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Asteroid Discovery and Characterization at Long Wavelengths

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Planetary Science

by

Emerson Afton Whittaker

2026

ABSTRACT OF THE DISSERTATION

Asteroid Discovery and Characterization at Long Wavelengths

by

Emerson Afton Whittaker

Doctor of Philosophy in Planetary Science

University of California, Los Angeles, 2026

Professor Jean-Luc Margot, Chair

The discovery and characterization of asteroids is necessary for understanding their demographics and their evolution within the Solar System. Diameter is an important asteroid property for assessing impact probabilities and uncovering clues about solar system origins. I assess how the accuracy of diameter estimates of asteroids based on infrared thermal modeling of imaging from the four-band Wide Field Infrared Survey Explorer (WISE) is dependent on the number of infrared bands used, as the NEO Surveyor mission will have only two infrared bands corresponding roughly to WISE bands W2 and W3. With a reparameterization of the near-Earth asteroid thermal model (NEATM), we found diameters modeled using observations from W2 and W3 bands to be systematically 10% smaller than their more accurate counterparts obtained with all four WISE bands. Obtaining high-quality orbits for faint main belt asteroids is foundational for many studies, particularly for projects related to solar system origins and asteroid families. I utilize the WISE mission dataset to detect and report new asteroid candidates using synthetic tracking, an algorithm which shifts and stacks pixels along a predicted on-sky motion template for an asteroid. We report astrometry to the Minor Planet Center for 8,223 discovery candidates and 13,575 known objects. We anticipate hundreds of discovery candidates will be linkable to asteroids discovered by the Vera Rubin Telescope, giving them excellent orbits arising from a > 15 -year orbital arc. Assessing the lifetime of a typical near-Earth asteroid binary system as its orbit evolves is useful for understanding

impact hazards. I use planetary radar delay-Doppler imaging taken of the 1999 KW4 near-Earth asteroid binary system with the Arecibo observatory and the Goldstone radar system between 2001 and 2019 to confirm the presence of libration for the secondary and measure the system's orbital expansion rate. I find a best fitting outward orbital expansion rate for the semimajor axis of $da/dt < 0.58 \text{ cm/yr}$, suggesting that the BYORP effect could play a role in evolving this system in combination with tidal evolution.

The dissertation of Emerson Afton Whittaker is approved.

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2026

TABLE OF CONTENTS

1	Introduction	1
2	Thermal models of asteroids with two-band combinations of WISE cryogenic data	3
2.1	Introduction	3
2.2	Methods	5
2.2.1	Data Selection	5
2.2.2	NEATM Reparameterization	6
2.2.3	Minimization Method	9
2.2.4	Bootstrap Trials for Uncertainty Estimation	10
2.2.5	Treatment of Low Pseudo-Temperature Solutions	11
2.2.6	Albedo Calculation	12
2.3	Results	14
2.3.1	W2–3 Results	14
2.3.2	W3–4 Results	16
2.4	Discussion	23
2.5	Conclusions	24
3	Discovery of Hundreds of Main-Belt Asteroids with Synthetic Tracking of WISE Data	26
3.1	Introduction	26
3.2	Methods	27
3.2.1	WISE Data	27

3.2.2	Object Detection with Synthetic Tracking	28
3.2.3	Image Collection, Elimination, and Registration	30
3.2.4	SALTAD Algorithm	32
3.2.5	SALTAD Settings	35
3.2.6	Evaluation of False Candidate Fraction	36
3.2.7	Evaluation of Completeness or Detectable Fraction of Real Asteroids	37
3.2.8	Additional Filtering of Hits	38
3.2.9	Merging of True Hits	43
3.2.10	Suspected Comet Candidates	44
3.2.11	Visual Inspection of Discovery Candidates	44
3.2.12	Submission of Astrometry and V Magnitude Estimate to MPC . . .	45
3.2.13	Orbit Determination	46
3.3	Results	47
3.3.1	False Candidate Fraction Results	47
3.3.2	Completeness Results	49
3.3.3	Sky Coverage	49
3.3.4	Filtering of Hits	51
3.3.5	Distribution of Apparent Velocities	52
3.3.6	Discovery Statistics	53
3.3.7	Astrometric Quality	54
3.3.8	MLEs of Submitted Candidates	57
3.3.9	Known Objects Detected	57
3.3.10	Candidates Near the S/N Cutoff	60
3.3.11	High S/N Discovery Candidate	62

3.3.12	Submission to MPC	62
3.3.13	Precoveries for Known Objects	65
3.3.14	Arc Lengths of Discovery Candidates	65
3.3.15	Comets and Suspected Cometary Candidates	65
3.4	Discussion	67
3.5	Conclusions	68
4	Measurement of the orbital expansion rate in the binary asteroid 1999	
	KW4 Moshup-Squannit	70
4.1	Introduction	70
4.2	The 1999 KW4 System	71
4.2.1	Shape and Libration Modeling	71
4.2.2	Mutual Orbit Modeling	74
4.2.3	Importance of Shape and Orbit Determinations	76
4.3	Methods	78
4.3.1	Libration Parameters and Squannit’s Shape	78
4.3.2	Rate of Orbital Expansion or Contraction	81
4.4	Results	87
4.4.1	Libration Parameters and Squannit’s Shape	87
4.4.2	Rate of Orbital Expansion or Contraction	92
4.4.3	Upper Bound on BYORP Coefficient	95
4.4.4	Tidal Expansion Rate	96
4.4.5	Lifetime of Moshup-Squannit	97
4.4.6	Moshup and Squannit’s Densities	98
4.5	Discussion	98

4.6	Conclusions	100
5	Conclusion	102
A	Chapter 1 Appendix	104
A.1	Emissivity in Two-band Fits	104
A.2	Goodness of Fit Metric and Light Curve Variations	105
A.3	Anomalous Low-Temperature Solutions and Distance to the Sun	108
A.4	Robustness with Respect to Minimization Technique	109
A.5	Systematic Error Introduced from Constant Emissivity	110
B	Chapter 2 Appendix	111
B.1	Detections Table	111
B.1.1	Detection Codes	113
B.2	Estimating V magnitude from MLE dB	113
C	Chapter 3 Appendix	117
C.1	Shape Model Figures	117
	References	123

LIST OF FIGURES

- 2.1 Best-fit pseudo-temperature and diameter for 200 bootstrap trials for example asteroid 10006. The median is shown as a red dot, with corresponding parameter values displayed in the legend. Standard deviations in diameter and pseudo-temperature amongst the bootstrap trials are displayed below the legend. **Left:** Results from two-band modeling. **Right:** Results from four-band modeling, where we replicate the results from [Myhrvold et al. \(2022\)](#) with the same set of observational clusters but a different set of random bootstrap trials. 10
- 2.2 Observed dichotomy in pseudo-temperature solutions for two-band fits illustrated with results from 200 bootstrap trials for asteroid 10125. **Top:** Planck spectral distributions of emitted light (dotted lines) and reflected light (dashed lines) calculated with the median pseudo-temperature T_1 and median diameter D obtained from the bootstrap trials. The orange curves correspond to the median of 80 trials with high pseudo-temperatures and the purple curves correspond to the median of 120 trials with low pseudo-temperatures. Note how two dissimilar pseudo-temperatures yield a similar total flux (solid lines) within the W2 band, while also yielding similar W3 fluxes. Colored symbols indicate the total (emitted + reflected) spectral flux integrated over the W2 bandpass for each of the 200 trials. The W2 bandpass is shown in a light blue solid line as the spectral response per photon ([Wright et al., 2010](#)). The grey circle represents the true median measured W2 flux of the object adjusted to a phase angle of zero, with the error bar given by the standard deviation of the phase-adjusted W2 fluxes. **Lower Left:** Histogram of emitted to reflected light (integrated over the W2 bandpass) for 200 bootstrapped trials. High pseudo-temperature solutions are shown in orange whereas low pseudo-temperature solutions are shown in purple. **Lower Right:** Reduced chi-squared vs. pseudo-temperature T_1 , showing similar values in the two temperature regimes. 13

2.3	Diameter vs. pseudo-temperature T_1 for 4542 cases calculated from a two-band (W2–3) least-squares solution and four-band (W1–4) regularized solution to a reparameterized NEATM model. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. The W2–3 data shown in this figure is available as the data behind the figure. The data also includes additional information that will allow the reader to recreate Figures 4–6. . . .	15
2.4	Histograms of the relative difference and absolute value of the relative difference in D and T_1 parameter estimates. Median values are represented with a vertical black line while a pink highlighted block is used to represent the 5 – 95% quantile range. Top Left: Relative difference in diameter. Top Right: Relative difference in pseudo-temperature T_1 . Bottom Left: Absolute value of relative difference in diameter. Bottom Right: Absolute value of relative difference in pseudo-temperature T_1	16
2.5	Histograms of D and T_1 parameter relative uncertainty, i.e., the standard deviation in accepted bootstrap trials for an object divided by the median value of the bootstrap trials. The median of the relative uncertainties for all objects is shown as a vertical line. Two-band results are shown in red whereas four-band results are shown in blue. Left: Relative uncertainty in diameter. Right: Relative uncertainty in pseudo-temperature T_1	17
2.6	Histogram of visible-band geometric albedos (p_v) as calculated using the median modeled diameter for each cluster and the absolute magnitude for the object, H . The median albedo for the two-band fits is shown as a dark red line, while the median albedo of the four-band fits is shown as a dark blue line.	17

2.7	Plot of occultation diameters versus our corresponding two-band (W2–3) modeled diameters. The orange line represents a perfect one-to-one ratio between diameters, whereas the black dotted lines represent a 10% error in the positive and negative directions.	18
2.8	Left: Plot of NEOWISE four-band diameters versus our corresponding two-band (W2–3) modeled diameters. The orange line represents a perfect one-to-one ratio between diameters. The color bar indicates our pseudo-temperature modeled with W2 and W3. Right: Plot of NEOWISE two-band (W2–3) diameters versus our corresponding two-band (W2–3) modeled diameters. . . .	18
2.9	Diameter vs. pseudo-temperature T_1 for 4685 cases calculated from a two-band (W3–4) and four-band (W1–4) reparameterized NEATM model. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. The W3–4 data shown in this figure is available as the data behind the figure. The data also includes additional information that will allow the reader to recreate Figures 10–12.	19
2.10	Histograms of the relative difference and absolute value of the relative difference. Median values are represented with a vertical black line while a pink highlighted block is used to represent the 5 - 95% quantile range. Top Left: Relative difference in diameter. Top Right: Relative difference in pseudo-temperature. Bottom Left: Absolute value of relative difference in diameter. Bottom Right: Absolute value of relative difference in pseudo-temperature.	20

2.11	Histograms of relative uncertainty, where relative uncertainty represents the standard deviation in accepted bootstrap trials for an object divided by the median value of the bootstrap trials. The median of the relative uncertainty for all objects is shown as a vertical line. Two-band results are shown in green while four-band results are shown in blue. Left: Relative uncertainty in diameter. Right: Relative uncertainty in pseudo-temperature.	21
2.12	Histogram of visible-band geometric albedos (p_v) as calculated using the median modeled diameter for each cluster and the absolute magnitude for the object, H . The median albedo for the two-band fits is shown as a dark green line, while the median albedo of the four-band fits is shown as a dark blue line. . .	21
2.13	Plot of occultation diameters versus our corresponding diameters modeled with W3 and W4. Red points represent objects with a higher quality occultation diameter with a quality code of 3, while green objects have a slightly lower quality occultation diameter with a quality code of 2. The orange line represents a perfect one-to-one ratio between diameters, while black dotted lines represent a 10% error in the positive and negative directions.	22
2.14	Left: Plot of NEOWISE four-band diameters versus our corresponding diameters modeled with W3 and W4. The orange line represents a perfect one-to-one ratio between diameters. The color bar indicates our pseudo-temperature modeled with W3 and W4. Right: Plot of NEOWISE two-band (W3-4) diameters versus our corresponding diameters modeled with W3 and W4.	22
3.1	Histogram of the on-sky angular speeds for all numbered asteroids with perihelion distances $q > 1.3$ au observed during the WISE cryogenic mission.	29

3.2	Plot of the sky coverage of our collection of subfields searched by SALTAD. The x-axis is ecliptic longitude in degrees whereas the y-axis is ecliptic latitude in degrees. The outline of each subfield is shown, where the color of the outline corresponds to the number of frames stacked to create the subfield. Regions within 10 degrees of the ecliptic poles were excluded due to absence of main belt objects expected in this region. Other gaps in coverage represent regions of the sky where there was an insufficient number of good overlapping frames to create a subfield or smaller gaps between adjacent subfields. Large gaps surround the galactic plane (solid black line) and regions of the sky in the WISE field of view during reaction wheel desaturation.	31
3.3	On-sky velocities corresponding to SALTAD hits, with ecliptic latitude and longitude rates of motion shown in arcseconds per hour. The light blue dots show all main belt asteroids observed during the cryogenic portion of the WISE survey, with the purple shape outlining the 2σ contour for the on-sky velocity distribution of these main belt asteroids. Orange stars represent objects detected by SALTAD and known to Skybot. Green triangles represent SALTAD hits matching known objects with astrometry reported by the NEOWISE team, whereas pink triangles represent SALTAD hits matching objects known to Skybot with no corresponding WISE astrometry. Most objects observed by the NEOWISE team are in Skybot's catalog. Black triangles represent hits of previously unknown objects that survived all the pipeline filters while black dots represent SALTAD hits that survived all the pipeline filters with the exception of the 2σ velocity cut or were eliminated during the merging process or later. The majority of SALTAD hits fall within the 2σ contour. Those that do not are eliminated by our pipeline filters, including some potentially false negatives, as around 5% of main-belt objects are expected to fall outside of the contour. For details about figure construction, see 3.3.5.	42

3.4	Diagram of the asteroid shift-and-stack detection pipeline. The detection culling pipeline has been simplified here for easier visualization. For a detailed breakdown of all steps in the detection culling pipeline, see Section 3.2.8.	46
3.5	Left: Histogram of the number of known objects expected within all subfields combined, with bins of 0.5 in V magnitude, as reported by the Skybot system. Objects expected in multiple subfields and/or detected by SALTAD in multiple subfields are included more than once. Four instances of an object in a subfield with Skybot estimated V magnitude = 0, representing 3 unique comets, are assumed to be inaccurate values and are excluded. Skybot-reported objects are shown in pink, whereas the fraction of those detected by SALTAD at any MLE level are shown in green. As expected, SALTAD appears to preferentially detect brighter objects. Right: Known object completeness as a function of MLE, i.e., percentage of known objects detected that have an MLE (i.e., S/N) above the MLEs shown on the x-axis. Our chosen cutoff at 8.3 dB yields an 83% completeness.	50
3.6	Ecliptic coordinates for all detections submitted to the MPC, excluding identified false positives. Blue dots represent detections that are part of a confirmed candidate and orange dots represent detections that are part of a discovery candidate.	55
3.7	Astrometric residuals for all detections of known objects with listed orbital elements. Ephemeris values are found using the JPL HORIZONS service. Left: Residuals for all detections reported to the MPC in this work as well as residuals for NEOWISE team detections submitted to the MPC. Axes are clipped to RA and Dec residual values between 10^{-2} and 10 arcseconds for clarity. Right: Histogram of the residual values for all detections reported to the MPC in this work and for all NEOWISE team detections submitted to the MPC. Residuals below 10^{-4} are clipped for clarity.	56

3.8	Histogram of MLE (dB) for all candidates submitted to the MPC, excluding identified false positives,	58
3.9	Orbital element plots for all confirmed candidates for which we reported detections to the MPC and for which orbital elements were available in MPCORB.DAT. Orbital elements for comets are not shown as they are not listed in MPCORB.DAT. Most objects are main-belt asteroids, but we also report several Jupiter Trojans and NEAs. Left: Semi-major axis vs. eccentricity. Right: Semi-major axis vs. inclination.	58
3.10	Example detections for known MBAs that have at least 3 previously unreported observations. Images show subframes of registered WISE images after SALTAD processing. Black and white images show the object in individual frames whereas the image in color shows the shifted and stacked image of the object, where the color conveys the number of standard deviations above the noise. We stack the images by summing the signal in each frame and dividing by the sum of squares of the noise for each frame. Red dots are located at the SALTAD detection centroid, an orange “+” appears at the location of the HORIZONS ephemeris, and a blue “x” at any locations previously reported by the NEOWISE team. The annotations at the bottom of each frame report the S/N of the central pixel only, which is not equivalent to the MLE dB of the true hit, which is listed at the top.	61
3.11	Images of two candidates with MLE = 8.3 dB at the beginning, middle, and end of SALTAD processing. The top three images show the detections of asteroid 198452. The top row shows the raw frames as read by SALTAD with any non-data values replaced by the mean value of the subframe. The middle row shows the same images after the image equalization step. The bottom image shows the same frames after convolution with the WISE PSF, as the frames are when they are shifted and stacked by SALTAD. The bottom three rows show a discovery candidate. Top, middle, and bottom rows as before.	63

3.12	Images of discovery candidate with high S/N at the beginning, middle, and end of SALTAD processing. The top row shows the raw frames as read by SALTAD with any non-data values replaced by the mean value of the subframe. The middle row shows the same images after the image equalization step. The bottom image shows the same frames after convolution with the PSF, as the frames are when they are shifted and stacked by SALTAD.	64
3.13	Number of good discovery candidates vs. arc length in days of our submitted detections. The average arc length is 17 hours. Note that the y-axis is scaled logarithmically.	66
4.1	Principal-axis views of Moshup (top) and Squannit (bottom) shape models (reproduction of Figure S4 from Ostro, S. J., Margot J. L., Benner, L. A. M., et al. 2006, <i>Science</i> 314 (5803), 1276-1280. Reprinted with permission from AAAS). Yellow regions highlight regions of less reliable shape reconstruction as they were not viewed by Arecibo at incidence angles below 70°.	73
4.2	Example delay-Doppler images taken by the Arecibo observatory. The x-axis shows Doppler frequency increasing to the right while the y-axis shows range from the observer increasing downward. Delay resolution for these images is 15 m and Doppler resolution is 0.12 Hz. Left: Sum of the 21 radar images (63 looks) taken by Arecibo on May 26th, 2001 from 6:06:03 - 06:31:22 AM UTC. Each image is 3 looks. The secondary can be seen moving in its orbit around the primary. Right: Cutout of the secondary from the radar image taken at 06:07:11 (3 looks).	79

4.3	Reduced chi squared as a function of tested libration angles and libration amplitudes of the secondary. The top plot shows the reduced chi squared values for the ellipsoidal models of the secondary while the bottom plot shows the reduced chi squared values for the spherical harmonic models of the secondary. The black contour delineates the region within one standard deviation of the best fit. Each model has 612,822 degrees of freedom.	89
4.4	Shape model for Squannit with libration angle = 4° and libration phase = 230° . Yellow regions were not viewed by Arecibo at incidence angles below 70° , indicating poorer coverage.	90
4.5	Reduced chi squared for the libration and no libration models as a function of the rotation period of the secondary, with the libration model shown plotted with blue dots and the no libration model plotted with orange triangles. The ellipsoid models are shown on the top and the spherical harmonic models are shown on the bottom. The best fitting spherical harmonic model with libration is located at a rotation period of 17.36 hrs, with an uncertainty in the best fitting period of 0.2 hrs.	91
4.6	Reduced chi squared vs. tangential acceleration for a mutal orbit model that includes Moshup's oblateness ($J_2 = 0.05569$) and our best estimate of the total system mass based on fits to the 2001 range-Doppler separations ($M = 243.19 \times 10^{10}$ kg) held constant. The sum of squares of residuals is minimized at $T = 9.12 \times 10^{-15}$ m/s ² , with a reduced chi squared value of 0.799.	95
4.7	Reduced chi squared for various combinations of J_2 , system mass, and tangential acceleration. The reduced chi squared is minimized at $J_2 = 0.047541$, $M = 244.948358 \times 10^{10}$ kg, $T = 1.99 \times 10^{-16}$ m/s ² , with a value of $\chi/\text{DOF} = 0.973$ and $\text{DOF} = 260$. Size of the points is proportional to the inverse of the reduced chi squared. Color of the points indicates the tangential acceleration, where red are positive accelerations leading to orbital expansion, and blue are negative accelerations leading to orbital contraction.	96

A.1	Left: Histogram of emissivity values obtained with the floating-emissivity model and W2–3. Right: Diameter vs. pseudo-temperature two-band estimates for the constant-emissivity model and the floating-emissivity model.	105
A.2	Reduced chi-squared vs. pseudo-temperature. Pseudo-temperatures and reduced chi-squared values are both represented by the median value of 200 bootstrapped trials. Two-band values are shown in red whereas four-band values are shown in blue.	106
A.3	Logarithmic plot of lightcurve peak-to-peak amplitude vs. reduced chi-squared for all remaining clusters with a corresponding lightcurve fit. W2–3 clusters are shown in red whereas W1–4 clusters are shown in blue. Lightcurve amplitudes were obtained from Lam et al. (2022)	107
A.4	W1–4 flux values vs. asteroid-Sun distance for asteroid 1182. Black points and error bars indicate the WISE fluxes and uncertainties, whereas blue lines indicate the best four-band fit to the data. Error bars incorporate the correction factors.	107
A.5	Histograms of distance from the Sun (Left) and phase angle (Right) at the time of observations. The blue bars represent all observational clusters, while the orange bars represent clusters that had a low median pseudo-temperature ($T_1 < 273$ K) from the two-band bootstrapped fits, before implementation of our low-temperature solution removal process.	108

A.6	Comparison of results from two-band W2–3 fits to results from four-band fits with ϵ fixed to 0.9. Left: Diameter vs. pseudo-temperature T_1 for 4533 clusters. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. Middle: Histogram of the relative difference in diameter. Median values are represented with a vertical black line while a pink highlighted block is used to represent the 5 - 95% quantile range. Right: Histogram of relative difference in pseudo-temperature.	109
A.7	Comparison of results from Myhrvold et al. (2022) four-band fits to results from four-band fits with ϵ fixed to 0.9. Left: Diameter vs. pseudo-temperature T_1 for 4672 clusters. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. Middle: Histogram of the relative differences in diameter. Median values are represented with a vertical black line and the 5 - 95% quantile range is represented with a region highlighted in pink. Right: Histogram of relative differences in pseudo-temperature.	110
B.1	Description of each detection code listed in our detections SQL table. Only detections with codes 0, 1 or 4 are submitted to the MPC.	115
B.2	Skybot-estimated V band magnitude vs. the MLE divided by the number of frames in a subfield. Blue points indicate hits matching with objects known to Skybot while the black line indicates the line of best fit given in equation B.1. We attempt to use the line of best fit to estimate a conversion rate from MLE to V magnitude for our discovery candidates. $\sqrt{MLE}$ is divided by the number of frames since $\sqrt{MLE}$ is given by the sum of the S/N of each frame, so $\frac{\sqrt{MLE}}{N}$ is equivalent to the average S/N for one frame of the hit.	116

C.1	Observed echo (left), modeled echo (center), and on-sky rendering (right) of Squannit at each observation epoch. The modeled echo and on-sky renderings use the best-fit shape model, which has a libration amplitude = 4° , libration phase = 75.1° at time of pericenter passage on MJD = 52055.4132, and rotation period = 17.36 hr. The spin vector is shown in magenta and the long and intermediate principal axes are shown in red and green respectively. The center of mass is marked with a '+' and the subradar point is marked with an 'X'. Figure is continued on subsequent pages.	118
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LIST OF TABLES

3.1	Detection Nomenclature	30
3.2	False Positive Rates	48
3.3	Selection of Known Objects with ≥ 3 New Observations	60
4.1	Squannit Shape Model Results	92
4.2	Secondary - Primary Separations	93
B.1	Sample of SALTAD Detections Table	114

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CHAPTER 1

Introduction

Asteroids, primarily located at present in the asteroid belt between Mars and Jupiter, are small rocky bodies whose history is closely intertwined with the history of the Solar System's formation. These bodies can exist as lone objects or as binary or multiple systems, where two or more asteroids may exist in orbit around each other. Many asteroids are made of primordial material that has undergone little heating and pressure since the beginning of the solar system (Urey and Craig, 1953; Lauretta et al., 2024). The distribution of the asteroids has been dramatically shaped by the presence of resonances with the giant planets, and likely contains clues about the movements of these giant planets during or following Solar System formation (Walsh et al., 2011). However, subsequent non-gravitational effects as well tidal evolution can act to obscure information about solar system formation if they are not properly accounted for. By discovering asteroids, characterizing asteroid sizes, albedos, and orbits, and by understanding all of the forces that act on these bodies, we gain a clearer picture of the present which can be used to better disentangle what happened in the past during Solar System formation.

In addition to acting as time capsules for the early Solar System, we also wish to understand asteroids because they pose an imminent risk to the Earth via the impacts of near-Earth asteroids (NEAs). The near-Earth asteroids are a dynamic and ever-changing group of bodies with an average lifetime of about 10 million years once they reach near-Earth space before experiencing collision with one of the terrestrial planets, disintegration by close approach to the Sun, or ejection from the Solar System (Gladman et al., 2000). Size is the most fundamental predictor of expected damage when considering a potential

asteroid impact, and is thus the most critical value to constrain. Additionally, a significant fraction of near-Earth asteroids are binary systems, with an estimate of 15% of Earth crossing asteroids based on the prevalence of doublet craters on Earth and Venus paired with numerical modeling of tidal forces (Bottke and Melosh, 1996; Richardson et al., 1998).

Important to Solar System studies and impact hazard assessment alike is the characterization of asteroid sizes and albedos. While asteroids vary greatly in visible albedo, making some more discoverable in visible light than others, all asteroids share a similar emissivity in the infrared. By observing asteroids in the infrared, surveys like the Wide Field Infrared Survey Explorer (WISE) eliminate the demographic bias towards high albedo asteroids such that lower albedo asteroids can be discovered and their sizes and albedos can be estimated. In my first project, I determine how the number of WISE infrared bands an object is observed in effects the random and systematic error in the estimate of its diameter.

Again fundamental to Solar System formation studies and planetary defense efforts is the discovery of new asteroids and a precise characterization of their orbits. In my second project, I explore using synthetic tracking, a relatively new method of asteroid discovery, on the WISE dataset in order to obtain measurements of potentially new objects using data from the first portion of the WISE mission.

Understanding the lifetime of near-Earth binary systems is necessary to properly accounting for their impact threat. Shape modeling and mutual orbit modeling of an individual asteroid system can be used to untangle its history, allowing the system to act as a case study for how binary asteroids evolve under non-gravitational effects and tidal evolution. In my third project, I use radar ranging data to create an improved shape model for an asteroid secondary that includes libration and to measure the mutual orbit expansion rate of the asteroid system. By measuring the orbital expansion rate, I provide a constraint for the strength of the non-gravitational forcing this binary system experiences and provide a rough estimate of its lifetime.

CHAPTER 2

Thermal models of asteroids with two-band combinations of WISE cryogenic data

2.1 Introduction

The Wide Field Infrared Survey Explorer (WISE) is a space-based telescope that conducted a full-sky survey in the infrared in four wavelength bands centered at 3.4, 4.6, 12, and 22 μm (called W1, W2, W3, and W4, respectively). This survey yielded observations of over 100,000 asteroids and the opportunity to provide estimates of size, temperature, albedo, and sometimes even rotation period for many asteroids (Wright et al., 2010).

The massive task of analyzing these asteroid observations was pioneered by the NEOWISE team, who used the Near-Earth Asteroid Thermal Model (NEATM) (Harris (1998), but see also Myhrvold (2018b)) to fit for a variety of parameters for each asteroid such as diameter, temperature, and albedo depending on the number, quality, and band distribution of the flux measurements available for each object. A curated subset of these observations were reanalyzed by Myhrvold et al. (2022), who used a reparameterization of the NEATM to eliminate unnecessary degrees of freedom within the model, avoid the assumption of constant emissivities, improve the goodness-of-fit to observables, and obtain more accurate parameter estimates.

Of the asteroid observations analyzed by the NEOWISE team, asteroids with observations in only W3 and W4 outnumber asteroids with observations in all four bands by 19:1 (Myhrvold, 2018a). Additionally, of all possible two-band combinations, W3 and W4 are expected to give the best results because these wavelengths are near the peak of the spectral

energy distribution for asteroid thermal emission and experience less contamination from reflected sunlight, which peaks in the visible range. Therefore, it is important to compare the W3–4 results with the four-band results to quantify any uncertainties and potential biases for this large data set.

The anticipated NASA Near-Earth Object Surveyor Mission (NEO Surveyor) is a 50 cm telescope designed to expand the inventory of asteroids, including the identification of potentially hazardous asteroids (PHAs). Unlike WISE, NEO Surveyor may operate with two wavelength bands (4–5.2 μm and 6–10 μm) (Mainzer and NEOCam Science Team, 2016), which correspond roughly to WISE bands W2 and W3. While W2 and W3 do not span exactly the same wavelengths as the anticipated NEO Surveyor bands, they are similar enough that they can provide an estimate of what to expect from NEO Surveyor’s future observations. Our sample of data from the WISE cryogenic mission includes observation of 4420 asteroids. Approximately 5% of the asteroids in this sample were observed in two distinct observational sequences separated by more than 30 days. Separate solutions were calculated for the two clusters. Our sample includes 4685 clusters of observations.

In this work, we conducted two sets of two-band (W2–3 and W3–4) thermal fits of the 4685 clusters of asteroid observations analyzed with four bands by Myhrvold et al. (2022) to quantify the uncertainties and potential biases associated with two-band fits. Understanding the strengths and weaknesses of two-band instruments in estimating asteroid parameters may assist the planetary defense community in predicting what new information NEO Surveyor is likely to provide about the PHA population, while also giving us the opportunity to anticipate weaknesses and blind spots that may hinder a PHA’s characterization.

2.2 Methods

2.2.1 Data Selection

In this work, we used the same dataset that was used by Myhrvold et al. (2022), but we focused on two wavelength bands, either W2–3 or W3–4, instead of all four infrared bands. We provide a brief description of how this data set was selected; for details, see Myhrvold et al. (2022).

Photometric data were obtained from the WISE full cryogenic mission phase, between January 7 and August 6 of 2010. First, a “moving object search” was performed on the WISE All-Sky Single Exposure Level 1b Source Catalog for all asteroids in the Minor Planet Center Orbit Database, returning a list of observation dates as Modified Julian Date (MJD) values (WISE Team, 2020). The JPL HORIZONS service was used to obtain the relevant parameters for each asteroid on the given observation dates including H and G values, positions, distances to the Sun and to the WISE spacecraft, and Sun-Target-Observer (phase) angles. This step yielded about 3.2 million records. A catalog cone search with a radius of 10 arcsec was performed around each of these positions, and all detected sources were stored for future processing.

After this initial selection, multiple filters were applied to the data. First, we removed asteroid conjunctions, where two or more presumed-asteroid sources were identified within 10 arcsec of another. A total of 12,000 conjunctions were found.

Next, we removed 419,039 observations with low (<2) signal-to-noise ratio in W3 or W4, where asteroids are typically brightest.

A further 96,685 observations were eliminated on the basis of five quality indicators reported within the WISE database, which were applied on a per-band basis. Observations were discarded if they had nonexistent (“Null”) or poor (“ph_qual” $\neq$ “A” or “B”) flux determinations, if they were susceptible to contamination or confusion (“cc_flags” $\neq$ 0), if they had a low signal-to-noise ratio, (“wksnr” < 4), if they were saturated (“wksat” $\neq$ 0), or if they had poor point-spread-function fits (“wkrchi2” > 2) ($1 \leq k \leq 4$).

The fourth filter was designed to eliminate potential confusion between asteroids and other astronomical sources listed in the AllWISE (co-added) Source Catalog. Every AllWISE source within 10 arcsec of an ephemeris position from one of the remaining potential asteroids was listed as a potentially confusing background source and further examined. This step eliminated approximately 13,000 observations.

The fifth filter was based on limiting the allowable distance between the WISE observed position and the predicted ephemeris position to specific thresholds (Myhrvold et al., 2022, Table 1).

The sixth filter split the observations for each asteroid into distinct clusters of observations if consecutive observations were more than 30 days apart. Following NEOWISE practice, we determined independent solutions for each cluster of observations. 94% of asteroids were observed in a single cluster. Any cluster that did not contain at least three observations in each of the four WISE wavelength bands was removed.

Lastly, any observation ≥ 1.5 mag away from all other fluxes within its cluster was removed.

After filtering, we were left with 82,548 observations grouped into 4685 clusters, representing 4420 asteroids.

2.2.2 NEATM Reparameterization

The original derivation of the NEATM hypothesizes a temperature distribution across the asteroid according to the following expression (Harris, 1998; Harris and Lagerros, 2002):

$$T_{NEATM}(\theta, \phi, r_{as}) = T_{ss}(r_{as})(\sin \theta \cos \phi)^{0.25}, \quad (2.1)$$

where θ and ϕ are the zenithal and azimuthal angles in a spherical coordinate system and r_{as} is the asteroid-Sun distance in astronomical units (au). In NEATM, the maximum

temperature across the surface is the subsolar temperature:

$$T_{ss}(r_{as}) = \left(\frac{S(1 - p_v q)}{\epsilon_B \sigma \eta r_{as}^2} \right)^{0.25}, \quad (2.2)$$

where S is the solar constant (1360.8 W/m²), p_v is the visible geometric albedo, q is the phase integral, ϵ_B is the emissivity, σ is the Stefan-Boltzmann constant, and η is the beaming parameter, defined as “the inverse of the enhancement in total thermal flux at zero solar phase angle over that of a uniformly radiating sphere” (Lebofsky et al., 1986). See Myhrvold (2018b) for an extensive discussion of the NEATM model derivation, assumptions, and limitations.

Results obtained by the NEOWISE team and archived at the PDS (Mainzer et al., 2016) were obtained with 10 different combinations of adjustable parameters. For a list of model names used by the NEOWISE team, see Table 1 of Myhrvold (2018a). The NEOWISE team assumed that the emissivity is 0.9 in each wavelength band.

Many studies arbitrarily set $\epsilon = 0.9$ (Delbo, 2004). In NEOWISE work, the emissivity is held at 0.9 while the albedo is allowed to vary. However, holding the emissivity constant while allowing the albedo to vary violates Kirchhoff’s law, which may introduce systematic errors into the model (Myhrvold, 2018b). For an estimate of these errors, see Appendix A.5.

In the presence of reflected sunlight, the flux equation involves both emitted and reflected light components (Myhrvold, 2018b):

$$F_{obs}(\lambda, \alpha, r_{as}, r_{ao}) = \frac{D^2}{4\text{au}^2 r_{ao}^2} \times \left(\epsilon(\lambda) F_{model}(\alpha, \lambda) + p(\lambda) \frac{\psi_{HG}(\alpha, G)}{r_{as}^2} F_{Sun}(\lambda) \right), \quad (2.3)$$

where λ is the wavelength, α is the phase angle, r_{ao} is the asteroid-observer distance in au, ψ_{HG} is the phase function, $F_{model}(\alpha, \lambda)$ is the emitted flux specific to the adopted asteroid thermal model, and $F_{Sun}(\lambda)$ is the solar flux incident on an asteroid at 1 au from the Sun, with $S = \int_0^\infty F_{sun}(\lambda) d\lambda$. In our formulation, the geometric albedo $p(\lambda) = (1 - \epsilon(\lambda))/q$ obeys Kirchhoff’s Law.

Because F_{model} for the NEATM depends on integrating the Planck distribution over all visible angles for an object, and the Planck distribution depends on T_{NEATM} , it is important to consider how this temperature is included in the modeling process. Using equation 2.2 in the thermal modeling process, as used by the NEOWISE team in [Masiero et al. \(2011\)](#), allows for the visible albedo p_v and the beaming parameter η to vary. While the beaming parameter can be conceptually useful, it can cause problems in a least-squares fit that simultaneously solves for η and the visible albedo p_v because η and p_v appear in the numerator and the denominator of equation (2.2), respectively. One can avoid these problems with a reparameterization by introducing a pseudo-temperature

$$T_1 = T_{ss}\sqrt{r_{as}}, \quad (2.4)$$

which represents the subsolar temperature of an asteroid with $r_{as} = 1$ au ([Myhrvold, 2018b](#)). By instead fitting for T_1 in the thermal model, this reparameterization eliminates two parameters from the model, η and p_v , and restores the NEATM model (without reflected sunlight) to the two-parameter model that it should be (D and T_1 for fixed ϵ), thus improving the chance that numerical fitting algorithms will retrieve an accurate result.

[Myhrvold et al. \(2022\)](#) showed that the assumption of a constant emissivity ($\epsilon = 0.9$) across all four WISE bands is likely responsible for the fact that up to 50% of NEOWISE model fits miss the data completely in one or more WISE bands. For this reason, they modified the fitting procedure and allowed the emissivity ϵ to vary between 0.5 and 1 for all four wavelength bands. However, they used a regularization term to penalize models that yielded a large dispersion in emissivities or emissivities that differed strongly from the canonical value of 0.9. For details about this regularization term, see [Myhrvold et al. \(2022, Section 3.6\)](#). In their four-band fits, the adjustable parameters were $(D, T_1, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$, where the emissivity subscripts represent the WISE bands.

2.2.3 Minimization Method

A smaller number of parameters is required to fit two-band data. The diameter and pseudo-temperature are sufficient to model the data, and the introduction of a floating emissivity is superfluous, as demonstrated by an analysis of variance (Appendix A.1). Therefore, we set $\epsilon = 0.9$ for both wavelength bands and fit for D and T_1 . Because the regularization term is zero when $\epsilon = 0.9$, the minimization term used in this work is simply the sum of squared residuals (SSR):

$$SSR = \sum_{i=1}^N (x_{i,model} - x_{i,obs})^2 \quad (2.5)$$

where $x_{i,model}$ are the modeled photometric values, $x_{i,obs}$ are the observed photometric values, and N is the number of flux measurements for an object. Following Myhrvold et al. (2022), we chose to discard the flux uncertainties from the calculation of this minimization term because they are similar across flux measurements and can be much smaller than light curve variations due to the asteroid’s rotation. For example, the median flux uncertainty of all median fluxes for each cluster in the W2–3 dataset is 0.14 mag while the median light curve amplitude for the asteroids with an associated calculated light curve (35% of objects) is 0.50 mag. For an examination of the impact of lightcurve amplitude on the goodness-of-fit metric, see Appendix A.2.

Because the minimization methods are different between the two-band models with fixed emissivity and the four-band models with variable emissivity, we sought to verify that differences in parameter estimates were due to the number of bands used in the fits rather than the minimization method. To that end, we also ran a set of fits where we fixed the emissivity at 0.9 and used the SSR loss term in the four-band fits, replicating the two-band fitting procedure exactly. The results strongly suggest that the observed differences between two-band and four-band fits is due to the consequences of eliminating two bands from the fits rather than any difference between fitting methods (Appendix A.4).

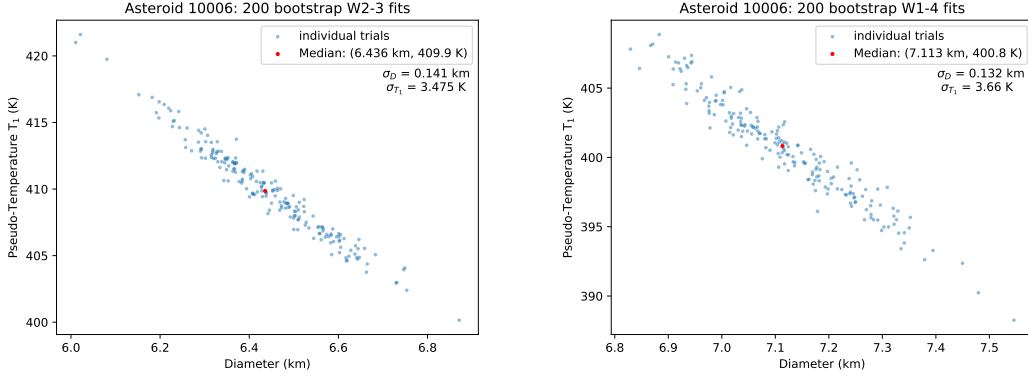


Figure 2.1: Best-fit pseudo-temperature and diameter for 200 bootstrap trials for example asteroid 10006. The median is shown as a red dot, with corresponding parameter values displayed in the legend. Standard deviations in diameter and pseudo-temperature amongst the bootstrap trials are displayed below the legend. **Left:** Results from two-band modeling. **Right:** Results from four-band modeling, where we replicate the results from Myhrvold et al. (2022) with the same set of observational clusters but a different set of random bootstrap trials.

2.2.4 Bootstrap Trials for Uncertainty Estimation

We used a bootstrap method to obtain estimates of the parameter uncertainties. We performed 200 bootstrap trials for each cluster, where, for each trial, we randomly sampled the asteroid flux measurements with replacement such that the number of data points was the same as it was originally. If this resampling resulted in fewer than 3 datapoints in either band, then we repeated the random sampling until there were at least 3 datapoints in each band included in the fit. A model was fit to each resampled data set, resulting in 200 solutions per cluster. The final model parameters were obtained by taking the median of the 200 parameter estimates, and the parameter uncertainties were obtained from the standard deviations of the parameter values within the 200 trials. An example of the bootstrap procedure with 200 model fits is shown in Figure 2.1 for asteroid 10006, which was observed in only one cluster.

For a random subset of 50 asteroids, we expanded the number of trials to 2000 and compared the 2000- and 200-trial results. We found that median values were essentially unchanged, whereas estimates of uncertainties differed by up to 10%.

2.2.5 Treatment of Low Pseudo-Temperature Solutions

For individual bootstrap trials of both W2–3 and W3–4, the modeled solutions occasionally yielded an unrealistically low pseudo-temperature, with $T_1 < 273$ K. Most solutions yielded more plausible pseudo-temperatures, with $T_1 > 273$ K. Specifically, the median pseudo-temperature of the 200 bootstrapped trials for the two-band W2–3 model was < 273 K for 5.4% of the 4685 cases. For the W3–4 model, all median pseudo-temperatures were > 273 K. In each instance among the 5.4% of anomalous W2–3 cases, we observed a dichotomy of solutions, with some of the trials yielding a high pseudo-temperature and small diameter and other trials yielding a low pseudo-temperature and large diameter (Figure 2.2, Lower Right).

We suspect that the dichotomy in pseudo-temperatures is related to the fact that it is possible to achieve approximately the same W2 flux with vastly different contributions of emitted and reflected light (Figure 2.2, Top), while also maintaining approximately the same W3 flux. For instance, the low-temperature fits (median ratio of integrated emitted to reflected light of 0.014) for asteroid 10125 tend to have comparable chi-squared values to the high-temperature fits (median ratio of integrated emitted to reflected light of 7.4) (Figure 2.2, Lower Left and Lower Right). With noisy data, the fitting algorithm will occasionally land on an unphysical low-temperature solution because its chi-squared value is marginally lower than that of the high-temperature solution. These anomalous cases tended to have smaller heliocentric distances and larger phase angles than most objects in our sample. These trends are further explored in Appendix A.3.

In the case of the W3–4 models, there were no asteroids with pseudo-temperatures below 273 K, but there were 18 clusters with pseudo-temperatures between 273 K and 300 K. These clusters do not present a dichotomy of solutions in the bootstrap trials, instead having all pseudo-temperatures clustered within the same pseudo-temperature range. The median distance to the Sun for these asteroids was 1.06 au, as opposed to the median for all clusters of 2.37 au, suggesting that NEOs were again implicated. In fact, 17 of the 18 cases are indeed NEOs, representing $\sim 20\%$ of all NEOs in our dataset. NEOs may

be more susceptible to inaccurate fits due to their higher observation phase angles, which exacerbate NEATM limitations and suboptimal phase-curve corrections. This hypothesis is supported by a median phase angle of 70° among the 18 low pseudo-temperature cases, in contrast to the median of 25° among all clusters.

It was important to detect and remove the anomalous low-temperature solutions. For the four-band models, each case was modeled with seven different starting values of emissivity, which provided a natural way of discarding low-temperature solutions. The maximum T_1 was identified as $T_{1,max}$, and any solution with $T_1 < 0.95 \times T_{1,max}$ was discarded. For the two-band models where emissivity was not varied, a different approach had to be used. If fewer than 10% of the bootstrap trials yielded a high-temperature ($T_1 > 273$ K) solution, the entire cluster was thrown out. Otherwise, the trials with $T_1 < 273$ K were thrown out. For the W2–3 results, this process eliminated 3.05% of two-band cases, leaving 4542 cases. For the W3–4 results, this process eliminated no two-band cases. We focused on these 4542 W2–W3 cases and 4685 W3–4 cases for the remainder of the analysis, where each case was compared to its four-band counterpart from [Myhrvold et al. \(2022\)](#).

2.2.6 Albedo Calculation

The visible-band geometric albedo was calculated for each observation cluster according to the equation

$$p_v \equiv \left(\frac{1329}{D} \right)^2 10^{-\frac{2H}{5}}, \quad (2.6)$$

where D is the median diameter estimate from the bootstrap trials and H is the absolute magnitude of the asteroid. We used the H values from [Vereš et al. \(2015\)](#) if available¹. Otherwise, we used the H value published by the Minor Planet Center/HORIZONS plus 0.26 to account for the mean systematic offset reported in [Vereš et al. \(2015\)](#). For further details, see [Myhrvold et al. \(2022, Section 4.2\)](#).

¹Except for asteroid 19761, whose H value was off by ~ 5 in the 2015 table.

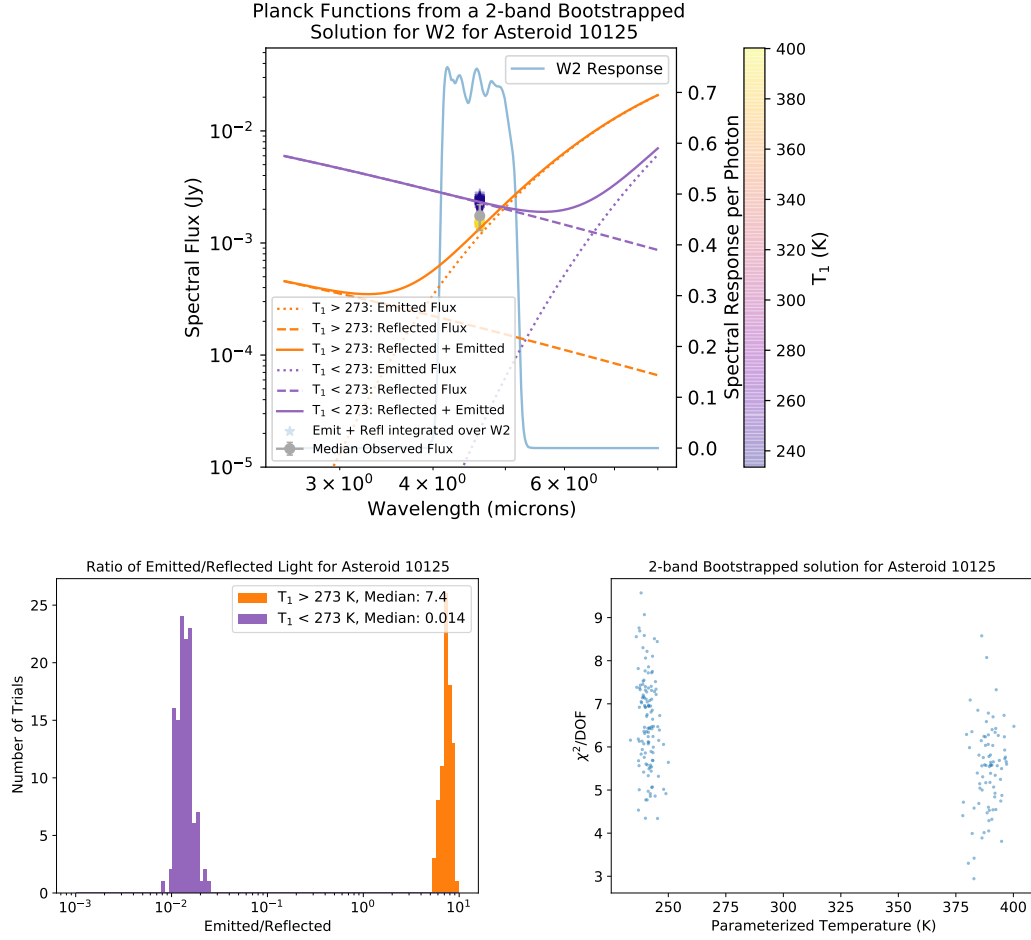


Figure 2.2: Observed dichotomy in pseudo-temperature solutions for two-band fits illustrated with results from 200 bootstrap trials for asteroid 10125. **Top:** Planck spectral distributions of emitted light (dotted lines) and reflected light (dashed lines) calculated with the median pseudo-temperature T_1 and median diameter D obtained from the bootstrap trials. The orange curves correspond to the median of 80 trials with high pseudo-temperatures and the purple curves correspond to the median of 120 trials with low pseudo-temperatures. Note how two dissimilar pseudo-temperatures yield a similar total flux (solid lines) within the W2 band, while also yielding similar W3 fluxes. Colored symbols indicate the total (emitted + reflected) spectral flux integrated over the W2 bandpass for each of the 200 trials. The W2 bandpass is shown in a light blue solid line as the spectral response per photon (Wright et al., 2010). The grey circle represents the true median measured W2 flux of the object adjusted to a phase angle of zero, with the error bar given by the standard deviation of the phase-adjusted W2 fluxes. **Lower Left:** Histogram of emitted to reflected light (integrated over the W2 bandpass) for 200 bootstrapped trials. High pseudo-temperature solutions are shown in orange whereas low pseudo-temperature solutions are shown in purple. **Lower Right:** Reduced chi-squared vs. pseudo-temperature T_1 , showing similar values in the two temperature regimes.

2.3 Results

2.3.1 W2–3 Results

Figure 2.3 shows the two-band (W2–3) and Myhrvold et al. (2022) four-band (W1–4) estimates of pseudo-temperature T_1 vs. diameter for 4542 clusters representing 4277 asteroids. While the overlap between the two-band and four-band fits is substantial, there are also some differences worth noting. For example, the two-band fits sway towards higher pseudo-temperatures and smaller diameters overall in comparison to their four-band counterparts (Figure 2.4). These percentage differences were obtained by calculating $(D_{W2-3} - D_{W1-4})/D_{W1-4}$ for each cluster. The median diameter difference is -10%, with the 5% and 95% quantiles of the distribution at -32% and -1.5%, respectively. The median pseudo-temperature difference is 2.4%, with 5% and 95% quantiles at -0.30% and 14% respectively). Of these results, 85 of the asteroids were near-Earth objects, representing 92 clusters. Looking only at the near-Earth objects, the median diameter difference is 8.9%, with the 5% and 95% quantiles of the distribution at -20% and -2.4%, respectively. The near-Earth object median pseudo-temperature difference is 2.1%, with 5% and 95% quantiles at 0.57% and 5.5%, respectively.

We found that two-band and four-band fits generally have comparable parameter uncertainties, although the median diameter uncertainty is somewhat larger for two-band fits (Figure 2.5).

Using equation 2.6, we also calculated the albedo for the two-band fits. We eliminated any unphysical albedo values ($p_v > 1$) from the dataset used to calculate the median albedo, resulting in 14 eliminated two-band clusters and 1 eliminated four-band cluster. The results (Figure 2.6) demonstrate that the median albedos appear higher for the two-band fits than the four-band fits by about 35%.

In addition to comparing two-band and four-band fit results, we were also interested in comparing two-band fit results to independent diameter estimates. Short of visiting an asteroid with a spacecraft, the most accurate diameter measurements are obtained with

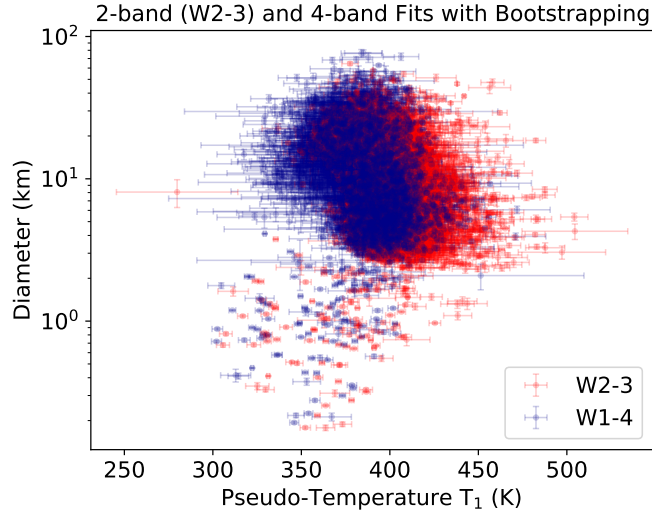


Figure 2.3: Diameter vs. pseudo-temperature T_1 for 4542 cases calculated from a two-band (W2–3) least-squares solution and four-band (W1–4) regularized solution to a reparameterized NEATM model. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. The W2–3 data shown in this figure is available as the data behind the figure. The data also includes additional information that will allow the reader to recreate Figures 4–6.

radar observations or stellar occultation observations. Here, we used the stellar occultation diameter estimates with quality codes of 2 or 3 (Herald et al., 2019) to obtain 23 diameter comparisons for 22 distinct asteroids (Figure 2.7). The two-band W2–3 results give generally smaller diameters than the occultation data, with the median percent difference $(D_{W2-3} - D_{occ})/D_{occ}$ at -15% (5–95% quantiles at -39% and 6.2%). This comparison further confirms our finding that two-band diameters are $\sim 10\%$ smaller than their true diameters.

Finally, we also compared our two-band diameter estimates to two-band and four-band diameter estimates from NEOWISE (Mainzer et al., 2019). For the four-band results, we found 2632 asteroids in common, where 137 asteroids were represented in two different epochs, which therefore resulted in 2769 diameter comparisons (Figure 2.8, left). The median percent difference $(D_{W2-3} - D_{W1-4,N})/D_{W1-4,N}$ was -7.8% (5–95% quantiles at -22% and 11%), suggesting that the NEOWISE four-band diameter estimates

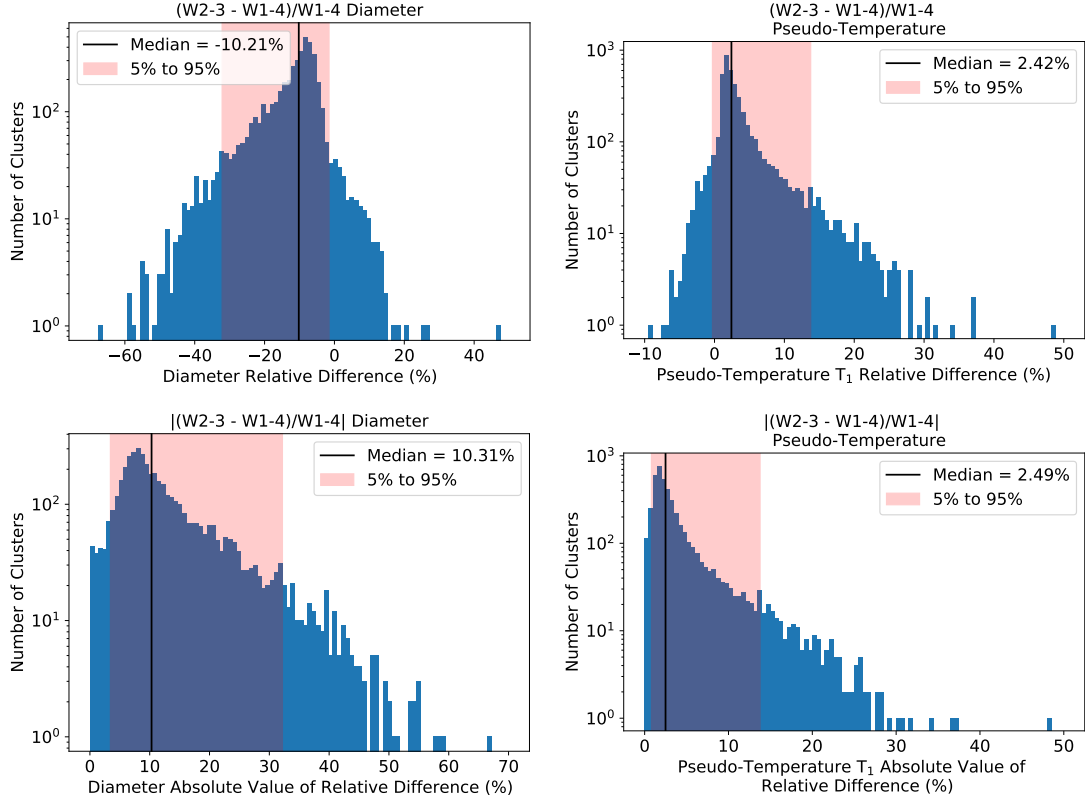


Figure 2.4: Histograms of the relative difference and absolute value of the relative difference in D and T_1 parameter estimates. Median values are represented with a vertical black line while a pink highlighted block is used to represent the 5 – 95% quantile range. **Top Left:** Relative difference in diameter. **Top Right:** Relative difference in pseudo-temperature T_1 . **Bottom Left:** Absolute value of relative difference in diameter. **Bottom Right:** Absolute value of relative difference in pseudo-temperature T_1 .

are also underestimates of the true diameter, but to a lesser degree than our two-band fits. This trend remained true when we compared our two-band results to the NEOWISE two-band (W2–3) results. With 42 asteroids in common and 0 asteroids represented in two epoch clusters, giving 42 diameter comparisons, the median percent difference $(D_{W2-3} - D_{W2-3,N})/D_{W2-3,N}$ was -9.1% (5–95% quantiles at -25% and 15%).

2.3.2 W3–4 Results

Figure 2.9 shows the two-band (W3–4) and Myhrvold et al. (2022) four-band (W1–4) estimates of pseudo-temperature T_1 vs. diameter for 4685 clusters, representing 4420

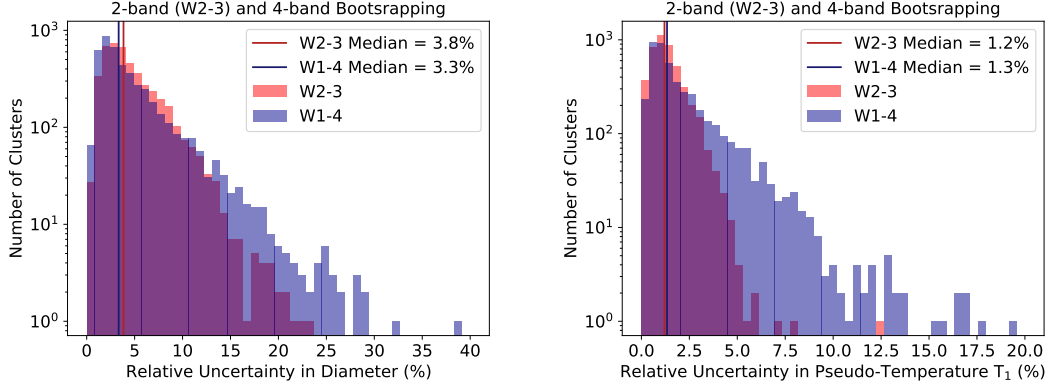


Figure 2.5: Histograms of D and T_1 parameter relative uncertainty, i.e., the standard deviation in accepted bootstrap trials for an object divided by the median value of the bootstrap trials. The median of the relative uncertainties for all objects is shown as a vertical line. Two-band results are shown in red whereas four-band results are shown in blue. **Left:** Relative uncertainty in diameter. **Right:** Relative uncertainty in pseudo-temperature T_1 .

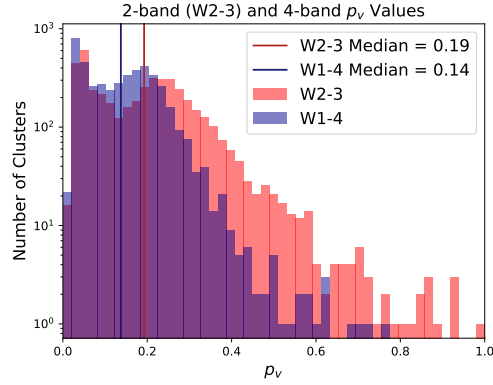


Figure 2.6: Histogram of visible-band geometric albedos (p_v) as calculated using the median modeled diameter for each cluster and the absolute magnitude for the object, H . The median albedo for the two-band fits is shown as a dark red line, while the median albedo of the four-band fits is shown as a dark blue line.

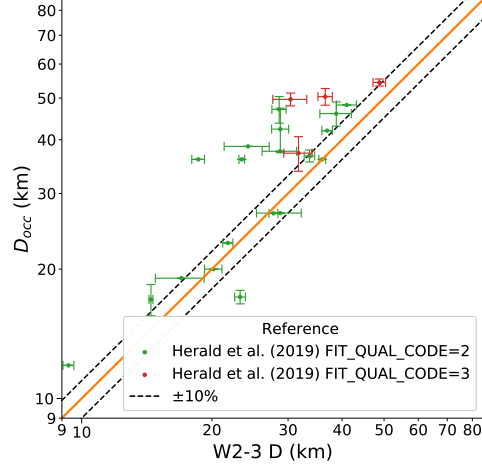


Figure 2.7: Plot of occultation diameters versus our corresponding two-band (W2–3) modeled diameters. The orange line represents a perfect one-to-one ratio between diameters, whereas the black dotted lines represent a 10% error in the positive and negative directions.

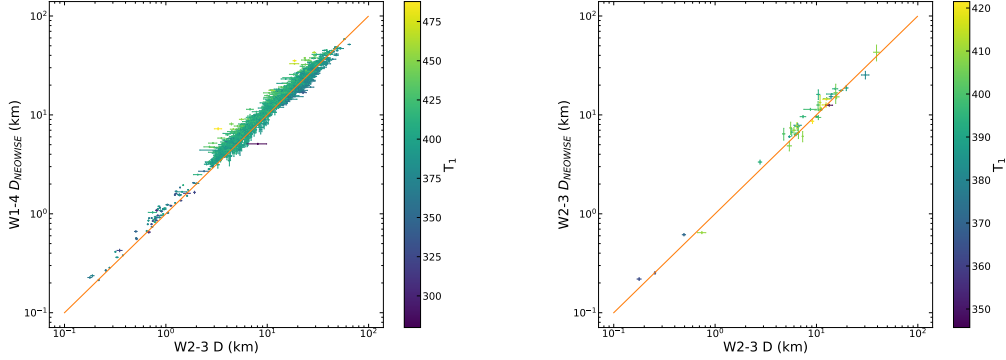


Figure 2.8: **Left:** Plot of NEOWISE four-band diameters versus our corresponding two-band (W2–3) modeled diameters. The orange line represents a perfect one-to-one ratio between diameters. The color bar indicates our pseudo-temperature modeled with W2 and W3. **Right:** Plot of NEOWISE two-band (W2–3) diameters versus our corresponding two-band (W2–3) modeled diameters.

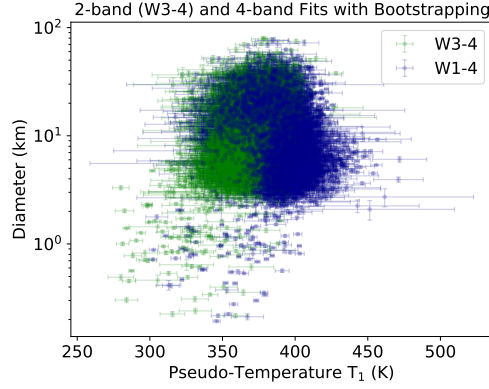


Figure 2.9: Diameter vs. pseudo-temperature T_1 for 4685 cases calculated from a two-band (W3–4) and four-band (W1–4) reparameterized NEATM model. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. The W3–4 data shown in this figure is available as the data behind the figure. The data also includes additional information that will allow the reader to recreate Figures 10–12.

asteroids. The overlap between the two-band (W2–3) and four-band fits is substantial. However, the W3–4 fits appear to deviate from the four-band fits in the opposite directions as that of W2–3. For example, the two-band (W3–4) fits tend towards smaller pseudo-temperatures and larger diameters overall in comparison to their four-band counterparts (Figure 2.10), in contrast to the trends observed in W2–3. The median diameter difference is 11%, with the 5% and 95% quantiles of the distribution at -2.1% and 26%, respectively. The median pseudo-temperature difference is -5.6%, with the 5% and 95% quantiles of the distribution at -13% and -0.15%, respectively. Of these results, 87 of the asteroids were near-Earth objects, representing 95 clusters. Looking only at the near-Earth objects, the median diameter difference is 14%, with the 5% and 95% quantiles of the distribution at 3.9% and 24%, respectively. The median pseudo-temperature difference is -8.3%, with the 5% and 95% quantiles of the distribution at -13% and -3.5%, respectively.

We found that two-band fits have a slightly larger spread in parameter uncertainties compared to four-band fits (Figure 2.11).

When obtaining the albedos for the W3–4 fits, there were no unphysical values which

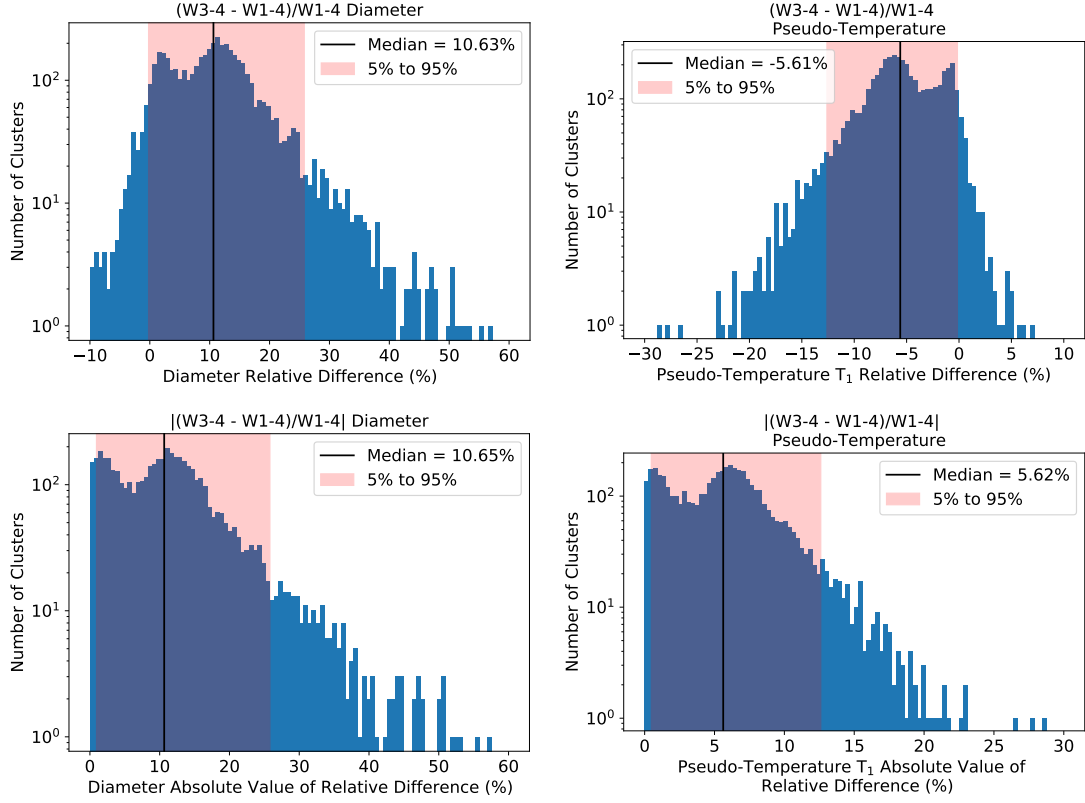


Figure 2.10: Histograms of the relative difference and absolute value of the relative difference. Median values are represented with a vertical black line while a pink highlighted block is used to represent the 5 - 95% quantile range. **Top Left:** Relative difference in diameter. **Top Right:** Relative difference in pseudo-temperature. **Bottom Left:** Absolute value of relative difference in diameter. **Bottom Right:** Absolute value of relative difference in pseudo-temperature.

needed to be eliminated. However, one unphysical albedo ($p_v > 1$) was eliminated from the four-band dataset before calculating the median albedo. The albedo distribution for W3–4 fits (Figure 2.12) indicate that the median albedos for the two-band (W3–4) fits differ from the four-band fits by about -16%.

We compared our diameter estimates to stellar occultation diameter estimates with quality codes of 2 or 3 (Herald et al., 2019). With 23 diameter comparisons for 22 distinct asteroids (Figure 2.13), we found that the W3–4 results usually give larger diameters than the occultation data, with a median percentage difference ($((D_{W3-4} - D_{occ})/D_{occ})$ of 6.1% (5%–95% quantiles at -17% and 27%).

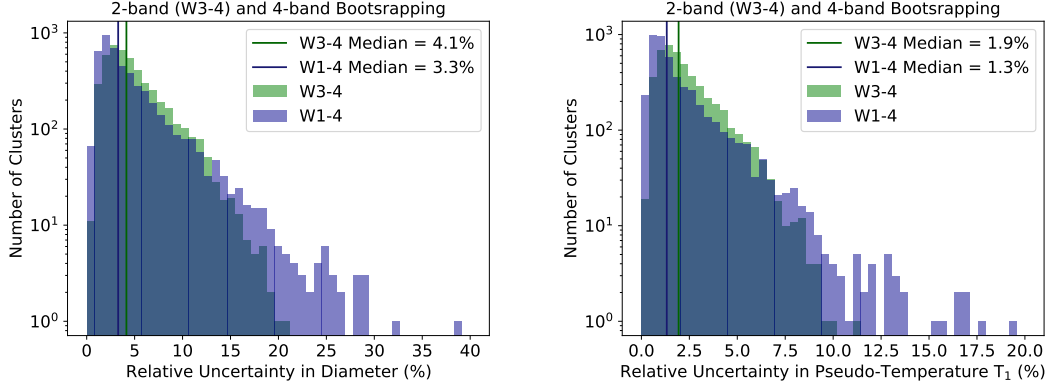


Figure 2.11: Histograms of relative uncertainty, where relative uncertainty represents the standard deviation in accepted bootstrap trials for an object divided by the median value of the bootstrap trials. The median of the relative uncertainty for all objects is shown as a vertical line. Two-band results are shown in green while four-band results are shown in blue. **Left:** Relative uncertainty in diameter. **Right:** Relative uncertainty in pseudo-temperature.

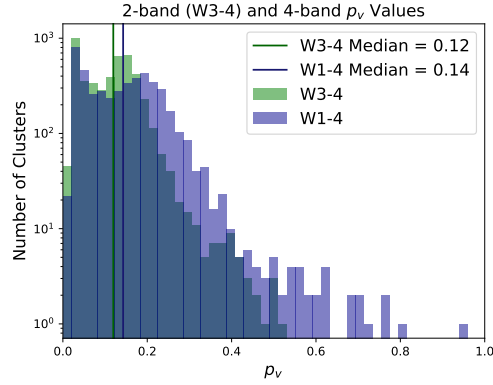


Figure 2.12: Histogram of visible-band geometric albedos (p_v) as calculated using the median modeled diameter for each cluster and the absolute magnitude for the object, H . The median albedo for the two-band fits is shown as a dark green line, while the median albedo of the four-band fits is shown as a dark blue line.

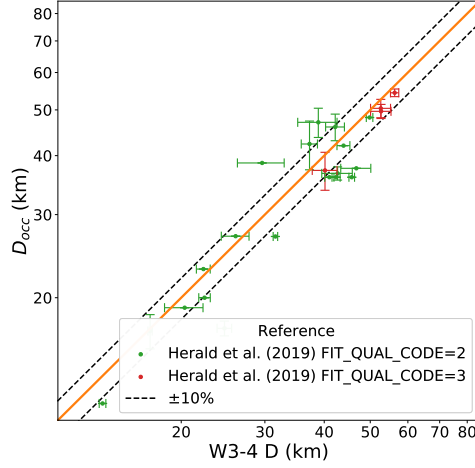


Figure 2.13: Plot of occultation diameters versus our corresponding diameters modeled with W3 and W4. Red points represent objects with a higher quality occultation diameter with a quality code of 3, while green objects have a slightly lower quality occultation diameter with a quality code of 2. The orange line represents a perfect one-to-one ratio between diameters, while black dotted lines represent a 10% error in the positive and negative directions.

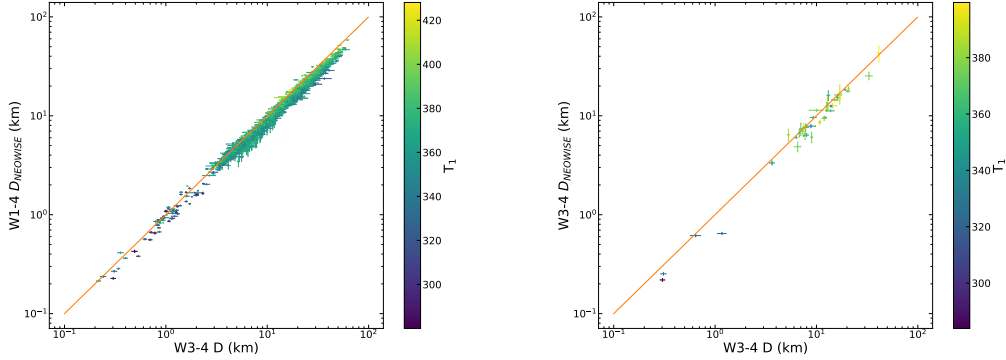


Figure 2.14: **Left:** Plot of NEOWISE four-band diameters versus our corresponding diameters modeled with W3 and W4. The orange line represents a perfect one-to-one ratio between diameters. The color bar indicates our pseudo-temperature modeled with W3 and W4. **Right:** Plot of NEOWISE two-band (W3–4) diameters versus our corresponding diameters modeled with W3 and W4.

We also compared our two-band (W3–4) diameters to the diameters resulting from the four-band fits done by NEOWISE for the same set of asteroids. We found 2651 asteroids in common, where 138 asteroids were represented in two different epochs, which therefore resulted in 2789 diameter comparisons plotted in Figure 2.14. The median percent difference $(D_{W3-4} - D_{W1-4,N})/D_{W1-4,N}$ was 14% (5%–95% quantiles at 2.0% and 25%). which suggests that the NEOWISE four-band results underestimate the diameter whereas our W3–4 results overestimate the diameter.

For the two-band (W3–4) NEOWISE results, we found 42 asteroids in common and 0 asteroids with two different clusters of epochs, giving 42 total comparisons. The median difference $(D_{W3-4} - D_{W3-4,N})/D_{W3-4,N}$ was 9.5% (5%–95% quantiles at -11% and 38%), which suggests that the NEOWISE two-band results also underestimate the diameter while our W3–4 results overestimate the diameter.

2.4 Discussion

The total flux from an object scales as D^2 (Equation 2.3) and the emitted component scales as T^4 (Stefan-Boltzmann Law). When emitted light dominates, we expect D to scale approximately with T^2 . Therefore, we expect a small difference in T_1 to produce a larger difference in D . This trend is apparent in Figure 2.4, where the median absolute value relative difference in pseudo-temperature is 2.42%, whereas the median absolute value relative difference in diameter is larger, at 10.2%.

We found that analysis of W2–3 observations tended to yield underestimates of asteroid diameters by about 15% in comparison to occultation diameters, which we take to be the most accurate diameter measurements for this data set. Such underestimates could potentially affect planetary defense efforts by underestimating the kinetic energy of potential impactors or by affecting Yarkovsky drift calculations. This limitation affects analyses of existing W2–3 observations and likely of data from future missions with similar bands.

Additionally, as seen in Figures 2.5 and 2.11, we found a slightly larger median spread in bootstrap diameter estimates for the two-band fits (W2–3 and W3–4) than the four-band fits. This larger spread indicates that two-band instruments may yield slightly larger uncertainties than analogous measurements obtained with four bands.

Our results are based on a curated subset of observations obtained during WISE’s cryogenic mission phase. This subset includes only asteroids that were detected in all four bands and may therefore exclude smaller or darker asteroids that were too dim to yield detections in W1–2. Likewise, it excludes objects that resulted in saturated observations and may therefore exclude larger or brighter objects. With these potential biases in mind, our sample covers a large range of diameters from 0.2 to 80 km as well as a large range of visible albedos. As a result, it is expected to have broad applicability and relevance for future surveys.

We found that the two-band results for the near-Earth objects within our sample exhibit roughly the same characteristics as those for main belt objects. However, our results required removal of anomalous low-temperature trial solutions, which were slightly more likely to originate from the NEO population due to their larger phase angles. Our work suggests that future NEO surveys may benefit from the removal of unreasonable thermal model solutions for NEOs observed at large phase angles.

2.5 Conclusions

In this study, we compared the results of the reparameterized Near-Earth Asteroid Thermal Model for a curated set of asteroids observed by WISE in two wavelength bands (W2–3 and W3–4) to previous results from all four wavelength bands (W1–4) calculated by Myhrvold et al. (2022). This comparison was done with the goal of elucidating unique aspects of determining asteroid parameters using modeling with two infrared bands so that any potential biases or shortcomings could be predicted and corrected for with the upcoming NEO Surveyor mission, which will observe asteroids in two-bands similar to W2 and W3,

or with similar future missions.

We found that the two-band W2–3 fits tended to have slightly higher pseudo-temperatures and smaller diameters than the four-band fits. By comparison with the more accurate occultation diameters, we found that the W2–3 fits are likely underestimating the size of the asteroids by about 15%. We also found that the W2–3 fits tend to have about the same median spread in pseudo-temperatures, but a slightly larger spread in diameters across the bootstrapped fits, indicating a greater uncertainty in size of the asteroid as opposed to the four-band fits. The W3–4 fits tended to overestimate the diameters by about 6.4% in comparison to the occultation results, and tended to have a larger diameter and lower pseudo-temperature than the corresponding four-band fits. The W3–4 results show a larger median spread in diameters and pseudo-temperatures than the four-band fits, indicating greater uncertainty in size and pseudo-temperature than the four-band fits.

Based on all of our results, we have concluded that NEATM modeling of asteroids is worth doing even if there are data only in two of the four WISE wavelength bands, at least in the case of W2–3 and W3–4, because this modeling is still able to constrain asteroid sizes and albedos reasonably well. Although the diameters appear to have a greater systematic error for W2–3 and W3–4 than for the four-band fits, these values are still valuable for understanding the asteroid population as a whole, and for being able to predict the relative danger posed by a potential impact event in a way that is more precise than using visible photometry alone.

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Software used: NumPy ([Harris et al., 2020](#)), SciPy ([Virtanen et al., 2020](#)), pandas ([Wes McKinney, 2010](#)), Matplotlib ([Hunter, 2007](#))

CHAPTER 3

Discovery of Hundreds of Main-Belt Asteroids with Synthetic Tracking of WISE Data

3.1 Introduction

The Wide Field Infrared Survey Explorer (WISE) conducted a full-sky survey in the infrared (IR) in four wavelength bands centered at 3.4, 4.6, 12, and 22 μm (called W1, W2, W3, and W4, respectively), yielding observations of more than 100,000 asteroids (Wright et al., 2010). During the cryogenic phase of the mission, WISE observed 23,616 previously unknown main belt asteroids (MBAs) with arc lengths longer than one day (Masiero et al., 2011).

Wright et al. (2010) estimated that previously unknown MBAs as small as 2.4 km could be detected. The detections reported by the NEOWISE team are based on point spread function (PSF) fits to single-exposure images, where the PSFs have been estimated based on the observations of bright stars (Cutri et al., 2020). However, shift-and-stack techniques allow detection of fainter asteroids, as demonstrated for 0.03% of the sky with WISE data by Mazur et al. (2018). SALTAD (Software and Algorithmic Testbed for Asteroid Detection) is an algorithm that uses a shift-and-stack technique to discover asteroids with linear or non-linear moving object hypothesis templates (Gural and Larsen, 2004; Gural et al., 2005, 2018). The stacking of images increases the signal-to-noise ratio (S/N) by approximately $\sqrt{N}$, where N is the number of images where the asteroid is present. Here, we use SALTAD to search for previously undiscovered objects in the W3 data set.

Following Mazur et al. (2018), we stack at least 4 images and at most 9 images, which

ideally gives a S/N improvement of 2–3 and the ability to detect asteroids that are up to 1.2 magnitudes fainter than those discovered by the NEOWISE team. As part of the shift-and-stack detection algorithm, we obtain right ascension (RA) and declination (DEC) measurements of the centroids in individual images. We submit this astrometry for good-quality candidates to the International Astronomical Union’s Minor Planet Center (MPC), the central location for receiving and distributing astrometry for small bodies and for identifying, designating, and computing orbits for them (Marsden, 1980). Much of this astrometry should be linkable to expected discovery detections from the Vera Rubin Telescope’s Legacy Survey of Space and Time (LSST) or other surveys, enabling precise orbit calculations for these bodies. The precise orbits for these newly discovered faint asteroids could then be used in future scientific investigations, including studies of asteroid families, non-gravitational forces, dynamical evolution of the main belt, and solar system formation.

In Section 2, we describe our shift-and-stack detection pipeline. In Section 3, we describe the results at each step of our pipeline, our candidates, and the astrometry reported to the MPC. In Section 4, we discuss our sky coverage, our discovery candidates, and potential for future linkages. We present our conclusions in Section 5.

3.2 Methods

3.2.1 WISE Data

We have downloaded approximately 1.5 million W3 WISE All-Sky Release Single-Exposure images from the NASA/IPAC Infrared Science Archive (2020).

The W3 band is the most important band for this investigation because the dominant flux received from asteroids originates from their thermal emission, which peaks near the W3 central wavelength for MBAs. The archived FITS images cover a $47' \times 47'$ field of view with 1016×1016 pixels and a scale of 2.757 arcseconds per pixel. Most of the sky was observed in WISE W3 ~ 16 times. However, stacking all of these images for use by

SALTAD is not possible because they do not create a large enough subfield for objects with MBA on-sky velocities, which will exit the subfield before reaching the last frame. Objects must be present on all frames of a subfield in order to be detectable with SALTAD. We search patches of sky with a $31.1' \times 31.1'$ area (2/3 of an image across) to create subfields covering at least an area equivalent to $27' \times 27'$. These choices of search area and minimum subfield area were guided by the results of a test of 18 patches near the ecliptic: these values provided the best balance of maximizing the fraction of sky patches with results, the number of frames stacked, and the number of hits to known objects, while minimizing the runtime per on-sky area covered. The WISE FITS headers include World Coordinate System (WCS) values, which we use to co-register all images covering each subfield. This WCS was derived by the WISE team on the basis of the 2MASS point source catalog, which was used as the primary astrometric reference for WISE positional reconstruction (Cutri et al., 2020).

3.2.2 Object Detection with Synthetic Tracking

We use the CPU implementation of SALTAD 170, the most up-to-date version of the software. SALTAD enhances images with background equalization, the removal of the temporal mean of each pixel, and noise whitening. Then, the parameter space for different hypothesized motions is explored to form image stacks and search for moving objects. We use linear motion templates, which is an excellent approximation to asteroid sky motion as seen by WISE during the time span of the 4–9 observations, generally < 1 day. The WISE observing geometry precluded observations near opposition, where apparent retrograde motion can occur. The solar elongation was always $\sim 90^\circ$, yielding phase angles of $14\text{--}38^\circ$ for asteroids between $\sim 1.8\text{--}3.6$ au (Masiero et al., 2011). We verified the linear motion approximation with the worst-case scenario of NEA motion. Any deviation from linear motion amounted to a $\sim$ tenth of a pixel.

To determine the typical sky motions for main belt asteroids as viewed by WISE, we downloaded all astrometric data for numbered asteroids observed by WISE during the

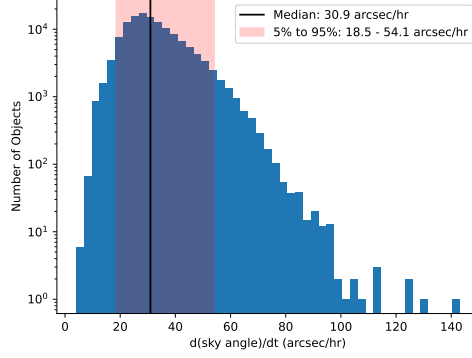


Figure 3.1: Histogram of the on-sky angular speeds for all numbered asteroids with perihelion distances $q > 1.3$ au observed during the WISE cryogenic mission.

cryogenic period and used this data to find the sky angle speeds for each asteroid with perihelion distance $q > 1.3$ au. We found that the 5–95% percentiles of the distribution of angular rates of motion correspond to 18.5–51.4″/hour (Figure 3.1) and we adopted this range for our search.

SALTAD shifts and stacks all images in a subfield according to a finite number of possible motion templates and provides a list of hits ranked in decreasing order of signal-to-noise ratio (S/N). Because the WISE PSF is relatively stable and the noise is approximately constant across all images in a stack, the expected S/N improvement for the hits is $\sim \sqrt{N}$, where N is the number of images where the asteroid is present.

The descriptions of the hits are output to an ASCII text file, including the S/N and centroid locations in fractional pixels in each image (Gural et al., 2018), which we can immediately convert to RA and DEC positions with the WCS information in the stacked image headers. This WCS was generated from the co-registration of the stack of WISE images in the subfield. We confirmed that the WCS solution is accurate enough for our purposes by obtaining a plate solution for subfield 060013509 using sources from the Gaia catalog (Vallenari et al., 2023). The plate solution improved astrometric accuracy by ~ 0.1 arcsec.

The nomenclature associated with detections at various stages of processing are shown

Table 3.1: Detection Nomenclature

Term	Meaning
A <i>detection</i>	A single centroid of the N centroids within a hit
A <i>hit</i>	A set of N centroids representing a single detected pixel in the stack of shifted images within a subfield
A <i>false hit</i>	A hit that does not survive all our filters
A <i>true hit</i>	A hit that survives all our filters
A <i>candidate</i>	A true hit that has been merged with 0 or more hits (Section 3.2.9)
A <i>confirmed candidate</i>	A candidate matching a previously known object
A <i>discovery candidate</i>	A candidate matching no previously known object

NOTE— N refers to the number of images in a stack.

in Table 3.1.

3.2.3 Image Collection, Elimination, and Registration

We created a software pipeline that collects existing W3 WISE images that cover a particular patch of sky, eliminates problematic images, registers the remaining images, configures the necessary settings for SALTAD, and runs SALTAD to shift and stack the images.

3.2.3.1 Image Collection

We split the celestial sphere into 269,496 equally sized *patches* using an LSST Software [HealpixSkyMap](#) object. This sky map includes 12 *tracts*, 8 of which contain 20,178 patches and 4 of which contain 27,018 patches. The sky map is rotated 68.4° to orient it along (the normal to) the ecliptic plane and thus provide better alignment with the pattern of consecutive WISE scans.

An illustration of the sky coverage is shown in Figure 3.2. We use a Structured Query Language (SQL) database to store all metadata about the available images, and we query the database for frames with centers located within the equatorial coordinate limits of the patch corners.

We check that this query returns at least 4 images, otherwise the patch is skipped.

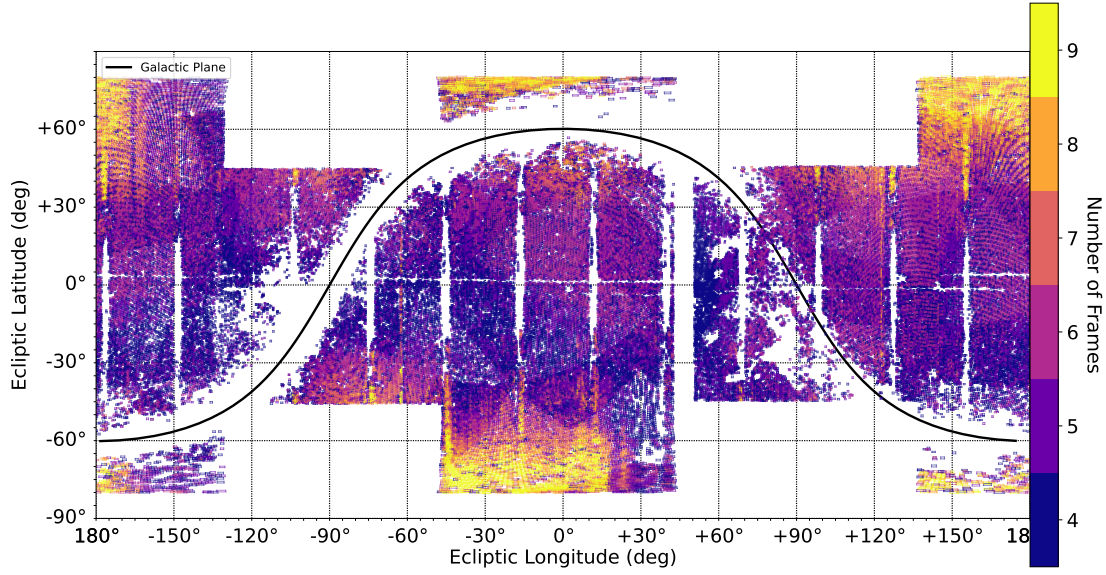


Figure 3.2: Plot of the sky coverage of our collection of subfields searched by SALTAD. The x-axis is ecliptic longitude in degrees whereas the y-axis is ecliptic latitude in degrees. The outline of each subfield is shown, where the color of the outline corresponds to the number of frames stacked to create the subfield. Regions within 10 degrees of the ecliptic poles were excluded due to absence of main belt objects expected in this region. Other gaps in coverage represent regions of the sky where there was an insufficient number of good overlapping frames to create a subfield or smaller gaps between adjacent subfields. Large gaps surround the galactic plane (solid black line) and regions of the sky in the WISE field of view during reaction wheel desaturation.

Next, we sort the images by distance from the patch center and use a recursive algorithm to find a way to stack as many of the images as possible such that the unshifted subfield area remains larger than 0.2025 square degrees, which is equivalent in area to a $27' \times 27'$ subfield. The images used to create this subfield move on to a quality assurance check.

3.2.3.2 Image Elimination

We exclude images that were flagged by the NEOWISE team as poor or questionable quality. Specifically, we exclude images with any of the following events listed in their header: ‘SAA’ (within pre-defined South Atlantic Anomaly boundary), ‘MOON’ (within Moon keep-out zone), ‘MOM’ (momentum dumping, aka reaction wheel desaturation), or ‘IMCAL’ (image calibration). We also exclude images that are within 0.5 degrees of Mars,

Jupiter, Saturn, or the South Atlantic Anomaly, and we exclude images within 20 degrees of the Moon, regardless of the value of the MOON flag as we find that SALTAD results may be compromised due to close proximity to the Moon even when the ‘MOON’ header flag is not present.

We exclude images that have $> 50\%$ non-data pixel values. We also test for linear features in the image that could be caused by things like satellite tracks by fitting a line to all pixels with values $3\text{--}4\sigma$ above the noise floor and exclude images with an r^2 regression value greater than 0.1.

We test for images with uneven backgrounds by splitting the image into a 3×3 grid and taking the median of each grid square, then subtract the maximum of these medians by the minimum of these medians and divide the whole quantity by the median of the medians. If this operation yields a value greater than 0.008, meaning that the background is varying by greater than 0.8%, we exclude the image.

3.2.3.3 Image Registration

If at least 4 good quality images remain, we continue the processing of the patch. Otherwise, we abandon the patch. We use the Montage software package (Berriman et al., 2003) to reproject and coadd all images. Then, we identify the area that is common to all images in the stack, and all images are trimmed to this common area. These reprojected, cropped images, which we call a subfield, become the input to SALTAD. Because the WISE space telescope is an orbiting platform with a rotating scan pattern, the images in our stack have been “dithered”, fulfilling an MPC recommendation regarding synthetic tracking.

3.2.4 SALTAD Algorithm

SALTAD uses a matched filtering algorithm (see Mohanty (1981) and Gural et al. (2005) for details), which tests many motion template hypotheses in an attempt to detect targets of unknown speed and direction. We start with a sequence of image frames $\mathbf{V}(k)$ for time

index $k = 1, \dots, N$. First, SALTAD removes hot pixels from each frame and equalizes the frames to provide uniform background levels. Then, it subtracts a mean image $\langle \mathbf{V} \rangle$ from each frame to remove stationary and nonfluctuating image components. The mean-subtracted images are $\mathbf{V}(k) - \langle \mathbf{V} \rangle$. To deal with locally changing noise variance, a whitening step is applied. With the estimated noise covariance matrix given by $\mathbf{R}$, each mean-subtracted, whitened image frame is calculated as $\mathbf{R}^{-1/2}[\mathbf{V}(k) - \langle \mathbf{V} \rangle]$. Additionally, each mean-subtracted, whitened frame is convolved with the Gaussian PSF before the matched filter operation.

During the matched filter operation, frames are shifted and stacked along the hypothesized motion template. The template has the same dimensions as the data frames but contains a single nonzero element per frame. The set of templates $\mathbf{T}(k)$ captures the linear spatial displacement as a function of the time index k . In our case, each motion hypothesis follows a constant on-sky velocity. The signal template must also be whitened, giving $\mathbf{R}^{-1/2}\mathbf{T}(k)$. Dropping the temporal index and flattening all arrays to one-dimensional vectors, the matched filter output is given by $M = \mathbf{T}'\mathbf{R}^{-1}(\mathbf{V} - \langle \mathbf{V} \rangle)$, where the prime is the transpose operator. Because the mean-subtracted, whitened image frame pixels approximate the individual-frame signal strength after mean subtraction and whitening, the matched filter operation yields an estimate of the sum of the individual-frame signal,

$$M \simeq \sum_{k=1}^N signal_k. \quad (3.1)$$

To assess whether a detection is present, the value of the matched filter output is scaled by the normalization term $\mathbf{T}'\mathbf{R}^{-1}\mathbf{T}$:

$$TRT = \frac{M}{\mathbf{T}'\mathbf{R}^{-1}\mathbf{T}}. \quad (3.2)$$

If TRT exceeds a user-defined value that yields an acceptable false-alarm rate, one can conclude that a candidate object is present.

However, for its first-pass detection processing, SALTAD uses a maximum likelihood estimate (MLE) of the form

$$MLE = \frac{M^2}{Var(M)}, \quad (3.3)$$

where M is the matched filter output and $Var(M)$ is the local variance estimate of the matched filter statistic in a donut region between 5 and 15 pixels from the pixel under test. This MLE value is equal to the square of the sum of the individual-frame SNRs

$$MLE = \left(\sum_{k=1}^N SNR_k \right)^2. \quad (3.4)$$

Interpreting the MLE criterion as the true maximum likelihood of the matched filter requires the assumptions that the noise is well represented by a Gaussian noise model and that the likelihood is a Whittle likelihood, the approximate likelihood for a stationary Gaussian time series.

For each motion hypothesis, the algorithm sorts the hits in descending order of MLE and keeps track of the total number of hits with MLE values above a specific threshold. This threshold is adaptively adjusted by the SALTAD algorithm in an attempt to produce a roughly constant number of hits for each motion hypothesis. It is important to note that the value of the MLE threshold used by the SALTAD algorithm to report a detection is not identical to the MLE value that we use to report a detection. Instead, we examine the hits proposed by SALTAD and apply our own detection threshold (Sections 3.2.6 and 3.3.1).

Because the image registration process and the shifting and stacking process are both computationally intensive, we must accelerate our computing pipeline in order to search the entire sky within a reasonable time frame. While the GPU implementation of SALTAD can in theory greatly accelerate this process, the GPU acceleration is only applied to the shifting and stacking of each individual set of images within a patch, not to the other SALTAD processes or our co-registration process. Because of the small sizes of each patch and the need to co-register each set of images, the GPU functionality of SALTAD does not

give sufficient acceleration to complete our search of the entire sky. Therefore, we choose instead to use the Python multiprocessing pool function to split our computations across 18 CPUs.

3.2.5 SALTAD Settings

We made a couple of small adjustments to the SALTAD software in order to better suit the WISE dataset. First, we changed the run number from a four-digit number to a nine-digit number to allow for the entire patch number and tract number to be included in the run number, which we refer to as the patch ID. For example, the 234th patch within tract 6 is denoted with patch ID 060000234. We use 32-bit floating point file format images for input into SALTAD, where non-data values in the WISE images are given a NaN pixel value. As such, we also modified the function `HotPixelRemoval` to replace non-data valued pixels with the image-wide mean value of the valid pixels. We add a function called `FitsSave` to allow FITS images to be saved at any point in the SALTAD processing. Because WISE W3 pixels capture $\sim 2.8''$ of the sky, several times larger than those of optical instruments, we also modify the asteroid centroiding software to choose its optimal centroid value for a detection from a 3×3 pixel grid around the pixel that was stacked along a motion hypothesis rather than a 7×7 pixel grid.

Our choices of user-defined settings for SALTAD were guided by a desire to maximize processing speed without compromising our ability to detect novel moving objects. We chose to use a global starless mean for the background processing rather than a local mean since it provided much higher computing speeds with similar results. We believe this speedup is possible because our images generally contain relatively uniform background noise. We use a Gaussian analytic model for PSF processing with a template extent of 3 pixels and a full-width at half maximum (FWHM) of 2.4 pixels, where the pixel scale is 2.757 arcseconds per pixel (Cutri et al., 2020). Our choice for the FWHM is based on averaging the FWHM for the major and minor axes of the central W3 PSF, i.e., 7.36 arcsec and 6.08 arcsec, respectively (Cutri et al., 2020). As different observations of an

object appear in different parts of the frame and the shape of the PSF varies slightly across the focal plane, we expect the axial difference to be effectively blurred when combining all detections for one hit. For the removal of the mean, we use the temporal mean of each pixel in the set of frames. We choose the raster scan search option for hypothesis processing, with a trailed pixel limitation of 1 pixel.

We use bilinear interpolation for shifting and stacking, and we set the SNR constancy to 6 dB. Noise is estimated based on local variance of pre shifted-and-stacked images, where the noise variance is shifted-and-stacked along with the images themselves.

We set the minimum TRT value of 0.7 to flag potential detections. We choose a starting MLE threshold for a hit of 8.5 dB but observed that this threshold was promptly modified by SALTAD to values closer to 6 or 7 dB. Our own MLE threshold for detection is set to 8.3 dB based on completeness and false discovery rate results (Section 3.3.1). SALTAD computes MLE in dB as $5 \times \log_{10}(MLE)$. Therefore, the SNR of the stack at our detection threshold corresponds to $\sqrt{10^{8.3/5}} = \sqrt{45.7} = 6.76$. The MLE and TRT statistics are described in detail in Section 3.2.4 and Mohanty (1981) and Gural et al. (2005).

We set the maximum number of hits per subimage to 5000, with a maximum of 0.0005 hits per motion hypothesis per pixel squared, which gives a maximum of around 173 hits per hypothesis for a subfield near the minimum allowed area. These settings allow us to recover almost all known and recoverable objects with reasonable computational cost.

Examples of SALTAD configuration files are provided in SOM.

3.2.6 Evaluation of False Candidate Fraction

There is a tradeoff between the completeness of our survey and the rate of false discoveries we make. To find the MLE cutoff that gives the desired balance, we run SALTAD for a subset of patches with the images for each patch arranged out of sequence.

We use a subset of the sky consisting of 1000 patches, with the first patch located at ecliptic longitude = 180° and latitude = 0° . Our test area includes subfields between

160° and 197° ecliptic longitude and -37° and 38° ecliptic latitude. Of these 1000 patches, 802 result in viable subfields with ≥ 4 images. We co-registered the images as we did previously, but we scrambled the order of the images presented to SALTAD in order to ensure that no real objects could be recovered. We scramble the image order randomly, scrambling again if three or more images remain in chronological order or antichronological order. We process these scrambled images through SALTAD.

3.2.7 Evaluation of Completeness or Detectable Fraction of Real Asteroids

To evaluate the completeness of our search, we compare our real detections in correctly ordered images to ephemerides or astrometry of known objects.

We use the Skybot system¹ to compare the locations of our hits to the locations of known solar system objects. For each hit, we calculate the angular separation between the detection in each frame and the known object and compute the average angular separation between the hit and the known object. If this average separation is less than 3", we assume that the detection corresponds to the known object. We can detect a sizable fraction of the objects in the field with SALTAD (Figure 3.5, left). Some bright objects with V magnitude < 15 are not detectable because the central pixel or pixels where they are located in each frame are saturated and replaced with non-data values within the WISE single exposure images (Cutri et al., 2020). In the cases where saturation is not an issue, their signal still bleeds across multiple pixels such that PSF fits with the 2.4 pixel FWHM are inadequate. Even if SALTAD manages to produce a hit related to a bright object, the poor astrometric accuracy resulting from bleeding may yield an average discrepancy with the ephemeris in excess of 3" such that the hit does not get associated with the known object.

We also compare our observations to astrometry reported by the NEOWISE team (described further in 3.2.8.2), where a hit with two or more detections within 3" of

¹<https://ssp.imcce.fr/webservices/skybot/>

NEOWISE-reported astrometry of a known object is considered a match.

3.2.8 Additional Filtering of Hits

To minimize false positives, we apply a number of additional filters to the list of hits obtained with SALTAD, which themselves have passed multiple filters, including S/N constancy, signal constancy, and bounds on the ratios of S/N values at the middle frame relative to the first and last frames. The limit for the S/N constancy and the signal constancy filters is 6 dB. The limit for the first/middle and middle/end ratios is 4 dB. This 4 dB middle frame filter is only applied if there are less than 6 frames in the stack.

Importantly, the S/N (MLE cutoff) filter is not the first filter we apply because we want to identify observations of known objects that may fall below the S/N threshold.

3.2.8.1 Removal of duplicate hits

We first check for duplicate hits within the same subfield. We check each detection against each other detection in the given subfield. One or more hits are considered duplicates of a given hit if the mean of the angular separations between the corresponding detections in each hit is less than $3''$, where the mean is computed over all frames where the source is detected. If duplicate hits are found, we keep only the hit with the highest MLE. This step insures that all reported astrometry comes from independent frames, with no duplicates.

Of the remaining hits that are above the MLE threshold, we also check for individual SALTAD detections that are within $5''$ of another SALTAD detection within the subfield. We wish to avoid submitting such detections because the MPC designates them as near duplicates and does not process them.

This situation can occur, for instance, when two different objects cross paths on the sky (a conjunction). If a duplicate is found, we keep only the detection with the highest MLE and remove the other detection(s) from their respective hits. This process can result

in the loss of an entire hit if the number of detections drops below the required minimum. We find that 0.65% of hits are eliminated at this step of the pipeline. Such duplicate detections need to be eliminated only if they occur at the same epoch. However, a mistake in our software resulted in a comparison of all detections above the MLE threshold within a subfield to each other, regardless of epoch. We estimate that we erroneously eliminated fewer than 2% of all valid detections, with a small impact on our overall results.

3.2.8.2 Non-reporting of previously reported astrometry

We determine whether a hit corresponds to an asteroid already reported by the NEOWISE team. We do not submit previously reported astrometry, but we do submit astrometry for detections unreported by the NEOWISE team.

Based on the MPC catalogs of numbered objects and multi-opposition unnumbered objects (Oct. 20, 2024 versions) with observatory code C51, we compare the NEOWISE-reported detections to the SALTAD detections in each frame. If they are within 3" of each other in two or more frames, we consider our hit to correspond to the NEOWISE-reported object. We also consider a hit to correspond to the NEOWISE-reported object if it corresponds to an object known to Skybot that has astrometry reported by the NEOWISE team anywhere in the same frame.

3.2.8.3 Non-reporting of conjunctions with previously reported astrometry

If the hit is not a NEOWISE-reported object but there is a NEOWISE-reported detection within 5" of at least one of the detections in the hit, the hit is eliminated. This step is taken to avoid reporting near-duplicates of NEOWISE-reported detections.

While this can occur for a conjunction of two separate objects, in which case we would be throwing out a good hit, we find that the discarded hits usually have similar on-sky velocities as the objects associated with NEOWISE-reported detections, but evidently without approaching within 3" of the NEOWISE-reported detections in two or more

frames. Hits thrown out for conjunctions with NEOWISE-reported detections combined with hits thrown out due to a designation conflict make up 0.094% of eliminated hits, where a designation conflict can include two or more hits corresponding to the same object designation or one hit with two or more corresponding object designations. This suggests a negligible loss of good hits due to this filter.

3.2.8.4 Non-reporting of single detections

We submit astrometry of known objects in frames where the NEOWISE team did not report astrometry, but only if we find at least two unreported detections, as the MPC does not accept submissions of single detections on a given “night”. Note that all hits identified by SALTAD begin with at least 4 detections. However, individual detections are frequently removed by our additional filters (Section 3.2.8), usually due to being previously reported by the NEOWISE team. Hence, we occasionally report only 2 or 3 detections, frequently 4 detections, and fairly often more than 4 detections.

3.2.8.5 S/N filter

We use an MLE cutoff filter to exclude all hits with an $\text{MLE} < 8.3$ dB.

3.2.8.6 Near bright object filter

We found that multiple hits near bright objects generally indicate spurious detections, and we use a filter to eliminate such hits. We label detections as “bright objects” if they have a Skybot estimated V magnitude of < 16 , if they are labeled as a ‘Comet’ class by Skybot, or if they are not a previously known object but have an $\text{MLE} > 11$ dB. We check whether more than 4 remaining hits have a mean separation from a bright object within 400 arcseconds or a mean separation from the estimated position of any potential ghost image of the bright object as specified in [Cutri et al. \(2020\)](#) within 90 arcseconds. If so, we discard the corresponding hits.

3.2.8.7 Edge effect filter

Hits that have either a vertical or horizontal extent of 1 pixel across all frames and have a median x or y pixel value that is within 5 pixels of any edge of the subfield are eliminated to ensure that edge effects do not produce spurious hits.

3.2.8.8 Apparent velocity filter

We estimate the on-sky velocity associated with each hit and test whether it lies within the two-sigma region of a Gaussian kernel density estimation (KDE) for the distribution of ecliptic rates of apparent motion (Figures 3.1 and 3.3). Hits that do not lie within this region are eliminated, including a fraction of hits that likely correspond to real objects.

3.2.8.9 Background source filter

We check whether any detection in a stack is within $5''$ of a stationary source listed in the AllWISE database (Wright et al., 2013). Any such detection is eliminated. If four or more detections remain in the stack, we recalculate the MLE and eliminate the hit if the updated MLE falls below the 8.3 dB threshold. If fewer than four detections remain in the hit, the hit is also eliminated.

3.2.8.10 Dominant frame filter

We eliminate hits where the signal is dominated by bright pixels in a single frame that somehow escaped SALTAD’s S/N constancy filter. Specifically, we identify the brightest pixel in a 3×3 pixel region around the detection centroid in each of the N frames. If the brightest pixel among all N frames minus the median of the N brightest pixels exceeds 70% of the brightest pixel among all N frames, we discard the hit. We assume that such a single bright pixel in only a single frame is indicative of noise rather than a real object.

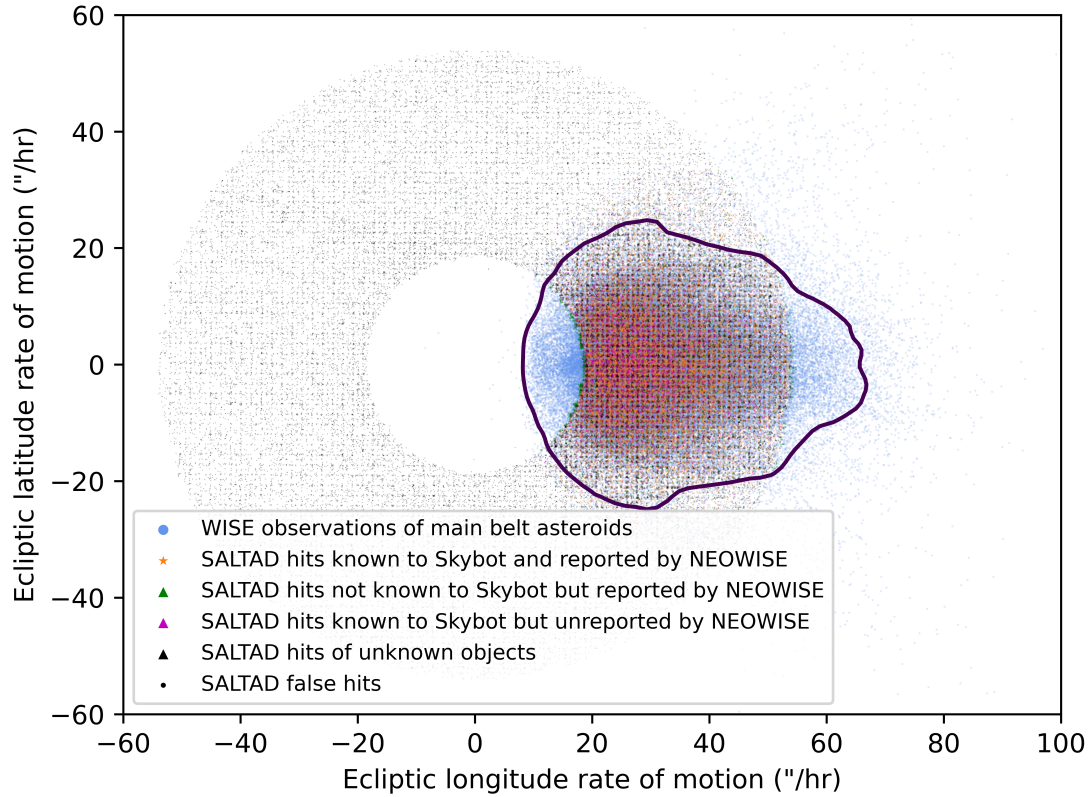


Figure 3.3: On-sky velocities corresponding to SALTAD hits, with ecliptic latitude and longitude rates of motion shown in arcseconds per hour. The light blue dots show all main belt asteroids observed during the cryogenic portion of the WISE survey, with the purple shape outlining the 2σ contour for the on-sky velocity distribution of these main belt asteroids. Orange stars represent objects detected by SALTAD and known to Skybot. Green triangles represent SALTAD hits matching known objects with astrometry reported by the NEOWISE team, whereas pink triangles represent SALTAD hits matching objects known to Skybot with no corresponding WISE astrometry. Most objects observed by the NEOWISE team are in Skybot’s catalog. Black triangles represent hits of previously unknown objects that survived all the pipeline filters while black dots represent SALTAD hits that survived all the pipeline filters with the exception of the 2σ velocity cut or were eliminated during the merging process or later. The majority of SALTAD hits fall within the 2σ contour. Those that do not are eliminated by our pipeline filters, including some potentially false negatives, as around 5% of main-belt objects are expected to fall outside of the contour. For details about figure construction, see 3.3.5.

3.2.9 Merging of True Hits

Because our search proceeds via tracts and patches, it is possible that an object detected in one patch will also be detected in another patch. Therefore, we search for matching objects in any subfields that have overlapping areas. A true hit is considered a match of another true hit if there are at least two instances where a detection from one hit is located in the same WISE frame and is within 3" of a detection from the other hit. A match is also identified if two true hits have the same asteroid number or MPC designation. Matches without an asteroid number or designation are merged for reporting into a single internal designation, with astrometry reported for the true hit with the highest MLE value in each frame. However, true hits associated with the same known object are not merged prior to reporting. To avoid submitting duplicate detections from different tracts, we search previously submitted detections of the same object. When we encounter two detections of the same object in the same frame – e.g., from two distinct *candidates* – we submit only the detection from the candidate with the highest MLE. If we already reported the other detection to the MPC, i.e., the other detection was in a previously searched tract, the detection from the current tract being searched is not submitted. True hits that have undergone the merging process and true hits to known objects are referred to as *candidates*, whether matching hits were found or not.

3.2.9.1 Too many hits and near duplicates

During the merging process, we include a few additional filtering steps for issues that occur between hits in different subfields or issues that are only evident after detection culling of an entire subfield is complete. We eliminate true hits not matched to a known object in subfields that contain more than 10 true hits not matched to a known object, as we suspect these hits to be spurious and caused by issues in one or more frames that evaded our frame filtering system. For reference, there are on average 1.6 true hits per subfield that contains at least one true hit. We also eliminate any near-duplicate detections, i.e., a

detection that is $< 5''$ from another detection in the detections table where the true hit doesn't overlap another true hit. See Appendix B.1.1 for details.

3.2.10 Suspected Comet Candidates

We find a number of high S/N candidates ($\text{MLE} > 11$ dB) with intriguing characteristics unrelated to known objects. They are surrounded by multiple nearby hits. Because we expect most candidate main belt asteroids with an $\text{MLE} > 11$ to be known objects, we suspect that these candidates are comets, where the nearby bright hits may be indicative of cometary debris. If we find more than 4 hits within 400 arcseconds of the putative comet or within 90 arcseconds of a potential optical ghost of the putative comet, we flag the bright candidate with a 'cometary candidate' ('CC') code. All other hits associated with the bright hit are removed from the pipeline at the bright object filter (see 3.2.8.6), as we suspect them to be image artifacts or fragments. These suspected cometary candidates were each inspected visually to determine whether they are real or are latents from bright known objects caused by image persistence of the CCD. If the cometary candidate was found to be of non-cometary origin, the flag for these objects was then changed to 'fake comet' ('FC'), and detections were not submitted to the MPC.

3.2.11 Visual Inspection of Discovery Candidates

We visually inspect the 21×21 pixel region surrounding the centroid in each frame and in the stacked frames for each of our remaining discovery candidates to catch any false positives which may have slipped through our series of filters. We eliminate candidates with the following qualities:

- A nearby bright object, such as a bright star or asteroid, is artificially inflating the signal strength for the candidate.
- The candidate appears to be picking up signal from a ghost image of a bright asteroid.

- The candidate is located within a bright streak caused by a comet or satellite.
- The candidate is located within a region that appears to have an uneven background in one or more frames.
- The signal for the candidate appears dominated by one or more frames where the central signal appears to be coming from a non-extended bright non-Gaussian source, resembling hot pixels or cosmic ray hits.
- The signal in the stacked frame creates an apparent plus-sign shape, potentially indicative of hot pixels.
- The signal in one or more frames has a very extended, non-Gaussian shape that does not resemble a comet tail.

3.2.12 Submission of Astrometry and V Magnitude Estimate to MPC

We coordinated extensively with the MPC prior to submitting astrometry. We submitted a detailed astrometric submission proposal that underwent MPC review, answered follow-up questions in a Zoom call with the MPC team, and submitted a test batch that underwent critical MPC review. A few minor issues were detected in the test batch regarding duplicate detections or mislabeling that we addressed in subsequent submissions.

We submit the astrometry to the MPC after all astrometry has been merged, one tract at a time, where each tract covers 1/12th of the sky. For each tract, we create files for submission of astrometric observations in MPC-readable ADES PSV format; one file for confirmed candidates and one file for discovery candidates. For the discovery candidates, we roughly estimate a magnitude for each candidate based on the MLE (see Appendix Section B.2 for details).

Figure 3.4 shows the complete asteroid detection pipeline.

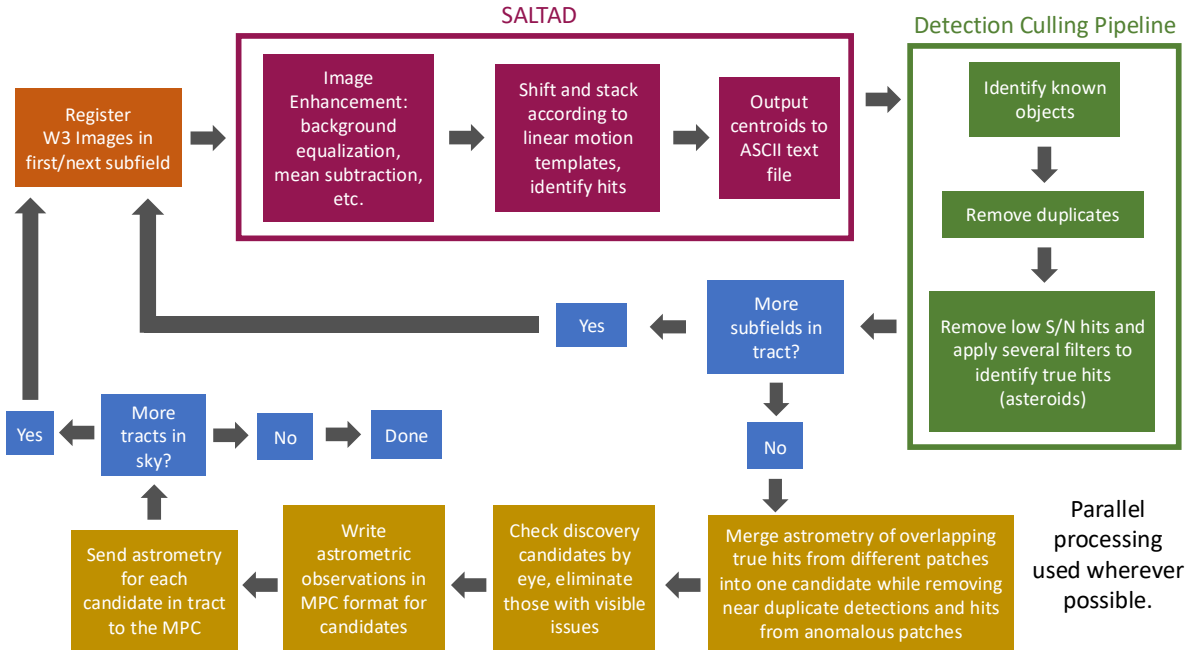


Figure 3.4: Diagram of the asteroid shift-and-stack detection pipeline. The detection culling pipeline has been simplified here for easier visualization. For a detailed breakdown of all steps in the detection culling pipeline, see Section 3.2.8.

3.2.13 Orbit Determination

Asteroids can typically be detected with WISE in 16 frames spanning 1.5 days (Lam et al., 2022). For discovery candidates, the median number of detections reported was 4 with a median arc length of 17 hours. Due to the short astrometric arcs, orbital element uncertainties are large, which makes recovery challenging. However, we submitted all astrometric positions to the MPC, and we anticipate that some of our detections will be linked by MPC staff or others to known objects or other observations in the isolated tracklets file.

Additionally, the Vera Rubin Telescope is rapidly approaching nominal operations (Robinson et al., 2025) and will discover millions of main belt asteroids (Željko et al., 2019). Our detections with WISE data from 2010 and arcsecond astrometric accuracy will immediately extend the arcs of any of these objects rediscovered by LSST from days/weeks/months to over 15 years, greatly improving the orbit solutions and eliminating

the possibility of losing these objects.

Our detections of confirmed candidates have served as precoveries for at least 346 known objects (see Section 3.3.13 for details). In addition, 66 of our discovery candidates have also been identified by the MPC as matching a known object (see Section 3.3.6 for details), and in 44 of these cases our detections constitute precovery observations. These links strongly suggest that additional links between our detections and future detections from Rubin/LSST and other telescopes will be made.

3.3 Results

3.3.1 False Candidate Fraction Results

Our evaluation of the false discovery rate relies on detecting hits in out-of-sequence images (Section 3.2.6).

We run SALTAD’s suggested hits through the filters described in Section 3.2.8, and we quantify the total number surviving hits, all of which are false (i.e., fortuitous collections of detections that do not correspond to a simulated or real object). For the statistical analyses of the hits that have gone through these additional filters, we consider the list of hits returned by SALTAD to be the total population.

There are many more hypotheses that SALTAD accepts and recognizes as hits than there are expected detectable asteroids within a subfield. For this reason, we find that the *fraction of false positives per subfield* provides a more meaningful performance metric than the false positive rate. We find the number of false positives from our testing at a variety of MLE cutoffs. For the chosen MLE cutoff of 45.7 (8.3 dB), 49 hits labeled as “true hits” are apparent false positives. After visually inspecting each false detection using the same criteria used for visual inspection of candidates from the real pipeline, we find and eliminate one plus-sign shaped hit, giving a final false positive count of 48 false positives. This result yields a rate of $48/802 = 0.060$ false candidates per subfield.

Considering that we find 119,165 total viable subfields across the sky, if we assume that our test region of the sky is representative of the entire survey region, we expect $\sim 7,100$ false candidates. Given that we observe 66,511 known objects and 8,223 discovery candidates in total across these viable subfields in the actual survey, we estimate a false candidate fraction of $7,100/(66,511 + 8,223) = 9.5\%$. Because most of our candidates are previously known objects, and thus are decidedly not false positives, this means that a significant fraction of our discovery detections will be false positives, with around 1,100 expected to be real. Therefore, these discovery candidates should not be considered to be confirmed until they are linked by the Minor Planet Center to additional observations.

Table 3.2: False Positive Rates

MLE Cutoff (dB)	Negative Hits	Surviving Hits (False)
7.9	1924026	730
8.0	1924380	376
8.1	1924577	179
8.2	1924659	97
8.3	1924707	49
8.4	1924738	18

NOTE— Number of hits not surviving our automatic filters and number of false automatic-filter-surviving hits as a function of MLE cutoff for the 1924756 total hits across 802 subfields in a large test region (see text).

We observed that certain systematic effects affect performance, including latent images of moving objects and gradients across the images – such as those caused by proximity to the bright Moon – and linear features – such as those due to comet trails, satellite trails, and their latent images. While we have attempted to design filters to eliminate most of these false hits, some may still get through. Time-correlated systematics include any features which vary non-randomly over time relative to the background stars, including latent images of moving objects.

To calculate the number of false positives that we would expect due to random Gaussian noise, we can multiply the number of hypotheses tested for a subfield by the p-value corresponding to our MLE cutoff value. With our MLE cutoff of 8.3 dB (45.7), the SNR

of the stack is ~ 6.76 , and the probability of exceeding this SNR by chance alone is 1.38×10^{-11} . Taking one subfield (patch ID = 060013508) as a representative example, we test 10,384 motion hypotheses at each one of the 626×593 pixels, resulting in a total of 3.85×10^9 hypotheses tested. Therefore, the number of false positives we expect in this subfield is $3.85 \times 10^9 \times 1.38 \times 10^{-11} = 0.053$. Across the 802 viable subfields, there are 5.55×10^{12} tests, yielding approximately 77 false positives ($5.55 \times 10^{12} \times 1.38 \times 10^{-11} = 76.5$). This value is not far off from the measured value of 49 false positives for this area.

3.3.2 Completeness Results

Following the methods outlined in Section 3.2.7, we assess the completeness of our search by comparing the list of objects known to Skybot and/or NEOWISE that were detected by SALTAD and quantify how many of the SALTAD hits had an MLE greater than or equal to a test MLE threshold (Figure 3.5, right). Here, we define completeness as the number of known asteroids detected by SALTAD whose corresponding hit is at or above the MLE cutoff divided by the number of known asteroids detected by SALTAD.

At our chosen MLE cutoff of 8.3 dB, we maintain a completeness near 83% based on the fraction of the sky covered by our full search (Figure 3.5, left).

3.3.3 Sky Coverage

We ran our pipeline for the entire sky one tract at a time, where the sky was split into 12 tracts.

The image elimination step (Section 3.2.3.2) reduces the sky coverage of our search. Our elimination of frames taken during momentum dump operations yield four rectangular-shaped gaps near the ecliptic poles (Figure 3.2). The momentum dumping zones account for 10.2% of the sky. Our uneven background filter primarily eliminates images in a large swath along the galactic plane, with regions within approximately 7° of the galactic plane

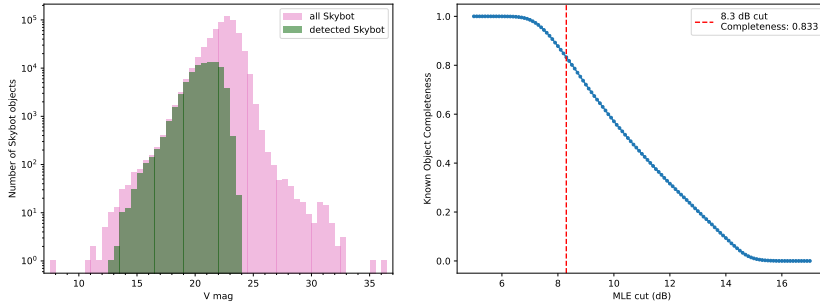


Figure 3.5: Left: Histogram of the number of known objects expected within all subfields combined, with bins of 0.5 in V magnitude, as reported by the Skybot system. Objects expected in multiple subfields and/or detected by SALTAD in multiple subfields are included more than once. Four instances of an object in a subfield with Skybot estimated V magnitude = 0, representing 3 unique comets, are assumed to be inaccurate values and are excluded. Skybot-reported objects are shown in pink, whereas the fraction of those detected by SALTAD at any MLE level are shown in green. As expected, SALTAD appears to preferentially detect brighter objects. Right: Known object completeness as a function of MLE, i.e., percentage of known objects detected that have an MLE (i.e., S/N) above the MLEs shown on the x-axis. Our chosen cutoff at 8.3 dB yields an 83% completeness.

being lost. This area accounts for 12.2% of the sky. Other regions where we lose sky coverage are roughly vertical bands due to proximity to the bright partial or full Moon and uneven coverage from WISE survey strategy to avoid the bright Moon, and thin, roughly horizontal strips near the ecliptic due to background variation. We estimate that these regions amount to roughly 5% of the sky. Finally, we intentionally exclude patches within 10 degrees of the ecliptic poles (1.5% of the sky) because these regions are unlikely to host main belt asteroids and contain many frames, making them time-consuming for image registration. After eliminating regions near the moon and ecliptic poles, momentum dumping zones, and the galactic plane, approximately 71% of the sky remains.

We find that 119,165 of the 269,496 patches (44.2%) have enough good quality overlapping frames to create a subfield searchable by SALTAD. All viable patches are shown in Figure 3.2. The median number of overlapping frames in the viable patches is 5. We lose a substantial fraction of the sky because many patches do not yield at least four overlapping images that are of good quality and meet our minimum acceptable area for a subfield.

Subfields are, on average, slightly smaller than patches (94% in surface area) and do not necessarily center perfectly over their patch centers. The union of the areas of the viable subfields covers 39.8% of the sky.

3.3.4 Filtering of Hits

Before the filtering of hits (Section 3.2.8), SALTAD identified a total of 254,884,956 hits. Duplicate removal eliminates 0.65% of the hits. Next, 0.094% of the remaining hits are removed due to overlap of only a single detection with a known object or due to conflicting designations associated with the hit. The S/N filter ($\text{MLE} > 8.3 \text{ dB}$) eliminates 99.94% of the remaining hits. Approximately 1.1% of the remaining hits are eliminated by the near-bright-object filter due to proximity to a bright moving object. Only 0.097% of the remaining hits are eliminated by the edge effect filter. Approximately 31% of the remaining hits get eliminated by the apparent velocity filter. The background source filter eliminates 0.16% of remaining hits due to having a new MLE below the 8.3 dB threshold after elimination of detections too close to a background source, and then eliminates 4.1% of remaining hits because fewer than 4 frames remained after elimination of detections near a background source. Lastly, 0.73% of the remaining hits were eliminated by the dominant frame filter.

In total, our pipeline culls the list of hits from 254,884,956 to 100,557 hits, giving a 99.96% elimination rate, where most of these were eliminated due to an insufficient S/N. We label these remaining hits as true hits. Among those, 90,675 correspond to previously known objects and the other 9,882 are attributed to previously undiscovered objects. At this stage, a hit that overlaps with at least two detections listed in the isolated tracklet file is considered a “known object” within our table. After removing the 15 hits with detections matching tracklets in the isolated tracklet file from our list of known objects and accounting for known objects identified in more than one patch, we find that the 90,675 true hits are due to 66,511 unique known objects.

During the merging process for the 9,882 hits not attributed to known objects, we

find 3 instances where a pair of hits must be merged into one discovery candidate. After eliminating hits in subfields with > 10 hits not attributed to known objects in one subfield, hits with fewer than 4 non-duplicates with detections from another true hit, false comet candidates, and hits from a subfield that accidentally included a NaN image, we have 8,773 discovery candidates.

During the visual inspection of the remaining discovery candidates, we eliminate 550 total discovery candidates. Each was eliminated for one or more of the reasons listed in Section 3.2.11. Of these, 167 included images near a bright object, 111 included ghost images, 76 included one or more detections located in or near a streak, 105 included images with background variation, 87 included a non-extended bright non-Gaussian source, 33 had a plus sign shape, and 9 included an extended source. This elimination process leaves 8,223 discovery candidates.

3.3.5 Distribution of Apparent Velocities

Figure 3.3 shows how hits are distributed in on-sky velocity space, with velocity split into ecliptic latitudinal and longitudinal components. To make this plot, we temporarily reordered our filters, with the apparent velocity filter applied last. As a result, we show only a small fraction of false hits, namely those that were eliminated by the apparent velocity filter or eliminated during the merging process. The purple contour is the region within 2σ of the apparent velocity distribution. Black triangles represent true hits whereas black dots represent false hits that passed through all filters of our pipeline except for the velocity filter or were eliminated during the merging process. 75% of hits making it through all filters except for the apparent velocity filter fall naturally within the 2σ contour of the apparent velocity distribution, where only 25% of remaining hits fall outside of the 2σ contour, despite the area within the 2σ contour making up $< 30\%$ of the parameter space searched. This fact demonstrates that our detection culling pipeline is effective at identifying candidates with asteroid-like motions.

3.3.6 Discovery Statistics

We identify 66,511 unique known objects in our results, where we call these objects confirmed candidates.

For comparison, there are 424,552 total objects known to Skybot and/or the NEOWISE team present in the area of the sky covered by viable subfields, with 111,818 of these having been reported by the NEOWISE team. Among the 66,511 confirmed candidates, we find 13,575 candidates with more than one good quality observation not previously reported to the MPC by the NEOWISE team. We report these new observations.

We find 8,223 discovery candidates that we submit to the MPC, 3 of these being candidates that are formed by merging two or more hits via the merging process. 15 additional candidates are linked to other hits via overlapping with detections we previously submitted to the MPC that are present in the isolated tracklet file. The ecliptic coordinates of all detections submitted to the MPC are shown in Figure 3.6, where blue dots represent detections that are part of a confirmed candidate and orange dots represent detections that are part of a discovery candidate. There is a greater proportion of discovery candidates at ecliptic latitudes above 20° than for confirmed candidates. Specifically, 95.6% of the submitted confirmed candidate detections are within 20° of the ecliptic, and only 43.7% of discovery candidate detections are within 20° of the ecliptic. For comparison, 37.3% of subfield centers are within 20° of the ecliptic. In terms of north-south differences, we find 51% of discovery candidates are above the ecliptic while 54% of confirmed candidates are above the ecliptic, suggesting a slight survey bias towards objects north of the ecliptic. These values suggest that the discovery candidate detections are more abundant closer to the ecliptic than would be expected if they were spread evenly across the search area, but not as strongly as for the confirmed candidates. At least two factors may explain this pattern. First, search programs may have preferentially favored regions of the sky near the ecliptic, resulting in a bias in the population of known objects. For instance, (Granvik et al., 2018) found that one of the most prolific asteroid discovery stations (Mt. Lemmon, IAU code G96), “concentrates primarily on the ecliptic”. Our survey, which

is closer to an all-sky survey, may therefore preferentially detect faint, high-inclination asteroids missed by previous surveys. Second, it is possible that our false positive rate is higher than we estimated. Because we use synthetic tracking, we are vulnerable to false positives coming from stacking the signal along any sort of linear feature that may appear in the SALTAD-processed images for the subfield.

For this reason, some caution about the discovery candidates is warranted, and the MPC has placed them in the isolated tracklets file. In the future, it will be possible to better evaluate the fraction of candidates that are successfully linked to detections from other surveys, including the Rubin Observatory.

So far, 66 of our discovery candidates have been identified by the MPC as previously known objects or received a provisional designation after our submission. In 5 cases, 2025 FX39, 2025 NZ210, 2025 OK26, 2025 UT66, and 2026 ET99, the objects were only identified and given a preliminary designation at the MPC after we submitted our detections. For the other 61 objects, each candidate presumably had a $> 3''$ average distance from the Skybot ephemeris and was not matched to any previous NEOWISE-observed objects and was consequently not labeled as a known object by our pipeline. Among the 66 discovery candidates that were identified as a previously known object by the MPC, 44 objects are cases where our observations provide the earliest astrometry, with a median observation arc extension of 4.7 years. The longest arc extension is 13 years for 2023 TH102. Therefore, in total, we find 8,157 discovery candidates that have yet to be linked to a known object or observed by others.

3.3.7 Astrometric Quality

We assess the accuracy of the SALTAD astrometry of known objects by computing the residuals with respect to the JPL HORIZONS ephemeris. We use HORIZONS rather than the MPC ephemeris as its API allows for higher precision in observation time. We submit observations to the MPC only when the rms of the residuals for a given true hit is $< 3''$. The residuals for all detections of confirmed candidates with listed orbital elements that

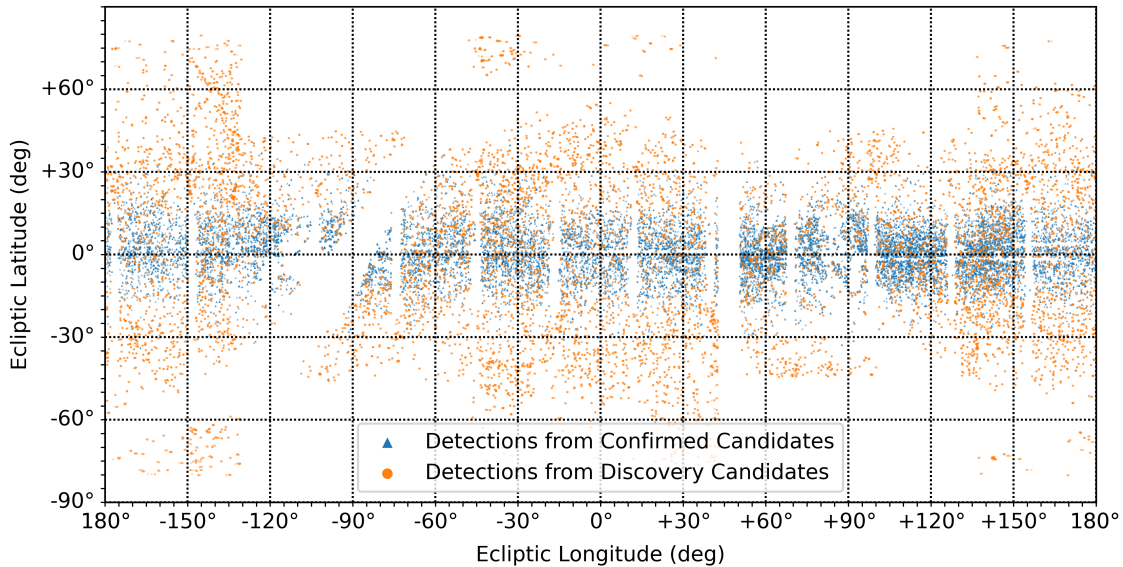


Figure 3.6: Ecliptic coordinates for all detections submitted to the MPC, excluding identified false positives. Blue dots represent detections that are part of a confirmed candidate and orange dots represent detections that are part of a discovery candidate.

were submitted to and accepted by the MPC in this work are shown in Figure 3.7. For all confirmed candidates included, we also show all detections for which astrometry has been submitted to the MPC by the NEOWISE team.

The median residual for the NEOWISE team submitted detections with a corresponding SALTAD true hit was $0.59''$, while the median residual for our submitted confirmed candidates was $1.7''$. Because astrometric accuracy depends on S/N, we expect higher astrometric accuracies for the high-SNR detections reported by the NEOWISE team and lower accuracies for our low-SNR detections.

We also compare the median residual for observations by the NEOWISE team and our team when the objects and observing epochs are identical, e.g., detections from confirmed candidates that we did not submit to the MPC because they had been previously reported by the NEOWISE team. In this case, our median residual error is $1.1''$ compared to NEOWISE’s median residual error of $0.6''$. However, for some of these objects, especially those with short astrometric arcs and/or few follow-up observations, the NEOWISE team observations play a significant role in fitting the ephemeris against which we are evaluating

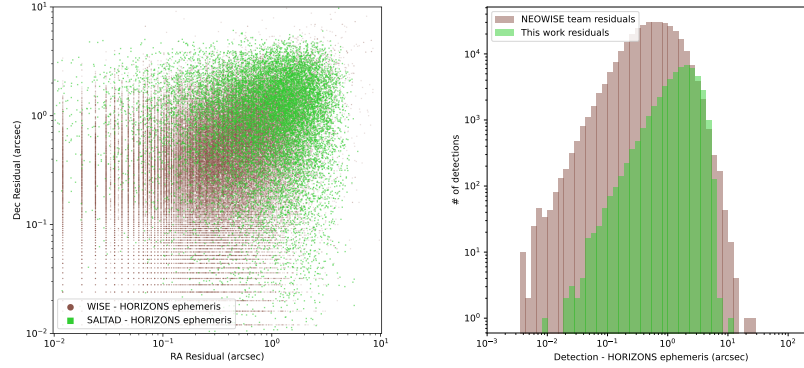


Figure 3.7: Astrometric residuals for all detections of known objects with listed orbital elements. Ephemeris values are found using the JPL HORIZONS service. Left: Residuals for all detections reported to the MPC in this work as well as residuals for NEOWISE team detections submitted to the MPC. Axes are clipped to RA and Dec residual values between 10^{-2} and 10 arcseconds for clarity. Right: Histogram of the residual values for all detections reported to the MPC in this work and for all NEOWISE team detections submitted to the MPC. Residuals below 10^{-4} are clipped for clarity.

our astrometry. Therefore, we fully expect the NEOWISE team residuals with respect to a NEOWISE-infused solution to be better than ours. If the ephemeris had been fitted to our data, the NEOWISE team astrometry would likely exhibit worse residuals. Other contributors to the high residuals are related to the SALTAD detection process, including the Gaussian blur imposed by convolution with a model PSF. SALTAD requires images in a subfield to be registered to a common WCS to avoid the need to track WCS positions as it shifts and stacks pixels, which introduces some astrometric error. To quantify this error, we considered five W3 single-exposure images from patch 06001310 and compared the astrometry of stationary sources extracted using SExtractor (Bertin and Arnouts, 1996) from the original, unregistered images and from the registered images. For matching sources (within $3''$ of each other), we calculated the mean RMS of the angular separation to be $0.42''$, suggesting that registration could be a significant error source. Therefore, our larger uncertainties can be attributed to a combination of factors: observation of typically fainter objects than NEOWISE, blurring by a model PSF, possible centroiding errors on faint objects, error introduced by co-registration of the images to a common

WCS, calculation of our residuals with respect to a NEOWISE-infused ephemeris, and perhaps other error sources that are not fully understood. Our astrometric accuracy does meet the principal requirement of enabling linking with other observations of the discovery candidates, as demonstrated by the 66 links established so far.

3.3.8 MLEs of Submitted Candidates

Our candidates submitted to the MPC span MLEs ranging from 5.8 to 15.6 dB for the confirmed candidates and 8.3 to 13.2 dB for the discovery candidates, shown in Figure 3.8. The median MLE of the submitted confirmed candidates was 8.2 dB, equivalent to $\text{SNR} = 6.6$, while the median MLE of the discovery candidates was 8.4 dB, equivalent to $\text{SNR} = 6.9$. Discovery candidates fall off faster with increasing MLE than the confirmed candidates, which is to be expected since brighter objects are more likely to have been previously discovered.

We identify NEOWISE discoveries as those objects that were not reported by us and appear in the MPCAT-OBS catalog with an observatory code equal to C51 and a discovery asterisk (*) in column 13 of the 80-column record. The median MLE for the moving objects discovered by the NEOWISE team is about 9.8 dB, giving an SNR of about 9.5. Therefore, our new detections of discovery candidates are on average 1.4 times fainter than the detections from the NEOWISE team’s analysis. However, our discoveries are artificially truncated at the faint end by the 8.3 dB detection threshold, so this comparison is imperfect. Because we stacked 4 or 5 frames in most subfields, we expect an increase in sensitivity of ~ 2 according to the $\sqrt{N}$ factor, where N is the number of frames stacked.

3.3.9 Known Objects Detected

We detected 66,511 known objects in our test area, with 13,575 of these confirmed candidates having at least 2 previously unreported detections, making their detections reportable to the MPC per MPC best practices. Among those, 13,461 have orbital

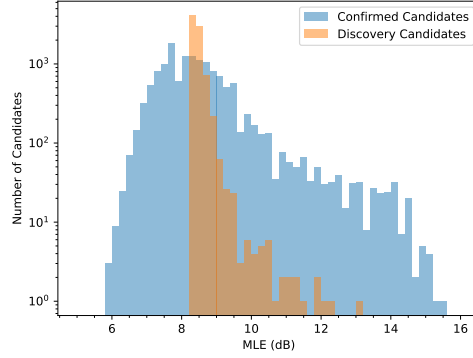


Figure 3.8: Histogram of MLE (dB) for all candidates submitted to the MPC, excluding identified false positives,
with confirmed candidates shown in blue and discovery candidates shown in orange.

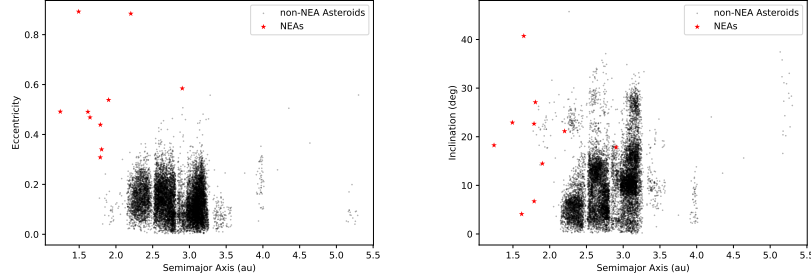


Figure 3.9: Orbital element plots for all confirmed candidates for which we reported detections to the MPC and for which orbital elements were available in MPCORB.DAT. Orbital elements for comets are not shown as they are not listed in MPCORB.DAT. Most objects are main-belt asteroids, but we also report several Jupiter Trojans and NEAs. Left: Semi-major axis vs. eccentricity. Right: Semi-major axis vs. inclination.

elements specified in MPCORB.DAT², and 10 of those have a perihelion < 1.3 au, making them Near-Earth Objects (NEOs). Figure 3.9 shows orbital elements for all confirmed candidates for which we reported detections and for which orbital elements were listed in MPCORB.DAT.

As a representative example, we provide details about three MBAs that have at least 3 new observations (Table 3.3).

Each of these confirmed candidates are shown in Figure 3.10. These images show the

²<http://www.minorplanetcenter.org/iau/MPCORB.html>

detections in each frame, as well as their shifted and stacked sum image in color, where the color corresponds to the number of standard deviations above the noise for the stacked image. We calculate the S/N associated with each pixel in each frame using the equation

$$(S/N)_{\text{pixel}} = \frac{\text{signal} - \text{background}}{\text{noise}}, \quad (3.5)$$

where signal is the pixel value and background is the mean value of the entire frame, excluding pixels ≥ 3 standard deviations above the mean and excluding pixels < 0 for not-yet demeaned images (see frames from the beginning and middle of SALTAD processing in the first and second rows of frames, respectively, for each candidate in Figures 3.11 and 3.12). Noise is the standard deviation of the frame with the same regions excluded as were excluded for calculating the background.

We stack each pixel using the equation

$$(S/N)_{\text{stack}} = \frac{\sum_f \text{signal}_f}{\sqrt{\sum_f \text{noise}_f^2}}, \quad (3.6)$$

where f is frame number and we sum over each frame in the stack. Each subframe is placed with the SALTAD-derived centroid of the PSF rounded to the nearest integer values as the center pixel, with the red dot located at the exact centroid. This stacking may differ slightly from SALTAD native stacking and give slightly different signal to noise values as SALTAD allows the centroid to be anywhere in a 3×3 box surrounding the stacked pixel (see section 3.2.5). We placed an orange “+” at the HORIZONS JPL ephemeris location and a blue “x” at the location previously reported by the NEOWISE team, if there is one. Frames without a blue “x” signify that the astrometry was not reported by the NEOWISE team.

The SALTAD centroids appear to remain close to the Skybot and HORIZONS ephemerides, demonstrating the accuracy of the SALTAD astrometry. The residuals for all confirmed candidates are shown in Figure 3.7.

Table 3.3: Selection of Known Objects with ≥ 3 New Observations

Patch ID	Hit #	MPC Number	MLE (dB)	H mag	Diameter (km)	Albedo	V mag
060013509	560	399850	7.1	18.56 ^a	-	-	22.5 ^c
060013535	1	529749	9.9	16.86 ^d	3.4 ± 0.5 ^b	0.027 ± 0.009 ^b	22.5 ^c
060013527	2	559187	8.7	16.64 ^e	-	-	21.9 ^c

References—^aMPO 745323, ^bMasiero et al. (2011), ^cBerthier et al. (2006), ^dMPO 731814, ^eMPEC 2023 X165

NOTE— Parameters of a selection of known objects detected by SALTAD with at least 3 previously unreported observations. The known object name associated with each SALTAD hit as well as the V magnitude are derived from the Skybot cone search, whereas H magnitude was extracted from MPC records and diameter and albedo were derived by the NEOWISE team.

SALTAD’s software includes image cleaning techniques such as hot pixel removal, background subtraction, mean processing, and whitening, and it also convolves each image with the instrument PSF before searching for detections, so the data will appear less noisy and smoother in Figure 3.10 than in the input images. Our detection of these and other known objects and their small residuals relative to the HORIZONS ephemerides demonstrates our ability to successfully detect main belt asteroids with sufficient astrometric accuracy.

3.3.10 Candidates Near the S/N Cutoff

We also highlight two candidates with an MLE of 8.3 dB, just above the S/N cutoff: a confirmed candidate at hit 3 in patch 060013192 showing known asteroid 198452, and a discovery candidate at hit 1 in patch 060013580, to which we assign the internal ID of AZ4eejR. Asteroid 198452 has an absolute magnitude of 16.18 as listed in MPO 683733, towards the faint end of what the NEOWISE team was able to observe within the cryogenic survey data as shown in Figure 7 of Masiero et al. (2011). We detect this object in six frames and find that no astrometry from these frames was reported to the MPC by the NEOWISE team. Therefore, we submit our detections of 198452 to the MPC. Both candidates are shown in Figure 3.11. In each set of rows, the top row shows the registered

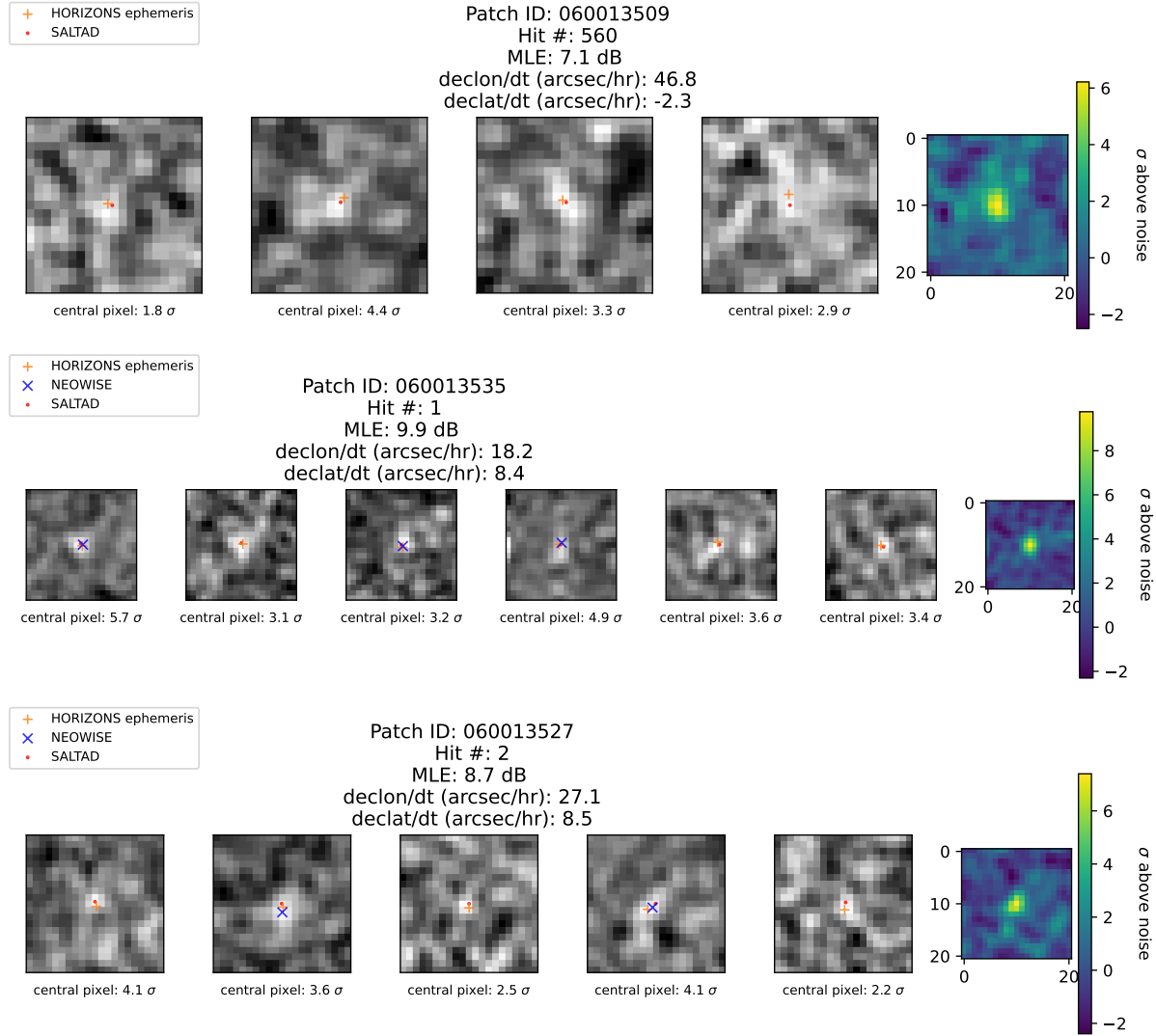


Figure 3.10: Example detections for known MBAs that have at least 3 previously unreported observations. Images show subframes of registered WISE images after SALTAD processing. Black and white images show the object in individual frames whereas the image in color shows the shifted and stacked image of the object, where the color conveys the number of standard deviations above the noise. We stack the images by summing the signal in each frame and dividing by the sum of squares of the noise for each frame. Red dots are located at the SALTAD detection centroid, an orange “+” appears at the location of the HORIZONS ephemeris, and a blue “x” at any locations previously reported by the NEOWISE team. The annotations at the bottom of each frame report the S/N of the central pixel only, which is not equivalent to the MLE dB of the true hit, which is listed at the top.

images for each hit before all SALTAD processing but with non-data values replaced with the mean value of the subframe, as is done in Figure 3.10; the middle row shows the images of each candidate after hot pixel removal, background estimation, and equalization of the frames by SALTAD; and the bottom row shows the frames at the latest stage of SALTAD processing, after image demeaning, whitening, and convolution with the PSF. While these sources are not obvious to the eye in the registered images or the equalized images, they become more apparent after convolution with the 2.4 pixel FWHM Gaussian PSF. The similar appearance of the candidates in both sets of images, with one being a confirmed candidate and the other a discovery candidate, provides additional evidence beyond our false discovery rate that hits at this MLE cutoff provide generally reliable detections.

3.3.11 High S/N Discovery Candidate

Finally, we highlight a high S/N discovery candidate, one of the merged discovery candidates, hit number 1 in patch 030010814 with an $\text{MLE} = 9.3$ dB (Figure 3.12).

3.3.12 Submission to MPC

We submit 85,235 detections to the MPC with the IAU observatory code C51 for WISE, representing 23,002 distinct hits and one comet (C/2007 Q3) observed by eye. Using the MPC’s summary of the “Where are my observations?” feature (SWAMO)³ to observe submissions for the WISE observatory code during our time of submissions, we find that, as of April 28, 2026, 47,715 of our detections were published in the Minor Planet Circulars (MPC 181535-183282, MPC 183283-184784, MPC 184785-185966, MPC 185967-187040, MPC 187041-188964, MPC 188965-191900), 34,816 of our detections were included in the isolated tracklet file⁴, 77 of our detections were discarded as near-duplicates, and 2627 of our detections were deleted. Most of these deletions were caused by our visual inspection

³<https://data.minorplanetcenter.net/swamo/>

⁴<https://minorplanetcenter.net/iau/ITF/itf.txt.gz>

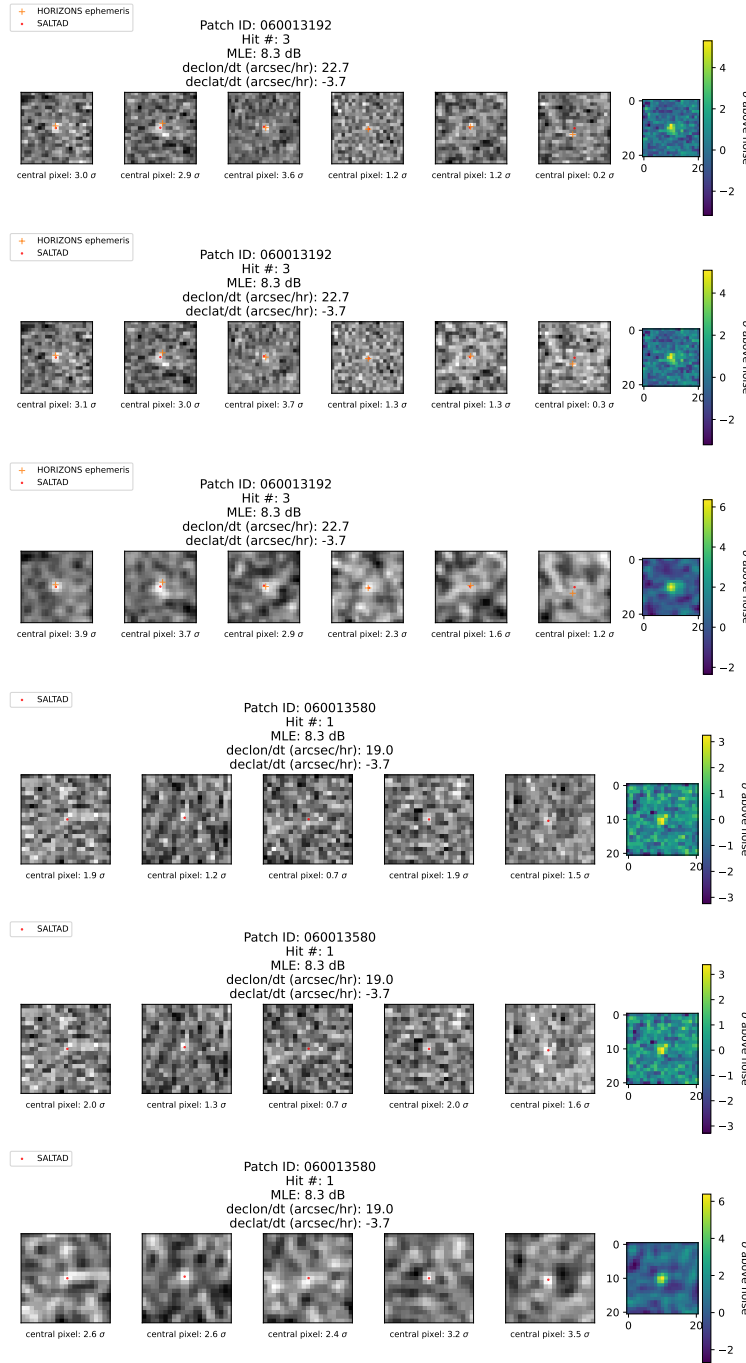


Figure 3.11: Images of two candidates with $\text{MLE} = 8.3 \text{ dB}$ at the beginning, middle, and end of SALTAD processing. The top three images show the detections of asteroid 198452. The top row shows the raw frames as read by SALTAD with any non-data values replaced by the mean value of the subframe. The middle row shows the same images after the image equalization step. The bottom image shows the same frames after convolution with the WISE PSF, as the frames are when they are shifted and stacked by SALTAD. The bottom three rows show a discovery candidate. Top, middle, and bottom rows as before.

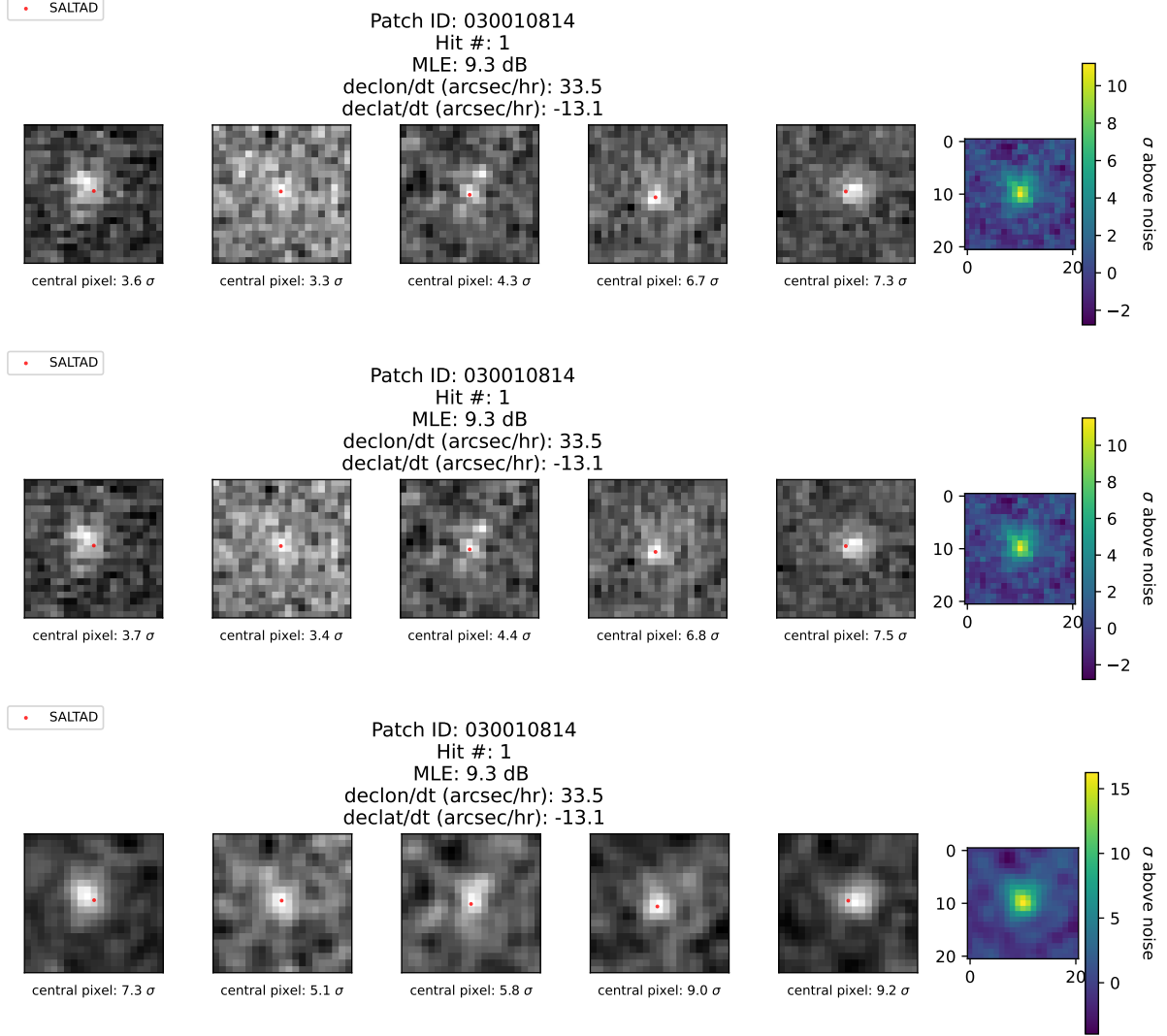


Figure 3.12: Images of discovery candidate with high S/N at the beginning, middle, and end of SALTAD processing. The top row shows the raw frames as read by SALTAD with any non-data values replaced by the mean value of the subframe. The middle row shows the same images after the image equalization step. The bottom image shows the same frames after convolution with the PSF, as the frames are when they are shifted and stacked by SALTAD.

of each discovery candidate being conducted after we had submitted our observations, necessitating the deletion of detections from visually identified false positives.

3.3.13 Precoveries for Known Objects

We search for instances where our confirmed candidate detections provide “precoveries”, i.e., the earliest astrometry for an object, thereby lengthening its orbital arc. We only search for precoveries on objects whose first detections listed by the MPC were made in 2011 or later. Therefore any precoveries for objects with discovery detections taken in 2010 are not included in the following results. Using MPCORB data from July 8, 2025, six days after we had finished submitting all of our detections, we find that our detections lengthen the orbital arc for 346 known objects, with a median time between our last observation and the first MPC-listed observation of 4.6 years. The longest time between our latest observation and the first MPC-listed observation is 9.7 years for asteroid 2020 CT5. None of these precoveries were for comets or NEOs.

3.3.14 Arc Lengths of Discovery Candidates

Our discovery candidates have an average arc length of 17 hours, with a minimum arc length of 3.2 hours (excluding one singlet candidate for which other submitted detections were later deleted) and a maximum arc length of 4.6 days. The distribution of the arc lengths of all discovery candidates is shown in Figure 3.13, showing a bimodal distribution with peaks around 0.5 days and 2.9 days. Detections from all discovery candidates were placed in the isolated tracklet file by the MPC, regardless of arc length.

3.3.15 Comets and Suspected Cometary Candidates

From SALTAD processing, we find at least two new detections for 4 comets: 145P, 230P, 248P, and C/2010 B1, where these detections were submitted to the MPC. While debugging our pipeline’s process for saving object names known to Skybot in the field, we also observed

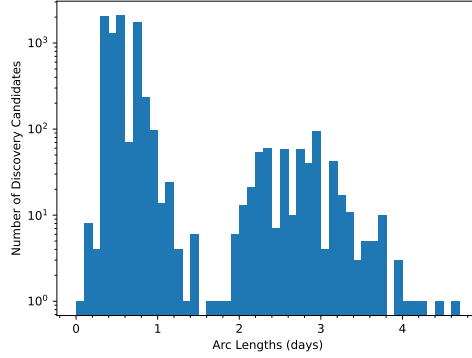


Figure 3.13: Number of good discovery candidates vs. arc length in days of our submitted detections. The average arc length is 17 hours. Note that the y-axis is scaled logarithmically.

C/2007 Q3 in the images. Because this comet appears very bright and diffuse, SALTAD was not able to detect it and instead obtained numerous false hits within the coma that were all eliminated due to there being more than 10 new object candidates within a single patch (Section 3.2.9.1). Detections from this object were not reported by the NEOWISE team, so we determined the centroid locations of the comet in each frame by eye and submitted the detections to the MPC. There were no other comets in the field known to Skybot that had a V mag suggesting they would be visible by eye, so no other objects were submitted with by-eye locations from this work. None of our detections lengthened a comet’s orbital arc.

In Figure 3.3, a region of slightly higher density of false hits can be seen extending in a linear fashion from around $(-40, 30)$ to $(50, 0)$. These false hits occur along the trajectory of comet 10P in several subfields. Its perihelion occurred on July 4th, 2010, in the middle of the cryogenic portion of the WISE mission. The comet itself was not detected by SALTAD.

Upon inspection, all hits flagged as suspected cometary candidates were discovered to be latent images created by large, bright main-belt objects appearing nearby on the sky but out of the frame. The flag for these objects was then changed to ‘FC’ for ‘fake comet’, and detections were not submitted for processing by the MPC.

3.4 Discussion

Our survey covers 39.8% of the sky after elimination of areas within 10 degrees of the ecliptic poles and frames within 20 degrees of the Moon and only using good quality frames to stack at least 4 images within a patch. While this coverage is less than ideal, the most severe loss of coverage occurs closer to the ecliptic poles, where fewer main belt asteroids reside. While most of the sky was observed in WISE W3 on average ~ 16 times during the cryogenic mission, stacking all of these images is precluded because they do not create a large enough subfield for objects with MBA on-sky velocities to be visible in each frame within the subfield. In other words, the asteroid exits the subfield when more than 4–9 images are stacked. The faster the on-sky velocity, the larger the subfield that is needed to be able to observe the body in all frames across the subfield. Discovery of near-Earth objects with synthetic tracking using WISE images is challenging because they tend to have large on-sky velocities that preclude detection across multiple frames in our subfields.

We detect 8,223 discovery candidates, i.e., 35% of the number of previously unknown MBAs with arc lengths > 1 day reported by the NEOWISE team with data from the WISE cryogenic mission. We suspect that hundreds of the discovery candidates will likely be detected by LSST such that our astrometry will extend the observation arcs of these objects from days or months in the early period of the LSST survey to around 15 years. Because some of our discovery candidates have already been successfully linked to known objects, including an object not recovered until 13 years after the WISE cryogenic mission, there is good evidence that our detections will be linkable to future discoveries from LSST and other surveys. The availability of much more precise orbital elements will enable more expedient numbering as well as multiple science investigations.

Based on the latitudinal distribution of our submitted astrometry (Figure 3.6), it is possible that there are a high number of false positives with an MLE cutoff of 8.3 dB. However, we find that the fraction of discovery candidate detections within 20° of the ecliptic increases only slightly to 50.6% if the MLE cutoff is increased to 8.5 dB while

the number of detections drops dramatically from 34,862 to 8,875. Therefore, the cost of using a higher S/N cutoff would be high, and the benefit would be limited. Additionally, all of our discovery candidate astrometry was placed in the isolated tracklet file, so only detections that are linkable will be assigned preliminary designations, which essentially nullifies the negative impact of any false positive.

Conducting a dedicated search for TNOs within the WISE dataset was beyond the scope of this investigation. However, such a search could prove fruitful, as their lower on-sky velocities would allow stacking of ~ 15 images in a subfield and enable detection of fainter objects. However, considering the short average arc lengths, the lower on-sky velocities of TNOs would mean that orbital uncertainties would be quite large, making for more challenging orbital linking. Using synthetic tracking to search for objects in the longer W4 wavelengths may be worth investigation as well.

3.5 Conclusions

We use synthetic tracking with SALTAD in order to find new moving objects in the WISE W3 dataset, where shifting and stacking images increases the signal-to-noise ratio by the square root of the number of images stacked. Following [Mazur et al. \(2018\)](#), we co-register and stack at least 4 images and at most 9 images from the cryogenic mission phase in subfields covering 39.8% of the sky. We intentionally exclude low-quality images, images with variable background, and images acquired during spacecraft momentum dumps, within 20° of the Moon, and within 10° of the ecliptic poles. We eliminate hits with a maximum likelihood estimate (MLE) below 8.3 dB and implement a series of quality filters that result in candidates with an estimated false discovery rate of 9.5%. We search for asteroids with on-sky velocities between 18.5 and 54.1 arcsec/hr, corresponding to the 5 – 95% percentiles for the distribution of main belt asteroid on-sky velocities. We submitted the astrometry from each frame to the Minor Planet Center for both newly discovered and previously known objects with at least three new objects detected. We find and submit

astrometry for 8,223 discovery candidates, where 8,157 of these are as yet unidentified. So far, 66 of these candidates have been identified as known objects by the MPC, where our observations extend the orbital arcs for 44 of them. As for previously known objects, we also submit astrometry for 13,575 confirmed candidates and have extended the orbital arcs for 346 of these objects. We expect that many of the discovery candidates will be recovered by the Rubin Observatory or other surveys and that our astrometry will greatly extend the observation arcs for hundreds of these small bodies.

This publication makes use of data products from the Wide-field Infrared Survey Explorer, which is a joint project of the University of California, Los Angeles, and the Jet Propulsion Laboratory/California Institute of Technology, funded by the National Aeronautics and Space Administration. First author was funded in part by Nathan P. Myhrvold Graduate Fellowship. Dr. Myhrvold provided the initial idea and motivation for this work in 2021. We thank the International Astronomical Union’s Minor Planet Center for their support and guidance. We also thank our three anonymous reviewers and our statistics editor whose helpful comments clarified and strengthened our paper.

Software used: SALTAD (Gural and Larsen, 2004; Gural et al., 2005, 2018), LSST Science Pipelines (Vera C. Rubin Observatory Science Pipelines Developers, 2025), Montage (Berriman et al., 2003), Skybot (Berthier et al., 2006), JPL Horizons (Giorgini et al., 1996), NumPy (Harris et al., 2020), SciPy (Virtanen et al., 2020), Pandas (Wes McKinney, 2010), Matplotlib (Hunter, 2007), Astropy (Astropy Collaboration et al., 2022) Cartopy (Met Office, 2015) Spherical Geometry (Simon et al., 2025) Healpy (Zonca et al., 2019)

CHAPTER 4

Measurement of the orbital expansion rate in the binary asteroid 1999 KW4 Moshup-Squannit

4.1 Introduction

Earth’s history has been punctuated by impacts from small bodies. While both near-Earth asteroids (NEAs) and comets pose a background risk to Earth, it is easier to observe and predict the locations of near-Earth asteroids, making them good targets of study. These asteroids are kicked out of the main asteroid belt by orbital drift due to the Yarkovsky effect and interaction with gravitational resonances of Jupiter and Saturn (Bottke et al., 2002). Based on dynamical simulations, a single near-Earth object is expected to have a lifetime of about 10 million years once it reaches near-Earth space before colliding with one of the terrestrial planets, disintegrating by close approach to the Sun, or ejecting from the Solar System (Gladman et al., 2000).

A significant fraction of near-Earth asteroids are binary systems, with estimates of 17% of NEAs plus or minus a factor of 2 based on lightcurve observations (Pravec and Harris, 2000), 16% of NEAs based on radar observations (Margot et al., 2002) and 15% of Earth crossing asteroids based on the prevalence of doublet craters on Earth and Venus paired with numerical modeling of tidal forces (Bottke and Melosh, 1996; Richardson et al., 1998). It is believed that most near-Earth asteroid binaries are formed via a rotational fission event (Margot et al., 2002) via a torque caused by uneven re-radiation of sunlight known as the YORP effect (Rubincam, 2000; Bottke et al., 2002; Jacobson and Scheeres, 2011). The origin of the fission event was once thought to be related to close gravitational

encounters with the planets (Bottke and Melosh, 1996; Asphaug and Benz, 1996; Merline et al., 2002; Margot et al., 2002), although gravitational encounters are not predicted to be the main formation mechanism based on modeling by Walsh and Richardson (2008).

One possible method of destruction for a binary system is for a collision to destroy one of the two components. Since smaller objects tend to have a shorter collisional lifetime, we will consider the case of a roughly 100 m secondary being collisionally disrupted. The collisional lifetime of a 100 m object in the main belt is about 70 Myr based on modeling displayed in Figure 14 of Bottke et al. (2005). However, there may be processes that disrupt binaries on timescales shorter than 70 Myr, including tidal evolution, BYORP evolution, and close planetary flybys. Torques that act on the binary system and cause it to expand or contract at a rate faster than the collisional lifetime or the near-Earth space lifetime of the system could cause the system to split into an asteroid pair or collapse back into a single asteroid body while in near-Earth space. However, close flybys of the inner planets may also act to split apart a binary system (Merline et al., 2002), so probabilities of such encounters should be considered as well. Confirming the predictions of how these systems form and the timescales over which they evolve is interesting physics in its own right, but is also critical for predicting when and how these bodies can pose an impact threat to Earth. We attempt to constrain the orbital expansion rate and lifetime of the 1999 KW4 system, one of the most well-studied NEA binary systems. In doing so, we hope to gain a glimpse into the workings of one of many NEA binary asteroid systems as it evolves in time.

4.2 The 1999 KW4 System

4.2.1 Shape and Libration Modeling

The near-Earth asteroid (66391) Moshup-Squannit, previously known by its preliminary designation of 1999 KW4, is a binary system with an orbital period of 17.42 ± 0.04 hours. Moshup is the primary and Squannit is the secondary. It is classified as a

potentially hazardous asteroid by the Minor Planet Center, meaning that its minimum orbital intersection distance (MOID) with respect to Earth is less than 0.05 au. High resolution radar images taken in 2001 provided the opportunity to use the shape modeling software (Hudson, 1994; Magri et al., 2007) to obtain a three-dimensional shape model of the asteroid and its binary companion, shown in Figure 4.1 (Ostro et al., 2006). This dataset also made it possible to estimate masses and densities of components via reflex motion of the secondary, allowing detailed study of the dynamics of the system (Scheeres et al., 2006).

The distinctive top shape with equatorial ridge, high spin rate, and low estimated density paired with dynamical simulations confirmed the hypothesis of asteroid fission via rotational spin-up by YORP and/or tidal effects due to close encounters with the inner planets. However, Moshup was found to have a lower density at 1.97 ± 0.24 g/cm³ than Squannit at $2.81 (+0.82, -0.63)$ g/cm³. Although these densities technically agree within their large error bounds, the discrepancy is unexpected because two bodies formed by rotational fission are expected to have similar compositions and possibly similar porosities, resulting in similar densities. As part of this work, we attempted to improve the density estimate of Squannit.

Squannit was observed to be tidally locked, with a rotation period very close to the mutual orbital period, but a single spin period at the exact synchronous rate did not fit the data well, and the long axis of Squannit was occasionally observed to be oriented several degrees away from the Moshup-Squannit line. Ultimately, the rotation period of Squannit could not be constrained on the basis of radar data alone beyond the 17.3 – 17.5 hr range, and fitting the fine structure of Squannit’s shape remained challenging (Ostro et al., 2006). These observations, modeling difficulties, and additional evidence from dynamical simulations suggested that Squannit might exhibit a noticeable libration in longitude, i.e., a periodic back and forth rocking of the body superimposed on a synchronous or resonant spin state (Ostro et al., 2006; Scheeres et al., 2006).

Dynamical numerical simulations using the Ostro et al. (2006) shape models of Moshup

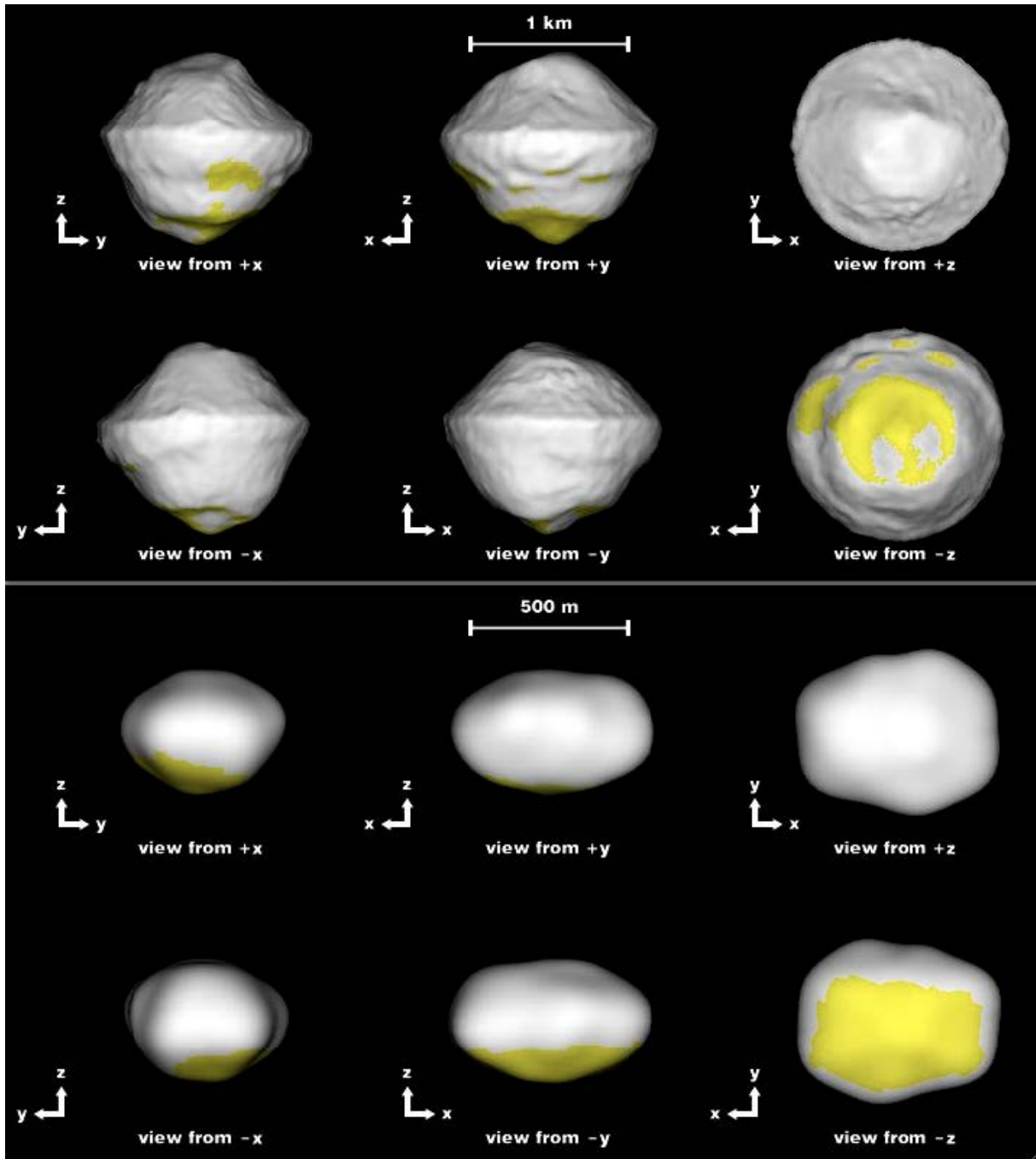


Figure 4.1: Principal-axis views of Moshup (top) and Squannit (bottom) shape models (reproduction of Figure S4 from Ostro, S. J., Margot J. L., Benner, L. A. M., et al. 2006, *Science* **314** (5803), 1276-1280. Reprinted with permission from AAAS). Yellow regions highlight regions of less reliable shape reconstruction as they were not viewed by Arecibo at incidence angles below 70°.

and Squannit (Figure 4.1) covering a two-week simulated time period gave a maximum libration angle of 8° (Scheeres et al., 2006). Libration amplitude modeling done by Naidu and Margot (2015) suggests that the relaxed mode libration amplitude for the Moshup-Squannit system is less than 1° . However, the gravitational perturbations from the Sun as the system passes perihelion in its elliptical heliocentric orbit may keep the system in an excited energy state (Scheeres et al., 2006), resulting in larger librations. It is also possible that the system is experiencing excited mode librations if it is not a fully tidally evolved system (Naidu and Margot, 2015). Known limitations of the numerical simulations include the fact that the shape of Squannit is uncertain. In addition, Naidu and Margot (2015) approximated the shapes with triaxial ellipsoids, which may not capture the detailed dynamics of the system.

The current radar shape model for Squannit is likely in error because it fails to account for its longitudinal libration, which has never been properly estimated. We set out to reanalyze the 2001 data set to perform a joint estimation of the shape and libration of Squannit, with the goal of refining estimates of its shape, volume, density, spin period, libration amplitude, and center-of-mass locations at each observation epoch. We were interested in these improved COM estimates to enable high-precision modeling of the orbital expansion rate in Moshup-Squannit using data from 2001, 2002, 2017, 2018, and 2019. To conduct this reanalysis, we used the improved version of `shape` that enables the modeling of librations (Naidu et al., 2015).

4.2.2 Mutual Orbit Modeling

The mutual orbit of Moshup and Squannit is subject to change via a number of processes including tidal evolution, the BYORP effect, and the binary Yarkovsky effect (e.g., Jewitt, 2025). The first two of these effects are expected to dominate the orbital evolution of the 1999 KW4 system for the following reasons. The binary Yarkovsky effect is split into two parts: the binary Yarkovsky effect on the primary asteroid (BYP) and the effect on the secondary asteroid (BYS). The BYS is only expected to play a significant role in the

case of a secondary with a rotation period not equal to the mutual orbital period, and therefore is expected to have a negligible role in the 1999 KW4 system (Zhou et al., 2024). While the BYP may play a role for 1999 KW4, it is expected to cause at most about a -0.19 cm/yr drift in the semimajor axis of the mutual orbit, which is only a third of our measured orbital drift (Section 4.4.2.3) and in the wrong direction. Therefore, we conclude that the BYP effect will not dominate in this case.

In the Moshup-Squannit system, where the rotation period of the primary is smaller than that of the secondary, the secondary will be pulled by the tidal bulge of the primary, causing a gain in angular momentum for the mutual orbit with a corresponding decrease in Moshup’s spin angular momentum. This tidal evolution leads to mutual orbital expansion, unless counteracted by stronger torques that lead to mutual orbit contraction (Taylor and Margot, 2010). Binary YORP, shortened to BYORP, is an effect where non-uniform thermal emission from an asteroid creates a torque on the binary system that acts to change the orbit of a tidally locked secondary asteroid around its primary. The BYORP effect can either provide a gain or loss in angular momentum of the mutual orbit depending on the small scale topography and location of the center of mass of the secondary. If the secondary re-radiates more light in a direction parallel to its orbital motion rather than the anti-parallel direction, the system will gain angular momentum, leading to orbit expansion, whereas greater re-radiation in the anti-parallel direction will lead to loss of angular momentum (Cuk and Burns, 2005). The strength of this effect can be modified by thermal effects, including eclipses, thermal inertia, and mutual radiation. Thermophysical modeling of binary system 1996 FG3 suggests that secondary eclipses and thermal inertia can reduce the strength of the BYORP effect by several percent, whereas other thermal effects have a more negligible effect on BYORP (Sorli et al., 2025). Because of model limitations and imperfect knowledge of thermal and topographic properties, it is not possible to predict which effects dominate in the mutual orbit evolution of Moshup-Squannit. Measuring the orbital expansion rate accurately is an important first step towards understanding binary asteroid system evolution.

Previous work on the evolution of the mutual orbit in Moshup-Squannit has yielded conflicting results. [McMahon and Scheeres \(2010\)](#) predicted an expansion by 7 cm/year based on the radar-derived shape model of [Ostro et al. \(2006\)](#). [Scheirich et al. \(2021\)](#) estimated an expansion of 1.2 ± 0.3 cm/year based on photometric observations and a number of simplifying assumptions. Whether the BYORP effect is making a positive or a negative contribution to the total expansion rate is still contested ([McMahon and Scheeres, 2010](#)). In addition, BYORP contraction/expansion can be greatly reduced if the secondary librates significantly ([Scheirich et al., 2021](#)).

Here, we used the 2001 radar data and additional radar images obtained in 2002, 2017, 2018 and 2019 to measure the Moshup-Squannit mutual orbit expansion rate. Our improved shape model provides high-quality center of mass (COM) locations of the secondary at each imaging epoch, allowing for improved measurements of the secondary-to-primary separations, improved solutions to the system’s mutual orbit with few assumptions, and, ultimately, a measurement of the time rate of change of semimajor axis. Accurate knowledge of the orbital expansion rate can be used to place bounds on the estimated lifetime of the 1999 KW4 system. An exact lifetime remains elusive if the mutual orbital evolution is BYORP-dominated, because BYORP evolution can proceed in a stochastic manner that depends on changes in the surface distribution of rubble and the center of gravity of the secondary.

4.2.3 Importance of Shape and Orbit Determinations

Squannit’s density estimate ($2.81^{+0.82}_{-0.63}$ g/cm³) is significantly higher than Moshup’s (1.97 ± 0.24 g/cm³) ([Ostro et al., 2006](#)). If these density estimates are accurate, they pose a difficulty with the idea that the secondary was formed through rotational spinup and mass shedding from the primary, the widely accepted formation mechanism for small binary asteroids where the primary is top-shaped with a fast rotation period ([Margot et al., 2002](#); [Walsh and Richardson, 2008](#)). In such a scenario, one expects that both components were formed from the same source material and should thus have similar physical properties,

including similar densities. The radar properties of Moshup and Squannit are indeed indistinguishable (Ostro et al., 2006). Scheeres et al. (2006) suggest that the density difference may be caused by a porosity difference, where the primary’s porosity is high due to its fast rotation rate and the secondary’s porosity is low due to librational shaking acting to compact the body. However, it is not clear whether this density discrepancy is due to errors caused by Squannit’s suboptimal shape model. Therefore, improving the knowledge of Squannit’s shape would enable confirmation of the density discrepancy, which would require elucidation.

There are very few measured orbital expansion rates of binary asteroid systems, and most have large uncertainties. Measuring the mutual orbit expansion rate for 1999 KW4 enables comparison with previous estimates and enhances our ability to model the evolution of binary asteroids more generally. Additionally, pinning down the mutual orbit expansion rate is the first step in disentangling the contribution to expansion coming from the BYORP effect versus the contribution coming from tidal evolution. While it has been speculated that the BYORP effect might dominate the mutual orbit expansion in this system, there remain large uncertainties in the orbital evolution due to tidal dissipation in rubble pile asteroids. Predictions of tidal expansion for Moshup-Squannit currently span two orders of magnitude, with values between 0.0158 cm/year and 1.813 cm/year (Taylor and Margot, 2011; Scheirich et al., 2015, 2021).

An improved estimate of the libration amplitude of the secondary is also needed to improve estimates of the strength of the BYORP effect, as the presence of significant librations is expected to decrease the BYORP effect’s contribution to the orbital expansion rate of the secondary. Altogether, an improved shape model and an orbital expansion rate measurement will help to create a clearer picture of the orbital evolution of the 1999 KW4 system.

4.3 Methods

4.3.1 Libration Parameters and Squannit’s Shape

We used the `shape` software version 2.10.11 to create an improved shape model for Squannit that accounts for its longitudinal libration amplitude, frequency, and phase. We started with the dataset and settings used to produce the [Ostro et al. \(2006\)](#) shape model. Because non-linear least squares fitting with a large number of free parameters tends to become trapped in local minima without exploring the full parameter space, we conducted a grid search to find the best values of the librational parameters. Following the technique used in [Naidu et al. \(2015\)](#) for modeling the secondary of near-Earth asteroid (185851) 2000 DP107, we followed a multi-step iterative procedure to find the best fitting shape model.

The Squannit data set consists of 249 delay-Doppler images split into 47 sets, where ~ 10 consecutive images were grouped in each set of Arecibo images and most Goldstone images were in sets of 1 - 3 images. A sum of 21 images taken at a 15 m resolution during a 25-minute interval on May 26, 2001 is shown in Figure 4.2 along with a cutout of the secondary from a single image taken within the interval. The grouping of images into sets is required because the Keplerian motion is noticeable and is not approximated well by the polynomial representation built in `shape`. For each set, we fit polynomial coefficients to estimate the motion of the center of mass of the body. The number and time span of frames per set was chosen so that the polynomial can properly account for the frame-to-frame motion.

The data were obtained on May 21–29, 2001 at Arecibo Observatory at 2380 MHz and May 26–29, 2001 and June 1–2, 2002 at the Goldstone DSS-14 Station at 8560 MHz. The radar images vary in resolution, with resolutions as low as $1/4 \mu\text{s}$ (37.5 m) for Goldstone images and as high as $0.1 \mu\text{s}$ (15 m) and 0.057 Hz for Arecibo images. Additional information about the radar data set can be found in the supplemental material of [Ostro et al. \(2006\)](#). We made one change from the original dataset by removing set 5 due to overlap between the signal from the primary and secondary, as the software is only

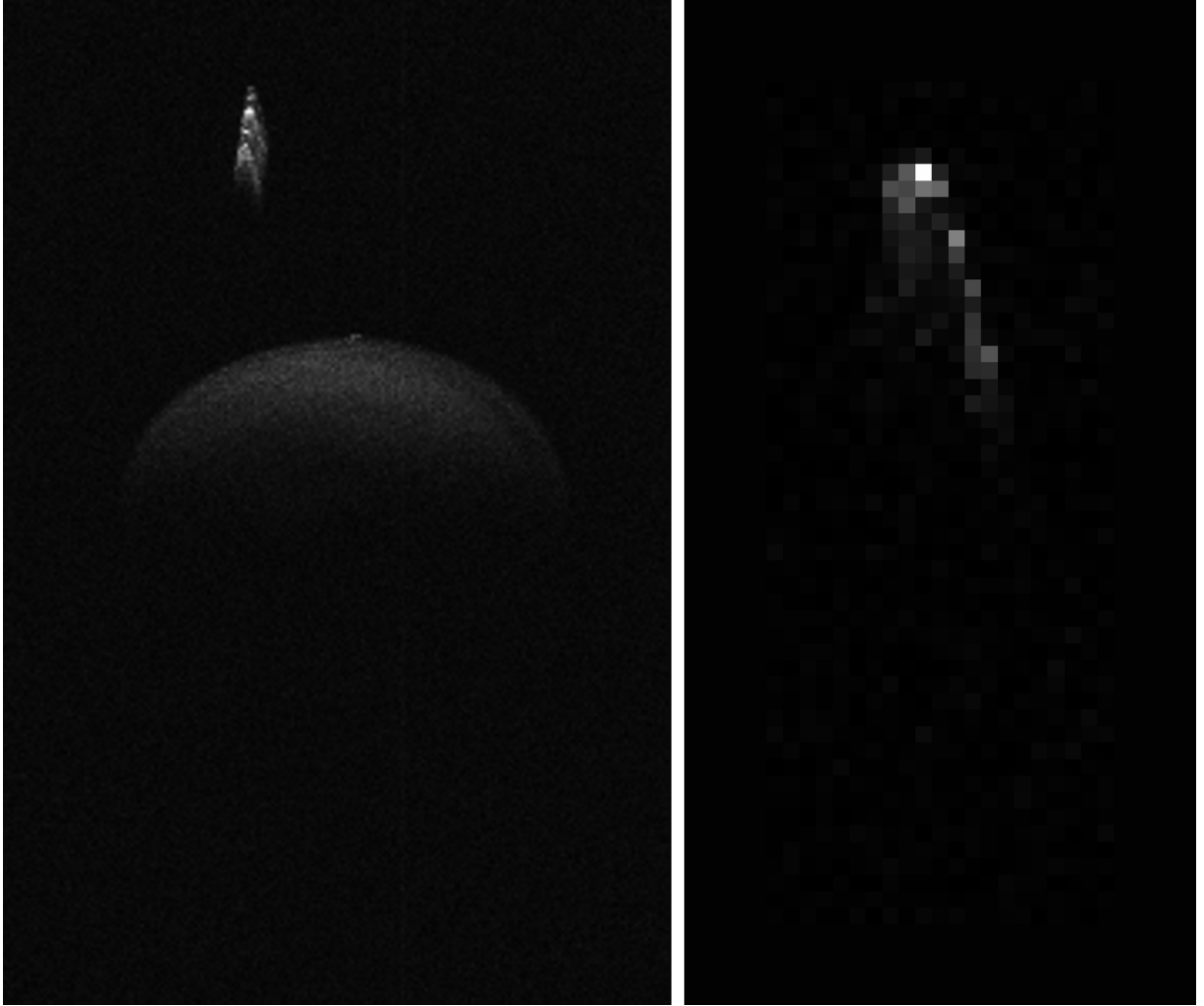


Figure 4.2: Example delay-Doppler images taken by the Arecibo observatory. The x-axis shows Doppler frequency increasing to the right while the y-axis shows range from the observer increasing downward. Delay resolution for these images is 15 m and Doppler resolution is 0.12 Hz. **Left:** Sum of the 21 radar images (63 looks) taken by Arecibo on May 26th, 2001 from 6:06:03 - 06:31:22 AM UTC. Each image is 3 looks. The secondary can be seen moving in its orbit around the primary. **Right:** Cutout of the secondary from the radar image taken at 06:07:11 (3 looks).

able to model one body at a time and requires that the signal from each body in a given radar delay-Doppler image does not overlap. This change left us with 246 images split into 46 sets. We also changed the weighting of the images such that each delay-Doppler image was given an equal weight within the fitting scheme. We found that this lowered the reduced chi squared for the nominal libration-free model from 0.367 to 0.354. The reduced chi squared uncertainty is 0.2% of the reduced chi squared based on the sum of chi squared variances within each set divided by the total number of degrees of freedom. Therefore, this 3.5% reduction in reduced chi squared represents a 10σ difference in reduced chi squared, which is highly significant.

We kept most of the fitting parameters identical as those used in [Ostro et al. \(2006\)](#)'s shape modeling effort. For the secondary, we begin with an ellipsoid model fit where the longest diameter, the elongation, and the flattening, also referred to as $2a$, a/b , and b/c , are allowed to float. We then progress to an 8th degree spherical harmonic fit with 81 free parameters plus an overall scale factor. For the purpose of generating modeled delay-Doppler images to be compared to the data, both the ellipsoid and spherical harmonic shape models are realized within the `shape` code using a vertex-based model with 30 theta steps and 1,148 total vertices, giving roughly a $180^\circ/30^\circ = 6^\circ$ spacing between vertices. We relaxed the penalty functions `inertiadev.uni` and `comdev` from weights of 100 each to weights of 10 each, where `inertiadev.uni` encourages the principal axes to coincide with the body-fixed axes and `comdev` encourages the COM to remain close to origin of the body-fixed coordinates. We made these changes because `shape` realizes the body as a faceted vertex model whose facets are never perfectly symmetrically distributed, so the third principal axis and the spin axis cannot be perfectly aligned and the body's center of mass will not be located perfectly at the origin. Therefore, we did not wish to hinder the algorithm's ability to find the best fit to the shape of the body by giving either of these penalties too much weight in the objective function. Even with these lower weights, our model realizations have COM locations that are only ~ 0.4 m from the origin of coordinates and a third principal axis that is only $\sim 0.5^\circ$ from the spin axis.

We performed an ellipsoid shape model fit followed by a spherical harmonic shape model fit of the secondary for each point on a grid of values for libration amplitude and libration phase, where both parameters were held constant at each grid point and rotation period and shape parameters were allowed to vary. We assumed that the libration frequency is the forcing frequency given by the mean orbital motion found in [Ostro et al. \(2006\)](#). We tested all possible phase values between 0 and 360 degrees in increments of 10 degrees, and we tested libration amplitudes between 0 and 10 degrees in increments of 1 degree. At the grid point that gave the lowest chi-squared value, we visually confirmed that the simulated observations for the modeled shape are similar to the real observations.

4.3.2 Rate of Orbital Expansion or Contraction

After solving for Squannit’s improved shape model and obtaining improved COM locations, we measured the secondary-to-primary COM separations on radar images spanning 2001–2019. We used these measurements to determine the mutual orbit and the orbital expansion rate with few assumptions. This method should provide a more accurate measurement of the orbital expansion rate than that achieved by analysis of light curve data by [Scheirich et al. \(2021\)](#) because their analysis required the following assumptions: (1) the spin period of Squannit was assumed to be equal to the orbital period; (2) the libration amplitude of Squannit was assumed to be zero; (3) the obliquity of Moshup was assumed to be zero; (4) the orbital plane was assumed to be fixed; (5) the orbital eccentricity was assumed to be zero; (6) the component albedos were assumed to be identical; (7) the component densities were assumed to be identical; (8) the components were modeled as spheres in the initial search; and (9) the components were modeled as ellipsoids only after local minima were identified.

4.3.2.1 Improved COM Estimates

We obtained improved estimates of COM locations by fitting a shape model to the radar images. We group our data based on the observation year and model each year separately.

We kept the shape parameters at their best-fit solutions, and allowed only the delay correction polynomial and the rotation phase of each body to vary. Moshup’s shape was modeled following the vertex shape model of [Ostro et al. \(2006\)](#) with 4,586 vertices and we used our libration-aware spherical harmonic shape model for Squannit obtained using the 2001 dataset. Ancillary values needed for the fit, such as the right ascension, declination, and range at each epoch were obtained from JPL Horizons. We vignetted the images to separate the primary and secondary components where possible and to estimate their COM locations separately.

The rotation pole direction is held constant at $(325.95^\circ, -64.95^\circ)$ in ecliptic coordinates for the primary and $(324.028^\circ, -63.0666^\circ)$ for the secondary. The initial condition for the rotational phase about the body-fixed z-axis is -71.1° for the primary and 50.3° for the secondary.

The shape software requires initial estimates of the range and Doppler locations of the bodies to ensure convergence. We provided a list of range-Doppler pixel locations for each component in each image. To estimate these initial values, the COMs of the primary and secondary for each radar delay-Doppler image were estimated by finding the leading edge of each body based on a 3σ threshold in the manner described by [Naidu et al. \(2015, Section 2.2\)](#), where σ is the standard deviation of the background noise. The range coordinate of the COM can be estimated to be R meters behind the leading edge, where R represents the equivalent radius of the body, which is known from the shape model. We used the [Ostro et al. \(2006\)](#) value of 658.5 m for the radius of Moshup, whereas for Squannit, we updated the initial estimate by using the equivalent radius of a sphere of the model’s volume from our best-fitting model that includes libration, $R = 227$ m. In the frequency dimension, the minima and maxima of the Doppler coordinates of the leading edge of Moshup were estimated, and the midpoint of the Doppler extent was reported as the COM location. For Squannit, the Doppler coordinates of the center of the leading edge were recorded. For the initial range estimate, we set the row location to $0.7 \times$ the leading edge range coordinate $+ 0.3 \times$ the COM range coordinate, which tends to visually

align with the brightest row in the image for a body.

4.3.2.2 Secondary-to-Primary Range-Doppler Separations

Our mutual orbit fits require estimates of the range-Doppler separations between primary and secondary at all epochs. For the 2001 Arecibo data, the secondary model tends to use images each created from a single transmit-receive cycle we call a “run” while the primary model used images created by stacking data from ~ 3 runs, as the secondary has a larger on-image motion than the primary in the delay-Doppler space of the images. In contrast, Goldstone images used for the primary and the secondary modeling are all created from stacks of multiple runs. The images are grouped by similar observation times into sets for input into `shape`, where a separate set of delay correction polynomials is fit for each set. Therefore, images of the primary used for `shape` modeling and images of the secondary used for `shape` modeling do not always align in time. For this reason, we only included images where the primary and the secondary both have the same midpoint of observation time for our range and Doppler difference calculations. Of the 246 secondary images and the 106 primary images, we used 84 images of the primary and 84 images of the secondary where the observation times of the midpoint frame are the same for the primary and secondary.

For the 2002 - 2019 dataset, we assigned an uncertainty to the range separations equal to

$$X \times \text{baud} \times \frac{c}{2}, \quad (4.1)$$

where the baud is the duration of a symbol in the pseudo-random code, which dictates the range resolution, c is the speed of light, and X is small integer constant, either 1, 2, or 3. We chose an integer constant of 3 for images with the highest delay resolution (≤ 75 m) to account for the increased uncertainty in the location of the COM in delay when the image of the body is stretched across a greater number of rows. We chose an integer constant of 2 for some of the intermediate resolution images with a 75 m resolution, and

an integer constant of 1 for the lowest resolution images (≥ 150 m). The speed of light is divided by 2 to account for the two-way nature of radar. We assigned an uncertainty to the Doppler separations corresponding to twice the frequency resolution. All Goldstone Doppler separations are converted to units of S-band (2380 MHz) Hertz by multiplication with 2380/8560. We do not vary the integer multiplier on the uncertainty with resolution because we do not expect the brightest column from the object to be offset from the center of mass in the same way that we expect the brightest row to be offset from the center of mass of the body. Uncertainties for the 2001 dataset use a simpler schema and are assigned a (60 m, 0.7 Hz) range and Doppler uncertainty for Goldstone imaging and a (45 m, 0.33 Hz) range and Doppler uncertainty from Arecibo imaging.

4.3.2.3 Mutual Orbit

To model the orbit, we used the orbital integrator Mercury ([Chambers, 1999](#)). We used the Bulirsch-Stoer integrator option, integrating with the primary as the central body containing the full system mass, and the Sun and the secondary as orbiting bodies, where the secondary is treated as a massless point source. While we integrated in cartesian coordinates, the integrator can also output orbital parameters for all modeled bodies at each integration time. We included the Sun to account for its expected third-body effects on the apsidal and nodal precession of the mutual orbit. We included the effects of primary oblateness through the J_2 parameter as defined in [Fang et al. \(2011\)](#),

$$J_2 = \frac{C - \frac{1}{2}(A + B)}{MR^2}, \quad (4.2)$$

where A , B , and C are the principal moments of inertia for the primary in kg m^2 , estimated with the shape model and a uniform density assumption, $M = (2.353 \pm 0.1) \times 10^{10}$ kg is the mass of the primary and $R = 658.5 \pm 19.8$ m is the equatorial radius of the primary given by [Ostro et al. \(2006\)](#). These values yield a unitless J_2 value of 0.05569 ± 0.016 . The uncertainty for J_2 was estimated by propagating uncertainties for the variables in the

equation for J_2 as listed in Table 4.1 using standard error propagation formulas (Ku, 1966; Bevington and Robinson, 1992). This uncertainty does not account for the uncertainty present in our uniform density assumption for Moshup, but this contribution to the uncertainty in J_2 is likely overshadowed by the existing uncertainties in the principal moments of inertia.

To model tidal expansion and/or BYORP expansion/contraction, we included a perturbing acceleration to the Mercury force model. The force per unit mass that describes the motion of the secondary with respect to the primary is given by

$$\mathbf{F} = \ddot{\mathbf{r}} = -\mu r^{-2} \hat{\mathbf{r}} + d\mathbf{F}, \quad (4.3)$$

where $\mu = GM$ and G is the universal gravitational constant, M is the system mass, and $d\mathbf{F}$ is a small disturbing force per mass, i.e., an acceleration. This specific disturbing force can be written

$$d\mathbf{F} = R\hat{\mathbf{e}}_R + T\hat{\mathbf{e}}_T + N\hat{\mathbf{e}}_N, \quad (4.4)$$

where the $\hat{\mathbf{e}}$ s are orthogonal unit vectors. The vector $\hat{\mathbf{e}}_R$ points radially outwards along $\hat{\mathbf{r}}$, the position vector of the secondary with respect to the primary, $\hat{\mathbf{e}}_T$ is transverse to the radial vector in the orbital plane and is positive in the direction of motion of the particle, and $\hat{\mathbf{e}}_N$ is normal to the orbital plane in the direction $\hat{\mathbf{e}}_R \times \hat{\mathbf{e}}_T$. Using these definitions, Burns (1976) described the perturbation equation for the rate of change of the semimajor axis as

$$\frac{da}{dt} = \frac{2a^{3/2}}{\sqrt{\mu(1-e^2)}} [Re \sin(\nu) + T(1 + e \cos(\nu))], \quad (4.5)$$

where a is the semimajor axis, e is the eccentricity, and ν is the true anomaly. Because the eccentricity of the mutual orbit is small ($e = 0.0004 \pm 0.0019$, Ostro et al., 2006), we used the simplified equation in the limit $e \rightarrow 0$. Solving for T , we found

$$T = \frac{1}{2} \frac{da}{dt} \sqrt{\mu/a^3}. \quad (4.6)$$

In practice, the numerical integrations proceeded in the following way. We assigned starting parameter values to nine parameters: the six orbital elements (a , Ω , ω , e , i , M), J_2 , the total system mass, and the perturbing specific force, also referred to as the perturbing acceleration. The orbit was integrated over 2001–2019 with a time step of 0.01 days, resulting in a series of modeled positions and velocities. These positions and velocities were converted into range and Doppler separations and the observed minus computed (O-C) residuals were calculated. The chi squared value was calculated as the sum of squares of the O-C residuals.

In a first step, we attempted to identify the total system mass appropriate for a mutual orbit model that includes an oblate primary. To do so, we held five orbital parameters constant (a , Ω , ω , e , and i) at the solution published by [Ostro et al. \(2006\)](#), set J_2 to its nominal value of 0.05569, set da/dt to zero, and we fit for the central mass and the mean anomaly M at epoch MJD = 52055.4132. For this step, we used only the 2001 secondary-to-primary separations, which are by far the most precise. These observations span 5.8 days, and the $da/dt = 0$ assumption is immaterial over such a short time interval. We fit for the mean anomaly and central mass using the non-linear least-squares minimizer MPFIT, which uses the Levenberg Marquardt technique to minimize a function value, where we minimized χ^2 [Markwardt \(2009\)](#). The minimization process continued until the relative error between two consecutive iterates was less than 10^{-6} or MPFIT reached its 20th iteration.

Once we established a best-fitting central mass using the 2001 separations, we conducted a series of fits to the 2001–2019 secondary-to-primary separations with a mutual orbit model that included both a nonzero J_2 for Moshup and a nonzero non-gravitational perturbing acceleration. Because finding the absolute minimum in a complex chi-square surface is non trivial, we hold all parameters constant and initiated the orbital integrations with combinations of initial conditions in system mass, J_2 , and perturbing accelerations.

We draw from a list of values of total mass in the range 237.58×10^{10} kg to 248.80×10^{10} kg. This range centers on the best-fit mass to the 2001 dataset, 243.19×10^{10} kg,

and extends 5.61×10^{10} kg up to the value of the mass found in [Ostro et al. \(2006\)](#) using $J_2 = 0$. Our lowest value in the range extends downwards 5.61×10^{10} kg from the best fit 2001 mass. Our values are spaced linearly. We draw from 60 linearly spaced values of J_2 in the range 0.03969 to 0.07169. This range covers values from 1 standard deviation below the nominal J_2 of 0.05569 to 1 standard deviation above the nominal J_2 . To set the range of da/dt values, we used as a guide the 7 cm/year expansion rate estimate of [McMahon and Scheeres \(2010\)](#), which yields a tangential scalar component $T = 1.108 \times 10^{-13}$ m/s² according to Equation (4.6). We tested 100 different values of perturbing accelerations logarithmically spaced between $-10^{-11} - -10^{-16}$ and $10^{-16} - 10^{-11}$ to encompass a wide range of possible accelerations. After the fit converged, we computed the mutual orbit expansion rate as the time derivative of the semi-major axis at the reference epoch of pericenter passage reported by [Ostro et al. \(2006\)](#), MJD = 52055.4132.

4.4 Results

4.4.1 Libration Parameters and Squannit's Shape

The angle between θ_x and the long axis of the secondary at any point in time is given by

$$\psi = \theta_z + \Omega_z(t - t_0) + \Psi \quad (4.7)$$

where

$$\Psi = A \sin(\omega(t - t_0) + \phi) \quad (4.8)$$

Ψ is the angle between the long axis of the body and the line connecting the primary and secondary known as the libration angle, where A is libration amplitude and ϕ is libration phase. θ_x , θ_y , and θ_z are the three Euler angles describing the initial orientation of the body-fixed coordinates relative to ecliptic coordinates. θ_x and θ_y represent the location of the spin pole of the secondary relative to the ecliptic, with $\theta_x =$ ecliptic pole longitude $+ 90^\circ = 54.0^\circ$ and $\theta_y = 90^\circ$ - ecliptic pole latitude $= 153.1^\circ$, and $\theta_z = 50.3^\circ$ is the initial

Euler rotation angle about the body-fixed z-axis. Ω_z is the rotation speed of the secondary in $^\circ/\text{day}$. We define t_0 as midnight on May 26, 2001 UTC.

Figure 4.3 shows the reduced chi squared values for the shape of Squannit for the tested grid of libration amplitudes and libration phases. Both the ellipsoidal and the spherical harmonic models of the secondary show a similar pattern in reduced chi squared across the grid of libration values, with the region showing amplitudes between 3° and 7° and phases between 200° and 300° being most favored, although libration amplitudes across the entire range covered have at least some acceptable models, with everything to the left of the black contour indicating models within one standard deviation of the best fit. The shape model of Squannit with the lowest reduced chi squared is the spherical harmonic model with a 4 degree libration amplitude and a libration phase of 230° . We convert this libration phase into the libration phase at the estimated time of pericenter passage from [Ostro et al. \(2006\)](#), $\text{MJD} = 52055.4132 \pm 0.88$ using Equation 4.9,

$$\phi_p = \phi_0 + \omega(t_p - t_0) \quad (4.9)$$

From this, we find that our best-fit libration phase is 75.1° at the time of pericenter passage. Together with the libration amplitude of 4° , this gives a libration angle of 3.87° at the time of pericenter passage $\text{MJD} = 52055.4132$. The shape model is shown from the positive and negative directions for each principal axis in Figure 4.4, while its properties are given in Table 4.1.

`shape` calculates the uncertainty in the reduced chi squared as the sum of total chi squared variance over the number of degrees of freedom, according to

$$\frac{\sum_f 2w^2 n_{\text{del}} n_{\text{Dop}}}{\sum_f w n_{\text{del}} n_{\text{Dop}}} \quad (4.10)$$

where f is the number of total number of frames, w is the statistical weight of the frame, n_{del} is the number of delay rows in the frame and n_{Dop} is the number of Doppler columns in the frame. For 612,822 degrees of freedom, `shape` gives the uncertainty in the reduced

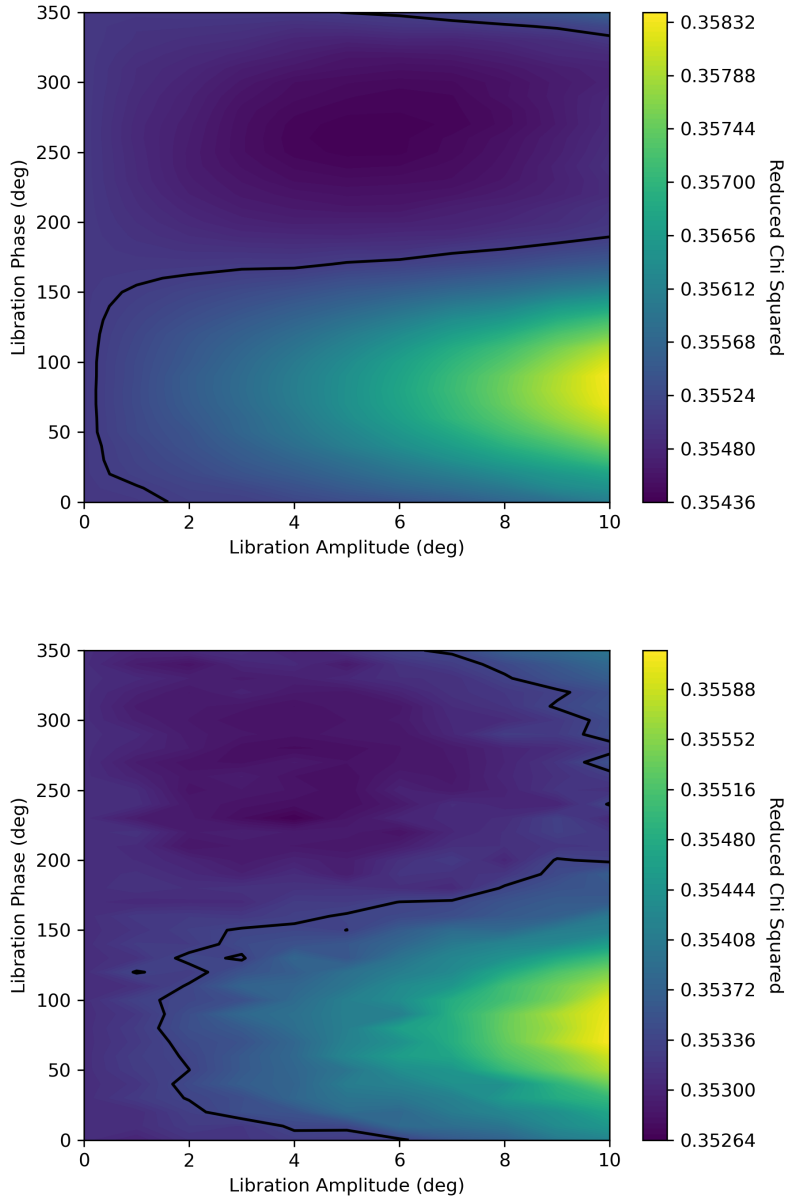


Figure 4.3: Reduced chi squared as a function of tested libration angles and libration amplitudes of the secondary. The top plot shows the reduced chi squared values for the ellipsoidal models of the secondary while the bottom plot shows the reduced chi squared values for the spherical harmonic models of the secondary. The black contour delineates the region within one standard deviation of the best fit. Each model has 612,822 degrees of freedom.

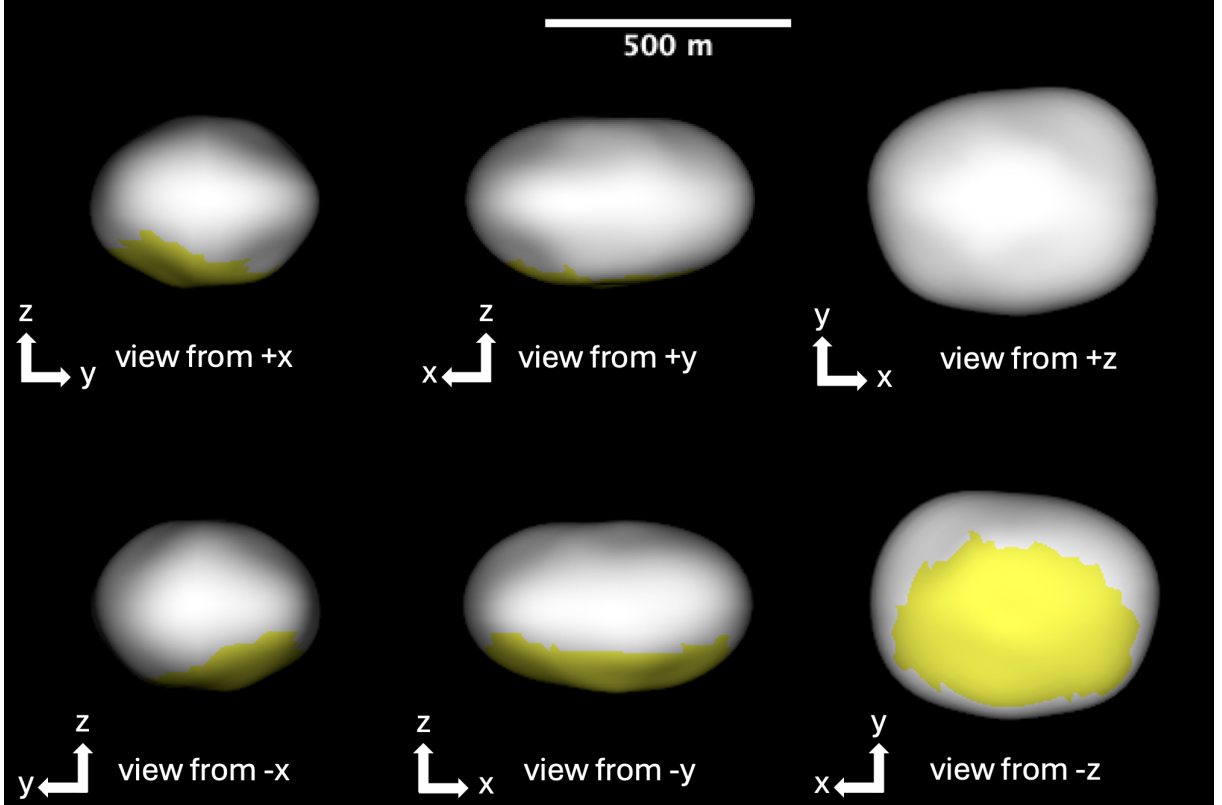


Figure 4.4: Shape model for Squannit with libration angle = 4° and libration phase = 230° . Yellow regions were not viewed by Arecibo at incidence angles below 70° , indicating poorer coverage.

chi squared as 0.2% of its value, or 0.0007. This is calculated based on the variance in the chi squared distribution between sets divided by the total number of degrees of freedom. Therefore, a libration amplitude of 0 degrees is still within agreement with the data even though a 4° libration amplitude gives the best fit. However, Figure 4.5 shows that the model of the secondary with libration included also provides a slightly better constraint on the rotation period of the secondary seen by a slightly narrower valley for the ellipsoid models and a lower minimum reduced chi squared for both the ellipsoid and spherical harmonic models. The best fitting rotation period of 17.36 hrs is $\sim 1.7 \sigma$ shorter than the orbital period of the system measured by [Ostro et al. \(2006\)](#), supporting with fairly good agreement that the secondary is tidally locked.

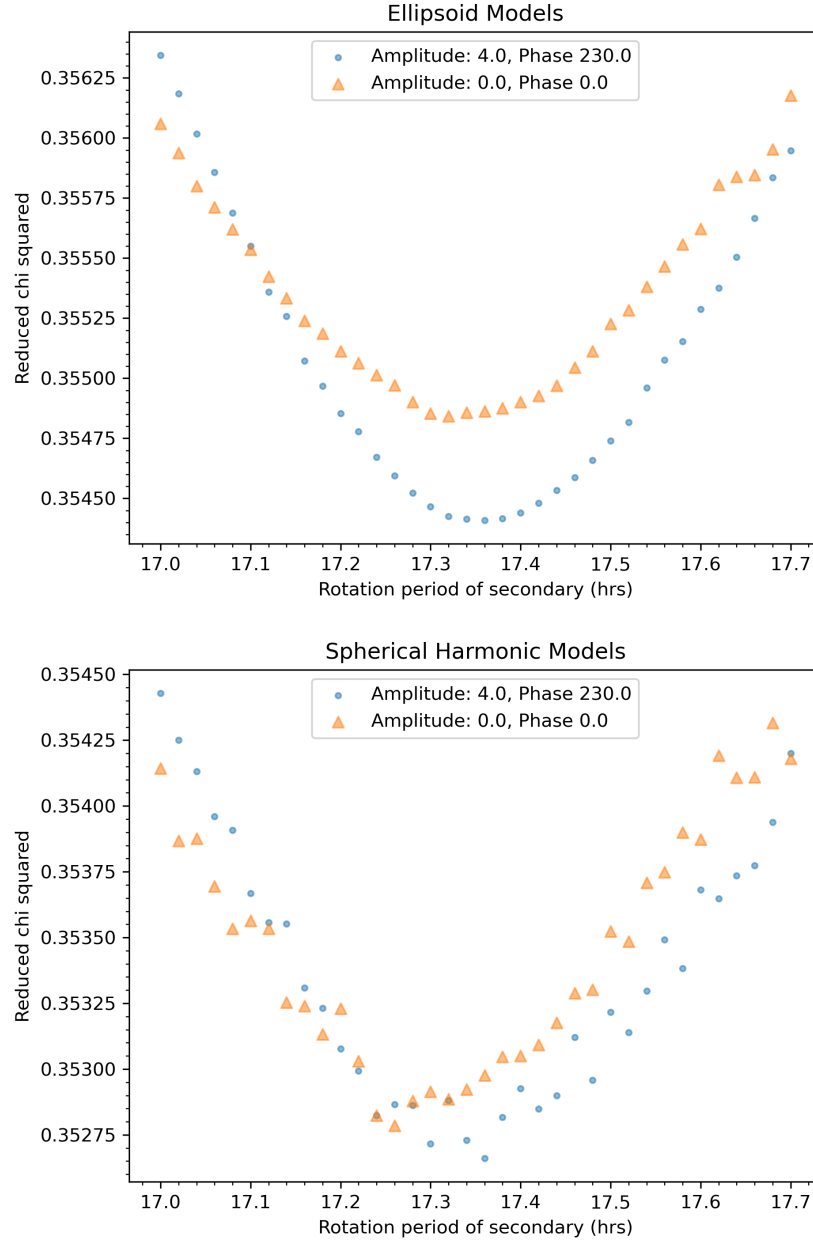


Figure 4.5: Reduced chi squared for the libration and no libration models as a function of the rotation period of the secondary, with the libration model shown plotted with blue dots and the no libration model plotted with orange triangles. The ellipsoid models are shown on the top and the spherical harmonic models are shown on the bottom. The best fitting spherical harmonic model with libration is located at a rotation period of 17.36 hrs, with an uncertainty in the best fitting period of 0.2 hrs.

Table 4.1: Squannit Shape Model Results

Extent Along Principal Axes (km)	x	0.578 ± 0.027
	y	0.457 ± 0.027
	z	0.344 ± 0.027
Area (km ²)		0.677 ± 0.079
Volume (km ³)		0.0489 ± 0.0086
Density (g/cm ³)		2.76 ± 0.69
Moments of Inertia/density (m ⁻¹)	A	$(8.08 \pm 2.06) \times 10^{11}$
	B	$(1.149 \pm 0.270) \times 10^{12}$
	C	$(1.376 \pm 0.311) \times 10^{12}$
Equivalent Diameter (km)		0.454 ± 0.027
DEEVE Extents (km)	x	0.592 ± 0.027
	y	0.459 ± 0.027
	z	0.344 ± 0.027
Rotation Period (hrs)		17.36 ± 0.2

NOTE— The uncertainties in the extents and the equivalent diameter are given by the mean edge length of the model facets, 27 m. The uncertainty in the rotation period is based on when the increase in reduced chi squared is estimated to exceed the reduced chi squared uncertainty based on Figure 4.5, where the reduced chi squared uncertainty is given by the total chi squared variance / DOF, which is equal to 0.0007 for these models. Uncertainty in the moment of inertia/density is found by standard error propagation of the formal uncertainties on the moments of inertia and density.

4.4.2 Rate of Orbital Expansion or Contraction

4.4.2.1 Improved COM Estimates

We used the best fitting spherical harmonic shape model of Squannit with a 4° libration amplitude, a 230° libration phase, and a rotation period of 17.36 hrs as well as the shape model of Moshup from [Ostro et al. \(2006\)](#) to refine our COM estimates for the 2002 - 2019 images. The rotational phases of the primary and secondary were allowed to float during the fitting process because the exact rotational phases in 2017, 2018, and 2019 are unknown. After the fit converged, we used the COM positions reported by `shape`.

4.4.2.2 Secondary-to-Primary Range-Doppler Separations

We calculated the secondary-to-primary separations at each observation epoch, a subset of which are shown in Table 4.2.

Table 4.2: Secondary - Primary Separations

Epoch (MJD)	Rng (m)	σ_{Rng} (m)	Dop (Hz)	σ_{Dop} (Hz)	RA (°)	DEC (°)	DIST
52052.445350	1332.4	60.0	-2.655	0.700	318.406452	-21.132972	0.044068
52052.466950	1618.0	60.0	-2.045	0.700	318.162804	-21.042776	0.043895
52052.490740	1895.9	60.0	-1.773	0.700	317.894453	-20.940003	0.043706
52052.525950	2061.3	60.0	-0.795	0.700	317.491056	-20.794124	0.043427
52052.544250	2133.7	60.0	-0.815	0.700	317.280240	-20.718220	0.043283
52052.559450	2148.4	60.0	-0.077	0.700	317.105136	-20.648908	0.043164
52052.578050	2132.2	60.0	0.684	0.700	316.890864	-20.564092	0.043018
52053.402280	1376.3	60.0	3.073	0.700	306.010621	-15.986678	0.037322

NOTE— Ranges (Rng) and Dopplers (Dop) are best fit center-of-mass differences, secondary - primary. Ranges are positive away from the observer and Dopplers are positive towards positive frequencies. Right ascension (RA), Declination (DEC), and geocentric distance (DIST) are ephemeris-derived.

The full table of measurements at 130 epochs is provided in the supplemental machine readable table “range_doppler_separations.txt”.

4.4.2.3 Mutual Orbit

Our first step consisted of estimating the total system mass using only the 2001 separations. We used Mercury to find a best fitting total system mass and mean anomaly at the reference epoch used by [Ostro et al. \(2006\)](#), i.e., time of pericenter passage at MJD = 52055.4132. We obtained a fit with a reduced chi squared of 0.30 with 166 degrees of freedom. The best fitting mean anomaly was at exactly 0° as expected. The best-fit system mass is $(243.19 \pm 0.15) \times 10^{10}$ kg, which is a little more than one standard deviation smaller than the primary mass listed in [Ostro et al. \(2006\)](#).

We attribute the difference in total mass to the fact that our mutual orbit model accounts for the effect of the oblateness of the primary (J_2), whereas the original fit by [Ostro et al. \(2006\)](#) did not. To confirm this attribution, we momentarily set $J_2 = 0$ and ran another fit for the mean anomaly and the system mass. We again found the best fitting mean anomaly at epoch to be exactly 0, and we found the best fitting system mass to be $(248.81 \pm 0.17) \times 10^{10}$ kg, which agrees with the mass published by [Ostro et al. \(2006\)](#) at the 40 ppm level.

With an updated range for the total mass of the system, we then integrated the mutual orbit over the 2001–2019 times span of observations. We set the time of pericenter passage and other orbital parameters to the values used by [Ostro et al. \(2006\)](#). We set the primary’s J_2 to 0.05569, the value we derived from the improved shape model of the secondary (Section 4.4.1), and we set the system mass to 243.19×10^{10} kg as found by fitting our 2001 separations with the inclusion of this J_2 value. Initially, we test a range of 102 tangential accelerations T spaced logarithmically from -10^{-11} to -10^{-15} and 10^{-15} to -10^{-11} while holding all other parameters constant. The results are shown in Figure 4.6. We found that the best fitting model, hereafter called the best acceleration array model, has a value for the tangential acceleration $T = (9.12^{+0.40}_{-3.42}) \times 10^{-15}$ m/s², with 260 degrees of freedom and a reduced chi squared of 0.799. From equation 4.6, this tangential acceleration is equivalent to $da/dt = 0.58^{+0.02}_{-0.22}$ cm/yr.

For a deeper exploration of the chi squared surface, we also conducted a Monte Carlo sampling, where we drew 1000 random samples of starting values for J_2 , system mass, and perturbing acceleration within the ranges specified in Section 4.3.2.3. We evaluated the sum of squares of residuals at these 600 starting values. As shown in Figure 4.7, we found that all 600 models have a reduced chi squared > 2 except for one model, which has a reduced chi squared of 0.973. The best-fit model among the 1000 models considered, hereafter called the best Monte Carlo model, has $J_2 = 0.047541$, system mass $M = 244.948358 \times 10^{10}$ kg, and perturbing acceleration $T = 1.99 \times 10^{-16}$, giving $da/dt = 0.0127$ cm/yr.

We were interested in finding out whether the data could be explained without orbital expansion. To test this idea, we used the same grid point ($J_2 = 0.047541$, $M = 244.948358 \times 10^{10}$ kg), but set the perturbing acceleration to 0, and we used MPFIT to fit for J_2 and M . We found that this model, hereafter called the zero acceleration model, with $J_2 = 0.04755 \pm 0.00002$ and $M = (244.948 \pm 0.002) \times 10^{10}$ kg, explains the data slightly better than the model identified in the previous paragraph, with a reduced chi squared of 0.714.

Taking the best acceleration array model based on the derived J_2 and best fitting

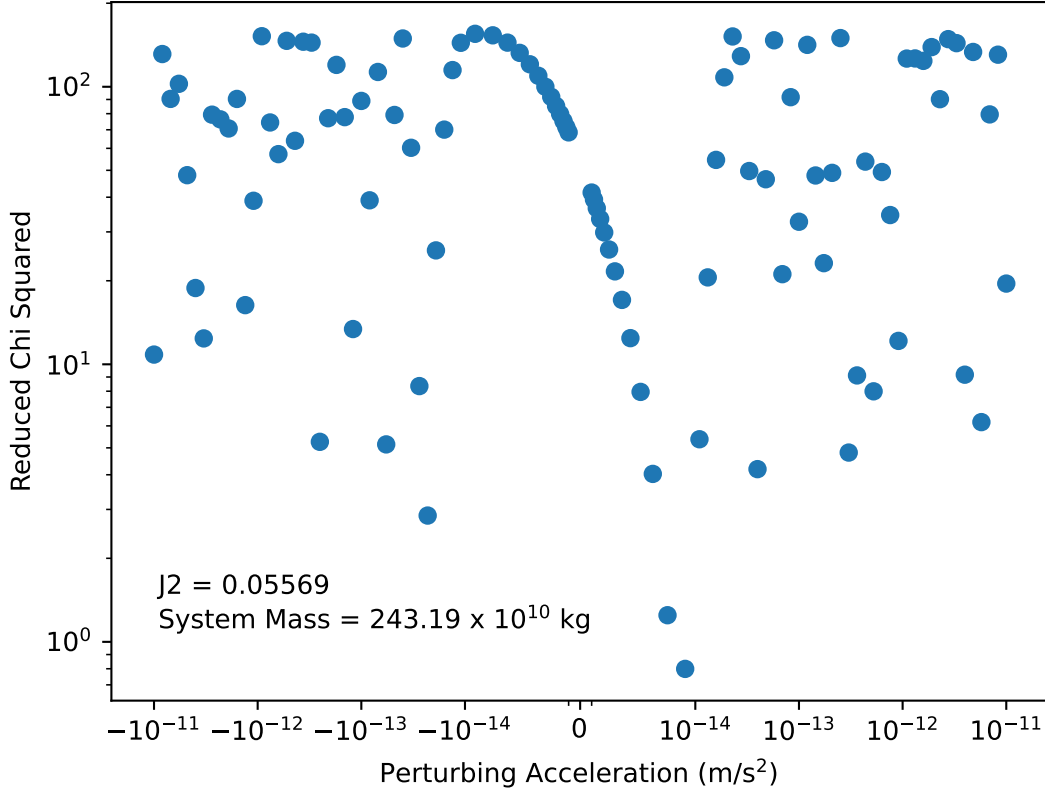


Figure 4.6: Reduced chi squared vs. tangential acceleration for a mutual orbit model that includes Moshup’s oblateness ($J_2 = 0.05569$) and our best estimate of the total system mass based on fits to the 2001 range-Doppler separations ($M = 243.19 \times 10^{10} \text{ kg}$) held constant. The sum of squares of residuals is minimized at $T = 9.12 \times 10^{-15} \text{ m/s}^2$, with a reduced chi squared value of 0.799.

system mass for the 2001 data as the upper limit for da/dt and the zero acceleration model with $da/dt = 0$ as the lower limit for da/dt , we find that the orbital expansion rate for this system is likely $< 0.58 \text{ cm/yr}$.

4.4.3 Upper Bound on BYORP Coefficient

If we assume that the orbital expansion rate of 0.58 cm/yr is dominated by BYORP forcing, we can calculate an upper bound on the BYORP coefficient for Moshup-Squannit. By adapting Equation 3 from [Scheirich et al. \(2021\)](#), we derive a maximum BYORP

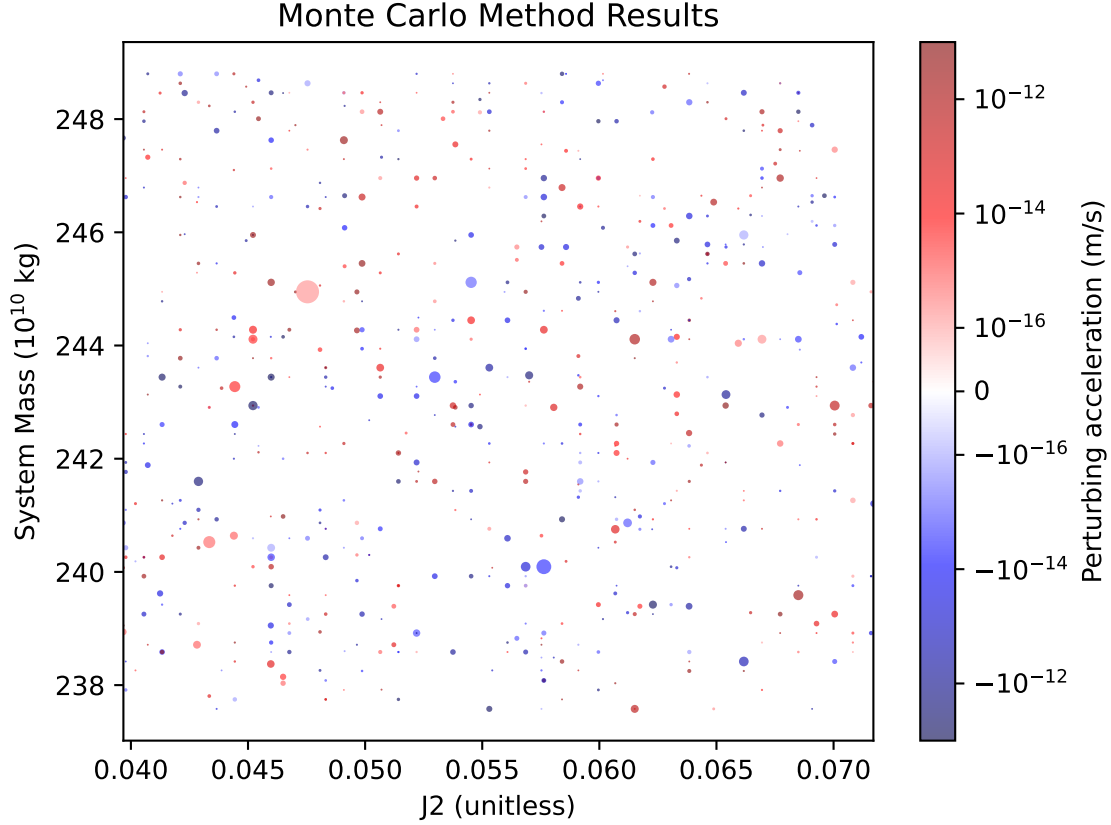


Figure 4.7: Reduced chi squared for various combinations of J_2 , system mass, and tangential acceleration. The reduced chi squared is minimized at $J_2 = 0.047541$, $M = 244.948358 \times 10^{10}$ kg, $T = 1.99 \times 10^{-16}$ m/s², with a value of $\chi/\text{DOF} = 0.973$ and $\text{DOF} = 260$. Size of the points is proportional to the inverse of the reduced chi squared. Color of the points indicates the tangential acceleration, where red are positive accelerations leading to orbital expansion, and blue are negative accelerations leading to orbital contraction.

coefficient of $< 1.60 \times 10^3$. This value would obviously need revision if part or all of the expansion was due to tides (Section 4.4.4).

4.4.4 Tidal Expansion Rate

We wish to estimate the orbital expansion rate due to tidal dissipation for the Moshup-Squannit system to determine whether tides may be a dominant contributor to the measured expansion rate. The orbital expansion due to tides raised by the secondary on

the primary is given by

$$\frac{da}{dt} = \frac{9\rho_n^2 G R_{eq}^7 M_s n}{19\alpha_p \mu_p Q_p M_p a^4} \quad (4.11)$$

where the subscripts p and s refer to the primary and secondary, respectively (Cuk and Burns, 2005; Murray and Dermott, 2000). In this expression, $\alpha_p = \frac{C}{M_p R_{eq}^2}$, where C_p is the largest principal moment of inertia, M_p is the mass of the primary, and R_{eq} the equatorial radius of the primary. The quantities μ_p and Q_p are estimates of the rigidity and material quality factor of the primary, respectively. These quantities are very poorly constrained for small rubble pile bodies. One possible estimate for the $\mu_p Q_p$ product was calculated by Taylor and Margot (2011) by assuming tidal evolution during the ~ 10 Myr lifetime for near-Earth binary systems. They found $\mu_p Q_p = 1.6 \times 10^9$. However, it is important to note that $\mu_p Q_p$ could be substantially lower if the binary formed recently or larger if the binary formed in the main belt. There are indications that Moshup-Squannit likely formed in near-Earth space (Cueva et al., 2026). With the $\mu_p Q_p$ corresponding to the 10 My timescale, the estimated orbital expansion due to tides is $da/dt = 0.006$ cm/yr. If the binary instead formed 100,000 years ago, then $\mu_p Q_p$ would be two orders of magnitude smaller, and tidal evolution would explain the observed da/dt with a negligible BYORP contribution.

4.4.5 Lifetime of Moshup-Squannit

To estimate the binary system's lifetime, we begin with Equation (4.6), where we made the simplifying assumptions that the mutual orbit is circular and the perturbing acceleration is entirely tangential. We solved the differential equation via integration and solved for time t to obtain

$$t = \frac{\sqrt{\mu}}{T} \left(\frac{1}{\sqrt{a_0}} - \frac{1}{\sqrt{a}} \right), \quad (4.12)$$

where a_0 is the current semimajor axis of the system and a is the semimajor axis of the system at time t . To find the lifetime of the system, we assume T remains constant and set a equal to the Hill radius of the primary relative to the Sun, $r_H = 71.8$ km given the

current heliocentric semimajor axis of the system. At a maximum expansion rate of 0.58 cm/yr, we expect the binary system to have a lifetime of $> 7.2 \times 10^5$ years, unless cut short by a close planetary encounter.

4.4.6 Moshup and Squannit’s Densities

We obtained a new estimate of Squannit’s density using the improved volume of the secondary from the libration-aware shape model and the improved estimate of the system mass from the J_2 -aware mutual orbit solution. We assumed that all changes to the system mass came out of the primary’s mass since the effect of J_2 is to enhance the gravitational effect of a body, making its gravitational effect similar to a spherical body of a larger mass. Therefore, we held Squannit at the mass of 13.5×10^{10} kg derived by [Ostro et al. \(2006\)](#). Squannit’s updated density is calculated by multiplying [Ostro et al. \(2006\)](#)’s density estimate by the ratio of initial to updated volume, $V_{si}/V_{su} = 0.981$. We found 2.76 g/cm³, slightly lower than the original estimate of 2.81 g/cm³.

We also calculated an updated density for Moshup as $\rho_{pu} = \rho_{pi} - (M_i - M_u)/V_p$, where M_i and M_u are the initial and updated system masses, respectively. We found 1.92 g/cm³, also slightly lower than the original measurement of 1.97 g/cm³.

The ratio of secondary/primary densities is approximately 1.43 for both the initial measurements of [Ostro et al. \(2006\)](#) and our updated measurements. Therefore, including librations in the shape model of Squannit did not resolve the apparent large density difference between the secondary and the primary.

4.5 Discussion

While we are able to successfully model the orbit of the 1999 KW4 system using range-Doppler data spanning 2001–2019, our initial exploration of the chi squared surface is not sufficiently thorough yet to claim that we have identify the global minimum with reasonable confidence. We found more than one solution with a reduced chi squared < 1 ,

which makes it challenging to pin down the orbital expansion rate with confidence. At this point in our exploration of the possible solutions, we adopt an upper bound on the mutual orbit expansion rate of $da/dt < 0.58$ cm/yr. A solution with zero orbital expansion gives an equivalently good fit. Our value is smaller than the theoretical/numerical estimate of 7 cm/year of [McMahon and Scheeres \(2010\)](#) and the lightcurve modeling derived estimate of 1.2 ± 0.3 cm/year by [Scheirich et al. \(2021\)](#), which requires a number of assumptions (Section 4.3.2) that may not accurately represent the binary system.

Because the tidal expansion rate could be smaller or larger than the nominal estimate of 0.006 cm/yr, we cannot definitively say whether any orbital expansion that may be occurring is dominated by tidal effects or by non-gravitational effects such as the BYORP effect.

In the case of an orbital expansion rate of 0.58 cm/yr, we expect that the 1999 KW4 system will expand until it reaches the Hill radius and splits into an asteroid pair in no less than 7.2×10^5 years. This statement holds for tidal expansion. However, if expansion is dominated by BYORP, any rubble movement on the primary or the secondary may lead to changes in the strength and direction of the BYORP effect, which could act to effectively slow down the expansion of the orbit or even change its direction towards orbit contraction. Additionally, any changes in heliocentric semimajor axis of the system over this timescale will change the Hill radius, which will also affect the lifetime of the system. The formation of another satellite by spinup of Moshup would also dramatically affect the system dynamics. Finally, any close planetary encounter could also cut the system lifetime short. However, apart from such dramatic events, the system lifetime is not expected to be particularly short.

Our refined shape model of the secondary and refined system mass estimate leave the secondary/primary density ratio virtually unchanged from the density ratio of 1.43 derived by [Ostro et al. \(2006\)](#), reinforcing that the secondary’s density is in fact higher than that of the primary. In the likely scenario of binary formation via YORP spin-up, one would expect the primary and secondary to be of similar composition, suggesting

that the density difference must be mainly due to a difference in porosity. [Scheeres et al. \(2006\)](#) suggests that the secondary’s libration could cause persistent shaking that acts to compact the body, decreasing porosity, while the primary is at a near-critical rotation rate, which may act to expand the body near the equator and increase its porosity. Squannit’s structure may also be dominated by one or a few large boulders, giving the secondary a higher density than the primary. Further testing via radar sounding could determine the structure of the secondary, but such measurements can presently only be made by a spacecraft in close proximity to the asteroid. While there are no planned missions to 1999 KW4, the ESA-led Hera mission will rendezvous with the Didymos - Dimorphos asteroid system, a near-Earth binary of similar size and morphology to the 1999 KW4 system, in November of 2026 and will conduct radar sounding observations of the binary system, providing an opportunity to study the interiors of bodies likely formed via YORP spinup ([Michel et al., 2022](#)).

4.6 Conclusions

We reanalyzed the high-quality planetary radar data set of Moshup-Squannit obtained in 2001 to create an improved shape model for Squannit that accounts for its longitudinal librations. We found that the data are consistent with a libration amplitude between 0° and 10° , with a best fit libration amplitude of 4° and a libration phase of 75.1° at time of pericenter passage on $\text{MJD} = 52055.4132$.

We measured the mutual orbit expansion rate by fitting the secondary-to-primary COM range-Doppler separations spanning 2001—2019 to an orbit model that includes both the oblateness of Moshup and a non-gravitational tangential acceleration term. We found a best fitting value for the tangential acceleration of $9.12 \times 10^{-15} \text{ m/s}^2$, equivalent to $da/dt = 0.58 \text{ cm/yr}$. This value is roughly two orders of magnitude larger than one possible value of the tidal expansion rate of $da/dt = 0.006 \text{ cm/yr}$ applicable if the system tidally expanded over 10 My. If the system expanded over only 100 ky, the expansion due

to tides would exceed the BYORP expansion.

With the measured expansion rate of 0.58 cm/yr, we can estimate a remaining lifetime no shorter than 7.2×10^5 years for the system.

We find that our updated secondary shape model and system mass estimates provide nearly the exact same secondary/primary density ratio as was found previously by [Ostro et al. \(2006\)](#), reinforcing the idea that the secondary is less porous than the primary.

Software used: Shape, ([Hudson, 1994](#); [Magri et al., 2007](#); [Naidu et al., 2015](#)), Orbitfit, ([Fang et al., 2011](#)), Mercury, ([Chambers, 1999](#)), MPFIT, ([Markwardt, 2009](#)), NumPy ([Harris et al., 2020](#)), SciPy ([Virtanen et al., 2020](#)), pandas ([Wes McKinney, 2010](#)), Matplotlib ([Hunter, 2007](#)), Astropy, ([Astropy Collaboration et al., 2022](#)), OpenCV, ([Bradski, 2000](#))

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CHAPTER 5

Conclusion

In this work, I have contributed to the discovery of asteroids and the characterization of their sizes and dynamics. First, I assess the accuracy of asteroid diameters estimated from the reparameterized near-Earth asteroid thermal model (NEATM) when two infrared bands are used versus when all four available infrared bands are used. We focus in particular on the W2 and W3 bands from the WISE mission, as these are most similar to the bands that will be available on the soon-to-be-launched NEO Surveyor telescope, and the W3 and W4 bands, as most asteroids that were only imaged in two WISE bands were imaged in these two longer wavelength bands. We found diameters modeled using observations from W2 and W3 bands to be systematically $\sim 10\%$ smaller than their more accurate counterparts obtained with all four WISE bands, while diameters modeled using observations from W3 and W4 bands were systematically $\sim 11\%$ larger than their four WISE band counterparts. In both the W2-3 case and the W3-4 case, the diameter uncertainties were larger for the two band fits than for the four bands fits, as is to be expected.

Next, I utilize the single-exposure images from the cryogenic portion of the WISE mission (January - August 2010) to detect and report asteroid discovery candidates using synthetic tracking. We report astrometry to the Minor Planet Center for 8,223 discovery candidates, of which 66 have so far been linked at the MPC to designated objects. We also report previously unreported astrometry for 13,575 known objects. The Vera Rubin Telescope is poised to discover millions of asteroids ([Željko et al., 2019](#)), and we expect hundreds of these discovery candidates to be linkable to these discoveries. When linked, we expect these objects to have excellent orbits arising from a > 15 -year orbital arc.

For the last portion of this work, I use high-quality planetary radar delay-Doppler imaging taken of the 1999 KW4 near-Earth asteroid binary system with the Arecibo observatory and the Goldstone radar system between 2001 and 2019. First, we fit for a new model of the secondary including terms for a possible longitudinal libration and we find a best fit that is suggestive of a $\sim 4^\circ$ libration, although the data is also in agreement with a libration-free model within the uncertainties. We use this model and the [Ostro et al. \(2006\)](#) model of the primary to refine the center-of-mass (COM) estimates for each body and thus the orbital separation estimates at each observation time. Using these refined separation estimates, I fit a mutual orbit for the system that includes the J2 contribution of the primary, the gravitational effects of the Sun, and a non-gravitational tangential acceleration component. I find a best fitting outward orbital expansion rate for the semimajor axis of $da/dt < 0.58 \text{ cm/yr}$, leading to an estimated system lifetime of $> 7.2 \times 10^5$ years. These results suggest that the system is not expected to be separated in the near future and that the BYORP effect could play a role in addition to tidal evolution in expanding the mutual orbit.

APPENDIX A

Chapter 1 Appendix

A.1 Emissivity in Two-band Fits

To examine the validity of using a constant emissivity ($\epsilon = 0.9$) in the two-band W2-3 fits, we added one degree of freedom to our model and performed additional fits with a single floating value of the emissivity, ϵ_{2-3} , which we applied to both bands. We used the same minimization term with regularization that was used for four-band fits. We ran a single model for each asteroid, using only the earlier cluster of observations when two clusters were available. We then eliminated asteroids for which either the constant-emissivity model or the floating-emissivity model gave a pseudo-temperature below 273 K, leaving a total of 4146 asteroids modeled. The results appear to cover a range of emissivity values (Figure A.1, Left), but tend to cluster around the starting values of the emissivity trials, which included 0.6, 0.7, 0.8, 0.85, 0.9, 0.95, and 0.99. This clustering indicates that the fitting algorithm generally does not experience sufficient pressure to depart from the initial condition on emissivity, and that this parameter is therefore not essential. The pseudo-temperature and diameter distributions are qualitatively similar (Figure A.1, Right).

We conducted an analysis of variance (also known as F-test) to determine whether the inclusion of the additional floating parameter (ϵ_{2-3}) was statistically justified. We found that the addition of the emissivity parameter cannot be deemed superfluous (p-value of 0.05) for only 5.5% of our solutions. In the other 94.5% of solutions, the additional parameter is superfluous. Because we expect about 5% of cases to reach the critical p-value by definition, the results of the F-tests strongly suggest that a fixed emissivity is sufficient

when fitting two infrared bands.

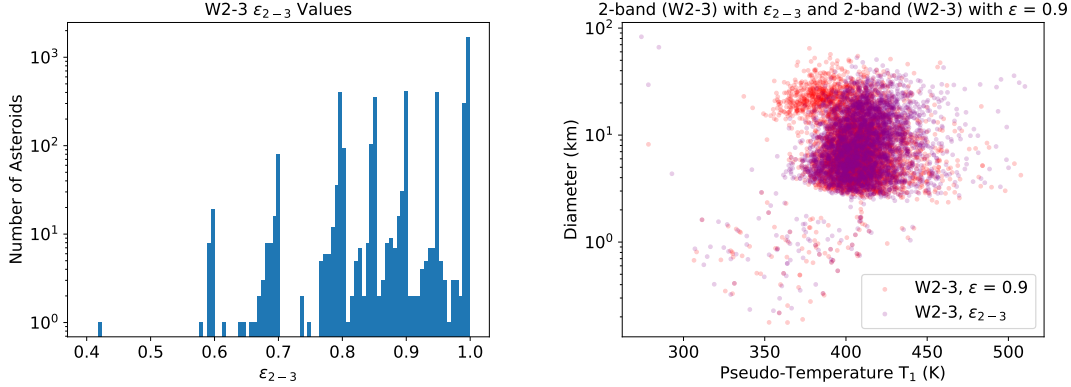


Figure A.1: **Left:** Histogram of emissivity values obtained with the floating-emissivity model and W2-3. **Right:** Diameter vs. pseudo-temperature two-band estimates for the constant-emissivity model and the floating-emissivity model.

A.2 Goodness of Fit Metric and Light Curve Variations

Although we did not include WISE flux uncertainties when minimizing the SSR, we were interested in evaluating the traditional reduced chi-squared value as an indicator of goodness of fit. For each cluster of observations, we calculated the median value over 200 bootstrap trials of

$$\chi_\nu^2 = \frac{\chi^2}{DOF} = \frac{1}{N-p} \sum_{i=1}^N \frac{(O_i - C_i)^2}{\sigma_i^2}, \quad (\text{A.1})$$

where N is the number of data points, p is the number of parameters being fit by the model, O_i is an observed flux, C_i is the corresponding modeled (computed) flux, and σ_i is the flux uncertainty (Figure A.2). The instances of the four-band bootstrap trials used to calculate χ_ν^2 , the reduced chi-squared, are different from those of Myhrvold et al. (2022), but we expect essentially the same results.

The WISE flux uncertainties have been found in previous research by Hanuš et al. (2015) and Myhrvold (2018a) to be underestimated. To account for this problem, Myhrvold et al. (2022) used “double detections” of thousands of objects to derive more accurate uncertainties. They found correction factors that can be multiplied by the WISE uncer-

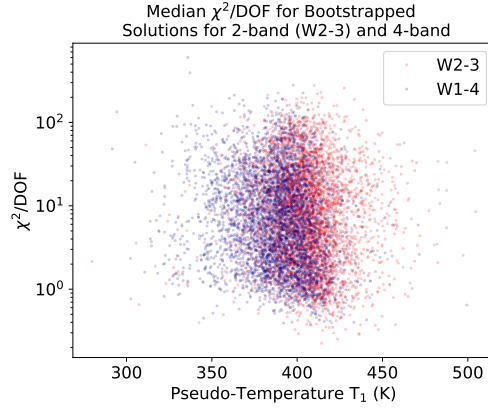


Figure A.2: Reduced chi-squared vs. pseudo-temperature. Pseudo-temperatures and reduced chi-squared values are both represented by the median value of 200 bootstrapped trials. Two-band values are shown in red whereas four-band values are shown in blue.

tainties to obtain more realistic uncertainty values. These correction factors are 1.224 for W1, 1.120 for W2, 1.479 for W3, and 1.218 for W4 (Lam et al., 2022), and they have been applied to the σ_i used in equation (A.1).

The median values of reduced chi-squared are larger than unity ($\chi^2_\nu = 7.6$ for W2–3 and $\chi^2_\nu = 6.4$ for W1–4), and some reduced chi-squared values exceed 100. We ascribe the large reduced chi-squared values primarily to lightcurve variations, which are not accounted for in our model. Reduced chi-squared values are indeed correlated with light curve amplitude, with a correlation coefficient of 0.48 between the logarithm of the amplitudes and the logarithm of the two-band reduced chi-squared values and a correlation coefficient of 0.50 between the logarithm of the amplitudes and the logarithm of the four-band reduced chi-squared values. (Figure A.3).

The observations of asteroid 1182 (Figure A.4) provide an example of this phenomenon. This asteroid has a light curve amplitude of 1.16 mag and a reduced chi-squared of 196 for the W2–3 model and 191 for the W1–4 model.

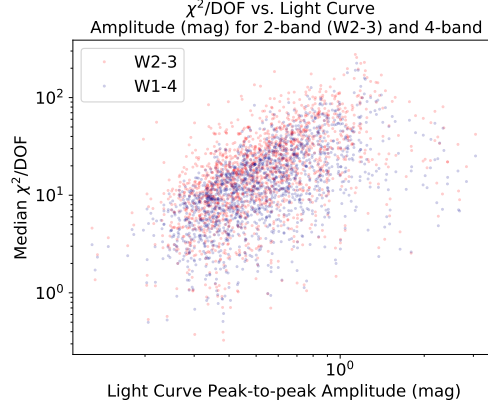


Figure A.3: Logarithmic plot of lightcurve peak-to-peak amplitude vs. reduced chi-squared for all remaining clusters with a corresponding lightcurve fit. W2–3 clusters are shown in red whereas W1–4 clusters are shown in blue. Lightcurve amplitudes were obtained from Lam et al. (2022).

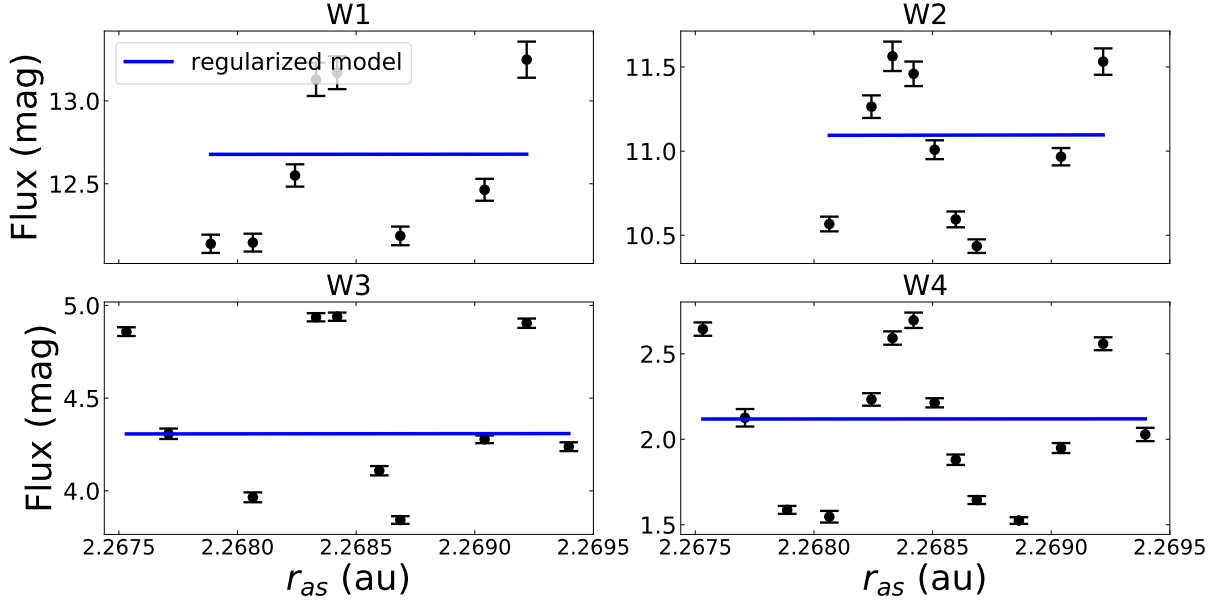


Figure A.4: W1–4 flux values vs. asteroid-Sun distance for asteroid 1182. Black points and error bars indicate the WISE fluxes and uncertainties, whereas blue lines indicate the best four-band fit to the data. Error bars incorporate the correction factors.

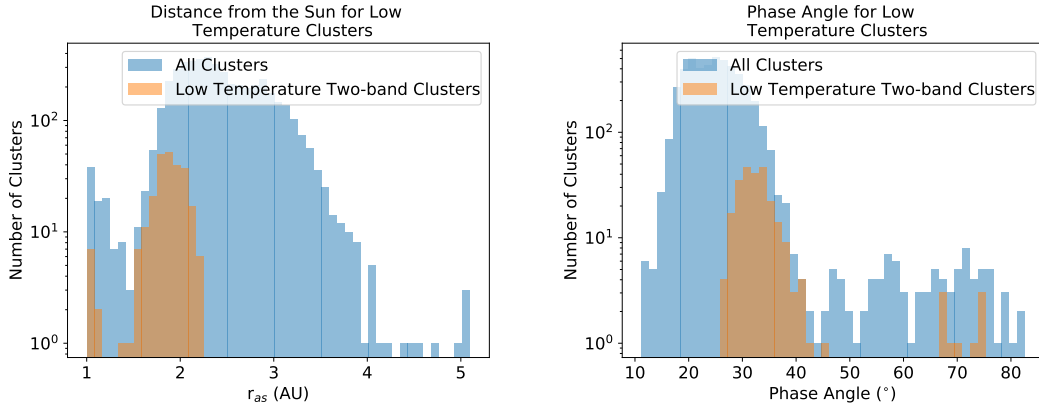


Figure A.5: Histograms of distance from the Sun (Left) and phase angle (Right) at the time of observations. The blue bars represent all observational clusters, while the orange bars represent clusters that had a low median pseudo-temperature ($T_1 < 273$ K) from the two-band bootstrapped fits, before implementation of our low-temperature solution removal process.

A.3 Anomalous Low-Temperature Solutions and Distance to the Sun

In our exploration of the factors contributing to low pseudo-temperature values in some W2–3 solutions, we found a correlation between the asteroid-Sun distance during observations and the median pseudo-temperature of that cluster from the bootstrapped trials, as well as a correlation between phase angle and median pseudo-temperature (Figure A.5). With the WISE observing geometry, asteroids observed at small distances from the Sun generally have larger phase angles than asteroids located further away. We suspect that anomalous thermal solutions are more likely for asteroids at large phase angles due to NEATM limitations, errors in the assumed slope parameter (G) value, and the fact that the HG phase function (Bowell et al., 1989, but see correction in Myhrvold et al. (2022)) does not adequately account for the true brightness expected at larger phase angles. We found that 4.8% of the W2–3 low pseudo-temperature clusters represent near-Earth objects, which is somewhat higher than the 2.0% of all W2–3 clusters that are near-Earth objects. Most of the low-temperature solutions are due to asteroids with small asteroid-Sun distances and relatively large phase angles, such as inner main belt objects or Mars-crossing objects.

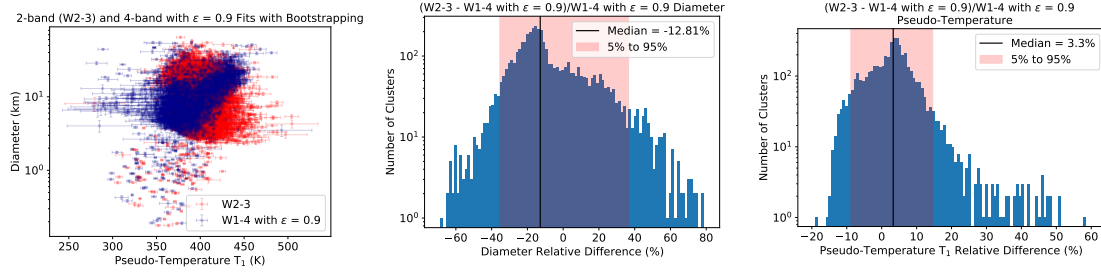


Figure A.6: Comparison of results from two-band W2-3 fits to results from four-band fits with ϵ fixed to 0.9. **Left:** Diameter vs. pseudo-temperature T_1 for 4533 clusters. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. **Middle:** Histogram of the relative difference in diameter. Median values are represented with a vertical black line while a pink highlighted block is used to represent the 5 - 95% quantile range. **Right:** Histogram of relative difference in pseudo-temperature.

A.4 Robustness with Respect to Minimization Technique

In addition to the nominal four-band fits, we also conducted four-band fits where we fixed the emissivity to 0.9 and minimized the SSR, in order to replicate the two-band fits as closely as possible. The only difference between the two-band fits and these fixed-emissivity four-band fits is the number of wavelength bands used in the fitting process. The distribution of W1-4 solutions with fixed emissivity (Figure A.6) does look significantly different from the distribution of nominal four-band fit results (Figure 2.3). However, the difference in median diameter between the two-band and four-band fits remains very similar, $(D_{W2-3} - D_{W1-4, \epsilon=0.9})/D_{W1-4, \epsilon=0.9} = -13\%$, compared to the -10% observed with the nominal four-band fits. The difference in median pseudo-temperatures is also similar, 3.3% vs. 2.4% with the nominal four-band fits. This consistency in results suggests that the median offsets in pseudo-temperature and diameter observed between the two-band and four-band fits is due to the consequences of eliminating W1 and W4 from the fitting procedure rather than any difference in fitting methods between the two-band and four-band fits.

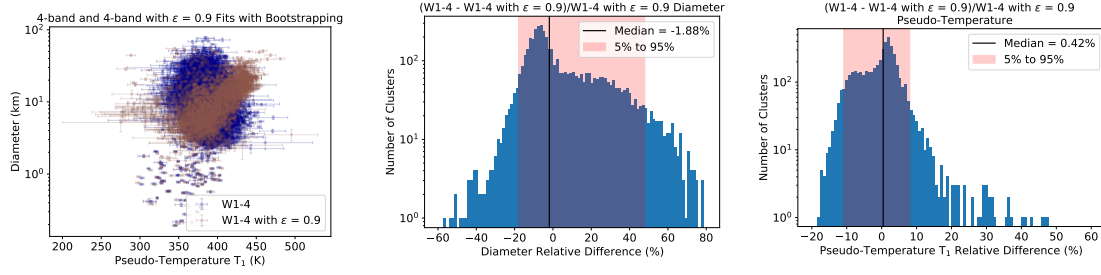


Figure A.7: Comparison of results from Myhrvold et al. (2022) four-band fits to results from four-band fits with ϵ fixed to 0.9. **Left:** Diameter vs. pseudo-temperature T_1 for 4672 clusters. Each point represents the median diameter and pseudo-temperature value among all accepted bootstrap trials for an object while error bars represent the standard deviations of the diameter and pseudo-temperature values among accepted bootstrap trials. **Middle:** Histogram of the relative differences in diameter. Median values are represented with a vertical black line and the 5 - 95% quantile range is represented with a region highlighted in pink. **Right:** Histogram of relative differences in pseudo-temperature.

A.5 Systematic Error Introduced from Constant Emissivity

Using a fixed emissivity at 0.9 in thermal models introduces systematic errors. To quantify this error, we compared results of the Myhrvold et al. (2022) four-band fits to four-band fits obtained with an emissivity fixed to 0.9 and with an SSR minimization term rather than a regularized minimization term. The diameter and pseudo-temperature estimates are noticeably different (Figure A.7). The median diameter difference $(D_{W1-4} - D_{W1-4, \epsilon=0.9})/D_{W1-4, \epsilon=0.9}$ is deceptively small at -1.88%, but the 5% and 95% quantiles of the distribution show diameter differences of -18% to 48%.

APPENDIX B

Chapter 2 Appendix

B.1 Detections Table

We create a “detections” SQL table containing all detections that exit our detection culling pipeline as true hits not already submitted to the MPC. We include here the first 9 lines of the table to demonstrate its layout. The full machine-readable table is located in the supplemental materials and is titled “detections.txt.” Columns represent:

- Internal Des: The internal designation we assign for the hit.
- Patch ID: The identifier for the sky patch in which the hit is located.
- Hit: The number of the hit within the list of hits output by SALTAD for the patch in which the hit is located. Hits are output for each patch in descending order of MLE.
- MLE (dB): The maximum likelihood estimate (MLE) of the hit in decibels.
- Submitted?: Specifies whether the detection has been submitted to the MPC (1 = yes, 0 = no).
- Detection Code: A code corresponding to the quality of the detection (see [Section B.1.1](#) for details).
- MPC Des: The MPC-assigned temporary designation or number for a candidate in the case that it was submitted as a discovery candidate but was found by the MPC to match a known object.

- RA (deg): Right Ascension of the detection, in degrees.
- Dec (deg): Declination of the detection, in degrees.
- MJD (d): Modified Julian Date at the midpoint of the exposure for the detection, in days.
- Object Match: The known object, if any, with astrometry sufficiently matching that of the confirmed candidate.
- Detection Frame ID: The internal designation of the hit with the zero-indexed frame number for the detection added to the end; represents a unique identifier for the detection.
- $\frac{d(eclon)}{dt}$ (deg/d): The longitudinal portion of the on-sky velocity in ecliptic coordinates, in degrees per day.
- $\frac{d(eclat)}{dt}$ (deg/d): The latitudinal portion of the on-sky velocity in ecliptic coordinates, in degrees per day.
- RMS Res ("): The root-mean-square of the residuals between the SALTAD detection and the HORIZONS ephemeris position for each detection in the hit, in arcseconds.
- RA Res ("): The longitudinal component in equatorial coordinates of the residual, in arcseconds.
- Dec Res ("): The latitudinal component in equatorial coordinates of the residual, in arcseconds.
- Com: Comment on the detection given by a two letter code. Codes are as follows:
 - CC: comet candidate
 - FC: false comet
 - NI: NaN image accidentally included in the subfield

- DK: Duplicate detection for a known object that was submitted on accident and has since been deleted at the MPC
- VI: Detection was eliminated after visual inspection of the candidate
- SNR (dB): Signal to noise ratio of the detection in decibels, as calculated by SALTAD.

B.1.1 Detection Codes

Figure B.1 describes all different detection codes used to differentiate different kinds of submittable and non submittable detections. Only detections with codes 0, 1 or 4 are submitted to the MPC. Our table includes 2670 detections representing 685 known objects for which the detection code equals 6 but the RMS of the residual is $\leq 3''$. Some of these detections were initially flagged as having large residuals due to errors in attributing hits to the correct object and/or errors in retrieving the correct ephemeris values which have since been resolved. Others were excluded due to a residual calculation error that has since been resolved. In these cases, the RMS of the residuals are no longer $> 3''$.

B.2 Estimating V magnitude from MLE dB

We obtain an approximate MLE-per-frame to V magnitude conversion equation by plotting all Skybot-estimated V magnitudes for 11,412 of the true hits to known objects in tract 6 versus their $\sqrt{MLE}$ divided by the number of frames in the subfield and fitting a line of best fit, as seen in Figure B.2. Our equation to estimate V magnitude from an MLE is given by

$$V_{mag} = -0.186961 \times 10 \log\left(\frac{\sqrt{MLE}}{N}\right) + 21.5288, \quad (\text{B.1})$$

where N is the number of frames in a subfield and V_{mag} is in magnitudes. We use the square root of MLE because the $MLE = S^2/\text{noise}^2$, where S is combined signal of the hit and noise is the square root of the variance between frames, so $\sqrt{MLE} = S/\text{noise}$. We estimate the uncertainty to be about ± 2 magnitudes. For each discovery candidate, we

Table B.1: Sample of SALTAD Detections Table

Internal Des	Patch ID	Hit	MLE (dB)	Submitted?	Detection Code	MPC Des	RA (deg)	Dec (deg)	MJD (d)	Object Match	Detection Frame ID	$\frac{d(eclon)}{dt}$ (deg/d)	$\frac{d(eclat)}{dt}$ (deg/d)	RMS Res (")	RA Res (")	Dec Res (")	Com	SNR (dB)
00083Tm	192	2	8.30	1	0	-	86.737895	-18.493861	55266.302619	-	00083Tm0	17.1046	-13.7585	-	-	-	-	0.70
00083Tm	192	2	8.30	1	0	-	86.753401	-18.505089	55266.434923	-	00083Tm1	17.1046	-13.7585	-	-	-	-	3.60
00083Tm	192	2	8.30	1	0	-	86.762804	-18.511208	55266.501138	-	00083Tm2	17.1046	-13.7585	-	-	-	-	1.50
00083Tm	192	2	8.30	1	0	-	86.778634	-18.523040	55266.633442	-	00083Tm3	17.1046	-13.7585	-	-	-	-	2.70
000aaTB	872	1	8.30	1	0	-	84.907324	-16.642693	55264.185755	-	000aaTB0	41.4828	-17.9257	-	-	-	-	2.40
000aaTB	872	1	8.30	1	0	-	84.946317	-16.658110	55264.318059	-	000aaTB1	41.4828	-17.9257	-	-	-	-	0.10
000aaTB	872	1	8.30	1	0	-	84.965780	-16.663619	55264.384274	-	000aaTB2	41.4828	-17.9257	-	-	-	-	1.50
000aaTB	872	1	8.30	1	0	-	85.004628	-16.677400	55264.516578	-	000aaTB3	41.4828	-17.9257	-	-	-	-	1.50
000aaTB	872	1	8.30	1	0	-	85.043557	-16.692035	55264.648882	-	000aaTB4	41.4828	-17.9257	-	-	-	-	0.40

NOTE— Table of all SALTAD detections that exit our detection culling pipeline as true hits not already submitted to the MPC. Only the first 9 lines are shown here. See detections.txt for the full table.

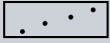
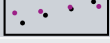
Detection code	Detection code number	Explanation	Picture (if applicable)	Submitted to MPC?
no_overlap	0	Hit does not overlap another hit in any frame		Yes
overlap_samepatch_highmle	1	Hit overlaps another hit in the same patch , but this detection is from the hit that had the highest MLE and is the only hit listed in the SQL table		Yes
overlap_differentpatch_highmle	2	Hit overlaps another hit in a different patch , with this detection having the highest MLE at this frame , not necessarily this frame that overlaps		No
overlap_differentpatch_lowmle	3	Hit overlaps another hit in a different patch , with this detection not having the highest MLE at this frame (for unknown or known objects)		No
merged_detection	4	A combination of two or more hits from two or more patches that includes the point from the highest MLE detection in each frame		Yes
lone_detection	5	Hit only has one detection with new astrometry, and therefore should not be sent to the MPC		No
large_res_detection	6	SALTAD residual RMS > 3" for the hit		No
toomany_unknown_hits	7	Unknown hit is one of > 10 for its patch, and is therefore unreliable		No
neardup_detection	8	Detection is < 5 arcsec from another detection in the detections table, but the hit doesn't overlap another hit		No
badbright_hit	9	Hit was flagged as a comet candidate, but is an artifact from a known object upon inspection		No
hand_elim	10	Hit was eliminated by hand after submission to the MPC for reason specified in comments column		Yes

Figure B.1: Description of each detection code listed in our detections SQL table. Only detections with codes 0, 1 or 4 are submitted to the MPC.

submit a single estimated magnitude rounded to the nearest tenth of a magnitude with an uncertainty of 2 magnitudes for the detection with the median SNR. However, when using our equation to calculate estimated V magnitudes, we did not properly convert the MLE dB for each discovery candidate from decibels to linear units before dividing by N and plugging it into equation B.1. Considering our submitted discovery candidates ranged in MLE from 8.3 to 13.2 dB and were from stacks of between 4 and 9 frames, our worst possible error would be the case of a 13.2 dB object detected in a stack of 4 frames, where the estimated V magnitude would be 20.2 and the reported V magnitude would have been 20.9. This maximum error of 0.7 magnitudes is within our given error bars of ± 2 magnitudes, and thus do not greatly effect our resulting submitted V magnitudes.

Additionally, because magnitudes scale logarithmically with a base of 2.5 while decibels scale use a logarithm with base 10, if we did it again, we would hold the slope constant at

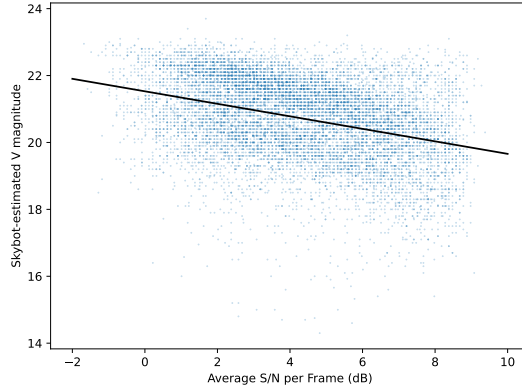


Figure B.2: Skybot-estimated V band magnitude vs. the MLE divided by the number of frames in a subfield. Blue points indicate hits matching with objects known to Skybot while the black line indicates the line of best fit given in equation B.1. We attempt to use the line of best fit to estimate a conversion rate from MLE to V magnitude for our discovery candidates. $\sqrt{MLE}$ is divided by the number of frames since $\sqrt{MLE}$ is given by the sum of the S/N of each frame, so $\frac{\sqrt{MLE}}{N}$ is equivalent to the average S/N for one frame of the hit.

-0.25 and fit for the intercept only. This gives the equation

$$V_{mag} = -0.25 \times 10 \log\left(\frac{\sqrt{MLE}}{N}\right) + 20.723101. \quad (\text{B.2})$$

While holding the slope constant is more technically correct, we see in this case that the objects with lower estimated V magnitudes tends to pull down the intercept value such that the the line of best fit provides an underestimate of the V magnitude for the majority of points being fit. In the case of a 13.2 dB object detected in a stack of 4 frames, the estimated V magnitude would be 18.9 while the reported V magnitude would have been 20.9, just reaching the 2 magnitude error bar for the worst case scenario.

APPENDIX C

Chapter 3 Appendix

C.1 Shape Model Figures

We include images of Squannit’s observed echoes, modeled echoes, and on-sky renderings of the three dimensional model in [Figure C.1](#).

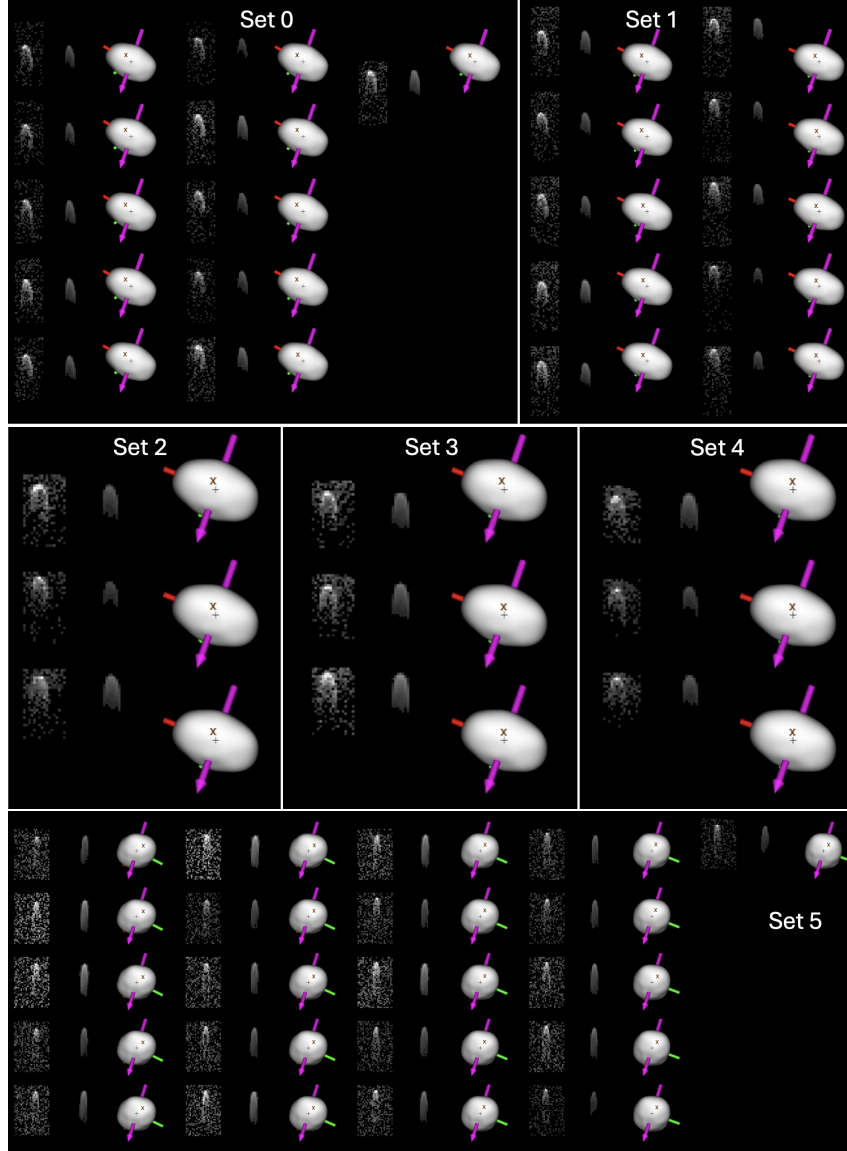
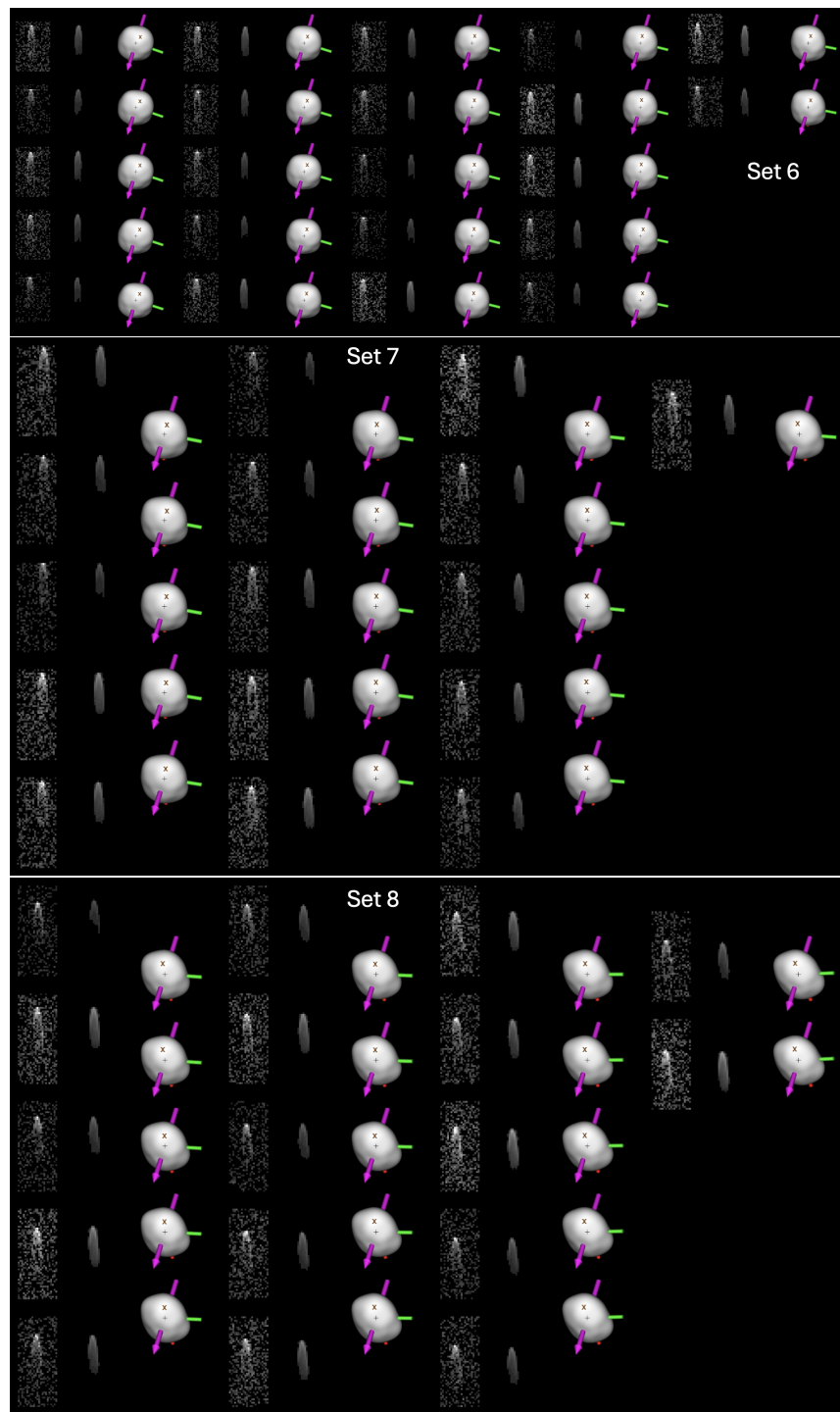
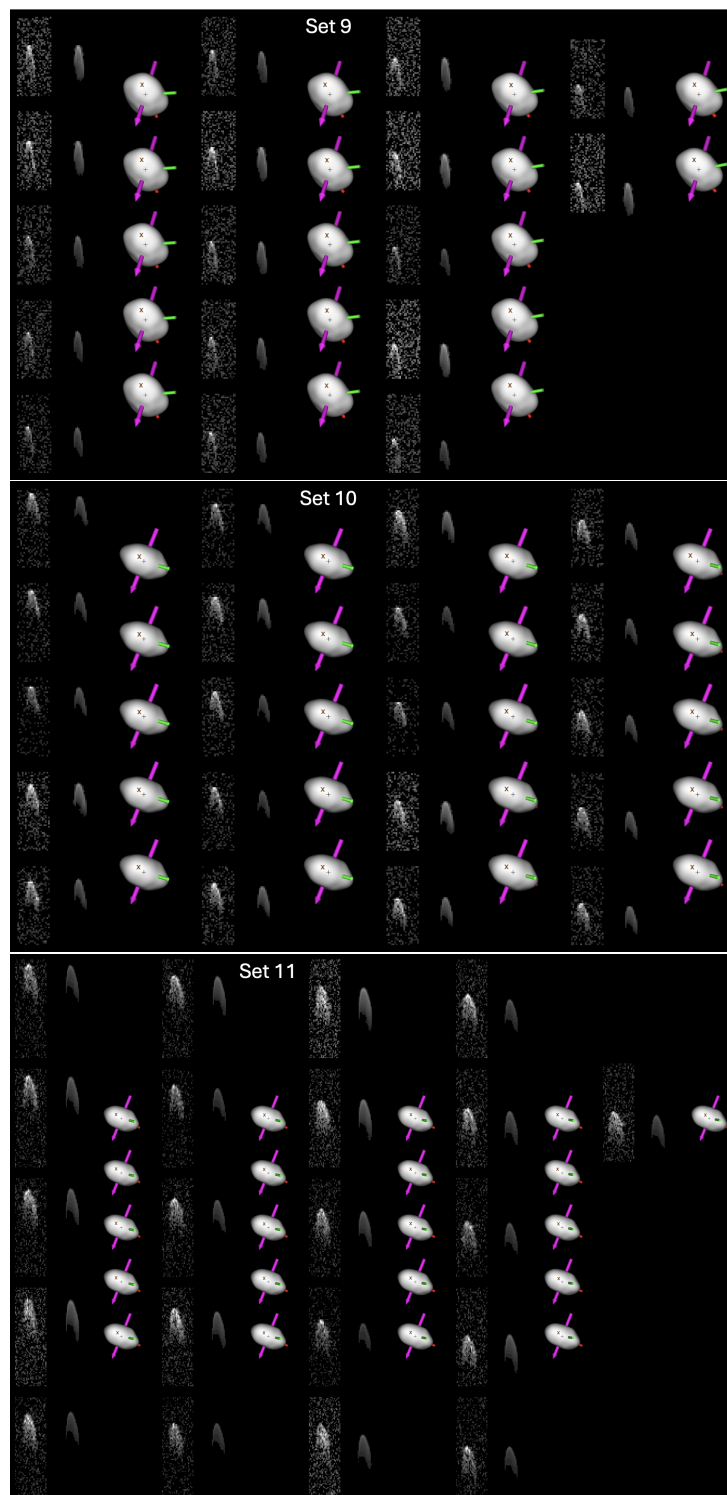
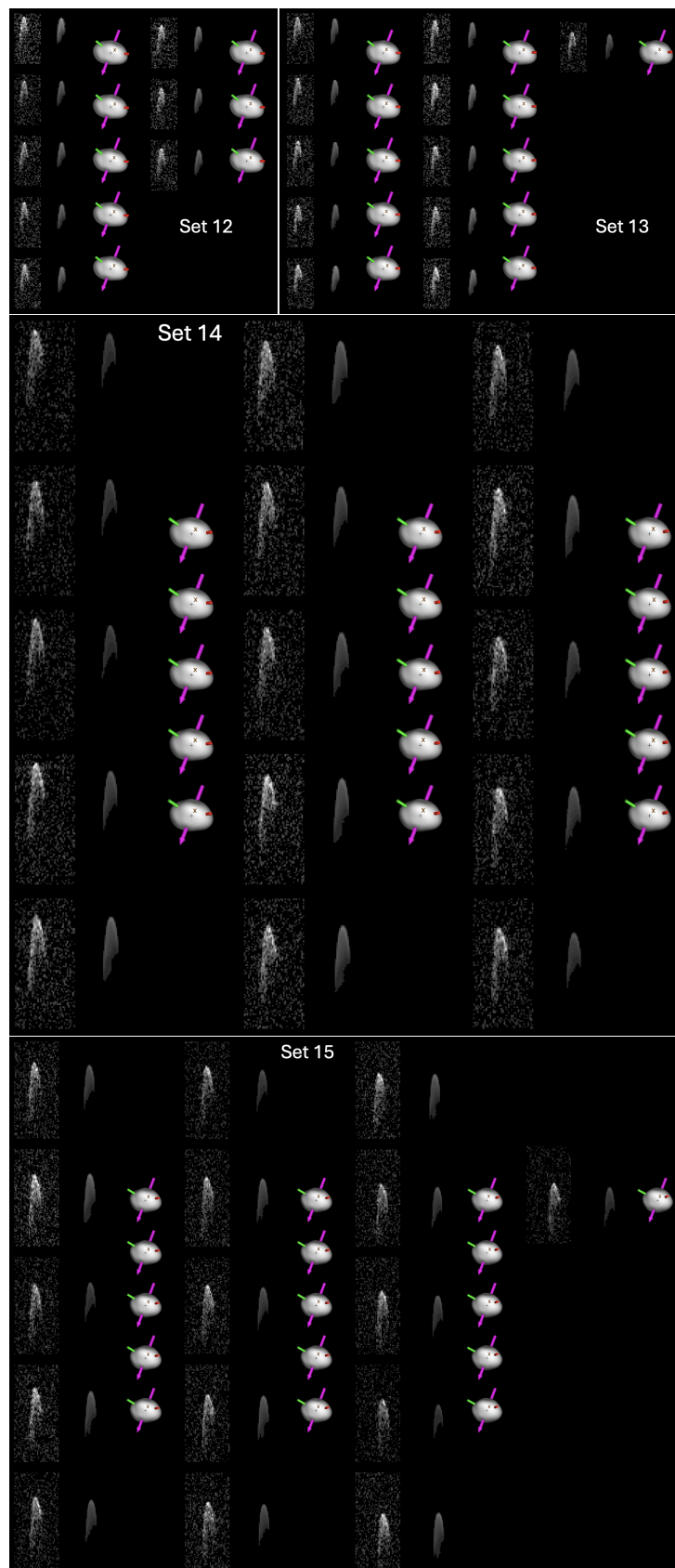
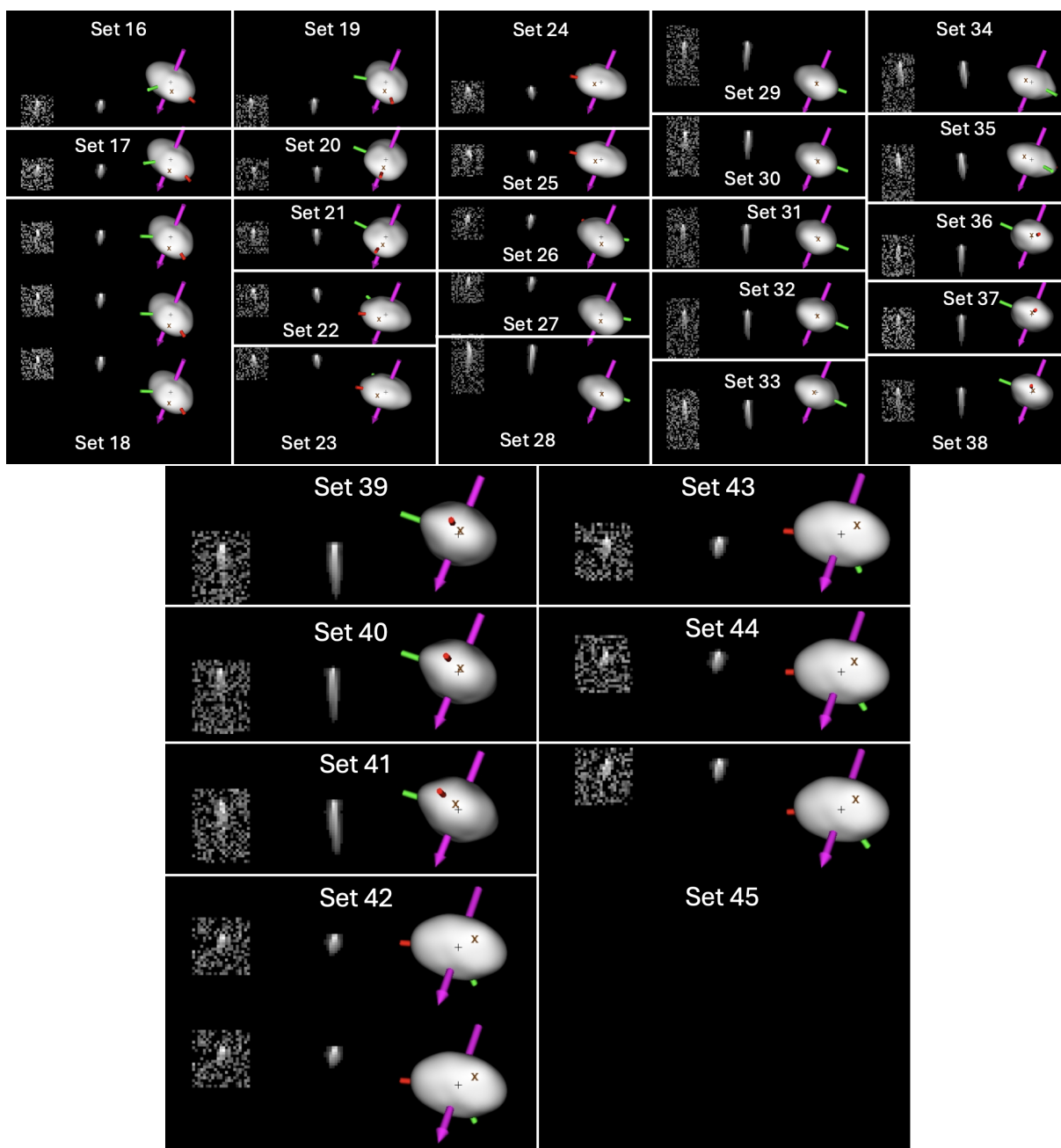


Figure C.1: Observed echo (left), modeled echo (center), and on-sky rendering (right) of Squannit at each observation epoch. The modeled echo and on-sky renderings use the best-fit shape model, which has a libration amplitude = 4° , libration phase = 75.1° at time of pericenter passage on MJD = 52055.4132, and rotation period = 17.36 hr. The spin vector is shown in magenta and the long and intermediate principal axes are shown in red and green respectively. The center of mass is marked with a '+' and the subradar point is marked with an 'X'. Figure is continued on subsequent pages.









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