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Publication Date

1964-01-31

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UNIVERSITY OF CALIFORNIA
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Berkeley, California
AEC Contract No. W-7405-eng-48

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NOTE ON THE Y_{40} DEFORMATION IN HEAVY NUCLEIKichinosuke Harada^{*)}Lawrence Radiation Laboratory
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Many theoretical analyses on the alpha-decay rates of deformed nuclei have shown that heavy nuclei should have Y_{40} deformations, though they are small, in addition to the well-established Y_{20} deformations. The deformation of the nuclear surface is assumed to be represented by

$$R = R_0 (1 + \beta_2 Y_{20} + \beta_4 Y_{40} + \dots) . \quad (1)$$

Fröman¹⁾ and Gol'din and others²⁾ have deduced the effective β_4 values from the experimental alpha decay rates based on the assumption that the alpha amplitude over the nuclear surface is constant, and they found that the β_4 values near Cm are negative. A theoretical calculation for the equilibrium Y_{40} deformation has been done by Kjällquist.³⁾ Treating the Y_{40} term as a small perturbation of Nilsson potential,⁴⁾ she obtained a result in which the β_4 values of nuclei in the actinide region are all positive.

Recently, Mang and Rasmussen⁵⁾ have developed a fundamental approach for the analysis of alpha decay. They have calculated the alpha amplitudes on the nuclear surface by making use of the nucleon configurations in the deformed potential, and obtained a fairly successful result. However, their

result shows that when they do not take into account the Y_{40} deformation the calculated alpha intensities decaying to 4_+ states of daughter nuclei are weaker than the experimental intensities, and when they adopt Kjällquist's values as β_4 the intensities decaying to 2_+ states become stronger than the experiment. If we could take the negative β_4 values suitably, we might be able to get a better agreement between the theory and the experiment.

It seems to us that there are two points to be re-examined in Kjällquist's calculation. The first is the single-particle level order. She used an old Nilsson level order,⁴⁾ which was corrected later by Mottelson and Nilsson.⁶⁾ The second is the use of the asymptotic wave functions in the limit of very strong deformation. Since the Y_{40} deformation is expected to be very small, it is not obvious whether the omission of $T \cdot \bar{S}$ and l^2 terms is allowable or not. In this note we try to check their effects on the equilibrium β_4 values by the simplest perturbation method.

We start from the potential

$$V = \frac{1}{2} m \omega_0^2 \alpha^2 \left[\left(1 - \frac{4}{3} \delta^2 - \frac{16}{27} \delta^3 \right)^{-1/3} \left(1 - \frac{4}{3} \delta \sqrt{\frac{4\pi}{5}} Y_{20} \right) r^2 + \delta_4 \frac{r^4}{R_0^2} Y_{40} \right] \quad (2)$$

If we put $\delta_4 = 0$ and $\alpha^2 = 1$, the above potential is identical with Nilsson's. From Eq. (2) the equipotential surface becomes

$$R(\theta) = \frac{R_0}{\alpha} \left[1 - \frac{4}{45} \delta^2 + \left(\frac{2}{3} \delta + \frac{4}{21} \delta^2 \right) \sqrt{\frac{4\pi}{5}} Y_{20} + \left(\frac{12}{35} \delta^2 \sqrt{\frac{4\pi}{9}} - \frac{1}{2} \delta_4 \right) Y_{40} \right], \quad (3)$$

neglecting higher-order terms in δ and δ_4 . Therefore β_4 is given by

$$\beta_4 = \frac{12}{35} \delta^2 \sqrt{\frac{4\pi}{9}} - \frac{1}{2} \delta_4 ; \quad (4)$$

α^2 is determined by the volume-conservation condition,

$$\int R^3(\theta) d\Omega = \int R_0^3 d\Omega ,$$

and becomes

$$\alpha^2 = 1 - \frac{16}{105} \frac{1}{\sqrt{\pi}} \delta^2 \delta_4 + \frac{1}{8\pi} \delta_4^2 . \quad (5)$$

It is to be noticed that α^2 has a negative linear term in δ_4 . Then the perturbation caused by δ_4 deformation becomes

$$V - V(\delta_4 = 0) = \frac{1}{2} m \omega_0^2 \delta_4 \frac{r^4}{R_0^2} Y_{40} + \left(\frac{1}{8\pi} \delta_4^2 - \frac{16}{105} \frac{1}{\sqrt{\pi}} \delta^2 \delta_4 \right) V(\delta_4 = 0) . \quad (6)$$

In the first-order perturbation theory the total energy of the nucleus is given by

$$E = \sum_i \epsilon_i + \sum_i \langle i | \frac{1}{2} m \omega_0^2 \delta_4 \frac{r^4}{R_0^2} Y_{40} | i \rangle + \left(\frac{1}{8\pi} \delta_4^2 - \frac{16}{105} \frac{1}{\sqrt{\pi}} \delta^2 \delta_4 \right) \sum_i \langle i | V(\delta_4 = 0) | i \rangle , \quad (7)$$

where ϵ_i and $|i\rangle$ are the single-particle eigenenergy and eigenfunction in Nilsson potential, respectively. The second sum is reduced effectively to the summation over the outer nucleons only, but the last one must be summed up over all nucleons.

To simplify the numerical calculations, we assume uniform density distribution for the nucleus. Then the last sum becomes

$$\begin{aligned} \frac{1}{2} C &\equiv \sum_i \langle i | V(\delta_4 = 0) | i \rangle \\ &\approx \sum_i \langle i | \frac{1}{2} m_{\omega_0}^2 (1 - \frac{4}{3} \delta \sqrt{\frac{4\pi}{5}} Y_{20}) r^2 | i \rangle \\ &= \frac{1}{2} m_{\omega_0}^2 \left(\frac{3}{5} - \frac{8}{15} \delta^2 \right) A R_0^2 > 0 . \end{aligned} \quad (8)$$

The equilibrium δ_4 value is determined by the condition

$$\frac{\partial E}{\partial \delta_4} = 0 , \quad (9)$$

and the result is

$$\delta_4 = \frac{64}{105} \sqrt{\pi} \delta^2 - \frac{8\pi}{C} \sum_i \langle i | \frac{1}{2} m_{\omega_0}^2 \frac{r^4}{R_0^2} Y_{40} | i \rangle . \quad (10)$$

The final expression for β_4 thus becomes

$$\beta_4 = - \frac{8}{105} \sqrt{\pi} \delta^2 + \frac{4\pi}{C} \sum_i \langle i | \frac{1}{2} m_{\omega_0}^2 \frac{r^4}{R_0^2} Y_{40} | i \rangle . \quad (11)$$

Since the first term is very small (≈ -0.007), the sign of β_4 is determined by the sign of $\sum_i \langle i | r^4 Y_{40} | i \rangle$.

For the purpose of illustration, we give two kinds of $\langle i | r^4 Y_{40} | i \rangle$ values calculated by the exact and by the asymptotic wave functions in Table 1. Differences in $\frac{1}{2} + [660]$, $\frac{3}{2} + [651]$, $\frac{1}{2} - [770]$, and $\frac{3}{2} - [761]$ are remarkable. Nevertheless, we can see that $\sum_i \langle i | r^4 Y_{40} | i \rangle$ is positive even for the heaviest nucleus. Then we may conclude that the signs of β_4 are still all positive in the actinide region, although we cannot call into question their true magnitude from this simple calculation.

If we neglect the δ dependence of $\sum_i \langle i | r^4 Y_{40} | i \rangle$ we can estimate β_4 values simply from Eq. (11). The result is shown in Fig. 1. In the course of the calculations we used the relations

$$\hbar\omega_0 = 41A^{-1/3} \text{ MeV}, \quad R_0 = 1.2A^{1/3} \text{ fermi},$$

and δ values in reference 7. The dotted curves are Kjällquist's. We should find satisfaction in that almost the same results were obtained from the two different methods.

I wish to thank Professor John O. Rasmussen for his kind interest in this work, for his stimulating discussions, and for the privilege of working at his laboratory.

Table 1. The expectation values of $r^4 Y_{40}$ calculated by exact and asymptotic wave functions [given in units of $(\frac{\hbar}{m\omega_0})^2$].

Proton			Neutron		
configuration	exact ($\eta=4$)	asymptotic ($\epsilon=0.207$)	configuration	exact ($\eta=4$)	asymptotic ($\epsilon=0.207$)
$\frac{1}{2}^-$ - [541]	10.39	11.95	$\frac{1}{2}^+$ + [651]	21.69	23.58
$\frac{3}{2}^-$ - [532]	- 1.186	- 4.264	$\frac{1}{2}^-$ - [770]	20.23	76.79
$\frac{11}{2}^-$ - [505]	6.350	4.273	$\frac{1}{2}^+$ + [640]	7.842	1.722
$\frac{1}{2}^+$ + [660]	14.07	55.53	$\frac{3}{2}^+$ + [642]	5.064	1.164
$\frac{3}{2}^+$ + [651]	4.862	23.58	$\frac{3}{2}^-$ - [761]	12.25	38.65
$\frac{1}{2}^-$ - [530]	5.423	- 3.706	$\frac{5}{2}^-$ - [752]	2.324	10.03
$\frac{5}{2}^-$ - [523]	-12.40	-10.95	$\frac{3}{2}^+$ + [631]	- 8.181	-10.61
$\frac{5}{2}^+$ + [642]	- 1.507	1.164	$\frac{13}{2}^+$ + [606]	9.521	6.795
$\frac{3}{2}^-$ - [521]	- 6.532	- 9.832	$\frac{5}{2}^+$ + [633]	- 7.272	-11.72
			$\frac{7}{2}^-$ - [743]	- 5.660	- 9.063
			$\frac{5}{2}^+$ + [622]	-12.31	-13.41
			$\frac{1}{2}^+$ + [631]	- 1.461	-10.61
			$\frac{7}{2}^+$ + [624]	-11.89	-15.08
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Total	19.47	67.75	Total	32.15	88.24

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MU-33233

Fig. 1. The estimated values of β_4 for the different isotopes are plotted. The dotted curves are the ones calculated by Kjällquist.

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