

Lawrence Berkeley National Laboratory

Lawrence Berkeley National Laboratory

Title

TRUBULENT THERMAL CONVECTION BETWEEN HORIZONTAL PLATES

Permalink

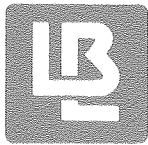
<https://escholarship.org/uc/item/72p280c7>

Author

Kaviany, M.

Publication Date

1981-03-01



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Engineering & Technical Services Division

RECEIVED
LAWRENCE
BERKELEY LABORATORY

APR 10 1981

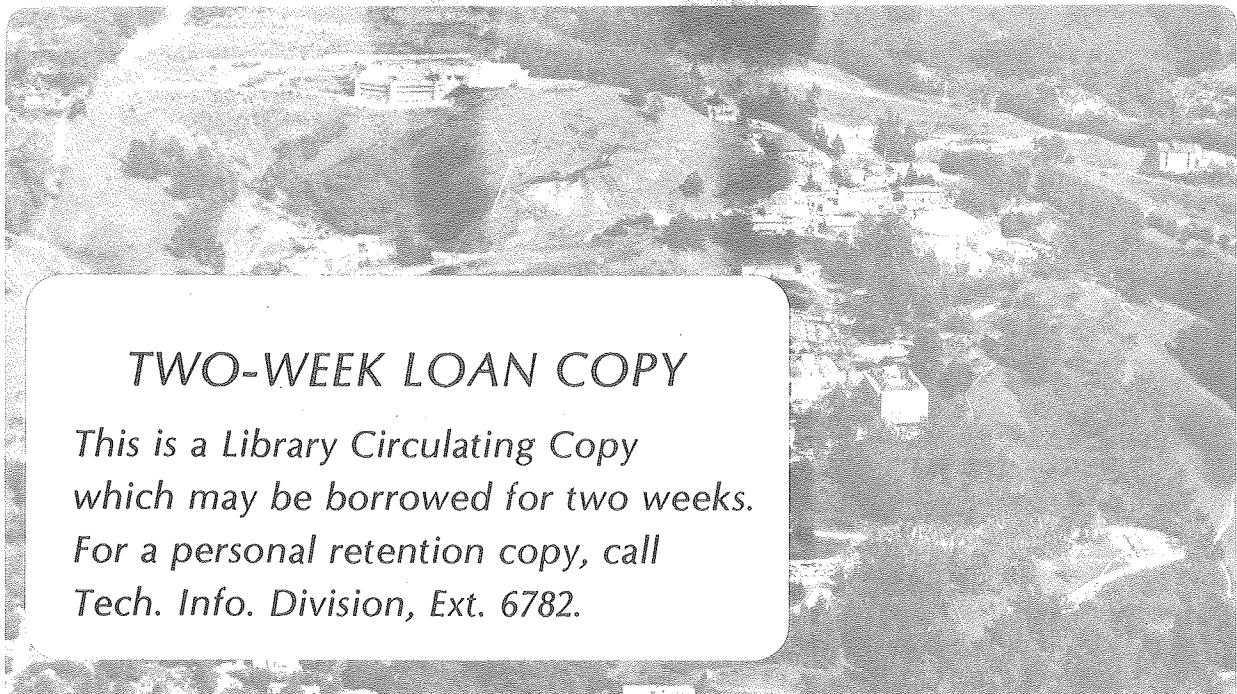
Submitted for publication

LIBRARY AND
DOCUMENTS SECTION

TURBULENT THERMAL CONVECTION BETWEEN HORIZONTAL
PLATES

M. Kaviany

March 1981



TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 6782.*

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

TURBULENT THERMAL CONVECTION BETWEEN HORIZONTAL PLATES

M. Kaviany

Department of Mechanical Engineering
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

ABSTRACT

Steady state turbulent thermal convection between horizontal plates is considered. A single equation of turbulence together with a specification of the distribution of mixing length is applied. A two layer model is assumed, a sublayer where only molecular transport takes place and a turbulent core where turbulent transport dominates. The thickness of the sublayer is determined from the linear stability theory. The governing equations are solved numerically. The predicted results are compared with the extensive measurements of Deardorff and Willis and the predictions of Kraichnan.

NOMENCLATURE

		<u>Greek Symbols</u>	
a	amplitude	α	thermal diffusivity
c, c_1, c_2	constants	β	thermal expansion coefficient
c_p	specific heat	γ	constant given in Equation (2)
D	dissipation	δ	sublayer thickness
g	gravitational acceleration	δ_c	thickness recommended by Chang
k	kinetic energy	δ_H	thickness recommended by Howard
K	thermal conductivity	δ_k	thickness recommended by Kraichnan
L	distance between plates	δ_s	thickness obtained from Linear Stability
ℓ	mixing length	ϵ	eddy diffusivity
Nu	Nusselt number	ν	kinematic viscosity
P	production	ρ	density
Pr	Prandtl number	τ	frequency
q_o	heat flux		
Ra	Rayleigh number		
T	temperature		
w	vertical component of velocity		
z	distance from the plate		
z_α	thermal boundary layer thickness		
z_ν	momentum boundary layer thickness		
z_ℓ	defined in Equation (9)		
z_u	spatial extent of integration		

<u>Subscripts</u>			<u>Subscripts</u>
c	cold		' fluctuating component
h	heat	_____	time averaged
k	kinetic energy	_____	non-dimensionalized
L	based on L		
δ	based on δ		
m	momentum		
D	dissipation		

Introduction

The problem considered here is steady state turbulent thermal convection in a horizontally infinite layer of fluid confined between rigid heat conducting plates, driven by a temperature difference between the plates. According to Turner [3] the transition to turbulence occurs at $Ra \approx 1.4 \times 10^4 Pr^{0.6}$, which has been verified for a large range of Prandtl number.

This problem has been treated both analytically and experimentally. Kraichnan [2] studied theoretically the dependence of turbulent convection on Prandtl number. His approach is that of Prandtl's mixing length and a similarity theory. Three distinguished regions were recognized; in the first, close to the boundary, molecular transport is dominant; in the second, both molecular and turbulent transport are important; finally still further out turbulent transport dominates. The thicknesses of the regions in which the molecular transport remains important, are estimated by taking their boundaries to be determined by characteristic values of local Reynolds and Peclet numbers. These characteristic values were given as 30 and 3, for $(\overline{w'^2})^{1/2}_z/\alpha$ and $(\overline{w'^2})^{1/2}_z/\nu$ respectively. This is essentially taking the mixing length to be the distance to the rigid boundary. Kraichnan's results for $Pr > 0.1$ are

$$\text{Nu}_L = 0.089 \text{ Ra}_L^{1/3}, \quad \text{Ra}_L = \frac{g\beta(T_H - T_C)L^3}{\alpha\nu}, \quad \text{Nu}_L = \frac{qL}{(T_H - T_C)K}$$

$$z_\alpha < z < z_\nu \quad (\bar{w}'^2)^{1/2} = 3.0\alpha z/z_\alpha^2$$

$$(\bar{T}'^2)^{1/2} = 0.17(T_H - T_C) z_\alpha/z$$

$$\bar{T} = (\bar{T}')^{1/2}, \quad z_\alpha = L/2 \text{ Nu}_L$$

$$z_\nu < z < L/2 \quad (\bar{w}'^2)^{1/2} = [\beta\alpha(T_H - T_C)^{1/3}](z/z_\alpha)^{1/3}$$

$$(\bar{T}'^2)^{1/2} = 0.18/(T_H - T_C) (z_\alpha/z)^{1/3}/\text{Pr}^{1/3}$$

$$\bar{T} = (\bar{T}'^2)^{1/2}, \quad z_\nu = 3.2 \text{ Pr}^{1/2} z_\alpha$$

(1)

Chang [4] applied the principle of wave motion and proposed a three layer model. Howard [5] suggested a theory similar to Chang's, based on slow accumulation and fast break away of bulbs of warm fluid in the boundary.

Somerscales and Gazda [6] carried measurements of the heat flux and the mean temperature distribution for $5 < \text{Pr} < 18$ and $10^7 < \text{Ra}_L < 4 \times 10^8$. Deardorff and Willis¹ measured the properties of turbulent thermal convection in an air layer. Rayleigh numbers of 6.3×10^5 , 2.5×10^6 and 10^7 were studied with a convection chamber designed to allow measurements to be taken along a horizontal path. Vertical profiles were presented of horizontally averaged temperature, r.m.s. fluctuations of temperature, horizontal and vertical velocities, total heat flux and other properties.

They confirmed the suggestion made previously by other workers, i.e.,

$$\text{Nu}_L = 0.069 \text{Pr}^{0.074} \text{Ra}_L^{1/3} \quad (2)$$

The extensive work of Deardorff and Willis will be used for comparison to the numerical prediction.

The goal of this study is the numerical prediction of distributions of the mean temperature and the turbulent kinetic energy. This is done by applying the one equation model of turbulence. A two layer model, namely a laminar (conduction) sublayer and a turbulent core, is proposed. The governing equations are nondimensionalized such that the results can be extended to all Rayleigh numbers. The numerical predictions are compared with the available experimental and analytical results.

System and Equations for One-Equation Model

Figure 1 is a sketch which indicates the steady state temperature distribution and some of the variables. The existence of a conduction (Laminar) sublayer is suggested by the experimental results. The region between the two sublayers is the turbulent core, where turbulent transport becomes important near the boundaries and dominates the molecular transport near the midplane. Only the upper half of the fluid layer will be considered and the solution can be extended to the lower half.

The temperature distribution is specified by the energy equation. In absence of any mean motion this equation is written as

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} K \frac{\partial T}{\partial z} - \rho c_p \overline{w' T'} \quad (3)$$

For the solution the turbulent heat flux $\overline{w'T'}$ has to be prescribed. The boundary conditions are

$$T = T_C \quad \text{at } z = 0$$

$$\frac{\partial T}{\partial z} = 0 \quad \text{at } z = \frac{L}{2} .$$

The eddy diffusivity for heat is defined such that, $-\epsilon_h \partial T / \partial z = \overline{w'T'}$ and ϵ_h is obtained from a prescribed mixing length and the kinetic energy of turbulence. The kinetic energy of turbulence is defined as $k = 1/2 \overline{u_i'^2}$ and is given by

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} (\nu + \epsilon_k) \frac{\partial k}{\partial z} + P - D . \quad (4)$$

The boundary conditions are

$$k = 0 \quad \text{at } z = 0$$

$$\frac{\partial k}{\partial z} = 0 \quad \text{at } z = \frac{L}{2} .$$

For a situation of no mean motion, the production term P is $-\beta g \overline{w'T'}$ or $\beta g \epsilon_h \partial T / \partial z$. The dissipation D is defined in terms of the kinetic energy of the turbulence and a mixing length

$$D = C_2 \frac{k^{3/2}}{\ell} . \quad (5)$$

The magnitude of ϵ_h is defined as

$$\epsilon_h = C_1 k^{1/2} \ell . \quad (6)$$

By assuming equilibrium between the production and the dissipation, and applying Equations (6) and (7) one obtains

$$\epsilon_h = \left(\frac{C_1}{C_2}\right)^{3/2} (\beta g)^{1/2} \ell^2 \left(\left|\frac{\partial T}{\partial z}\right|\right)^{1/2}, \quad (7)$$

which states the Prandtl's mixing length theory for turbulent transport of heat by the mean temperature field. This theory requires that $C_1^3/C_2 = 1$.

Conventionally, when there exists a momentum diffusivity, turbulent Prandtl numbers are defined such that

$$\epsilon_k = \frac{\epsilon_m}{Pr_k} = \frac{\epsilon_h Pr_h}{Pr_k}. \quad (8)$$

This much specification, together with a definition of the necessary constants and a prescription about the magnitude of the mixing length enables the solution of the problem. Here we assume that

$$\ell = z \text{ up to } z_\ell \text{ and a constant thereafter}, \quad (9)$$

which is identical to the assumption made by Kraichnan. The distance z_ℓ beyond which the mixing length is a constant has to be determined from the experimental results.

Nondimensionalization of the Governing Equations

For the case of steady state conditions considered here, Equation (3) when integrated once becomes

$$(\alpha + \epsilon_h) \frac{\partial T}{\partial z} = \frac{q_o}{\rho c_p}, \quad (10)$$

where q_0 is the total heat flux which does not vary with depth. Using Equation (10), the production becomes

$$P = \beta g \epsilon_h \frac{\partial T}{\partial z} = \frac{\beta g q_0 \epsilon_h}{\rho c_p (\alpha + \epsilon_h)} \quad (11)$$

Through q_0 , Equation (4) depends on the Rayleigh and Prandtl numbers. It is desirable to make the solution to Equation (4) independent of Rayleigh number. The experimental results of Deardorff and Willis indicates that at $z = L/2$ the turbulent kinetic energy is proportional to Ra_L and P and D are proportional to $Ra_L^{4/3}$. Therefore we define the following

$$\bar{z} = \frac{z}{L Ra_L^{1/6}}, \quad \bar{\ell} = \frac{\ell}{L Ra_L^{1/6}}, \quad \bar{k} = \frac{k}{\alpha^2/L^2 Ra_L},$$

$$\bar{D} = \frac{D}{\alpha^3/L^4 Ra_L^{4/3}} = C_2 \frac{\bar{k}^{3/2}}{\bar{\ell}}$$

$$\bar{\epsilon}_h = \frac{\epsilon_h}{Ra_L^{2/3}} = C_1 \bar{k}^{1/2} \bar{\ell}, \quad \bar{T} = \frac{T - T_C}{T_H - T_C} \quad (12)$$

Further we assume that in the turbulent core, where Equation (4) is assumed to hold, the molecular transport can be neglected. By making this assumption and applying Equations (5), (6), (8), (11) and (12) to Equation (4) one obtains

$$\frac{\partial}{\partial \bar{z}} \left(\frac{Pr_h}{Pr_k} C_1 \bar{k}^{1/2} \bar{\ell} \frac{\partial \bar{k}}{\partial \bar{z}} \right) + Pr Nu_L Ra_L^{-1/3} - \bar{D} = 0 \quad (13)$$

By applying Equation (2) to replace Nu_L by $C Ra_L^{1/3} Pr^\gamma$, Equation (13) becomes

$$\frac{C_1 Pr_h}{Pr_k} \frac{\partial}{\partial \bar{z}} \bar{k}^{1/2} - \bar{k} \frac{\partial \bar{k}}{\partial \bar{z}} + C Pr^{1+\gamma} - \bar{D} = 0 \quad , \quad (14)$$

where C and γ are the constants given in Equation (2). The boundary conditions for Equation (14) are the same as those for Equation (4). Equation (14) is independent of Ra_L . The production term does not vary with depth because the molecular transport is neglected.

With the above nondimensionalization one obtains a distribution for \bar{k} for a given Prandtl number. The resulting distribution should be applicable to all Rayleigh numbers in the turbulent regime.

The Conduction Sublayer

It has been recommended by Chang and later by Kraichnan and Somerscales et al., that there exists a region adjacent to the rigid boundary where molecular heat conduction is the dominant mode of transport.

Chang recommended an oscillatory conduction layer with thickness δ_c , amplitude a , and frequency τ_c given as

$$a = \delta_c = 5.06 \left(\frac{2\nu\alpha}{g\beta(T_H - T_C)} \right)^{1/3} \quad , \quad \tau_c = \frac{a^2}{4\alpha} \quad , \quad (15)$$

which when expressed in terms of Rayleigh number becomes

$$Ra_c = \frac{\beta g (T_H - T_C) \delta_c^3}{\nu\alpha} = 64.8 \quad .$$

Howard's theory which is based on an oscillatory conduction layer that extends to the midplane, leads to a frequency τ_H given by

$$\frac{g\beta(T_H - T_C) (\pi\alpha \tau_H)^{3/2}}{\nu\alpha} = 1000 .$$

If one assumes that the average thickness of the conduction layer is $(\pi\alpha \tau_H)^{1/2}$ then

$$Ra_H = \frac{g\beta(T_H - T_C)\delta_H^3}{\nu\alpha} = 1000 . \quad (16)$$

Kraichnan recommended that the thickness of the conduction sublayer be taken as $\delta_K = L/2Nu_L$. As is the case with Howard's theory, this assumption requires that all the temperature drop to take place in the two sublayers. However, through Equation (1) Kraichnan also recommended that for Prandtl number larger than 0.1, only 64 percent of the temperature drop takes place in the two sublayers. By applying Equation (2) we have the sublayer thickness recommended by Kraichnan as

$$Ra_K = \frac{g\beta(T_h - T_c)\delta_K^3}{\nu\alpha} = 380.5 Pr^{-0.222} . \quad (17)$$

However Somerscales and Gazda's observation led them to recommend a thickness about one hundredth of that suggested by Kraichnan.

Based on applications of linear stability theory to the case of two fluid layers with constant properties in each layer and with heavier fluid layer above the lighter fluid layer, we previously found (Kaviany) [7] a critical Grashof number given by

$$Gr_s = \frac{g\beta(T_H - T_C)\delta_s^3}{\alpha^2} = 864 \quad (18)$$

Since this prediction also assumes that all the resistance is in the sublayers, therefore it can only be used as the upper limit for the thickness of the sublayer.

In all of the above works δ/L is proportional to $Ra_L^{1/3}$, the constant of proportionality and Prandtl number dependency varies among the recommendations. Here we shall use a form

$$\frac{\delta}{L} = \frac{Ra_L^{1/3}}{Ra_L^{1/3}} \quad (19)$$

where Ra_δ is to be determined from the temperature distribution that will be obtained using the one equation model. These recommendations will be compared after obtaining the distribution of the eddy diffusivity.

Solution

Equation (14) was solved numerically as a time dependent and a steady state solution was found. The scheme of the solution was explicit finite difference. Equal spatial increments of $\bar{z} = 0.0005$ were used. The edge of the sublayer was taken to be at $\bar{z} = 0$. A total of 200 increments were used, thus $\partial\bar{k}/\partial\bar{z} = 0$ was applied at $\bar{z} = \bar{z}_u = 0.100$. For the linear mixing length distribution given by Equation (9) an analytical solution to Equation (14) exists. This solution indicates that \bar{z}_u appears in the argument of the hyperbolic tangent function which reaches its asymptotic value of unity for arguments larger than 3. This corresponds to \bar{z}_u of about 0.1, i.e., results of

Equation (14) are not sensitive to the value of \bar{z}_u as long as it is larger than 0.1. The reason for seeking a numerical solution to Equation (14) was that mixing length distributions other than linear were also considered for which an analytical solution did not seem feasible.

The values used for the constants are

C_1	C_2	\bar{z}_ℓ	Pr_h	Pr_k
0.73	0.39	0.022	0.59	1.0

constants; C_1 and C_2 are related through $C_1^3/C_2 = 1$ and along with \bar{z}_ℓ were chosen such that at a distance far away from the rigid boundary: $\bar{P} = \bar{D}$ and \bar{k} reaches its experimental value. These are different than the values 0.5 and 0.125 which are conventionally used for C_1 and C_2 respectively. The values for the turbulent Prandtl numbers, Pr_h and Pr_k are those used with success for prediction of transient turbulent thermal convection in a pool of water (Kaviany and Seban) [8].

Results for Turbulent Kinetic Energy

Figure 2 shows the numerical solution to Equation (14) for \bar{k} , also shown are the experimental results of Deardorff and Willis. The values of the parameters in their experiment were

Pr	Ra_L	Nu_L
0.713	6.3×10^5	5.8
0.713	2.5×10^6	9.1
0.713	1.0×10^7	14.4

The experimental results were non-dimensionalized according to Equation (12). The experimental values shown at $\bar{z} = 0$ are those taken from Deardorff and Willis at the edge of the sublayer, where a value equal to half of the sublayer thickness recommended by Chang in Equation (15) was used. Similar spatial transformation is applied to the other experimental data. The numerical results are shown with a solid wave. The experimental results are shown in symbols. The experimental data for all the three Rayleigh numbers reported by Deardorff and Willis are shown. According to the criterion suggested by Turner, for $Pr = 0.713$ even $Ra = 6.3 \times 10^5$ results in turbulent flow. However the trend of the distribution for the experimental data at $Ra = 6.3 \times 10^5$ is different than that for $Ra = 10$ [7].

The results show that the one equation model results in a distribution with a slope near the boundary that is not as large as the experimental data. For $\bar{z} > 0.06$ the turbulent kinetic energy takes on a constant value of 0.02.

Figure 3 shows the distribution of the production and the dissipation of the turbulent kinetic energy. The spatial distribution of the experimental data is adjusted such that $\bar{z} = 0$ is at the edge of the sublayer. The experimental results are shown in symbols and the numerical results are shown with the solid curve. As we have assumed, the production does not vary with depth and has a constant value of $CPr^{1+\gamma}$. For $\bar{z} > 0.06$ the dissipation and production are equal. The asymptotic value for \bar{D} is for $\bar{z} > 0.022$ which is the distance beyond which the prescribed mixing length takes on a constant value.

Figure 4 shows the distribution of the eddy diffusivity for heat and the distribution for the prescribed mixing length. The mixing length has a slope of unity and takes on a constant value of 0.022 for $\bar{z} > 0.022$. The distribution of the eddy diffusivity for heat is nearly linear up to $\bar{z} = 0.022$. It reaches a maximum value of 0.0023 at about $\bar{z} = 0.060$.

Figures 2 through 4 show the results for \bar{k} , \bar{D} and $\bar{\epsilon}_h$ that can be used for $Pr = 0.713$ at any Rayleigh number in the turbulent regime. These quantities can be converted to a dimensional form by applying Equations (12) through which the Rayleigh number dependency appears. Next we will use the Rayleigh number independent results for $\bar{\epsilon}_h$ given in Figure 4 to calculate the temperature distribution for a specific Rayleigh number and then the predictions will be compared with the experimental results.

Prediction of the Mean Temperature

The equation of thermal energy, i.e., Equation (10) can be written as

$$\left(1 + \frac{\epsilon_h}{\alpha}\right) \frac{\partial \bar{T}}{\partial \frac{z}{L}} = \frac{q_o L}{(T_H - T_C)K} = Nu_L, \quad (20)$$

where

$$\frac{\epsilon_h}{\alpha} = C_1 \bar{k}^{1/2} \bar{\lambda} Ra_L^{2/3} = \bar{\epsilon}_h Ra_L^{2/3}. \quad (21)$$

Equation (20) is integrated to obtain the mean temperature distribution, this results in

$$\int d\bar{T} = \int_0^{\frac{\delta}{L}} Nu_L d \frac{z}{L} + \int_{\frac{\delta}{L}}^{0.5} Nu_L \frac{d \frac{z}{L}}{1 + \epsilon_h} . \quad (22)$$

The value of the right hand side of Equation (22) depends on the thickness of the sublayer as well as the distribution of ϵ_h . The integrals on the right hand side of Equation (22) should result in a value of 0.5, if the thickness of the sublayer and the distribution of ϵ_h are specified properly.

Table 1 gives the values for the sublayer thickness resulting from Equation (15) through (18). The experimental conditions of Deardorff and Willis for air are used. Also shown in Table 1 are the values of the sublayer thickness obtained by requiring that the right hand side of Equation (22) to take a value of 0.5.

Pr = 0.713, Ra _L	δ/L					
	Eq. (15)	Eq. (16)	Eq. (17)	Eq. (18)	Eq. (22)	Eq. (23)
6.3×10^5	0.0609	0.117	0.0867	0.1111	0.0282	0.0300
2.5×10^6	0.0296	0.0737	0.0547	0.0702	0.0191	0.0189
1.0×10^7	0.0186	0.0464	0.0345	0.0442	0.0127	0.0119

When Equation (19) is used to find a Ra_δ that gives the results similar to that obtained by requiring the right hand side of Equation (22) to be 0.5, one finds a Ra_δ = 17. Thus Equation (19) becomes

$$\frac{\delta}{L} = \frac{2.6}{Ra_L^{1/3}} \quad (23)$$

Equation (23) does not include any Prandtl number dependency. No Prandtl number dependency was recommended by Kraichnan, even though the approximations he used in obtaining the sublayer thickness were different for high and low Prandtl numbers.

Figure 5 shows the distinction of the mean temperature for Rayleigh number of 10^7 and Prandtl number of 0.713. Solid curve represents the numerical results obtained here. The dashed line represents the prediction of Kraichnan given by Equation (1). It was necessary to change the value of the constant in Equation (1) from 0.18 to 0.078 in order to match the results at $z = z_v$. The experimental results of Deardorff and Willis are shown in symbols. Also shown are the sublayer thicknesses obtained from Equations (1) and (23). The results show that for $0.025 < z/L < 0.3$ the numerical predictions are lower than the experimental results. This is due to the relatively large value for ϵ_h in the region $0.025 < z/L < 0.1$ and relatively small value of ϵ_h for $z/L > 0.1$, predicted by the one equation model. The predictions of Kraichnan is in agreement with the experimental results up to $z/L = 0.07$ which is the value of z_v/L . Similar agreement was obtained for other Rayleigh numbers.

Some mixing length distributions other than linear were considered. An exponential distribution near the rigid boundary seemed to give satisfactory results, however due to the need for the specification of another constant in the exponent, this distribution was not followed further.

Conclusion

It has been shown that a single equation model of turbulence, together with a specification of the distribution of a mixing length similar to that recommended by Prandtl and Kraichnan, give a relatively adequate prediction of steady state distribution of the turbulent kinetic energy and the mean temperature. The turbulent kinetic energy was non-dimensionalized such that a general, Rayleigh number independent, solution

was found. The results suggest that the mixing length distribution near the rigid boundary should have distribution other than linear.

The numerical predictions are compared with the prediction of Kraichnan. The present results are less satisfactory near the sublayer, while those of Kraichnan give inadequate results away from the sublayer.

ACKNOWLEDGMENTS

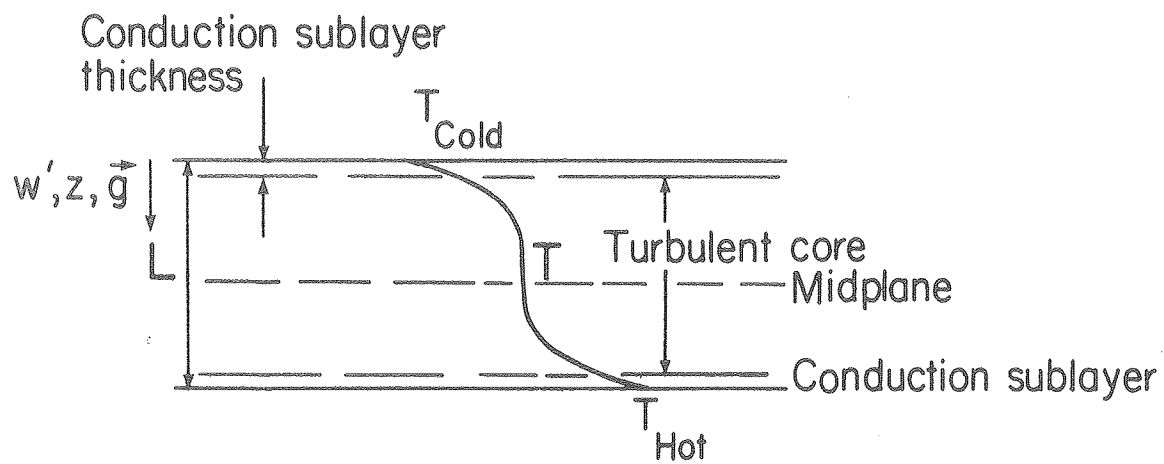
The author is indebted to Professor R. A. Seban for his valuable contributions. This work was supported in part by the U. S. Department of Energy under Contract No. W-7405-ENG-48.

REFERENCES

1. Deardorff, J. W., and Willis, G. E., "Investigation of Turbulent Thermal Convection between Horizontal Plates," J. Fluid Mechanics, Vol. 28, pp. 675-704 (1967).
2. Kraichnan, R. A., "Turbulent Thermal Convection at Arbitrary Prandtl Number," The Physics of Fluids, Vol. 15, No. 11, pp. 1374-1389 (1962).
3. Turner, J. S., "Buoyancy Effects in Fluids," The University Press, Cambridge, 1973.
4. Chang, Y. P., "A Theoretical Analysis of Heat Transfer in Natural Convection and in Boiling," Transaction of ASME, pp. 1501-1513 (1957).
5. Howard, L. W., "Convection at High Rayleigh Number," 11th Annual Meeting, Int. Congress of Applied Mechanics (1964).
6. Somerscales, E. F. C. and Gazda, I. W., "Thermal Convection in High Prandtl Number Liquids at High Rayleigh Numbers," Int. J. Heat and Mass Transfer, Vol. 12, pp. 1491-1511 (1969).
7. Kaviany, M., "Analysis of Turbulent Generation between Horizontal Plates," Ph.D. Thesis, University of California, Berkeley (1979).
8. Kaviany, M., Seban, R. A., "Transient Turbulent Thermal Connection in a Pool of Water," submitted to Int. J. Heat and Mass Transfer (1980).

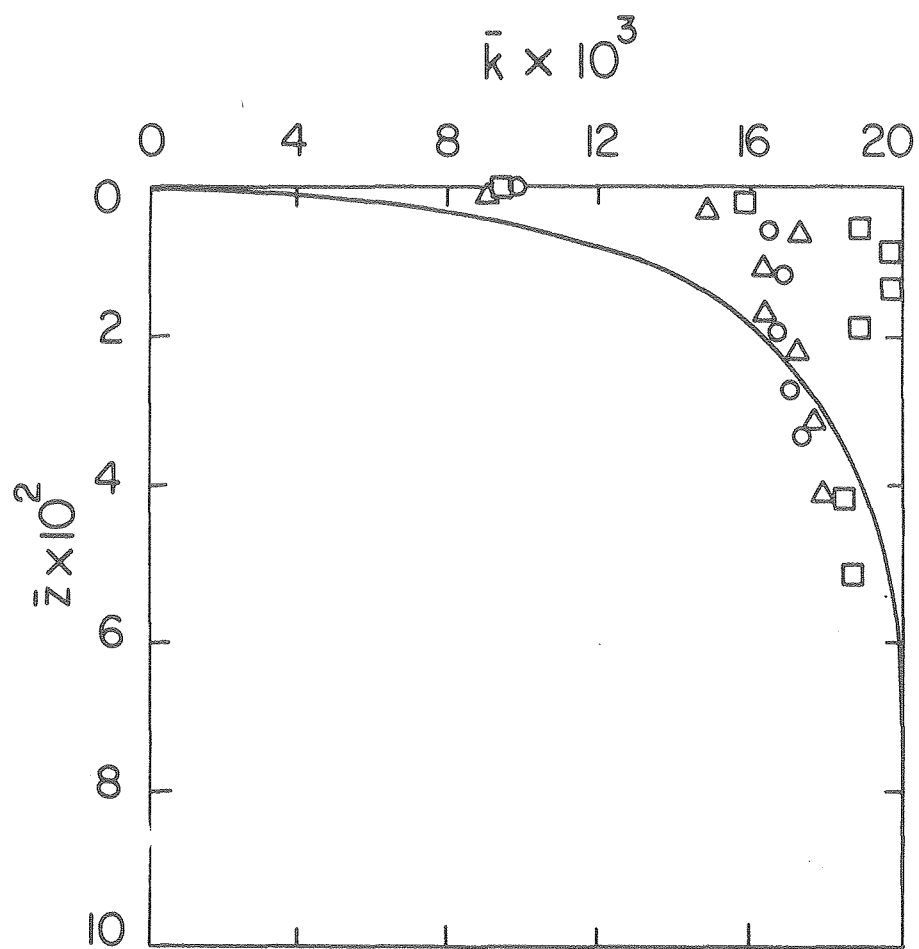
FIGURE CAPTIONS

- Fig. 1. Details of the problem considered.
- Fig. 2. Distribution of the turbulent kinetic energy, solid curve represents the numerical results of one equation model and the symbols represent the experimental results of Deardorff and Willis. $Pr = 0.713$, $\square \dots Ra_L = 6.3 \times 10^5$, $\Delta \dots Ra_L = 2.5 \times 10^6$ and $\circ \dots Ra_L = 10^7$.
- Fig. 3. The production and the dissipation of the turbulent kinetic energy. Solid curves represent the numerical results of one equation model and Symboles represent the experimental results of Deardorff and Willis. $Pr = 1.713$, $\square \dots Ra_L = 6.3 \times 10^5$, $\Delta \dots Ra = 2.5 \times 10^6$ and $\circ \dots Ra = 10^7$.
- Fig. 4. Distributions of the prescribed mixing length and eddy diffusivity for heat.
- Fig. 5. Mean temperature distribution for $Ra = 10^7$, $Pr = 0.713$. Solid curve represents the numerical results of one equation model, dashed curve represents the prediction by Kraichnan and the symboles indicate the experimental results of Deardorff and Willis. Also shown are δ/L as defined in this study and z_α/L and $z\sqrt{L}$ as defined by Kraichnan.



XBL 8012-2469

Fig. 1



XBL 8012 -2470

Fig. 2

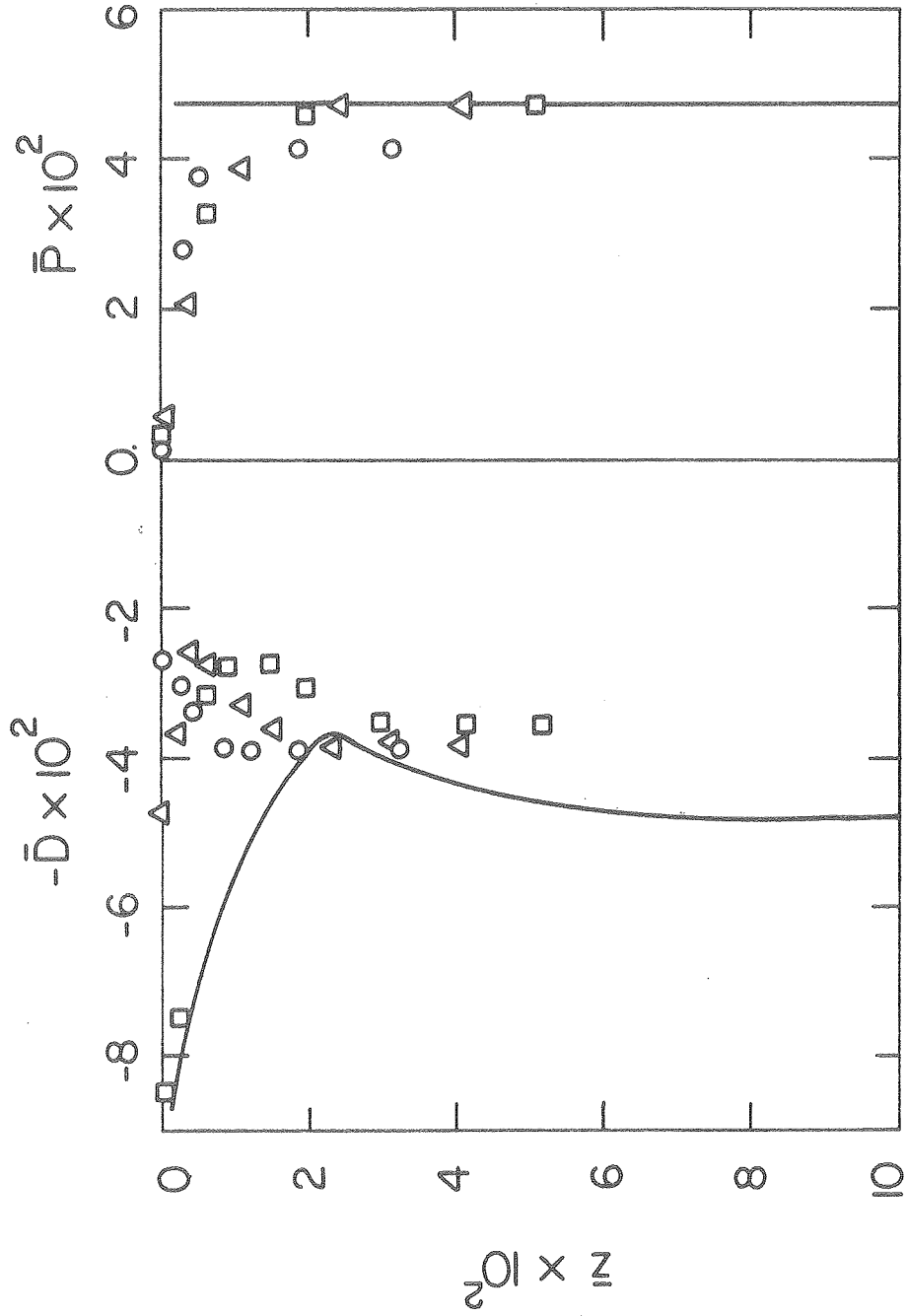
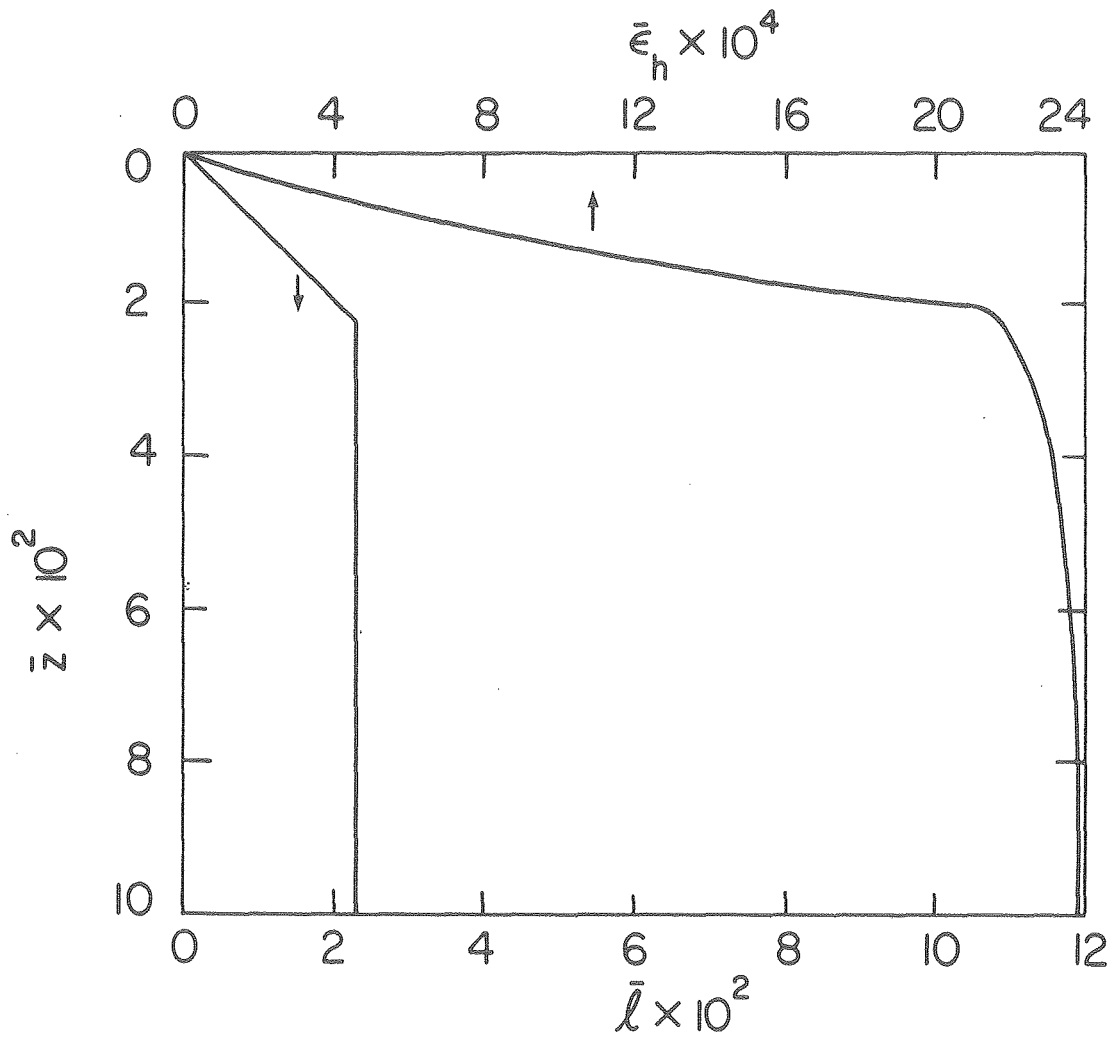


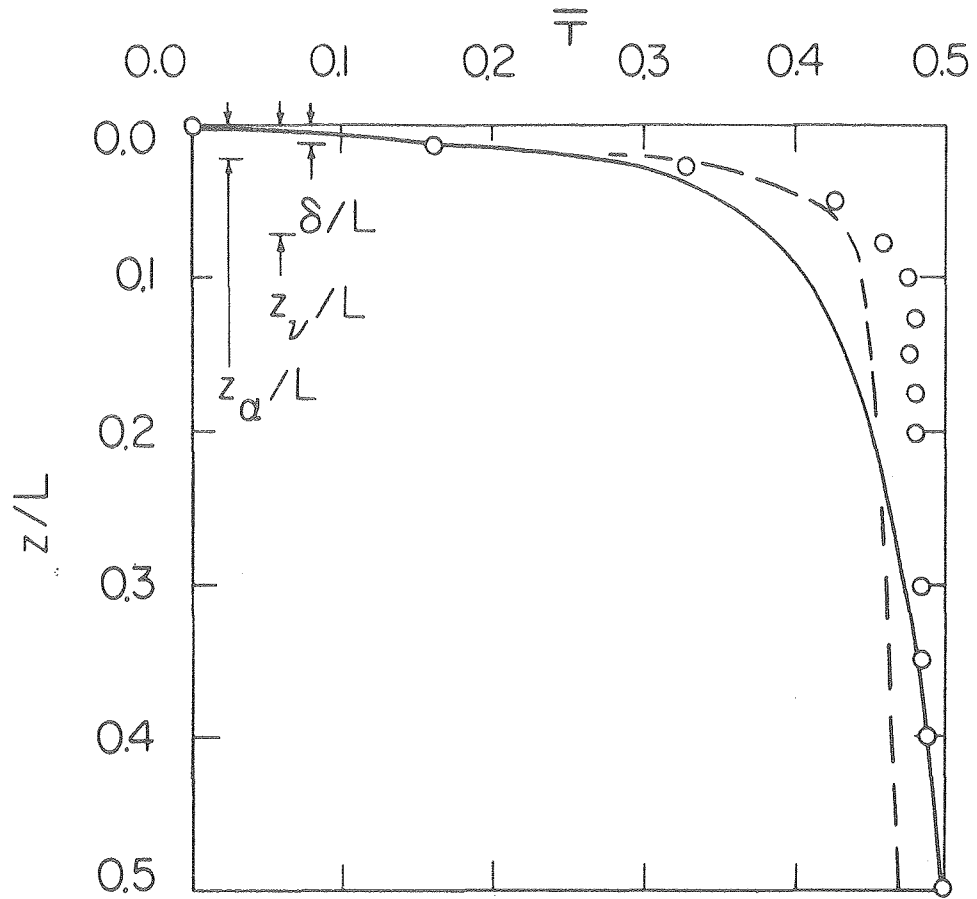
Fig. 3

XBL 8012-259I



XBL 8012-2472

Fig. 4



XBL 8012-2471

Fig. 5