data connected to the three themes were identified. Transcriptions were produced from video data for the selected episodes, including participants' talk, discernable gestures, and notes written on the teacher's whiteboard (in the case of lesson videos). Every transcribed episode, along with sections of written email exchanges was tagged with one or more of the three themes: distance learning, gender and/or race.

Code Definitions

From the transcriptions and email exchanges related to the three themes (distance learning, race, and gender), I developed a series of codes through an open-coding process to identify similarities and differences within the data and to facilitate comparison across participant talk (Corbin & Strauss, 2014; Maxwell, 2013). I used an iterative process of redefining codes to eliminate redundant codes and group similar codes together. Codes were intended to categorize what participants attended to and how they interpreted their observations. Once sorted by topic, references were grouped into specific observations, accounting for different wording but the same meaning. For example, the comments "I noticed the classroom is very male-dominate" and "the boys talk too much" were both coded as the same observation (i.e., male dominance). Patterns and observations related to race and/or gender were grouped together based on similarities and overlap in content, as shown in Table 3f.

Table 3f. Participants' Talk about Distance Learning, Gender, and/or Race

	Patterns	Observations
Distance Learning	Limited Access to Technology & Other Resources	Students needed stable internet connections
		Students needed well-functioning equipment
		Students needed quiet workspaces
	Interactional Challenges Related to Community-Building	Sharing information was harder
		Accessing one-on-on teacher support was harder
		Getting to know students was harder
	Heightened Anxiety and Motivational Struggles	Students' mental and emotional well-being were impacted
Gender and/or Race	Representation Issues Related to Gender and/or Race	Imbalanced enrollment in STEM courses
		Racial segregation in EHS's learning community system
		Too few female teachers and teachers of color in mathematics
	Gendered and/or Racialized Narratives about Participation and Mathematics	Feelings of not belonging in advanced mathematics classes
		Boys are smarter at math than girls
		Wednesday help sessions are for girls
	Classroom Participation Issues Related Mostly to Gender	(White) male dominance
		Girls not speaking up

Distance learning patterns related to access to technology, community-building constraints, and students' personal well-being. Gender and/or race patterns related to representation, narratives about participation in mathematics, and participation issues in their class. The parentheses in "(White) male dominance" were used because sometimes participants said, "the boys talk too much" and sometimes participants said, "the White boys dominate the talk."

Findings presented in Chapter 4 highlight similarities and differences across participants' perspectives, specifically related to their observations of how student participation was shaped by factors related to distance learning, gender, and/or race.

Chapter 5 – Participation in Whole-Class Discussions

There were three components of analysis in Chapter 5, each of which used different analytic methods and a different data source. This section is organized according to these three components: participants' perspectives (narrative analysis of interviews), contribution metrics (statistical analyses of EQUIP observations), and contribution solicitations (interaction analysis of a video-recorded whole-class discussion).

Participants' Perspectives

Analysis related to participants' perspectives on whole-class participation is presented in Part 1 of Chapter 5. This analysis builds off methods used in Chapter 4 to analyze participant talk data. In this chapter, the focus was on how classroom participants (i.e., the teacher, student teacher, and students) talked about the solicitation of student contributions during whole-class discussions, including comments about volunteering, calling on students, and motivation for participation. Relevant references in participant talk were identified by re-reading content logs from participant interviews. The student teacher did not make any comments about soliciting student contributions during whole-class discussions, so his perspective was not included in analysis presented in Chapter 5. Transcriptions were produced from video data for relevant episodes, from which quotations were selected to represent common themes and noteworthy connections to other analyses.

Contribution Metrics

Analysis related to individual student contribution metrics collected using the EQUIP tool is presented in Part 2 and in the beginning of Part 3 in Chapter 5. These metrics were collected during all observed whole-class discussions that occurred during the second semester. There were 30 lessons included in analysis for Period 1 and 32 lessons included for Period 2. Since EQUIP users essentially code in real-time (i.e., assigning discourse dimension values to each student contribution as the lesson occurs), data collection and data analysis happened simultaneously.

Variables. The discourse dimensions in EQUIP relevant to this chapter are shown in Table 3g. Descriptions of each code within the various dimensions are presented in Table 3h.

Table 3g. Discourse Dimensions Used in EQUIP Observation Tool

Discourse Dimension	Assigned Codes
Contribution Solicitation Method	Called On, Volunteered
Contribution Type	Responded to Question, Declined Question, Asked Question, Shared Problem Solution, Identified Mistake, Commented, Read Aloud, Shared Screen
Communication Mode	Spoke, Typed in Chat, Demonstrated and/or Acted
Length of Talk	0 words, 1-4 words, 5-20 words, 21+ words
Content of Contribution	What, How, Why, Other

Table 3h. Code Descriptions for EQUIP dimensions

Contribution	Contributions were defined broadly to include every time an individual student spoke, typed, or acted in a way that contributed an idea, a question, or a comment during a whole-class discussion, regardless of correctness or completeness. Contributions included student actions that supported the progression of the lesson, such as reading a problem out loud or sharing a computer screen with the class. If a student stated they did not want to answer a question asked of them, that statement was coded as a contribution even though no mathematical content was shared.
Contribution Solicitation Method	
Called On	Contributions that were preceded by an explicit invitation from the teacher or student teacher for a specific student to participate, without the student indicating a desire to share something - this process is also referred to as "cold-calling"
Volunteered	Contributions that resulted from a student signaling to the teacher that they had something to share (e.g., raising their hand) and then the teacher calling on them OR contributions that occurred when a student blirted out a response, comment, or question without being called on

Contribution Type	
Responded to a Question	Contributions in which a student responded to a question asked by the teacher, student teacher, or another student
Asked a Question	Contributions in which a student asked a question directed to the teacher, the student teacher, or another student
Declined to Answer	Contributions in which a student did not answer the question they were being asked, typically because they did not have a response they wanted to share
Shared Problem Solution	Contributions in which a student explained their full solution to a problem, often the problem was from the previous night's homework but occassionally it was a classwork problem
Offered a Comment	Contributions in which a student says something that is not a directed response to a question asked and is not a question itself, such as offering a suggestion for another way to approach a problem without being prompted to share
Read Aloud	Contributions in which a student reads a problem or question out loud to benefit the whole class, typically requested by the teacher
Shared their Screen	Contributions in which a student shares their computer screen to benefit the whole class, typically requested by the teacher when talking about the classwork and/or homework assignment for the day
Identified a Mistake	Contributions in which a student points out and explains a mathematical mistake made by the teacher, student teacher, or a student
Communication Mode	
Spoke	Contributions made through verbal (spoken) communication
Typed in Chat	Contributions made using the Zoom chat functionality that were "public" for everyone in the class to see
Demonstrated and/or Acted	Contributions made through non-spoken and non-typed communication, including body gestures, head nods, shared computer screens, or mathematics written on paper
Length of Talk	
0 Words	Contributions using zero spoken or typed words, corresponds with Demonstrated and/or Acted Contributions
1-4 words	Contributions using between one and four words
5-20 words	Contributions using between five and twenty words (approximately) - determined in real-time, so a "best guess" strategy was used
21+ words	Contributions using twenty one or more words (approximately) - determined in real-time, so a "best guess" strategy was used
Content of Contribution	
What	Contributions that contained information pertaining to a "What" question, such as "What do you get when you simplify this expression?" - typically short, answer-focused contributions
How	Contributions that contained information pertaining to a "How" question, such as "How did you get your answer?" - typically longer, process-focused contributions
Why	Contributions that contained information pertaining to a "Why" question, such as "Why might we want to differentiate this equation?" - typically longer, conceptually-focused contributions
Other	Contributions that are non-mathematical in nature, including assignment logistics, general class announcements, and social comments

Contribution-level data were exported from EQUIP to Excel and then converted into student-level data. Variables were reconfigured to display the total number of contributions by

contribution type, length, content category, and solicitation method for each student for the entire semester. Every row that is colored white in Table 3h was made into its own continuous variable (e.g., *sol_volunteer* for the number of times each student volunteered a contribution). A variable representing total contributions was also created, in addition to a student_id index variable and a categorical variable representing the class period for each student (i.e., 1, 2, …). Contribution data were then merged with categorical student demographic data, specifically, gender (female, male) and race group (student of color, White). The final step of data preparation was to import data as a .csv file into STATA. All statistical analyses in this dissertation were completed using STATA/BE 17 software.

Combining Period 1 and Period 2 Datasets. Data from the two class periods were combined into one dataset for analysis. There were qualitative and quantitative reasons supporting the choice to combine data (as opposed to analyzing the class periods separately). Qualitative reasons included: Students were taking the same course (AB Calculus) at the same school taught by the same teacher (Ms. B) and student teacher (Mr. K), my researcher role in the two classes was the same, methods for data collection were the same, and lesson observations did not give me any reason to anticipate significant differences in participation between the periods. Quantitative reasons included: student demographic data (gender and race groups) were similar between the two class periods (Tables 3i, 3j, 3k), histograms of total contributions by period were similar (Figure 3b), a two-sample t-test showed the means for total contributions for the two periods were similar (Figure 3c), and generally, a larger n size is preferable for statistical analyses.

Table 3i. Gender Distribution in Periods 1 & 2

Period	Gender F	Gender M	Total
1	8 28.57	20 71.43	28 100.00
2	12 37.50	20 62.50	32 100.00
Total	20 33.33	40 66.67	60 100.00

Table 3j. Race Distribution in Periods 1 & 2

Period	Race SOC	Race White	Total
1	11 39.29	17 60.71	28 100.00
2	12 37.50	20 62.50	32 100.00
Total	23 38.33	37 61.67	60 100.00

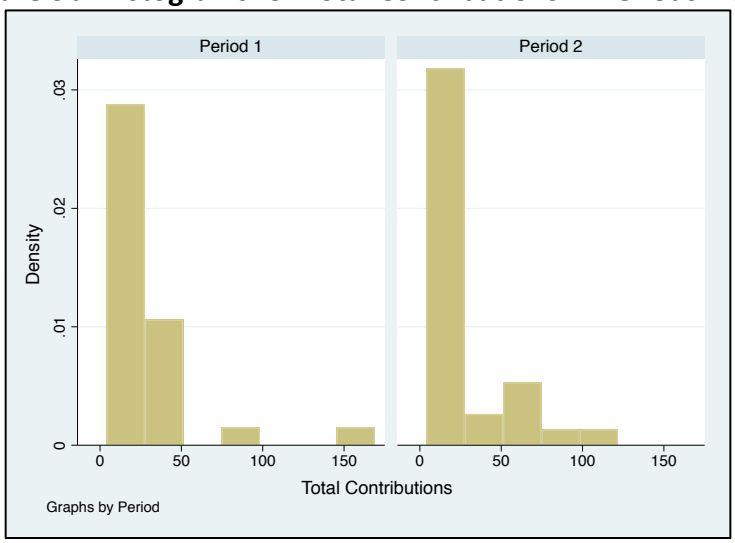
Table 3k. Gender-Race Distribution in Period 1 & 2

Period	Gen_Race F_SoC	Gen_Race F_White	Gen_Race M_SoC	Gen_Race M_White	Total
1	3 10.71	5 17.86	8 28.57	12 42.86	28 100.00
2	6 18.75	6 18.75	6 18.75	14 43.75	32 100.00
Total	9 15.00	11 18.33	14 23.33	26 43.33	60 100.00

Table 3i shows there were twenty males in each period, and there were four more females in Period 2 than Period 1 (12 vs. 8), but females represented a minority group in both class periods. Table 3j shows about 39% of students in Period 1 were students of color and 38% of students in Period 2 were students of color, which are similar to each other. Table 3k shows White females represented about the same proportion of students in the two periods (18% in Period 1, 19% in Period 2), as did White males (43% in Period 1, 44% in Period 2). However, there were more male students of color in Period 1 and more female students of color in Period 2. All four gender-race groups were represented in somewhat similar ways.

To get a sense of what the data looked like, I created histograms of total contributions for the class periods (Figure 3b).

Figure 3b. Histograms for Total Contributions in Periods 1 & 2



The graphs in Figure 3b both have a spike between 5 and 25 and then have lower density after that. The Period 2 spike is a bit higher than Period 1, but the overall shapes of these histograms are somewhat similar. In addition, before conducting a two-sample t-test, I did a robust equal variance test of total contributions in the two class periods (Figure 3c).

Figure 3c. Output from Variance test of Total Contributions by Period

```
. robvar totalcont, by(period)

                Summary of TotalCont
    Period |      Mean   Std. dev.       Freq.
-----------+------------------------------------
         1 |  27.392857   31.823983          28
         2 |   24.65625   26.516713          32
-----------+------------------------------------
     Total |  25.933333   28.893062          60

W0  =  0.23559799   df(1, 58)     Pr > F = 0.62923221

W50 =  0.00195085   df(1, 58)     Pr > F = 0.96492183

W10 =  0.06706992   df(1, 58)     Pr > F = 0.79656868
```

Since the F-statistic is greater than 0.05 (P = 0.6292), we fail to reject the null hypothesis that variance is equal. Using this additional information, I conducted a two-sample t-test with equal variances to see if there were significant differences in contributions between the periods. Output from the t-test is shown in Figure 3d.

Figure 3d. Output from Two-Sample T-Test of Total Contributions by Period

Two-sample t test with equal variances

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf.	interval]
1	28	27.39286	6.014167	31.82398	15.0528	39.73291
2	32	24.65625	4.687537	26.51671	15.09596	34.21654
Combined	60	25.93333	3.730078	28.89306	18.46946	33.3972
diff		2.736607	7.532404		-12.34114	17.81435

diff = mean(1) - mean(2) t = 0.3633
H0: diff = 0 Degrees of freedom = 58

Ha: diff < 0 Ha: diff != 0 Ha: diff > 0
Pr(T < t) = 0.6412 Pr(|T| > |t|) = 0.7177 Pr(T > t) = 0.3588

The null hypothesis was that the mean of total contributions in period 1 was equal to the mean of total contributions in period 2. Assuming $\alpha = 0.05$, the high P-value (0.7177) indicates failure to reject the null hypothesis. The null hypothesis is highly probable given the data. These quantitative results support the decision to merge Period 1 contribution data with Period 2 contribution data into a single dataset for analysis.

Quantitative Analyses. I examined patterns of student contributions for the course, as a whole (including Period 1 and Period 2), by student gender, by student race category, and by gender-race groupings, using a combination of descriptive summary statistics and two-sample t-tests to understand how student contributions varied among students in Ms. B's calculus course. I started by looking at contribution quantities for the whole class by contribution type, communication mode, length of talk, and content of contribution. Almost all contributions were spoken, so the Communication Mode variable was not considered in further analyses. Additionally, I looked at the method used to solicit each student contribution. Next, I completed contribution comparisons between groups of students. To get a better sense of how contributions varied between student groups and to prepare for two-sample t-tests, I created box plots for total contributions, contribution type, contribution length, contribution content, and contribution solicitation by gender, by race, and by gender-race groups. I also conducted robust equal variance tests for each variable to determine if I needed equal variance or unequal variance t-tests. Using that information, I conducted two-sample t-tests for total contributions, along with contribution type, length, content, solicitation between gender groups, race groups, and gender-race groups (e.g., White female students & female students of color). These t-tests allowed me to identify significant differences in students' participation patterns during whole class discussions. Effect sizes for the significant variables were then calculated using Hedges' *g* (as opposed to Cohen's *d*), since sample sizes for the comparison groups were relatively small

and not the same as each other. The relatively small sample sizes suggest the applicability of findings outside of this particular classroom context may be limited.

Contribution Solicitations

Analysis related to contribution solicitations is presented in Part 3 of Chapter 5. To complement the quantitative analyses examining individual student contribution metrics across an entire semester, one video-recorded whole-class discussion was selected for a detailed qualitative analysis. Using EQUIP contribution metrics as a guide, one representative whole-class discussion (a 40-minute excerpt from the Period 2 lesson on 3/1/21) was selected for analysis. The chosen whole-class discussion occurred during a lesson in the middle of the semester and had contribution metrics that seemed typical for Ms. B's calculus course. The discussion was transcribed, with the transcription including teacher and student talk, periods of silence, and the contents of the teacher's whiteboard.

Interaction analysis was conducted using the transcription of a whole-class discussion with a focus on understanding how Ms. B solicited contributions from students. The assumption is that the contributions students made (analyzed in Part 2 of Chapter 5 as individual contribution metrics) were shaped by the ways in which Ms. B invited those contributions during whole-class discussions (analyzed in Part 3 of Chapter 5 as contribution solicitations). Every contribution solicitation by Ms. B was flagged in the transcript. Codes were assigned to each solicitation as described in Table 3l.

Table 3l. Code Descriptions for Contribution Solicitations

Contribution Solicitation	Contribution solicitations were defined broadly to include every time Ms. B spoke in a way that invited a contribution from one or more students. If a student made a contribution without any solicitation from Ms. B, that event counted as a solicitation with "No Invitation."
Invitation Type	
Open Invitation	A contribution solicitation in which Ms. B asked a question open to anyone, without encouragement (e.g., asking if anyone has questions)
General Encouragement	A contribution solicitation in which Ms. B asked a question open to anyone, which included an attempt to convince students to respond (e.g., giving a hint, praising students)
Explicit Request	A contribution solicitation in which Ms. B asked a question directed at one or more specific people with a response expected (e.g., cold-calling a student by name)
No Invitation	A contribution made by a student without any solicitation from Ms. B (e.g., asking a question without being prompted by the teacher)
Intended Contributor	
Anyone	A contribution solicitation intended for any student in the class
Specific Student	A contribution solicitation intended for one student in particular
Whole Class	A contribution solicitation intended for all students in the class (e.g., asking all students to show the answer using their fingers)
No One	A contribution made by a student without any solicitation from Ms. B (e.g., asking a question without being prompted by the teacher)
Response	
None	A contribution solicitation to which no one responded (e.g., extended silence after asking if students have any questions)
Volunteer (*name*)	A contribution solicitation which resulted in a student making a contribution after volunteering to share
Called On (*name*)	A contribution solicitation which resulted in a student making a contribution after Ms. B called on a student to share who had not volunteered
Contribution Type (from EQUIP, see Table 3g)	

Interactional patterns were identified by looking at the type, frequency, timing, and content of contribution solicitations throughout the whole-class discussion. Particular attention was paid as to how solicitation patterns related to students' gender and race.

Chapter 6 – Participation in a Small-Group Task

There were two components of analysis in Chapter 6. The first component of analysis pertained to participants' perspectives on the task and used data from stimulated-recall interviews with the teacher, student teacher, and four focal students. The second component involved analyzing students' individual and interactional participation during the selected task, using data from a 10-minute videorecording of Yonas, Guadalupe, Hosein, and Elijah working on The Ladder Problem. This task was selected for analysis for several reasons. First, all four students in the group had consented to videorecorded observations and one-on-one interviews. Second, the task occurred relatively early in the semester (2/2/21), so the effects of "senioritis" (12th grade students "checking out" due to their looming graduation) were not evident. Third, this was a task the teacher had refined over her years of teaching. One goal was to see what could be learned by studying the participation of students engaging in a well-planned, cognitively

demanding task. Throughout analysis, particular attention is paid to how Guadalupe's experience compared to the experiences of her male peers.

Participants' Perspectives

Analysis related to participants' perspectives on the focal group's work on one small-group task is presented in Part 1 of Chapter 6. This part focused on how classroom participants (the teacher, student teacher, and four focal students) experienced The Ladder Problem task, including how and why students participated in the ways that they did. Videos of the six stimulated-recall interviews were transcribed for speech, salient expressions, and gestures. Transcripts from the interviews were then divided into *observations*, operationalized as speech in which an interviewee attended to and/or interpreted a particular part of the task video. When the subject, topic, or timeframe of the observation changed, a new *observation* was assigned. Codes were developed through an open-coding process to identify similarities and differences in data and to facilitate comparison across the participants' observations (Corbin & Strauss, 2014). Using an iterative process, codes were redefined to eliminate redundancy and group similar codes together. Final codes are described in Table 3m.

Table 3m. Code Descriptions for Video-Stimulated Interviews

Observation	Observations were defined as segments of speech in which an interviewee attended to and/or interpreted a particular part of the task video. When the subject, topic, or timeframe of the observation changed, a new observation was assigned.
Subject	
Self	An observation in which a person talked about themselves
[Student Name]	An observation in which a person talked about a specific student by name - the student's name was recorded as the subject
Teacher	An observation in which a person (other than Ms. B) talked about Ms. B
Student Teacher	An observation in which a person (other than Mr. K) talked about Mr. K
Researcher	An observation in which a person talked about Ms. Fink
Group	An observation in which a person talked about more than one person in the group collectively (e.g., "we", "they")
Task	An observation in which a person talked about the task, including the design of the task and/ or the mathematical content of the task
Topic	
Actions	An observation about what someone said or did, including collaborating, disengaging, and asking questions
Thinking	An observation about what someone was thinking or might have been thinking, including being confused, understanding, and being challenged
Feelings	An observation about how someone was feeling or might have been feeling, including comfortable, motivated, and disappointed
Relationships	An observation about student - student relationships or student - teacher relationships, including friendships, respect, and connections
Ways of Being	An observation about how people are, including being teenagers, being imperfect, and being social
Lesson(s)	An observation about the task or the lesson, including the level of challenge, the mathematical content, and the question prompts
Distance Learning	An observation about the distance learning context, including limitations and resources
Group Composition	An observation about how groups were composed, including gender distribution and randomness
Timeframe	
This Day	An observation about the day the task took place
Different Day	An observation about a specific day that was different from the day the task took place
This Year	An observation about the current school year that extended beyond what happened on the day of the task, not connected to any particular day
Previous Year(s)	An observation about a time prior to the current school year
Timeless	An observation that extends beyond specific days or years
Tone	
Positive	An observation positioned by the speaker as desirable
Neutral	An observation positioned by the speaker as neither desirable nor undesirable, or an observation positioned as being a mix of both desirable and undesirable
Negative	An observation positioned by the speaker as undesirable or unfortunate

Themes were identified from the coded data regarding what participants observed and how participants made sense of their observations.

Student Participation

Analysis related to student participation during The Ladder Problem task is presented in Part 2 and Part 3 of Chapter 6. Part 2 focuses on how each of the four students (Guadalupe, Yonas, Hosein, and Elijah) participated individually during the task, including what each student said and when they said it. Part 3 focuses on how participants interacted with one another during the task and how interactions between participants shaped various forms of participation for each student. Analysis includes qualitative coding of students' contributions, counting contributions by type, placing contributions along task completion timelines, and mapping interactions between participants.

The 10-minute video of the four students working on the task in their Zoom breakout room was transcribed for speech, actions, expressions, and gestures. The transcript (See Appendix D) was then divided into *contributions*, operationalized as uninterrupted speech by one person of a single contribution type (see description below). Sometimes a single talk-turn contained two contributions, such as when Yonas began reading the problem aloud (contribution 1), then went on to share mathematical reasoning (contribution 2). Every contribution was assigned a time of contribution based on when the contribution began, and each contribution was coded as either mathematical or social based on the topic of discussion or action. Mathematical contributions consisted of all contributions related to understanding and/or engaging in the assigned task, including logistical dialog such as locating the Googledoc file containing the task prompt. Social contributions consisted of all contributions not related to understanding or engaging in the assigned task, including greetings, jokes, and personal questions. Codes were assigned to every mathematical and social contribution based on *acts of positioning* (van Langenhove & Harré, 1999), operationalized into three categories: contribution types, contribution prompts and invitations, and contribution responses.

Contribution Types. Each contribution is an act of positioning (Gholson & Martin, 2014). The contributing student positions themself through the type and content of their contribution. Did the student offer a possible solution to the problem justified by reasoning? Or did they verbalize their confusion about what to do next? Contribution types were coded as: Asks a question, Makes a comment, Shares mathematical reasoning, Shares solution with reasoning, Shares solution without reasoning, Expresses agreement, Expresses disagreement, Expresses uncertainty, or Reads the problem aloud. Note these contribution types are similar yet somewhat different from the contribution types used in Chapter 5. These coding differences are due to differences in the natures of whole-class (Ch. 5) and small-group (Ch. 6) discussions and the limitations of data collection and analysis methods (real-time coding during whole-class discussions vs. micro-analysis of a videorecorded lesson).

Contribution Prompts and Invitations. Every invitation for a contribution is an act of positioning (Langer-Osuna, 2011; Radinsky, 2008), including explicit and implicit invitations. Not every contribution is invited. Students position themselves and each other depending on how contributions are prompted. Was a student called on by name to contribute a math idea? Or did they interrupt another student to share an idea with seemingly no invitation at all? Types of prompts included participant talk, action, gesture, and "nothing." Types of invitations included explicit individual, explicit group, implicit, open conversational floor, and interruption. The person whose actions prompted or invited the contribution was also coded. Details regarding

how codes for prompts and invitations were defined and related to each other are integrated with the analytic results in Chapter 6, Part 3.

Contribution Responses. Participants' responses to contributions are acts of positioning as well (Anderson, 2009; Hand, 2010). Students are positioned by their peers and teachers through the reactions they get to the contributions they make. Is the contribution met with explicit affirmation? Is the validity of the contribution challenged? Or is the contribution ignored? Contribution responses were coded as: Positive (verbal agreement or smile), Negative (verbal disagreement or interruption), Neutral (related comment expressing neither agreement nor disagreement), or No response (silence and no change in facial expression).

Chapter 4: Analyzing Participants' Perspectives on Participation

Introduction

Working toward participatory equity – cultivating classrooms with *fair* (not necessarily the *same*) opportunities to participate (Esmonde, 2009; Secada, 1989) – requires taking into account that calculus classes are historically White, male-dominated spaces, in which females and racially minoritized students face additional barriers to participation (Leyva et al., 2021). A fundamental goal is to create genuine opportunities for every student to participate in ways that support rich content knowledge and positive mathematical identities (Esmonde, 2009). Some students have easy access to meaningful participation in advanced mathematics classes; other students face a range of barriers that make it harder for them to engage in ways perceived to be compentent. While patterns in participation barriers exist, individual students respond differently to the barriers they encounter depending on their experiences, their beliefs about themselves and their classmates, and the resources readily available to them.

Pursuing participatory equity requires dismantling barriers and finding ways around the barriers that cannot be taken apart. But before we think about how best to address these obstacles, we need to first understand what the barriers are and how they impede students' opportunities to participate. Students participate in school as individuals who are members of classroom communities operating within broader social contexts. Classroom-level factors (e.g., how competence is defined, how tasks are designed, how participation is structured) and broader institutional and societal factors (e.g., narratives about race and gender, representation of females and people of color in advanced mathematics) shape expectations for the ways students can and should participate, privileging some students and marginalizing others. Interactions in mathematics classrooms play out in gendered and racialized ways (Esmonde & Langer-Osuna, 2013; Gholson & Martin, 2019; Sengupta-Irving & Vossoughi, 2019), creating additional barriers to participation for certain groups of students (Leyva et al., 2021). On top of these persisting challenges, students during the 2020-2021 school year faced new barriers to participation brought on by the Covid-19 global pandemic (Francom et al., 2021; Lister et al., 2021).

Creating genuine opportunities for every student to participate in meaningful ways requires a deep understanding of the factors that make it difficult, uncomfortable, and/or risky for students to contribute during mathematical discussions. Centering classroom participants' perspectives, especially students' voices, is essential for understanding the depth and nature of the obstacles students face on a daily basis in mathematics classrooms. The overarching question guiding analysis in this chapter is: How did participants talk about connections between participation and various classroom, institutional, and societal factors that shaped student engagement in the course? The underlying empirical research questions are: 1) How did participants articulate and make sense of factors that impeded student participation? 2) How did participants articulate and make sense of perceived student participation "issues" and "successes"?

Chapter Overview

This chapter is intended to be read within the context of the whole dissertation. Therefore, the bulk of the literature, theory, and methodology framing these analyses is provided in the previous chapters. Before sharing the findings that are focused on the barriers to participation students faced in Ms. B's calculus classes, I provide some information about the

community, the school, and Ms. B's classroom. Results from analysis are then presented in three parts. Part 1 focuses on participants' observations of barriers to participation related to distance learning, followed by participants' observations of barriers to participation related to gender and/or race in Part 2. Part 3 focuses on how participants talked about participation issues and participation successes in Ms. B's calculus classes. At the end of this chapter, findings are summarized, comparing and contrasting participants' experiences. Implications of these findings related to supporting more equitable opportunities for participation and learning in mathematics classrooms are presented in the Discussion section in Chapter 7.

Background on the Community, the School, and the Classroom

This study took place in a mid-sized city in the United States. The city was home to a large research university and one main public high school (Evergreen High School). In recent years home prices had risen, the percent of White residents had risen, and the overall city population had decreased slightly. Evergreen High School (EHS) had several thousand students enrolled during the 2020-2021 school year and over 150 full-time teachers. School demographics indicated about 40% of students identified as White, 20% as Hispanic, 15% as Black, 15% as 2 or more races, and 10% as Asian. About ¼ of the student body was categorized as "economically disadvantaged." Just as the city demographics had shifted toward Whiter residents, so too had the school demographics. During a lesson debrief session, the teacher (who had taught at EHS for 15 years) shared a story about some of her students from a while ago. She shared, "It was a Geometry class, and I ended up with a group of all Black boys, which is obviously a different campus from what we have now."

Students at EHS were divided among five learning communities (three smaller and two relatively larger) intended to meet the various needs of the school's diverse student body. Students in different learning communities had different requirements for graduation, and they typically took courses specific to their learning communities. Over the past decade or so, members of the broader school community have raised questions about the limitations of this system, specifically related to racial segregation, "achievement gaps", and access to advanced-level courses across the learning communities. The two larger learning communities have consisted of predominantly White students and offered the most rigorous STEM courses. The three small learning communities have consisted mostly of students of color and have offered few advanced or honors courses to students. Most of the students enrolled in the AB Calculus course featured in this study were members of the two larger learning communities.

Although the AB Calculus course is considered by many to be an advanced mathematics course, it is not part of the honors math track at this school. Students in 12th grade in the honors track take BC Calculus instead of AB Calculus. According to one of the focal students, acceptance into the honors math track was based on a test students took in 8th grade. Nate, a student in this study, explained his experience with math placement during a one-on-one interview.

> I always liked math … I wanted to get into the honors math, but I didn't. I still did well in the other math classes, but - … there was a test in 8th grade for me to take and there was some Algebra on the test that I wasn't exposed to in my 8th grade math class, so I was kind of confused, but you know. It's all good.

Most of the students in Ms. B's calculus classes took a regular track Math 3[4] course their junior year and then chose to take AB Calculus over an AP Statistics course for their senior year. However, some students in MS. B's classes were in the honors track the previous year but opted to take the AB Calculus course instead of the BC Calculus course their senior year. Typically, the AB Calculus class is intended to prepare students for the AB Calculus Advanced Placement (AP) exam at the end of the school year, but due to the pandemic, those expectations were changed by the school. Since students were meeting for roughly half as many class days due to the pandemic schedule, it seemed unreasonable to set the expectation that students would be prepared for the exam. There were several students who still planned on taking the AP exam in the spring, but those students did extensive preparation outside of the regular class curriculum. It is not known how many, if any, of the students in Ms. B's AB Calculus classes ended up taking the AP AB Calculus exam.

Ms. B enjoyed working with pre-service and other in-service teachers. She had hosted many student teachers and student observers from the local university over her years at EHS and welcomed me (the researcher) into her classroom. She saw these experiences as opportunities for her own growth as a teacher. For the spring semester of the 2020-2021 school year, Ms. B agreed to take on a full-time student teacher (Mr. K) who was working on his single-subject mathematics teaching credential through the local university. She told me initially that she was not planning on mentoring any full-time student teachers this year because of the additional work associated with the transition to distance learning. However, when Mr. K reached out to her to ask if she would be his mentor teacher for the semester, she decided to do it. She told me that if it had been anyone else, she would not have agreed. Mr. K had visited her classroom once a week for one semester the previous year for a university course he was taking at the time. Mr. K and Ms. B had both enjoyed that experience and had learned a lot from each other. Ms. B seemed happy with her decision; she regularly complimented Mr. K and talked about how lucky she was to have him. She shared, "I've had four spectacular [student teachers] and he's one of them. They were all really different, and they were all young people of color, every one of the really great ones. It makes such a difference." She regularly sought out his feedback and ideas for lesson improvements, she wanted to learn from his past experiences, and she appreciated his perspective which was often different from her own. She was impressed by his comfort with the mathematical content and comfort with teaching. Likewise, Mr. K also enjoyed working with Ms. B.

> I liked a lot of things about [this student teaching placement], and I felt fortunate and privileged to have it… I felt like a partner with her, and I felt like - she also has many more years of experience, but it felt like she treated me like equal.

Mr. K felt respected by Ms. B and appreciative of the opportunity he had to learn from her. The positive working relationship continued through to the end of the semester. Mr. K was offered a full-time teaching position at Evergreen High School for the following school year, but he decided

[4] Math 3 is a year-long course covering algebra, geometry, and statistics topics. It is the third course in the EHS math sequence, preparing students for AP AB Calculus or AP Statistics in 12th grade. Regular Math 3 and Honors Math 3 are offered at the school. Honors Math 3 requires students to earn a "B" or above in Honors Math 2.

to turn down the offer to pursue a different opportunity. He and Ms. B stayed in touch after his student teaching placement ended.

Part 1: Barriers to Participation - Distance Learning

This part addresses participants' observations about barriers to participation related to distance learning. The Covid pandemic brought on new challenges for both teachers and students, resulting in a school year unlike any other they had experienced. Students acknowledged the year was different from what they were used to, making references such as, "it's a little bit weird this year," "it's like school but not," and "back when real school was happening." Participants' talk about distance learning was generally negative in tone, though some comments were more intensely negative than others. For example, in a one-on-one interview one student shook her head and said, "this year has just been the worst…Zoom is so gross."

Through analysis, three patterns in participants' observations about distance learning were identified. First, distance learning created new barriers to participation that did not exist in previous (in-person) school years, specifically related to students' access to technology and other resources. Second, distance learning made it harder for class participants to interact effectively and efficiently with one another, making it harder to build community. Third, distance learning heightened students' pre-existing anxieties and motivational struggles related to class participation. Table 4a includes a summary of participants' distance learning observations made during formal interviews, informal documented conversations, lessons, and lesson debrief sessions. Observations are grouped by the three afore mentioned patterns.

Table 4a. Patterns in Participants' Distance Learning Observations

Pattern	Observations
Limited Access to Technology & Other Resources	Students needed stable internet connections
	Students needed well-functioning equipment
	Students needed quiet workspaces
Interactional Challenges Related to Community-Building	Sharing information was harder
	Accessing one-on-on teacher support was harder
	Getting to know students was harder
Heightened Anxiety and Motivational Struggles	Students' mental and emotional well-being were impacted

Overall, the teacher, student teacher, and students agreed it was harder for students to participate virtually in the Zoom environment than it would have been to participate in person in a physical classroom. However, there were some differences in what participants noticed and how they talked about the impacts of distance learning on student participation. Each of these patterns is explored in detail in the following three sections, using direct quotations from the teacher, student teacher, and numerous students, along with a few from the researcher, to convey participants' perspectives on distance learning at Evergreen High School.

Limited Access to Technology & Resources

To participate during Zoom classes, students needed access to technology and other resources that were not needed during previous school years. Students needed a stable internet

connection from home (or from wherever they were physically located), well-functioning equipment (computer, camera, microphone), and a quiet, well-lit workspace. The teacher began every class period with a formal greeting to her students. She would say, "Good morning, everyone!" and expected every student to unmute and greet her in return. Part of this beginning routine involved the teacher asking "blank-screen" students to turn on their cameras and to let her know if and why they were unable to turn them on. Typically, there were 3-5 students each day who offered various reasons for their blank screens, ranging from "I don't have a camera today" to "my internet is unstable." When a student said, "I'm still eating my breakfast," the teacher responded, "That's ok. Turn your camera on and eat your breakfast with us." Many of the technology and resource observations were elicited through the teacher's routine of holding students accountable for their visible presence in the class. In addition, there were observations made by the researcher as she tracked student absences and student tardiness each day at the request of the teacher. If a student arrived late to class, they would often send a private chat message to the researcher with a reason for their tardiness, such as, "I couldn't get connected" or "I was here on time but got booted off and had to log in again."

Stable Internet connections

The first group of technology and resource observations pertained to dealing with unstable internet connections. Most students told the teacher at least once during the semester that they had their cameras turned off due to internet connectivity issues. The researcher tracked who had their cameras off each day for one term (half the semester). She found that 19 out of the 28 students in the Period 1 class and 26 out of the 32 students in the Period 2 class had their cameras turned off for at least one class during that term. Period 1 had an average of 3.7 blank screens per class and Period 2 had an average of 4.6 blank screens per class. Unstable internet connections also resulted in students getting bumped out of class on a regular basis. Most students returned within minutes of dropping out of the class, but several times students were unable to rejoin so they texted a friend in class, who in turn sent a Zoom chat message to the teacher or researcher to notify them of the situation. A student's disappearance and re-entry to the class often went unnoticed by their peers and sometimes by the teacher if it happened during a whole-class discussion; their re-entry was relatively smooth. On the other hand, getting bumped out of the Zoom class during small-group breakout rooms was not so smooth, since upon re-entry the student would get stuck in the main room waiting to be reassigned to a group and the teacher would not always know the student was there.

The researcher and teacher experienced their own connectivity issues over the semester as well. The researcher missed one entire day of class due to a power outage and had to turn her camera off on several other days due to unstable internet warnings. Fortunately, the teacher's connection was usually strong and reliable. She logged on from her regular classroom each day at the high school and was able to teach every class except one without encountering any internet problems. The exception was one Period 2 class which did not go as planned. Right from the beginning of class, the teacher's video was lagging, making it impossible for students to follow what she was saying or writing on the white board. The lesson turned into an independent work session with the teacher supporting students through Zoom chat messages as they worked on their homework problems.

Well-Functioning Equipment

Internet connectivity seemed to be the focus of most technology-related observations, but a few students also faced challenges accessing functioning equipment. One student in the Period 1 class had his camera turned off for two weeks straight because he said he had lent his webcam to his cousin who really needed it. Another student in the class did not have her camera on for several days in a row because she had lent her laptop to her brother and was calling in on an iPad with a broken camera instead of her computer. There were also several students who experienced various microphone issues who had to resort to sending Zoom chat messages to the class after multiple failed attempts to unmute and speak. Equipment concerns were not as widespread as other distance learning concerns, but they were quite impactful for the students who encountered them.

Quiet Workspaces

The final group of technology and resource observations related to students' workspaces. Students did not offer any observations about challenges they faced finding quiet workspaces for class; these observations were made by the teacher, student teacher, and researcher during lesson debrief sessions. Most students when they logged into a class stayed in the same location for the duration of the class, and often it was the same space day after day. For some students, this space appeared to be a private bedroom with a desk and a closed door. For other students, this space appeared to be a kitchen table or living room couch in a space shared with other people who were also in Zoom meetings / classes or doing household activities. There were several times when a student was unmuted and in the process of sharing an idea but then had to mute themself temporarily because of background noise and then move to another location before they continued sharing. Other times students tried to talk over background noises in their homes. Mr. K called on his own experiences growing up to make sense of the workspace challenges that some students faced.

> I know what they're going through at home in a way. When I spoke to Carlos's parents, they seemed to be really supportive, but when he's in the zoom classes, you can't hear anything he says because all we hear is washing, cooking, talking. So, I feel like I went through the same thing in that my parents respected me going to school, but they didn't 100% support me to make sure I had space, quietness. I didn't have a desk until I got to college, and I bought that desk myself.

Access to stable internet connections, well-functioning equipment, and quiet workspaces varied from day to day and from student to student. The teacher repeatedly reached out to students whom she thought could benefit from support offered by the school district, but many technology and resource issues remained unresolved and were treated as unfortunate realities of the distance learning context. Many of these barriers to participation students faced were unpredictable and, in some cases, seemingly unavoidable.

Interactional Challenges Related to Building Community

When learning interactions are restricted to virtual settings, the resources available to teachers and students for community-building are different. While some people may argue for the benefits of online communications, participants in this study focused on the frustrations of

trying to interact with one another over Zoom and the lack of knowledge about and connection with members in their classroom community. Key components of productive classroom communities include effective communication, strong relationships, and access to one another, all of which were obstructed by distance learning.

During the 2020-2021 school year, teachers had no choice but to adjust their instructional practices to fit the confines of virtual learning environments. Fortunately for Ms. B, she was able to go to work every day as she had in previous years and teach from her regular classroom. The main difference was that she was alone in that room and was restricted to virtual interactions with her students, communicating through her laptop computer that sat on a table facing herself with an expansive white board in the background. In the virtual environment, Ms. B did her best to recreate the in-person learning experiences she had fine-tuned over years of teaching this course. She maintained a similar lesson structure, but the interactions between community members were necessarily different. Being part of Ms. B's in-person calculus class was not the same as being part of her virtual calculus class. The teacher was successful in accomplishing quite a lot in terms of creating meaningful learning experiences for students. However, participants talked more about challenges related to interacting with one another online than they did about successes. Specifically, they talked about how much harder it was to communicate effectively and efficiently, to build teacher-student and student-student relationships, and to access one-on-one support from Ms. B.

Effective and Efficient Communication

One common observation about distance learning was how difficult it was to communicate with other class members during lessons, both in terms of navigating online logistics and reading interpersonal cues. During whole-class discussions, the process of having to unmute oneself and time contributions appropriately to not overlap with someone else in a class of 30 people is tricky to say the least. That is on top of negotiating students' varying comfort levels with whole-class participation in general. Sarah, a student in Ms. B's Period 2 class, shared, "last year it was a lot easier in class to ask questions and participate. It's been really difficult over zoom for me personally. So, I guess I would participate if it felt like it was necessary for myself, but I wouldn't go out of my way." Another student, Sophia, shared an observation about Caleb, one of her talkative male classmates. She shared, "he's saying the answer because he knows the answer, but I'm sure half the class knows it but don't want to say it out loud or don't have the time because he says it too fast." Sophia acknowledged that other students had thoughts they were not sharing, sometimes because they did not want to but other times because it was just too hard to do. The researcher noticed this phenomenon as well. During a lesson debrief session, the following conversation took place between the researcher and teacher.

> **Researcher**: I noticed two times when Emily missed out on opportunities because of Caleb, and I think there may have been many. But there was one time when they both unmuted, and Emily unmuted slightly before Caleb, but he started talking. And I was like, 'ok, that was a missed opportunity for her.' There was another time because of her volume she was a lot quieter that day, because of her setup. They were talking at the same time, but nobody heard Emily. So, I found myself wondering about her experience.
> **Teacher**: I didn't even notice that.

> **Researcher**: Right, there's no way for you to notice that. There are so many other things that you're paying attention to.

Managing online whole-class participation effectively requires a teacher to have eyes on many different parts of the virtual room. Because this class had an extra set of the eyes (the researcher's), more aspects of student participation were noticed. But even after problematic patterns were noticed, it was still hard to address them, given the limits of the virtual environment.

Participants also talked about how hard it was to communicate while working in Zoom breakout rooms on small-group tasks. One of the expectations in this class was for students to work in small groups to investigate and solve challenging math problems together as teams. Students tried hard to work together, and they were successful in many ways. However, there was consensus among classroom participants that their workaround communication methods were much less efficient than in-person exchanges. Alma, a student in the Period 2 class, shared her experience.

> One of the negatives is definitely the communication, like people talking and interacting with each other. It's really hard on zoom. And it's like if we were in person, you could easily see someone's work and then they can just show you and then you can be like, 'Oh, I understand it now,' but sometimes I have to ask people, 'Wait, what did you do?' like three times because I didn't have anything down or I just couldn't envision what they were saying.

Alma's reflection highlights the need for students to be even more persistent when asking for help or clarification when working over Zoom. It is often hard for students to admit they are confused, especially in an advanced math class, but as this student pointed out, the need to ask for clarification was even greater in Zoom because it was that much harder for her to follow what the other person was saying. One of Alma's classmates, Alison, talked about how the lack of body language increased the inefficiency of communication and wasted time.

> You don't have body language to work with. You don't have any physical stuff to work with. You can tell what people are thinking more in-person. We waste a lot of time, like, 'Oh, should you share your screen?' 'I could open Desmos.' 'Do you want me to open Desmos?' Otherwise, someone would just be on their computer doing it and you'd see and you'd just turn the computer around and look at it. Now, someone's like, 'I've been doing this thing,' but you have no idea because you can't see it. So, I think Zoom is definitely much harder to learn on, because it's slower. There are some roadblocks. It's a lot more awkward.

Alison pointed out the inefficiency of relatively basic communications, such as what a student is writing or what computer program a student is using. During in-person classes, students would simply turn and look at a person's paper or at their computer; little to no talk would be needed to determine basic information about what a student was doing.

After giving a group quiz, the teacher and researcher reflected on the ideas shared by these students. The guidelines of the group quiz required groups of four students to work together to solve several challenging problems. The problems were more challenging than what the students would see on their individual tests several days later. All students were expected to write solutions to all problems on their individual papers. Then, toward the end of the class period, the teacher would collect one randomly selected problem from each student in the group. For example, problem 1 would be collected from Student D, problem 2 from Student A, and so on. The four students in the group would all get the same combined score from all problems. This grading structure made it even more important for students to communicate well since their grade depended on what their groupmates had written on their papers. Fortunately for the students, the teacher graded on a curve. During the group quiz lesson debrief session the teacher and researcher had the following exchange.

> **Teacher:** They would have finished [the group quiz] in-person. Because of the lack of skills and because of the incredible inefficiency of this collaboration thing where you're like, 'look, look at this (holds up paper to computer camera).' It's just impossible.
> **Researcher:** That was one of the observations that I made. The group that I was with was very collaborative and tried very hard, but the obstacles and the hurdles were much greater than in person. They did a lot of holding up to the screen and then there was Katie who made a comment about, 'So wait, I'm getting confused because of the flip of the video and I'm dyslexic so when I see all of this flipping - I'm so messed up.' They were all being very open about the struggles, and they continued to push through and really try. Some read it out loud, some showed it. But it just made me wish that they could lean over and look, see, and point.

In this group quiz situation and in daily collaborative classwork, students worked hard to overcome the communication obstacles presented by Zoom, but still their learning interactions were not as efficient as working together in person.

Classroom participants also talked about how the information they had about the people with whom they were interacting was incomplete, which may have made it harder for them to get to know each other and build strong connections. Of all the participants, the teacher talked about this issue most often. Sometimes her observations were about distance learning in general, and other times her observations were about interactions in breakout rooms specifically. Ms. B talked about how hard it was to manage the breakout rooms because when she was checking in with one group, she had no access to what all the other groups were doing. When debriefing lessons, she would make comments periodically like, "I never know what's going on in these breakout rooms." During in-person instruction, a teacher can keep a pulse on the rest of the room while checking in with groups individually, but that was not possible in Zoom. In addition to knowing nothing about the groups she *was not* with, the teacher talked about how little she knew about the group she *was* with. She shared the following reflection about a group she was observing.

> The silences were lifeless, but I'm sure there was stuff going on in there … I really, really wish I could have seen what they were doing when they were staring straight ahead. What

> were they drawing? What was happening? What were they thinking? I really miss that. Because what happens is that the silences contain no information. And that's just a huge loss.

Ms. B believed her students were thinking and working, but she could not tell what they were thinking about or what they were working on. There was no way for her to peek over the shoulder of a student to see what they were writing. She wanted more information to assess students' understanding than Zoom could give her.

The teacher also acknowledged that student to student interactions were more difficult due to the incomplete information students had about each other. She talked about how hard it was for students to read social cues in "this clueless environment" (Zoom).

> I think that students are very, very sensitive to the chemistry of each other. As teenagers especially, you're always feeling and responding. And part of that is a very generous thing where you don't want to tread on people's toes. Like if somebody is lost, you don't want to just blah, blah, blah explain the whole thing. And they can't pick up those cues in this clueless environment. And so, they're sort of lost because they want to participate socially in a way that's - you know, doesn't make them look like an idiot. There's all these different things that especially teenagers are aware of. Do you look like an idiot? Do you look like a jerk? Are you treading on your classmates who are confused? Are you asking a question that's so dumb people think you're foolish? And they don't have those clues in their group because of this digital interface has almost no clues at all. And so, they're just lost. It's almost like they're waiting for the clues to come, but they never arrive.

Communication about mathematical thinking was difficult, but so too was interpersonal communication. Connecting with community members in respectful and caring ways requires reading social cues from other people, something the teacher felt was not possible through Zoom. After the school year ended, the teacher reflected on her overall distance learning experience by talking about how much was lost due to missing information.

> The entire year, the experience of online learning, I had this sort of terror of all this information about students that was missing in this world of not being able to see them and feel them and smell them and touch them. And I think it is probably interesting to measure and account for that in some way, but I don't wanna - I don't even want to think about it. But it's an enormous, enormous loss… it's kind of like, if you take yourself back to thinking about a very small child and how important it is to be holding them. And there's a continuum. You don't hold your seniors, but they're still - it's part of that transition and you need to be in their physical presence, and I don't think people tend to even measure that or notice it. It has to be tremendously important.

Ms. B acknowledged what had been lost in terms of interpersonal communication and connections, but she did not want to dwell on it. The school year was over, and she seemed ready to put it behind her.

Relationship-Building

Distance learning also presented obstacles that made it difficult for community members to get to know each other and to build productive and supportive relationships, including student-teacher and student-student relationships. Multiple times the teacher talked about knowing some students better than other students, and she always attributed the difference to whether she knew the students before the pandemic or not. While reflecting on a group's participation in a breakout room, the teacher stated, "I don't know them as well because I haven't taught them in previous years. So, remember, the other two I know in person." Ms. B explained that she had taught two out of the four students in this particular group in previous school years, but she emphasized that knowing them "in person" was what made a difference in her ability to interpret the students' actions and mathematical understandings. Distance learning had not allowed Ms. B to get to know her students in the ways she had in previous years. Guadalupe[5], one of the two students Ms. B knew in person, shared a similar appreciation for the relationship she and her teacher had built prior to distance learning.

> I kind of built a connection at the end of the year with Ms. B last year. And I knew that I wanted to be in her class again this year just because it was going to be such a different environment online and everything. And I kind of wanted to end my high school career having some familiarity with the teacher.

Guadalupe pointed out that it was especially important to have an already established connection with her teacher going into her senior year online.

There were other times the teacher talked about specific students about whom she was concerned and wanted to know more. She expressed disappointment in not knowing these students as math learners and not being able to interpret their actions. She shared the following about two students during a formal interview in February, midway through the school year, while reflecting on the effects of distance learning. Both students were male students of color who rarely shared verbal contributions during class.

> I'd love to hear from Omar. I don't really know what he can do. A lot of his struggles are with internet. He occasionally writes a note about how much he loves the class. I don't think he's trying to butter me up. I think he's having a good time, but I'm not sure. I don't think he needs to butter me up, but I feel like he's- like we've lost him a bit.

> I'm annoyed with Diego because his attendance is so bad. I don't have a strong sense of him as a mathematician. He doesn't always turn in his HW. I don't know why. I still feel like I don't know him which is a shame.

The teacher's words "we've lost him a bit" and "which is a shame" indicate a sense of sadness and defeat. Ms. B wanted to know more about these two students, but it seemed she felt there was nothing she could do about it given the circumstances.

[5] Guadalupe is one of the four focal students featured in Chapter 6.

Students also shared observations about not knowing other students well due to the lack of interaction with peers. Sophia shared, “I don’t talk to a lot of people in the class, so I’m always curious, like, if they’re even alive. They’re just sitting in the back. It’s like school but not. It’s so weird.” Another student, Andy, talked about how his comfort in class was connected to getting to know his classmates.

> Freshmen year I was most active because I was most comfortable with my class. Sophomore year I was a little less but then overtime I got more comfortable, so I was more active. And then same with junior year, a little less at the start and then more. And I feel like a difficulty with online school is I can’t really get close to my classmates like that, and I can’t get comfortable in a way, so I feel like of all the years this one would be the least interactive.

Sophia seemed relatively unaffected by the lack of peer interaction; she was curious about other students but did not say her own learning was hindered in any way. Andy, however, indicated that not getting close to classmates meant he did not feel as comfortable, which resulted in less active participation on his part. Both students pointed out how this year was not like other years in school because they did not know their classmates as well as in years past.

Ms. B talked about how the relationships students formed in their small groups were not the same this year as in years past. During in-person instruction, the teacher would have students work together in the same group for weeks at a time, but due to the logistics of creating breakout rooms in Zoom, the teacher had resorted to assigning students to a different random group every day. After watching one of the researcher’s videos of four students working together during the teacher’s Period 1 class, the teacher reflected on what was lost during distance learning.

> I wish that those four could sit at a table for a whole month. You know, usually the teams don't move around. This new teams every day thing is a feature of zoom. Those four could do something. They could do some pretty impressive work, over a month, sitting together. That would be something to see. I guess I'm talking about the things- I see some loss there.

Although this comment was preceded by positive comments made by the teacher about how much the students in this group had accomplished, she felt the students could have done even more if they had the opportunity to work together in person over a longer period of time. The teacher referenced this same idea during a lesson debrief session when sharing her observations about a well-functioning group she had just observed. This group of four students often worked together on homework outside of class time.

> It was completely different than all the other [breakout] rooms… you recreated their study group. They already know each other. They’re not strangers. They have a relationship, and they can survive the zoom world and that’s what it’s usually like at the end of a unit. This is your old team, like your family. You’re sitting there teasing each other, saying, ‘remember, you got it before!’ The rest of the rooms were, ummm, (shakes head).

This group interacted successfully from the teacher's perspective; they were able to overcome distance learning obstacles because of their pre-existing relationships. Typically, these comfortable, supportive relationships formed over time through in-person interactions in Ms. B's classes, but this year that was not the case.

Access to One-on-One Teacher Support

Some students also shared challenges related seeking out one-on-one support from Ms. B. In addition to the optional Wednesday "help" sessions specific to each class period, the teacher offered office hours through Zoom on Thursday afternoons, open to all her students in all her classes. The student teacher also offered his own office hours on Monday afternoons through Zoom. Despite these offerings, some students still felt it was difficult to get the support from their teacher that they wanted. Mia shared the following.

> Sometimes if I would talk to my teachers in past years, they would one-on-one help me get through certain things, and I get that now it's a lot harder to do that cuz of covid and it's online and stuff like that. So, I feel like that definitely does not help that I can't - I don't have that one-on-one, cuz like sophomore year, I would go after class, during lunch periods, after school and I'd get that one-on-one time and they'd help me. Ok, you need to do this and this and this in order to be at a better grade, be able to understand better. So, I don't know. It's a little bit weird this year."

There were indeed fewer opportunities for students to interact with their teacher one-on-one. Email was always an option, and the teacher encouraged students to write questions to her on their homework papers (which many students took her up on), but no matter how available the teacher made herself, the options for support were still limited compared to previous years. In addition, the helping interactions themselves were different. Having a teacher support a student through Zoom is not the same as the two of them being together physically, looking at a single paper between them or standing at a whiteboard together.

Heightened Anxiety and Motivational Struggles

Some students talked about barriers to participation related to their emotional and mental health. They talked openly about how distance learning made their anxieties and motivational struggles worse than they would have been otherwise. Students could not always put into words why that was, but they certainly felt the added weight and stress the distance learning environment imposed on them. In addition, Ms. B mentioned on occasion reaching out to individual students to check on their well-being. There were a number of students whom she helped connect with mental health resources through the school district, more than she had in years past. To uphold student privacy, the details of these interactions were not shared with me, nor were they included in this study beyond this acknowledgement of their existence. The observations and reflections shared in this section came directly from students during one-on-one interviews. Due to the intense personal nature of these obstacles, the assumption is the few stories shared with the researcher were only the tip of the iceberg.

The stories students shared supplemented the teacher's necessarily vague accounts of students' pandemic-related mental health issues. One student, Alison, was open about her anxiety linked to the fear of having to speak in math classes.

> I actually really hate it. I really dislike talking in big classes, which I've worked on my entire life, and I've gotten better, but I get so, so stressed out that I won't be able to focus and then I learn less if I know she might call on me. Then it's just - I don't take in information. It's worse, it's so much worse on Zoom. I don't know why actually. I don't really understand. It's just different. I feel like it should be easier honestly, cuz you're like in your room. You're not actually with people, but somehow it just makes it so much worse. I think it's maybe just circumstance, like this year has just been the worst. In my math class last year, he would call on us randomly, I mean I didn't like it, but I could do it. I could handle it. But this year is just bad. But I also stopped caring as much recently. Just like, she calls on me and if I don't know the answer, I don't know the answer. Oh well. It's anxiety. Zoom has given me so much more anxiety than I've ever had.

Alison described her anxiety as limiting her participation and learning. She could not identify exactly what it was about Zoom that heightened her anxiety, but she was clear her anxiety was worse this year due to distance learning. Another student, Diego, attributed his lack of participation and poor attendance to a lack of self-discipline. This is the same student the teacher mentioned being frustrated with because "his attendance [was] so bad."

> Oh man, I wish I could - I feel like - I know I can do better. I feel like I should be doing better, so I've been kind of down about it, but with the whole online learning thing - I wish - The big thing is actually my sleep schedule. I'm late pretty often, and I wish I kind of interacted more, but uh, and I want to go to Wednesday classes, but I just can't find it within me. I just gotta, you know, I gotta get more discipline or something. Just gotta give my all into it, but yeah. I feel like I could definitely be doing better. I'm under-performing if anything. Yeah, I don't know. It's pretty much just not good … I mean, yeah, I'd be late to first period a couple times too back when real school was happening, but now it's definitely like ten times worse. If my alarm clock cold slap me or something, it would be great.

Diego used negative self-talk to explain his situation, sharing that he couldn't find it within himself to do better. Both Ms. B and Diego wanted his participation to improve but were unable to make it happen. They both blamed distance learning. Just like the first student, this student talked about how he had similar struggles before this year, but online learning made them worse. These two students' struggles were different in nature, but they resulted in a similar lack of readiness to participate in the calculus class.

There were other students who managed to participate on a semi-regular basis but who also acknowledged that participating in this virtual environment was harder than it was in-person, and it was harder for some people than others. Alma shared her firsthand experience of participating online versus in person.

> Because of online learning, distance learning, and everything, because of that, it's really hard to participate, even more than you know in person where everything is normal. Yeah, it's just - it's hard to participate and it's hard to have - what's the word - like it's

> hard to want to put in effort for the class or it's hard to be actively interested in the class because - I don't know. Online learning just isn't the same as seeing people in-person. I understand why a bunch of people - it's hard for a bunch of people to participate. I mean it's hard for me … Yeah, motivation, that's the word.

Sophia shared a similar observation, but took more of an outsider's perspective, commenting on her friends and not herself.

> I know that there's a lot of people, like I have a lot of friends who just turn off their cameras and don't participate at all. And talking with them, they just really don't want to have anything to do with the class at all. So, you know, like there's just a level of willingness and you kind of have to be on the same wavelength to be able to participate.

Alma described her struggles as related to "motivation" and Sophia attributed students' struggles to "willingness." Both students were talking about how an internal desire to participate was lacking. The four student examples presented in this section are just a few firsthand accounts of how distance learning heightened students' pre-existing anxieties and motivational struggles.

Part 2: Barriers to Participation - Gender and/or Race

This part addresses participants' observations about barriers to participation related to gender and/or race. Overall gender was talked about more often than race, and sometimes participants talked about gender and race together, for example when speaking about participation patterns associated with intersectional groups of students (e.g., "it's different with PoC girls", "White boys feel the most confident"). References to race and/or gender made by the teacher or student teacher occurred most often during lesson debrief sessions as they reflected together on students' participation during previous class periods, whereas students' references to race and gender typically occurred during one-on-one interviews with the researcher. Most interview questions were open-ended and sometimes resulted in participants bringing up race or gender on their own. For example, toward the end of an interview with Yonas[6] (Black male student) the researcher asked, "Is there anything else that you can think of related to your participation or your experiences in this class that you feel would be helpful for me to know?" Yonas responded, "Well, ok. I think - This is - ok, yeah. Um. This is not just this class. I'm back-tracking, but I was in the honors math classes the other years … and I was the only Black kid in the class. And it just felt like there's a big spotlight on me."

Three patterns in participants' observations about gender and/or race were identified through analysis. First, participants talked about representation issues in STEM related to gender and race in ways that were consistent with previous studies (e.g., Egalite et al., 2015; Leyva et al., 2021). Second, participants talked about how issues related to gender and race impacted their personal decisions regarding how and when to participate in class. Third, participants spoke often about problematic patterns in classroom participation related to gender, but rarely talked about problematic patterns related to race. Table 4b includes a summary of participants' gender and/or race observations made during formal interviews, informal documented conversations, lessons, and lesson debrief sessions. Observations are grouped by the three aforementioned patterns.

[6] Yonas is one of the four focal students featured in Chapter 6.

Table 4b. Patterns in Participants' Gender and/or Race Observations

Pattern	Observations
Representation Issues Related to Gender and/or Race	Imbalanced enrollment in STEM courses
	Racial segregation in EHS's learning community system
	Too few female teachers and teachers of color in mathematics
Gendered and/or Racialized Narratives about Participation and Mathematics	Feelings of not belonging in advanced mathematics classes
	Boys are smarter at math than girls
	Wednesday help sessions are for girls
Classroom Participation Issues Related Mostly to Gender	(White) male dominance
	Girls not speaking up

Participants drew on their firsthand experiences to make sense of the roles gender and race played in the context of this calculus class. Although both gender and race were talked about, there seemed to be a heightened awareness by the teacher, student teacher, and students about barriers to participation related to gender and much less attention paid to race (or at least less time spent talking about race). Each of the patterns in the table is explored in detail in the following three sections, using direct quotations from the teacher, student teacher, researcher, and numerous students to convey participants' perspectives on race and gender at Evergreen High School.

Underrepresentation by Gender and/or Race

For decades, research has documented the lack of females and people of color enrolling in STEM courses, pursuing STEM degrees, and working in STEM fields (e.g., Cheryan et al., 2017). Not only is poor representation problematic from an outcome standpoint (communities miss out on the unrealized potential of valuable contributors), but it is also problematic from an interactional process standpoint. When classroom communities are less diverse, the few non-male, non-White students who are there have a harder time engaging in meaningful interactions with community members. These students tend to feel less comfortable contributing to discussions and tend to have a harder time building strong relationships with other students and their teachers. Classroom participants reflected on how issues of representation shaped their experiences at Evergreen High School. In that sense the findings presented in this section are not surprising; similar experiences are well-documented in literature (e.g., Egalite et al., 2015; Leyva et al., 2021). However, hearing how Ms. B, Mr. K, and some students connected the representation of females and people of color at EHS to their own classroom experiences provides helpful context for understanding broader participation patterns in their class.

Enrollment in STEM Courses

Participants talked about the lack of female students and students of color in advanced mathematics courses and other STEM courses (i.e., economics and honors physics) at Evergreen High School. They talked about how imbalanced enrollment made it harder for students in minority groups to participate fully. Gender was always part of conversations about representation; sometimes participants talked about only gender, other times they talked about both gender and race. Many of students' assessments of diversity in the AB Calculus course were relative to what they had experienced in other classes at EHS.

On multiple occasions Guadalupe (female Mexican student) talked about how issues of representation affected her comfort with participating. One example occurred after she watched a video of herself working in a small group with three male students[7]. She shared the following.

> There's also a lot of guys which is normally the case. And so, you don't really like - I don't know - I talk to Mia about it. Like, we normally don't talk in breakout rooms unless we're asked to. Part of it is because we don't really feel like we have the space to. I don't know, when it's more girls than guys, we feel more comfortable.

Another example is seen in Guadalupe's response to the researcher's question, "If you could change one thing about this class, what would it be?"

> I really wish that it was more diverse … and it's something that starts your freshman year … and it just kind of has a snowball effect by the end. Like, we're graduating, and some White students still feel like they're on top of the world and we're left where we started.

She spoke about the school not supporting students like her, presumably students of color, to make progress and join the White students "on top of the world."

Mia (female White student) compared her experiences in the calculus class to her experiences the previous year in an economics class, which was also male dominated.

> I had an econ class last year that had 22 guys and 8 girls, and this is very similar to that, except maybe a little bit less bad because Ms. B is actually doing something about it. My teacher last year didn't really know what to do about it. So, it's not my first time dealing with people like this … My econ class last year really made me tougher. I was like, 'No, I actually get this. Don't let other people's comments do anything to you' … I think everything that happened in that class kind of prepared me. If this was to happen, I knew exactly what to do, because I didn't wait half a year until I bought it up to my teacher. I waited five days and brought it up to my teacher. I was in a better head space, so it didn't really affect me. It just pissed me off.

Mia gave Ms. B credit for successfully making her experience "a little bit less bad." She also gave herself credit for being "tougher" and for speaking up sooner. Her concerns with male dominance went beyond the enrollment numbers. Mia was also voicing concerns about the degrading comments made by male students in her classes.

In contrast to other assessments, Yonas (male Black student) was happy with the level of diversity in their AB Calculus class, but his frame of reference was different from Guadalupe's and Mia's.

> I was in the honors math classes the other years, and every year the amount of minorities just like got cut in half, every single year. And junior year, I think there were three Asian kids and then the rest were White, and I was the only Black kid in the class …

[7] This small-group video is the focus of Chapter 6.

> having a way more diverse class [this year] has boosted my morale, made me feel like there's a better community … And yeah, there's still some parts, like the male to female ratio. I don't even - there's like 6 girls in that class or something. So yeah, that's - I can't speak on that. And I feel like that is probably the same way that I felt before.

Yonas spoke explicitly about how his current calculus class was more racially diverse relative to his previous honors class, while also acknowledging the gender imbalance that still existed. He drew on his own experiences to imagine what female students may be feeling. Yonas's spirits were boosted this year; Guadalupe's and Mia's were not. Their perspectives were different and so too were the barriers to participation they faced.

Rebecca (female White student) also talked about how the calculus class was relatively more diverse than her other classes. She spoke about gender imbalance in her honors physics class.

> [Gender] is definitely on my radar. I'm interested in that too. And not just in this class, but in classes in general. And I know going into this class - typically with some classes at Evergreen, there's an underrepresentation of females in it, but I think in this class it's more balanced. But for example, I take physics too, and there's like 5 girls in that class only, which is really significant. And no one really talks, but when someone does talk it's not a girl.

Rebecca seemed thankful that the calculus class was not nearly as bad as her physics class. Ms. B also reflected on this same physics course. She talked about the participation of some outspoken boys who were enrolled in both her calculus course and the physics course. There was some overlap in content between the two courses, and according to Ms. B, the physics class moved quicker and taught shortcuts without connecting to deeper conceptual understanding.

> What's kind of concerning to me is this tension between people who are really smart and are mostly in this honors physics class who already know everything, which they don't - and then the normal experience of being expected to understand the chain rule after doing no math for 5 weeks and people feeling like complete idiots and then not being able to say it, including the people in physics who don't really understand it very well. That physics class I know, because my two sons were in it, is typically very, very disproportionately male.

Ms. B talked about how the physics students (mostly male) were perceived to be "really smart" because they had already supposedly learned the content, which made it harder for them to ask for help when they had questions. And the non-physics students were having a hard time because they thought everyone else understood the ideas better than they did. Students also had five weeks without any math due to the distance learning schedule. It was not a good situation for any of the students, but presumably the female students had it worse because it was mostly male students who were holding onto the title (even if it was tenuous) of being "really smart."

Racial Segregation in Evergreen High School's Learning Community System

Some students pointed out that the issue of racial representation in their calculus class extended beyond this one course. They connected the lack of racial diversity in their AB Calculus class to racial segregation in Evergreen High School's learning community system. Almost all students enrolled in Ms. B's AB Calculus course were part of one of the two large learning communities. Since those two large learning communities were predominantly White, so too were the students in Ms. B's AB Calculus course. In interviews with students, the researcher did not ask students about their experiences in their learning communities, but several students shared reflections unprompted. All reflections, except for one, came from students of color in one of the large (mostly White) learning communities. Hosein[8] (male student of color) was one of the only students in the AB Calculus class who was part of a *small* learning community. His learning community reflections offer a different perspective and point to work that was being done in his small learning community to acknowledge and address issues related to gender and race.

Sarah (female student of color) connected being one of the few non-White students in her learning community to her participation challenges.

> I'm not White and I've been in [large learning community #1] all of Evergreen High School. Those classrooms are mostly white, and so I've had a few years to adjust to that. It's definitely hard to participate in those settings.

Although Sarah had adjusted, it was still difficult for her to participate. Guadalupe (female student of color) was another student who talked explicitly about the Whiteness of her large learning community. She acknowledged the Whiteness of both the students and the teachers.

> I'm in [large learning community #2]. [Large learning community #2] is a little bit more diverse than [large learning community #1], but it's also still predominantly White. I think I've only had one non-White teacher, my whole time at Evergreen High … I feel like the most diverse my classes have ever gotten is 10 POC in one class that I've been in and that's like uhh, what? But I mean, the thing is, going into the real-world, I know that it's going to be like that, but that's why it's so important that we learn the skill set to empower us.

Guadalupe also connected the Whiteness of her learning community to affluence in this next reflection. She again suggested the school should be doing more to support students of color.

> All of the [large learning community #1] White kids are taking really advanced math … The [learning communities] are very segregated. And it's not like they intentionally do that, but they just have a certain rep. It's like [large learning community #1], they're the ones that can afford the private tutoring when the classes get really hard and whatever, right? And they have that extra support that a lot of us don't have … There should be more tutoring that the school provides, really, just access to resources … I feel like they put a lot of pressure on the students to bring up these issues. And I get it because it's a really

[8] Hosein is one of the focal students featured in Chapter 6.

> White - it's predominantly white teachers. But it should not be all up to the students. And that's why we're getting nowhere because they leave it up to us, but we don't even feel comfortable enough talking to administration.

Guadalupe felt the White kids in the large learning community had better access to resources than other students like herself, and the school should take responsibility for initiating efforts to fix these problems instead of leaving it up to students.

Hosein pointed out that the majority of students in the AB Calculus course came from the two large learning communities and blamed the small learning community math teachers for this outcome.

> Nearly all of the kids in the calc AB class come from those big schools because the math teachers in [small learning community] don't give enough instruction to prepare, so we have like the small-school calc. I was the only [small learning community] student in the honors math program.

Hosein also acknowledged positive aspects of the teachers' work in his small learning community, which was focused on addressing issues of diversity and participation in classrooms. Hosein's small learning community was more racially diverse than the large learning communities, but it was still "fairly White."

> [Teachers in small learning community] have come together with the goal of educating students and creating more White allies within [small learning community], cuz [small learning community] is fairly White … The teachers made a list of students in ranking order of who should speak first to promote more equality. And it started just in that group, but it has spread throughout all of [small learning community] now, so all the [small learning community] classes that I'm in are using that model as a guide. It's actually really been helpful in boosting participation by PoC students among others … At the beginning of the year when we were going over classroom norms, there was just a slide with like, 'this is something new that we're doing. We're trying this out to try to boost participation of underrepresented students.' And it literally had a list of like 5 POC people, queer people, women, like all the way down, which is really cool. And I've actually seen it helping a lot.

In contrast, Guadalupe, a member of a large learning community, felt the school administration was not doing enough to empower students of color at Evergreen High School, Hosein's experience in his small learning community was different. Hosein felt the efforts to "boost participation of underrepresented students" were successful within his small learning community. It is worth noting that each of EHS's learning communities had its own designated focus. Hosein's learning community was designed with an explicit focus on social justice. The other learning communities were not. The AB Calculus course was not part of Hosein's small learning community and therefore did not implement the participation protocol he described. It is not known if Ms. B was aware of this model being used in the small learning community classes.

Female Teachers and Teachers of Color in Mathematics

Most of the observations about teacher representation by gender and/or race were made by the teacher and student teacher. Their conversations were focused on the student teacher's race (Mexican) and the teacher's gender (female). Guadalupe was the only student to mention teacher gender or race; several times she referred to Ms. B being White and the general Whiteness of the teachers at EHS. Comments made by Ms. B, Mr. K, and Guadalupe highlight the importance of building strong teacher-student relationships, supported by common experiences and understandings often related to gender and/or race.

During a lesson debrief session, Ms. B complimented Mr. K by saying, "You will be a model for your Mexican-American students." She went on to ask him, "Did you have any Mexican-American teachers, math teachers?"

> No. I had mainly Caucasian math teachers. There was one teacher. He grew up in Oakland. He's Black and he was who kind of pushed me to want to be a teacher. I had him for honors pre-calc and he kicked our butts and for AP calc and he kicked our butts too.

Although Mr. K did not have any Mexican math teachers, he made a connection with one teacher of color, a Black man who held high expectations for him in his advanced mathematics classes. Mr. K's relationship with this teacher influenced his decision to pursue teaching as a career. Mr. K also talked about understanding the experiences of Ms. B's students from Mexican families.

> The way I relate to these students is that I know what they're going through at home in a way. When I spoke to Carlos's parents, they seemed to be really supportive, but when he's in the zoom classes, you can't hear anything he says because all we hear is washing, cooking, talking, right? So, I feel like I went through the same thing in that my parents respected me going to school, but that they didn't 100% support me to make sure I had space, quietness.

Both Ms. B and Mr. K positioned Mr. K's Mexican heritage as an asset which supported him in relating to his students in ways that other teachers may not have been able to do. Ms. B reflected on the genders of her former math teachers, but not their race.

> I've never had a female math teacher in my whole life. I didn't even realize until like a year ago. I thought through every single - never had one. And I think it kind of made me a little bit fierce. It gave me some skills.

The teacher's take was that her lack of female math teachers made her strong and forced her to adapt. She also noted that in her college math classes she was "usually the only girl in the room," adding that "the math building didn't even have a women's bathroom." Mr. K followed up by connecting Ms. B's gender to students' senses of themselves as competent learners and doers of mathematics.

> I think that you being a woman can empower some of the girls in the class to think, 'Oh, I can definitely be a mathematician' or 'I definitely have what it takes to -' or not even have

> what it takes but just that 'I can learn material simply because she did it, so why can't I do it?'

Ms. B agreed, "Yeah, I also sort of have the right to kick them, which would be hard - I think it would be trickier for you. Maybe you just do it in your own style." Both the teacher and student teacher acknowledged that being the same gender mattered for Ms. B's relationships with her female students and being a person of color mattered for Mr. K and his students of color.

Guadalupe did not mention teacher gender, but she did make several references to the abundance of White teachers at Evergreen High School. Guadalupe shared, "I think I've only had one non-White teacher my whole time at Evergreen High." In general, Guadalupe did not feel the mostly White administrators and teachers at her school were doing enough to support students of color. However, Guadalupe positioned Ms. B as an exception to this rule. Despite Ms. B's Whiteness, Guadalupe felt supported by her.

> I talked to her about some race stuff within the classroom, and I understand it may be uncomfortable sometimes when it's like a White teacher, and I'm not White, you know. And I was like, first of all, the girls aren't talking, but there's way - there's like three girls of color in here. And we're really not talking at all. And we would like - I would like for you to advocate for us as our teacher because it shouldn't just be all up to us. And she took that really to heart and brought it into the classroom. So, that was nice.

Guadalupe explained that her relationship with Ms. B began the year prior when she had Ms. B as her junior year math teacher.

> I also kind of built a connection at the end of the year with Ms. B last year. And I knew that I wanted to be in her class again this year just because it was going to be such a different environment online and everything. And I kind of wanted to end my high school career having some familiarity with the teacher … At the beginning, it was a little bit frustrating having Ms. B, just cuz she expects a lot out of us. And sometimes the concepts started to get a lot more difficult, and I wouldn't understand it always. But at the end, I actually valued how much pressure she put on me to learn.

Guadalupe talked about how Ms. B had high expectations for her and how they had built a strong relationship. Ms. B linked her high expectations for female students to the fact that she, too, was female and had "the right to kick them." Many students, both female and male, students of color and White, talked about feeling supported by Ms. B, just as Guadalupe did. None of the students mentioned Ms. B's gender, but presumably Ms. B's female identity supported her in relating to students in ways that other teachers may not have been able to do.

Gendered and Racialized Narratives about Participation and Mathematics

Research has shown students' opportunities to participate in rich mathematics can be stifled by sexist and racist narratives that position students from socially minoritized backgrounds in mathematics as less capable and less likely to succeed (Gresalfi & Hand, 2019; Martin, 2000). Female students and students of color in Ms. B's calculus class shared how oppressive narratives shaped their experiences at Evergreen High School, specifically with respect to their participation

in their calculus class. The three narratives presented here are ones about which multiple participants spoke. They pertained to feelings of not belonging in advanced math classes, the myth that boys are smarter at math than girls, and the claim that Wednesday help sessions were for girls. The first two narratives are commonly experienced in STEM classes across various classroom contexts, whereas the third narrative, about Wednesday help sessions, was specific to the context of this distance learning calculus course. All observations about the first two narratives came from students. The teacher and student teacher talked only about the narrative related to Wednesday help sessions.

Just as with issues of representation, the existence of gendered and racialized narratives about belonging and smartness in mathematics is not surprising or new (e.g., Gholson & Martin, 2014; Rainey et al., 2018; Sengupta-Irving & Vossoughi, 2019). However, hearing students' first-hand accounts of how these narratives impacted their decisions about when and how to participate provides another layer of complexity to classroom interactions and provides helpful context for understanding broader participation patterns in this class. In addition, the seemingly unique narrative about Wednesday sessions provides an interesting example of how a new narrative can form in a given context. The three narratives presented in this section are overlapping and deeply connected. Separating them into three sub-sections of findings is done for organization purposes but does not imply independence from one another.

Who Belongs in Advanced Mathematics Classes

Yonas, Sarah, Alma, and Guadalupe all talked about feelings of not belonging in their math classes. All four students were students of color. Three were also female; Yonas was the only male. The students' backgrounds were unique, yet their stories were remarkably similar, all including references to a common story that only certain people (White, male, affluent) belong in advanced mathematics courses. The students made explicit references to race and gender, in addition to implicit references to socioeconomic class (e.g., access to tutors) and family education (e.g., my family couldn't help me). They talked about how they felt different from their peers in class. They talked about how people tended to have low expectations of them and were surprised when they knew something. They talked about how they felt they needed to prove themselves and were afraid of being wrong.

During a one-on-one interview with the researcher, Yonas brought up the topic of race after being asked if there was anything else he wanted to share about his participation or experiences in the calculus class. He offered the following.

> Well, ok. I think - This is - ok, yeah. Um. This is not just this class. I'm back-tracking, but I was in the honors math classes the other years, and every year the amount of minorities just like got cut in half, every single year. And junior year, I think there were three Asian kids and then the rest were White, and I was the only Black kid in the class. And it just felt like there's a big spotlight on me … I would write my questions down on my notes and I went to my teacher, and I would ask her all my different questions. I didn't want to ask something stupid in front of the whole class and just have them go, 'Oh, wow. He shouldn't be in this class.'

Yonas hesitated before he began, seemingly trying to figure out if (or perhaps how) he should share. He reflected back to the years he spent in the honors math track, of which he was no

longer a part[9]. He opted out of that track midway through last year, switching from an "honors" Math 3 class to a "regular" Math 3 class.

Similar to Yonas, Sarah shared observations about feeling pressure to prove her worthiness of being in an advanced mathematics class. Sarah pointed both to gender and race as factors that played a role in her discomfort and anxiety related to classroom participation and fear of being wrong in front of peers. She shared the following.

> I'm not White and I've been in [large learning community #1] all of high school, and those classrooms are mostly white, and so I've had a few years to adjust to that. It's definitely hard to participate in those settings, but then also the dude aspect of it also plays a role. Yeah, I think both of them do … My parents stopped being able to help me with math pretty early on, so it was definitely hard at the beginning of high school to feel competent in my math or science skills. I guess it's a combination of all of those things … Scared to be wrong is my own anxiety. Also, something I've realized in some of these settings is it's felt like the expectations for me, on the outside, have not been very high, in like group projects and stuff like that. So, it's a weird kind of pressure - how do I establish that I know what's going on without - and I don't want to sound wrong because then it's like if I'm already down here (puts her hand low at chest level), I need to be correct to - (puts her hand up above her head). I need to work a little harder.

Sarah was not White, not a "dude," and her parents were unable to help her with math. She was not like most of her classmates. Although she claimed that she had "adjusted," her anxiety prevailed as her peers continued to position her as someone less capable. She felt she needed to work harder than her peers to prove that she belonged there.

Alma, another female student of color, was also enrolled in [large learning community #1] and talked about how white the students were and how she sometimes felt out of place.

> There's of course like you know, everyone has these prejudices and judgments already in their heads about people. Like, if you're a girl and doing math, sometimes it's like people judge you and they're like, 'Oh, you actually found out that answer? I didn't know.' Or like, 'Wow, you're really smart. I didn't know.' Or something like that. And you're just like, 'Ummm. Thanks?' A bunch of people in this class are from the honors math, I think. And also, I just found in my last math class there were a lot of [large learning community #1] kids and usually a lot of [large learning community #1] kids are White, so there's that difference too. There's not a lot of people of color in these hard AP calc classes. I don't know. Sometimes I can feel out of place. I haven't had that many problems with that, but sometimes you notice the small things.

Alma attributed people's surprise about her being smart to the fact that she was a girl. It is unclear if her race played a role in those judgements as well. She went on to acknowledge the

[9] Students in the honors math track at EHS took BC Calculus their senior year. Yonas and his classmates were taking AB Calculus. AB Calculus is considered "advanced" in many contexts, but it was not the most advanced course offered at EHS.

lack of students of color in advanced math classes and feelings of not belonging but said she had not had many problems with *that*, presumably referring to race.

The fourth student who shared feelings of not belonging was Guadalupe. She spoke extensively about how Evergreen High School compared to her middle school, how her fear of being wrong was connected to her peers thinking she was "less than," and how her access to resources was limited. She began by talking about transitioning to high school.

> I used to really like math when I was in middle school. I went to an under-served middle school. Not to toot my own horn, but I was like good at it, and we had such a great class. I would help out other students and stuff. I liked it because I understood it. Moving into Evergreen, it was a very, very different environment than what I was used to, and I kind of fell out. It wasn't just math, in other subjects too. I kinda fell out of touch with it because I would say something, and people would be surprised that I knew something … I was the only one out of all the freshmen - I got a 100% on my math final, but that's the type of thing that no one knew about me. The teacher just told me.

Guadalupe went from being a confident mathematics student eager to help her peers to a student who disconnected from mathematics. She was still very capable, although no one knew it except for her and her teacher. Guadalupe went on to explain how low expectations set by her peers led to a fear of being wrong.

> One of the things that really sets me back is that I'm super afraid of being wrong or sounding stupid. Like people kind of conceiving me as like I'm kinda less than because I don't understand an idea as fast. I would say it's the same for a lot of students, just in general. We're kind of scared of how others perceive us … At Evergreen High, there's this culture of like, if PoC students talk, they kind of look at us like, 'Oh, you're not stupid?' Like, 'Oh, you know what we know?' And I don't know - I don't want to bring it all about race, but it's kind of just my experience, and I know other students have experienced it too. So, it's like sometimes we - we refrain from talking because we don't want to bring up the wrong thing and have it be, like, what we're known for.

She attributed her peers' low expectations to her being a person of color and said these experiences are shared by other students of color as well. Just as with Yonas, Sarah, and Alma, Guadalupe felt pressure to prove she belonged and was fearful that making a mistake would solidify her position as someone less capable. Guadalupe also talked about how her parents were unable to offer her the same kinds of support that her peers received.

> Me, my brother, and my parents immigrated here when I was really little … through the whole college app process, my parents didn't really know where I was applying or what I was doing. They don't know how it works. I don't blame them … One of the things that makes it difficult is no one at home can help me. And sometimes you really just need someone to walk through the problem with you ... I don't think a lot of people obviously in my class understand the pressure and we all have different home circumstances. My parents don't understand how hard this class is, but they expect a certain grade at the

> end of the semester … And coming into Evergreen High, it's like people can afford private tutors just in general, like testing, SATs. When I was going through that whole thing, I was super lost. And I went back to my middle school teachers to ask for help. And they gave me these books and stuff, just like small things I didn't have or I wasn't used to over there that people had access to here. It kind of made me feel a little bit like imposter syndrome. Like, 'Oh, I don't know if I'm good enough to be here.'

Guadalupe felt it was harder for her to navigate being a high school student at EHS than it was for her peers, and her experiences made her question if she was really good enough to be there.

Boys are Smarter at Math than Girls

The idea that boys are smarter at math than girls dates back to … forever. While much progress has been made in debunking this myth, this narrative is still alive and well in advanced mathematics classrooms (e.g., Ernest et al., 2019; Robinson-Cimpian et al., 2014). These particular words were never uttered in the class, but female students felt the effects of this idea through daily classroom interactions. Students did not place full responsibility for the persistence of this narrative on male students, but rather, they blamed "society" and "the way they grew up." In the previous sub-section, Alma shared her experiences related to being a girl and doing math. She explained how she felt judged by other people and how people presumed she would not know things based on her gender. These observations were echoed by other female students in the calculus class. There was no mention of this narrative by male students in the class.

Emma was one of the few female students of color who spoke regularly in the calculus class during groupwork and whole-class discussions, yet she still felt pressure due to pre-conceived ideas about girls' mathematical abilities.

> I feel like I work better or feel more comfortable with female classmates just because like we already have something in common. I don't know. I always feel like there's this pre-conceived thing that guys are better at math than the girls or whatever. And I feel less pressure when I interact with the girls than interacting with the guys.

Despite Emma's discomfort, she still found ways to contribute to mathematical discussions. Alison's classroom participation was the opposite of Emma's; Alison spoke rarely during groupwork and only when absolutely necessary during whole-class discussions. She shared in great detail how this gendered narrative impacted her thinking and her interactions with peers.

> Then there's the whole girls and math. Oh god, I can't get over that either. I will think that the boys in my class know it better than I do, and they think they do too. And it's because we're just trained that way. They have this whole level of confidence. I've noticed it with my friend, Austen. He totally doesn't mean it. He'll think he knows the answer even when he doesn't. And I will always be sure that I don't know the answer even when I do. So, he always gets mad at me for being - he's like, 'Alison, you're smart. Just tell me what you have. It's ok if you're wrong.' … that's a common experience with girls, like having to prove yourself. I feel like there's less room to be wrong, like you can't really be wrong because if you're wrong then it just proves that girls are bad at math. Even though we all - even though all of us aren't really thinking that. It's just built in. It's society. We can't help it.

> It's so messed up too. Nobody really thinks that, I don't think, but we can't help it. I don't think I know anyone who really thinks girls are bad at math. I would be so surprised. I'd be like, 'Oh, you're terrible! What? You exist?' But it's definitely real.

It seems the effect of this narrative on Alison's participation was far greater than the effect on Emma's. Alison's comment, "I will always be sure that I don't know the answer even when I do" suggests that Alison has internalized this narrative in ways that Emma has not. There is also contradiction in what Alison said. On the one hand she said, "we're just trained that way," but she also said, "Nobody really thinks that." It seems she was pointing to deeply engrained (subconscious) sexist views, saying that on the surface, we don't think that way, but deep down we do. Alison's use of the word "we" in "we can't help it" placed responsibility for perpetuating this narrative on the shoulders of both male and female students.

Guadalupe's experience with this narrative was similar to Alison's. Guadalupe blamed students' upbringing for perpetuating the belief that boys are smarter than girls, and she talked about having to disprove this myth to other people and to herself.

> The reason why these boys are so condescending, it's because they genuinely - and it's not their fault, it's just the way that they grew up or whatever - it's like they think that you're not as smart as them and they don't even like, it's not like they say, 'Oh my God, she's not as smart as me.' It's like a natural thing for them, you know. So, it's like constantly trying to disprove something. Disprove it for them but prove that you're good enough for yourself. And so, first of all, as a woman, you need to really be on top of it. Then as a woman PoC, you're like double on top of it. And I'm not trying to devalue any experience of you as a woman. I'm just saying like, you have to be really, really on top of things and make sure that you don't lose any respect from anyone. And it plays into the smallest things like math class or clubs too. It's always like you don't want to be wrong, even though it's okay to be wrong. But if you're wrong, then they will remember that a lot more than they will remember when you were right.

Unlike the other students, Guadalupe referenced the compounding effects of being both female and a person of color, saying you need to be "double on top of it ... and make sure that you don't lose any respect from anyone."

Wednesday Help Sessions are for Girls

The third narrative that classroom participants talked about was one that came up in Ms. B's Period 1 calculus class regarding who was expected to attend the optional Wednesday help sessions. During a regular class breakout room discussion in January, Caleb (White male student) said something about how Wednesday sessions are for girls, eliciting surprise and confusion from other participants. This conversation was not recorded nor was it observed by the researcher. However, Mr. K was in the breakout room at the time and relayed the story to Ms. B, and eventually to the researcher as well. It remained unclear how widely accepted this narrative was. It may have been limited to only the one student who said it, or it could have been a commonly held belief across students, class periods, and courses. Data do not show anyone else propagating this message. Regardless, it is worth taking a closer look at the context surrounding this student's comment to make better sense of what was meant by it, how it might have come to be, and how

it may have affected students' opportunities to participate in the Period 1 calculus class and beyond.

Since the beginning of the school year, Ms. B had been holding optional sessions on Wednesdays during the regular class time (9-10 AM) for students in her Period 1 AB Calculus class to come ask questions and get help, per the school's distance learning policy. These Wednesday sessions were in addition to her Thursday afternoon office hours which were open to all her students in all her classes. Students were welcomed and invited to come to both Wednesday help sessions and Thursday office hours[10]. Attendance at the Wednesday sessions varied from week to week during the second semester, ranging from a minimum of three students (1/6/21, the first week back after winter break) to nine students (4/21/21, the session right before their final exam). Both male and female students attended every session, though females typically outnumbered males. The percent of female attendance ranged from 44% (4 female students out of 9 total) to 80% (4 female students out of 5 total) across the Wednesday sessions during the semester.

In one-on-on interviews, several female students said they appreciated the Wednesday sessions because the group of students was smaller. They felt more comfortable asking questions. Zoe (White female student) attended several of the help sessions during the semester and was asked how she decided which ones to attend.

> If I'm really confused and if I don't really feel confident with the math … The Wednesdays are actually really helpful because I think it's a lot easier to ask questions in a small group versus a big group. It's definitely sometimes scary when you're asking, like when you think, 'This is a dumb question. Everyone else knows this but me.' And it's a lot easier asking those types of questions in front of a smaller group of kids, even though they're not dumb questions.

Zoe, who earned an A in the AB Calculus course, was worried about her questions being perceived as "dumb," even though part of her knew they were not dumb questions. Guadalupe, who also earned an A in the class, talked about a similar fear of asking questions.

> I feel a lot more comfortable talking at Wednesday sessions just because it's such a small group and it's intended for us to get help. Sometimes I don't ask the smartest questions. I just say questions if I genuinely don't understand concepts. Like, what was it? Yesterday? Yeah, it was yesterday, right? When I was like - I didn't understand why we were writing a derivative in a certain way and it seemed like everyone else understood it, and I understood at the end, but I had to ask for her to repeat it multiple times in different ways. I feel a lot more comfortable going there and asking for help. Whereas with whole class, it's just - there's a lot of intimidating people that are moving really fast. And I don't want to slow them down just because I don't understand something.

[10] The researcher did not observe Thursday afternoon office hours, but she did observe Wednesday help sessions.

Both female students felt comfortable pursuing their questions during the Wednesday sessions because there were fewer students and the focus was on getting help, in contrast to the regular calculus lessons.

Given this context, Table 4c provides a timeline of the events leading up to, including, and following the statement by Caleb that Wednesday help sessions are for girls. Details of these events are provided after the table.

Table 4c. Timeline of Events Related to the "Wednesdays are for Girls" Comment

Date (2021)	Event
20-Jan	Eight students attended a Wednesday help session, 5 girls + 3 boys.
21-Jan	At the beginning of the regular calculus class, the teacher brought up the topic of Wednesday help sessions and asked one student (Mia) to share her experience attending past sessions. The teacher encouraged all students to attend future sessions.
21-Jan	During a breakout room discussion that same class, a male student (Caleb) made a comment about Wednesday sessions being for girls in response to a comment made by Mia. Caleb included a negative comment about another male student (Todd) who had attended a Wednesday session recently. The student teacher was in the breakout room and heard the exchange.
23-Jan to 25-Jan	The teacher and student teacher told the researcher about the "Wednesday sessions are for girls" comment through an email exchange.
25-Jan	The researcher interviewed Mia about the conversation that took place during the breakout room discussion. (The male student involved did not consent to interviews and therefore was not asked about the conversation.)
27-Jan	Caleb attended his first Wednesday help session of the semester. He then went on to attend every remaining Wednesday session of the semester. Todd, the male student about whom the negative comment had been made, did not attend another Wednesday session that school year.

On January 20, 2021, eight students attended the Wednesday session, five female students and 3 male students. Over the hour, students made a total of 42 contributions; they volunteered 83% of the time, as opposed to being called on by the teacher. The student who contributed the most was a White female student (Sophia: 13 times), followed by two White male students (Ethan: 9 times, Todd: 6 times), then two female students of color (Guadalupe & Mia: each 4 times). The remaining three students, one White female, one female student of color, and one White male, each contributed twice.

On Thursday January 21, 2021, the teacher brought up the topic of Wednesday help sessions with the whole class at the beginning of the regular calculus class period. She wanted to encourage more students to attend. The teacher addressed the class.

> I'd like to talk about what happens on Wednesdays. And what I'm going to ask is that a student who comes Wednesdays, if they can just explain so it's not coming from me. Mia, are you still up for it? Do you mind if I just call on you? Mia already offered, so, Mia, what do you think about Wednesdays?

The student selected to share was a female student of color who had attended most (if not all) Wednesday sessions so far that school year. She had contributed four times during the previous Wednesday session, once called on by the teacher and three times she volunteered. She shared her thoughts with the class.

> Basically, it's not like other courses, where if you're like behind or if you're missing assignments, you go. It's a lot more for if you need help with homework and we often just do the homework, so it's a lot easier. Also, if you want to do the extra credits[11] and you want to make sure you're doing it right. It's a perfect opportunity to do the extra credits. So, you have gone over it once already. It's super helpful.

The teacher added a few more thoughts.

> Thank you. I think some people think they have to be invited or have a special situation. I told some people this already, but this rate at which this class is moving is - basically what I've done is take five days and squished them into four and so if it feels quick on some days it's because I've compressed things a little bit … So, if it feels like the class is fast it's because we used to have those Wednesdays as a regular day, so Wednesday is for everybody. Ok? And I think they're pretty enjoyable. Any questions before we get going? Is everyone good?

Later that class period, students were assigned to breakout rooms to work together on a classwork problem. Mia was assigned to a breakout room with her female friend (Guadalupe) and two White male students (Caleb and Todd). During the groupwork, Caleb made a comment about how Wednesday sessions are for girls. The student teacher, Mr. K, was in the breakout room at the time of this conversation, along with the four students. The researcher did not observe this group's interactions firsthand, but later heard about the incident from other participants. In an email exchange between the teacher, student teacher, and researcher, the teacher brought up the conversation which she had heard about from the student teacher. On January 23, 2021, the teacher wrote the following.

> One issue I would love to look into is student collaboration by gender … Mr. K told me some funny story about someone (Caleb?) saying the Wednesday classes were for girls and the Thursday office hours were for boys and girls. Mr. K can perhaps remember it better!

The student teacher responded to the teacher's prompt in the email exchange, explaining what happened from his perspective.

> When I was in a breakout room with Caleb and a couple other people, Mia said that she enjoyed the Wednesday sessions since there are a lot of girls and she specifically said plus

[11] The teacher offered an extra credit point to each student who explained a homework solution at the beginning of class.

> Todd. Then Caleb said that Thursday office hours are for boys, and that Todd is weird for going to the Wednesday office hours. I told him not to shame anybody for getting help, and he apologized. But overall, it was kind of strange to know that they associated a certain office hour time with a certain gender.

In addition to Mr. K's account, Mia shared her version of what happened during a Zoom interview with the researcher.

> Mr. K was in my breakout group on Thursday, that I had with Guadalupe. And Caleb was making fun of people who were going to Wednesday sessions. And so, we're like, 'Dude, what are doing?' And then we mentioned that Todd had shown up and - 'Of course, it's like Todd with all the girls.' I was like, 'Dude, F off.' I don't exactly remember how it happened, but I think we were just talking about the Wednesday session, and he was just kind of smirking at it. I was like, 'Your friends joined too.' And that's how he reacted. And then Mr. K was kind of like, 'Dude, what the hell?', which was great. But it was just - that was that.

Based on these two accounts of the conversation, it seems Caleb made some kind of comment associating Wednesday sessions with female students and Thursday office hours with male students, including a negative comment about Todd who had attended a recent Wednesday session. It is unclear exactly what the words were that came out of Caleb's mouth, but the message received by both the student teacher and Mia was that Caleb was shaming students who attended Wednesday sessions, girls in general and Todd who was identified as having attended recently. Both the student teacher and Mia called out Caleb for his offensive and inappropriate comments.

After this breakout room conversation, Caleb went on to attend the next Wednesday session on January 27, 2021, and every Wednesday session for the remainder of the school year, whereas Todd, the student he made fun of for attending Wednesday sessions, did not attend a single session after January 20, 2021. It was unclear why attendance patterns changed so drastically for the two male students[12]. Mia continued to attend Wednesdays on a regular basis, though she was not too happy Caleb was now attending as well.

> Wednesday sessions - I'm much more inclined to speak up and ask my questions because it's usually just a few people. And I feel like those people who go there - except now Caleb shows up, which is so annoying, ummm - I feel much more comfortable being able to make mistakes. I find it funny that he's coming especially since he made fun of it.

Caleb's presence did not deter Mia from attending the Wednesday sessions; she still felt more comfortable asking questions and making mistakes on Wednesdays compared to the regular class days. However, it can be assumed Mia would have felt even more comfortable if Caleb was not there, based on previous comments she has made about this student's attempts to "try to poke at certain things and undermine your work." Nothing more is known about Todd, who attended

[12] Neither Caleb nor Todd participated in interviews for this study.

his last Wednesday session on January 20. It is not known whether he was made aware of the breakout room comments and because of them felt ashamed to attend further sessions. Or, if he chose not to attend further sessions for completely unrelated reasons. Regardless, the comments shaming his Wednesday presence most likely did not encourage his future attendance at those sessions.

Part 3: Participation Issues and Successes as Perceived by Participants

Findings in Parts 1 and 2 highlighted some of the barriers to participation that students in Ms. B's AB Calculus classes faced during the 2020-2021 school year. This part addresses how participants articulated and made sense of participation issues in the calculus classes and how they talked about participation successes. Issues and successes were identified and defined from the perspectives of the participants. For example, how did Ms. B talk about problems with participation in her classes? What seemed to bother students about their own or their peers' participation? In what ways was satisfaction with student participation expressed and about whom? Participants' perspectives on issues and successes are compared, highlighting areas of alignment and disparity.

Student Participation Issues

This part addresses participants' observations about classroom participation "issues" (i.e., observations about students' participation that were framed by participants as less-than-ideal), which often included references to gender and only occasionally race. The most frequent subject of reflection was the unequal distribution of verbal participation between male and female students. Participants reflected on this participation pattern with respect to both the Period 1 and Period 2 calculus classes, though the topic was mentioned more often with respect to Period 1. Several students in the Period 1 calculus class initiated a discussion of this issue with the teacher within several weeks of starting school in the fall, and the topic continued to be a regular focus of participant reflection through to the end of the school year. This issue of unequal talk distribution was most often talked about by participants as a gender issue (boy dominance) and occasionally as a race-gender issue (White boy dominance). At other times, the issue was framed with respect to the female students in the class (girls not speaking up) as opposed to the male students. The two framings for this issue are similar, in that they were based on observations of unequal talk; however, they are different in how they assigned responsibility for the issue. The girl-focused framing shifts responsibility from the boys who were talking too much to the girls who were not talking enough. This shift in framing has implications for how participants made sense of the root causes of (and potential solutions for) the unequal distribution of classroom talk. Findings related to the male-focused framing are presented first.

(White) Male Dominance

The teacher, student teacher, researcher, and multiple students (both female and male) shared observations throughout the semester about the distribution of student talk across gender and/or race categories. Most observations pertained to whole class discussions, but some participants also spoke about small-group discussions[13]. Sometimes participants spoke about the imbalanced verbal participation as a race-gender issue (White male dominance), sometimes as a

[13] Chapter 6 takes a close look at one small-group discussion, including participants' observations related to gender.

gender issue (male dominance), and hardly ever as a race issue (lack of contributions by students of color).

The teacher first told the researcher about the patterns of unequal participation in her calculus class during an informal one-on-one conversation between the two of them in September 2020. The researcher had not yet visited any of Ms. B's classes. The purpose of this conversation was to prepare the researcher to begin volunteering in one of Ms. B's other math classes. The plan was for the researcher to volunteer in Ms. B's Math 1 classes throughout the school year, do a few informal lesson observations in the two calculus classes during the fall semester, and then conduct formal observations in calculus Periods 1 and 2 during the spring semester. During this initial conversation Ms. B shared how the first month of her Period 1 calculus class had gone[14].

> This past term, online teaching was better than expected. With calculus, we did create some community. It was great, much better than expected… Each class had 5-6 students I knew well from last year. Some of them I had personally strongly encouraged them to take calculus. I told them they needed and deserved to be in that class… As a class we talked a lot about distribution of talk. The girls were saying that the boys were talking too much. Initially, they were saying that the White kids were talking more. The kids made a tally to keep track. We all talked about it as a class. The girls were calling me out on it, asking 'What are you going to do about it?'

The teacher described the girls in the class as being the people who initially identified the issue. The teacher's decision to speak about talk distribution as a class validated the girls' concerns and indicated the topic was worthy of taking class time to discuss it with everyone. Ms. B spoke about the "girls calling [her] out on it" with a tone of respect. It seemed she was proud of the girls for standing up for themselves. Through later conversations with Ms. B and students, the researcher learned that "the girls" who brought up their talk distribution concerns were Guadalupe and Mia.

The next conversation about student participation occurred through an email exchange between the teacher and researcher as they made plans for the researcher's first informal calculus lesson observation. In mid-October, the teacher emailed the researcher saying, **"**I would like you to observe in 1st period calculus where female students and students of color continue to raise the issue of unequal participation. I'm struggling with how to address this in a positive way." The researcher attended the next Period 1 calculus class and shared a write-up of her observations of who participated and in what ways with the teacher after class. One of the researcher's observations was that only one student of color, Guadalupe, had participated during the whole class discussion in "competent" mathematical ways, defined as answering or asking at least one "how" or "why" mathematical question. The teacher shared her reaction to this observation through email.

> One response you might record from me, is that this is making me feel a little defensive. I feel like I have worked hard to include students in discussion and that this is one of my

[14] This conversation was not videorecorded. The excerpt shared here was taken from the researcher's meeting notes.

> strengths as a teacher. I think I pay attention to this issue and try to address it. To see the data clearly showing that the few remaining, brave students of color in calculus do not have a strong voice in my classroom is making me feel sad. thanks for doing this work!

The teacher acknowledged feeling disappointed and "a little defensive," given the attention she had devoted to supporting her students of color in calculus. Her final exclamation, "thanks for doing this work!" indicated she appreciated this feedback and was committed to pursuing this work further. This conversation was one of the rare occasions when student participation was discussed with respect to race but not gender.

The researcher began formal lesson observations with debrief sessions in January 2021, along with one-on-one participant interviews. From January until May, participants talked about the on-going issues with male and/or White male dominance during class discussions, though most participants acknowledged the spring semester was somewhat "better" than the fall semester. Mia, who initially brought up concerns of unequal participation to Ms. B with Guadalupe, shared the following.

> It's definitely gotten I'd say better in some ways, but also, it's second semester senior year, so people care less. I think first term was the peak of the chaos in that class… my econ class last year really made me tougher… I knew exactly what to do, because I didn't wait half a year until I bought it up to my teacher. I waited five days and brought it up to my teacher. I was in a better head space, so it didn't really affect me; it just pissed me off… my other classmates seem to feel the same about the situation.

Mia described the issue as "better in some ways," but attributed at least part of the reason to the fact that it was the end of their senior year of high school and students did not care as much. She also gave herself credit for being in "a better head space" this year. She went on to describe more about the situation which initially prompted her to talk to her teacher.

> From first semester, Todd was consistently interrupting. Literally it was so annoying. I actually made a tally on my notes app one day and I sent it to Ms. B. And I was like, this is how many times every person has talked in our class today and it was like, Todd had like 12 times he had unmuted. Josh had like 6 or 7. Caleb was at like 8 or 9. And then Michelle once. Liliana once. I was like, look, 'This is so bad.' No one else is getting a chance to talk, and I think that Ms. B talked to Todd and since then it's kind of worked.

Mia collected participation data from their class, shared it with her teacher, and made the point that "no one else is getting a chance to talk." She talked about the issue as pertaining to three specific students, all White males (Todd, Josh, and Caleb). In response, Ms. B talked to at least one of the students who had been dominating the conversation. Mia spoke more about her overall satisfaction with the way Ms. B tried to address their concerns.

> I feel like Ms. B overall does a pretty good job making room for other people to speak and kind of not letting - I don't know exactly how to describe it but - the misogynistic stuff that has seeped through, because me and Guadalupe did make Ms. B aware of the issues

> we were feeling at least within a couple weeks into the class first term. So, she has known, so I think she kind of has an eye out for it. But before we brought it up to her, I don't think she'd really realized that was a situation that was bothering us so much.

In this excerpt, Mia connected the dominant students' behavior to misogyny. She also shared that although she felt Ms. B was responsive to their concerns, she did not think Ms. B would have known how they were feeling if they hadn't told her. Guadalupe also shared her appreciation for Ms. B's response to their concerns, though she did not talk about it in as much detail.

> And if you talk to [Ms. B], like the girls, Mia and I, we talked to her at the beginning of last semester… we told her that we felt really excluded from the classroom and that we weren't feeling comfortable. And she made sure to bring it up in class.

Guadalupe felt that if she brought up concerns to Ms. B, her teacher would listen and do something about them.

The teacher agreed with the two students that the unequal verbal participation issues got better over the course of the school year. In February, the teacher shared with the researcher after class one day, "At the beginning of the year there were students who were really complaining about [unequal talk], calling it out. And I feel like it created a healthier class." The teacher gave the students credit for calling out the problem and for contributing to a healthier class. In a March email exchange with the student teacher and researcher, Ms. B reflected in more detail on the change in participation patterns for Caleb, the most outspoken White male student in the Period 1 calculus class, who was also the student who said Wednesday sessions were for girls.

> I was thinking about Caleb's growth this year as a student. He started the year as being pretty dominating in class… We also know that he at one point identified Wednesday as the day that "girls" got help So now he comes on Wednesday, apparently is willing to be identified as a girl, and is behaving better than before: I can feel him holding back and giving others time to think. Guadalupe has at least told me that she feels better about the class participation and dynamics all around. Of course, she has also gained a lot of ground in terms of leadership and confidence.

Ms. B described Caleb as "behaving better than before." She also shared that Guadalupe felt better about the participation, but highlighted Guadalupe's gain in confidence as partially responsible for this shift in classroom dynamics. The teacher's email continued.

> Now the question might be: is Caleb still taking up too much airtime? I am not sure. I do feel like each of the other students left today's class with increased understanding and confidence. I think they each said that as they left. I also believe that Caleb's participation and enthusiasm for math can be a model for others, as long as his behavior is respectful. I do not want to suppress this kind of enthusiasm, and I think his classmates have helped him mature this year.

After acknowledging the dominant student was holding back, the teacher reflected on whether the change in his participation was enough. The teacher articulated a dilemma, the tension of opening up enough airtime for other students to make sense of the mathematics while not suppressing Caleb's enthusiasm for math. The student teacher responded to the teacher's email.

> I agree with everything that has been said! I definitely see him participating less than before, and I think he is actually holding back from saying more. He's getting better at 'reading the room' and is now allowing students to think and is also giving them space to participate. I think he comes to office hours because he likes the class, and also likes to talk. I don't see him coming in as that big of a deal, and he's a great student!

Whereas the teacher shared a bit of doubt as to whether the change in participation was enough, the student teacher seemed relatively certain that it was enough, as indicated by his assessment that his presence in the help session was not "that big of a deal" and his final statement, "he's a great student!"

Most of the talk about class participation was focused on student participation during whole-class discussions. However, some participants mentioned patterns of male-dominance when reflecting on a short video of four focal students (Yonas, Guadalupe, Hosein, Elijah) working on a small-group task, which is the focus of Chapter 6. The teacher, student teacher, and three out of the four students mentioned imbalanced participation by gender. The teacher noticed that Guadalupe spoke less than expected, saying, "I thought Guadalupe was a little bit - I wonder if she had something on her mind because I feel like she's usually more like Yonas where she's sort of leading the charge." The teacher noticed Guadalupe's lack of verbal participation but did not connect it to gendered participation patterns. The student teacher also noticed Guadalupe's lack of talk, but he connected it back to the initial conversation Guadalupe had with Ms. B about White male dominance in their class.

> This so far reminds me of the conversation that Ms. B had where you know in September with these set of girls of - you know, kind of it being male majority, especially in a breakout room where there's three guys and one girl. It's a three to one ratio. Ummm, and Guadalupe is a pretty confident person, yet we haven't heard her talk much about the problem.

Two of Guadalupe's male peers also noticed the unequal participation in this group, and they both connected it to broader gender patterns just as Mr. K had. When reflecting on the small-group video, Yonas shared, "It was mostly male-led convos … that's what often happens in our class," and during his interview, Hosein shared, "It's pretty usual that it's more male voices talking … the class in general and translated to the small breakout groups have a trend of more male-dominated conversation." These two students talked about how the interactions in this group were typical and how they were representative of talk patterns in their calculus class overall. Guadalupe also noted the imbalanced ratio of male to females, and the typicality of that situation.

> There's also a lot of guys which is normally the case … they were having like a whole conversation [without me] … When I'm in a situation like that and I know it's not going to get better, I kinds just shut down … because the way I think of it, if they don't want my input, then I'm not going to pay attention.

She described feeling left out of the conversation, which led to her shutting down and disengaging from the task. She spoke about this situation as if this was not a new experience for her.

While the topic of male students dominating classroom talk was brought up most often with respect to the Period 1 calculus class, the issue was spoken about regarding the Period 2 calculus class a few times as well. During a one-on-one student interview in May, Sarah reflected on participation patterns in the Period 2 class. She recalled several times during class when Ms. B explicitly called for female volunteers to share.

> I think it's definitely part of a gender thing. Ms. B said recently a couple times, like, 'Are there any girls?' or you know, something along those lines, which I have definitely noticed in school over the course of being in middle school and high school. So, yeah, I guess there's that aspect of it that I notice. It's normally a confidence thing. Or the male voices normally being more dominant in classroom conversations, which can suck, but also, I think there's a lot to unpack there.

Sarah likened the participation patterns in calculus to patterns she had noticed in previous years as well, connecting them to confidence and the dominance of male voices. Around the same time as this student interview in May, the teacher asked the researcher during a Period 2 lesson debrief, "Do you happen to remember the [gender] ratio in this class? It feels 50-50, but it's almost entirely boys speaking." The researcher responded, "It's not 50-50. I think it's maybe 2/3 and 1/3," and the teacher continued, "The voices are way more disproportionally male." The researcher agreed. The teacher's sense that there were relatively more female students in Period 2 than Period 1 was correct, but the ratio was still not very close to 50-50. There were 8 female students out of 28 students in Period 1 calculus (28.6% female), and 12 female students out of 32 students in Period 2 calculus (37.5%). Regardless of the number of female students enrolled in each class period, participants felt male students dominated the conversations in both classes.

Girls Not Speaking Up

Throughout the spring semester, the teacher expressed frustration that female students in her calculus classes were not speaking up. These female-focused conversations were separate from the conversations about male dominance, yet they were similarly prompted by observations that male students were speaking during class discussions and female students were not. Almost all observations about girls not speaking up were regarding Ms. B's Period 2 calculus class, whereas most of the comments about male dominance were regarding Ms. B's Period 1 calculus class. Typically, conversations about female students not speaking up were focused on student gender alone; rarely was race mentioned. However, the three students mentioned most often as not speaking up enough were three female students of color. Emma was recognized by the teacher as having made progress this year in terms of her participation,

whereas the teacher continued to feel frustrated by the lack of verbal participation by Sarah and Leah.

Even though most of the talk about girls not speaking up was focused on the Period 2 class and focused on gender alone, Guadalupe (from the Period 1 class) referenced both race and gender when recalling that first conversation she had with Ms. B in September about unequal student talk in their class.

> I was like, first of all, the girls aren't talking, but there's way - there's like three girls of color in here. And we're really not talking at all. And we would like - I would like for you to advocate for us as our teacher because it shouldn't just be all up to us. And she took that really to heart and brought it into the classroom. So, that was nice.

Guadalupe emphasized that female students of color were speaking even less than the White female students, asking Ms. B to take some responsibility for addressing this issue. Guadalupe seemed content with how Ms. B responded to her concerns. And it can be assumed that Ms. B was content with how the female students in the Period 1 class participated because she never complained about girls not speaking up in that class period.

The teacher's frustrations with girls not speaking up were focused on the girls in her Period 2 calculus class, in particular, "girls with skills." Sometimes the teacher made general comments and sometimes she made comments about specific female students. For example, during a January lesson debrief session the teacher shared, "Michelle has to speak up. These girls gotta speak up. I'm - I'm annoyed with them." This was the only time the teacher mentioned a White female student in this context; all other mentions of girls not speaking up were referring to female students of color. The teacher also made moves during whole-class discussions in Period 2 to elicit female voices through explicit prompts, as described by Sarah in the previous section. During a lesson on implicit differentiation in February, Ms. B wanted students to guess the equation to a line she had drawn tangent to a curve on a graph.

> I'm going to ask, who wants to guess the equation, just guess. Please don't do any calculus. Just guess. What do you think is the slope? What do you think is the intercept? Just guess. There are no wrong guesses. The only wrong is to refuse to guess. What-a you got? ... Estimate the equation of that red tangent line. I'm ready for some estimates. They don't even have to be close. What kind of slope? What kind of intercept?

Ms. B tried to make the students feel comfortable guessing, but initially no students offered guesses. After the teacher's repeated prompts, two male students volunteered possible equations for the line, which the teacher wrote on the whiteboard. The teacher wanted to add more guesses to the board before moving on.

> These are wonderful. Continue. Just guess. (10 seconds of silence) There are no wrong guesses. The only thing wrong is to refuse to guess. (3 seconds of silence) I mean I would go with a negative slope, but that would be embarrassing if you had a positive slope, but - (3 seconds of silence) Ok, sorry to be blunt, but can we get a girl to guess?

The teacher waited in silence and eventually offered one hint by telling students the slope should be negative, and then went on to ask explicitly for “a girl to guess.” Emma (female student of color) responded and offered another possible equation. The teacher thanked the student and asked, “Can we get one more? Sorry I had to be sort of blunt about that one, but - One more?” No one else responded and the teacher conceded, “I mean, these are pretty close.” She then continued on with the next part of the lesson. Periodically throughout the remainder of the semester, the teacher continued these explicit prompts. During a lesson in May, the teacher said, "Thank you boys, but I'd like to hear some female voices too. Boys can still talk too, but I want the girls to speak up." Emma spoke in response to this request made by the teacher, the same student who responded to the request for “a girl to guess” back in February.

During the last month of the school year the teacher reflected on the progress that some female students had made in terms of speaking up and the lack of progress other female students had made. In the second week of May, Ms. B shared these reflections with the researcher about three specific female students who were participating more than they had in the past. She talked about Zoe (White), Rebecca (White), and Emma (mixed race).

> I can actually see the work that I have put into the girls who speak up. Like I feel like there’s been some effort to develop those voices … Last year Zoe was - she was my student last year and she was often unsure in class, and I was often scolding her about that and pushing her. And you know, I wrote her college letter and I helped her get into schools. She’s someone who’s closer to me than a lot of other students who I don’t know as well. And I feel like with Rebecca as well, I’ve had those exchanges. Emma also I have helped this year. She’s been participating more. There’s like a little bit of - maybe they even feel sorry for me because no one’s participating, but they still have the confidence to step in. The word nurture feels too weak, too soft. It’s more like I’ve been kicking them. And so, it shows I guess that shows sort of that affirmative action piece, that you shove and it feels like pushing, not just checking in. Like, yelling, like sort of an emotional, ‘Come on. Let’s do it!’

The teacher took some credit for changing the girls’ behavior by “kicking,” “pushing,” and “yelling” at them to “do it.” However, she also gave these three students credit for having “the confidence to step in.” The teacher continued by talking about her frustration with two female students who still were not speaking up. She talked about Sarah (Mexican / Middle Eastern) and Leah (Filipino).

> I’m really annoyed with Sarah and Leah cuz they are the starlets in this class by far. They get everything right. They would have been in BC calculus, but they decided not to bother. And they’re not contributing to the class. They’re not supporting - I think that they - I mean that might just be their personality, but I also think that they might have a long history of being the one in the corner who just took good notes and tried to keep up in those fast-paced classes. But I write them notes, like, ‘Why don’t we hear from you?’ I put their names on the board, like, ‘Come on!’ and then they rarely actually take the bait. It’s interesting.

The teacher did not seem to understand why these two students who "get everything right" were not taking "the bait" and were "not contributing to the class." Ms. B suggested several possible explanations for why the girls were not speaking, none of which acknowledged any potential barriers to participation related to gender or race.

About a week later, the teacher and researcher had another conversation about the participation of these two female students of color who were not speaking up. They discussed further the possible reasons why Sarah and Leah may not be sharing their thoughts.

> **Teacher:** It still makes me sad that these girls with the skills don't want to share. That's a skill that they should be working on. Communication.
> **Researcher:** There are probably boys in the same position, right?
> **Teacher:** The strongest boys in this class do speak up. Nate and Kyle are important members of the class.

In this exchange, the teacher talks about the female students as not wanting to share, implying their decision not to speak up is connected to a lack of motivation or an intentional decision on their part. The teacher acknowledged the girls had "skills," presumably mathematical skills, but said they needed to work on developing communication skills. Ms. B's reference to Nate and Kyle as "important members of the class" seems to imply the boys are more important to the class because they speak up. Ms. B elaborated further on her wishes for the two female students.

> I think that they're thinking about [mathematical ideas], but I'm not sure. They certainly have the skills. But I wish it was somehow exciting for them. It's fun … I wish that they were curious at that level and able to express it. They might be, but I wish that they could express it … I know it's naïve of me (laughs).

The teacher's comment about being naive and her laugh indicate she knew she was offering an overly simplistic explanation for the situation. The teacher concluded this conversation by connecting the female students' unwillingness to speak to a lack of compassion.

> Sometimes I feel like it's almost a lack of compassion. We're in a breakout room and the other kids are lost. You could at least speak up with some support. And that is also worrying to me. Don't you see these as your colleagues? Cuz Sarah and Leah are able to do all these - everything I've given them; they can do it all. And there are lots of people who can't.

In these final thoughts, the teacher talked about how the two students' decisions to not speak up were hurting their peers. The teacher indicated there was a moral obligation for Sarah and Leah to support the other students in their breakout rooms who needed help.

Two days later after grading a project assignment, Ms. B shared another theory about why the female students may not be speaking.

> There's a bit of a gender gap in terms of this kind of video project where Nate did not do a good job … And then Emma did this beautiful thing. It's a slide show that she narrates

> with things appearing in colors. I mean it was really exquisite. And then Sarah did a beautiful job. And I got to hear her voice. I think those girls might just be shy.

It is unclear exactly how Ms. B made the jump from an exquisitely narrated project to shyness. Perhaps she was thinking that Sarah was comfortable talking in front of a camera but not comfortable talking in front of the class, so the difference in comfort level could be attributed to Sarah being shy. However, with this explanation it is still a bit unclear as to what being "shy" means and how shyness is connected to Ms. B's previous ideas about the girls lacking communication skills, curiosity, and compassion.

Ms. B was not the only person who wondered why some of the female students were not speaking up more. Zoe, one of the students Ms. B praised for having developed her voice over the year, shared during a one-on-one interview at the end of the school year that she, too, found it interesting that some female students chose not to speak up during class.

> One thing that I've definitely noticed is for the really hard problems, it'll be like these 3 or 4 girls who get it and they're the only girls in the class - or not the only girls, the only students in the entire class who get a really hard problem, but they don't really want to speak up and explain it or talk about it, which I've found kind of interesting. I mean, you've probably seen. Some of the strongest - like Emma, Sarah, Leah - they are some of the smartest kids in the class, but they never speak up. I don't know if that has to do with gender stuff, but I feel like maybe that's less true for guys.

Analysis showed the broad claim that girls were not speaking up in the Period 2 class was most often connected to specific frustrations that Emma, Sarah, and Leah were not speaking up, as opposed to girls in general. These were the girls whom Ms. B referred to as "having the skills" and Zoe described as "some of the smartest kids in the class." The desire to have Emma, Sarah, and Leah speak up seemed to be related to a desire to have female students share mathematically "beautiful" contributions with the class. These three female students were also all students of color, although that fact was never included in the articulation of the participation issue. Ms. B talked about this as a gender issue, not a gender-race issue.

The topic of girls needing to speak up was also discussed in a lesson debrief conversation in May connected to the composition of small-group breakout rooms in Period 2. The researcher had (intentionally) created a breakout room that day that contained four female students, the first time an all-female group had occurred in Ms. B's calculus classes that school year. Each day breakout rooms were assigned randomly and with so few female students in the class, a random group of all girls had never happened. All-boy groups happened daily. During the debrief conversation after the lesson that day, Ms. B shared that she did not support "segregated" groups of students as a general philosophy, referring to the all-girl group the researcher had created that day. The teacher explained, "I don't believe in segregation … but you're experimenting locally with - like, you gotta learn to speak up (shrugs shoulders). Everyone has to learn it together. I don't know. Mr. K, what do you think?" The student teacher had a different perspective.

> I guess I don't find gender separation to be a form of segregation because there are still some on-campus dorms [at my university] that are all girls for example. And I think that if

> it's a safe space for them and it helps them learn, I think they can do the whole growing to speak in a room full of guys - I think they can do that at a different time.

Mr. K pointed out that having a safe space for female students to learn mathematics was more important at that point in time than having female students speak up in front of male students. The teacher acknowledged the student teacher's position. She responded, "I mean, that's the argument for all-black colleges for sure." She saw value in what he was saying but did not seem convinced enough to change her mind. The researcher added her perspective by noting the societal context of this advanced mathematics course.

> I feel like a high school calculus class is a White, male-dominated space just by definition of the course itself, and so I feel like asking people who aren't of that group - like, it's not a level playing field in some ways, so when people enter that space, it's complicated.

The teacher acknowledged the researcher's position by nodding and saying, "No, that's a good point. Yeah, it's interesting." Even though no official resolution was reached by the end of this conversation, the three participants contemplated the urgency of the need for girls to speak up and the reality of what that might mean for them given the historical (and current) sexist and racist nature of high school calculus courses. This was the only time, other than Guadalupe's initial complaint to Ms. B about participation, that race was part of the discussion about girls not speaking up in calculus.

The only documented case of a classroom participant reflecting specifically on the differences between the experiences of White female students and female students of color in Ms. B's calculus classes occurred during a one-on-one interview with Guadalupe. She shared the following with the researcher.

> Within calculus and the girls, I feel like we should be supporting each other more than some of us do … it's different with PoC girls and the White girls in the class … It's a little bit more hostile, Emily is very hostile toward me. And so, I'll be like, 'Oh, okay.' I don't want to overstep. And with Mia it's not like that … I understand the PoC girls more and so they're a bit more supportive … a lot of times [White female students] are a little bit more reserved about their work, which is fine. You don't have to help us or share your work with us, but we don't necessarily act that way towards them … it's not the same. It's just different.

Guadalupe did not speak about differences in class participation patterns in this excerpt, but rather about differences in how the two groups of female students acted toward one another. She felt more supported by "the PoC girls" than she did by "the White girls." In the Period 2 calculus class there were six female students of color, but in the Period 1 class there were only three.

Student Participation Successes

Students had varying levels of success participating during Ms. B's AB Calculus classes. The 2020-2021 school year was far from easy for any of the participants in Ms. B's classes, but there were some individuals who were perceived by their teacher and/or by themselves as having

achieved some level of participatory success. Overcoming (or avoiding) various barriers to participation related to distance learning, gender and/or race seemed to be relatively easier for some students than it was for others. The goal of participatory equity is to construct genuine opportunities for every student to engage deeply with mathematics in ways that support rich content development and positive mathematical identities. To assess progress toward this goal, it is helpful to hear participants' perspectives regarding who they felt participated in productive and meaningful ways. It is also helpful to understand which students encountered which barriers and what factors seemed to support "successful" students in overcoming the barriers they faced.

Successful Participators: Guadalupe, Emma, Zoe, Hosein, Yonas, and Nate

This section contains details about six students who were considered to be successful participators in Ms. B's AB Calculus course. Students were selected for inclusion in this third findings section based on whose second semester classroom participation was assessed most positively by their teacher, themselves, and/or their peers, given the available lesson and interview data.

Guadalupe (Mexican female) was one student who the teacher brought up quite often as an example of a participation success story. The teacher shared from her perspective, "Guadalupe's done it. I know I say her as an example too many times, but she has. She's figured out how to speak up and her math skills are wonderful." Ms. B often shared how proud she was of Guadalupe, using statements such as, "Guadalupe contributed a lot [today]. I am very proud of Guadalupe as I have really encouraged her to take calculus and speak up." In this statement, the teacher referenced a supportive relationship she and Guadalupe had built, suggesting her encouragement had helped Guadalupe enroll in the calculus course and participate in the daily lessons. Despite Guadalupe's overall success "figuring out how to speak up" and developing her math skills, there were a couple times when Ms. B talked about how Guadalupe's participation did not quite meet expectations. One example was when Ms. B watched a video of Guadalupe working on a small-group task[15] and reflected on her participation.

> I thought Guadalupe was a little bit - I wonder if she had something on her mind because I feel like she's usually more like Yonas where she's sort of leading the charge. So, I think that could have been a tiny, just a slightly, like not the perfect sample of Guadalupe's engagement. But she was engaged.

The teacher referred to this sample of Guadalupe's participation as atypical, but still concluded she was engaged. When Guadalupe reflected on her own participation during this task, she agreed with Ms. B that she did not say very much. She also admitted to disengaging from the task, which Ms. B had not picked up on. Watching the video of herself prompted Guadalupe to talk about gender. She shared, "I talk to Mia about it. We normally don't talk in breakout rooms unless we're asked to because we don't really feel like we have the space to. When it's more girls than guys, we feel more comfortable." Guadalupe did indeed participate more than most other female students in the class and based on these patterns, Ms. B had given Guadalupe the benefit of the doubt and had assumed she was engaged in this task even when she was not. In this situation Guadalupe was continuing to struggle with a barrier that Ms. B did not see. Guadalupe's

[15] Analysis of this small-group task is featured in Chapter 6.

reputation as a successful participator had perhaps made it more difficult for Ms. B to see the gender barrier with which Guadalupe continued to struggle.

Guadalupe formed supportive relationships with both students and adults in the class. She mentioned female students of color specifically ("I understand the PoC girls more and so they're a bit more supportive"). She got advice from the student teacher ("I talk to Mr. K about college stuff, and he'll give me advice about life experiences"). And she inquired about the researcher's experiences ("I've never met someone that's going through the process of doing a PhD. It's like really empowering to see you do it and it's super cool. And I would like to know more"). Guadalupe also spoke in-depth about the teacher-student relationship that she and Ms. B formed beginning the year prior to this calculus class.

> Ms. B and I, we were in the same class my junior year too. And at the beginning of junior year, I hated math. I was like, 'I really don't want to be here in class, Ms. B.' And by the end of junior year, we were on very good terms. And so, she wrote me a letter of rec and everything. I think it's math in combination with the teacher and how she teaches. She very much cares that we understand and grasp every concept. So, knowing that she cared that we learned and that she was willing to put in her extra time and we were able to match that. That felt like, we can do it, you know. And so, yeah, it was a difficult class, but it was also a really fun class, and it was nice to go talk to her at the beginning of class and everything … On a scale of one to ten, I think I was a solid 8 [in math confidence this year] … by the end, it was definitely a solid 8-9. At the beginning, it was like, 'Oh, I don't really know what's going on.'

Guadalupe talked about going from hating math at the beginning of her junior year to having fun and rating herself an 8-9 out of 10 in mathematical confidence by the end of her senior year. She attributed this shift to Ms. B's teaching methods, care, and willingness to "put in her extra time." Guadalupe also spoke about how her teacher supported her non-academic needs as well.

> There are different parts that I like about Ms. B. For one, it's like, if she sees that you're putting in effort to building like a teacher student relationship with her, she'll put that same effort back. And it's not always about math like this past semester, I was going through a hard time just personally. And I was able to come and talk to her about it. I was able to kinda like explain like the pressures that I had and she, she - oh it was while I was applying to college. My situation is like, my parents, they cannot afford to pay for literally any part of my college. So, I had everything riding on this one scholarship, which I got, so it's all okay now, but it is just thinking about all of this kind of wore off on me and she noticed it. So, she talked to me about it. She told me to just focus on one thing at a time, and that I would be okay no matter what … so that was really nice.

Guadalupe acknowledged the role that Ms. B had played in her success, both mathematically and personally. Guadalupe's scholarship, to which she referred, was a full-ride, 4-year scholarship to a top-ranked university in the US, evidence that the university admissions people considered Guadalupe to be a successful high school student as well.

From both Guadalupe's perspective and her teacher's, Guadalupe made noteworthy progress in the development of her mathematical skills, confidence, and willingness to participate in math class since the beginning of her junior year. That said, Guadalupe talked about how she continued to struggle with barriers to participation related to race and gender; she did not mention obstacles related to distance learning. She referenced feelings of not belonging ("if PoC students talk, they kind of look at us like, 'Oh, you're not stupid?'"), boys thinking they were smarter than girls ("[boys] think that you're not as smart as them … it's like constantly trying to disprove something), and being uncomfortable asking questions ("there's a lot of intimidating people that are moving really fast. And I don't want to slow them down"). She also talked about how affluent White kids have better access to resources (e.g., tutors), acknowledged issues of representation ("I've only had one non-White teacher my whole time at Evergreen High"), and wished for more diversity in her calculus class. Guadalupe spoke up and made Ms. B aware of her concerns related to White male dominance early in the school year, and in both of their minds, the situation improved. Guadalupe had figured out some ways to participate. Despite Guadalupe's apparent participation success, she continued to battle issues of gender and race through to the end of the school year. The barriers to participation Guadalupe faced had not disappeared, but the presence of other female students in her groups and the positive relationships she formed with other students and adults in the class helped her feel more comfortable and confident, which gave her the strength she needed to push through some of those barriers.

Emma (mixed race female) was another student who Ms. B talked about as a participation success story. Ms. B complained often about the girls not speaking up in the Period 2 class, but Emma was one of the two girls who spoke up most often (Zoe was the other). Ms. B talked about how she had built a relationship with Emma focused on supporting her participation and how she felt it had paid off. "Emma also I have helped this year. I told you, and she's been participating more." Emma was the student who spoke up voluntarily both times in response to Ms. B's pleas for female voices during the whole class discussions shared previously in this chapter. Ms. B also spoke about the high quality of the work Emma produced. "And then Emma did this beautiful thing. It's a slide show that she narrates with things appearing in colors. I mean it was really exquisite." Just as with Guadalupe, Emma's success was reinforced through the college admissions process, which Ms. B shared during a lesson debrief session.

> The girls are really, really strong, and Emma is going to go off and be a math major at [university] and she's gonna be great. She loves it. In the little notes about the homework, she says how much fun she's having in class.

In this passage, not only did Ms. B talk about Emma's strong mathematical skills, but she also talked about Emma's enjoyment of math, another important component of mathematical "success" in the teacher's eyes.

Emma confirmed her enjoyment of math in an interview with the researcher. After watching a video of herself working on a group task, the researcher asked Emma, "Do you get enjoyment from that kind of math discussion?"

> Kind of. Yeah, because it wasn't frustrating … [I felt] pretty comfortable … I didn't feel nervous or pressure ... I felt like I could express my thoughts, like they would help me if I was wrong ... I was pretty motivated to participate. I remember doing a HW problem like this one. I felt confident that I could do it and could explain it ... It was a pretty challenging problem.

Emma shared that she enjoyed the small-group discussion because she felt comfort and could express her thoughts. She felt motivated to participate and confident in her abilities. Emma's account of this small-group experience was consistent with her teacher's description of her as "participating more" and as a "really strong" student.

In this same interview, Emma also acknowledged there were times when she did not feel as comfortable, and gender of her groupmates seemed to play a role.

> I feel like I work better or feel more comfortable with female classmates just because like we already have something in common, I don't know. I always feel like there's this like pre-conceived thing that guys are better at math than the girls or whatever. And I feel less pressure when I interact with the girls than interacting with the guys.

Despite Emma's overall success with participation, gender still acted as a barrier to participation for her; she felt more comfortable working with female classmates. In this passage, Emma referenced the "boys are smarter than girls" narrative which she said caused her to feel additional pressure when interacting with male peers. Although Emma did not reference gender when reflecting on the positive small-group experience mentioned previously, it is worth noting that group in which she "didn't feel nervous or pressure" was composed of three female students and one male student. Emma identified as a student of color (she described herself as "mixed"), but when asked by the researcher if she felt race shaped classroom participation, Emma responded, "I don't think I've ever really noticed anything different about my interactions based on race." She did not mention any obstacles related to distance learning either. Although Emma did not talk about her relationship with Ms. B, the interactions Ms. B described suggest the two of them had formed a strong, positive relationship. Presumably, that teacher-student relationship and the presence of other female students in her work groups supported Emma in pushing through the gender barriers she encountered.

Zoe (White female) was one of the students in Period 2 who Ms. B considered to be a successful participator (in addition to Emma). Ms. B talked about her role in supporting Zoe's growth, starting the previous school year.

> I can actually see the work that I have put into the girls who speak up. I feel like there's been some effort to develop those voices … Last year Zoe was - she was my student last year and she was often unsure in class, and I was often scolding her about that and pushing her and you know I wrote her college letter and I helped her get into schools and she's my, you know, someone who's closer to me than a lot of other students who I don't know as well.

Ms. B talked about the close relationship she had formed with Zoe and how Zoe had gone from being "often unsure in class" last year to one of "the girls who speak up" in class this year. The researcher also noticed Zoe's verbal participation, sharing during a lesson debrief session in May that "All the voluntary participation today was boys, except for Zoe." Ms. B agreed, "Right. Zoe's a little bit like me because I would also be the only girl participating, and I think she and I both know that. You know, we can sort of feel it." Ms. B compared Zoe to how she was as a student, again referencing a strong, personal connection between the two of them.

Even though Zoe was recognized by others as a someone who spoke out frequently during class, she shared some hesitations about participating during a one-on-one interview with the researcher.

> For whole class, obviously you know, it's pretty hard to participate. I definitely like - I mean, sometimes I'll ask questions, if I have questions, probably 50% of the time. I guess I choose to ask them or not ask them just depending - I don't really know what goes on in my head. There definitely will be times when I have questions that I would ask them if it was a normal classroom, and I could just like one-on-one quickly talk to the teacher. But, because it's in front of the whole class, I don't. But I guess I do when I think it's a good question and other people will have that question … There are definitely exceptions, but I would say if I'm in a breakout room with all girls or mostly girls or one other girl, it definitely makes me a lot more likely to speak up actually.

Zoe talked about how she only asked about half of the questions she had, attributing some of her hesitation to the distance learning format which required her to ask her questions in front of the whole class. She also said that she found it easier to speak in breakout rooms if there was at least one other girl in the group, indicating that the gender of her peers influenced how and when she participated. Zoe faced barriers related to distance learning and to gender throughout the school year. However, Zoe was supported in overcoming these barriers by the presence of other female students in her breakout rooms and by the close relationship she had formed with Ms. B starting the previous school year when instruction was still in-person.

Hosein (mixed race male) was designated as a student participation success story based on his own personal account of his participation during class and based on his awareness of his peers. He talked about regulating his own participation in support of his peers' participation.

> I probably participate too much in the whole group. I've been working on taking up a lot less space in classroom discussions, especially on zoom. I'm definitely the person who will raise my hand and answer every single question. I wrote a college essay on it actually. Just because I feel really curious about things, and I want to have my questions answered. And so, it comes from a really benign place, but what ends up happening is that I am denying other kids in the classroom the chance to do the same thing. I've been consciously trying to not step up as much. And you know, I think I'm definitely - probably still stepping up a little too much, at least compared to other people. I notice the classroom is very male-dominated, so I'm also trying to be cognizant of that.

Hosein felt he participated too much during whole class discussions; however, no other class participants ever mentioned Hosein's participation as an issue. He shared that he felt he was "still stepping up a little too much, at least compared to other people." When Hosein said "other people" he may have been referring to female students in this case, seeing as how his next comment acknowledged the male-dominated nature of participation in their class. Hosein had been in an honors math class the previous year but had opted to leave the honors track to take AB Calculus instead of BC Calculus his senior year. He did not talk much about this shift, but it is possible that he felt he was over-participating in Ms. B's class because he was participating more than he did last year in the honors class. Or it is possible he was participating relatively more than his peers in Ms. B's class, whereas he participated relatively the same as his peers last year.

Hosein did not reference any specific barriers to participation that he personally faced, but he did spend time talking about how barriers affected the participation of "marginalized groups" of students. He talked about an anti-racist group he joined and how he thought about his role in helping to break down barriers for others.

> This anti-racist support group that I'm a part of in [small learning community] implemented these new rules around stepping up and stepping back and letting five PoC people talk first and people in other marginalized groups before calling on White people. And then really asking the White people and other less marginalized people to hold back a little bit and give space and allow there to be silence ... And it's worked and it's kind of been - it's triggered some sort of more active realization in my head. It's always been there in the back, like, 'Ah, Hosein, step back more. Stop asking so many questions.' Now it's like, 'Yeah, just stop talking and let other people ask their questions.' … At the beginning of the year when we were going over classroom norms, there was a slide with like, 'This is something new that we're doing. We're trying this out to try to boost participation of underrepresented students.' And it literally had a list of like five PoC people, queer people, women, like all the way down, which is really cool. And I've actually seen it helping a lot.

Hosein demonstrated awareness that people of color, queer people, and women face additional barriers to participation. Despite self-identifying as "mixed race," Hosein positioned himself as one of the "White people and other less marginalized people" who needed to hold back and give space to others. The only barrier to his own participation seemed to be the voice in his head telling him to stop asking so many questions, which he felt was a necessary barrier in order to provide other students more opportunities to participate.

Ms. B talked about being proud of Yonas (Black male) for his engagement with mathematics this year. She spoke often about his success in her class, specifically with respect to his understanding of the mathematics, his dedication, and his love of calculus.

> I'm very proud of Yonas. I mean, he was my only African American male student in all of my calculus classes … he does demonstrate a lot of understanding … And he really enjoys it. He says he watches these youtube videos to try to get a deeper understanding. He told me he would watch videos about things we hadn't learned yet just for fun. So, he's a real nerd, which is lovely.

Ms. B did not speak about Yonas's participation during class, but rather, spoke about his participation outside of class, specifically his choice to watch math videos for fun. On another occasion, Ms. B shared, "Yonas has told me that on the months off when he's not in class he does calculus anyway," referencing the distance learning class schedule that resulted in students attending math classes every other term. She also shared the following.

> [Yonas] said he was going to come to office hours last term just because he loved calculus so much. He didn't actually come, but that expression of his I think was really genuine. He's a wonderful kid. His cousin lives three doors from me, and I think he went to school with my children.

Ms. B appreciated Yonas's love of math and was happy that he found enjoyment in her class. Although Ms. B had not taught Yonas before this school year, she noted a personal connection to his family.

Yonas talked about how feelings of not belonging in his honors math class had restricted his participation the previous year but not this year. Consistent with Ms. B's assessment, Yonas indicated he felt he was successful in Ms. B's calculus class. He explained, "This class is way more diverse than last year, which probably helped me perform because I'm thinking now, like even in awkward stages, I was still participating more in this class than I was [last year]." Presumably, the "awkward stages" to which Yonas referred were times when he felt confused or unsure what to do. Yonas talked about the significant barriers to participation he faced related to race the previous year ("I was the only Black kid in the class … it just felt like there's a big spotlight on me"), but this year these barriers were not a problem for him. The relative increase in diversity in his math class had lowered the racial barriers to a point where Yonas felt much more comfortable.

Just as Hosein had talked about barriers faced by students other than himself, so too did Yonas. Yonas shared his observation that the conversations in calculus class this year were "mostly male-led" and male students greatly out-numbered female students. Connecting to his experiences last year, Yonas speculated that female students "probably [felt] the same way that I felt before." Although barriers related to gender did not apply to him, he was still aware they existed.

It was not often that participants talked about the participation of White male students in positive ways; Nate (White male) was one exception. During a lesson debrief session, Ms. B stated, "The strongest students in this [Period 2] class are pretty much all girls, except for Nate, in terms of the work." In a different conversation about girls not speaking up, Ms. B said, "The strongest boys in this class do speak up. Nate and Kyle are important members of the class." This assessment of Nate as a strong student is consistent with how Nate viewed himself. In a one-on-one interview, Nate seemed content with what he had accomplished in his calculus class, and he shared that he felt the distance learning environment had supported him to participate more this year than last. Nate was the only participant who claimed distance learning *helped* his participation

> I think I probably actually participate more in this Zoom learning. I don't know why - it's weird. I think the ability to use breakout groups is like really powerful because it actually forces the kids to talk, but like if we were at school, I could easily not do anything. It's easier to not raise your hand in an entire class of students than it is to not talk in a breakout group. So, I think that has positively affected my participation. That being said, I would talk in front of the entire class in my previous math classes too.

He also claimed he participated quite a bit in previous years, but without any additional data, it is difficult to know how much the distance learning structure actually affected his participation. However, it seems safe to say that distance learning did not present any notable barriers to participation for Nate, and it may have been an asset for him. Nate did not encounter barriers related to race or gender either. In fact, Nate did not mention any factors that impeded in participation in any way.

Summary of Student Participation Success Stories

Some students perceived to be successful participators did not seem to encounter any notable barriers (like Nate), but most successful participators talked about encountering considerable barriers to their participation at some point in their high school math classes. They talked about some obstacles using the present tense and others using the past tense.

There were several factors that seemed to support students in pushing through the particular barriers they each faced. Students who formed strong relationships with Ms. B likely had an advantage in addressing barriers to participation, as did students who had moved out of the honors math track into the relatively lower-level (but still advanced) AB Calculus course. Table 4d offers a summary of the barriers to participation and the supports for participation for each of the six focal students.

Table 4d. Barriers to and Supports for Participation

	Identity		Barriers to Participation			Supports for Participation	
	Race	Gender	Distance Learning	Race	Gender	Honors Math Last Year	Strong Relationship w/ Ms. B
Guadalupe	Mexican	Female		YES	YES		YES
Emma	Mixed	Female			YES		YES
Zoe	White	Female	Minimal		YES		YES
Hosein	Mixed	Male		Other people	Other people	YES	
Yonas	Black	Male		Last year	Other people	YES	YES
Nate	White	Male					

None of the students considered to be successful talked about major barriers to their participation related to distance learning. Zoe talked about how there were times when she would have asked questions "if it was a normal classroom," but Zoom made it more difficult. Other than that, none of these six successful students talked about challenges related to distance learning. There were quite a few students who talked about barriers related to distance learning, like Alison, Sarah, and Diego, but they were not successful participators. Three students of color mentioned barriers to participation related to race. Guadalupe talked about how these barriers

continued to impact her decisions about how and when to participate. Yonas talked about how barriers related to race hampered his participation in the past, but he said the relatively more (racially) diverse class this year made it easier for him to participate. Hosein talked about barriers to participation that students of color faced in general at the school, but he positioned himself as "less marginalized" and therefore relatively unaffected by these barriers himself. All three of the female students talked about how they continued to face barriers related to gender, specifically related to feeling pressure to not make mistakes and the need to prove their worthiness of being in an advanced mathematics class. Hosein and Yonas both acknowledged the gender-related barriers their female peers faced.

Yonas, and Hosein were both part of the honors math track during the previous school year and both opted to move out of the honors track and take AB Calculus their senior year instead of BC Calculus. Both are students of color, yet they seemed relatively unaffected by the racial barriers faced by their peers. These three students likely entered the AB Calculus class with already-established academic status (their teacher and their peers knew they had taken honors math in the past), meaning they may not have felt the same kind of pressure to prove their mathematical worthiness to themselves or other people. These two also seemed relatively more aware of their peers' experiences compared to the other students. Hosein talked about how his participation may be taking away opportunities from his peers, and both Hosein and Yonas acknowledged the male-dominated nature of class discussions. In addition, Yonas talked about it being relatively easier for him to participate this year compared to last because he felt more comfortable. For Yonas and Hosein, it seemed their past experiences had given them mathematical confidence and a different perspective that allowed them to overcome the barriers they faced and understanding the barriers their peers continued to encounter.

Four out of the six successful students formed strong relationships with Ms. B either prior to or during the school year. Guadalupe and Zoe had Ms. B as their math teacher their junior year. Ms. B had encouraged both to take the AB Calculus course this year (as opposed to Statistics), and Ms. B had written letters of recommendation for their college applications. Yonas knew Ms. B because he had family members who lived next door to her. He had known her for years outside of school. Emma and Ms. B did not know each other well before this year but had managed to get to know each other over the year through Zoom conversations and notes written on homework assignments. Emma kept Ms. B updated on her college plans to major in mathematics; she planned to attend the same university where Ms. B's son was currently enrolled in school. Guadalupe, Emma, and Zoe all talked about the barriers to participation they continued to face throughout the school year. However, the strong relationships they formed with their teacher likely contributed to the students' success in finding ways to overcome those barriers and participate successfully in the eyes of their teacher.

Nate was the only focal student not to talk about *any* barriers related to classroom participation, not his own participation or those of his peers. He did not have the advantages of being in the honors math class last year or having formed a strong relationship with Ms. B. But he seemed unphased by the transition to distance learning, he was White, and he was male. Therefore, he was able to avoid the obstacles that many of his peers were forced to face.

Summary and Chapter 4 Take-Aways

The overarching question guiding analysis in this chapter was: How did participants talk about connections between participation and various classroom, institutional, and societal

factors that shaped student engagement in the course? Addressing this question required examining how participants talked about participation challenges and successes and how they drew on context to make sense of their own and each other's participation. There is only so much that can be learned by observing classroom participation from an outside perspective. Through one-on-one interviews with classroom participants the researcher gained a more comprehensive understanding of how barriers related to distance learning, gender, and race shaped students' day-to-day learning experiences.

Despite the school district's efforts to increase access to technology and other distance learning resources, many students and the teacher talked about how distance learning introduced new barriers to participation that did not exist for them during in-person instruction. Participants found it harder to build community in the online environment due to ineffective and inefficient communications. There were also a few students who talked about how distance learning increased pre-existing personal barriers to participation related to their anxieties and motivational struggles. On top of distance learning barriers, students from marginalized groups talked about how barriers related to race and gender affected their participation and the participation of their peers. Students talked about how underrepresentation of females and people of color in classes put a spotlight on them, increasing their anxiety and the pressure they felt to not make mistakes and to always be seen as smart.

A number of students fell short of Ms. B's participation expectations, such as Sarah (female student of color), Alison (White female student), and Diego (male student of color). All three of these students spoke about how distance learning barriers impeded their participation. Sarah also talked about race and gender, and Alison talked about gender and her lack of mathematical preparation for the course. Both female students shared that it had always been hard for them to participate in math classes, but distance learning made it even harder. Sarah also talked about how she was comfortable not participating because she was getting straight A's, whereas Alison was not happy with her grades and wished she was able to participate more. Ms. B's feelings were in fact opposite; she talked frequently about wanting desperately for Sarah to speak up more but did not say much about Alison's limited contributions. Diego spoke only about barriers to participation related to distance learning, but it is possible he experienced additional barriers that he did not mention during his interview. Ms. B wanted Diego to participate more, but her assessment of him was not nearly as critical as his assessment of himself. These three students faced substantial barriers to participation due to distance learning, gender, and/or race, but it seems they did not have the supports that some of their peers had. None of them were in the honors math class last year and none of them spoke about having a strong relationship with Ms. B.

Ms. B did not seem concerned about the six "successful" students highlighted in this chapter. In her eyes these students had figured out how to participate. However, hearing the students talk about their participation struggles and worries indicated many of them were still facing on-going obstacles impeding full participation. Interestingly, none of these strong participators talked about major hinderances due to distance learning. It is possible the positive relationships they had established with Ms. B prior to the distance learning year might have helped them circumvent and/or overcome some of the interactional challenges introduced by the virtual environment. But even the successful participators could not avoid all barriers. Students' identities had not changed. Sexism and racism had not gone away. It was just that some

students, like Guadalupe, Emma, and Yonas, had found ways to push through some of the barriers they faced, at least enough so they were able to contribute in seemingly productive ways.

When classroom participants talked about specific participation issues with their calculus classes, they talked primarily about problematic patterns related to gender. They talked most often about how male students contributed more often than female students. Sometimes this issue was framed with respect to the male students as "male dominance" (Period 1) and other times the issue was framed with respect to the female students as "girls not speaking up enough" (Period 2). It is not completely clear why the frame of reference was different for the two class periods, but it may have been connected to the individual students mentioned most often in observations about participation. In Period 1, Caleb (White male student) was the student spoken about most often during debrief sessions between the teacher and student teacher. Both Ms. B and Mr. K initiated conversations about him. Caleb was also mentioned by female students during their interviews as someone who talks too much during class. In Period 2, Sarah and Leah (both female students of color) were the students mentioned most often. Conversations revolved around Ms. B's frustrations that Sarah and Leah were not sharing their ideas. These were the girls whom Ms. B referred to as "having the skills." Zoe also identified Sarah and Leah during her interview as students who did not contribute as much as she thought they should. If these gendered claims were indeed prompted by observations of just a few students, it is then worth asking to what extent were these general claims true? Did male students dominate class discussions in Period 1 in general or was it just that Caleb dominated discussions? Were girls in Period 2 not speaking up enough in general or was it just that Leah and Sarah were not speaking up enough?

The difference in focus between male and female students may not at first seem like a big deal, but this shift in language positions students differently in terms of who is responsible for creating and solving the issue. Male dominance implies the responsibility lies with male students and the male students are the ones who need to change their behavior. On the other hand, girls not speaking up implies responsibility lies with the female students and they are the ones who need to change their behavior. There is a third option to consider. If participation issues are thought about as evidence of inequitable opportunities to participate which are co-constructed through interactions, then responsibility for creating and solving these issues lies on the shoulders of all participants engaged in classroom interactions, including verbal and non-verbal participants. Instead of having the issue reside with either male students or female students, responsibility for creating and solving the issues resides with the entire classroom community.

It is especially important to take note when participation issues are framed with respect to students from historically marginalized backgrounds because it is not fair to blame marginalized students for the sexist and racist systems in which they live and operate. In the case of Sarah and Leah, one question that comes to mind is if the teacher's participatory expectations for Sarah and Leah were higher than they were for other students because these two girls understood the content. And if so, is that fair to them? What responsibility do Sarah and Leah have to act in support of their peers or to give back to a system that has not necessarily treated them well? Given that their identities are still forming (which may be the case for decades to come), is it reasonable to ask Sarah and Leah to put others' needs ahead of their own? If they

knew the material and were confident in their abilities, not speaking up is in fact a reasonable and rational decision to make.

Participants seemed comfortable talking about participation patterns related to gender. Male and female students brought up this topic in one-on-one interviews without hesitation. The teacher talked about gendered patterns on a regular basis, during interviews, lesson debriefs, and with the students during class time (e.g., “Can we get a girl to guess?”). Ms. B often invoked her own personal experiences of sexism in mathematics in relation to the experiences of her female students. The student teacher reflected on the participation of female students as well. The dominance of male voices was a central theme in conversations between Ms. B and Mr. K as they brainstormed ways to restrain the voices of male students while also bolstering the voices of females. Gender was referenced regularly when articulating both participation issues and successes.

Compared to gender, participants rarely talked about participation patterns related to race. Guadalupe and Mia spoke to Ms. B at the beginning of the school year about female students of color not talking much during class discussions. The attention to unequal talk continued through to the end of the school year, but the “students of color” part was largely dropped as the primary focus of conversation became gender and not race. It was not completely clear whether the lack of mention was due to discomfort with the topic or the fact that participants did not notice patterns worth mentioning. The assumption is that both discomfort with the topic and lack of noticing contributed to the lack of mentions. Guadalupe and Hosein brought up the topic easily and without pause, but they were the only students who did so. When Yonas brought up the topic of race with the researcher, his verbal hesitation (“Well, ok. I think - This is - ok, yeah. Um.”) indicates discomfort with initiating talk about this topic. It seemed he wanted to share but was not sure if or how he should. Ms. B brought up the topic of supporting her students of color a few times, but when she did, she acknowledged not always knowing how best to support them. She seemed much more comfortable talking about supporting her female students because she could relate to their gendered experiences. Likewise, Ms. B suggested that Mr. K would be a great support for his students of color given his Mexican identity. It may not be accurate to say that Ms. B was uncomfortable talking about race, but it seemed to be harder for her to talk about the topic because she did not have firsthand experiences from which to draw. That being said, Ms. B shared her observations freely with Mr. K and the researcher, so it is quite likely that if she had seen problematic patterns based on race, she would have shared them. In addition, twice the researcher asked participants during interviews if they thought race influenced participation patterns in groupwork. Both Mr. K and Emma responded by saying they did not think race had played a role.

Chapter 5: Assessing Participatory Equity in Whole-Class Discussions

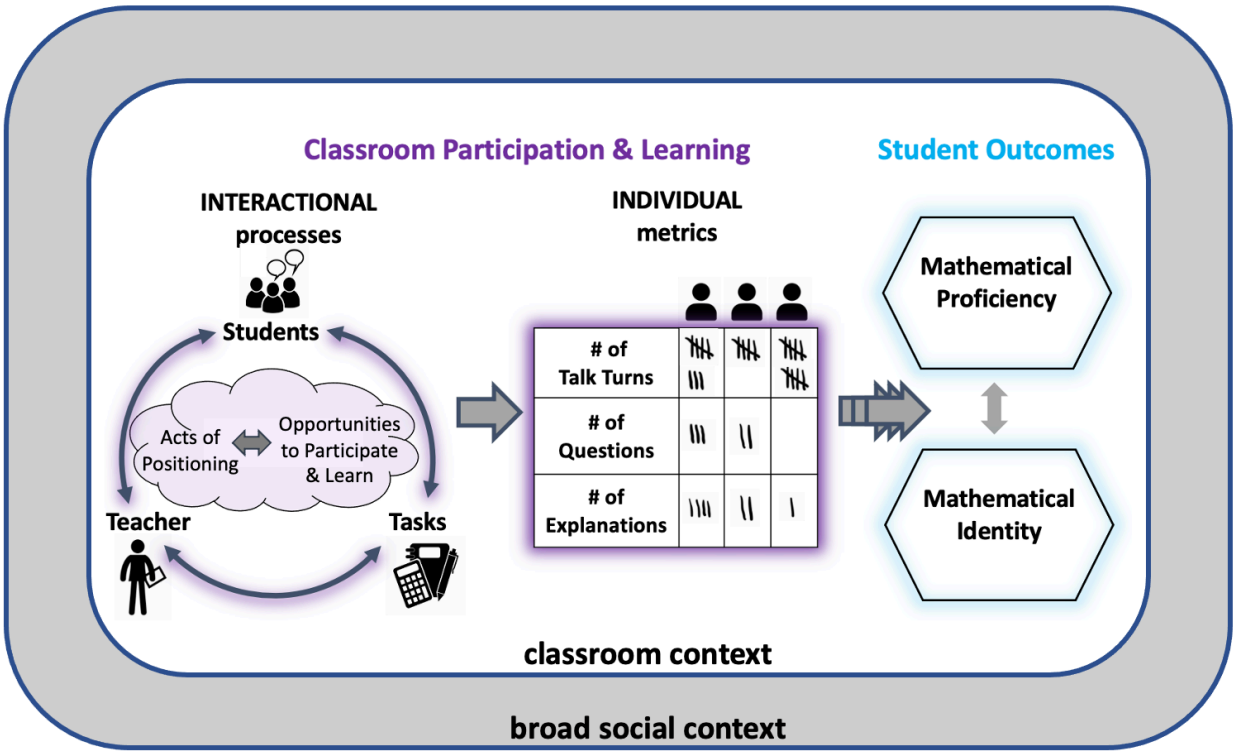
Introduction

Highly dialogic classrooms hold great potential as learning environments that encourage students to engage deeply with mathematics and with each other (e.g., Wells, 1999). But even in classrooms deemed as "active learning" or "inquiry-based," opportunities for participation can be constructed and distributed in ways that privilege some students over others (e.g., Esmonde, 2009; Reinholz et al., 2022), leading to inequitable learning experiences. Gendered and racialized patterns of participation in mathematics classrooms have been well-documented and continue to persist (Esmonde & Langer-Osuna, 2013; Gholson & Martin, 2019; Sengupta-Irving & Vossoughi, 2019), both in terms of representation (Cheryan et al., 2017) and in terms of classroom interactions (Langer-Osuna, 2011). There are teachers, like Ms. B, who are aware of these inequities and who work to address them through the pedagogical choices they make. But often it's hard to understand in what ways students' experiences are equitable (or inequitable) without taking a close look at students' contribution patterns over time and without hearing students' perspectives on classroom participation. Teachers' perceptions of what happens in their classes can be clouded by their own unconscious biases (Reinholz & Shah, 2018). The teacher may think male students are dominating class discussions, but are they? If so, in what ways? The teacher may think she is supporting contributions from all students, but is she? If so, in what ways?

The analyses presented in this chapter are intended to paint a clearer picture of how students in Ms. B's calculus course participated during whole-class discussions over one semester. The goal is to assess in what ways, to what extent, and from whose perspectives participation was equitable. In other words, who seemed to have opportunities to participate during whole-class discussions in ways that likely supported rich content knowledge and positive mathematical identities? By "who" I mean, which individual students and which groups of students (e.g., female students, male students of color). Participation in mathematics classrooms is more complicated than what can be captured through unilateral questions, such as: Did whole-class discussions provide equitable opportunities for female students (or for students of color, or …) to contribute? Intersectional forms of oppression are common in advanced mathematics classrooms (Leyva et al., 2021) and need to be understood in more detail. In addition, it is important to remember that students within the same intersectional group can experience the same classroom in different ways (Zavala, 2014), so it is important to think about students as individuals with varied backgrounds and beliefs who share some common experiences related to marginalization but who also have their own unique experiences.

Analyses in this chapter focus on individual participation metrics and interactional participation processes with implications related to students' mathematical proficiency and identity, as conceptualized in Figure 2a.

Figure 2a. Conceptual Framework for Studying Participatory Equity



Opportunities to participate and learn are constructed through classroom interactions involving students, teachers, and tasks. Students' contributions during mathematical discussions are a function of the opportunities they have to participate (Gresalfi et al., 2009), which can be quantified as individual participation metrics. Over time, classroom participation shapes how students understand content (mathematical proficiency) and how students come to see themselves and each other as learners and doers of mathematics (mathematical identity). Aspects of classroom context are addressed in Parts 1, 2, and 3 of this chapter. Participants' perspectives are the focus of Part 1, individual participation metrics are the focus of Part 2, and interactional participation processes are the focus of Part 3.

The overarching question guiding analysis in this chapter is: In what ways, to what extent, and from whose perspectives was classroom participation equitable during whole-class discussions in one high school calculus course? The specific empirical research questions are: 1) How did participants assess their experiences with student contributions during whole-class discussions? 2) How did students contribute during whole-class discussions over one semester? 3) How were students' opportunities to participate constructed through interactions? Together, the answers to these questions paint a more comprehensive picture of how individual students and groups of students contributed over the course of one semester in Ms. B's calculus classes. Taking a close look at participation patterns over time highlights consequential differences in students' opportunities to construct rich content understandings and to build positive mathematical identities. The multiple-perspective examination of student participation supports a more comprehensive understanding of participatory equity than found in the literature.

Chapter Overview

This chapter is intended to be read within the context of the whole dissertation. Therefore, the bulk of the literature, theory, and methodology framing these analyses are provided in the previous chapters. Before sharing the findings that are focused on whole-class contributions over one entire semester, I provide some information about the typical lesson structure for Ms. B's Zoom-based calculus classes. Findings are then presented in three parts. Part 1 is focused on classroom participants' experiences with whole-class contributions. Part 2 is focused on individual student participation metrics (e.g., # of total contributions, # of questions asked, # of questions declined) logged using the EQUIP tool (Reinholz & Shah, 2018). Part 3 is focused on interactional participation processes (e.g., how contributions were solicited during whole-class discussions). At the end of this chapter, findings are summarized, providing a significantly nuanced understanding of participatory equity in the course. Implications of these findings related to supporting more equitable opportunities for participation and learning in mathematics classrooms are presented in the Discussion section in Chapter 7.

Observations of "regular" class lessons served as the primary data source for this chapter, although data from participant interviews are also used to supplement observational findings. Wednesday help sessions were excluded from analysis because the nature of the interactions that took place during those sessions differed substantially from regular lessons, and help sessions were not the focus of this investigation. There were 62 lessons included in the Chapter 5 analyses (30 lessons from Period 1, 32 lessons from Period 2), which included contributions from 60 students (28 students from Period 1, 32 students from Period 2).

Typical Lesson Structure

Ms. B had taught this Calculus AB course numerous times before but had never taught it in a distance learning format. Despite the virtual mode, she tried her best to stay true to the structure she had used in the past. This year 60-minute "regular" class lessons were held on Mondays, Tuesdays, Thursdays, and Fridays. Ms. B's calculus lessons used the same basic structure on most days. Every day she greeted the class, "Good morning, everyone!", the sign that class was beginning. She expected every student to unmute and greet her back. After the greeting, she made announcements and then proceeded with homework sharing. Typically, Ms. B would have several homework problem numbers written on the board, signaling that these were the problems that would be discussed that day. If a student wanted to share their solution for one of the problems (and earn one extra credit point), they let Ms. B know, either before class or during this first part of the lesson. If no one volunteered for a specific problem, Ms. B did her best to solicit a volunteer. After each homework problem was explained, Ms. B would prompt the class to ask questions. Once all questions had been addressed, they moved on to the main lesson of the day.

The main lesson usually included a period of whole class discussion, followed by small-group work in Zoom breakout rooms, and then a second whole-class discussion to solidify and clarify the intended learning goals before the end of class. This was the standard lesson structure, although there was some variation depending on the content and lesson goals for any given day. For example, occasionally there would be days without homework sharing if no assignment was due that day (e.g., the day after a unit test), or some days had only a few minutes of whole-class time if the lessons were intended to be review time or group project days. There were also a few

days when students alternated between whole-class and small-group several times. In this chapter, the only "regular" lessons excluded from analysis were ones that did not include any whole-class student contributions, such as when students took individual tests that lasted the entire class period.

Part 1: Participants' Perspectives on Whole-Class Contributions

This part addresses the question: How did participants assess their experiences with student contributions during whole-class discussions? The perspectives of the teacher (Ms. B) and various focal students are included. Because the student teacher did not talk about this topic during interviews, his perspective is not shared in this section. During one-on-one interviews with the teacher, she was asked to share her thoughts on how, when, and why she solicited student contributions during whole-class discussions. During one-on-one interviews with students, they were asked to share their thoughts on how, when, and why they contributed during whole-class discussions.

Teacher's Perspective

Student contributions were an important part of discussions in Ms. B's 60-minute calculus lessons. Students made 767 whole-class contributions during the 30 observed lessons in Period 1 and 789 whole-class contributions during the 32 observed lessons in Period 2, averaging 25.1 contributions per lesson across the two class periods. Ms. B wanted her students to speak up and to engage in the lessons together as a class. In her words, "engagement equals learning… I really believe that. It's almost the same event for a person. It's almost indistinguishable." Ms. B talked about wanting students' voices to be more prominent in the discussions than her own. She wanted students to share their ideas and be the ones making mathematical decisions. When asked to describe herself as a teacher during an interview, the teacher shared the following.

> I would describe myself as an anarchist, which is a word that means a lot of things. People think it means violence, which is not what I mean at all. I mean, as in who is in charge and who is making decisions. Does it come from above (makes motion with hands above head) or does it come from here (makes motion with hands lower at chest level)? … where decision making is bubbling up from the teams of students about mathematics. And I think that's a wonderful experience for people. It helps them reason. It gives them strength.

Ms. B mentioned the benefit of including students' voices in discussions for both content learning ("it helps them reason") and student agency ("it gives them strength"). She felt there was a need for students to share their ideas publicly, explaining, "For almost everyone, if they have an opportunity to show their skills in front of the whole room, it really makes them feel a little god-like and they remember that forever."

However, Ms. B did not buy into district-prescribed methods that forced students to share at random times. She wanted students to feel comfortable speaking up on their own, as demonstrated by this interview excerpt.

> I feel very torn, and I always feel resentful of the propaganda that I get from the professional development at my school, where we are supposed to be random calling on people. They call it "equity" (makes air quotes with fingers), and I get defensive about

> that. I want the people who don't feel empowered - they kind of have to - like, Guadalupe's done it. I know I say her as an example too many times, but she has. She's figured out how to speak up, and her math skills are wonderful.

As she did in this example, Ms. B often referenced Guadalupe as a student who felt empowered enough to speak in class. Ms. B connected Guadalupe's speaking up with her having wonderful math skills. And more generally, Ms. B connected verbal contributions with students having ideas worth sharing. Even though she wanted students to speak up on their own, Ms. B also talked about how she solicited contributions by calling on students who had not volunteered. She explained the following.

> Ideally, you would call on students to share things if you knew they had something interesting to share, something valuable … Hopefully the lesson structure is posing a problem that is interesting enough and rich enough that there are multiple ways to approach it, and so valuable means that it's an idea that is productive but maybe not everyone has seen it. So, it's worth sharing because of its content and maybe giving people clues or ideas or directions. And it could be because it's correct or because a person is headed in a direction that is not that correct but is still illuminating.

Ms. B cared about facilitating productive mathematical discussions, but she also cared about who was participating in those discussions. She felt it was important to include a wide range of student voices.

> You can also out kids who are doing something interesting who are not necessarily visible to the class, which can be an issue of making sure the girls get to speak, making sure the students of color get to speak. If everything is working correctly, then you're doing all those things at the same time and you're able to notice enough that you can do both. You're creating two narratives. One is the mathematical reasoning narrative and the other is that there are all of these interesting ideas coming from unexpected corners of the classroom.

In addition to wanting her students to engage in rich mathematical discussions, Ms. B wanted to expand students' ideas regarding who the authors of insightful mathematical ideas were. She mentioned explicitly wanting to highlight the voices of female students and students of color who might be less "visible" in the class.

Soliciting student contributions through cold-calling can support students' content knowledge and identity development by giving students opportunities to put their thinking into words and demonstrate their mathematical competencies in front of their teacher and their peers. However, cold-calling students can be risky. Calling on an unsuspecting student to reveal their current thinking in front of their whole class can put that student in an uncomfortable situation and may not result in a viable opportunity for them to share their thinking. Classroom context shapes how the act of declining to answer a question positions students with respect to perceived mathematical competency. If discussions are focused primarily on correctness and efficiency, declining to share in response to a teacher's request can have a negative effect on

content and identity development and can leave the student feeling more insecure and less likely to share their ideas in the future. On the other hand, if discussions are focused primarily on generating an abundance of ideas from everyone in the class and exploring new and creative solving strategies without a set path in mind, declining to answer a question may hold less weight and have more of a neutral effect on perceptions of students' mathematical competencies.

Ms. B's classroom could be classified as somewhere in between these two characterizations. She cared about students developing deep mathematical understandings and positive mathematical identities more than efficiency; she believed having students build mathematical connections on their own and struggle through confusion was worth the extra time. She found joy in students' unexpected questions and novel solution strategies. She also cared about the rigor of the course and was forced to work within the constraints of the distance learning environment. Due to the reduced-load pandemic schedule, Ms. B had about half as many class sessions with her students as she had in previous years, but she felt it was important to at least introduce specific topics to her AB Calculus students before the end of the school year. During a lesson debrief session in April, Ms. B explained to her student teacher that she was trying to give students extra help that day because she felt the material was very challenging. Ms. B shared, "I was pretty happy that they were getting anywhere. Were you happy Mr. K or did you think it was unfair?" Mr. K replied, "I think that it's a lot of work for a Monday coming back [from spring break]." The teacher responded, "Yeah, the next few weeks are going to feel like that. I don't want to switch it. I just want them to see integrals before they graduate from high school." Ms. B felt it was important to keep pushing ahead with curriculum.

Ms. B tried to balance the need for productive curriculum trajectory with the desire to incorporate a variety of student voices, while also being aware of the dangers of cold-calling students. She talked about how it was much harder to know who to call on and when during distance learning, since she didn't have the same access to students' work as she did during in-person classes. In the following interview excerpt, Ms. B reflected on a time when Alison declined a question that she had called on her to answer.

> In person, [declining to answer] basically never happens. In that case [with Alison], I probably would have gotten enough information from her that I wouldn't have called on her because that declining is a very low - that's a bad - it leaves a student disabled more. It doesn't give them any strength at all.

From Ms. B's perspective, there were tradeoffs between the various methods for soliciting student contributions. Ideally, she wanted students to feel empowered enough to volunteer to share their ideas with the class. However, she knew that some students were not ready to do that on their own. She knew there were great ideas that would not be shared unless she called on students to share them. She was also aware that often those great ideas originated with female students and students of color.

Students' Perspectives

When students were asked during interviews about their contributions during whole-class discussions, most talked about presenting their homework solutions to the class for extra credit. Routinely, at the beginning of each class period, Ms. B asked for one or more volunteers to share their solutions for specific problems from the previous night's homework assignment.

Ms. B gave each student who shared an extra credit point. When interviewed in February, Andy (male, Vietnamese) could only recall one time so far that school year that he had offered a mathematical contribution during a whole-class discussion, and that contribution was related to homework sharing.

> This year I think the only time that I talked in front of the whole class was … I needed help. I couldn't figure out what to do with the homework, so I searched it up … Ms. B asked me about it, and I told her what I did in front of the class. But besides that, I don't really think there's been any time when I've talked in front of the whole class besides just reading out questions or something like that this year.

Andy acknowledged other contributions he had made, but described them as "just reading out questions," implying those contributions did not hold as much weight in his mind as sharing math solutions. Andy did not mention extra credit, but many other students did. Rebecca (female, White) shared, "The only time I participate in the whole class is if I'm doing the extra credit problem. And I think having the incentive of the extra credit pushes me to do it. Otherwise, I really wouldn't." Similarly, Sarah (female, Mexican & Middle Eastern) shared, "I think there was once, maybe twice, when I said that I'd do a homework problem, but that was because of the extra credit that I felt I needed." Mia (female, Iranian-American) wanted the extra point so badly that she orchestrated her homework sharing ahead of time. "I asked Ms. B the day before if I could get that one, but yeah, I'm glad I got it because I'm kind of worried about my grade."

The incentive of one extra point was enough to get some students to volunteer. For others, it was Ms. B's behind-the-scenes encouragement that was a critical factor. Alma (female, Nicaraguan & White) shared how Ms. B's words of support convinced her to volunteer.

> I did one of the extra credit problems … it was nice. Actually, Ms. B recommended - she was like, 'You should probably do this extra credit.' And I'm like, 'Ok, why not?' cuz I got it right, and I felt pretty confident about it, so it was - I don't know. After sharing with the class, I felt really proud of myself.

Alma's response to sharing in front of the class was what Ms. B was hoping for. It's what Ms. B described as feeling "a little god-like" and leaving a lasting impression. Sarah shared a similar experience, relaying how Ms. B's comments on her homework made her feel seen.

> I think when she writes on my homework, like, 'You should share this with the class tomorrow,' it definitely does feel nice, the recognition from her. I'm like, 'Ah, thank you. You see that I get it.' … She has been one of the first teachers, if not *the* first, to directly tell me, 'You know what you're doing, please share.' And I think in terms of participation for everyone, I think that kind of support is really cool. I think it's difficult over zoom, but yeah, I wish I had other teachers that did that.

This story is another indication that Ms. B's intentional actions to encourage students, specifically two female students of color in these cases, to share their ideas with the rest of the class were, at least to some degree, successful.

Nate (male, White) was the only student who talked about volunteering on a regular basis; he was also the student with the highest total number of contributions in the Period 2 class during the semester (113). “If I was really confident on the homework last night, I’m willing to share … I think I share a decent amount. I quite regularly volunteer to do a problem from the last night’s homework.” Nate shared quite often, but he described his sharing as being contingent on his feeling confident. The presumption is that if he did not feel confident with the material then he did not volunteer to share his ideas. Interestingly, Nate did not contribute at all during the whole-class lesson featured in the next section of this chapter, though he was present during the class.

One of the students who did contribute during the featured lesson was Zoe (female, White). She volunteered once to respond to one of Ms. B’s questions and volunteered a second time to ask a clarifying mathematical question. During an interview with Zoe, she talked about asking questions in class.

> For whole class, obviously you know, it’s pretty hard to participate. I mean sometimes I’ll ask questions, if I have questions, probably 50% of the time. I guess I choose to ask them or not ask them just depending - I don’t really know what goes on in my head. There definitely will be times when I have questions that I would ask them if it was a normal classroom and I could just like one-on-one quickly talk to the teacher, but because it’s in front of the whole class, I don’t. But I guess I do when I think it’s a good question and other people will have that question.

Zoe was the most frequent female contributor in the class (53 contributions for the semester); yet she still thought it was “pretty hard to participate … in front of the whole class.” However, asking questions “50% of the time” was relatively often, compared to how infrequently other students asked questions. In fact, some students, like Alison (female, White), said they never asked questions. Alison shared, “If I get lost, I will not ask a question. I’ve never been able to do that. It’s too - It’s so - It’s so incredibly hard to raise my hand in class.” Alison did her best to get her questions answered by talking to friends after class and using the internet, but for her, asking a question during class time was not a viable option.

Another topic related to whole-class participation that students talked about during their interviews was Ms. B’s use of cold-calling methods to solicit student contributions. A few students, like Sarah (female, Mexican & Middle Eastern), were comfortable enough with the strategy. Sarah explained, “I’ve been understanding [the material] the past couple of times she’s called on me, so I’ve had the answers and I felt totally fine about it.” Alma (female, Nicaraguan & White) was also fine with Ms. B calling on her, if she knew the answer.

> I remember a few days ago, Ms. B was calling on random people to share an equation or something … it was like the only time that I was like, please pick me because I actually know what to do … It was like the one time that I was really sure that I had everything right. I had figured it out with my group, and she didn’t call on me, which was fine. I’m fine with it, but I was like, ‘I’m ready to say something if you call on me.’ … I was at the point of volunteering if it was possible.

Alma was not just ok being called on in this case; she really *wanted* Ms. B to call on her. She wanted the opportunity to share her understanding with the rest of the class. Remember, Alma is the student who described feeling proud of herself after sharing an extra credit homework problem. Rebecca (female, White) was the only student who shared a balanced assessment of soliciting student contributions through cold-calling, expressing both pros and cons of the strategy.

> I have mixed feelings about [cold-calling], because if she calls on people randomly, then that insures that everyone is paying attention because they don't want to be called on and have no idea what's going on ... But at the same time, it can be scary to have it not be volunteer, because even if I am paying attention, sometimes I still don't understand and then that can be kind of embarrassing if I were to be called on and I don't know what to say. So, I understand both - I think both methods make sense, and I don't really know what I would prefer overall.

Rebecca acknowledged the value of the strategy in helping students stay focused, but also explained the discomfort involved in being called on and not knowing what to say.

Most students, however, were not in favor of Ms. B randomly calling on students to share during whole-class discussions. In fact, some students shared rather traumatic descriptions of their experiences of being called on. Despite expressing confidence with participating in math in general, Sophia (female, White) shared, "Randomly calling on people makes my anxiety go crazy. Every time my name is called makes my brain explode, even if I know the answer." Diego (male, Filipino) described the situation similarly by saying, "It's gut-wrenching when she calls on me." Alison (female, White) had the most dramatic description of all.

> I get so, so stressed out that I won't be able to focus and then I learn less if I know she might call on me. Then it's just - I don't take in information … In the classroom, if you call on me, I literally - I might start crying. I have started crying. It was this year. I had to leave the room. She called on me. That's it. And I didn't know it.

Alison's experience being called on, or even just the fear of being called on, was more intense than mere annoyance or discomfort. She described the emotional and physical toll it took on her and how her ability to learn was severely hampered. Alison is one of the students who was called on during the lesson featured in the next section. She was unable to answer the question posed to her by Ms. B and ultimately declined to answer the question. It is unclear if the story Alison recounted about leaving the room to cry was related to this particular lesson or a different lesson this school year. Alison shared this story during an interview that took place four days after the featured lesson.

Alison's anxiety related to being called on is likely shaped by her experiences in mathematics classes from years ago, new obstacles created through the distance learning environment, and the current classroom culture in Ms. B's AB calculus course. Ms. B wanted her students, especially female students, to "show their skills in front of the whole room." She wanted them to experience success with mathematics. Ms. B also shared that, "Ideally, you would call on students to share things if you knew they had something interesting to share,

something valuable." While soliciting students' brilliant ideas through cold-calling might seem likely to support content and identity development, this perspective also prompts the question, *What happens when students do not have ideas perceived as "interesting" or "valuable" to share?* Ms. B explained, "In that case [with Alison], I probably would have gotten enough information from her [in person] that I wouldn't have called on her." So, if the class was meeting in person, Alison might have avoided the embarrassment of having to decline the question, but she would not have had opportunities to contribute in mathematically meaningful ways.

Part 2: Individual Student Contributions (One Semester)

This part addresses the question: How did students contribute during whole-class discussions over one semester in Ms. B's two calculus periods? Student contributions were examined by documenting the total contributions and the type, length, and general content of each contribution made by groups of students (e.g., females vs. males, White female students vs. female students of color). Contribution analyses are presented in the first section at the course level (combining Period 1 and Period 2 together) to give a sense of the overall nature of student participation during Ms. B's two AB Calculus classes. The second section presents contributions by gender (male students vs. female students) to identify gender-related participation patterns. During interviews and lesson debrief sessions, classroom participants spoke often about their perceptions that male students dominated conversations and girls did not speak up enough. The gender group contribution analysis offers a data-driven perspective using contribution metrics to complement participants' perspectives on how student gender was related to student participation during whole-class discussions.

Although classroom participants focused much more on gender than race when talking about participation differences in their class, there were some references to White boys talking more and female students of color talking less. Regardless of what participants noticed and chose to share, it is worth looking at how contributions were distributed across racial groups as well as gender, since patterns of White dominance in mathematics classes are well documented in the literature (e.g., Martin, 2019). The second section in Part 2 presents contribution analyses that examined participation patterns in Ms. B's classes by racial category (students of color vs. White students). To acknowledge and investigate the complexity and multi-dimensionality of student identity, the third section in Part 2 presents contribution analyses by gender-race groups (e.g., male students of color vs. White male students). The goal of this final series of comparisons was to tease out participation differences between subgroups and to get a better idea of how gender and racial patterns interacted with one another.

Analysis indicated that Ms. B orchestrated highly dialogic whole-class discussions. Generally speaking, students had numerous opportunities to contribute in ways that likely supported the development of rich content knowledge and positive mathematical identities. A closer look at contributions by gender, by race, and by gender-race groups showed significant differences in how contributions were distributed between students. White dominance superseded gender dominance, with White males having the most robust opportunities to participate, followed by White females, and female students of color. Male students of color contributed far less often than any of the other gender-race groups. There were a few notable individual student exceptions to these patterns as well.

Course-Level Contribution Metrics

This section contains participation metrics aggregated at the course level (Period 1 + Period 2) to give a general sense of how students contributed during whole-class discussions. This information is mostly contextual in nature, helping to paint a picture of what student participation looked like in Ms. B's AB Calculus course. Patterns in contribution types, communication modes, contribution lengths, and contribution content are presented, along with presumed connections to student learning and identity development.

Student contributions during whole-class discussions were quantified by totaling the number of contributions made by students in both AB Calculus class periods during the second semester (spring 2021), shown in Table 5a.

Table 5a. Total and Mean Contributions for Both Class Periods

	Total # of Students	Total # of Contributions	# of Lessons Observed	Mean Contributions per Lesson	Mean Contributions per Student
Period 1 & Period 2	60	1556	62	25.1	25.9

Together, the 60 students made a total of 1556 contributions during whole-class discussions in semester 2, resulting in about 25 contributions per lesson and 26 contribution per student. The 28 students enrolled in the Period 1 calculus class made a total of 767 contributions during 30 observed lessons, and the 32 students enrolled in the Period 2 calculus class made a total of 789 contributions during 32 observed lessons. Both class periods included a relatively high number of student contributions; on average, students contributed about 25 times during every whole-class lesson discussion. These numbers are consistent with how the teacher talked about wanting students to share their ideas and be the ones "running the room."

Table 5b shows the break-down of student participation by contribution type for the combined class periods. The percentages in Table 5b represent the proportion of the contributions made during second semester whole-class discussions that were of that contribution type. For example, 72% of contributions made were responses to questions and 13% were asking questions. The table also includes the *projected* directional correlation with the developmental trajectories of students' mathematical content knowledge and mathematical identity for each contribution type. For example, if a student shared a solution (+++), the student's mathematical understandings are *assumed* to be developing in a strong positive direction (toward the teacher's content goals) and the student's mathematical identity is *assumed* to be developing in a strong positive direction (toward the goal of seeing themself and being seen by others as a valuable mathematical contributor). These assumptions are based on findings from previous studies of participation and learning in mathematics classrooms (e.g., Sengupta-Irving & Enyedy, 2015; Shah & Crespo, 2018; Turner et al., 2013).

Table 5b. Contribution Types for Both Class Periods

	Total # Students	Total # of Contributions	Contribution Types															
			Asked a Question		Responded to a Question		Shared a Solution		Identified a Mistake		Declined to Answer		Read Out Loud		Shared their Screen		Offered a Comment	
Period 1 & Period 2	60	1556	209	13%	1119	72%	87	6%	28	2%	53	3%	20	1%	12	1%	28	2%
			+		++		+++		+++		--		neutral		neutral		?	

Most student contributions were of types that had projected positive correlations with student learning and identity. Responding to questions was the prevalent contribution type (72%), with asking questions coming in second (13%). There were 87 contributions (6%), which included a student sharing and explaining a solution to a problem on the previous night's homework. Typically, one or two students shared at the beginning of each whole-class discussion. This type of contribution had a strong positive presumed correlation with content and identity development (+++), since the contributing student was positioned by the teacher as having a complete and mathematically valid solution worth sharing with the whole class. Identifying mistakes is another type of contribution presumed to have strong positive correlation with content and identity development (+++), since pointing out an error in their own or someone else's work positions the contributing student as having mathematical understandings that others may not have and as having the confidence to speak out about it.

The only type of contribution seen as having a negative correlation with learning and identity was a student declining to answer a question asked explicitly of them. Declined answers represented a small percentage of contributions (3%), though these events likely had a strong (negative) effect on students. From participants' reflections, it seems choosing not to answer a question in this class positioned students as lacking mathematical competence and lacking confidence. There were the fewest reading out loud contributions and screen sharing contributions, but those types of contributions can be considered neutral and relatively inconsequential in terms of student learning and identity development. And finally, there were 28 unprompted comments by students. The content varied (e.g., course logistics, math suggestions), and therefore, so did the relevancy of these comments for student learning and identity.

Table 5c contains information about the modes of communication used to make contributions, the length of contributions, and the general content of contributions for both class periods. Just as in Table 5b, the percent of total contributions is included for each category, as are projected correlations with learning and identity.

Table 5c. Contribution Mode, Length, and Content for Both Class Periods

	Total # Students	Total # of Contributions	Communication Mode						Contribution Length								Content of Talk							
			Spoke		Typed in Chat		Showed or Acted		0 Words		1-4 Words		5-20 Words		21+ Words		What		How		Why		Other	
Period 1 & Period 2	60	1556	1513	97%	21	1%	22	1%	22	1%	471	30%	703	45%	360	23%	695	45%	478	31%	252	16%	131	8%
			neutral						neutral		+		++		+++		+		++		+++		?	

Almost all contributions in both class periods were spoken (97%). Due to the distance learning format, this is not surprisingly. It is much harder for students to share ideas and explain their

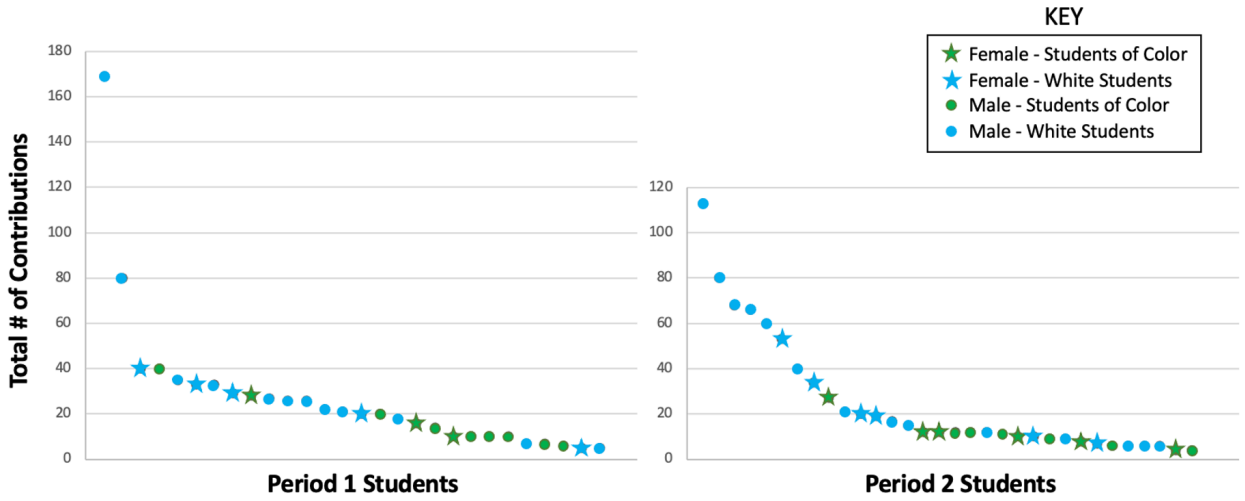
thinking by showing their work or writing out a solution if the class is using Zoom, as opposed to learning together in classrooms with document cameras and whiteboards. It is assumed the mode of communication chosen by the student had a neutral relationship with student learning and identity. For example, there was no discernable difference between a student choosing to say a response out loud, type a response in the Zoom chat window, or respond through non-verbal means. Mode of Communication is not included in analyses presented later in this chapter since there is so little variation in the data. It is included here to signal non-spoken modes of communication are worth investigating, but in these two distance learning classes, non-spoken communication was not documented often enough for there to be noteworthy comparisons between groups of students.

In general, longer student contributions are better than shorter contributions (Michaels et al., 2010), as represented in Table 5c by the increasing number of "+" for Contribution Length categories. Longer student explanations support higher levels of cognitive demand (Henningsen & Stein, 1997), which lead to more opportunities for students to solidify mathematical understandings and to see themselves and by seen by others as competent learners and doers of mathematics. For the reasons shared in the previous paragraph, non-verbal (0 word) contributions are assumed to have a neutral relationship with student learning and identity. Most contributions in both class periods were between 5 and 20 words in length (45%), while an additional 23% of contributions were even longer. The relatively high number of extended verbal contributions suggest students' voices (as opposed to the teacher's voice) occupied a considerable amount of talk time during whole class discussions. Ms. B's classes were not typical IRE (Initiate, Respond, Evaluate) classes (Cazden, 2001); students were encouraged to explain their ideas thoroughly. Data presented for Content of Talk (Table 5c) supports this claim as well. Nearly half of student contributions pertained to "how" or "why" questions (Braaten & Windschitl, 2011), implying that students were engaging with underlying mathematical concepts and not just procedural steps and final answers.

Overall, the frequencies and distributions across contribution types, lengths, and content suggest that Ms. B orchestrated highly dialogic whole-class discussions. Students (generally speaking) had numerous opportunities to contribute in ways that likely supported the development of rich content knowledge and positive mathematical identities. But "generally speaking" is not good enough. In pursuit of equitable participation, it is important to understand how opportunities for participation are distributed among students. Who is contributing and in what ways? How are opportunities to develop deep content understandings and positive mathematical identity distributed among students? In particular, it is important to understand the quantities and types of opportunities students of color and female students have to participate in mathematics classrooms. Students who identify as members of historically marginalized groups need to have as many, if not more, genuine opportunities to engage in rich mathematics as their privileged peers.

Figure 5a presents the total number of contributions for each student in the two class periods. Students are distinguished from one another both in terms of gender (female = star, male = dot) and racial category (students of color = green, White students = blue).

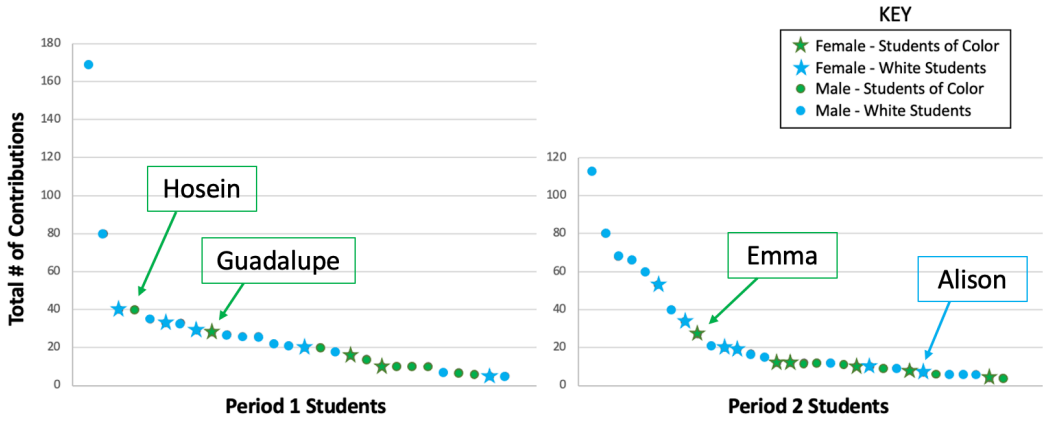
Figure 5a. Distribution of Total Contributions by Student and by Gender-Race Groups



A quick glance at the two charts in Figure 5a reveals an abundance of blue on the left sides and an abundance of green on the right sides. This color pattern indicates that, generally speaking, White students contributed more often than students of color in both class periods. In fact, 13 out of the 15 top contributors in Period 1 and in Period 2 were White students. The top two in each class were White males. Female students (represented by star symbols) seem to be spread out relatively evenly across the class periods, aside from the first few leading contributors.

There were a few student exceptions to this pattern of White dominance with whole-class contributions, highlighted in Figure 5b.

Figure 5b. Distribution of Total Contributions – A Few Exceptions to White Dominance



In Period 1, there was one male student of color (Hosein) who contributed 40 times during the semester, tying him for third-highest contributor in the class. Also in Period 1, there was one female student of color (Guadalupe) who contributed 28 times during the semester, the 9th highest contributor in the class. Both Hosein and Guadalupe are featured in the small-group task analysis in Chapter 6 and were included as participation success stories in the previous chapter. In Period 2, the one student of color that stands out as an exception is a female student of color (Emma) who contributed 27 times during the semester, the 9th highest contributor in the class.

Emma was also included as a participation success story in the previous chapter. These were the only three students of color who contributed more than 20 times during whole-class discussions in the second semester, compared to 21 White students.

Exceptions to the White dominance pattern came in the form of White students who contributed rarely during the semester. Three White students in Period 1 and four White students in Period 2 contributed seven or fewer times. Thoughts from one of these students, a White female student in Period 2 (Alison), were included in the previous chapter. She talked about how barriers related to distance learning, gender, and content preparation hindered her participation in the class. Alison is also featured later this chapter in Part 3 (Whole-class interactional analysis), as the one student who declined to answer a question during the focal discussion.

To get a better understanding of how opportunities for participation were distributed among sub-groups of students, the following sections include statistical analyses of contributions by student groups. For each grouping type (i.e., by gender, by race category, by gender-race group), findings related to total contributions are presented first, followed by findings related to contribution type, contribution length, and contribution content. To provide an overview of contributions, comparisons of *total contributions* for sub-groups are included with detailed explanations regardless of statistical significance. To focus attention on noteworthy patterns, summarized statistical results for only those additional variables found to be significant are included for *contribution type, length, and content*.

Contributions By Gender Groups

Overall, school enrollment was split about 50-50 between female and male students; however, female students accounted for only 33% of the students in Ms. B's Period 1 and Period 2 calculus classes. Enrollment numbers show female students were underrepresented in both class periods, but the questions being address in this section are, for the female students who were enrolled in the course, what kinds of opportunities did they have? How was their participation different (if at all) from their male peers?

During interviews and lesson debrief sessions, the teacher, student teacher, and students from both class periods shared observations about participation being unequally distributed between female and male students in their math classes. In the Period 1 class, unequal participation was usually framed as an issue of boys dominating the conversation. In contrast, unequal participation in the Period 2 class was often framed as an issue of girls not speaking up enough. Both framings indicate attention to and concern about male students contributing more to whole-class discussions than female students.

The analyses that follow address the question: To what extent and in what ways was participation during whole-class discussions equitably distributed between female[16] and male students in Ms. B's AB Calculus course? Findings indicate there were no significant differences of mean total contributions between female and male students, though there were four male outliers who contributed far more often than any female student, two from each class period. The only contribution type variables found to have significant differences in means between

[16] I choose to write "female and male," as opposed to "male and female," intentionally. Typically, the dominant group is written first to ground comparisons (Gutiérrez, 2018); my purpose in writing "female" first is to center the experiences of students from non-dominant groups.

female and male students were *Responded to Question* and *Declined to Answer*. Female students responded to questions less often than male students, and they declined to answer questions more often than male students. In addition, female students made fewer short (1 to 4 word) contributions than male students.

Total Contributions

Figures 5c (box plots) and 5d (descriptive statistics) show visually and numerically how total contributions compared for female and male students in Ms. B's two AB Calculus class periods. Observations are at the student-level and include all contributions made over the course of the second semester by each of Ms. B's 60 students.

Figure 5c. Box Plot of Total Contributions by Gender

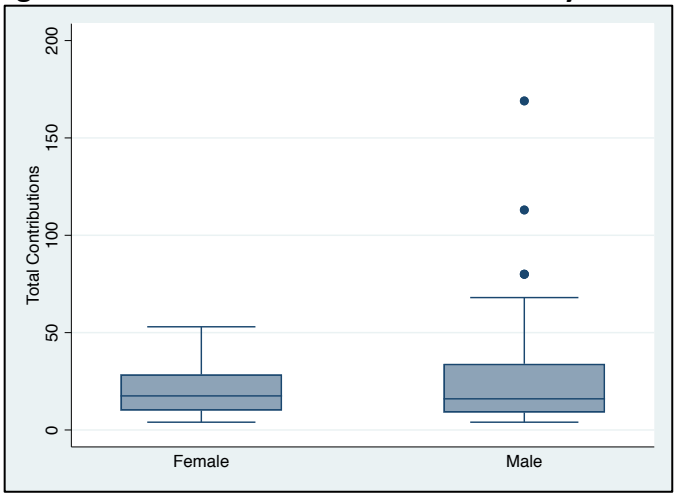


Figure 5d. Descriptive Statistics for Total Contributions by Gender

```
. tabstat totalcont, stat(n mean median sd min max) by(gender)

Summary for variables: totalcont
Group variable: gender (Gender)
```

gender	N	Mean	p50	SD	Min	Max
Female	20	19.85	17.5	13.15205	4	53
Male	40	28.975	16	33.91429	4	169
Total	60	25.93333	16.5	28.89306	4	169

Total contributions for female students ranged from 4 to 53, and for male students ranged from 4 to 169. Figure 5a shows there were three male outlier values. These three points represent four students: Caleb – 169 (Pd.1), Nate – 113 (Pd.2), Josh – 80 (Pd.1), and Chris – 80 (Pd.2). The outliers drove the mean higher for male students, 20 contributions for females vs. 29 contributions for males. However, the median (shown by the horizontal line in the shaded boxes in Figure 5c and the p50 values in Figure 5d) for females was in fact higher than the median for males. Without the outliers, total contributions for male students do not appear that different from total

contributions for female students, though the upper half of male contributions stretch slightly higher than female.

Before conducting a two-sample t-test to check for significant differences in the means, a robust equal variance test of total contributions by gender (Figure 5e) confirmed the two groups (females and males) had unequal variances.

Figure 5e. Robust Equal Variance Test for Total Contributions by Gender

```
. robvar totalcont, by(gender)

                |        Summary of TotalCont
         Gender |        Mean   Std. dev.       Freq.
----------------+------------------------------------
         Female |       19.85   13.152046          20
           Male |      28.975   33.914287          40
----------------+------------------------------------
          Total |   25.933333   28.893062          60

W0  =  5.1613839   df(1, 58)     Pr > F = 0.02681989

W50 =  1.9776441   df(1, 58)     Pr > F = 0.16497407

W10 =  2.4516987   df(1, 58)     Pr > F = 0.12283799
```

Since the F-statistic is less than 0.05 (P = 0.0268), we reject the null hypothesis that variances are equal. Output from a two-sample t-test with unequal variances is shown in Figure 5f.

Figure 5f. Two-Sample T-Test for Total Contributions by Gender

```
. ttest totalcont, by(gender) unequal

Two-sample t test with unequal variances

   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
  Female |      20       19.85    2.940887    13.15205    13.69465    26.00535
    Male |      40      28.975     5.36232    33.91429    18.12868    39.82132
---------+--------------------------------------------------------------------
Combined |      60    25.93333    3.730078    28.89306    18.46946     33.3972
---------+--------------------------------------------------------------------
    diff |            -9.125    6.115823               -21.37814    3.128145
------------------------------------------------------------------------------
    diff = mean(Female) - mean(Male)                              t =  -1.4920
H0: diff = 0                     Satterthwaite's degrees of freedom =  55.6542

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0707         Pr(|T| > |t|) = 0.1413          Pr(T > t) = 0.9293
```

The null hypothesis was that the mean total contributions for female students was equal to the mean total contributions for male students. Assuming $\alpha = 0.05$, the relatively high P-value (0.1413) indicates failure to reject the null hypothesis. The null hypothesis of the means being equal is highly probable given the data. These quantitative results indicate no significant difference in the mean total contributions between female and male students.

Contribution Types, Lengths, and Content

Table 5d summarizes the statistical output for the contribution variables found to be significant when comparing the means between female students and male students in Ms. B's calculus course.

Table 5d. Summary Statistics for Significant Contribution Variables by Gender

VARIABLE		Females			Males			Mean	Effect	95% CI		T-Value	df	P-Values		
		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
Type	Responded to Q	13.1	9.2	20	21.4	25.5	40	-8.3	0.38	-17.3	0.8	-1.82	54.3	0.0370	0.0740	0.9630
	Declined to Answer	1.3	1.3	20	0.68	0.9	40	0.62	0.59	-0.06	1.31	1.88	28.2	0.9645	0.0711	0.0355
Length	1 to 4 Words	4.7	4.8	20	9.4	13.5	40	-4.7	0.41	-9.49	0.0	-1.99	54.0	0.0261	0.0521	0.9739

Three contribution variables were found to have significant differences in means between female and male students, the number of times students responded to questions, declined to answer, and made short (1 to 4 word) contributions. The P-values for all three of these variables for Ha: diff !=0 (0.0740, 0.0711, 0.0521) did not meet statistical significance at the $\alpha = 0.05$ level, but the lower P-values for the one-sided tests (0.0370 for Ha: diff < 0, 0.0355 for diff > 0, 0.0261 for diff < 0) did reach statistical significance. These results suggest acceptance of the alternative hypotheses that the differences in means (female mean – male mean) are less than zero for *Responded to Question* and *1 to 4 Words* and more than zero for *Declined to Answer*. Female students were less likely than their male peers to respond to questions and make short (1 to 4 word) contributions. They were also more likely to decline to answer a question. The effect sizes for these variables fell within the medium range, with *Declined to Answer* having the largest effect out of the three.

Contributions By Race Groups

According to demographic statistics reported for 2021 in U.S. News and World Report, students of color at Evergreen High School accounted for 58% of student enrollment, including students who identified as "Hispanic," "Black," "Asian," and "Two or More Races." Eleven out of the 28 students in the Period 1 class (39.3%) and 12 out of the 32 students in the Period 2 class (37.5%) identified as students of color. Enrollment numbers show students of color were underrepresented in both classes, but the questions addressed in this section are, for the students of color who were enrolled in the course, what kinds of opportunities did they have? How was their participation different (if at all) from their White peers?

During interviews and lesson debrief sessions, the teacher, student teacher, and students from both class periods occasionally mentioned race, but it was typically mentioned with respect to issues of representation as opposed to issues of participation in the class. There were a few mentions of "White boys" dominating and of "female students of color" not talking enough, but there were no statements made about students of color (in general) versus White students (in general). No one said, "White students are talking a lot and students of color are not." But that doesn't mean these patterns did not exist, or even that no one noticed them. It simply means participants were not talking about them (with me).

The analyses that follow address the question: To what extent and in what ways was participation during whole-class discussions equitably distributed between students of color[17] and White students in Ms. B's AB Calculus course? Findings indicate there were significant differences in the means of total contributions between race categories. Specifically, it is highly probable that the mean for total contributions by students of color is lower than the mean for total contributions by White students. There were also statistical differences between students of color and White students in the means for some contribution types (i.e., Responded to Question, Identified Mistake), contribution lengths (i.e., 1 to 4 words, 5-20 words, 21+ words), and contribution content (i.e., what, how, why). The contribution numbers were lower for students of color than White students for all of these contribution categories.

Total Contributions

Figures 5g (box plots) and 5h (descriptive statistics) show visually and numerically how total contributions compared for students of color and White students in Ms. B's two AB Calculus class periods. Observations are at the student-level and include all contributions made over the course of the second semester by each of Ms. B's 60 students.

Figure 5g. Box Plot of Total Contributions by Race Category

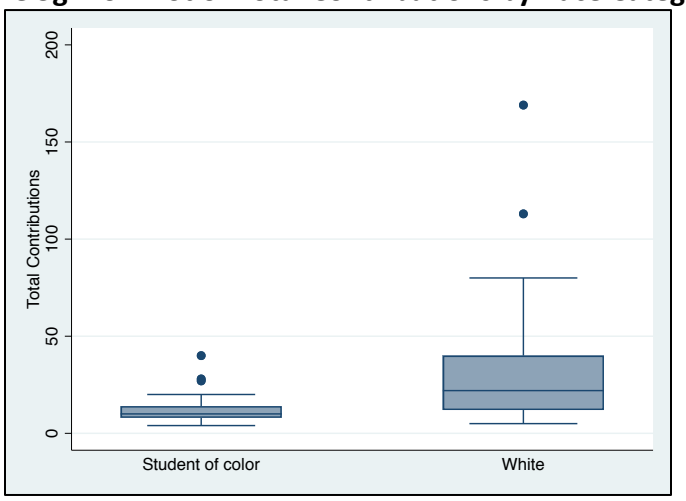


Figure 5h. Descriptive Statistics for Total Contributions by Race Category

```
. tabstat totalcont, stat(n mean median sd min max) by(race)

Summary for variables: totalcont
Group variable: race (Race)
```

race	N	Mean	p50	SD	Min	Max
SoC	23	12.95652	10	8.525247	4	40
White	37	34	22	33.901	5	169
Total	60	25.93333	16.5	28.89306	4	169

[17] I choose to write "students of color and White students," as opposed to "White students and students of color," intentionally. Typically, the dominant group is written first to ground comparisons (Gutiérrez, 2018); my purpose in writing "students of color" first is to center the experiences of students from non-dominant groups.

Total contributions for students of color ranged from 4 to 40, and for White students ranged from 5 to 169. Figure 5g shows there were three outlier values in the students of color group: Hosein – 40 (Pd.1), Guadalupe – 28 (Pd.1), and Emma – 27 (Pd.2). The White student outliers were the same two mentioned previously (Caleb and Nate). The outliers drove the means higher for both groups, but the mean total contributions for students of color was still considerably lower than for White students, 13 contributions vs. 34 contributions. The median was lower for students of color as well, 10 contributions vs. 22 contributions. The box plots in Figure 5g for the two groups look quite different from each other. Total contributions made by students of color are represented by a very thin squashed-looking box plot, while total contributions made by White students are represented by a much wider box with longer whiskers. In fact, the median total contributions for White students is about equal to the end of the top whisker for students of color.

Before conducting a two-sample t-test to check for significant differences in the means, a robust equal variance test of total contributions by race category (Figure 5i) confirmed the two groups (students of color and White students) had unequal variances.

Figure 5i. Robust Equal Variance Test for Total Contributions by Race Category

```
. robvar totalcont, by(race)

                |        Summary of TotalCont
           Race |        Mean   Std. dev.       Freq.
----------------+------------------------------------
            SoC |   12.956522   8.5252473          23
          White |          34      33.901          37
----------------+------------------------------------
          Total |   25.933333   28.893062          60

W0  = 11.2133378   df(1, 58)     Pr > F = 0.00143144

W50 =  6.8407183   df(1, 58)     Pr > F = 0.01133849

W10 =  8.9997462   df(1, 58)     Pr > F = 0.00397475
```

Since the F-statistic is less than 0.05 (P = 0.0014), we reject the null hypothesis that variances are equal. Output from a two-sample t-test with unequal variances is shown in Figure 5j.

Figure 5j. Two-Sample T-Test for Total Contributions by Race Category

```
. ttest totalcont, by(race) unequal

Two-sample t test with unequal variances

   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
     SoC |      23    12.95652    1.777637    8.525247    9.269928    16.64312
   White |      37          34     5.57329      33.901    22.69684    45.30316
---------+--------------------------------------------------------------------
Combined |      60    25.93333    3.730078    28.89306    18.46946     33.3972
---------+--------------------------------------------------------------------
    diff |           -21.04348    5.849919               -32.84121   -9.245751
------------------------------------------------------------------------------
    diff = mean(SoC) - mean(White)                                t =  -3.5972
H0: diff = 0                     Satterthwaite's degrees of freedom =  42.9697

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0004         Pr(|T| > |t|) = 0.0008          Pr(T > t) = 0.9996
```

The null hypothesis was that the mean total contributions for students of color was equal to the mean total contributions for White students. Assuming $\alpha = 0.05$, the extremely low P-value (0.0008) for Ha: diff !=0 indicates we should reject the null hypothesis that means between these two groups are equal. Further, the extremely low P-value (0.0004) for Ha: diff < 0 indicates we should accept the alternative hypothesis that the difference in means (SoC mean – White mean) is less than zero. There were significant differences in the means of total contributions between race categories. Specifically, it is highly probable that the mean for total contributions by students of color is lower than the mean for total contributions by White students.

Contribution Type, Length, and Content

Table 5e summarizes the statistical output for the contribution variables found to be significant when comparing the means between students of color and White students in Ms. B's calculus course.

Table 5e. Summary Statistics for Significant Contribution Variables by Race Category

VARIABLE		Students of Color			White Students			Mean	Effect	95% CI		T-Value	df	P-Values		
		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
	Total Contributions	13.0	8.5	23	34.0	33.9	37	-21.0	0.77	-32.8	-9.2	-3.6	43.0	0.0004	0.0008	0.9996
Type	Responded to Q	8.6	4.8	23	24.9	25.6	37	-16.3	0.80	-25.1	-7.6	-3.78	40	0.0003	0.0005	0.9997
	Identified Mistake	0.17	0.49	23	0.65	1.03	37	-0.5	0.55	-0.87	-0.08	-2.39	55	0.0101	0.0201	0.9899
Length	1 to 4 Words	2.9	2.7	23	10.9	13.7	37	-8.0	0.73	-12.7	-3.3	-3.46	40.4	0.0006	0.0013	0.9994
	5 to 20 Words	6.3	5.1	23	15.1	16.7	37	-8.8	0.65	-14.6	-2.8	-2.96	45.7	0.0024	0.0049	0.9976
	21+ Words	3.3	2.9	23	7.6	5.9	37	-4.3	0.86	-6.6	-2	-3.8	55.5	0.0002	0.0004	0.9998
Content	What	5.0	3.1	23	15.6	18.3	37	-10.6	0.73	-16.8	-4.4	-3.45	39.3	0.0007	0.0014	0.9993
	How	4.7	4.0	23	10.0	9.3	37	-5.3	0.69	-8.9	-1.9	-3.08	52.6	0.0016	0.0033	0.9984
	Why	1.7	2.3	23	5.8	6.4	37	-4.1	0.78	-6.4	-1.7	-3.51	48.8	0.0005	0.0010	0.9995

There were larger differences between race groups than between gender groups, and the differences between students of color and White students had higher significance values and effect sizes than between females and males. The means for total contributions and all the types,

lengths, and content listed in Table 5e were significantly lower for students of color than White students. For example, students of color responded to 8.6 questions on average during the semester, compared to 24.9 questions for White students, and students of color contributed a "why" explanation only 1.7 times, compared to 5.8 for White students. The effect sizes for all these variables fell into the medium to large range, with *Responding to Question* and *21+ Words* having the largest effects.

Contributions By Gender-Race Groups

Students' whole-class contributions were initially examined by gender groupings because classroom participants spoke most often about noticing gendered differences in student participation. However, some participants made references to participation patterns that aligned with intersectional race-gender categories (i.e., White boys talking more and female students of color talking less). This section examines how contributions made during whole-class discussions were distributed among four groupings of students: female students of color, White female students, male students of color, and White male students.

The analyses that follow address the question: To what extent and in what ways was participation during whole-class discussions equitably distributed among these four groups of students in Ms. B's AB Calculus course? Findings indicate there were no significant differences in the mean *Total Contributions* between female and male students within the two race categories, but there were significant differences between students of color and White students within both gender groups. Total contributions were significantly lower for female students of color when compared to White female students, and total contributions were significantly lower for male students of color when compared to White male students as well. There were also statistical differences between female students of color and White female students in the means for one contribution type (i.e., Responded to Question), one contribution length (i.e., 1 to 4 words), and one contribution content (i.e., what). Comparing male students of color with White male students showed that there were statistical differences for contribution types (i.e., Responded to Question, Identified Mistake), contribution lengths (i.e., 1 to 4 words, 5-20 words, 21+ words), and contribution content (i.e., what, how, why). There were also some differences between White student groups. Means were statistically different between White female students and White male students for some contribution types (i.e., Responded to Question, Declined to Answer), one contribution length (i.e., 1 to 4 words), and one contribution content (i.e., why). There were no statistical differences between the means of any of the contribution variables for female students of color and male students of color.

Total Contributions

Figures 5k (box plots) and 5l (descriptive statistics) show visually and numerically how total contributions compared for the four gender-race groups in Ms. B's two AB Calculus class periods. Observations are at the student-level and include all contributions made over the course of the second semester by each of Ms. B's 60 students.

Figure 5k. Box Plot of Total Contributions by Gender-Race Groups

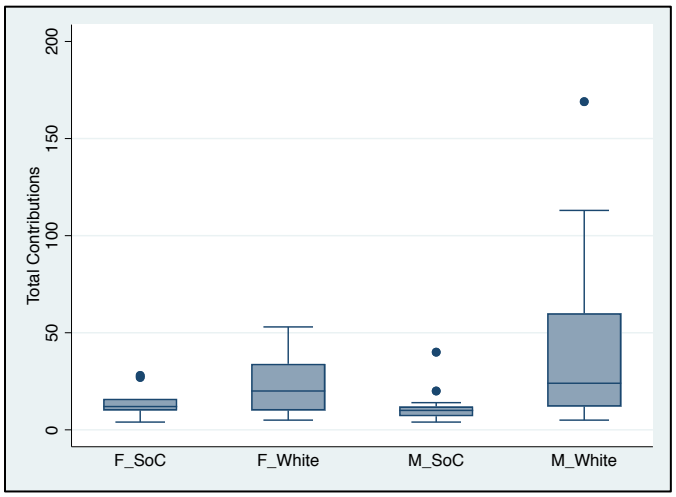


Figure 5l. Descriptive Statistics for Total Contributions by Gender-Race Groups

```
. tabstat totalcont, stat(n mean median sd min max) by(gen_race)

Summary for variables: totalcont
Group variable: gen_race (Gen_Race)
```

gen_race	N	Mean	p50	SD	Min	Max
F_SoC	9	14.11111	12	8.252946	4	28
F_White	11	24.54545	20	14.84159	5	53
M_SoC	14	12.21429	10	8.919974	4	40
M_White	26	38	24	38.87004	5	169
Total	60	25.93333	16.5	28.89306	4	169

All four groups contained at least one student who contributed as few as 4 or 5 times during the semester. Female students of color had the lowest maximum at 28 total contributions, followed by male students of color with a maximum of 40. White female students came next with a maximum of 53, and White male students had the highest maximum at 169. Figure 5k shows outliers in three out of the four groups, although the outlier values in the female and male students of color groups are all within the box (between the first and third quartile values) for the White male group. Even though the female students of color group had the lowest maximum, it was the male students of color group that had the lowest mean (12 total contributions) and the lowest median (10 contributions). The White male group had a mean considerably higher than all other groups (38 contributions) and a median only a little higher than the White female group (24 contributions vs. 20). The box plots in Figure 5k for the two students of color groups look similar to each other with short whiskers and very thin boxes around their relatively low medians. One the other hand, the two groups of White students look somewhat similar to each other, yet quite different from the students of color groups. The boxes for the White student groups are much wider and the upper whiskers longer than the students of color groups, with the White male group being wider and longer than the White female groups. In fact, the median

values for both the White female group and White male group are higher than the high end of the whiskers for both students of color groups.

Before conducting a two-sample t-test to check for significant differences in the means, a robust equal variance test of total contributions by gender-race groups (Figure 5m) confirmed the four groups (female students of color, White female students, male students of color, and White male students) had unequal variances.

Figure 5m. Robust Equal Variance Test for Total Contributions by Gender-Race Groups

```
. robvar totalcont, by(gen_race)

                |        Summary of TotalCont
   Gen_Race     |        Mean   Std. dev.       Freq.
----------------+------------------------------------
      F_SoC     |   14.111111   8.2529456           9
    F_White     |   24.545455   14.841588          11
      M_SoC     |   12.214286   8.9199739          14
    M_White     |          38    38.87004          26
----------------+------------------------------------
      Total     |   25.933333   28.893062          60

W0  =  6.9822272   df(3, 56)     Pr > F = 0.00045082

W50 =  3.3924568   df(3, 56)     Pr > F = 0.02405512

W10 =  4.4742978   df(3, 56)     Pr > F = 0.00692778
```

Since the F-statistic is less than 0.05 (P = 0.0005), we reject the null hypothesis that variances are equal. Output from a series of two-sample t-tests with unequal variances are shown in Figure 5n (students of color by gender), Figure 5o (White students by gender), Figure 5p (female students by race category), and Figure 5q (male students by race category).

Figure 5n. Two-Sample T-Test for Total Contributions: Students of Color by Gender

```
.  ttest totalcont if race=="SoC", by(gender) unequal

Two-sample t test with unequal variances

   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
  Female |       9    14.11111    2.750982    8.252946    7.767336    20.45489
    Male |      14    12.21429    2.383963    8.919974    7.064046    17.36453
---------+--------------------------------------------------------------------
Combined |      23    12.95652    1.777637    8.525247    9.269928    16.64312
---------+--------------------------------------------------------------------
    diff |            1.896825    3.640217               -5.744728    9.538378
------------------------------------------------------------------------------
    diff = mean(Female) - mean(Male)                              t =   0.5211
H0: diff = 0                     Satterthwaite's degrees of freedom =  18.2081

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.6957         Pr(|T| > |t|) = 0.6086          Pr(T > t) = 0.3043
```

Figure 5o. Two-Sample T-Test for Total Contributions: White Students by Gender

```
. ttest totalcont if race=="White", by(gender) unequal

Two-sample t test with unequal variances

   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
  Female |      11    24.54545    4.474907    14.84159    14.57474    34.51617
    Male |      26          38    7.623042    38.87004    22.30005    53.69995
---------+--------------------------------------------------------------------
Combined |      37          34     5.57329      33.901    22.69684    45.30316
---------+--------------------------------------------------------------------
    diff |           -13.45455    8.839432               -31.40227    4.493179
------------------------------------------------------------------------------
    diff = mean(Female) - mean(Male)                              t =  -1.5221
H0: diff = 0                     Satterthwaite's degrees of freedom =  34.8521

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0685         Pr(|T| > |t|) = 0.1370          Pr(T > t) = 0.9315
```

Figure 5p. Two-Sample T-Test for Total Contributions: Female Students by Race Category

```
. ttest totalcont if gender=="Female", by(race) unequal

Two-sample t test with unequal variances

   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
     SoC |       9    14.11111    2.750982    8.252946    7.767336    20.45489
   White |      11    24.54545    4.474907    14.84159    14.57474    34.51617
---------+--------------------------------------------------------------------
Combined |      20       19.85    2.940887    13.15205    13.69465    26.00535
---------+--------------------------------------------------------------------
    diff |           -10.43434    5.252875               -21.56374    .6950488
------------------------------------------------------------------------------
    diff = mean(SoC) - mean(White)                                t =  -1.9864
H0: diff = 0                     Satterthwaite's degrees of freedom =  16.1105

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0321         Pr(|T| > |t|) = 0.0643          Pr(T > t) = 0.9679
```

Figure 5q. Two-Sample T-Test for Total Contributions: Male Students by Race Category

```
. ttest totalcont if gender=="Male", by(race) unequal

Two-sample t test with unequal variances

   Group |     Obs        Mean    Std. err.   Std. dev.   [95% conf. interval]
---------+--------------------------------------------------------------------
     SoC |      14    12.21429    2.383963    8.919974    7.064046    17.36453
   White |      26          38    7.623042    38.87004    22.30005    53.69995
---------+--------------------------------------------------------------------
Combined |      40      28.975     5.36232    33.91429    18.12868    39.82132
---------+--------------------------------------------------------------------
    diff |           -25.78571    7.987118               -42.10719   -9.464243
------------------------------------------------------------------------------
    diff = mean(SoC) - mean(White)                                t =  -3.2284
H0: diff = 0                     Satterthwaite's degrees of freedom =   29.585

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0015         Pr(|T| > |t|) = 0.0030          Pr(T > t) = 0.9985
```

The null hypothesis was that the mean values for total contributions for the two selected groups were equal. Assuming $\alpha = 0.05$, the relatively high P-values for the first two t-tests (0.6086 for female and male students of color, 0.1370 for White female and male students) indicate failure to reject the null hypothesis. The null hypotheses that means were equal for students of color by gender and White students by gender are highly probable given the data.

However, the last two t-tests tell a different story. The P-value (0.0643) for Ha: diff !=0 for *female* students by racial category did not meet statistical significance at the $\alpha = 0.05$ level, but the low P-value (0.0321) for Ha: diff < 0 did. These results indicate we should accept the alternative hypothesis that the difference in means for female students (female SoC mean – female White mean) was less than zero. For *male* students by racial category, the P-values for Ha: diff !=0 (0.0030) and Ha: diff < 0 (0.0015) both met statistical significance at the $\alpha = 0.05$ level. These results indicate rejection of the null hypothesis and acceptance of the alternative hypothesis that the difference in means for male students (male SoC mean – male White mean) was less than zero.

These results indicate there were no significant differences in the mean total contributions between female and male students within the two race categories, but there were significant differences between students of color and White students within both gender groups. Total contributions were significantly lower for female students of color when compared to White female students, and total contributions were significantly lower for male students of color when compared to White male students as well.

Contribution Type, Length, and Content

Just as with the *Total Contribution* t-tests in the previous section, gender-race analyses for *Contribution Types, Lengths, and Content* were conducted as four separate comparisons: female students of color compared to White female students, male students of color compared to White male students, female students of color compared to male students of color, and White female students compared to White male students. There were no statistically significant differences in the means of contribution variables between female students of color and male students of color, so the differences that presented as "gender differences" (e.g., female students declined to answer more often than male students) were only significant within the White student sub-group. Results for each of the comparisons are presented separately, excluding the comparison between female students of color and male students of color since there is nothing further to report.

Table 5f summarizes the statistical output for the contribution variables found to be significant when comparing the means between female students of color and White female students in Ms. B's calculus course.

Table 5f. Summary Statistics for Significant Contribution Variables, Females by Race Category

VARIABLE		Female						Mean	Effect	95% CI		T-Value	df	P-Values		
		Students of Color			White Students											
		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
	Total Contributions	14.1	8.3	9	24.5	14.8	11	-10.4	0.84	-21.6	0.7	-1.99	16.1	0.0321	0.0643	0.9679
Type	Responded to Q	9.2	4.4	9	16.4	10.9	11	-7.2	0.83	-14.9	0.6	-1.98	13.6	0.0341	0.0682	0.9659
Length	1 to 4 Words	2.2	2.4	9	6.7	5.3	11	-4.5	1.06	-8.3	-0.7	-2.51	14.6	0.0122	0.0245	0.9878
Content	What	4.7	3.3	9	10.5	8.2	11	-5.8	0.89	-11.7	-0.1	-2.18	13.6	0.0238	0.0476	0.9762

The P-values for *Total Contributions* (0.0643, discussed previously) and *Responded to Question* (0.0682) for Ha: diff !=0 did not meet statistical significance at the $\alpha = 0.05$ level, but the lower P-values for the one-sided tests (0.0321, 0.0341) for Ha: diff < 0 did reach statistical significance. These results suggest acceptance of the alternative hypotheses that the differences in means (female SoC mean – female White mean) were less than zero. The P-values for *1 to 4 Words* and *What* contributions did meet statistical significance at the $\alpha = 0.05$ level. Female students of color were less likely than their White female peers to contribution in general, to respond to questions, to make short (1 to 4 word) contributions, and to make "what" contributions. Effect sizes for all four variables were greater than 0.8, and therefore can be considered large effects, with short contributions having the largest effect.

Table 5g summarizes the statistical output for the contribution variables found to be significant when comparing the means between male students of color and White male students in Ms. B's calculus course.

Table 5g. Summary Statistics for Significant Contribution Variables, Males by Race Category

VARIABLE		Male						Mean	Effect	95% CI		T-Value	df	P-Values		
		Students of Color			White Students											
		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
	Total Contributions	12.2	8.9	14	38.0	38.9	26	-25.8	0.81	-42.1	-9.5	-3.23	29.6	0.0015	0.0030	0.9985
Type	Responded to Q	8.1	5.2	14	28.5	29.2	26	-20.4	0.85	-32.5	-8.3	-3.47	27.8	0.0009	0.0017	0.9991
	Identified Mistake	0.07	0.27	14	0.62	1.06	26	-0.6	0.63	-1.0	-0.1	-2.47	30.4	0.0096	0.0192	0.9904
Length	1 to 4 Words	3.4	2.8	14	12.7	15.7	26	-9.3	0.72	-15.8	-2.8	-2.94	27.9	0.0032	0.0065	0.9968
	5 to 20 Words	6.0	5.6	14	16.7	19.3	26	-10.7	0.67	-19.0	-2.5	-2.64	31.9	0.0064	0.0127	0.9936
	21+ Words	2.6	2.2	14	8.2	6.5	26	-5.6	1.03	-8.5	-2.8	-4.01	34.1	0.0002	0.0003	0.9998
Content	What	5.3	3.1	14	17.8	12.0	26	-12.5	0.73	-21.1	-3.9	-2.98	27.0	0.0030	0.0059	0.9970
	How	4.3	4.1	14	11.1	10.7	26	-6.8	0.76	-11.6	-2.0	-2.87	35.4	0.0034	0.0068	0.9966
	Why	1.1	2.2	14	6.8	7.2	26	-5.7	0.95	-8.8	-2.5	-3.67	32.7	0.0004	0.0008	0.9996

There were more significant differences between students of color and White students for males than there were for females. The contribution variables found to be significant when comparing means for male students of color and White male students were the same variables that were significant in the race category comparison of all students of color compared to all White students (Table 5e). The means for total contributions and all the types, lengths, and content listed in Table 5g, were significantly lower for male students of color than White male students. For example, male students of color responded to 8.1 questions on average during the semester, compared to 28.5 questions for White male students, and male students of color contributed a "why" explanation only 1.1 times, compared to 6.8 for White male students. All these variables had

medium-large to large effect sizes, with *21+ words* and *Why* contributions having the largest effects.

Table 5h summarizes statistical output for the contribution variables found to be significant when comparing the means between White female students and White male students in Ms. B's calculus course.

Table 5h. Summary Statistics for Significant Contribution Variables, White Students by Gender

VARIABLE		White Students						Mean Diff.	Effect Size	95% CI		T-Value	df	P-Values		
		Female			Male											
		Mean	SD	N	Mean	SD	N			LL	UL			diff < 0	diff!=0	diff > 0
Type	Responded to Q	16.4	10.9	11	28.5	29.2	26	-12.1	0.48	-25.6	1.2	-1.84	34.8	0.0369	0.0738	0.9631
	Declined to Answer	1.64	1.3	11	0.58	0.90	26	1.1	1.03	0.2	2.0	2.49	14.3	0.9871	0.0258	0.0129
Length	1 to 4 Words	6.7	5.3	11	12.7	15.7	26	-6.0	0.44	-13.0	1.1	-1.72	34.1	0.0474	0.0948	0.9526
Content	Why	3.3	2.8	11	6.8	7.2	26	-3.5	0.55	-6.9	-0.2	-2.14	34.9	0.0197	0.0393	0.9803

There were no significant differences between White female and White male students for Total Contributions, but there were a few significant differences with other contribution variables. Just as with the overall gender comparison, the P-values for *Responded to Question* and *1 to 4 Words* for Ha: diff !=0 (0.0738, 0.0948) did not meet statistical significance at the $\alpha = 0.05$ level, but the lower P-values for Ha: diff < 0 (0.0369, 0.0474) did reach statistical significance. These results suggest acceptance of the alternative hypotheses that the differences in means were less than zero for *Responded to Question* and *1 to 4 Words*. Declined to Answer was statistically significant at $\alpha = 0.05$, unlike for the general gender comparison, meaning White females in particular were much more likely to decline to answer when compared to White males. The effect size of 1.03 supports this conclusion. In addition, there were also significant differences in means for "why" contributions; White female students were less likely to make why contributions than their White male peers.

Contribution Metric Summary

Classroom participants (teacher, student teacher, and students) talked about problematic participation patterns in the calculus classes related to gender but rarely mentioned participation patterns related to race. The quantitative analyses of student contribution metrics presented in this chapter tell a somewhat different story about whole-class participation in Ms. B's calculus classes, highlighting patterns related to race more than gender. Table 5i summarizes findings from the preceding sections in Part 2. For each comparison group, the table highlights the contribution variables found to have significantly different means, the types of significance, and the relative effect sizes.

Table 5i. Summary of Statistically Significant Contribution Variables by Comparison Group

VARIABLE		COMPARISON GROUPS: All Students by Gender	All Students by Race	Female Students by Race	Male Students by Race	Students of Color by Gender	White Students by Gender
	Total Contributions		White > SoC	White > SoC	White > SoC		
Type	Responded to Q	Male > Female	White > SoC	White > SoC	White > SoC		Male > Female
	Declined to Answer	Female > Male					Female > Male
	Identified Mistake		White > SoC		White > SoC		
Length	1 to 4 Words	Male > Female	White > SoC	White > SoC	White > SoC		Male > Female
	5 to 20 Words		White > SoC		White > SoC		
	21+ Words		White > SoC		White > SoC		
Content	What		White > SoC	White > SoC	White > SoC		
	How		White > SoC		White > SoC		
	Why		White > SoC		White > SoC		Male > Female

KEY	
	significant at α=0.05 for Ha: diff < 0 or Ha: diff > 0, but not Ha: diff !=0
(no border)	significant at α=0.05 for Ha: diff !=0
	small to medium effect size (0.3 <= Effect Size < 0.5)
	medium to large effect size (0.5 <= Effect Size < 0.8)
	large effect size (0.8 <= Effect Size)

Findings suggest *White* dominance superseded *male* dominance. The top contributors in both class periods were indeed male, but they were also White. When considering the total contributions made by all students in both classes, the Whiteness of contributors was statistically significant but not the maleness. There were some statistical differences by gender, but there were more contribution variables found to have significantly different means when comparing race categories. In addition, the variables identified as significant in race comparisons had greater significance levels and larger effect sizes than for gender comparisons.

Analysis of gender-race groups showed a more nuanced, and consequentially different, participation story. The contribution differences identified in the "All Students by Gender" comparison were found to be significant only for *White* students by gender. For example, White female students (like Alison) were more likely to decline to answer questions than their White male peers. There were no significant differences by gender for students of color. The contribution differences identified in the "All Students by Race" comparison were found to be significant for both female and male students, though there were more differences for males.

Overall, these quantitative contribution analyses indicate White male students had the most robust opportunities to contribute during whole-class discussions, followed by White female students, female students of color, and finally male students of color. These findings highlight the importance of going beyond unilateral analyses examining only gender or race, and instead conducting intersectional analyses of students' experiences in mathematics.

Part 3: Interactional Participation Processes (One Semester + One Discussion)

This part addresses the question: How were students' opportunities to participate constructed through whole-class interactions? Analyses shared focus on how interactional participation processes played out among classroom participants and how individual student participation metrics were shaped by these interactional participation processes. Specifically, Part 3 examines how interactions among participants supported (or inhibited) students'

contributions through the construction of opportunities to participate. Opportunities to participate were examined by looking at how the teacher solicited student contributions (i.e., through cold-calling students by name and accepting volunteers) over the course of the semester, in addition to looking closely at how contributions were solicited during a representative 40-minute whole-class discussion in one of the calculus classes.

From the analyses of individual participation metrics in Part 2, we know there were consequential differences in the numbers and types of contributions between genders, racial categories, and gender-race groups. The question now is: What can we learn about how and why this happened? This is a huge, complicated question, which cannot be answered fully or evenly nearly within the scope of this project. However, we can address part of the question by looking at how contributions were solicited during whole-class discussions. Findings suggest that Ms. B broadened some students' opportunities for participation by encouraging students (either generally or specifically) to volunteer and by calling on individual students to contribute to whole-class discussions. Despite Ms. B's intentional efforts to diversify student contributors, White voices, and to some degree male voices, still dominated discussions due in large part to the overwhelming number of voluntary contributions they made, with only a few exceptions. Just as in Part 2, particular attention is paid to how interactional participation patterns were aligned to students' gender and racial identities.

Spring Semester Overview

Over the semester, Ms. B used a combination of accepting volunteers and cold-calling students (i.e., calling on a specific student by name who did not volunteer to share) to solicit student contributions during whole class discussions in her calculus classes. The distributions of solicitations through calling on students versus asking for volunteers were similar across the two calculus class periods. Table 5j displays the distribution of solicitations by solicitation type (Called On vs. Volunteered) for each class period. As a reminder, "contributions" include responses regardless of perceived mathematical correctness and responses by students who declined to answer the question. Table 5j includes all solicitations that resulted in a student contribution. Solicitations made by the teacher to which no one responded (e.g., "Does anyone want to ask a question?" followed by silence), did not result in student contributions and were not included in this table.

Table 5j. Contribution Solicitations for Both Class Periods

	Total # of Students	Total # of Contributions	Contribution Solicitations			
			Called On		Volunteered	
Period 1 & Period 2	60	1556	414	26.6%	1142	73.4%

Considerably more contributions were made by volunteers than by students who were called on by their teacher. Ms. B solicited contributions from volunteers about 2.5 to 3 times more often than she did through cold-calling individual students. Students (generally speaking) had numerous opportunities to contribute through voluntary solicitations. But as noted earlier, "generally speaking" is not good enough. In pursuit of equitable participation, it is important to understand how opportunities for participation are constructed through interactions and

distributed among students. Who is being solicited to contribute and in what ways? How do contribution solicitation patterns correlate with gender and/or race groups? How are students' opportunities to participate shaped by Ms. B's solicitation methods?

To get a better understanding of how opportunities for participation were distributed between subgroups of students, statistical analyses of contribution solicitations by student groups were performed. Findings related to *Called On* solicitations are presented first, followed by findings related to *Volunteered* solicitations. To provide an overview of solicitations by sub-group, box plots and descriptive statistics are included for each solicitation type (i.e., Called On, Volunteered). To focus attention on noteworthy patterns, summarized statistical results are included only for the comparison groups for which the solicitation variables were found to be significant. In the case of *Called On* solicitations, significant differences were found between female groups; White female students were called on more often than female students of color. In the case of *Volunteered* solicitations, significant differences were found between race categories (i.e., White students volunteered more than students of color), male groups (i.e., White males volunteered more than male students of color), and White student groups (i.e., White males volunteered more than White females).

In alignment with previous analyses, Figures 5r (box plots) and 5s (descriptive statistics) show visually and numerically how *Called On* contributions compared for the four gender-race groups in Ms. B's two AB Calculus class periods. Observations are at the student-level and include all contributions that resulted from *Called On* solicitations made over the course of the second semester by Ms. B's 60 students.

Figure 5r. Box Plot of Called-On Solicitations by Gender-Race Groups

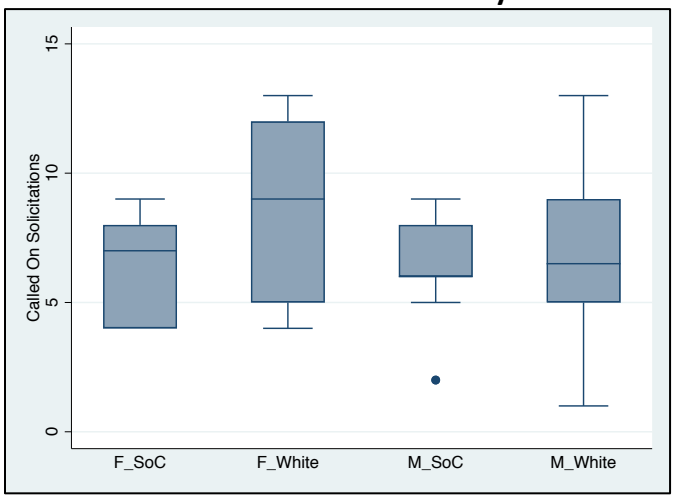


Figure 5s. Descriptive Statistics for Called On Solicitations by Gender-Race Groups

```
. tabstat sol_calledon, stat(n mean median sd min max) by (gen_race)

Summary for variables: sol_calledon
Group variable: gen_race (Gen_Race)
```

gen_race	N	Mean	p50	SD	Min	Max
F_SoC	9	6.222222	7	1.986063	4	9
F_White	11	8.454545	9	3.474583	4	13
M_SoC	14	6.428571	6	1.785165	2	9
M_White	26	6.730769	6.5	2.807956	1	13
Total	60	6.9	6.5	2.685176	1	13

Total *Called On* solicitations ranged from 4 to 13 for female students for the semester and ranged from 1 to 13 for male students. For students of color, the range was 2 to 9 and for White students the range was 1 to 13. The White males have the longest range, shown by the long whiskers in Figure 5r, and the White females have the biggest interquartile range, shown by the long box. The mean (8.5 *Called On* solicitations) and median (9 *Called On* solicitations) were highest for the White female group of students.

Just as with the contribution analyses presented in Part 2, a series of t-tests were run on contributions by solicitation type for the various comparison groups (e.g., females vs. males, White female students vs. female students of color) to see if any significant differences existed between the mean values. Table 5k summarizes the statistical output for the comparison of *Called On* contributions for female students of color and White female students, the only group for which there were notable differences.

Table 5k. Summary Statistics for Called On Solicitations, Female Students by Race

		Female													
		Students of Color			White Students			Mean	Effect	95% CI		T-Value	df	P-Values		
VARIABLE		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
Solicitation	Called On	6.2	2.0	9	8.5	3.5	11	-2.2	0.79	-4.9	0.4	-1.80	16.3	0.0451	0.0901	0.9549

The P-value for *Called On* (0.0901) for Ha: diff !=0 did not meet statistical significance at the $\alpha = 0.05$ level, but the lower P-value for the one-sided test (0.0451) for Ha: diff < 0 did reach statistical significance. These results suggest acceptance of the alternative hypotheses that the difference in means (female SoC mean – female White mean) was less than zero. Despite the marginal significance level, the effect size of 0.79 can be considered relatively large. Whole-class contributions were solicited through cold-calling from White female students more often than from female students of color.

Figures 5t (box plots) and 5u (descriptive statistics) show visually and numerically how *Volunteered* contributions compared for the four gender-race groups in Ms. B's two class periods. Observations are at the student-level and include all contributions that resulted from *Volunteered* solicitations made over the course of the second semester by Ms. B's 60 students.

Figure 5t. Box Plot of Volunteer Solicitations by Gender-Race Groups

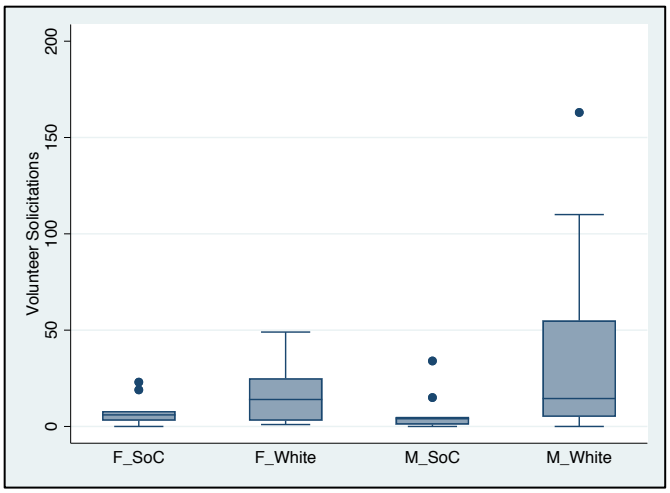


Figure 5u. Descriptive Statistics for Volunteer Solicitations by Gender-Race Groups

```
. tabstat sol_volunteer, stat(n mean median sd min max) by (gen_race)

Summary for variables: sol_volunteer
Group variable: gen_race (Gen_Race)
```

gen_race	N	Mean	p50	SD	Min	Max
F_SoC	9	7.888889	6	7.94425	0	23
F_White	11	16.09091	14	14.2229	1	49
M_SoC	14	5.785714	4	8.954402	0	34
M_White	26	31.26923	14.5	38.89839	0	163
Total	60	19.03333	7.5	28.80676	0	163

Visual differences in *Volunteered* contributions between gender-race groups are apparent in Figure 5t. The boxes for both female and male students of color are very narrow and hover just above the zero line, although each group does have several outliers. With a mean of 16.1 contributions, a median of 14.2 contributions, and a maximum of 49, the White female group stretches higher, but not nearly as high as the White male group. The White male group has a mean of 31.2, median, of 14.5 (just above White females) and a maximum well over 100.

Three group comparisons had significant differences in their means for Volunteered contributions. Table 5l summarizes the statistical output for the comparison between students of color and White students.

Table 5l. Summary Statistics for Volunteer Solicitations by Race

VARIABLE		Students of Color			White Students			Mean	Effect	95% CI		T-Value	df	P-Values		
		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
Solicitation	Volunteered	6.6	8.5	23	26.8	34.0	37	-20.1	0.74	-32.0	-8.3	-3.44	42.8	0.0007	0.0013	0.9993

The P-value for *Volunteered* (0.0013) for Ha: diff !=0 indicates clear statistical significance at the $\alpha = 0.05$ level, with 95% confidence the mean difference in voluntary contributions would fall between -32.0 and -8.3. The mean for *Volunteered* contributions was considerably lower for

students of color (6.6 volunteered contributions) than for White students (26.8 volunteered contributions), and the effect size for this variable can be considered medium to large. White students made voluntary contributions much more often than students of color.

Table 5m summarizes the statistical output for the comparison between male students of color and White male students.

Table 5m. Summary Statistics for Volunteer Solicitations, Male Students by Race

		Male													
		Students of Color			White Students			Mean	Effect	95% CI		T-Value	df	P-Values		
VARIABLE		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
Solicitation	Volunteered	5.8	9.0	14	31.3	38.9	26	-25.5	0.80	-41.8	-9.1	-3.19	29.6	0.0017	0.0034	0.9983

The P-value for *Volunteered* (0.0013) for Ha: diff !=0 indicates clear statistical significance at the $\alpha = 0.05$ level, with 95% confidence the mean difference would fall between -32.0 and -8.3. The mean for the number of *Volunteered* contributions was considerably lower for students of color (6.6 volunteered contributions) than for White students (26.8 volunteered contributions), and the effect size for this variable (0.8) can be considered large. White male students made voluntary contributions much more often than male students of color.

Table 5n summarizes the statistical output for the comparison between White female students and White male students.

Table 5n. Summary Statistics for Volunteer Solicitations, White Students by Gender

		White Students													
		Female			Male			Mean	Effect	95% CI		T-Value	df	P-Values		
VARIABLE		Mean	SD	N	Mean	SD	N	Diff.	Size	LL	UL			diff < 0	diff!=0	diff > 0
Solicitation	Volunteered	16.1	14	11	31.3	38.9	26	-15.2	0.45	-33.0	2.6	-1.73	34.6	0.0459	0.0917	0.9541

The P-value for *Volunteered* (0.0917) for Ha: diff !=0 did not meet statistical significance at the $\alpha = 0.05$ level, but the lower P-value for the one-sided test (0.0459) for Ha: diff < 0 did reach statistical significance. These results suggest acceptance of the alternative hypotheses that the difference in means (White female mean – White male mean) was less than zero. The effect size of 0.45 can be considered small to medium. White male students made voluntary contributions more often than White female students.

The quantitative analyses of contribution solicitations indicate few differences in the distribution of cold-calling solicitations but greater differences in responses to voluntary solicitations. And again, White dominance seems to supersede male dominance, although both are present. Table 5i summarizes findings from the spring semester overview of whole-class contribution solicitations. For each comparison group, the table highlights the solicitation type found to have significantly different means, the types of significance, and the relative effect sizes.

Table 5o. Summary of Statistically Significant Solicitation Variables by Comparison Group

		COMPARISON GROUPS					
VARIABLE		All Students by Gender	All Students by Race	Female Students by Race	Male Students by Race	Students of Color by Gender	White Students by Gender
Solicitation	Called On			White > SoC			
	Volunteered		White > SoC		White > SoC		Male > Female

KEY	
	significant at α=0.05 for Ha: diff < 0 or Ha: diff > 0, but not Ha: diff !=0
(no border)	significant at α=0.05 for Ha: diff !=0
	small to medium effect size (0.3 <= Effect Size < 0.5)
	medium to large effect size (0.5 <= Effect Size < 0.8)
	large effect size (0.8 <= Effect Size)

Even though the teacher did not use any formal mechanisms to dictate her decisions about who to call on, the distribution of cold-calling across gender and race groups was relatively balanced, with one exception. Ms. B tended to call on White female students a little more often than female students of color. The distribution of voluntary contributions across groups was much less balanced. The most significant differences were between students of color and White students, and more specifically between male students of color and White male students. White males volunteered contributions much more often than male students of color. The one case where gender was found to be significant was when comparing the voluntary contributions of White female and male students. White males volunteered more often than White females. Ms. B wanted her students to "feel empowered" to share their ideas. She wanted them to "show their skills in front of the whole room… [and] feel a little god-like." But the reality was that only a subset of students in her classes volunteered to share their ideas, and those students were most often White males, followed by White females.

To include more voices in class conversations, Ms. B called on students by name to share their thoughts consistently throughout the semester. Almost all whole-class discussions during the second semester included at least one *Called On* solicitation (56 out of the 62 lessons). However, some of these solicitations did not have the intended result. When called on, students declined to answer 13% of the time. About half of the students in Ms. B's calculus course declined to answer at least once (31/60 students). Out of the 31 students, most declined only once, but some declined multiple times. In fact, there were three students who each declined as many as four times (1 White female student, 1 female student of color, and 1 White male student). Alison, whose thoughts on cold-calling were shared earlier in this chapter was that one White female student who declined to answer four times over the semester.

A Closer Look at One Whole-Class Discussion

The quantitative analysis of contribution solicitations in the previous section provides a sense of how students were prompted to contribute during whole-class discussions in Ms. B's classes. More can be learned by taking a closer, qualitative look at how cold-calling and volunteering played out through classroom interactions. To provide a close examination of how opportunities for participation were constructed through contribution solicitations in Ms. B's calculus classes, one whole-class discussion was selected for analysis. This lesson excerpt is intended to be representative of the types of interactions that took place daily during whole-class discussions in Ms. B's classes. The excerpt was taken from a Period 2 lesson in the beginning of March. This lesson was selected because it followed the typical lesson structure, beginning with a whole-class discussion including the presentation of homework solutions and an introduction to new content material, followed by small-group work time, concluding with a second whole-class discussion to summarize and formalize take-aways from the lesson. Roughly

2/3 of contributions were made in response to solicitations for volunteers; the others were made in response to cold-calling. As was typical, most contributions were responses to teacher questions, followed by questions asked by students. This lesson also included a student presenting their homework solution, students reading out loud, a student declining to answer, and a student identifying a mistake. A variety of students contributed during the lesson. The observations and reflections shared by the teacher and student teacher during the lesson debrief session immediately following this lesson suggest this lesson was representative in their minds.

There were two whole-class discussions in this lesson, one at the beginning and one at the end of the class period. Analysis in this section is based on the classroom interactions that took place during the first whole-class discussion, approximately 40 minutes of class time. This discussion features Alison, one of the students with the highest number of declined responses during the semester (4); she declined once during this discussion. In addition, several students featured in this excerpt, including Alison, reflected on Ms. B's contribution solicitations (shared in Part 1 of this chapter). Taken together, the interaction analysis and the students' reflections provide a more robust understanding of how students' opportunities for participation were shaped by the teacher's solicitations for contributions.

The focal lesson introduced the topic of logarithmic differentiation. It was the last lesson in a unit about the chain rule and its uses. As was typical, the lesson began with a few announcements made by the teacher. Then the class discussed solutions for several homework problems the teacher had chosen ahead of time. The teacher talked about two problems and solicited a volunteer to present their solution for a third problem. This beginning part of the lesson took about 15 minutes. After announcements and homework discussion, the teacher shifted the students' attention to the main lesson for the day. The following three problems were written on the teacher's classroom white board and shared with students as a Google document (Figure 5v).

Figure 5v. Logarithmic Differentiation Classwork

3.8B Logarithmic Differentiation

1) First, review these laws of logarithms:

$ln(ab) =$ $ln(\frac{a}{b}) =$ $ln(a^b) =$

2) Next we will find y' if $y = x^x$ Guess an analytical representation first!

3) Find y' if $y = \frac{sin^2 x tan^4 x}{(x^2+1)^2}$

The lesson excerpt selected for analysis was divided into three segments: 1. Announcements & Homework Sharing (15 minutes), 2. Classwork Problem #1 & Logarithmic Review (10 minutes), and 3. Classwork Problem #2 (12 minutes). Classwork Problem #3 was completed by students working in Zoom breakout rooms and was therefore not included in this analysis. Details of the whole-class discussion analysis can be found in Appendix E, which includes descriptions of each

lesson segment, along with tables containing solicitation dialogue, invitation types, and contribution explanations. A summary of findings from the interaction analysis is presented next, highlighting which students participated and in what ways. Its focus is on how opportunities for participation were constructed through interactions.

Ms. B's solicitations across the three parts of this whole-class discussion resulted in 30 student contributions over about 40 minutes of class time. Table 5p summarizes these contribution solicitations. This table highlights how student contributions were distributed across groups of students according to gender and race. It includes all invitations for individual student contributions regardless of outcome, meaning that invitations from Ms. B that were met by silence and did not result in a student contribution *are* included in this table. However, explicit invitations for whole-class responses *are not* included (e.g., "What is the log of 100? Show me with your fingers").

Table 5p. Summary of Individual Student Contribution Solicitations

	Row	Invitation Type	Intended Contributor	Response	Contribution Type	Contributor			
						Male, White	Female, White	Female, SoC	Male, SoC
Announcements & Homework Sharing	1	general encouragement	anyone	none					
	2	open invitation	anyone	volunteer (Chris)	Asked a Q	1			
	4	general encouragement	anyone	none					
	5	open invitation	anyone	volunteer (Emma)	Shared a solution			1	
	7	open invitation	anyone	volunteer (Kyle)	Identified Mist	1			
	9	general encouragement	anyone	none					
Classwork Problem #1 & Logarithm Review	10	explicit request	specific student	called on (Andy)	Read out loud				1
	12	explicit request	specific student	called on (Leah)	Read out loud			1	
	16	explicit request	specific student	called on (Alison)	Declined		1		
	18	explicit request	specific student	called on (Rebecca)	Responded to Q		1		
	21	explicit request	specific student	called on (Chris)	Responded to Q	1			
	23	explicit request	specific student	called on (Alma)	Responded to Q			1	
	28	general encouragement	anyone	volunteer (Colin)	Responded to Q	1			
	30	explicit request	specific student	called on (Joe)	Responded to Q	1			
	32	explicit request	specific student	called on (Ellie)	Responded to Q		1		
Classwork Problem #2	34	general encouragement	anyone	volunteer (Zoe)	Responded to Q		1		
	36	open invitation	anyone	volunteer (Kyle)	Responded to Q	1			
	38	general encouragement	anyone	volunteer (Chris)	Responded to Q	1			
	40	general encouragement	anyone	volunteer (Max)	Responded to Q	1			
	42	general encouragement	anyone	none					
	43	open invitation	anyone	volunteer (Graham)	Responded to Q	1			
	45	general encouragement	anyone	volunteer (Joe)	Responded to Q	1			
	47	open invitation	anyone	volunteer (Max)	Responded to Q	1			
	49	open invitation	anyone	volunteers (Max & Kyle)	Responded to Q	2			
	51	open invitation	anyone	none					
	52	explicit request	specific student	called on (Sarah)	Responded to Q			1	
	54	open invitation	anyone	none					
	55	explicit request	specific student	called on (Ethan)	Responded to Q	1			
	57	No invitation	no one	volunteer (Zoe)	Asked a Q		1		
	58	explicit request	specific student	called on (Sarah)	Responded to Q			1	
	60	explicit request	specific student	called on (Kevin)	Responded to Q				1
	62	open invitation	anyone	none					
	63	explicit request	specific student	called on (Mateo)	Responded to Q				1
	65	open invitation	anyone	volunteers (Colin & Chris)	Responded to Q	2			
	68	open invitation	anyone	volunteer (Emma)	Asked a Q			1	
	70	open invitation	anyone	none					
					TOTAL Contributions	16	5	6	3

Rows are color coded according to the invitation type and response: gray = open invitation or general encouragement with no response, purple = explicit request with a response, white = open invitation with a response. The gray rows highlight the 8 invitations that got no response; some were general encouragement solicitations (e.g., “Number 114 is still open to the first speaker and there are a lot of people who can explain it. So, speak up at any time. You can interrupt me if you want to.”), and others were open invitations (e.g., “Questions on logarithmic differentiation?”). The purple rows highlight the 13 times Ms. B cold-called individual students through explicit requests (e.g., “Should I call on somebody? You can pass if you feel - Sarah. You wanna help differentiate?”). The white rows highlight the 15 times voluntary contributions were solicited (e.g., “More guesses? I would like at least like four guesses”).

The color coding shows how Ms. B's solicitation methods changed throughout the lesson. She started by seeking volunteers at the beginning of the lesson with homework sharing and announcements, then shifted to almost entirely cold-calling for classwork problem #1, back to volunteers during the first part of Problem #2 when eliciting guesses for the answer, then to cold-calling when going through the step-by-step differentiation procedure, and eventually back to volunteers for reflections and questions at the end of the discussion. Most student contributions, regardless of solicitation method, were responses to questions the teacher asked. Zoe made the only contribution without an invitation; she asked a question partway through Classwork Problem #2.

The numbers in the four right-hand columns indicate the gender and race category for each contributing student. Ms. B solicited *Called On* contributions from students across all four gender-race groups, represented by the "1"s in the purple rows are scattered across the contributor columns. On the other hand, most *Volunteered* contributions came from White male students, represented by the numbers found mostly in the far left-hand contributor column (i.e., "Male, White"). Emma and Zoe were the exceptions. Both female students volunteered twice. Emma volunteered to share the homework solution at the beginning of class and asked one mathematical question. Zoe asked a math question too, in addition to volunteering a guess for y'. Overall, 16 contributions were made by White male students, 5 contributions were made by White female students, 6 by female students of color, and 3 by male students of color.

Table 5q presents some of the information contained in Table 5p, but this time organized by student. Each "V" in the table represents one time that student made a voluntary contribution. Each "C" in the table represents one time that student made a contribution after being cold-called by Ms. B. Student names are listed vertically in the order of their first contribution.

Table 5q. Summary of Student Contributors

Contributors							
Male, White		Female, White		Female, Students of Color		Male, Students of Color	
Chris	V, C, V, V	Alison	C	Emma	V, V	Andy	C
Kyle	V, V, V	Rebecca	C	Leah	C	Kevin	C
Colin	V, V	Ellia	C	Alma	C	Mateo	C
Joe	C, V	Zoe	V, V	Sarah	C, C		
Max	V, V, V						
Graham	V						
Ethan	C						

V = Volunteered
C = Called on

Eighteen of the 32 students in the class contributed at least one time. White male students dominated the voluntary contributions, as indicated by the large number of "V"s next to the names of White male students. Chris made four contributions, Kyle and Max each made three, and Colin and Joe each made two. Ethan was the only White male student who contributed without volunteering. The only non-White male students who made more than one contribution were Zoe and Emma who each made two *Volunteered* contributions and Sarah who made two *Called On* contributions. Sarah was the only student who was called on twice by Ms. B. She was first called on to explain part of the logarithmic differentiation process to the class and then called on a second time to answer a student question re-directed to her by Ms. B. The three contributions from male students of color came in response to explicit requests by Ms. B, one of

which was an invitation to read a problem. Andy's voice was audible as he read out loud, but his mathematical ideas were not part of the discussion.

Without cold-calling, almost all student voices during the lesson would have belonged to White students, and most student voices would have been male. Ms. B's deliberate actions to call on non-White and non-male students to contribute resulted in some diversity in student voices, but not enough to reflect the distribution of the students enrolled in the class. White male voices were notably overrepresented through contributions and the voices of male students of color were notably underrepresented.

Analysis suggests that the dominance of White male voices in this lesson was due in large part to the tremendous number of voluntary solicitations made by Ms. B. As a group, White male students seemed more comfortable and/or more willing to offer voluntary contributions during the discussion. General encouragement solicitations and open invitations seemed to offer genuine opportunities for White male students to contribute, but not for other student groups. Zoe and Emma seemed to be the two exceptions. All other contributions from female students and students of color came in response to explicit requests made by the teacher.

Although Ms. B created opportunities for student contributions across all gender-race groups by calling on students, the opportunities created through called on solicitations were consequentially different in nature from the ones created through voluntary solicitations. The teacher asked for volunteers to explain homework solutions, to make mathematical conjectures, and to articulate patterns they saw. These solicitations for voluntary contributions created rich opportunities for positive identity development, and because White male students were the ones volunteering most often, they were the ones who benefitted from these rich opportunities. On the other hand, the teacher called on students to read the problem, to share answers for "review" problems, and to share the next steps in procedural solving processes. Perhaps Ms. B thought these were "safer" questions to ask non-volunteering students. These solicitations for cold-called contributions extended opportunities to students to contribute, but not in ways that necessarily supported deep content understandings and strong positive identity. In the case of Alison, calling on her likely had a strong (unintended) negative effect on her identity as a learner and doer of mathematics.

The teacher called on students from all groups in frequencies proportional to their enrollment. She believed students had ideas that were not being shared and she wanted the discussions to include more diverse voices. But she did not always call on students *within* a gender-race group equally. For example, in Period 2, Ms. B called on one White female student 13 times to contribute during the semester and a second White female student 4 times. The first female student had a total of 40 contributions during the semester and the second had a total of 5. This difference in *Called On* contributions could be explained by the fact that often Ms. B intentionally called on students who she thought knew the answer or at least had something productive to share. This was consistent with how she encouraged only certain students to present their homework solutions to the class, students she knew had written a thorough and mathematically valid solution on their homework paper.

Ms. B's decision to call on students who she thought could share something "valuable" was well intentioned (she acknowledged having to decline a question was a terrible thing for a student to experience), but her focus on sharing correct ideas might have unintentionally narrowed opportunities for participation by making students more anxious to share for fear of

being wrong. This explanation is supported by students' comments about only volunteering to share homework solutions when they felt confident and knew they were correct. Findings presented here suggest Ms. B's well-intentioned pattern of calling on students across gender-race groups helped make participation a bit more equitable but was not enough to compensate for the White male domination of voluntary contributions, which was perhaps exacerbated by Ms. B's focus on having students share "productive" ideas.

Summary of Findings

The overarching question guiding analysis in this chapter was: In what ways, to what extent, and from whose perspectives was classroom participation equitable during whole-class discussions in a high school calculus course? Answering this question required integrating data from multiple sources and perspectives. First, we needed to know how classroom participants experienced whole-class contributions (Part 1). How important were student contributions to the teacher? What were the teacher's intentions when calling on students to participate? How did the students feel about contributing during discussions in class? Second, we needed to know how students contributed during whole-class discussions over an extended period of time (Part 2). Who contributed and in what ways over the course of one semester? How were students' total contributions distributed across various types of contributions? How did other aspects of contributions correlate with students' genders and racial identities? Third, we needed to know how opportunities for participation were constructed through contribution solicitations (Part 3). How were students invited to contribute during whole-class discussions? How did the teacher's pedagogical decisions support (or inhibit) students' opportunities for participation? How did responses to contribution solicitations differ between groups of students? Together, findings helped us determine to what extent groups of students (and individuals within those groups) had opportunities to participate in whole-class discussions, and if the opportunities they had likely supported content proficiency and positive mathematical identities. Findings were based on analyses of one-on-one interviews with the teacher and focal students, student contribution data logged over one semester using the EQUIP lesson observation tool (Reinholz & Shah, 2018), and a representative 40-minute videorecording of a whole-class discussion.

Assessing Equitable Participation in Whole-Class Discussions

The intent of this chapter was to integrate multiple perspectives to tell an in-depth and comprehensive story of what happened for students in Ms. B's two calculus classes during whole-class discussions - to determine the extent to which opportunities for participation were equitable, specifically for female students and for students of color. To operationalize and assess equitable participation, the definition for equity was broken down into three primary questions: 1) Which students had opportunities to participate? 2) Which students had opportunities to participate in ways that likely supported rich content understandings? 3) Which students had opportunities to participate in ways that likely supported positive mathematical identities? Each of these three questions was broken down further into three secondary questions that could be answered empirically through the presented analyses. Figure 5w summarizes findings focused on assessing participatory equity, organized according to the primary and secondary equity-related questions.

Figure 5w: Assessing Equitable Participation during Whole-Class Discussions

Assessing Equitable Participation	Female SoC	Female White	Male SoC	Male White
To what extent did students have opportunities to participate?	✓ (Somewhat)	✓+ (mostly)	✓-- (very limited)	✓++
Did total contributions during whole-class discussions meet expectations based on course enrollment?	No, fell short	Yes	No, fell way short	Yes, exceeded
Did the teacher's pedagogical moves support these students' voices in whole-class discussions?	Mostly	Yes	Limited	Yes
Did students in this group express comfort contributing during whole-class discussions?	Mixed	Mixed	Mostly No	Mostly Yes
To what extent did students have opportunities to participate in ways that supported rich content understandings?	✓-- (very limited)	✓ (Somewhat)	✓-- (very limited)	✓+ (mostly)
Did students ask questions during whole-class discussions?	No	Mixed	No	Yes
Were students' contributions mathematically rich?	Limited	Mostly	No	Yes
Did students feel comfort taking mathematical risks?	No	No	No	Mixed
To what extent did students have opportunities to participate in ways that supported positive mathematical identities?	✓ (Somewhat)	✓ (Somewhat)	✓- (limited)	✓++
Were students positioned as valuable mathematical contributors?	Yes	Yes	Mixed	Yes
Did students feel supported and valued by their teacher?	Mostly	Mostly	Mixed	Yes
Did students volunteer to share their ideas during whole-class discussions?	No	Mixed	No	Yes

Female students of color in Ms. B's calculus classes had somewhat limited opportunities to participate. Their opportunities were better than male students of color, but worse than their White peers. As a group, female students of color had fewer total contributions than expected based on course enrollment numbers. However, Ms. B's focus on supporting female voices appears to have broadened the opportunities female students of color had to participate during whole-class discussions. Through cold-calling and explicit words of encouragement during class, Ms. B repeatedly positioned female students of color as valuable contributors. She seemed to genuinely believe they had ideas worth sharing with their classmates. Ms. B's behind-the-scenes compliments written on assignments and suggestions to share homework solutions in class made a lasting (positive) impression on some students, such as Alma and Sarah. Yet, the contribution patterns indicate that female students of color remained hesitant to take mathematical risks and had limited opportunities to engage deeply with mathematical content. The assumption is that many of their questions went unanswered and many of their ideas remained hidden because students did not feel comfortable enough sharing during whole-class discussions. Overall,

findings indicate that these students had more opportunities to participate than they would have had without Ms. B's concerted efforts, but these opportunities were still not rich enough or plentiful enough to suggest participation was equitable for female students of color, as a group, in Ms. B's classes.

There were some individual exceptions, such as Guadalupe (Period 1) and Emma (Period 2). Both students ranked 9th in their classes in terms of total contributions for the semester; Guadalupe contributed a total of 28 times and Emma contributed 27 times. And both students had a high number of voluntary contributions, which increased as the semester progressed. Guadalupe made 19 voluntary contributions (13 during the second half of the semester) and Emma made 23 voluntary contributions (14 during the second half of the semester). Both students talked about feeling supported by Ms. B and about engaging deeply with the mathematical content in the class, though both still expressed some hesitation with sharing at times. Given the available data, it appears whole-class participation was equitable to a certain extent for these two young women of color. However, their 9th place rankings (as opposed to higher) and their hesitation sharing ideas in class indicate that participation was not as equitable as it could have been.

White female students in Ms. B's calculus classes had opportunities to engage deeply with mathematics, and these opportunities were relatively more robust than the opportunities students of color had. Most White female students were in the top half of the contributors in their class in terms of total number of contributions, and their contributions tended to be longer. Ms. B talked about the strong relationships she had formed with a number of female students in the class and how she wanted the girls to be proud and speak up. She often referenced her own experiences being female and being outnumbered in advanced mathematics courses. Typically, Ms. B did not distinguish between White female students and female students of color in her talk, but since she is a White woman herself, the omission of race may suggest that Ms. B was relating, more specifically, to the dominant (White) female experience. Although Ms. B's distribution of cold-calling students was remarkably close to the course enrollment distribution, she tended to call on White female students slightly more often than other groups of students. In addition, there were times when she asked publicly and explicitly for more female voices during whole-class discussions.

Even though White female students had relatively higher numbers of contributions and consistent support from Ms. B, this group of students still exhibited signs of discomfort taking academic risks. They volunteered less often than their White male peers, they were hesitant to ask questions, and they declined to answer more than any other group. Zoe, the most prolific contributor in this group (53 total contributions, 47 of which were voluntary) admittedly that she only asked about half of the questions she had. But at least Zoe was asking some of her questions. Other students, like Alison were barely engaging in whole-class discussions at all, and when they did engage, it was usually not a positive experience. Alison contributed 7 times over the semester, but 4 of those times she declined to answer the question she was asked, which likely hurt her more than it helped her. It seems Ms. B's efforts were somewhat successful in supporting more equitable participation for this group. However, it was evident that participation was more equitable for some students in this group than for others.

Male students of color in Ms. B's calculus classes had very limited opportunities to participate. This gender-race group of students had the fewest opportunities and the weakest

opportunities in terms of content and identity development. Their total contribution numbers were much lower than expected based on enrollment. In fact, Hosein was the only male student of color across the two class periods who exceeded the mean number of contributions per student, which was about 26 contributions per student for the semester. Hosein contributed 40 times and he was considered a statistical outlier within this group. Yonas was next in line with 20 contributions. The numbers of contributions made by their male peers of color dropped off considerably after that; the numbers with respect to voluntary contributions were especially low. In interviews, Ms. B talked about wanting to support the voices of her male students of color. She reiterated the importance of their mathematical ideas, which was supported through her efforts to solicit their ideas through cold-calling. But her actions with male students of color did not seem to be as effective as her actions with the female students in her class. Overall, this group of students, except for Hosein and possibly Yonas, did not have enough opportunities to engage deeply with mathematics in ways that supported rich content and/or positive identity development. Overall, whole-class participation was not equitable for this group of students.

White male students, as a group, had more frequent and richer opportunities to participate than any of the other gender-race groups of students. The most prevalent student voices in the class belonged to White male students, although there was some indication that students, like Nate, were only comfortable sharing when they felt confident about the mathematics. But still, the group dominated almost every category of contributions. This group's dominance was especially evident when Ms. B solicited voluntary contributions during whole-class discussions. White male students accounted for 70% of the voluntary contributions, though they represented only 43% of the students in the classes. Since the teacher solicited student contributions by asking for volunteers 73% of the time (as opposed to cold-calling), White male students had more genuine opportunities to participate because they were more likely to take up the teacher's numerous invitations for voluntary contributions. Some White males received *more* than their fair share of opportunities to participate (Caleb - 169 total contributions, Nate - 113, Josh - 80, Chris - 80), while other White males received far fewer. There were 12 students across the two class periods who contributed fewer than eight times during the semester. Five out of these 12 students were White males and none of these five White male students participated in one-on-one interviews for this study, meaning their perspectives on participation were not included in the analysis. One danger in highlighting participation patterns for groups of students is that the experiences of some students may get lost amidst the experiences of others. To say if participation was equitable for White male students is more complicated than it might seem at first glance. Yes, White males as a group had an abundance of opportunities to participate in ways that supported content and identity development, but within this group the distribution of opportunities was not equitable. The excessive number of contributions made by just a few White male students took opportunities away from their peers, including their female peers, their peers of color, and some of their White male peers as well.

Chapter 5 Take-Aways

The fact that these differences in participation metrics and interactions occurred between gender and racial groups is not a surprise. There is an abundance of literature documenting similar inequities in student participation (e.g., Langer-Osuna, 2011; Leyva et al., 2021; Mack, 2012; Tatum et al., 2013). That being said, there are new insights we can take from the findings in this chapter.

The participation issues classroom participants (the teacher, student teacher, and students) talked about related to gender were generally supported by the individual student contribution analysis. Some differences in participation by gender existed, participants noticed them, and participants talked about them with each other and with me. However, there were not significant differences in total contributions by gender, which is counter to participants' talk. The fact that female and male participants were keenly aware of participation differences is noteworthy. In addition, evidence suggests the teacher's concerted efforts to support female participation broadened opportunities for female students, specifically for Guadalupe, Zoe, Emma, and Sarah. Overall, there were some gender inequities, but presumably they would have been worse without the teacher's interventions.

Unlike with gender, participants did not talk about participation differences between racial groups as an issue in the class. They talked about one or two White boys who talked too much, but the issue seemed to be tied to these specific students, as opposed to being a bigger issue pertaining to racialized experiences more holistically. And no one centered male students of color in any comments made about participation. No one commented on the lack of verbal participation by male students of color and/or ways to support this group of students. Just because participants did not talk about racial inequities does not necessarily mean they did not notice racial inequities. My guess is the teacher and student teacher did not notice them, because I believe they would have said something if they did. I admit that I did not notice them while observing the class, and I was observing for the primary purpose of identifying inequitable patterns in participation. The teacher and I are both White women and therefore likely have a harder time recognizing and relating to race-related micro-aggressions. On the other hand, the student teacher, a Mexican man, has experienced marginalization in mathematics classes due to race. However, when asked in an interview if he thought race was impacting the way a group of students interacted with each other, he responded with the following:

> I don't think race played a big factor in it. I think that in a classroom, especially in public high school, people don't look at race that much. People just kind of look at personalities, especially when they are more mature there towards the end of the school year. I don't think race mattered in this, but I think race mattered in the classroom as a whole. Like, for example, I think Caleb, yeah. You know, Caleb was white. He was a guy. He spoke too much, and I made sure to tell him, 'You know, Caleb, relax for a second." He kind of listened. So, I think overall it matters, especially because I think there was a lot more White people than people of color in these classrooms.

Mr. K acknowledged race was a factor, but only with respect to Caleb's participation and with respect to the low representation of people of color. There was no indication that he noticed more about the racial inequities than Ms. B did or than I did. It is unclear if students did not notice participation differences by race or if they saw them and chose not to talk about them. It is certainly possible that students of color noticed racial inequities in participation but chose not to share them with me. Despite the seemingly strong relationships I built with students that year, the fact remained that I was a White woman researcher asking students of color to share with me their personal observations. Students shared a lot, but I cannot assume they shared everything they noticed or experienced.

Chapter 6: Assessing Participatory Equity in a Small-Group Task

Introduction

This chapter focuses on participatory equity, which I define as a fair distribution of opportunities for students to participate and learn, including opportunities to develop content proficiency and positive mathematical identity (Esmonde, 2009). A major goal for classroom interactions is for every student to have genuine opportunities to participate in ways that lead to rich mathematical understandings and support students to see themselves and each other as capable learners and doers of mathematics (Schoenfeld, 2014). Given this goal of participatory equity, now comes the question … how do we assess the ways in which and the extent to which participation was equitable for a given task and a given group of students? Assessing participatory equity requires operationalizing the goal in ways that are tangible and quantifiable (at least to some degree) – and determining if every student had fair opportunities to participate in ways that supported content and identity development is not a simple feat. How did students feel while working on the task? How did students' assessments of their shared experience align or not align with each other's and with their teacher's? What counted as a *genuine* opportunity to participate and for whom? In this chapter I present analyses that address empirical questions related to participatory equity in a small-group task for one group of four students (Yonas, Guadalupe, Hosein, and Elijah). In the Discussion section (Chapter 7), I revisit the question of how to assess participatory equity more broadly, connecting empirical findings to theoretical considerations and priorities.

Studies of equity and student participation in mathematics typically rely on classroom observation data and reflect researchers' perspectives on whether participation is equitable or not (e.g., Shah & Crespo, 2018). Occasionally, studies highlight teachers' perspectives regarding student participation (e.g., Wager, 2014) or students' perspectives on their own or their peers' participation (e.g., Esmonde et al., 2009), but rarely do studies consider more than one perspective at a time. This chapter, in contrast, examines equity and student participation in a distance learning high school calculus class from multiple perspectives, comparing students' firsthand accounts of their learning experiences to observations made by their peers, their teacher, their student teacher, and a researcher.

This chapter combines interaction video analysis of one 10-minute, small-group task (The Ladder Problem) with video-stimulated observations of that task made by one group of students, their teacher, and their student teacher. It examines the learning experiences of the four students, paying particular attention to what each perspective adds to the complex story of what happened during this task and what the implications are for assessing participatory equity. The overarching question guiding analysis in this chapter is: In what ways, to what extent, and from whose perspectives was classroom participation equitable for one group of four students working on one calculus task? This general question is addressed by examining the following underlying empirical research questions: 1.) How did participants assess their experiences with The Ladder Problem task? 2.) How did each student contribute during the task? 3.) How were students' opportunities to participate constructed through interactions?

Together, the answers to these three questions paint a multivalent picture of what transpired during this 10-minute small-group task for these four students. A close look at what happened for one group through their engagement with one task and how participants made sense of these happenings highlights consequential differences in students' opportunities to

construct rich content understandings and build positive mathematical identities. The multiple-perspective examination of classroom interactions supports a more comprehensive assessment of participatory equity.

Chapter Overview

This chapter is intended to be read within the context of the whole dissertation. Therefore, the bulk of the literature, theory, and methodology framing these analyses is provided in the previous chapters. Before sharing results from analysis focused on one small-group task, I provide some information about how the selected lesson fits within the trajectory of this year-long AB Calculus course and how the focal task was embedded in the lesson on that day. Findings are then presented in three parts. Part 1 is focused on participants' assessments of their experiences with The Ladder Problem task. Part 2 is focused on individual student participation metrics (e.g., # contributions, timing of contributions, distribution between mathematical and social contributions). Part 3 is focused on interactional participation processes (e.g., how contributions were invited (or not) through talk and action). At the end of this chapter findings are summarized, highlighting areas of alignment and disparity within and across the varied perspectives, paying particular attention to Guadalupe's experiences. Implications of these findings related to supporting more equitable opportunities for participation and learning in mathematics classrooms are presented in the Discussion section in Chapter 7.

The group featured in this chapter was comprised of four students: Yonas, Guadalupe, Hosein, and Elijah. Their teacher described these four students collectively as "strong math students who are skilled and interested." All four students earned A's and B's consistently in this class. Guadalupe was the only student in this group who identified as female; she also identified as Mexican. Yonas identified as Black, Hosein as mixed, and Elijah as White. The teacher identified as a White female, the student teacher as a Mexican male, and the researcher as a White female. Figure 6a contains a Zoom screenshot of the four students, the teacher (Ms. B), the student teacher (Mr. K), and the researcher (Ms. F) during the small-group discussion featured in this chapter. (Participants shown with permission.)

Figure 6a. Participants in the Focal Group on Zoom

The primary data sources for this chapter were a 10-minute videorecording of the group working on the task and stimulated-recall interviews with the four students, the teacher, and the student teacher.

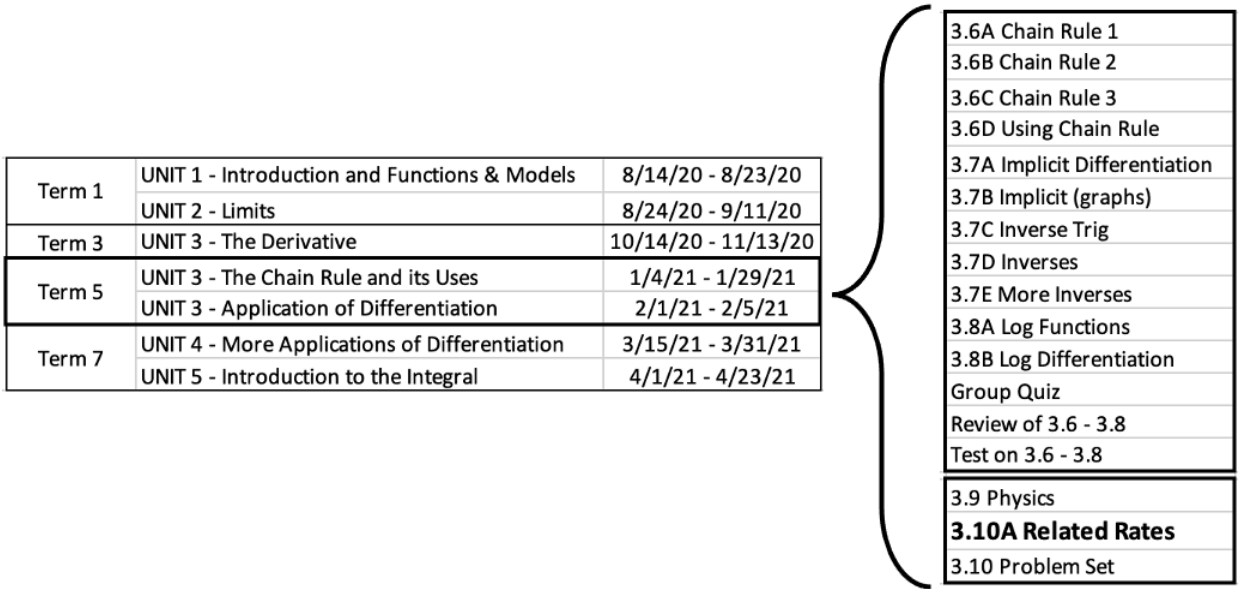
Background on the Course, the Lesson, and the Task

The analyses in this chapter focus on understanding student participation during a small-group calculus task for one group of four 12th grade students. The focal task was part of a lesson that took place via Zoom on 2/2/21 in Ms. B's Period 1 AB Calculus class during the Covid

pandemic. Ms. B had taught this course numerous times before, but she had always taught the course in person. Mr. K had joined this class at the beginning of January as a student teacher. He had responsibilities teaching one of Ms. B's other math classes; in this class he was primarily an observer and instructional helper when needed.

Ms. B worked hard to keep the rigor of this calculus course high but had to cover less content due to the condensed pandemic schedule (Period 1 met only during terms 1, 3, 5, and 7). Students in this course typically took the AB Calculus Advanced Placement exam in the spring, but this year that was not the expectation. Ms. B began the year with lessons related to functions and limits and then moved on to derivatives and spent the bulk of the year on differentiation. She concluded by introducing the concept of integrals during the last few weeks of the course. Figure 6b shows the content progression for the course.

Figure 6b. Content Progression for Ms. B's Period 1 - AB Calculus Course



The lesson featured in this chapter (3.10A Related Rates) came from *Unit 3 – Application of Differentiation* and occurred during the last week of Term 5. Students spent the first four weeks of the term working on the mechanics of differentiation, taking a test the week before this lesson. Students then built on this lesson by working on a problem set containing a series of differentiation application problems to finish out the term.

The 3.10A Related Rates lesson revolved around a task which Ms. B referred to as "The Ladder Problem." Prior to this lesson Ms. B had mentioned this was one of her favorite lessons, and after teaching the lesson she commented repeatedly about how proud she was of how she had designed the task. During post-lesson debrief sessions she explained that the ladder problem is "a classic" related rates calculus problem, but usually students are shown the problem and then shown how to solve it. Instead, she preferred to give the problem to students to work on in small groups without any formal instruction on this topic and prompt them to figure it out for themselves by including stimulating questions. Ms. B wanted students to think about the underlying ideas before formalizing solving strategies.

On Tuesday, 2/2/21, the lesson began as most other lessons did, with "Good morning, everyone!" greetings leading into homework sharing. About 20 minutes into the 60-minute class period, Ms. B introduced The Ladder Problem. Students had access to the task through a Google doc link (see Figure 6c). Ms. B had also written the problem on her white board shown in the background of her Zoom screen.

Figure 6c. The Ladder Problem Task as Presented in a Google Doc to Students

3. 10A Introduction to Related Rates

1) A ladder leans against a wall. It begins to slide down the wall. Does the top of the ladder move at the same rate as the bottom of the ladder?

2) Suppose the bottom slides away from the wall at a rate of 1 ft/sec. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall.

Assume the ladder is 10 feet

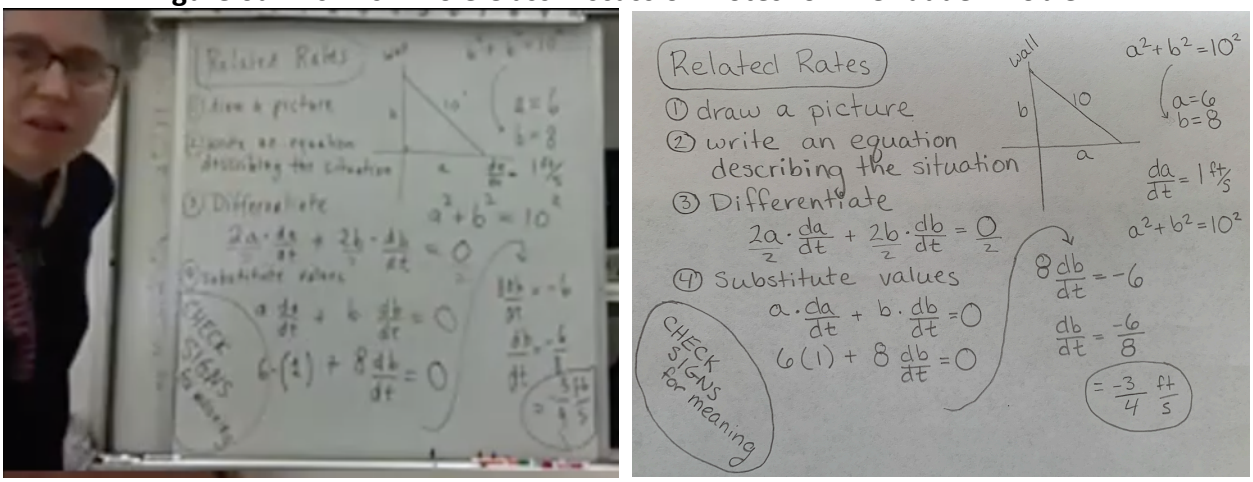
Your homework is to begin the 3.10 problem set. We will work on this during class on Thursday and also tomorrow (wednesday).

Ms. B read the problem out loud and asked if anyone had questions. After a brief exchange with one student, Ms. B invited students to join their randomly assigned breakout rooms and work with their teams to answer the questions. This chapter focuses on what happened with Group 7 (Yonas, Guadalupe, Hosein, and Elijah) during that small-group work time. While students were in their groups, Ms. B and Mr. K (the student teacher) popped in and out of different groups to check on progress. Students worked in their groups for approximately ten minutes before Ms. B closed the breakout rooms, forcing all students back into the whole-class Zoom room.

The class spent the remaining 25 minutes of time sharing their thinking about this specific problem and formalizing an approach to solving related rates problems more generally. Ms. B began the whole class discussion by asking students to vote Yes or No in response to the first question in the task (Does the top of the ladder move at the same rate as the bottom of the ladder?). Every student voted by raising their hand; all but one student voted for No, indicating they thought the rates were different. Alice, the one student who voted Yes, was a seemingly confident female student who earned A's consistently on her math assessments. Ms. B capitalized on this opportunity to resolve the disagreement and encourage students to take ownership over their own mathematical thinking by saying, "I personally am applauding for Alice. Right? You guys know exactly why. So, we have to convince Alice. And Caleb has already tried, I know. So, does anyone else want to try? Can you convince Alice? Alice, don't be convinced until you're convinced. But you don't need me to say it, you got it." Over the next four and a half minutes, three students shared three different ways of justifying the difference in rates, at which point, Alice shared that she was convinced they were correct. Ms. B followed this discussion by asking for estimates for the answer to the second question (How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?). Multiple students shared ideas, including "twice as fast" and "half as quickly." Without indicating whose estimate was closest, Ms. B worked out the problem on the board. She documented general steps one

should take when solving related rates problems as she went through the process of solving this problem, resulting in a final answer of -3/4 feet per second. An image of Ms. B's white board notes, including a completed solution for the Ladder Problem, are included in Figure 6d, along with my rewritten version of the white board contents provided for clarity.

Figure 6d. Ms. B's Whole Class Discussion Notes for The Ladder Problem



In preparation for the upcoming related rates problem set, the class spent the final few minutes of class brainstorming formulas that may be useful for solving problems involving round objects. The problem set was the final assignment of Term 5 for the Period 1 AB Calculus class.

The Ladder Problem was selected for analysis because it was a well-planned lesson that Ms. B had continued to refine over the years. The mathematics was rich, students' ideas were centered, and the lesson was a success in many ways. There were no obvious issues or places for improvement with the design of this lesson. The reason for taking a closer look was to see how four individual students experienced this seemingly successful mathematics task. What more could be learned about students' experiences with this task through stimulated recall interviews and micro-analysis of videorecorded group interactions? How did students' experiences with the task differ from one another and differ from their teachers' experiences? To what extent and in what ways did each student have genuine opportunities to engage in rich mathematics?

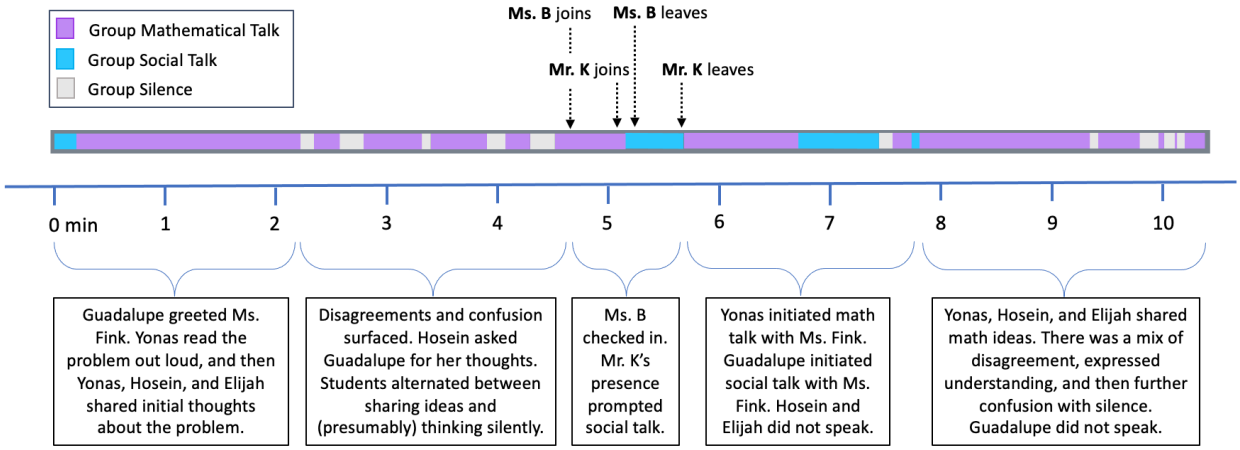
Summary of the Focal Group's Work on the Task

Before getting into the analytic details, it may be helpful to have a broad sense of what took place during the ten minutes Yonas, Guadalupe, Hosein, and Elijah worked on The Ladder Problem. Videorecording of the group's work began about 30 seconds after the students joined their Zoom breakout room. Upon entering the Zoom room, Ms. Fink was immediately greeted by Guadalupe. After a short social exchange, the group shifted back to mathematical talk. Students spent most of the ten minutes talking about the task; they shared ideas, asked questions, and changed their minds about how to solve the problem. There was back-and-forth dialogue between students and relatively little silence. When there was silence, students appeared to be deep in thought. Ms. B and Mr. K joined the group to check in about halfway through the work time, but neither stayed in the group very long. The group continued to make progress with the task but did not come to full group consensus on a solution before being sent back into the whole-

class Zoom room. All four students voiced agreement that the ends of the ladder moved at different rates, but they had not all agreed on what those relative rates were. Most of the talk was about the math content, but there were also a few periods of social talk. Overall, Yonas spoke the most, and Guadalupe spoke the least. Guadalupe seemed eager to make social connections with other people in the group, to which Mr. K, Ms. Fink, Yonas, and Hosein were at least somewhat receptive. Elijah seemed to focus solely on the mathematics.

Figure 6e displays a timeline of the group's mathematical talk, social talk, and silence over the course of completing The Ladder Problem. Also included in this timeline are the moments when the teacher (Ms. B) and student teacher (Mr. K) joined and left the group's Zoom breakout room and summaries of what transpired during specified periods of time.

Figure 6e. A Timeline of the Group's Mathematical Talk, Social Talk, and Silence



This figure is discussed in more detail in Part 2 of this chapter. A complete transcript of the group's work on the task is provided in Appendix D.

Part 1: Participants' Perspectives on the Small-Group Task

This section addresses the question, How did participants assess their experiences with The Ladder Problem task? The perspectives of the teacher (Ms. B), the student teacher (Mr. K), and the four focal students (Yonas, Guadalupe, Hosein, and Elijah) were explored in individual interviews. During the interviews, participants watched the 10-minute video recording of the focal group working on the task one time through and then responded to open-ended questions. Each participant was interviewed separately and asked to share what they found noteworthy about the focal group's engagement with The Ladder Problem task.

Teacher's Perspective

Ms. B's assessment of The Ladder Problem was overwhelmingly positive. Findings related to Ms. B's perspective are shared in three sections. During the formal video-stimulated interview and multiple times during informal debrief sessions, Ms. B expressed pride in the decisions she made regarding the design of the mathematics task, presented in the first section. In addition, Ms. B complimented the way the focal group worked together and engaged with the mathematics as they worked through the problem, presented in the second section. Ms. B's negative comments were focused on what she felt was lost by conducting this lesson through Zoom as opposed to in-person, presented in the final section.

Task Design

The most salient aspect of The Ladder Problem for Ms. B appeared to be task design. About two weeks after this lesson was taught to the focal group (and three days before the formal video-stimulated interview took place), Ms. B brought up The Ladder Problem during an informal debrief session after class. She shared that this is a "famous" math problem commonly taught in calculus courses but that she teaches it differently from how it is usually taught.

> This is where I'm very proud of my teaching style. So, I start by asking these questions, 'How is the ladder moving?' 'Is one end moving differently than the other?' and then 'How quickly is it moving?' without giving any related rates propaganda … there will be students who will figure out the whole thing without any instruction on related rates … then the process, the algorithm that I show them is just validating what people have already figured out in a way that's nice and structured. And it's - you know it's that little different approach … So, for me, I always look at it as, 'What was the lesson structure?' Even participation is about, 'What questions were you asking?', 'Could you have asked it differently?'

Ms. B intentionally gave this calculus problem to her students without any formal instruction on related rates ahead of time. She wanted her students to make sense of the underlying ideas before introducing them to more structured processes, and she thought the question prompts were the key to students' success with the problem.

These ideas were reiterated when Ms. B reflected on the focal group and at several other times throughout the semester. After watching the small-group video during the video-stimulated interview, Ms. B expressed satisfaction with the students' discussion.

> What a lovely discussion! … Well, the first thing is that I just really want to emphasize- I'm bragging a little bit, but the fact that I put the question this way made such a huge difference. So, I told you, normally you just present this problem and then you tell people how to solve it. But instead, I asked these two questions, which I feel like are so, so interesting questions.

Ms. B brought up The Ladder Problem questions again after The Ladder Problem was taught to a different class period during the following term. In the lesson debrief session with the student teacher, Ms. B shared the following.

> I'm going to say one more time, look how asking the right question changes the dynamics of the class. Ms. Fink has had to hear me say this over and over again, but normally we pose this problem and then we just show them how to do it, but instead I ask them first, 'Are both ends moving at the same rate?' And it created a completely different discussion where even [Period 2 Student] spoke.

Ms. B continued by offering advice to the student teacher about the general process of refining lessons but concluded by saying that The Ladder Problem did not need any further revisions.

> One of the other axioms of teaching that's really, really brutal but it's really important is that when you think a day went badly, and that can include really bad class behavior, you're always supposed to critique your lesson plan … How could I have structured the lesson so they would have behaved better? … but I'm not going to improve this lesson more. I love it (smiles and laughs).

Ms. B mentioned the design of the Ladder Problem one final time toward the end of the semester. During a lesson debrief session in May, she reiterated some of these same thoughts.

> I've told you this before - I feel like in a lot of analysis of classrooms we don't focus enough on the mathematics in the lesson and that's where sort of 90% of the action is, like how is the lesson structured and is it something people can access? Does it catch their fancy? ... I think I said this to you already but um, when you share The Ladder Problem, I want you to remember to say that I structured the lesson in this way … these tiny little moves if you do them right can make a big difference.

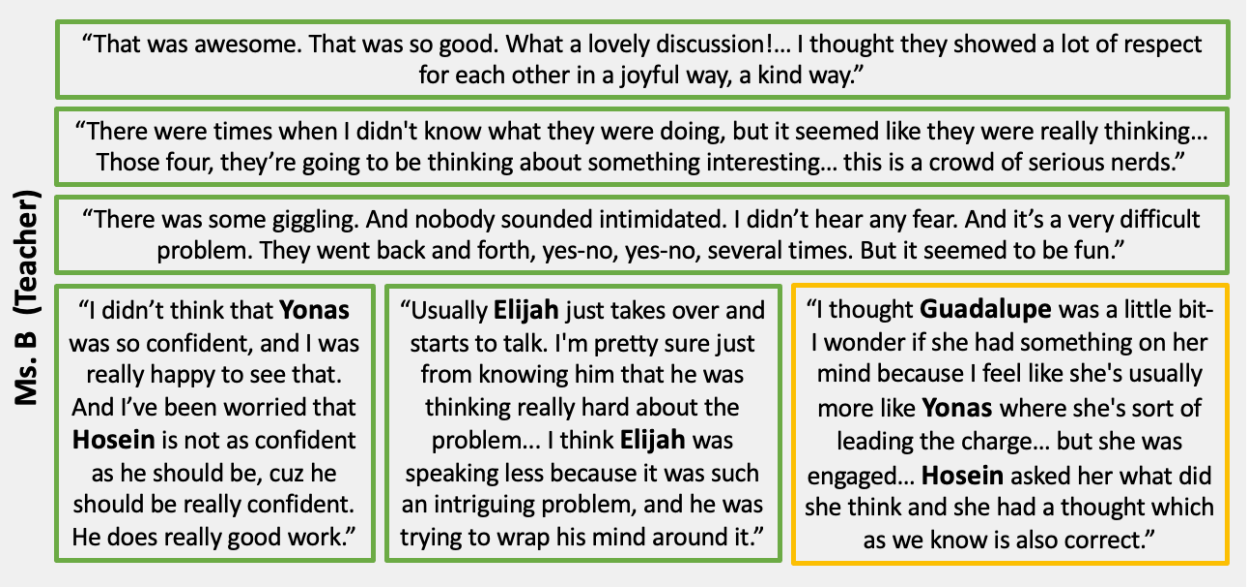
In this last passage, Ms. B acknowledged my role as a researcher, as someone who would eventually be sharing what I learned about the dynamics of The Ladder Problem with other people. She explicitly asked me to talk about the decisions she made when crafting the lesson, indicating task design was the most prominent feature of The Ladder Problem lesson for Ms. B and that she was very happy with the outcome.

Focal Group Engagement

As shown in the previous quotations, when Ms. B talked about the design of The Ladder Problem, she connected task design to student participation, behavior, and access, talking about how the questions she asked led to positive discussions for her students. Ms. B's satisfaction with the Ladder Problem design was reinforced through her experience watching the focal group's engagement in the task. She was proud of the way the students worked together, impressed by their confidence, and happy to see smiles.

Ms. B watched the entire 10-minute video segment of Yonas, Guadalupe, Hosein, and Elijah working on The Ladder Problem before sharing her thoughts. As soon as the video stopped, Ms. B exclaimed, "That was awesome. That was so good. What a lovely discussion!", a strong positive response to the student interactions she just watched. Ms. B went on to share what she noticed about these students working on The Ladder Problem. Figure 6f contains direct quotations from the interview with Ms. B. These quotations were selected to represent Ms. B's assessments of the group as a whole and to represent her main take-aways for each of the four students.

Figure 6f. Ms. B's Observations of the Focal Group Working on The Ladder Problem



Almost all the comments Ms. B made about the students were positive, indicated by the green outlines in Figure 6f. She described the interactions as fun, joyful, kind, and respectful. She was impressed with their commitment to the mathematics ("they were really thinking"), spoke highly of the students' capabilities ("this is a crowd of serious nerds"), seemed happy with their displays of confidence ("I didn't hear any fear"), and complimented their collaboration ("they went back and forth, yes-no, yes-no, several times"). Ms. B referred to Yonas as "leading the crowd" and acknowledged that he was more confident than she realized. Ms. B was also happy to see Hosein sharing his ideas since she had been worried about his confidence. With Elijah, Ms. B was convinced that he was speaking less than usual because he was engaged in deep thought and "trying to wrap his mind around it." The only less-than-positive comment Ms. B made was in reference to Guadalupe, indicated by the yellow outline in Figure 6f. Ms. B routinely spoke very positively about Guadalupe's participation and her "strong mathematical skills." In this case, Ms. B noted Guadalupe's participation seemed out of character for her, suggesting Guadalupe might have had something on her mind. Ms. B noticed that something did not seem quite right with Guadalupe, but Ms. B concluded she was engaged and making mathematically correct statements.

Overall, Ms. B observations indicated that she was quite satisfied with the focal students' experiences with The Ladder Problem. The discussion was fluid, every student demonstrated some verbal understanding of the target mathematical content, students appeared to be thinking hard, students were taking risks and venturing tentative ideas, students joked and smiled then returned quickly to talking about the mathematics. All indications were that this group had engaged in this task in the ways Ms. B had intended.

Distance Learning Limitations

During the video-stimulated interview Ms. B was asked, "Is there anything that you wish had gone differently in this group?" Ms. B responded by talking about various limitations, obstacles, and frustrations brought on by distance learning. One of Ms. B's primary frustrations while watching the video of this lesson was not being able to see what students were doing, which made it hard for her to get a clear sense of what was going on.

> I really, really wish I could have seen what they were doing when they were staring straight ahead. What were they drawing? What was happening? What were they thinking? I really miss that. Because what happens is that the silences contain no information. And that's just a huge loss … The silences were lifeless, but I'm sure there was stuff going on in there. Like what was Guadalupe looking at? She was staring at something, and I think she was concentrating but I couldn't tell. What was Elijah looking at? Elijah is always thinking about something. So, I miss that. A lot.

Ms. B's comments about "missing that" suggest that she used to have access to information about what students looked at and wrote on their papers but now, because of Zoom, she does not.

In addition, Ms. B talked about the limitations of distance learning related to students' interactions with the mathematics and with each other. Ms. B reflected on how she had taught this problem in years past, connecting in-person interactions with a deeper understanding of the mathematics and more ways to participate.

> In in-person classes, this problem gets understood even more deeply in this uncontrolled form where I hadn't really shown them anything. And usually - and I have them interacting with the board. So, typically at the board, if you have any estimates for the rate at the end of the ladder, then go put them on the board. So, everyone is just writing stuff on the board, so there's much more richness in terms of ways to participate.

She also noted that working together in a randomly assigned Zoom breakout room is not the same as working in-person with a consistent group of peers.

I wish that those four could sit at a table for a whole month. You know, usually the teams don't move around. This new teams every day thing is a feature of zoom. Those four could do something. They could do some pretty impressive work, over a month, sitting together. That would be something to see. I guess I'm talking about the things - I see some loss there.

She added additional frustrations related to the logistics of managing breakout rooms, including not being able to spend enough time listening to students and the abrupt nature of transitioning back to a whole class discussion.

> I don't get time in breakout rooms. I mean, that was my visit. Which I just try to put out any fires and move on. I don't get to listen … [Closing breakout rooms] is terrible. The kind of central command atmosphere is really - I mean, you're literally ripped from your team.

Ms. B's negative comments about distance learning limitations were spread throughout the interview; she made similar comments frequently during informal lesson debrief sessions as well. However, as Ms. B pointed out, she felt her students were successful despite these obstacles.

> I'm really pleased at how much my students are learning. I'm really proud of them, that they can switch it up and do something. And they just try it and then they do it. And they were in good cheer. They're not sitting feeling depressed. 'Oh, if only we were sitting together.' They're adaptable.

Ms. B talked often about the obstacles she and her students faced in trying to build community, to engage in mathematics together, and to address personal needs during this year of distance learning. Ms. B seemed to be looking at this group of four students as representative of her students in general. She was proud of her students' learning, their "good cheer," and their adaptability given all that they were facing with the pandemic. All things considered, she was impressed by the mathematical conversation these four students engaged in together.

Student Teacher's Perspective

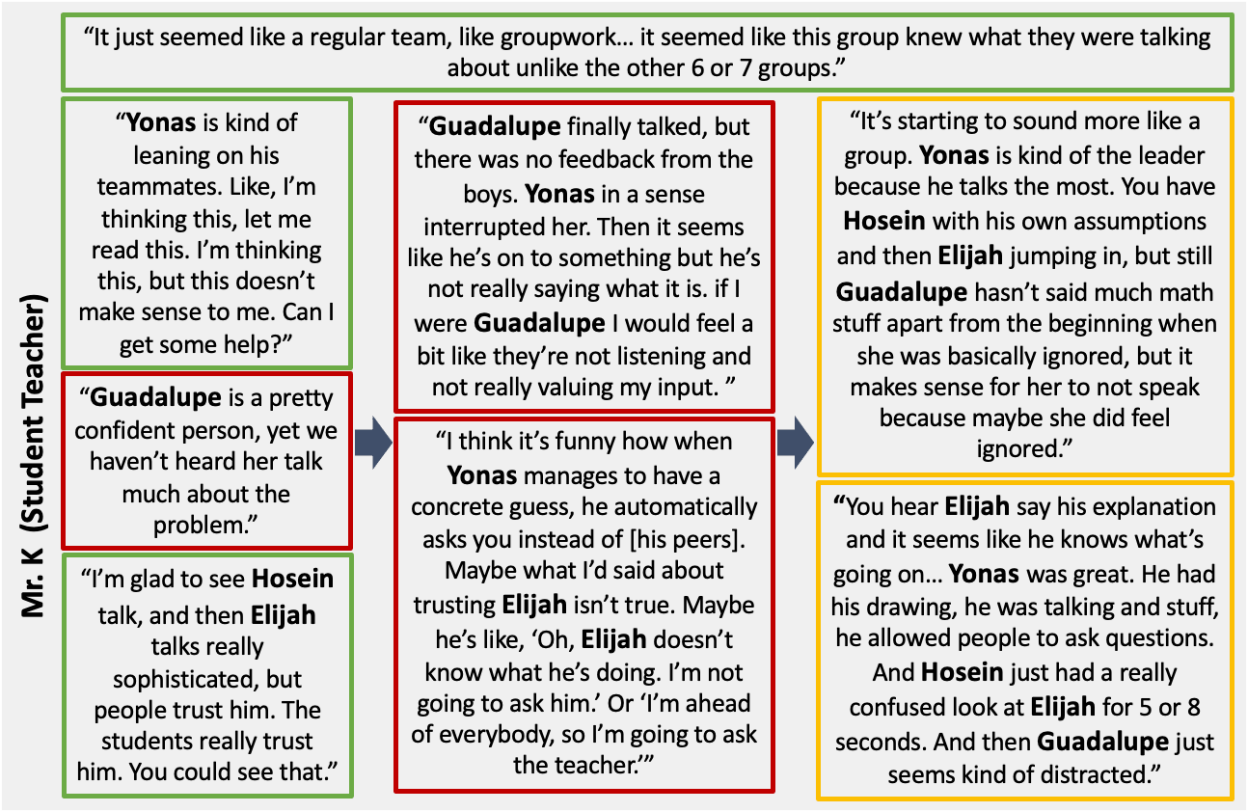
Mr. K's assessment of students' experiences with The Ladder Problem was mixed. For many reasons, he thought students' participation exemplified typical groupwork behavior; he identified strengths in the ways the students interacted with one another and also areas that warranted further consideration. Specifically, Mr. K focused on concerns related to Guadalupe's lack of verbal participation, connecting problematic participation patterns to unsupportive group interactions and larger issues related to gender imbalances. He also wondered about the roles teachers and students should play in supporting productive small-group discussions, acknowledging there is no simple solution for addressing inequitable small-group participation.

Findings related to Mr. K's perspective are shared in two sections. The first section contains Mr. K's observations about the four individual students in the group, including his take on who did what and when. The second section contains a more in-depth examination of how Mr. K made sense of Guadalupe's participation, connecting what happened in the video to ideas and experiences that extended beyond the 10-minute video clip.

Focal Group Engagement

During the video-stimulated interview, Mr. K chose to share his observations periodically while watching the video. When he thought of something he wanted to share, he asked, "Can you pause the video?" All participants were given this option (to share thoughts throughout the video), but Mr. K was the only one who chose to share in this way. Mr. K asked to pause the video eleven times during the 10-minute video. He shared additional observations at the end of the video as well. Because of his in-the-moment observations, Mr. K's reflections were more sequential in nature as opposed to holistic, giving insight into how Mr. K's impressions developed from the beginning of the task to the end. Figure 6g contains direct quotations from the interview with Mr. K. These quotations were selected to represent Mr. K's reflections of the group over the course of watching the video, indicated by the arrows.

Figure 6g. Mr. K's Observations of the Focal Group Working on The Ladder Problem



Unlike Ms. B, Mr. K commented more on individual students than the group as a whole, which may have resulted from the in-the-moment nature of his observation sharing. Mr. K did not mention anything particularly noteworthy about this group, saying, "it just seemed like a regular team, like groupwork." However, in comparison to the other groups in the class, he stated that this group seemed to know "what they were talking about" in terms of the intended mathematical content.

Mr. K's observations about the individual students were mixed. He talked about some aspects of interactions that made him proud and other aspects that concerned him. His initial comments about the participation of Yonas, Hosein, and Elijah were positive. He was happy to see these students sharing their thoughts with the group. In contrast, he pointed out that despite the confidence Guadalupe usually portrays, she was not talking with her peers during the beginning part of the task. This observation is consistent with Ms. B's assessment of Guadalupe's participation as atypical. Midway through the 10-minute task, Mr. K identified two problematic interactional sequences involving Yonas. The first occurred after Guadalupe "finally talked." He noted that none of Guadalupe's peers provided feedback when she ventured a response to the first task question (i.e., Does the top of the ladder move at the same rate as the bottom of the ladder?). In addition, he interpreted Yonas's unrelated comment immediately following Guadalupe's comment as Yonas interrupting Guadalupe without any clear connection to what was just said. The second problematic interaction, according to Mr. K, occurred when Yonas asked the researcher / observer for feedback on his idea instead of asking his peers. About 5 ½ minutes into the task, Yonas asked, "This is going to be x-squared - Ms. Fink, this is going to be x-squared plus y-squared, right?" Ms. Fink responded by redirecting Yonas's question back to the

group by saying, “So, why don’t you - if you have an idea, why don't you maybe even draw a picture and share it with your group and see what they say.” In both interactional sequences, Mr. K imagined himself in the position of one of the students. In the first case, he imagined what it might feel like from Guadalupe’s position, and in the second case, he imagined what Yonas might have been thinking. At the end of the video, Mr. K summarizes with generally positive comments about Yonas and Elijah and somewhat neutral comments about Hosein. His comments about Guadalupe have a negative tone, referencing her lack of math talk and apparent distraction; it is worth noting that Mr. K seemed to blame the interactional environment (and not Guadalupe) for her less than ideal participation, saying, “but it makes sense for her to not speak, because maybe she did feel ignored.”

A Closer Look at Guadalupe’s Experience

During the video-stimulated interview, Mr. K spent more time reflecting on Guadalupe than he did reflecting on the other students. His reflections, all of which have implications for assessing participatory equity, connected Guadalupe’s participation with The Ladder Problem to ideas and experiences beyond this specific task. He talked about gender imbalance, teachers’ incomplete (and sometimes inaccurate) views of small-group participation, the importance of students (specifically Guadalupe) feeling supported and comfortable, teachers’ roles in facilitating student participation, and responsibility for supporting equity. Some of Mr. K’s comments were made during pauses in the video and others were made after he had watched the entire video.

A couple minutes into the video, Mr. K asked to pause it. He reflected on the fact that Guadalupe, the only girl in the group, was not talking.

> Maybe I’m looking into it too much, but this so far reminds me of the conversation that Ms. B had in September with this set of girls about kind of it being male majority, especially in a breakout room where there’s three guys and one girl. It’s a three to one ratio. And Guadalupe is a pretty confident person, yet we haven’t heard her talk much about the problem … It’s kind of sad that there is such a difference, but also the groups were random. You can’t really do much about it.

The “three to one ratio” reminded Mr. K of a conversation Ms. B had recounted to him and Ms. Fink that had taken place at the beginning of the school year. Several girls in the class had brought up concerns to Ms. B regarding how the boys in the class were dominating class discussions. Ms. B listened to their concerns and addressed the issue with the whole class back in September. In this reflection, Mr. K connected Guadalupe’s lack of talk to the “male majority,” implying boys were dominating the conversation in this group, just as they had done previously in the class. He was not happy with this pattern, but since the groups were assigned randomly through Zoom, he felt there wasn’t much that could be done about it.

Several minutes later, Mr. K reflected on Ms. B’s presence in the breakout room, suggesting that Guadalupe may have been empowered by Ms. B to represent the group’s thinking. In response to Ms. B’s question, “Are you guys saying yes or no?” Guadalupe responded without hesitation, “We’re saying they’re going to be at different rates.” Mr. K reflected on Guadalupe’s participation in comparison to Yonas.

> I think that Ms. B kind of empowers the girls, or at least Guadalupe, a lot. So, the moment she comes in, I think that Guadalupe is kind of like - Guadalupe seems like kind of a leader of the group, even though the last 4 or 5 minutes of the video don't match up with that. It seems like if anything Yonas is kind of the leader because he talks the most.

Mr. K pointed out that the image Guadalupe was portraying to Ms. B of being a leader in the group was not consistent with how she had been interacting with her peers before Ms. B joined. He highlighted a common issue teachers face when facilitating groupwork, that of not knowing what happens in student groups before or after they join the group. Later during the interview, Mr. K reflected further on Guadalupe's inconsistent verbal participation.

> Guadalupe is usually willing to talk math and usually speaks up about it. In office hours, she was usually one of the people who talked the most about math, not about other stuff. So [her participation in this group] is atypical … maybe I'm wrong, maybe this was her everyday behavior and maybe I was fooled in office hours.

Mr. K began questioning his earlier assessments of Guadalupe. He went from thinking her limited verbal participation with The Ladder Problem was atypical, to wondering if perhaps he had been "fooled."

After watching the entire video, Mr. K brought up Guadalupe again. The researcher said, "You've shared a lot already. Is there anything else that you're thinking about that is noteworthy or stands out to you?" Mr. K replied with the following questions.

> Something that I'm wondering about is how Guadalupe is feeling and what she's thinking about the problem as well, but I think that's not as important as how she's feeling. Is she comfortable in the group? Does she feel like she can speak her mind in the group? Does she feel supported?

Mr. K brought up the idea that student thinking and student feelings of comfort are both important. Yet, in his mind, at that time Guadalupe's feelings were more important than her thinking. Although Mr. K did not say it explicitly, he implied the answers to his questions were "no." The interviewer probed, "Do you think there's something that could have been different to address those concerns or to support her more?" Mr. K shared some ideas.

> I think that something that could have been different is having more female representation in the first place. But I also think that even though you may not have felt it was your place during that time, maybe you saying, 'Oh, Guadalupe, what are you thinking?' 'Do you have any reply to Hosein's question?' 'Were you able to interpret what Elijah is saying?' Maybe some kind of sentence starters, because I think the moment the students hear you talk, they'll listen to Guadalupe.

Mr. K brought up female representation and suggested the researcher (Ms. Fink) could have interjected and prompted Guadalupe's verbal participation. Neither of his initial suggestions placed responsibility for addressing the concerns with Guadalupe in the hands of the four

students in the group. However, later in the interview he pointed out how Guadalupe's peers could have acted differently by listening to her ideas.

> The group seemed equitable for the boys, but not necessarily for Guadalupe. Equitable in the sense of the most basic rule of group work - if someone says something, you should hear them out even if you don't agree with them. But that didn't seem to be the case for Guadalupe.

Mr. K felt that Yonas, Hosein, and Elijah all had enough opportunities to have their thoughts and ideas heard, but Guadalupe did not. He hypothesized that Guadalupe felt unheard and undervalued by her peers, presumably due to the gender imbalance in the group and unsupportive interactions among group members. Mr. K continued to brainstorm ideas for addressing inequitable participation.

> Responsibility of equity should fall on the students, especially in group work. But students are not that perceptive. But at the same time, if you're the teacher and say you had said, 'Oh you guys should include Guadalupe more.' Then now it's kind of like a task, 'Oh, what a drag. We need to include her because she's not talking.' I don't have the answer to this, but it's definitely something interesting to think about. It makes me wonder should there be some sort of ghosts in the classroom that go around and say, 'Guadalupe, you should talk more,' or 'Hosein, ask Guadalupe something.'

Earlier in the interview, Mr. K suggested that Ms. Fink could have prompted Guadalupe's verbal participation through direct questions, but in this passage, he countered that idea by acknowledging how teacher interventions could come across as annoying and artificial to students. His final comment imagining "ghosts in the classroom" suggested responsibility for shifting participation patterns should be shared across students; Guadalupe should talk more, and her peers should ask her more questions.

Like Ms. B, Mr. K noticed Guadalupe's lack of verbal participation and, at least initially, judged her behavior as atypical for her in class. However, he seemed less certain about his assessments of Guadalupe and wondered out loud if he had been fooled by her. Mr. K did not comment on the mathematical content, like Ms. B did. He focused more on making sense of the interactions between students and what individual students may have been thinking and feeling. His age, only several years older than the four calculus students, and his identity as a student of color, may have made it more natural for him to reflect on the lesson through the students' eyes as opposed to through the teacher's eyes. This was the first time he had seen Ms. B teach this lesson, so he probably did not have clear expectations for students' mathematical conversations. The last time he had seen this material taught, he had experienced it as a student himself.

Mr. K's noticing of this group task was different from Ms. B's. Some observations they made were similar to each other and some were quite different. Mr. K's sensemaking was likely shaped by his relatively recent experiences learning this same content material as a student of color in a predominantly White high school classroom. Ms. B's sensemaking was likely shaped by her extensive experience refining and implementing this lesson for years with countless groups of students. In addition, Ms. B knew these students in ways that Mr. K did not. She had taught

Guadalupe the previous year in person, and she had known the families of Yonas and Elijah for years. Mr. K had only just met these students (through Zoom) one month prior to this lesson. This is not to say that one perspective is better or more accurate than the other. Their perspectives were complementary, offering different insights and wonderings worth exploring further.

Students' Perspectives

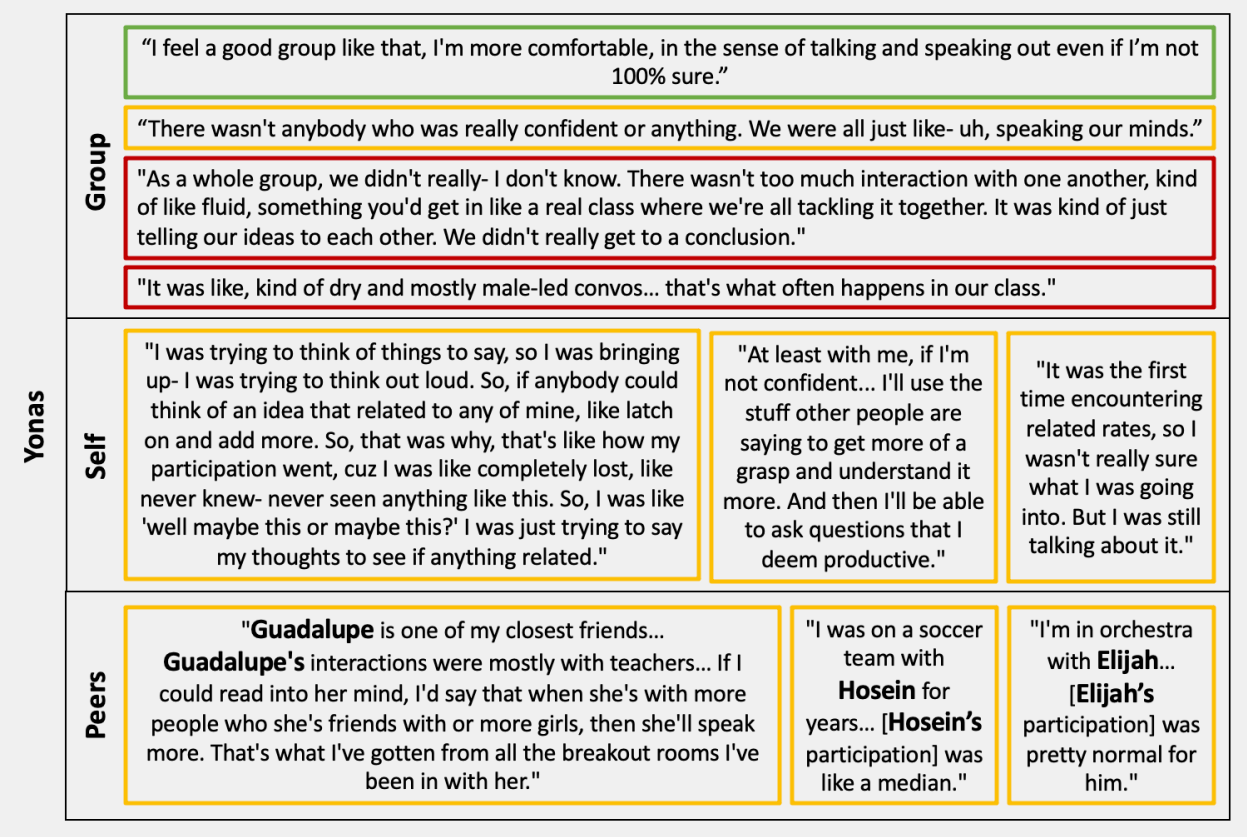
Yonas, Guadalupe, Hosein, and Elijah were in the same group, and all watched the same 10-minute video of their group working on The Ladder Problem. Yet, each student's assessments of the experience were unique, and in some cases contradicted each other's accounts. All four students opted to watch the entire video before sharing their reflections. Findings related to the students' perspectives are divided into two sections. The first section contains a summary of each student's observations, including what they noticed and how they made sense of their group, themselves, and their peers in the context of this small-group task. The second section organizes students' accounts of working on The Ladder Problem in response to a series of hypothetical questions. These questions were not asked explicitly during the participant interviews; rather, answers to these questions were presumed through analysis of students' comments made in response to more open-ended interview questions. The differences in students' presumed answers highlight consequential differences in the students' experiences working on The Ladder Problem.

Summary of Each Student's Observations

Each student's observations about student engagement with The Ladder Problem were divided into three categories (Group, Self, and Peers), depending on the subject of the student's observation. For example, if the student made a comment about their own participation, that was categorized as "Self." If they made a comment about one of their peers, that was categorized as "Peer." Representative quotations from the student interviews were selected to illustrate the breadth and depth of what students shared with respect to each of the three categories. Each observation was also color coded to indicate whether the comment had a positive tone (green), neutral or mixed tone (yellow), or negative tone (red). The level of detail, the content, and the tone of student observations varied considerably among the four students.

Yonas's observations and assessments of the students' work on The Ladder Problem are presented in Figure 6h.

Figure 6h. Yonas's Observations and Assessments of The Ladder Problem

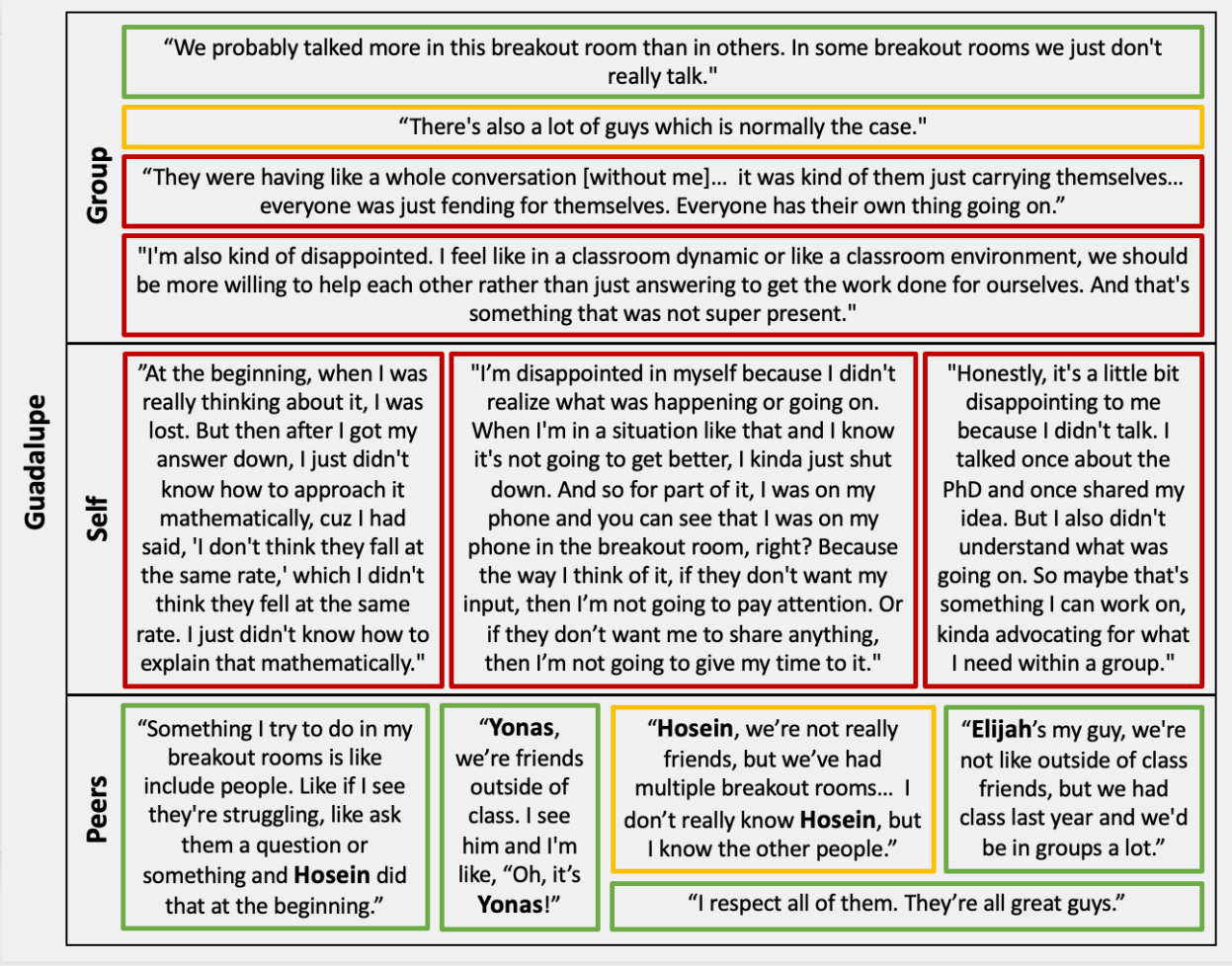


Yonas's overall assessment of the group was mixed; he shared positive, negative, and neutral comments about the group as a whole. He referred to the group as "a good group like that," saying he was comfortable sharing his ideas. However, he expressed disappointment that "there wasn't much interaction with one another" and that they did not reach a final solution. He also acknowledged the conversation was "dry and mostly male-led." He seemed to position all of the students on the same level in terms of understanding, saying they were all just "speaking [their] minds."

Unlike the mixed tones of his group comments, Yonas's comments about his own participation and his peers' participation were relatively neutral. He described how he was "trying to think out loud" because he was "completely lost." He was talking even though he was not confident in the hope of sparking a conversation that would help him "get more of a grasp and understand it more." The only aspect of his peers' participation that stood out to him was the fact that "Guadalupe's interactions were mostly with teachers." He extended this observation and suggested the reason Guadalupe did not speak as much during the student conversations was because she was not with "people who she's friends with or with more girls." Yonas also acknowledged knowing all three peers outside of class; Guadalupe was one of his "closest friends," he played soccer with Hosein, and was in orchestra with Elijah.

Guadalupe's observations and assessments of the students' work on The Ladder Problem are presented in Figure 6i.

Figure 6i. Guadalupe's Observations and Assessments of The Ladder Problem



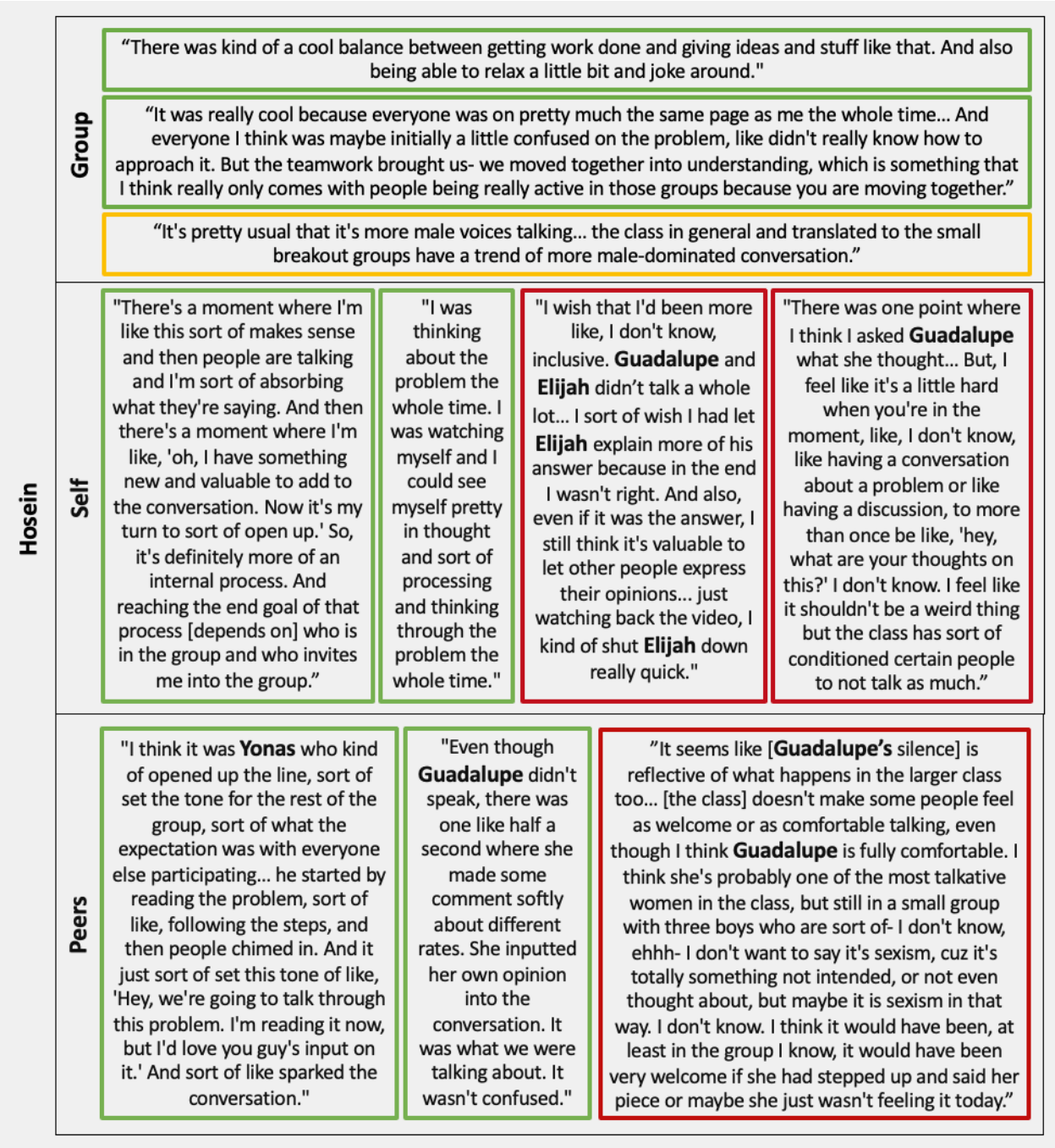
Like Yonas, Guadalupe's overall assessment of the group was mixed. In a positive tone, she shared, "we probably talked more in this breakout room than in others." However, after noting that there were "a lot of guys" in the group, she went on to describe how "they were having like a whole conversation" and were "just carrying themselves." Her first comment used the word "we," so she was including herself as part of the relatively talkative group, but she used the words "they" and "them" in other comments about the group, positioning herself as outside of the group and separate from her peers. She also expressed disappointment in the group for not being "more willing to help each other."

Guadalupe's assessments of her own participation were consistently negative, whereas her assessments of her peers were generally positive. She talked about being disappointed in herself multiple times. She was disappointed that she did not "know how to approach [the problem] mathematically," "didn't realize what was happening or going on," and "didn't talk." She explained that she "kinda just shut down" because she felt her groupmates were not interested in her input and she did not think it would get any better. She did not blame her peers completely, though. Toward the end of the interview, she took some responsibility for her lack of talk as well, saying, "maybe that's something I can work on, kinda advocating for what I need within a group." Despite feeling like her peers didn't want her to share her ideas, Guadalupe described them as "great guys" and said she respects all of them. She does not know Hosein very

well, but she gets excited to see her friend Yonas and described Elijah as “my guy.” She acknowledged the interactions were not supportive of her participation but did not seem to hold that against the individual students in her group in a personal way.

Hosein’s observations and assessments of the students’ work on The Ladder Problem are presented in Figure 6j.

Figure 6j. Hosein’s Observations and Assessments of The Ladder Problem

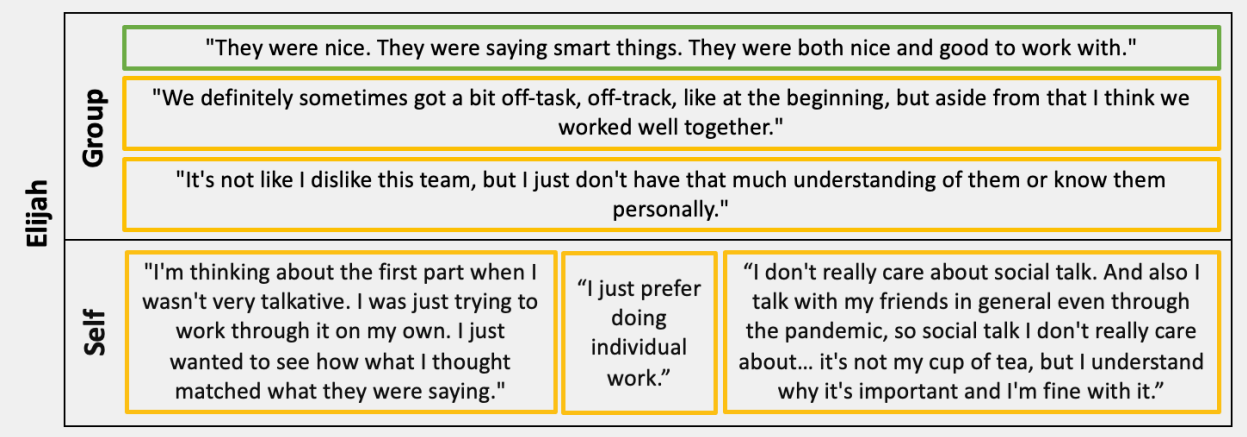


Hosein's tone when talking about the group was generally positive. He observed there were "more male voices talking," but still, he felt there was a "cool balance between getting work done and giving ideas … and being able to relax a little bit and joke around," "everyone was on pretty much the same page," and the group "moved together into understanding."

When reflecting on the details of what he and his peers did, Hosein made both positive and negative observations, many of which focused on Guadalupe's lack of talk. He described his "internal process" of "absorbing what [his peers were] saying" and then feeling ready to "add something new and valuable" to the conversation. He claimed he was deeply engaged in the task and "thinking through the problem the whole time." He also recognized that "Guadalupe and Elijah didn't talk a whole lot" and connected their lack of talk to his not being "inclusive." Specifically, he took responsibility for not letting "Elijah explain more of his answer" and for "shut[ting] Elijah down." He acknowledged asking Guadalupe at one point "what she thought," but said it is hard to do that "more than once" when you're "having a conversation about a problem." He then blamed this "weird" situation on the class environment, saying, "the class has sort of conditioned certain people to not talk as much." After detailing how Yonas "opened up the line" and "sparked the conversation" in their group, Hosein expanded further on the idea that the class did not "make some people feel as welcome or as comfortable talking." He noted that Guadalupe "inputted her own opinion into the conversation" at one point and "it wasn't confused." He also presumed Guadalupe was "fully comfortable" and described her as "one of the most talkative women in the class." Yet, he connected her silence in this group on this day to "not intended … sexism." He also placed some responsibility for not talking on Guadalupe's shoulders; his comment, "if she had stepped up and said her piece…" implied that Guadalupe had agency to speak but chose not to.

Elijah's observations and assessments of the students' work on The Ladder Problem are presented in Figure 6k.

Figure 6k. Elijah's Observations and Assessments of The Ladder Problem



Elijah had the shortest responses to the open-ended questions asked during the student interview and consequently spent the least amount of time reflecting on The Ladder Problem (5 minutes for Elijah vs. 13 minutes for Yonas, 16 minutes for Guadalupe, and 18 minutes for Hosein). Although Elijah seemed generally content with the overall experience, most of Elijah's

comments were delivered in a neutral tone with little emotion. Without much elaboration, Elijah shared that the other students "were both nice and good to work with," but Elijah didn't know them personally." Names of peers were not used during the interview; rather, Elijah referred to them collectively as "this team." Elijah explained that at times the team "got a bit off-task" but they "worked well together," even though Elijah preferred "doing individual work." Elijah explained, at the beginning "I was just trying to work through it on my own" and also shared that groupwork (and social talk) were not "my cup of tea." However, Elijah seemed open to peer communication and willing to participate in ways that benefitted other students.

Comparison of Students' Experiences

Students' accounts of working on The Ladder Problem are organized in this section according to a series of hypothetical questions. These questions were not asked explicitly during the participant interviews; rather, answers to these questions were presumed through analysis of students' comments made in response to more open-ended interview questions. If students had been asked these questions during their interviews, the answers presented here are their anticipated responses based on what they said during their interviews. Support for each presumed answer is given in the form of direct quotations from the stimulated-recall interviews. The questions and presumed answers are intended to offer a summarized view of students' experiences by representing aspects of student engagement that are generally thought to support student learning and identity development during small-group tasks in mathematics classrooms (e.g., mathematical challenge, enjoyment, inclusion). The differences in students' presumed answers and supporting narrative observations highlight consequential differences in the students' experiences working on The Ladder Problem.

A summary of questions and attributed answers about the experiences of Guadalupe, Yonas, Hosein, and Elijah working on The Ladder Problem is given in Figure 6l.

Figure 6l. Questions & Answers (Summarized) about Students' Experiences with the Task

	Guadalupe	Yonas	Hosein	Elijah
Was I challenged by this math problem?	Yes	Yes	Yes	Yes
Was I motivated to think and/or talk about this problem?	Somewhat	Yes	Yes	Yes
Was I comfortable sharing my ideas?	No	Yes	Yes	Yes
Did I enjoy this math experience?	No	Somewhat	Yes	Yes
Did the group work together to solve the problem?	No	No	Yes	Yes
Was everyone included in the conversation?	No	No	No	I think so

The answers to these questions suggest that each of the four students experienced The Ladder Problem differently from their peers. Guadalupe's experience was relatively negative compared to her peers, while Hosein and Elijah's experiences were relatively positive. Yonas's experience fell somewhere in the middle with both positive and negative aspects. Direct quotations from students' interviews justifying the presumed answers to the student experience questions are provided in Figure 6m.

Figure 6m. Questions & Answers (with Support) about Students' Experiences with the Task

	Guadalupe	Yonas	Hosein	Elijah
Was I challenged by this math problem?	**YES** "At the beginning, when I was really thinking about it, I was lost. But then after I got my answer down, I just didn't know how to approach it mathematically cuz I had said like, I don't think they fall at the same rate, which I didn't think they fell the same way. I just didn't know how to explain that mathematically."	**YES** "It was the first time encountering related rates, so I wasn't really sure what I was going into... I was like completely lost, like never knew, never seen anything like this."	**YES** "Everyone was maybe initially a little confused on the problem, like didn't really know how to approach it... I was thinking about the problem the whole time. I was- I was watching myself and I could see myself pretty in thought and sort of processing and thinking through the problem the whole time."	**YES** "This problem was a bit more challenging for me than lots of them."
Was I motivated to think and/or talk about this problem?	**SOMEWHAT** "At the beginning, when I was really thinking about it, I was lost. But then after I got my answer down, I just didn't know how to approach it mathematically... when I'm in a situation like that and I know it's not going to get better. I kinda just like shut down."	**YES** "I was trying to think of things to say, so I was bringing up- I was trying to think out loud. So, if anybody could think of an idea that related to any of mine, like latch on and add more... I was like 'well maybe this or maybe this?' I was just trying to say my thoughts to see if anything related... With more participation you can get better answers. So, I try to get myself as well as others to participate."	**YES** "I was really motivated to participate. When there are people who have their cameras on, who are talking, and who are making progress on a problem, there's an urge, at least on my- for me, to want to get the next step of the problem, to be the person to be like, 'Hey, what about doing this next?'"	**YES** "There were interesting things to think about with it... I was just trying to work through it on my own. I just wanted to see how what I thought matched what they were saying."
Was I comfortable sharing my ideas?	**NO** "[My female friend and I] normally don't talk in breakout rooms unless we're asked to. Part of it is because we don't really feel like we have the space to. I don't know, when it's more girls than guys, we feel more comfortable... And sometimes I'm gonna be honest, it's hard to ask for help partially because of what we were talking about last time. I don't like seeming stupid, or like I need help or like I don't understand it. That just makes you feel very not empowered."	**YES** "With a good group like this, I'm more comfortable, in that sense of talking and speaking out even if I'm not 100% [sure]... It's easier [this year]. If I have a question or- it's also easier to admit being lost as well in here. The environment doesn't feel as toxic [as last year]."	**YES** "I'm comfortable in most groups... it took a while for me to jump into the conversation, at least longer than Elijah and Yonas, but um, yeah... there's a moment where I'm like this sort of makes sense and then people are talking and I'm sort of absorbing what they're saying and then there's a moment where I'm like 'oh, I have something new and valuable to add to the conversation. Now it's my turn to sort of open up.'"	**YES** "In general, it's fairly easy to participate."
Did I enjoy this math experience?	**NO** "I enjoyed talking to you about your PhD. That was the only reciprocated conversation. But in terms of math, not really. I just felt really lost and not smart the whole time."	**SOMEWHAT** "I'm in orchestra with Elijah. I was on a soccer team with Hosein for years, and Guadalupe's one of my close friends. With a good group like this, I'm more comfortable... I kind of didn't like how we didn't get anywhere, like in the end. We just sort of circled around and there wasn't really any like, 'Well, that's a good idea, let's work with that.' So, I would say I didn't like that part."	**YES** "I enjoyed all of it... I really enjoy understanding the problem at least on some level that I can have a good conversation that actually feels like it's going somewhere, that I'm problem solving with other people. It's like a generally good feeling. Yeah. I really- really enjoy when- this is why this conversation stuck to me so much."	**YES** "I enjoyed that. It was a fun problem to work with... Math is definitely one of the things I enjoy more. It's this nice, clear logical thing and it's fun to explore and work through things like that."

	Guadalupe	Yonas	Hosein	Elijah
Did the group work together to solve the problem?	**NO** "It was kind of them just carrying themselves… we should be more willing to help each other rather than just answering to get the work done for ourselves… everyone was just fending for themselves."	**NO** "There wasn't too much interaction with one another, kind of like fluid, something you'd get in like a real class where we're all tackling it together. It was kind of just telling our ideas to each other. We didn't really get to a conclusion."	**YES** "I really, really enjoy the whole collaboration and throwing back and forth ideas, all this kind of stuff to come to an answer as a team, like truly as a team… The teamwork brought us- we moved together into understanding."	**YES** "We definitely sometimes got a bit off-task, off-track, like at the beginning, but aside from that I think we worked well together."
Was everyone included in the conversation?	**NO** "It seems like they were having a whole conversation and I was obviously very lost… Honestly, it's a little bit disappointing to me because I like didn't talk- I talked once about the PhD and once shared my idea… there's also a lot of guys which is normally the case."	**NO** "It's like kind of dry and mostly like male-led convos, so that's like the aspect that I was getting to- that's what- often times in our class, that's what happens."	**NO** "I wish that I'd been more like, I don't know, inclusive. Guadalupe and Elijah didn't talk a whole lot… watching back the video I was like I kind of shut Elijah down really quick… it seems like [Guadalupe's silence] is reflective of what happens in the larger class too… the class in general and translated to the small breakout groups have a trend of more male-dominated conversation."	**I THINK SO?** "They were nice. They were- they were saying smart things. They were both nice and good to work with."

The quotations provided in each cell of the table are intended to support the presumed answers (e.g., YES, NO).

The only issue on which all four students agreed was about whether the students were challenged by this math problem. All four agreed the problem was challenging. Guadalupe and Yonas felt "lost," Hosein spoke about being "confused," and Elijah said, "this problem was a bit more challenging for me than lots of them." Though all were challenged by the math, the way this challenge affected their motivation and comfort was different. Guadalupe was initially motivated to think about the problem but disengaged and "shut down" when she did not know how to move forward. She felt lost but was not comfortable asking questions because she did not like "seeming stupid" or like she needed help. She attributed not understanding to feeling "very not empowered." In contrast, the challenge of the math problem seemed to encourage deep thinking by Yonas, Hosein, and Elijah without discouraging their verbal participation. All three of them seemed to be motivated to participate and seemed to feel comfortable sharing their ideas. Yonas said he was "trying to think out loud … [and] get [him]self and others to participate." He shared that he was comfortable "talking and speaking out even if [he was] not 100%." Hosein said he was "really motivated to participate," he had an "urge" to "get the next step of the problem," and that he was generally "comfortable in most groups." Elijah seemed pleased that there were "interesting things to think about" and expressed no hesitation with sharing his ideas, saying, "it's fairly easy to participate."

Enjoyment of the task seemed to be tied to different aspects of the experience for each of the four students. Guadalupe seemed to enjoy the mathematical experience the least. She connected enjoyment to "reciprocated conversations" and feeling "smart." She said she enjoyed a conversation with me (researcher / observer) about my PhD but did not enjoy the math because she "just felt really lost and not smart the whole time." The "reciprocated conversation" to which Guadalupe referred came seven minutes into the group's work on the task. Seemingly out of the

blue, Guadalupe asked me, “Ms. Fink, are you gonna be a math teacher? I forgot.” I responded by saying that I used to be a middle school math teacher, but I am now working on a PhD. Guadalupe responded, “That’s so cool! How hard is a PhD? It’s like research-based, right?” To which I said, “Yeah. Well, this is my sixth year at Berkeley, so -,” implying a PhD is indeed hard and takes time. Guadalupe laughed and said, “Oh, hell no! I always tell my mom that I want a PhD, then I think about how much school it is. Oh my god. That’s crazy. Props to you, Ms. Fink.” It is unclear what exactly constitutes a “reciprocated conversation” in Guadalupe’s mind. I answered her direct questions and returned her smiles, but I did not ask her any questions or offer additional information to extend the conversation. Perhaps the shared smiles and shared understanding of my personal endeavor was what equated to enjoyment for her.

Guadalupe’s peers seemed to enjoy the experience more than she did, especially Hosein and Elijah. Both Hosein and Elijah stated explicitly that they enjoyed the experience. Hosein said he “enjoyed all of it,” connecting enjoyment to understanding the problem enough to “have a good conversation that actually feels like it’s going somewhere.” Hosein’s comment, “this is why this conversation stuck to me so much,” implied his Ladder Problem experience stood out in his mind as even more enjoyable than most other conversations in class. Elijah acknowledged “it was a fun problem to work with,” but connected enjoyment of this experience to math in general, saying, “math is definitely one of the things I enjoy more … it’s this nice, clear logical thing.” Unlike Hosein and Elijah’s expressions of pure enjoyment, Yonas conveyed mixed feelings related to enjoyment. He talked about knowing all his groupmates outside of class and how he’s more comfortable with “a good group like that,” but he went on to say he did not like how they “didn’t get anywhere” and how they just “circled around” without consensus on how to move forward.

The students were split on whether the group worked together on the task or not; Guadalupe and Yonas felt people were acting independently, while Hosein and Elijah thought people worked well together. Guadalupe felt they were “just answering to get the work done” and “everyone was just fending for themselves.” Similarly, Yonas observed “there wasn’t too much interaction with one another,” describing the experience as “kind of just telling our ideas to each other” without reaching “a conclusion.” In contrast, Hosein was excited about the “collaboration,” claiming the group reached “an answer as a team, like truly as a team,” and teamwork moved them “together into understanding.” Elijah’s assessment was positive though not quite as enthusiastic as Hosein’s. Elijah acknowledged the group “sometimes got a bit off-task,” but still they “worked well together.”

Elijah did not seem to register any imbalance in verbal participation, whereas Guadalupe, Yonas, and Hosein all noticed the conversation was dominated by male voices, implying that Guadalupe’s voice was missing or at least lacking. Elijah did not say much about the other students, saying generally that they were “nice” and “saying smart things.” Since Elijah did not mention anyone being excluded, the assumption is from Elijah’s perspective, everyone was included. The other three students all noticed that Guadalupe, the only female in the group, was left out of the conversation. They also all noted that this pattern of there being “a lot of guys,” “male-led convos,” and “male-dominated conversations” was “normally the case” and was “reflective of what happens in the larger class too.” So, in their eyes, everyone was included except for Guadalupe.

In pursuit of participatory equity, one important goal is to design and implement small-group learning tasks that support every student to engage deeply with the mathematics in ways

that lead to content proficiency and positive mathematical identities. Robust learning environments require students to have equitable access to the mathematics and to have opportunities to exercise agency and take ownership over their learning (Schoenfeld, 2014). In this case with The Ladder Problem, students' accounts of their learning experiences suggest the three male students, to varying degrees, had access to the content, exercised agency, and took ownership over their learning. Guadalupe did not. Initially, she engaged with the mathematics, but ultimately, she disengaged and stopped thinking about the mathematics. She did not feel she had the space to share her ideas or get her questions answered. Yonas, Hosein, and Elijah all seemed to have relatively easy access to the math; they were challenged, motivated, and comfortable sharing their ideas. Hosein and Elijah also enjoyed the experience in ways that likely affirmed their identities as competent learners and doers of mathematics. Yonas enjoyed it less but still had opportunities to contribute in mathematically competent ways. Hearing the students' perspectives provides insight into how each of them experienced this task personally, but it also provides insight into what they noticed about their peers' experiences. Hosein and Yonas's awareness of Guadalupe's lack of verbal participation, and Elijah's lack thereof, raises questions about the roles students should play in supporting equitable participation for their peers. Whose responsibility is it to notice and address inequitable participation?

Part 2: Individual Student Contributions

This section addresses the question, How did each student contribute during the task? Data shared here pertains to *individual* student participation metrics, as opposed to *interactional* participation processes (shared in the next section, Part 3: Interactional Participation Processes). Student contributions are examined by looking at the number of contributions made by each student, the type of contributions, and the timing of when contributions were made. Two main categories of contributions were identified (mathematical contributions and social contributions). There were consequential differences in the quantity, types, and timing of contributions made by each student, with Yonas dominating mathematical contributions and Guadalupe dominating social contributions. Particular attention is paid to the nature of Guadalupe's contributions and how her contributions compared to those of her peers.

Quantity and Types of Contributions

Student contributions were quantified by counting the number of words spoken and the number of contributions made by each student, shown in Table 6a. All words spoken were first divided into contributions, then all contributions were categorized as either mathematical or social depending on the presumed function. The word metrics provide insight into the amount of airtime occupied by each student for their mathematical and social contributions.

Table 6a: Number of Mathematical & Social Words and Contributions by Student

	Yonas	Guadalupe	Hosein	Elijah
# of **Mathematical** Words Spoken	454	55	227	213
# of **Social** Words Spoken	9	130	1	0
TOTAL # of Words Spoken	**463**	**185**	**228**	**213**
# of **Mathematical** Contributions	23	4	10	10
# of **Social** Contributions	2	9	1	0
TOTAL # of Contributions	**25**	**13**	**11**	**10**

Yonas spoke 463 words over the course of the 10-minute task, more than twice as many words as his peers, with the vast majority (98%) related to the mathematical task. Yonas's words resulted in a total of 25 contributions, 23 of which were mathematical. Hosein and Elijah both spoke over 200 words, resulting in 10 mathematical contributions each. Hosein had a one-word social contribution as well, whereas Elijah spoke zero social words and consequently had zero social contributions. Overall, Guadalupe spoke the least out of the four students, and unlike her peers, Guadalupe's social words and corresponding social contributions more than doubled her math words and math contributions. She spoke 185 words resulting in a total of 13 contributions; 70% of her words and contributions were social. Looking at the group as a whole, Yonas offered the majority of mathematical contributions and Guadalupe offered the majority of social contributions.

To provide a bit more detail about each student's participation, mathematical and social contributions were coded and organized by type. Table 6b shows the number of contributions made by each student by contribution type.

Table 6b: Number of Mathematical & Social Contributions by Type by Student

Number of Contributions by Type			Yonas	Guadalupe	Hosein	Elijah
	Mathematical	Asks a question	1	1	3	
		Makes a comment	6		2	1
		Shares mathematical reasoning	7		3	1
		Shares solution with reasoning	2	1	1	2
		Shares solution without reasoning		1		
		Expresses agreement	3	1	1	4
		Expresses disagreement	1			1
		Expresses uncertainty				1
		Reads the problem aloud	3			
	Social	Asks a question		2		
		Makes a comment	2	7	1	

In terms of mathematical contributions, all students shared at least one solution supported by mathematical reasoning, and each student expressed verbal agreement with a peer at least once. Yonas and Elijah were the only students who verbalized disagreement, Elijah was the only student who verbalized uncertainty, and everyone except Elijah asked at least one question. In terms of social contributions, everyone except Elijah made at least one social comment. Guadalupe made 7 comments and asked two questions, all of which received positive responses from at least one other person in the group.

Mathematical Reasoning Trajectory

To make sense of who contributed and in what ways, it is helpful to understanding the flow of mathematical ideas within the group. The sections that follow present data on how and when each student contributed, both mathematically and socially, over the span of the task. In contrast, this section focuses on the group's overall mathematical progress with The Ladder Problem. From the beginning to the very end of their work time, students in this group shared mathematical ideas and built off each other's thinking. Their mathematical talk time was divided between discussing the first question of the task (Q1: Does the top of the ladder move at the same rate as the bottom of the ladder?) and discussing the second question (Q2: How fast is the

top of the ladder sliding down the wall when the bottom of the ladder is 6 feet from the wall?). Table 6c presents the group's trajectory of mathematical reasoning in chronological order, including each student's proposed answer (if there was one), the student's reasoning, and the response the student received after sharing their ideas.

Table 6c. The Group's Trajectory of Mathematical Reasoning

Student	Task Question	Answer	Reasoning	Response
Yonas	Q1	different	"they start at different points"	no response
Yonas	Q1	same	none	agreement from & interruption by Elijah
Elijah	Q1	same	"they move the same amount"	no response
Hosein	Q1	different	"if you have instead of a ladder on a wall, you have a really long stick or like a whip or something. Like, you're not moving it very far, but it becomes faster and faster towards the end... my gut feeling is they have different rates."	neutral repsonse from Elijah
Elijah	Q1	unsure	none	no response
Guadalupe	Q1	different	"if it's like falling, it falls this distance but normally it pushes out a lot faster. It would be flat at the end."	interruption by Yonas
Yonas	Q1	?	"It's going to be like a triangle"	neutral response from Elijah
Hosein	Q1	different	"It has to move a greater distance, in the same amount of time."	agreement from Yonas
Yonas	Q1	different	"this is gonna be implicit differentiation. So... we have to solve for each one."	agreement from Guadalupe
Guadalupe	Q1	different	none	neutral response from Ms. B
Yonas	Q2	?	"Ms. Fink, this is going to be x-squared plus y-squared, right?..."	neutral response from Ms. Fink, redirected back to group
Yonas	Q2	?	"So, we can do implicit differentiation to find the rate of change by when it's falling and-"	no response, interruption by Hosein
Hosein	Q2	top falls twice as fast as the bottom	"it's ten feet long and it's at like eight feet up the wall. Y is 8. It's six feet from the wall. So, the ladder has to slide an additional four feet to be completely flat. But the top of the ladder has to slide down 8 feet."	disagreement from Elijah
Elijah	Q2	top does *not* fall twice as much as the bottom	"because it's six verses eight."	disagreement from Hosein
Hosein	Q2	top falls twice as fast as the bottom	"it has to move an additional four on the bottom compared to the eight that it has to move at the top."	agreement from Elijah
Elijah	Q2	top falls twice as fast as the bottom	"if we assume it started out flat against the wall then for it to get to eight six then it would be moving two feet at the top and four feet at the bottom."	no response
Elijah	Q2	top falls three times as fast as the bottom	"Because if it's only going out to ten-"	interruption, bumped back into the whole-class Zoom room

All four students shared answers for Question 1, supported by reasoning. Yonas was the first to propose the ends of the ladder fell at different rates, but before anyone could respond, he changed his mind and proposed the opposite. Elijah interrupted with agreement, then Hosein proposed an unrelated idea suggesting the opposite answer. Guadalupe was the first one to say with confidence that the rates were different. Yonas interrupted her explanation but eventually went on to agree with Guadalupe's answer, as did Hosein and Elijah. After Guadalupe shared the answer to the first question with Ms. B, the group's focus shifted to the second question about relative rates. Yonas did a lot of thinking out loud, sharing ideas about how he was approaching the problem without stating a definitive answer to the question. Hosein interrupted Yonas's

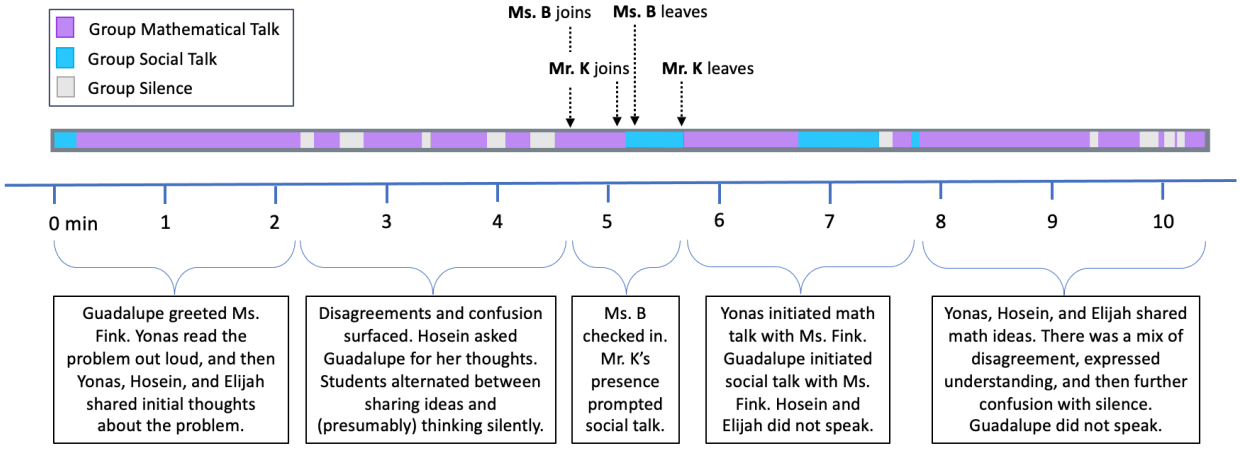
think-aloud to propose that the top fell twice as fast as the bottom. At first Elijah disagreed with Hosein's proposal, but after further clarification from Hosein, Elijah agreed, stating his own reasoning for this conclusion. But Elijah continued to think about the question and ended up disagreeing with his own conclusion just as they were all bumped back into the whole-class Zoom room. Although Yonas started out sharing ideas about how he was trying to answer the second question, he never shared an answer to the question or gave any indication that he agreed or disagreed with the answers shared by Hosein and Elijah. Guadalupe did not contribute to the group's reasoning about question 2, so her thinking about the related rates question was unknown as well. It should be noted that neither Hosein nor Elijah's proposed answers to the second question were correct, but their engagement with the task likely set them up to make sense of the solution when everyone reconvened and discussed the problem together as a class.

Contribution Timelines

To get a better sense of how the students' mathematical and social contributions played out over the course of the task, it is helpful to see the group's talk mapped out over time. An overall timeline for the group's work on the task is provided, along with individual timelines for each student showing when each of their mathematical and social contributions was made. A closer look at Guadalupe's contributions, showing what she said and when, is included as well.

The videorecording of the task lasted 10 minutes and 19 seconds, after which the students were abruptly bumped back into the whole-class Zoom room. For the duration of the recording, time was divided into periods of group mathematical talk, group social talk, and group silence. Group silence included non-talk periods of more than three seconds. Non-talk periods of three seconds or less were assumed to be natural pauses in an ongoing conversation and, therefore, were subsumed within the most recent talk designation. Figure 6n displays a timeline of the group's mathematical talk, social talk, and silence over the course of working on The Ladder Problem. Also included in this timeline are the moments when the teacher (Ms. B) and student teacher (Mr. K) joined and left the group's Zoom breakout room, and summaries of what transpired during specified periods of time.

Figure 6n. A Timeline of the Group's Mathematical Talk, Social Talk, and Silence



Most of the group's working time was spent on mathematical talk, broken up by brief periods of silence and several short social conversations. Aggregated, the group spent a total of

7.4 minutes (72%) on mathematical talk, 1.5 minutes (15%) on social talk, and 1.4 minutes in silence (13%). After a short social greeting, the group spent about two minutes engaged in mathematical talk as Yonas read the problem out loud and three of the four students (everyone except Guadalupe) offered some initial thoughts about whether the ends of the ladder move at the same or different rates.

The next segment of time was characterized by disagreement, confusion, silence, and more ideas being surfaced. During this segment, Hosein asked Guadalupe what she was thinking, ending a 14-second period of silence just before the 3-minute mark. Yonas interrupted Guadalupe's explanation with another thought of his own. Everyone except Guadalupe continued to share ideas.

The third segment of time included the presence of both the teacher (Ms. B) and the student teacher (Mr. K). Ms. B asked a couple mathematical questions to check in on the progress of the group, to which Guadalupe and Yonas responded. Mr. K observed silently. Guadalupe noted the presence of "all three teachers" in the breakout room (Ms. B, Mr. K, and Ms. Fink), which prompted Ms. B to leave. Guadalupe continued with a period of social talk about Mr. K looking "hecka bored." After smiling in response to the comments, Mr. K left the group as well without saying a word.

The next two minutes involved talk with Ms. Fink. First, Yonas sought confirmation from Ms. Fink about an idea he had. In response, Ms. Fink suggested Yonas draw a picture and ask his groupmates for their opinions. Next, Guadalupe posed some questions to Ms. Fink about her future teaching and research plans as Yonas figured out how to access online drawing tools. Yonas then shared his screen and started to draw a ladder, prompting a very brief social exchange between Guadalupe and Yonas about the ladder sketch. Hosein and Elijah were both silent during this segment.

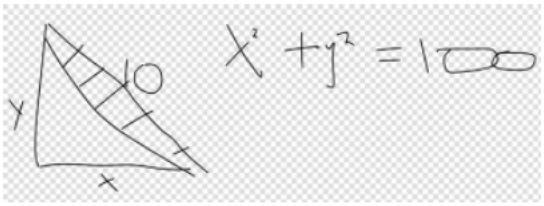
During the final two and a half minutes, Yonas, Hosein, and Elijah continued to share more ideas as they sorted out disagreements and offered clarifications of their thinking about the relative rates of the ladder ends. The math talk was mixed with short periods of silence as students considered their peers' ideas. At 9:54, Elijah stated, "Yeah, definitely moves twice as fast on the bottom," but Elijah continued to think. Just before the Zoom breakout room was closed, Elijah retracted his previous statement, saying, "Or - wait. Hang on - (4 seconds of silence). Wait, it moves three times as fast on the bottom as on the top. Because if it's only going out to ten -." He did not get a chance to finish his thought because the breakout rooms were closed, and all students were forced back into the whole class Zoom room.

The timelines in Figures 6p – 6s display each student's mathematical and social contributions to the group task over time. Arrows indicate the time a contribution began. Mathematical and social contributions are numbered and labeled with an "M" or "S" accordingly. The four figures are presented together on a single page to facilitate visual comparisons among them.

As stated earlier, Yonas had by far the highest number of mathematical contributions. As shown in Figure 6p, Yonas spread these contributions relatively evenly across the course of the task. His longest period of silence (about 90 seconds) was between M2 and M3, after he had read the problem out loud and shared an initial thought; Elijah and Hosein followed suit and shared ideas of their own following Yonas's sharing. Yonas was the first person to offer a mathematical contribution after each of the four periods of social talk, shifting focus back to the task at hand.

M1, M13, M17, and M19 effectively ended the social talk periods and initiated new periods of mathematical talk, including the two social talk periods Yonas took part in. In addition, some of Yonas's mathematical contributions occurred after periods of silence (M9, M10, M22, M23). There was only one non-verbal contribution made during this task, and it was Yonas's mathematical contribution (M17) made during a period of silence around the 7.5-minute mark. This contribution consisted of Yonas sharing his screen and drawing a diagram of The Ladder Problem during a period of silence (see Figure 6o).

Figure 6o. Yonas's Diagram of The Ladder Problem



Guadalupe had the highest number of social contributions, and she was the one who initiated all four periods of group social talk, starting with a greeting for Ms. Fink just as she entered the Zoom breakout room (S1). Besides her initial social contribution, Guadalupe was silent for the first several minutes of the task, while Yonas, Hosein and Elijah all shared mathematical ideas. She offered her first mathematical contribution just before the 3-minute mark after an explicit question from Hosein. Guadalupe's next four contributions (M2, M3, S2, S3) were made while the teacher and/or student teacher were present. Guadalupe's final episode of talk about 7 minutes into the task involved mostly social talk, with one brief mathematical question (M4) toward the end. She ended the task with another long period of silence.

Hosein began the task and ended the task with periods of silence lasting over a minute each time. His 10 mathematical contributions were split into two groups, one toward the beginning and one toward the end of the task. The first set of mathematical contributions stretched over a period of about 2 minutes when the group was discussing the first question about whether the ends of the ladder moved at the same rate. Hosein exchanged ideas with his peers and asked Guadalupe a question. His second set of mathematical contributions occurred about 4.5 minutes later toward the end of the task when the group was addressing the second question having to do with the difference in the rates of the ladder ends. The only other contribution Hosein made was his one, single-word social contribution ("Never") in response to a story told by Guadalupe 5.5 minutes into the task.

Elijah was the only student who made zero social contributions. Like Hosein, Elijah's 10 mathematical contributions were divided into two sets, one set occurred toward the beginning of the task and one toward the end. Elijah alternated between speaking and seemingly thinking by himself and listening to his peers. He frequently assessed his peers' ideas, expressing uncertainty at the beginning (M2), offering verbal agreement 4 times (M4, M5, M7, M9), and disagreeing once (M10), right at the end. Elijah's talk dominated the last minute and a half of the group's work time as his thinking changed. He went from thinking the top moved twice as fast as the bottom, to thinking the top moved three times as fast as the bottom. His final contribution (M10) was cut short due to the abrupt closing of the breakout rooms.

Figure 6p. A Timeline of Yonas's Mathematical and Social Contributions

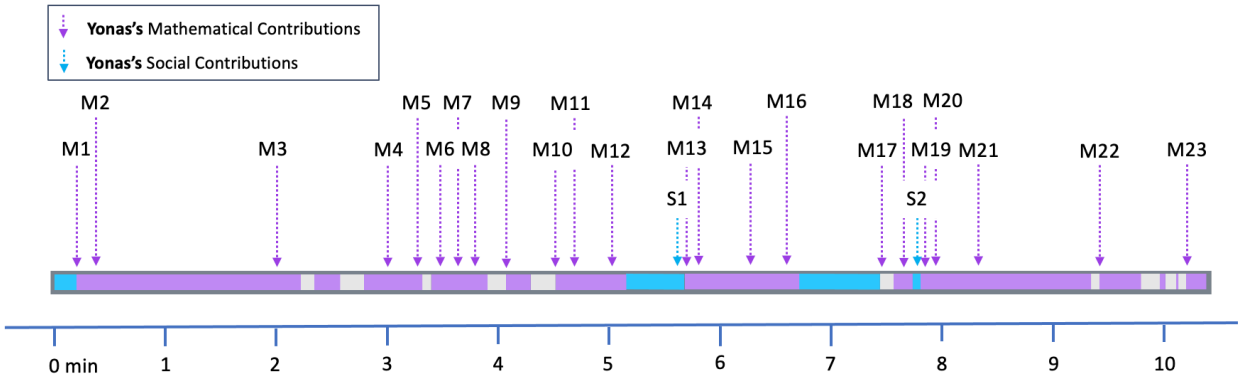


Figure 6q. A Timeline of Guadalupe's Mathematical and Social Contributions

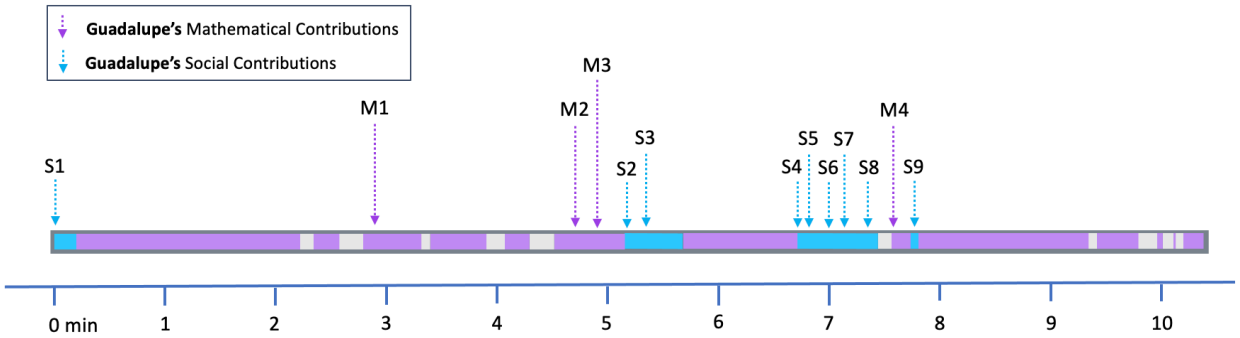


Figure 6r. A Timeline of Hosein's Mathematical and Social Contributions

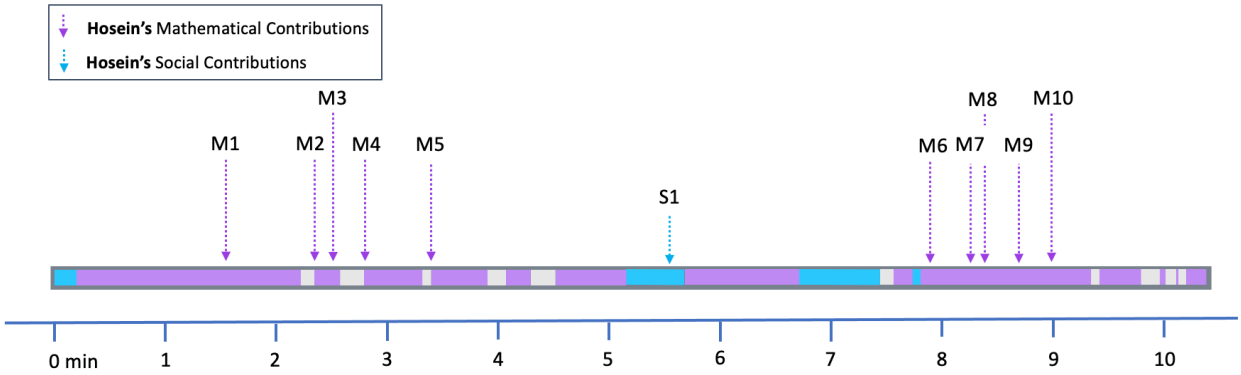
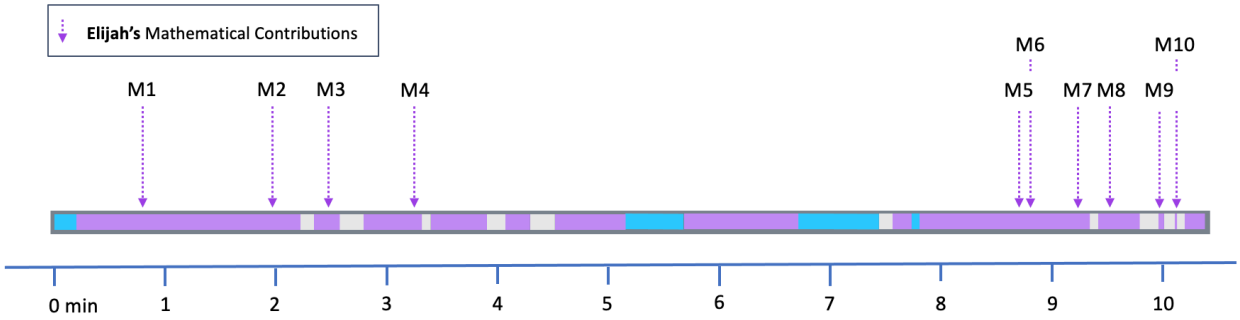


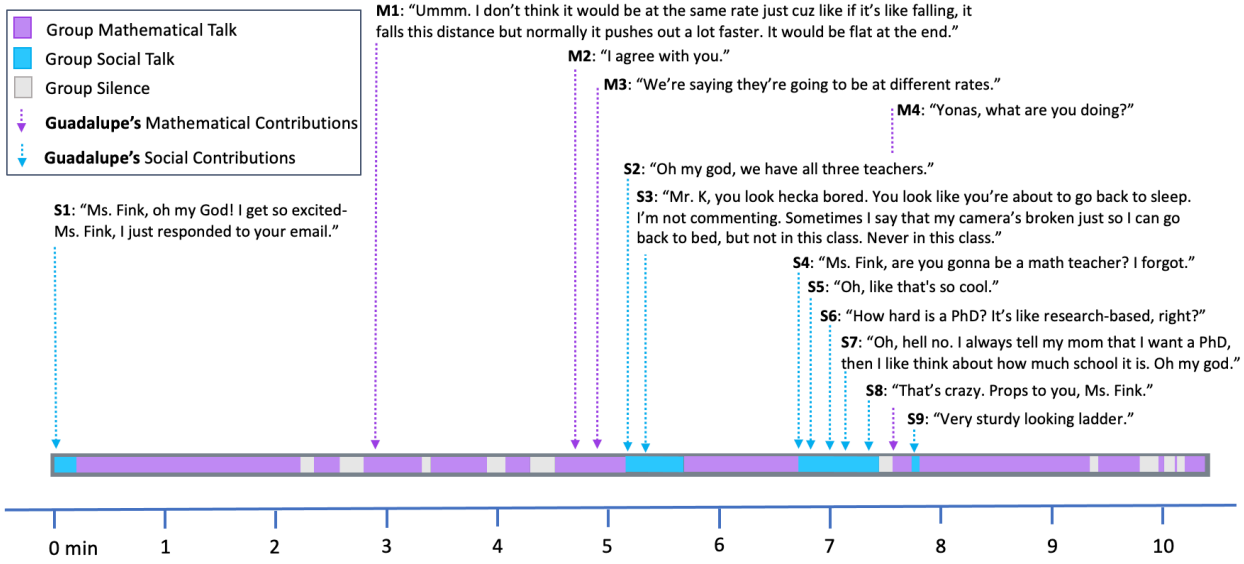
Figure 6s. A Timeline of Elijah's Mathematical and Social Contributions



A Closer Look at Guadalupe's Contributions

Guadalupe's participation in The Ladder Problem was notably different from that of her peers, in terms of total number of contributions and in terms of the distribution between mathematical and social. Knowing that Guadalupe was a student of color and the only female in the group adds to the importance of understanding her participation story more deeply. Elaborating on Figure 6q, Figure 6t displays the content of each of Guadalupe's mathematical and social contributions. Every word Guadalupe spoke during the 10-minute task is included in Figure 6t.

Figure 6t. A Timeline of Guadalupe's Mathematical and Social Contributions (A Closer Look)



Guadalupe spoke a total of 185 words (55 mathematical and 130 social) and made 13 contributions (4 mathematical and 9 social).

Since Guadalupe made the fewest mathematical contributions, it is worth taking a closer look at the four mathematical contributions she did make to get a better sense of her engagement with the intended mathematical content. After Yonas read the problem out loud at the beginning of the task, he, Hosein, and Elijah each shared a possible answer to the first question (Does the top of the ladder move at the same rate as the bottom of the ladder?) supported by reasoning; however, their answers did not all align. This sharing of ideas was followed by an expression of uncertainty by Elijah ("Yeah. I'm unsure.") and an expression of disagreement by Yonas ("I mean, the wall kind of like keeps the friction. I get what you're saying. When it's like sliding down, it's always going to be touching, though. That's the thing."). After 14 seconds of silence, Hosein responded to the difference in opinions by soliciting Guadalupe's input. He asked, "Guadalupe, what are you thinking about it?" She responded, "Ummm. I don't think it would be at the same rate just cuz like, if it's like falling, it falls this distance (motions with her arms) but normally it pushes out a lot faster. It would be flat at the end." That solution with reasoning was Guadalupe's *first* mathematical contribution, and it occurred almost three minutes into the task. Her suggestion that the rates were different was correct, although she said

the bottom "pushes out a lot faster," which seems to imply she thought the bottom was moving faster than the top, which is not correct.

About a minute and a half later in the conversation, Yonas was still working through the solution by talking out loud. He was saying, "The bottom of the ladder is 6 feet from the wall. The bottom of the ladder is 6 feet from the wall. Oh, Ok. Yeah, they will be at different rates. That makes sense." Guadalupe responded to Yonas with her *second* mathematical contribution by expressing agreement, "I agree with you." A few seconds later Ms. B appeared in the breakout room and asked, "Are you guys saying yes or no?" Both Yonas and Guadalupe began talking at the same time. They both said, "We're saying -" and then Yonas stopped, allowing Guadalupe to continue and offer her *third* mathematical contribution, a solution without reasoning. She went on to say, "they're going to be at different rates." Ms. B asked one more mathematical question, to which Yonas responded, before she left the group.

About seven and a half minutes into the task, Yonas shared his screen with the group was drawing a diagram of a ladder leaning against a wall using an online drawing tool. Before the drawing was complete Guadalupe offered her *fourth* and final mathematical contribution by asking, "Yonas, what are you doing?" Yonas went on to explain to the group what his diagram was representing. All 55 of Guadalupe's mathematical words comprising her 4 mathematical contributions were uttered in the last two paragraphs. She offered one mathematically correct solution with questionable reasoning, offered one correct solution without reasoning to the teacher, made one expression of agreement with Yonas's answer, and asked Yonas one clarifying question which prompted Yonas to explain his thinking to the group. While these contributions indicate some level of engagement with the mathematics for Guadalupe, it is impossible to tell how her thinking changed over the course of the conversation and if she understood how to justify the claim that the ends of the ladder were moving at different rates.

Since most of Guadalupe's contributions were social (9 out of 14 contributions), it is worth taking a closer look at her social contributions to get a better sense of how Guadalupe's social talk contributed to the students' experiences with the task. There were four periods of social talk during the 10-minute task, all of which were initiated by Guadalupe. The first words recorded during the group task were social. As soon as Ms. Fink joined the group, about 30 seconds after the students joined the breakout room, Guadalupe greeted her. "Ms. Fink, oh my God! I get so excited - Ms. Fink, I just responded to your email." Ms. Fink responded, "Ok. Thank you!" and then Yonas began reading The Ladder Problem out loud. This social comment seemed to serve the function of welcoming Ms. Fink into the group and following up on an outside-of-class connection. (The email had to do with setting up a one-on-one interview for this research project.)

The second set of social words occurred about 5 minutes into the task. The teacher had joined a few moments earlier and the student teacher had just popped in too. Guadalupe exclaimed, "Oh my god, we have all three teachers!" Ms. B said goodbye and left the group; Mr. K stayed a little longer. Guadalupe continued, "Mr. K, you look hecka bored (see Figure 6u). You look like you're about to go back to sleep. I'm not commenting. Sometimes I say that my camera's broken just so I can go back to bed, but not in this class. Never in this class." Smiling, Hosein responded, "Never," implying he did not believe her, and laughing, Yonas followed up with, "Jeez, does that actually happen?" Everyone in the group smiled, except for Elijah (see Figure 6v). Guadalupe's comment, "Never in this class" prompted Hosein's one and only social contribution

and Yonas's first of two social contributions. These social comments by Guadalupe seemed to serve the function of lightening the mood and building personal connections with the student teacher and her peers by revealing some vulnerability.

Figure 6u. Mr. K Looking "Hecka Bored"

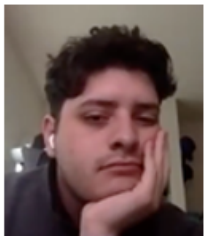


Figure 6v. Smiles from Everyone Except Elijah

The third set of social words occurred between Guadalupe and Ms. Fink about 6.5 minutes into the task during a pause in the group's conversation. Yonas had just said, "Let me get drawing tools." Into the silence, Guadalupe asked her first social question. "Ms. Fink, are you gonna be a math teacher? I forgot." This was the conversation referenced earlier about Ms. Fink's PhD, the one Guadalupe said she enjoyed, the "only reciprocated conversation." Guadalupe asked a second social question and offered additional social comments during this exchange. The combination of social questions and comments by Guadalupe seemed to serve the function of building a personal connection between herself and Ms. Fink.

The fourth and final set of social words occurred soon after the previous conversation between Guadalupe and Ms. Fink. Yonas had started using the online drawing tools, Guadalupe asked what he was doing (identified earlier as one of her mathematical contributions), Yonas explained, and then Guadalupe commented on Yonas's drawing. With a smile she said, "Very sturdy looking ladder." Yonas smiled and responded, "Oh yeah, it's nice," which was his second of two social contributions during the task. This brief social exchange between Guadalupe and Yonas was followed by Yonas continuing his mathematical explanation of what his drawing represented. Guadalupe's final social comment seemed to serve the purpose of interjecting some humor into the situation to lighten the mood.

Part 3: Interactional Participation Processes

This section addresses the question, How were students' opportunities to participate constructed through interactions? Data shared in this section focuses on *interactional* participation processes, as opposed to *individual* student participation metrics. PART 3 examines how interactions among participants supported (or inhibited) students' mathematical and social contributions through the construction of opportunities to participate. Opportunities to participate are examined by looking at what seemed to prompt students' contributions, how contributions were invited (or not) by other participants, and with whom each student interacted. Analysis suggests that what constituted a *genuine* opportunity to participate differed from student to student; students exhibited different participation thresholds, meaning some students were more likely to contribute in response to certain types of invitations than others. For some students, an open conversational floor served as an opportunity to participate, but not for others. Particular attention is paid to how Guadalupe's opportunities to participate compared to those of her peers.

Contribution Invitations

Participants' Prompts

A code was assigned to each mathematical and social contribution depending on what seemed to prompt the contribution. The timing and content of contributions were used to speculate connections between contributions and other participants' talk, actions, and gestures. If a contribution was seemingly connected to something another participant said previously (e.g., building on a peer's idea, responding to a question), the contribution was coded as prompted by *Participant Talk*. If a contribution was seemingly connected to another participant's non-verbal action or gesture (e.g., someone joining the breakout room, someone making a facial expression), the contribution was coded as prompted by *Participant Action or Gesture*. If a contribution did not seem to be connected to any specific previous action or talk, then the contribution was coded as *Seemingly Unprompted*. The number of mathematical and social contributions prompted in each of the three manners for each student is presented Table 6d.

Table 6d: Number of Mathematical & Social Contributions by Type of Prompt

		Yonas	Guadalupe	Hosein	Elijah
Mathematical	Prompted by Participant Talk	12	3	5	8
	Prompted by Participant Action or Gesture	0	1	0	0
	Seemingly Unprompted	11	0	5	2
	Total	23	4	10	10
Social	Prompted by Participant Talk	2	4	1	0
	Prompted by Participant Action or Gesture	0	4	0	0
	Seemingly Unprompted	0	1	0	0
	Total	2	9	1	0

Prompts for Yonas's and Hosein's mathematical contributions were relatively balanced between participant talk and being seemingly unprompted (Yonas: 12 vs. 11; Hosein: 5 vs. 5). The relatively high number of unprompted mathematical contributions suggest that Yonas and Hosein were comfortable offering new ideas to the group by initiating mathematical contributions that were unconnected to previous comments. Elijah's math contributions were most often connected to verbal contributions made by other participants, though he did make two mathematical contributions without being prompted. This pattern suggests Elijah was not necessarily steering the conversation in new directions, but he was an active passenger who offered input and shared his thoughts along the way. All of Guadalupe's math contributions were prompted by other participants' talk or actions. Data suggest Guadalupe was not as comfortable as her peers at sharing new ideas with the group, as indicated by the lack of unprompted mathematical contributions. She was, however, the only student to make a mathematical contribution prompted by a non-verbal participant action. Guadalupe's question, "Yonas, what are you doing?" was prompted by Yonas sharing his screen and drawing a diagram of The Ladder Problem. Her response to Yonas's actions suggests she was paying attention to her peers and attempted to actively engage in mathematical conversation at certain points, despite her relatively low number of mathematical contributions.

The two social contributions made by Yonas and the one made by Hosein were all prompted by participant talk, specifically by Guadalupe's talk, and as noted earlier, Elijah made no social contributions. Most social contributions were made by Guadalupe, split evenly between participant talk and participant action prompts. She responded both to what people said and what they did, indicating again, an awareness of the other people in her group. She took her cues to participate, both mathematically and socially, from other participants in the group (including both students and adults). Participants' actions prompting Guadalupe's social contributions included Ms. Fink joining the Zoom breakout room and Mr. K looking sleepy. In addition, Guadalupe made one unprompted social contribution, suggesting she was more comfortable initiating unrelated social talk than mathematical talk.

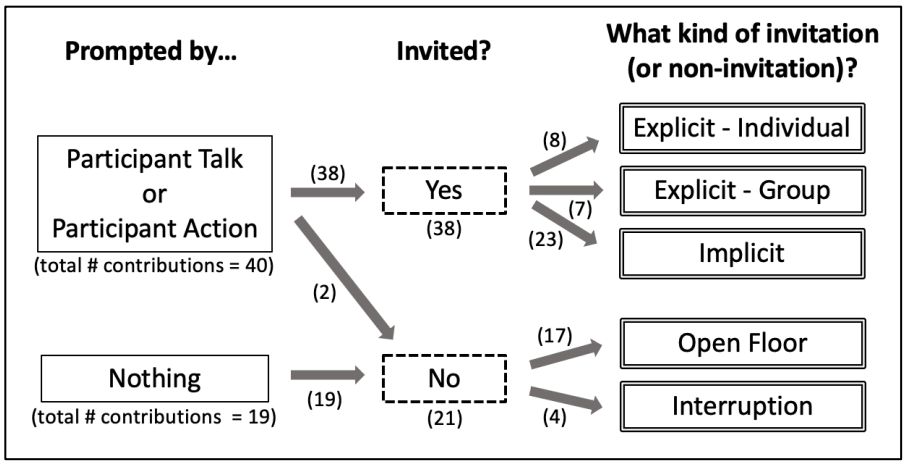
After determining what seemingly prompted each contribution, another code was assigned to each mathematical and social contribution depending on how contributions were invited (or not) by other participants. First, it was determined if a contribution received an invitation. If a contribution was unprompted, it was assigned a "No invitation" code (i.e., *No – Interruption* or *No – Open Floor*). If someone else was speaking at the time of the unprompted contribution, the contribution was coded as an *Interruption*. If no one else was speaking at the time of the unprompted contribution, the conversational floor was considered "open," and the contribution was coded as such. Most contributions that were prompted either by participant talk, actions, or gestures were considered as having an invitation. The only exceptions were two cases where the prompted contributions occurred while someone else was speaking, interrupting the other student's sharing, cutting the prior contribution short. Interruptions, prompted or not, were considered as having No invitation.

All non-interruption prompted contributions were assigned a Yes invitation code (*YES – Implicit, YES – Explicit Group*, or *YES – Explicit Individual*). Implicit invitations occurred when a response to a previous contribution was socially appropriate but not necessary. For example, if one student shared a mathematical idea and a second student referenced and built on that idea, the second student's contribution would be coded as having an *Implicit* invitation. On the other hand, if the one student shared an idea and then at the end of their contribution said, "Does that make sense?" and a second student responded affirmatively, the second student's contribution would be coded as having an *Explicit - Group* invitation. The question "Does that make sense?" indicated that someone in the group was expected to respond. Explicit invitations occurred when a response to a previous contribution was expected based on traditionally norms of socially appropriate behavior. An *Explicit - Individual* invitation occurred when a response was expected from a particular person, either by reference to a person's name or based on the flow of conversation.

Group-Level Invited and Non-Invited Contributions

Figure 6w displays a map of how invitation codes were assigned to prompted and unprompted contributions. The group-level quantities for each code are also included in the figure.

Figure 6w. A Coding Map of Contribution Invitations with Code Quantities



A total of 59 contributions were made by the four students over the course of task completion; approximately 2/3 of contributions were prompted by another participant and 1/3 were seemingly unprompted. Fifteen contributions were in response to explicit invitations, with 8 directed at individual students and 7 directed at the group. Sixty percent of invited contributions (23 out of 38) were the result of implicit invitations. Most of the contributions made without an invitation were made during pauses in conversation when the conversation floor was open (17 out of 21). The remaining four non-invited contributions were interruptions. These data indicate that within the group there was a combination of students prompting each other and building off each other's ideas and students sharing unprompted "new" ideas with their peers. Some invitations were directed at specific people, but most invitations were either implicitly or explicitly open to everyone in the group. It is not surprising that some interruptions occurred, given that coordinating conversation turns through online platforms, like Zoom, is typically more difficult than when working together in-person.

Student-Level Invited and Non-Invited Contributions

The previous figure displayed numbers of contribution invitations for both mathematical and social contributions, aggregated across all four students in the group. Table 6e shows a breakdown of contribution invitations by contribution category (mathematical or social) and by student. This table presents how each student's mathematical and social contributions were explicitly, implicitly, or not invited by other group participants.

Table 6e: Mathematical & Social Contribution Invitations (and Non-Invitations) by Student

		Yonas	Guadalupe	Hosein	Elijah
Mathematical	YES - Explicit Individual Invitation	5	1	1	0
	YES - Explicit Group Invitation	2	1	1	3
	YES - Implicit Invitation	5	2	3	3
	NO - Open Floor	10	0	4	2
	NO - Interruption	1	0	1	2
	Total	23	4	10	10
Social	YES - Explicit Individual Invitation	1	0	0	0
	YES - Explicit Group Invitation	0	0	0	0
	YES - Implicit Invitation	1	8	1	0
	NO - Open Floor	0	1	0	0
	NO - Interruption	0	0	0	0
	Total	2	9	1	0

All four students made mathematical contributions that were invited explicitly. Every student but Elijah was called on, individually, to contribute mathematically at least once; Yonas was called on the most (5 times). Guadalupe's one explicit invitation came from Hosein when he asked her, "Guadalupe, what are you thinking about it?" several minutes into the task. She responded by sharing her one and only mathematical solution supported by reasoning. Every student responded to at least one explicit invitation for a mathematical contribution from the group. Guadalupe's one explicit group invitation came from the teacher when she joined the breakout room to check-in on the group's progress. Ms. B asked, "Are you guys saying yes or no?" to which Guadalupe responded, "We're saying they're going to be at different rates." Guadalupe's other two mathematical contributions were in response to implicit invitations from Yonas, one prompted by Yonas's talk and one prompted by his drawing. Yonas, Hosein, and Elijah all had implicitly invited contributions as well (Yonas – 5, Hosein – 3, and Elijah – 3).

Guadalupe did not make any non-invited mathematical contributions, while all three of the other students did. Yonas offered 10 mathematical contributions during pauses in conversation without any kind of invitation, by far the most of any student, followed by Hosein with 4 and Elijah with 2. All three of these students also had a mathematical contribution that interrupted someone else at least once. Yonas and Hosein each interrupted once, but both students apologized for the interruption and in both cases the interrupting student seemed to think they had a new idea worth sharing. When Guadalupe was sharing a mathematical idea around the 3-minute mark, Yonas seemed to be processing the problem in his head. Guadalupe was still speaking when Yonas said, "Oh my god. Wait. Sorry, sorry. I just had a theory." He went on to suggest a new idea about the problem, without any clear connection to what Guadalupe was saying. Guadalupe did not respond. Hosein's interruption came about 8 minutes into the task when Yonas was drawing his diagram and talking through his solving steps while sharing his screen. Hosein said, "Um, Yonas? I'm sorry to interrupt. I have an idea." Yonas responded, "Yeah. No, no. Say yours. It's always good," suggesting Yonas welcomed the interruption and was happy to handoff the responsibility of talking to someone else. The nature of Elijah's two interruptions was somewhat different from Yonas's and Hosein's, in that Elijah did not acknowledge that he was interrupting one of his peers. When the students were sharing their initial thoughts about the problem, Elijah interrupted Yonas mid-sentence. Yonas was saying, "it seems like the same

rate because I can imagine it falling somewhat -" Elijah jumped in with, "Yeah, so like my initial thought was the same. Like if we have …" Elijah went on to share his ideas for the next 37 seconds. Elijah connected to what Yonas was saying but did not give Yonas a chance to finish his thought. Elijah interrupted Yonas a second time toward the end of the task. This time Yonas seemed to be talking through his thoughts out loud, saying, "Hmmm. Let's see. It's one foot on the bottom per second -" Elijah jumped in with, "Oh yeah, and that does make sense cuz if we assume…" He went on to continue his thought without any acknowledgement that Yonas had been speaking. Elijah also made the same number of contributions when the floor was open as when someone else was speaking, suggesting that if Elijah had a new thought to share, he shared it regardless of what other people were doing.

Most social contributions were made by Guadalupe and invited implicitly through participation talk and action. Guadalupe was invited through talk with Ms. Fink and actions by Ms. Fink, Mr. K, and Yonas. Guadalupe also had one social contribution without an invitation. About 6.5 minutes into the task, Guadalupe asked, "Ms. Fink, are you gonna be a math teacher? I forgot." This open floor uninvited social contribution suggests Guadalupe was more comfortable contributing socially than mathematically, since all her mathematical contributions were invited. The opposite is true of her peers; Yonas, Hosein, and Elijah made uninvited mathematical contributions but no uninvited social contributions, suggesting they were all more comfortable contributing mathematically than socially. Yonas offered only two social contributions, Hosein offered one, and Elijah did not offer a single social contribution.

Invited Contribution Interactions

Most student contributions were invited, either explicitly or implicitly, by other participants' talk or actions (38 out of 59, 64%). These are identified in Table 6f by the dark blue rounded rectangle. To better understand interactional participation processes it would be helpful to know whose contributions were invited by whom.

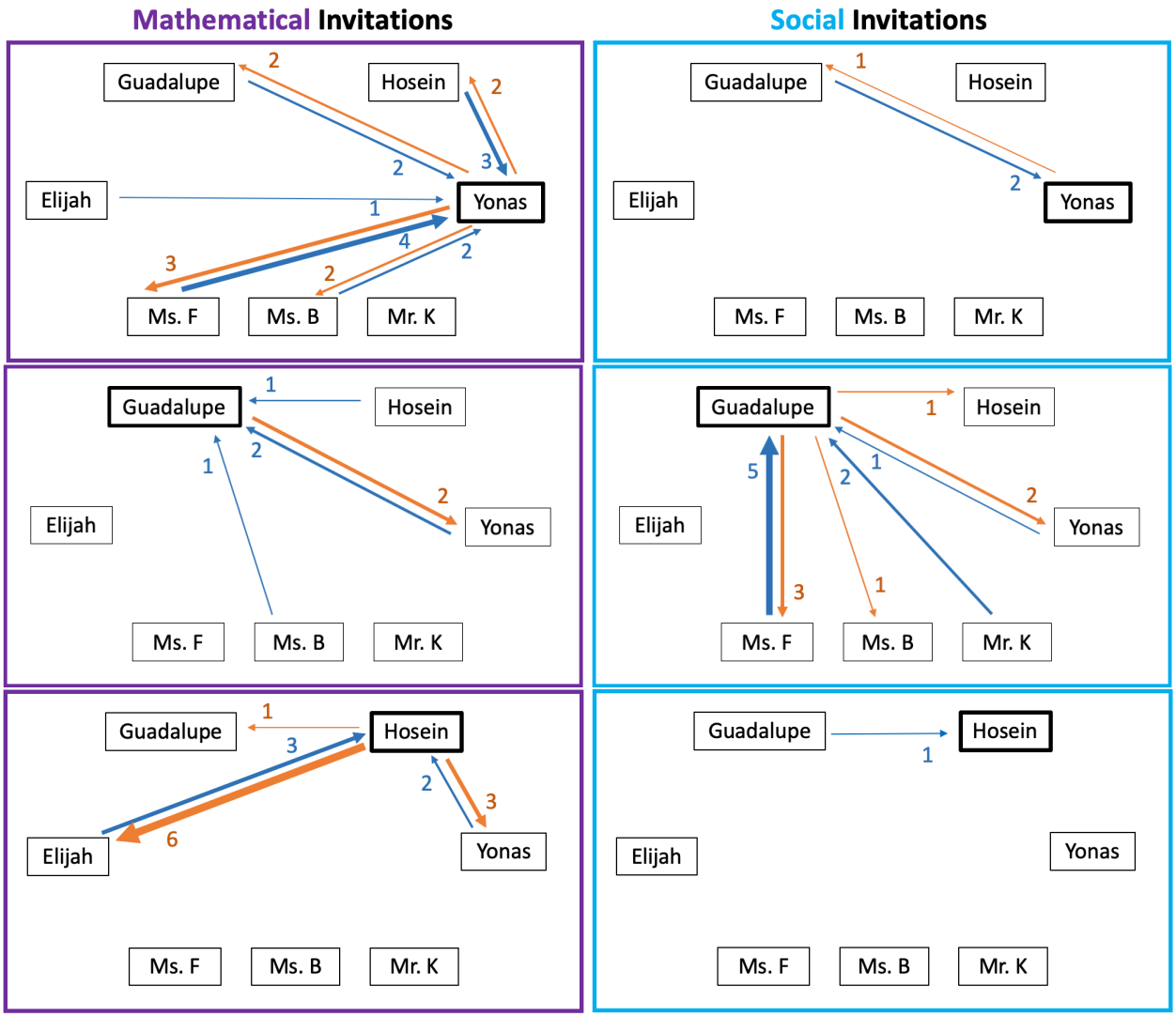
Table 6f: Invited Mathematical & Social Contributions by Student

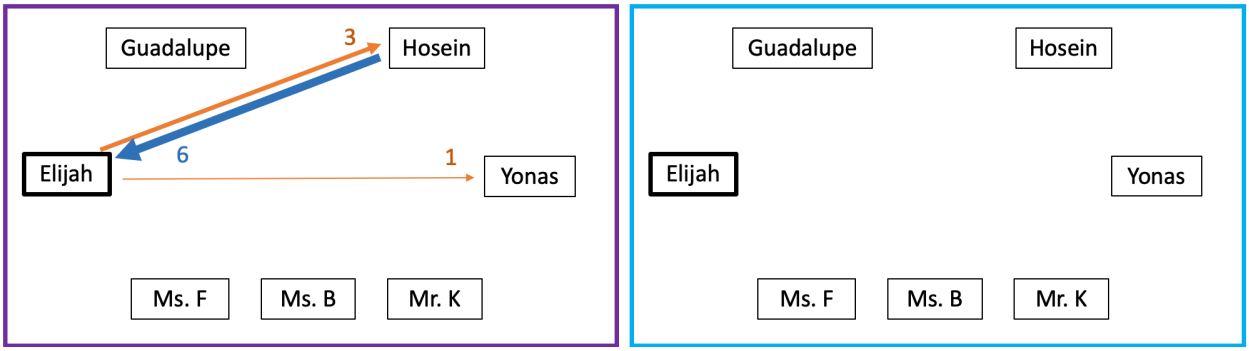
		Yonas	Guadalupe	Hosein	Elijah
Mathematical	YES - Explicit Individual Invitation	5	1	1	0
	YES - Explicit Group Invitation	2	1	1	3
	YES - Implicit Invitation	5	2	3	3
	NO - Open Floor	10	0	4	2
	NO - Interruption	1	0	1	2
	Total	23	4	10	10
Social	YES - Explicit Individual Invitation	1	0	0	0
	YES - Explicit Group Invitation	0	0	0	0
	YES - Implicit Invitation	1	8	1	0
	NO - Open Floor	0	1	0	0
	NO - Interruption	0	0	0	0
	Total	2	9	1	0

Each invited mathematical and social student contribution was coded according to which participant made the invitation. Participants included the four students plus the three adults who

spent time with this group during the task (Ms. Fink, Ms. B, and Mr. K). Ms. Fink was with the group for the entire time. Ms. B visited the group once for 28 seconds, and Mr. K visited the group once for 33 seconds. Ms. B's and Mr. K's visits overlapped by 8 seconds. Figure 6x provides a closer look at the student contributions that were invited by another participant. The figure contains two rectangles per student, one for mathematical invitations (purple) and one for social invitations (blue). The top two rectangles show data for Yonas (bold outlined name). Orange arrows pointing away from Yonas represent the number of times Yonas invited a contribution from someone else. For example, the orange arrow from Yonas to Hosein in the top left rectangle shows that Yonas invited two math contributions from Hosein. Blue arrows pointing toward Yonas represent the number of times someone else invited a contribution from Yonas. For example, the blue arrow pointing from Hosein to Yonas shows that Hosein invited three of Yonas's mathematical contributions. The thickness of arrows corresponds to the number of invitations, also shown as a number next to each arrow.

Figure 6x: Mathematical & Social Invitations *by* and *to* each Student





The top two rectangles show that Yonas connected through contribution invitations with everyone except Mr. K. Interactions between Yonas and others were relatively balanced and reciprocal; he interacted roughly the same amount with each person, and invitations *by* and *to* each person were relatively even. Elijah was an exception, with just one interaction with Yonas, and Yonas interacted socially only with Guadalupe.

The next two rectangles, highlighting Guadalupe's interactions, show that she was connected to everyone except Elijah, and most of her interactions were social. She invited only two mathematical contributions from her peers; both were invitations for Yonas. Guadalupe was involved in all social contributions that took place during this task; she either made the social contribution or she invited someone else to make a social contribution. Most of Guadalupe's social interactions involved adults, especially Ms. Fink, and can be characterized as friendly, casual, and often humorous. For example, when Ms. Fink first entered the breakout room, Guadalupe greeted her with, "Ms. Fink, oh my God! I get so excited!" Ms. Fink's action of joining the group invited Guadalupe's contribution. In response to Mr. K's sleepy appearance, Guadalupe teased, "Mr. K, you look hecka bored." Mr. K's action of resting his chin in his hand invited Guadalupe's contribution. And, in response to Yonas's virtual whiteboard drawing, Guadalupe commented with sarcasm, "Very sturdy looking ladder!" Yonas's action of drawing a wobbly-looking ladder invited Guadalupe's contribution. Guadalupe made only four mathematical contributions, the fewest in the group. Her first mathematical contribution was invited by Hosein's explicit question, "Guadalupe, what are you thinking about [the problem]?" The second contribution was an expression of agreement ("I agree with you") in response to an explanation shared by Yonas just after Ms. B joined the room. The third contribution was invited by Ms. B's question to the group, "Are you guys saying yes or no?" Guadalupe's final mathematical contribution was a question she asked Yonas about what he was doing, invited by Yonas's virtual white board drawing. Guadalupe was also the only student to have contributions invited through participant talk *and action*; all other students responded only to participant talk.

The third row of rectangles shows that most of Hosein's interactions occurred with Yonas and Elijah, a total of 9 interactions with Elijah and 5 with Yonas. Three of Hosein's math contributions were invited by Elijah's talk and two by Yonas's talk. Hosein's interactions with Guadalupe were limited; Hosein invited one mathematical contribution from Guadalupe, and she invited one social contribution from Hosein. Hosein had no interactions with adults.

The bottom two rectangles highlight the very limited scope of Elijah's interactions. Elijah had no social interactions with anyone, and his mathematical interactions occurred almost exclusively with Hosein; six of Elijah's contributions were invited by Hosein's talk. There were

several back-and-forth math conversations between Hosein and Elijah which sometimes included Yonas peripherally, but never Guadalupe. One example occurred toward the end of the discussion when Hosein asked the group, “So, does that mean that the top falls twice as fast as the bottom?” Elijah responded right away, “Um. I don't think it's twice as fast because it's six verses eight.” Hosein explained further, “No, but it has to move an additional four on the bottom compared to the eight that it has to move at the top.” Elijah contemplated Hosein’s response, saying, “Hmmm. True. Hmmm. Interesting.” This 2-person exchange illustrates the type of back-and-forth conversation Elijah engaged in with only Hosein. Elijah and Hosein’s questions and comments invited each other’s contributions.

Opportunities to Participate

Invitations (and Non-Invitations)

One way to examine opportunities to participate is to look at the kinds of invitations (and non-invitations) that led to mathematical and social contributions for each student. Students contribute when opportunities to participate are created through invitations (and non-invitations) that make them feel comfortable and willing to participate. Students need different kinds of opportunities to feel ready and motivated to participate. Some students need an explicit invitation to contribute mathematically while other students contribute regardless of what their peers are doing. In addition, some students need a lot of support to participate mathematically but not socially, or vice versa. When working on The Ladder Problem, Yonas, Guadalupe, Hosein, and Elijah responded differently to the various forms of invitations and non-invitations offered, meaning opportunities to participate varied from student to student. For some students, opportunities to participate were created through an open conversational floor but not for everyone. And opportunities to participate mathematically looked different from opportunities to participate socially.

The Data presented in Table 6g and Table 6h show how often each of the four students contributed during The Ladder Task after receiving various forms of invitations and non-invitations. The numbers in these tables are based on Table 6e. The main difference is that the two types of explicit invitations (i.e., individual and group) are combined into one category in the tables presented here. Also, note the order of the invitation and non-invitation types from top to bottom. These are listed in the table corresponding to the presumed likelihood of a generic person’s participation, based on generally accepted norms of classroom behavior. For example, it is presumably more likely a student will participate in a small-group discussion if they have an explicit invitation to contribute, and it is relatively less likely a student will participate if someone else is talking. Table 6g shows the number of invitations and non-invitations each student received for their mathematical contributions.

Table 6g: Invitations (and Non-Invitations) for Mathematical Contributions by Student

	How many times did a student contribute **mathematically** with…	Yonas	Guadalupe	Hosein	Elijah
More Likely	an Explicit Invitation	7	2	2	3
	an Implicit Invitation	5	2	3	3
	an Open Conversational Floor	10	0	4	2
Less Likely	Someone Else Talking	1	0	1	2

Yonas, Hosein, and Elijah contributed in response to all types of invitations and non-invitations. The highest number of contributions by Yonas and Hosein occurred when the conversational floor was open, suggesting they were comfortable offering ideas when the breakout room was quiet and there was no obvious path forward. For them, it seemed just as likely they would participate with an invitation as without, unless someone else was already speaking. They both interrupted a peer one time, so having someone speaking did not stop them from participating, but it made them less likely to participate. Both of their interruptions included an apology, indicating hesitation to participate. Elijah, on the other hand, contributed twice when someone else was speaking and did not offer apologies or acknowledge the interruption either time. Elijah's contributions were spread rather evenly across all forms of invitations and non-invitations, suggesting other people's actions did not make much of a difference; Elijah contributed whenever he wanted to contribute.

All three male students seemed comfortable sharing their ideas out loud, but Yonas shared much more often than the others. Many of Yonas's contributions were mathematical comments and bits of reasoning that did not contain solutions to the problem. Of all the students, Yonas seemed most comfortable talking through partially formed ideas. He spoke as he thought. In his own words, "I was trying to think out loud. So, if anybody could think of an idea that related to any of mine." On the other hand, Hosein and Elijah seemed to be thinking through their ideas silently in their heads and shared only when they felt they had something new to add. In Hosein's words, "I'm sort of absorbing what they're saying. And then there's a moment where I'm like, 'oh, I have something new and valuable to add to the conversation. Now it's my turn to sort of open up'."

Guadalupe's invitational patterns were quite different from those of her peers. Guadalupe contributed in response to only explicit and implicit invitations; she did not offer any contributions if the conversational floor was open or if someone else was talking. When Hosein offered Guadalupe an explicit invitation to share her thinking toward the beginning of the task, Guadalupe seemed comfortable sharing her proposed answer supported by mathematical reasoning. This contribution seemed relatively easy for her. Guadalupe also contributed in response to an explicit invitation by the teacher. In this case, the teacher directed a mathematical question to the group and Guadalupe responded, "We're saying they're going to be at different rates," representing the group's thinking. There was no hesitation by Guadalupe and her use of the word "we're" implies she was comfortable speaking on behalf of the group. This contribution seemed relatively easy for her as well. Her other two mathematical contributions were in response to implicit invitations, and interestingly, both implicit invitations came from Yonas and both occurred during periods in the conversation when Hosein and Elijah were not speaking. There had been a minute and a half without either Hosein or Elijah speaking before Guadalupe's first implicitly invited contribution and two minutes before Guadalupe's second implicitly invited contribution without either speaking. These data suggest perhaps not all implicit invitations are the same. Yonas, Hosein, and Elijah all shared numerous mathematical ideas to which Guadalupe could have responded, but she chose to respond only to ideas shared by Yonas at times when the other students were not involved in the conversation. Guadalupe was more likely to participate in response to Yonas's implicit invitations than to Hosein's and Elijah's. In addition, since Guadalupe did not contribute at all without an invitation, which could have been because she found it difficult or because she simply had no desire to participate in that way.

Table 6h shows the number of invitations and non-invitations each student received for their social contributions.

Table 6h: Invitations (and Non-Invitations) for Social Contributions by Student

	How many times did a student contribute socially with…	Yonas	Guadalupe	Hosein	Elijah
More Likely	an Explicit Invitation	1	0	0	0
	an Implicit Invitation	1	8	1	0
	an Open Conversational Floor	0	1	0	0
Less Likely	Someone Else Talking	0	0	0	0

Guadalupe dominated social contributions, most of which were in response to implicit invitations from other people, especially from the adults in the group. Guadalupe seemed comfortable interacting with the adults in the room and seemed to be actively seeking opportunities to interact with them. The appearances of Ms. Fink and then later Mr. K in the breakout room served as implicit invitations for social contributions from Guadalupe. Typically, it may be challenging and less likely for social contributions to be offered during an open conversational floor, however, Guadalupe did do so at one point. As Yonas was getting his "drawing tools" figured out, without any apparent prompt Guadalupe asked, "Ms. Fink, are you going to be a math teacher? I forgot." This one non-invited social contribution led to a series of other social contributions between Guadalupe and Ms. Fink, accounting for 4 out of Guadalupe's 8 implicitly invited social contributions.

Only one social contribution occurred in response to an explicit invitation, and that was when Yonas responded to Guadalupe's sarcastic comment about his ladder drawing looking "very sturdy." It might have been awkward if Yonas had not said anything. He replied, "Oh yeah, it's nice," a brief acknowledgement of Guadalupe's comment. Yonas and Hosein both had one social contribution in response to an implicit invitation from Guadalupe. Guadalupe's story about how she sometimes turns her camera off in class so she can go back to bed elicited the implicitly invited social contributions from Yonas and Hosein. Both students seemed comfortable responding to Guadalupe as they smiled and laughed along with her. This social exchange prompted smiles from Ms. Fink and Mr. K as well, but not from Elijah. Having the five other people in the group smiling together was not enough to prompt even a non-verbal response from Elijah. In fact, not only did Elijah not make any social contributions, Elijah also did not acknowledge any social contributions made by other people in the group through gesture or expression or other non-verbal means. Elijah appeared to be deep in thought and seemingly oblivious to the social interactions that occurred within the group.

Participation Thresholds and Genuine Opportunities

Another way to examine opportunities to participate is to think about student contributions as being connected to participation thresholds for each student at a given time in a given context. Students will contribute when they have opportunities to participate that meet their personal participation thresholds; the potential gain of their contributing needs to outweigh the potential loss. Thresholds for participation vary from student to student and from day to day depending on the circumstances, which means that what counts as a *genuine* opportunity varies

as well. Thresholds are conceptualized as flexible and socially constructed, shifting as students interact with one another. Over time (e.g., ten minutes working on a task, during one whole semester) threshold patterns emerge.

Yonas, Guadalupe, Hosein, and Elijah working together on this one day experienced different thresholds for participation, meaning genuine opportunities to participate for one student looked different from genuine opportunities for another. And genuine opportunities to participate mathematically looked different from genuine opportunities to participate socially. Table 6i contains a key for Tables 6j and 6k, which present the mathematical and social participation thresholds for each student as they related to various types of invitations (and non-invitations) during The Ladder Problem.

Table 6i: Participation Threshold Key

Genuine Opportunity	
Tentative Opportunity	
Not an Opportunity	

Invitations that created *genuine* opportunities to participate for a given student are colored green; green was assigned when there were at least two contributions made and no hesitation or restriction on contributions was noted. Invitations that created *tentative* opportunities to participate for a given student are colored yellow; yellow was assigned when there was only one contribution made and/or contributions were restricted in some way. Invitations that did not create opportunities to participate for a given student are colored pink; pink was assigned when there were no contributions made by a student.

Table 6j displays the mathematical participation thresholds for each of the four students while working on The Ladder Problem.

Table 6j: Mathematical Participation Thresholds by Invitation Type and Student

Opportunities to participate mathematically through…	Yonas	Guadalupe	Hosein	Elijah
an Explicit Invitation				
an Implicit Invitation				
an Open Conversational Floor				
Someone Else Talking				

Yonas, Guadalupe, Hosein, and Elijah contributed multiple times in response to explicit invitations without notable hesitation, and therefore, explicit invitations created genuine opportunities to participate mathematically for all four students. Implicit invitations and an open conversational floor constituted genuine opportunities for Yonas, Hosein, and Elijah to contribute mathematically as well, but not Guadalupe. Although Guadalupe contributed twice in response to implicit mathematical invitations, she only responded to implicit invitations from Yonas and only when neither of the other students were involved in the conversation. Due to these restrictions, implicit invitations were considered tentative opportunities to participate for Guadalupe. If someone else was talking, that too counted as an opportunity to participate mathematically for everyone except Guadalupe. However, Yonas's and Hosein's interruption

apologies suggest the opportunity was not as clear as it was for Elijah. Therefore, someone else talking was considered a genuine opportunity for Elijah to contribute mathematically, but only a tentative opportunity for Yonas and Hosein.

Table 6k displays the social participation thresholds for each of the four students.

Table 6k: Social Participation Thresholds by Invitation Type and Student

Opportunities to participate socially through…	Yonas	Guadalupe	Hosein	Elijah
an Explicit Invitation				
an Implicit Invitation				
an Open Conversational Floor				
Someone Else Talking				

No one interrupted another person to offer a social contribution, so if someone else was talking, there were no opportunities for social participation. Guadalupe was the only person to offer a social contribution during an open conversational floor and she only offered one, indicating an open floor was a tentative opportunity for Guadalupe to participate socially but not an opportunity for anyone else. Guadalupe offered quite a few social contributions in response to other participants' talk and actions, so implicit invitations were considered genuine opportunities for Guadalupe's social participation. Although Guadalupe did not experience any explicit invitations for social contributions, it is assumed that since implicit invitations were genuine opportunities then explicit invitations would be genuine opportunities to participate for her as well. Yonas and Hosein each responded to the same social comment made by Guadalupe, indicating implicit invitations were tentative opportunities for them to participate socially. Yonas had one explicit social invitation and Hosein had none. It is assumed that since they both contributed in response to an implicit social invitation, an explicit invitation would elicit a similar response from them. Explicit invitations were considered tentative opportunities for Yonas and Hosein, though it is certainly possible they could be genuine opportunities. There just was not data to support that claim. Since Elijah did not have a single social contribution, it is unclear what a genuine opportunity for social participation might look like for Elijah, or even what a tentative opportunity would be.

These data indicate that Guadalupe's threshold for mathematical participation was much higher than her peers. She needed either an explicit invitation or an implicit invitation from Yonas to share her mathematical ideas. However, Guadalupe's threshold for social participation was low; an implicit invitation such as the appearance of an adult was enough to prompt a social greeting or a light-hearted joke from her. The opposite was true for Elijah. Elijah's threshold for mathematical contributions was the lowest of all four students. Elijah seemed ready to share a mathematical thought no matter what anyone else was doing, even if other people were speaking. Yet, Elijah's threshold for social contributions was the highest of all four students. In fact, the threshold was so high that it was never reached in this episode. Participation thresholds for Yonas and Hosein were seemingly the same as each other. Their thresholds for mathematical participation were relatively low. Both were eager to share ideas and expressed hesitation when interrupting someone else talking, but were still willing to participate if they had an idea they believed was worth sharing. Their thresholds for social participation were relatively high, but they

both offered small social contributions that acknowledged Guadalupe's social contributions to the group.

Summary of Findings

The overarching question guiding analysis in this chapter is: In what ways, to what extent, and from whose perspectives was classroom participation equitable for this group of four students working on this calculus task? Answering this question required integrating data from multiple sources and from different perspectives. First, we needed to know how classroom participants experienced the task (Part 1). Was the teacher happy with the way students engaged with the mathematical content? What insight did the student teacher share from his observational perspective? Did each student feel their ideas were valued and included in the mathematical conversation? Second, we needed to know how each student contributed during the task (Part 2). Who contributed most often and least often? How were students' contributions distributed across various types of mathematical and social contributions? When and how did students contribute throughout the 10-minute task? Third, we needed to know how opportunities to participate were constructed through participant interactions (Part 3). How were students invited (or not) to contribute? With whom did students interact? How did opportunities to participate differ among the students? Together, analysis can help determine to what extent every student had genuine opportunities to participate, and if the opportunities they each had were enough to support content proficiency and positive mathematical identity. Conclusions are based on analyses of video-stimulated interviews with the teacher, the student teacher, and four students and a 10-minute videorecording of the four students working on the small-group task.

Assessing Equitable Participation in a Small-Group Task

The intent of this chapter was to integrate multiple perspectives to tell an accurate and comprehensive story of what happened for this group working on this task to determine the extent to which participatory equity was achieved for these four students. To operationalize and assess participatory equity, the definition for equity was broken down into three primary questions: 1) Which students had genuine opportunities to participate? 2) Which students had opportunities to participate in ways that supported rich content understandings? 3) Which students had opportunities to participate in ways that supported positive mathematical identities? Each of these three questions was broken down further into three or four secondary questions that could be answered empirically through the presented analyses. Answers to the secondary questions were drawn from one or more perspective (teacher, student teacher, student, and/or researcher). Figure 6y provides a summary of compiled findings focused on assessing participatory equity, organized according to the primary and secondary equity-related questions.

Figure 6y: Equity Assessment for the Focal Group's Participation in The Ladder Task

Assessing Equity		Yonas	Guadalupe	Hosein	Elijah
To what extent did students had genuine opportunities to participate?		✓+	✓- (limited)	✓+	✓+
Perspective					
Teacher & Student Teacher	Did the student participate in ways that aligned with class norms and teacher expectations?	yes	limited	yes	yes
Researcher	Was the student given opportunities to participate that met their personal participation thresholds?	yes	math - limited social - yes	yes	math - yes social - n/a
Student	Did the student feel comfortable participating?	yes	math - no social - yes	yes	yes
To what extent did students have opportunities to participate in ways that supported rich content understandings?		✓+	✓- (limited)	✓+	✓+
Perspective					
Teacher & Student Teacher	Did the student engage with the mathematical content as intended by the teacher?	yes	limited	yes	yes
Researcher	Did the student's contributions provide evidence of mathematical understandings?	yes	limited	yes	yes
Student	Did the student feel mathematically challenged by the task?	yes	yes	yes	yes
Student	Did the student feel supported to take mathematical risks and offer new ideas?	yes	no	yes	yes
To what extent did students have opportunities to participate in ways that supported positive mathematical identities?		✓ (mostly)	✓-- (very limited)	✓+	✓ (mostly)
Perspective					
Researcher & Student	Was the student positioned as a valuable mathematical contributor by themself and by other participants?	yes	limited	yes	yes
Student	Was this task a positive mathematical experience for the student?	mostly	no	yes	yes
Student	Did this experience align with the student's personal goals and/or needs?	mostly	no	yes	mostly

As the student who made the most contributions, Yonas had an abundance of genuine opportunities to participate. The teacher and student teacher expressed their satisfaction with the way Yonas shared his thoughts with the group and talked through his tentative ideas. His relatively low threshold for mathematical participation was met without an issue, and although his threshold for social participation was a bit higher, he contributed socially in ways that were appropriate and responsive toward Guadalupe. Yonas also expressed that he was very comfortable sharing in a "good group like that." In addition, Yonas's opportunities to participate seemed to support the development of rich content understandings. Yonas's thinking evolved over the course of the task as he wondered out loud, eagerly shared new ideas, and drew a diagram to help solve the problem, all of which pleased the teacher. Yonas admitted to being confused and lost but still felt comfortable enough to take risks, throw out incomplete ideas, and readjust his thinking along the way. Lastly, Yonas's opportunities to participate also seemed to support positive mathematical identity for the most part. He positioned himself as a valuable mathematical contributor by taking initiative to share ideas. Hosein, Guadalupe, Ms. Fink and Ms.

B also positioned Yonas as a valuable mathematical contributor through their interactions with him, specifically the explicit invitations they made for his mathematical participation. Yonas said he felt comfortable with this group of students, yet he perceived there to be a lack of interaction among them. He also pointed out that the conversation was male-dominated and did not result in a final solution to the problem. The group did not function perfectly in his mind; however, he still had opportunities to see himself as a contributing member of this math community.

Of the four students, Hosein had the most positive response to this task during his interview, using the word "cool" multiple times to describe the experience. With Hosein's relatively low threshold for mathematical participation, he seemed to have plenty of genuine opportunities to participate. The teacher and student teacher were happy with how they saw Hosein participating; the teacher even commented that she had been worried Hosein's confidence was low, but what she saw here made her feel better about him. Hosein also expressed his comfort and ease with this group. He appreciated the social interactions and participated socially enough to feel connected with his peers. In addition, Hosein's opportunities to participate seemed to support the development of rich content understandings. He too expressed feeling lost at times, but he continued to throw out ideas and questions to the group. His thinking changed in response to things his peers said as their collective understandings developed. Lastly, Hosein's opportunities to participate also seemed to support positive mathematical identity, more so than any of the other students. He positioned himself as a mathematical contributor by offering new ideas and by challenging his peers' ideas. His peers also positioned him as a mathematical contributor by engaging in back-and-forth mathematical conversations with him; Elijah, in particular, looked to Hosein for feedback and support. Although Hosein seemed to get personal enjoyment and satisfaction from this mathematical experience, he acknowledged that it was not ideal for some of his peers. He reflected on how Elijah and Guadalupe may have felt excluded and what he could have done differently, using the video-watching exercise as an opportunity to reflect and grow as a supportive peer.

Elijah had the least to say during the video-stimulated interview; Elijah showed little emotion, but the words spoken conveyed satisfaction with the experience tied to the "interesting" math problem. Elijah had the lowest mathematical participation threshold and, consequently, an unlimited number of genuine opportunities to participate. It did not matter if someone else was speaking or if the group was silent, Elijah shared mathematical thoughts without hesitation. Both the teacher and student teacher were happy with Elijah's engagement, attributing Elijah's periods of silence with periods of deep thinking and mathematical reflection. In addition, Elijah's opportunities to participate seemed to support the development of rich content understandings. Just as with Yonas and Hosein, Elijah found the problem challenging and his thoughts about the problem shifted over the course of the ten minutes. Elijah stayed focused on the task, determined to figure it out right up until the last moment when the students were abruptly bumped back into the whole-class Zoom room. Lastly, Elijah's opportunities to participate also seemed to support positive mathematical identity. Elijah positioned himself as a valuable and confident mathematical contributor by sharing tentative ideas and by challenging ideas that were previously presented in the group. Elijah's peers supported this positioning by considering Elijah's ideas and engaging in mathematical conversations together. Elijah chose not to contribute socially, but that did not seem to affect overall satisfaction with the experience. Elijah noted a preference for individual work and not caring about social talk, so the collaborative

nature of the task was not ideal. However, Elijah enjoyed the experience for the most part due to the intriguing math problem.

Guadalupe's experience with The Ladder Problem was decidedly less positive than that of her peers. Her mathematical contributions were notably less frequent, and her own assessment of the experience was predominantly negative. Guadalupe's relatively high threshold for mathematical participation resulted in a limited number of genuine opportunities for her to contribute mathematically. She expressed that she was not comfortable sharing ideas or asking questions about the task, although she was comfortable initiating social interactions with her peers and the adults in the room. The teacher noticed that Guadalupe did not speak as much as she expected, but she assumed Guadalupe was thinking about the problem. Although the teacher did not pick up on Guadalupe's disengagement from the task and feelings of exclusion, the student teacher did. Mr. K hypothesized that Guadalupe's lack of verbal contributions may have been connected to being interrupted by Yonas and feeling ignored by her peers. Both the teacher and student teacher wished Guadalupe had spoken more. In addition, Guadalupe's opportunities to participate seemed only to support limited development of content understandings. Toward the beginning, Guadalupe shared a mathematically correct solution supported by tentative (and not completely accurate) reasoning in response to Hosein's explicit invitation, and Guadalupe offered that same solution to the teacher when she checked in on the group. However, there was no evidence of where Guadalupe's thinking went from there. It is unclear if and how Guadalupe could justify the "different rates" solution and if she understood the relative rate comparison. Guadalupe felt challenged by this task, so there was potential for mathematical growth, but she did not feel comfortable taking mathematical risks and therefore the potential was never realized. Lastly, Guadalupe's opportunities to participate seemed only to support very limited development of positive mathematical identity. Although Hosein positioned Guadalupe as a valuable mathematical contributor by asking for her thoughts, Yonas's interruption as she shared those thoughts undermined that positioning. Guadalupe positioned herself as a mathematical contributor in front of the teacher by responding to the teacher's question during the group check-in, but Guadalupe's lack of verbal mathematical contributions and abundance of social contributions throughout the rest of the task positioned Guadalupe as more of a social contributor than a mathematical contributor among her peers. This task was not a positive mathematical experience for Guadalupe; she talked about feeling lost, disappointed in herself, not empowered, and not smart. Guadalupe was seeking reciprocated conversations; she said the only part of the task she enjoyed was the social exchange about Ms. Fink's PhD.

Chapter 6 Take-Aways

Classroom participants' assessments of the small-group task differed in important ways. Ms. B's account was overwhelmingly positive and focused on the mathematical content discussed by the students. Mr. K's account was mixed; he made positive comments about how the students worked together and expressed concerns about how Guadalupe might have felt in the group, connecting her participation to gender imbalance. The students' assessments ranged from Elijah's almost entirely positive account to Guadalupe's mostly negative account. Each perspective adds another dimension to the story of what happened for this group of students working on this task.

When examined closely from multiple perspectives, small-group interactions that meet a teacher's primary goals may still be problematic in terms of the participation of individual

students. The Ladder Problem task was appropriately challenging and prompted the group of students to engage collectively in a rich conversation about the targeted mathematical content. Ms. B's enthusiasm for the task design was justified. This group did not reach a final solution to the problem; in fact, none of the groups did. However, they explored relevant ideas and relationships, setting them up for the whole-class discussion that followed. In many ways, this task was successful in terms of the mathematical discussion that was generated. However, a closer look at individual student contributions and interactions between students suggests the task was less successful in terms of the distribution of participation. Each student's experience working on the task was unique, and Guadalupe's experience differed from that of her peers in consequential ways. Analysis suggests Yonas, Hosein, and Elijah all had numerous genuine opportunities to participate in ways that supported rich content understandings and positive mathematical identities. Guadalupe did not. Her threshold for mathematical participation was high, she made few mathematical contributions, and she admitted to disengaging from the task.

Guadalupe started out engaged in the task; she followed along, shared some ideas, and responded to the teacher. But Guadalupe started feeling uncomfortable partway through the task, seeming to feel that she didn't have space to contribute. At that point she retreated from the group and disengaged mathematically. Guadalupe's shift from mathematical engagement to disengagement suggests a shift in her threshold for mathematical participation. Overall, her threshold was high, but it seemed to be relatively lower toward the beginning of the task when she readily responded to Hosein's prompt, verbally agreed with Yonas's statement, and responded immediately to the teacher's question. It is conceivable that given different group interactions, Guadalupe's mathematical threshold could have gotten lower as the task progressed, as opposed to higher. It is worth considering what alternative actions, taken by Guadalupe, her peers, and/or the adults in the room, might have lowered Guadalupe's threshold for mathematical participation, leading to broader and richer opportunities for her to contribute to this task.

For the duration of the task, it seemed social interactions might have been more important to Guadalupe than mathematical interactions. It was not because Guadalupe did not care about the mathematics. She cared a lot, and she worked very hard to earn A's in this class. It could have been because this was one of the few times during the day when Guadalupe had opportunities to interact socially with other people. During interviews she talked about how she felt isolated during the pandemic and how much she enjoyed connecting with people in class. She also talked about how she often looked up mathematical topics on the internet after class when she had questions. She was able to meet her mathematical needs outside of class time on her own, but not her social needs. Guadalupe's experience is a reminder that students' readiness to engage fully in mathematics is a function of the extent to which their other human needs are being met.

In reviewing the episode as a whole, we see that participation was not equitable for this group of four students. Opportunities to participate and learn seemed to be distributed fairly among Yonas, Hosein, and Elijah, but Guadalupe did not get her fair share. Analysis suggests some possible reasons for this unfair distribution of opportunities, including specific interactional moments (when Yonas interrupted Guadalupe's mathematical contribution), differing participation thresholds (Guadalupe's threshold was considerably higher than that of her peers), and gender imbalance (Guadalupe was the only female in the group). Questions remain about

how Guadalupe's participation thresholds, her contribution patterns, and her personal assessments might have been different given a different task with a different group on a different day. Additionally, analysis in this chapter prompts questions about the interplay between gender and broader interactional patterns in the class. Guadalupe was one of the students who initiated the conversation with Ms. B at the beginning of the school year about boys dominating class discussions. The teacher was aware of these issues and talked about them frequently during class time with students and during debrief sessions with Mr. K. Yet these issues persisted, and in this case, Ms. B did not see them. Guadalupe's experience highlights larger concerns regarding how to identity, understand, and ultimately address such persistent patterns.

Chapter 7: Discussion

This dissertation set out to add nuance and depth to the field's understanding of "equitable participation" in mathematics classrooms through a multifocal investigation of participatory outcomes and processes in a distance learning high school calculus class. The study examined how inequities were constructed and played out through whole-class and small-group interactions, how classroom-level and societal-level storylines shaped participants' expectations for and assessments of participation, and how participants' perceptions of participation compared to observable participation metrics. The study aimed to improve the field's articulation and understanding of participatory equity, including what it feels like from students' perspectives, and to offer insight into how educators and researchers might go about working toward this goal.

The three guiding principles that informed the design of this study – a focus on participatory outcomes and processes, consideration of multiple levels of context, and integration of outsider and insider views – have been helpful in articulating claims and implications for this study as well. This discussion is organized according to the three principles. However, it is worth noting the principles are interconnected, and therefore comments are organized according to the most salient principle.

Participatory Outcomes and Processes

Claim: Participatory issues were articulated by participants as related to participation *outcomes*, whereas barriers to participation were enacted through interactional *processes*. Specifically, classroom participants voiced concerns about the unequal distribution of talk, framed as "boys talking too much" or "girls not speaking up." However, when students talked about factors inhibiting participation, they talked about ineffective and inefficient communications, feelings of not belonging, and having their contributions not valued by others. Issue articulation (and interventions) focused on symptoms of the problem (outcomes) as opposed to underlying causes (processes). Analysis revealed that the underlying causes of unfairly distributed opportunities to participate were structural in nature and carried out through interactions. For example, the routine of having only students with complete and correct homework solutions present to the class differentially shaped opportunities for participation in consequential ways.

Implications for Practice: A focus on classroom participation issues as outcomes only (without attention to interactional processes) implies that responsibility for the issue is held by the identified students, in effect blaming the victims for the injustices (McDermott & Varenne, 1995). The goal is then "fixing" problematic students (e.g., the boys should speak less or the girls should speak more), rather than addressing problems in the interactional environment of the classroom. Students had valid reasons for not sharing (e.g., females and students of color felt they needed to prove their worthiness and were therefore scared to take risks for fear of being wrong), but those reasons were not the primary focus of interventions. Interventions need to acknowledge and address the inequitable processes through which underlying barriers to participation are enacted by asking and addressing questions such as, why are female students feeling uncomfortable or unwilling to share? And what can we do *as a community* to support students of color to feel comfortable enough to take academic risks? Putting pressure on individuals or groups of students to talk more will not solve the problem. That messaging does

not reduce the obstacles students face. In fact, the messaging could have the opposite effect by increasing students' anxieties about performing in front of judgmental others without error.

Addressing persistent inequities in mathematics classrooms requires challenging deeply entrenched assumptions about who is expected to be good at math and what it means to do math successfully. Through her words and actions, Ms. B communicated her belief that female students, as a group, were just as capable mathematically as male students. Ms. B seemed to believe the same about students of color, although her actions did not challenge assumptions about race in the ways they did about gender. Ms. B wanted all her students to participate and be successful, but it appeared she thought that was out of reach for some of her students. Ms. B had a relatively narrow view of what it meant to do math successfully. She sought out and celebrated students' "brilliant ideas", but those ideas came from only a select few students. Her references to "girls with skills" and her strategy to only call on people when she was confident they knew the answer reinforces the idea that only some people have ideas worth contributing. What about the "girls without skills"? To be clear, this is not a phrase Ms. B used. I am using it here to make a point. I believe every student in that class (female, male, Mexican, Black, Vietnamese, gay, straight, poor, rich, etc.) had mathematical skills and knowledge worth contributing, but I do not think they were fully leveraged. Ms. B was (and still is) an amazing teacher who had a hugely positive impact on the lives and mathematical futures of many students. I do not want to undermine the work she has done. Rather, I am pointing out a direction for future thought and future growth, not just for Ms. B but for teachers in general, especially teachers of advanced mathematics courses like calculus. The shift I am suggesting is a huge shift. It requires designing and implementing math tasks that truly honor multiple mathematical strengths. It requires getting to know students well enough to be able to identify and leverage individual students' mathematical strengths, especially students who had a hard time in previous math classes, lack confidence, and may lack some presumed prerequisite skills. If educators do not attend to the underlying interactional processes and inequitable structural patterns that construct barriers and raise students' participation thresholds, participatory inequities will persist, and marginalized students will be further marginalized.

Implications for Research: By analyzing participation outcomes (e.g., participation metrics captured through EQUIP) together with participation processes and structures (e.g., how opportunities to participate were differentially constructed and became part of the classroom environment), this dissertation responds to calls for mixed methods approaches to understanding participatory equity in classroom contexts. Reinholz & Shah, the creators of EQUIP, recognize the value and the limitations of quantitative participation counts, stating, "Tools like EQUIP are most effective when used in conjunction with deeper qualitative analyses that can contextualize and add depth to quantitative data" (Reinholz & Shah, 2018, p. 169). This study builds on recent studies focused of participatory equity in mathematics classrooms (e.g., Ernest et al., 2019; Shah & Lewis, 2019) by connecting sometimes opaque concepts like "opportunities to participate" (Gresalfi et al., 2009) to observable analytic measures that account for what students do *and* how students feel. In addition, "participation thresholds" are introduced as a novel way to conceptualize and operationalize consequential differences in students' opportunities to participate. Through these methods, students' perspectives are centered (a key aspect of assessing the extent to which opportunities are *fairly* distributed) and both content proficiency and identity outcomes are considered (Esmonde, 2009). There is no one

simple reality in classrooms. Further research is needed to add more textured characterizations of students' learning experiences. One possible next step is to explore how various contextual factors (e.g., task design, group composition, content, norms and their origins) shape students' opportunities to participate.

Multiple Levels of Context

Claim: Intertwined classroom-level and societal-level storylines about mathematical competence (e.g., whose contributions are valued and supported) shaped participants' experiences with classroom participation in unfair ways. Activated storylines from the larger context included, a) boys are better at math than girls, b) students of color do not belong in advanced mathematics courses, and c) students with skills are expected to share their ideas. Specifically, these storylines affected how barriers to participation functioned (e.g., seemingly "successful" students like Guadalupe faced ongoing obstacles related to not feeling valued due to being female and a student of color), how participatory expectations were assigned (e.g., students who understood were expected to share, especially female students "with skills"), and how teacher-student interactions played out (e.g., Ms. B felt she had the right to push her high-achieving female students since she related to their experiences).

I**mplications for practice:** Students do not enter classrooms on an even playing field, even if instructors attempt to provide instruction that is fair and balanced. Societal-level storylines about gender and race are ever-present for female students and students of color in mathematics classes. A female student in Ms. B's class explained, "Even though all of us aren't really thinking that. It's just built in. It's society. We can't help it." Oppressive narratives about who is presumed to be smart at math affect the moment-to-moment decisions students make (e.g., Should I raise my hand to ask a question? Is it worth the risk of sounding stupid?) in ways not always recognized (and therefore not addressed) by teachers. Most K-12 teachers are White. Many teachers, like Ms. B, want to empower their students of color, but they may not know how. Ms. B was aware of racial inequities in math education, but compared to gender, her attention to race in mathematics was newer and her relationship with race-related issues was not as personal. The reality is that White women hold the majority of teaching positions in the United States, yet the student body they teach is growing increasingly more diverse, specifically less White. Analysis of Ms. B's classroom underscores the importance of representation in the teaching force, having teachers who can identify with and relate to their students' cultures, challenges, and triumphs. All teachers need to be prepared to teach students with identities and experiences different from their own. Teachers need to be supported in building connections and relationships with their students. Ms. B felt empowered to push her female students to achieve more (and analyses suggest Ms. B did indeed broadened opportunities for her female students), but she was unable to do the same for her male students of color.

A first step in moving forward is for teachers to learn how to notice inequities that play out in their classrooms (Louie, 2018) by talking with and listening to students and by examining classroom participation from multiple perspectives (Hinestroza, 2022; Reinholz & Shah, 2018). Teachers need to recognize and appreciate the daily struggles students face, including students with whom they easily identify and those with whom they do not. It is not fair to students for teachers to stay in their comfort zones. Building relationships and finding ways to identify with each individual student is crucial. Teachers also put aside concerns about those students who appear to be doing well, such as in the case of Guadalupe. Even though it seemed "she had

figured it out," Guadalupe continued to fight against oppressive narratives. In addition, teachers need to be able to reflect on classroom participation in relatively objective and holistic ways, which will be touched on further in the next section.

Analysis of Ms. B's classroom-level context highlights a dilemma that teachers sometimes face when facilitating dialogic classrooms with attention to equitable participation: Encouraging reluctant students to share their brilliant ideas while protecting students who are unable to share from public embarrassment. Ms. B chose to cold-call students to share ideas, and she consciously tried to call on students whom she thought had something worthwhile to share, but sometimes students ended up declining to answer. Ms. B's intent was to support the development of positive mathematical identities, but when students declined to answer, they experienced the opposite. As stated previously, one way to address this dilemma is to build classroom cultures that truly value perspectives and ideas shared by all students. Creating genuine opportunities for every student to contribute to the advancement of collective understandings in diverse and meaningful ways mitigates against fear of participation rather than reinforcing it.

Implications for research: This dissertation highlights the importance of examining classroom participation in light of multiple levels of context, especially given a focus on equity. Classrooms are not neutral spaces (Leyva et al., 2021). The actions, thoughts, and feelings of classroom participants are tied closely to how students are positioned with respect to racialized and gendered storylines (Langer-Osuna, 2011), whether consciously or not. There is no responsible way to separate analyses of student participation from classroom or societal context because without context, there is no way to even begin to attribute reasons for how and why students participate in the ways that they do (Hinestroza, 2022). Analyses also highlight the twisted nature of classroom-level and societal-level context, as in the case of negotiated meanings of mathematical competence (Gresalfi et al., 2009). Aspects of gender were incorporated explicitly into a presumably classroom-level storyline about what it meant to be productive and smart in Ms. B's calculus class. There seemed to be a higher participatory burden placed on female students who were positioned as competent, as opposed to male students. Ms. B referenced "the girls with skills" repeatedly. She was not thinking about these students as simply competent students who were not sharing; she was thinking about them as competent *female* students who were not sharing. It can also be assumed that aspects of race, while perhaps less explicitly than gender, were also incorporated into classroom-level ideas of competence, given what is known about racial microaggressions in classrooms (Battey & Leyva, 2016) and what was shown through analysis of whole-class participation metrics. Analyses suggest the separate green and blue straight lines representing classroom-level and societal-level storylines from the conceptual framework shared in Chapter 2 - Figure 2b (see Figure 7a), might be better represented by the braided, intertwined curved lines as shown in Figure 7b. The resulting braid could represent a storyline about competence and the numerous colored strands composing that braid would be sub-stories about gender, race, social status, wealth, language, etc.

Figure 7a. Representation of Storylines in Conceptual Framework (Excerpt from Figure 2b)

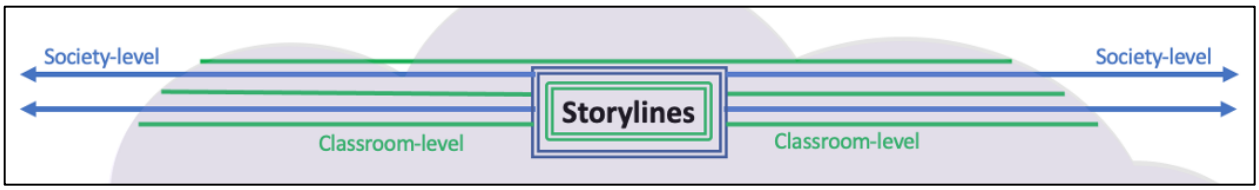
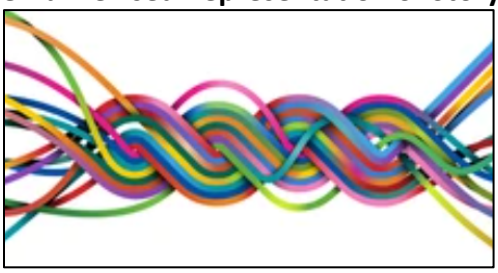


Figure 7b. Revised Representation of Storylines



While this dissertation moves in the direction of documenting and unpacking the interplay of multi-context storylines, further work to operationalize and conceptualize positioning theory with respect to classroom participation is needed to untangle and understand the extremely complex nature of student participation (Herbel-Eisenmann et al., 2015).

Outsiders' and Insiders' Views

Claim (Insider vs. Insider): Insiders' views of shared learning experiences were consequentially different from each other. Specifically, working on an appropriately challenging small-group task that met the teacher's expectations was perceived to be a generally positive experience for the three male students and a decidedly negative experience for the only female student. Elijah, Yonas, and Hosein had numerous genuine opportunities to participate in mathematically meaningful ways, supporting both rich content understandings and positive positional identity; Guadalupe did not. In addition, participants' perceptions and understandings of other participants' experiences during the small-group task varied considerably. Specifically, the student teacher was the only one who correctly sensed that Guadalupe felt left-out and unappreciated by her group. The teacher and two of the three male students commented on Guadalupe's lack of talk, but none of them connected Guadalupe's participation to problematic interactions that should be addressed. The third male student did not mention Guadalupe at all.

Claim (Insiders vs. Outsider): Insiders' views of participation issues were consistent with each other with regard to whole-class participation patterns, and yet were consequentially different from an outsider's view as represented by participation metrics. Specifically, throughout the semester numerous participants talked about the issue of male students dominating conversations but said little about race, whereas semester-long participation metrics indicated White dominance superseded male dominance, with male students of color having the fewest opportunities for mathematically meaningful participation.

Implications for Practice: Hearing participants' varied perspectives on The Ladder Problem is a reminder not to lump students together into a single group. Instead of thinking generally, "What seems best for my students?" It is likely more helpful to think specifically, "What seems best for Guadalupe? For Hosein? For Yonas? And for Elijah?" And even better yet, instead of presumably what is best for them, why not ask them directly? The degree and nature of

Guadalupe's discontent with The Ladder Problem would not have been known if I had not asked her to share her experiences with me. Ms. B noticed that Guadalupe seemed to be less verbal than usual, but Ms. B did not know that she felt left out of the conversation and felt her ideas were not valued. Similarly, Hosein and Yonas noticed Guadalupe's lack of talk, but they attributed that to the gender imbalance in the group, something over which they had no control, as opposed to anything they had done. For inequities to be addressed, they must be known to exist, and they must be understood to a certain extent. This dissertation underscores the importance of actively seeking and listening to students' perspectives on their learning experiences. In addition, this study highlights challenges associated with teaching for equity (e.g., How can teachers organize instruction to accommodate students' varied perspectives on fair opportunities?). Given the historical marginalization in mathematics classrooms of students identifying as Black, Brown, indigenous, queer, poor, etc., does it perhaps make sense to prioritize the perspectives and needs of students who have been marginalized over others who have not?

The disconnect between what participants noticed (and/or talked about) and what observable participation metrics indicated is eye-opening, to say the least. It is a reminder that unconscious biases are hard at work (Reinholz & Shah, 2018) and that social and cultural processes shape what and how teachers and students notice participation (Louie et al., 2021; Wager, 2014). In particular, this study emphasizes the obstacles a White teacher faced when seeing, understanding, and addressing racial inequities in her classroom. Talking about gender is different from talking about race, especially for a White female teacher working with a White female researcher. Ms. B had thought about gender and mathematics for decades. When reflecting on the participation in her classes, she repeatedly drew from her own experiences being marginalized as a woman who studied advanced mathematics. She noticed gender patterns and she was comfortable talking about them with me (the researcher), with her student teacher, and with her students. She even made explicit invitations during class time trying to recruit more female voices to share (e.g., "Ok, sorry to be blunt, but can we get a girl to guess?"). It is hard to imagine Ms. B making a similarly explicit call for a student of color to share during class. It's just not the same, especially coming from a White teacher. As a female teacher she felt she had the right to push her female students, but she did not feel she had the same right to push her students of color. She did, however, tell Mr. K that he would serve as a great role model for his students of color, implying that he, as a man of color, would have the right to push (and inspire) them.

Learning to notice for equity is a skill and an awareness that teachers develop over time, but it does not happen automatically (Louie, 2018). Teachers draw on their own personal experiences to make sense of their students' experiences, and therefore have certain biases that shape what and how they notice (Goodwin, 1994; Sherin & van Es, 2009). Teachers need to seek to continually deepen their understandings of equity through intentional and sustained efforts to recognize perspectives different from their own. Perspectives from their students are of utmost importance, as are perspectives from outsider observers. Teachers need opportunities to reflect on consistencies and inconsistencies across multiple perspectives with colleagues who share similar equity goals. Noticing is a first step, but attention also needs to be paid to supporting conversations about race, conversations among teachers and between teachers and students. Data indicated that race was rarely mentioned, but it is not clear who noticed racial patterns but chose not to speak about them. I can honestly say that I did not notice the racial differences in

whole-class participation until I conducted the quantitative analyses, but I am White and my perspective in undoubtedly different from students of color participating in that class. Given how obvious the pattern was in the data, it is hard to believe that no one in the class noticed. Some students must have at least felt those inequities. Teachers need to be supported to notice and address inequitable participation patterns, but this needs to be done without putting the burden on students. If students speak up, yes, teachers need to listen and respond, but that is not enough. Responsibility for identifying issues and advocating on behalf of marginalized students needs to rest on the shoulders of teachers, administrators, and researchers, not on the students themselves.

Implications for research: This dissertation demonstrates how a mixed methods approach that compares and contrasts insider and outsider perspectives provides a more comprehensive and contextualized assessment of participatory equity than either method could offer alone (Clark, 2019; Creswell & Creswell, 2017). The picture of participation in Ms. B's classroom would have been consequentially different had not all perspectives been thoroughly explored and juxtaposed with each other. Racial inequities in student contributions would not have been identified without outsider observations of whole-class participation, and the in-depth and nuanced understandings of how barriers to participation were enacted and how opportunities to participate were constructed would have been missed without insider reflections. Reliable assessments of students' opportunities for positive mathematical identity development would have also been particularly difficult without students' firsthand accounts of their learning experiences.

Identity studies often incorporate student perspectives (e.g., Langer-Osuna, 2011; Leyva, 2017; Nasir & Hand, 2008), but many studies of participation and learning do not (e.g., Ing et al., 2015; Otten & Soria, 2014). Studies that seek to explain "how" or "why" questions related to student participation or seek to assess the impact or success of participatory interventions must include students' perspectives. It is inaccurate, unfair, and even oppressive for researchers to presume to know how or why students participate in the ways that they do or how classroom interactions affect students' perceptions of themselves or others as learners and doers of mathematics based on observation alone. This study highlights what can be learned from students if researchers ask them to reflect on their experiences and share their thoughts. Participatory inequities are felt by students but often not seen by their teachers, peers, or researchers, playing out through microaggressions that position certain students (e.g., Black, female, Indigenous, recent immigrant, poor, etc.) as less competent and/or less worthy of contributing to mathematics (Gholson & Martin, 2014; Sengupta-Irving & Vossoughi, 2019). By engaging students in dialogue about their experiences, not only will educators and researchers learn more about the nature and impact of participatory inequities, but they will also learn more about how to build from students' strengths and leverage the resources they bring to their communities (Agarwal & Sengupta-Irving, 2019). Further research is needed to understand how best to leverage the power of integrating outsider and insider perspectives of classroom participation for teachers' learning.

Another next step is for researchers to explore various operational definitions for participatory equity across different contexts. Connections between classroom participation, content development, and identity development are complicated, multi-dimensional, and contextual. This study offers a more comprehensive way to operationalize participatory equity,

but there are surely alternative ways that would highlight other important aspects of equity related to participatory outcomes and processes. That being said, students' experiences should be centered when assessing "equitable participation", not only what students do but also how students feel.

Limitations

This dissertation combined multiple data sources and a wide array of analyses to shed light on classroom participation in Ms. B's distance learning calculus class. Limitations of the study include which analyses were conducted, whose perspectives were documented, and how students were labelled / grouped for analyses.

Micro-interactional analyses were conducted on only one whole-class discussion and one group of four students working on one small-group task. Although these episodes were determined to be representative, they were both short episodes taken from a semester's worth of lessons. Analyzing more whole-class discussions and more groups working on small-group tasks would add valuable insight into classroom participation outcomes and processes, as would analyses of the weekly "help sessions." Further analysis would add texture and robustness to the findings.

Out of the 60 students enrolled in Ms. B's two calculus classes, 27 students consented to one-on-one interviews, 17 of whom were interviewed at least once. Unfortunately, students who do not consent to research are often members of marginalized communities whose perspectives are of utmost importance to consider and understand. More work is needed to develop ethical and inclusive research practices that center issues of marginalization (Chen & Horn, 2022; Fox et al., 2021).

Students in Ms. B's classes represented numerous ethnic and racial identities, with no identities dominating any others in terms of representation. As described by students, identities included: Black, Chinese, Filipino, Indian, Iranian-American, Korean, Mexican, Mexican / Middle Eastern, Mixed, Nicaraguan / White, Vietnamese, and White. For some analyses, I chose to compare "students of color" with "White students." I used this method of categorization with hesitation, knowing that grouping students of color together would foreground certain patterns and hide others. In addition, any grouping (e.g., "White male" or "Black Female") necessarily clusters people who act or are treated differently. In-depth intersectional analyses of individual students would complement the analyses presented in this dissertation (e.g., Leyva, 2021).

My hope is that despite its limitations, this dissertation paves the way for future studies to explore the consequential issues it has uncovered.

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Appendices

Appendix A. Lesson Observation Templates

Regular Lesson Observation Template Date: _____

School: Evergreen High School
Teacher: _____ (student teacher = _____)
Period: ____
Absent students: _____
Tardy students: _____
Lesson / classwork: _____

Time
WC _____ - _____
SG _____ - _____
WC _____ - _____
SG _____ - _____

Homework problems discussed

# _____	# _____	# _____	# _____
Expl by _____	Expl by _____	Expl by _____	Expl by _____
a) T writes on board	a) T writes on board	a) T writes on board	a) T writes on board
b) S shares screen	b) S shares screen	b) S shares screen	b) S shares screen
c) ST uses ipad	c) ST uses ipad	c) ST uses ipad	c) ST uses ipad
d) other _____	d) other _____	d) other _____	d) other _____

whole class observations

group members

_____

_____

_____

_____

small-group observations

additional notes / observations / summary

Wednesday "Help Session" Observation Template Date: _____

School: Evergreen High School
Teacher: _____ (student teacher = _____)
Period: ____

Students present: _____

_____

Lesson: _____

Homework problems discussed

# _____	# _____	# _____	# _____
Req by _____	Req by _____	Req by _____	Req by _____
Expl by _____	Expl by _____	Expl by _____	Expl by _____
a) T writes on board	a) T writes on board	a) T writes on board	a) T writes on board
b) S shares screen	b) S shares screen	b) S shares screen	b) S shares screen
c) ST uses ipad	c) ST uses ipad	c) ST uses ipad	c) ST uses ipad
d) other _____	d) other _____	d) other _____	d) other _____
# _____	# _____	# _____	# _____
Req by _____	Req by _____	Req by _____	Req by _____
Expl by _____	Expl by _____	Expl by _____	Expl by _____
a) T writes on board	a) T writes on board	a) T writes on board	a) T writes on board
b) S shares screen	b) S shares scree	b) S shares screen	b) S shares screen
c) ST uses ipad	c) ST uses ipad	c) ST uses ipad	c) ST uses ipad
d) other _____	d) other _____	d) other _____	d) other _____

whole class observations / summary

Appendix B. Student Interview Protocols

Student Interview (Beginning of Semester)

Students will be asked about their thoughts regarding math generally and about their learning

experiences in their calculus class this year in particular. This first interview will help me get to know a little bit about students' backgrounds and their previous experiences with math. I also hope to get a sense of how they are thinking about expectations for participation and success in their calculus class. The results of these first interviews will help me select focal students for this study.

Introduction

Thank you so much for talking with me today. I really appreciate you taking the time to do this. Through my research project I am hoping to learn more about student participation and learning in virtual settings, and your thoughts are going to be a really important part of that process for me. So, thank you! I'm guessing this will take about 20-30 minutes. Does that work for you?

I am going to start by asking you to tell me a little bit about your experiences related to math in general. Then we'll talk about your calculus class in particular, and finally, we will take a look at a video that I recorded recently during one of your calculus classes. I would like to hear what you have to say about the video, what you notice or find interesting, what you were thinking or feeling at the time the video was taken, and any other thoughts you can share with me to help me understand what was going on and why. Please feel free to skip any questions that I ask that you'd rather not talk about. That's perfectly fine. I also want you to know that I will not share your responses with Ms. A. or with any other students in this class.

Math Experiences

- Can you start by telling me a little bit about your experiences related to math? (e.g., what do you like/dislike about math?)
    - How have your feelings about this subject changed through elementary, middle and now high school?
    - Are you planning on pursuing math in college and beyond?
    - Would you say that you like math? What do you like about it?
- What does it mean to you to be "smart at math"?
    - In what ways are you smart at math?

Calculus Class / Math

- Can you tell me a little bit about how you decided to take this class?
    - What do your friends and family think about you taking this class?
- How would you describe yourself as a student in this class? How is that similar or different from how you are in other classes?
- Do you usually do homework by yourself or with friends? Who do you work / study with?
- Can you tell me about your strengths and your challenges in this class?
    - What are your strengths in other classes?
    - What are your strengths outside of school?
- When you encounter challenges in this class, what do you do?
- What do you think it means to be successful in this class?
    - Do you have a sense of who is successful in this class? How do you know?
    - How successful do you feel you've been in this class so far this year? 1 (not at all) to 8 (very)
- If you could change one thing about this class, what would it be and why?

Perceptions of Participation

- How would you describe ideal participation in order for students to be success in this class?
    - Whole class participation vs. small-group participation?
- How would you describe your participation during whole-class, small-group, and Wednesday help sessions (if you've attended any)?
    - Your participation relative to your peers?
    - Changes in your participation over the course of this school year? What do you think prompted those changes?
- When and why do you speak in class? … When you're confused? When you're not confused?
- What are some similarities and differences in how you participate in this (and other) virtual classes vs. in-person classes?
- If you could magically change something about how you or other people participate in this class, what would you change?
- Thinking about how you feel right now in this calculus class, where would you place yourself along this mathematical confidence scale and why?

    - Thinking about your math class last year, would your placement have been any different? Why?
- Do you have any goals for yourself for the last term of the school year?
    - Grades? Learning? Confidence? Participation?

Wrap Up

- Are there any other thoughts that you'd like to share with me regarding your experiences with math in general or with your calculus class this year?
- And then finally, I have a few quick background questions…
    - How old are you?
    - How do you describe your gender?
    - How do you describe your race, ethnicity, culture?
    - How many years have you attended BUSD schools?
    - Have you had Ms. A as a math teacher before this year?
    - Is there anything else that you think might be helpful for me to know about you?

Student Interview (Stimulated-Recall)

The second student interview will give me an opportunity to ask follow-up questions about ideas shared during the previous interview. During this interview I will also inquire about any changes to students' participation during class and if there have been any noteworthy events related to participation in this class that they want to share with me. The bulk of the interview time will be spent watching and reflecting on a short videoclip previously recorded during small-group work in this class. The interview will help me make sense of what happened during the selected episode of classroom interactions and will help me understand how students' participation is evolving.

Introduction

Thank you for agreeing to talk with me again. I really enjoyed talking with you last time, and I appreciate you taking the time to share more of your thoughts with me today. I expect our conversation to last 20-30 minutes. Does that work for you?

I'm going to start this interview by asking a few clarifying questions related to things we talked about last time. Then we will talk a little bit more about participation in this class. And finally, we will watch a short video clip that I recorded of you working with a few other students in a small group. I'm curious to hear your thoughts about what you were experiencing and thinking while working on that assignment.

Do you have any questions before we get started?

Clarifying ideas from the previous interview

- Last time you said, _____. Can you say a little more about _____?
- Last time you said, _____. I'm wondering how you feel about _____.

Perceptions of Participation since the previous interview

- Do you feel that ways you participate in class have changed at all since we last spoke (in the last few weeks/months)? If so, in what ways? Why do you think that is?
- Are there any recent moments that stand out in your mind as being interesting or different in terms of your participation or your classmates' participation in class? Any particular days or activities that you felt participation was atypical?
- Do you have any concerns right now about your participation or your classmates' participation in this class?
- If you had to choose three other students to work with in a group in this class, who would you choose and why?
- Do you have any other thoughts about math in general or about participation in this class that you'd like to share with me?

Video Reflections

I will have selected a video clip ahead of time that contains some interesting interactions with the student's group, something that I need the student's help to understand better. This episode could be an example of common interactional patterns I've seen repeated numerous times, could be an example of outlier interactions, or confusing interactions that do not seem to make sense to me.

My plan is for us to now watch a short video clip from class the other day. I've chosen this particular video clip because I think it's really interesting and I would love to get your help trying to understand it better. I'm going to start the video and then I'll stop it when I have questions for you. But I also want you to request a stop when you have thoughts to share (so when you see something interesting or confusing or think of something that might be helpful for me to know). The goal of this activity is for me to get a better understanding of what happened here. I want to hear your personal thoughts and ideas. I see you as the expert right now, since you were an important part of this group discussion.

Do you have any questions about this activity?

(during the video watching session)

When the student requests a stop…

- Why have you requested this stop?
- Can you describe to me a little more about what you see?
- Do you remember what you were thinking or feeling while this was happening?
- Do you remember *why* you [did that thing]?

When I stop the video…

- What do you notice here?
- Do you remember what you were thinking or feeling while this was happening?
- Do you remember *why* you [did that thing]?

(at the conclusion of the video watching)

- Does this groupwork seem typical to you? How is this similar or different from how you usually participate in groups? What about when you're working with a different group of students? Or working in another class?
- What happened during this groupwork that worked well for you, or in other words, what was something that you think helped you learn? Was there anything that you feel held you back from learning or specifically did not help you learn?
- Is there anything else that you want to tell me about this video clip that will help me understand better what happened during this groupwork?

Closing

Thank you so much for taking the time to do this. Your thoughts are really important to me. At this point I do not know when we will be talking again, but I do look forward to it.

Student Interview (End of Semester)

The third student interview will give me an opportunity to ask follow-up questions about ideas shared during the previous interviews and ask some final questions related to the student's experiences in this calculus class and their mathematical identity, focusing on students' perceptions of themselves and their peers' mathematical competency and success in this class. This interview will take place toward the end of the semester.

Introduction

Thank you for agreeing to talk with me once again. I appreciate you taking the time to share more of your thoughts with me today. This will be our last official interview. I expect our conversation to last about 15 minutes. Does that work for you?

I'm going to start this interview by asking a few clarifying questions related to things we talked about last time, and then I'll ask a few concluding questions about the class overall.

Do you have any questions before we get started?

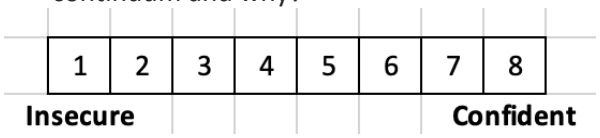
Clarifying ideas from the previous interview

- Last time you said, _____. Can you say a little more about _____?
- Last time you said, _____. I'm wondering how you feel about _____.

Final Reflections

- Thinking back over this school year, what stands out to you as noteworthy?
- What factors helped you succeed in this class this year?
    - What obstacles did you face?
    - What are some things that you think you got better at over the year?
- I asked you about this mathematical confidence scale the first time we spoke. Thinking about how you feel now that you've finished the course, where would you place yourself along this

continuum and why?



- In our first interview you said that being smart in math means …[excerpt]. Do you still agree with that? Has your thinking about being smart changed at all?
    - In what ways would you say that you are smart at math?
    - Can you tell me about a student who is smart at math in ways that are different from you?
- What is one thing that you really liked about this class? What is one thing that you would have changed?
- Was there anything that you found surprising about your math class this year?
- Do you know yet what your plans are for next year?
    - Do you plan to pursue mathematics courses in college?
- Is there anything else at all that you want to share with me about your experiences in calculus this year?

Thank you again for talking with me. I've really enjoyed getting to know you and to learn more about your experiences. I wish you the best of luck next year.

Appendix C. Teacher / Student Teacher Interview Protocols

Teacher / Student Interview (Beginning of Semester)

Introduction

Thank you so much for talking with me today. I really appreciate you taking the time to do this. Through my research project I am hoping to learn more about student participation and learning in virtual settings, and your thoughts are going to be a really important part of that process for me. So, thank you! I'm guessing this will take about 20-30 minutes. Does that work for you?

Background

- Can you tell me a little bit about yourself and your background with mathematics?
- How long have you been teaching?
    - With this school district / school?
    - How many years have you taught calculus?
- How would you describe yourself as a math teacher? Priorities? Teaching style?
- Do you have any specific goals for this particular group of students?
- Do you have any specific goals for yourself professionally this year?

Success / Smartness

- What does "being smart at math" mean to you?
    - What are some ways that students can show that they are smart at math?
- What do you think it takes for a student to be successful in this calculus class?

Student Participation

- How would you describe ideal student participation?
    - Whole class participation?

    - small-group participation?
- How successful do you think the Wednesday help sessions have been? And for whom?
- What are some similarities and differences in how students participate in this *virtual* class vs. *in-person* classes?
- Do you have any concerns right now about student participation in this class, either generally or with respect to specific students?
- If you could magically change something about student participation in this class, what would you change?

Perceptions of Focal Students

I'd like you to look at the list of students that I gave you and use that list to help you respond to the next few questions.

- What do you see as the strengths and weaknesses of each of these students in terms of their mathematical participation in this course?
- Have you noticed any changes in the participation of any of these students since the beginning of the school year? For the students you've taught before, any changes in their participation from previous years to this year?

Conclusion

- Is there anything else that you can think of related to your participation or your experiences in this class that you feel would be helpful for me to know?
- How do you describe your gender? Your race, ethnicity, culture?

Teacher / Student Teacher Interview (Stimulated-Recall)

Video Reflections

I will have selected one or more short video clips that contain interesting interactions. These will be the same videos that I'm reflecting on with students.

During this conversation, my plan is for us to watch one or more short videos of students working in small groups. I've chosen these particular video clips because I think they're really interesting and I would love to get your help trying to understand them better. I'm going to start the video and then I'll stop it when I have questions for you. But I also want you to request a stop when you have thoughts to share (so when you see something interesting or confusing or think of something that might be helpful for me to know).

Do you have any questions about this activity?

(during the video watching session)

When the teacher requests a stop…

- Why have you requested this stop?
- Can you describe to me a little more about what you are seeing?
- Do you have thoughts on why ___ might have happened?
- What questions do you have about what's going on here?

When I stop the video…

- What do you notice here?
- Do you have thoughts on why ___ might have happened?

- What questions do you have about what's going on here?

(at the conclusion watching the first video)

- How would you assess the work that these students are doing?
- Does this groupwork seem typical to you? How is this similar or different from how these students usually participate in class?
- What happened during this groupwork that you think worked well for these students in terms of their learning (and for whom)? Was there anything that you noticed that you think hindered their learning (and for whom)?
- Do you have any thoughts to share about this video clip?

Repeat with video #2.

Teacher / Student Teacher Interview (End of Semester)

The last interview will give me an opportunity to ask follow-up questions about ideas shared during the previous interviews and ask some final questions related to the teacher's experiences this year. This interview will take place toward the end of the semester.

Introduction

Thank you for agreeing to talk with me once again. I appreciate you taking the time to share more of your thoughts with me today. This will be our last official interview. I expect our conversation to last about 15 minutes. Does that work for you?

Do you have any questions before we get started?

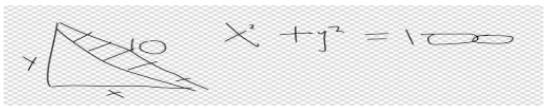
- Thinking back over this past school year, what stands out to you as noteworthy?
- What do you feel are some successes that you experienced with your calculus classes?
- What are some frustrations/challenges?
- What are some things that you learned this year?

- What changes did you notice in particular students throughout the course of the year? (related to their participation? Mathematical understanding?)
- Are there any students who surprised you?
- Are there any students whom you are particularly proud of or concerned about?

- Do you have any specific goals for your calculus classes next year?
- Are there any other thoughts that you'd like to share with me regarding reflections on this school year and/or student participation in your calculus classes?

Appendix D. Transcript for Small-Group Task (2/2/21)

Index	Time	Speaker	To Whom	Episode	Math / Social	Transcript
1	0:00	Guadalupe	Ms. Fink	Guadalupe greets Ms. Fink	Social	Ms. Fink, oh my God! I get so excited- Ms. Fink, I just responded to your email. (Guadalupe & Ms. Fink smile.)
2	0:09	Ms. Fink	Guadalupe			Ok. Thank you. (Guadalupe & Ms. Fink smile.)
3	0:11	Yonas	group	Hosein, Yonas and Elijah exchange initial ideas about possible solutions supported by mathematical reasoning	Math	A ladder leans against a wall, begins to slide down the wall, does the top of the ladder move at the same rate as the bottom of the ladder?
4	0:23	Yonas	group			No? because they start at different points? Like is it saying like a ladder. The wall's like ten feet high. It leans against it. But it only it might only be like four feet off on the ground. I think it might be- it seems like the same rate because I can imagine it falling somewhat-
5	0:50	Elijah	Group			Yeah, so like my initial thoughts was the same like if we have a like, like and it's just flat against the wall, the top that is going to be at like point x. Right? And then if you have it flat against the floor the bottom of ladder is going to be like that same distance x away from the wall. And so, they move the same amount. like the top of the ladder moving from point x to the floor and the bottom of the ladder moving from against the wall to point x on the floor is the same amount of movement.
6	1:27	Hosein	Group			What if you have like instead of a ladder on a wall, you have a really long stick or like a whip or something like you're not moving it very far, but it becomes faster and faster towards the end, right? (Raises hand like he's holding a whip) Like, you know what I'm talking about? Like I feel like I don't I don't have anything to like, like back it up, but my gut feeling is like they have different, different rates.
7	1:59	Elijah	Hosein			Yeah. I'm unsure.
8	2:01	Yonas	Hosein			I mean, the wall kind of like keeps the friction. I get what you're saying. When it's like sliding down, it's always going to be touching, though. That's the thing.
9	2:20	Hosein	Group	Hosein tries to find the assignment online		Sorry, where are you guys finding the um, lesson?
10	2:23	Elijah	Hosein			3.10? 3.10.
11	2:30	Hosein	Group			I just went there, and it like deleted it. Maybe just because it kicked me out. (Guadalupe, Yonas, Ms. Fink all give quick smile.)
12	2:48	Hosein	Guadalupe	Hosein asks Guadalupe for her thoughts		Guadalupe, what are you thinking about it?
13	2:49	Guadalupe	Group			Ummm.. I don't think it would be at the same rate just cuz like if it's like falling, it falls this distance but normally it pushes out a lot faster. It would be flat at the end.
14	3:01	Yonas	group	Guadalupe, Hosein, and Yonas agree on an answer		Oh my god. Wait. Sorry, sorry. I just had a theory. (Guadalupe smiles, Yonas smiles, Ms. Fink smiles.) It's going to be like a triangle, right? (Hosein nods then holds up his arm in a diagonal position.)
15	3:13	Elijah	Yonas			Yeah.
16	3:14	Yonas	himself			So, x.
17	3:24	Hosein	Group			It has to move a greater distance... in the same amount of time.
18	3:26	Yonas	Hosein			Oh, yeah. Okay.
19	3:35	Yonas	group			So it slides away, so the bottom slides away at 1 ft per second, how fast does the top ladder sliding down?
20	3:41	Yonas	group			Oh, this is gonna be implicit differentiation. So, they are different, and we have to solve for each one, okay. That makes sense.
21	4:04	Yonas	group			It slides away. So, x is going to be 1 foot squared. The bottom of the ladder is 6 ft from the wall. The bottom of the ladder is 6 feet from the wall. Oh, Ok. Yeah, they will be at different rates. That makes sense.
22	4:48					**(Teacher joins the breakout room)**
23	4:49	Guadalupe	Yonas			I agree with you.
24	4:51	Yonas				Yeah, you guys were right.

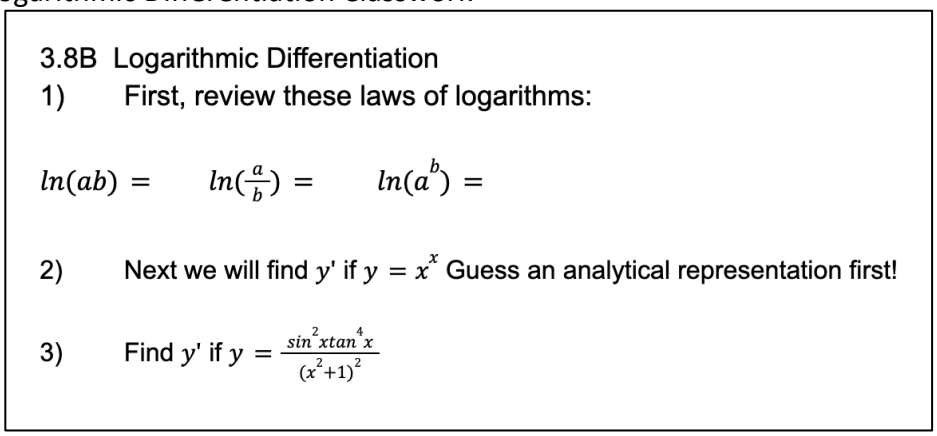
25	4:52	Teacher	Group			Are you guys saying yes or no? (T shows thumbs up or thumbs down)
26	4:54	Yonas	Teacher			We're saying-
27	4:54	Guadalupe	Teacher			We're saying they're going to be at different rates.
28	4:57	Teacher	Group			Has anyone done what I suggested? make a model?
29	5:02	Yonas				I dropped my pencil off of my computer.
30	5:04					**(Student Teacher joins the breakout room)**
31	5:06	Teacher	Yonas	The teacher and student teacher join the group to check-in on progress		OK.
32	5:08	Guadalupe	Group		Social	Oh my god, we have all three teachers. (Guadalupe smiles, Ms. Fink smiles, Hosein smiles.)
33	5:10	Teacher	Group			Yeah, I'm leaving.
34	5:12					**(Teacher leaves breakout room)**
35	5:13	Guadalupe	Mr. K			Mr. K, you look hecka bored. You look like you're about to go back to sleep. I'm not commenting. Sometimes I say that my camera's broken just so I can go back to bed, but not in this class. Never in this class. (Guadalupe, Ms. Fink, Mr. K, Yonas, Hosein all smile.)
36	5:28	Hosein	Guadalupe			Never.
37	5:31	Yonas	Guadalupe			(laughing) Jeez, does that actually happen?
38	5:33					**(Student Teacher leaves breakout room)**
39	5:39	Yonas	Ms. Fink	Yonas seeks feedback from Ms. Fink about his idea that there is a possible connection to the Pythagorean Thm.	Math	This is going to be x-squared- Ms. Fink, this is going to be x-squared plus y-squared, right?
40	5:48	Ms. Fink	Yonas			Can you say- Can you say more?
41	5:50	Yonas	Ms. Fink			c-squared. Sorry, sorry. It's going to be, it's going to be because like the hypotenuse is ten, it's going to be x squared plus y squared equals a 100 squared because it's just like a triangle. So, and then the x is going to be like the bottom of the- like the bottom of the ladder. So, y's going to be the top of it.
42	6:16	Ms. Fink	Yonas			So, you're talking about the Pythagorean Theorem?
43	6:18	Yonas	Ms. Fink			Yeah, yeah. And how like there's an X and Y, there's X and Y values from the ladder. So,
44	6:26	Ms. Fink	Yonas			So, why don't you, if you have an idea, why don't you maybe even draw a picture and share it with your group and see what they say.
45	6:36	Yonas	Ms. Fink			Let me get drawing tools.
46	6:40	Guadalupe	Ms. Fink	Guadalupe asks Ms. Fink about her PhD program and job aspirations.	Social	Ms. Fink, are you gonna be a math teacher? I forgot.
47	6:43	Ms. Fink	Guadalupe			Um, I was a middle school math teacher for a long time, but I'm now, but I'm working on a PhD. So now it's going to be more on research. And maybe- like a- and maybe like a college, like teacher preparation teacher, a college instructor. That's my goal.
48	6:52	Guadalupe	Ms. Fink			Oh, like that's so cool. (Guadalupe smiles, Ms. Fink smiles.)
49	7:00	Guadalupe	Ms. Fink			How hard is a PhD? It's like research-based, right?
50	7:05	Ms. Fink	Guadalupe			Yeah. Well, this is my sixth year at Berkeley, so- (Ms. Fink smiles.)
51	7:08	Guadalupe	Ms. Fink			Oh, hell no. I always tell my mom that I want a PhD, then I like think about how much school it is. Oh my god. (Guadalupe smiles, so do Yonas, Ms. Fink and Hosein.)
52	7:21	Guadalupe	Ms. Fink			That's crazy. Props to you, Ms. Fink. (Guadalupe and Ms. Fink smile.)
53	7:23	Yonas	group	Guadalupe comments on Yonas's drawing	Math	
54	7:30	Guadalupe	Yonas			Yonas, what are you doing?
55	7:31	Yonas	Guadalupe			I'm trying to, I'm trying to draw the line. So, this is the ladder right here, on the wall.
56	7:40	Guadalupe	Yonas		Social	Very sturdy looking ladder. (Guadalupe smiles.)
57	7:42	Yonas	Guadalupe			Oh yeah, it's nice. (Yonas smiles.)

58	7:45	Yonas	group	Hosein and Elijah discuss the relative rates of the top and bottom of the ladder, initially disagreeing, then agreeing, then disagreeing again.	Math	And then this, and then, so we already know this is ten…
59	7:50	Hosein	Yonas			Right.
60	7:51	Yonas	group			But then this is going to be x and y. So, this is going to be x squared plus y squared equals ten squared. Oops. So, we can do implicit differentiation to find the rate of change by when it's falling and-
61	8:15	Hosein	Yonas			Um, Yonas, I'm sorry to interrupt. I have an idea.
62	8:21	Yonas	Hosein			Yeah… No, no, say yours. It's always good
63	8:23	Hosein	group			So, so, we have a ladder, extending it- it's ten feet long and it's at like eight feet up the wall. Y is y is 8. It's six feet from the wall. So, the ladder has to slide an additional four feet to be completely flat. But the top of the ladder has to slide down 8 feet.
64	8:47	Elijah	Hosein			Yeah. Yeah… Uh-hum.
65	8:48	Hosein	group			So, does that mean that the top falls twice as fast as the bottom?
66	8:51	Elijah	Hosein			Um. I don't think it's twice as fast because it's six verses eight. Right?
67	8:59	Hosein	Elijah			No, but it has to move an additional four on the bottom compared to the eight that it has to move at the top.
68	9:08	Elijah	Hosein			Hmm… True. Hmm… Interesting. So-
69	9:21	Yonas	group			Hmm… Let's see. It's one foot on the bottom per second.
70	9:30	Elijah	Hosein			Oh yeah and that does make sense cuz if we assume it started out flat against the wall then for it to get to eight six then it would be moving two feet at the top and four feet at the bottom. Yeah, you're right.
71	9:54	Elijah	group			Yeah, definitely moves twice as fast on the bottom.
72	10:08	Yonas	Elijah			It's weird. Yeah. There's a there's a bunch of different parts.
73	10:12	Elijah	Group			Or- wait. Hang on… Wait, it moves three times as fast on the bottom as on the top. Because if it's only going out to ten- (cuts off, students sent back to whole-class)

Appendix E. Interaction Analysis of Whole-Class Discussion

A summary of this analysis is included in Chapter 5: Part 3. This section contains the details underlying that summary. The focal lesson introduced the topic of logarithmic differentiation. It was the last lesson in a unit about the Chain Rule and its uses. The lesson began with a few announcements made by the teacher, and then the class discussed solutions for several homework problems. The teacher talked about two problems and solicited a volunteer to present their solution for a third problem. After announcements and homework discussion, the teacher shifted the students' attention to the main lesson for the day. The following three problems were written on the teacher's classroom white board (Figure A1).

Figure A1. Logarithmic Differentiation Classwork



The first problem was intended to be review, but the teacher sensed quickly that many students were not comfortable with the basic laws of logarithms. The class spent about 10 minutes reviewing logarithms and talking about the first classwork problem during this second part of the lesson. The class then talked about the second classwork problem for about 12 minutes before going into Zoom breakout rooms to work on the third problem in small groups. The whole class reconvened for about 14 minutes of discussion after the small-group work before the class period ended. The lesson excerpt selected for analysis contains whole-class discussion from the first three parts of the lesson just described: 1. Announcements & Homework Sharing (15 minutes), 2. Classwork Problem #1 & Logarithmic Review (10 minutes), and 3. Classwork Problem #2 (12 minutes). Analysis is organized according to these three parts of the lesson. Descriptions of each part of the lesson are provided, including detailed tables with solicitation dialogue, invitation types, and contribution explanations. Analysis focuses on how opportunities to participate were constructed through interactions.

Announcements & Homework Sharing

The first part of the lesson was devoted to class announcements and homework sharing. Ms. B started the class period by trying to recruit a student volunteer to share their solution for homework problem #114. Her attempt was unsuccessful, so she moved on to class announcements. She solicited comments on the announcements and then tried a second time to recruit a volunteer for #114. Again, she was unsuccessful, so she went on to talk about two other problems from the previous night's homework. Ms. B was finally successful in finding a student to explain #114 on her third attempt. Emma volunteered to share. Midway through Emma's explanation, Ms. B got confused as she was scribing Emma's solving process on the whiteboard. Ms. B solicited support from the student teacher and then the class in general. Kyle volunteered and clarified the confusion. Ms. B ended up giving both Emma and Kyle an extra credit point for explaining the selected homework problem.

Table A1 provides more detail regarding this first part of the whole-class discussion. A new row in the table was created each time the teacher invited a student contribution and each time a student contribution was made. Some of Ms. B's contribution invitations resulted in a student contribution being made and others did not. Contribution invitations were categorized as *open invitation* (i.e., asking a question open to anyone), *general encouragement* (i.e., asking a question open to anyone, including an attempt to convince someone to respond), or *explicit request* (i.e., asking a question directed at one or more people with a response expected). Additional information is included in the table regarding the types of contributions made and the students who made the contributions.

Table A1: Contribution Solicitations during Announcements & Homework Sharing

Row	Time	Summary	Solicitation & Contribution Dialogue (excerpts from the video transcription)	Invitation Type	Contribution Description	Contributor
1	0:00	Ms. B tries, unsuccessfully, to recruit a student volunteer to share a solution for homework problem #114. She then goes on to share some class announcements, including addressing students' responses to "check-in" questions she posed as part of the previous night's homework.	**Ms. B:** Number 114 is still open to the first speaker and there are a lot of people who can explain it. So, speak up at any time. You can interrupt me if you want to.	general encouragement		
2	4:00	Ms. B asks students for quick comments about the announcements she just made.	**Ms. B:** Does anyone need to say anything briskly right now? Chris, go ahead.	open invitation		
3	4:04	Chris asks a logistical question about the check-in questions, and Ms. B responds.	**Chris:** Oh, I just had a quick question. What's the best way to answer the questions that you sent out? **Ms. B:** Did you put them in your text somewhere and I didn't see them? **Chris:** No, I forgot to answer them. **Ms. B:** If you just send me an email. Now that's easier for me to see because those pop right up into my screen. **Chris:** Ok.		Individual volunteered, Asked a Question	Chris: male, White
4	4:30	Ms. B tries again to solicit a volunteer to share a solution for problem #114, but no one volunteers. Ms. B goes over solutions for two other homework problems. She explains as she writes on the whiteboard in her classroom.	**Ms. B:** Has anyone volunteered for 114 while I was talking away? No? Nobody wants to volunteer? You guys got 114. (3 seconds of silence) Nope? Ok. I'm going to give a little more time to talk about two problems and then we'll get to that. I know someone is going to speak up.	general encouragement		
5	6:40	Ms. B tries a third time to solicit a volunteer for problem #114.	**Ms. B:** Who's doing 114? What the heck? I'm doing all the talking. (4 seconds of silence) I'm going to wait.	open invitation		
6	6:48	Emma volunteers to share a solution for problem #114. She begins by reading the problem out loud and then shares how she started the problem. Ms. B writes on the whiteboard as Emma explains her process.	**Ms. B:** Emma, is that you? **Emma:** Yeah. **Ms. B:** Great. Thank you, Emma. **Emma:** (reads the problem and begins explaining her solution process)		Individual volunteered, Shared a Solution	Emma: female, student of color
7	12:17	Ms. B notices something confusing about how Emma is explaining the solution and seeks other voices to help clarify what is going on. She first asks the student teacher, but then without giving him a chance to respond, she opens the question up to the rest of the class.	**Ms. B:** Wait, something went wrong here. Mr. K, can you help me? I'm stuck. I'm getting lost. Or can anyone help?	explicit request (Mr. K), open invitation		
8	12:26	Kyle volunteers to help clarify by pointing out a mistake in what is written on the board. Emma agrees with Kyle's proposed change. Ms. B and Kyle continue back and forth until Ms. B understands and agrees with what Kyle is proposing. Ms. B makes the necessary changes to what is written on the whiteboard and continues to talk through the final steps of the solution. She then concludes by praising the two students for sharing.	**Kyle:** Ms. B, I think- **Ms. B:** Yeah. **Kyle:** The formula for the derivative of the inverse function is f prime of f inverse of x, not f inverse prime of f inverse of x. **Emma:** Oh yeah. **Kyle:** So on the first equation- **Ms. B:** This here? **Kyle:** So, she's trying to solve for the derivative of e to the x…		Individual volunteered, Identified a Mistake	Kyle: male, White
9	15:06	Ms. B prompts the students to ask questions about the problem solution, but no one says anything. They move on to the main lesson of the day.	**Ms. B:** Does anyone want to ask a question? We had to ravel and unravel that a few times, so I imagine there might be some questions. (4 seconds of silence) Or you guys want to leave that right there?	general encouragement		

Ms. B spent the first few minutes of class doing all the talking. At the end of the class announcements, she solicited student contributions through an open invitation ("Does anyone need to say anything briskly right now?"). The use of the words "need" and "briskly" may have inhibited opportunities to participate by implying that student contributions should be necessary and should be quick. Chris, a White male, volunteered and asked a clarifying question about how to submit part of his homework. Ms. B's thorough and polite response indicate Chris's question was indeed appropriate.

The next ten minutes of the class were focused on homework sharing. Ms. B had unsuccessfully tried to solicit a volunteer for one of the problems at the very beginning of class through general encouragement. She said, "Number 114 is still open to the first speaker and there are a lot of people who can explain it. So, speak up at any time. You can interrupt me if you want to." Her phrases "there are a lot of people who can explain it," "speak up at any time," and "you can interrupt me" likely supported opportunities to participate by emphasizing students' mathematical capabilities and by sanctioning student interruptions. After the announcements, Ms. B tried a second time to solicit a volunteer, again using general encouragement. "Nobody wants to volunteer? You guys got 114… I know someone is going to speak up." Ms. B seemed surprised that no one had volunteered, and she communicated confidence that someone eventually would. Ms. B's third and final attempt to solicit a volunteer for problem #114 was a firm open invitation. Her previously supportive tone had shifted and sounded somewhat annoyed. "What the heck? I'm doing all the talking. I'm going to wait." Ms. B's persistence paid off. Emma, a female student identifying as "mixed" race, volunteered to share problem #114. Partway through Emma's explanation, Ms. B got confused and solicited support from the class. She initially asked Mr. K for help, but immediately opened the invitation to the entire class, which created a potential opportunity for a second student to contribute to homework sharing (row 7). Kyle, a White male, volunteered, identified a mistake, and talked through the last few steps of the process. At the end of the homework sharing segment, Ms. B provided one more opportunity for participation through general encouragement when she prompted for questions. She added, "We had to ravel and unravel that a few times, so I imagine there might be some questions." By highlighting the complexity of the path they took to solve the problem and allowing wait time, Ms. B likely made it easier for students to ask questions, but still, no one did.

Classwork Problem #1 & Logarithmic Review

The second part of the lesson was devoted to the first classwork problem (see Figure 2e) and general logarithmic review. Before digging into the first classwork problem, Ms. B called on Andy and Leah to read the first classwork problem and second classwork problem. Ms. B then read the third classwork problem herself and asked the whole class how they would feel about solving it, showing with thumbs up, down, or sideways. Ms. B directed students' attention back to the first classwork problem which was focused on reviewing three laws of logarithms: $\ln(ab) =$, $\ln\left(\frac{a}{b}\right) =$, $\ln(a^b) =$. She gave students time to think and prepare their answers. After a few minutes, Ms. B called on Alison to share her answer, but Alison said she didn't know and then declined to answer the question. Ms. B called on Rebecca next to share her answer because Rebecca had indicated earlier by displaying a yellow thumb symbol that she was ready to discuss this problem. Ms. B followed Rebecca's correct answer with a review of logarithms to make sure everyone understood how Rebecca had gotten her answer. Ms. B had the class work through several base ten logarithm problems that were not originally part of the classwork plan, sometimes calling on specific students to share their ideas (Chris and Alma) and other times asking for all students to respond using their hands (e.g., "Show with your fingers"). After a few minutes of review, Ms. B returned to the original classwork problems and called on Joe and Ellie to share responses for the remaining two parts of problem one. Both students shared correct answers.

Table A2 is an extension of Table A1, providing more detail regarding the second part of the whole-class discussion, focusing on Ms. B's contribution invitations and the resulting student contributions.

Table A2: Contribution Solicitations during Classwork Problem #1 & Logarithmic Review

Row	Time	Summary	Solicitation & Contribution Dialogue (excerpts from the video transcription)	Invitation Type	Contribution Description	Contribution By
10	16:15	Ms. B orients the students to the structure of the lesson and directs students to a shared Google doc that contains the classwork, which is also written on her whiteboard. Ms. B calls on Andy to read the first part of the problem.	**Ms. B:** Let's have someone else read just so that we can all kind of hear this together. Um, Andy, would you mind reading those three? Don't answer them. Just read them.	explicit request (Andy)		
11	16:39	Andy reads the first problem.	**Andy:** Sure. Ummm. Natural log of ab equals, natural log of a over b equals, and then natural log of a to the b power equals.		Individual called on, Read out loud	Andy: male, student of color
12	16:52	Ms. B calls on Leah to read the second problem.	**Ms. B:** Thank you. Someone else read question two. Ummm. Leah, if you can see it? What are we doing in question two?	explicit request (Leah)		
13	16:58	Leah reads the second problem. Ms. B emphasizes the importance of the guesses in the second problem. She then goes on to read problem three by herself.	**Leah:** Next we will find y prime if y equals x to the x. Guess an analytical representation first.		Individual called on, Read out loud	Leah: female, student of color
14	17:32	Ms. B asks students to look at problem three and share how they would feel about trying to solve it. She prompts students to use their thumbs (up, sideways, down) to convey their feelings through zoom. Students show a mix of thumbs. Ms. B shows her thumb pointing down.	**Ms. B:** If you were given this problem to differentiate (points to question three), how would you feel?... How do you feel about this derivative problem?	explicit request (whole class)	Whole class response, Responded to a Question	most students
15	18:29	Ms. B asks students to think about the first problem and then tells them she will pick people to share. She asks students to use the "thumbs up" reaction on Zoom to indicate when they are ready, but eventually goes back to using human thumbs.	**Ms. B:** So, a couple minutes to think about just question one... How about put up that big orange thumb when you're ready to talk about these?... We've got one big orange thumb... Ok, how about human thumbs? I think the yellow thumb is- who is ready with one? Ok, you guys need more time, a little bit more time.	explicit request (whole class)	Whole class response, Responded to a Question	most students
16	20:51	Ms. B calls on Alison to share her answer to the first problem.	**Ms. B:** Ok, let's see how we're doing. You can pass if you're stuck. Is Alison here? I feel like I didn't see her enter. **Alison:** Yeah, yeah, I'm here. **Ms. B:** Oh good, Alison. Sorry. Alison, do you have the natural log of a times b, or a guess?	explicit request (Alison)		
17	21:08	Alison initially declines to answer the question. Ms. B offers a hint, but Alison is still unable to answer the question. Ms. B reassures Alison that it is ok to say 'I don't know.'	**Alison:** Oh, the *natural* (extra emphasis) log, oh wait. I don't think I have a guess. I was doing it different. **Ms. B:** That's fine. You're allowed to- Alison, would it help if I told you that it doesn't matter what base log you're in? The laws are the same regardless of the base? **Alison:** Um, ok. Is it just like product rule? **Ms. B:** We're not differentiating. We're just reviewing laws of logarithms. **Alison:** Oh, yeah, I don't know what- **Ms. B:** That's ok. You can say, 'I don't know.' That's a perfectly good answer.		Individual called on, Declined to Answer	Alison: female, White
18	21:45	Ms. B calls on Rebecca to share an answer for the first problem.	**Ms. B:** Um, I'm going to ask Rebecca because she had the yellow thumb first. **Ms. B:** What have you got, Rebecca?	explicit request (Rebecca)		
19	21:49	Rebecca shares a correct answer to the first part of problem one.	**Rebecca:** For the first one? **Ms. B:** Yes please. Read the whole thing as a sentence. **Rebecca:** Ummm. Ln of the quantity ab equals ln a plus ln b.		Individual called on, Responded to a Question	Rebecca: female, White
20	22:31	Ms. B surveys the class to see who wants an explanation for the answer Rebecca just shared. A few students raise their hands. Ms. B goes on to do another problem with the class using base ten. She asks students to show answers using their fingers.	**Ms. B:** What is the log of 100? Show me with your fingers.	explicit request (whole class)	Whole class response, Responded to a Question	some students
21	22:46	Ms. B calls on Chris to explain his reasoning to the class.	**Ms. B:** Interesting. Chris, can you explain the two? Because a lot of people are not remembering how to do this.	explicit request (Chris)		

22	22:51	Chris shares how he got his answer.	**Chris:** Umm. There's- because it's base ten, it's like ten to the power of two. Er, ten to the power of *what* (extra emphasis) equals 100?		Individual called on, Responded to a Question	Chris: male, White
23	23:10	Ms. B asks the class to respond to another base ten problem by showing the answer with their fingers. She calls on Alma to share her reasoning.	**Ms. B:** I put the ten there, so what's the answer to that? It's worth a quick review. Ok, can someone say- Alma, why?	explicit request (Alma)		
24	23:18	Ms. B writes out an equation on the whiteboard as Alma explains.	**Alma:** Um, because ten to the power of one is equal to ten.		Individual called on, Responded to a Question	Alma: female, student of color
25	23:33	Ms. B continued by presenting another log base ten problem, asking students to show answers with their fingers.	**Ms. B:** Let's do log base ten of 1000. Show me with your fingers.	explicit request (whole class)	Whole class response, Responded to a Question	most students
26	23:55	Ms. B asks students to share answers to another problem.	**Ms. B:** Log base ten of ten times a hundred. Answer? What's the answer? Show with your fingers. That's not a lot of people showing fingers.	explicit request (whole class)	Whole class response, Responded to a Question	only a few students
27	24:09	After seeing that not many students are showing an answer with their fingers, Ms. B reframes the problem and asks the question in a different way.	**Ms. B:** What exponent gets you to the answer of this question? Show with your fingers.	explicit request (whole class)	Whole class response, Responded to a Question	some students
28	24:22	Still not getting the response she wants, Ms. B suggests students think about the strategy Rebecca shared earlier.	**Ms. B:** Can anyone finish the pattern that Rebecca- it was you, right?- reviewed just now. That's the same as saying, what can you do with the ten? What can you do with the hundred?	general encouragement		
29	24:32	Colin volunteers an answer to the teacher's question.	**Colin:** Log ten plus log 100.		Individual volunteered, Responded to a Question	Colin: male, White
30	24:44	Ms. B summarizes what they just did and connects base ten problems to natural log problems. She shifts the focus back to the classwork and calls on Joe to share an answer to the second part of problem one.	**Ms. B:** Natural log a over b- Joe, try it.	explicit request (Joe)		
31	25:09	Joe makes an educated guess about the answer to the problem.	**Joe:** Ah, it's probably natural log of a minus natural log of b.		Individual Called on, Responded to a Question	Joe: male, White
32	25:29	Ms. B moves on to the third part of problem one and calls on Ellie to give it a try.	**Ms. B:** Does anyone remember what happens with that b? Natural log of a to the b power. Does anyone want to try? Ellie, it's yours.	explicit request (Ellie)		
33	25:38	It is unclear if Ms. B cold-calls Ellie or if Ellie volunteers in some way. Regardless, Ellie tries to respond, but her voices breaks up due to a bad connection. She tries again and is able to say her answer. Ms. B summarizes the logarithmic rules.	**Ellie:** It's b ln a.		Individual Called on, Responded to a Question	Ellie: female, White

Unlike the announcements and homework sharing segment, during this part of the lesson Ms. B solicited student contributions almost entirely through explicit requests. The only exception was an open prompt Ms. B made toward the end of their logarithmic review that included a hint to think about the pattern Rebecca had reviewed (row 28), to which Colin, a White male, responded. Some of Ms. B's explicit requests were directed at the class as a whole, and others were directed at specific students. The whole class requests elicited non-verbal participation from numerous students in the class simultaneously. The intent seemed to be for all students to participate, though Ms. B's reactions suggested levels of participation varied depending on the questions she asked. Whole class solicitations prompted students to use their hands to communicate their feelings about a problem (row 14), their readiness to talk about a problem (row 15), and their answers to base ten logarithm problems (rows 20, 25, 26, 27).

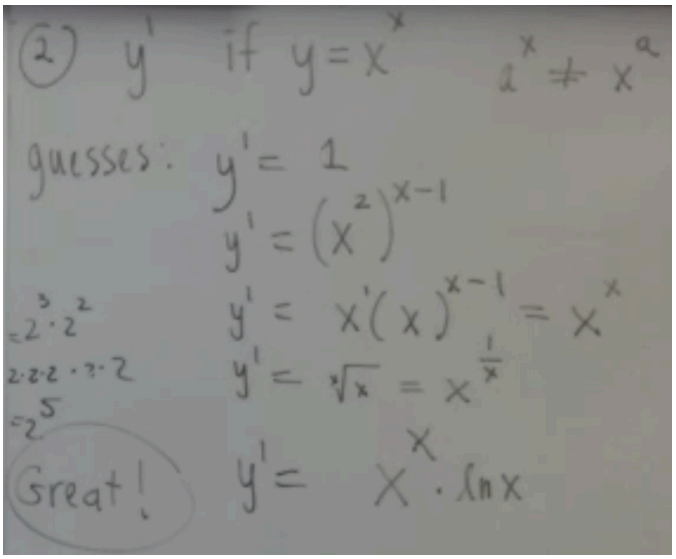
Most of Ms. B's solicitations for student contributions related to the first classwork problem and the logarithm review came in the form of explicit requests to specific students. Ms. B called on eight different students during this part of the lesson. To get the classwork discussion started, Ms. B called on two students to read the problems out loud (rows 10 & 12), which provided opportunities for the voices of Andy, a Vietnamese male student, and Leah, a Filipino female student, to be heard. These students read mathematical statements but did not have opportunities to share their own mathematical ideas. Ms. B first attempted to solicit mathematical ideas about the first problem by calling on Alison, a White female student who had low confidence with respect to her mathematical capabilities in this calculus class (and in previous classes). Ms. B called on Alison to share her answer to the first part of the problem which Ms. B assumed would be review for the students, asking, "Alison, do you have the natural log of a times b, or a guess?" Ms. B broadened the opportunity for Alison to participate by adding "or a guess" at the end, which served to expand the scope of what would count as an appropriate contribution. Even after an additional hint from Ms. B, Alison was unable to give an answer.

All of Ms. B's five remaining explicit student requests during this part of the lesson resulted in mathematically valid contributions from individual students, including two White female students (rows 18 & 32), one mixed race female student (row 23), and two White male students (rows 21 & 30). Some of these requests solicited students to share answers to problems posed by Ms. B and other requests sought explanations for how students got their answers. Since these explicit requests of individual students were interspersed with Ms. B's whole class requests, Ms. B could draw on the information she gained from whole class responses to help her decide who to call on next. Ms. B referenced this process directly when calling on Rebecca, who had indicated she was ready to discuss one of the problems by showing a thumb symbol on Zoom. Ms. B said, "I'm going to ask Rebecca because she had the yellow thumb first." After having one student decline to answer, which Ms. B believed was not ideal ("it doesn't give them any strength at all"), it is reasonable to conclude Ms. B opted for a safer path and called on students she felt would be more comfortable sharing. These students included students who were quick to indicate they had an answer, like Rebecca, or perhaps students who had demonstrated correct answers to the logarithm problems by holding up the right number of fingers.

Classwork Problem #2

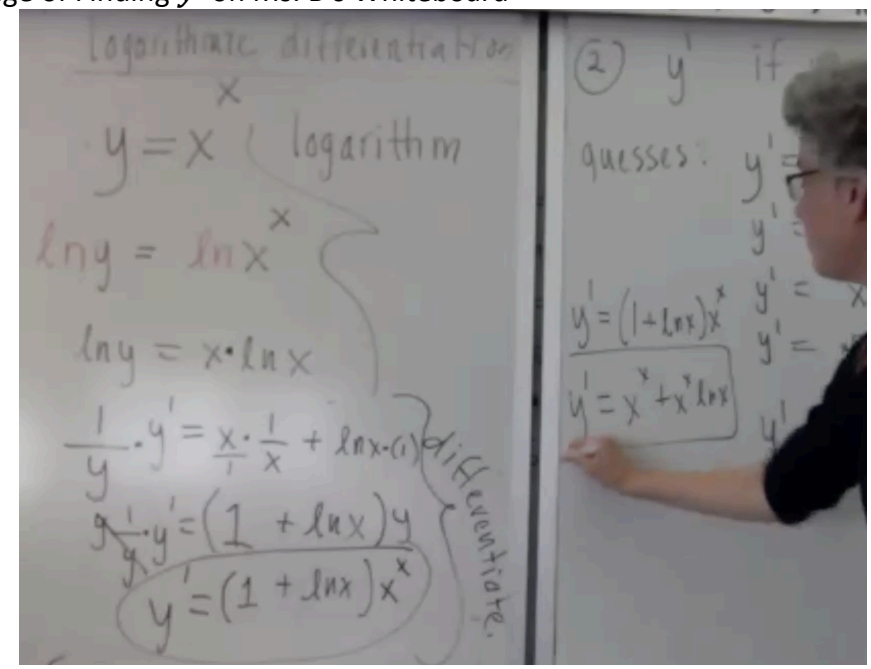
The third part of the lesson was devoted to the second classwork problem (see Figure 2e), which involved soliciting guesses for analytical representations before calculating y'. Ms. B began discussion of the second classwork problem by soliciting guesses for y', given $y = x^x$. Prompted by numerous invitations by Ms. B, students ultimately volunteered five different guesses for y' (Figure 2f). Zoe volunteered her guess first, followed by Kyle, Chris, Max, and Joe. Graham volunteered to simplify Kyle's guess.

Figure 2f. Image of $\boldsymbol{y'}$ Guesses on Ms. B's Whiteboard



After generating the list of guesses recorded on the whiteboard, Ms. B told the class that none of the guesses were correct, but she continued to underscore the importance of going through the guessing process. Ms. B then led the class through the steps required to find y', using a combination of cold-calling students and asking for volunteers. Ms. B called on Sarah, Ethan, Kevin, and Mateo to share their ideas, whereas Zoe, Max and Kyle volunteered on their own to share their ideas and questions along the way. Together, they found that $y' = x^x + x^x lnx$ (Figure 2g).

Figure 2g. Image of Finding y' on Ms. B's Whiteboard



Once they had reached an answer for y', Ms. B invited students to compare the final answer with their original guesses. Colin and Chris responded to Ms. B's open invitation to share their

observations, with Chris pointing out that y' was a combination of the power rule and the chain rule. The discussion concluded with Emma asking a clarifying question before Ms. B sent students off into Zoom breakout rooms to work on the last classwork problem.

Table A3 is an extension of Table A1 and Table A2, providing more detail regarding the third part of the whole-class discussion, focusing on Ms. B's contribution invitations and the resulting student contributions related to classwork problem #2.

Table A3: Contribution Solicitations during Classwork Problem #2

Row	Time	Summary	Solicitation & Contribution Dialogue (excerpts from the video transcription)	Invitation Type	Contribution Description	Contribution By
34	26:22	Ms. B moves to the section of the whiteboard with the original classwork problems. She asks students to guess what the answer is to the second classwork problem.	**Ms. B:** Let's guess an analytical representation. I really want some good guesses. We have x to the x. What's y prime? There are no wrong guesses now. Your guesses are *really* (extra emphasis) important for this one.	general encouragement		
35	26:40	Zoe offers a guess without being called on.	**Zoe:** One.		Individual volunteered, Responded to a Question	Zoe: female, White
36	26:43	Ms. B records Zoe's guess on the whiteboard and prompts students to share more guesses.	**Ms. B:** Thank you. More guesses. I would like at least like four guesses.	open invitation		
37	26:57	Kyle volunteers another guess. Ms. B clarifies the guess as she writes it on the whiteboard.	**Kyle:** x squared to the power of x-1. **Ms. B:** x squared to the power of x-1, like that? **Kyle:** uh-hmm.		Individual volunteered, Responded to a Question	Kyle: male, White
38	27:09	Ms. B encourages students to continue guessing. She suggests they use derivative laws.	**Ms. B:** Ok. Great. Make more guesses. There's some really traditional guesses that I haven't seen yet. These are really interesting guesses that I wasn't expecting. There's some traditional guesses that I want to see. (3 seconds of silence) Use some of the derivative laws that you know. Chris, go ahead.	general encouragement		
39	27:28	Chris volunteers to share another guess, using the power rule.	**Chris:** Like the power rule, x times x to the power of x-1. **Ms. B:** Ok, does everyone see that Chris used the power rule. Ok. Good.		Individual volunteered, Responded to a Question	Chris: male, White
40	27:42	Ms. B tells the students she wants at least two more guesses. There are extended periods of silence while she waits for students to volunteer and share guesses.	**Ms. B:** Continue. These are wonderful. I want more guesses. I want at least two more. (10 seconds of silence) I will wait. (3 seconds of silence) What's another rule that you might use if you see exponents? (6 seconds of silence)	general encouragement		
41	28:07	Max speaks up and shares another guess.	**Max:** Uh, maybe like x under a root sign but then it's x- I don't know what it's called, but- **Ms. B:** So, x root- **Max:** Yeah.		Individual volunteered, Responded to a Question	Max: male, White
42	28:32	Ms. B tells students there is still at least one more guess that she's looking for.	**Ms. B:** Thank you. Great. Ok, I want some more guesses. There's another famous one that's not up there yet. (10 seconds of silence)	general encouragement		
43	28:47	After a period of silence, Ms. B suggests that a student simplify one of the previous guesses written on the whiteboard.	**Ms. B:** Ok, while we're waiting, does anybody want to take Kyle's and simplify it? You have x to the x-1 power and you're multiplying by another x. What do you get? (4 seconds of silence)	open invitation		
44	28:59	Graham speaks up and simplifies the answer.	(Student response was not recorded due to permissions)		Individual volunteered, Responded to a Question	Graham: male, White
45	29:23	Ms. B pushes again for a particular guess she wants on the board. She prompts students by encouraging them to think about derivative laws.	**Ms. B:** There's another guess that I want up there. I will not move on until you make this guess. Think of derivative laws that involved exponents. (2 seconds of silence)	general encouragement		
46	29:35	Joe volunteers another guess. It was the guess Ms. B wanted.	**Joe:** You could try x to the x times the natural log of x. **Ms. B:** That's what I was fishing for. Thank you, Joe.		Individual volunteered, Responded to a Question	Joe: male, White
47	29:58	Ms. B tells students that none of their guesses are correct. She then prompts them to think about why that might be, asking, 'What's the problem?'	**Ms. B:** You'll see none of these are correct. Can someone say why we're truly stuck with this? Why are we truly stuck? Why can we not use this rule? Why can we not use power rule? What's the problem?	open invitation		
48	30:11	Max responds to Ms. B's question by pointing out there are two x's in the expression. Ms. B builds on that comment and explains the process of logarithmic differentiation to the class.	**Max:** There's two x's.		Individual volunteered, Responded to a Question	Max: male, White

49	31:27	Ms. B asks students what they think the next step might be in the solving process.	**Ms. B:** Now, what can we do with that (points to ln y = ln x^x written on the whiteboard) that will make things better?	open invitation		
50	31:32	Speaking at the same time, two students suggest the same next step.	**Max:** You can move the x- **Kyle:** You can do the power rule		Individual volunteered, Responded to a Question	Max, Kyle: male, White
51	32:11	Ms. B continues to walk students through the process of logarithmic differentiation. She tells them they've done the logarithm part and now it's time to do the differentiation part.	**Ms. B:** Just remember we want y prime. I'm ready. I'm ready. Differentiate. (6 seconds of silence)	open invitation		
52	32:33	No one volunteers to differentiate the problem, so Ms. B calls on Sarah to share.	**Ms. B:** Should I call on somebody? You can pass if you feel- Sarah. You wanna help differentiate?	explicit request (Sarah)		
53	32:40	Sarah responds by explaining the next step with differentiation.	(Student response was not recorded due to permissions)		Individual called on, Responded to a Question	Sarah: female, student of color
54	32:56	Ms. B asks for a volunteer to pick up where Sarah left off with differentiating, but no one responds.	**Ms. B:** Someone take over and simplify. (7 seconds silence) Come on. This is a nice step.	open invitation		
55	33:07	Ms. B asks Ethan to suggest a next step.	**Ms. B:** Ok, I'll choose somebody randomly. Ethan, simplify.	explicit request (Ethan)		
56	33:14	Ethan responds with hesitation. At first he says he doesn't know what to do, but after Ms. B's hint, he provides the correct answer.	**Ethan:** I'm not really sure. **Ms. B:** Ethan, I'll give you a hint. **Ethan:** Well, I can see x times one over x is one, but I'm not really sure what to do with the ln of x of one. **Ms. B:** It's ln x times one. Does that make you feel better? **Ethan:** Is it just ln x? **Ms. B:** Great.		Individual called on, Responded to a Question	Ethan: male, White
57	33:51	Before they move on to the final solving step, Zoe speaks up and asks Ms. B to explain one of the previous steps.	**Zoe:** Can you explain how you got one over y, times y prime?	No invitation	Individual volunteered, Asked a Question	Zoe: female, White
58	33:57	Ms. B redirects the question to Sarah who had previously explained that step to the class.	**Ms. B:** Yes. Sarah, that was your step. Do you want to explain it?	explicit request (Sarah)		
59	33:40	Sarah explained the step again and Zoe confirmed her question was answered.	(Student response was not recorded due to permissions)		Individual called on, Responded to a Question	Sarah: female, student of color
60	34:14	Ms. B picks up where they left off and asks Kevin to share the next step.	**Ms. B:** Ok, so what do we do to find y prime? This is a math one step. Kevin, what would you do to find y prime?	explicit request (Kevin)		
61	34:21	Kevin explains what to do next.	(Student response was not recorded due to permissions)		Individual called on, Responded to a Question	Kevin: male, student of color
62	34:26	Ms. B continues to prompt students to finish off the problem.	**Ms. B:** Ok. Y prime equals- anyone see a nice thing to do? Instead of having the y, what do you want to call it? (4 seconds of silence)	open invitation		
63	34:39	When no one volunteers, Ms. B asks Mateo to share.	**Ms. B:** Mateo, go. What would you call y, so we have everything in terms of x?	explicit request (Mateo)		
64	34:42	Mateo finishes off the last step of the problem and Ms. B announces they now have the answer.	(Student response was not recorded due to permissions)		Individual called on, Responded to a Question	Mateo: male, student of color
65	35:09	Ms. B directs students to compare the answer they just got with the guesses on the whiteboard from earlier in class. She asks if they notice anything funny.	**Ms. B:** We get x to the x, plus x to the x, natural log x. Notice anything funny? (4 seconds of silence) Someone say it. (4 seconds of silence)	open invitation		
66	35:30	Colin responds to Ms. B's prompt, but offers an incorrect answer.	**Colin:** y prime equals x. **Ms. B:** y prime equals this whole thing. If you can simplify this more, tell me, because I don't know how to.		Individual volunteered, Responded to a Question	Colin: male, White
67	35:42	Chris makes the connection between their answer and the combination of two of their guesses from earlier. Ms. B goes on to talk about how the patterns are generalizable.	**Chris:** It's like the power rule plus the change rule on the right.		Individual volunteered, Responded to a Question	Chris: male, White

68	36:20	After summarizing what they just did, Ms. B asks if there are any questions.	**Ms. B:** Ok, I'm going to stop and answer questions… Questions on this process?	open invitation		
69	36:55	Emma volunteers a question about one of the steps written on the whiteboard. Ms. B clarifies what she had done.	**Emma:** Wait, how did you get rid of the x to the x that was being added?		Individual volunteered, Asked a Question	Emma: female, student of color
70	37:50	Ms. B prompts for further questions, but no one says anything. She then directs students' attention to the third classwork problem and instructs students to work in groups to solve it. Ms. B assigns students to breakout rooms.	**Ms. B:** Ok, do you guys have any questions? You're very quiet today. Questions on logarithmic differentiation?	open invitation		

This third part of the whole-class discussion included 18 student contributions, twice as many as during the previous part of the lesson. The first half of these contributions were made voluntarily and were solicited by Ms. B through a mix of general encouragement and open invitations. Ms. B spent 3-4 minutes eliciting guesses for what y' might be, using strategies to encourage students to participate and share tentative ideas. She emphasized the importance of students' guesses and showed appreciation for the guesses along the way ("There are no wrong guesses now. Your guesses are *really* (extra emphasis) important for this one." "These are wonderful." "Great."). Ms. B also gave hints along the way to steer students in productive directions ("Use some of the derivative laws that you know." "What's another rule that you might use if you see exponents?" "Think of derivative laws that involved exponents."). The class ended up with five different guesses for y' on the board.

Ms. B then guided the class through the process of logarithmic differentiation so they could find out the actual value for y'. To start the process, Ms. B solicited a few ideas through open invitations (rows 47 & 49). Both solicitations resulted in contributions by White male students. To continue the process, Ms. B called on Sarah, a half Mexican and half Middle Eastern female student who performed well on class assignments but did not typically volunteer in class. Ms. B said, "You can pass if you feel - Sarah. You wanna help differentiate?" This explicit request was not as firm as some of the others Ms. B had made. She softened it by giving Sarah the option to pass and by asking if she wants to help, as opposed to telling her to provide the next step. However, Sarah did not hesitate. She took the opportunity presented to her and explained the next step in the differentiation process. Ms. B shifted back to an open invitation to try to elicit a volunteer for the next step, saying, "Someone take over and simplify (7 seconds of silence). Come on. This is a nice step." But she was unsuccessful. She ended up calling on Ethan, a White male student, to continue (row 55). After receiving a hint from Ms. B, Ethan was able to come up with the next step. At which point, Zoe asked a unprompted question about one of the steps, the only unsolicited student contribution that occurred during this discussion (row 57). Instead of answering the question herself, Ms. B redirected the question to Sarah, opening up another opportunity for Sarah to contribute in mathematically rich ways, both in terms of content and identity development (row 58). Ms. B then called on two additional students to share the final two steps in the process, Kevin, a student described by the teacher as "Asian," and Mateo, a student described by the teacher as "Korean." When calling on Kevin, the teacher mentioned, "This is a math one step," which may have given Kevin a hint as to what to do, but it could have also put a bit more pressure on him to be correct, since this was something he supposedly learned several years ago.

The last few minutes of this discussion were spent having the students compare the final value of y' with the class's original five guesses and then reflect on the overall process of

logarithmic differentiation. Ms. B solicited students' observations through an open invitation, which resulted in thoughts from Colin and Chris, both White male students (row 65). Ms. B's final to solicitations were open invitations intended to prompt questions (rows 68 & 70). Emma, who had volunteered to share the homework solution at the beginning of class, volunteered again. This time she asked to have one of the differentiation steps explained again. Ms. B asked a second time for questions but got no response. From here, the class transitioned into Zoom breakout rooms to work on the third classwork problem.