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Prepared for the U. S. Atomic Energy Commission
under Contract W-7405-ENG-48

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FACTORIZATION AND THE COUPLINGS OF THE VACUUM TRAJECTORY*

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April 10, 1973

ABSTRACT

The decoupling theorems associated with an isolated factorizable Pomeron pole of unit intercept are re-examined. It is found that the coupling of three such poles, $\Gamma(t,t,0)$, need not vanish, precisely at the point $t = 0$. This is demonstrated by summing only over states in the appropriate unitarity sum, and sum rule, which are consistent with the M^2, $s/M^2 \to \infty$ limit. The triple-Regge region then makes a constant contribution to σ_{total}, instead of the $\ln \ln s$ result obtained if the isolated pole is assumed to couple also to states such that s/M^2 = constant. The physical implications regarding factorization and the pole-cut relationship are discussed. The relationship between higher order optical theorems (Mueller discontinuities) and particular terms in the unitarity sum for the two $\to$ two absorptive part A_{22} is exploited. Consistent contributions to the triple-Regge region contribute constant vertex corrections to pure pole behavior in A_{22}. There is no cut contribution and the magnitude of the vertex corrections reflects the relative amount of diffractive production. The analysis is extended to multiple fireball production where pure multipole structures emerge. The series naturally terminates if the diffractive component is sufficiently small. The implications for the behavior of the total cross section at machine energies are discussed.

* This work was supported by the U. S. Atomic Energy Commission.

1. Introduction

There now appears considerable theoretical evidence that multi-Regge analyses of reactions where factorizable poles of unit intercept may contribute, lead to an inconsistency with direct channel unitarity unless subsidiary decoupling assumptions are made[1-3]. Based on experience with two-to-two collisions, where simple factorizable pole dominance seems experimentally acceptable at even relatively low energies, it is natural to assume that the same singularities enter in inelastic reactions in appropriate kinematic domains.

When such appropriate kinematic regions are defined for processes described as diffraction dissociation, however, it is found that the usual simple Pomeron pole dominance assumption leads to a contribution to the total cross section which grows with increasing energy. This is in conflict with the fact that the same Pomeron pole leads, through the optical theorem applied to elastic processes, to a constant total cross section. A commonly accepted escape from this conclusion is that the coupling $\Gamma(t_1,t_2,t_3)$ of three such vacuum trajectories must vanish when the associated trajectory masses t_i go to zero from below. This has a number of unpleasant consequences[1-3], including the decoupling of the Pomeron from diffractive processes, and perhaps from elastic scattering[4].

Another possibility is that the Pomeron intercept is below one. This allows a connection to be made between the relative amount of diffractive inelastic processes and the nonvanishing magnitude of $\Gamma(0,0,0)$. However, this dynamical possibility requires that the total cross section go to zero asymptotically. Although an opposite trend seems apparent in the ISR data[5], such a rise in σ_{total} can be seen as a temporary threshold effect[6].

It shall be our concern here to present an alternative interpretation of the decoupling problem which clarifies the nature of the states to which the factorizable, unit intercept, Pomeron pole may couple without inconsistencies arising. In the triple and di-triple Regge limits we show that if one is only slightly more conservative about the kinematic region in which the mathematical limits guarantee dominance of an isolated factorizable pole, then there is no inconsistency with intercept-one poles. For example, if one restricts singly diffractive processes $A + B \to A' + X$ to states X which do not include particles occupying the fragmentation region of particle A, then the contribution of these states to σ_{total} is constant (and not $\propto \ell n\, \ell n\, s$). A similar result applies to the di-triple Regge limit.

This suggests that the inconsistent contributions, $\ell n\, \ell n\, s$ and $(\ell n\, \ell n\, s)^2$ in the di-triple case, are built from specific kinematic regions in which the assumption of an isolated factorizable pole breaks down. The way in which this happens is indeed delicate and involves the relationship between the leading pole and cuts in the region $t < 0$. We present some plausibility arguments concerning the way in which these singularities must be related in order to lead to a consistent picture.

The exclusive processes of diffraction dissociation involved in the triple and di-triple Regge limits correspond, by the Mueller analysis[7], to terms in the unitarity sums associated with the discontinuities of (continued) six- and eight-point functions, in certain kinematic regions. That is, there is a direct relationship between particular subsets of intermediate state contributions to the two $\to$ two absorptive part A_{22} and the full untruncated sum over states in

higher order optical theorems. This has an elegant formulation in terms of inclusive sum rules[8]. The failure to consistently satisfy unitarity with Pomeron contributions to inelastic channels in the condition $A_{22} \propto \Sigma\, T_{2n}\, T^*_{n2}$ shows up as violations of the sum rules which impose higher order unitarity constraints on these reactions. The more conservative restriction on the states to which the factorizable pole may couple in two $\to$ n reactions leads to consistency with the sum rules.

Consistent contributions in the triple Regge regions are found to be associated with vertex corrections to pure factorizable pole exchange in the output two $\to$ two absorptive part. That is, those states in the two $\to$ n amplitude which give consistency with the inclusive sum rules, and hence with unitarity, simply modify the output pole residues and do not shift its position or involve the "cut." There are, however, other classes of kinematically distinct contributions to the two $\to$ two unitarity sum which do not have a simple connection with a finite higher order optical theorem. One such class is that involving diffraction dissociation into two asymptotically growing masses. These processes give rise to a much more serious $\ell n\, s$ contribution to σ_{total}. These terms correspond, in the two $\to$ two absorptive part, to Pomeron loop corrections to the Pomeron propagator. These corrections, and the vertex corrections above, are clearly related to the conditions on consistency between input and output for the leading pole, as they reflect on the vertex and wave function renormalization for composite systems.

In parts 2 and 3 of this paper we explore the triple-Regge limits and suggest conditions on the exclusive production regions in which consistent factorizable pole assumptions may be made. In section 4 we consider the nature of the factorization assumptions made and suggest how the breakdown of these assumptions may occur in the necessary kinematic regions. These arguments indicate that the quantitative measure and qualitative nature of the breaking of factorization may be very different matters. In section 5 we examine the relationship between exclusive and inclusive views of diffractive processes and show how the vertex corrections to pure pole behavior in A_{22} arise. The j-plane analysis is particularly transparent. In section 6, the analysis is extended to higher order vertex corrections and to pole-dominated multiple-fireball contributions to the A_{22} unitarity sum. The inconsistencies associated with latter can be removed only by further physical input since the higher order optical theorem conditions are lacking. Suggestions as to this input are made. Section 7 is devoted to conclusions.

2. Triple Regge Region

We consider a process as in figure 1.a., where particle B dissociates into mass M^2, larger than some resonance mass $M_R^{\ 2}$, but with the same quantum numbers as B. In the limits M^2, $s/M^2 \to \infty$, an $O(2,1)$ decomposition of the connected three $\to$ three amplitude[9] indicates an energy dependence*

$$\frac{d^2\sigma}{dt\, dM^2} \underset{\substack{M^2 \to \infty \\ s/M^2 \to \infty}}{\sim} s^{-2}\, \gamma(t) (M^2)^{\alpha_P(0)} \, (s/M^2)^{2\alpha_P(t)} \tag{2.1}$$

where, adopting the normalizations of reference (3),

$$\gamma(t) = \frac{1}{16\pi} |\beta_{PAA}(t)|^2 \ \tilde{\beta}_{PBB}(0)\, \Gamma(t,t,0). \tag{2.2}$$

This normalization is such that

$$\frac{d\sigma_{AB}}{dt} \sim \frac{1}{16\pi} |\beta_{PAA}(t)|^2 \ |\beta_{PBB}(t)|^2 \ s^{2(\alpha_P(t)-1)} \tag{2.3}$$

and

$$\sigma^{AB}_{total} \sim \tilde{\beta}_{PAA}(0)\, \tilde{\beta}_{PBB}(0)\, s^{\alpha_P(0)-1} . \tag{2.4}$$

The two-to-two absorptive part in the forward direction is related to σ_{total} by

$$A_{22}(s,0) = \Delta^{\frac{1}{2}} (s,m^2,m^2)\, \sigma_{total}(s) \sim s\, \sigma_{total}(s) . \tag{2.5}$$

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* Throughout this paper, we drop the common Regge energy scale s_0, as well as inessential signature factors.

The $(M^2)^{\alpha_P(0)}$ behavior in equation (2.1) is associated with the leading Regge singularity $\alpha_P(0)$ in the $B\bar{B}$ channel for $\ell n\, M^2 \to \infty$. The leading helicity singularities for the situation $s \to \infty$, $\eta \equiv \ell n\, s/M^2 \to \infty$, give rise to the behavior $(s/M^2)^{2\alpha_P(t)}$. This requires an identification of poles in complex helicity with ordinary Regge poles. The triple limit can be regarded as the $s/M^2 \to \infty$ limit of the ordinary Regge decomposition of the three $\to$ three amplitude $(s, M^2 \to \infty,\ s/M^2,\ t$ fixed).

In order to determine the contribution to the total cross section coming from these processes, we must integrate over regions of t and M^2 which are consistent with the physical and mathematical assumptions made. The latter have to do with the regions of validity of the double expansion above. It is usually assumed, by analogy with two-body reactions or on the basis of models, that the choice $\eta = \ell n\ s/M^2 > \lambda$, where λ is a large constant*, is sufficient to insure Regge behavior. The physical assumption then usually made is that a factorizable, isolated Pomeron pole dominates when this condition is fulfilled. If this pole has intercept one and unless $\Gamma(0,0,0) \equiv 0$, this leads to an integrated contribution to σ_{total} which grows like $\ell n\, \ell n\, s$. As indicated above, this is inconsistent with the contribution of the same factorizable pole to σ_{total} as given by the optical theorem.

We now show that it is the very liberal interpretation of the domain of validity of the triple-Regge expansion, coupled with the factorization assumption, which leads to inconsistency. In

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* In phenomenology, λ is usually taken to be of order two.

integrating expression (2.1), we are usually integrating over rapidity gaps $\Delta \equiv \ell n\, s/M^2$ such that $\Delta_{max} \geq \Delta \geq \lambda$ (see figure 1.b.). The maximum gap may be taken as $\Delta_{max} = \ell n\, s/M_R^2$, or more conservatively as $\Delta_{max} \approx \gamma\, \ell n\, s$, where γ is a constant near one. The important point is that Δ_{max} scales with $\ell n\, s$ while the lower limit on Δ does not. It might be thought that the unitarity problem (the inconsistency above) arises because the gap Δ may become too large. It is rather that it may become too small. Let us separate the integration into two parts, $\epsilon\, \ell n\, s \geq \Delta \geq \lambda$ and $\Delta_{max} \geq \Delta \geq \epsilon\, \ell n\, s$, where ϵ is a constant which is not asymptotically as small as $(\ell n\, s)^{-1}$. The latter region guarantees the validity of the asymptotic expansion in $\eta = \Delta = \ell n\, s/M^2$ as $s \to \infty$ which is necessary to isolate the relevent helicity (Regge) pole. In this region we are integrating over masses M^2 such that $M_R^2 \leq M^2 \leq s^{1-\epsilon}$ where M^2 becomes very large if ϵ is not too close to one. The contributions to both regions comes from an integral of the form

$$\sigma_{T.P.} \sim \frac{\gamma(0)}{32\pi} \left[\int_{\lambda}^{\epsilon Y} d\Delta + \int_{\epsilon Y}^{\Delta_{max}} d\Delta \right] (a + \alpha'\Delta)^{-1} \tag{2.6}$$

where we have taken $\alpha_P(t) = 1 + \alpha' t$, $\gamma(t) = \gamma(0)\exp(2at)$ and integrated over t. The contribution from the second region, where the minimum gap is growing with $Y = \ell n\, s$, thus insuring the strict mathematical limit, is

$$\sigma'_{T.P} \sim \frac{\gamma(0)}{32\pi\alpha'}\ \ell n\, (1/\epsilon). \tag{2.7}$$

This is a constant unless $\epsilon \sim \lambda/Y$, in which case we have the whole integral in (2.6) and recover the usual $\ln \ln s$ result. This is very similar to the result when the Pomeron intercept is below one[3]. For ϵ fixed > 0, the contribution from the first term, which we shall refer to as the fixed gap region, grows with energy like $\ln \ln s$. The inconsistency thus arises from the assumption that the triple-Regge formula, with isolated factorizable poles, can be extended down into the constant gap region. This assumption corresponds mathematically to using an asymptotic expansion in $\eta = \ln s/M^2$ as an approximate expansion for finite η.

This result suggests that one has isolated factorizable poles only in the strict $(\eta \to \infty)$ triple-Regge region. If there are non-factorizable contributions for $\lambda \leqslant \eta \leqslant \infty$, they must destructively interfer with the pole in this region in order to restore consistency. Since this must occur over an appreciable rapidity range, it is likely that this entails a major qualitative change in the nature of the leading singularity near $j = 1$ as $\Delta \to$ constant in the $s \to \infty$ limit. This question will be clarified in section 4.

These assertions about the factorization properties have a very natural physical interpretation. The region $M^2 \leqslant s^{1-\epsilon}$ includes only dissociation states M^2 of particle B which do not contain particles lying in the fragmentation region of particle A. That is, the left-most particle of M^2 in the rapidity space decomposition of figure 1.b., has a Feynman x-value which goes to zero like $s^{-\epsilon}$. Nonfactorizable exchange would then be required for those produced states which overlap both fragmentation regions. This seems a natural physical definition of factorization, but it is at odds with the usual multi-Regge assumption of local factorization over constant rapidity

lengths in production processes. This clearly matters most in the case of intercept one since the unitarity condition then gives us an obvious constraint on consistency.

The contribution of equation (2.7) is then a measurable lower bound on diffractive contributions to σ_{total} since the Pomeron residues in $\gamma(0)$ are normalized to this quantity (and to $\sigma_{elastic}$). That is, $\Gamma(0) \ln(1/\epsilon)$ is a measure of the relative amount of diffraction. If ϵ has a reasonable value, $\epsilon \approx \frac{1}{2}$ say, then the contributing region $\Delta = \ln s/M^2 \geqslant \epsilon Y$, will encompass very nearly the same kinematic region at current machine energies $(4 \leqslant \ln s \leqslant 8)$ as the usual choice $s/M^2 \geqslant 6 \to 7$. Triple-Regge phenomenology need not then be altered, except that it becomes unnecessary to assume that the triple coupling vanishes at $t = 0$.

There is some energy dependence however. If the integrated diffractive contribution is constant (and appreciable), then this constant contribution must come from a region in the leading proton x-distribution $(1 - x \approx M^2/s)$ which shrinks slowly toward $x = 1$ as s increases. The "quasi-elastic" peak near $x = 1$ should then rise with energy. Whether it is this mechanism which is responsible for the dip near $x \approx 0.88$ depends on the dynamics of the fixed gap region.

3. Di-Triple Regge Region

We can now proceed, in an entirely similar fashion, to the di-triple Regge region. Processes like those in figure 2 are involved in the combined limits s, M^2, s_1/M^2, $s_2/M^2 \to \infty$ with $M^2 s/s_1 s_2 = 1$ fixed. Taking $1 - x_i \approx M^2/s_i$, the latter condition becomes $(1 - x_1)(1 - x_2) = M^2/s$. Introducing the gap parameters $\Delta_i \equiv \ell n\, s_i/M^2$, this requires $\Delta_1 + \Delta_2 = \ell n\, s/M^2$. Neglecting azimuthal dependences (we are interested in the $m_{\text{Toller}} = 0$ part) we have,

$$s^2 \frac{d\sigma}{ds_1 ds_2 dt_1 dt_2} \sim \frac{1}{s} (M^2)^{\alpha(0)} (s_1/M^2)^{2\alpha(t_1)} (s_2/M^2)^{2\alpha(t_2)} \gamma(t_1)\gamma(t_2) . \tag{3.1}$$

In terms of the gap variables this becomes (with $\alpha(0) \equiv 1$)

$$\frac{d\sigma}{d\Delta_1 d\Delta_2 dt_1 dt_2} \sim \exp[2(\alpha(t_1) - 1)] \exp[2(\alpha(t_2) - 1)\Delta_2]\, \gamma(t_1)\, \gamma(t_2). \tag{3.2}$$

This clearly factors into $f(t_1,\Delta_1)\cdot f(t_2,\Delta_2)$. Only the limits are coupled when we integrate over the di-triple region. First, integrate over the t_i, assuming $\gamma(t_i) = \gamma(0)\exp[2at_i]$. This yields (with $\alpha(t_i) = 1 + \alpha' t_i$),

$$\frac{d\sigma}{d\Delta_1 d\Delta_2} \sim [\gamma(0)]^2\, [2(a + \alpha'\Delta_1)]^{-1}\, [2(a + \alpha'\Delta_2)]^{-1} . \tag{3.3}$$

It is usually assumed that this expression is valid when $\Delta_i \geqslant \lambda_i$ constant, and $\Delta_1 + \Delta_2 = \ell n\, s/M^2 \leqslant \ell n\, s/M_R^2$. Integrating over these regions gives a contribution to σ_{total} which grows like $[\ell n(\ell n\, s)]^2$.

Let us now proceed, as in the triple Regge case, to break up the integrations over gaps into regions $\lambda_i \leqslant \Delta_i \leqslant \epsilon_i\, \ell n(s/M_R^2)$ and $\epsilon_i\, \ell n(s/M_R^2) \leqslant \Delta_i$, with $\Delta_1 + \Delta_2 \leqslant \ell n(s/M_R^2) \equiv Y_R$. The quantities ϵ_i are constants such that $1 > \epsilon_i > 0$. Then, the contribution to σ_{total} is

$$\sigma_{\text{tot}}^{\text{DTR}} = \frac{1}{64\pi} \left[\int_{\lambda_1}^{\epsilon_1 Y_R} d\Delta_1 + \int_{\epsilon_1 Y_R}^{Y_R} d\Delta_1 \right] \times \left[\int_{\lambda_2}^{\epsilon_2 Y_R} d\Delta_2 + \int_{\epsilon_2 Y_R}^{Y_R} d\Delta_2 \right] \times \gamma^2(0)[a + \alpha'\Delta_1]^{-1}[a + \alpha'\Delta_2]^{-1}\, \theta(Y_R - \Delta_1 - \Delta_2). \tag{3.4}$$

This now contains four terms. The term where both gaps are allowed to take constant values behaves asymptotically as $[\ell n(\ell n\, s)]^2$. The two terms where only one gap is allowed to take constant values go like $[\ell n(\ell n\, s)]$. The term where both end gaps take only asymptotically <u>growing</u> values is <u>constant</u>. This last term corresponds to the strict limits s_1/M^2, $s_2/M^2 \to \infty$ as $s \to \infty$ and is a sum over missing masses which contain <u>no</u> particles in <u>either</u> fragmentation region. By our earlier arguments, both end gaps must become large (∞) to isolate factorizable poles. The sum over states M^2 is then confined to the pionization region. This leads to an output which is consistent with positivity. We may thus assert that the pionization region builds the factorizable pole <u>in the output</u>. The other regions, and the boundaries between the regions above, are then associated with nonfactorizable input and output. The factors $\gamma^2(0)$ may then be identified with the couplings of three factorizable poles and need not vanish. In the other kinematic regions, where factorization does not seem to work consistently, it seems difficult to define a corresponding object. In the next section we consider this factorization question in greater detail from a slightly unconventional generalization of the usual point of view.

4. Factorization

There are several general statements which can be deduced from the preceding analysis concerning the way in which factorization may be broken. Our dynamical knowledge of nonfactorizable Regge singularities is very limited. Apart from simple j-plane speculations, we know of the AFS[10)] and the very similar Mandelstam[11)] cuts which arise from s-channel pole iterations. Feynman diagram models lead to a hierarchy of cuts of this type and, in addition, suggest new j-plane structures[12)]. Consistency in the above picture may provide a way of testing these prescriptions for the manner in which pure factorizable pole behavior must be modified.

Consider, for example, the triple-Regge case, in which the gap $\Delta \equiv \ell n\, s/M^2$ appears. We are interested in the extent to which there exist factorizable Regge singularities associated with this variable in the region $\epsilon\, \ell n\, s/M^2 \gtrsim \Delta \gtrsim$ constant in the limit $s \to \infty$. According to the usual ideas, this region may receive contributions not only from a pole at $j = 1$, but from cuts as well. As is also customary, let us assume for the moment that the contributions from these singularities may be separated. That is, that there is no ambiguity due to the collision of singularities near $j = 1$ and that we can consequently isolate separate residues. We also argue that since the pole over-estimates the dynamics in this region, leading to inconsistency between input and output, that these other singularities must, on the whole, destructively interfer with the pole. Let us further assume that the cut residues have the usual energy dependence $[\ell n\, s/M^2]^{-n}$ relative to the pole. We now have an example of the way in which factorization might be broken. If we take $s, M^2 \to \infty$ with

s/M^2 fixed, it is clear that the relative cut to pole strength does not change. This limit does not isolate the pole from the attendant cuts. If, on the other hand, we take $s, M^2 \to \infty$ such that $s/M^2 \to \infty$, then the cut discontinuities die away, isolating the factorizable pole.

Now, it is a simple matter to show that this simple additive arrangement of factorizable and nonfactorizable singularities is not sufficient to restore consistency, although it does allow the emergence of a factorizable pole as $s/M^2 \to \infty$. This suggests that even if the cuts disappear asymptotically in s/M^2 that the nature of the combined singularities for finite s/M^2 is more complicated than in this simple scheme. This implies that we cannot simply isolate the residues in an additive fashion.

One way in which the relationship between pole and cuts may be complicated is revealed by considering the t-dependence of the exclusive single diffraction dissociation process corresponding to the triple-Regge limit. For fixed s and M^2, there is a minimum momentum transfer, $t_{min} \approx -(M^4/s^2)m^2$ which goes to zero in the limit $s/M^2 \to \infty$. In this limit, we are studying the coincidence of pole and branch points at $j = 1$. For $t_{min} < 0$, however, pole and branch points $\alpha_C^{(n)}(t) = n\,\alpha_P(t/n^2) - n + 1 \approx 1 + (\alpha'/n)t$, are separated, with branch points to the right of the pole (see figure 3.a,b.). Whereas it would be convenient and natural to believe that the pole dominates at $t = 0$, this becomes less tenable when the pole lies to the left of the (infinitely many) branch points, under the associated cuts. The pole may strongly distort the cut discontinuities. It may even do so in a way which preserves quantitative factorization to a degree which does not reflect the qualitative differences.

An electrostatic analogy of point and line charges provides a heuristic approach to the possible collision of singularities. In figure 3.a., point and line charges represent the arrangement of pole and branch cuts for $t < 0$. The independence of the discontinuities associated with these singularities is equivalent to the simple addition of electrostatic fields due to the independent charge distributions. The sign of the relevant cut discontinuity may be associated with the sign of the analogous charge. Suppose now that we relax the static assumption, letting the charges move to equilibrium situations on the cuts. Charges (discontinuities) with the same sign as the pole move away from the pole, accumulating near the branch point, or at infinity. Charges of the same sign as the pole accumulate near it in the j-plane. Thus the pole may not only induce a higher lying effective pole, to the right in j, but also an effective pole (of the opposite sign) nearby itself. The right-most branch points may have sufficiently localized charge density to approximately factorize. The energy-dependence associated with this situation will be much stronger than that associated with independent cuts.

This situation obtains when the momentum transfer is less than zero--that is, far below elastic threshold. In this region such hard cuts are allowed according to the recent work of Creutz, Paige and Wang[13] and contrary to Bronzan and Jones[14]. If we move closer to $t = 0$, and hence toward elastic threshold, there is less induced accumulation of charge to the right of the pole and the pole ultimately dominates. There may be an attendant negative effective pole near $j = 1$ at $t = 0$, if there are negative cuts. We do not wish to speculate further on the possibilities of this "dipole" structure. The important point is that the relationship of pole and cuts may depend strongly upon their relationship in the j-plane, a relationship which is a function of t. The easy quantitative separation of factorizable and nonfactorizable contributions reflects the underlying qualitative arrangement of singularities only if one can simply separate independent residues.

There are, of course, many other ways in which factorization may be broken. It may prove impossible to isolate independent s/M^2 and M^2 limits. This would happen, for example, in the case of absorptive cuts in the three → three amplitude. Another possibility is suggested by unitary "multiperipheral" models[12] in which a unitarity cut moves offensive poles onto unphysical sheets when there is danger of inconsistency. This cut is generated by a larger spectrum of bound state poles than is usually considered in multi-Regge models.

The question of factorization arises in two-body reactions in a similar form. Tests at $t = 0$ require total cross section measurements for three reactions (e.g., p-p, π-p, π-π). This requires a knowledge of the dynamics only at zero momentum transfer. Other tests, for $t \neq 0$, indicate that factorization is adequate to the 10 - 20% level. It is not known whether this ensures a dominant isolated pole or only a simple feature of an otherwise complicated situation. Inclusive reactions, proceeding at or near $t = 0$, should provide better tests. In the triple-Regge region, however, the point $t = 0$ is difficult to reach. Indeed, $t_{min} \approx (1 - x)^2 m^2/x$ with $(1 - x) \approx M^2/s$. Our assertion that factorization and isolated pole dominance ought only to be trusted for $M^2/s \xrightarrow[s,M^2\to\infty]{} 0$, is equiv-

alent to the assumption of isolated pole dominance only in the limit $t_{min} \to 0$. As long as the approach to this limit is properly understood, so that asymptotic inconsistencies are removed, factorization may be quite adequate at finite energies. This may be required if an appreciable triple-Pomeron contribution is required to fit the ISR data[15]. Vanishing of the triple coupling at $t = 0$, to remove inconsistencies, would remove a potentially major source of scaling contributions.

All of these problems are more extreme if the Pomeron trajectory is flat $(\alpha' \equiv 0)$. In this case the usual cuts would have the same energy dependence (in s/M^2) as the pole and hence would not die away as $s/M^2, M^2 \to \infty$. The flat Pomeron may then not be factorizable anywhere in the triple limit. Equivalently, the triple coupling must vanish for all t. If it did not, we would find a more serious $\ell n\, s$ inconsistency instead of the $\ell n\, \ell n\, s$ above. The only way to have a nonvanishing diffractive contribution with $\alpha'_P \equiv 0$ is in the ordinary Regge limit $s \to \infty$, M^2, t fixed. That is, the flat Pomeron may couple only to states of finite mass in the limit $s \to \infty$.

5. Inclusive and Exclusive Processes

Until now we have been incautious in freely interchanging references to diffraction dissociation as inclusive and as exclusive processes. In this section we shall examine the conditions for the equivalence of these views. The basic separation between these descriptions lies in whether one is making an asymptotic decomposition of an n-point function with n small (inclusive, generalized optical theorem for discontinuity) or n large (exclusive, amplitude itself). We will use the single diffraction dissociation process associated with the triple-Regge region as an illustration. Asymptotically, in this case, both views seem equivalent. In section 6, we will consider situations which are not equivalent.

In the triple Regge case, we can reach the appropriate kinematic region in two ways. In the exclusive view, we consider the ordinary Regge limit, $s \to \infty$, M^2 and t fixed (see figure 1). Pomeron exchange presumably dominates this single dissociation amplitude. These processes make a positive contribution to the sum over intermediate states in the two → two unitarity condition. This means that we have made a very specific exclusive statement about a potentially small part of the sum over intermediate states. For s very large, we can take M^2 large so that there are many states involved in the truncated unitarity sum over M^2. The number of states we are leaving out depends upon what is meant by M^2 large. If we take $M^2 < \lambda s$ $(\lambda < 1)$, this means that we have left out only those inelastic channels corresponding to a fixed rapidity gap. That is, if production occurs uniformly in rapidity space, a fixed gap in the limit $Y \to \infty$ corresponds to relatively few particles not being

produced. The Pomeron in the production amplitude then communicates with a major share of the $2 \to n$ inelasticities. The condition $M^2 < \lambda s^{1-\epsilon}$ is a more severe truncation of the unitarity sum. In both cases, however, M^2 gets "large" as $s \to \infty$. In the two → two absorptive part A_{22}, we then argue that we can isolate the Pomeron contribution dual to this set of states (in either case). That is, that we can find the pole contribution to the discontinuity of the forward Pomeron-particle amplitude. The question we have raised in earlier sections is a question of which definition of "M^2 large" leads to consistency between input and output. The ordinary Regge arguments do not tell us. We must go to a higher order (three → three) optical theorem or invoke some additional physical principle to find out.

Before proceeding to this, we should like to observe a connection between this situation and some very old ideas about the nature of the Pomeron. This singularity was intended to dominate the absorptive parts of two → two elastic reactions in such a way as to give constant total cross sections through the optical theorem. Because of this, one could regard its role in two → two reactions as diffractive scattering, reflecting the shadow of the principal inelasticities in hadronic collisions. The question then arises as to which are the principal inelasticities which build this singularity. If, at a given energy we allow the "Pomeron" to couple to these important inelasticities in the two → n production amplitudes, then we are in danger of a subtle form of multiple counting[16]. This manifests itself in multi-Regge bootstraps as a stronger output singularity (higher j) than input. For $j = 1$, this is disastrous.

The bootstrap computations utilize the two → two unitarity condition as a calculational device, rather than as a constraint. Further input is necessary to achieve consistency. This input has usually been of the form of a decoupling assumption. When the pole goes through 1 at $t = 0$, a residue vanishes. As noted above, another possibility is to go to a higher order optical theorem for a clue as to which inelastic couplings are allowed.

When we proceed to try to understand the elastic three to three amplitude in terms of these old ideas we are faced with an immediate ambiguity. The "Pomeron" should represent the shadow of the inelasticities in the M^2, elastic three-body channel. If a pair of these particles $(\bar{A}'A)$ has small relative invariant mass, we might be correct in regarding this as a quasi-two body elastic process in which the unconstrained sum over states in M^2 generates the ordinary two-body Pomeron. This seems the situation relevant to the triple-Regge case if a small continuation is allowed (to negative invariant mass squared for $A\bar{A}$, $t \lesssim -(M^2/s)^2 m^2$). More complicated cases seem more difficult to interpret on these grounds.

The Mueller-Regge analysis, with suitable technical assumptions about the identification of helicity with Regge poles and about factorization, provides a way to proceed. The continuations from physical to unphysical regions links the $2 + n \to 2 + n$ amplitude discontinuity to the n-particle inclusive cross sections $2 \to n + X(M^2)$. This connection requires a certain consistency between two → two and $n \to n$ absorptive parts. This requirement has an elegant formulation in terms of the inclusive sum rules. Satisfaction of these sum rules is presumably a necessary condition to full

unitarity[17]. In the analysis of section 2, the contribution from the region $s/M^2 \geqslant s^\epsilon$ gives a constant contribution to the inclusive sum which is bounded from above by σ_{total} = constant. If this is the only region in which factorization in the three → three analysis is achieved (and if the nonfactorizable, and hence incalculable, contributions from the other regions behave themselves) then the energy-momentum sum rule may be satisfied. This is a minimal condition on the role of the factorizable Pomeron in the three → three discontinuity in a particular kinematic region, or equivalently in exclusive diffraction dissociation into a particular class of states (those not overlapping the fragmentation region of the other particle), to its role in σ_{total} for two-body reactions. If this is all that is necessary for consistency then it would appear that the factorizable pole may couple, in an exclusive sense, to a large class of inelastic states, although not as large as multi-Regge models for production would allow. This may not be sufficient, as we shall see in the next section.

This discussion establishes a parallel between a particular truncation of the unitarity sum in the two → two optical theorem and the unconstrained unitarity sum in the three → three amplitude in a particular (continued) kinematic region for the external particles. Let us now consider this class of contributions to the two → two absorptive part A_{22}. First note that this class may be enlarged to consider states consisting of one large (growing) mass fireball resulting from the dissociation of particle B (figure 1.a.) and one finite mass cluster A' of mass M' containing n' particles. The arguments of section 2 go through as before. This process is then related to a term in the Mueller discontinuity in M^2 of an $\bar{n}' + 2 \to \bar{n}' + 2$ amplitude, where the $\bar{n}'$ particles have finite invariant mass M' and we are interested in the limit $s/M^2,\ M^2 \to \infty$ with M'^2, t fixed. With our kinematical constraints on the region of validity of the Regge and helicity pole expansions (which correspond to restrictions on the states to which the factorizable Pomeron may couple) the terms which contribute a constant amount to $\sigma^{22}_{total} \propto (1/s)A_{22}(s,t{=}0)$ then have the form shown in figure 4.a. Closure in the sum over A' is indicated. Note that the states A' may partially fill the fragmentation region of particle A. That is, the particles in A' may have finite longitudinal fractions $x_i \approx 2p_{i\parallel}/\sqrt{s}$, and still have finite total invariant mass in the limit. This argument is easily extended to the di-triple region. The corresponding structure is shown in figure 4.b. The ordinary Regge contribution ($s \to \infty$, M'^2, M^2, t fixed) is involved in the generation of the usual cut, figure 4.c.

We now recognize the contribution from "factorizable" pole exchange in the input as providing vertex corrections to the output factorizable pole. The output pole represents the shadow of high mass (multiplicity) inelastic channels which may be reached by factorizable pole input. Note that these vertex structures would vanish (at t = 0) if the triple Pomeron coupling vanished. The coupling of the Pomeron to finite mass diffractive states is related to the Pomeron vertex renormalization. As we shall see, there are higher order terms in this sequence associated with higher order inclusive processes in slightly different kinematic regions.

We may study these objects in the j-plane by a simple Mellin

transform. For the single vertex (triple-Regge) we had, in equation (2.3),

$$A_{22}^{T.P.}(s) \approx s\ \sigma_{tot}^{T.P.} \approx s\ \frac{\gamma(0)\ \ell n(1/\epsilon)}{32\ \pi\ \alpha'}\ . \tag{5.1}$$

The Mellin transform is again a simple pole with residue related to the integral over the restricted triple-Regge region. That is, the Pomeron coupling to a rather large (but still restricted) class of inelastic states only increases the residue of the pole through the unitarity bootstrap. The pole position is not changed. We can see how this arises by transforming without doing the t-integral. The result is more transparent if we scale M^2 by M_R^2. Doing so, we have

$$A_{22}^{T.P.}(j) \propto \sum_{A'} \int_{-\infty}^{0} dt\ \gamma_{A'}(t) \int_{0}^{\infty} dY\ \exp(-jY)$$

$$\times \int_{0}^{(1-\epsilon)Y} d\ \ell n\ M^2\ \frac{1}{s}\ (s/M^2)^{2\alpha_P(t)} (M^2)^{\alpha_P(0)+1}\ . \tag{5.2}$$

The last two integrals give

$$A_{22}^{T.P.}(j) \propto \sum_{A'} \int_{-\infty}^{0} dt\ \frac{\gamma(t)}{D(j,t)[\ j + 1 - 2\alpha_P(t)]} \tag{5.3}$$

where, with $\alpha_P(t) = 1 + \alpha' t$,

$$D(j,t) = j - 1 - 2\alpha'\ \epsilon\ t\ . \tag{5.4}$$

This gives two poles in the t-integral and hence coupled branch points in j

$$A_{22}^{T.P.}(j) \sim \int_{-\infty}^{0} dt \frac{\gamma(t)}{[j-1-2\alpha' t][j-1-2\alpha'\epsilon t]} . \qquad (5.5)$$

The usual result is obtained simply by taking $\epsilon \to 0$, so that

$$A_{22}^{U.T.P.}(j) \underset{\epsilon=0}{\sim} [j-1]^{-1} \int_{-\infty}^{0} dt \frac{\gamma(t)}{[j-1-2\alpha' t]} . \qquad (5.6)$$

The integrand has the behavior $\gamma(t)/t$ near $j = 1$. It is usually assumed that $\gamma(t) \xrightarrow[t \to 0]{} 0$ to remove this dependence. Otherwise the j-plane structure, near $j = 1$, is of the singular form $\gamma(0)[j-1]^{-1} \ln(j-1)$, which corresponds to $A_{22}^{U.T.P.}(s) \sim s \ln \ln s$.

In our case, with $\epsilon \neq 0$, we have a pair of coupled branch points, appearing as poles in the t-integrand. Both poles approach $j = 1$ from below as $t \to 0$, with the pole at $j = 1 + 2\alpha'\epsilon t$ leading. This removes the bad behavior near $t = 0$ by a cancellation. This is most easily seen by rewriting the double pole in the t-integral using the identity $(AB)^{-1} = (B-A)^{-1}(A^{-1} - B^{-1})$. We can invert the Laplace transform in $Y = \ln s$ (Mellin transform in s) so that

$$A_{22}(s) \sim \frac{1}{2\pi i} \int_{-\infty}^{0} \frac{dt \quad \gamma(t)}{(1-\epsilon)2\alpha' t} \int_{c-i\infty}^{c+i\infty} dj \quad e^{jY}$$

$$\times \left[(j-1-2\epsilon\alpha' t)^{-1} - (j-1-2\alpha' t)^{-1} \right]$$

$$= e^{Y} \int_{-\infty}^{0} \frac{dt \ \gamma(t)}{(1-\epsilon)2\alpha' t} \left[\exp(2\epsilon\alpha' tY) - \exp(2\alpha' tY) \right] . \qquad (5.7)$$

The lowest surviving term in the expansion of the bracket is proportional to t, removing the bad dt/t behavior without requiring

$\gamma(0) \equiv 0$. For $\gamma(t) = \gamma(0) \exp(2at)$, the integral is of the Frullani type and is easily done

$$A_{22}(s) \sim \frac{s\ \gamma(0)}{2\alpha'} \ln \left| \frac{a + \alpha' Y}{a + \alpha' Y \epsilon} \right| . \qquad (5.8)$$

This is just our earlier result. When $\epsilon = 0$ (or $O(1/\ln s)$), we recover the usual $\ln \ln s$. In general then, it is not necessary to assume $\gamma(0) = 0$.

As noted earlier, the output is a simple pole at $j = 1$ with residue $\gamma(0) \ln(1/\epsilon)/(32\pi\alpha')$ instead of the more complicated singularity $\ln(j-1)/(j-1)$. In both cases, the singularity structure is that associated with a sum over particular subsets of states in the two-body unitarity condition. The simple vertex renormalization is associated with a smaller class of states than that responsible for the $\ln(j-1)$ behavior. The latter is intimately related to the generation of the two-Pomeron cut. That is, the cut contributions are generated at the boundary[18)] between the fragmentation and pionization regions (the region, constant $\leqslant s/M^2 \leqslant s^{\epsilon}$) in the production process. Simply eliminating factorizable Pomeron couplings to states which occupy both regions removes the inconsistency. In short, those states in the unitarity sum which allow consistency between input and output are those which lead to vertex corrections which modify the pole residue and do not contribute to the cut. We hasten to add that we do not have an understanding of the dynamics associated with the highly ambiguous boundaries between the kinematic regions. We have earlier argued that since the pole overestimate these regions that there must be destructive interference from some other mechanism which restores consistency. This is probably in the form of cut contributions but they cannot be merely additive. In section 4 we presented a plausibility argument for the necessary pole-cut collision. The conclusion is that there are states in the two → two unitarity sum which do not consistently satisfy an exclusive cluster decomposition with clusters separable by an isolated Pomeron pole. These states are those which do not correspond to a limit of a (finite) higher order optical theorem. Many models based on either fragmentation or central region dynamics have been introduced. It seems, from our arguments, crucial to understand the dynamics of the interface between the kinematical regions where these pure ideas physically obtain.

We have thus far considered only a limited set of states in the unitarity sum. There are, of course, other exclusive classes of states involved in the two → two absorptive part. Furthermore, one might imagine making an exclusive decomposition of the states involved in M^2 in the three → three absorptive part. In the next section it will be shown that all exclusive decompositions which are not simply related to a higher order optical theorem are connected with a new j-plane structure in the two → two absorptive part.

6. Higher Order Terms -- the Asymptotic Behavior of σ_{total}

In the last section the relationship between few-particle inclusive reactions and many-particle exclusive processes involving the production of a single asymptotically growing fireball was explored. This very limited class of contributions to the two → two absorptive part was involved in the development of vertex corrections to factorizable pole exchange. This suggests that one might also consider exclusive $2 \to n$ processes which involve other rapidity space decompositions.* Before proceeding to consider multiple fireball processes we indicate how higher order vertex corrections come about.

Suppose one were to decompose the M^2 unitarity sum in the Pomeron-particle forward absorptive part (figure 1), and consider contributions of the form indicated in figure 5. The gap variables

* A comprehensive program for this exclusive decomposition has been undertaken by Bishari, Chew and Koplik[6,19], based on multi-Regge models. Input poles in the multiple-cluster production amplitude are distinguished from output poles generated by the sums over clusters in the two → two unitarity sum. The output singularity is below one, so that σ_{total} ultimately falls. This is necessary for consistency since pure pole behavior in the production amplitudes is assumed to obtain for rapidity gaps $\Delta_i \geqslant$ constant. This also insures that poles and cuts emerge together in the output. The separation of these singularities at finite energies depends on the number of cluster thresholds available. The two-Pomeron cut may then have a negative discontinuity at a given energy, while still emerging from the asymptotic analysis with a positive sign. This requires a sum over an infinite number of terms and the proper isolation of the ordinary (AFS) cut contributions.

are $Y\delta_1 \equiv \Delta_1 \equiv \ell n\, s/M^2$ and $Y\delta_2 \equiv \Delta_2 \equiv \ell n\, M^2/M'^2$. The Feynman variables associated with particles A' and C are $x_{A'} \approx (1 - e^{-\Delta_1})$ and $0 < x_C \approx e^{-\Delta_1}(1 - e^{-\Delta_2})$. Equivalently $1 - x_{A'} \approx M^2/s$, $1 - x_{A'} - x_C \approx M'^2/s$. This is clearly an interesting region since in the limit $s/M^2 \to \infty$, particle C is trapped between the fragmentation and pionization regions.

Assuming no Toller angle dependence, and that the Pomeron-particle-Pomeron vertex may be factored into $V(t_1)V(t_2)$, we have the contribution to the forward two → two absorptive part:

$$A_{22}(s) \approx s \int dt_1 dt_2\, d\Delta_1 d\Delta_2\, \exp[2(\alpha(t_1) - 1)\Delta_1]\, \exp[2(\alpha(t_2) - 1)\Delta_2]$$

$$\times\ \bar{V}^2(t_1)\, V^2(t_2)\, \Gamma(t_2,t_2,0) \tag{6.1}$$

where $\bar{V}^2(t_1) = \beta^2_{PAA'}(t_1)V^2(t_1)$ and $\alpha(0) \equiv 1$. This clearly factors into two terms, with only the limits on the integrals potentially coupled. Let us rescale the integrals so that

$$A_{22}(s,t_1,t_2) \propto s\, \bar{V}^2(t_1)V^2(t_2)\Gamma(t_2)Y^2 \int_{\gamma_{10}}^{\gamma_1} d\delta_1\, \exp[2(\alpha(t_1) - 1)\delta_1 Y]$$

$$\times \int_{\gamma_{20}}^{\gamma_2} d\delta_2\, \exp[2(\alpha(t_2) - 1)\delta_2 Y]$$

$$\approx s\, G(t_1,t_2) \left[\int_{\gamma_{10}}^{\gamma_1} d\delta_1 \frac{\partial}{\partial\delta_1} \exp[2(\alpha(t_1) - 1)\delta_1 Y]\right]$$

$$\times \left[\int_{\gamma_{20}}^{\gamma_2} d\delta_2 \frac{\partial}{\partial\delta_2} \exp[2(\alpha(t_2) - 1)\delta_2 Y]\right] \tag{6.2}$$

where

$$G(t_1,t_2) = \frac{\bar{V}^2(t_1)\, V^2(t_2)\, \Gamma(t_2,t_2,0)}{4(\alpha(t_1) - 1)(\alpha(t_2) - 1)}. \tag{6.3}$$

If $\alpha(t_i) = 1 + \alpha' t_i$ we have the usual nonintegrable divergences near $t_1, t_2 = 0$. This can be removed by making the residues vanish like some power of t_i. This is necessary in the case in which the gap integrals include the region $\Delta_i \approx$ constant (or $\gamma_{10} \approx 0$ in the scaled integrals). It is unnecessary if we assume that isolated factorizable poles appear only in the asymptotic $\Delta_i \to \infty$ limit. In this case we take $\gamma_{10} > 0$ corresponding to $\Delta_i > \gamma_{10}\, \ell n\, s$. Taking the Laplace transform with respect to Y we have

$$A(j,t_1,t_2) = G(t_1,t_2) \int d\delta_1 \frac{\partial}{\partial\delta_1} \int d\delta_2 \frac{\partial}{\partial\delta_2} \int dY\, \exp[-(j - 1)Y]$$

$$\times\ \exp[2(\alpha(t_1) - 1)\delta_1 Y + 2(\alpha(t_2) - 1)\delta_2 Y]$$

$$= G(t_1,t_2) \int_{\gamma_{10}}^{\gamma_1} d\delta_1 \int_{\gamma_{20}}^{\gamma_2} d\delta_2 \frac{\partial}{\partial\delta_1} \frac{\partial}{\partial\delta_2}$$

$$\times\ [j - 1 - 2(\alpha(t_1) - 1)\delta_1 - 2(\alpha(t_2) - 1)\delta_2]^{-1} \tag{6.4}$$

yielding a sum over four poles if the limits on δ_i are independent. To obtain the leading behavior, we can always choose these limits small enough that they are independent. To stay in the appropriate kinematic region we need $\delta_1 + \delta_2 < 1$. The contributions from these regions then lead to t_i-integrals of the Frullani type rather than the usual exponential integrals. The latter would give $(\ell n(j - 1))^2$ near

$j = 1$. That is, the usual result would be

$$A(j) \approx [j - 1]^{-1} \int_{-\infty}^{0} \frac{dt_1 \; V^2(t_1)}{j - 1 - 2\alpha' t_1} \int_{-\infty}^{0} \frac{dt_2 \; V^2(t_2)\gamma(t_2)}{j - 1 - 2\alpha' t_2} \tag{6.5}$$

which yields a leading $s[\ell n \; \ell n \; s]^2$ behavior.

Making stricter limitations on the regions in which the limits are valid so that the gap integrals scale with Y leads again to a constant renormalization of the output $j = 1$ pole vertex. The integration region corresponds to $0 < x_C < 1 - x_{A'}$, with the limit $1 - x_{A'} \approx M^2/s \lesssim s^{-\epsilon} \to 0$. This allows satisfaction of the sum rule[20]

$$\lim_{M^2,\; s/M^2 \to \infty} \left[(1 - x_{A'}) \frac{d\sigma}{dP_{A'}} > \frac{1}{2} \int dP_{C\perp} \int_{>0}^{1-x_{A'}} dx_C \; x_C \frac{d\sigma}{dP_{A'} dP_C} \right] . \tag{6.6}$$

In the limit, the kinematic zero $1 - x_{A'} \to 0$ on the left-hand side is accompanied by a collapse of the integration region on the right. Using equations (2.1) and (6.1) this may be rewritten as

$$\lim_{\Delta_1 \to \infty} \left[\gamma(t_1) > \int dt_2 \, d\Delta_2 \exp\{[2\alpha(t_2) - \alpha(0) - 1]\Delta_2\} \; f(t_1, t_2) \right] . \tag{6.7}$$

Now, to reach the point $t_1 = t_1^{min} \approx -\exp(-2\Delta_1)m^2 \to 0$, we require the limit $\Delta_1 \to \infty$. The left-hand side, by the arguments above, need not vanish in this limit and thus neither must the right. The usual proof corresponds to taking $\Delta_1 \approx$ constant while $t_1 \to 0$ to isolate $\gamma(0)$. These limits are not independent. The sum rule will not then be satisfied.

We note that in the above analysis[20] we could pick nonvacuum quantum numbers for C and sum over restricted gaps Δ_2 dominated by an ordinary Regge pole. We then isolate the couplings $g_{PCR}(t_1, t_2)$ and $\gamma_{RRP}(t_2, 0)$. In the region $\Delta_1 \gtrsim O(\epsilon Y)$, it is not necessary for these couplings to vanish since the left-hand side of the sum rule need not vanish. A similar statement obtains for the Schwarz inequality proof of Abarbanel, Gribov and Kanchelli[2] where the correct M^2, $s/M^2 \to \infty$ limit is implemented. As established earlier, it is not necessary for the triple coupling, which is put in as an upper bound in the Schwarz inequality, to vanish in this limit.

In the preceding we have considered only those terms in the unitarity sum for A_{22} which have a simple connection with the Mueller analysis for $n \to n$ processes in certain (fragmentation) kinematic regions. These terms build the vertex corrections to factorizable pole exchange in a physically intuitive fashion. That is, the vertex substructure involves only the fragmentation products of the external particles which may be reached by factorizable pole exchange in the production amplitude. There are many other exclusive contributions to A_{22} which have neither a simple connection with the Mueller analysis nor with the vertices.

An important class of such processes will be referred to as diffraction dissociation into large masses and involves the production of two asymptotically growing fireballs. These processes were considered in an elegant paper by Abarbanel, Chew, Goldberger and Saunders[3]. The approach advocated there is implicitly multiperipheral. Consequently, there is a fundamental counting ambiguity if one attempts to isolate the contribution to σ_{total} from such processes[3,18,21].

We shall avoid such ambiguity by considering a more limited class of states, indicated in figure 6. By summing only over fireball masses such that $\ell n\ M_i^2 \leqslant \frac{1}{2}\ \ell n\ s - \lambda(s)$, there is a uniquely defined rapidity gap which always crosses the center of the rapidity axis. In this way, no event is counted twice and the sum over missing masses leads to a contribution to σ_{total}. We have*

$$\frac{d\sigma}{dM_1^2\ dM_2^2\ dt} \sim \left(\gamma^2(t)/s^2\right)\left(s/M_1^2\ M_2^2\right)^{2\alpha(t)}\left(M_1^2\ M_2^2\right)^{\alpha(0)} \qquad (6.8)$$

in the limit $M_1^2,\ M_2^2,\ s/M_1^2\ M_2^2\ \to\ \infty$. If we assume that this expression is valid for $M_i^2\ \geqslant\ M_R^2$ and $s/M_1^2\ M_2^2\ \geqslant$ constant, then the sum over states satisfying these constraints gives a contribution to σ_{total} which behaves like $\gamma^2(0)\ell n\ s \cdot \ell n\ \ell n\ s$ for $\alpha(t) = 1 + \alpha' t$. This $\ell n\ s$ behavior is a much more serious inconsistency with the "unitarity" constraint σ_{tot} = constant.

In the work of Abarbanel et al.[3)], it was assumed that this $\ell n\ s$ factor was removed when one corrected for the multiple counting involved when one isolated a particular link of the approximately $\ell n\ s$ links of the multi-Regge chain. This is not the case here. The $\ell n\ s$ is an essential consequence of the dipole structure of figure 6.c. This appears in the Mellin transform as $[j - \alpha_P(0)]^{-2}$. The loop in figure 6.c. gives rise to the $\ell n(\ell n\ s)$ factor. The latter factor is removed by our arguments that the rapidity gap associated with the loop should have a minimum value $\propto$ Y. The dipole structure remains.

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* The quantity $\gamma(t)$ is here defined as $\gamma(t) = \beta_{PAA}(0)\Gamma(0,t,t)$.

To study the associated j-plane structure we introduce the scaled gap and mass variables $\delta = (\ell n\ \ s/M_1^2 M_2^2)/Y$, $\eta_i = (\ell n\ M_i^2)/Y$. We then have $1 = \delta + \eta_1 + \eta_2$ with the further constraint $\eta_i < \frac{1}{2}$. It is convenient to introduce a δ-function $\int_{\delta_0}^{1} d\delta\ \delta(1 - \delta - \eta_1 - \eta_2)$ so that

$$d\sigma \approx dt\ d\eta_1\ d\eta_2 \int_{\delta_0}^{1} d\delta\ \ \gamma^2(t)\ Y^2\ \delta(1 - \eta_1 - \eta_2 - \delta)$$

$$\times\ \exp[\alpha(0)Y]\ \ \exp\{[2\alpha(t) - \alpha(0) - 1]\delta Y\}. \qquad (6.9)$$

We then take the Laplace transform of $s\ d\sigma/dt$ to compute $A_{22}(j) \propto \int dt\ A(j,t)$, where

$$A(j,t) \approx \frac{-\gamma^2(t)}{f(t)} \int_{\delta_0}^{1} d\delta\ \frac{\partial}{\partial\delta}\ \frac{\partial}{\partial j} \int_0^{\infty} dY\ \exp\{[-j + \alpha(0) + f(t)\delta]Y\}$$

$$\times \int_{\eta_{10}}^{\frac{1}{2}} d\eta_1 \int_{\eta_{20}}^{\frac{1}{2}} d\eta_2\ \ \delta(1 - \delta - \eta_1 - \eta_2)\ , \qquad (6.10)$$

with $f(t) \equiv 2\alpha(t) - \alpha(0) - 1$ and the derivatives $\left(-1/f(t)\right)(\partial/\partial\delta)(\partial/\partial j)$ yield the original factor Y^2. The last integrals are just the line integral in two dimensions. In general, there will appear the integral over an $n - 1$ dimensional surface in n dimensions, where n is the number of produced fireballs. Call this bounded function $I(\delta)$. Now, with $\eta_i < 1/2$, $I(\delta) \propto \theta(\delta)\cdot(1 - \eta_{10} - \eta_{20})$. Integrating only over scaled gaps $\delta \geqslant \delta_0 > 0$ makes this a trivial factor. If the mass integrals go down to a constant M_R^2, so $\eta_{i0} \to 0$, then $I(\delta) \equiv 1$. We have

$$A(j,t) = \frac{-\gamma^2(t)}{f(t)} \frac{\partial}{\partial j} \int_{\delta_0}^{1} d\delta \left[\frac{\partial}{\partial\delta} [j - \alpha(0) - f(t)\delta]^{-1}\right] I(\delta)$$

$$= \frac{-\gamma^2(t)}{f(t)} \frac{\partial}{\partial j} \left\{ [j - 2\alpha(t) + 1]^{-1} \right.$$

$$\left. - [j - (1 - \delta_0)\alpha(0) - \delta_0(2\alpha(t) - 1)]^{-1} \right\}$$

$$\approx \frac{\gamma^2(t)}{2\alpha' t} \frac{\partial}{\partial j} \left\{ [j - 1 - 2\alpha'\delta_0 t]^{-1} - [j - 1 - 2\alpha' t]^{-1} \right\}. \tag{6.11}$$

The usual result corresponds to $\delta_0 \to 0$. However, one must be careful to keep $I(\delta)$ in this case. The leading behavior then corresponds to

$$A(j) \underset{\text{usual}}{=} \left(\frac{1}{j - \alpha(0)}\right)^2 \int \frac{dt \quad \gamma^2(t)}{j - 2\alpha(t) + 1}$$

$$= \left[\frac{\partial}{\partial\alpha(0)} [j - \alpha(0)]^{-1}\right] \int \frac{dt \quad \gamma^2(t)}{j - 2\alpha(t) + 1}. \tag{6.12}$$

Near $j = 1$, the last factor corresponds to $\ln(j - 1)$. Just as in the vertex correction, taking $\delta_0 > 0$ removes the latter factor, as may be seen by doing the t-integral in equation (6.11). Near $j = 1$ we have

$$A(j) \approx \left[\frac{\partial}{\partial\alpha(0)} [j - \alpha(0)]^{-1}\right] \gamma^2(0) \ \ln(1/\delta_0) \tag{6.13}$$

where $\lim_{\delta_0 \to 0} (\delta_0^{-1}) \sim (j - 1)$.

We can now proceed to consider production of multiple fire-balls. The production of n fireballs contributes $(n - 1)$ bubble insertions with

$$A^n(j) \sim \frac{1}{n!} \beta^2_{PAA}(0) \left[\Gamma^2(0) \ln(1/\delta_0)\right]^n \frac{\partial^n}{\partial\alpha(0)^n} (j - \alpha(0))^{-1}. \tag{6.14}$$

If δ_0 were very small, we could consider the asymptotic sum of such insertions. It would be of the form

$$A(j) \sim \beta^2_{AAP}(0) \exp[\Gamma^2(0)\ln(1/\delta_0)(\partial/\partial\alpha(0))] [j - \alpha(0)]^{-1} \tag{6.15}$$

$$= \beta^2_{AAP}(0) [j - \alpha(0) - \Gamma^2(0)\ln(1/\delta_0)]^{-1}. \tag{6.16}$$

The exponential is a simple shift operator on the position of the output pole. Taking the limit $\delta_0 \to 0$ means that $\ln(1/\delta_0) \rightleftarrows -\ln(j-1)$ so that we recover the usual Fredholm denominator of the multi-Regge model. It is only in this limit that the sum over an infinite number of terms has any meaning.

The parameter δ_0, which is related to the relative amount of diffractive production, provides a natural limitation on the series represented by equation (6.14). The maximum number of terms is of order $1/\delta_0$. Since $\Gamma(0)\ln(1/\delta_0) \propto \sigma_{DD}/\sigma_{total}$ [3], a small relative value of the diffractive cross section would be reflected in a larger value of $0 < \delta_0 < 1$, and hence a smaller number of bubble insertions. If, as suggested by the triple-Regge analysis of section 2, δ_0 is of order 1/2, then at most one insertion is allowed. This then avoids conflict with the Froissart bound since the fastest allowed growth of

σ_{total} would be $\ell n\ s$, corresponding to a dipole contribution near $j = 1$.

As a function of energy, there may also be important threshold effects. The scaled relationships above are asymptotic in the sense that they have little meaning for $Y_R \equiv \ell n\ s/M_R^2$ too small. Ordinary Regge arguments indicate that a rapidity gap of two to three units may be necessary to isolate leading singularities. Since $\Delta_{min} = \delta_0\ Y_R$, this requires that the first threshold (that associated with a single bubble insertion) occurs for $\delta_0\ \ell n(s/M_R^2) = 2 \rightarrow 3$, or with $\delta_0 \approx 1/2$, for values of s in the ISR range. This threshold would be responsible for the onset of a $\ell n\ s$ rise in σ_{total}. It is important to note that the same parameter is responsible for both the termination of the series describing the asymptotic behavior and the thresholds associated with this series. The scale of the increase in σ_{total} is set by $\Gamma^2(0)\ell n(1/\delta_0)$, while that of the diffractive contribution is measured by $\Gamma(0)\ell n(1/\delta_0)$. If $\Gamma(0)$ is small, these two quantities may be rather different in magnitude.

The threshold argument requires a relaxation of the consistency requirements observed in the earlier sections, unless the effective δ_0 is such that no bubble insertions can occur. In this case, there is no propagator renormalization whose presence can be deduced through use of the direct channel unitarity relation. Any increase in σ_{total}, within this context, would have to arise from the nature of the pole-cut relationship in the boundary regions. This situation, as discussed in section 4, is a particularly uncertain matter.

We have been primarily interested, in this section, in exploring the implications for a consistent pole bootstrap of certain exclusive cluster decompositions which do not have a simple correspondence with a higher order optical theorem where a single pole dominates. By scaling the gap integrals we were again able to isolate the pure (multiple) pole behavior, without the contamination of cut contributions from fixed gap regions. These terms in the absorptive part A_{22} are particularly well-defined. It is difficult to see how consistency arguments (due to positivity) can avoid dealing with them. Lacking the higher order constraints which appeared for other contributions, we then have only the choices outlined above.

7. Conclusions

We have re-examined the origins of the decoupling phenomena usually associated with the occurence of an isolated factorizable Pomeron pole of unit intercept. It has been shown that the proof of the vanishing of the triple-Pomeron coupling depends on an assumption which is inessential to the asymptotic arguments required for the associated limit. This assumption is that an isolated Pomeron pole dominates even constant rapidity gaps even in the limit in which the total rapidity $Y = \ell n\ s$ becomes infinite. By separating the contributions to σ_{total}, and to the inclusive sum rules, the inconsistent contributions are seen to arise entirely from this fixed gap ($\Delta \equiv \ell n\ s/M^2 \approx$ constant) region. In terms of the Mueller analysis of the three → three reaction, the usual assumption corresponds to assuming that the poles which arise in the triple limit (s, M^2, $s/M^2 \to \infty$) also dominate in the ordinary Regge limit (s, $M^2 \to \infty$, s/M^2 fixed).

We then argued, with regard to the asymptotic limit $Y \to \infty$, that the emergence of an isolated factorizable pole requires the asymptotic limit $s/M^2 \to \infty$. In the nonasymptotic region, it was further suggested, the nature of the leading singularity is less than simple, involving the delicate relationship between pole and cuts. A simple analogy for the necessary collision of singularities was proposed which indicated that the quantitative measure of factorization may not reflect the complexity of the underlying arrangement of singularities.

When the j-plane analysis of those terms in the two → two unitarity sum for which one could consistently assume pole dominance was performed, it was found that they correspond to simple vertex

corrections to pure factorizable pole exchange (figure 4). The usual cut effects (associated with the constant gap region) were absent. The nonintegrable $\int dt/t$ behavior, which is usually removed by the vanishing of the triple coupling--or perhaps equivalently[1] by the vanishing of a nonsense wrong signature residue associated with the usual two-Pomeron cut--disappears in a more trivial fashion. It is worth remarking again that since $t \leqslant t_{min} \approx -(M^2/s)^2 m^2$, one may attain the kinematic point $t = 0$ only in the limit $s/M^2 \to \infty$. It is precisely at this point that the triple-coupling need not vanish. It is then the oversimplified dynamics for $t < 0$ that gives the trouble.

The new conditions on the region of validity of the simple triple-Regge formula overlap the old ones at current machine energies if the amount of diffraction dissociation has the proper size relative to σ_{total}. Only the asymptotic definitions of the regions where pure pole dominance obtains need change. Phenomenologically, one need not have a vanishing contribution near $t = 0$. This will be important if the ISR (and NAL!) data show the need for such a contribution.

The vertex corrections described above are easily associated with the Mueller analysis of $n \to n$ discontinuities in the fragmentation region. The simple cluster decomposition properties which establish a connection between these higher order optical theorems and particular terms in the two → two unitarity sum suggest the application of these techniques to multiple cluster terms in the $2 \to n$ unitarity sum. When this is done, a multipole expansion for the output Pomeron in the absorptive part A_{22} is obtained. Cut contributions are seen to arise from the Pomeron loop insertion in A_{22} (figure 6) when the loop spans a finite length in rapidity as $Y \to \infty$. Scaling the minimum gaps removes the cut contributions leaving pure multipole contributions. The number of such allowed bubble insertions is determined by the minimum gap scale $\epsilon(\equiv \delta_0)$ which is measured in the triple-Regge region. If diffraction dissociation is sufficiently small, no bubble insertions are allowed and full consistency with positivity for σ_{total} is achieved. Otherwise, to be consistent with the Froissart bound, at most two insertions are allowed (corresponding to the production of three fireballs, in the pionization and fragmentation regions). Since there do not appear to be higher order optical theorem constraints on these terms and since there are no cut effects, perhaps the two-two unitarity sum is providing new information on the nature of the vacuum singularity.

Acknowledgments

I wish to thank J. D. Jackson, C. Rosenzweig and especially J. Koplik for discussions. I have also profited from conversations with R. Blankenbecler, R. Cahn and M.-S. Chen. Gratitude is due S. Cosslett for a critical reading of the manuscript.

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FIGURE CAPTIONS

1.) a.) Single diffraction dissociation processes contributing in the triple-Regge region; b.) Rapidity space decomposition of the final state. Infinitesimal rapidity displacements, due to energy conservation in producing particle masses, are neglected; c.) Corresponding form of the three $\rightarrow n(M^2)$ amplitude with the $A\bar{A}$ system continued below threshold. The spectrum of bound states of this system lie on the Pomeron trajectory $\alpha(t)$.

2.) a.) Processes contributing in the di-triple Regge region with invariants defined; b.) Rapidity space decomposition with gap variables Δ_i indicated; c.) Corresponding region of the continued four $\rightarrow n(M^2)$ amplitude.

3.) Pole-cut relationships in the j-plane conjugate, by Mellin transform, to the variable s/M^2. a.) Relationship of intercept-one pole and branch points when $t < 0$ indicating possible collision of singularities; b.) Situation for $t = 0$; c.) Simplified situation for $\alpha_P(0) < 1$ and $t \lesssim 0$. There is no possible collision of singularities in this case.

4.) a.) Vertex correction to the simple pole in the two $\rightarrow$ two absorptive part A_{22} arising from the restricted triple-Regge region. b.) Same for di-triple region. The sum over contributing finite mass states A' and B' is indicated by the loop integrals. c.) Contribution to the two-Pomeron cut from the ordinary Regge region.

5.) Two-particle inclusive processes in the fragmentation region of A. Particle C must have vacuum quantum numbers if $\alpha(t_2)$ is also the Pomeron. a.) Rapidity space decomposition of the corresponding exclusive two $\to$ n(M^2, M'^2) process. b.) Corresponding exclusive M'^2 decomposition of the continued three $\to$ n'(M^2) amplitude; c.) Higher order vertex structure in A_{22} arising from these contributions.

6.) Diffraction dissociation into two large masses. a.) Rapidity space decomposition. Only those states with the gap $\Delta \equiv \ell n\ s/M_1^2\ M_2^2$ crossing the center of the rapidity axis are included; b.) Resulting dipole structure in A_{22} from states with $\Delta_{min} = \delta_0 Y$, $0 < \delta_0 < 1$. The variable Δ is independent of the variable s, in which the Mellin transform is performed. This loop contribution, and all higher order terms, will not occur if only states $M_i^2 > \epsilon_i\ \ell n\ s$ are included and if $\delta_0 + \epsilon_1 + \epsilon_2 > 1$.

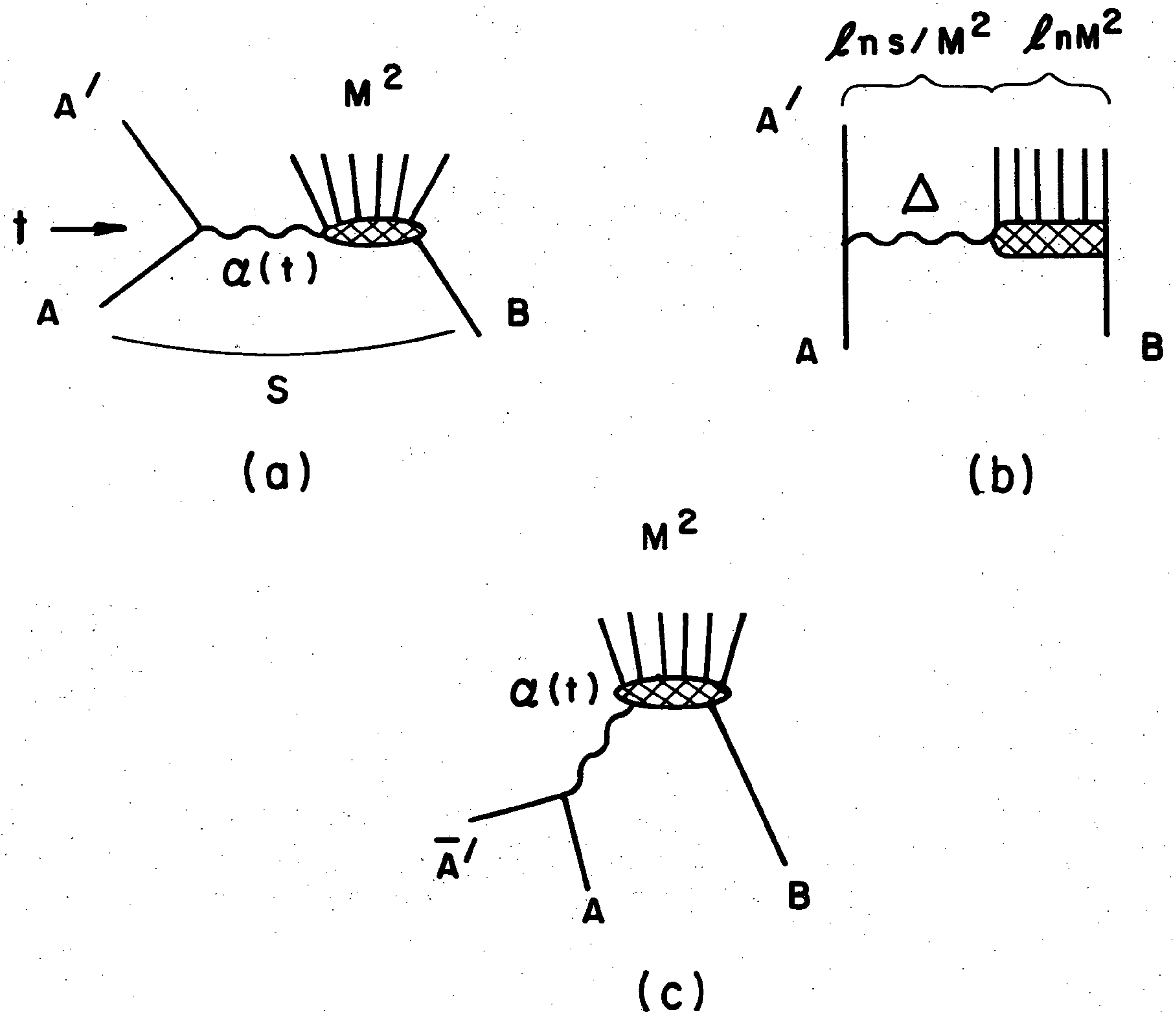

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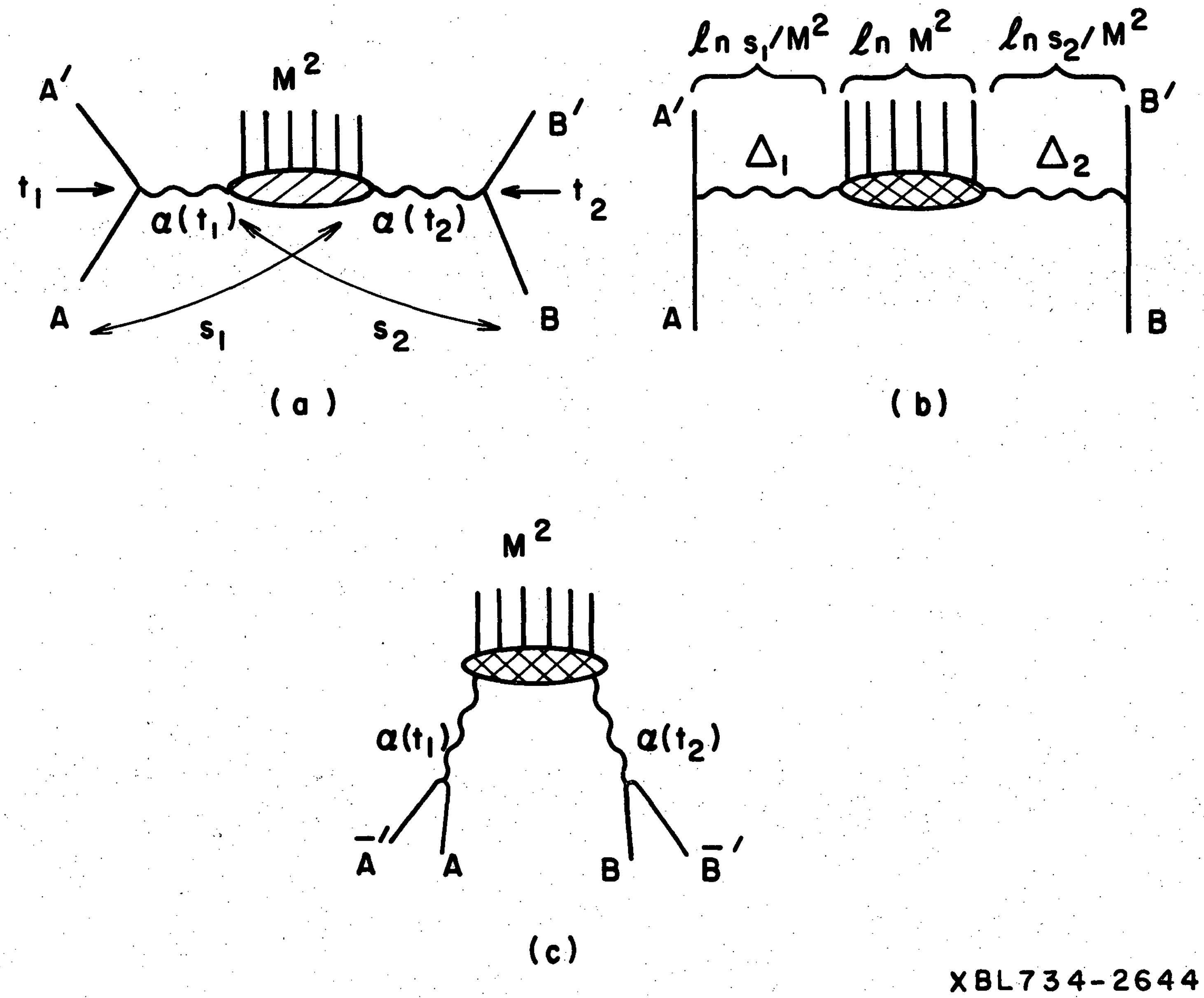
Fig. 1



A'
M^2
B'
t_1
t_2
$\alpha(t_1)$
$\alpha(t_2)$
A
B
s_1
s_2
(a)
$\ell n\ s_1/M^2$
$\ell n\ M^2$
$\ell n\ s_2/M^2$
A'
B'
Δ_1
Δ_2
A
B
(b)
M^2
$\alpha(t_1)$
$\alpha(t_2)$
$\bar{A}'$
A
B
$\bar{B}'$
(c)
XBL734-2644

Fig. 2

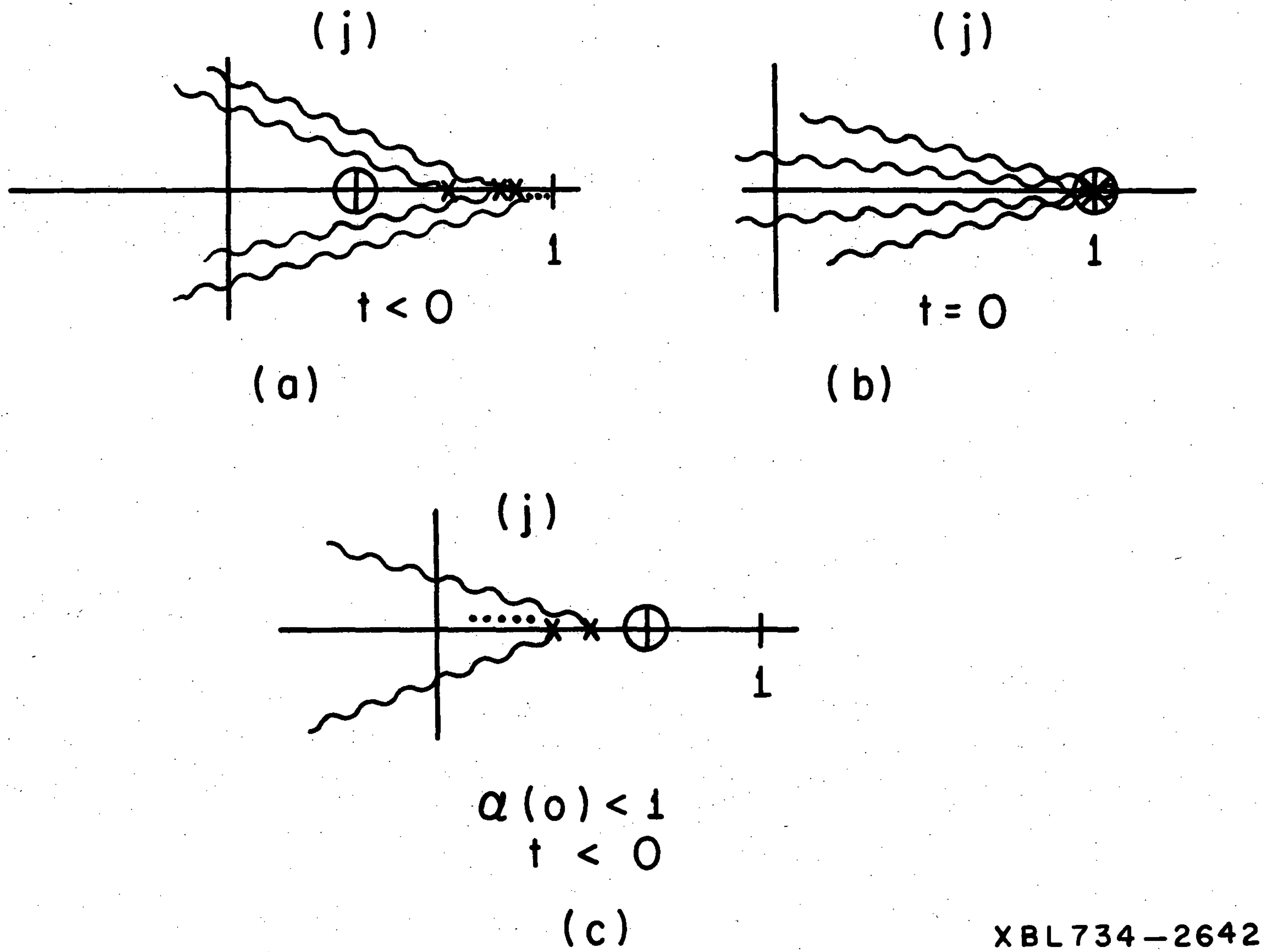


Fig. 3

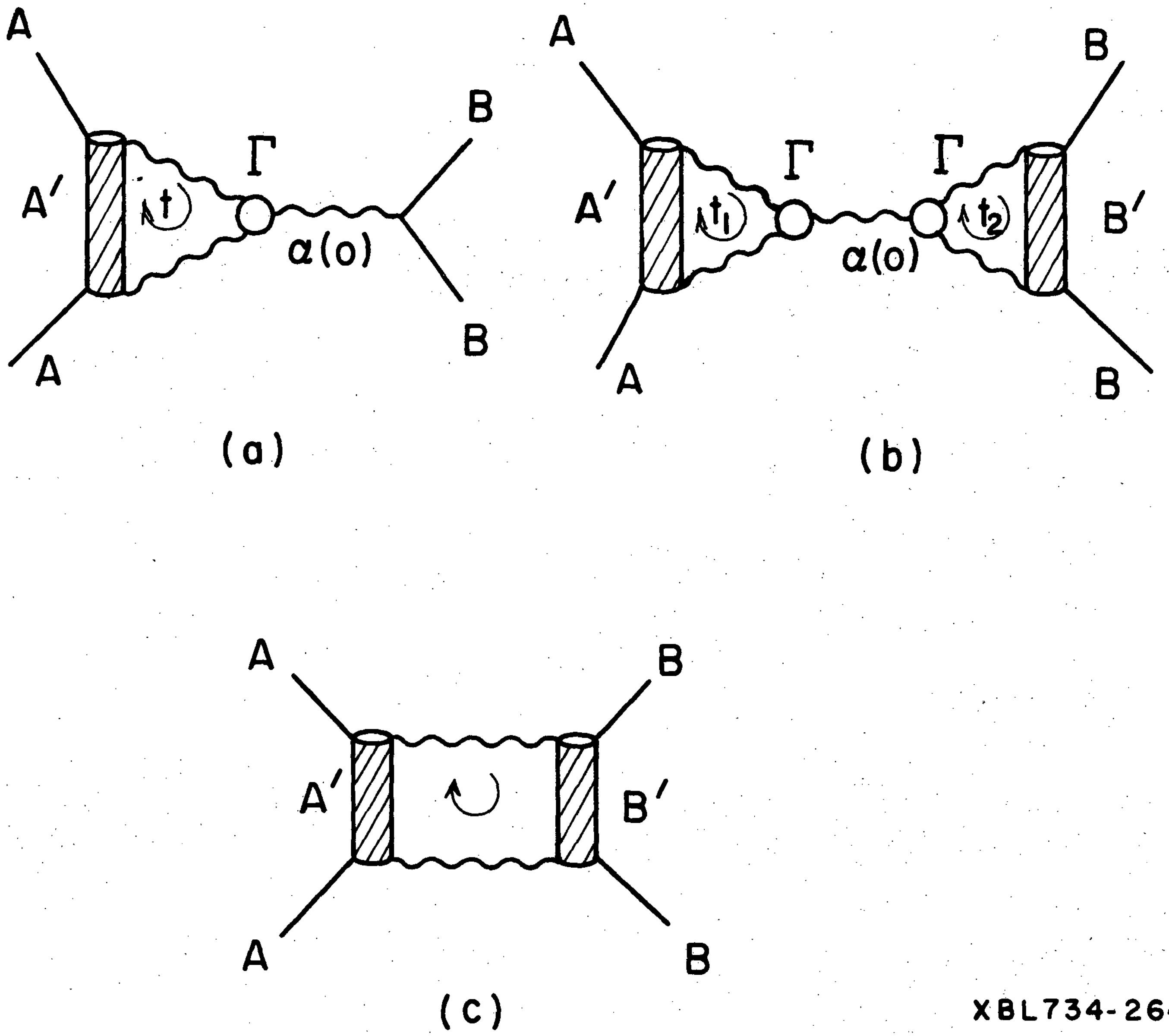


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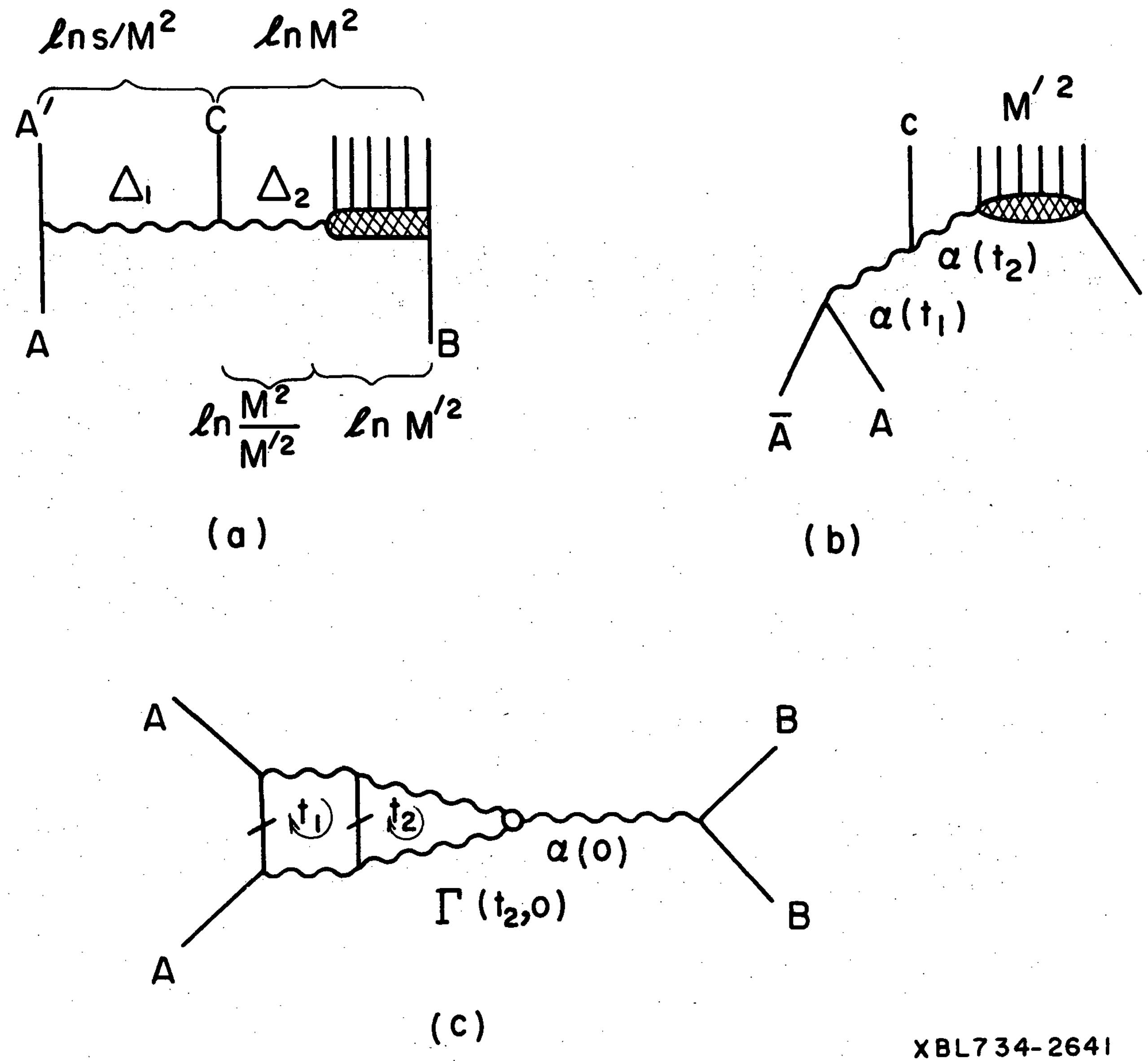
Fig. 4



ℓn s/M²
ℓn M²
A′
C
Δ₁
Δ₂
A
B
ℓn M²/M′²
ℓn M′²
(a)
c
M′ 2
α (t₂)
α (t₁)
Ā
A
(b)
A
B
t₁
t₂
α (0)
Γ (t₂,0)
A
B
(c)
XBL734-2641

Fig. 5

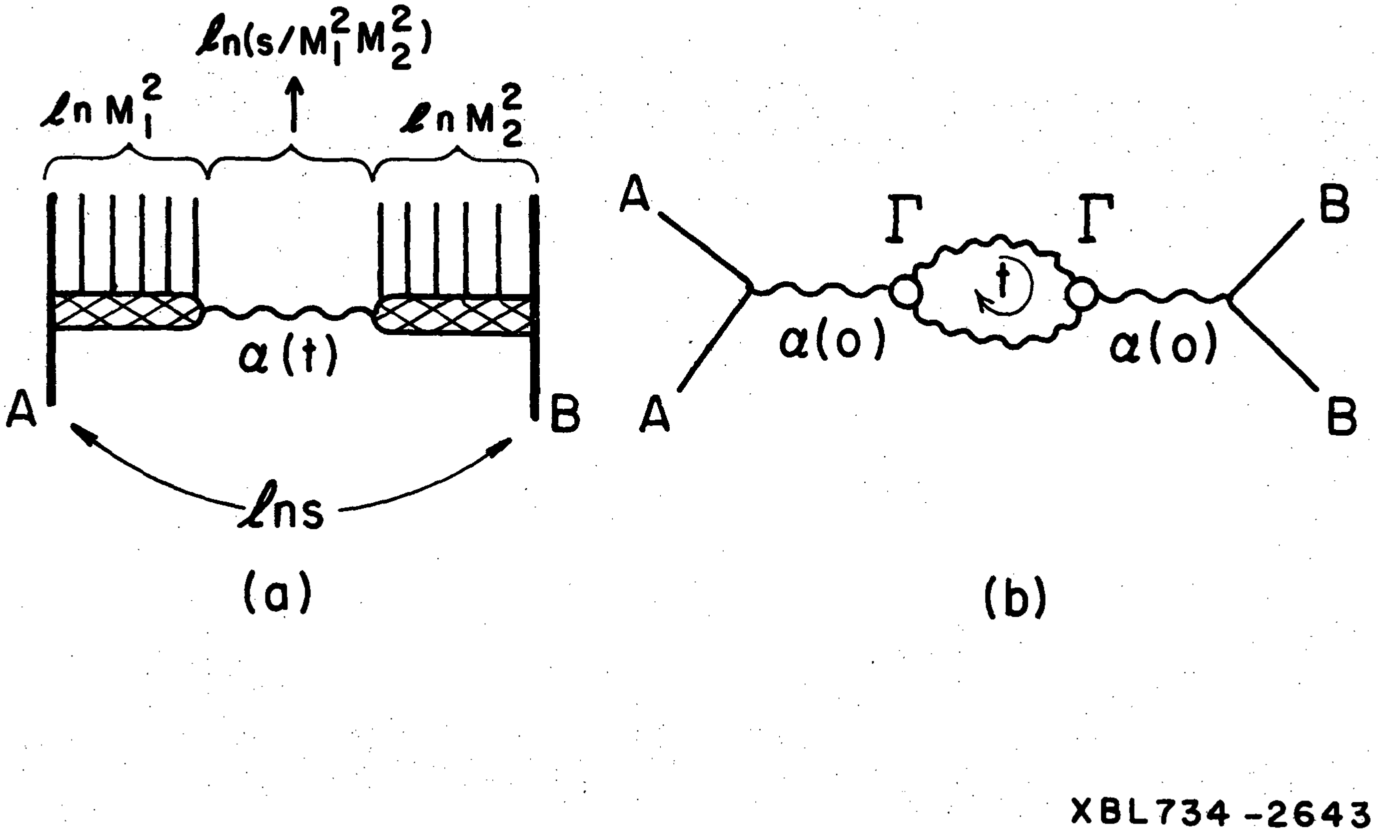


Fig. 6

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