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Obtaining Closure for Fin-and-Tube Heat Exchanger Modeling Based on Volume Averaging Theory (VAT)

Modeling a fin-and-tube heat exchanger as porous media based on volume averaging theory (VAT), specific geometry can be accounted for in such a way that the details of the original structure can be replaced by their averaged counterparts, and the VAT based governing equations can be solved for a wide range of heat exchanger designs. To complete the VAT based model, proper closure is needed, which is related to a local friction factor and a heat transfer coefficient of a representative elementary volume. The present paper describes an effort to model a fin-and-tube heat exchanger based on VAT and obtain closure for the model. Experiment data and correlations for the air side characteristics of fin-and-tube heat exchangers from the published literature were collected and rescaled using the "porous media" length scale suggested by VAT. The results were surprisingly good, collapsing all the data onto a single curve for friction factor and Nusselt number, respectively. It was shown that using the porous media length scale is very beneficial in collapsing complex data yielding simple heat transfer and friction factor correlations and that by proper scaling, closure is a function of the porous media, which further generalizes macroscale porous media equations. The current work is a step closer to our final goal, which is to develop a universal fast running computational tool for multiple-parameter optimization of heat exchangers. [DOI: 10.1115/1.4004393]

Keywords: *volume averaging theory, closure, heat exchanger, length scale*

Introduction

Fin-and-tube heat exchangers are widely used in thermal engineering applications, such as power stations, chemical engineering, automobiles, HVAC&R (Heating, Ventilating, Air Conditioning, and Refrigeration) applications, aircrafts, etc. A schematic diagram of a fin-and-tube heat exchanger is shown in Fig. 1. Extensive investigations on the performance of fin-and-tube heat exchangers have been done, either experimentally or numerically. In the past, the emphasis was on experimental work due to the absence of today's computational power. The experimental methods are expensive and time consuming, as many different models must be fabricated and tested. In the last 20 yr, computational fluid dynamics (CFD) has been widely used to simulate the flow and heat transfer processes in fin-and-tube heat exchangers for design and optimization purposes. Numerical methods are more efficient, but many are specific to the type of geometry that is being tested and direct numerical simulation of the full 3D structure is often not feasible. If one wants to find the optimum configurations for these kinds of heterogeneous hierarchical heat transfer devices, which require many parameters to describe their geometries, experiment or CFD simulation by itself is out of the question. In the case of a fin-and-tube heat exchanger, 15 parameters are required for its description: overall length, width and height, fin thickness, fin pitch, tube diameter, tube wall thickness, tube pitch in x and y directions, flow rates of fluids 1 and 2, initial temperatures of fluids 1 and 2, material of construction, and heat source.

If one wants to optimize such a device, simple equations are the only answer but they need to be made more rigorous. It is proposed that volume averaging theory (VAT) be used to develop the simple equations allowing clear rigorous statements to be made that define how the friction factor and heat transfer coefficient are to be determined. By modeling fin-and-tube heat exchangers as porous media, specific geometry can be accounted for in such a way that the details of the original structure can be replaced by their averaged counterparts, and the governing VAT equations can be solved for a wide range of heat exchanger designs. This "porous media" model, which is a function only of porous media morphology, represented by porosity and specific surface area, and its closure, can easily be adapted to many different structures.

The porosity and specific surface area are geometrically defined terms. The closure terms, which are related to a local friction factor and a heat transfer coefficient, can be obtained from the experimental data reported for fully developed flow, using the porous media length scale suggested by VAT. Whitaker [1] collected the experimental data and illustrated that a proper choice of the characteristic length and velocity for packed beds and tube bundles could lead to a single correlation, which satisfactorily predicts heat transfer rates in randomly packed beds and staggered tube bundles. Travkin and Catton [2,3] showed that choosing the correct length scale, a hydraulic diameter based on scaling of the VAT porous media equations, allows one to collapse true capillary flow and flow in a bed of spheres. This is a significant accomplishment, since it spans the physical description from globular to capillary geometry with a single length scale. In the present paper, published experimental measurements of friction factor and heat transfer performance for the air side of fin-and-tube heat exchangers were collected and rescaled by using the VAT length scale, leading to two much simpler correlations. In the following, some well-known literature of experimental studies on fin-and-tube heat exchangers is reviewed.

During the past 40 years, a large amount of experimental data and their resulting correlations on the air side flow and heat transfer

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characteristics of fin-and-tube heat exchangers have been published. Rich [4] presented experimental results for six coils, with the number of tube rows in the direction of air flow varying from 1 to 6. It was concluded that the pressure drop per row is independent of the number of tube rows. McQuiston [5] proposed the first well-known correlation for plate fin-and-tube heat exchangers with tube row number being in the range of 1–4. Based on a superposition model, which was initially proposed by Rich [6], Gray and Webb [7] gave an updated correlation for fin-and-tube heat exchangers, that is, superior to McQuiston's [5]. It should be noted that the correlations were based on the experimental data of four-row fin-and-tube heat exchangers. Kang et al. [8] presented experimental data and correlations for a three-row fin-and-tube heat exchanger core in a wide range of Reynolds number. Most recently, Wang et al. [9] proposed the most precise correlations of friction factor and Colburn j factor for the air side of fin-and-tube heat exchangers. The correlations are based on a total of 74 samples and the proposed heat transfer correlation can describe 88.6% of the database within an error of $\pm 15\%$, while the proposed friction correlation can correlate 85.1% of the database within an error of $\pm 15\%$.

In the following presentation, a fin-and-tube heat exchanger is first modeled based on volume averaging theory. Then, suitable experimental data and correlations are selected and rescaled using the porous media length scale. Scaling factors for Reynolds number, friction factor, and Nusselt number are deduced and presented for readers' convenience. In the end, two simple correlations for friction factor and Nusselt number are proposed.

VAT Based Modeling

The general geometric arrangement of a plate fin-and-tube heat exchanger is shown in Fig. 1. Usually, there are three or more rows of tubes, which are arranged in-line or staggered. Generally, air flows through the fins perpendicular to the tubes, while water flows through the tubes. This is a problem of conjugate heat transfer within a heterogeneous hierarchical structure. As described in the Introduction, it is quite difficult to optimize this kind of problem since many parameters are required to describe the geometry. Simple equations are the only answer, if one wants to find the optimum configuration for these kinds of conjugate heat transfer devices.

VAT Based Governing Equations. Based on rigorous averaging techniques developed by Whitaker [10], who focused on solving linear diffusion problems, and Travkin and Catton [3,11] who focused on solving nonlinear turbulent diffusion problems,

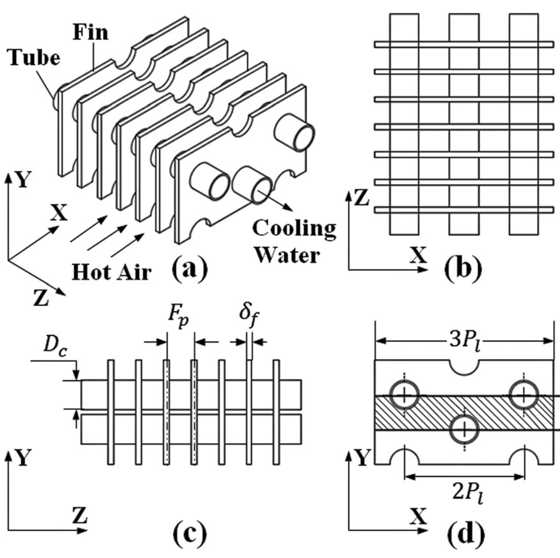

Fig. 1 A schematic diagram of a fin-and-tube heat exchanger

the thermal physics and fluid mechanics governing equations in heterogeneous porous media were developed from the Navier–Stokes equation and the thermal energy equations. This is the starting point for studying flow and heat transfer in porous media and also the basis of the present work.

In this section, a model based on volume averaging theory is developed to describe transport phenomena in fin-and-tube heat exchangers. The air flow and water flow are considered as "porous flow" in which the term "porous" is used in a broad sense.

The momentum equation for the air side is

$$-\frac{1}{\rho_1}\frac{\partial\langle\bar{p}_1\rangle_f}{\partial x}+\frac{\partial}{\partial z}\left(\langle m_1\rangle(\tilde{\nu}_{T_1}+\nu_1)\frac{\partial\tilde{\bar{u}}_1}{\partial z}\right)+c_{d_1}S_{w_1}\frac{\tilde{\bar{u}}_1^2}{2}=0 \quad (1)$$

and for the water side is

$$-\frac{1}{\rho_2}\frac{\partial\langle\bar{p}_2\rangle_f}{\partial z}+\frac{\partial}{\partial x}\left(\langle m_2\rangle(\tilde{\nu}_{T_2}+\nu_2)\frac{\partial\tilde{\bar{w}}_2}{\partial x}\right)+c_{d_2}S_{w_2}\frac{\tilde{\bar{w}}_2^2}{2}=0 \quad (2)$$

Because we are dealing with a conjugate type of problem, the thermal energy equations for both the solid and fluid states are required. For the air side, the VAT based energy equation is

$$\langle m_1\rangle\rho_1\tilde{\bar{u}}_1c_{p_1}\frac{\partial\tilde{\bar{T}}_1}{\partial x}=h_1S_{w_1}\left(\tilde{T}_s-\tilde{\bar{T}}_1\right) \quad (3)$$

and for the water side is

$$\langle m_2\rangle\rho_2\tilde{\bar{w}}_2c_{p_2}\frac{\partial\tilde{\bar{T}}_2}{\partial z}=h_2S_{w_2}\left(\tilde{T}_s-\tilde{\bar{T}}_2\right) \quad (4)$$

For the solid phase, the VAT based energy equation is

$$\frac{\partial}{\partial x}\left[(1-\langle m_1\rangle-\langle m_2\rangle)k_s\frac{\partial\tilde{T}_s}{\partial x}\right]+\frac{\partial}{\partial z}\left[(1-\langle m_1\rangle-\langle m_2\rangle)k_s\frac{\partial\tilde{T}_s}{\partial z}\right]$$
$$=h_1S_{w_1}\left(\tilde{T}_s-\tilde{\bar{T}}_1\right)+h_2S_{w_2}\left(\tilde{T}_s-\tilde{\bar{T}}_2\right) \quad (5)$$

here, $(1-\langle m_1\rangle-\langle m_2\rangle)$ can be considered as the averaged "blockage."

Closure Terms of the VAT Equations. Closure theories for transport equations in heterogeneous media have been the primary measure of advancement and for measuring success in research on transport in porous media. It is believed that the only way to achieve substantial gains is to maintain the connection between porous media morphology and the rigorous formulation of mathematical equations for transport.

To complete the VAT based model, four closure terms need to be closed. Two of them, the averaged porosity and the specific surface area, are geometrically defined and are given in the section "Closure Using Experimental Results" for fin-and-tube heat exchangers. The other two, which are the local drag coefficient, c_d, in the VAT momentum equations and the local heat transfer coefficient, h, in the VAT energy equations, were rigorously derived from lower scale governing equations by Travkin and Catton [11].

The closure term in the VAT momentum equation, c_d, has the form as

$$c_d=2\frac{\int_{\partial S_w}\bar{p}\cdot d\vec{s}}{\rho_f\tilde{\bar{u}}^2A_{wp}}\frac{S_{wp}}{S_w}+2\frac{\int_{\partial S_w}\tau_{wL}\cdot d\vec{s}}{\rho_f\tilde{\bar{u}}^2A_w}+2\frac{\int_{\partial S_w}\tau_{wT}\cdot d\vec{s}}{\rho_f\tilde{\bar{u}}^2A_w}$$
$$-\frac{\frac{\partial}{\partial x_j}\langle\hat{u}_i\hat{u}_j\rangle_f}{\frac{1}{2}\rho\tilde{\bar{u}}^2}+\frac{\frac{\partial}{\partial x_j}\left(\left\langle\tilde{\nu}_T\frac{\partial\tilde{\bar{u}}_i}{\partial x_j}\right\rangle_f\right)}{\frac{1}{2}\rho\tilde{\bar{u}}^2} \quad (6)$$

The first three terms are form drag, laminar and turbulent contributions to skin friction, respectively. The fourth term represents the spatial flow oscillations, which are a function of porous media morphology and tells one how flow deviates from some mean value over the representative elementary volume (REV). The fifth term represents flow oscillations that are due to Reynolds stresses and are a function of porous media morphology and its time averaged flow oscillations.

The closure term in the VAT energy equation, h, can be defined in various ways and, in general, will depend on how many of the integrals appearing in the VAT equation one uses and lumps into a single transport coefficient, see Travkin and Catton [11]. The nature of the equation shows that the energy transferred from the surface is integrated over an area and then divided by the chosen REV volume; therefore, the heat transfer coefficient is defined in terms of porous media morphology, usually described by specific surface and porosity.

The complete form of the closure term h is

$$h = \frac{\frac{1}{\Delta\Omega}\int_{\partial S_w}(k_f + k_T)\nabla T_f \cdot dS}{S_w(\tilde{T}_s - \tilde{T}_f)} - \frac{\rho_f c_{pf}\nabla \cdot \left(\langle m\rangle \widetilde{\hat{u}_f \hat{T}_f}\right)}{S_w(\tilde{T}_s - \tilde{T}_f)} + \frac{\nabla \cdot \left(\frac{k_f}{\Delta\Omega}\int_{\partial S_w} T_f dS\right)}{S_w(\tilde{T}_s - \tilde{T}_f)} \tag{7}$$

In most engineered devices, the geometry is regular and a well-chosen REV will lead to only the first term being needed. However, when in doubt, one should use the complete form given by Eq. (7).

Closure Using Experimental Results

The closure terms could be evaluated either by rescaling heat transfer and friction factor data from experiment that reports the values for fully developed flow or from CFD solutions by integrating closure terms over a REV. In the present paper, only the first method to obtain closure for the VAT based model of fin-and-tube heat exchangers is presented. Demonstration of the second method is saved to another paper.

Porosity and Specific Surface. The porosity and specific surface are determined by the geometry of the porous media, and it is quite easy to define them if one selects the REV correctly. The selection for a fin-and-tube heat exchanger, see Fig. 2, is seen to repeat in both the cross-stream and flow directions.

The porosity for the air side of the fin-and-tube heat exchanger is

$$\langle m_1\rangle = 1 - \frac{\delta_f}{F_p} - \frac{\pi D_c{}^2(F_p - \delta_f)}{4P_l P_t F_p} \tag{8}$$

and for the water side is

$$\langle m_2\rangle = \frac{\pi D_i{}^2}{4P_l P_t} \tag{9}$$

The specific surface area for the air side is given by

$$S_{w_1} = \frac{2P_l P_t - 2\pi\left(\frac{D_c}{2}\right)^2 + \pi D_c(F_p - \delta_f)}{P_l P_t F_p} \tag{10}$$

and for the water side is

$$S_{w_2} = \frac{\pi D_i}{P_l P_t} \tag{11}$$

Friction Factor and Heat Transfer Coefficient. Before obtaining the closure of friction factor and heat transfer coefficient, it is interesting to note that using a particular length scale leads to a parameter that is very beneficial when scaling heat transfer and friction factor results. It was shown by Travkin and Catton [11] that globular media morphologies can be described in terms of S_w, $\langle m\rangle$, and d_p and can generally be considered to be spherical particles with

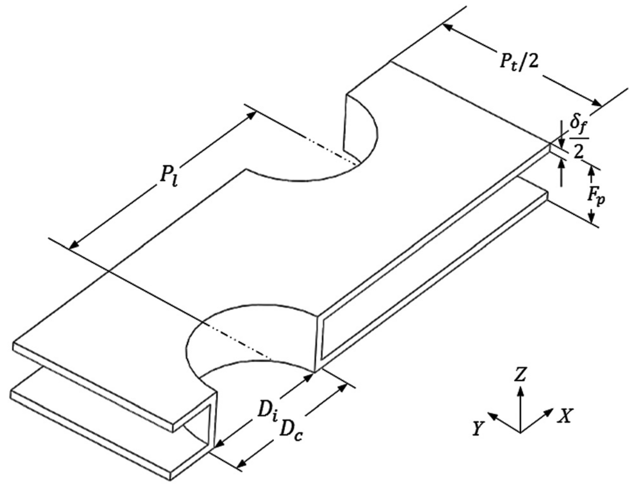

Fig. 2 REV for a fin-and-tube heat exchanger

$$S_w = \frac{6(1 - \langle m\rangle)}{d_p} \tag{12}$$

$$D_h = \frac{2}{3}\frac{\langle m\rangle}{(1 - \langle m\rangle)} d_p \tag{13}$$

This expression has the same dependency on equivalent pore diameter as found for a one diameter capillary morphology leading naturally to

$$S_w = \frac{6(1 - \langle m\rangle)}{d_p} = \frac{6(1 - \langle m\rangle)}{\frac{3}{2}\frac{(1 - \langle m\rangle)}{\langle m\rangle} D_h} = \frac{4\langle m\rangle}{D_h} \tag{14}$$

This observation leads to defining a simple "universal" porous media length scale

$$D_h = \frac{4\langle m\rangle}{S_w} \tag{15}$$

that meets the needs of both morphologies: capillary and globular. This was also recognized by Whitaker [1] when he used a very similar (differing by a constant) length scale to correlate heat transfer for a wide variety of morphologies. In the following, how to use this porous media length scale to recorrelate friction factor and heat transfer coefficient for a fin-and-tube heat exchanger is shown.

From a literature review, it can be concluded that Wang et al. [9] proposed the most precise correlations of friction factor and Colburn j factor for the air side performance of plain fin-and-tube heat exchangers. They were scaled with the fin collar outside diameter, D_c, and the maximum velocity, $u_{\max}$. The friction factor correlation is

$$f = 0.0267\mathrm{Re}_{D_c}^{F_1}\left(\frac{P_t}{P_l}\right)^{F_2}\left(\frac{F_p}{D_c}\right)^{F_3} \tag{16}$$

$$F_1 = -0.764 + 0.739\frac{P_t}{P_l} + 0.177\frac{F_p}{D_c} - \frac{0.00758}{N} \tag{17}$$

$$F_2 = -15.689 + \frac{64.021}{\log_e(\mathrm{Re}_{D_c})} \tag{18}$$

$$F_3 = 1.696 - \frac{15.695}{\log_e(\mathrm{Re}_{D_c})} \tag{19}$$

The definition of Reynolds number in the above correlation is

$$\mathrm{Re}_{D_c} = \frac{u_{\max} D_c}{\nu} \tag{20}$$

while to obtain closure for the VAT based model, the length scale given by Eq. (15) and the averaged velocity over the selected REV (Fig. 2), $\tilde{\tilde{u}}$, should be used. The Reynolds number using the VAT length scale is defined as

$$\mathrm{Re}_{D_h} = \frac{\tilde{\tilde{u}} D_h}{\nu} \tag{21}$$

For fin-and-tube heat exchanger morphology, it is not difficult to get the relationship between Re_{D_c} and Re_{D_h}, which is

$$\mathrm{Re}_{D_h} = \frac{\tilde{\tilde{u}} D_h}{u_{\max} D_c} \mathrm{Re}_{D_c} = \frac{1}{\langle m \rangle} \frac{A_{\min}}{A_{fr}} \frac{D_h}{D_c} \mathrm{Re}_{D_c}$$
$$= \frac{4\left(1 - \frac{D_c}{P_t}\right) \cdot \left(1 - \frac{\delta_f}{F_p}\right)}{S_w D_c} \mathrm{Re}_{D_c} \tag{22}$$

in which

$$\alpha = \frac{4\left(1 - \frac{D_c}{P_t}\right) \cdot \left(1 - \frac{\delta_f}{F_p}\right)}{S_w D_c} \tag{23}$$

is defined as the scaling factor for Reynolds number.

The friction factor defined by Wang et al. [9] is

$$f = \frac{A_{\min}}{A_o} \frac{2\Delta p \rho}{G_{\min}^2} \tag{24}$$

must be rescaled using the VAT length scale. According to Eq. (24), the pressure drop can be written in the following form:

$$\Delta p = \frac{A_o}{A_{\min}} \frac{1}{2} \rho u_{\max}^2 f \tag{25}$$

The friction factor using VAT length scale is defined as

$$f' = \frac{\Delta p}{\frac{1}{2}\rho \tilde{\tilde{u}}^2} \cdot \frac{D_h}{L} \tag{26}$$

Substitute Eq. (25) into Eq. (26) leads to

$$f' = \left[\frac{A_o}{A_{\min}} \frac{D_h}{L} \left(\frac{u_{\max}}{\tilde{\tilde{u}}}\right)^2\right] f = \left[\frac{A_o A_{fr}^2 \langle m \rangle^2}{A_{\min}^3} \frac{D_h}{L}\right] f \tag{27}$$

allowing the scaling factor for friction factor to be defined as

$$\beta = \frac{A_o A_{fr}^2 \langle m \rangle^2}{A_{\min}^3} \frac{D_h}{L} \tag{28}$$

In the following, the friction factor results using Wang's length scale, followed by the rescaled results, are shown for comparison.

Just as Fig. 3 shows, for fin-and-tube heat exchangers with different dimensions, see Table 1, friction factor results given by Wang's correlation [9] are scattered, leading to six different $f - \mathrm{Re}_{D_c}$ curves. However, if the data were rescaled with the universal porous media scale, given by Eq. (15), and velocity averaged over the selected REV (Fig. 2), the six curves collapse to a single curve, shown in Fig. 4, clearly demonstrating the value of the VAT based length scale.

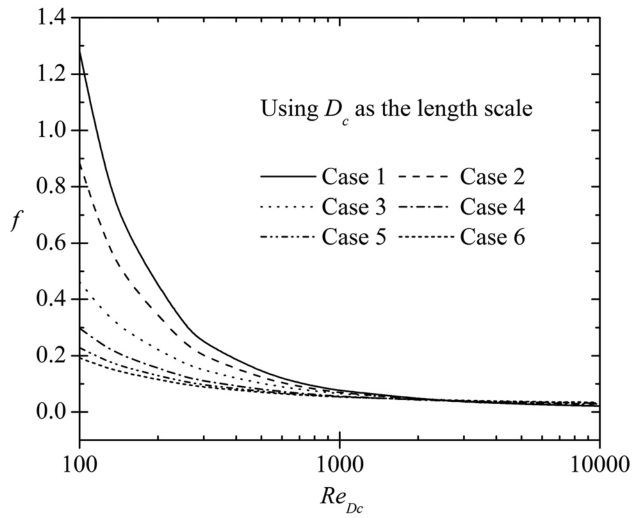

Fig. 3 Overlay plot of friction factor using D_c as the length scale

With the data being collapsed onto a single curve, a correct form needs to be chosen to correlate the rescaled data. Travkin and Catton [2] showed that using the proper scaling, like the one presented by Eq. (15), enables one to write the friction factor of porous media in the following form:

$$f_f = \frac{A}{\mathrm{Re}_{D_h}} + B \tag{29}$$

The constants A and B correspond to different types of morphologies of porous media, with $A = 100/3$ and $B = 7/12$ for the Ergun equation [12] for packed bed porous media, $A = 50$ and $B = 0.145$ for the pin fin array [13].

Furthermore, Travkin and Catton [2] stated that the friction factor is related to the closure of the VAT momentum equations and they showed that

$$c_d \cong f_f \tag{30}$$

Note that the closure equation (Eq. (6)) is an exact definition of friction factor and for fully developed flow Eq. (30) is more strictly defined as

$$c_d = f_f = \frac{A}{\mathrm{Re}_{D_h}} + B \tag{31}$$

With the help of JMP 8 [14], an available statistical analysis tool, the collapsed data enabled us to develop a simple correlation of friction factor for the air side

Table 1 Geometric dimensions of the fin-and-tube heat exchangers shown in Figs. 3 and 4

Legend	Case	N	D_c (mm)	F_p (mm)	P_t (mm)	P_l (mm)	δ_f (mm)
solid	1	6	10.23	3.16	25.4	22	0.13
dash	2	4	10.23	1.23	25.4	22	0.115
dot	3	5	10.55	2.2	30	28	0.2
dash-dot	4	4	10.23	1.55	25.4	22	0.115
dash-dot-dot	5	6	8.51	3.16	25.4	22	0.13
short dash	6	6	7.53	3.16	25.4	22	0.13

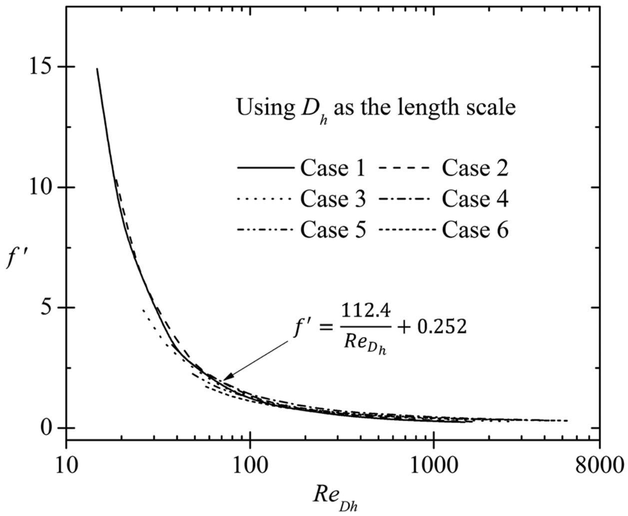

Fig. 4 Overlay plot of friction factor using D_h as the length scale

$$c_{d_1} = f' = \frac{112.4}{\mathrm{Re}_{D_h}} + 0.252 \tag{32}$$

A comparison of values of A and B with other morphologies is shown in Table 2.

It should be noted that the number of tube rows has different effects on the pressure drop and the heat transfer characteristics of fin-and-tube heat exchangers. Figure 5, which was plotted according to the correlations by Wang et al. [9], illustrates the difference. As can be seen, for fin-and-tube heat exchangers with multiple-row tubes, the friction factors are almost independent of the number of tube rows. This is why we could rescale Wang's [9] correlation of friction factor and collapse the data to a single curve, although it is only suitable for small tube row number (from 1 to 6). Figure 5 also indicates that the Nusselt number decreases with the increasing of tube row number, which means that the entrance effect plays an important role in the air side heat transfer coefficient of fin-and-tube heat exchangers, while to obtain closure for h, the experimental data for fully developed flow is required.

It was reported by earlier work [15–18] that the heat transfer characteristic of fin-and-tube heat exchangers is independent of the number of tube rows when $N > 6$. So it is taken for granted that there is no need to do any research on fin-and-tube heat exchangers with tube row number larger than six, which causes the scarcity of available experimental data for fully developed flow. As far as the authors know, the only available experimental data for $N > 6$ was reported by Tang et al. [16,17]. Fin-and-tube heat exchangers with tube row number of 6, 9, and 12 were tested experimentally. Unfortunately, all the tested samples have the same dimensions of D_c, F_p, P_t, and P_l, so we cannot use Tang's data to show how the rescaled data could be collapsed. Xie et al. [18] tested fin-and-tube heat exchangers with tube row number varying from 1 to 7 numerically, but the seven-row cores, which also had the same geometrical parameters, were simulated only when the authors tried to present the effect of fin material. While when the effects of geometrical parameters were studied, the tube row number was kept to be three.

Table 2 Closure coefficients for different morphologies

Morphology	A	B	Porosity range
Packed bed	100/3	7/12	0.3–0.72
Pin fins-inline	50	0.145	0.65–0.91
Pin fins-staggered	50	0.145	0.65–0.91
Plain fin-and-tube HX (staggered)	112.4	0.252	0.65–0.9

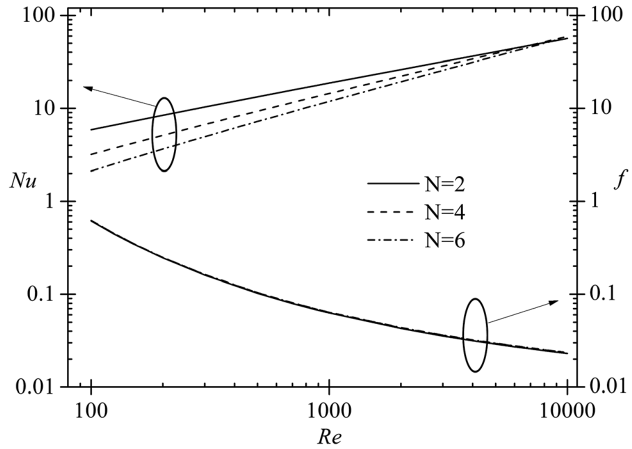

Fig. 5 Effect of tube row number on f and Nu according to Wang correlations [9]

It was demonstrated by Tang et al. [16] that the difference between Nu − Re curves for $N = 6$, 9, and 12 is negligible, which means when $N = 6$ the flow could be considered as fully developed. Thus, hopefully we could still use Wang's correlation of the Colburn j factor [9] to show whether the rescaled data collapse or not by keeping the tube row number to be six while changing the other dimensions. Wang's Colburn j factor correlation ($2 \le N \le 6$) is

$$j = 0.086\mathrm{Re}_{D_c}^{P1} N^{P2} \left(\frac{F_p}{D_c}\right)^{P3} \left(\frac{F_p}{D_h}\right)^{P4} \left(\frac{F_p}{P_t}\right)^{-0.93} \tag{33}$$

$$P1 = -0.361 - \frac{0.042N}{\log_e(\mathrm{Re}_{D_c})} + 0.158\log_e\left(N\left(\frac{F_p}{D_c}\right)^{0.41}\right) \tag{34}$$

$$P2 = -1.224 - \frac{0.076\left(\frac{P_l}{D_h^*}\right)^{1.42}}{\log_e(\mathrm{Re}_{D_c})} \tag{35}$$

$$P3 = -0.083 + \frac{0.058N}{\log_e(\mathrm{Re}_{D_c})} \tag{36}$$

$$P4 = -5.735 + 1.21\log_e\left(\frac{\mathrm{Re}_{D_c}}{N}\right) \tag{37}$$

$$D_h^* = \frac{4A_{\min}L}{A_o} \tag{38}$$

and Wang's definition of Colburn j factor [19] is

$$j = \frac{h}{\rho u_{\max} c_p}\mathrm{Pr}^{2/3} \tag{39}$$

which leads to

$$h = \frac{j\rho u_{\max} c_p}{\mathrm{Pr}^{2/3}} = \left(\frac{\mathrm{Re}_{D_c} k}{D_c}\mathrm{Pr}^{1/3}\right) j \tag{40}$$

Using D_c as the length scale, we find the Nusselt number to be

$$\mathrm{Nu} = \frac{hD_c}{k} = \left(\frac{\mathrm{Re}_{D_c} k}{D_c}\mathrm{Pr}^{1/3}\right) j \cdot \frac{D_c}{k} = \left(\mathrm{Re}_{D_c}\mathrm{Pr}^{1/3}\right) j \tag{41}$$

Using D_h as the length scale, the rescaled Nusselt number is

$$\mathrm{Nu}' = \frac{hD_h}{k} = \left(\frac{\mathrm{Re}_{D_c} k}{D_c}\mathrm{Pr}^{1/3}\right) j \cdot \frac{D_h}{k} = \left(\mathrm{Re}_{D_c}\frac{D_h}{D_c}\mathrm{Pr}^{1/3}\right) j \tag{42}$$

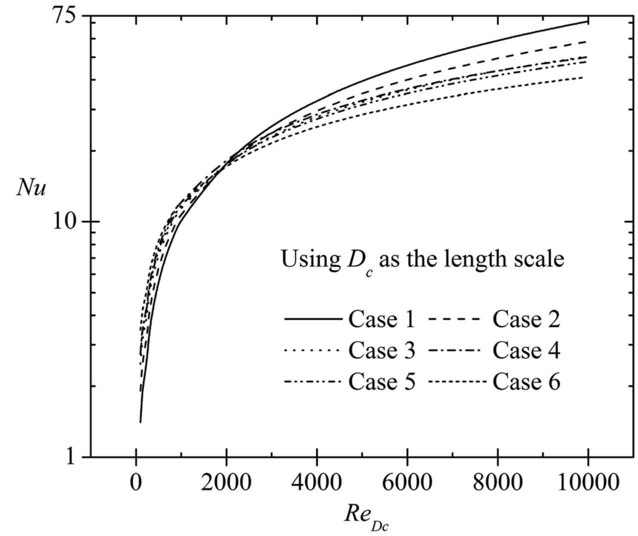

Fig. 6 Overlay plot of Nu number using D_c as the length scale

Figures 6 and 7 compare the variations of Nusselt number as a function of Reynolds number, using two different length scales. Obviously, the original data are scattered on Fig. 6 while the rescaled data collapses to one single curve. The dimensions of fin-and-tube heat exchangers used to show this are tabulated in Table 3. Using JMP 8, a correlation of the rescaled Nusselt number data was found and is

$$\mathrm{Nu}_1 = \mathrm{Nu}' = 0.24\mathrm{Re}_{D_h}^{0.6}\mathrm{Pr}^{1/3} \tag{43}$$

It is argued by some researchers [16–18, 20] that Wang's correlations [9] have certain application ranges like $1 \leq N \leq 6$, 6.35 mm $\leq D_o \leq$ 12.7 mm, which are usually used in the HVAC&R engineering, and are not applicable to some applications of large industry, such as the intercooler of multistage compressor, in which the number of tube rows might be much larger and the outside diameter of tubes might be larger than 13 mm. As a result, investigations on the heat transfer and friction characteristics of fin-and-tube heat exchangers with large number of tube rows and large tube diameter were carried out either numerically [18] or experimentally [16, 17, 20].

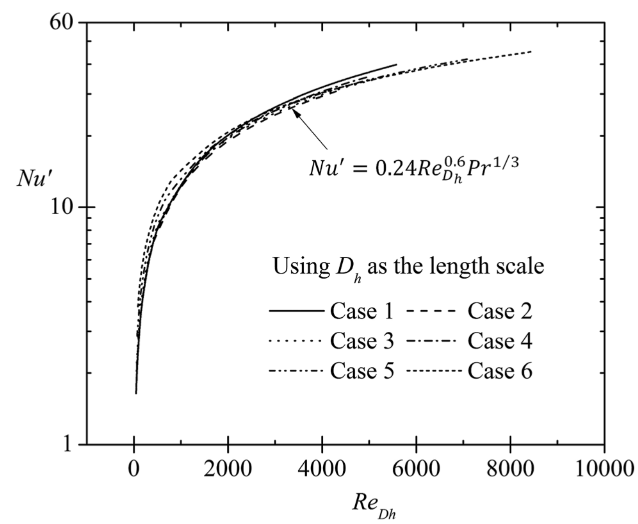

Fig. 7 Overlay plot of Nu number using D_h as the length scale

Table 3 Geometric dimensions of the fin-and-tube heat exchangers shown in Figs. 6 and 7

Legend	Case	N	D_c (mm)	F_p (mm)	P_t (mm)	P_l (mm)	δ_f (mm)
	1	6	10.23	4.5	25.4	22	0.13
	2	6	10.23	3.5	25.4	22	0.115
	3	6	10.55	4.5	30	28	0.2
	4	6	10.23	4	25.4	22	0.115
	5	6	8.51	4.5	25.4	22	0.13
	6	6	7.53	4.5	25.4	22	0.13

To verify the applicability of the correlations given by Eqs. (32) and (43) to large tube diameters, and at the same time to show that it is the right way to use Wang's j factor correlation for $N=6$ as fully developed flow, the experimental data by Tang et al. [16] for fin-and-tube heat exchangers with 12 rows of tubes, which is also the only available data in published literature to the best of the authors' knowledge, are rescaled and compared with the correlations. It should be noted that the definition of friction factor given by Tang et al. [16] is different from that used by Wang et al. [9]. This requires a different scaling factor be used

$$\gamma = \left(\frac{\langle m\rangle A_{fr}}{A_{\min}}\right)^2 \cdot \frac{D_h}{D_c} \tag{44}$$

It is shown in Fig. 8 that the rescaled experimental data by Tang et al. [16] and the rescaled correlations agree well, showing that by proper scaling, closure is only a function of the porous media morphology, which further generalizes macroscale VAT based equations.

For closure of the water side, all the scaling factors are equal to one and the friction factor and Nusselt number correlations for fully developed flow in a pipe are applicable to close the water side VAT equations, due to the reason that the hydraulic diameter of the water side could be simplified to

$$D_{h_2} = \frac{4\cdot\langle m_2\rangle}{S_{w_2}} = \frac{4\cdot\frac{\pi D_i^2}{4P_lP_t}}{\frac{\pi D_i}{P_lP_t}} = D_i \tag{45}$$

Techo et al. [21] correlated the friction factor for turbulent pipe flow as follows:

$$\frac{1}{\sqrt{f}} = 1.7372\ln\left[\frac{\mathrm{Re}_{D_h}}{1.964\ln(\mathrm{Re}_{D_h}) - 3.8215}\right] \tag{46}$$

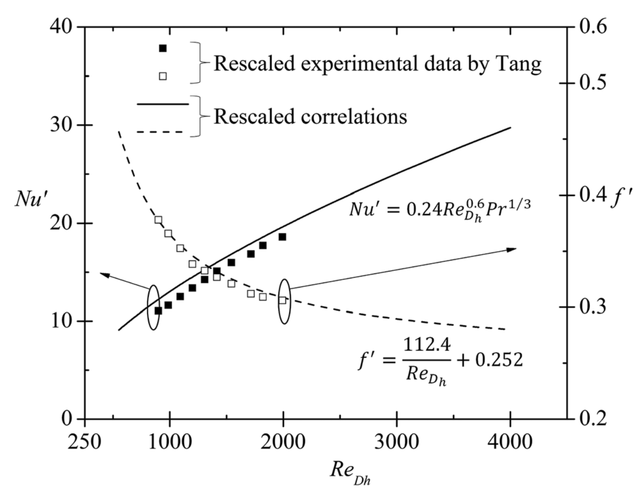

Fig. 8 Comparison between rescaled correlations and experimental data by Tang et al. [16]

which leads to

$$c_{d_2} = \frac{1}{\left\{1.7372\ln\left[\frac{\mathrm{Re}_{D_h}}{1.964\ln(\mathrm{Re}_{D_h}) - 3.8215}\right]\right\}^2} \tag{47}$$

As for the heat transfer coefficient, h_2, Whitaker [1] showed that the experimental data of Nusselt number from a number of investigators for turbulent pipe flow is quite nicely recorrelated by the expression

$$\mathrm{Nu}_2 = 0.015\mathrm{Re}_{D_h}^{0.83}\mathrm{Pr}^{0.42}\left(\frac{\mu_b}{\mu_0}\right)^{0.14} = \frac{h_2 D_{h_2}}{k_2} \tag{48}$$

in which, the ratio μ_b/μ_0 represents the ratio of the viscosity evaluated at the mean bulk temperature to the viscosity evaluated at the mean wall temperature. For air, the variation in the viscosity is negligible.

At this point, the VAT based model of fin-and-tube heat exchangers is fully closed. With the closure correlations, the governing equation set is relatively simple and could be solved discretely in a minute. With the help of a statistical tool for design of experiments (DOE), the performance of a fin-and-tube heat exchanger could be optimized in an hour, instead of days of CFD or experimental work. How to optimize the fin-and-tube heat exchangers based on volume averaging theory with the help of design of experiments tool will be presented in another paper.

Concluding Remarks

Volume averaging theory is little more than a judicious application of Green's and Stokes' theorems to carry out the integration needed to average the point-wise conservation equations in a rigorous way. Many everyday engineered devices are hierarchical and heterogeneous and can be effectively treated by application of VAT. It is an approach that can be applied to many different types of transport phenomena, see Travkin and Catton [11].

The present paper describes the development of a VAT based hierarchical model for a fin-and-tube heat exchanger and closure for the model by rescaling available experimental data. Wang's correlations [9] of friction factor and Colburn j factor for fin-and-tube heat exchangers were rescaled using the VAT based universal length scale. The results were surprisingly good, collapsing all the data onto a single curve for friction factor and Nusselt number, respectively. Two much simpler correlations of friction factor and Nusselt number were established. Tang's experimental data [16] for fin-and-tube heat exchangers with large number of tube rows and large tube diameter was rescaled and compared with the established simple correlations to verify them. It should be noted that these correlations are not necessarily the most accurate available; however, they have wide application, are easy to use, and are quite satisfactory for most design calculations [1]. Also, for optimization, extreme accuracy is not vital because variation in the parameter being optimized can be as much as an order of magnitude.

With closure of the friction factor and the heat transfer coefficient, the problem is closed and the porous media governing equations derived from VAT are

$$\tilde{M}(\langle m\rangle, S_w, c_d) \tag{49}$$

$$\tilde{T}_s(\langle m\rangle, S_w, h) \tag{50}$$

$$\tilde{T}_f(\langle m\rangle, S_w, h) \tag{51}$$

where $\tilde{M}$ stands for averaged momentum equation variables and $\tilde{T}_s$ and $\tilde{T}_f$ stand for averaged energy equation variables for solid and fluid phases.

From the statements above, the macroscale equations are functions only of porous media morphology, represented by porosity and specific surface area, and its closure. Furthermore, it was shown that by proper scaling, closure is a function of the porous media as well, which further generalizes macroscale porous media equations.

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Nomenclature

$A_{\min}$ = minimum flow area (m^2)
A_{fr} = frontal area (m^2)
A_o = total surface area (m^2)
A_w = wetted surface (m^2)
A_{wp} = the cross flow projected area (m^2)
c_d = drag coefficient
c_p = specific heat [J/(kg·K)]
D_i = inner diameter of the tube (m)
D_o = outer diameter of the tube (m)
D_c = fin collar outside diameter, $D_c = D_o + 2\delta_f$ (m)
D_h = porous media hydraulic diameter (m)
D_h^* = hydraulic diameter defined by Wang [19] (m)
d_p = diameter of the spherical particles (m)
∂S_w = internal surface in the REV (m^2)
F_p = fin pitch (m)
f = friction factor
f_f = fanning friction factor
$G_{\min}$ = mass flux of the air based on the minimum flow area [kg/(m^{2}s)]
h = heat transfer coefficient [W/(m^2·K)]
j = Colburn factor
k_f = thermal conductivity of the fluid [W/(m·K)]
k_s = thermal conductivity of the solid [W/(m·K)]
k_T = turbulent heat conductivity [W/(m·K)]
$\langle m\rangle$ = Average porosity
N = the number of tube rows
Nu = Nusselt number
p = pressure (Pa)
Pr = Prandtl number
P_t = transverse tube pitch (m)
P_l = longitudinal tube pitch (m)
Re_{D_c} = Reynolds number based on fin collar outside diameter and maximum velocity $\mathrm{Re}_{D_c} = u_{\max}D_c/\nu$
Re_{D_h} = Reynolds number based on hydraulic diameter and average velocity $\mathrm{Re}_{D_h} = \tilde{\bar{u}}D_h/\nu$
S_w = specific surface of a porous media, $S_w = \partial S_w/\Delta\Omega$ (1/m)
S_{wp} = the cross flow projected area per volume (1/m)
T = fluid temperature (K)
T_s = solid temperature (K)
u = x-direction velocity term (m/s)
w = z-direction velocity term (m/s)

Greek

α = scaling factor for Reynolds number
β = scaling factor for friction factor defined by Wang [19]
γ = scaling factor for friction factor defined by Tang [16]
δ_f = thickness of a fin (m)
μ = viscosity (Pa·s)
ν = kinematic viscosity (m^2/s)
ρ = density (kg/m^3)

$\tau_{wL} =$ laminar shear stress (Pa)
$\tau_{wT} =$ turbulent shear stress (Pa)
$\Delta\Omega =$ the volume of the REV (m^3)

Subscripts and Superscripts

$\sim =$ a value averaged over the representative volume
$- =$ an average of turbulent values
$\wedge =$ fluctuation of a value
$\langle f \rangle_f =$ means the superficial average of the function f
$T =$ turbulent
$1 =$ a value in the air side
$2 =$ a value in the water side
$0 =$ evaluated at the wall or surface
$b =$ evaluated at the bulk temperature

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